

Quantum Random Number Generators

A bunch of BS (beamsplitters...)

P. Kwiat

Kwiat's Quantum Clan (2012)

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Outline

1. Motivation

2. Location, location

3. It's all a matter of timing

- how I know when I have a good idea...
- post-processed randomness
- pre-processed randomness
- a very high-rate version, not
- a very high-rate version
- application to QKD

4. Gerd's CV

5. DIQRNG

- "Heinz Ketchup"
- "when it absolutely has to be there overnight"

6. DIQKD via VLPC?

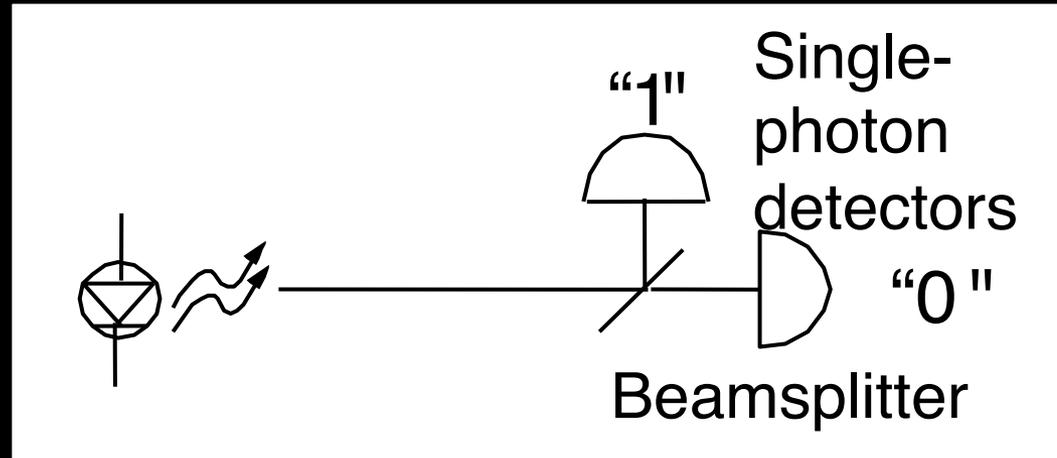
Motivation

- Base security statement of QKD on pseudo-random number generator? NO
- Use physical, non-quantum RNG? better not (bad coin, or bad flipper...)
- Need randomness in several places for QKD:
 - Transmitter basis choice
 - Transmitter bit choice
 - Receiver basis choice
- This can sometimes be achieved passively (i.e., BS at the receiver).
- Entanglement can also give random bit values

Best Motivation: Renato said so...

Commercial Quantum Information: Random Number Generation

God *does*
play dice
with the
Universe!



- Beamsplitter Approach

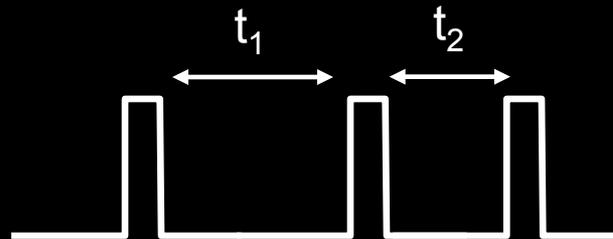
- Problems: Inefficient (< one bit per detection)

- Detector saturation
 - Different detector efficiencies (possibly over time)
 - Multiple (expensive) detectors

- Currently Available: ID Quantique

- 4 MHz (one pair of detectors); 16 MHz (4 pairs of detectors)

Time-based Implementation



- Time-based Approach

[M. Stipcevic, B. Rogina. “Quantum random number generator based on photonic emission in semiconductors”, Rev. Sci. Inst. 78, 045154 (2007)]

- Measure relative intervals between pulses
- If $t_1 > t_2$ then record a “0” bit, else a “1” bit
- Only one detector needed

- Problems
 - Maximum of 1/2 bit per detection
 - Slow output rate (1 MHz)

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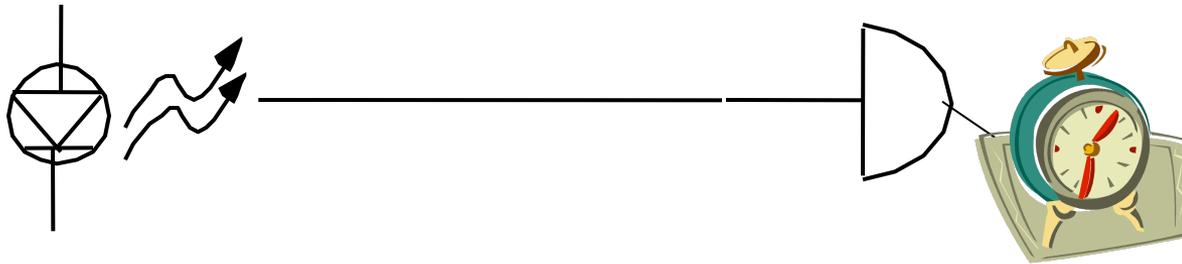
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Our Approach

~~[P. Kwiat et al., United States Patent Application Number 20060010182 (2006)~~
N. Lutkenhaus, J. Cohen, H.K. Lo, United States Patent Number 7197523 (2002)]

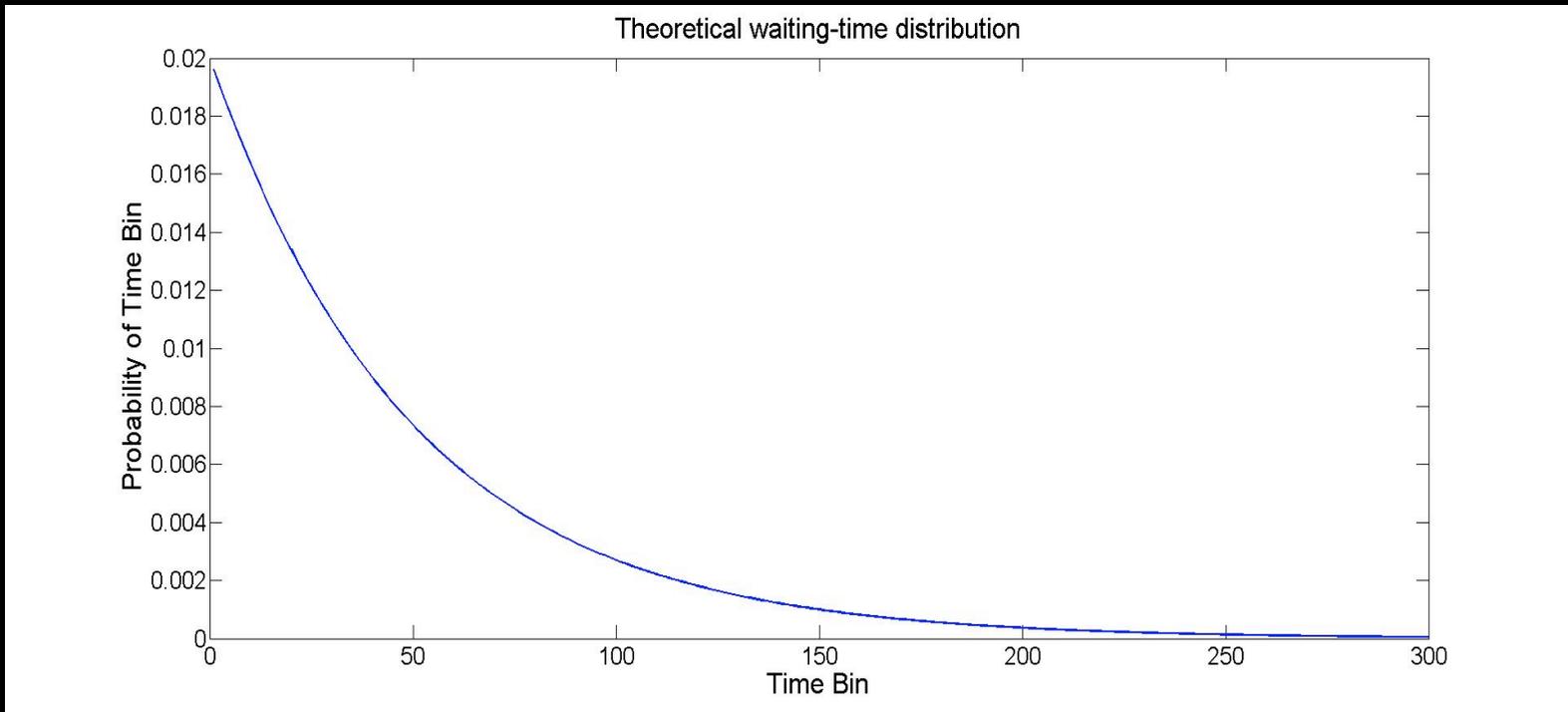
Time-based RNG



- Only *one* detector required
- Split time between successive photon detections into “time-bins”; measure inter-photon arrival time
- E.g., 1024 resolvable time-bins gives up to $\log_2 1024 = 10$ bits per detection (8 after hashing)

