Quantum dynamics in ultracold atoms

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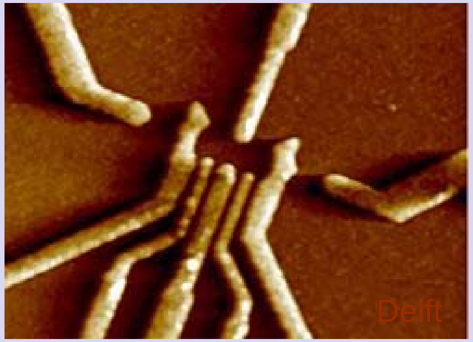
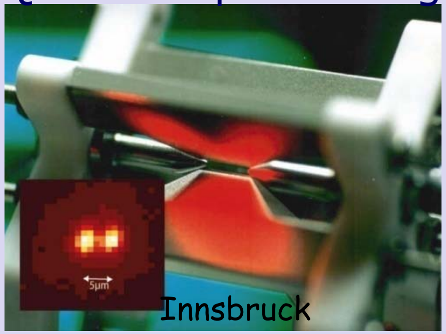
Quantum dynamics

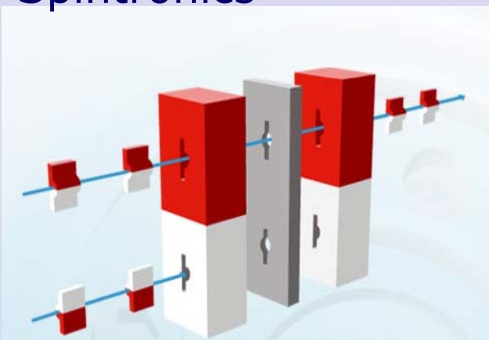
Nanostructures



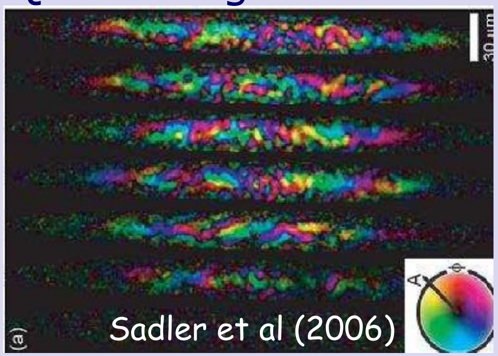
Quantum processing



Spintronics

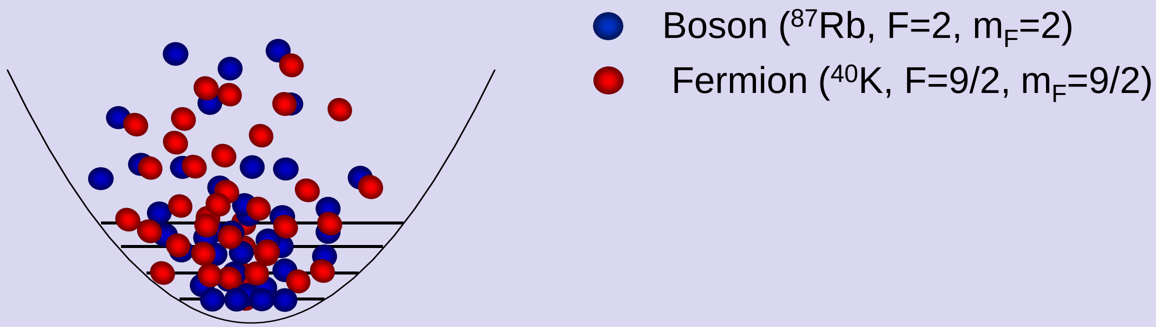


Quantum gases



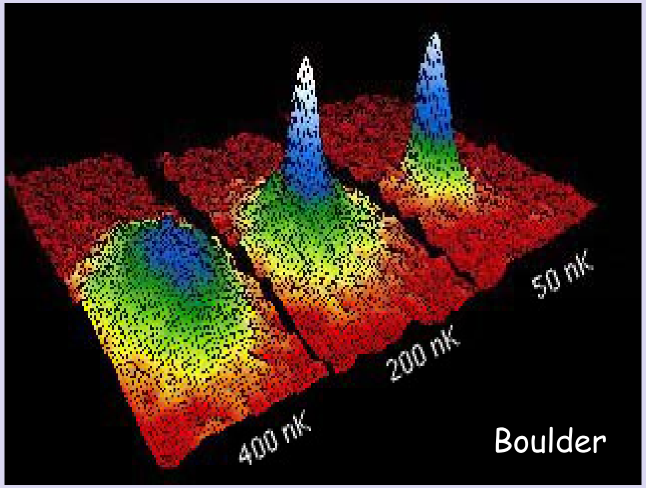
Preparing ultracold atoms

laser cooling T~100 μK
evaporative cooling T~100nK



magnetic trap

Bose-Einstein condensate



Dilute gases: $n \approx 10^{14}\ cm^{-3}$

Ultracold: $T_{degeneracy} \approx 100\ nK$

Weak interactions: $n^{1/3}a << 1$

Strong interactions in quantum gases

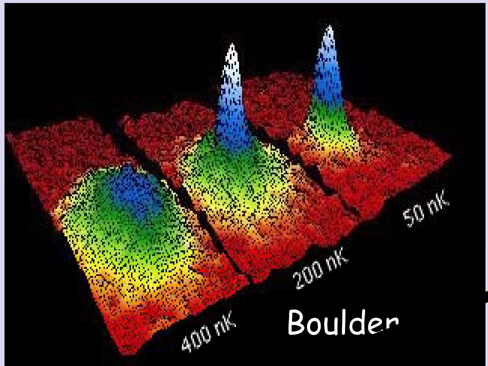
strongly interacting

2004 Fermi gases in optical lattices

2004 BEC- BCS crossover
Fermi gases at unitarity

2003 low dimensional gases

2002 superfluid to Mott insulator transition



2001 Nobelprize BEC:
Cornell, Ketterle, Wieman

1999 Quantum degenerate Fermi gases

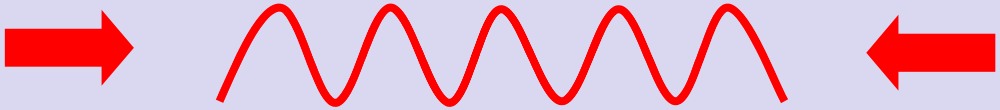
1995 BEC created in dilute atomic gases

1924 predicted by Bose and Einstein

weakly interacting

Optical lattices

standing wave laser field -> periodic potential for atoms

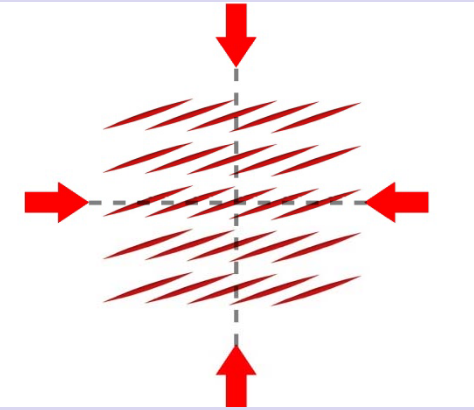


•intensity of laser -> strength of potential

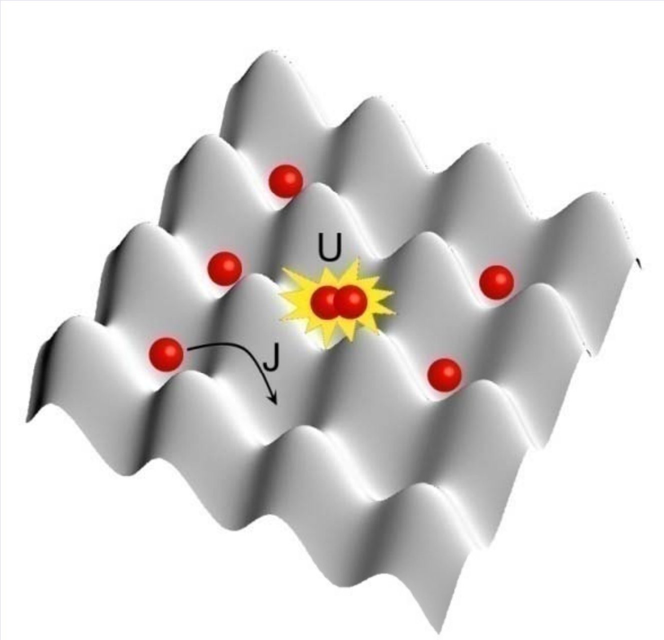
•wavelength/2 -> lattice spacing

•different geometries possible

2D lattice:



Bosons in an optical lattice



kinetic energy

interaction energy

$$H = -J \sum_{<ij>} (b_i^+ b_j + h.c.) + U/2 \sum_j n_j (n_j - 1)$$

Jaksch et al. (1998)

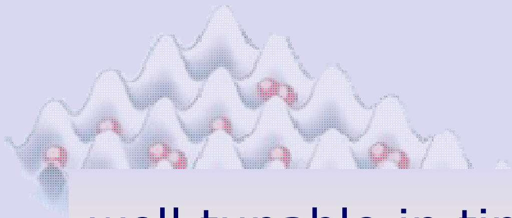
U and J related to lattice height

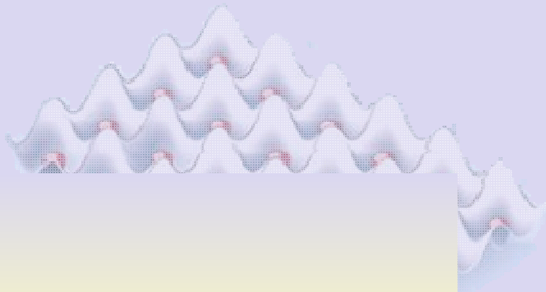
U tunable by Feshbach resonances

Cold gases as quantum simulators

superfluid

Mott-insulator



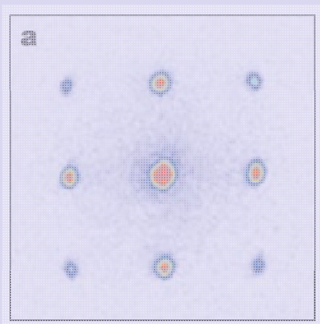


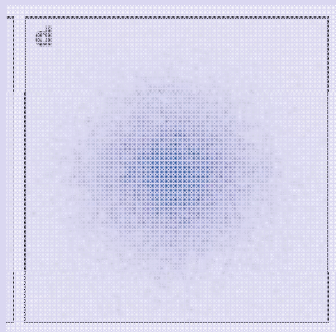
well tunable in time

well decoupled from environment

-> quantum dynamics in isolated system

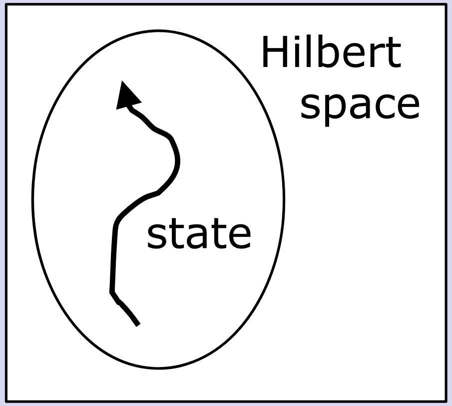
time-of-flight imag
~ momentum distribution





Greiner et al (2001)

