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FLUID AND DRIFT-KINETIC THEORY FOR MACROSCOPIC PLASMA DYNAMICS AT LOW COLLISIONALITY*

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INTRODUCTION

- REDUCED DESCRIPTIONS ARE CENTRAL TO PLASMA THEORY:

  LOWER DIMENSIONALITY

  ELIMINATION OF UNNECESSARY SCALES

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- THE CONVENTIONAL, SHORT-MEAN-FREE-PATH FLUID CLOSURE DOES NOT APPLY TO THE LOW-COLLISIONALITY REGIMES OF MAIN INTEREST IN SPACE AND MAGNETIC FUSION PLASMAS

- **IN A MAGNETIZED PLASMA, A VALID FLUID DESCRIPTION OF LOW-FREQUENCY, MACROSCOPIC PHYSICS ($\omega \ll \Omega_{c\iota},\ L \gg \rho_\iota$) PERPENDICULAR TO THE MAGNETIC FIELD IS POSSIBLE REGARDLESS OF COLLISIONALITY**

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- A CONSISTENTLY CLOSED LOW-COLLISIONALITY THEORY CAN BE BASED ON A HYBRID FLUID AND KINETIC SYSTEM, WITH DRIFT-KINETIC EQUATIONS FOR THE GYRO-AVERAGED DISTRIBUTION FUNCTIONS TO GET PARALLEL CLOSURE. SUCH A THEORY WILL BE PRESENTED HERE, EMPHASIZING:

    RIGOROUS TREATMENT OF HIGH-ORDER FINITE LARMOR RADIUS EFFECTS

    PRECISE CONSISTENCY BETWEEN FLUID AND DRIFT-KINETIC SIDES

    EXACT CONSERVATION LAWS

MAXWELL-BOLTZMANN SYSTEM FOR WEAKLY COUPLED PLASMAS

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} , \qquad \nabla \cdot \mathbf{B} = 0 , \qquad \nabla \times \mathbf{B} = \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} , \qquad \nabla \cdot \mathbf{E} = c^2 \rho$$

$$\frac{\partial f_s(\mathbf{v}, \mathbf{x}, t)}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{e_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \sum_{s'} C_{ss'}[f_s, f_{s'}]$$

$$n_s(\mathbf{x}, t) = \int d^3\mathbf{v} \; f_s(\mathbf{v}, \mathbf{x}, t) , \qquad \mathbf{u}_s(\mathbf{x}, t) = \frac{1}{n_s(\mathbf{x}, t)} \int d^3\mathbf{v} \; \mathbf{v} \; f_s(\mathbf{v}, \mathbf{x}, t)$$

$$\rho = \sum_s e_s n_s , \qquad \mathbf{j} = \sum_s e_s n_s \mathbf{u}_s$$

MAXWELL-BOLTZMANN SYSTEM FOR WEAKLY COUPLED PLASMAS

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$$\frac{\partial f_s(\mathbf{v},\mathbf{x},t)}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{e_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \sum_{s'} C_{ss'}[f_s, f_{s'}]$$

$$n_s(\mathbf{x},t) = \int d^3\mathbf{v}\ f_s(\mathbf{v},\mathbf{x},t) , \qquad \mathbf{u}_s(\mathbf{x},t) = \frac{1}{n_s(\mathbf{x},t)} \int d^3\mathbf{v}\ \mathbf{v}\ f_s(\mathbf{v},\mathbf{x},t)$$

$$\rho = \sum_s e_s n_s , \qquad \mathbf{j} = \sum_s e_s n_s \mathbf{u}_s$$

IN WHAT FOLLOWS, ASSUME SINGLE ION SPECIES OF UNIT CHARGE:

$$s \in \{\iota, e\} , \qquad e_\iota = -e_e = e$$

LOW-FREQUENCY SYSTEM FOR QUASINEUTRAL PLASMAS ($\omega \ll \omega_p,\ L \gg \lambda_D$)

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}\,, \qquad \nabla \cdot \mathbf{B} = 0\,, \qquad \nabla \times \mathbf{B} = \mathbf{j}$$

$$n_e = n_\imath = n\,, \qquad \mathbf{u}_e = \mathbf{u}_\imath - \frac{\mathbf{j}}{en}\,, \qquad \frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}_s) = 0$$

$$(m_\imath + m_e)n\left[\frac{\partial \mathbf{u}_\imath}{\partial t} + (\mathbf{u}_\imath \cdot \nabla)\mathbf{u}_\imath\right] = \mathbf{j} \times \mathbf{B} - \nabla \cdot (\mathsf{P}_\imath + \mathsf{P}_e) - \frac{m_e}{e}\left[\nabla \times (\nabla \times \mathbf{E}) - \nabla \cdot \left(\mathbf{u}_\imath \mathbf{j} + \mathbf{j}\mathbf{u}_\imath - \frac{1}{en}\mathbf{j}\mathbf{j}\right)\right]$$

$$\mathbf{E} + \frac{m_\imath m_e}{e^2 n (m_\imath + m_e)}\nabla \times (\nabla \times \mathbf{E}) = -\mathbf{u}_\imath \times \mathbf{B} + \frac{m_\imath}{en(m_\imath + m_e)}(\mathbf{j} \times \mathbf{B} - \nabla \cdot \mathsf{P}_e) +$$
$$+ \frac{m_e}{en(m_\imath + m_e)}\nabla \cdot \mathsf{P}_\imath + \frac{m_\imath m_e}{e^2 n (m_\imath + m_e)}\nabla \cdot \left(\mathbf{u}_\imath \mathbf{j} + \mathbf{j}\mathbf{u}_\imath - \frac{1}{en}\mathbf{j}\mathbf{j}\right) + \frac{1}{en}\mathbf{F}_e^{coll}$$

$$\frac{\partial f_s(\mathbf{v}, \mathbf{x}, t)}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{e_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \sum_{s'} C_{ss'}[f_s, f_{s'}]$$

$$\mathsf{P}_s = m_s \int d^3\mathbf{v}\ (\mathbf{v} - \mathbf{u}_s)(\mathbf{v} - \mathbf{u}_s)\ f_s\,, \qquad \mathbf{F}_e^{coll} = m_e \int d^3\mathbf{v}\ (\mathbf{v} - \mathbf{u}_e)\ C_{e\imath}[f_e, f_\imath]$$

THE LOW-FREQUENCY QUASINEUTRAL SYSTEM, WITH FOKKER-PLANCK-LANDAU OPERATORS FOR BINARY COULOMB COLLISIONS,

$$C_{ss'}[f_s, f_{s'}] = -\ \frac{c^4 e_s^2 e_{s'}^2 \ln \Lambda_{ss'}}{8\pi m_s}\ \frac{\partial}{\partial \mathbf{v}} \cdot \int d^3\mathbf{w}\ \mathsf{U}(\mathbf{v},\mathbf{w}) \cdot \left[\frac{f_s(\mathbf{v},\mathbf{x},t)}{m_{s'}}\ \frac{\partial f_{s'}(\mathbf{w},\mathbf{x},t)}{\partial \mathbf{w}}\ -\ \frac{f_{s'}(\mathbf{w},\mathbf{x},t)}{m_s}\ \frac{\partial f_s(\mathbf{v},\mathbf{x},t)}{\partial \mathbf{v}} \right]$$

$$\mathsf{U}(\mathbf{v},\mathbf{w})\ =\ \frac{|\mathbf{v}-\mathbf{w}|^2 \mathsf{I}\ -\ (\mathbf{v}-\mathbf{w})(\mathbf{v}-\mathbf{w})}{|\mathbf{v}-\mathbf{w}|^3}\ ,$$

CAN BE CONSIDERED THE "STANDARD MODEL" AT THE ROOT OF A LARGE PART OF THEORETICAL PLASMA RESEARCH

LOW-FREQUENCY, LARGE-SPATIAL-SCALE SYSTEM FOR MAGNETIZED PLASMAS

ASYMPTOTICS BASED ON THE EXPANSION PARAMETER $\delta \sim \rho_\iota / L \ll 1$

MASS AND TEMPERATURE RATIOS ORDERED AS $m_e/m_\iota \sim \delta^2, \quad T_e/T_\iota \sim 1$

TWO DYNAMICAL ORDERINGS TO BE CONSIDERED:

1. **COLLISIONLESS FAST DYNAMICS, FAR FROM MAXWELLIAN**

   $$\nu_\iota = \nu_e = 0, \qquad \omega \sim \delta \Omega_{c\iota}, \qquad u_\iota \sim u_e \sim v_{th\iota}$$

   First-order accuracy in δ. Apt for space applications.

2. **LOW-COLLISIONALITY SLOW DYNAMICS, NEAR MAXWELLIAN**

   $$\nu_\iota \sim \delta \nu_e \sim \delta^2 \Omega_{c\iota}, \quad \omega \sim \delta^2 \Omega_{c\iota}, \quad u_\iota \sim u_e \sim \delta v_{th\iota}, \quad f_\iota - f_{M\iota} \sim \delta f_{M\iota}, \quad f_e - f_{Me} \sim \delta^2 f_{Me}$$

   Second-order accuracy in δ. Apt for application to confined magnetic fusion experiments.

