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Quantum advantage in learning from experiments and notes on dequantization

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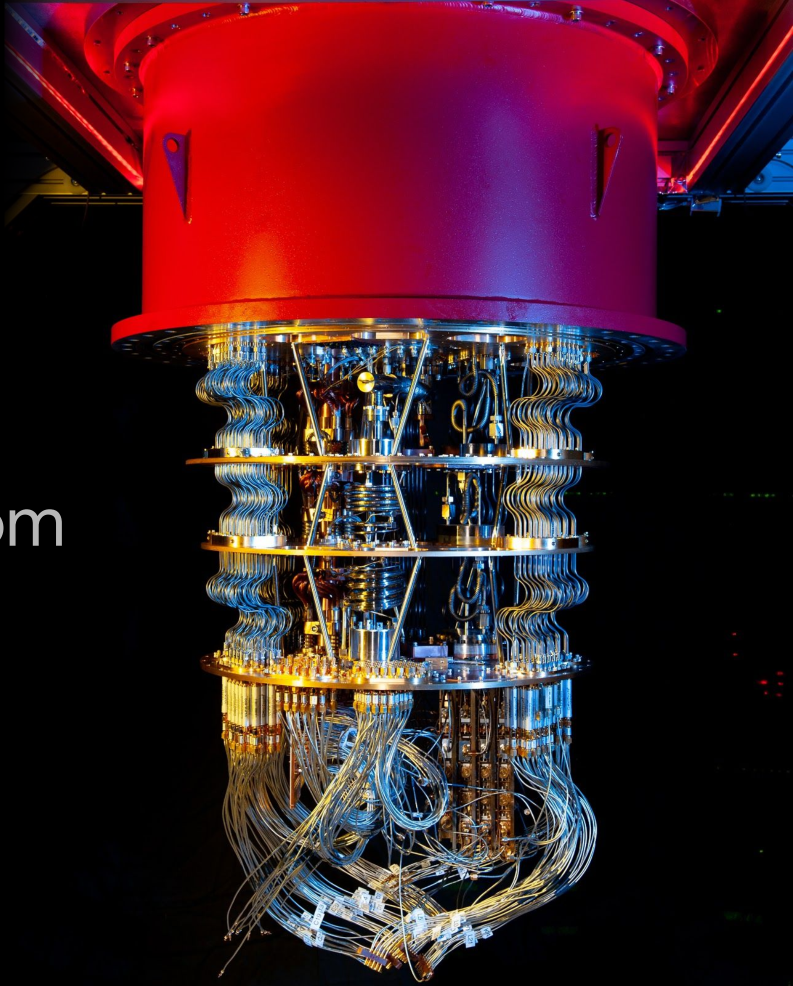
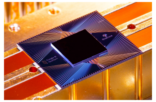


Quantum machine learning & data advantage

Computationally limited problems - Simple inputs, known computational procedure

x

Key to factor
Hamiltonian to simulate
...



Compute ~ n

Compute ~ exp(n)

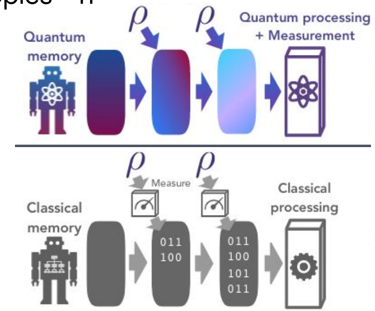
Data limited problems - Limited by availability of data, no computation possible to overcome lack of data

ρ (limited copies)

Transduced quantum state
Analog simulation state
Output of computation
...

Copies ~ n

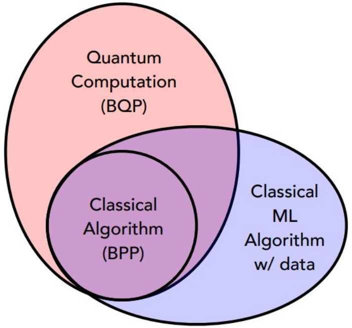
Compute ~ ?



Copies ~ exp(n)

Compute ~ ?

Data assisted problems - Known computational procedure, complexity can change with available data (~advice)



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A motivating example for data assisted problems

Given circuit U_{QNN} & some input $\vec{x}_i \rightarrow |x_i\rangle = \sum_{k=1}^{p} x_i^k |k\rangle$

Task - compute $y_i = f(x_i) = \langle x_i | U_{\text{QNN}}^\dagger O U_{\text{QNN}} | x_i \rangle$

Some data $\{(x_i, y_i)\}_i$

Arbitrary length quantum circuit

Hermitian operator

Direct simulation at least as hard as BQP, must be a powerful function of x_i!

$$f(x_i) = \left(\sum_{k=1}^{p} x_i^{k*} \langle k|\right) U_{\text{QNN}}^\dagger O U_{\text{QNN}} \left(\sum_{l=1}^{p} x_i^l |l\rangle\right)$$
$$= \sum_{k=1}^{p} \sum_{l=1}^{p} B_{kl} x_i^{k*} x_i^l,$$

At most quadratic function on entries of x_i with p^2 coefficients!

No data - hard quantum circuit
With data - Almost trivial learning task!

