Topological Quantum Computation with Majorana Zero Modes

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Microsoft Station Q

IPAM, 08/28/2018

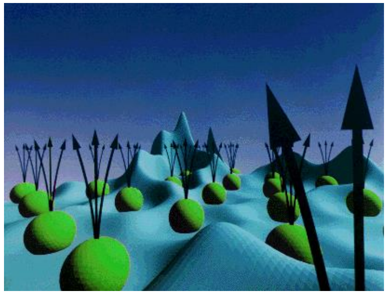
Outline

- Majorana zero modes in proximitized nanowires
- Experimental and material science progress
- Topological quantum computing with Majoranas
    - Majorana box qubits and scalable designs
    - Coherence times of Majorana-based qubits

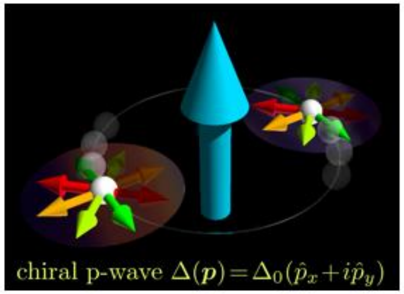
Proposed physical realizations of solid-state Majoranas

Intrinsic origin

$\nu = \frac{5}{2}$ FQHE

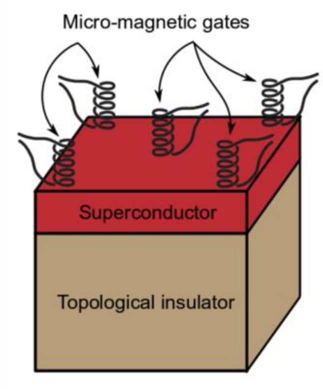


2D $p_x + ip_y$ SC

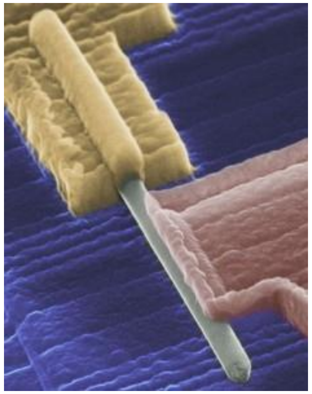


Synthetic materials

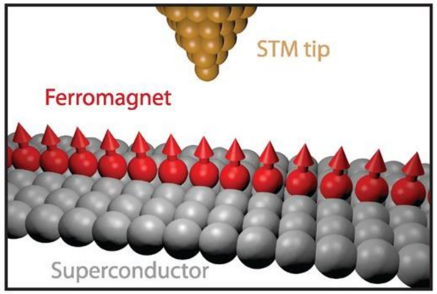
TI/SC



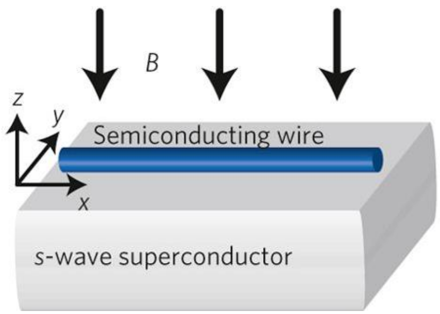
SM/SC



atomic chains



Generic model for Majorana wires

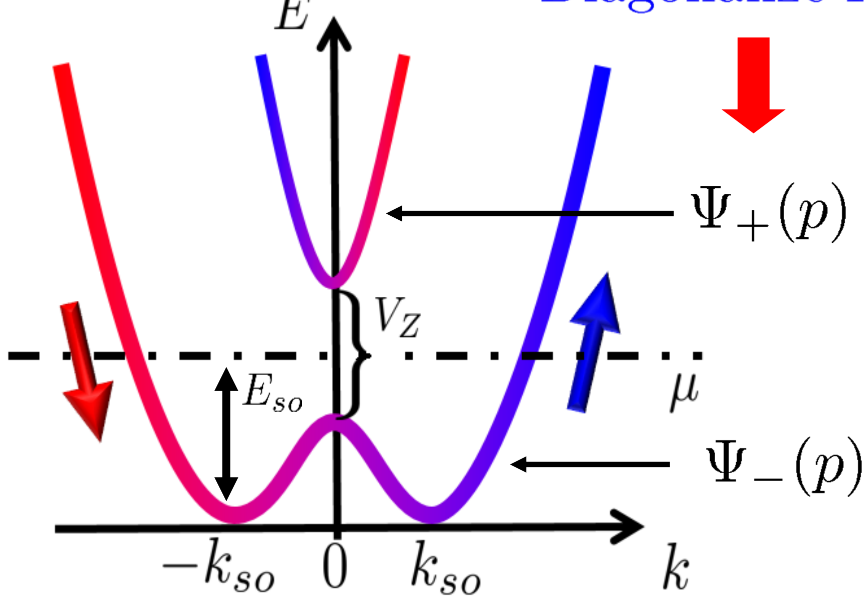


$$H_{\rm MW} = \int_{-L}^{L} dx \left[\psi^\dagger_\sigma \left(-\frac{\partial_x^2}{2m^*} - \mu + i\alpha\sigma_y\partial_x + V_x\sigma_x \right)_{\sigma\sigma'} \psi_{\sigma'} + \Delta_0^* \psi_\uparrow \psi_\downarrow + \Delta_0 \psi^\dagger_\downarrow \psi^\dagger_\uparrow \right]$$

Rashba spin-orbit+in-plane field

Proximity-induced superconductivity

Diagonalize H_0



$$H_{\rm SC} = \begin{cases} \Delta_{--}(p)\Psi^\dagger_-(p)\Psi^\dagger_-(-p) \\ \Delta_{+-}(p)\Psi^\dagger_+(p)\Psi^\dagger_-(-p) \\ \Delta_{++}(p)\Psi^\dagger_+(p)\Psi^\dagger_+(-p) \end{cases}$$

$\Delta_{--}(p) \propto \Delta_0 p_x$

large spin-orbit ($\alpha \sim 0.1 eV\AA$)

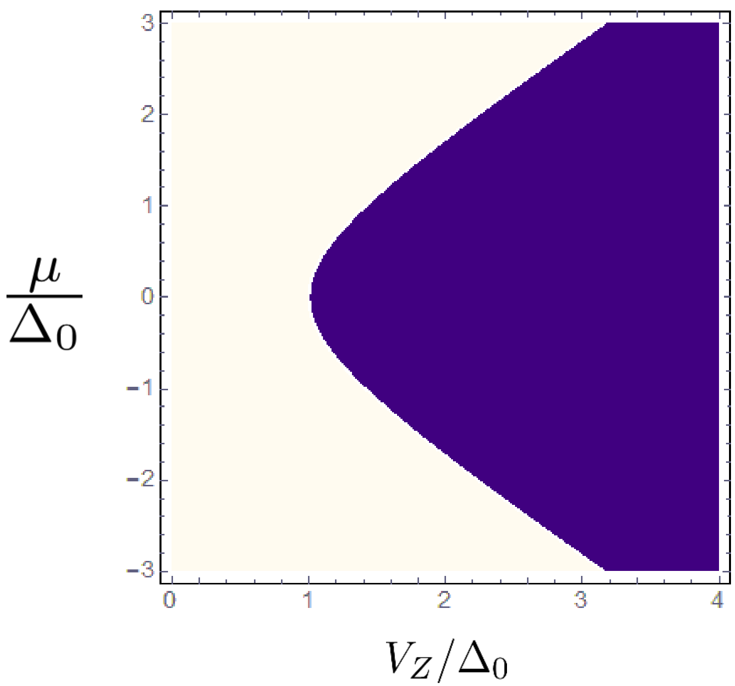
large g-factor ($g \sim 10 - 50$)

good contacts with metals

Lutchyn, Sau, Das Sarma, PRL(2010)
Oreg, Refael, von Oppen, PRL(2010)

