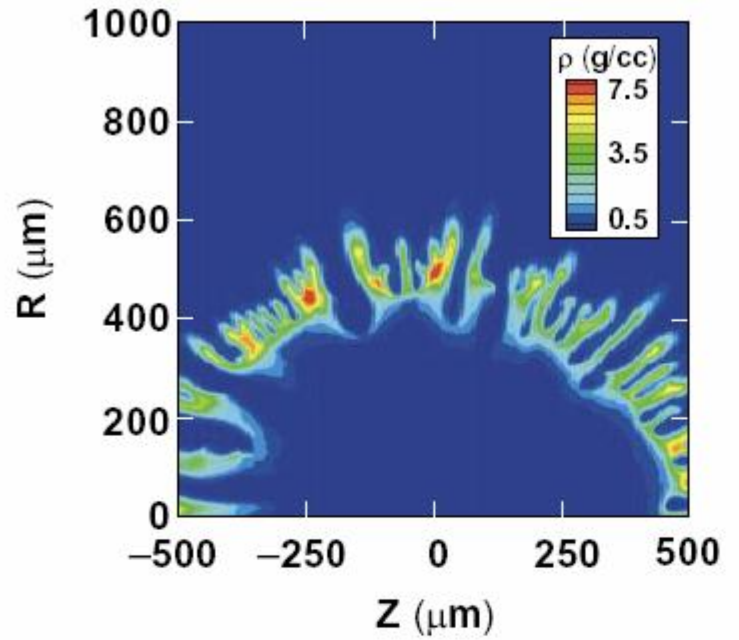
Theory and simulations of hydrodynamic instabilities in inertial fusion



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Ryan Nora (UR-Physics Dept)

Outline

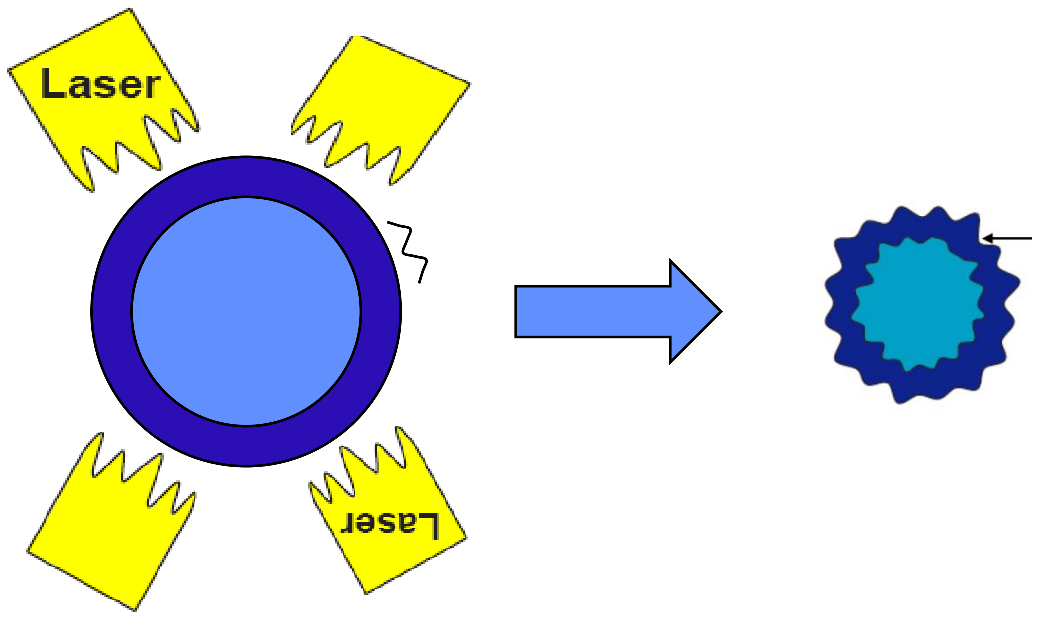
→ **Theory of the Rayleigh-Taylor instability in inertial fusion**

- Ablation fronts in laser-driven targets
- Classical linear Rayleigh-Taylor instability
- Ablative linear Rayleigh-Taylor instability
- Single-Mode Nonlinear Theory
- Multimode ablative RT

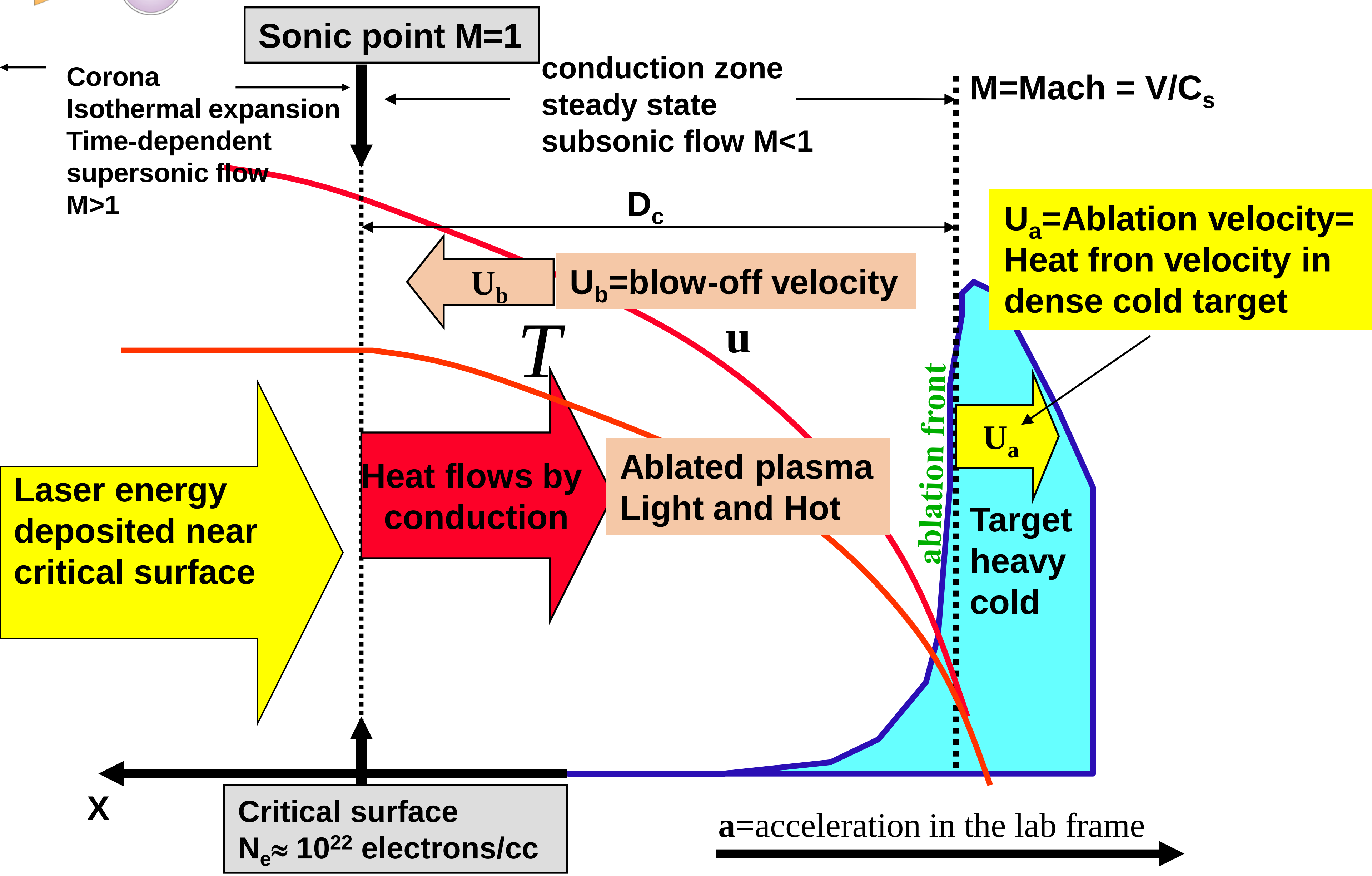
→ **Hydrodynamic simulations**

- Eulerian and Lagrangian hydrodynamics
- Two-fluids, nonlocal heat conduction, radiation transport, laser absorption
- Single mode and multimode simulations of hydrodynamic instabilities

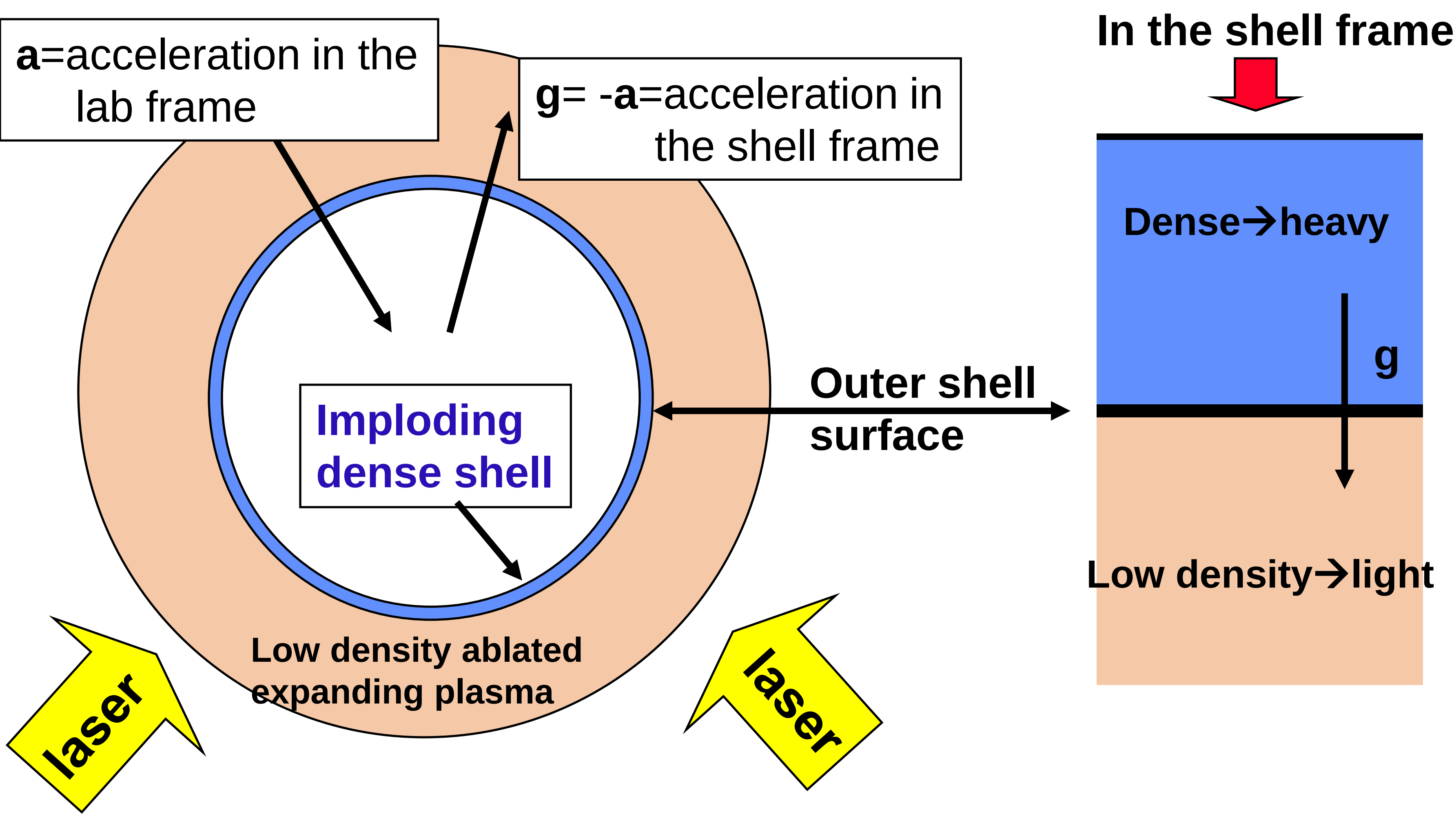
Theory of the Rayleigh-Taylor instability in inertial fusion



The laser energy deposition generates a thermal conduction zone and an ablative flow

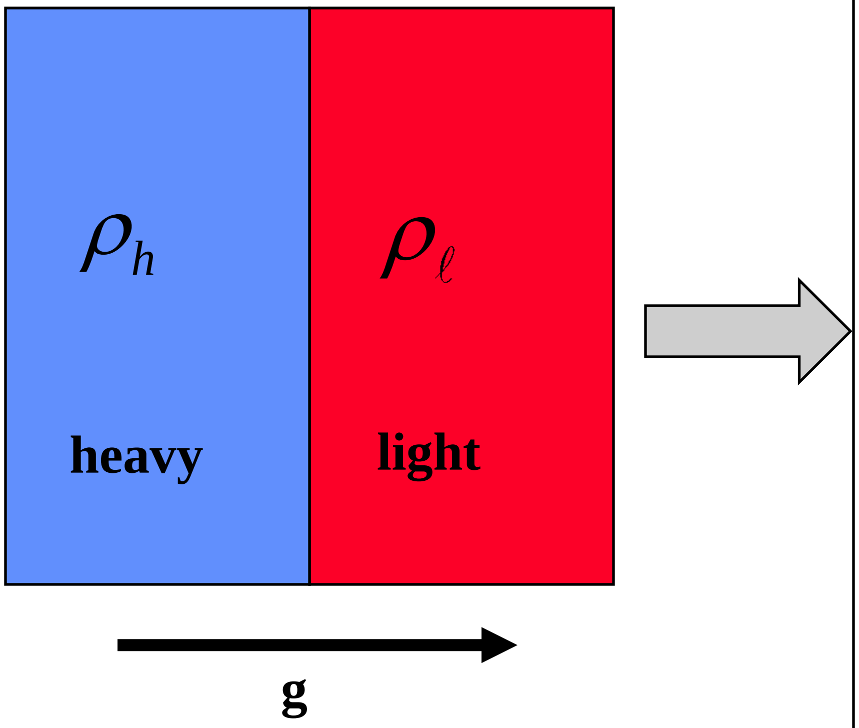


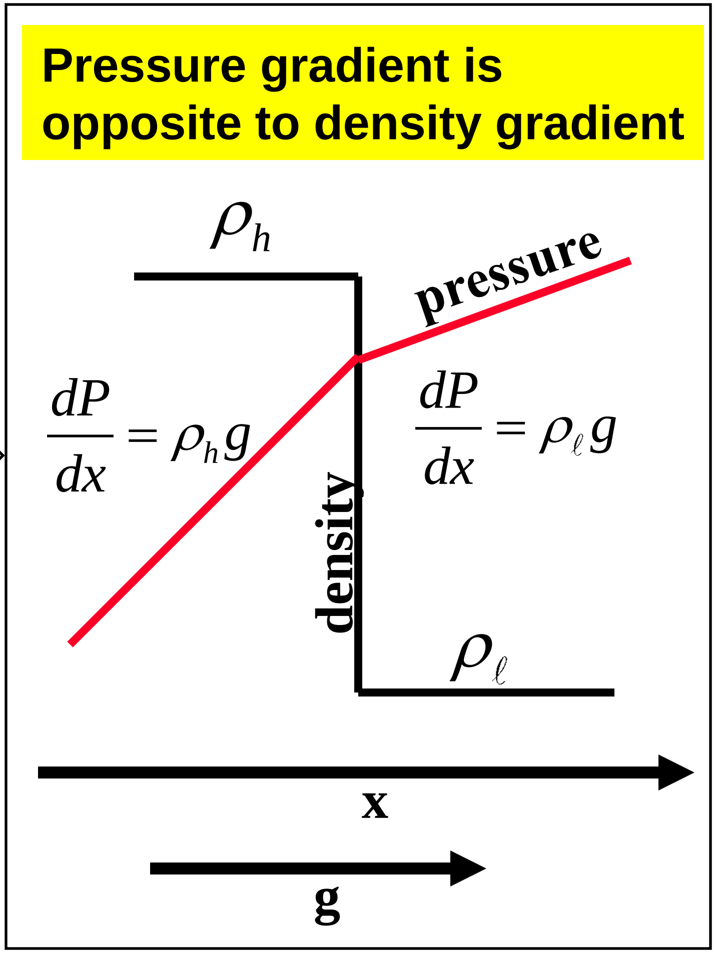
The outer surface of an imploding capsule separates a dense fluid supported by a lighter one



