

Finite bootstrap percolation

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A bootstrap percolation on a finite graph $G = (V, E)$ with (neighborhood) parameter ℓ is a nested sequence of subsets of V , $V_0 \subset V_1 \subset \dots$, such that for $t > 0$ a vertex $v \in V$ belongs to V_t iff either $v \in V_{t-1}$ or v has at least ℓ neighbours in V_{t-1} . The set V_t is the set of sites occupied at time t . Note that the entire percolation is determined by V_0 , the set of sites occupied at time 0. If eventually every site is occupied then we say that the starting set V_0 percolates. Choosing the vertices of V_0 at random, with probability p , we get a random bootstrap percolation with parameter ℓ and probability p . One of the main objects of study is the critical probability p_c , below which percolation is unlikely and above which it is very likely. In the talk I shall review a number of the known results and shall report on my work in progress with József Balogh.