Topological phase diagram

Drive topological phase transition
by changing V_x or μ

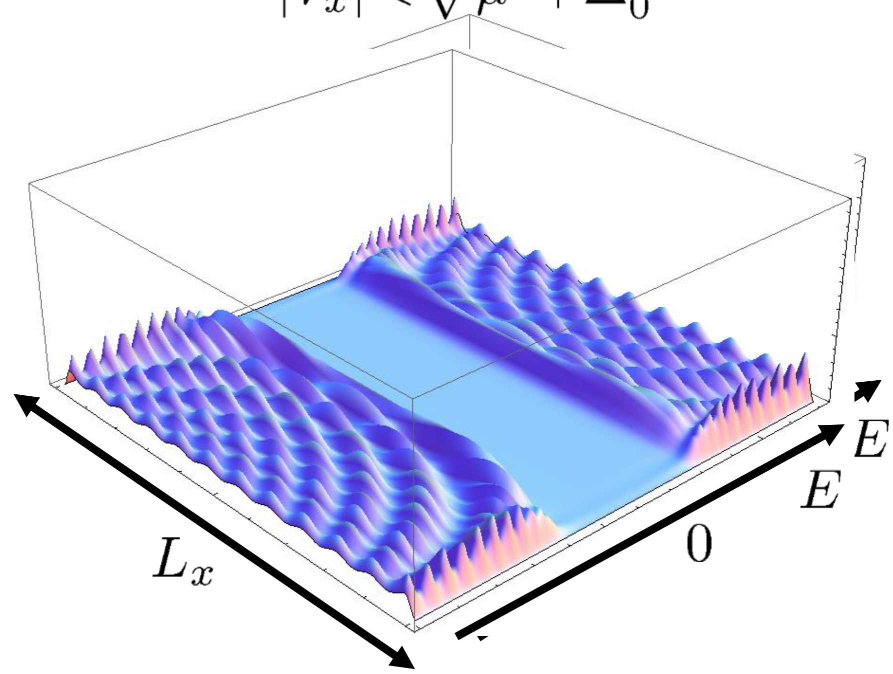


DoS in topologically non-trivial phase

DoS in topologically trivial phase

$|V_x| > \sqrt{\mu^2 + \Delta_0^2}$

$|V_x| < \sqrt{\mu^2 + \Delta_0^2}$



Sau, Lutchyn, Das Sarma, PRL(2010)

Experimental progress

- Zero-bias tunneling conductance:

First-generation experiments

- Mourik *et al*. (Delft), Science 336, 1003 (2012)
- Deng *et al*. (Lund/Peking), Nano Lett. 12, 6414 (2012)
- Das, A. *et al*. (Weizmann), Nature Phys. 8, 887 (2012)
- Finck *et al*. (UIUC), PRL 110, 126406 (2013)
- Churchill *et al*. (Harvard), PRB 87, 241401(R) (2013)

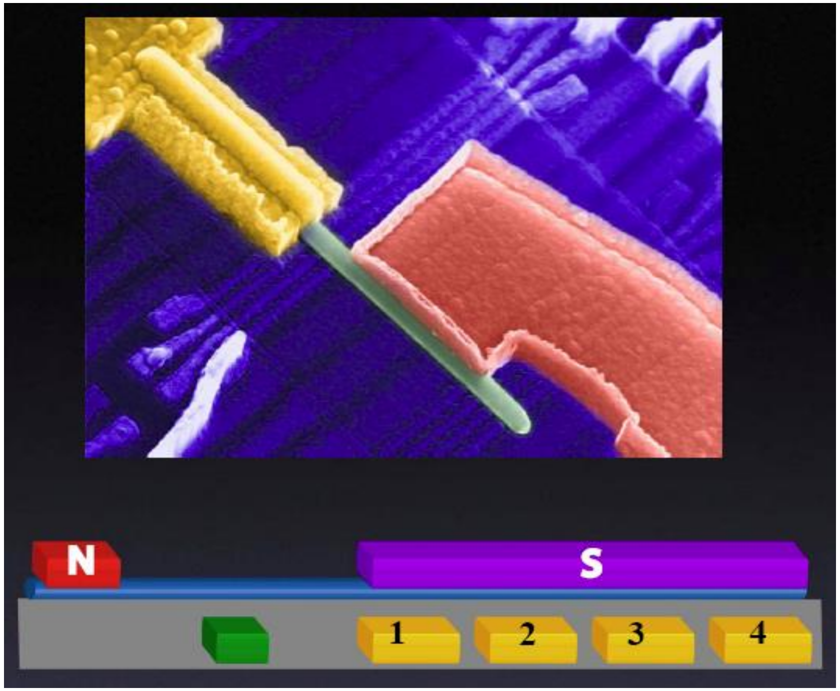
Second-generation experiments

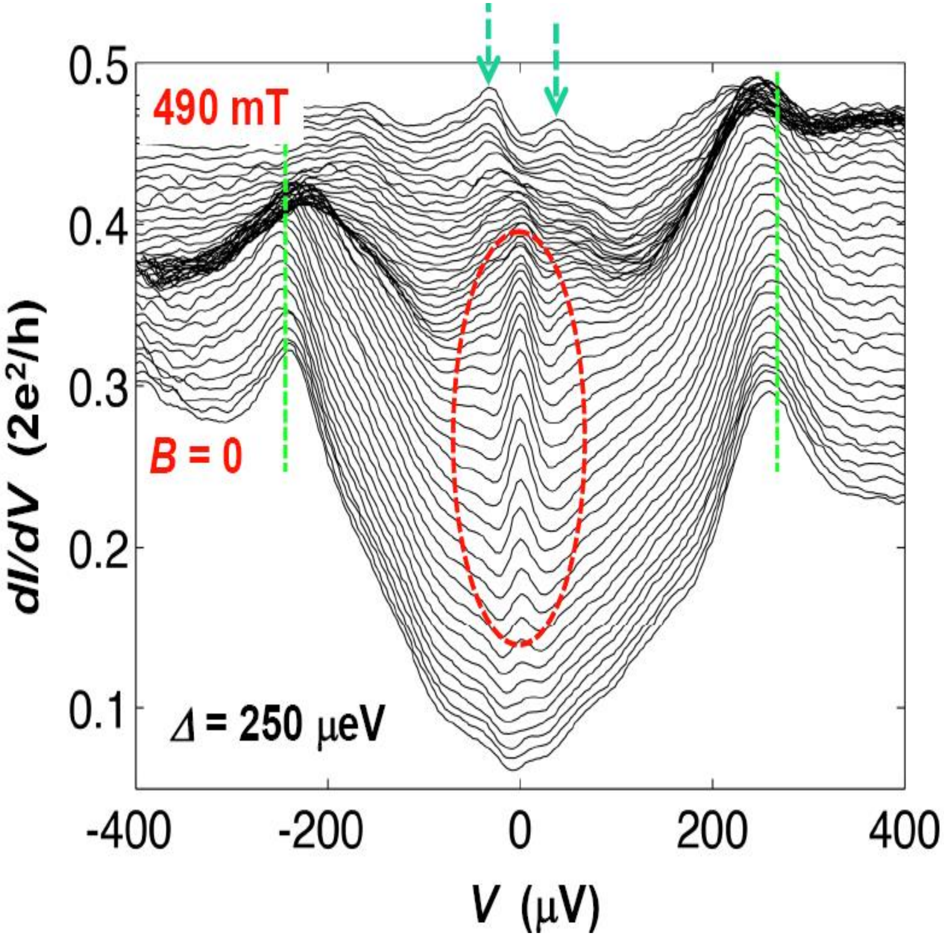
- Zhang et al. (Delft, InSb/NbTiN), Nature Comm. (2017)
- Deng et al. (NBI, InAs/Al), Science 354, 1557 (2016)
- Zhang et al. (Delft, InSb/Al), Nature 556, 74 (2018)

