Radiative Transfer in Early Structure Formation

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Outline

- Motivation
  - First Stars
  - Reionization & the Intergalactic Medium
  - Galaxies
- More formally (equations)
- Methods
- What's next?

Early Structure Formation

- Universe becomes neutral at z=1090
- no photon drag -> gravitational collapse of baryons possible
- H_2 molecules allow to cool
- baryons collapse in dark matter micro-halos and form isolated very massive stars

Pure Physics Problem

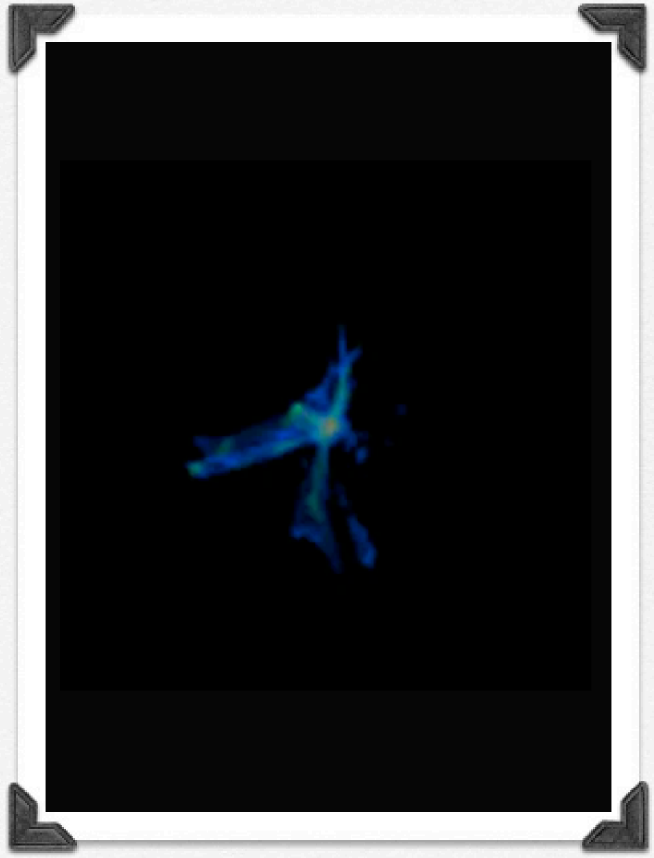
- Initial conditions well calibrated observationally to be Gaussian random field described only by a power spectrum
- Theory provides extension to the smallest as yet unobserved scales
- Gravity, hydrodynamics, chemistry and cooling physics well understood
- First Structure Formation is well defined problem

However, range of scales...

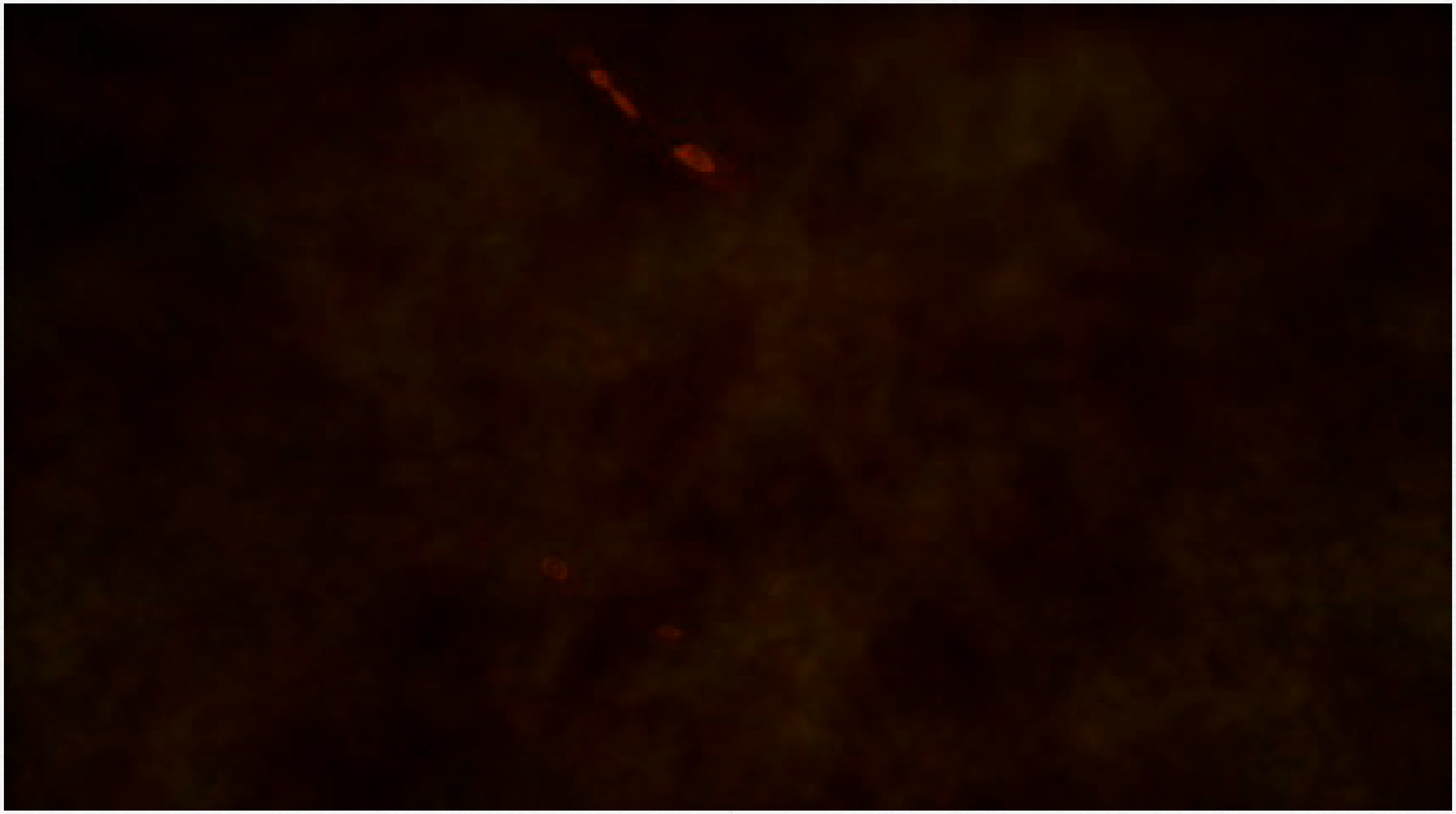
- Stars are less then a trillion times smaller than galaxies and evolve on quite different time scales

$$\frac{\mathrm{R}_\odot}{\mathrm{R}_{\mathrm{Milky\,Way}}} \approx 10^{-12}$$

$$\frac{\mathrm{P}_{\odot,\mathrm{Kepler}}}{\mathrm{t}_{\mathrm{Hubble}}(\mathrm{z}=30)} \approx 1$$



First stars:



Simulation: Tom Abel (KIPAC/Stanford), Greg Bryan (Columbia), Mike Norman (UCSD)
Viz: Ralf Kähler (AEI, ZIB), Bob Patterson, Stuart Levy, Donna Cox (NCSA), Tom Abel (KIPAC/Stanford)
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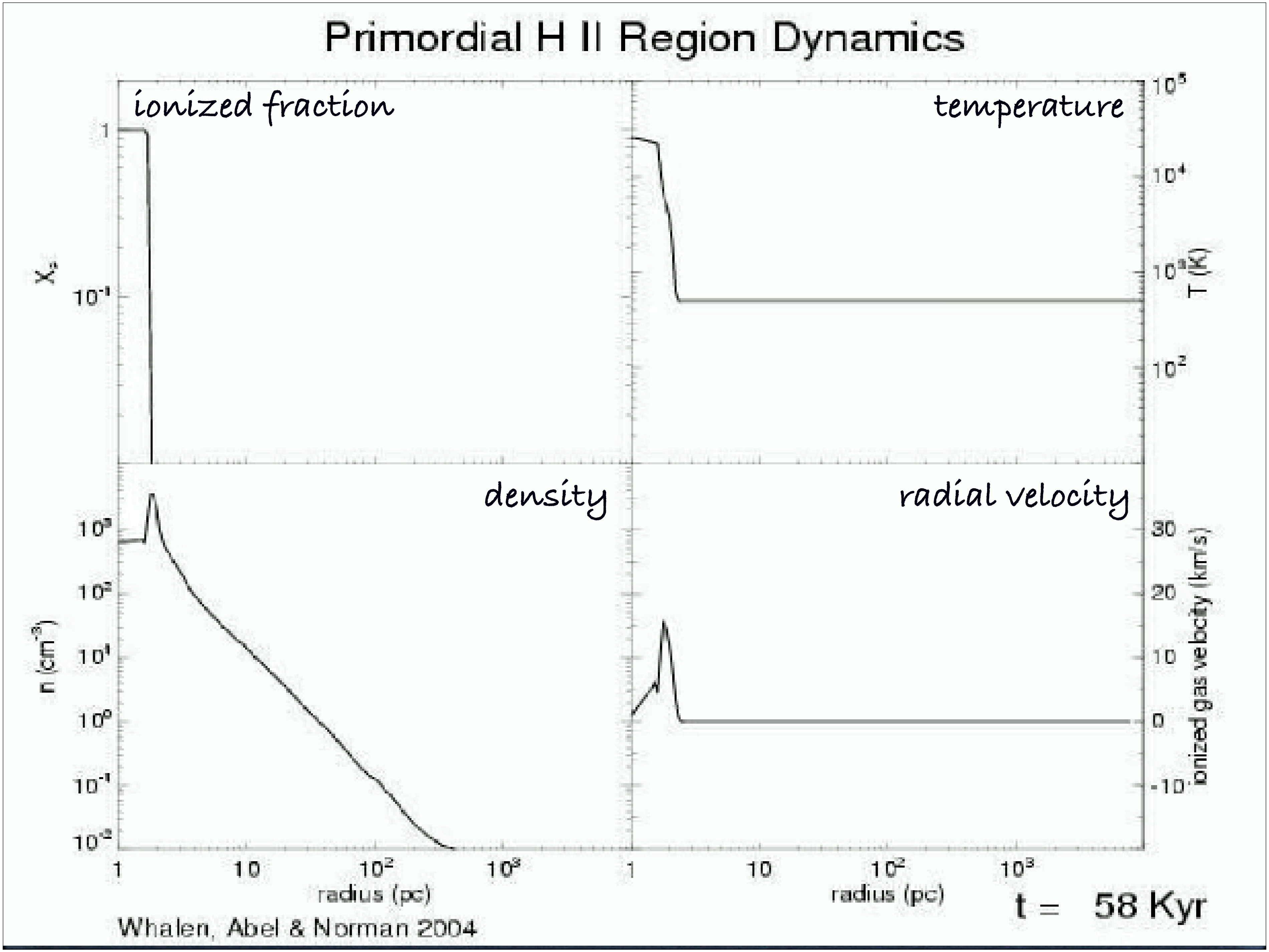
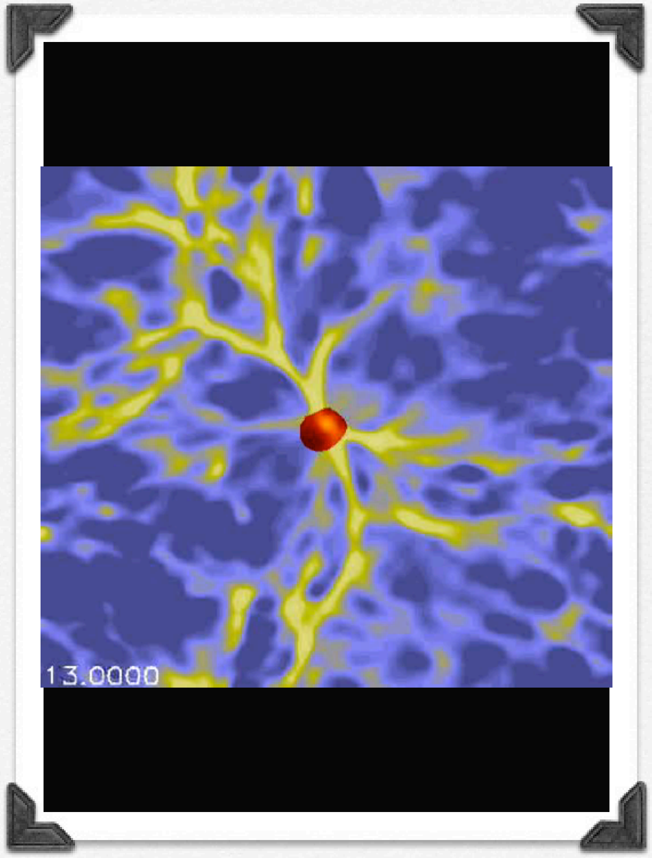
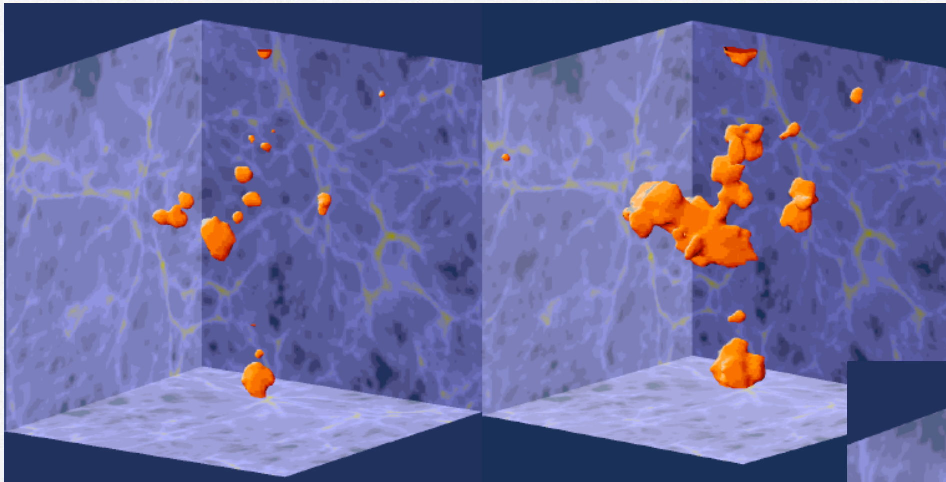


Primordial H II Region Dynamics
ionized fraction
temperature
density
radial velocity
X_i
T (K)
n (cm^{-3})
ionized gas velocity (km/s)
radius (pc)
radius (pc)
Whalen, Abel & Norman 2004
t = 58 Kyr

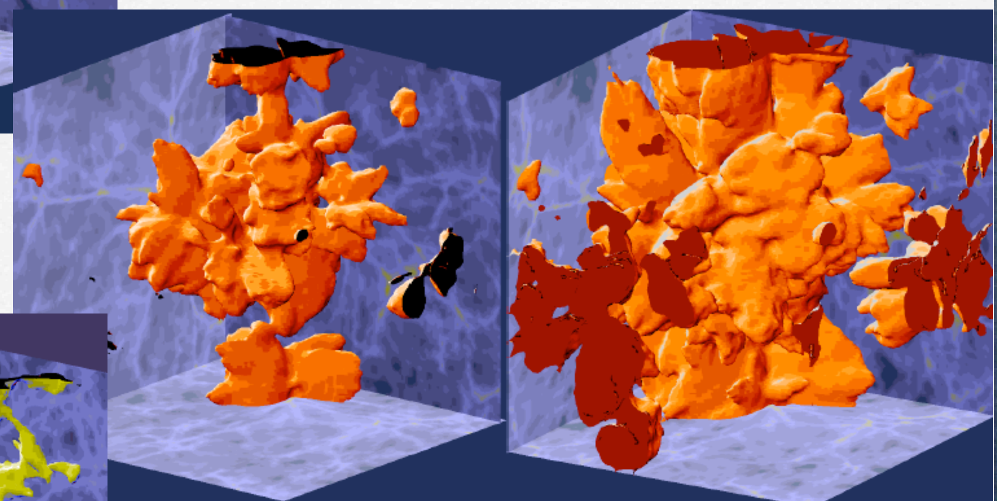
Stars ionize their surroundings

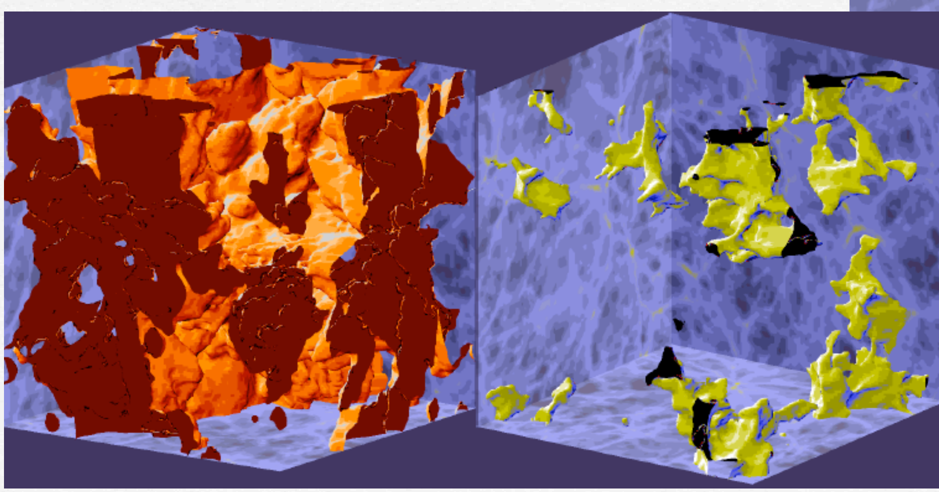
- Inherently 3D problem.
- Highly anisotropic radiation field.
- Galaxies too.



