**On Some Computational Challenges & Novel Solutions** in the Kinetic Modeling of Beams and Plasmas

Novel Simulation Methods in the Particle-In-Cell Code-Framework Warp



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> **Computational Methods in High Energy Density Plasmas** Workshop I: Computational Challenges in Hot Dense Plasmas IPAM, UCLA, Los Angeles, USA – March 26-30, 2012

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## Outline

- Warp overview
  - origins (Heavy Ion Inertial Fusion)
  - geometry, particle pushers, field solvers, etc
- Computational challenges *novel algorithms* 
  - large time-step in magnetic fields "Drift-Lorentz" particle pusher
  - E+VxB cancellation in Boris pusher *Lorentz invariant particle pusher*
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- wideband filtering *"strided" digital filters*
- Summary





#### The Heavy Ion Inertial Fusion (HIF) program is studying the science of ionheated matter, as well as drivers & targets for inertial fusion energy



## NDCX-II is our new platform for studies of

- -space-chargedominated beams
- -Warm Dense Matter physics
- -beam-target energy coupling



#### Space/time scales span 8 orders of magnitude: from <mm to km>/<ps to 100 ms>

Simulation goal – integrated self-consistent predictive capability



...to target

#### including:

- beam(s) generation, acceleration, focusing and compression along accelerator,
- loss of particles at walls, interaction with desorbed gas and electrons,
- neutralization from plasma in chamber,
- target physics and diagnostics.

#### => Need large-scale multiphysics computing

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# Main code is Warp - a parallel framework combining features of plasma (Particle-In-Cell) and accelerator codes



Reference frame:	lab	moving-window	Lorentz boosted	
	Z	z-vt	γ(z-vt); γ(t-vz/c <sup>2</sup> )	

• Bends: "warped" coordinates (no "reference orbit")

see A. Friedman, D. P. Grote, and I. Haber, Phys. Fluids B 4, 2203 (1992)



#### Field solvers based on Finite Difference methods

#### • Field solvers

- Electrostatic/Magnetostatic FFT, multigrid; AMR; implicit
- Fully electromagnetic Yee mesh, PML, MR

Automatic meshing around HCX ion beam source emitter from Warp (r,z) simulation



 $\underbrace{E}_{\infty} = 0.04 \qquad -0.004 \qquad -0.003 \\
0.02 \qquad -0.002 \qquad -0.002 \\
0.000 \qquad 0.005 \qquad C(m) \qquad 0.010 \qquad 0.015 \\
 = 0.011 \qquad -0.011 \qquad -0.011 \\
 = 0.011 \qquad -0.$ 

$$\vec{E} \approx \left\{ \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \left(1 + \beta^2\right) \frac{\partial \phi}{\partial z} \right\}$$
$$\vec{B} \approx \left\{ \frac{v_z}{c^2} \frac{\partial \phi}{\partial y}, -\frac{v_z}{c^2} \frac{\partial \phi}{\partial x}, 0 \right\}$$

Only one Poisson solve with stretched z coordinate gives all E and B field components.

\*J.-L. Vay, Phys. Plasmas 15, 056701 (2008)





0.06

0.007

0.006

0.005

#### Support for internal conductors & accelerator lattice elements

• Boundaries: "cut-cell" --- no restriction to "Legos" for electrostatic solvers



- Accelerator lattice: general; non-paraxial; can read MAD files
  - solenoids, dipoles, quads, sextupoles, linear maps, arbitrary fields, acceleration.



#### Particle pusher based on leapfrog method

• Position push

$$x^{n+1/2} = x^{n-1/2} + v^n \Delta t$$

#### • Velocity push (u=γv)

$$u^{n+1} = u^n + \frac{q}{m} f(E^{n+1/2}, B^{n+1/2}, u^n, u^{n+1}, \Delta t)$$

Four options

- standard Boris,
- Boris with "tan( $\alpha$ )/ $\alpha$ " rotation angle correction,
- "drift-Lorentz"<sup>1</sup> (for taking large time steps > cyclotron period),
- Lorentz invariant<sup>2</sup> (provides proper treatment of self **E**+**v**×**B** for relativistic particles).