(More generally, need ~$\left(\frac{p^2}{\epsilon^2}\right)$ data pts)

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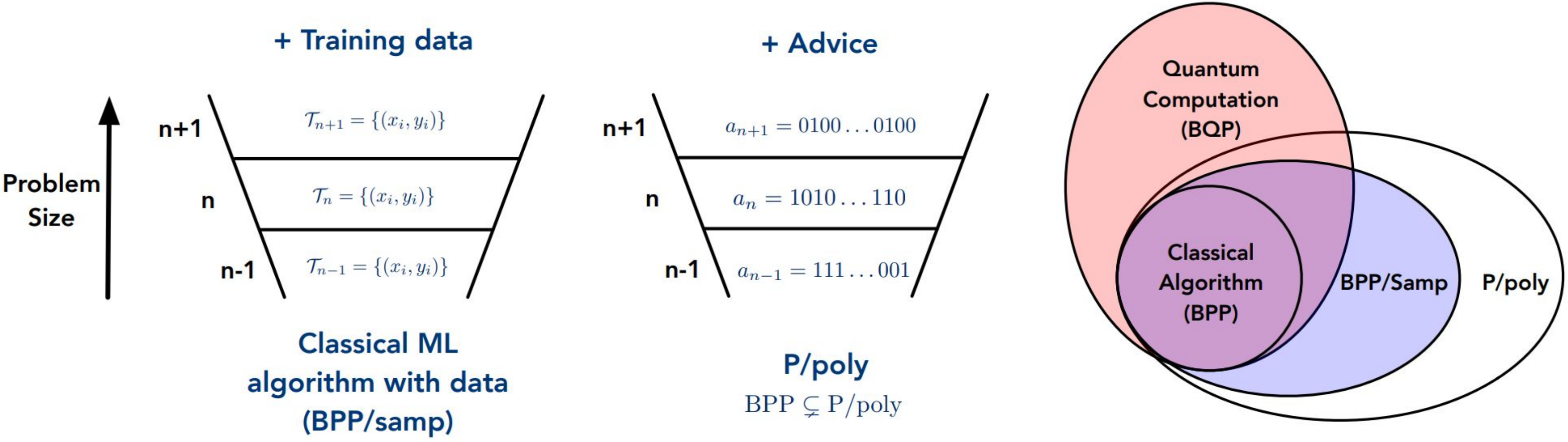
The power of data in quantum machine learning*



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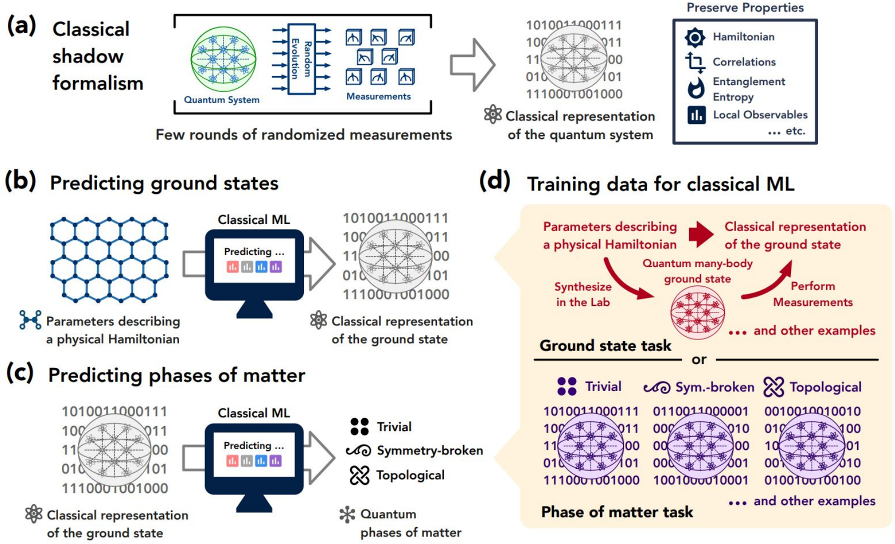
* Hsin-Yuan (Robert) Huang, Michael Broughton, Masoud Mohseni, Ryan Babbush, Sergio Boixo, Hartmut Neven, **Jarrod R. McClean** "Power of data in quantum machine learning" Nature Communications, Vol.12, No. 2631 (2021)

What kinds of problems are learnable from a little data?

Provably efficient machine learning for quantum many-body problems

Hsin-Yuan Huang,[1] Richard Kueng,[2] Giacomo Torlai,[3] Victor V. Albert,[4] and John Preskill[1,3]

arXiv:2106.12627 (2021)



Is the fate of quantum computers to provide training data for classical models?

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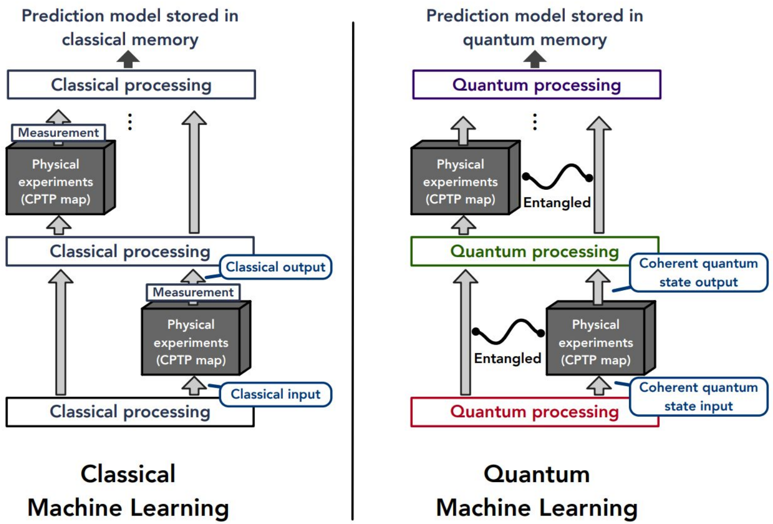
So what's left for a quantum computer?