Quantum dynamics in a closed system

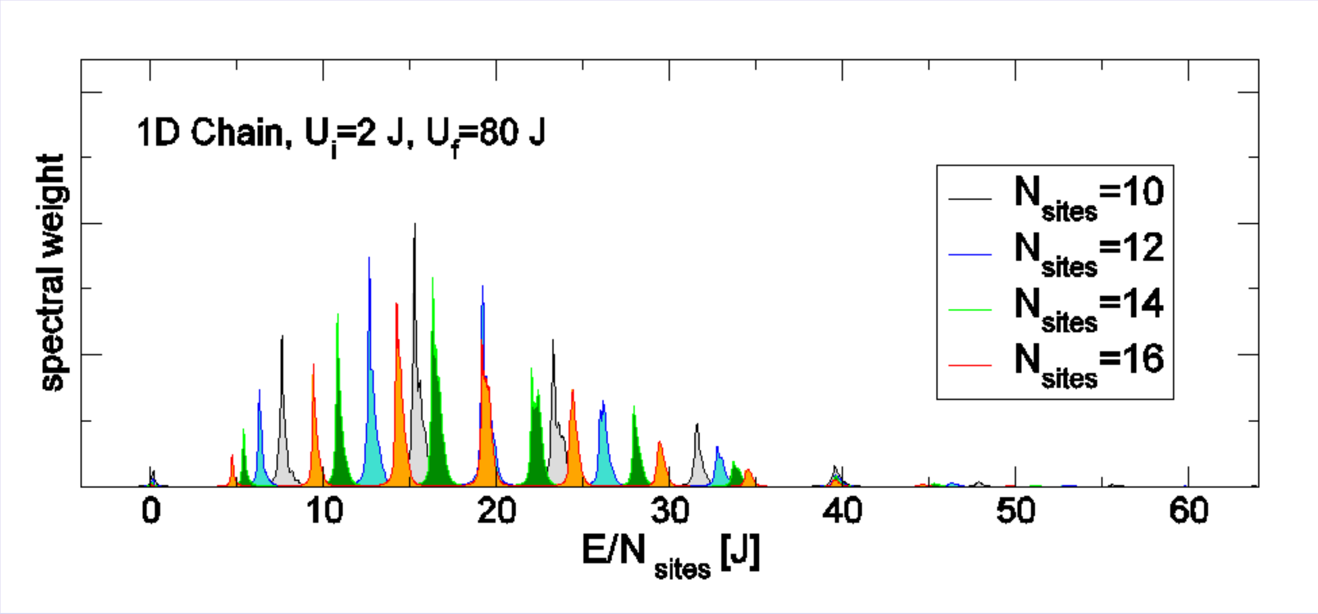


time evolution by Schrödinger equation

small time step:

$$|\psi(t+\Delta t)\rangle \approx e^{-i\Delta t H(t)} |\psi(t)\rangle = \sum_n e^{-i\Delta t E_n} c_n |n\rangle$$

ex: quench across superfluid to Mott-insulator transition



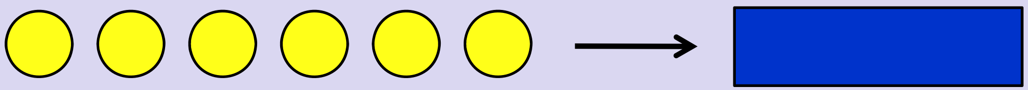
methods: exact diagonalization and time-dependent DMRG

The idea of DMRG: reduced Hilbert space

S. White (1992)

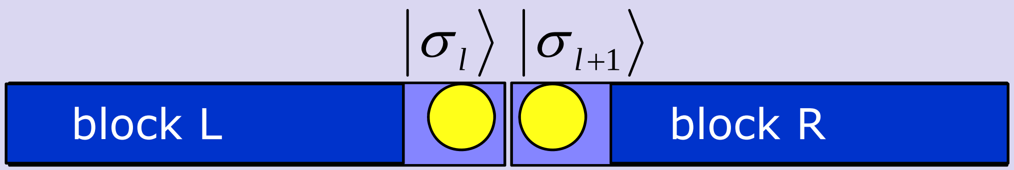
problem: too large Hilbert space

idea: construct <u>effective Hilbert space</u>



Variational method in matrix product state space

$$\left|L_l\right\rangle \approx \sum \left(A_l[\sigma_l]\cdots A_1[\sigma_1]\right)\left|\sigma_l \cdots \sigma_1\right\rangle$$

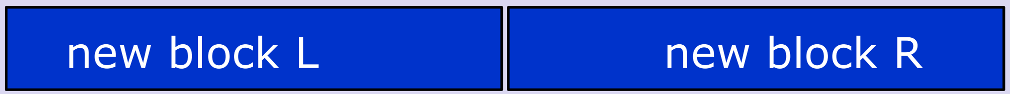


• Schmidt decomposition of 'wanted' state

$$\left|\psi\right\rangle = \sum_{\alpha} \lambda_\alpha \left|L_{l-1}\sigma_l\right\rangle_\alpha \left|R_{l+2}\sigma_{l+1}\right\rangle_\alpha$$

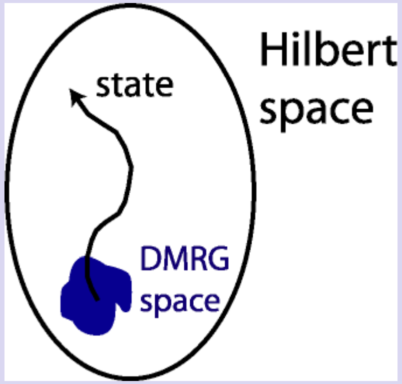
• keep only M highest λ

$$\left|\psi\right\rangle \approx \sum_{\alpha=1}^{M} \lambda_\alpha \left|L_l\right\rangle_\alpha \left|R_{l+1}\right\rangle_\alpha$$



• sweep free sites

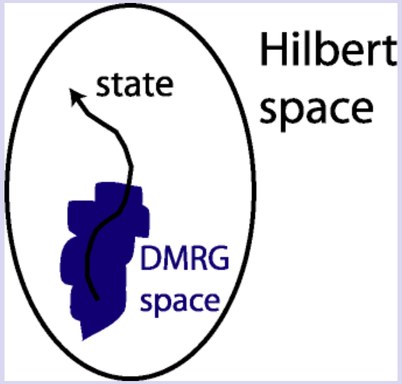
idea: time-dependent DMRG



static:

breakdown after
short time

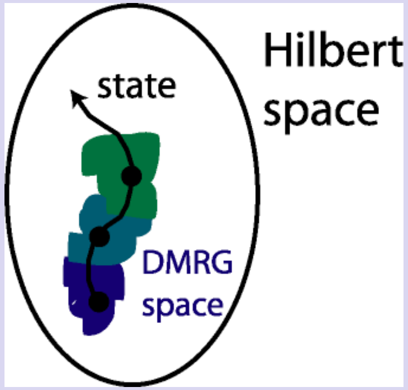
(Cazalilla,Marston)



enlarged:

numerically
very expensive

(Luo,Xiang,Wang;
Schmitteckert)



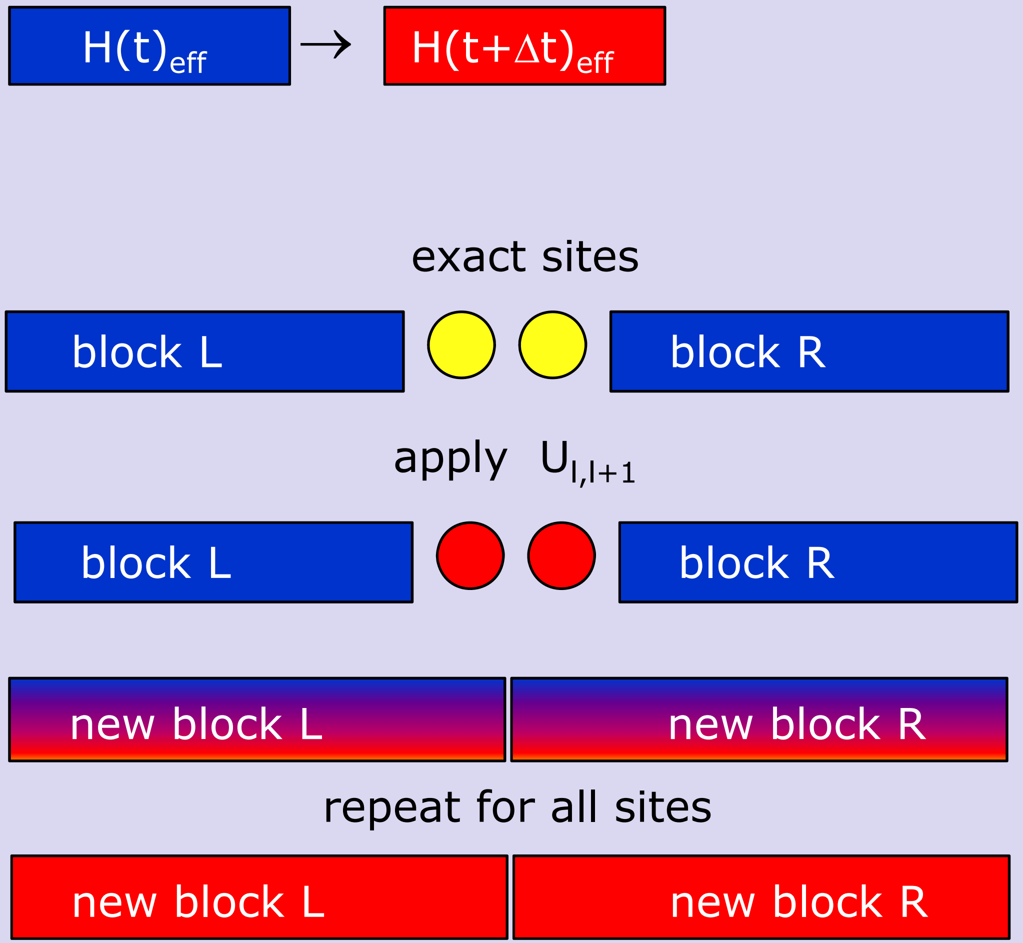
adaptive:

numerically cheap
long times

(Vidal;
Daley,CK,Schollwöck,Vidal;
White,Feiguin)

Algorithm: time-step

time-evolution (Schrödinger eq) $|\psi(t)\rangle_{eff} \rightarrow |\psi(t+\Delta t)\rangle_{eff}$