- Coulomb blockade experiments

- Higginbotham et al. (NBI, InAs/Al), Nat Phys. 2015
- Albrecht et al. (NBI, InAs/Al), Nature 531, 206 (2016)

First zero-bias peak observation in proximitized nanowires



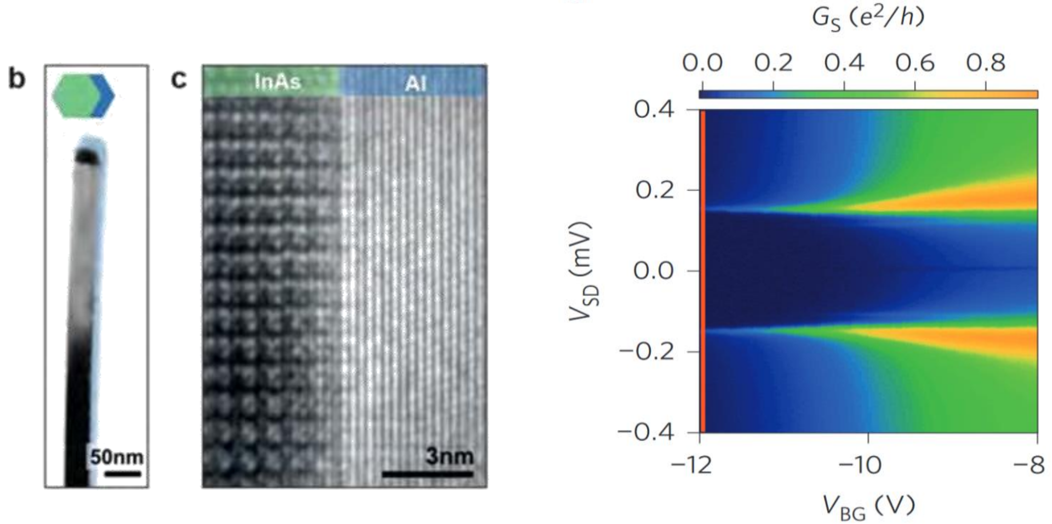


Mourik *et al*. (Delft), Science 336, 1003 (2012)

High quality proximitized nanowires

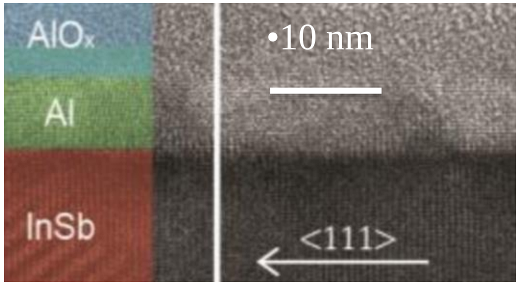
InAs/Al

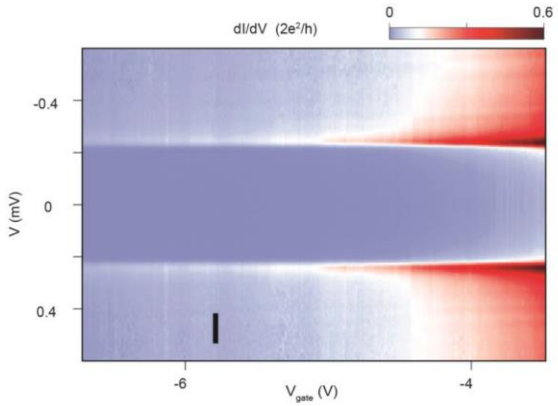
- Hard Gap
- Clean epitaxial interface
- Can withstand large critical fields
- Larger g-factor
- Lower effective mass
- Larger spin-orbit strength



Krogstrup et al. Nature Mat. (2015)

InSb/Al



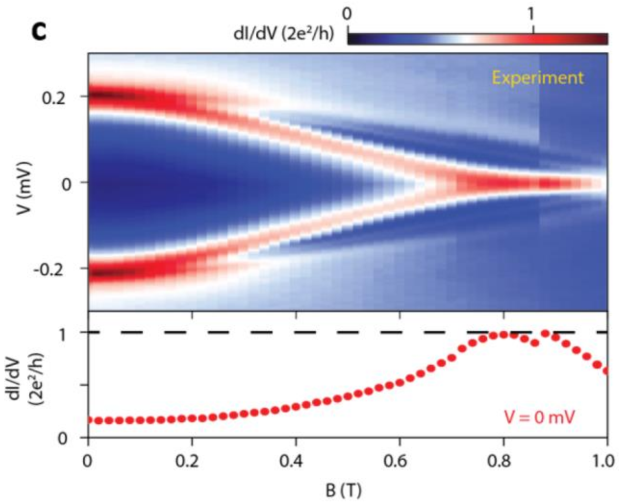


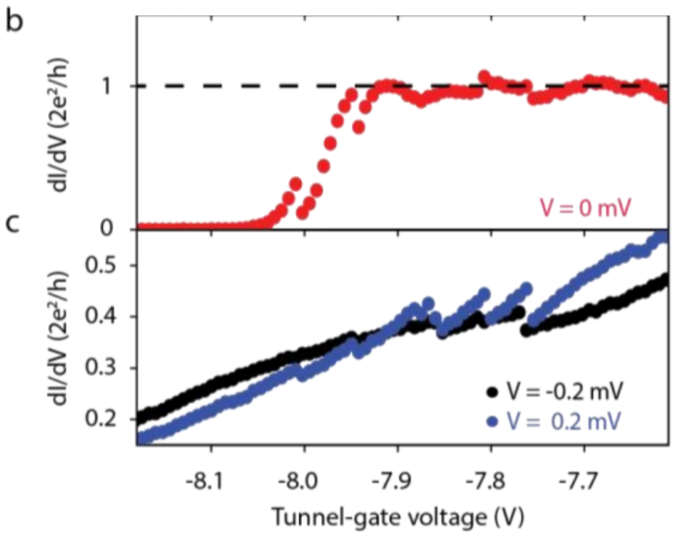
Gazibegovic et al., Nature (2017)

Second-generation experiments

Quantized conductance

(InSb/Al nanowires)

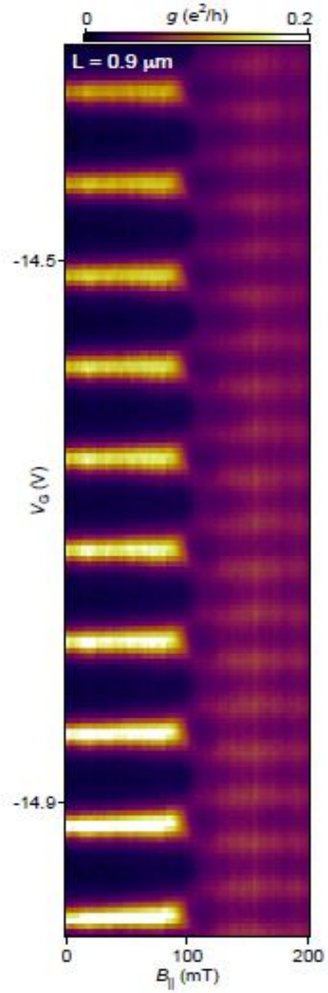


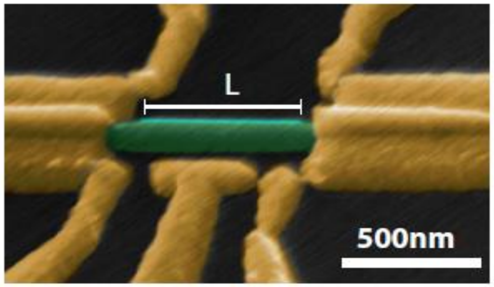


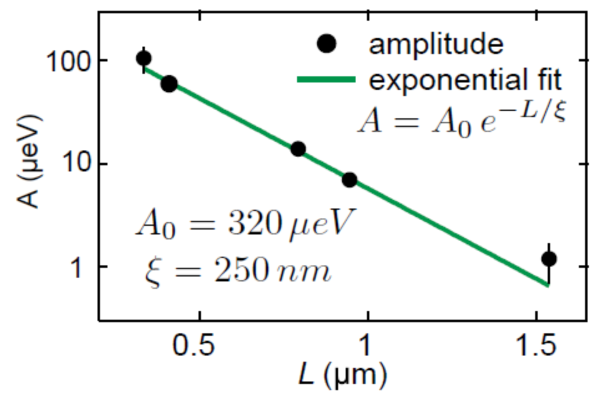
Zhang et al., Nature (2018)