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}\ , \qquad \nabla \cdot \mathbf{B} = 0\ , \qquad \nabla \times \mathbf{B} = \mathbf{j}$$

$$n_e = n_\iota = n\ , \qquad \mathbf{u}_e = \mathbf{u}_\iota - \frac{\mathbf{j}}{en}\ , \qquad \frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}_s) = 0$$

$$m_\iota n \left[\frac{\partial \mathbf{u}_\iota}{\partial t} + (\mathbf{u}_\iota \cdot \nabla)\mathbf{u}_\iota\right] = \mathbf{j} \times \mathbf{B} - \nabla \cdot (\mathsf{P}_\iota + \mathsf{P}_e)$$

$$\mathbf{E} = -\mathbf{u}_\iota \times \mathbf{B} + \frac{1}{en}(\mathbf{j} \times \mathbf{B} - \nabla \cdot \mathsf{P}_e) + \frac{1}{en}\mathbf{F}_e^{coll}$$

$$\mathsf{P}_e = nT_e\mathsf{I} + (p_{e\|} - p_{e\perp})(\mathbf{bb} - \mathsf{I}/3), \qquad \mathsf{P}_\iota = nT_\iota\mathsf{I} + (p_{\iota\|} - p_{\iota\perp})(\mathbf{bb} - \mathsf{I}/3) + \mathsf{P}_\iota^{GV}$$

$$\frac{3n}{2}\left(\frac{\partial T_e}{\partial t} + \mathbf{u}_e \cdot \nabla T_e\right) = -\mathsf{P}_e : (\nabla \mathbf{u}_e) - \nabla \cdot \mathbf{q}_e + \frac{2\nu_e n m_e}{(2\pi)^{1/2} m_\iota}(T_\iota - T_e) + \frac{\mathbf{j} \cdot \mathbf{F}_e^{coll}}{en}$$

$$\frac{3n}{2}\left(\frac{\partial T_\iota}{\partial t} + \mathbf{u}_\iota \cdot \nabla T_\iota\right) = -\mathsf{P}_\iota : (\nabla \mathbf{u}_\iota) - \nabla \cdot \mathbf{q}_\iota + \frac{2\nu_e n m_e}{(2\pi)^{1/2} m_\iota}(T_e - T_\iota)$$

THE LOW-FREQUENCY, LARGE-SPATIAL-SCALE, MAGNETIZED PLASMA SYSTEM CONSERVES EXACTLY PARTICLE NUMBER, MOMENTUM AND ENERGY:

$$\frac{\partial n}{\partial t} \; + \; \nabla \cdot (n\mathbf{u}_s) \; = \; 0$$

$$\frac{\partial}{\partial t}\Big(nm_\iota \mathbf{u}_\iota\Big) \; + \; \nabla \cdot \left(nm_\iota \mathbf{u}_\iota \mathbf{u}_\iota + \mathsf{P}_e + \mathsf{P}_\iota + \frac{1}{2}B^2\mathsf{I} - \mathbf{BB}\right) \; = \; 0$$

$$\frac{\partial}{\partial t}\left[\frac{1}{2}nm_\iota u_\iota^2 + \frac{3}{2}n(T_e + T_\iota) + \frac{1}{2}B^2\right] \; +$$

$$+ \; \nabla \cdot \left[\frac{1}{2}nm_\iota u_\iota^2 \mathbf{u}_\iota + \frac{3}{2}n(T_e\mathbf{u}_e + T_\iota\mathbf{u}_\iota) + \mathsf{P}_e \cdot \mathbf{u}_e + \mathsf{P}_\iota \cdot \mathbf{u}_\iota + \mathbf{q}_e + \mathbf{q}_\iota + \mathbf{E} \times \mathbf{B}\right] = 0.$$

1. FLUID-KINETIC CLOSURE OF THE LOW-FREQUENCY, LARGE-SPATIAL-SCALE, MAGNETIZED PLASMA SYSTEM FOR COLLISIONLESS FAST DYNAMICS, FAR FROM MAXWELLIAN

$$\nu_\imath = \nu_e = 0, \qquad \omega \sim \delta\Omega_{c\imath}, \qquad u_\imath \sim u_e \sim v_{th\imath}$$

first-order accuracy in δ

THE NON-GYROTROPIC (PERPENDICULAR) CLOSURES ARE DERIVED ALGEBRAICALLY AFTER δ-EXPANSION OF THE FLUID MOMENTS OF THE FULL KINETIC EQUATION:

$$\mathbf{P}^{GV}_{\iota,jk} \;=\; \frac{1}{4}\,\epsilon_{jlm} b_l\; \mathbf{K}_{\iota,mn}\;(\delta_{nk} + 3 b_n b_k)\;+\;(j \leftrightarrow k)$$

$$\mathbf{K}_{\iota,jk} \;=\; \frac{m_\iota}{eB}\left[\, p_{\iota\perp}\,\frac{\partial u_{\iota,k}}{\partial x_j} \;+\; 2(p_{\iota\|} - p_{\iota\perp})\; b_j b_l\,\frac{u_{\iota,k}}{\partial x_l} \;+\; \frac{\partial(q_{\iota T\|} b_k)}{\partial x_j} \;+\; (2 q_{\iota B\|} - 3 q_{\iota T\|})\; b_j \kappa_k \right] \;+\;(j \leftrightarrow k)$$

$$\mathbf{q}_{\iota\perp} \;=\; \frac{\mathbf{b}}{eB}\times\Big[\; p_{\iota\perp}\nabla\left(\frac{p_{\iota\|} + 4 p_{\iota\perp}}{2n}\right) \;+\; \frac{p_{\iota\|}(p_{\iota\|} - p_{\iota\perp})}{n}\,\boldsymbol{\kappa} \;+$$

$$+\;2 m_\iota (q_{\iota B\|} + 2 q_{\iota T\|})\;(\mathbf{b}\cdot\nabla)\mathbf{u}_\iota \;+\; m_\iota q_{\iota T\|}\;\mathbf{b}\times(\nabla\times\mathbf{u}_\iota) \;+\; \nabla \tilde{r}_{\iota\perp} \;+\; (\tilde{r}_{\iota\|} - \tilde{r}_{\iota\perp})\;\boldsymbol{\kappa}\;\Big]$$

$$\mathbf{q}_{e\perp} \;=\; -\,\frac{\mathbf{b}}{eB}\times\Big[\; p_{e\perp}\nabla\left(\frac{p_{e\|} + 4 p_{e\perp}}{2n}\right) \;+\; \frac{p_{e\|}(p_{e\|} - p_{e\perp})}{n}\,\boldsymbol{\kappa} \;+\; \nabla \tilde{r}_{e\perp} \;+\; (\tilde{r}_{e\|} - \tilde{r}_{e\perp})\;\boldsymbol{\kappa}\;\Big]$$

THE GYROTROPIC (PARALLEL) CLOSURES ARE THE FOLLOWING MOMENTS OF THE GYROPHASE-AVERAGED DISTRIBUTION FUNCTIONS IN THE REFERENCE FRAMES OF THE MEAN FLOWS, TO BE SOLVED FOR KINETICALLY:

$$p_{s\parallel} = 2\pi m_s \int dv'_\parallel \; dv'_\perp v'_\perp \; v'^2_\parallel \; \bar{f}_s \; , \qquad p_{s\perp} = \pi m_s \int dv'_\parallel \; dv'_\perp v'_\perp \; v'^2_\perp \; \bar{f}_s$$

$$q_{s\parallel} = \pi m_s \int dv'_\parallel \; dv'_\perp v'_\perp \; v'_\parallel \; v'^2 \; \bar{f}_s$$

$$q_{\iota B\parallel} = \pi m_\iota \int dv'_\parallel \; dv'_\perp v'_\perp \; v'^3_\parallel \; \bar{f}_\iota \; , \qquad q_{\iota T\parallel} = q_{\iota\parallel} - q_{\iota B\parallel}$$

$$\tilde{r}_{s\parallel} = \pi m_s^2 \int dv'_\parallel \; dv'_\perp v'_\perp \; v'^2_\parallel \; v'^2 \; (\bar{f}_s - f_{2Ms}) \; , \qquad \tilde{r}_{s\perp} = \frac{\pi m_s^2}{2} \int dv'_\parallel \; dv'_\perp v'_\perp \; v'^2_\perp \; v'^2 \; (\bar{f}_s - f_{2Ms})$$

where

$$\mathbf{v}' = \mathbf{v} - \mathbf{u}_s(\mathbf{x},t) = v'_\parallel \; \mathbf{b}(\mathbf{x},t) + v'_\perp \; [\cos\alpha \; \mathbf{e}_1(\mathbf{x},t) + \sin\alpha \; \mathbf{e}_2(\mathbf{x},t)]$$

$$\bar{f}_s(v'_\parallel, v'_\perp, \mathbf{x}, t) = (2\pi)^{-1} \oint d\alpha \; f_s(v'_\parallel, v'_\perp, \alpha, \mathbf{x}, t)$$

$$f_{2Ms}(v'_\parallel, v'_\perp, \mathbf{x}, t) = \left(\frac{m_s}{2\pi}\right)^{3/2} \frac{n^{5/2}}{p_{s\parallel}^{1/2} p_{s\perp}} \; \exp\left[-\frac{m_s n}{2}\left(\frac{v'^2_\parallel}{p_{s\parallel}} + \frac{v'^2_\perp}{p_{s\perp}}\right)\right]$$