Suzuki Trotter decomposition

$$U \approx \prod_{l \in odd} U_{l,l+1} \prod_{l \in even} U_{l,l+1}$$

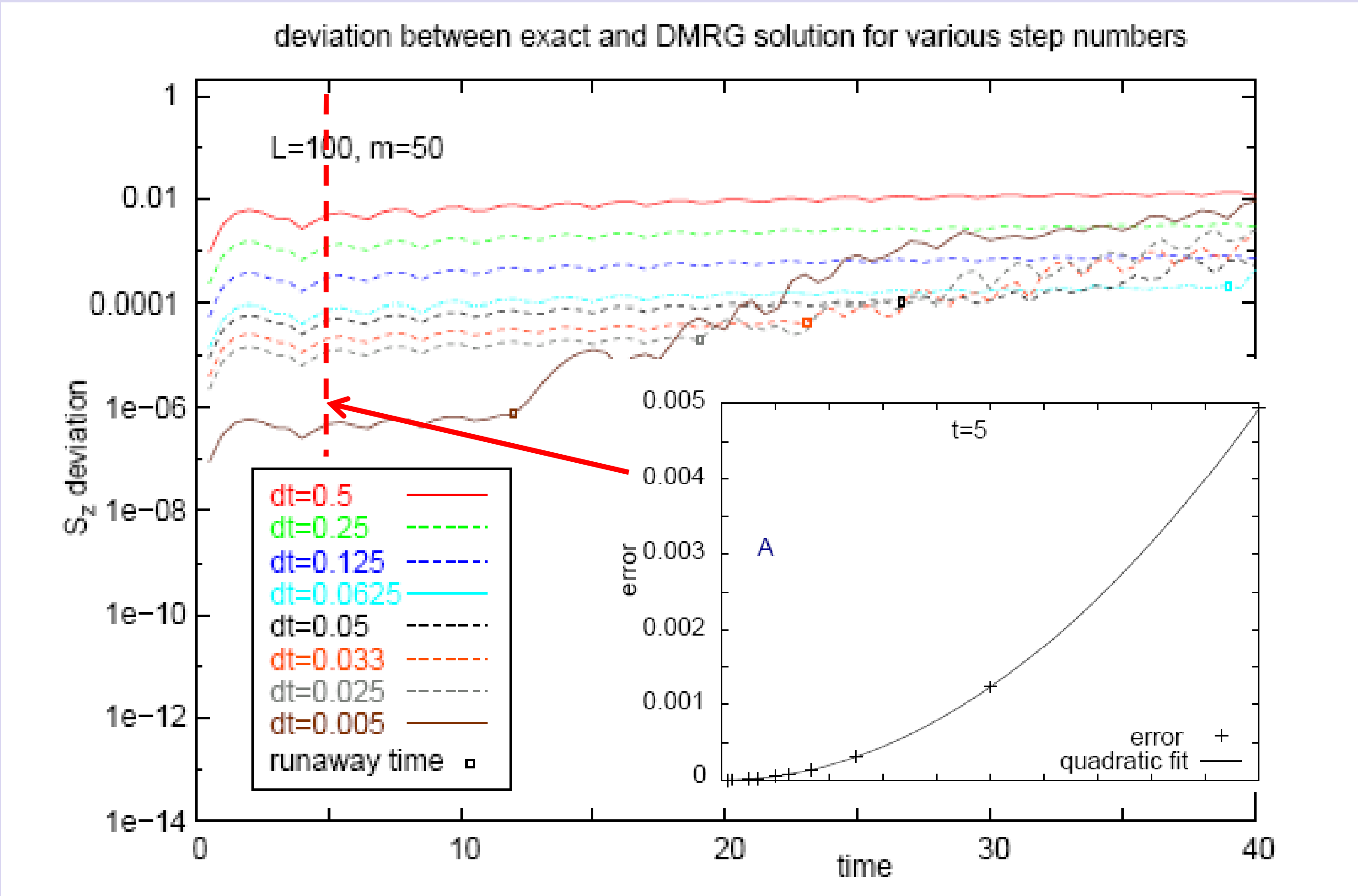
with

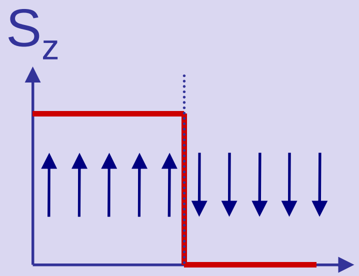
$$U_l \approx \exp(-ih_{l,l+1}\Delta t)$$

errors:

- Trotter-Suzuki error
- truncation error

Trotter error ~$L\Delta t^n$

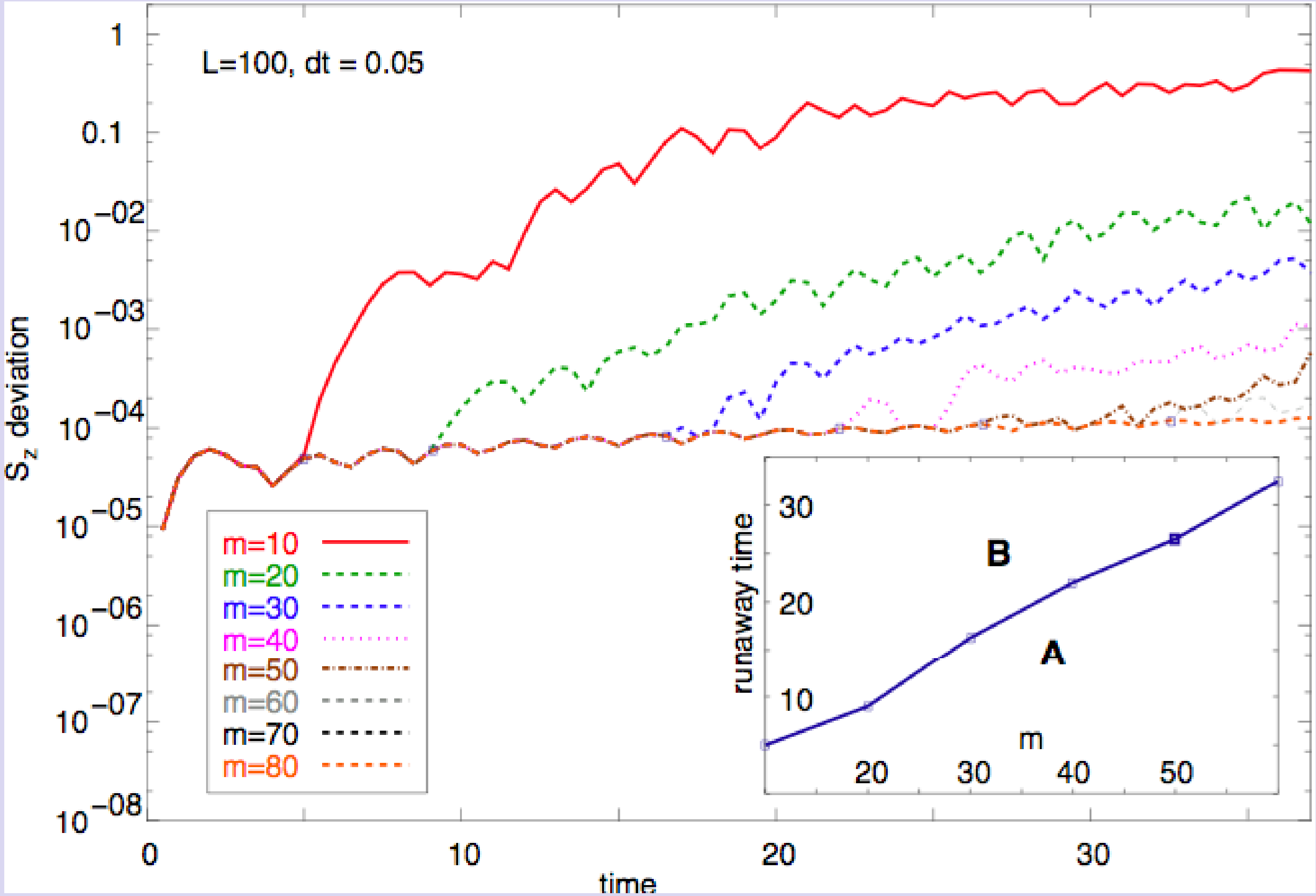




- dominating at short time
- well controlled by Δt

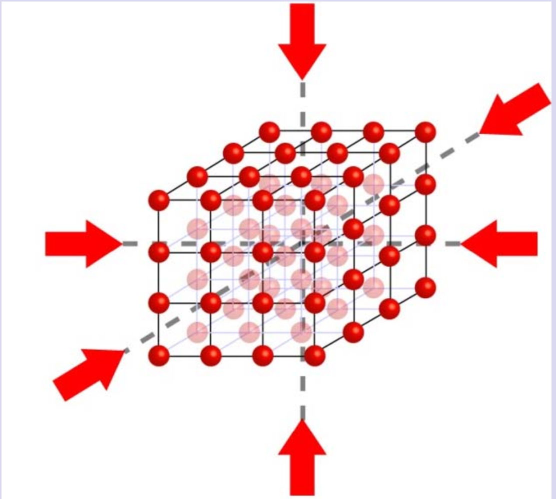
Gobert, CK, et al PRE (2004)

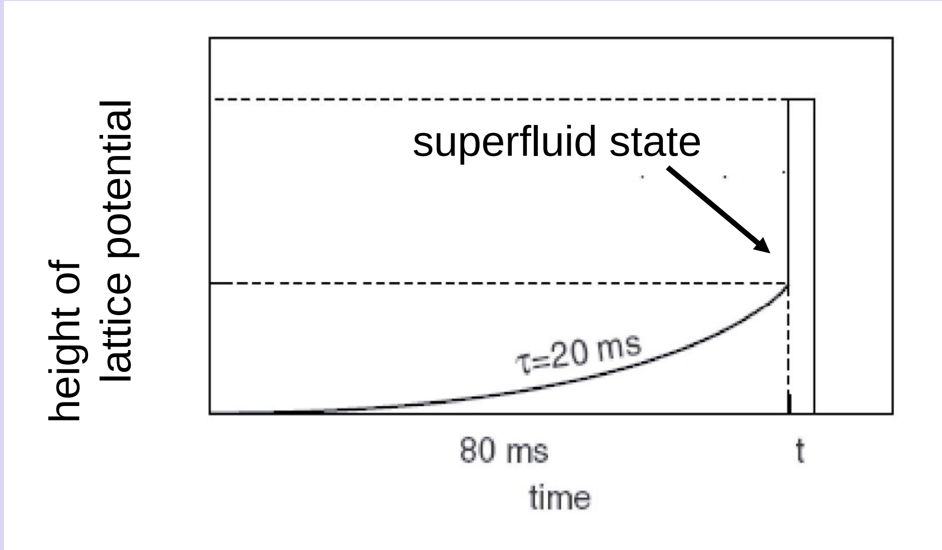
Truncation error



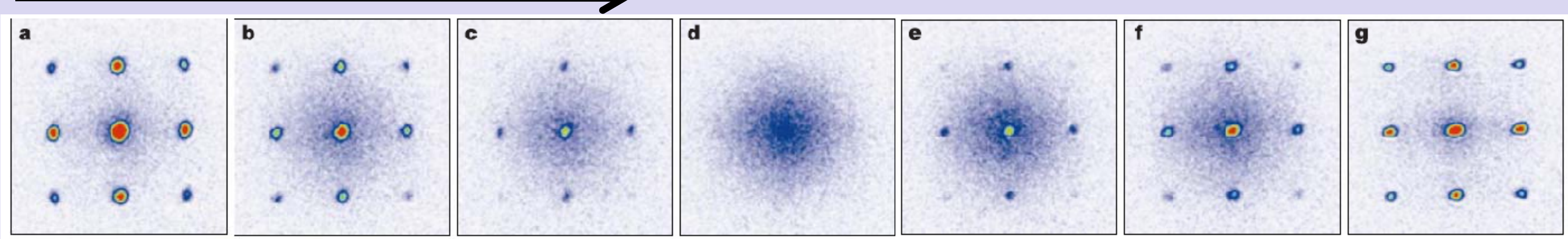
runaway time: crossover between Trotter error and truncation error
errors well controlled

Experiment: abrupt change from superfluid to Mott-insulator





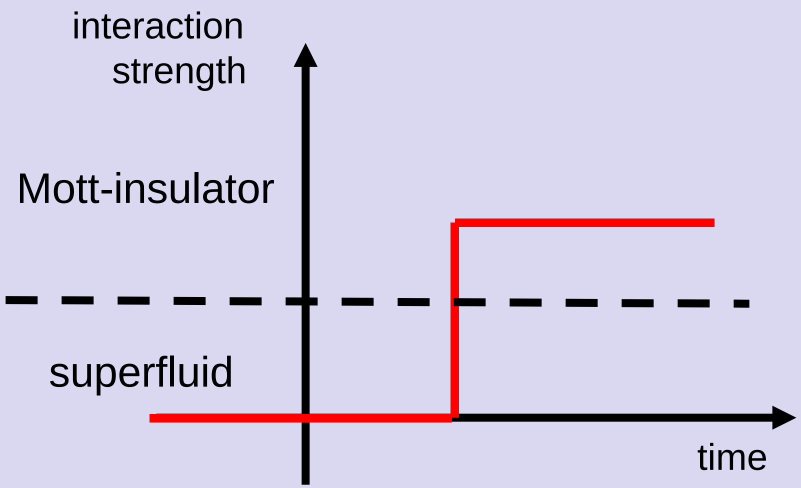
time after quench



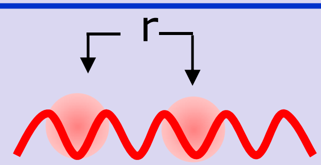
time-of-flight images
~ momentum distribution