<sup>1</sup>R. Cohen et. al., *Phys. Plasmas* **12**, 056708 (2005) <sup>2</sup>J.-L. Vay, *Phys. Plasmas* **15**, 056701 (2008)



### Surface/volume physics

- Particle emission: space charge limited, thermionic, hybrid, arbitrary
- Secondary electrons: Monte-Carlo with energy and angular dependence (Posinst; M. Furman)



- Ion impact- or photo-induced electron emission
- Ion impact-induced gas emission, tracking
- Monte Carlo Collision: ionization, capture, charge exchange



#### Warp is parallel, combining modern and efficient programming languages

• Parallellization: MPI (1, 2 and 3D domain decomposition)



• Python and FORTRAN\*: "steerable," input decks are programs



\*http://hifweb.lbl.gov/Forthon (wrapper supports FORTRAN90 derived types)



### Warp's versatile programmable framework allows great adaptability



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#### "Drift-Lorentz" particle pusher\* relaxes the problem of short electron timescales in magnetic field



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#### Lorentz invariant particle pusher

• Boris pusher introduces error in cancellation of self E and v×B

$$u^{n+1} = u^n + \frac{q\Delta t}{m} \left( E^{n+1/2} + \frac{u^{n+1} + u^n}{2\gamma^{n+1/2}} \times B^{n+1/2} \right) \qquad u = \gamma v$$

**issue: E**+**v**×**B**=0 implies **E**=**B**=0 => large errors when **E**+v×**B**≈0 (e.g. relativistic beams).

- Solution - Velocity push:  $u^{n+1} = u^n + \frac{q\Delta t}{m} \left( E^{n+1/2} + \frac{v^{n+1} + v^n}{2} \times B^{n+1/2} \right)$
- Looks implicit but solvable analytically\*

$$\begin{cases} \gamma^{i+1} = \sqrt{\frac{\sigma + \sqrt{\sigma^2 + 4(\tau^2 + u^{*2})}}{2}} & \text{(with } \mathbf{u} = \gamma \mathbf{v}, \quad \mathbf{u}' = \mathbf{u}^{\mathbf{i}} + \frac{q\Delta t}{m} \left( \mathbf{E}^{i+1/2} + \frac{\mathbf{v}^i}{2} \times \mathbf{B}^{i+1/2} \right), \quad \tau = (q\Delta t/2m) \mathbf{B}^{i+1/2}, \\ \mathbf{u}^{i+1} = [\mathbf{u}' + (\mathbf{u}' \cdot \mathbf{t})\mathbf{t} + \mathbf{u}' \times \mathbf{t}]/(1+t^2) & u^* = \mathbf{u}' \cdot \tau/c, \quad \sigma = \gamma'^2 - \tau^2, \quad \gamma' = \sqrt{1 + u'^2/c^2}, \quad \mathbf{t} = \tau/\gamma^{i+1}). \end{cases}$$

\*J.-L. Vay, Phys. Plasmas 15, 056701 (2008)



### Single particle test of Lorentz invariant pusher





#### Example of application to LHC-like beam interacting with e-cloud

Calculation of e-cloud induced instability of a proton bunch\*

- Proton energy: g=500 in Lab
- L=5 km, continuous focusing



- beam was lost after a few betatron oscillations with Boris pusher,
- accurate result was obtained with new pusher.

Pusher also used for modeling of

- astrophysical highly magnetized relativistic electron-ion flow (L. Sironi & A. Spitkovski, Princeton U.)
- particle trajectory integration in plasma with QED effects (N. Elkina, LMU, Germany)
- \*J.-L. Vay, Phys. Rev. Lett. 98, 130405 (2007)



electron

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### **Coupling of AMR to PIC/Vlasov/MHD: issues**

Mesh refinement implies a jump of resolution and some procedure for coupling the solutions at the interface.