Theory

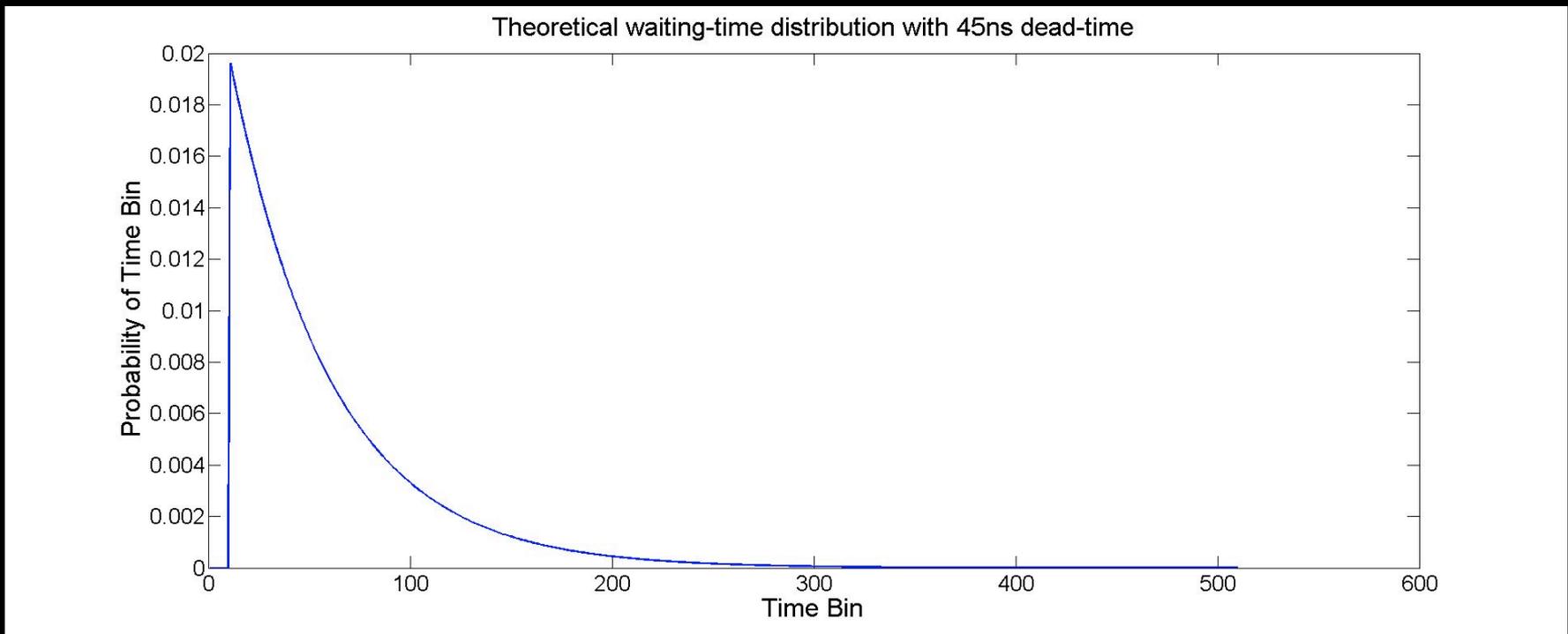
- Behavior of incoming photons is a Poisson process, and waiting-time distribution between events behaves as a decaying exponential.
 - Assume simple model: $P(t, \Delta t) = R e^{-Rt} \Delta t$ (R = average rate)
 - Discrete version: $P(i) = R e^{-(R \Delta t i)} \Delta t$ for a given rate, time-bin resolution Δt , and time-bin i .



- Not all time bins are equally likely
⇒ smaller values occur more often ⇒ need to *hash* (SHA256)

Theory - Modified

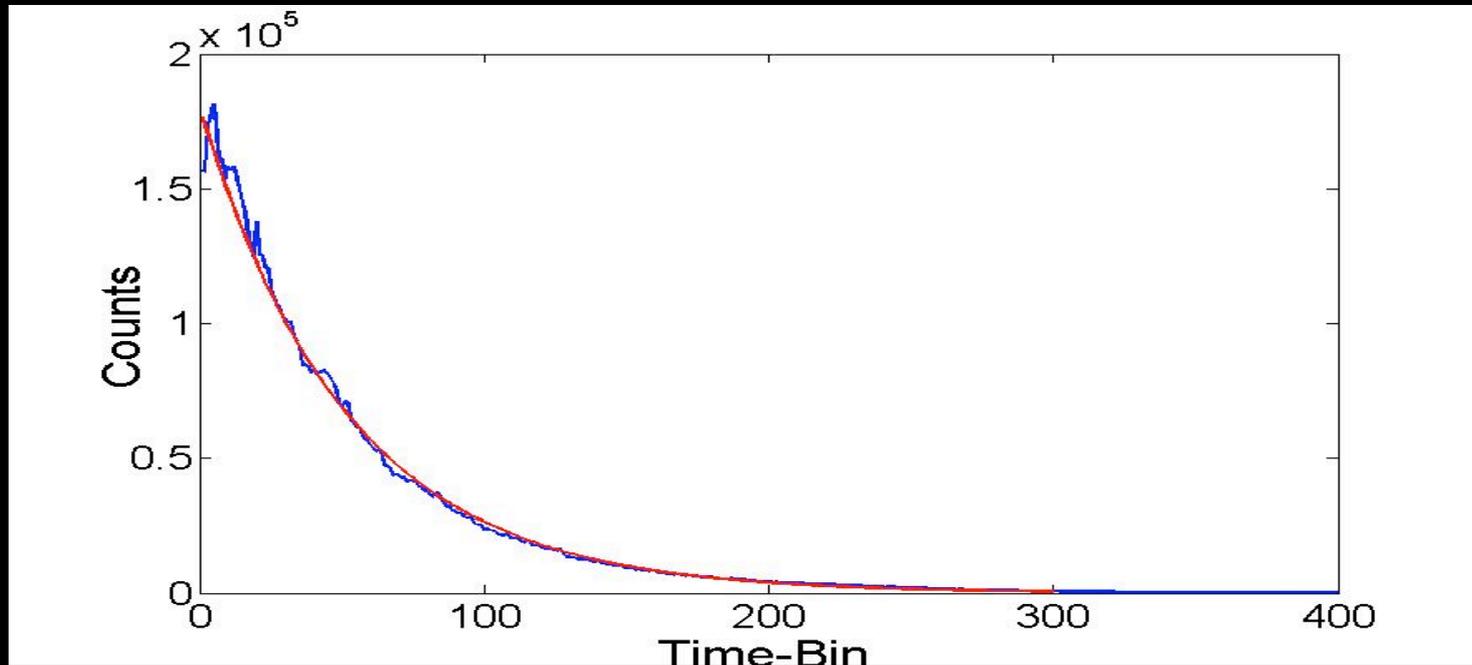
- All real detectors have a deadtime: after a detection event, the detector cannot register another photon for some recovery interval (e.g., 45 ns). *Solution: Ignore initial bins*



- Detectors can also produce “afterpulses”: echoes of a detection (~1% for our detectors). *Solution: Slightly more hashing needed.*