Greiner et al. Nature (2002)

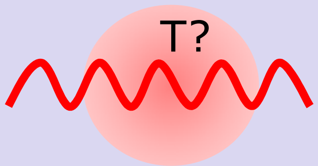
Theoretical description



questions:



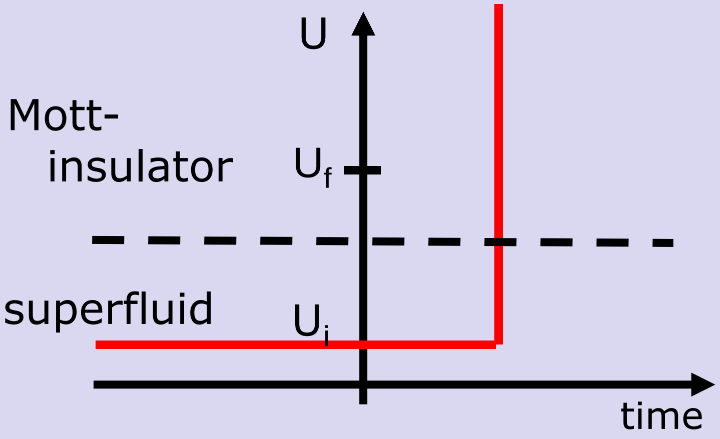
- response on short times?
- how do correlations build up/decay?
- entanglement evolution?
- speed?



if stationary state:

- what are its properties?
- subsystems ‘thermalized’?

Total revival of the wave function



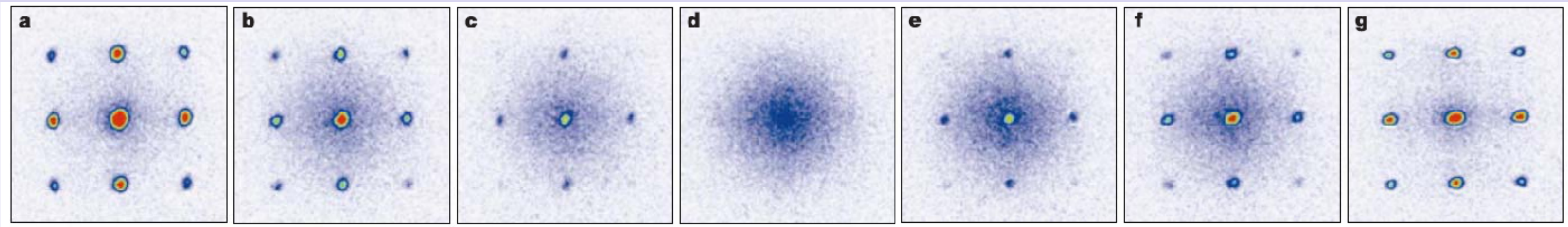
only interaction term:

time evolution operator

$$\exp[-\tfrac{it}{\hbar} U_f \underbrace{\sum_j \tfrac{1}{2} \hat{n}_j (\hat{n}_j - 1)}_{\text{integer value}}]$$

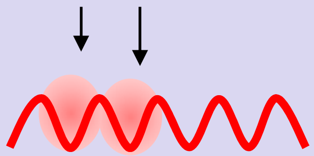
all Fock states revive latest at T=h/U

-> wave function evolves periodically in time

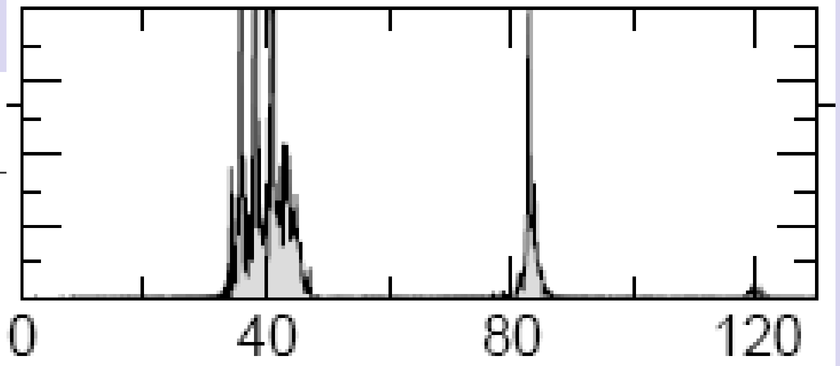


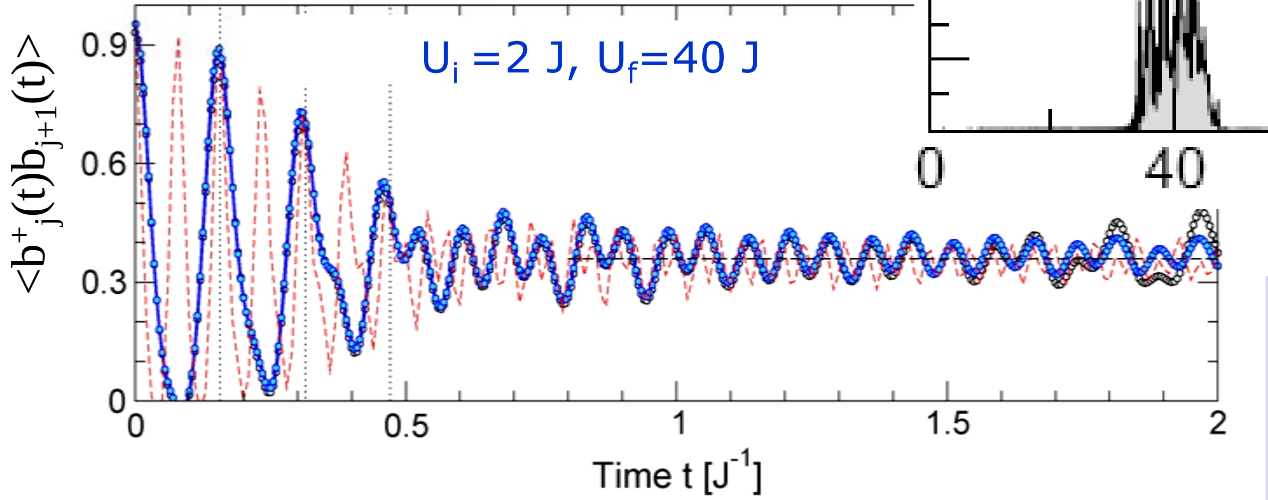
T=h/U

Relaxation with finite hopping



Fouriertransformation <$b^+_j(t) b_{j+1}(t)$>





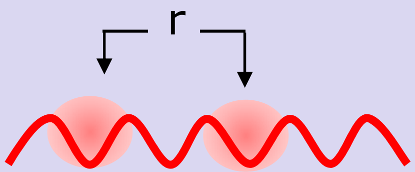
- quasi-particle frequency bands

-> beating

-> relaxation ~1/(zJ)

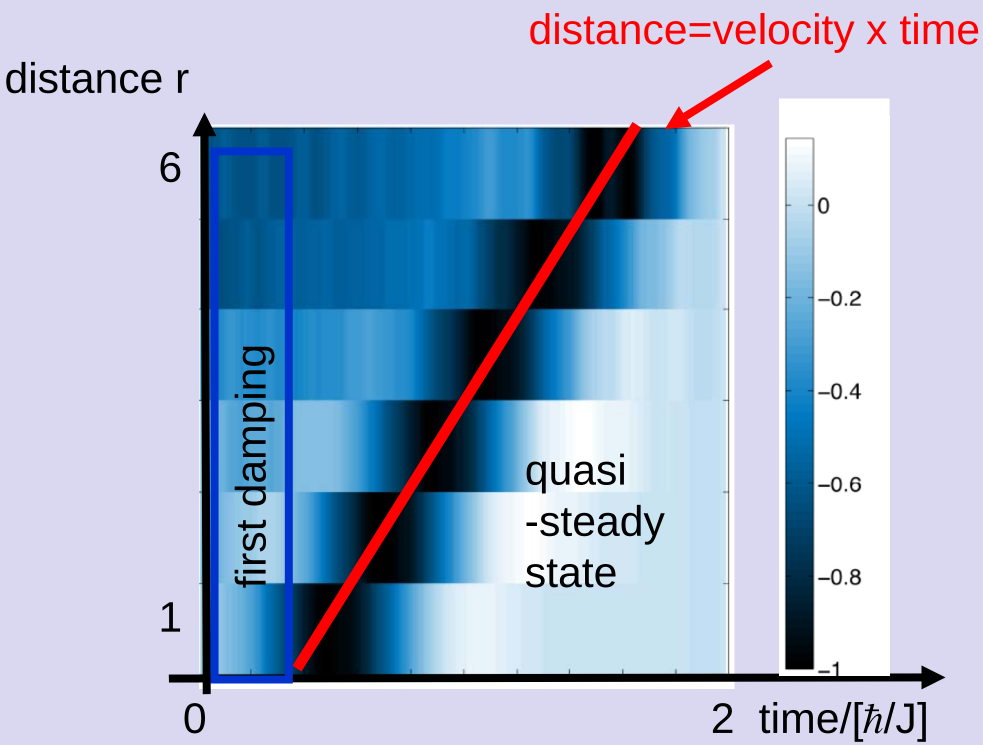
C. Kollath, A. Läuchli, E. Altman, PRL 98, 180601 (2007)