#### Consequences:

- loss of symmetry: self-force<sup>(1,2)</sup>,
- loss of conservation laws<sup>(1,2)</sup>,
- EM: waves reflection<sup>(3)</sup>.
- (1) Vay et al., *Laser Part. Beams* 20 (2002)
  (2) Vay et al., *Phys. Plasmas* 11 (2004)
  (3) Vay, *J. Comput. Phys.* 167 (2001)



#### Electrostatic mesh refinement method: spurious self-force



#### Spurious self-force mitigated in Warp using guard cells



(2) Vay et al., Phys. Plasmas 11 (2004)

#### Example of application to AMR simulations of ion source

#### -- speedup from AMR x10



#### 1-D AMR-EM: illustration of instability\*



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\*J.-L. Vay, J. Comput. Phys. 167, 72 (2001)

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#### Warp's Electromagnetic MR uses PML & substitution to prevent reflections

#### Warp's electromagnetic MR solver

- Termination of patches with PML avoids spurious reflections
- Guard cell used for mitigating spurious self-force



\*Vay, Adam, Heron, Comp. Phys. Comm. 164 (2004)



#### Electromagnetic MR simulation of beam-induced plasma wake



Speedup x10 in 3D (using the same time steps for all refinement levels).



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#### Berkeley Lab Laser Accelerator (BELLA) under construction

Laser Plasma Accelerators (LPAs) promise acceleration over short distances

10 GeV stages Staging Ш aser BERKELEY LAB Energy spread & **Emittance** preservation H PW laser **Radiation sources** 40 J / 40 fs Positron acceleration + PWFA expt.'s n<sub>e</sub>/n<sub>0</sub> 7 510 6 Warp simulation of 5 10 GeV stage 3 Energy 2 0 Warp – 2D **BELL** Collaboration with C. Geddes/LOASIS Group - LBNL The Heavy Ion Fusion Science SciDAC-II

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### LPAs difficult to model because of large separation of scales

Simulations in a Lorentz "boosted frame" reduces numerical difficulty by orders of magnitude\*



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![](_page_27_Figure_0.jpeg)

![](_page_27_Picture_3.jpeg)

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![](_page_28_Picture_13.jpeg)

![](_page_28_Picture_14.jpeg)

#### Warp electromagnetic solver is based on Non-Standard Finite-Difference (NSFD) stencil

![](_page_29_Figure_1.jpeg)

<sup>3</sup>J.-L. Vay, et al., J. Comput. Phys. **230** (2011) 5908

<sup>1</sup>J. B. Cole, IEEE Trans. Microw. Theory Tech. **45** (1997).

J. B. Cole, IEEE Trans. Antennas Prop. 50 (2002).

<sup>2</sup>M. Karkkainen et al., Proc. ICAP, Chamonix, France (2006).

#### NSFD-based solver offers tunability of numerical dispersion

![](_page_30_Figure_1.jpeg)

Solver can be tuned to better fit particular needs.

### Examples of tuning of numerical dispersion on unit pulse in 2D

![](_page_31_Figure_1.jpeg)

Note: Cole-Karkkainen (CK) is sometimes improperly referred to as "dispersionless solver".

![](_page_31_Picture_3.jpeg)

#### Perfectly Matched Layer<sup>1,2</sup> (PML) implemented with NSFD solver - for absorption of outgoing waves -

$$\begin{aligned} (\Delta_t + \sigma_x) B_{zx} &= -\Delta_x^* E_y & \text{NSFD} \\ (\Delta_t + \sigma_y) B_{zy} &= \Delta_y^* E_x & \text{NSFD} \\ (\Delta_t + \sigma_x) E_y &= -c^2 \Delta_x (B_{zx} + B_{zy}) & \text{FD} \\ (\Delta_t + \sigma_y) E_x &= c^2 \Delta_y (B_{zx} + B_{zy}) & \text{FD} \end{aligned}$$

Example:

Reflection of circular pulse using 5 cells PML  $\Delta = 5\Delta x$ with quadratic progression and standard coefficients  $\sigma_i$ or improved coefficients<sup>2</sup>  $\sigma_i^*$ 

$$\sigma_{i} = \sigma_{m} \left( i\Delta x / \Delta \right)^{2}$$
  

$$\sigma_{m} = 4 / \Delta x$$
  

$$\sigma_{i}^{*} = \left( e^{-\sigma_{i+1/2}\Delta t} - e^{\sigma_{i}\Delta t} \right)$$