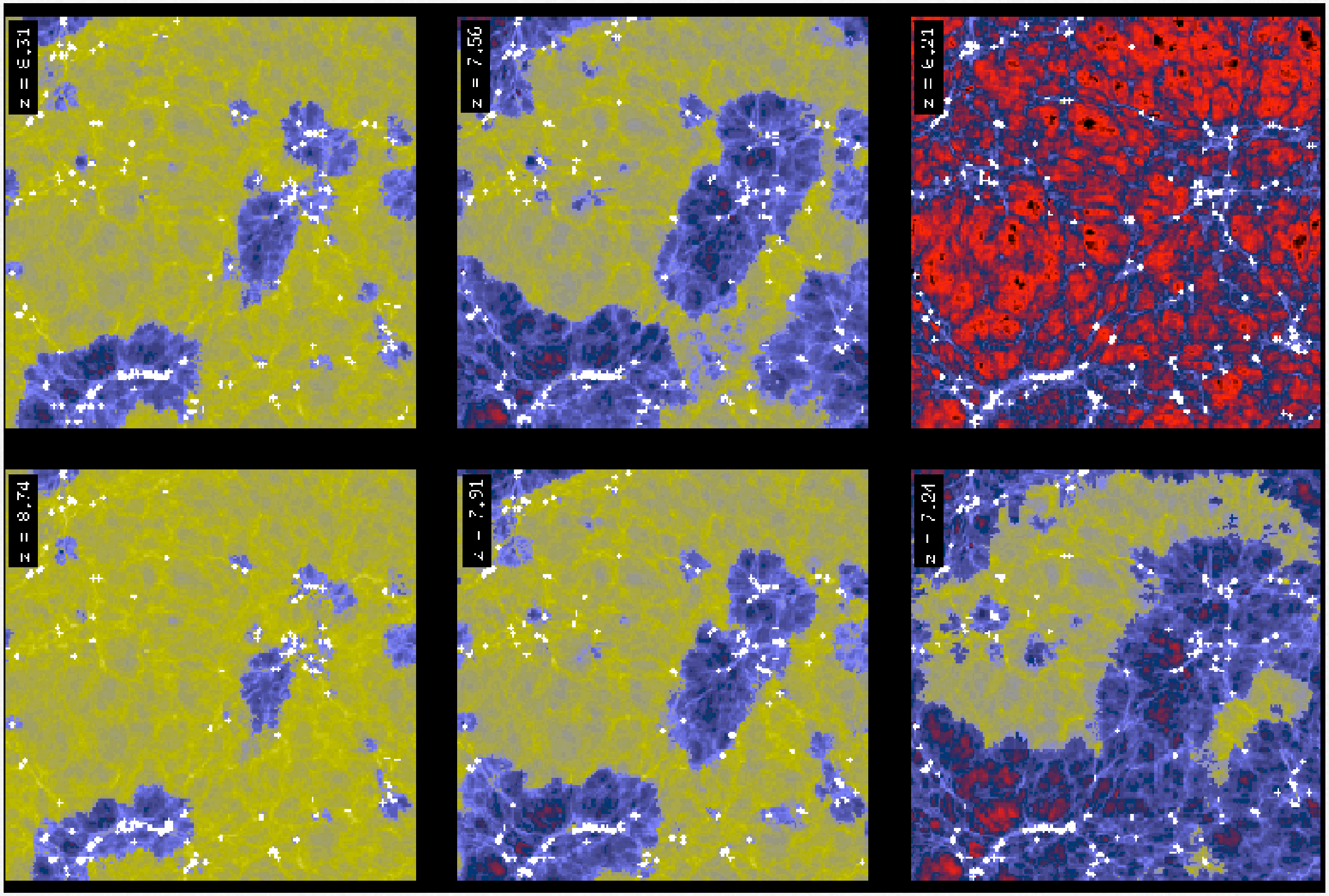


There's a bunch of them!





Abel, Norman & Madau 1998



Sokasian, Abel & Hernquist 00-03

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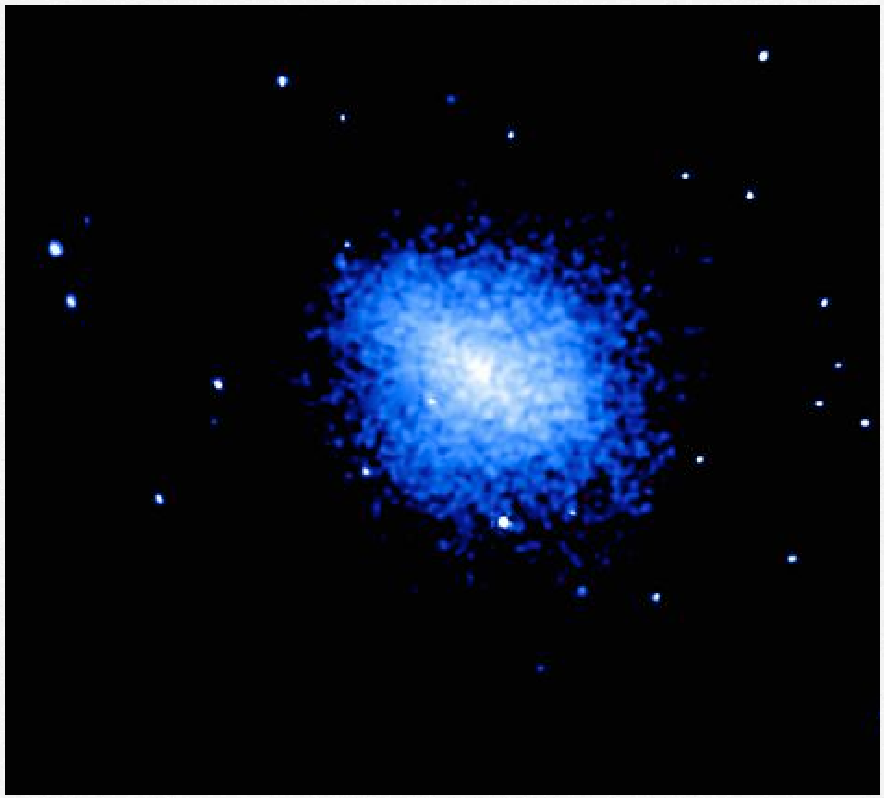
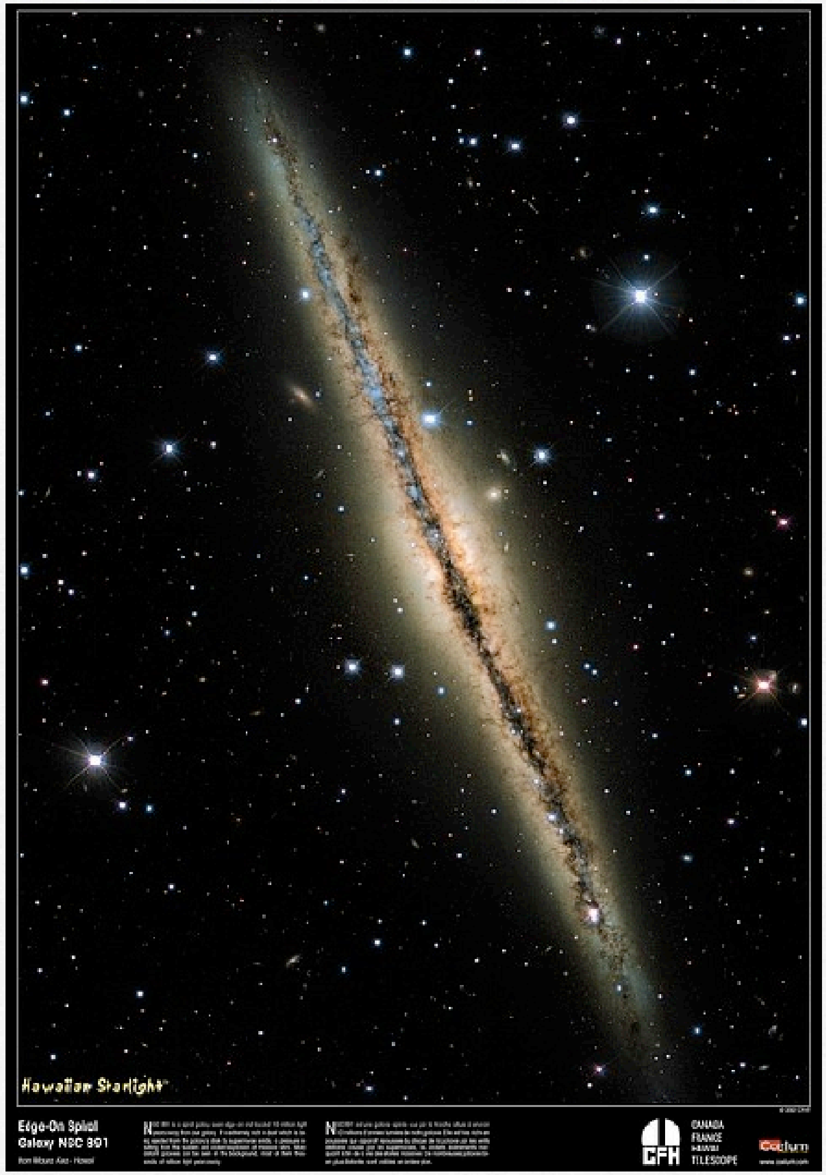
Point + diffuse sources

