IPAM/UCLA, Sat 24th Jan 2009

Numerical Approaches to Quantum Many-Body Systems

QS2009 tutorials

lecture:

Entanglement Renormalization

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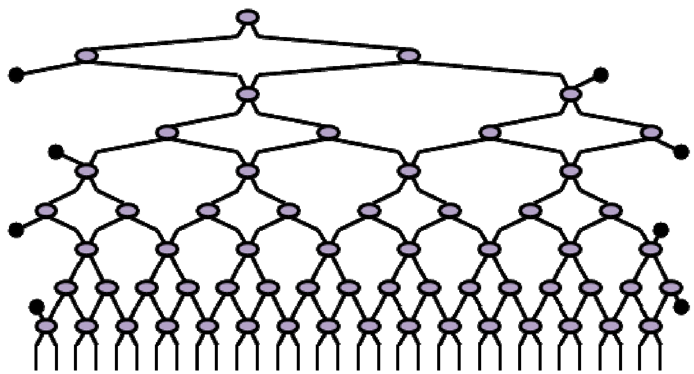
Australia

Outline

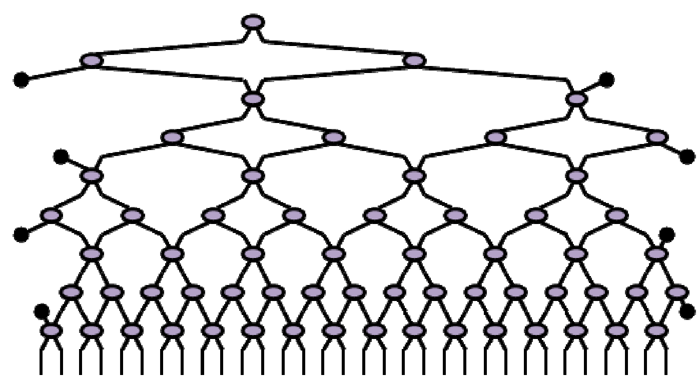
- MERA (multi-scale entanglement renormalization ansatz)
- RG transformation
- Computation of expected values



• MERA (multi-scale entanglement renormalization ansatz)

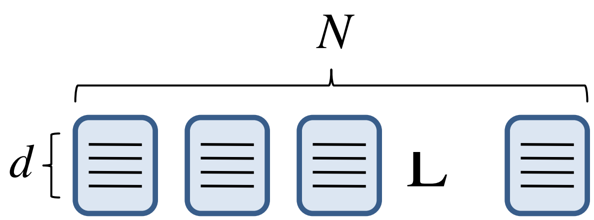
• RG transformation

• Computation of expected values



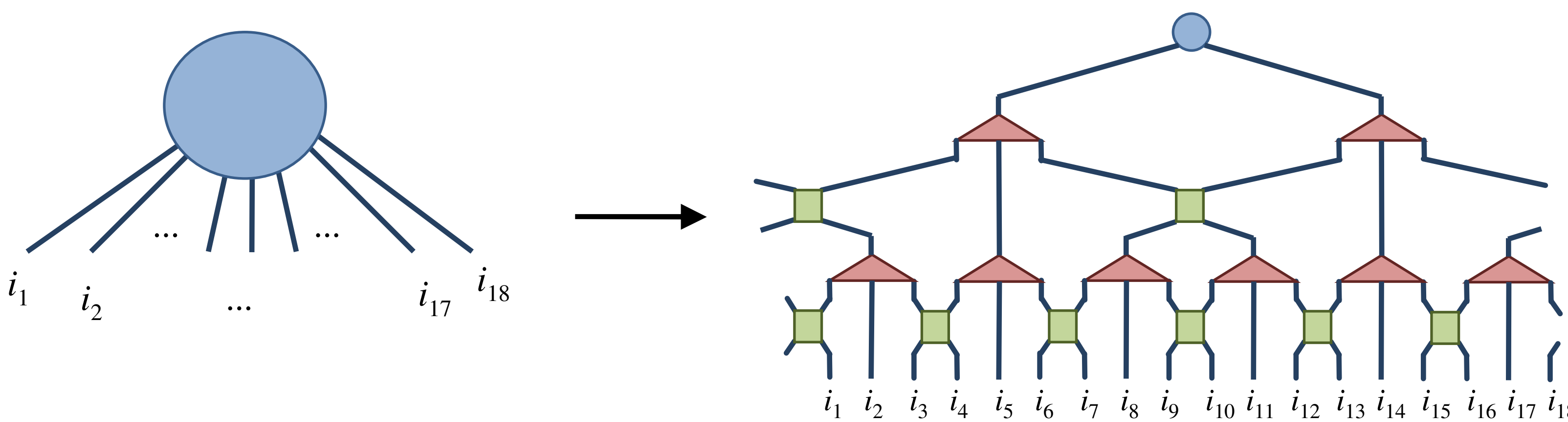
MERA (multi-scale entanglement renormalization ansatz)

- Lattice with N sites

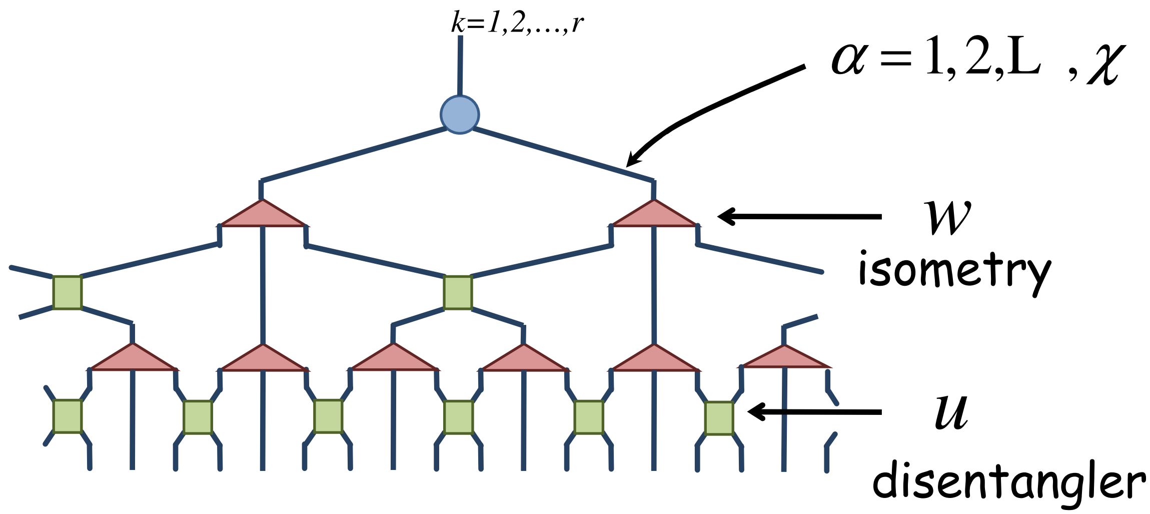


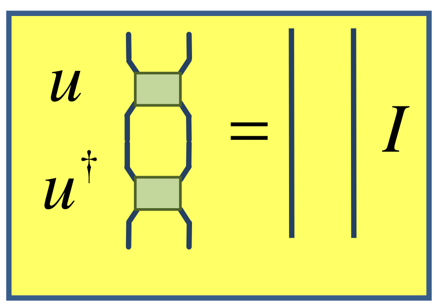
$$H_1 \otimes H_2 \otimes \ldots \otimes H_N$$