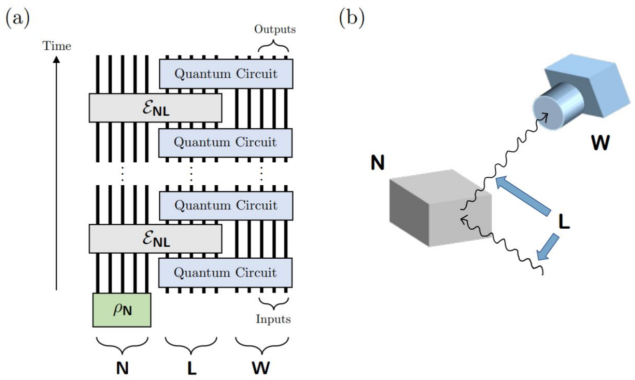
Information-theoretic bounds on quantum advantage in machine learning

Hsin-Yuan Huang,[1,2] Richard Kueng,[3] and John Preskill[1,2,4,5]

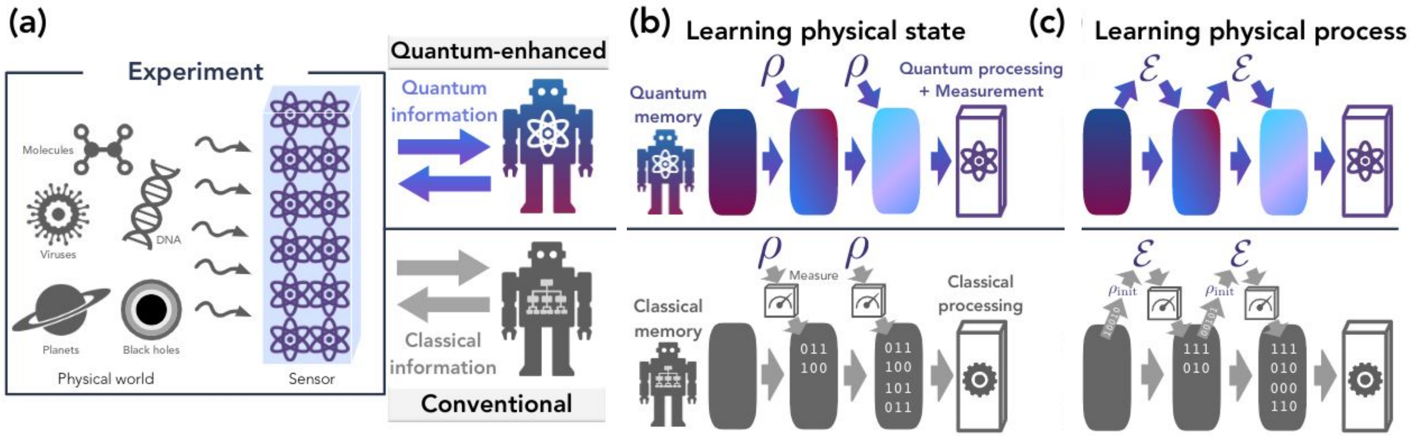


Quantum Algorithmic Measurement

Dorit Aharonov[1,a], Jordan Cotler[2,3,b], Xiao-Liang Qi[3,c]



Quantum memory and quantum-enhanced experiments



This work - Exponential advantage with exactly 2 copies on 2 different tasks and efficient classical compute (and additional proofs)

Quantum advantage in learning from experiments
Huang, Broughton, Cotler, Chen, Li, Mohseni, Neven, Babbush, Kueng, Preskill, McClean
arXiv:2112.00778 (2021)

What's the simplest task we can have an advantage on?

Setup -

N copies of $\rho \propto (I + \alpha P)$

$$\alpha \in (-1, 1)$$

P Is a general n-qubit pauli operator e.g.

$$Z \otimes X \otimes I \otimes Z \otimes \ldots = Z_1 X_2 Z_4 \ldots$$

Form of state is known

α, P unknown

Note -
State is un-entangled but **not** factorizable
Can be realized in depth 1 Clifford circuit

Task -

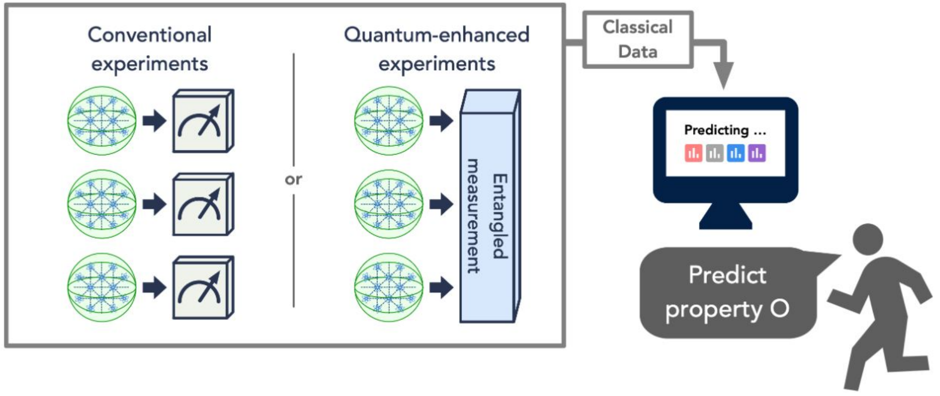
Take any measurements you want on the N copies

Collect classical data signature of state

Given new Pauli operator O, predict $|\mathrm{Tr}[\rho O]|$

(**Alternatively**) Given 2 candidate Pauli operators Q_1, Q_2

Determine $|\mathrm{Tr}[Q_1 \rho]| > |\mathrm{Tr}[Q_2 \rho]|$?



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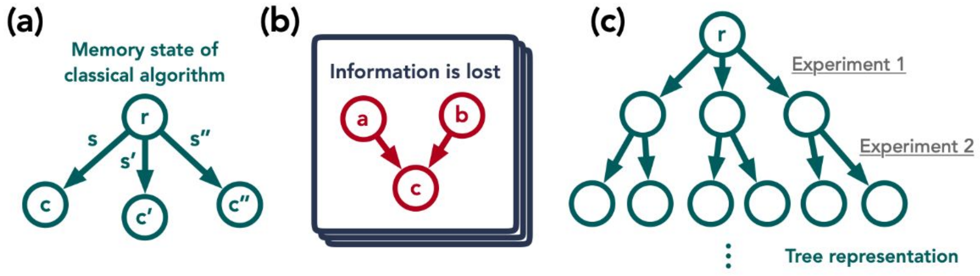
The best possible conventional experiments



Result summary

Best conventional strategy requires N ~ 2^n

to predict $|\mathrm{Tr}[\rho O]|$ to additive error < .25

with probability > .8

Sketch of proof

Reduction to discrimination task

Null hypothesis

$\rho = I/2^n$

O random

Alternate hypothesis

$\rho \propto (I + \alpha P)$

$\alpha \in \{+.9, -.9\}$ $O = P$

-Optimal discriminating POVM can be bounded using hypothesis structure
-Is independent of previous measurements
-Gives exp vanishing returns