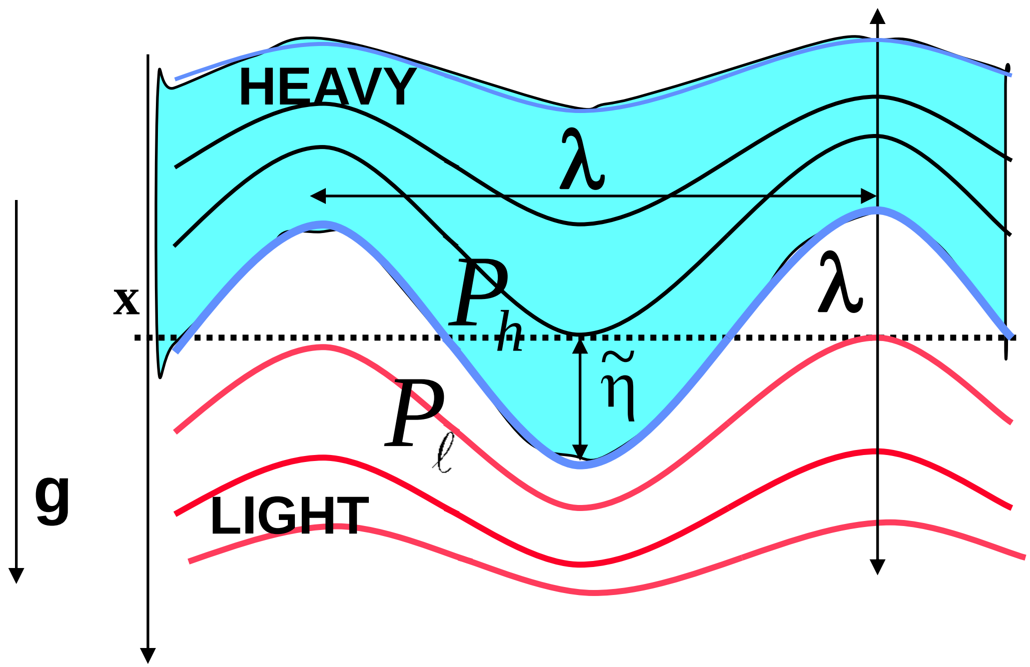
The outer surface of an imploding capsule is unstable to the Rayleigh-Taylor instability

EQUILIBRIUM CONDITIONS





Derive the classical R-T from Newton's law



$$k = 2\pi/\lambda$$

$$F = S(P_h - P_\ell) = ma = \rho_h \lambda S \ddot{\tilde{\eta}}$$

$$\frac{dP_0}{dx} = \rho_0 g = \begin{cases} \rho_h g & heavy \\ \rho_\ell g & light \end{cases}$$

$$P_\ell = P_0 + \left[\frac{dP_0}{dx}\right]_\ell \tilde{\eta} = P_0 + \rho_\ell g \tilde{\eta}$$

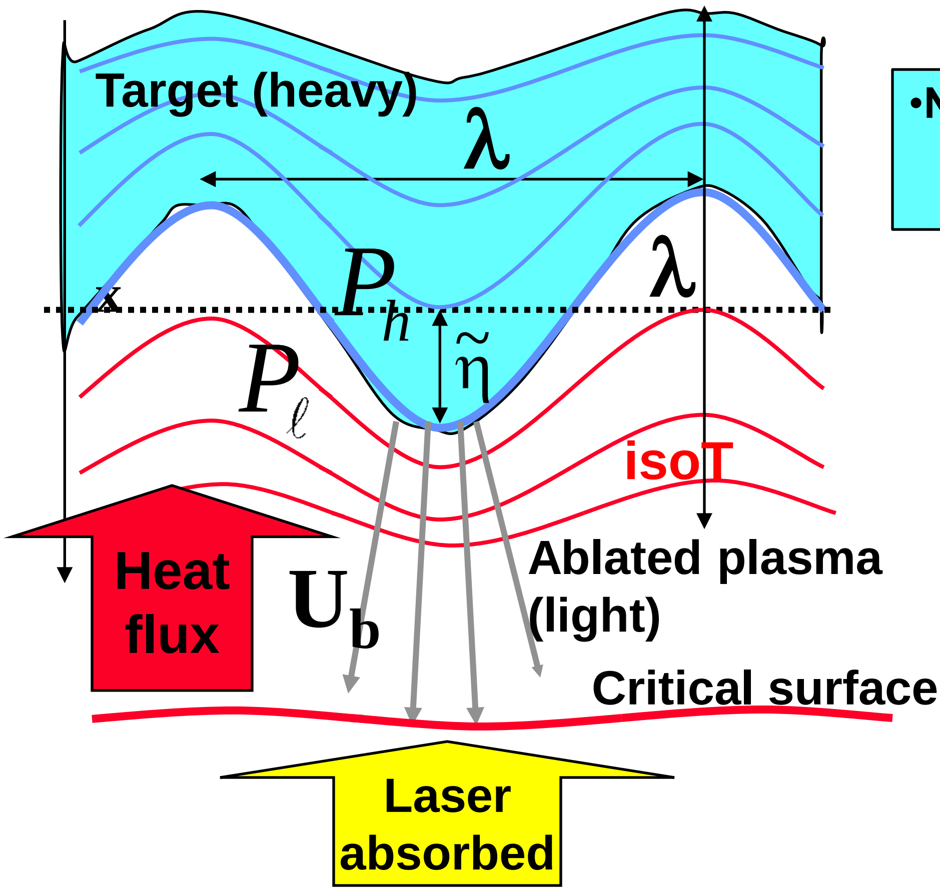
$$P_h = P_0 + \left[\frac{dP_0}{dx}\right]_h \tilde{\eta} = P_0 + \rho_h g \tilde{\eta}$$

$$F = ma \rightarrow S\rho_h g\tilde{\eta} = \rho_h \lambda S \ddot{\tilde{\eta}} \rightarrow \ddot{\tilde{\eta}} = kg\tilde{\eta} \rightarrow \tilde{\eta} \sim e^{\gamma t} \rightarrow \gamma = \sqrt{kg}$$

Classical growth rate

Rayleigh, Proc. London Math. Society, 1883
Taylor, Proc. Royal Soc. of London, 1950

The ABLATIVE R-T is just Newton's law at work again but with a restoring force: the dynamic pressure.



•Newton's law

$$S[P_h - (P_\ell + \rho_\ell u_b^2)] = \rho_h \lambda S \ddot{\tilde{\eta}}$$

Dynamic pressure

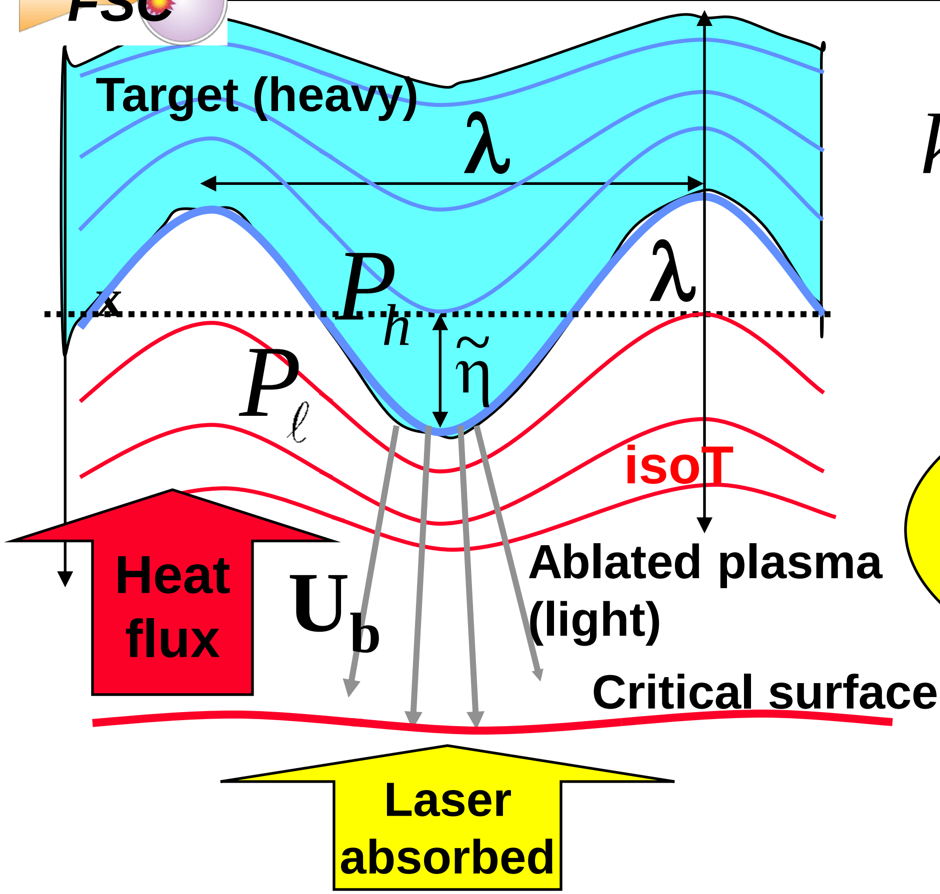
• Perturbed dynamic pressure

$$\widetilde{\rho_\ell u_b^2} \sim \rho_\ell U_b \tilde{u}_b$$

Ablation rate = $\dot{m}_a$

$$u_b = U_b + \tilde{u}_b$$

The perturbed dynamic pressure is stabilizing



$$k(\rho_h g\tilde{\eta} - \dot{m}_a \tilde{u}_b) = \rho_h \gamma^2 \tilde{\eta}$$

Energy flow balance
(see Appendix)

$$\frac{5}{2} p u_b \approx q_{heat} \Rightarrow \tilde{u}_b \approx k U_b \tilde{\eta}$$

$$\gamma = \sqrt{kg - k^2 U_a U_b}$$

Ablation introduces a cutoff (wave number) in the unstable spectrum

S. Bodner, Phys. Rev. Lett. 33, 761 (1974)
H. Takabe et al, Phys. Fluids 28, 3676 (1985)
J. Sanz, Phys. Rev. Lett. 73, 2700 (1994)
V. Goncharov PhD Thesis, U. Rochester (1996)
R. Betti, et al, Phys. Plasmas 3, 2122 (1996)
R. Piriz. J. Sanz, L. Ibanez, Phys Plasmas 4, 1118, (1996)
R. Betti et al, Phys. Plasmas 5., 1446 (1998)

Appendix: Perturbed blow-off velocity

Start from flux balances

$$\frac{5}{2}pu_b = -q_{heat} = \kappa\nabla T$$

$u_b = u_b^0 + \tilde{u}_b$

$T = T^0 + \tilde{T}$

← **Linearize: equilibrium** ^{0} **+ perturbation** $\tilde{}$

Use equilibrium fluxes are → balanced

$$\frac{5}{2}p(u_b^0 + \tilde{u}_b) = \kappa\nabla T^0 + \kappa\nabla\tilde{T} \Rightarrow \frac{5}{2}p\tilde{u}_b = \kappa\nabla\tilde{T}$$

Use

$$\frac{5}{2}p\tilde{u}_b = \kappa k\tilde{T}$$

find ← $\tilde{\nabla} \sim k$

Define → temperature perturbation

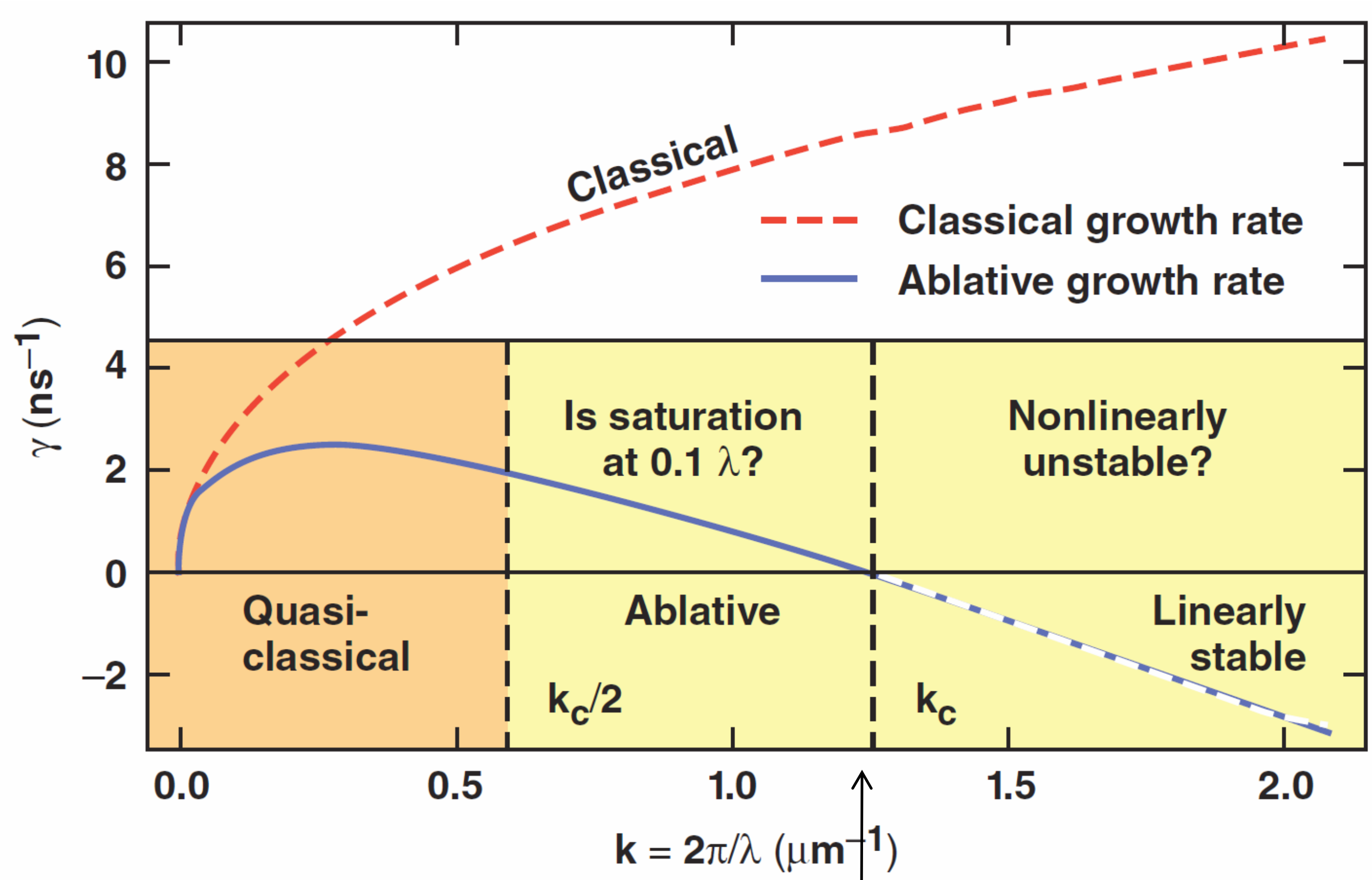
$$\tilde{T} \equiv T^0(\tilde{\eta}) - T^0 \text{ since } T(\tilde{\eta}) = T^0 + \frac{dT^0}{dx}\tilde{\eta} \Rightarrow \tilde{T} = \frac{dT^0}{dx}\tilde{\eta}$$

Taylor expansion

$$\frac{5}{2}p\tilde{u}_b = k\kappa\frac{dT^0}{dx}\tilde{\eta} = \frac{5}{2}pu_b^0 k\tilde{\eta} \Rightarrow \tilde{u}_b = u_b^0 k\tilde{\eta}$$