$$\left|\Psi_{GS}\right\rangle = \sum_{i_1=1}^{d} \sum_{i_2=1}^{d} \mathrm{L} \sum_{i_N=1}^{d} c_{i_1 i_2 \ldots i_N} \left|i_1\right\rangle \left|i_2\right\rangle \ldots \left|i_N\right\rangle$$

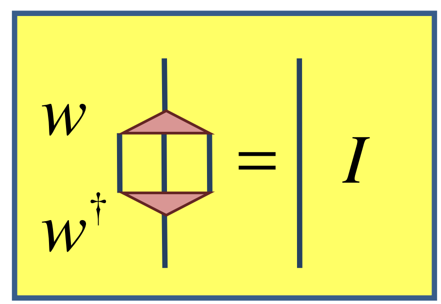


MERA (multi-scale entanglement renormalization ansatz)





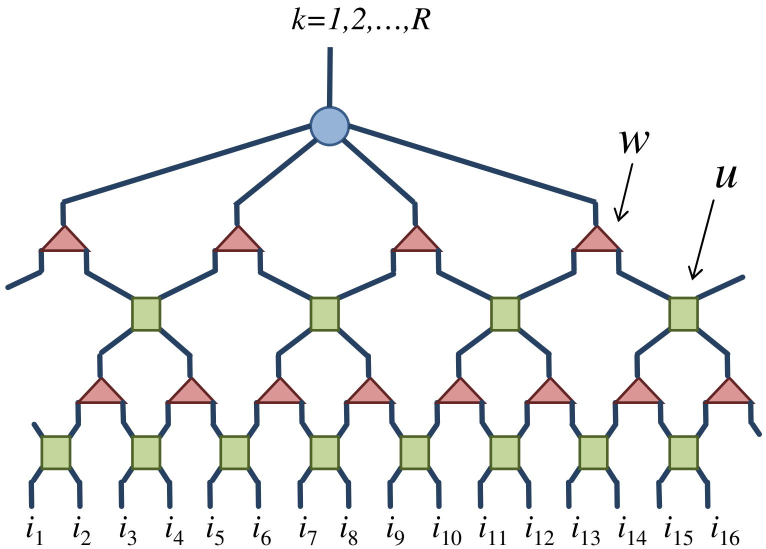
disentangler



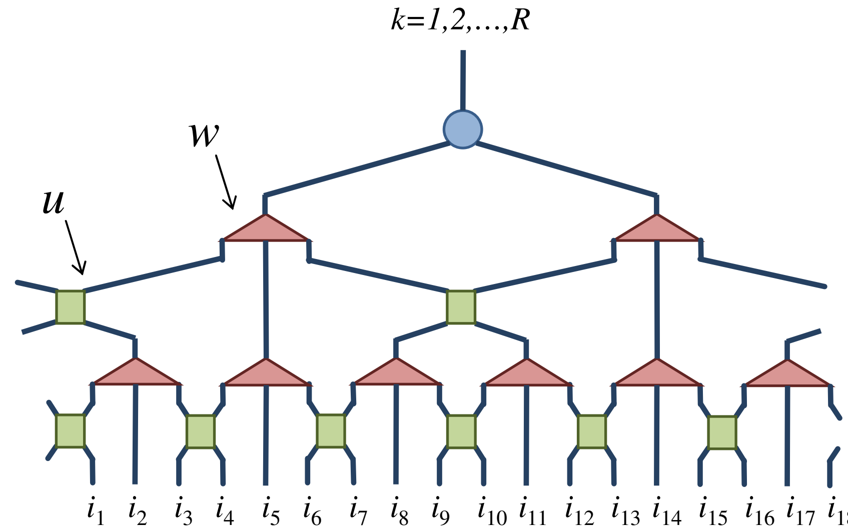
isometry

- Many possibilities

examples: binary 1D MERA

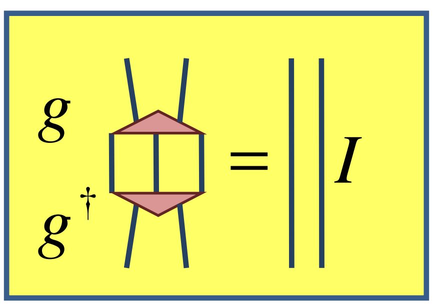


ternary 1D MERA

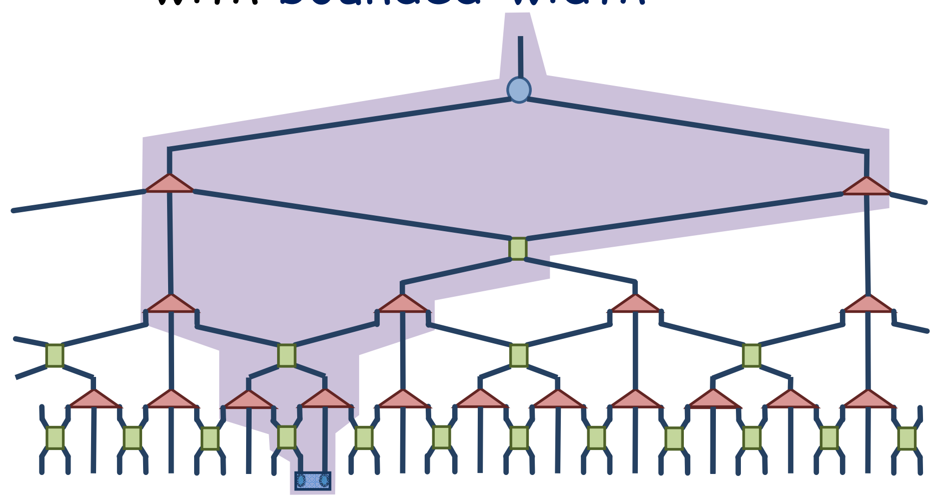


- Defining properties:

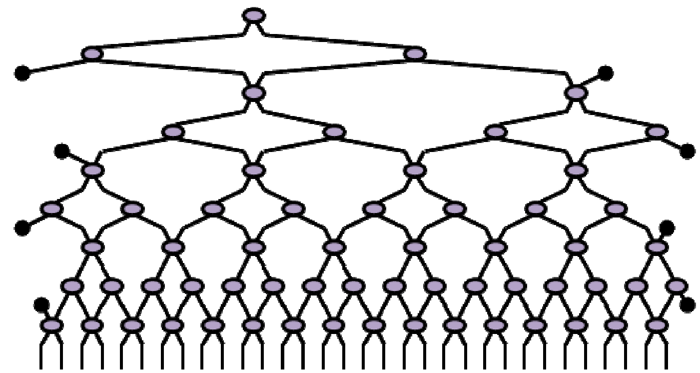
1) isometric tensors



2) past causal cone with bounded 'width'



What is the MERA useful for?



- Ground states/low energy subspaces in 1D, 2D lattices
- polynomial correlators, energy gaps

Simulation cost:

$\chi^p N$ inhomogeneous → $\chi^p \log N$ translation invariance → $\chi^p \log \xi$ finite correlation length; → χ^p scale invariance

In the workshop (Wednesday afternoon) I will explain:

- How to extract critical exponents/CFT data at a quantum critical point

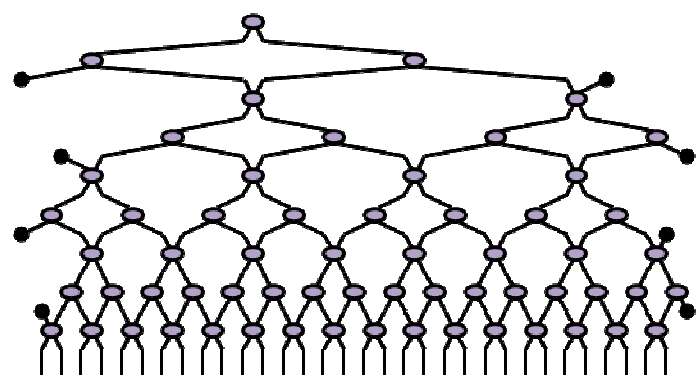
- How to represent 2D ground states with topological order

- Simulation of 2D frustrated Heisenberg antiferromagnets

- MERA (multi-scale entanglement renormalization ansatz)