THE DRIFT-KINETIC EQUATIONS FOR THE GYROPHASE-AVERAGED DISTRIBUTION FUNCTIONS IN THE REFERENCE FRAMES OF THE MEAN FLOWS, $\bar{f}_s(v'_\parallel, v'_\perp, \mathbf{x}, t)$, ARE:

$$\frac{d_s \bar{f}_s}{dt} \equiv \frac{\partial \bar{f}_s}{\partial t} + \dot{\mathbf{x}}_s \cdot \frac{\partial \bar{f}_s}{\partial \mathbf{x}} + \dot{v}'_{\parallel s} \frac{\partial \bar{f}_s}{\partial v'_\parallel} + \dot{v}'_{\perp s} \frac{\partial \bar{f}_s}{\partial v'_\perp} = 0$$

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with the ion coefficient functions

$$\dot{\mathbf{x}}_\iota = \mathbf{u}_\iota + v'_\parallel \mathbf{b} + \frac{v'^2_\perp}{2} \nabla \times \left(\frac{\mathbf{b}}{\Omega_{c\iota}}\right) - \frac{\mathbf{b}}{\Omega_{c\iota}} \times \left[\frac{\nabla \cdot \mathsf{P}_\iota}{m_\iota n} - 2v'_\parallel (\mathbf{b} \cdot \nabla)\mathbf{u}_\iota - \left(v'^2_\parallel - \frac{v'^2_\perp}{2}\right)\boldsymbol{\kappa}\right]$$

$$\dot{v}'_{\parallel\iota} = \frac{\mathbf{b} \cdot (\nabla \cdot \mathsf{P}_\iota)}{m_\iota n} - v'_\parallel \mathbf{b} \cdot [(\mathbf{b} \cdot \nabla)\mathbf{u}_\iota] - \frac{v'^2_\perp}{2} \mathbf{b} \cdot \nabla \ln B + \frac{v'^2_\perp}{2} \nabla \cdot \left[\frac{\mathbf{b}}{\Omega_{c\iota}} \times [(\nabla \times \mathbf{u}_\iota) \times \mathbf{b} + v'_\parallel \boldsymbol{\kappa}]\right] +$$
$$+ \left[\frac{\mathbf{b}}{\Omega_{c\iota}} \times [(\nabla \times \mathbf{u}_\iota) \times \mathbf{b} + v'_\parallel \boldsymbol{\kappa}]\right] \cdot \left[\frac{\nabla \cdot \mathsf{P}_\iota}{m_\iota n} - 2v'_\parallel (\mathbf{b} \cdot \nabla)\mathbf{u}_\iota - v'^2_\parallel \boldsymbol{\kappa}\right] - \frac{v'^2_\perp}{4\Omega_{c\iota}} \mathsf{M}_\times : (\nabla \mathbf{u}_\iota)$$

$$\dot{v}'_{\perp\iota} = \frac{v'_\perp}{2} \Big\{ \mathbf{b} \cdot [(\mathbf{b} \cdot \nabla)\mathbf{u}_\iota] - \nabla \cdot \mathbf{u}_\iota + v'_\parallel \mathbf{b} \cdot \nabla \ln B + \nabla \cdot \left[\frac{\mathbf{b}}{\Omega_{c\iota}} \times \left(\frac{\nabla \cdot \mathsf{P}_\iota}{m_\iota n} - 2v'_\parallel (\mathbf{b} \cdot \nabla)\mathbf{u}_\iota - v'^2_\parallel \boldsymbol{\kappa}\right)\right] +$$
$$+ 2\left[\frac{\mathbf{b}}{\Omega_{c\iota}} \times [(\nabla \times \mathbf{u}_\iota) \times \mathbf{b}]\right] \cdot \left[(\mathbf{b} \cdot \nabla)\mathbf{u}_\iota + v'_\parallel \boldsymbol{\kappa}\right] - \left(\frac{\mathbf{b}}{\Omega_{c\iota}} \times \boldsymbol{\kappa}\right) \cdot \left[\frac{\nabla \cdot \mathsf{P}_\iota}{m_\iota n} - 4v'_\parallel (\mathbf{b} \cdot \nabla)\mathbf{u}_\iota\right] \Big\}$$

and the electron coefficient functions

$$\dot{\mathbf{x}}_e \;=\; \mathbf{u}_e + v'_\parallel \mathbf{b} \;+\; \frac{v'^2_\perp}{2}\nabla\times\Big(\frac{\mathbf{b}}{\Omega_{ce}}\Big) \;-\; \frac{\mathbf{b}}{\Omega_{ce}}\times\Big[\frac{\nabla\cdot\mathsf{P}_e}{m_e n} - \Big(v'^2_\parallel - \frac{v'^2_\perp}{2}\Big)\boldsymbol{\kappa}\Big]$$

$$\dot{v}'_{\parallel e} \;=\; \frac{\mathbf{b}\cdot(\nabla\cdot\mathsf{P}_e)}{m_e n} \;-\; v'_\parallel \mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}_e] \;-\; \frac{v'^2_\perp}{2}\mathbf{b}\cdot\nabla\ln B \;+\; \frac{v'_\parallel v'^2_\perp}{2}\nabla\cdot\left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{ce}}\right) \;+\; v'_\parallel\frac{(\mathbf{b}\times\boldsymbol{\kappa})\cdot(\nabla\cdot\mathsf{P}_e)}{m_e n\Omega_{ce}}$$

$$\dot{v}'_{\perp e} \;=\; \frac{v'_\perp}{2}\left\{\mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}_e] - \nabla\cdot\mathbf{u}_e + v'_\parallel\mathbf{b}\cdot\nabla\ln B + \nabla\cdot\left[\frac{\mathbf{b}}{\Omega_{ce}}\times\left(\frac{\nabla\cdot\mathsf{P}_e}{m_e n} - v'^2_\parallel\boldsymbol{\kappa}\right)\right] - \frac{(\mathbf{b}\times\boldsymbol{\kappa})\cdot(\nabla\cdot\mathsf{P}_e)}{m_e n\Omega_{ce}}\right\}$$

and the electron coefficient functions

$$\dot{\mathbf{x}}_e \;=\; \mathbf{u}_e + v'_\parallel \mathbf{b} \;+\; \frac{v'^2_\perp}{2}\nabla\times\Big(\frac{\mathbf{b}}{\Omega_{ce}}\Big) \;-\; \frac{\mathbf{b}}{\Omega_{ce}}\times\left[\frac{\nabla\cdot\mathsf{P}_e}{m_e n} - \Big(v'^2_\parallel - \frac{v'^2_\perp}{2}\Big)\boldsymbol{\kappa}\right]$$

$$\dot{v}'_{\parallel e} \;=\; \frac{\mathbf{b}\cdot(\nabla\cdot\mathsf{P}_e)}{m_e n} \;-\; v'_\parallel \mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}_e] \;-\; \frac{v'^2_\perp}{2}\mathbf{b}\cdot\nabla\ln B \;+\; \frac{v'_\parallel v'^2_\perp}{2}\nabla\cdot\left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{ce}}\right) \;+\; v'_\parallel\frac{(\mathbf{b}\times\boldsymbol{\kappa})\cdot(\nabla\cdot\mathsf{P}_e)}{m_e n\Omega_{ce}}$$

$$\dot{v}'_{\perp e} \;=\; \frac{v'_\perp}{2}\left\{\mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}_e] - \nabla\cdot\mathbf{u}_e + v'_\parallel\mathbf{b}\cdot\nabla\ln B + \nabla\cdot\left[\frac{\mathbf{b}}{\Omega_{ce}}\times\left(\frac{\nabla\cdot\mathsf{P}_e}{m_e n} - v'^2_\parallel\boldsymbol{\kappa}\right)\right] - \frac{(\mathbf{b}\times\boldsymbol{\kappa})\cdot(\nabla\cdot\mathsf{P}_e)}{m_e n\Omega_{ce}}\right\}$$