← **Use equilibrium**

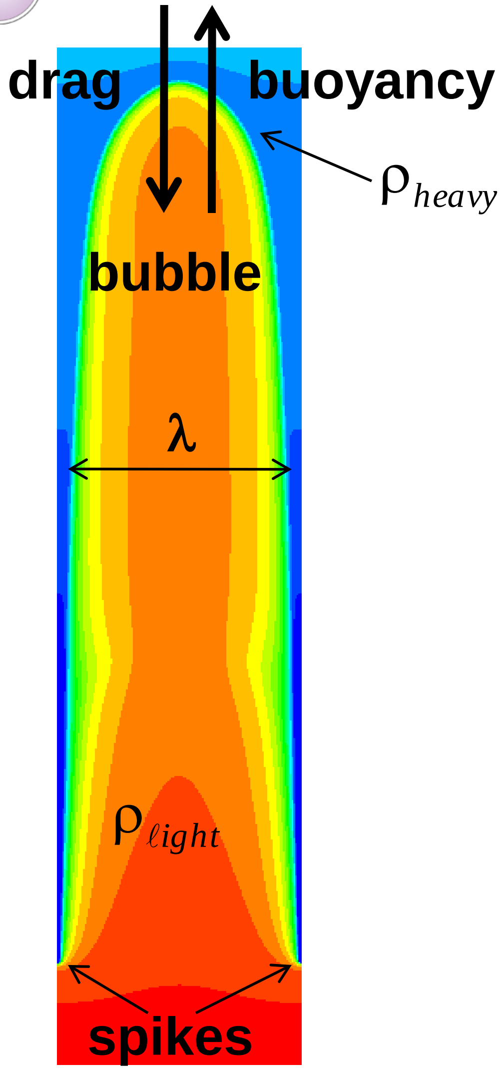
The ablative growth rate is significantly less than the classical value. Modes with k> k_c are stable



Cutoff wave number

u_a=3.5μm/ns
g=100 μm/ns^2

Nonlinear classical RT: the bubble velocity saturates when the bubble amplitude is ~0.1λ. The bubble amplitude does not saturate



Buoyancy ~ $(\rho_h - \rho_\ell) S\lambda g$

Drag ~ $\rho_h U^2 S$

Saturation → buoyancy=drag

$$U_{bubble}^{sat} \sim \sqrt{\lambda g\left(1-\frac{\rho_\ell}{\rho_h}\right)}$$

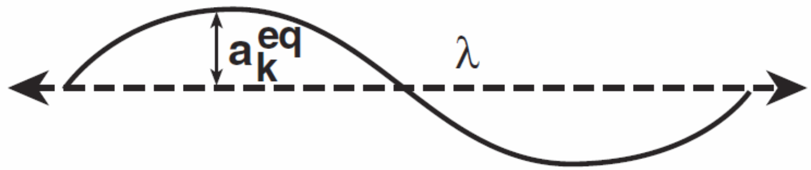
$$U_{bubble}^{sat(2D)} \approx \sqrt{\frac{g}{3k}\left(1-\frac{\rho_\ell}{\rho_h}\right)} \qquad U_{bubble}^{sat(3D)} \approx \sqrt{\frac{g}{k}\left(1-\frac{\rho_\ell}{\rho_h}\right)}$$

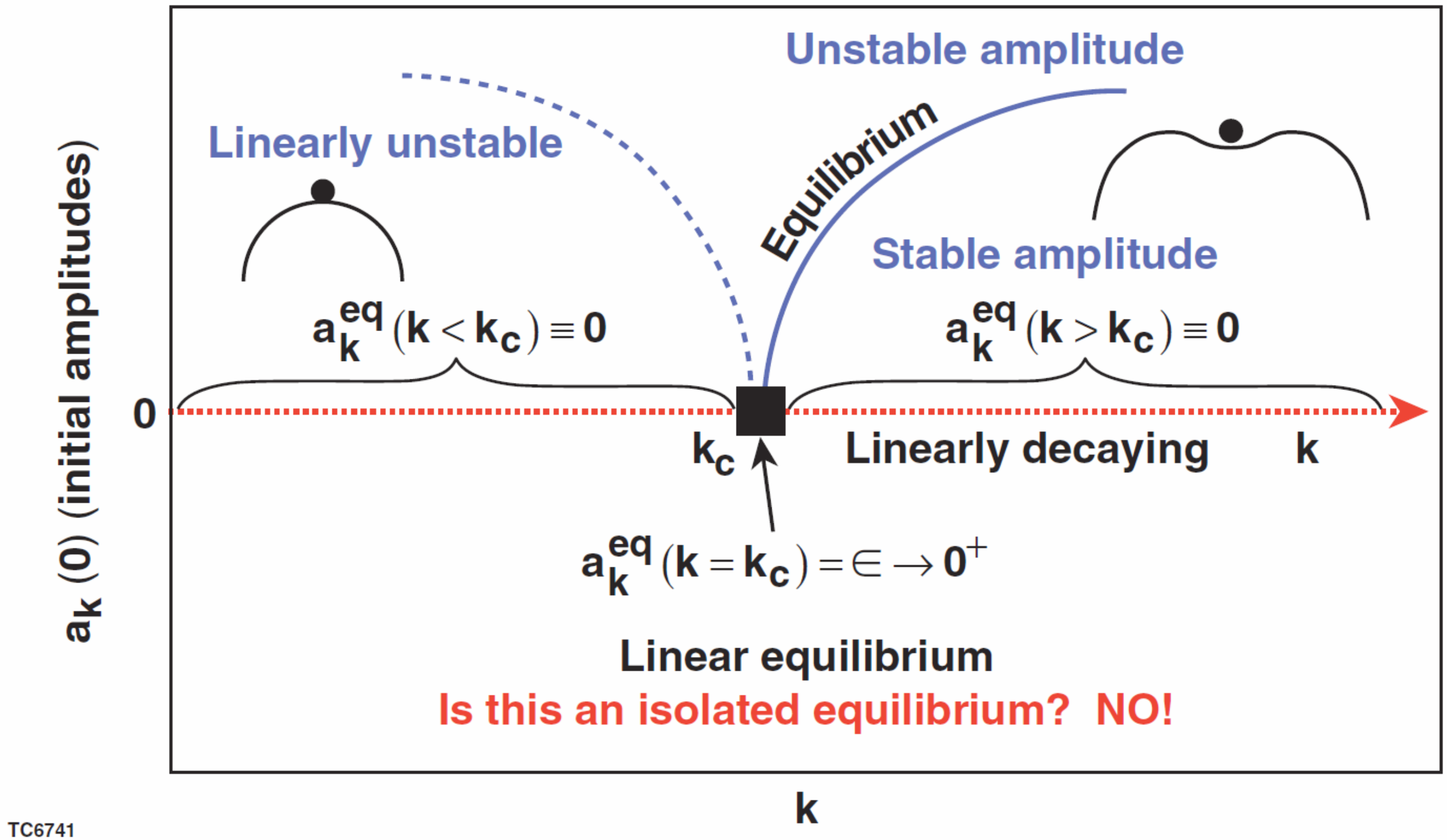
Transition to saturation:
linear bubble velocity = saturated velocity

$$\tilde{\eta} = \tilde{\eta}(0)e^{\gamma t} \qquad \dot{\tilde{\eta}} = \gamma\tilde{\eta} \approx U_{bubble}^{sat} \qquad \tilde{\eta}_{sat}^{2D} \approx 0.1\lambda$$

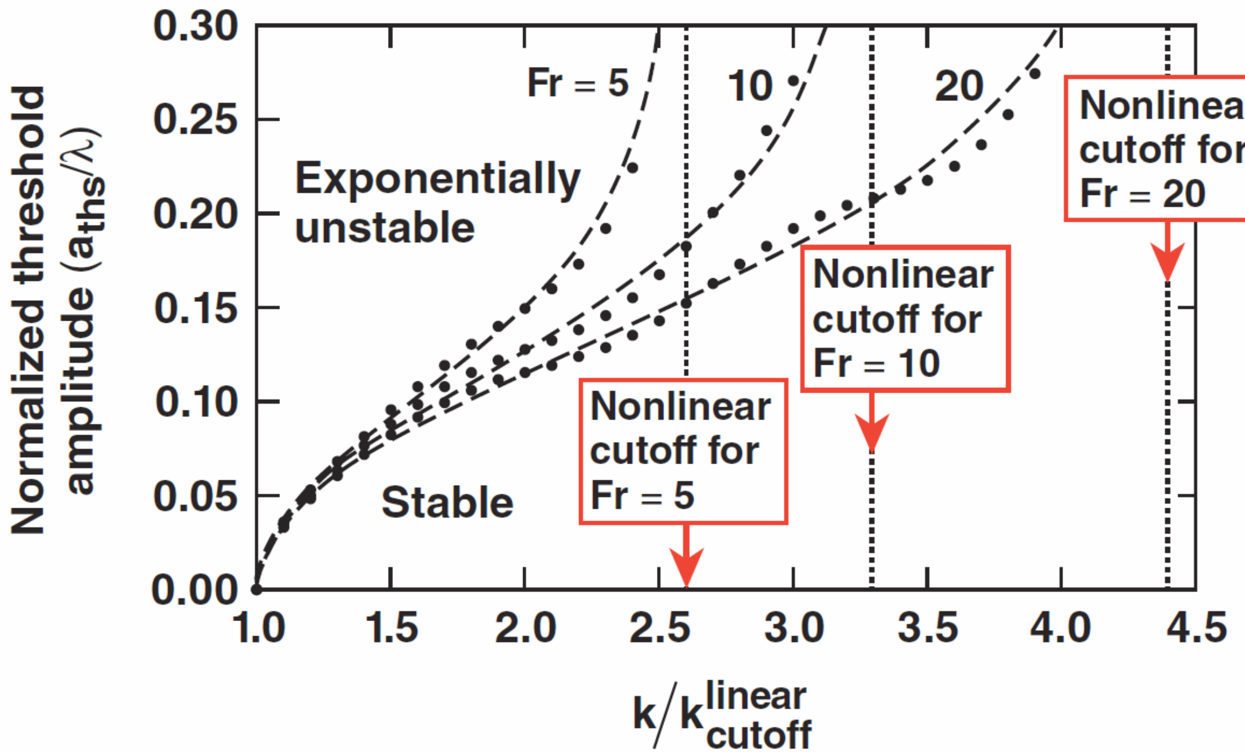
D. Layzer, Astrophys. Journal 122, 1 (1955)

What can we infer about the nonlinear ablative RT by simply looking at the linear spectrum?





The theory predicts full nonlinear stability only for wave numbers exceeding a nonlinear cutoff beyond the linear cutoff



TC6742

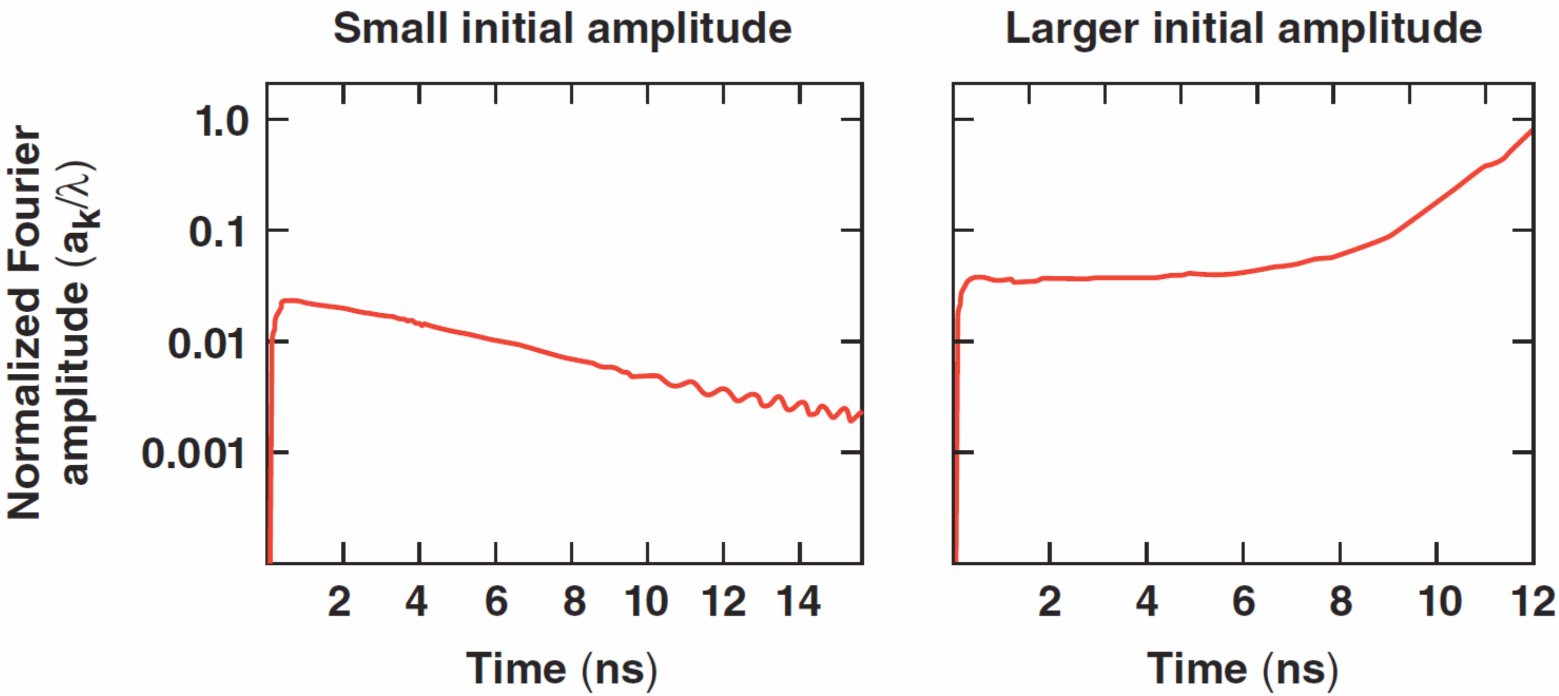
$$Fr = \frac{V_a^2}{gL_0}$$

$$k_{cutoff}^{nonlinear} = \frac{g}{3V_a^2}\left(\frac{2A}{1+A}\right)$$

J. Sanz, J. Ramirez, R. Ramis, R. Betti, RPJ Town, Phys. Rev. Lett. 89, 195002 (2002)
J. Sanz, R. Betti, R. Ramis, R. Ramirez 12B, B368-B370, Plasma Phys. Cont. Fus. (2004)

A linearly stable perturbation ($k > k_{cutoff}$) becomes unstable for a sufficiently large initial amplitude (as predicted by theory)