Entropy in our (first) system

$R = 11 \text{ MHz}$; $\Delta t = 5 \text{ ns}$

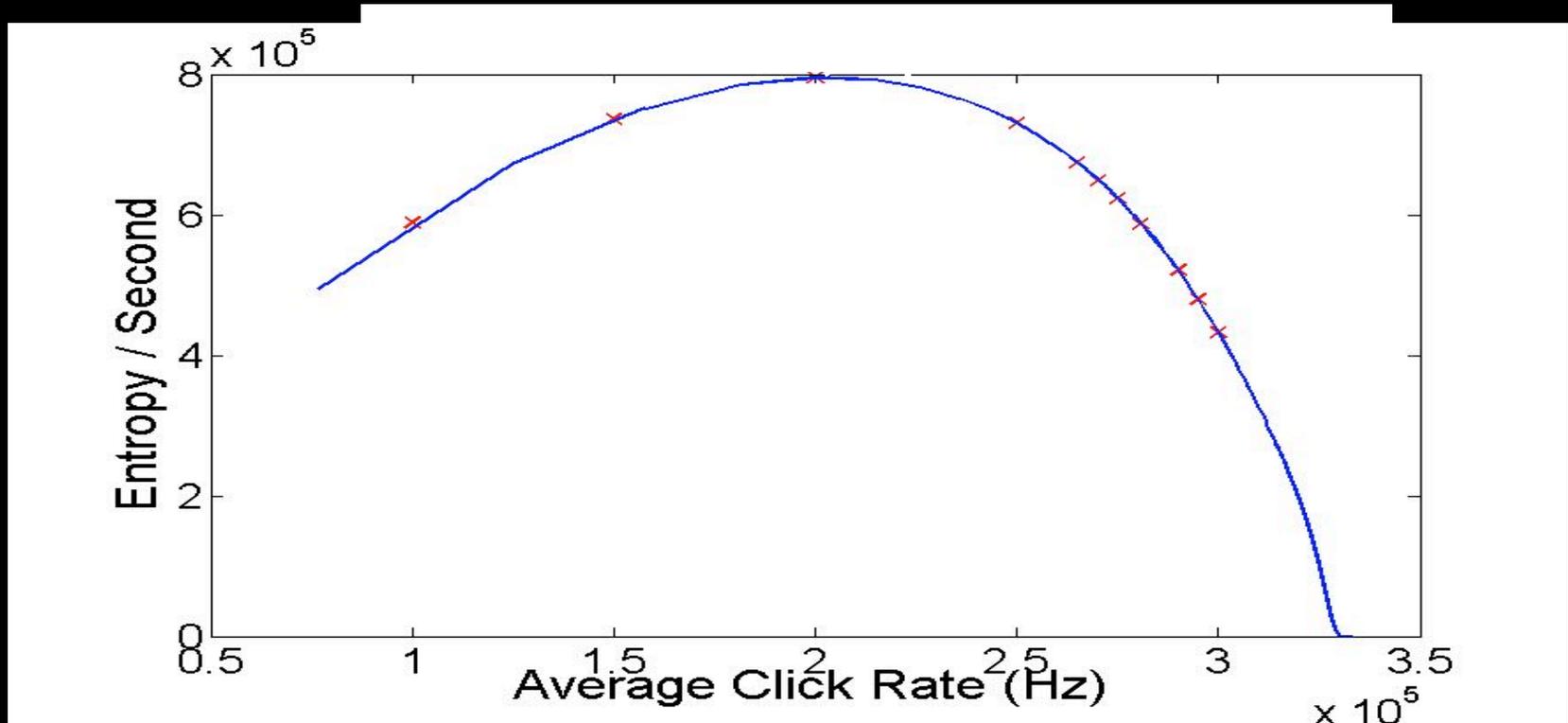


- $S_{\text{approx}} = 7.08 \text{ bits / detection}$
- $S_{\text{actual}} = 7.02 \text{ bits / detection}$
- Min-entropy (after hashing) = 0.997 bits/bit
- Passed all FIPS tests, χ -squared, auto-correlation analysis

Final randomness
rate: 40 MHz

Entropy Saturation

- As count rate increases, the interval between detections will decrease and the smaller valued time-bins will have more counts, decreasing the entropy per detection.



Attenuation

- Strong random deletion ‘washes out’ minor correlations due to non-quantum system characteristics or bunching.
- Attenuation achieved using standard neutral density filter (**ESSENTIALLY A VERY REFLECTIVE BS**), as well as a series of crossed polarizers.
- No observable effect on output randomness, i.e., attenuation mechanism does not seem to introduce any correlations.

Hashing

- “Whiten” the raw waiting-time data to give every value approximately same chance of occurring
 - SHA hash functions
 - Performs a series of complex logical bit operations upon the input string
 - Takes a variable-length string and reduces it to a shorter fixed-length string
 - Every output bit depends on every input bit
 - Relatively fast process, compatible with high detection rates
- Approximately 10 extra bits “overhead” of input entropy required to saturate the hash output
- E.g., 266 bits into SHA-256: Attacker guesses 50.1% of bits correctly
 - Other ‘randomness extractors’ could be used, e.g, bitwise XOR, Toeplitz-hashing extractor, Trevisan's extractor
 - “Postprocessing for QRNG: Entropy evaluation and randomness extraction”, Ma...HKLo, q-ph 1207.1473v1
 - **Use min-entropy, not Shannon**

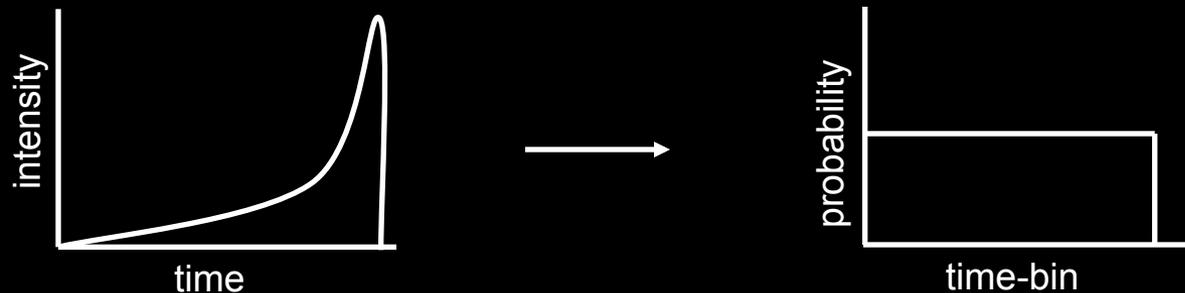
Min-Entropy

$$S_{\min} = \log_2(\max\{P_i\})$$

- The *min-entropy* describes the worst-case scenario; what is the maximum amount of information that could be learned by an adversary?
- For our decaying-exponential probability distribution, the min-entropy is always determined by the first time-bin, as it occurs the most often ($P_1 = R\Delta t = 1/\lambda$).
 - Smaller time-bin resolution will cause first bin's contribution to be proportionally smaller, bringing the min-entropy closer to the Shannon Entropy

Shaped Pulses

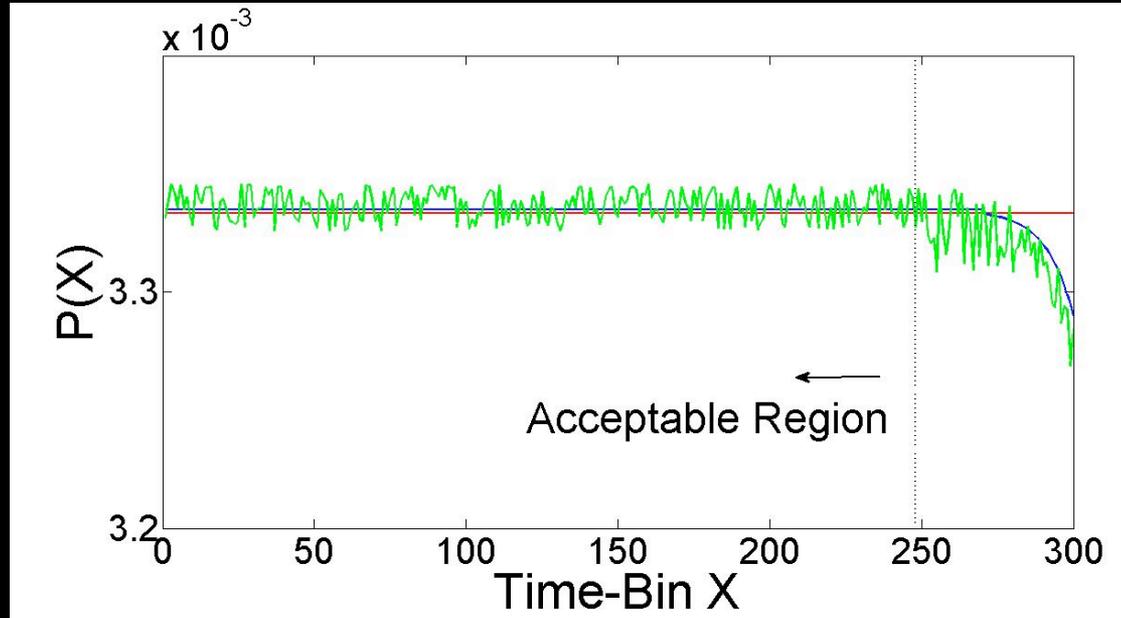
- By driving the laser diode with a shaped pulse such that every time-bin has an equal probability of occurring, we can increase min-entropy and reduce or eliminate the need for hashing.



- Optimal pulse shape $\propto 1/(t-T)$
- For a flat waiting-time distribution, the min-entropy is equal to the Shannon entropy.

Shaped Pulse Source

By shaping the current to approximate $1/(T-t)$, the waiting-time distribution can be tailored to fit the ideal uniform case.



- Resulting waiting-time distribution: min-entropy ~ 0.90 bits/bit
- Discard counts outside the dotted line: $S_{\min} \rightarrow 0.9984$ bits/bit.
- Post-hashed data: Passed all NIST RN test suite.
- Final entropy generation rate: $S_{\min} = 112$ Mbit/s

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Ultra-high speed QRNG, not quite

APPLIED PHYSICS LETTERS 98, 171105 (2011)

An ultrafast quantum random number generator with provably bounded output bias based on photon arrival time measurements

Michael Wahl,^{1,a)} Matthias Leifgen,^{2,b)} Michael Berlin,² Tino Röhlicke,¹ Hans-Jürgen Rahn,¹ and Oliver Benson²

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(Received 22 December 2010; accepted 25 March 2011; published online 26 April 2011)