THESE FULFILL THE PHASE-SPACE VOLUME CONSERVATION CONDITIONS

$$\frac{\partial}{\partial\mathbf{x}}\cdot\dot{\mathbf{x}}_s \;+\; \frac{\partial\dot{v}'_{\parallel s}}{\partial v'_\parallel} \;+\; \frac{1}{v'_\perp}\frac{\partial(v'_\perp\dot{v}'_{\perp s})}{\partial v'_\perp} \;=\; 0$$

so that

$$\frac{d_s\bar{f}_s}{dt} \;=\; \frac{\partial\bar{f}_s}{\partial t} \;+\; \frac{\partial}{\partial\mathbf{x}}\cdot(\bar{f}_s\dot{\mathbf{x}}_s) \;+\; \frac{\partial}{\partial v'_\parallel}\left(\bar{f}_s\dot{v}'_{\parallel s}\right) \;+\; \frac{1}{v'_\perp}\;\frac{\partial}{\partial v'_\perp}\left(v'_\perp\bar{f}_s\dot{v}'_{\perp s}\right)$$

THE FIRST THREE MOMENTS OF THE GYROPHASE-AVERAGED DISTRIBUTION FUNCTIONS ARE:

$$2\pi \int dv'_{\|} \; dv'_{\perp} v'_{\perp} \; \bar{f}_s(v'_{\|}, v'_{\perp}, \mathbf{x}, t) \;=\; n$$

$$2\pi \int dv'_{\|} \; dv'_{\perp} v'_{\perp} \; v'_{\|} \; \bar{f}_s(v'_{\|}, v'_{\perp}, \mathbf{x}, t) \;=\; 0$$

$$2\pi m_s \int dv'_{\|} \; dv'_{\perp} v'_{\perp} \; v'^2 \; \bar{f}_s(v'_{\|}, v'_{\perp}, \mathbf{x}, t) \;=\; p_{s\|} + 2p_{s\perp} \;\equiv\; 3p_s \;\equiv\; 3nT_s$$

THE FIRST THREE MOMENTS OF THE GYROPHASE-AVERAGED DISTRIBUTION FUNCTIONS ARE:

$$2\pi \int dv'_{\|} \ dv'_{\perp} v'_{\perp} \ \bar{f}_s(v'_{\|}, v'_{\perp}, \mathbf{x}, t) \ = \ n$$

$$2\pi \int dv'_{\|} \ dv'_{\perp} v'_{\perp} \ v'_{\|} \ \bar{f}_s(v'_{\|}, v'_{\perp}, \mathbf{x}, t) \ = \ 0$$

$$2\pi m_s \int dv'_{\|} \ dv'_{\perp} v'_{\perp} \ v'^2 \ \bar{f}_s(v'_{\|}, v'_{\perp}, \mathbf{x}, t) \ = \ p_{s\|} + 2p_{s\perp} \ \equiv \ 3p_s \ \equiv \ 3nT_s$$

AND THE CORRESPONDING MOMENTS OF THE DRIFT-KINETIC EQUATIONS YIELD:

$$\frac{\partial n}{\partial t} \ + \ \nabla \cdot (n\mathbf{u}_s) \ = \ 0$$

$$0 \ = \ 0$$

$$\frac{3}{2}\left[\frac{\partial p_s}{\partial t} + \nabla \cdot (p_s \mathbf{u}_s)\right] \ + \ \mathbf{P}_s : (\nabla \mathbf{u}_s) \ + \ \nabla \cdot \mathbf{q}_s \ = \ 0$$

CONSISTENT WITH THE FLUID PART OF THE SYSTEM

2. FLUID-KINETIC CLOSURE OF THE LOW-FREQUENCY, LARGE-SPATIAL-SCALE, MAGNETIZED PLASMA SYSTEM FOR LOW-COLLISIONALITY SLOW DYNAMICS, NEAR MAXWELLIAN

$$\nu_\imath \sim \delta\nu_e \sim \delta^2\Omega_{c\imath}, \quad \omega \sim \delta^2\Omega_{c\imath}, \quad u_\imath \sim u_e \sim \delta v_{th\imath}, \quad f_\imath - f_{M\imath} \sim \delta f_{M\imath}, \quad f_e - f_{Me} \sim \delta^2 f_{Me}$$

for consistency, $\quad \mathbf{b}\cdot\nabla T_\imath \sim \delta\ T_\imath/L\ , \quad \mathbf{b}\cdot\nabla T_e \sim \delta^2\ T_e/L$

second-order accuracy in δ

THE NON-GYROTROPIC (PERPENDICULAR) CLOSURES ARE:

$$\mathbf{P}^{GV}_{\iota,jk} \;=\; \frac{1}{4}\,\epsilon_{jlm} b_l\;\mathbf{K}_{\iota,mn}\;(\delta_{nk} + 3 b_n b_k) \;+\; (j \leftrightarrow k)$$

$$\mathbf{K}_{\iota,jk} \;=\; \frac{m_\iota}{eB}\left[\; nT_\iota\,\frac{\partial u_{\iota,k}}{\partial x_j} \;+\; \frac{\partial (q_{\iota T\|} b_k)}{\partial x_j} \;+\; (2q_{\iota B\|} - 3q_{\iota T\|})\; b_j \kappa_k \;+\; \frac{\partial}{\partial x_j}\left(\frac{nT_\iota}{eB}\;\epsilon_{klm}\; b_l\;\frac{\partial T_\iota}{\partial x_m}\right)\right] \;+\; (j \leftrightarrow k)$$

$$\mathbf{q}_{\iota\perp} = \frac{\mathbf{b}}{eB} \times \left\{\frac{5}{2} nT_\iota \nabla T_\iota + \frac{5}{6} T_\iota \nabla (p_{\iota\|} - p_{\iota\perp}) + T_\iota (p_{\iota\|} - p_{\iota\perp})\left[\frac{1}{3}\nabla \ln(nT_\iota) - \frac{5}{2}\boldsymbol{\kappa}\right] + \nabla \hat{r}_{\iota\perp} + (\hat{r}_{\iota\|} - \hat{r}_{\iota\perp})\boldsymbol{\kappa}\right\}$$

$$\mathbf{q}_{e\perp} \;=\; -\;\frac{5nT_e}{2eB}\;\mathbf{b}\times\nabla T_e$$

$$\mathbf{F}^{coll}_{e\perp} \;=\; \frac{2\nu_e m_e}{3(2\pi)^{1/2} e}\;\mathbf{j}_\perp \;-\; \frac{\nu_e m_e n}{(2\pi)^{1/2} eB}\;\mathbf{b}\times\nabla T_e$$

THE GYROTROPIC (PARALLEL) CLOSURES ARE THE FOLLOWING MOMENTS OF THE NON-MAXWELLIAN PARTS OF THE GYRO-AVERAGED DISTRIBUTION FUNCTIONS:

$$(p_{s\|} - p_{s\perp}) \;=\; \pi m_s \int dv'_\| \; dv'_\perp v'_\perp \; (2v'^2_\| - v'^2_\perp) \; \bar{f}_{Ns}$$

$$q_{s\|} = \; \pi m_s \int dv'_\| \; dv'_\perp v'_\perp \; v'_\| \; v'^2 \; \bar{f}_{Ns}$$

$$q_{\iota B\|} = \; \pi m_\iota \int dv'_\| \; dv'_\perp v'_\perp \; v'^3_\| \; \bar{f}_{N\iota} \; , \qquad\qquad q_{\iota T\|} = \; q_{\iota\|} - q_{\iota B\|}$$

$$\hat{r}_{\iota\|} = \; \pi m_\iota^2 \int dv'_\| \; dv'_\perp v'_\perp \; v'^2_\| \; v'^2 \; \bar{f}_{N\iota} \; , \qquad\qquad \hat{r}_{\iota\perp} = \; \frac{\pi m_\iota^2}{2} \int dv'_\| \; dv'_\perp v'_\perp \; v'^2_\perp \; v'^2 \; \bar{f}_{N\iota}$$

$$F^{coll}_{e\|} \;=\; \frac{2\nu_e m_e}{3(2\pi)^{1/2} e} \, j_\| \;-\; 2\pi \nu_e m_e v^3_{the} \; \int dv'_\| \; dv'_\perp v'_\perp \; \frac{v'_\|}{v'^3} \; \bar{f}_{Ne} \; ,$$

where

$$\mathbf{v}' \;=\; \mathbf{v} \;-\; \mathbf{u}_s(\mathbf{x}, t) \;=\; v'_\| \; \mathbf{b}(\mathbf{x}, t) \;+\; v'_\perp \; [\cos\alpha \; \mathbf{e}_1(\mathbf{x}, t) + \sin\alpha \; \mathbf{e}_2(\mathbf{x}, t)]$$

$$\bar{f}_{Ns}(v'_\|, v'_\perp, \mathbf{x}, t) \;=\; (2\pi)^{-1} \oint d\alpha \; [f_s(v'_\|, v'_\perp, \alpha, \mathbf{x}, t) - f_{Ms}(v', \mathbf{x}, t)]$$

$$f_{Ms}(v', \mathbf{x}, t) \;=\; \left(\frac{m_s}{2\pi}\right)^{3/2} \; \frac{n}{T_s^{3/2}} \; \exp\left(-\frac{m_s v'^2}{2T_s}\right)$$