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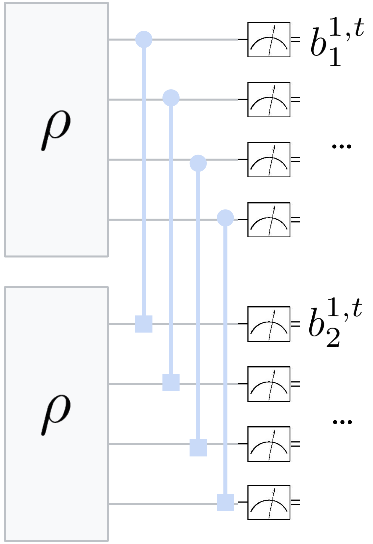
The simplest quantum-enhanced experiment

n qubits



n qubits

2n classical bits for N rounds creates Bell sketch of state $\{b_i^{k,t}\}$

$$O = \sigma_1 \otimes \sigma_2 \ldots \otimes \sigma_n$$

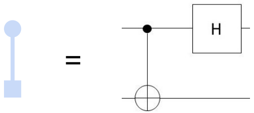
$$|s_k^{(t)}\rangle = \frac{1}{\sqrt{2}} I \otimes Z^{b_1^{k,t}} X^{b_2^{k,t}} (|00\rangle + |11\rangle) \qquad S_k^{(t)} = |s_k^{(t)}\rangle\langle s_k^{(t)}|$$

$$\hat{a}(O) = \frac{1}{N} \sum_{t=1}^{N} \prod_{k=1} \mathrm{Tr}\left[(\sigma_k \otimes \sigma_k) S_k^{(t)}\right] \longrightarrow$$ Estimate of $|\mathrm{Tr}[\rho O]|^2$

Depth 1
clifford gates

$$\rho \propto (I + \alpha P)$$

Bell measurements



Samples ~ $\frac{1}{\epsilon^4}$ Computation time ~ nN

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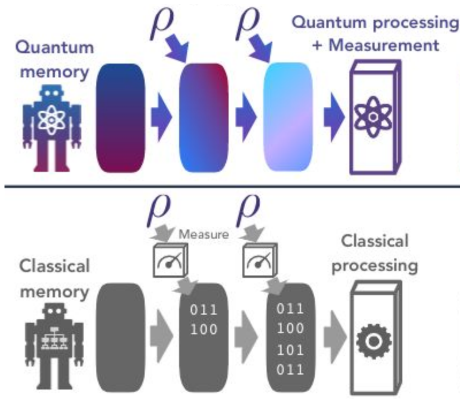
Summarizing the scale of the separation

N copies of $\rho \propto (I + \alpha P)$



Take any measurements you want on the N copies

Collect classical data signature of state

Given new Pauli operator O, predict $|\mathrm{Tr}[\rho O]|$ to error ϵ

Copies ~ $\dfrac{1}{\epsilon^4}$ Compute ~ n x Copies

Copies $> (2^n + 1)/.85$

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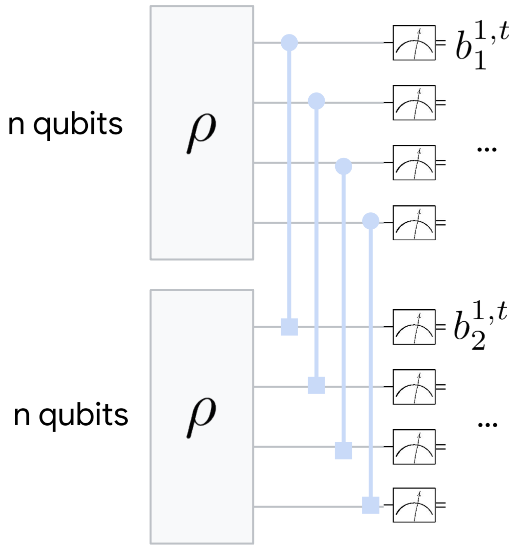
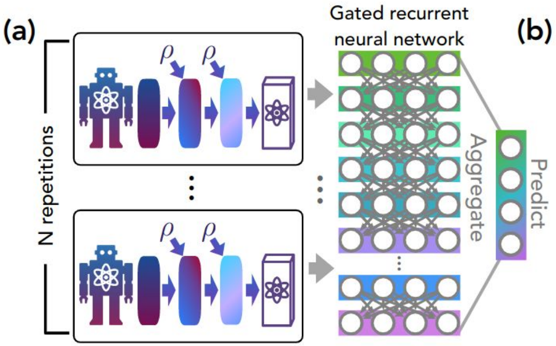
Bell measurements as a feature in learning



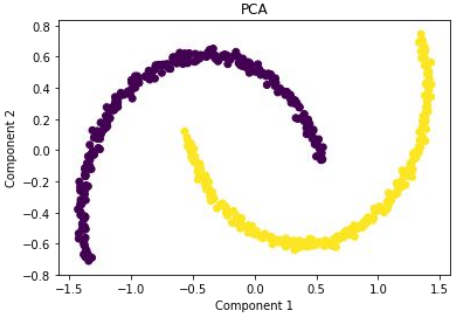
2n classical bits for N rounds creates Bell sketch of state $\{b_i^{k,t}\}$

Could we use this "**feature**" of a quantum state to learn this task?

Can state specific noise or features boost the performance? Or the performance of an adversary?