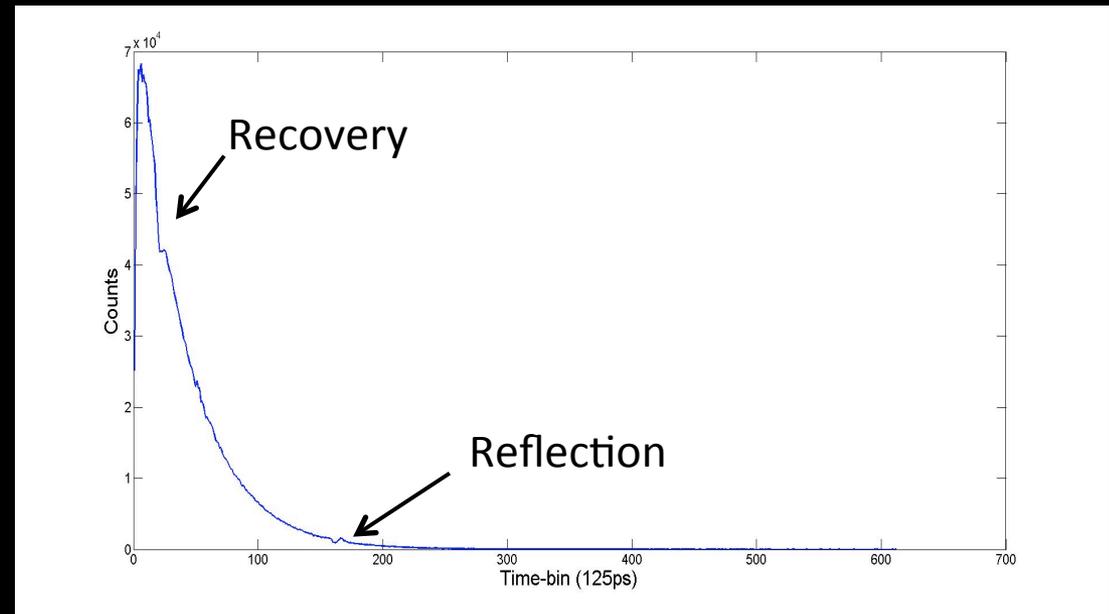
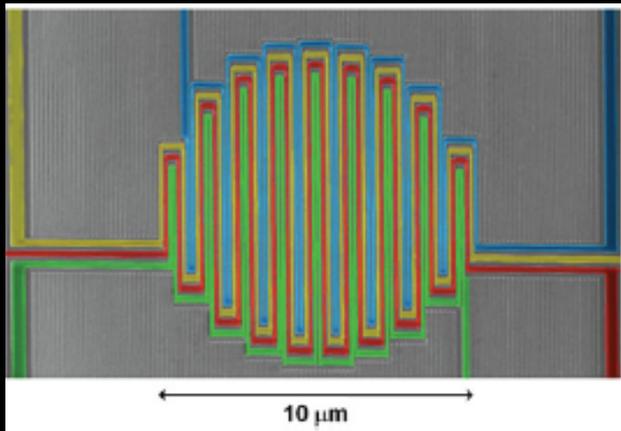
- Claim 16 bit/photon \rightarrow 150 Mb/s !
- Detector: PMT (Hamamatsu H5783)
 - risetime = 0.8 ns

For the analysis of the photon arrival times we use time tagging electronics with a resolution of 1 ps and a throughput of 12.5 Mcps.⁶ In order to achieve real-time performance we

- Detector jitter?? 30-150 ps \rightarrow 10 or 9 bit/photon \rightarrow \sim 100Mb/s

(Preliminary) Ultra-high speed QRNG

- Superconducting nanowire detectors allow fast rate of detection \rightarrow up to ~ 400 Mc/s, with $< \sim 100$ ps jitter
- Xilinx Vertex-6 FPGA + 600MHz Clock + 1:16 DeMux \rightarrow 104-ps bins
- Final entropy rate (after post-processing): **1.86 Gbit/s!**



Eric Dauler
Andrew Kerman
Danna Rosenberg

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- application to QKD – foreshadowing...

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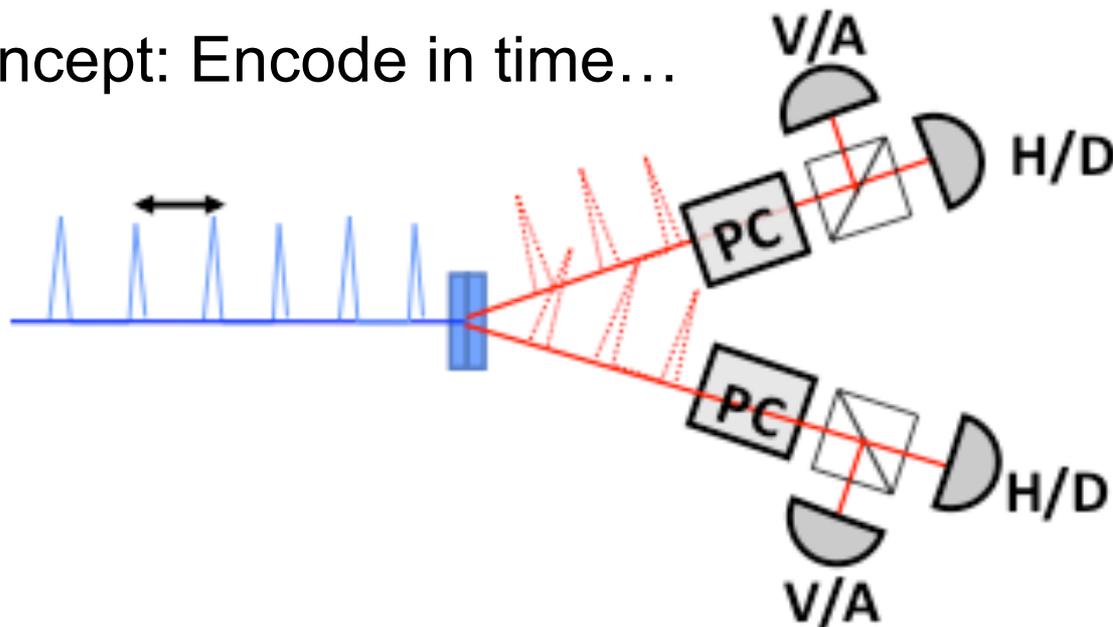
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Hyper-entanglement Enhanced QKD

Central Concept: Encode in time...



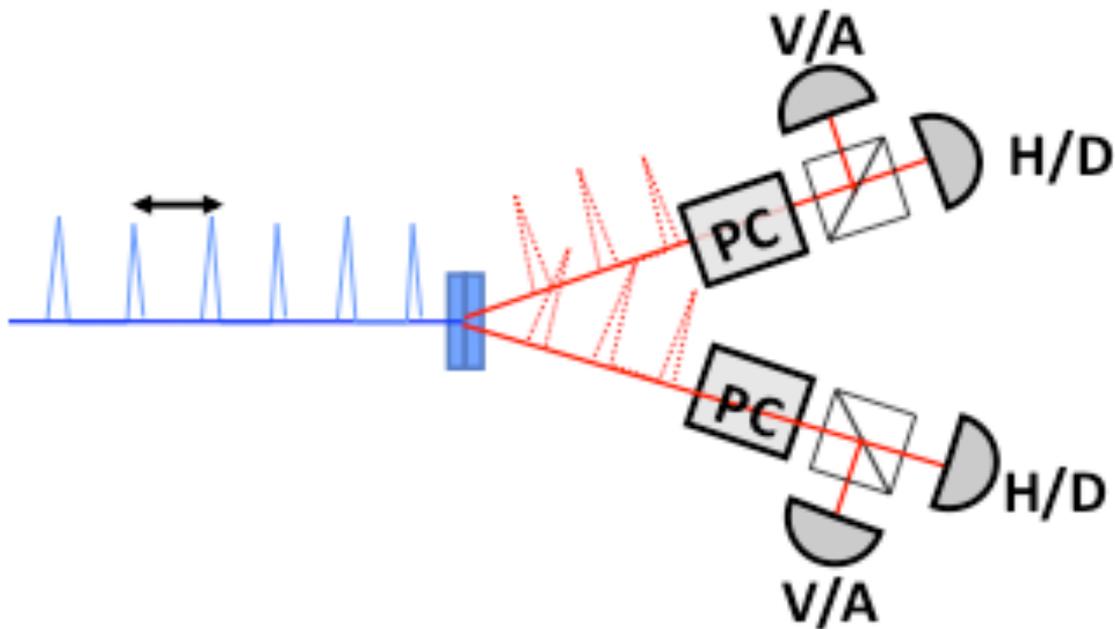
$$|\psi\rangle \propto (|t_0 t_0\rangle + |t_1 t_1\rangle + |t_2 t_2\rangle + \dots + |t_N t_N\rangle)$$

Bin spacing: Δt $\Delta t \sim 130$ ps

Code "length": $\sim N\Delta t$

bits/photon $\sim \log_2 N$ $N \sim 1,024 \rightarrow 10$ bit/pair

Central Concept: Encode in time...