$k > k_{cutoff}$: Linearly stable 10-μm perturbation

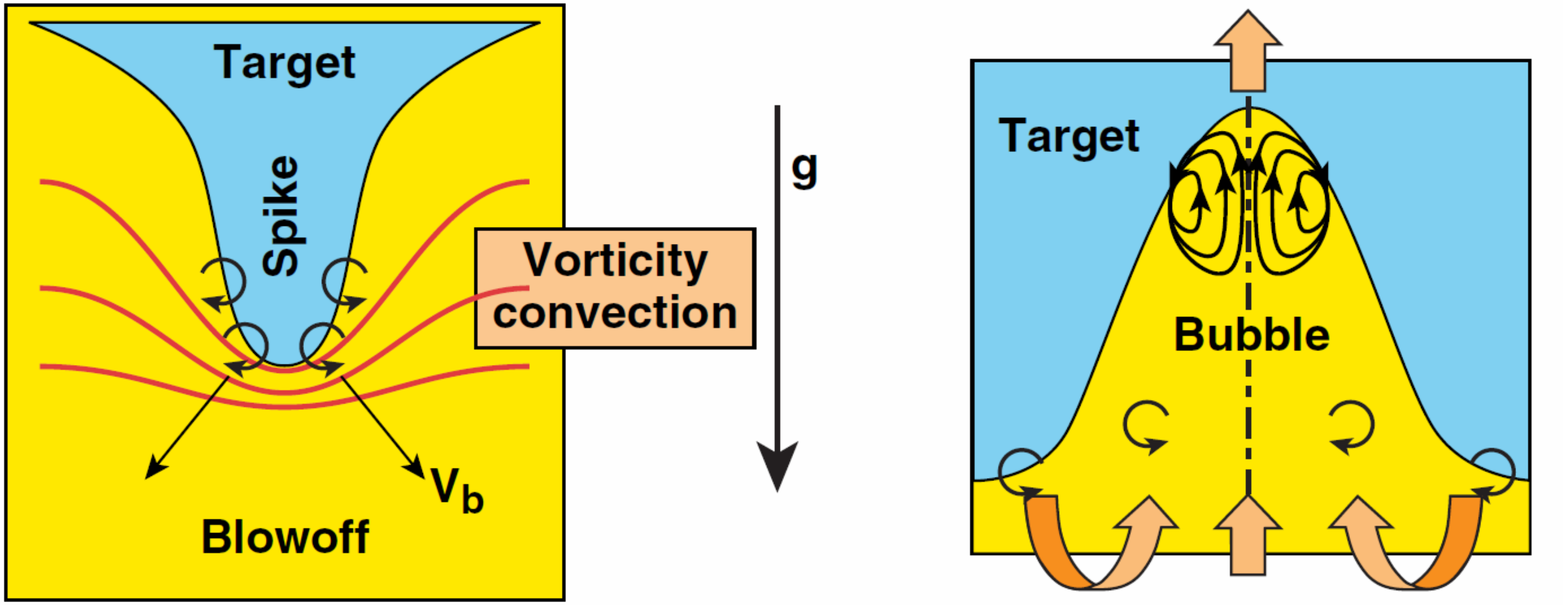


R. Betti, J. Sanz, APS-DPP Bulletin (2004)

In the deeply nonlinear phase, the vorticity accumulates inside the bubble raising the bubble terminal velocity

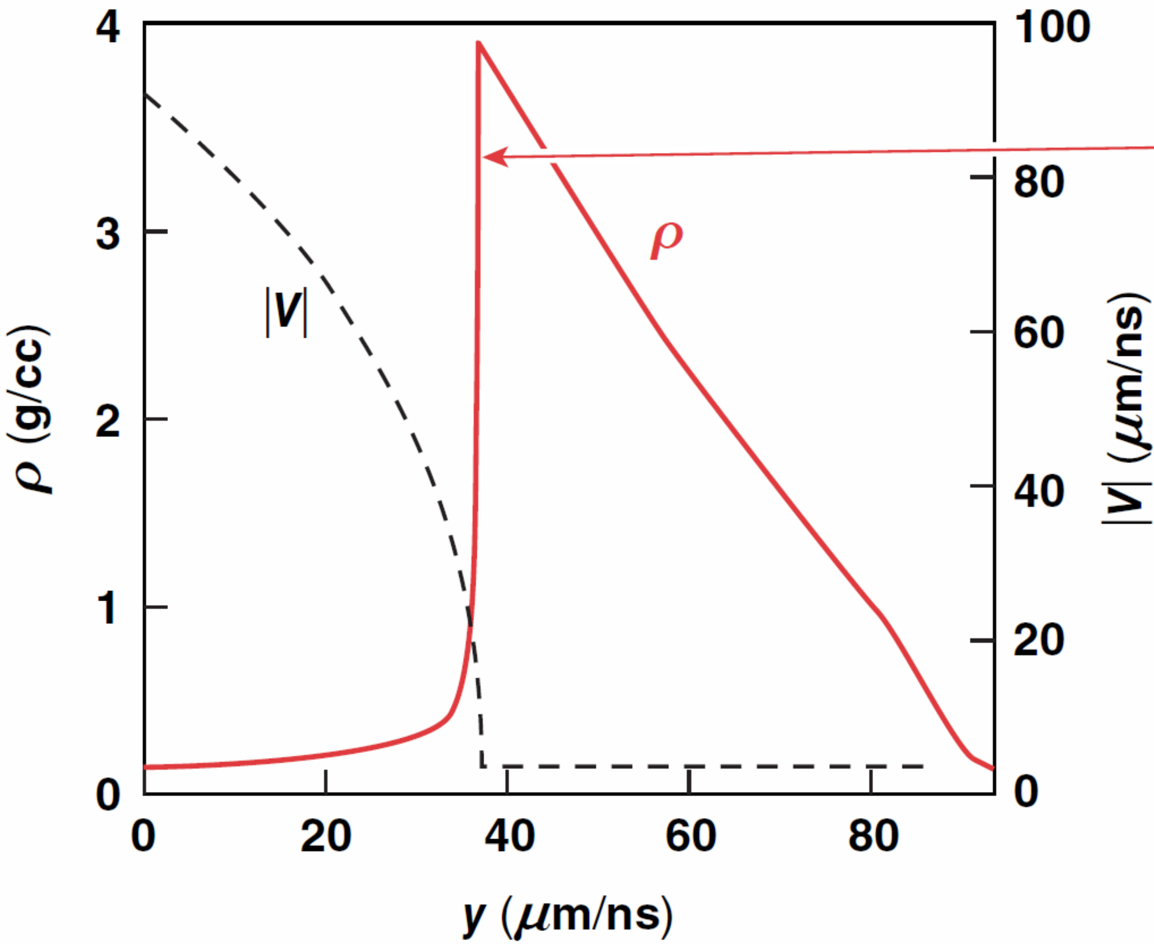
Vorticity is convected from the ablation front.

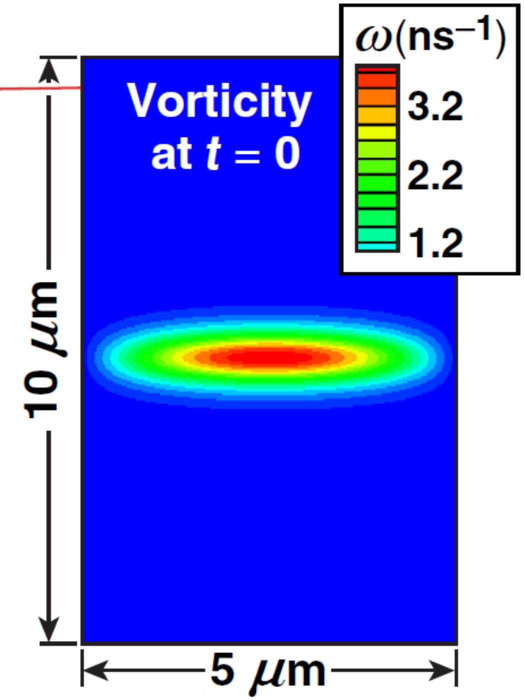
Vorticity accumulates in the bubble. The bubble accelerates.



R. Betti and J. Sanz, Phys. Rev. Lett. 97, 205002 (2006)

Single-mode simulation of the deeply nonlinear ablative Rayleigh-Taylor instability

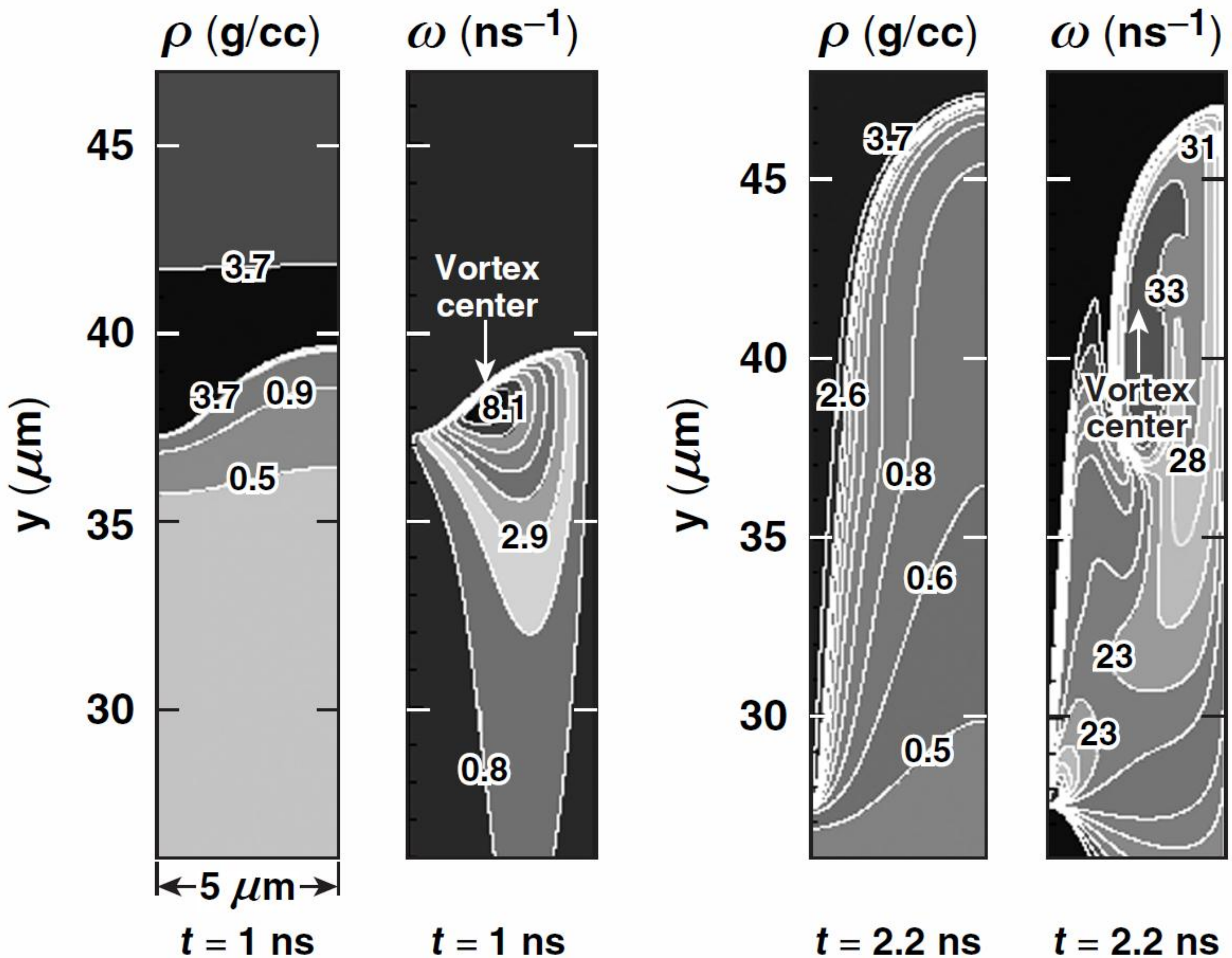




A weak vortex is used as initial perturbation.

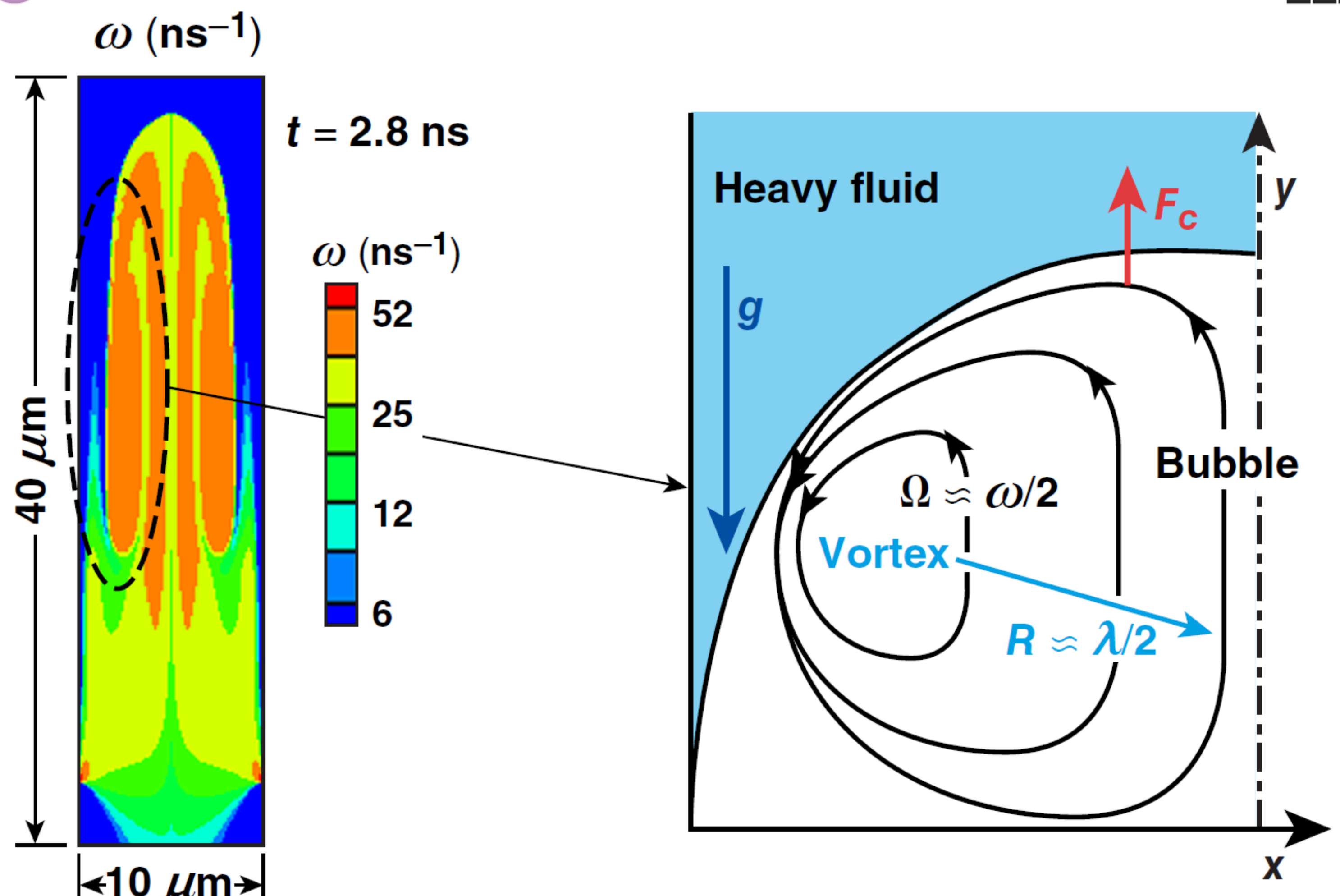
TC7556

Simulations show vorticity convection and accumulation

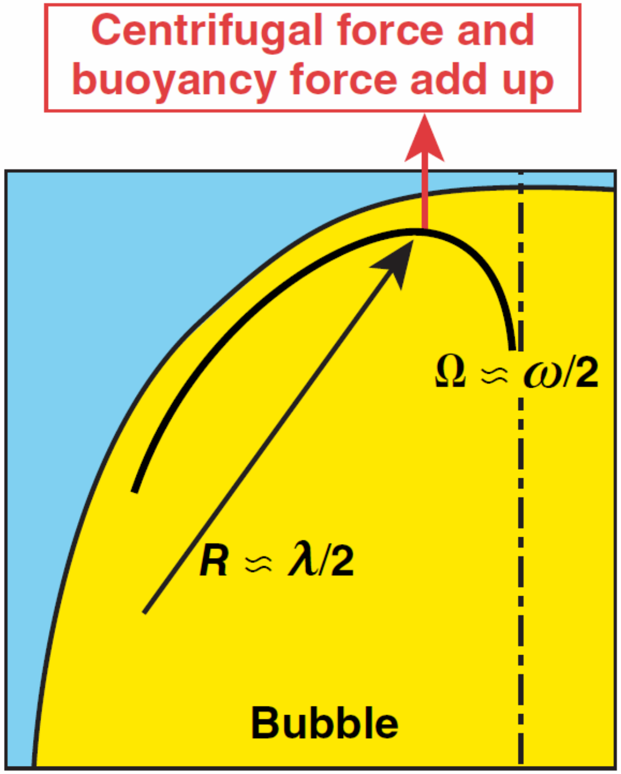


TC7557

A large vortex forms inside the bubble; the vortex generates a centrifugal force (F_c) pushing on the bubble tip