Majorana splitting energy

(InAs/Al nanowires)



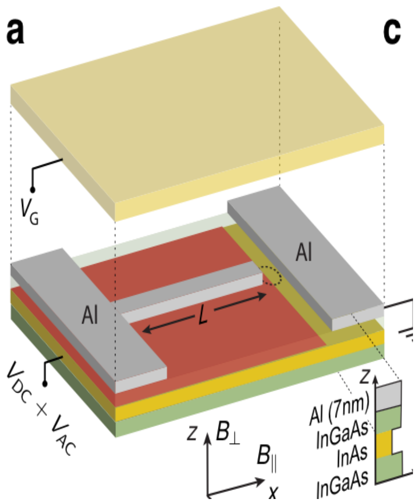




Albrecht et al., Nature (2016)

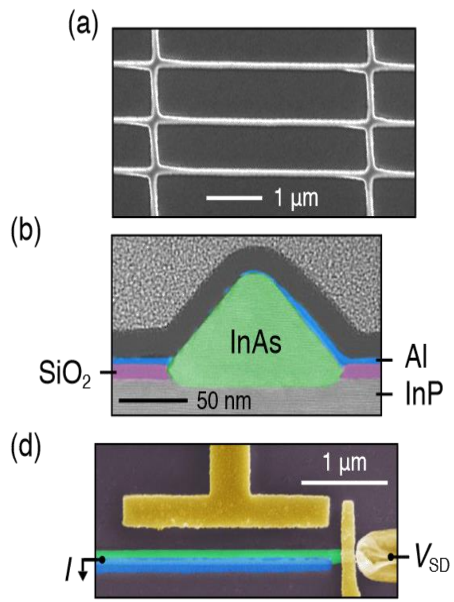
Next steps: Scalability and Control

2DEG

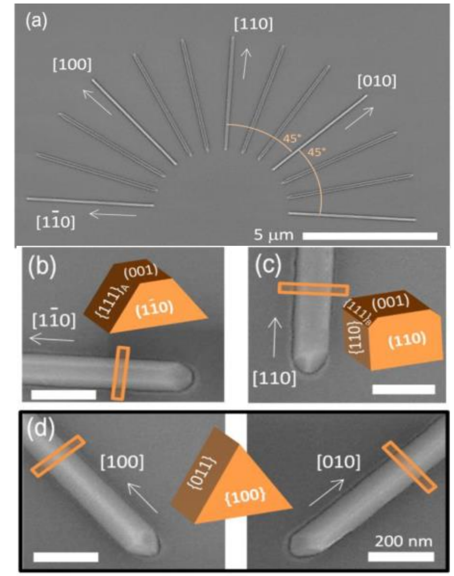


Suominen et al. (2017)

Selective Area Growth

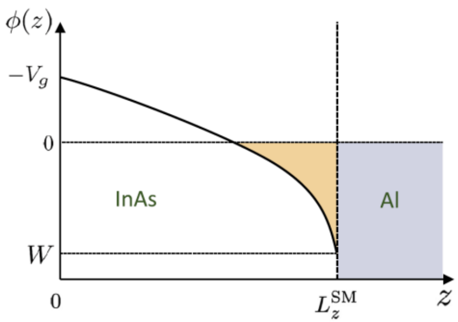


S. Vaitiekėnas et al. (2018)

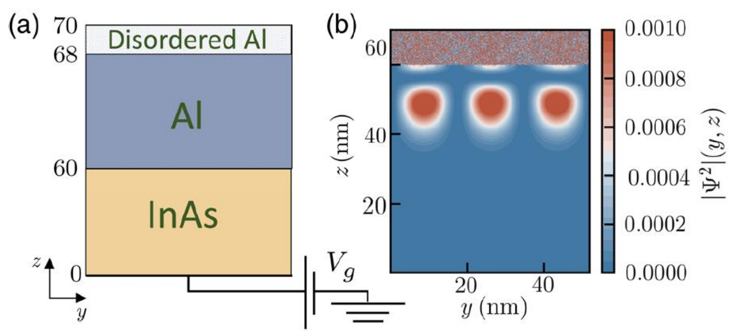


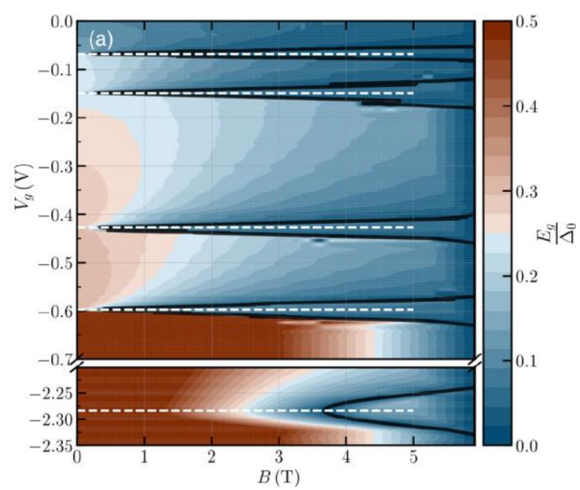
Joon Sue Lee et al. (2018)

Control via electric and magnetic fields



Antipov et al., (2018)





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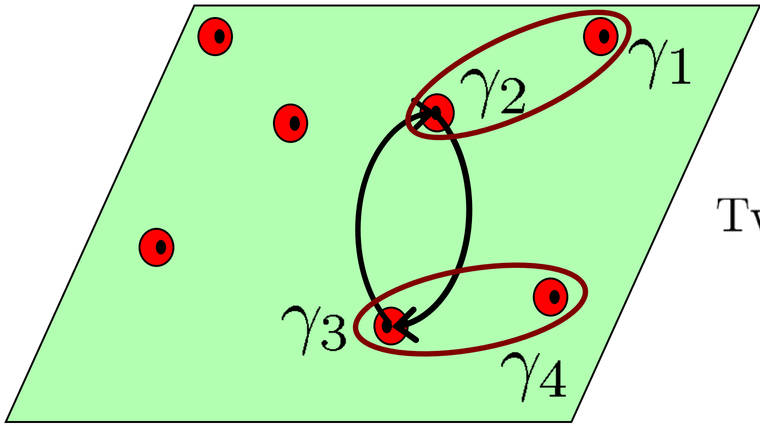
Topological quantum computation with Majoranas



Kitaev'03

Dirac fermion $\longrightarrow$ $c = \gamma_1 + i\gamma_2$

Two degenerate states $|0\rangle$ and $c^\dagger |0\rangle$ $\longrightarrow$ 1 qubit