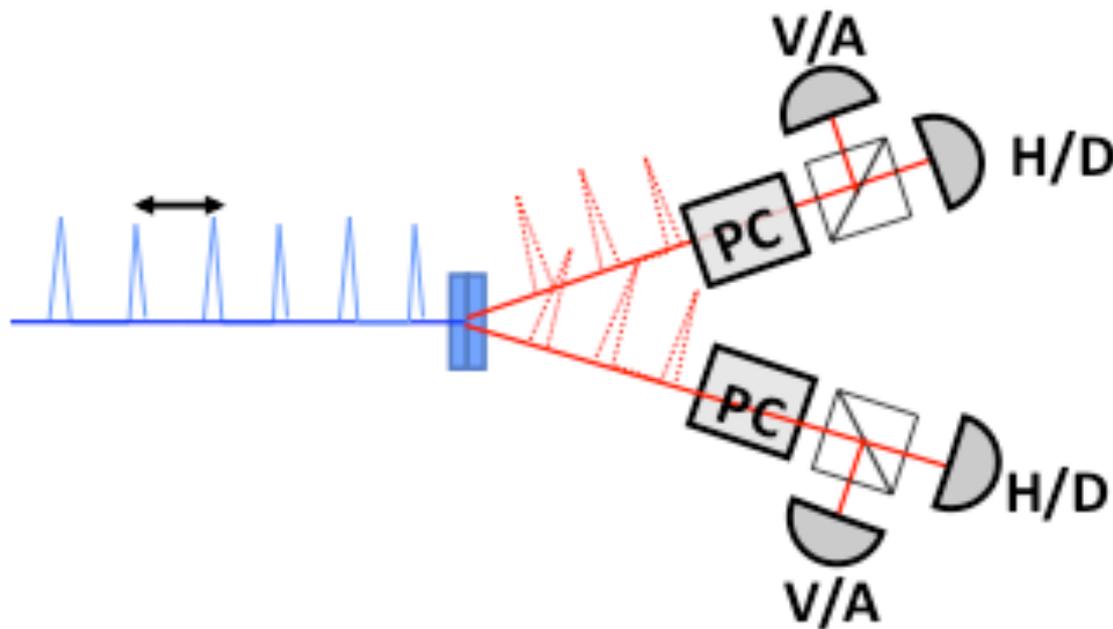


$$|\psi\rangle \propto (|t_0 t_0\rangle + |t_1 t_1\rangle + |t_2 t_2\rangle + \dots + |t_N t_N\rangle)$$

*Alice and Bob use which time bin they detect a photon in to generate multiple bits per click.**

*Ali-Khan, Broadbent, Howell, Phys. Rev. Lett. **98**, 060503 (2007)

Central Concept: Encode in time, verify in polarization



$$|\psi\rangle \propto (|t_0 t_0\rangle + |t_1 t_1\rangle + |t_2 t_2\rangle + \dots + |t_N t_N\rangle) \otimes (|HH\rangle + |VV\rangle)$$

Alice and Bob use which time bin they detect a photon in to generate multiple bits per click.* Get extra bpp from BB84 with polarization. They can constantly check for an eavesdropper using the D/A polarization basis (assuming no QND capability for Eve). “Future security” Perform standard error detection/correction and privacy amp. Eventually measure in MUB.

Current status: >5bit/photon, >2 Mb/s End goal: 10 bpp, >1Gb/s

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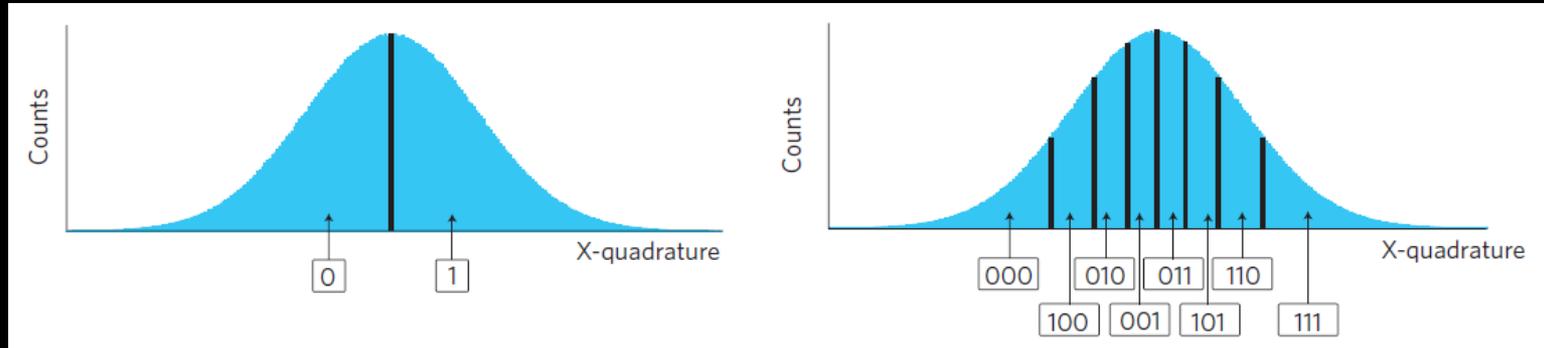
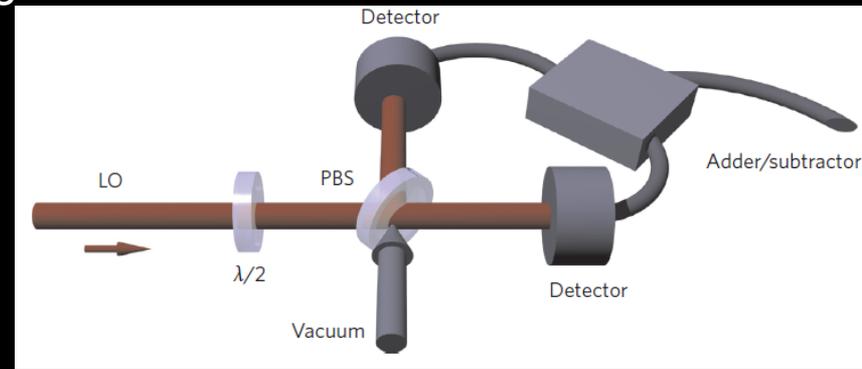
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Continuous Variable QRNG

- uses quantum uncertainty in quadrature amplitudes of the vacuum state as a source of randomness
- uses homodyne detection to measure the position quadrature of the vacuum state
- multiple bits may be extracted by dividing the quadrature into equal probability sections
- numerical hashing methods are required to eliminate classical sources of randomness and experimental biases
- demonstrated rates of 6.5Mbps [1] , 2 Gbps [2]



1. C. Gabriel ...G. Leuchs, *Nature Photonics* **4**, 711-715 (2010).
2. T. Symul, S.M. Assad, and P.K. Lam, *Appl. Phys. Lett.* **98**, 231103 (2011).

Laser Noise QRNG

F. Xu, ...and HK Lo, Opt. Expr. 20, 12366 (2012)

Abstract: A quantum random number generator (QRNG) can generate true randomness by exploiting the fundamental indeterminism of quantum mechanics. Most approaches to QRNG employ single-photon detection technologies and are limited in speed. Here, we experimentally demonstrate an ultrafast QRNG at a rate over 6 Gbits/s based on the quantum phase fluctuations of a laser operating near threshold. Moreover, we consider

noise) of a laser, which yields a speed of 500 Mb/s [18,19]. Instead of directly measuring weak quantum effects, this scheme measures the enhanced quantum noise (amplified spontaneous emissions) and thus can be realized by *conventional photodetectors* at a high-speed and with a

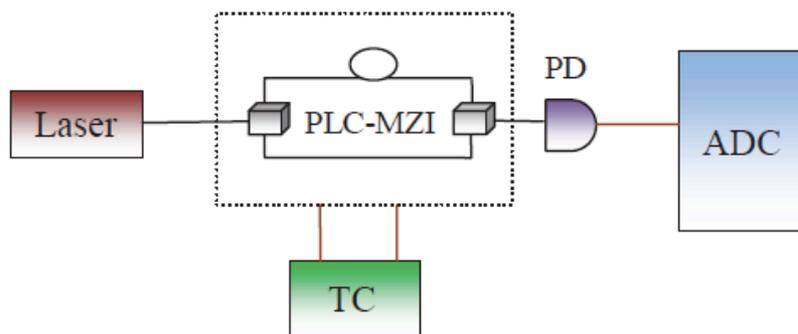


Fig. 1. Experimental setup. Laser, 1550nm cw DFB laser diode (ILX Lightwave); PLC-MZI, planar lightwave circuit Mach-Zehnder interferometer with a 500ps delay difference (manufactured by NTT); TC, temperature controller (PTC 5K from Wavelength Electronics Inc.); PD, 5GHz InGaAs photodetector (Thorlabs SIR5-FC); ADC, 8-bit analog-to-digital convertor inside an oscilloscope (Agilent DSO81204A).

“We finally remark that our implementations of randomness extractors [Toeplitz-hashing Trevisan’s extractor] with Matlab on a standard PC are not fast enough for a real-time QRNG.”

Don’t know what the rate is, but < 6 Gb/s.