THE DRIFT-KINETIC EQUATIONS FOR THE NON-MAXWELLIAN PARTS OF THE GYRO-AVERAGED DISTRIBUTION FUNCTIONS IN THE REFERENCE FRAMES OF THE MEAN FLOWS, $\bar{f}_{Ns}(v'_{\|}, v'_{\perp}, \mathbf{x}, t)$, ARE:

$$\frac{d_s \bar{f}_{Ns}}{dt} \;\equiv\; \frac{\partial \bar{f}_{Ns}}{\partial t} \;+\; \dot{\mathbf{x}}_s \cdot \frac{\partial \bar{f}_{Ns}}{\partial \mathbf{x}} \;+\; \dot{v}'_{\|s}\, \frac{\partial \bar{f}_{Ns}}{\partial v'_{\|}} \;+\; \dot{v}'_{\perp s}\, \frac{\partial \bar{f}_{Ns}}{\partial v'_{\perp}} \;=\; D_s\; f_{Ms} \;+\; \mathcal{Q}_s^{coll}$$

THE DRIFT-KINETIC EQUATIONS FOR THE NON-MAXWELLIAN PARTS OF THE GYRO-AVERAGED DISTRIBUTION FUNCTIONS IN THE REFERENCE FRAMES OF THE MEAN FLOWS, $\bar{f}_{Ns}(v'_\parallel, v'_\perp, \mathbf{x}, t)$**, ARE:**

$$\frac{d_s \bar{f}_{Ns}}{dt} \equiv \frac{\partial \bar{f}_{Ns}}{\partial t} + \dot{\mathbf{x}}_s \cdot \frac{\partial \bar{f}_{Ns}}{\partial \mathbf{x}} + \dot{v}'_{\parallel s} \frac{\partial \bar{f}_{Ns}}{\partial v'_\parallel} + \dot{v}'_{\perp s} \frac{\partial \bar{f}_{Ns}}{\partial v'_\perp} = D_s \, f_{Ms} + \mathcal{Q}_s^{coll}$$

with the coefficient functions for the electron collisionless advection operator

$$\dot{\mathbf{x}}_e = v'_\parallel \mathbf{b}$$

$$\dot{v}'_{\parallel e} = \frac{T_e}{m_e} \mathbf{b} \cdot \nabla \ln n - \frac{v'^2_\perp}{2} \mathbf{b} \cdot \nabla \ln B$$

$$\dot{v}'_{\perp e} = \frac{v'_\perp v'_\parallel}{2} \mathbf{b} \cdot \nabla \ln B$$

and the coefficient functions for the ion collisionless advection operator

$$\dot{\mathbf{x}}_\iota \;=\; v'_\parallel \mathbf{b} \;+\; \mathbf{u}_\iota - \mathbf{u}_{D\iota} \;+\; \frac{v'^2_\perp}{2} \nabla \times \left(\frac{\mathbf{b}}{\Omega_{c\iota}} \right) \;+\; \left(v'^2_\parallel - \frac{v'^2_\perp}{2} \right) \frac{\mathbf{b} \times \boldsymbol{\kappa}}{\Omega_{c\iota}}$$

$$\dot{v}'_{\parallel\iota} \;=\; \frac{\mathbf{b} \cdot (\nabla \cdot \mathsf{P}^{CGL}_\iota)}{m_\iota n} \;-\; \frac{v'^2_\perp}{2} \mathbf{b} \cdot \nabla \ln B \;-\; v'_\parallel \mathbf{b} \cdot [(\mathbf{b} \cdot \nabla)(\mathbf{u}_\iota - \mathbf{u}_{D\iota})] \;+\; \frac{v'_\parallel v'^2_\perp}{2} \nabla \cdot \left(\frac{\mathbf{b} \times \boldsymbol{\kappa}}{\Omega_{c\iota}} \right)$$

$$\dot{v}'_{\perp\iota} = \frac{v'_\perp}{2} \left\{ v'_\parallel \mathbf{b} \cdot \nabla \ln B \;+\; \mathbf{b} \cdot [(\mathbf{b} \cdot \nabla)(\mathbf{u}_\iota - \mathbf{u}_{D\iota})] \;-\; \nabla \cdot (\mathbf{u}_\iota - \mathbf{u}_{D\iota}) \;-\; v'^2_\parallel \nabla \cdot \left(\frac{\mathbf{b} \times \boldsymbol{\kappa}}{\Omega_{c\iota}} \right) \right\}$$

where

$$\mathbf{u}_{D\iota} \;=\; \frac{\mathbf{b} \times \nabla (n T_\iota)}{m_\iota n \Omega_{c\iota}} \qquad \textbf{and} \qquad \mathsf{P}^{CGL}_\iota = n T_\iota \mathsf{I} + (p_{\iota\parallel} - p_{\iota\perp})(\mathbf{bb} - \mathsf{I}/3)$$

and the coefficient functions for the ion collisionless advection operator

$$\dot{\mathbf{x}}_\iota \;=\; v'_\parallel \mathbf{b} \;+\; \mathbf{u}_\iota - \mathbf{u}_{D\iota} \;+\; \frac{v_\perp'^2}{2}\nabla\times\left(\frac{\mathbf{b}}{\Omega_{c\iota}}\right) \;+\; \left(v_\parallel'^2 - \frac{v_\perp'^2}{2}\right)\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{c\iota}}$$

$$\dot{v}'_{\parallel\iota} \;=\; \frac{\mathbf{b}\cdot(\nabla\cdot\mathsf{P}_\iota^{CGL})}{m_\iota n} \;-\; \frac{v_\perp'^2}{2}\mathbf{b}\cdot\nabla\ln B \;-\; v'_\parallel \mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)(\mathbf{u}_\iota - \mathbf{u}_{D\iota})] \;+\; \frac{v'_\parallel v_\perp'^2}{2}\nabla\cdot\left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{c\iota}}\right)$$

$$\dot{v}'_{\perp\iota} = \frac{v'_\perp}{2}\;\left\{v'_\parallel \mathbf{b}\cdot\nabla\ln B \;+\; \mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)(\mathbf{u}_\iota - \mathbf{u}_{D\iota})] \;-\; \nabla\cdot(\mathbf{u}_\iota - \mathbf{u}_{D\iota}) \;-\; v_\parallel'^2\nabla\cdot\left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{c\iota}}\right)\right\}$$

where

$$\mathbf{u}_{D\iota} \;=\; \frac{\mathbf{b}\times\nabla(nT_\iota)}{m_\iota n\Omega_{c\iota}} \qquad \textbf{and} \qquad \mathsf{P}_\iota^{CGL} = nT_\iota\mathsf{I} + (p_{\iota\parallel} - p_{\iota\perp})(\mathbf{bb} - \mathsf{I}/3)$$

THESE FULFILL THE PHASE-SPACE VOLUME CONSERVATION CONDITIONS

$$\frac{\partial}{\partial\mathbf{x}}\cdot\dot{\mathbf{x}}_s \;+\; \frac{\partial\dot{v}'_{\parallel s}}{\partial v'_\parallel} \;+\; \frac{1}{v'_\perp}\frac{\partial(v'_\perp\dot{v}'_{\perp s})}{\partial v'_\perp} \;=\; 0$$

so that

$$\frac{d_s\bar{f}_{Ns}}{dt} \;=\; \frac{\partial\bar{f}_{Ns}}{\partial t} \;+\; \frac{\partial}{\partial\mathbf{x}}\cdot(\bar{f}_{Ns}\dot{\mathbf{x}}_s) \;+\; \frac{\partial}{\partial v'_\parallel}\left(\bar{f}_{Ns}\dot{v}'_{\parallel s}\right) \;+\; \frac{1}{v'_\perp}\;\frac{\partial}{\partial v'_\perp}\left(v'_\perp\bar{f}_{Ns}\dot{v}'_{\perp s}\right)$$

The collision-independent driving terms, $D_s f_{Ms}$, are lengthy but explicit functions, given in Phys. Plasmas 17, 082502 (2010) and Phys. Plasmas 18, 102506 (2011)

In them, the fluid continuity and temperature equations were substituted for the time derivatives of the Maxwellians