10 Mpc scales

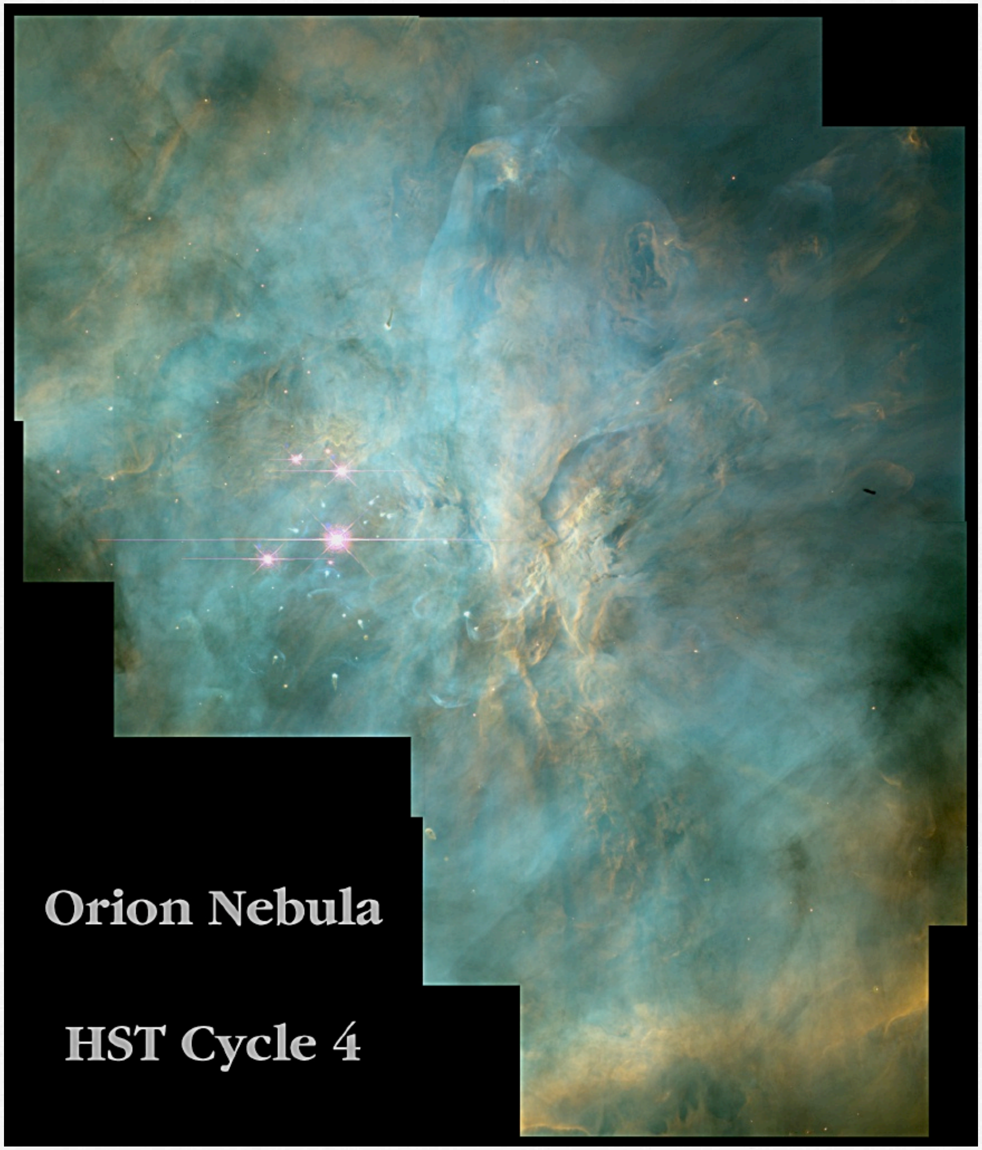


A2104 - Chandra image

10kpc scales



pc scales



sub AU scales

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Formally

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \frac{\hat{n}\cdot\nabla I_\nu}{\bar{a}} - \frac{H(t)}{c}\left(\nu\frac{\partial I_\nu}{\partial \nu} - 3I_\nu\right) = \eta_\nu - \chi_\nu I_\nu \qquad (1)$$

where $I_\nu \equiv I(t, \mathbf{x}, \mathbf{\Omega}, \nu)$ is the monochromatic specific intensity of the radiation field, $\hat{n}$ is a unit vector along the direction of propagation of the ray; $H(t) \equiv \dot{a}/a$ is the (time-dependent) Hubble constant, and $\bar{a} \equiv \frac{1+z_{em}}{1+z}$ is the ratio of cosmic scale factors between photon emission at frequency ν and the present time t. The remaining variables have their traditional meanings (e.g, Mihalas 1978.) Equation (1) will be recognized as the standard equation of radiative transfer with two modifications: the denominator $\bar{a}$ in the second term, which accounts for the changes in path length along the ray due to cosmic expansion, and the third term, which accounts for cosmological redshift and dilution.

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \frac{\hat{n}\cdot\nabla I_\nu}{\bar{a}} - \frac{H(t)}{c}(\nu\frac{\partial I_\nu}{\partial \nu} - 3I_\nu) = \eta_\nu - \chi_\nu I_\nu \tag{1}$$

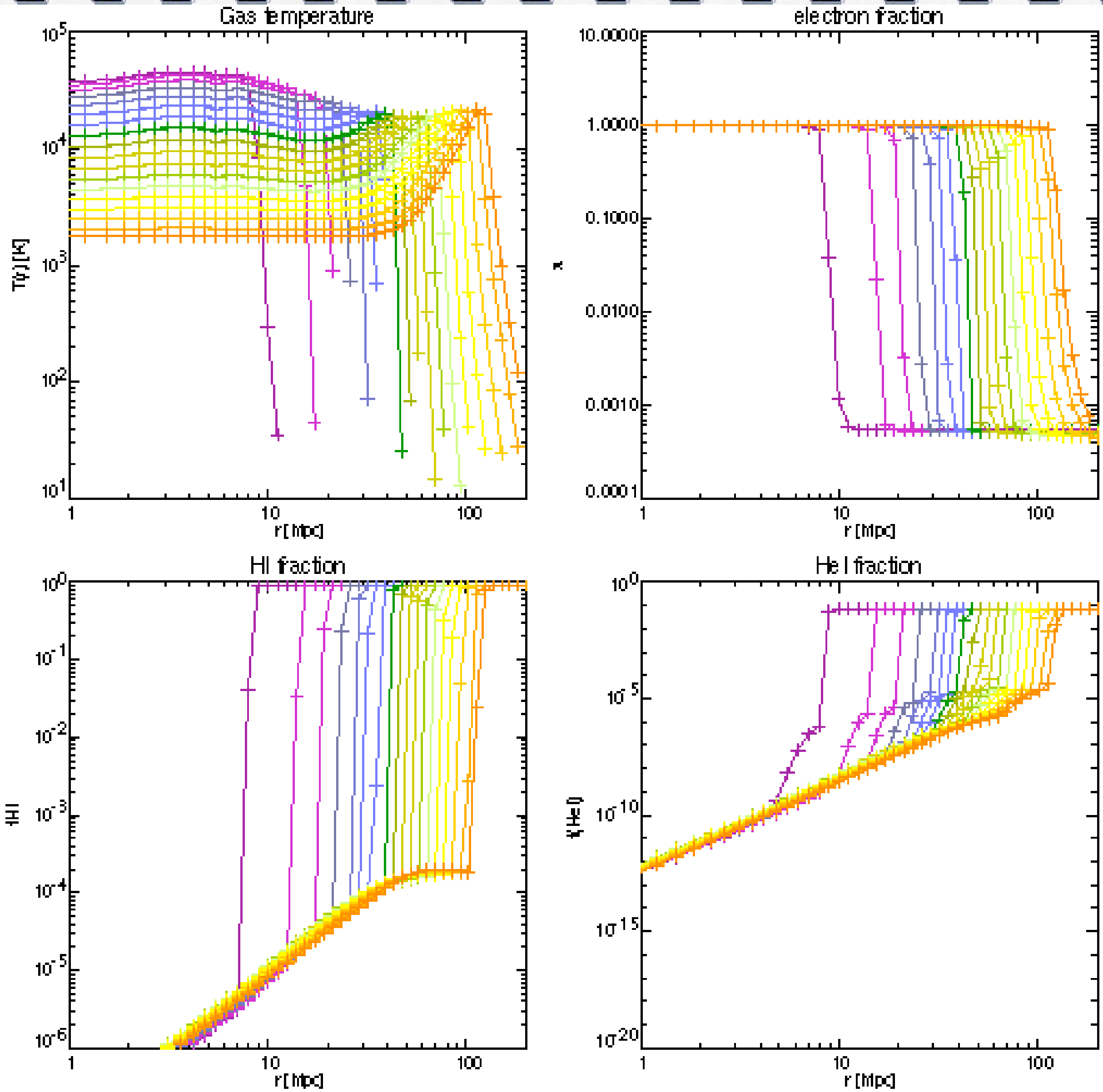
If our simulation box is of side length L and λ_p is the photon mean free path, then by construction $\lambda_p \ll L$. The ratio of the third to the second terms in equation (1) is $HL\bar{a}/c \ll 1$, and hence the third term can safely be ignored. Now, let us consider the factor $\bar{a}$ in equation (1). For a photon which is emitted at time t on one side of the box and absorbed on the other side at time $t + L/c$, $\bar{a} = (\frac{t+l/c}{t})^\eta \sim 1 + \eta L/ct = 1 + \eta L/L_H$, where η is the logarithmic expansion rate of the universe (2/3 for $\Omega_o = 1$) and L_H is the Hubble horizon scale. For $L \ll L_H, \bar{a} \doteq 1$, and $\nu_{em} \doteq \nu$. In practice, our dynamical timesteps are much longer than a photon crossing time. However, even in this case accuracy limits our dynamical timesteps such that $\Delta a/a \ll 1$, and hence $\bar{a} \doteq 1$ in any given timestep. Therefore, setting $\bar{a} \equiv 1$, equation (1) reduces to its standard, non-cosmological form:

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \hat{n}\cdot\nabla I_\nu = \eta_\nu - \chi_\nu I_\nu \tag{2}$$

where now ν is the instantaneous, comoving frequency.

ν

multifrequency important



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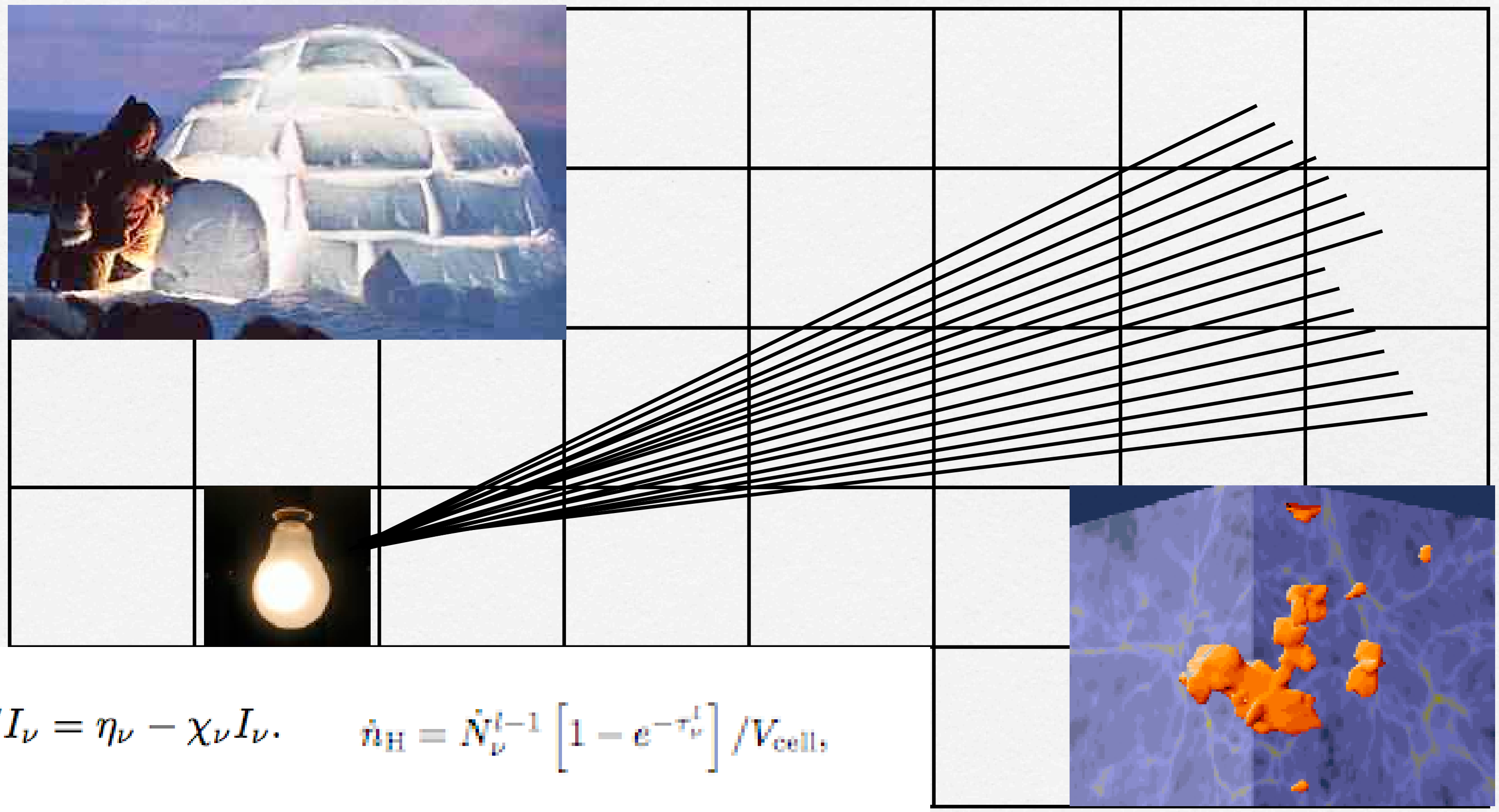
Strategy

- Split Point source radiation from diffuse radiation
  - Compact sources of light cause most dramatic shadows
  - Diffuse photons in many applications are close to high density regions and are absorbed
- Different methods best for different types offers mutliple optimization strategies
- Use multi-group techniques

First attempts



$$\hat{n} \cdot \nabla I_\nu = \eta_\nu - \chi_\nu I_\nu .$$

$$\dot{n}_{\rm H} = \dot{N}_\nu^{l-1}\left[1 - e^{-\tau_\nu^l}\right]/V_{\rm cell},$$

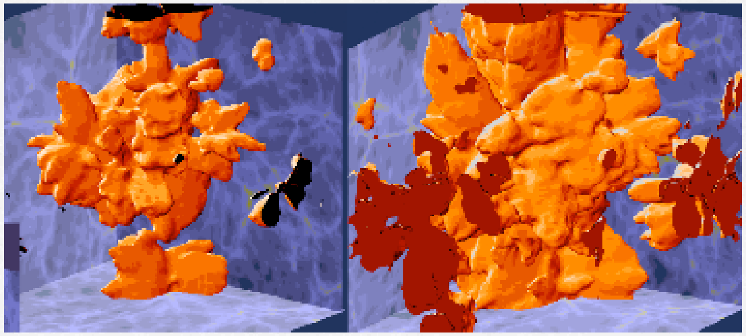
Abel, Norman & Madau, 2002, ApJ

Time varying opacity

$$\frac{dn_H}{dt} = k_{rec}\, n_p\, n_e - n_H\, n_e \; - n_H \mathbf{k_{PH}}$$

$$\mathbf{k_{PH}} = \sum_{rays} \frac{I_r[1 - \exp(-\delta\tau_r)]}{V_{cell}}$$

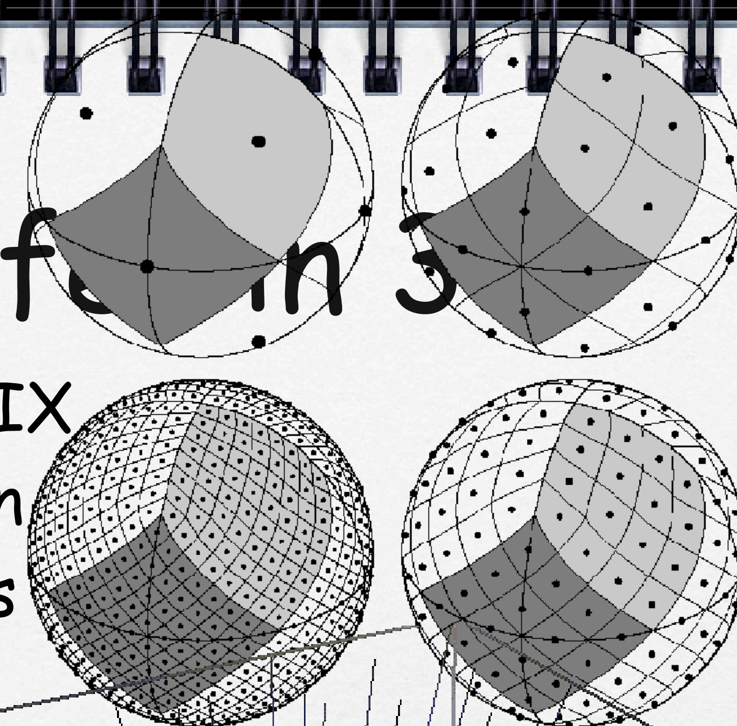
explicitly photon conserving



Loop for Ray Tracing:

- loop over sources
  - choose angles
  - loop over angles
    - casting rays
    - compute rates

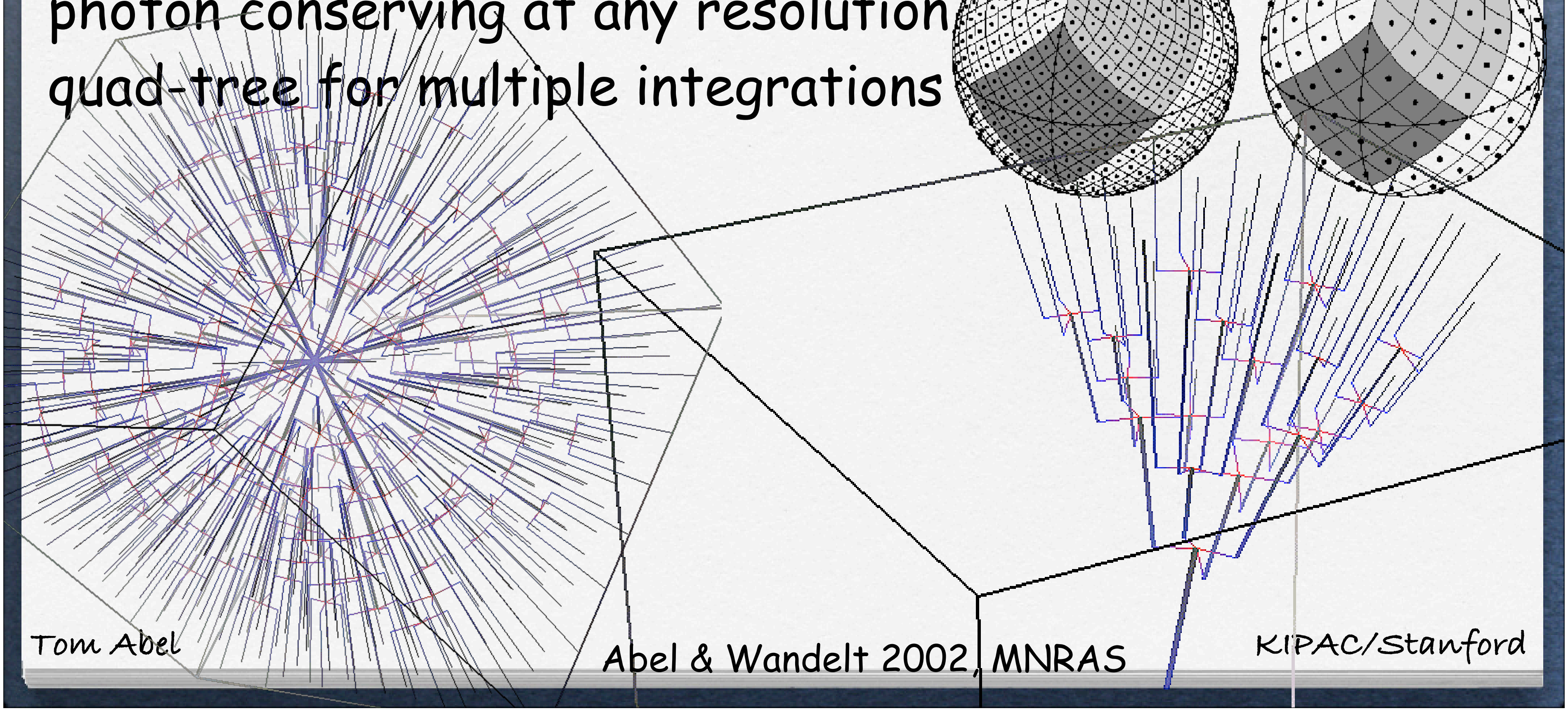
Radiative Transf in 3



adaptive ray-tracing using HEALPIX

photon conserving at any resolution

quad-tree for multiple integrations



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Abel & Wandelt 2002, MNRAS

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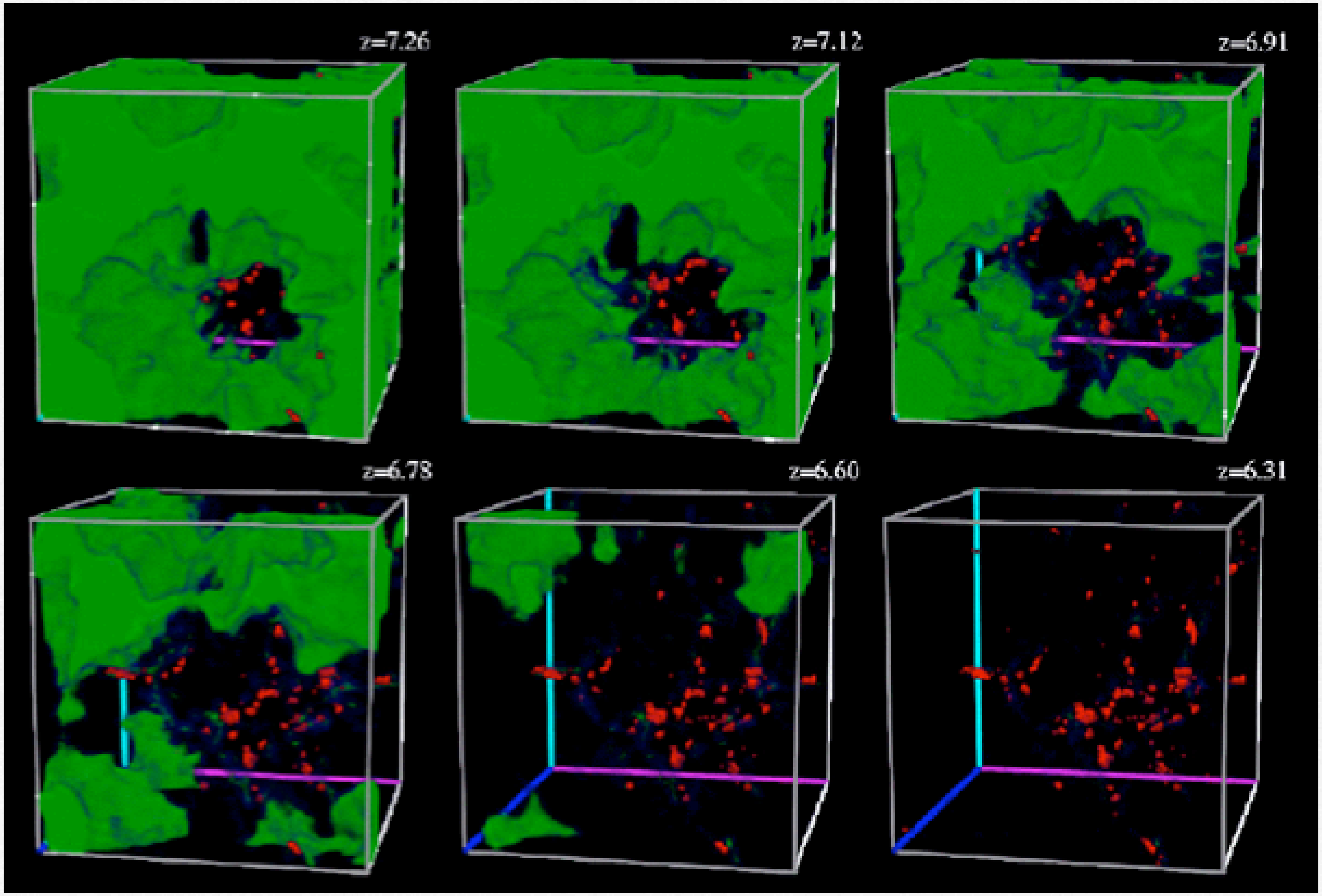
Hydrogen Reionization