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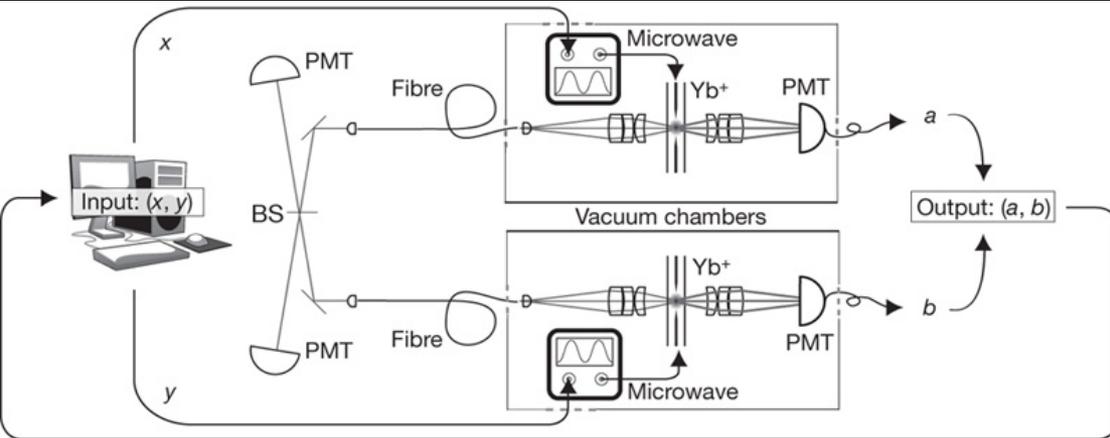
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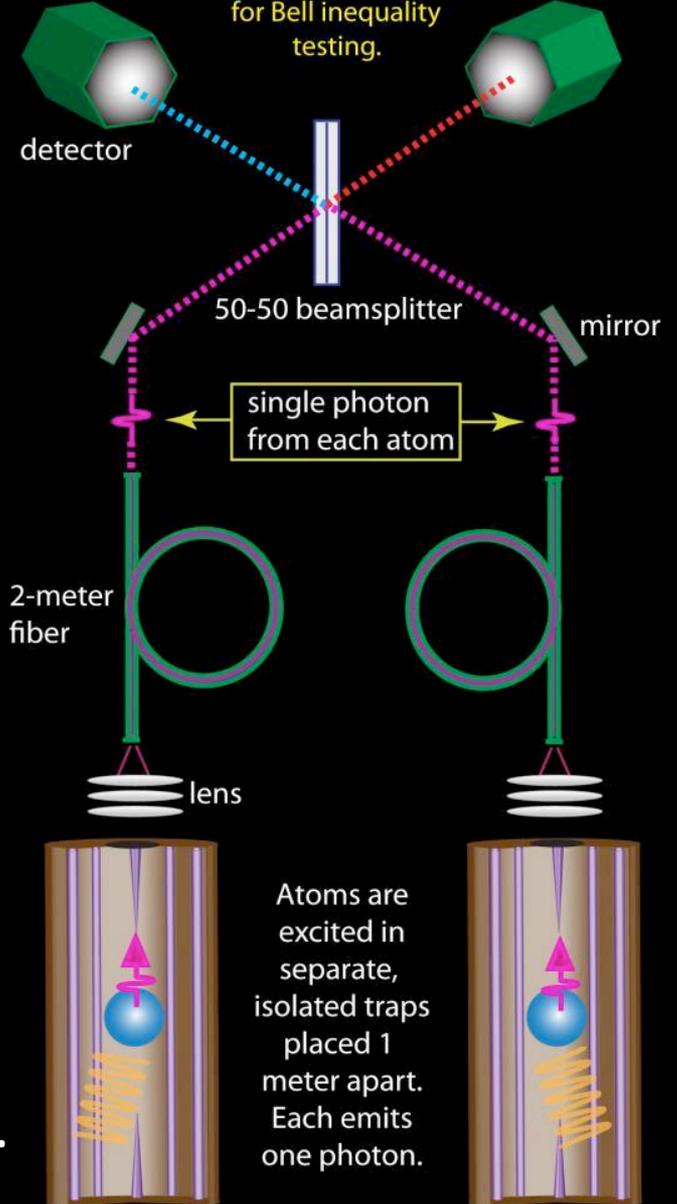
Advantages of Entanglement (for QKD)

- Automatic *randomness* of key
- Longer distances accessible (since Bob knows *when* to look for a photon) [But decoy states...]
- Established methods to verify security of key
- Any leakage of info to other DOF (from source)
⇒ increased bit error rate (BER)
[cf 4-diode source of Canary Island experiment...]
- System can be automatically verified (even if “sold” by Eavesdropper!). If you can make a “loophole-free” Bell inequality violation → “device-independent QKD”

Device-Independent QRNG



The trapped atoms are excited. Single photons, one from each atom, interact at a beamsplitter. If both detectors record a photon simultaneously, the ions are entangled and ready to rotate for Bell inequality testing.



Random numbers certified by Bell's theorem

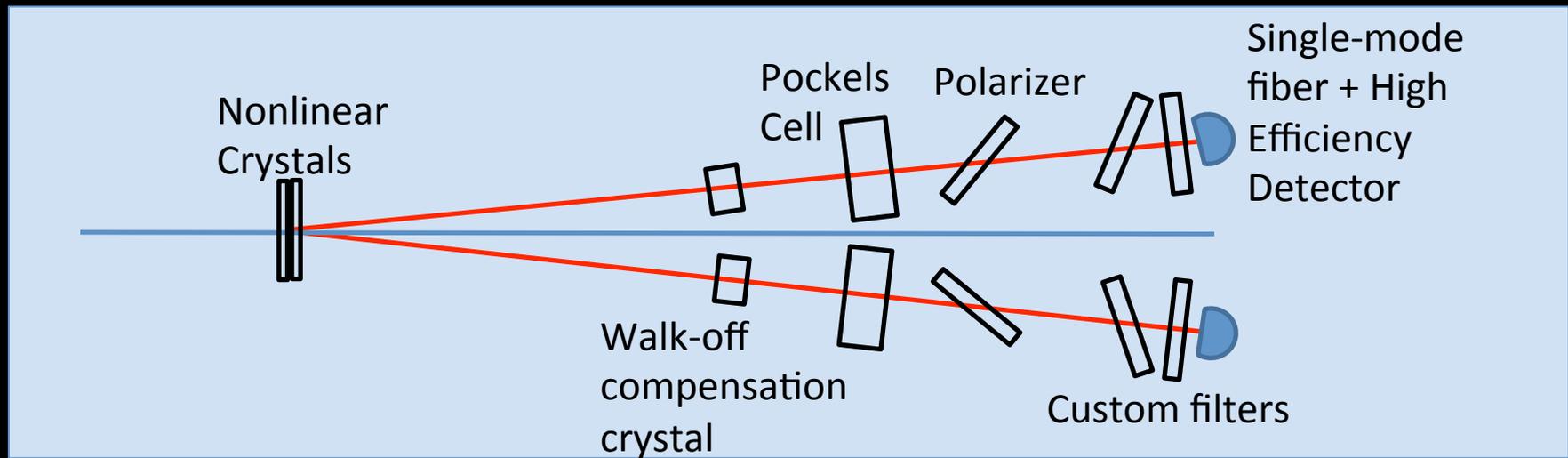
S. Pironio^{1,2,7}, A. Acín^{3,4,7}, S. Massar^{1,7}, A. Boyer de la Giroday⁵, D. N. Matsukevich⁶, P. Maunz⁶, S. Olmschenk⁶, D. Hayes⁶, L. Luo⁶, T. A. Manning⁶ & C. Monroe⁶

Random, but Not by Chance: A Quantum Random-Number Generator for Encryption, Security

ScienceDaily (Apr. 19, 2010) — Researchers have encrypted communications and other uses, that is cryptographically secure, inherently private and -- most importantly -- certified random by laws of physics.

Pironio, *et al* observed a Bell violation of $SCHSH = 2.414$ with 3016 events per month. Corresponding to ~ 45 bits per month.

SPDC Device-Independent QRNG...



Instead of atoms, we can/should be able to violate a Bell inequality using a downconversion source with 81% heralding efficiency (assume detector efficiency of ~92%)

Our source can run at $>10^6$ events per second

Data TODAY: → 2.6 MHz coincidence rate with 4 detectors

→ should have >5 MHz coincidence rate with 8 detectors

With 95% efficiency detectors, we will have an estimated ~ 30 kb/s. (10⁹ improvement...)

Detection Loophole in tests of Nonlocality

Clauser-Horne Inequality

Phys. Rev. D 10, 526 (1974)

$$C(A_1, B_1) + C(A_1, B_2) + C(A_2, B_2) - C(A_2, B_1) \leq S(A_1) + S(B_1)$$

Works with non-maximally entangled state:

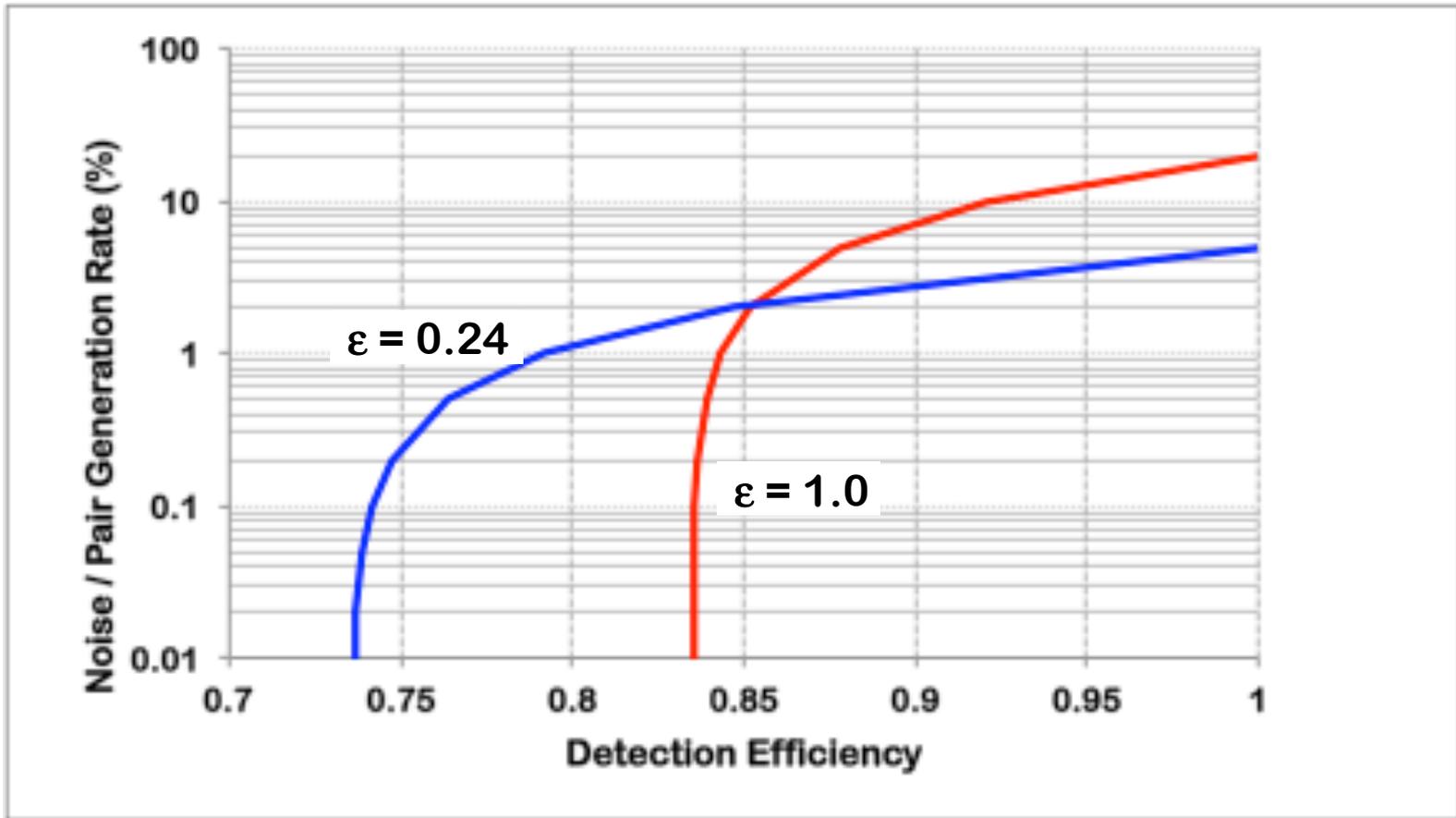
$$|HH\rangle + \varepsilon |VV\rangle \quad (\text{Eberhard, PRA 47, 747 (1993)})$$

→ choose A_1 and B_1 to minimize singles contribution

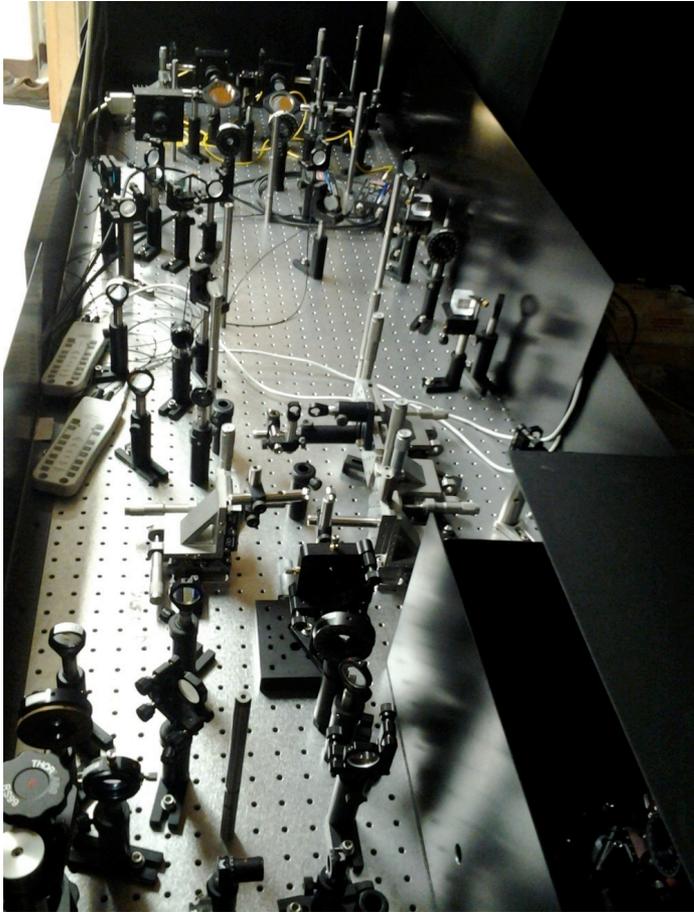
→ choose A_2 and B_2 to enable violation

→ reduce required detector efficiency, 83% → 67%,
*assuming no background or analyzer crosstalk,
and perfect quantum state.*

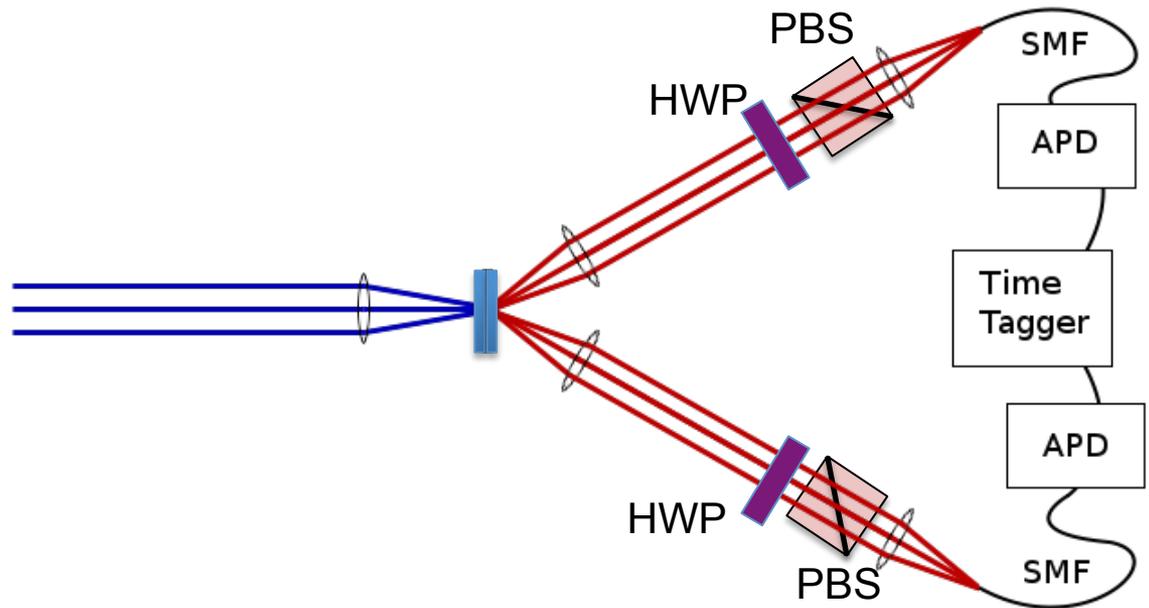
Allowable noise vs efficiency, with measured crosstalk, for measured ϵ_{HH+VV} states



- for our measured noise (0.1%), need $\eta > 74\%$
- how are we doing...