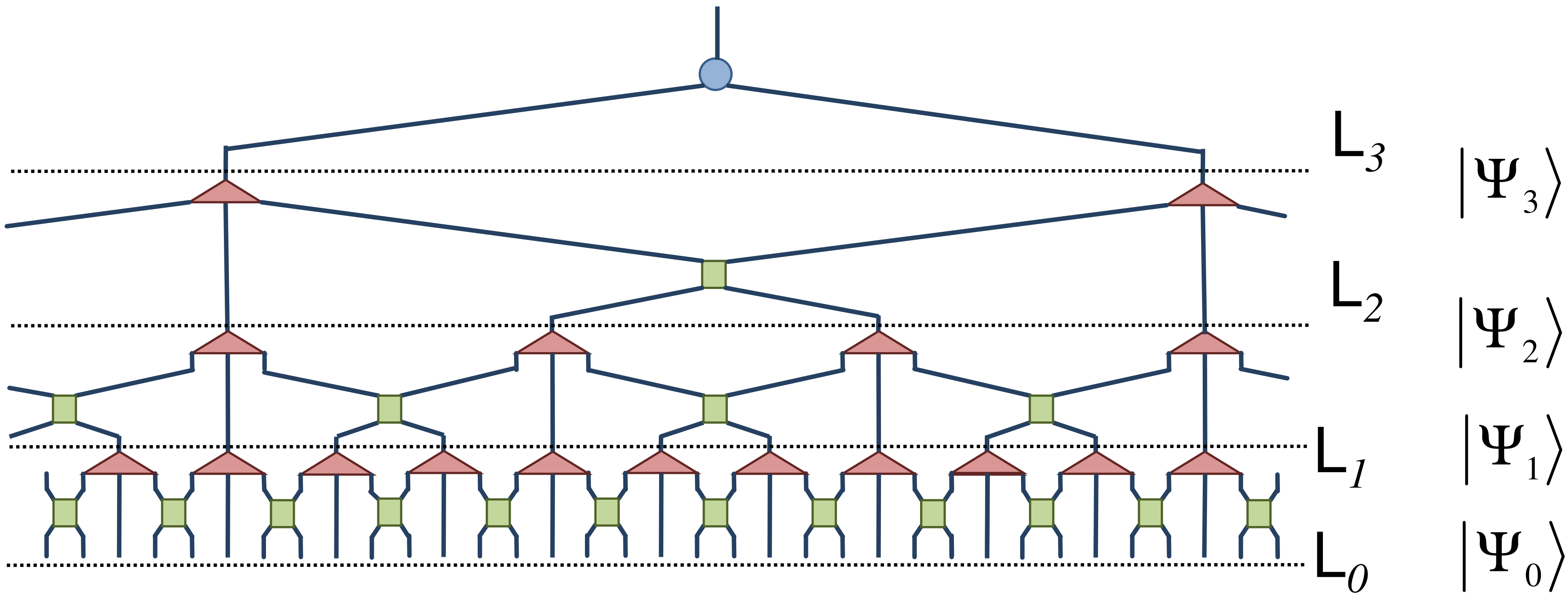
- RG transformation

- Computation of expected values

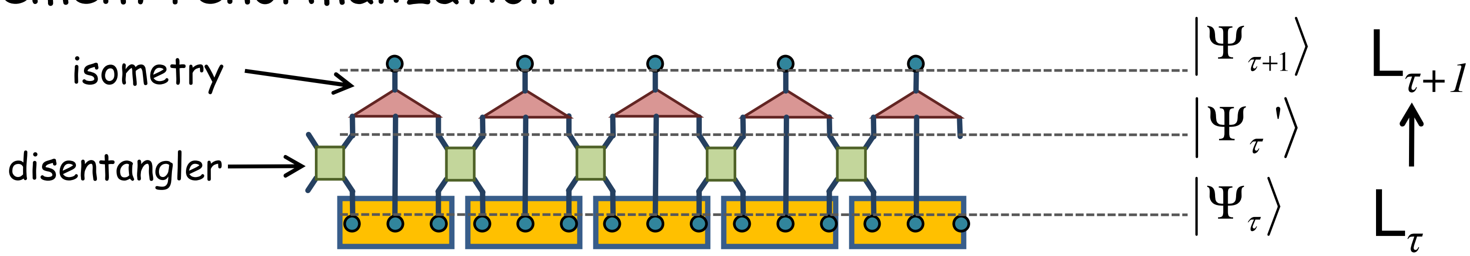


RG transformation

The MERA defines a coarse-graining transformation



Entanglement renormalization



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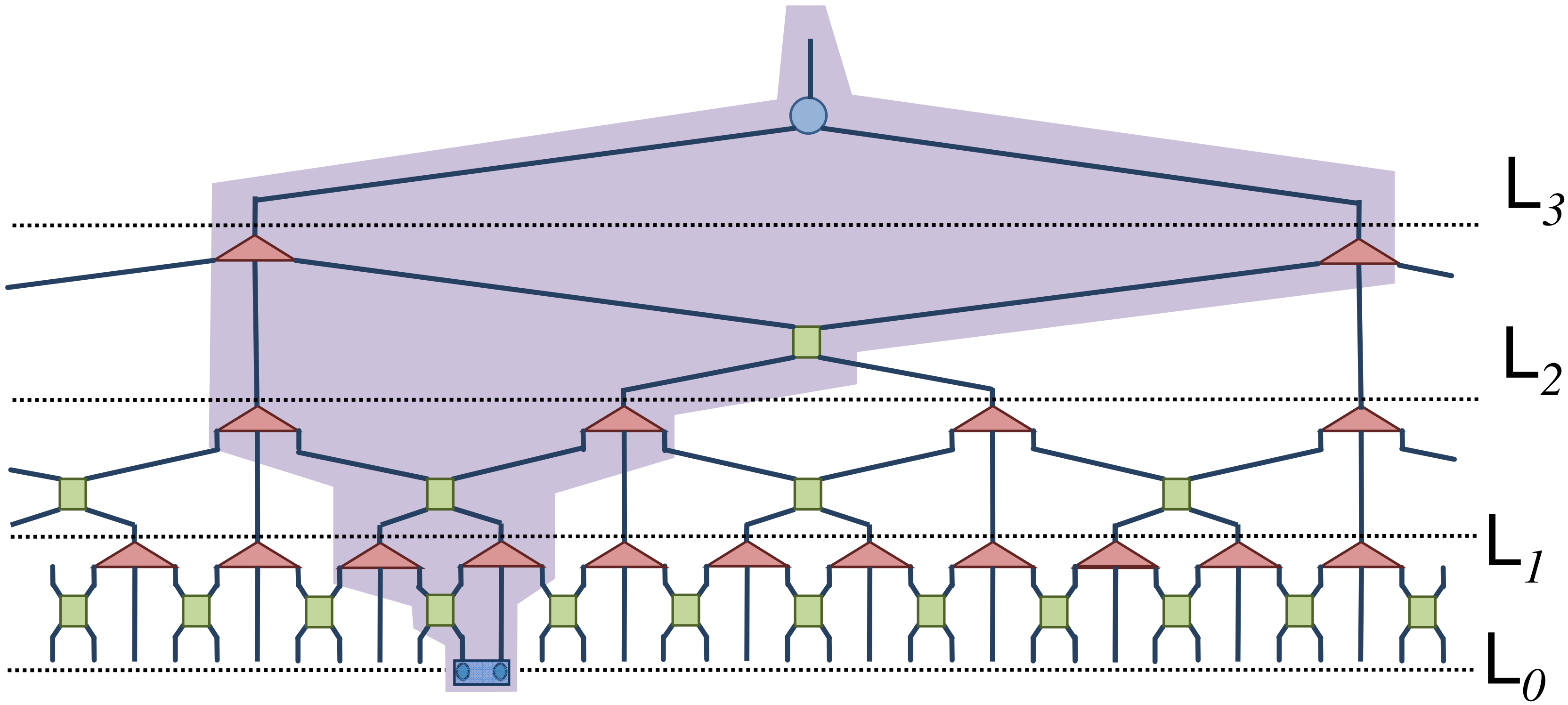
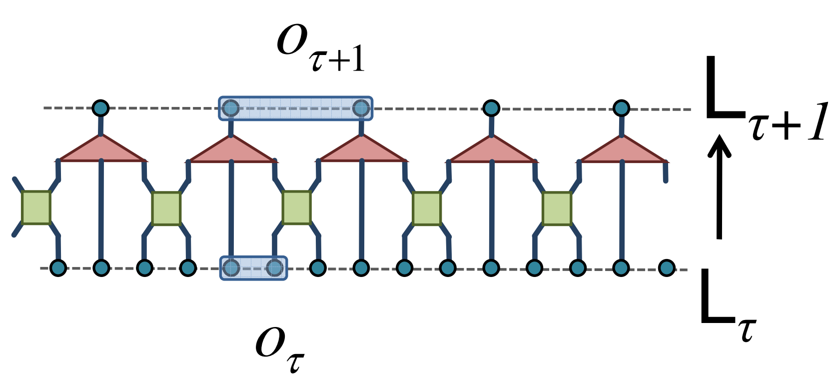