The collision-independent driving terms, $D_s f_{Ms}$, are lengthy but explicit functions, given in Phys. Plasmas 17, 082502 (2010) and Phys. Plasmas 18, 102506 (2011)

$$\frac{\partial \bar{f}_{NMe}}{\partial t} + \cos\chi\left(v'\mathbf{b}\cdot\frac{\partial \bar{f}_{NMe}}{\partial \mathbf{x}} + v_{the}^2\mathbf{b}\cdot\nabla\ln n\frac{\partial \bar{f}_{NMe}}{\partial v'}\right) - \frac{\sin\chi}{v'}\left(v_{the}^2\mathbf{b}\cdot\nabla\ln n - \frac{v'^2}{2}\mathbf{b}\cdot\nabla\ln B\right)\frac{\partial \bar{f}_{NMe}}{\partial\chi}$$
$$= \Bigg\{\cos\chi\frac{v'}{2T_e}\left(5-\frac{v'^2}{v_{the}^2}\right)\mathbf{b}\cdot\nabla T_e + \cos\chi\frac{v'}{nT_e}\mathbf{b}\cdot\left[\frac{2}{3}\nabla(p_{e\parallel}-p_{e\perp}) - (p_{e\parallel}-p_{e\perp})\nabla\ln B - \mathbf{F}_e^{\mathrm{coll}}\right]$$
$$+ P_2(\cos\chi)\frac{v'^2}{3v_{the}^2}(\nabla\cdot\mathbf{u}_e - 3\mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}_e]) + \frac{1}{3nT_e}\left(\frac{v'^2}{v_{the}^2}-3\right)[\nabla\cdot(q_{e\parallel}\mathbf{b}) - G_e^{\mathrm{coll}}]$$
$$+ \frac{1}{6eB}\left[2P_2(\cos\chi)\frac{v'^2}{v_{the}^2}\left(\frac{v'^2}{v_{the}^2}-5\right) + \frac{v'^4}{v_{the}^4} - 10\frac{v'^2}{v_{the}^2} + 15\right](\mathbf{b}\times\boldsymbol{\kappa})\cdot\nabla T_e$$
$$+ \frac{1}{6eB}\left[-P_2(\cos\chi)\frac{v'^2}{v_{the}^2}\left(\frac{v'^2}{v_{the}^2}-5\right) + \frac{v'^4}{v_{the}^4} - 10\frac{v'^2}{v_{the}^2} + 15\right](\mathbf{b}\times\nabla\ln B)\cdot\nabla T_e$$
$$+ P_2(\cos\chi)\frac{v'^2}{3eBv_{the}^2}(\mathbf{b}\times\nabla\ln n)\cdot\nabla T_e\Bigg\}f_{Me} + (C_{ee}[f_e,f_e] + C_{ei}[f_e,f_i])_{\sigma}. \qquad (26)$$

where P_l denotes the Legendre polynomials, i.e., $P_2(z) = 3z^2/2 - 1/2$. Proceeding along the same lines to obtain the gyrophase-dependent part $\tilde{f}_{NMe}$ neglecting $O(\delta^4 f_{Me})$, the result is simply

$$\tilde{f}_{NMe} = f_{Me}\frac{v'\sin\chi}{2eBv_{the}^2}\left(\frac{v'^2}{v_{the}^2}-5\right)(\cos\alpha\,\mathbf{e}_2 - \sin\alpha\,\mathbf{e}_1)\cdot\nabla T_e. \qquad (27)$$

The drift-kinetic equation (26) has some unconventional yet desirable features that are worth commenting on. First, it is referred to the moving-frame velocity coordinates (v',χ), which give rise to the driving term proportional to $\nabla\cdot\mathbf{u}_e - 3\mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}_e]$. In return, the fluid closure moments (12)–(15) are evaluated directly without the need of a cumbersome subtraction of the mean velocity. Second, the moving-frame derivation incorporates exactly the contribution of the electric field, consistent with the momentum conservation equation or generalized Ohm's law (10). Some pieces of the electric field are subject to cancellations, in particular, the electron inertia piece is canceled by an inertial force from the transformation to the moving frame. In addition, there is the above discussed cancellation of terms proportional to the parallel density gradient from free-streaming and parallel electric field acceleration on the Maxwellian. Equation (26) includes just the residual part of the electric field after these cancellations are taken into account. Finally, Eq. (26) is automatically consistent with the required conditions that the 1, $v'_\parallel$, and v'^2 velocity moments of $\bar{f}_{NMe}$ vanish because under these conditions, the 1, $v'_\parallel$, and v'^2 moments of Eq. (26) are exact identities.

The collision operators in Eq. (26) are needed only in their lowest nonvanishing order, $C_{ee}[f_e,f_e] \sim C_{ei}[f_e,f_i] \sim \delta^3(v_{the}/L)f_{Me}$. Therefore, it is sufficient to use the following linearized forms:

$$C_{ee}[f_e,f_e] = C_{ee}[f_{Me},f_{NMe}] + C_{ee}[f_{NMe},f_{Me}] \qquad (28)$$

and

$$C_{ei}[f_e,f_i] = C_{ei}^{(3)}[f_{Me},f_i] + C_{ei}^{(3)}[f_{NMe},f_{Mi}], \qquad (29)$$

where the superscripts indicate that only the leading parts of order $\delta^3(v_{the}/L)f_{Me}$ need to be retained. The details of these collision operators will be discussed in the next section.

V. COLLISION OPERATORS

The linearized collision operators (28) and (29) will be taken in their complete Fokker–Planck–Landau form.[22] In accordance with the present drift-kinetic derivation they must be expressed in the reference frame of the electron mean flow velocity, but this does not pose any difficulty by virtue of their Galilean invariance. Only some care has to be exercised to account for the different electron and ion mean velocities and to retain some electron-ion collision terms that produce leading-order effects as a result of contributions to the electron distribution function structure on the ion thermal velocity scale. For the sake of completeness, it is worth revisiting these collision operator expressions in detail.

The linearized electron-electron collision operator is completely standard and, by Galilean invariance, its laboratory frame expression applies equally to the moving reference frame. Thus, dropping the $(\mathbf{x},t)$ arguments and with the electron collision frequency defined as

$$\nu_e = \frac{c^4 e^4 n\ln\Lambda_e}{4\pi m_e^2 v_{the}^3} \qquad (30)$$

in the rationalized electromagnetic system of units being used here, the Maxwellian-test part is the well-known integral operator,[25,26]

$$C_{ee}[f_{Me},f_{NMe}](\mathbf{v}') = \frac{\nu_e v_{the}}{n}f_{Me}(v')\{4\pi v_{the}^2 f_{NMe}(\mathbf{v}') - \Phi[f_{NMe}](\mathbf{v}') + v_{the}^{-2}\Xi[f_{NMe}](\mathbf{v}')\}, \qquad (31)$$

where Φ and Ξ are the velocity space convolutions

operator acting on it Eq. (34) retains only the first-order accuracy, $d_i/dt = O(v_{thi}/L) + O(\delta v_{thi}/L)$. So, Eq. (34) is just a special case of the general first-order result of Ref. 12, namely its slow flow and close to Maxwellian limit:

$$\frac{d_i}{dt} = \frac{\partial}{\partial t} + \dot{\mathbf{x}}\cdot\frac{\partial}{\partial\mathbf{x}} + \dot{v}'_\parallel\frac{\partial}{\partial v'_\parallel} + \dot{v}'_\perp\frac{\partial}{\partial v'_\perp}, \qquad (36)$$

where the coefficient functions are

$$\dot{\mathbf{x}} = v'_\parallel\mathbf{b} + \mathbf{u}_i - \mathbf{u}_{Di} + \frac{v_\perp'^2}{2}\nabla\times\left(\frac{\mathbf{b}}{\Omega_{ci}}\right) + \left(v_\parallel'^2 - \frac{v_\perp'^2}{2}\right)\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{ci}}, \qquad (37)$$

$$\dot{v}'_\parallel = \frac{\mathbf{b}\cdot(\nabla\cdot\mathbf{P}_i^{CGL})}{m_i n} - \frac{v_\perp'^2}{2}\mathbf{b}\cdot\nabla\ln B - v'_\parallel(\mathbf{bb}):[\nabla(\mathbf{u}_i - \mathbf{u}_{Di})] + \frac{v'_\parallel v_\perp'^2}{2}\nabla\cdot\left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{ci}}\right), \qquad (38)$$

and

$$\dot{v}'_\perp = \frac{v'_\perp}{2}\left\{v'_\parallel\mathbf{b}\cdot\nabla\ln B + (\mathbf{bb}-\mathbf{I}):[\nabla(\mathbf{u}_i - \mathbf{u}_{Di})] - v_\parallel'^2\nabla\cdot\left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{ci}}\right)\right\} \qquad (39)$$

with $\mathbf{u}_{Di} = \mathbf{b}\times\nabla(nT_i)/(m_i n\Omega_{ci})$, the lowest-order diamagnetic drift velocity. It is immediately verified that Eqs. (37)–(39) fulfill the phase-space volume conservation condition

$$\frac{\partial}{\partial\mathbf{x}}\cdot\dot{\mathbf{x}} + \frac{\partial\dot{v}'_\parallel}{\partial v'_\parallel} + \frac{1}{v'_\perp}\frac{\partial(v'_\perp\dot{v}'_\perp)}{\partial v'_\perp} = 0, \qquad (40)$$

so the phase-space advection of $\bar{f}_{NMi}$ can be expressed in Liouville theorem form:

$$\frac{d\bar{f}_{NMi}}{dt} = \frac{\partial\bar{f}_{NMi}}{\partial t} + \frac{\partial}{\partial\mathbf{x}}\cdot(\bar{f}_{NMi}\dot{\mathbf{x}}) + \frac{\partial}{\partial v'_\parallel}(\bar{f}_{NMi}\dot{v}'_\parallel) + \frac{1}{v'_\perp}\frac{\partial}{\partial v'_\perp}(v'_\perp\bar{f}_{NMi}\dot{v}'_\perp). \qquad (41)$$