$2N$ Majoranas $\rightarrow$ N qubits

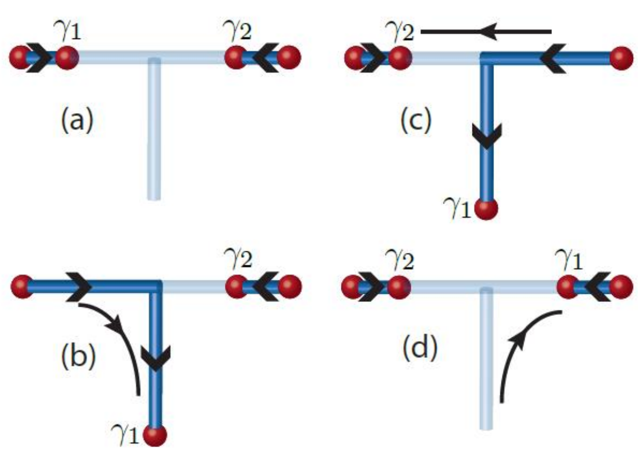
4 Majorana fermions $\longrightarrow$ 4 degenerate states

$$c_A = \gamma_1 + i\gamma_2$$
$$c_B = \gamma_3 + i\gamma_4$$

$\longrightarrow$

$|0_A, 0_B\rangle \quad |0_A, 1_B\rangle$

$|1_A, 0_B\rangle \quad |1_A, 1_B\rangle$

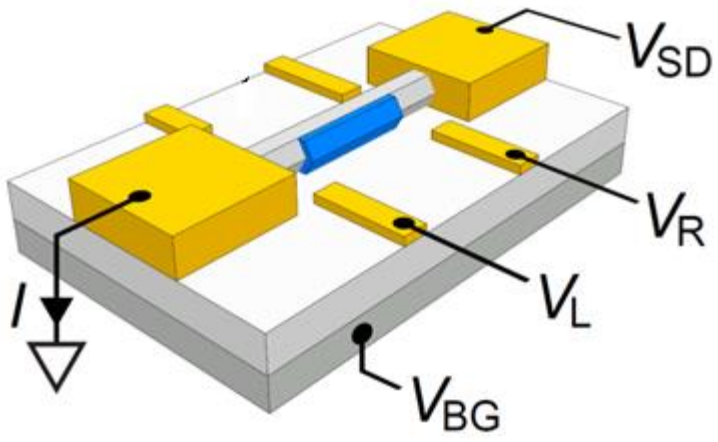


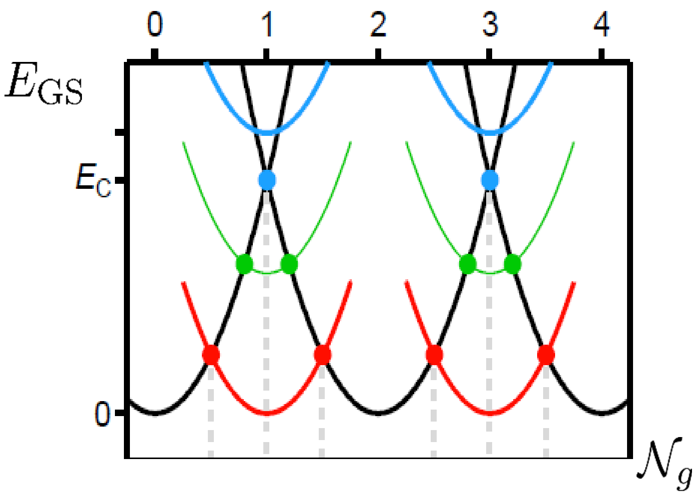
Alicea'11

Quantum state changes !

$$|0_A, 0_B\rangle \rightarrow \frac{1}{\sqrt{2}} \left(|0_A, 0_B\rangle + |1_A, 1_B\rangle \right)$$

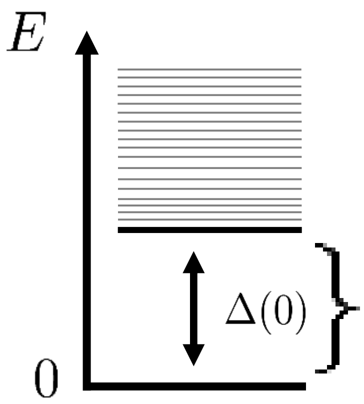
Majorana islands



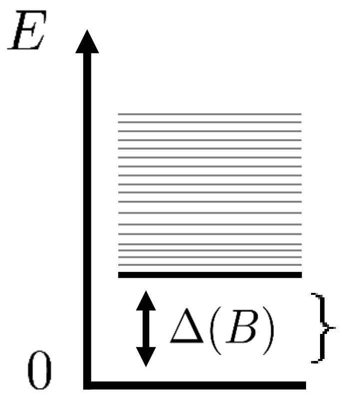


weak coupling analysis $g \to 0$: L. Fu (2010)