The asymptotic bubble velocity is higher than the classical value due to the vorticity accumulated inside the bubble



$$r_d = \frac{\rho_{\text{bubble}}}{\rho_{\text{dense}}}$$

Centrifugal force:

$$\rho_{\text{bubble}} R\Omega^2 = \rho_{\text{bubble}} \frac{R\omega^2}{4}$$

Buoyancy force:

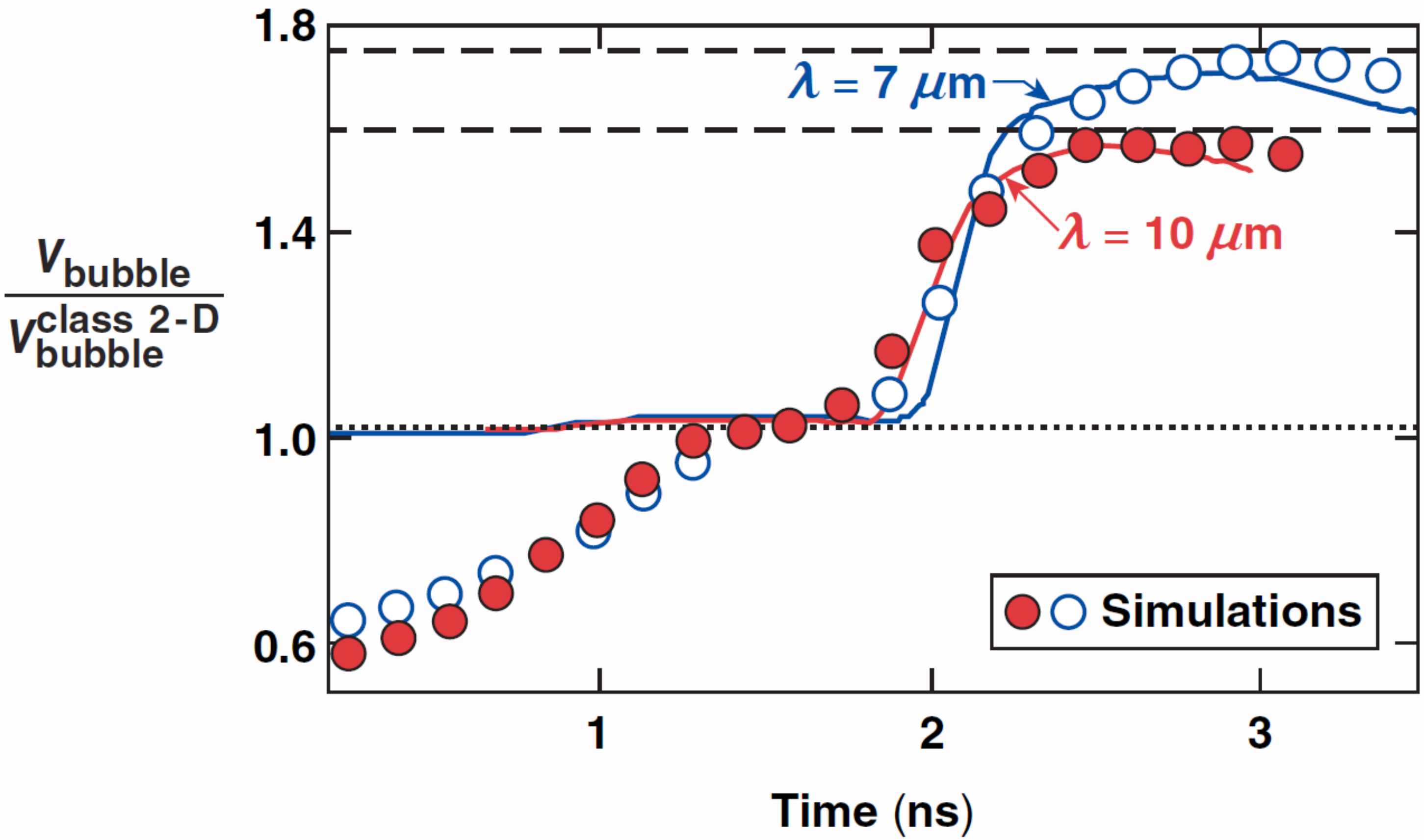
$$\left(\rho_{\text{dense}} - \rho_{\text{bubble}}\right) g$$

$$V_{\text{bubble}}^{\text{vort}} = \sqrt{\frac{g}{3k}\left(1 - r_d\right) + r_d \frac{\omega^2}{4k^2}}$$

Bubble-velocity enhancement

TC7561

The bubble accelerates to final velocities well above the classical value and in agreement with the theory

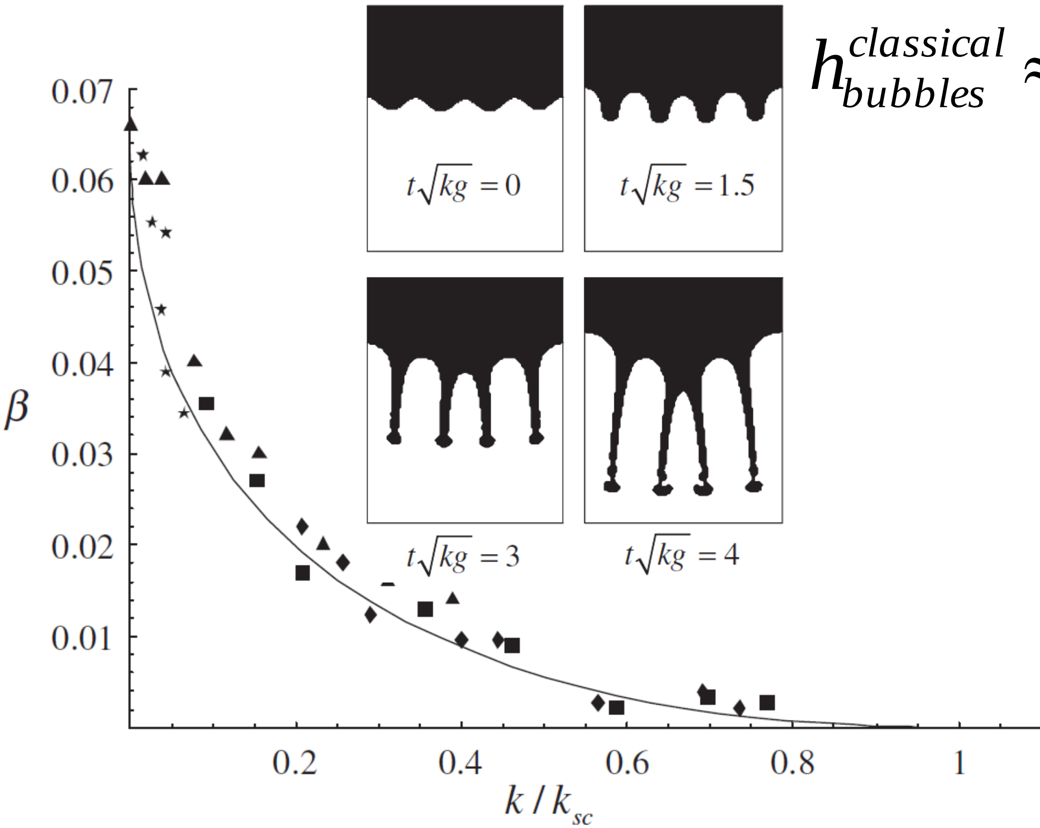


— $\sqrt{\frac{g}{3k}(1-r_d)+r_d\frac{\omega(t)^2}{4k^2}}$ - - $\sqrt{\frac{g}{3k}(1-r_d)+\frac{V_a^2}{r_d}}$

TC7564

R. Betti and J. Sanz, Phys. Rev. Lett. 97, 205002 (2006)

Multimode nonlinear interaction leads to an envelop growth of the bubble front $h=\beta g t^2$ with β dependent on the box size



$$h_{bubbles}^{classical} \approx \beta_{cl} g t^2 \Rightarrow \beta_{cl} \sim 0.05$$

$$h_{bubbles}^{ablative} \approx \beta_{abl} g t^2$$

$$\beta_{abl} = \beta_{cl}\left(1 - \sqrt{\frac{k_{box}}{k_{nonl-cutoff}}}\right)^2$$

This conclusion does not include the effect of bubble acceleration

J. Sanz, R. Betti, R. Ramis, R. Ramirez 12B, B368-B370, Plasma Phys. Cont. Fus. (2004)

Hydrodynamic Simulations

Excellent tutorial at http://hedpschool.lle.rochester.edu/1000_proc2011.php by Radha Bahukutumbi

Hydrodynamic codes use a combination of Eulerian and Lagrangian grids (ALE = arbitrary Lagrangian-Eulerian)

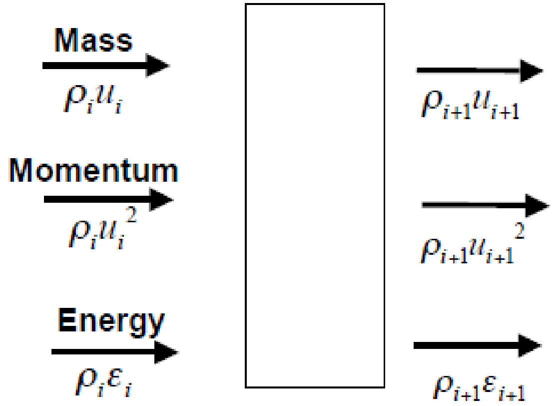
In the frame of the box
Eulerian form of fluid equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(P + \rho u^2)}{\partial x} = 0$$

$$\frac{\partial(\rho \varepsilon)}{\partial t} + \frac{\partial([P + \rho\varepsilon]u)}{\partial x} = 0$$

$$P = P(\rho, \varepsilon)$$



Equation-of-State (EOS)

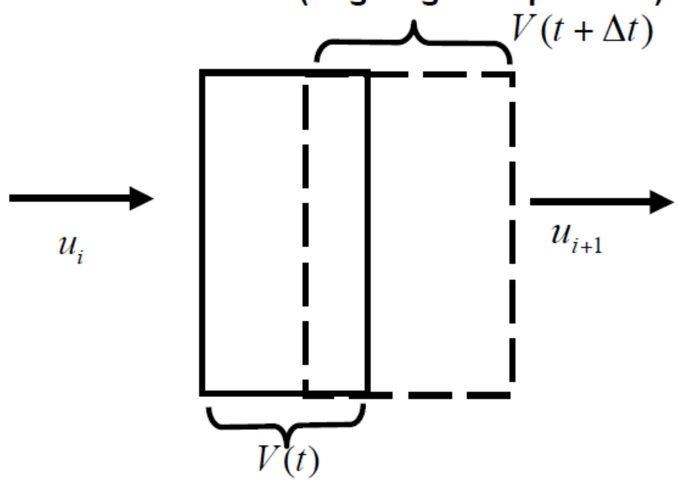
Replace to get flow equations in the fluid frame (Lagrangian equations)

$$\frac{d\rho}{dt} + \rho\frac{\partial u}{\partial x} = 0$$

$$\rho\frac{du}{dt} = -\frac{\partial P}{\partial x}$$

$$\frac{d\varepsilon}{dt} = \frac{P}{\rho^2}\frac{d\rho}{dt}$$

$$P = P(\rho, \varepsilon)$$



Shocks are treated by adding an artificial viscous term to the ion pressure

- Dissipative mechanisms like viscosity and heat conduction introduce a thin transition layer instead of a sharp discontinuity.

- Use an "artificial viscosity"

$$Q = a_o{}^2 \rho (\delta u)^2 \qquad \delta u < 0$$

$$= 0 \qquad \delta u \geq 0$$

- $P \rightarrow P + Q$ in fluid equations

- Originally proposed by Richtmyer and von Neumann

- No special internal boundary conditions required
- Shocks as approximate discontinuities in ρ, ε, P
- Obeys the basic conservation laws

The energy equation is simply extended to ions and electrons

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Lagrangian form of energy equation written in terms of two temperatures

$$\rho C_{ve}\frac{dT_e}{dt}=\frac{\partial}{\partial x}\left(\kappa_e\frac{\partial T_e}{\partial x}\right)-P_e\frac{\partial u}{\partial x}-\tau_{ei}^{-1}(T_e-T_i)+S_e$$ **Electrons**

$$\rho C_{vi}\frac{dT_i}{dt}=\frac{\partial}{\partial x}\left(\kappa_i\frac{\partial T_i}{\partial x}\right)-P_i\frac{\partial u}{\partial x}+\tau_{ei}^{-1}(T_e-T_i)+S_i$$ **Ions**

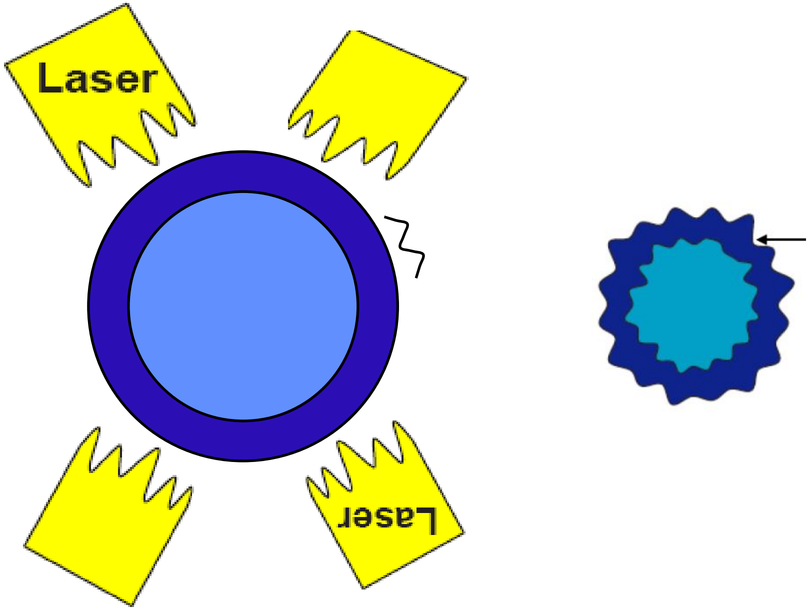
Specific heat

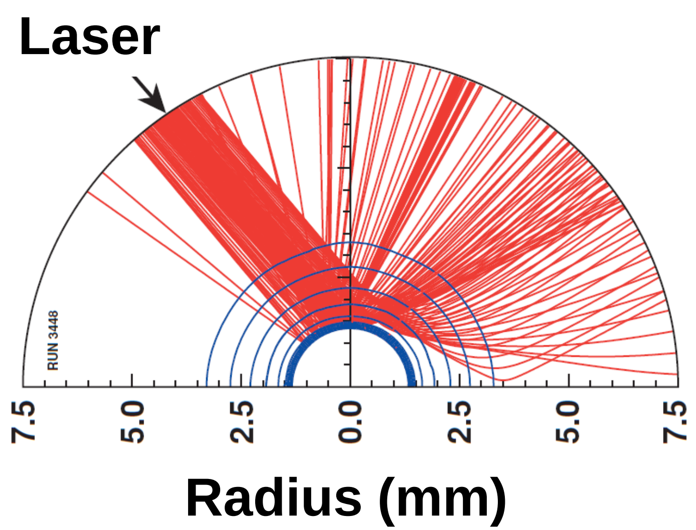
conductivity

Electron-ion relaxation time

Source terms
Ions – shock, alphas
Electrons – radiation, laser energy, alphas

Deposition of laser energy into a low density plasma is typically modeled using a ray trace algorithm