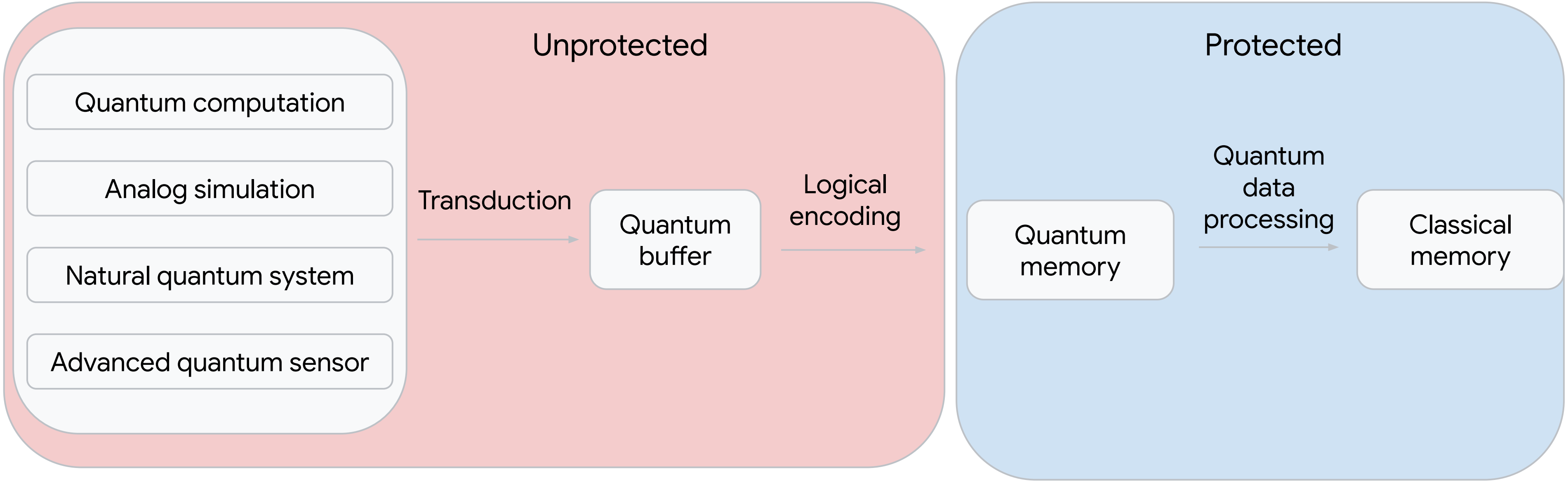


Supervised



Unsupervised

Imagining and emulating a quantum data pipeline

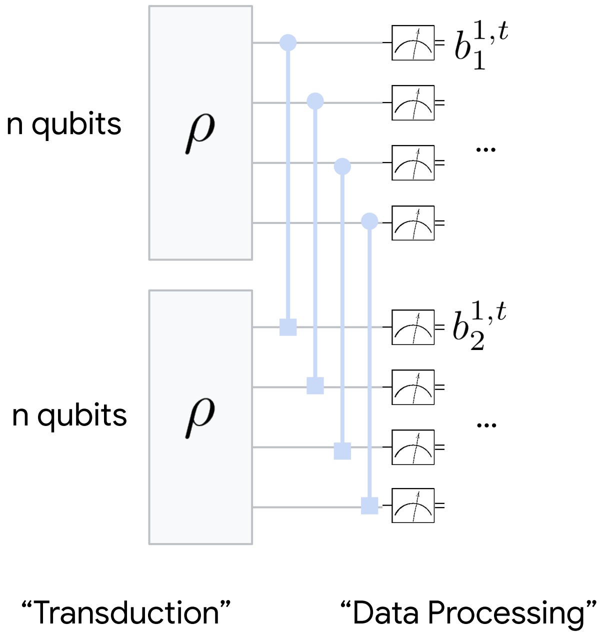


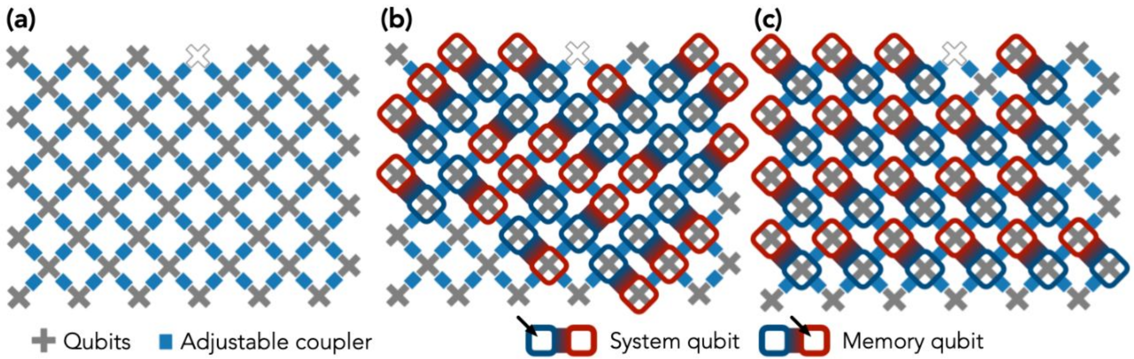
Always(?) unprotected

Proofs have some experimental flexibility but not unlimited - ultimate test is empirical

Noise in state prep and computation here let us test separation on simple tasks in more realistic conditions