• transformation of local operators

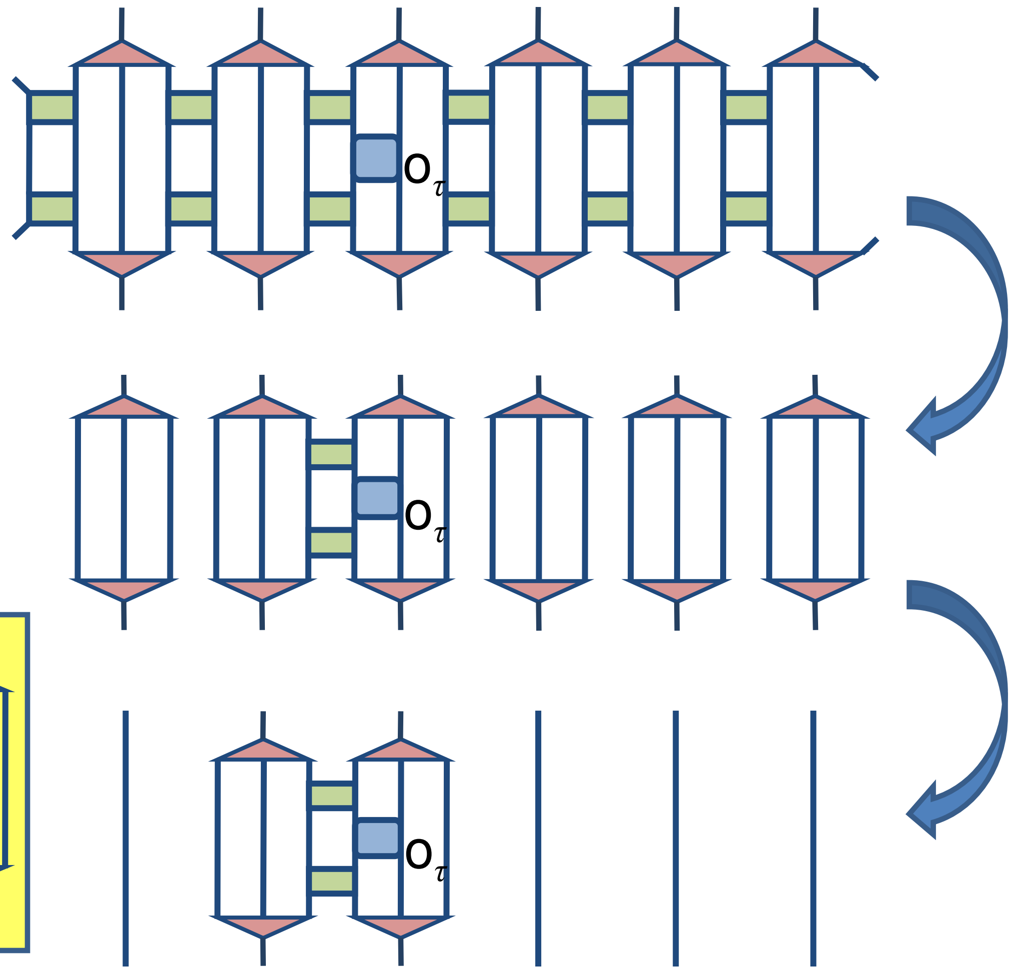


RG transformation

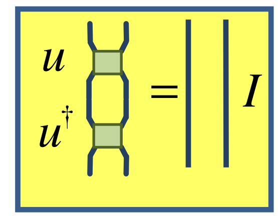
The MERA defines a coarse-graining transformation