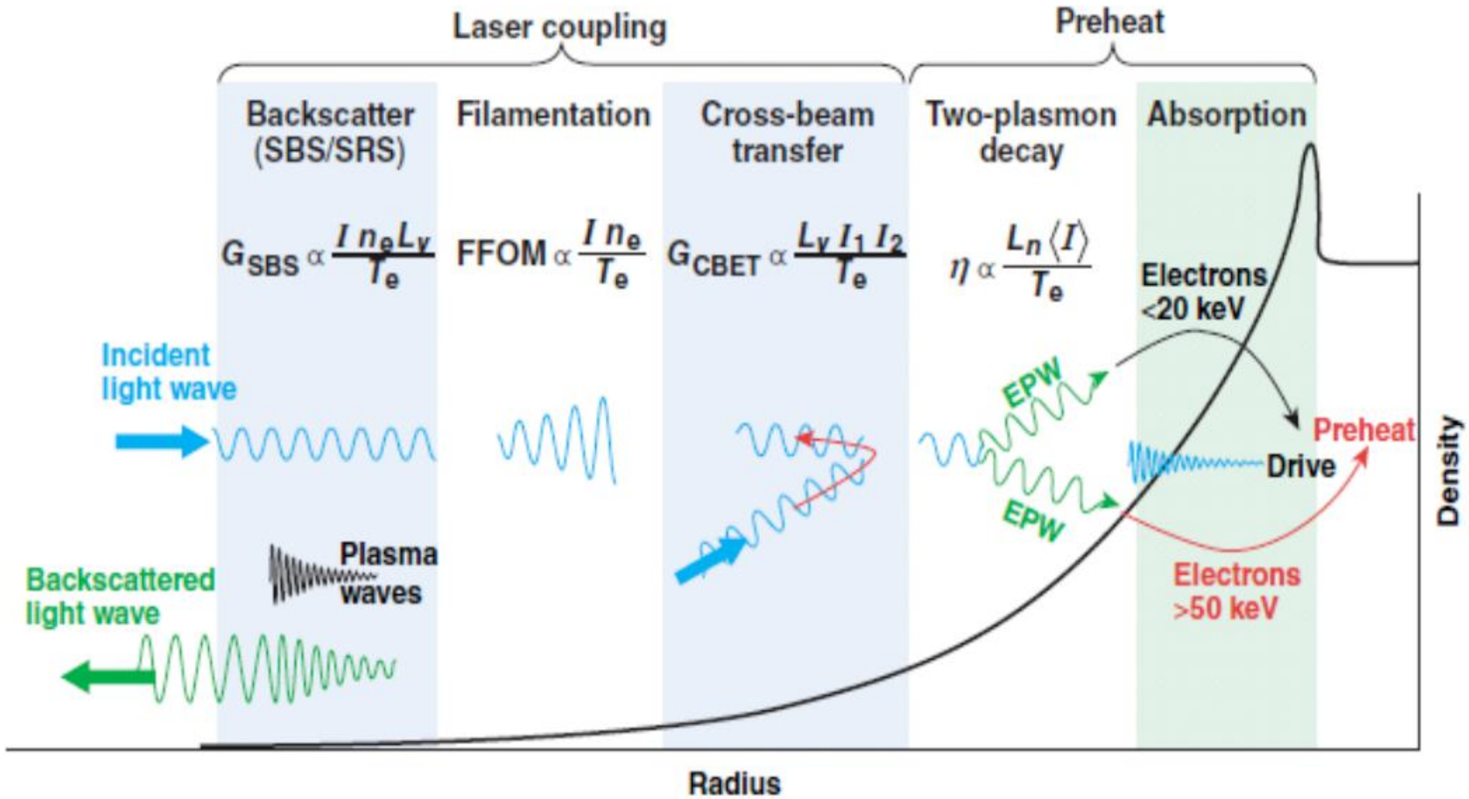


- Geometrical optics approximation is used with Inverse Bremsstrahlung as the mechanism for energy deposited in cell j

$$W_j = E_{j+1}\left(1 - e^{-\int_{r_o}^{r_1} K_{IB}\,ds}\right)$$

$$K_{IB} \sim \frac{n_e^{\,2} < Z^2 > \ln\Lambda}{\omega_o^{\,2}(m_e kT_e)^{3/2} < Z >}\left(1 - \frac{\omega_p^{\,2}}{\omega_o^{\,2}}\right)^{-1/2}$$

Geometrical optics ignores various coronal laser-plasma interactions



- Typically thresholds for evaluated to identify if they are exceeded
- Ultimately empirical evidence is the best indicator of the adequacy of the raytrace approximation

Most fluid codes approximate radiative transfer using multi-group diffusion

- The frequency spectrum is divided into G bins.

$$\nu_g < \nu < \nu_g + \Delta\nu_g, g = 1, 2, \ldots G$$

- For each group solve the diffusion equation.

emission absorption

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + \frac{\partial u}{\partial x}\right)U_\nu = c\rho\kappa_\nu(B_\nu - U_\nu) - \frac{\partial q_\nu}{\partial x} - P_\nu\frac{\partial u}{\partial x}$$

Radiation energy density

opacity

Planck function

Radiative flux

Radiative pressure

- The energy removed or added to each zone is summed over all energy groups and added as a source term to the heat equation.

Empirical studies indicate that heat flux inhibition is required in laser driven plasmas

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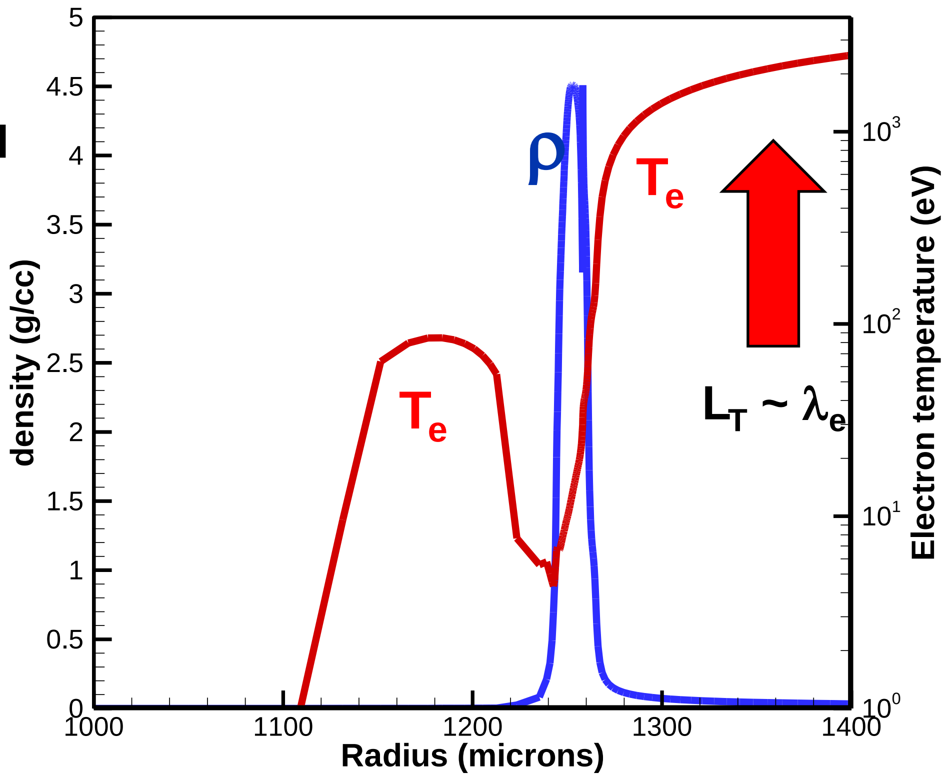
$L_T = |T/\nabla T|$ = temperature gradient scale length

λ_e = electron mean free path

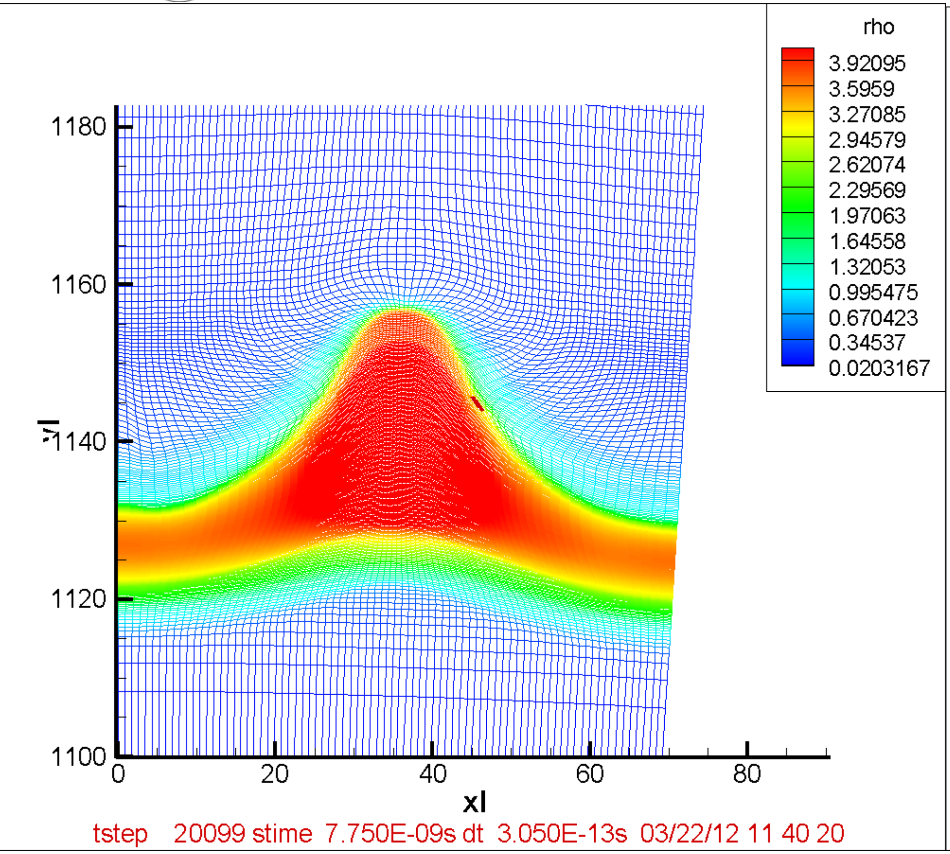
$\lambda_e \ll L_T \rightarrow$ diffusive heat transport SH

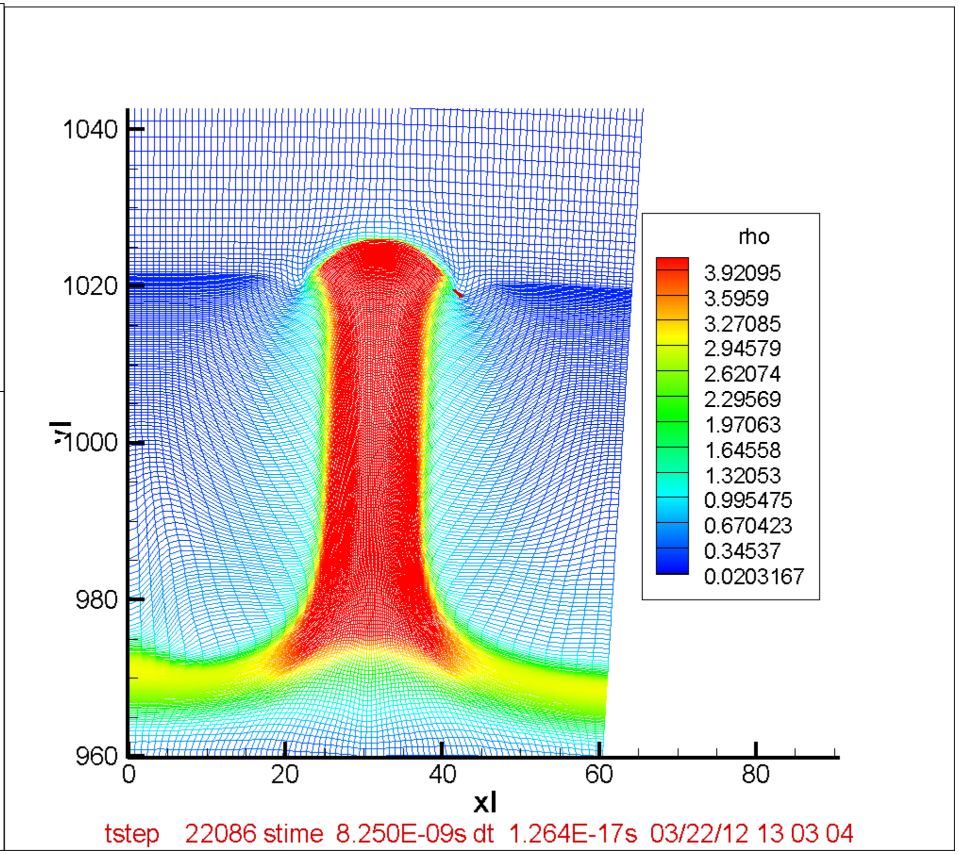
$\lambda_e \geq L_T \rightarrow$ nonlocal heat transport FS

- $q_{SH} = -\kappa \nabla T \quad q_{FS} = nTV_T$
- Sharp cutoff $\quad q_{eff} = \min(q_{SH}, f q_{FS})$
- $0.04 < f < 0.1$

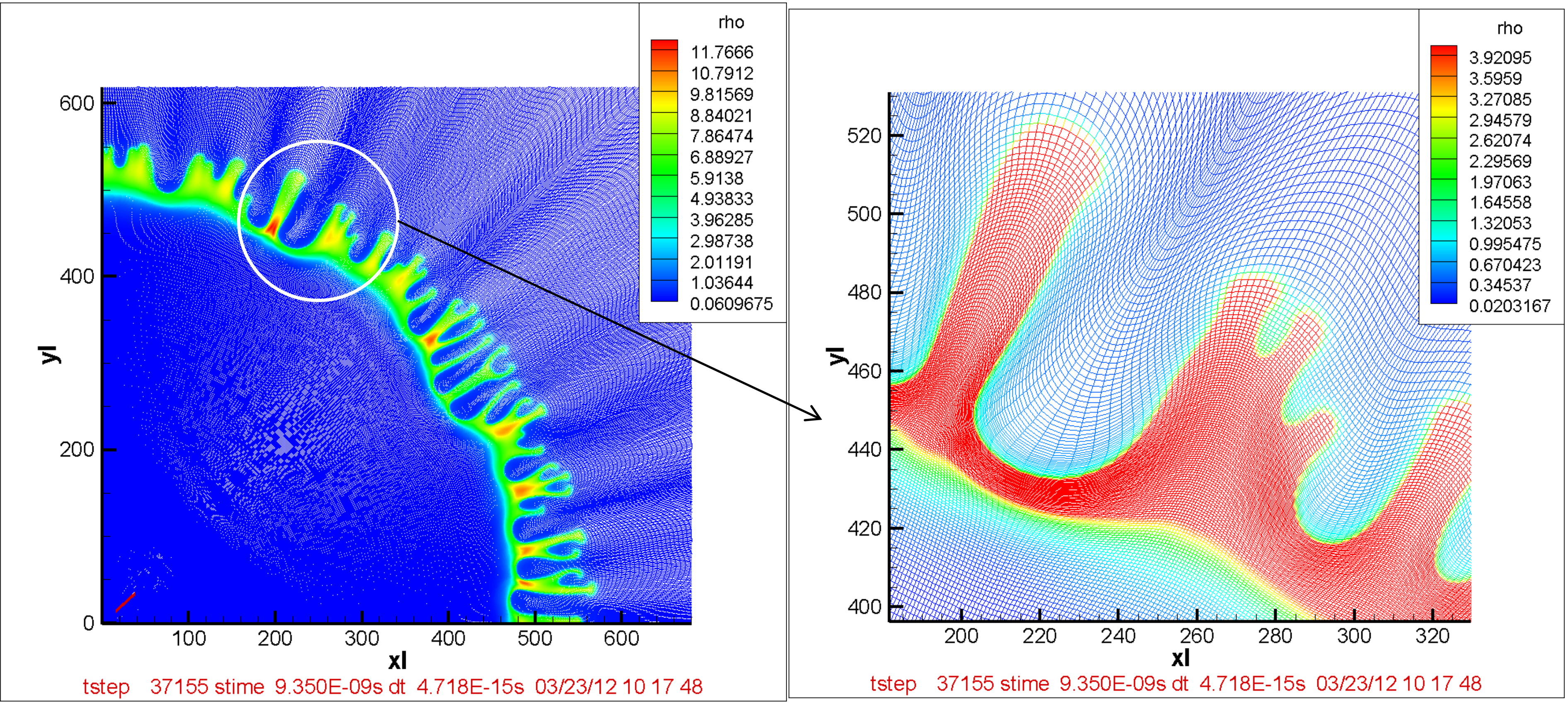


Single mode simulations are carried out to assess the suitability of the ALE (arbitrary Lagrangian-Eulerian) grid