Using our chip to understand performance on real data





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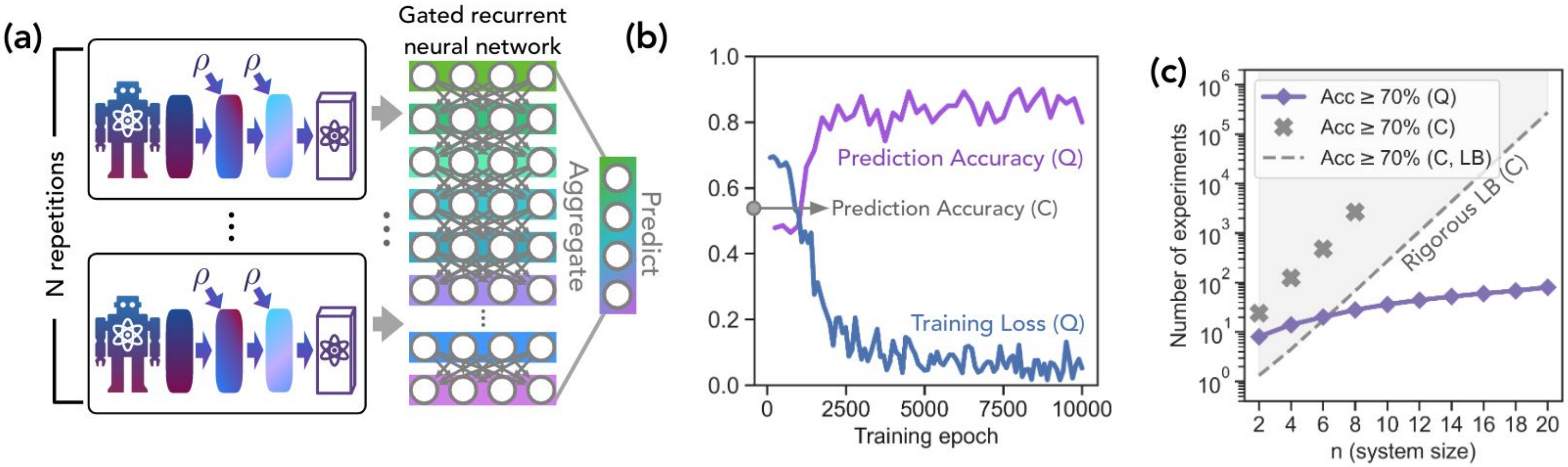
Experimental demonstration of advantage

N copies of $\rho \propto (I + \alpha P)$

Given 2 candidate Pauli operators Q_1, Q_2

Determine $|\mathrm{Tr}[Q_1 \rho]| > |\mathrm{Tr}[Q_2 \rho]|?$



Trained on noiseless data n < 8

Test on n up to 20 (= 40 physical qubits)

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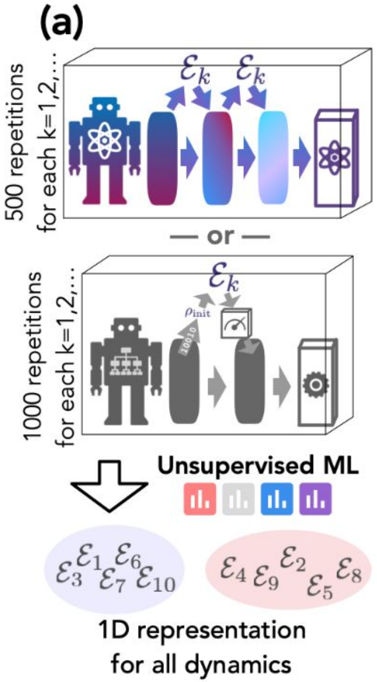
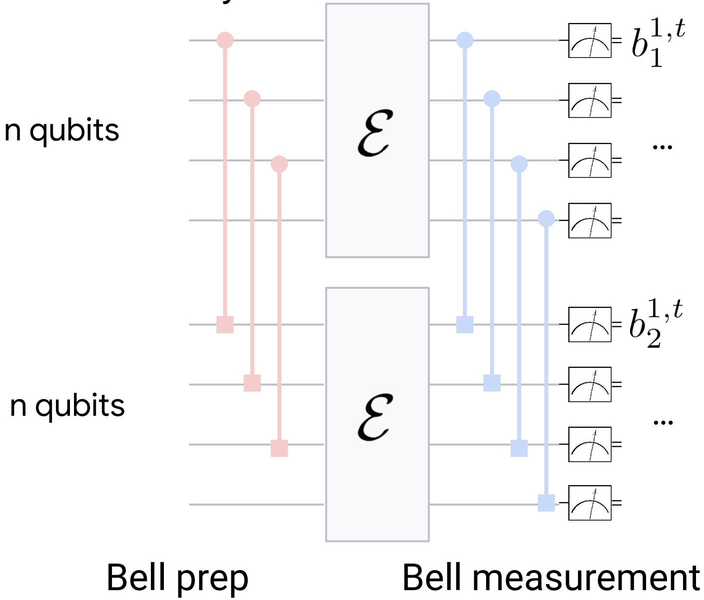
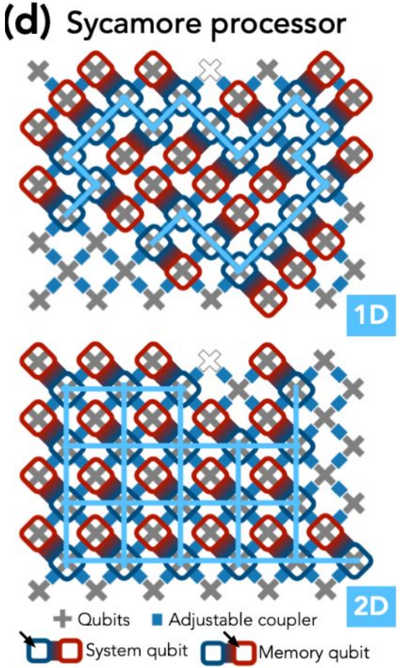
We've learned about states... how about processes?



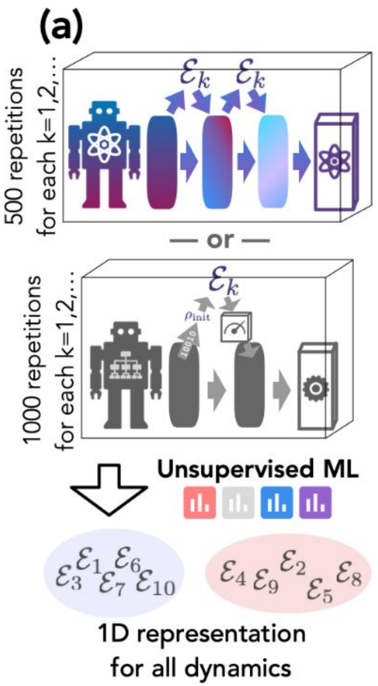
Given access to a process $\mathcal{E}$ N times, determine if
(a) random time-reversal symmetric evolution
(b) random unitary