$$L_\tau \longrightarrow L_{\tau+1}$$

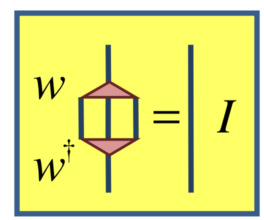
$$o_\tau \rightarrow o_{\tau+1}$$

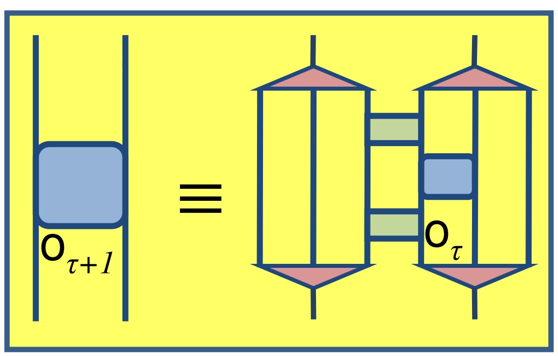


disentangler



isometry

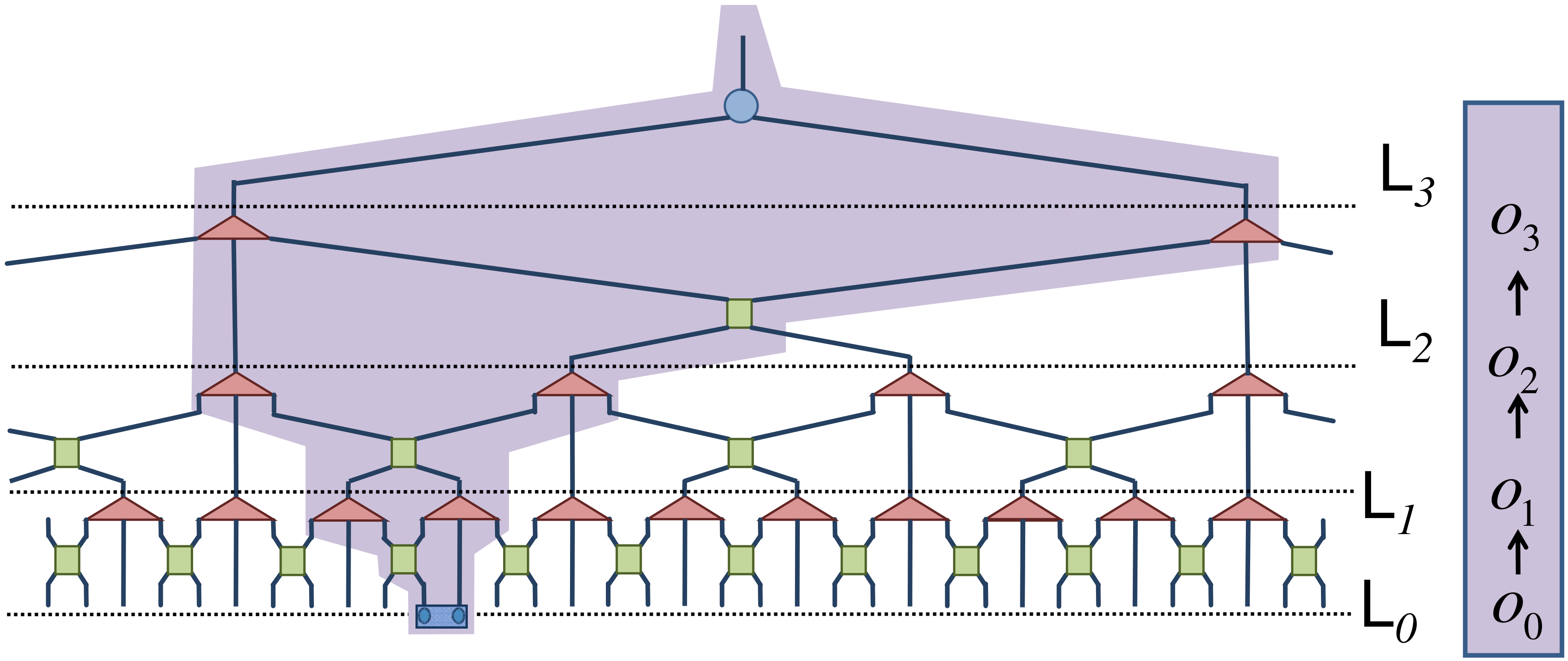




Ascending superoperator

RG transformation

The MERA defines a coarse-graining transformation

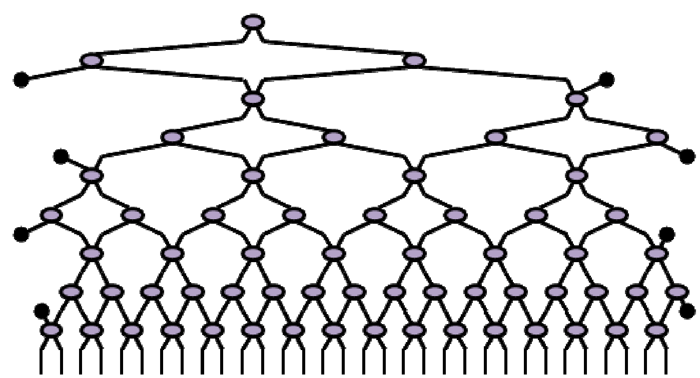


past causal cone
with bounded width ➡ Local operators
remain local

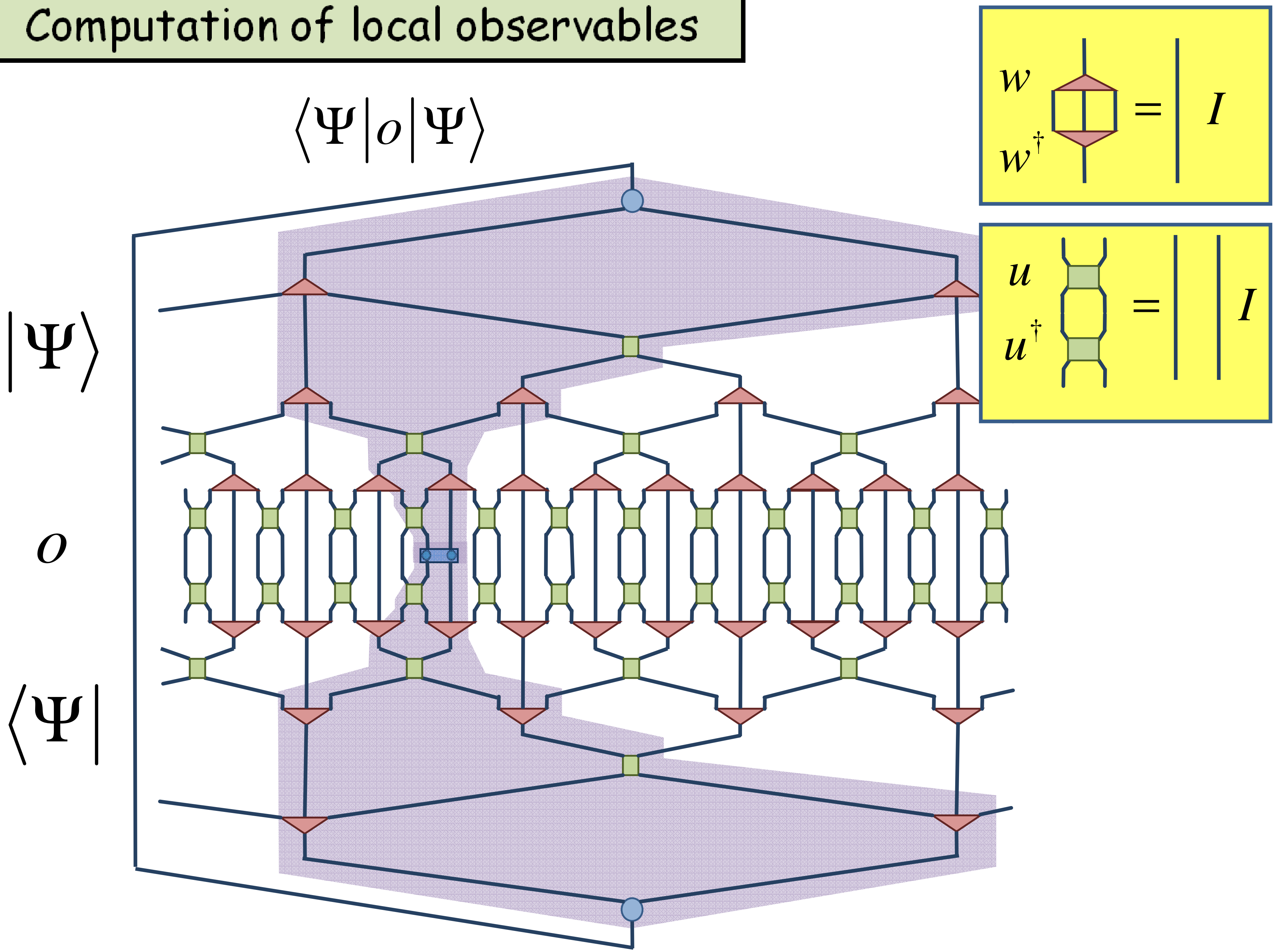
- MERA (multi-scale entanglement renormalization ansatz)