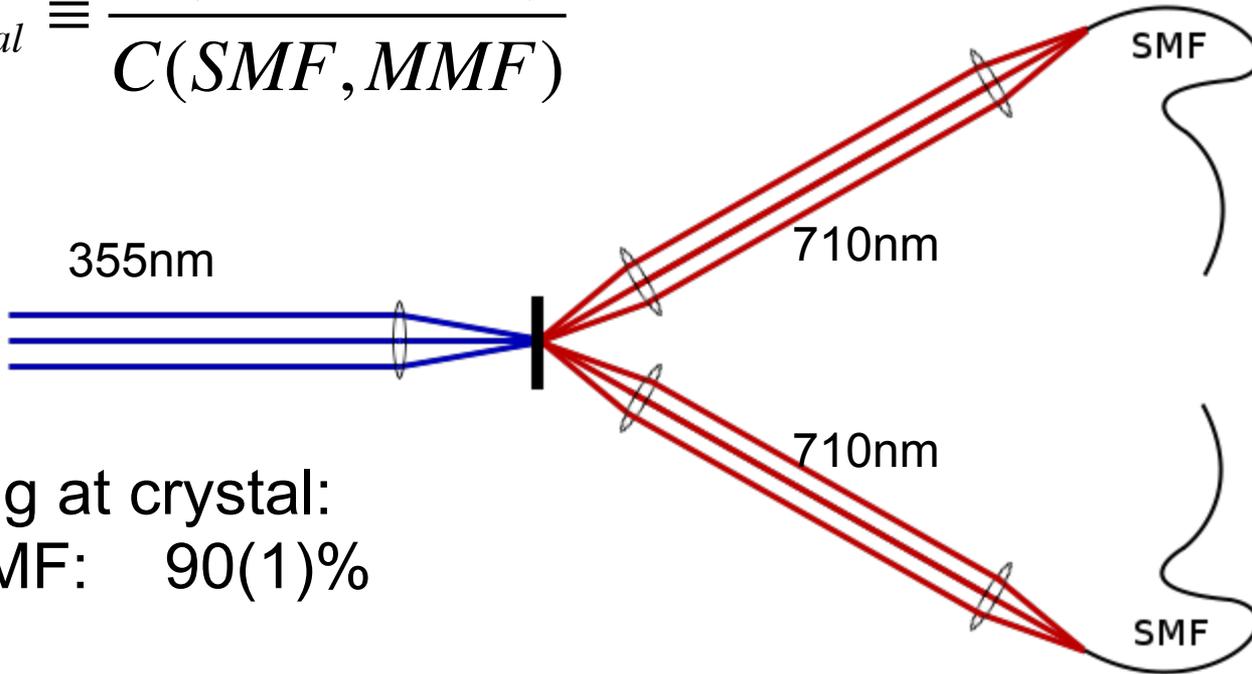


- 4 W, 120 MHz, 5 ps, 355 nm
- BiBO double-crystal setup



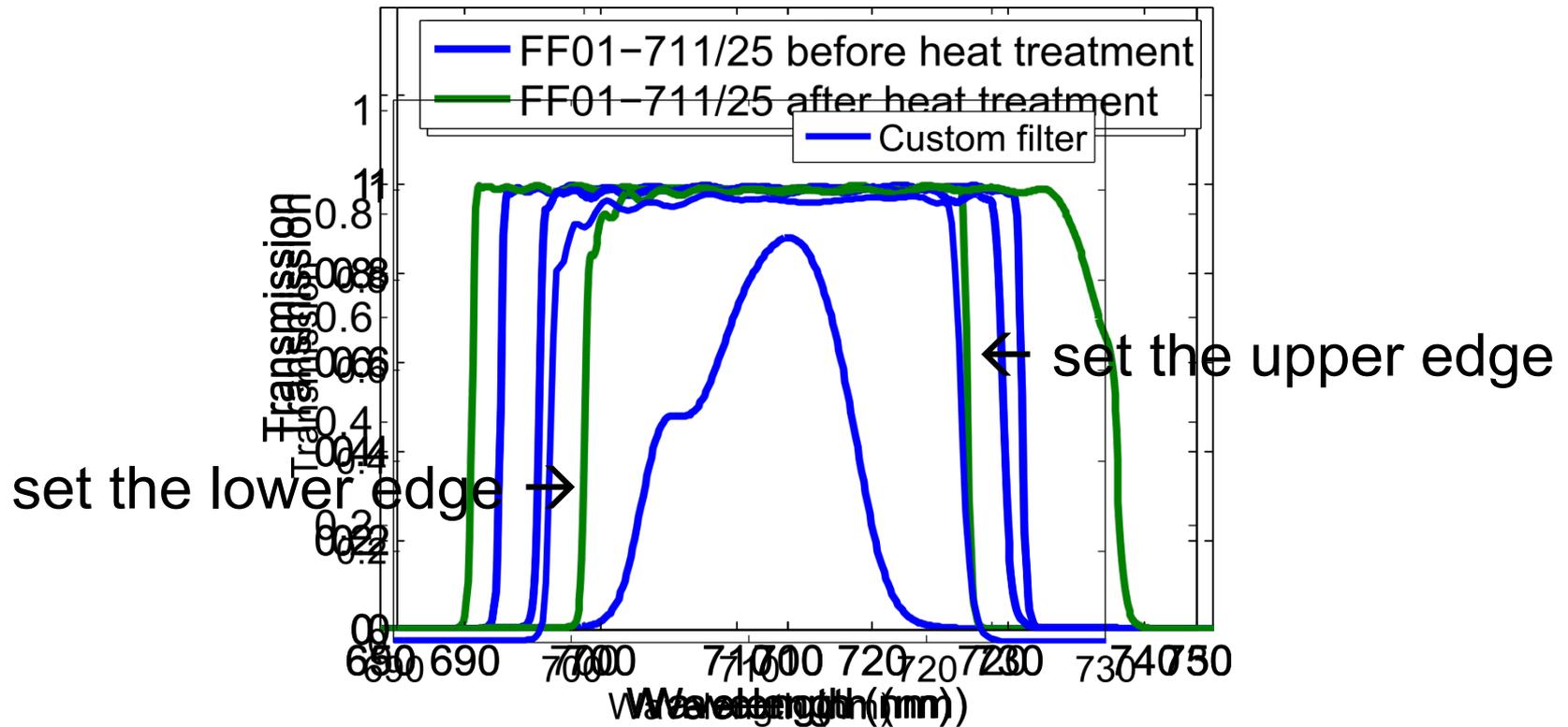
Spatial Heralding Efficiency

$$\eta_{spatial} \equiv \frac{C(SMF, SMF)}{C(SMF, MMF)}$$



Focusing at crystal:
SMF-SMF: 90(1)%

Spectral Heralding Efficiency



355 nm → 710 nm + 710 nm
 Heralding efficiency: 50% → 95%

Results

Predicted:

$$\eta = \eta_{\text{spatial}} * \eta_{\text{spectral}} * \eta_{\text{optics}} * \eta_{\text{detector}}$$

$$= 0.9 * 0.95 * 0.9 * 0.65$$

$$= 0.50$$

0.94 (AR-coated fibers)

0.95 (TES)

Measured (C/S): $\eta = 0.507(5)$ [with one crystal]

0.53

0.80

**Exceeds
required
75%!**

Record two-way heralding

BUT η_{spatial} drops to ~ 0.6 [for two crystals]

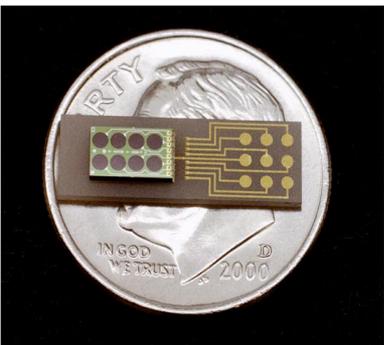
$\rightarrow \eta = 0.32$

\rightarrow need polarization-dependent focusing/collection

Preliminary data from yesterday \rightarrow now up to $\eta_{2\text{xtal}} = 45\%$

(Other) High-Efficiency Single-Photon Detectors

- *Solid State Photomultipliers (SSPMs)*
- *Visible Light Photon Counters (VLPCs)*



VLPCs

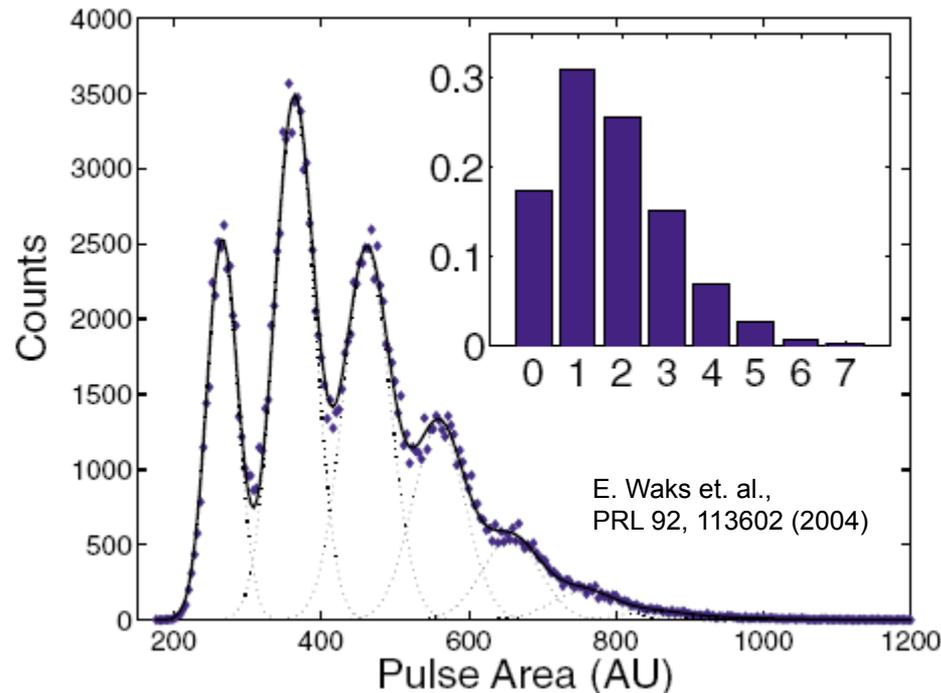


SSPMs

- *SSPMs originally developed by Rockwell for IR military applications*
- *VLPCs are their IR-desensitized successors (used by FermiLab)*
- *High measured efficiency (~88%)*
- *Very high inferred efficiency (~95%)*
- *Multi-photon detection capability*
 - [Takeuchi et al. APL 74, 1063 (1999)]*
- *~Fast (~300ps jitter)*
- *~6K operation*
- *“Big” – 1 mm → good for turbulent spatial modes*

Photon Number Resolving Capability

- VLPC can resolve photon number
 - Localized avalanche allows multiple parallel detection events
 - Low multiplication noise (low gain dispersion)
 - Pulse height proportional to incident photon number
 - Photon number resolution of up to ~20 photons



PNR useful for entanglement sources or heralded single-photon sources – determine there aren't two pairs.

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Questions?

You wouldn't attack someone holding a puppy would you?