<sup>1</sup>JP Berenger, J. Comput. Phys. **127** (1996) 363 <sup>2</sup>J.-L. Vay, J. Comput. Phys. **183** (2002) 367

![](_page_32_Figure_6.jpeg)

#### Same high efficiency as with Yee.

![](_page_32_Picture_8.jpeg)

![](_page_32_Picture_10.jpeg)

![](_page_32_Picture_11.jpeg)

#### Instability mostly insensitive to tuning of numerical dispersion... ...but very sensitive to time step!

![](_page_33_Figure_1.jpeg)

- Tunable NSFD solver allows  $c\delta t = \delta z/\sqrt{2}$  time step for (near) cubic cells
  - $c\delta t = \delta z/V2$  time step restricted to "pancake" cells in 3D using Yee FDTD solver

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Use of special time step was helpful but not sufficient for large γ boost

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![](_page_34_Picture_13.jpeg)

![](_page_34_Picture_14.jpeg)

#### Digital filtering of current density and/or fields -- commonly used for improving stability and accuracy

Multiple pass of bilinear filter + compensation routinely used

100% absorption at Nyquist freq.

![](_page_35_Figure_3.jpeg)

Wideband filtering difficult in parallel (footprint limited by size of local domains) or expensive

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Example: wideband filters using N repetitions of bilinear filter

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![](_page_35_Figure_6.jpeg)

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### "Strided" bilinear filters enable efficient and versatile filtering<sup>1</sup>

Using a stride N shifts the 100% absorption frequency to F<sub>nyquist</sub>/N

![](_page_36_Figure_2.jpeg)

![](_page_36_Figure_3.jpeg)

Combination of filters with strides allows for more efficient filtering:

- $G_1G_2 \equiv 20^*B+C$ ; speedup ×2
- $G_1G_2G_3 \equiv 50^*B+C$ ; speedup ×3.5
- $G_1G_2G_4 = 80^*B+C$ ; speedup ×5.5

![](_page_36_Figure_8.jpeg)

<sup>1</sup>J.-L. Vay, et al., J. Comput. Phys. 230 (2011) 5908.

![](_page_36_Picture_10.jpeg)

### Does physics allow to use wideband filtering?

Time history of laser spectrum (relative to laser  $\lambda_0$  in vacuum)

![](_page_37_Figure_2.jpeg)

#### Spectrum very different in lab and boosted frames

More filtering possible without altering physics\*.

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\*J.-L. Vay, et al., PoP Lett. 18 (2011).

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![](_page_38_Figure_0.jpeg)

### Numerical instability under control with tunable EM solver & filtering

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+ new laser/particle injection and diagnostics through planes<sup>1</sup>:

![](_page_39_Figure_2.jpeg)

![](_page_39_Figure_3.jpeg)

![](_page_39_Figure_4.jpeg)

Diagnostics

![](_page_39_Figure_6.jpeg)

<sup>1</sup>J.-L. Vay, et al., *J. Comput. Phys.* **230** (2011).

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![](_page_39_Figure_8.jpeg)

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<sup>2</sup>J.-L. Vay, et al., *PoP* **18** (2011)

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## Summary

- Warp is a parallel Particle-In-Cell simulation framework for the modeling of accelerators, beams, plasmas and laser-plasma interaction.
- Challenges such as space and time scale disparities, discretization errors, numerical dispersion, have pushed toward the development of new methods:
  - "Drift-Lorentz" particle pusher for large time steps of magnetized particles,
  - Lorentz invariant particle pusher,
  - mesh refinement for PIC,
  - boosted frame methods (including laser injection through moving plane),
  - tunable electromagnetic field solver,
  - strided filtering.
- The novel algorithms are quite general and have broad applicability, including to the modeling of high energy density plasmas.

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For questions regarding Warp, email to

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![](_page_40_Picture_14.jpeg)

![](_page_40_Picture_15.jpeg)