- RG transformation

- Computation of expected values

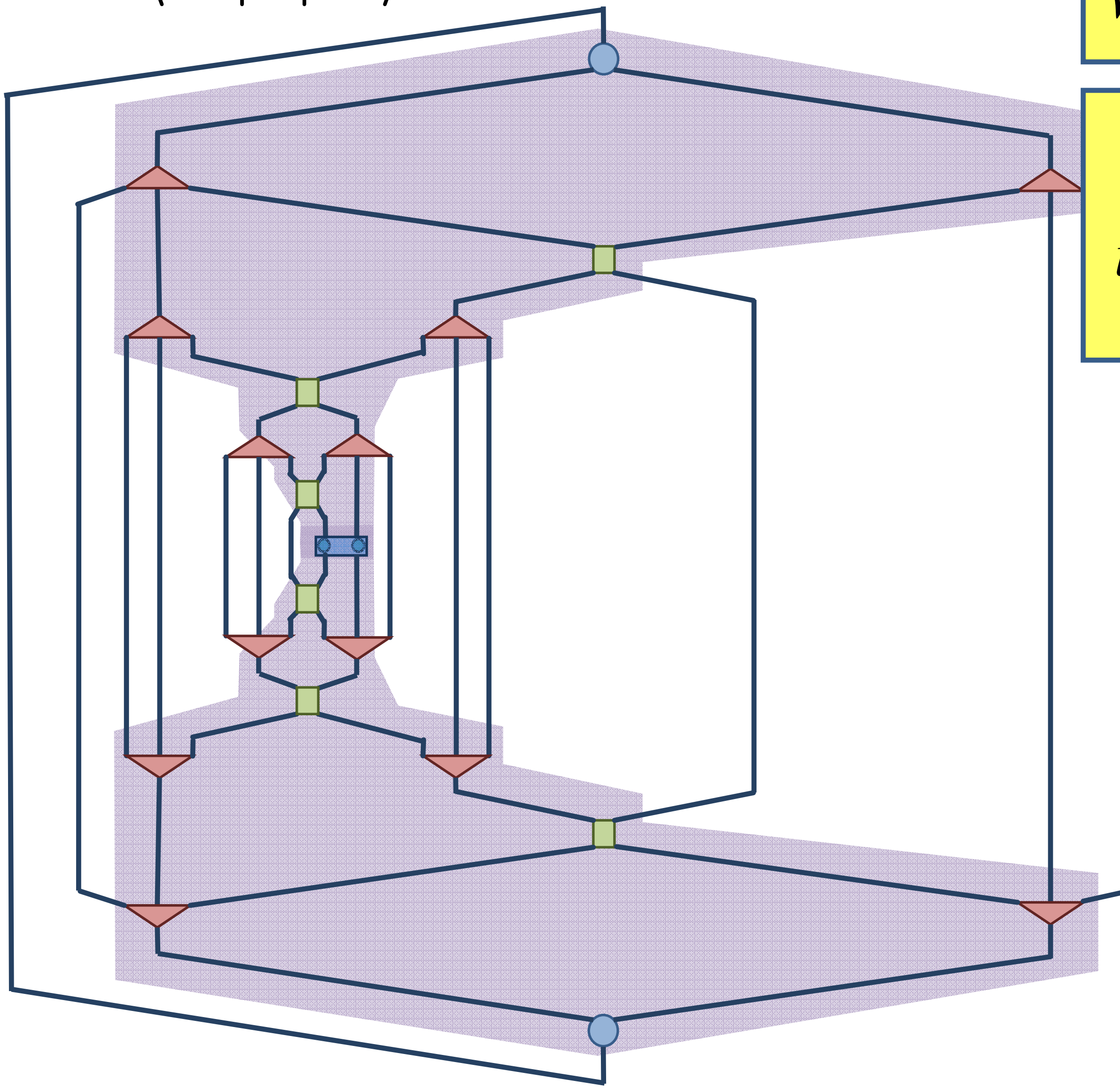


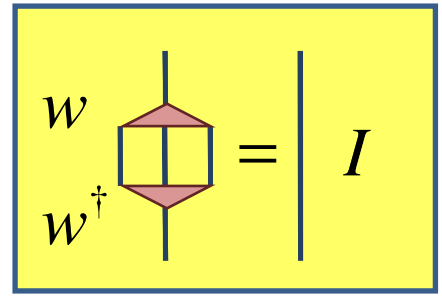
Computation of local observables

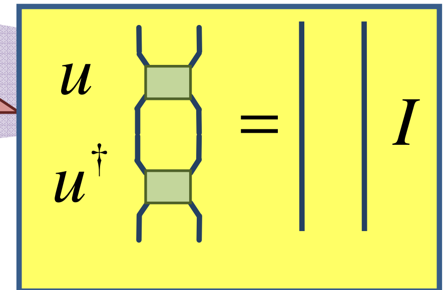


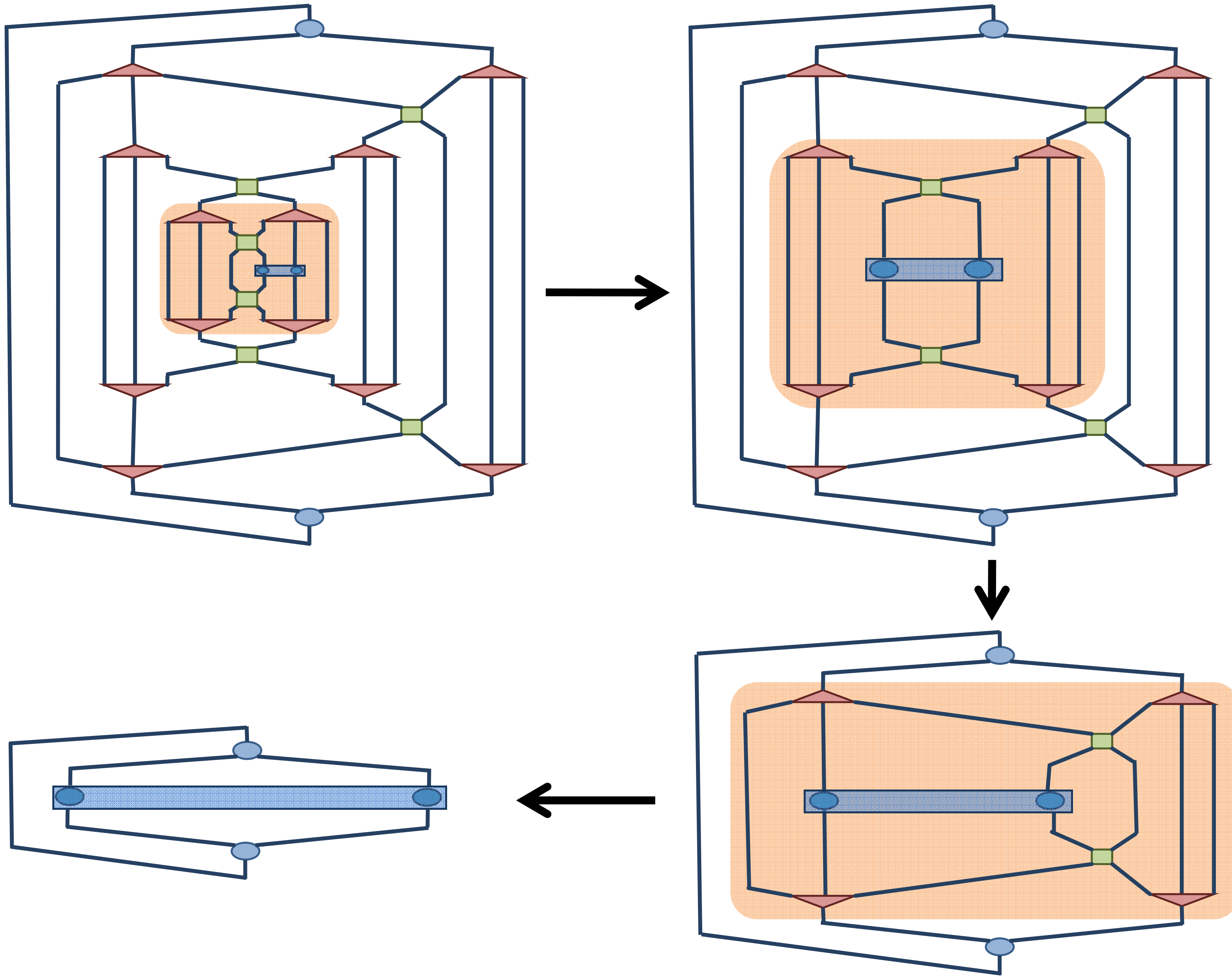
Computation of local observables

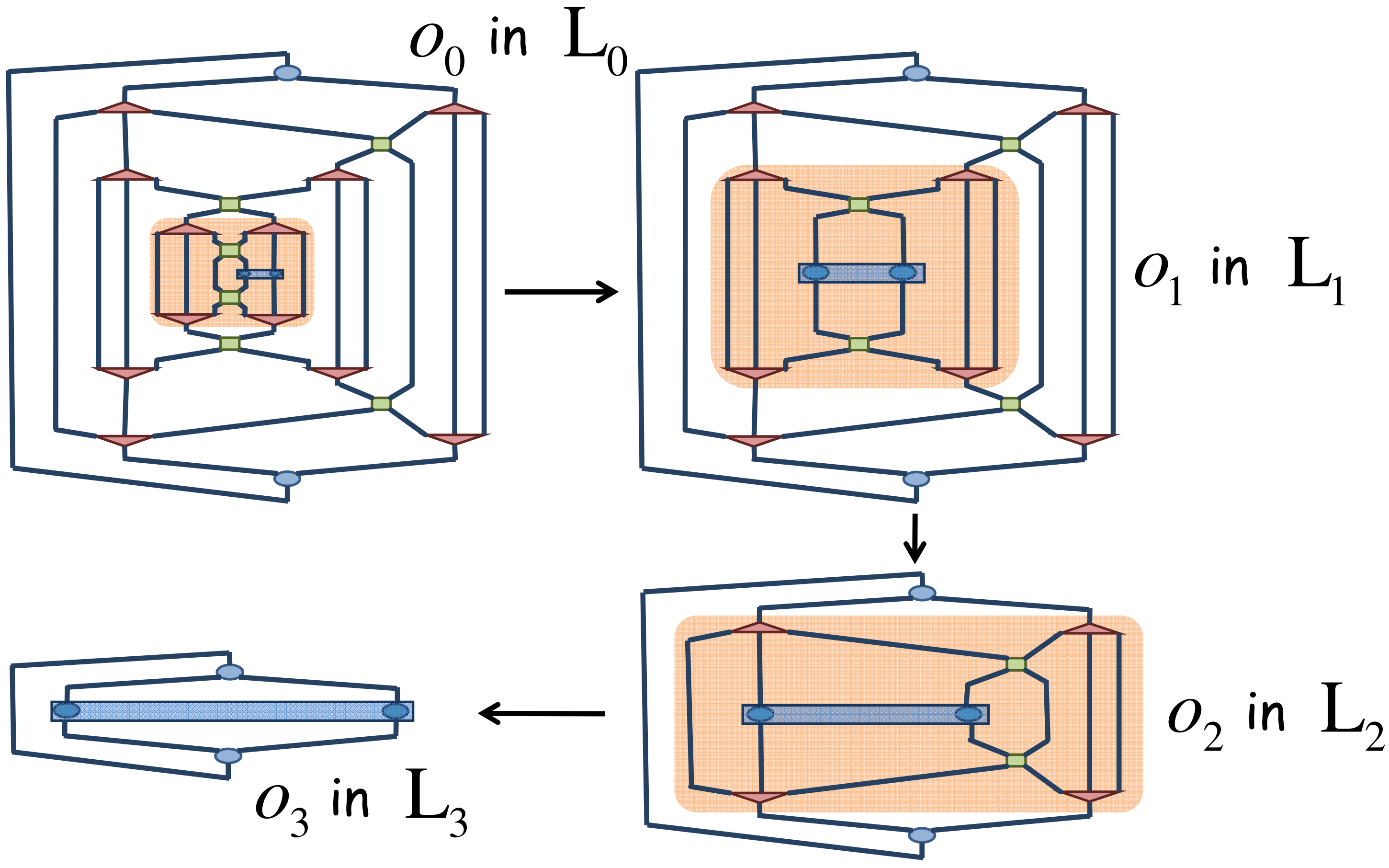
$$\langle \Psi | o | \Psi \rangle$$









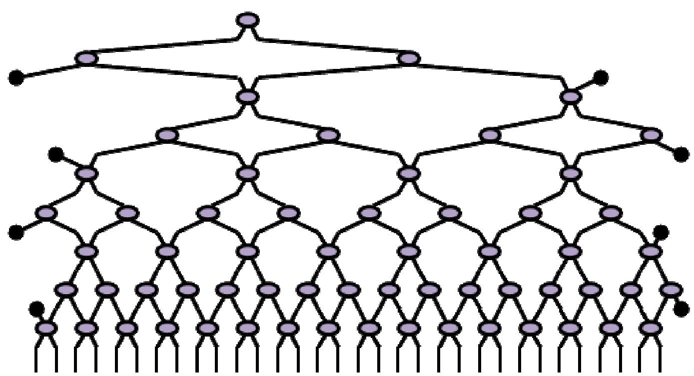


$$o_0 \rightarrow o_1 \rightarrow o_2 \rightarrow o_3 \qquad \langle \Psi | o | \Psi \rangle = tr(o_3 \rho_3)$$