 Razoumov, Norman, Abel & Scott, 2002, ApJ

Faster then Light Ionization Fronts

$$4\pi\, n_H\, R_I^2\, v_I = \dot{N}_{PH}; \quad [\dot{N}_{PH}] = \#/s$$

$$v_I = \dot{N}_{PH}/(4\pi\, n_H\, R_I^2)$$

For large N or small R one gets into trouble with a pure attenuation equation

→

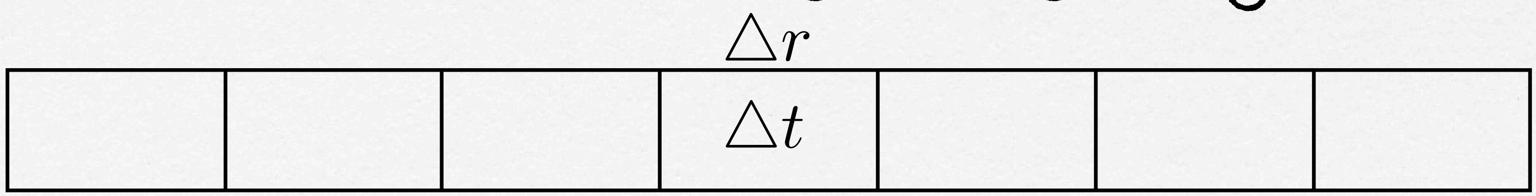
Include time dependent term or force R<c(t-t_0)

Jump condition for Ionization Fronts

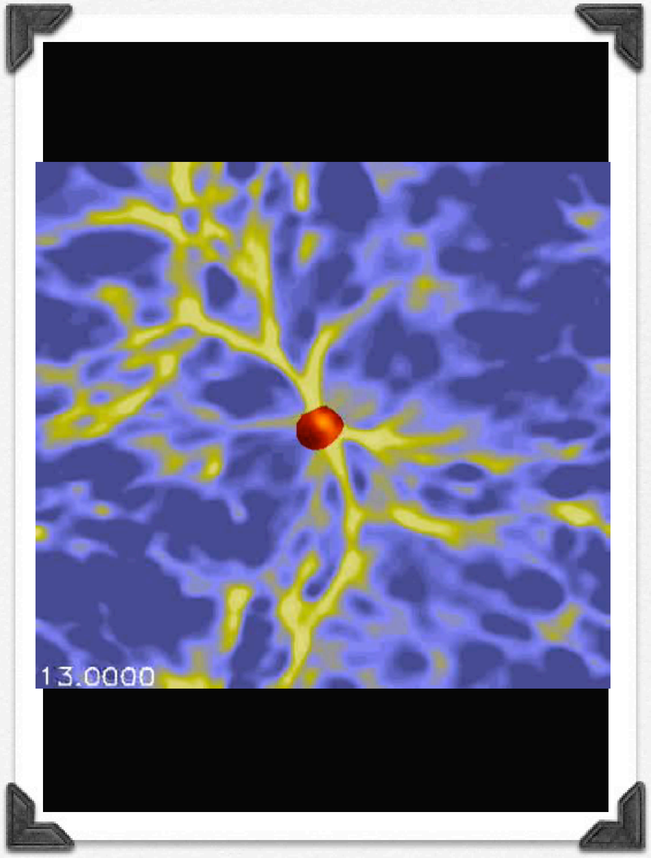
$$4\pi\, n_H\, R_I^2\, v_I = \dot{N}_{PH} - \frac{4\pi}{3} R_I^3\, n^2 k_{rec}$$

$$n_H\, \frac{dR_I}{dt} = \frac{\dot{N}_{PH}}{4\pi R_I^2} - \int_0^{R_I} k_{rec} n^2 dr$$

Picture this along a single ray:



- Store arrival times
- movie shows contours of constant time
- Gives quick rough idea



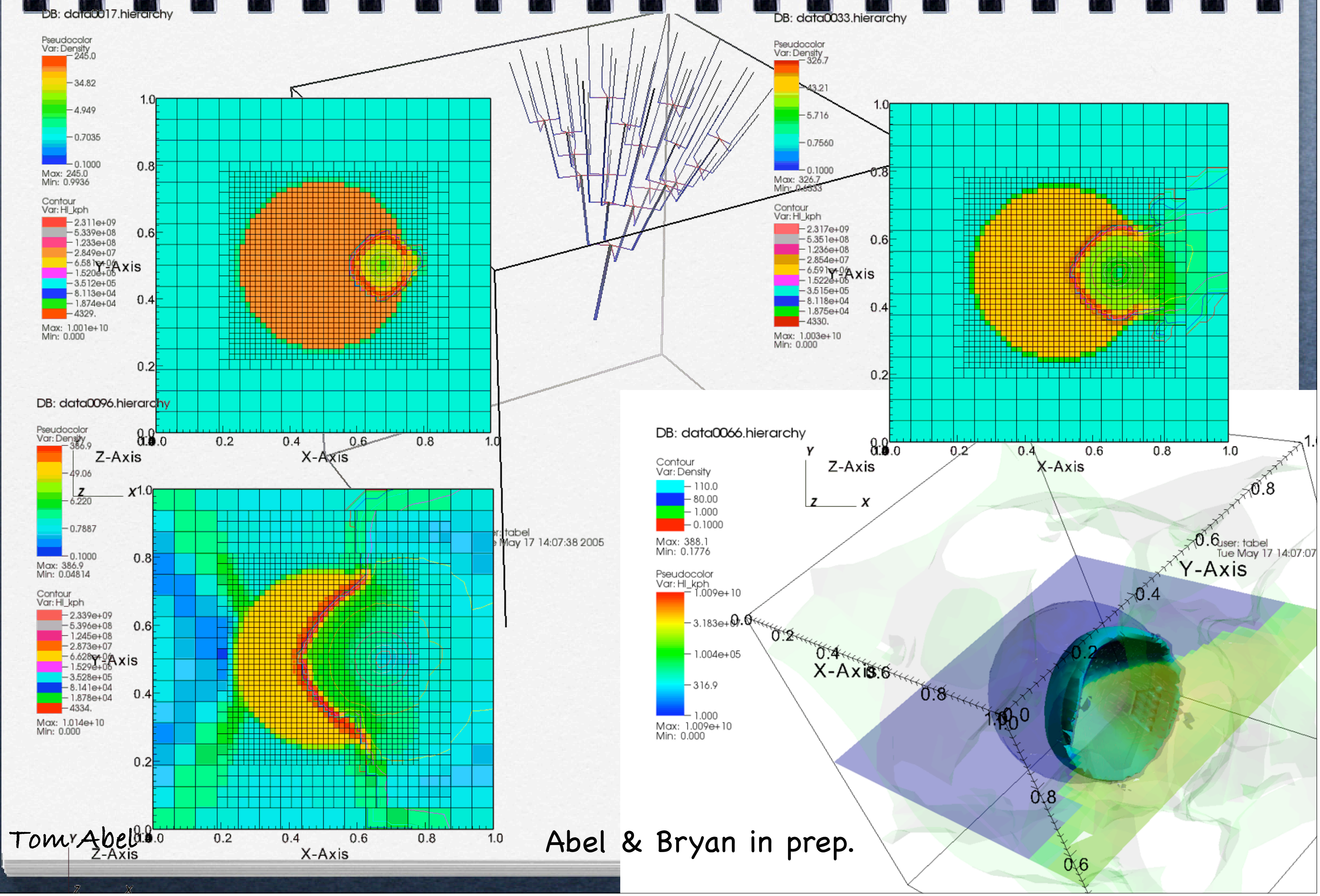
Arrival time technique

- Can be extended to multiple sources
- To capture some aspects of the time-dependent transfer equation
- Has been used with 100reds of thousand of sources in 3D:
  - Sokasian, Abel & Hernquist 2000-04: Studied helium & hydrogen reionization, 21cm emission and absorption and nature of UV background at z~3.

MoRay

- Combines ray tracing and Monte Carlo aspects.
- No random numbers
- Self splitting PhotonPackages emitted and traced through AMR grid hierarchy for multiple energy groups
- Energy conserving at any spatial resolution
- Unfortunately no adaptive time steps (yet)
- Be careful with moving and accelerating sources
- Same parallelization issues as MC methods

3d transfer in AMR without approximations?



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Abel & Bryan in prep.