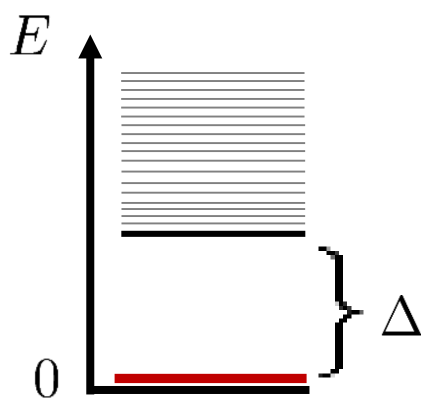
Non-topological phase
$(B = 0)$



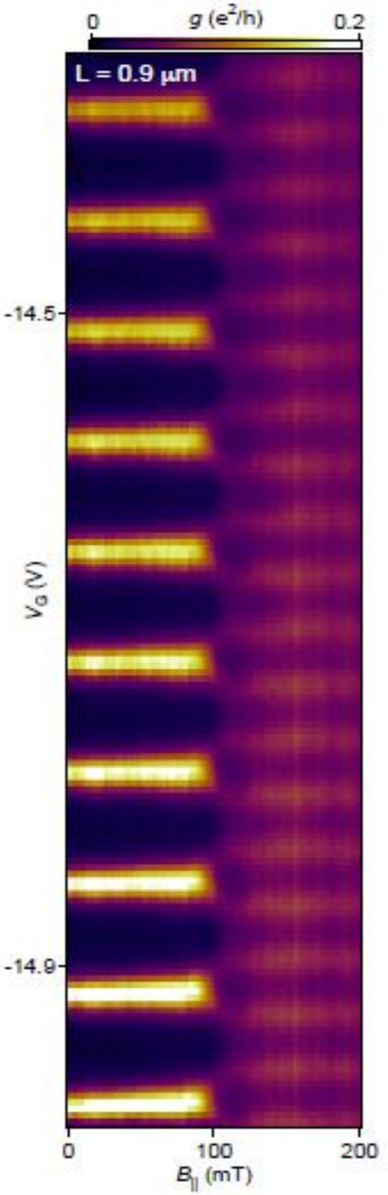
Non-topological phase
$(B_c > B > 0)$

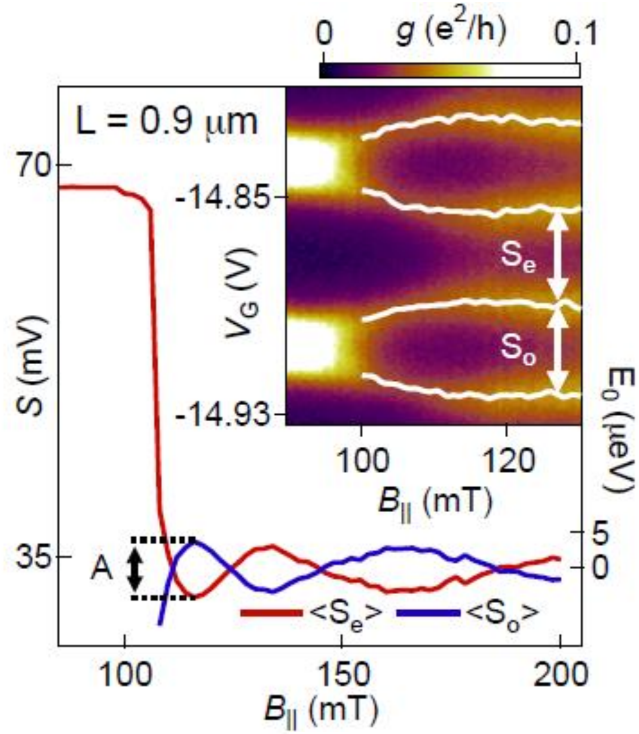


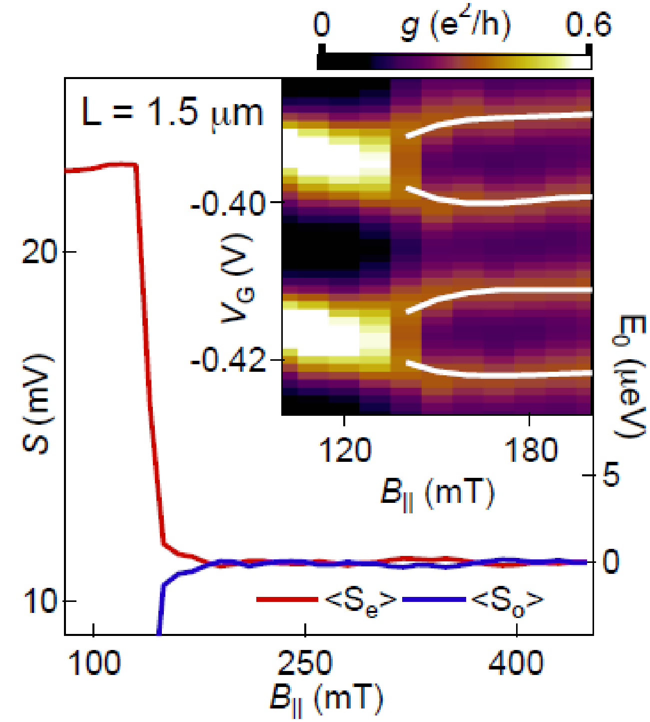
Topological phase
$(B > B_c)$

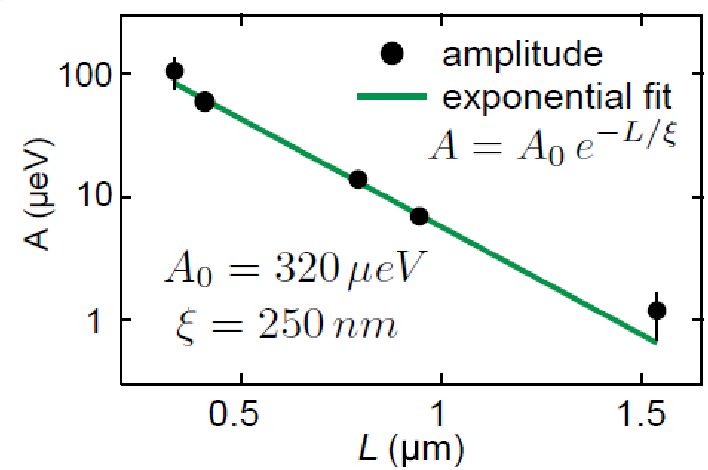


Energy splitting of Majorana zero modes









Albrecht et al., Nature, (2016)

Selected for a Viewpoint in *Physics*

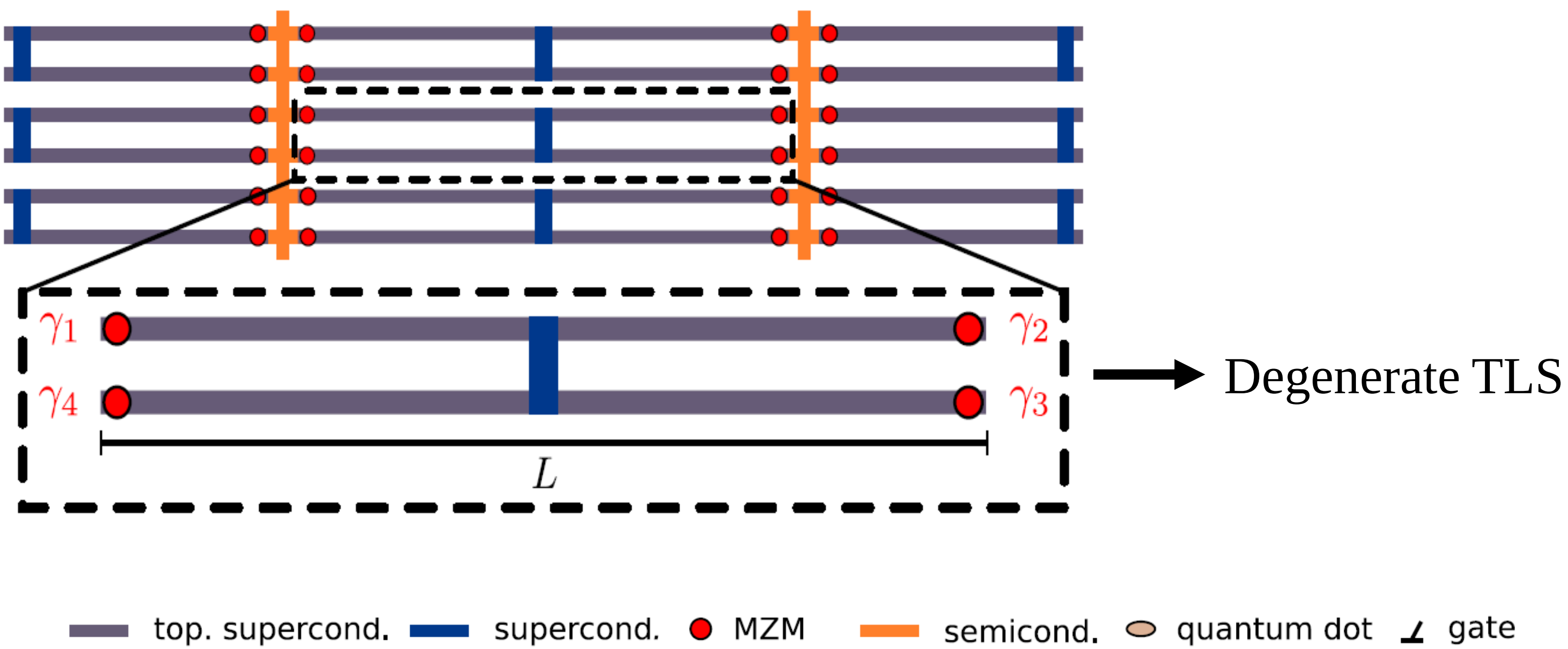
PHYSICAL REVIEW B **95**, 235305 (2017)

Scalable designs for quasiparticle-poisoning-protected topological quantum computation with Majorana zero modes

Torsten Karzig,[1] Christina Knapp,[2] Roman M. Lutchyn,[1] Parsa Bonderson,[1] Matthew B. Hastings,[1] Chetan Nayak,[1,2] Jason Alicea,[3,4] Karsten Flensberg,[5] Stephan Plugge,[5,6] Yuval Oreg,[7] Charles M. Marcus,[5] and Michael H. Freedman[1,8]

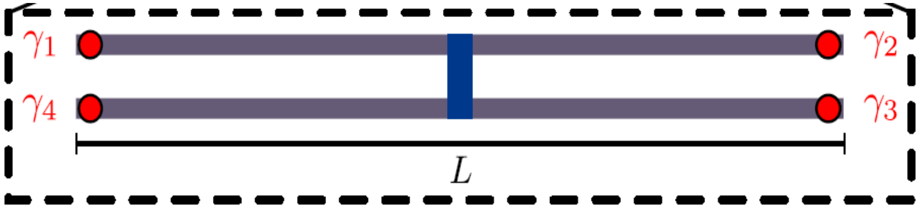
Majorana box qubits – SC islands with multiple Majorana pairs