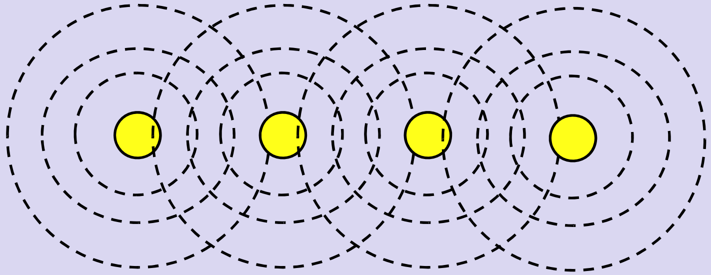
Light-cone like evolution to quasi-steady state



density-density correlations

$$\langle n_0 n_r \rangle(t) - \langle n_0 \rangle \langle n_r \rangle(t)$$





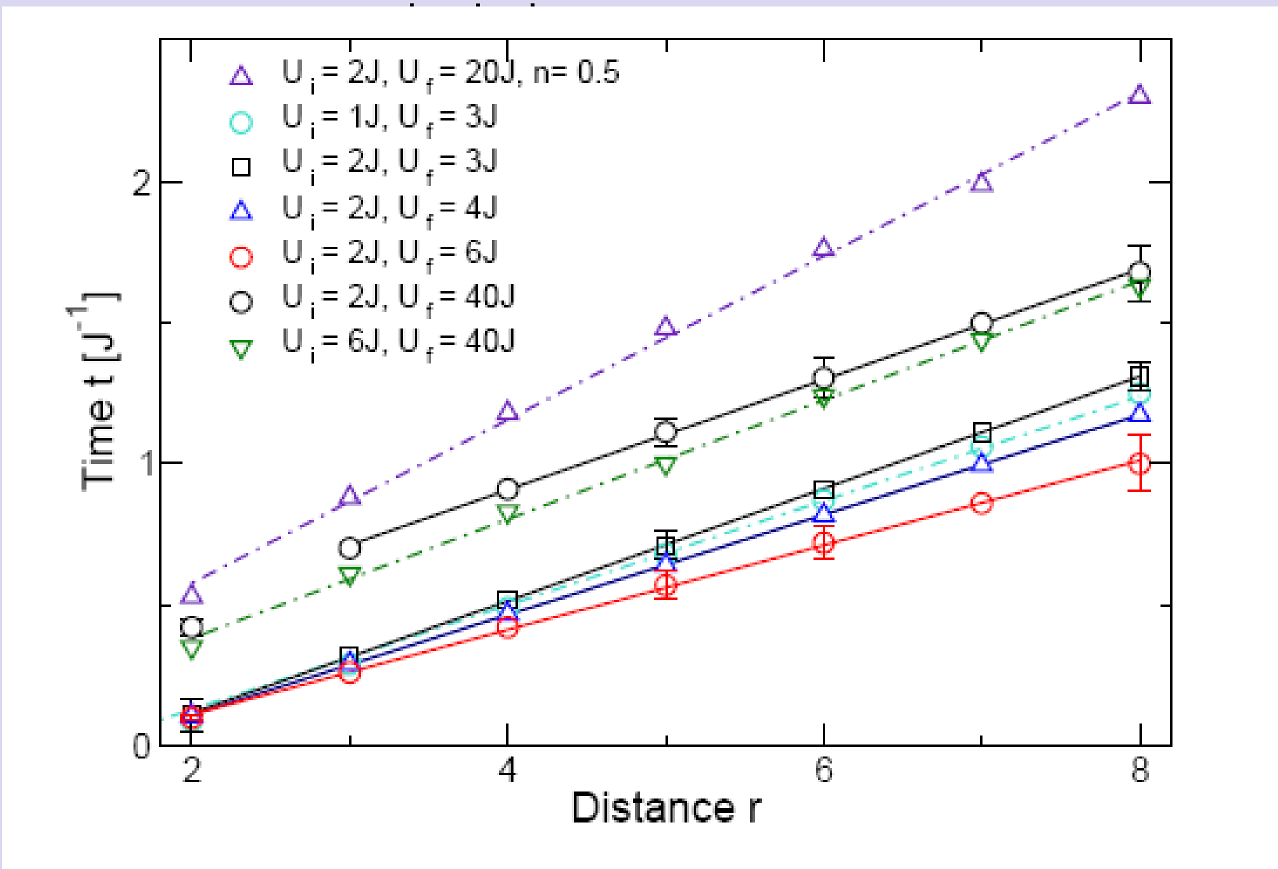
light cone like evolution in different models:

- Lieb and Robinson (1972) spin models
- Igloi and Rieger
- D. Gobert, CK, U. Schollwoeck, G. Schütz (2005)
- Calabrese and Cardy (2006) conformal field theory
- specific exactly solvable models…

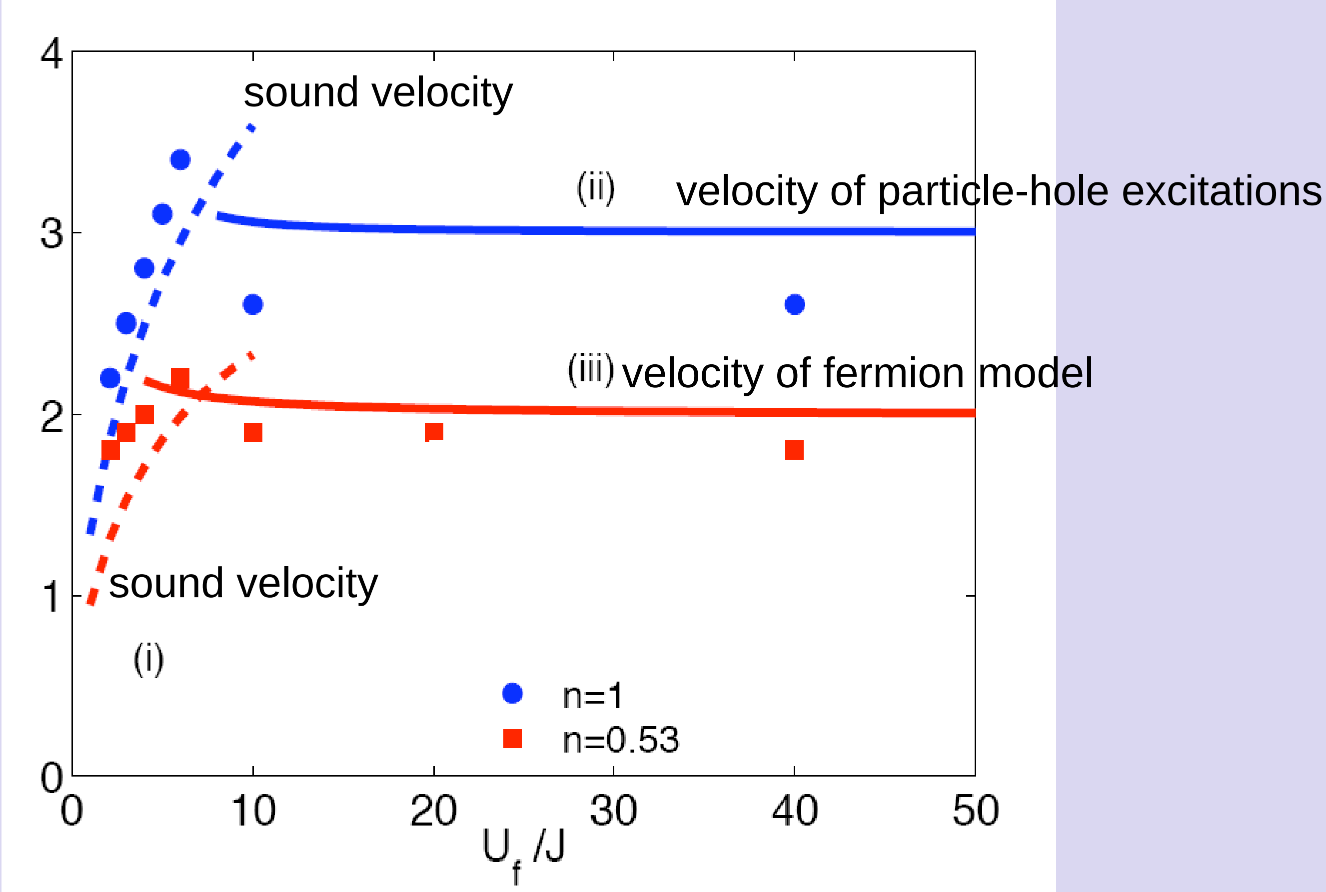
A. Läuchli and C. Kollath, J.Stat. Mech. (2008)

Speed of correlations

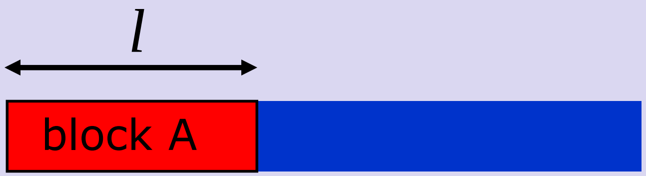
position of dip in density-density correlation



Speed of correlations



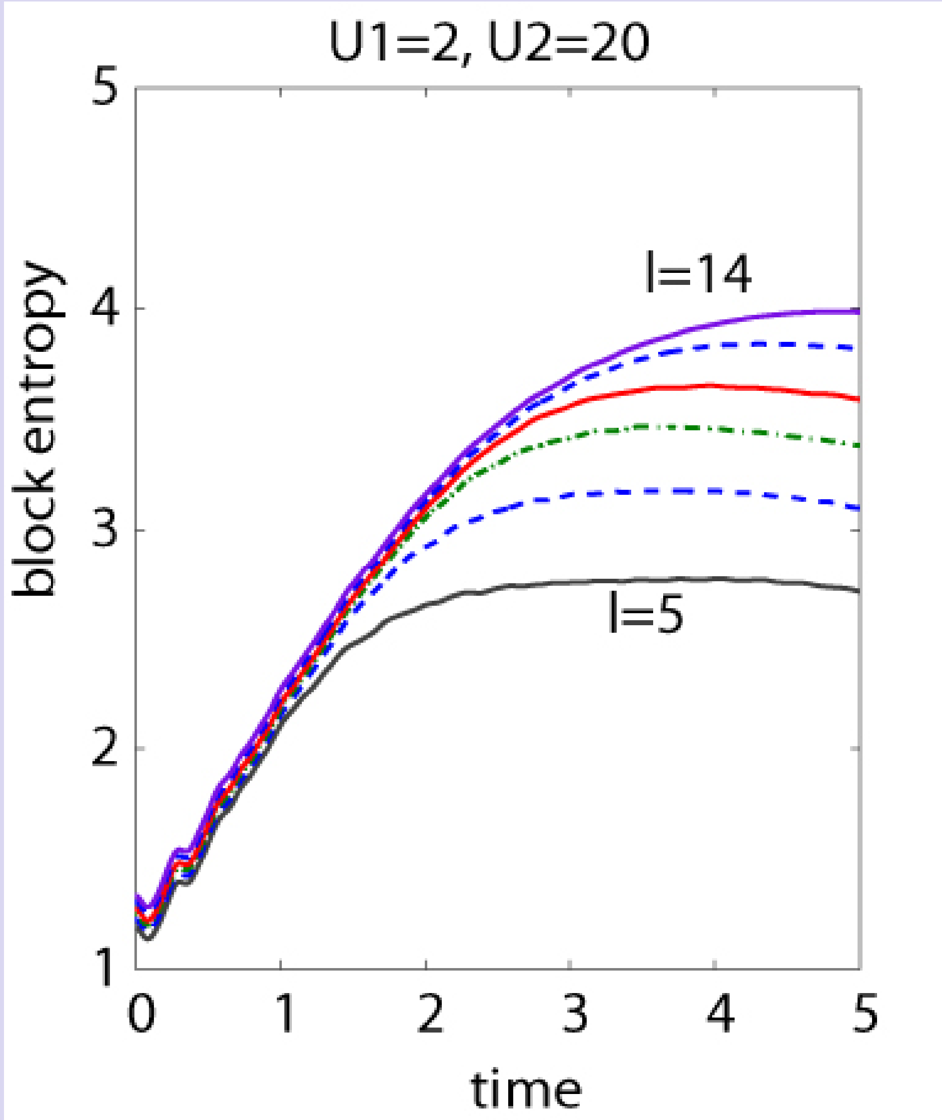
Entanglement evolution



von Neuman entropy of block A

$$S_A = -Tr_A \rho_A \log \rho_A$$

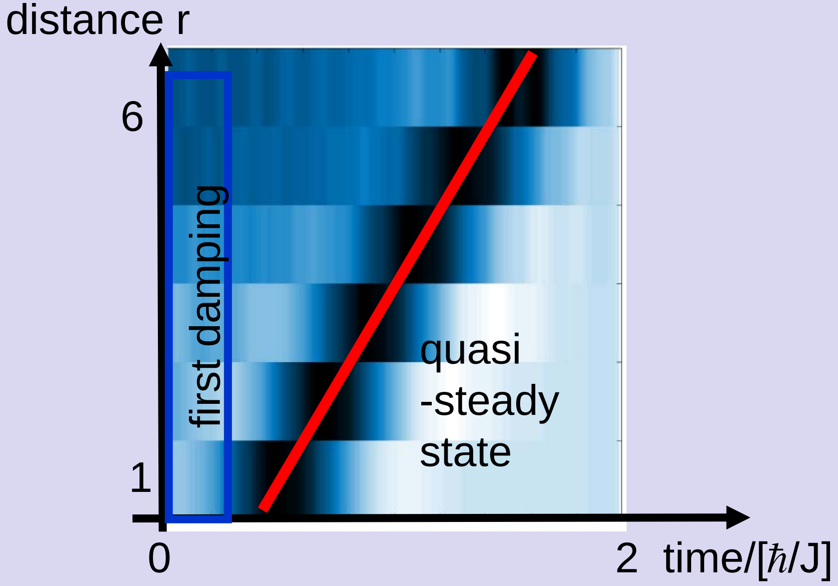
- saturation after different times
  t~ v l (open boundary conditions)