Turning now to the collision-independent driving term, the result from Eq. (35) has the desired accuracy of $D_i = O(\delta v_{thi}/L) + O(\delta^2 v_{thi}/L)$. It is convenient to write $D_i = D_i^{even} + D_i^{odd}$, splitting it into its even and odd parts with respect to $v'_\parallel$. The even part is

$$D_i^{even} = \frac{(2v_\parallel'^2 - v_\perp'^2)}{2v_{thi}^2}(\mathbf{I}/3 - \mathbf{bb}):(\nabla\mathbf{u}_i) + \frac{1}{3nT_i}\left(\frac{v'^2}{v_{thi}^2}-3\right)\nabla\cdot(q_{i\parallel}\mathbf{b}) - \frac{(2v_\parallel'^2 - v_\perp'^2)}{6\Omega_{ci}T_i}(\mathbf{b}\times\nabla\ln n)\cdot\nabla T_i$$
$$- \frac{1}{2m_i\Omega_{ci}}\left[\frac{(v_\parallel'^4 + v_\parallel'^2 v_\perp'^2)}{v_{thi}^4} - \frac{5(4v_\parallel'^2 + v_\perp'^2)}{3v_{thi}^2} + 5\right](\mathbf{b}\times\boldsymbol{\kappa})\cdot\nabla T_i - \frac{1}{2m_i\Omega_{ci}}\left[\frac{(v_\parallel'^2 v_\perp'^2 + v_\perp'^4)}{2v_{thi}^4} - \frac{5(2v_\parallel'^2 + 5v_\perp'^2)}{6v_{thi}^2} + 5\right](\mathbf{b}\times\nabla\ln B)\cdot\nabla T_i$$
$$+ \frac{1}{6n}\left[\frac{(5v_\parallel'^2 + 2v_\perp'^2)}{v_{thi}^2} - 15\right]\nabla\cdot\left\{\frac{\mathbf{b}}{m_i\Omega_{ci}}\times\left[\frac{1}{3}\nabla(p_{i\parallel} - p_{i\perp}) - (p_{i\parallel} - p_{i\perp})\boldsymbol{\kappa}\right]\right\}$$
$$+ \frac{1}{3nT_i}\left(\frac{v'^2}{v_{thi}^2}-3\right)\nabla\cdot\left\{\frac{\mathbf{b}}{m_i\Omega_{ci}}\times[\nabla\hat{r}_{i\perp} + (\hat{r}_{i\parallel} - \hat{r}_{i\perp})\boldsymbol{\kappa}]\right\} + \frac{(2v_\parallel'^2 - v_\perp'^2)}{6\Omega_{ci}T_i}\left[\mathbf{b}\times\left(\frac{1}{3}\nabla\ln n - \boldsymbol{\kappa}\right)\right]\cdot\nabla\left(\frac{p_{i\parallel} - p_{i\perp}}{n}\right)$$
$$+ \frac{1}{3nT_i}\left(\frac{v'^2}{v_{thi}^2}-3\right)(p_{i\parallel} - p_{i\perp})(\mathbf{bb} - \mathbf{I}/3):[\nabla(\mathbf{u}_i - \mathbf{u}_{Di})] \qquad (42)$$

and the odd part is

$$D_i^{odd} = \frac{v'_\parallel}{2}\left(5 - \frac{v'^2}{v_{thi}^2}\right)\mathbf{b}\cdot\nabla\ln T_i + \frac{v'_\parallel}{nT_i}\mathbf{b}\cdot\left[\frac{2}{3}\nabla(p_{i\parallel} - p_{i\perp}) - (p_{i\parallel} - p_{i\perp})\nabla\ln B\right]$$
$$+ \frac{v'_\parallel}{nT_i}\mathbf{b}\cdot(\nabla\cdot\mathbf{P}_i^{GV}) - \frac{v'_\parallel v_\perp'^2}{2nT_i v_{thi}^2}\nabla\cdot\left\{\frac{nT_i}{\Omega_{ci}}\mathbf{b}\times[2(\mathbf{b}\cdot\nabla)\mathbf{u}_i + \mathbf{b}\times(\nabla\times\mathbf{u}_i)]\right\}$$
$$- \frac{v'_\parallel}{\Omega_{ci}v_{thi}^2}\left\{(v_\parallel'^2 - v_\perp'^2)(\mathbf{b}\times\boldsymbol{\kappa})\cdot[2(\mathbf{b}\cdot\nabla)\mathbf{u}_i + \mathbf{b}\times(\nabla\times\mathbf{u}_i)] + \frac{v_\perp'^2}{4}\mathbf{M}_*:(\nabla\mathbf{u}_i)\right\}$$
$$+ \frac{v'_\parallel}{\Omega_{ci}}\left\{\left(1 - \frac{v_\perp'^2}{2v_{thi}^2}\right)[\mathbf{b}\times\nabla\ln(nT_i)]\cdot[2(\mathbf{b}\cdot\nabla)\mathbf{u}_i + \mathbf{b}\times(\nabla\times\mathbf{u}_i)] + \left(\frac{v'^2}{v_{thi}^2} - 5\right)(\mathbf{b}\times\nabla\ln T_i)\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}_i]\right\}$$
$$- \frac{v'_\parallel v_\perp'^2}{2nT_i^2}\nabla\cdot\left\{\frac{nT_i}{\Omega_{ci}^2}\mathbf{b}\times[(\mathbf{b}\cdot\nabla\ln n)\mathbf{b}\times\nabla T_i]\right\} + \frac{v'_\parallel v_\perp'^2}{4nT_i^2}\left(\frac{v'^2}{v_{thi}^2} - 5\right)\nabla\cdot\left(\frac{nT_i\tau}{\Omega_{ci}^2}\mathbf{b}\times\nabla T_i\right)$$
$$+ \frac{v'_\parallel}{\Omega_{ci}^2}\Bigg\{(\mathbf{b}\cdot\nabla\ln n)\left[(v_\perp'^2 - v_\parallel'^2)\mathbf{b}\times\boldsymbol{\kappa} + \left(v_{thi}^2 - \frac{v_\perp'^2}{2}\right)\mathbf{b}\times\nabla\ln n + \left(\frac{v_\perp'^2 v'^2}{4v_{thi}^2} + \frac{v_\parallel'^2}{2} - \frac{7v_\perp'^2}{4} - \frac{3v_{thi}^2}{2}\right)\mathbf{b}\times\nabla\ln T_i\right]$$
$$- \frac{\tau}{2}\left[\left(\frac{v'^2}{v_{thi}^2} - 5\right)(v_\parallel'^2 - v_\perp'^2)\boldsymbol{\kappa} + \left(\frac{v_\perp'^2 v'^2}{2v_{thi}^2} - v_\parallel'^2 - \frac{9v_\perp'^2}{2} + 5v_{thi}^2\right)\nabla\ln n\right]\Bigg\}\cdot(\mathbf{b}\times\nabla\ln T_i)$$
$$- \frac{v'_\parallel v_\perp'^2}{8\Omega_{ci}^2}\mathbf{M}:\left\{\left(\frac{v'^2}{v_{thi}^2} - 5\right)\frac{\Omega_{ci}}{nT_i^2}\nabla\left(\frac{nT_i}{\Omega_{ci}}\nabla T_i\right) + \left[\left(\frac{v'^4}{2v_{thi}^4} - \frac{8v'^2}{v_{thi}^2} + \frac{49}{2}\right)\nabla\ln T_i - \left(\frac{v'^2}{v_{thi}^2} - 7\right)\nabla\ln n\right]\nabla\ln T_i\right\}. \qquad (43)$$