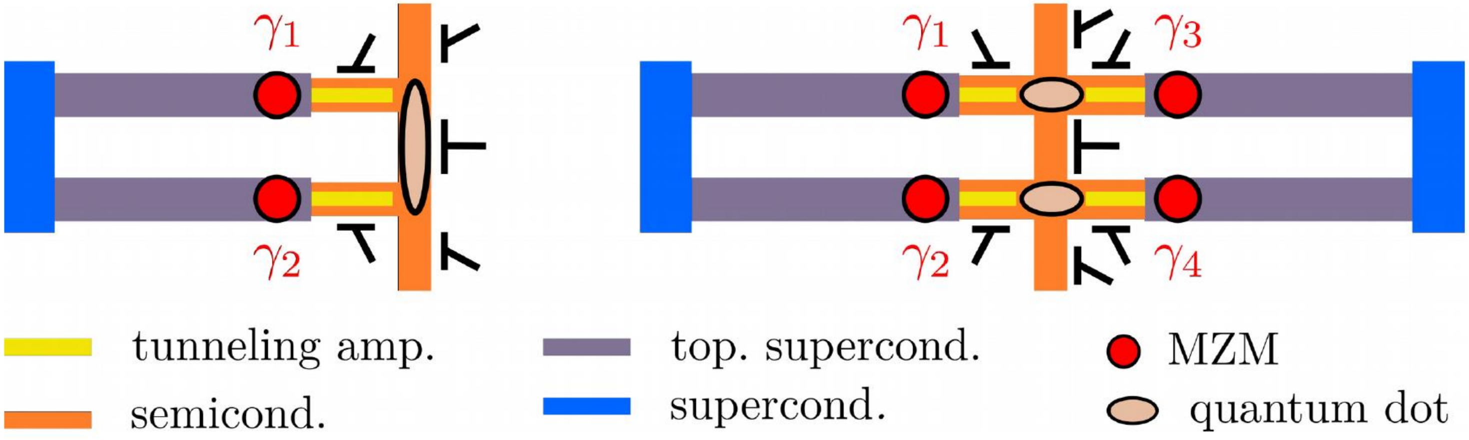
- Protected from external quasiparticle errors (i.e. fixed charge)
- Electrostatic control of Majorana couplings



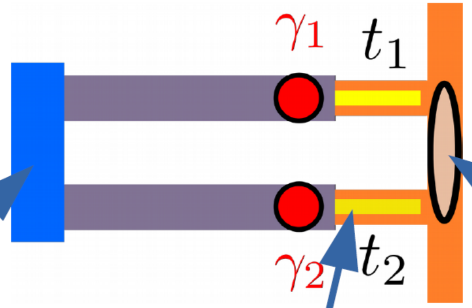
Measurement-centric approach

- TQC by measurements
- Parity measurements of
  - 2 MZMs (moving, braiding, single qubit gates)
  - 4 MZMs (entangling, multi qubit gates)





Parity-dependent energy-level shift



$$H = E_c\left(\hat{N}_S - N_g\right)^2 + \frac{e^{-i\phi/2}}{2i}\left(t_1 d^\dagger \gamma_1 + t_2 d^\dagger \gamma_2 - \text{h.c.}\right) + h d^\dagger d + \varepsilon_C\left(d^\dagger d - n_g\right)^2$$

hybridizing $|N_S = 0, n_d = 1\rangle$ with $|N_S = 1, n_d = 0\rangle$

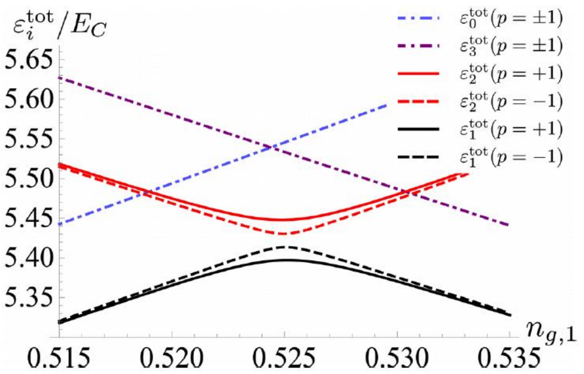
$$\delta\varepsilon^{\text{tot}} = -\frac{|t_1|^2 + |t_2|^2 - 2p\,\text{Im}(t_1 t_2^*)}{4\left[E_c(1-2N_g) - h - \varepsilon_C(1-2n_g)\right]} \qquad p = i\gamma_1\gamma_2$$

parity-dependent energy shift

What to measure

- Energy

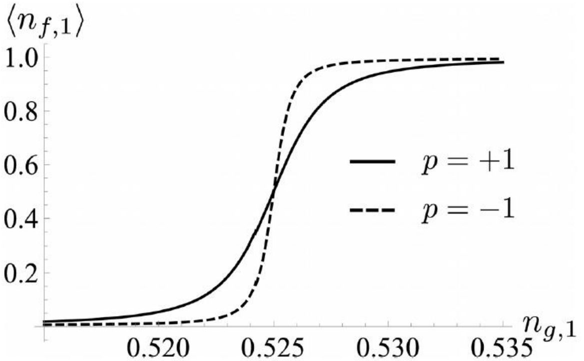
e.g. using frequency shift in transmission line resonator



$$\Delta\omega \sim \frac{g^2}{4\delta\omega^2}(\varepsilon_+ - \varepsilon_-)$$

- Charge

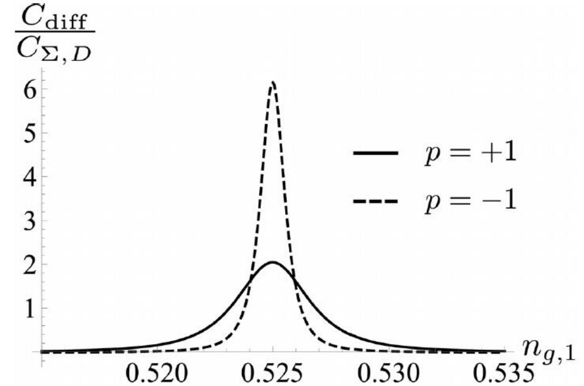
e.g. using reflectometry at nearby SET



$$\langle n_{d,1}\rangle \approx n_{g,1} - \frac{1}{2\varepsilon_C}\frac{\partial E_{GS}}{\partial n_{g,1}}$$

- Capacitance

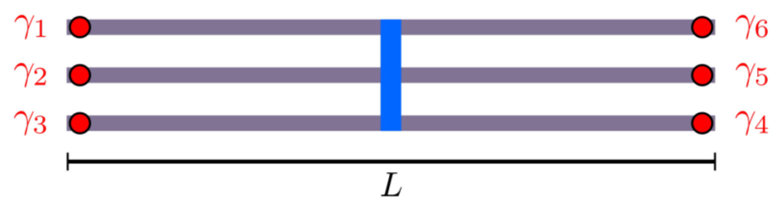
e.g. using reflectometry of rf signal in $V_{g,1}$



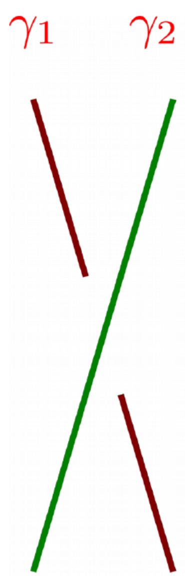
$$\frac{C_{\text{diff}}}{C_{\text{geom}}} = -\left(\frac{C_g}{C_{\text{geom}}}\right)^2 \frac{\partial(\langle n_{d,1}\rangle - n_{g,1})}{n_{g,1}}$$

Measurement-based Majorana braiding

Bonderson et al, PRL 101, 10501 (2008).