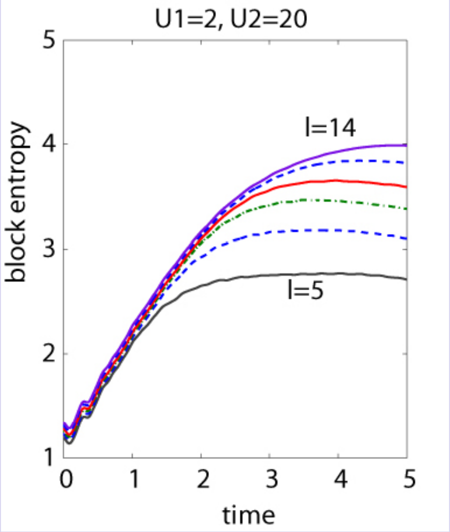


P. Calabrese and J.Cardy (2005) for CFT

A. Läuchli and C. Kollath (2008)

Summary: quench



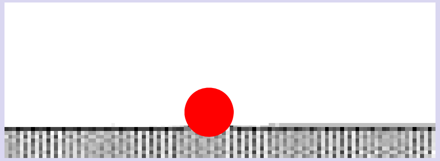


- what determines speed of light-cone?
- deviations from light-cone?
- general understanding of speed?
- Long-time limit?

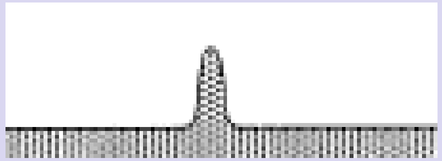
- S. Manmana et al. (2007) non-integrable fermionic model
- specific exactly solvable models (Luttinger model, Ising model, …) M. Rigol et al. PRL 98, 50405 (2007), M. Cazalilla PRL (2007), P. Calabrese and Cardy PRL (2006), Barthel and Schollwöck (2008), Roux(2008),Flesch et al (2008)...

Dynamic of local excitations

single particle excitations



density perturbations



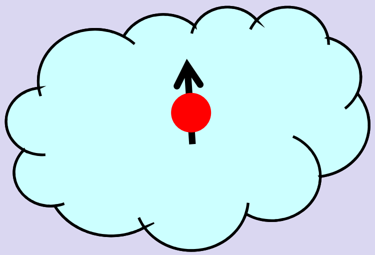
- characteristics of systems
- transport through nanostructures
- information transfer

here:
spin-charge separation in real time

Dynamics of single particle excitations

3D Fermi liquid

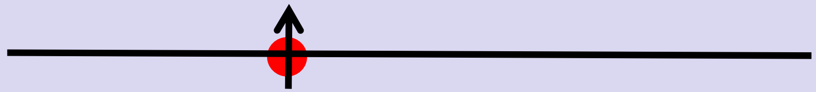
- quasi-particle
  with spin and charge



1D Luttinger liquid

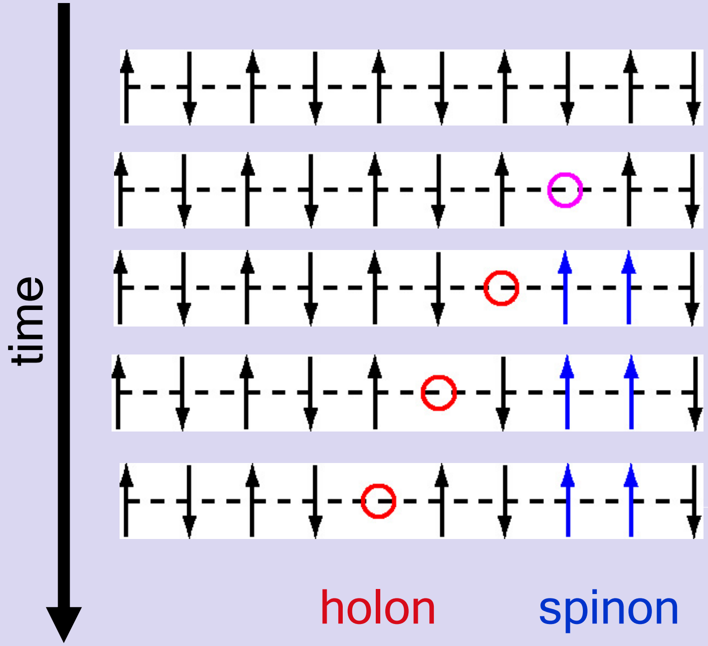
- separation of spin and charge

→ ***key feature***



Spin-charge separation: simple sketch

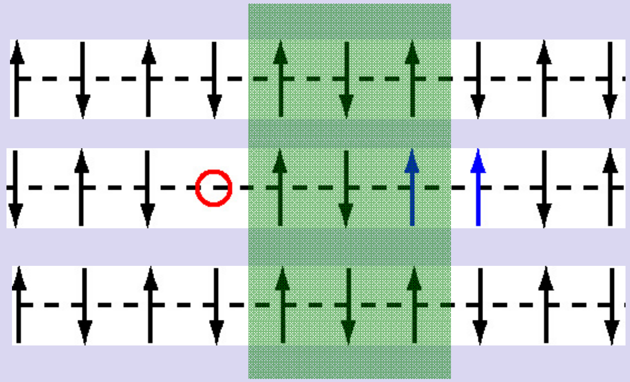
one-dimension



holon spinon

move separately

two-dimensions

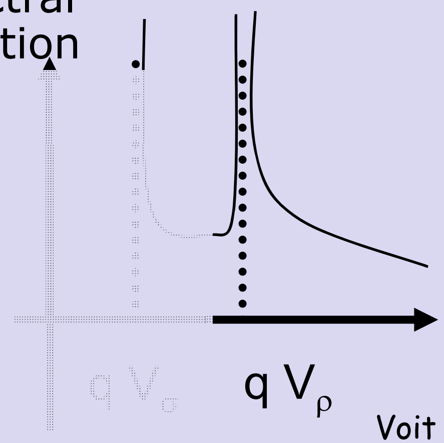


are held together

Condensed matter physics

$H = H_{\downarrow} + H_{\uparrow} + H_{interaction}$

introducing
charge: $\rho(x) \sim \rho_{\downarrow}(x) + \rho_{\uparrow}(x)$ bosonization valid at low energy
and spin: $\sigma(x) \sim \rho_{\downarrow}(x) - \rho_{\uparrow}(x)$

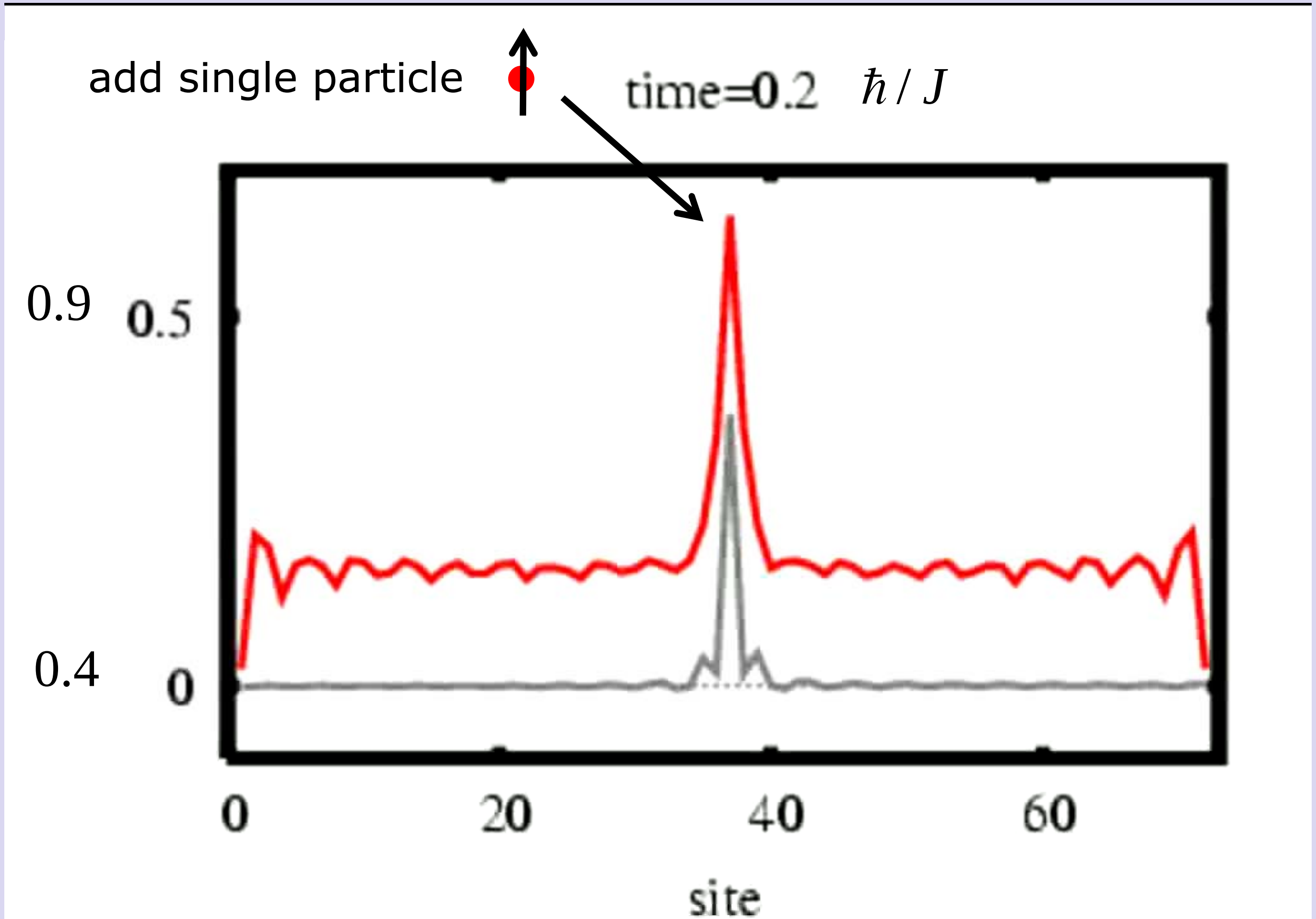


using bosonic (amplitude and phase) fields
⇨ $H = H_{\rho} + H_{\sigma}$

short times?
strong perturbations?
interfaces?

no interaction!