computational cost : $\log(N)\chi^q$

Conclusions

- MERA: efficient representation
- RG transformation: entanglement renormalization
- Computation of expected values is related to this RG transformation

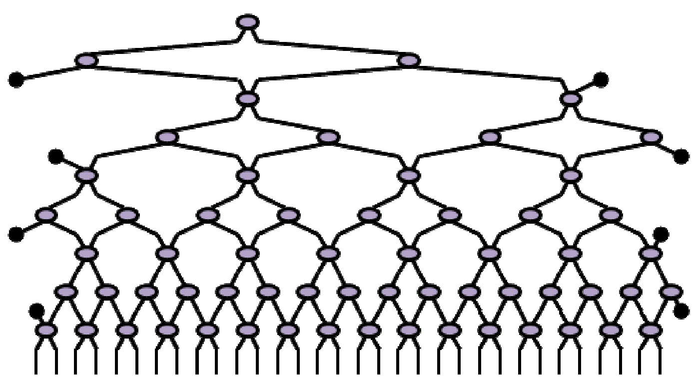


Using similar manipulations one can

- compute correlators

- minimize expected value of local Hamiltonian H

- simulate time evolution



Talk on Wednesday

- critical systems: scale invariant MERA   CFT

- exact MERA for 2D quantum double models and string-net models

- results for 2D frustrated antiferromagnets