	Number of qubits	Number of gates	Circuit depth
1D dynamics	40	842	40
2D dynamics	40	1388	54

Unsupervised discovery



Trials

$b_i^{1,t}$

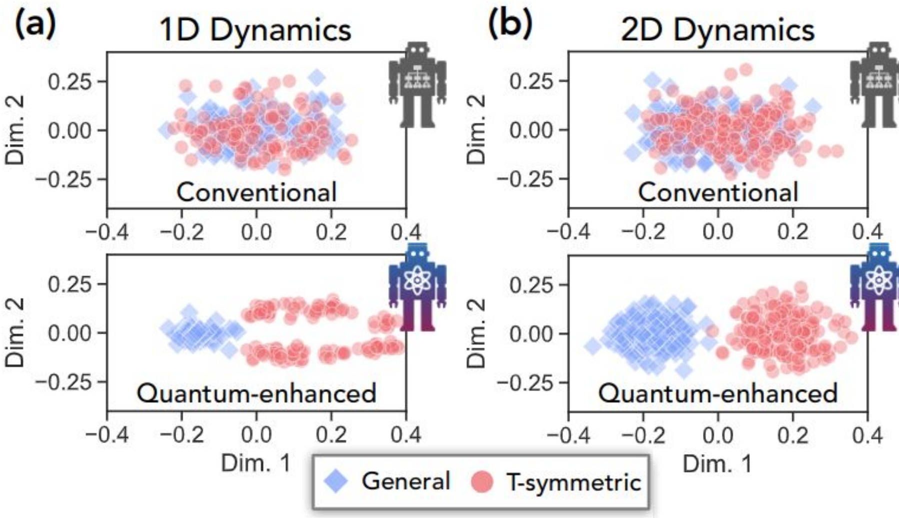
...

Avg and Variance

$(\mathbb{E}[b_i^1], \mathrm{Var}[b_i^1])$

...

Feature vector for each process $\vec{b}_i$



Kernel PCA

Squared exp kernel

$$K(\vec{b}_i, \vec{b}_j) = \exp(-\gamma||\vec{b}_i - \vec{b}_j||^2)$$

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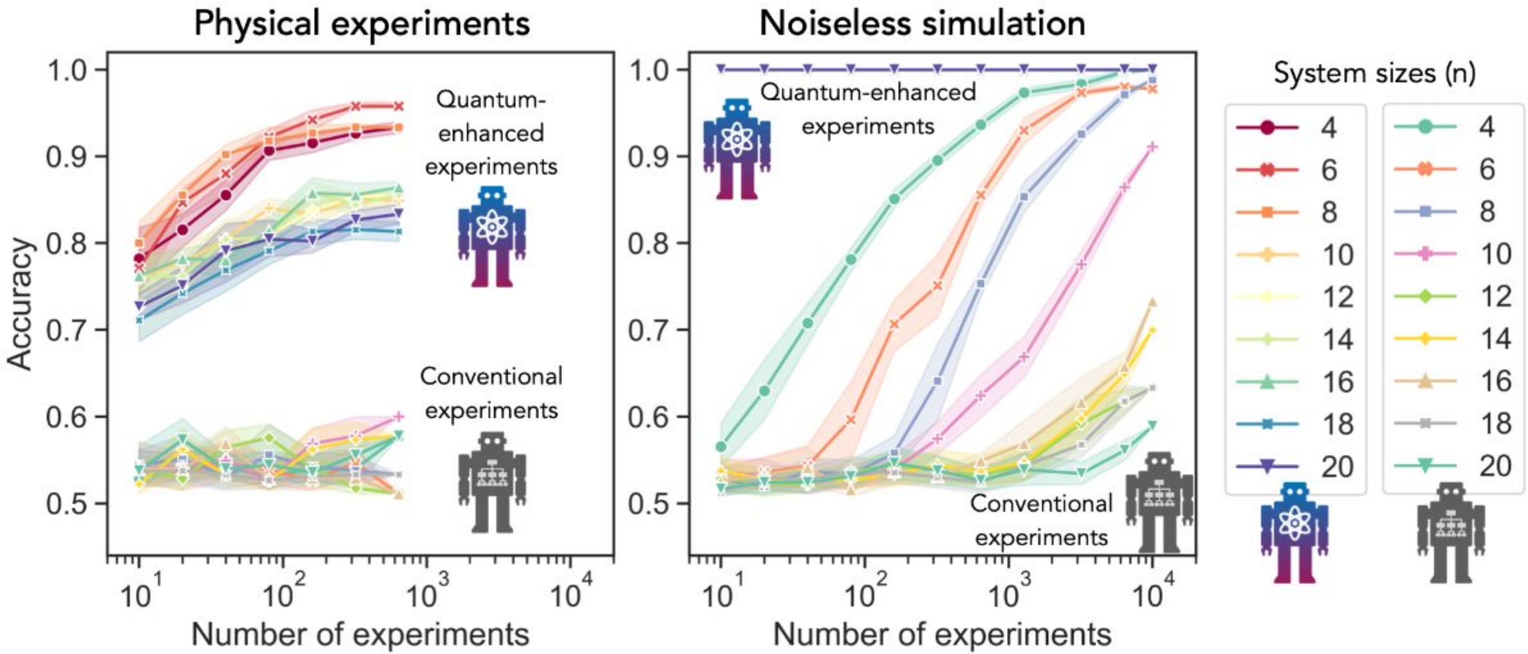
Unsupervised classification of processes

(1D scrambling data)



(40 physical qubits)

	Number of qubits	Number of gates	Circuit depth
1D dynamics	40	842	40
2D dynamics	40	1388	54

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SWAPs and virtual distillation to the quantum PCA