Single particle excitation two component fermions

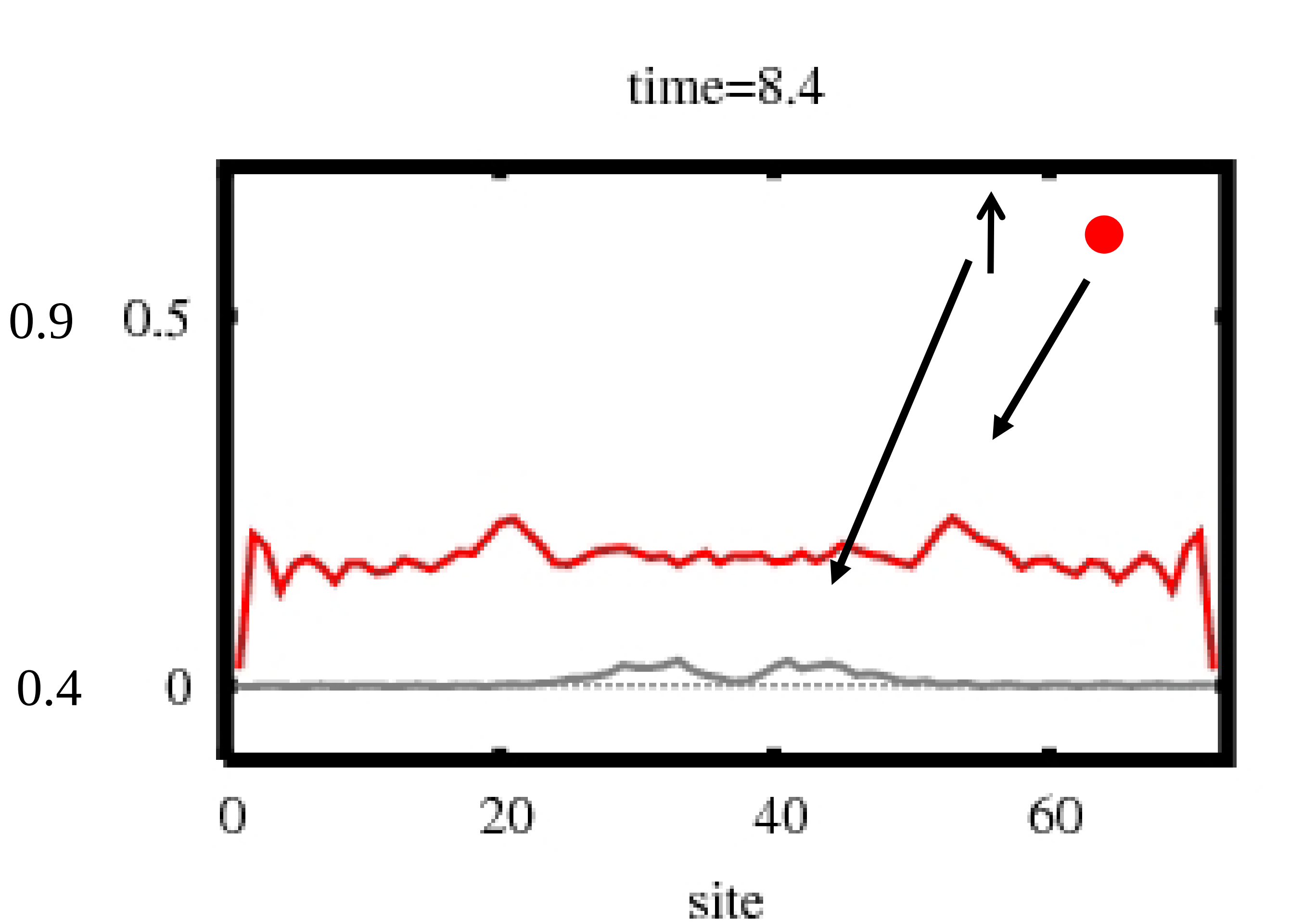


$n_{charge} = n_{\uparrow} + n_{\downarrow}$

$n_{spin} = n_{\uparrow} - n_{\downarrow}$

C.Kollath, U. Schollwöck, and W. Zwerger PRL **95**, 176401

Single particle excitation



$n_{charge} = n_{\uparrow} + n_{\downarrow}$

$n_{spin} = n_{\uparrow} - n_{\downarrow}$

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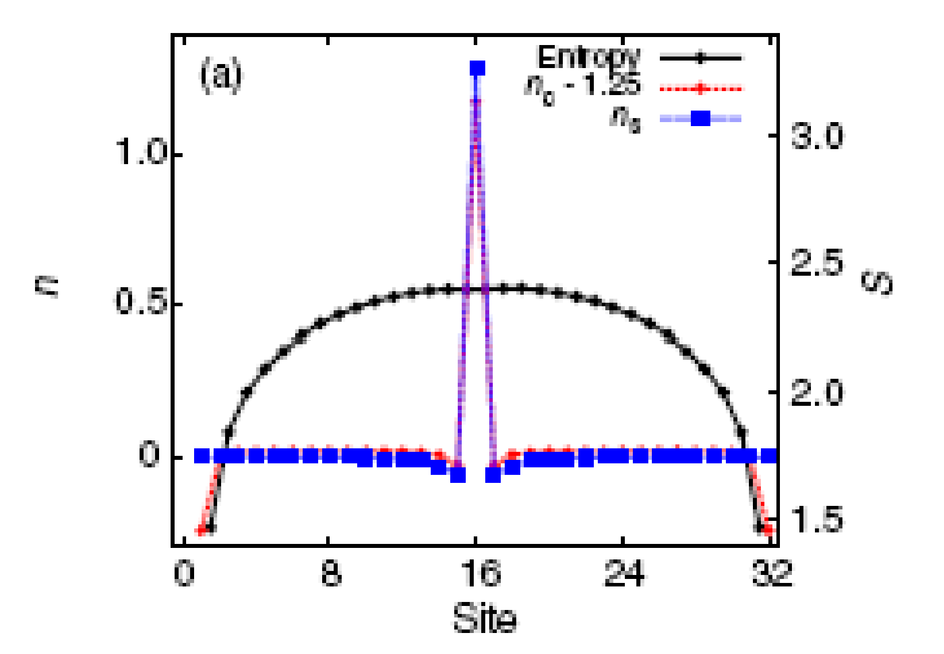
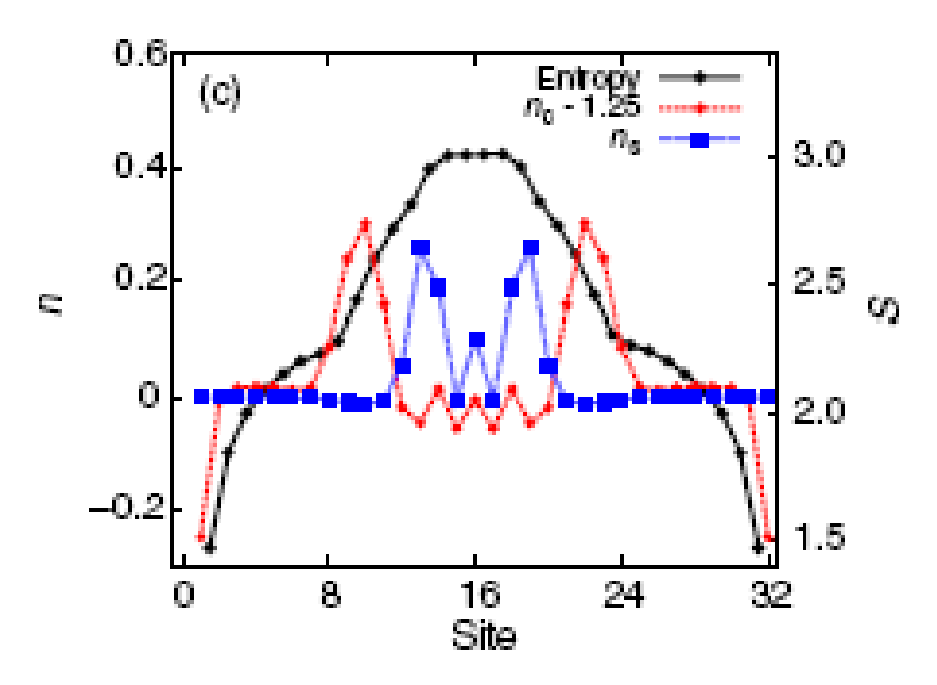
Single particle excitation and entropy growth