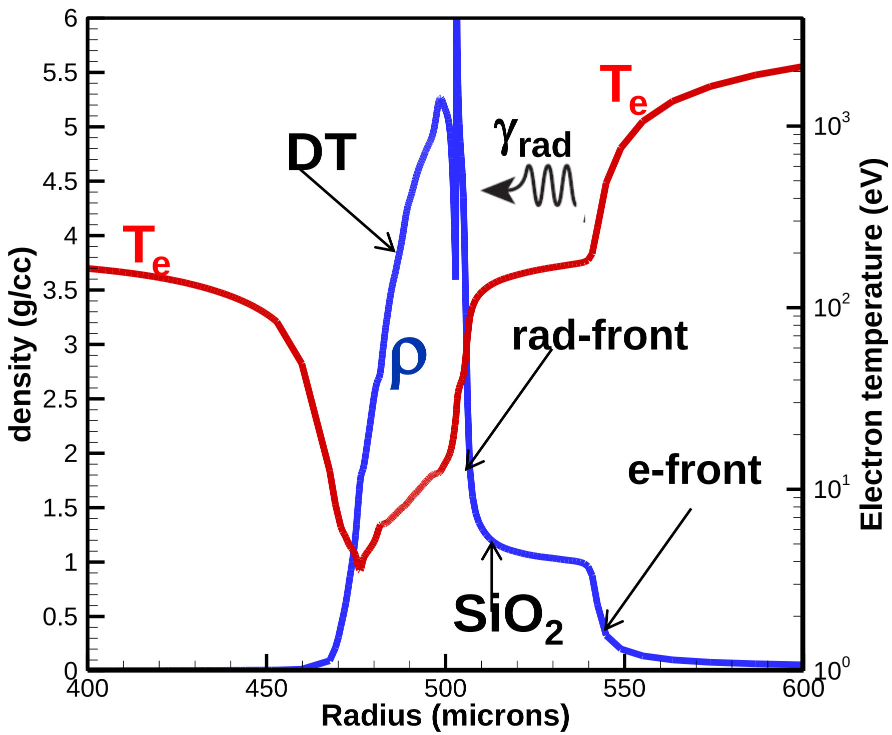
Multimode laser-imprinting simulations of CH+DT targets show Rayleigh-Taylor instability growth at the ablation front



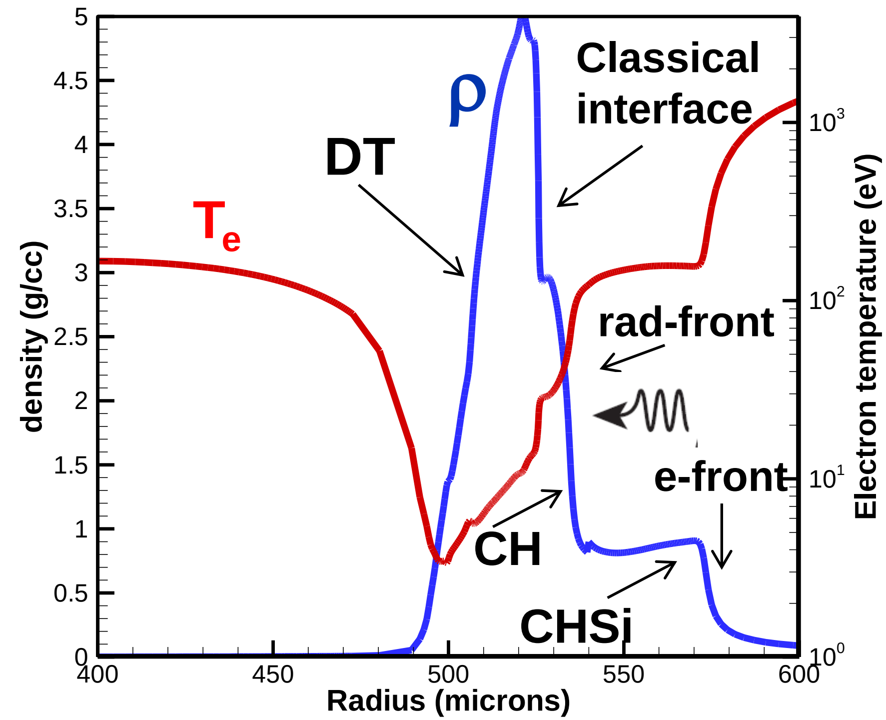
Simulations by R. Nora (LLE)

Imploding shells can develop multiple unstable interfaces depending on the material opacities

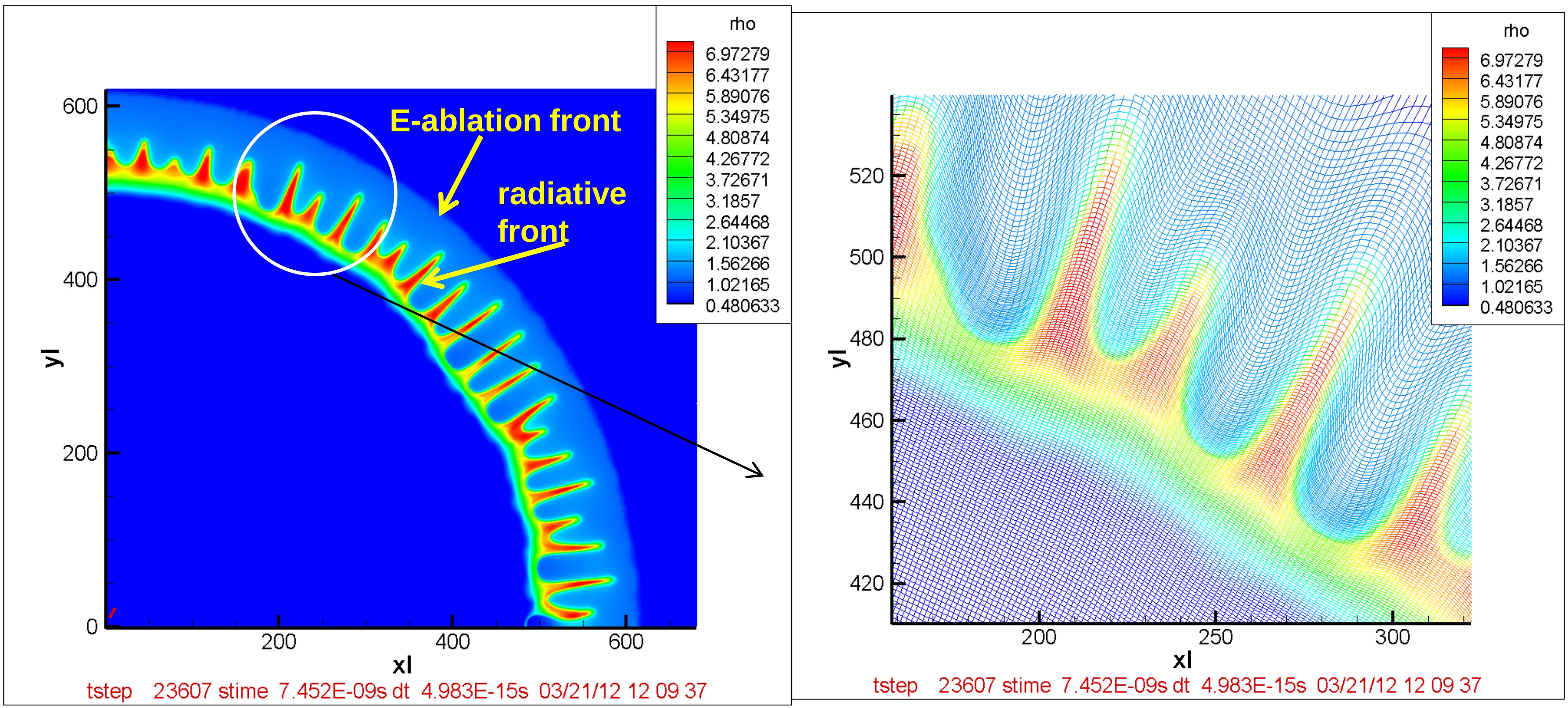
DT ice + SiO_2
Double ablation front



DT ice + CH+CHSi(5%)+Si
Double ablation front + Classical interface



Multimode laser-imprinting simulations of SiO_2 +DT targets show Rayleigh-Taylor instability growth at the radiative/classical front



Simulations by R. Nora (LLE)

conclusions

Hydrodynamic instabilities in ICF are generally understood but challenges remain in developing an accurate predictive capability for 2D-3D multimode interactions

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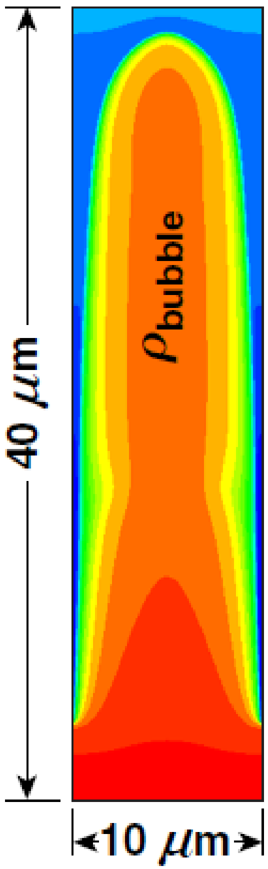
- **The linear theory of the ablative RT is fully developed and the physics is well understood. Ablation is stabilizing in the linear phase.**
- **The nonlinear single-mode evolution is well understood and ablation is destabilizing in the deeply nonlinear phase (bubble acceleration)**
- **The effect of the initial conditions on the nonlinear multimode ablation front dynamics is not well understood**
- **An accurate evaluation of the instability seeds from laser non-uniformities (imprinting) is difficult**
- **Three dimensional simulations are computational expensive and great difficulties remain in developing accurate nonlinear multimode simulations (this also applies to 2D simulations).**

BACK UP SLIDES ON NONLINEAR ARTI

The density in the bubble is the same as predicted by the linear theory* and a significant fraction of the dense target density



$$\rho_{bubble}^{linear} \approx (0.1\, kL_m)^{2/5} \rho_{dense}$$

$$\rho_{dense} \approx 4 \text{ g/cc}$$

$$L_m \approx 0.18\ \mu\text{m} \qquad \lambda \approx 10\ \mu\text{m}$$

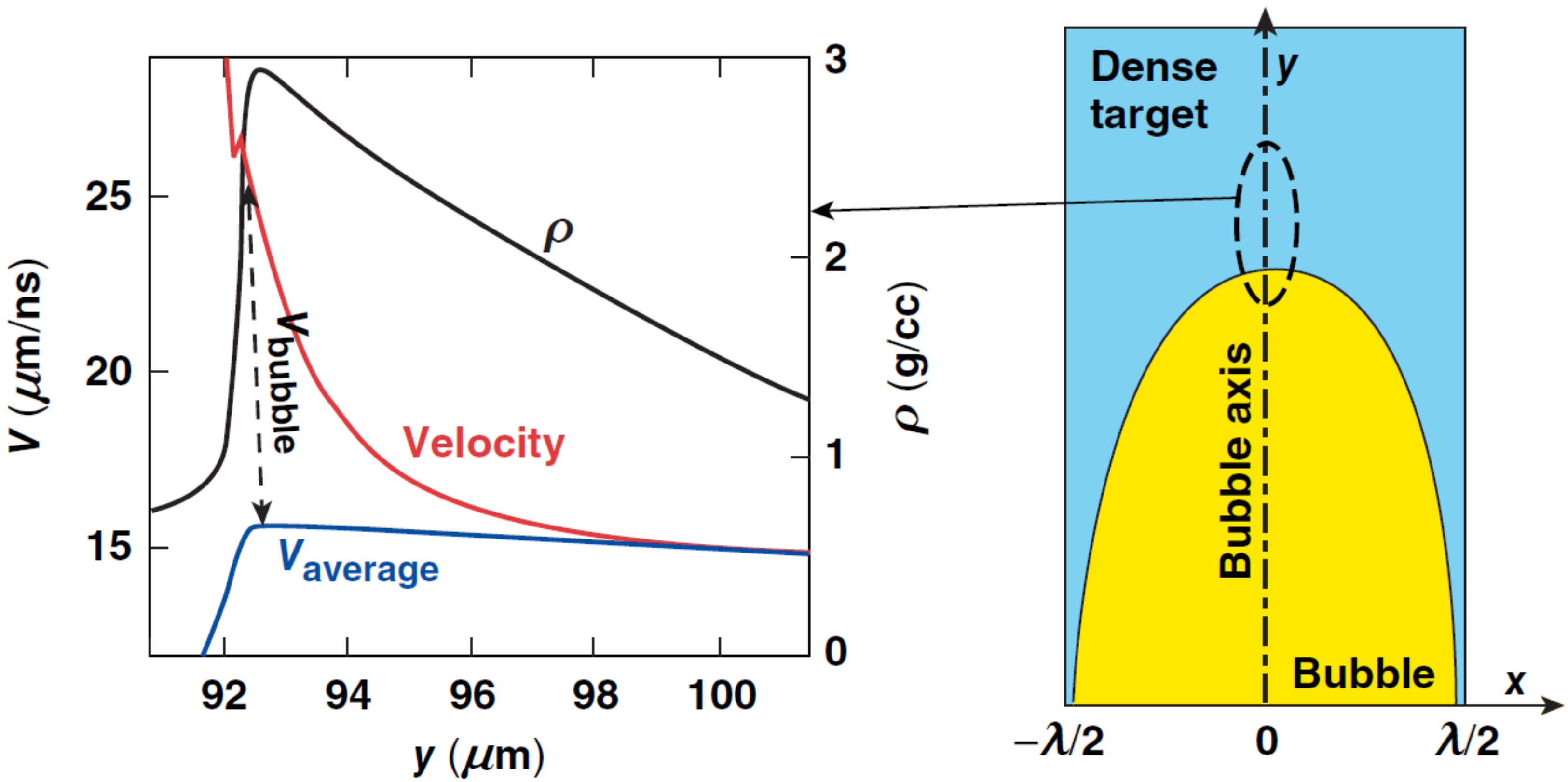
$$\rho_{bubble}^{linear} \approx 0.66 \text{ g/cc}$$

$$\rho_{bubble}^{simulation} \approx 0.65 \text{ g/cc}$$

L_m = the minimum density-gradient scale length
k = mode wave number

*R. Betti *et al.*, Phys. Plasmas 2, 2122 (1996).