Corrupted quantum data

$$\rho = f\rho_0 + \sum_i p_i \rho_i$$

Uncorrupted data

Orthogonal errors

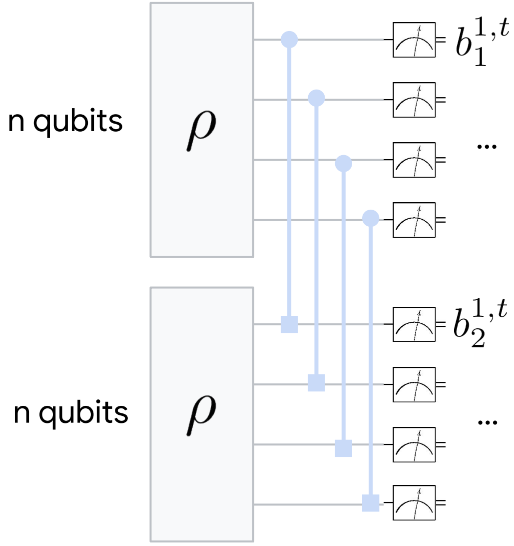
1 SWAP ⟶ Virtual distillation

SWAP as a generator ⟶ $\exp\left(-i\delta t\mathrm{SWAP}\right)$ ⟶ Quantum PCA

This work: Proof in a conventional scenario that exponential number of copies are required to learn about principal component vs constant in quantum enhanced setting.

Recall virtual distillation $\rho \to \rho^2$

Uses destructive SWAP



(What about dequantization?)

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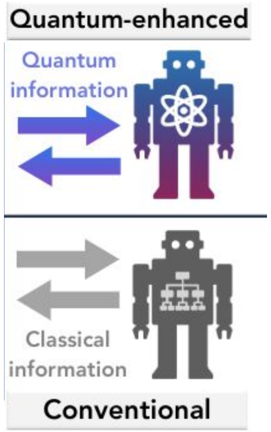
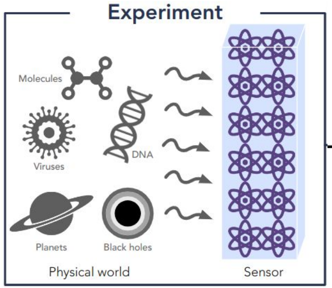
Recall dequantization



Quantum linear solution of equations
Quantum PCA
Quantum recommendation systems

$$Ax = b$$



$$\rightarrow \rho$$

Quantum access models

Explicit

$$\{x_i\}_{i=1\ldots 2^n} \rightarrow \frac{1}{||x||_2} \sum_i x_i |i\rangle$$

Implicit

$$U|0\ldots\rangle \rightarrow \sum_i c_i |i\rangle$$

Sample-and-query (SQ) access model (Classical)

Sample - The oracle outputs i with probability $|x_i|^2 / \sum_j |x_j|^2$

Query(i) - The oracle outputs x_i to arbitrary precision

QueryN - The oracle outputs $||x||_2$

Dequantization (informal) - If SQ access to data allows classical algorithms to match the advertised scaling of quantum algorithms up to poly overhead, the algorithm is said to be dequantized.

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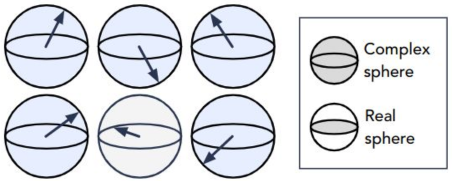
Take care when invoking SQ access - sometimes too powerful

Explicit

Prepare $\{x_i\}_{i=1\ldots 2^n} \rightarrow \frac{1}{||x||_2}\sum_i x_i|i\rangle$



Real vector search problem

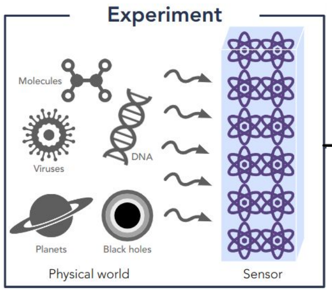
(Quantum) Requires ~2^n calls to **Prepare**

(Classical) Requires 1 call to **Query(0)**

(Other quantum - builds SQ reversibly)

Implicit

$U|0\ldots\rangle \rightarrow \sum_i c_i|i\rangle$



$\rightarrow \rho$

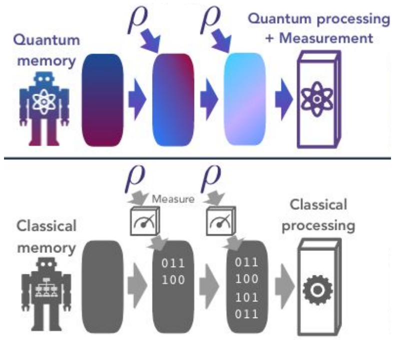
Query(0) enables strong simulation
→ solves #P-Complete problems efficiently

Reading state to accuracy required for **Query(i)** has exponential cost

(ArXiv:2112.00811)

Summary & Outlook

Punchline - IF we could find a suitable data source, our cloud quantum devices **today** allow us to learn things that are otherwise inaccessible.



(Recall computational vs data advantage)

This work

- Proofs of advantage in state learning, process learning, and quantum PCA
- Experimental demonstration of state and process learning using up to 40 physical qubits & 1300 gates

Outlook

- Inspire work on quantum data sources & sensors (beyond quadratic)
- Deeper connection to physics? Interferometry?
- Other tasks with 2-copy + Clifford advantage?
- Beyond Bell features?
- Can these proof techniques tell us something about existing learning tasks or quantum techniques?

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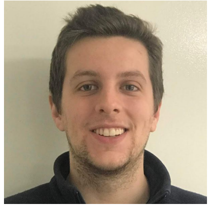
Acknowledgements

Hsin-Yuan (Robert) Huang



Michael Broughton

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Quantum AI

Jordan Cotler
Sitan Chen
Jerry Li
Masoud Mohseni
Hartmut Neven
Ryan Babbush
Richard Kueng
John Preskill
QCS, physics, calibration, and entire hardware team

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Quantum advantage in learning from experiments
Huang, Broughton, Cotler, Chen, Li, Mohseni, Neven, Babbush, Kueng, Preskill, McClean
arXiv:2112.00778 (2021)