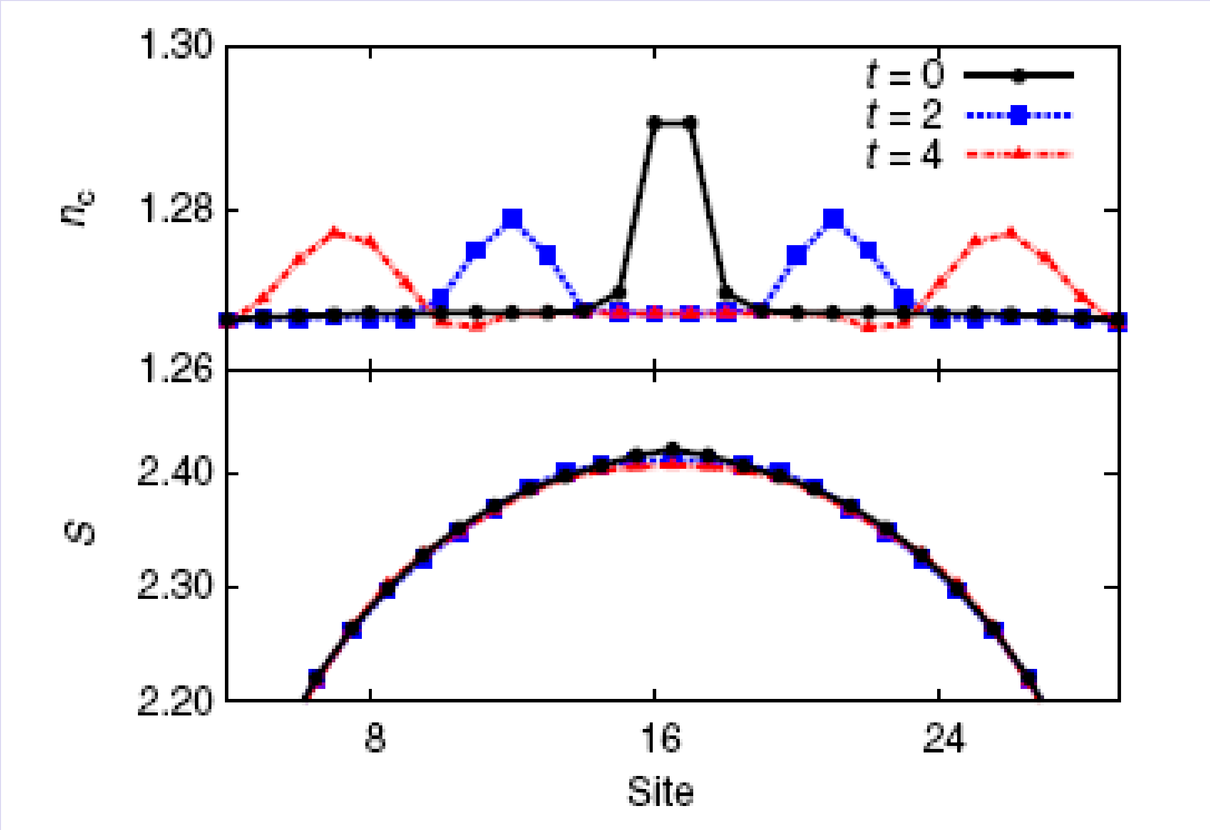
two component bosons



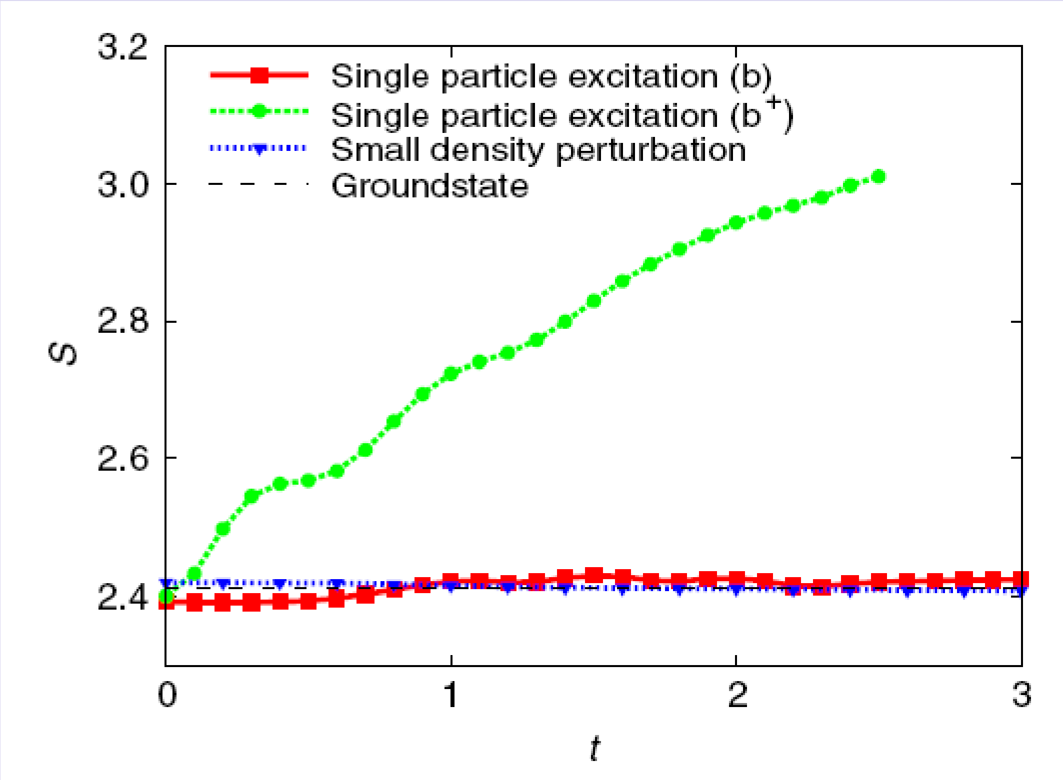
- separation of spin and charge
- strong growth of entropy with time
- contribution of spin and charge part

A. Kleine, CK, I. McCulloch, U. Schollwöck (2008)

Density excitation and its entropy growth



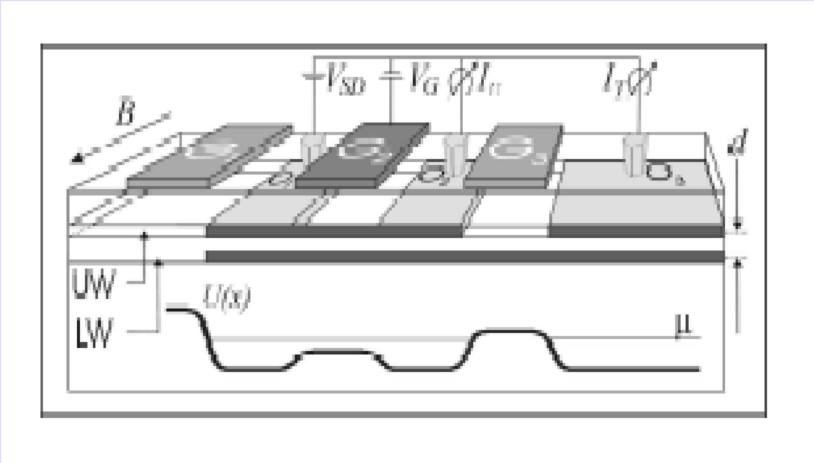
Comparison of maximum entropy growth



- separation of spin and charge
- strong growth of entropy with time for single particle excitation (numerically difficult)
- slow growth of entropy with time for density excitation

Experimental observations

condensed matter:

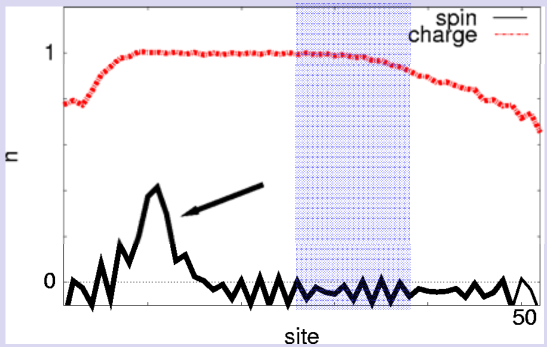


Auslaender et al. (2005)

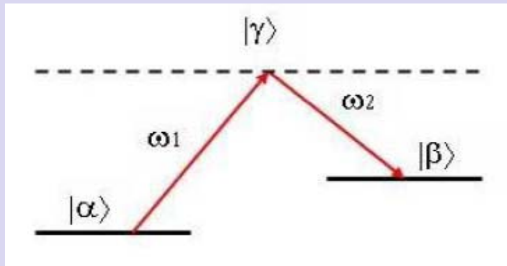
tunneling between parallel wires

cold atoms:

- detection in real time
  measure of density
  average over several lattice sites



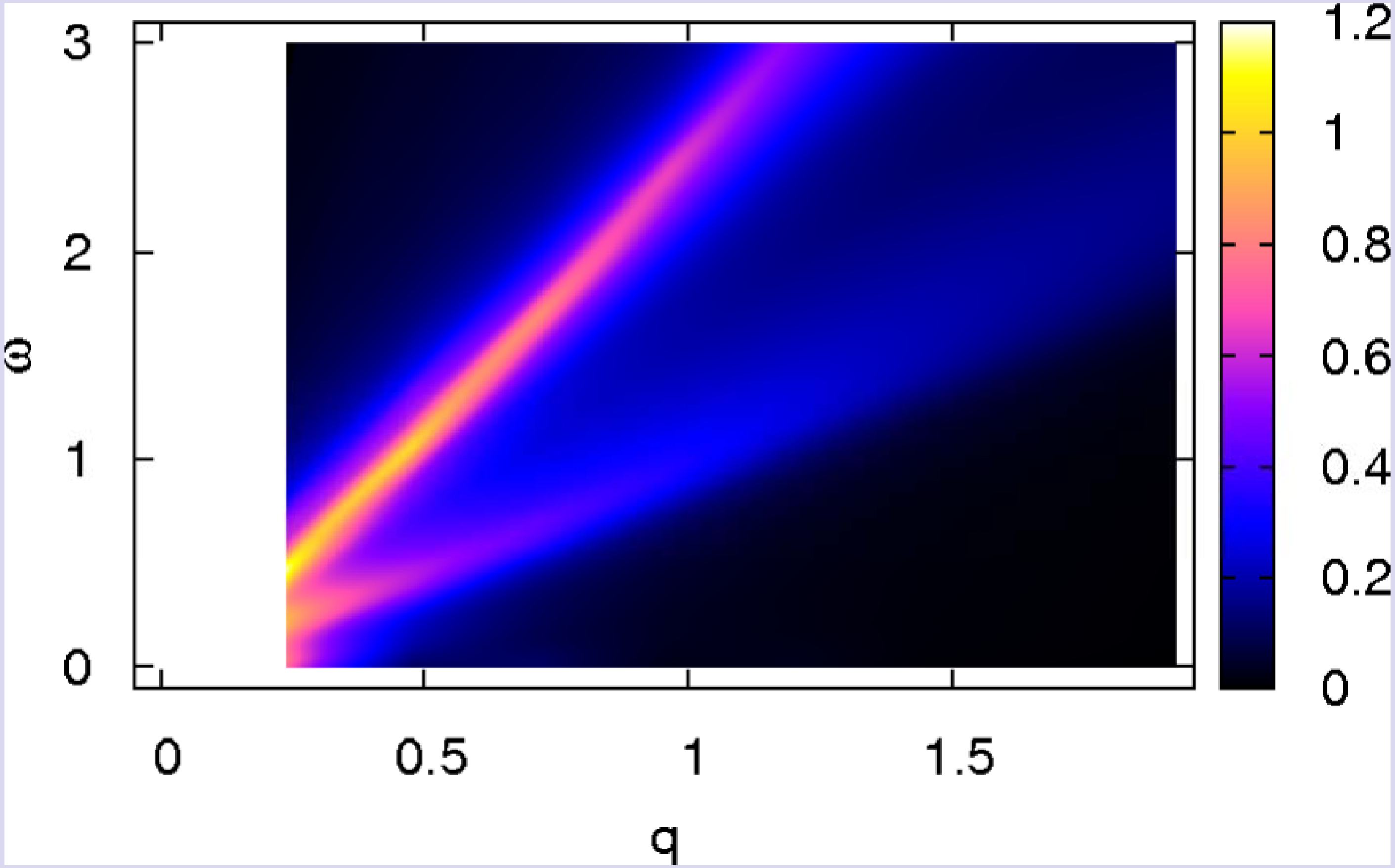
- Raman spectroscopy
  spectral function



Dao et al. PRL 98, 240402 (2007)
Stewart et al. Nature 454 (2008)

Spectral function

two component mixture of bosons in one-dimension

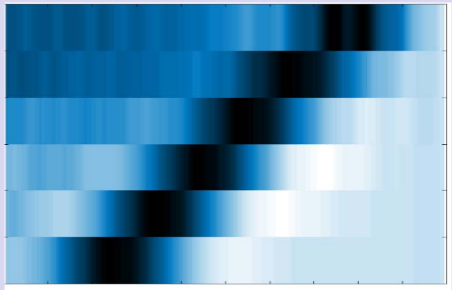


n_{charge}=0.63,
u=3, u_{12}=2.1

A. Kleiner, C.Kollath, I.McCulluoch, T. Giamarchi, U. Schollwöck, (2008)

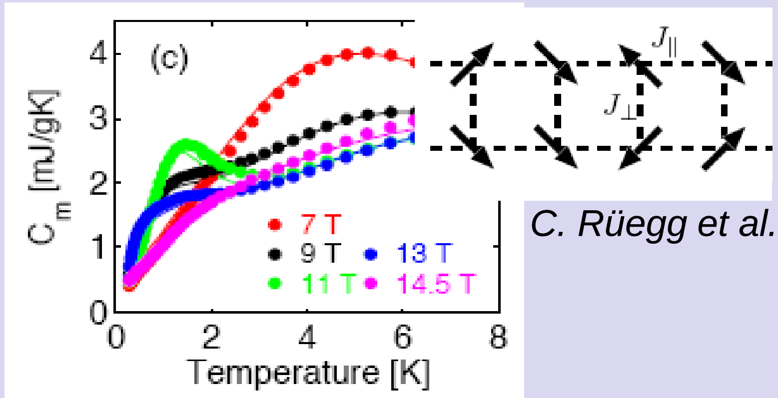
Applications of DMRG variants

•non-equilibrium situations
dynamics across quantum phase transition



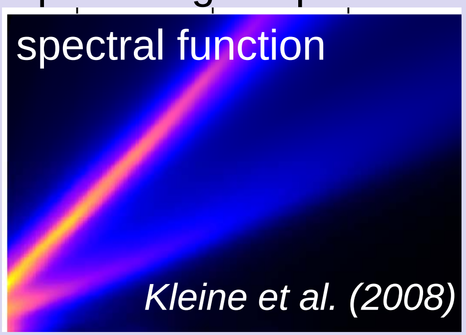
A. Laeuchli and CK (2008)

•finite temperature
thermodynamics in spin-ladders



C. Rüegg et al. (2008)

•local excitations & dynamic properties
spin-charge separation



Kleine et al. (2008)

•higher dimensions
•...

Postdoc position available