The first three moments of the collision-independent driving terms are:

$$\int d^3\mathbf{v}'\ D_e\ f_{Me}\ =\ 0$$

$$m_e \int d^3\mathbf{v}'\ v'_{\|}\ D_e\ f_{Me}\ =\ \mathbf{b}\cdot\left[\frac{2}{3}\nabla(p_{e\|}-p_{e\perp})-\left(p_{e\|}-p_{e\perp}\right)\nabla\ln B\right]$$

$$\frac{m_e}{2}\int d^3\mathbf{v}'\ v'^2\ D_e\ f_{Me}\ =\ \nabla\cdot(q_{e\|}\mathbf{b})$$

$$\int d^3\mathbf{v}'\ D_\iota\ f_{M\iota}\ =\ -\nabla\cdot\left\{\frac{\mathbf{b}}{m_\iota\Omega_{c\iota}}\times\left[\frac{1}{3}\nabla(p_{\iota\|}-p_{\iota\perp})-(p_{\iota\|}-p_{\iota\perp})\boldsymbol{\kappa}\right]\right\}$$

$$m_\iota \int d^3\mathbf{v}'\ v'_{\|}\ D_\iota\ f_{M\iota}\ =\ \mathbf{b}\cdot\left[\frac{2}{3}\nabla(p_{\iota\|}-p_{\iota\perp})-\left(p_{\iota\|}-p_{\iota\perp}\right)\nabla\ln B\right]\ +$$

$$+\ \nabla\cdot\left\{\frac{\mathbf{b}}{\Omega_{c\iota}}\times\left[\nabla q_{\iota T\|}+2(q_{\iota B\|}-q_{\iota T\|})\boldsymbol{\kappa}\right]\right\}\ +\ \left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{c\iota}}\right)\cdot\nabla q_{\iota T\|}$$

$$\frac{m_\iota}{2}\int d^3\mathbf{v}'\ v'^2\ D_\iota\ f_{M\iota}\ =\ \nabla\cdot(q_{\iota\|}\mathbf{b})\ +\ \nabla\cdot\left\{\frac{\mathbf{b}}{m_\iota\Omega_{c\iota}}\times\left[\nabla\hat{r}_{\iota\perp}+(\hat{r}_{\iota\|}-\hat{r}_{\iota\perp})\boldsymbol{\kappa}\right]\right\}\ +$$

$$+\ (p_{\iota\|}-p_{\iota\perp})\left\{\mathbf{b}\cdot\left[(\mathbf{b}\cdot\nabla)(\mathbf{u}_\iota-\mathbf{u}_{D\iota})\right]-\frac{1}{3}\nabla\cdot(\mathbf{u}_\iota-\mathbf{u}_{D\iota})\right\}$$

The collisional driving terms, $\mathcal{Q}_s^{coll}$ are

$$\mathcal{Q}_\iota^{coll} \;=\; (\;2\pi)^{-1}\oint d\alpha\{C_{\iota\iota}[f_{N\iota}, f_{M\iota}] + C_{\iota\iota}[f_{M\iota}, f_{N\iota}]\} \;\equiv\; \mathcal{C}_\iota[\bar{f}_{N\iota}]$$

and

$$\begin{aligned}
\mathcal{Q}_e^{coll} \;&=\; (2\pi)^{-1}\oint d\alpha\{C_{ee}[f_{Ne}, f_{Me}] + C_{ee}[f_{Me}, f_{Ne}] + C_{e\iota}[f_{Ne}, f_{M\iota}]\} \;+ \\
&+\; (2\pi)^{-1}\oint d\alpha\; C_{e\iota}[f_{Me}, f_{M\iota}] \;-\; \left[\frac{v'_\parallel}{nT_e}\; F_{e\parallel}^{coll} \;+\; \frac{2\nu_e m_e}{3(2\pi)^{1/2} m_\iota}\left(\frac{T_\iota}{T_e} - 1\right)\left(\frac{m_e v'^2}{T_e} - 3\right)\right] f_{Me} \;\equiv \\
&\equiv\; \mathcal{C}_e[\bar{f}_{Ne}] \;+\; \mathcal{I}_e^{coll}
\end{aligned}$$

The collisional driving terms, $\mathcal{Q}_s^{coll}$ are

$$\mathcal{Q}_\iota^{coll} \;=\; (\,2\pi)^{-1} \oint d\alpha \{C_{\iota\iota}[f_{N\iota}, f_{M\iota}] + C_{\iota\iota}[f_{M\iota}, f_{N\iota}]\} \;\equiv\; \mathcal{C}_\iota[\bar{f}_{N\iota}]$$

and

$$\begin{aligned}
\mathcal{Q}_e^{coll} \;&=\; (2\pi)^{-1} \oint d\alpha \{C_{ee}[f_{Ne}, f_{Me}] + C_{ee}[f_{Me}, f_{Ne}] + C_{e\iota}[f_{Ne}, f_{M\iota}]\} \;+ \\
&+\; (2\pi)^{-1} \oint d\alpha \; C_{e\iota}[f_{Me}, f_{M\iota}] \;-\; \left[\frac{v'_\parallel}{nT_e}\; F_{e\parallel}^{coll} \;+\; \frac{2\nu_e m_e}{3(2\pi)^{1/2} m_\iota} \left(\frac{T_\iota}{T_e} - 1\right) \left(\frac{m_e v'^2}{T_e} - 3\right)\right] f_{Me} \;\equiv \\
&\equiv\; \mathcal{C}_e[\bar{f}_{Ne}] \;+\; \mathcal{I}_e^{coll}
\end{aligned}$$

These have particle, momentum and energy conservation properties:

$$\int d^3\mathbf{v}' \;(1, v'_\parallel, v'^2)\; \mathcal{Q}_s^{coll} \;=\; 0$$

FROM THE DEFINITION OF $\mathbf{v}'$ AS RELATIVE TO THE MEAN FLOWS AND THE CONDITION THAT THE FLUID DENSITY AND TEMPERATURES BE THOSE IN THE MAXWELLIANS:

$$\int d^3\mathbf{v}' \; (1, v'_\|, v'^2) \; \bar{f}_{Ns} \; = \; 0$$

FROM THE DEFINITION OF v' AS RELATIVE TO THE MEAN FLOWS AND THE CONDITION THAT THE FLUID DENSITY AND TEMPERATURES BE THOSE IN THE MAXWELLIANS:

$$\int d^3\mathbf{v}' \ (1, v'_{\|}, v'^2) \ \bar{f}_{Ns} \ = \ 0$$

THEN, USING THE EXPRESSIONS OF THE COLLISIONLESS ADVECTION OPERATORS, IT IS VERIFIED THAT

$$\int d^3\mathbf{v}' \ (1, v'_{\|}, v'^2) \ \frac{d_s\bar{f}_{Ns}}{dt} \ \equiv \ \int d^3\mathbf{v}' \ (1, v'_{\|}, v'^2) \ (D_s \ f_{Ms} \ + \ \mathcal{Q}_s^{coll})$$

SO, CONSISTENT WITH THE CONDITIONS $\int d^3\mathbf{v}' \ (1, v'_{\|}, v'^2) \ \bar{f}_{Ns} = 0$, THE $(1, v'_{\|}, v'^2)$ MOMENTS OF THE DRIFT-KINETIC EQUATIONS ARE SATISFIED IDENTICALLY

SUMMARY

A RIGOROUS FINITE-LARMOR-RADIUS FLUID AND DRIFT-KINETIC THEORY HAS BEEN DEVELOPED, SUITABLE TO DESCRIBE MACROSCOPIC DYNAMICS OF MAGNETIZED PLASMAS IN COLLISIONLESS OR LOW-COLLISIONALITY REGIMES. IT FEATURES:

- LOW-FREQUENCY, QUASINEUTRAL EQUATIONS FOR THE ELECTROMAGNETIC FIELDS
- FLUID EQUATIONS FOR THE DENSITY, ION FLOW VELOCITY AND TEMPERATURES
- PARTICLE, MOMENTUM AND ENERGY CONSERVATION IN THE FLUID SYSTEM
- EXPLICIT EXPRESSIONS FOR THE NON-GYROTROPIC (PERPENDICULAR) CLOSURES
- GYROTROPIC (PARALLEL) CLOSURES AS MOMENTS OF THE GYROPHASE-AVERAGED DISTRIBUTION FUNCTIONS

- DRIFT-KINETIC EQUATIONS FOR THE GYROPHASE-AVERAGED DISTRIBUTION FUNCTIONS IN THE REFERENCE FRAMES OF THEIR MEAN FLOWS

- PHASE-SPACE VOLUME CONSERVATION BY THE DRIFT-KINETIC COLLISIONLESS ADVECTION OPERATORS

- FOR FAR-FROM-MAXWELLIAN DISTRIBUTION FUNCTIONS, THE $v'_{\|}$ MOMENTS OF THE DRIFT-KINETIC EQUATIONS ARE IDENTITIES, AND THE 1 AND v'^2 MOMENTS REPRODUCE THE FLUID CONTINUITY AND TEMPERATURE EQUATIONS

- FOR NEAR-MAXWELLIAN DISTRIBUTION FUNCTIONS WITH THE FLUID CONTINUITY AND TEMPERATURE EQUATIONS SUBSTITUTED FOR THE TIME DERIVATIVES OF THE MAXWELLIANS, THE 1, $v'_{\|}$ AND v'^2 MOMENTS OF THE DRIFT-KINETIC EQUATIONS ARE IDENTITIES