Measurement based exchange



$$R^{(12)} = \frac{1+\gamma_1\gamma_2}{\sqrt{2}}$$

Parity projector

$$\Pi_0^{(ij)} = \frac{1-i\gamma_i\gamma_j}{2}$$

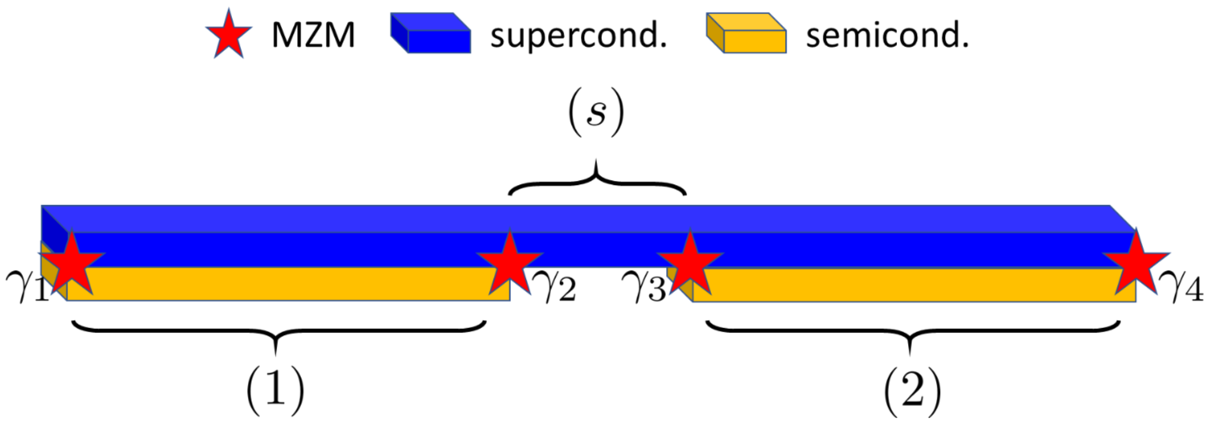
Measurement-based

exchange

$$\Pi_0^{(34)}\Pi_0^{(13)}\Pi_0^{(23)}\Pi_0^{(34)}$$

$$\propto R^{(12)}\Pi_0^{(34)}$$

Dephasing of Majorana-based qubits



In finite length wires, it may be difficult to satisfy $L/\xi \gg 1$ requirement. Therefore, we need to study effect of a finite Majorana splitting energy.

How does the environment couple to the qubit when Majorana modes have finite energy ?

Does charge noise couple to Majorana degrees of freedom ?

To address this question, one has to go beyond BCS mean-field theory!

Splitting energy within fluctuating SC model

spinless semiconductor — fluctuating superconductor — Cooper pair tunneling

$$S_{\text{eff}} = \int d\tau \frac{L}{2\pi} \left\{ \frac{K}{v} \left(\partial_\tau \theta_1 - i \frac{v}{K} k_F\right)^2 + \frac{K_\rho}{v_\rho} \left(\partial_\tau \theta_\rho - i \frac{v_\rho}{K_\rho} k_F^{(\rho)}\right)^2 - \frac{\Delta_P}{\xi} \cos\left(\sqrt{2}\theta_\rho - 2\theta_1\right) \right\}$$

Introduce coordinates $\theta_\pm = \frac{1}{\sqrt{2}}\theta_\rho \pm \theta_1$ and integrate out θ_+ mode:

$$S_{\text{eff}} = \frac{L}{2\pi} \int d\tau \left\{ \frac{1}{\tilde{v}} \left(\partial_\tau \theta_- + i\mu_-\right)^2 - \frac{\Delta_P}{\xi} \cos\left(2\theta_-\right) \right\}$$

$$\tilde{v} = v_\rho / 2K_\rho + v/K$$

$$\mu_- = \frac{v}{K} k_F - \frac{v_\rho}{\sqrt{2} K_\rho} k_F^{(\rho)}$$

θ_- instantons give exponential splitting

$$\varepsilon_{\text{hyb}} = A \cos\left(\frac{L\mu_-}{\tilde{v}}\right) \exp\left\{ -\frac{L}{\xi_{\text{MF}}} \frac{2\sqrt{2}}{\pi} \sqrt{\frac{\xi_{\text{MF}}}{\xi}} \right\}$$

couples to charge difference

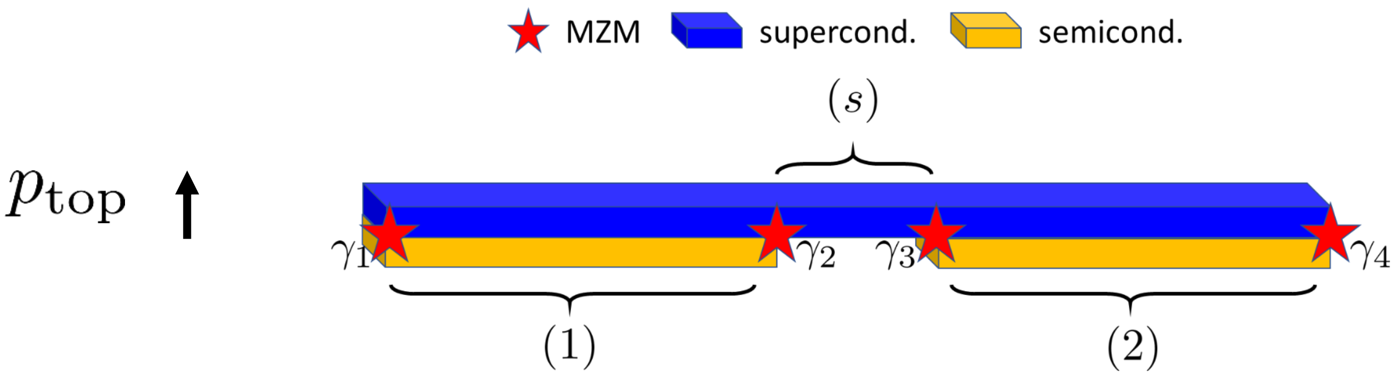
$$\Delta Q_{\text{MZM}} = \frac{\partial}{\partial \mu_-} \varepsilon_{\text{hyb}}$$

C. Knapp, T. Karzig, R. Lutchyn, C. Nayak, arXiv:1711.03968 (2017)

Dipole moment due to finite Majorana splitting energy

BCS mean-field theory predicts that final Majorana splitting induces total charge in the island.

However, mean-field theory breaks charge conservation and does not take into account superfluid contribution to screening. By using fluctuating SC model and then taking bulk SC limit, we find that total charge is decoupled from the Majorana degrees of freedom and instead finite splitting induces a dipole moment



$$p_{\text{top}} = \Delta Q_{\text{MZM}} \cdot d$$

distance between semiconductor and superconductor charge density

Bulk SC limit $K_\rho \to \infty$

$$\Delta Q_{\text{MZM}} = \frac{L}{\xi} \sin(k_F L) e^{-L/\xi}$$

Estimates for Majorana qubit dephasing times

charge noise: $T^*_{2,E} = c\frac{\xi}{L}e^{L/\xi}$

$c \sim 40\text{ns}$

finite temperature: $T^*_{2,\beta} = \tau_0 e^{\beta\Delta}$

$\tau_0 \sim 50\text{ns}$

L/ξ	5	10	20	30
$T^*_{2,E}$	600 ns	30 μs	100 ms	10 min
$T^*_{2,\beta}$	20s	20s	20s	20s
T^*_2	200 ns	30 μs	100 ms	20s

includes charge noise, excited quasiparticles, effect of phonons

Using the estimate from Albrecht et al. (2016) for the coherence length $\xi = 250\text{nm}$

we conclude that wires longer than $5\mu m$ to have a long coherence time $T^*_2 > 100\text{ms}$

C. Knapp, T. Karzig, R. Lutchyn, C. Nayak, arXiv:1711.03968 (2017)

Summary

- Majorana zero modes in proximitized nanowires
- Experimental progress
- Topological quantum computing with Majoranas
  - Majorana box qubits and scalable designs
  - Dephasing of Majorana-based qubits

T. Karzig et al., Phys. Rev. B 95, 235305 (2017)

C. Knapp et al., Phys. Rev. B 97, 125404 (2018)

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