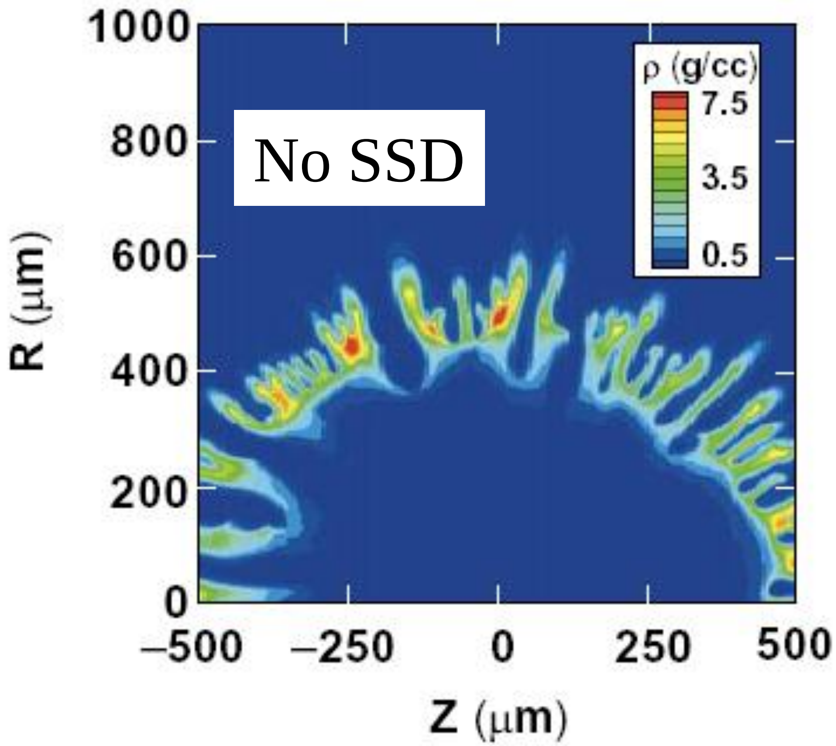
The bubble velocity is defined as the penetration velocity inside the overdense target

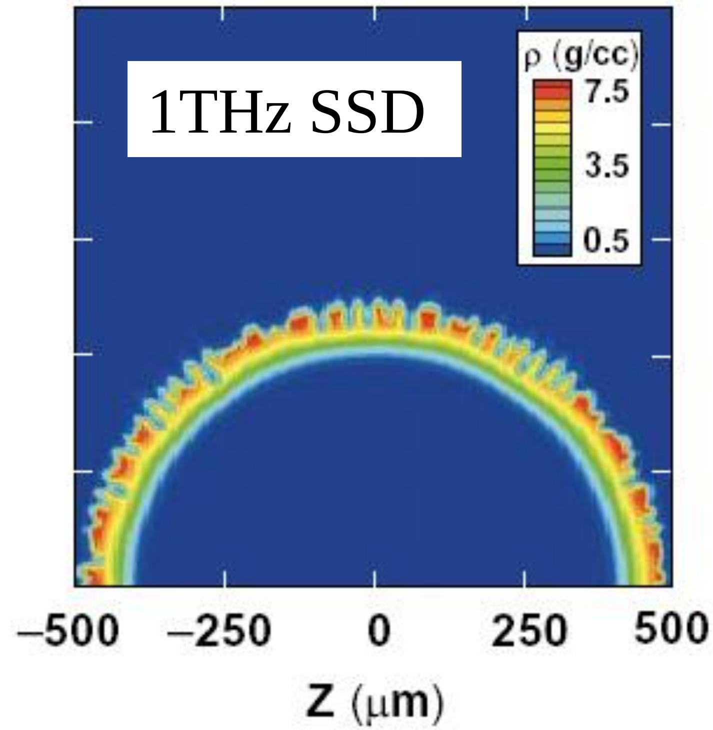


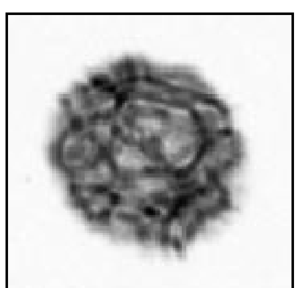
Mitigation techniques for the Rayleigh-Taylor instability in laser accelerated targets:

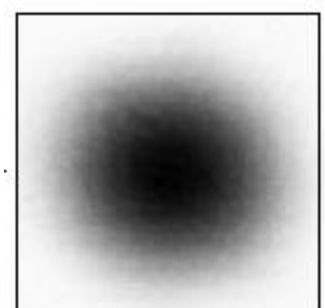
1. **Reduce the seeds**
2. **Reduce the growth rates**

Reducing the seeds for the RT (by making uniform laser beams) improve the integrity of the imploding shell









P. McKenty et al, Phys. Plasmas 2000

The growth rates can be reduced by shaping the entropy of the imploding shell

Stabilize the RT by increasing the ablation velocity

$$\gamma \approx 0.94\sqrt{kg} - 2.7ku_a$$

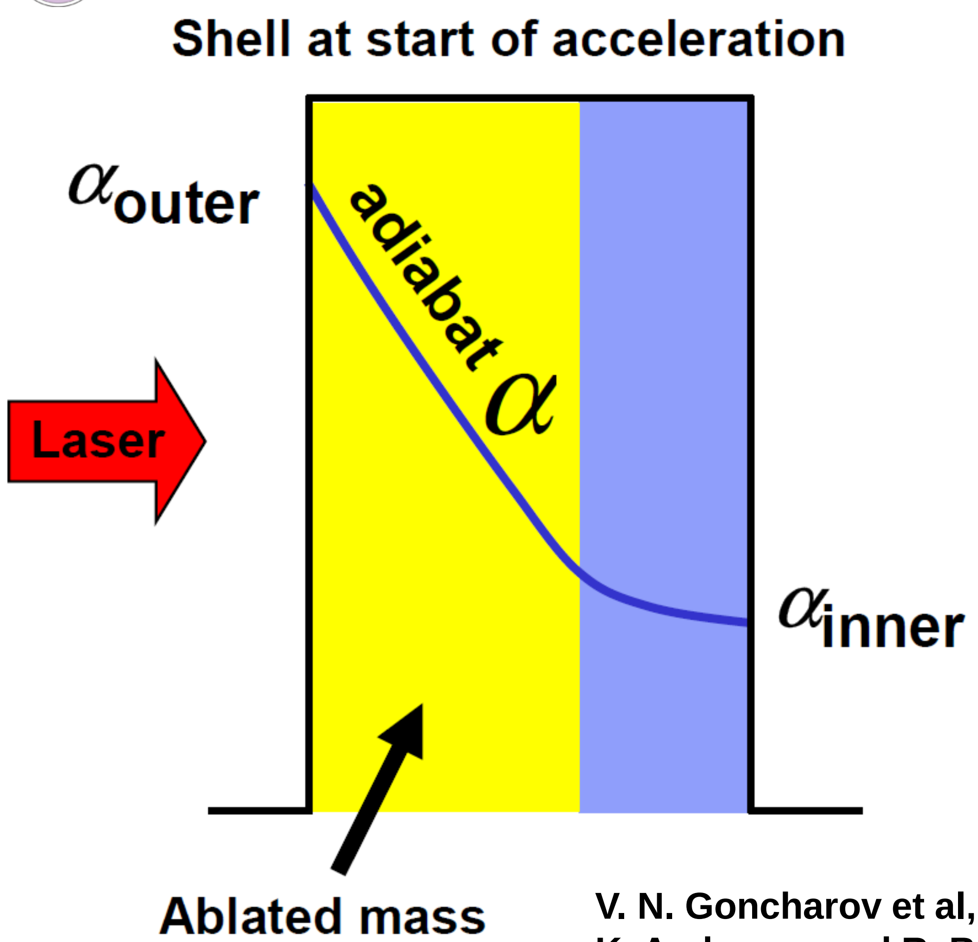
$$P_{app} \sim \alpha\rho^{3/5}$$

$$\alpha = \text{measure of entropy}$$

$$u_a = \frac{\dot{m}_a}{\rho} \sim \frac{\dot{m}_a}{P_{app}^{3/5}}\alpha^{3/5}$$

Increase the entropy (α) at the ablation front while keeping α low in the unablated material

Adiabat shaping reduces the RT growth without degrading the final compression



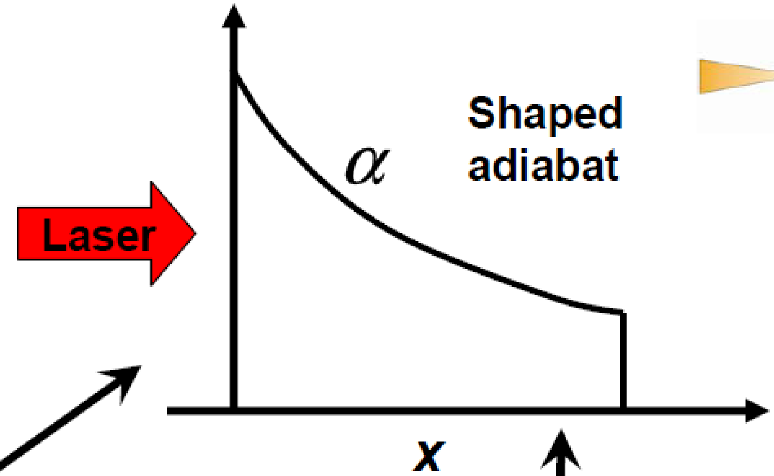
$$\alpha \sim \frac{P}{\rho^{5/3}}$$

Adiabat shaping can be done by shaping *P* or ρ or both.

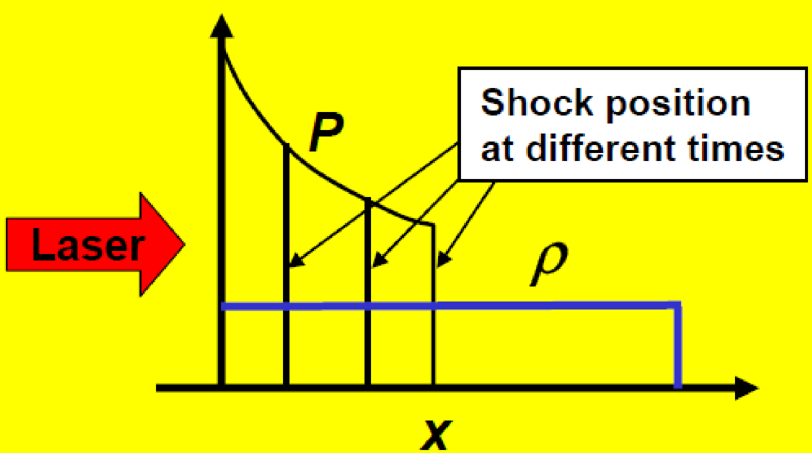
V. N. Goncharov et al, Phys. Plasmas 10, 1906 (2003)
K. Anderson and R. Betti, Phys. Plasmas 10, 4448 (2003)
K. Anderson and R. Betti, Phys. Plasmas 11, 5 (2004)
R. Betti et al, Phys. Plasmas 12, 042703 (2005)

Adiabat shaping can be induced by a decaying pressure or by a relaxed density

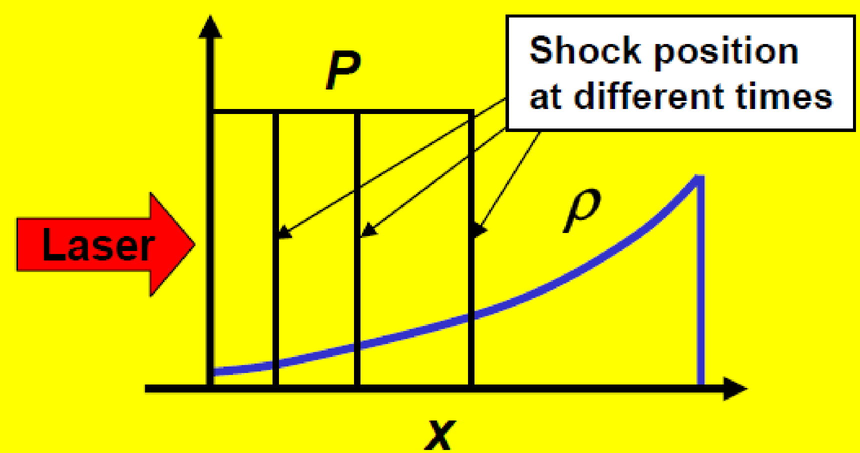
$$\alpha \sim \frac{P}{\rho^{5/3}}$$



Shaping *P* with a decaying shock



Shaping ρ by relaxation



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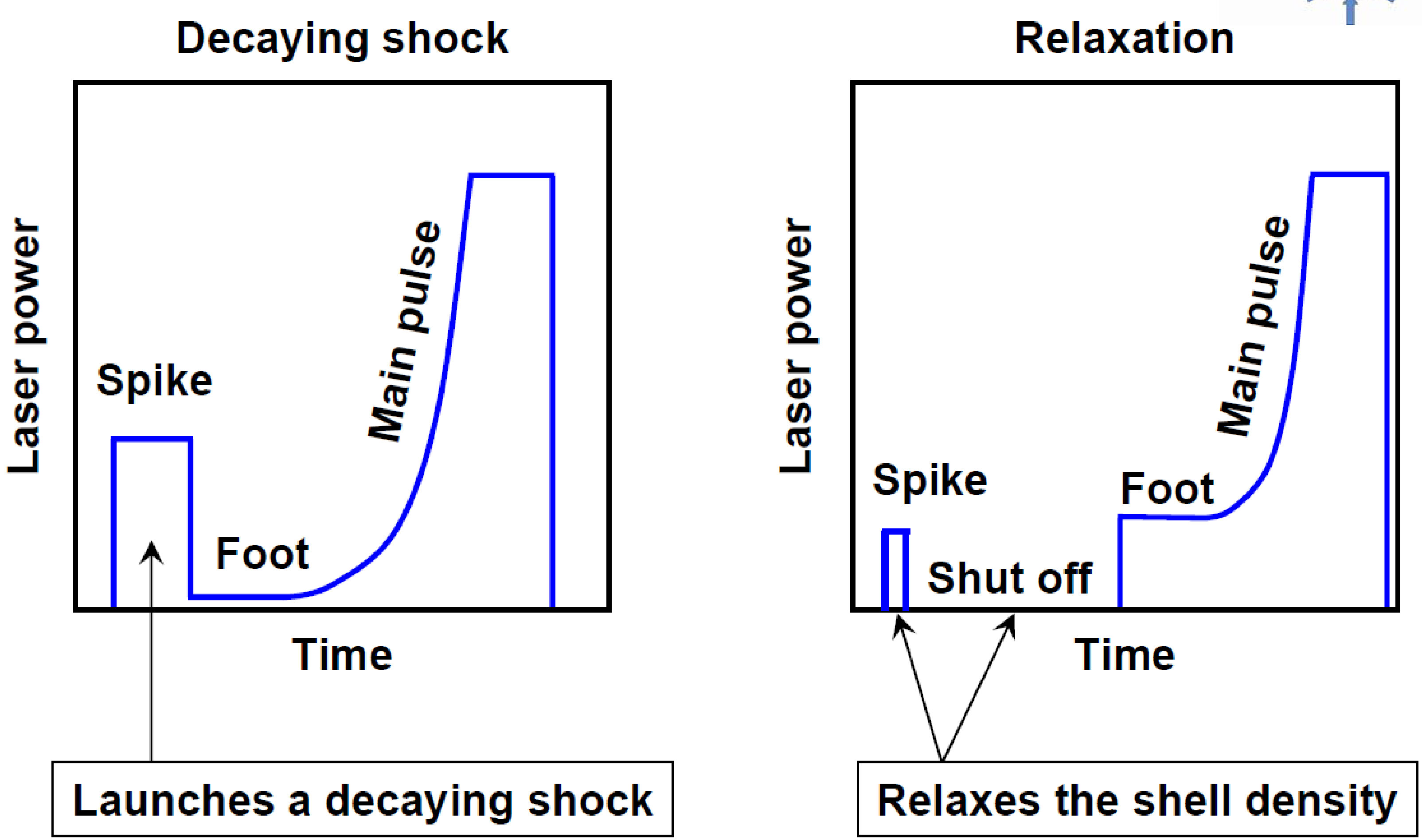
The adiabat is shaped by adding an intensity spike to the main laser pulse



V. Goncharov et al, Phys. Plasmas 10, 1906 (2003)
K. Anderson and R. Betti, Phys. Plasmas 10, 4448 (2003)

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