Moments

- Energy density $$E_\nu(t,\vec{x}) \equiv \frac{1}{4\pi}\int d\Omega\, f_\nu(t,\vec{x},\vec{n})$$
- Flux $$F^i_\nu(t,\vec{x}) \equiv \frac{1}{4\pi}\int d\Omega\, n^i f_\nu(t,\vec{x},\vec{n})$$
- Eddington Tensor $$E_\nu(t,\vec{x})h^{ij}_\nu(t,\vec{x}) \equiv \frac{1}{4\pi}\int d\Omega\, n^i\vec{n} f_\nu(t,\vec{x},\vec{n}).$$
- Transfer equ:

$$\frac{a}{c}\frac{\partial E_\nu}{\partial t} + \frac{\partial F^i_\nu}{\partial x^i} = -\hat{\kappa}_\nu E_\nu + \psi_\nu$$

$$\frac{a}{c}\frac{\partial F^j_\nu}{\partial t} + \frac{\partial}{\partial x^i}E_\nu h^{ij}_\nu = -\hat{\kappa}_\nu F^j_\nu.$$

Optically thin solution

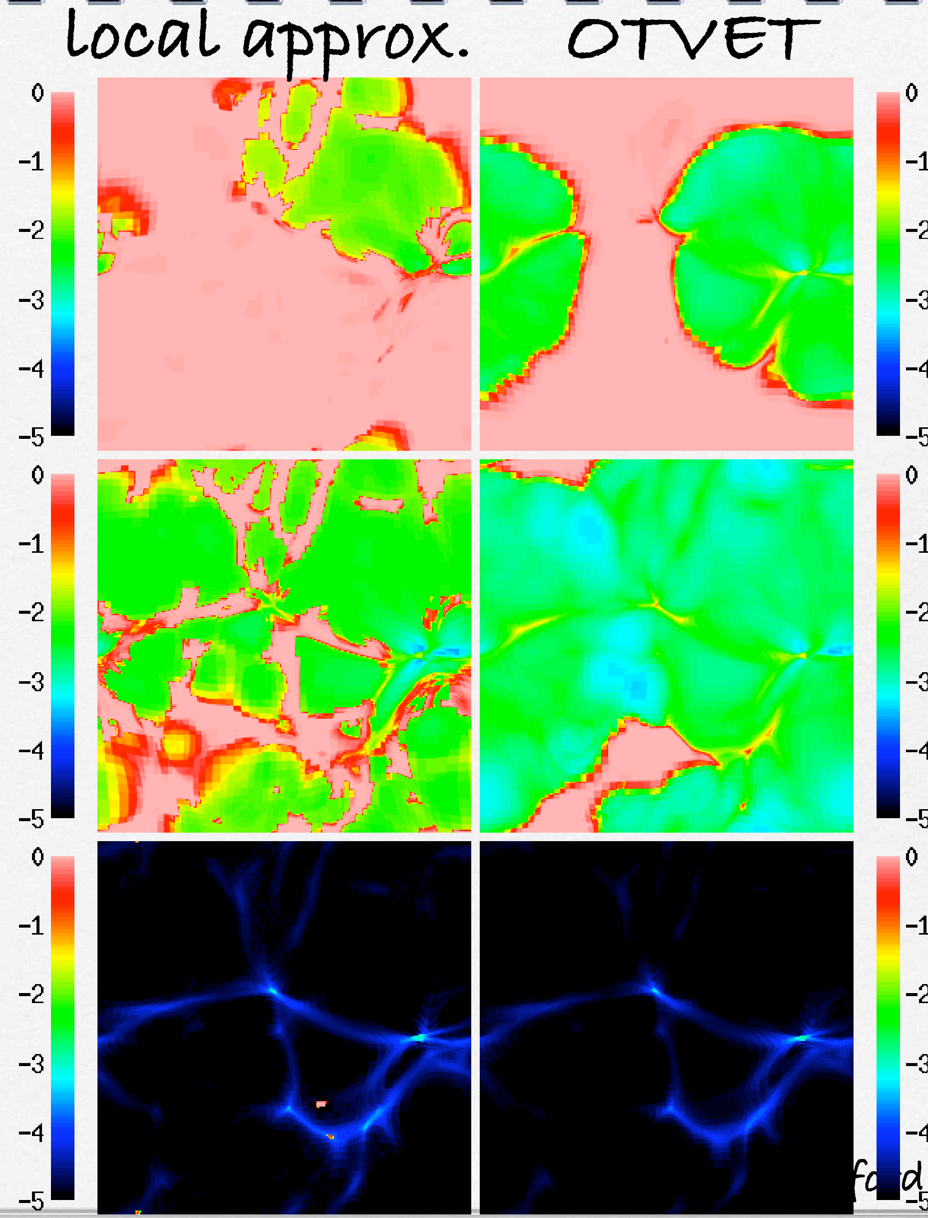
$$\langle J_\nu \rangle_\Omega(\vec{x}) = \bar{J}_\nu + \frac{a}{4\pi c} \int d^3x_1 \frac{S_\nu(x_1^i) - \bar{S}_\nu}{(\vec{x} - \vec{x}_1)^2}.$$

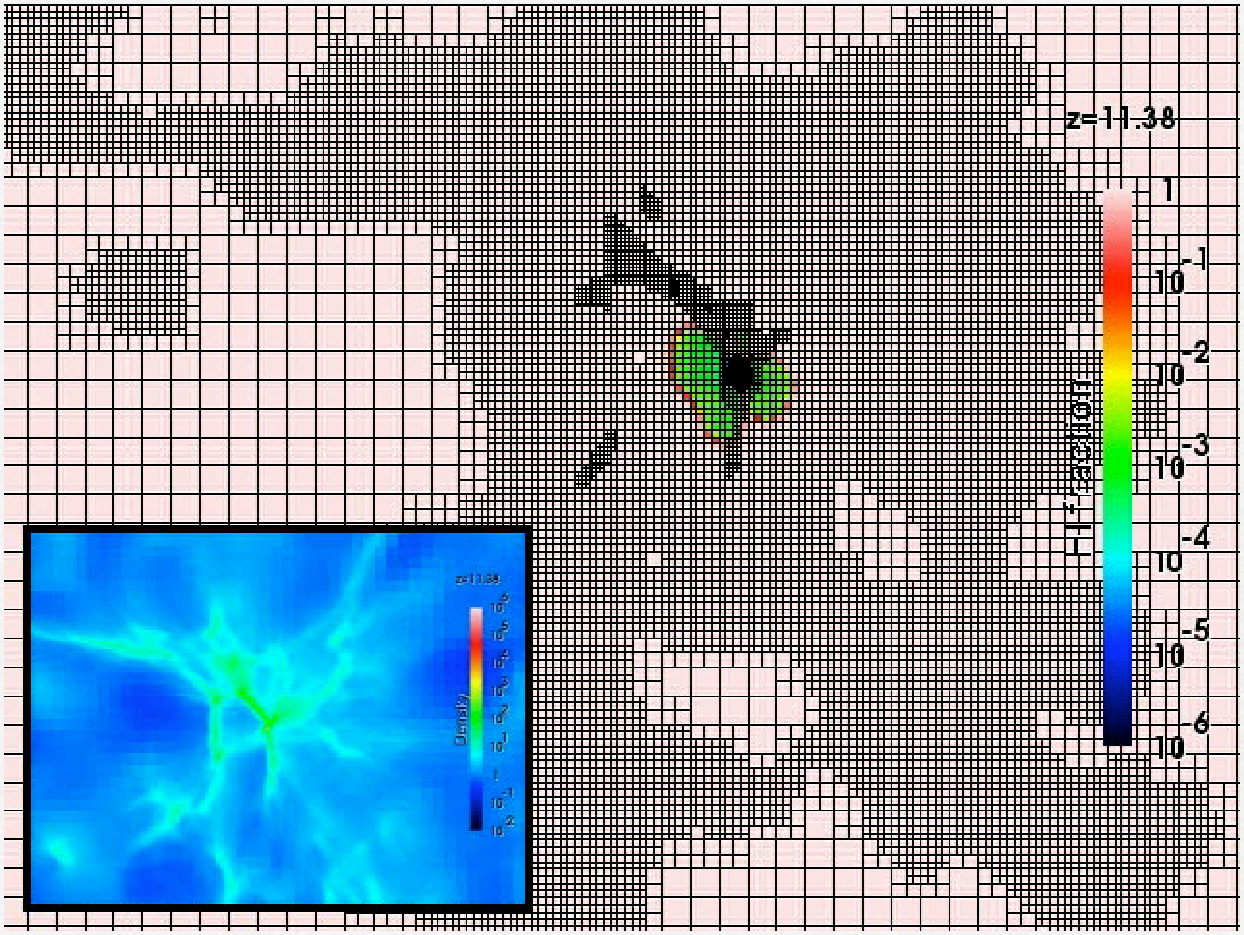
- Straight forward to compute
- Looks like gravity over numerous points of different mass -> many numerical methods available to compute this very rapidly

OTVET

- Abel 1999, PhD thesis
  Gnedin & Abel, 2001, NewA
  OTVET:
  Optically Thin Variable Eddington Tensor formalism
  independent of Nsources
  very fast moment solver
  multi-frequency
  all cosmological terms
  Adequate 3D-RT for apps. in
  cosmology,
  ISM physics,
  star formation

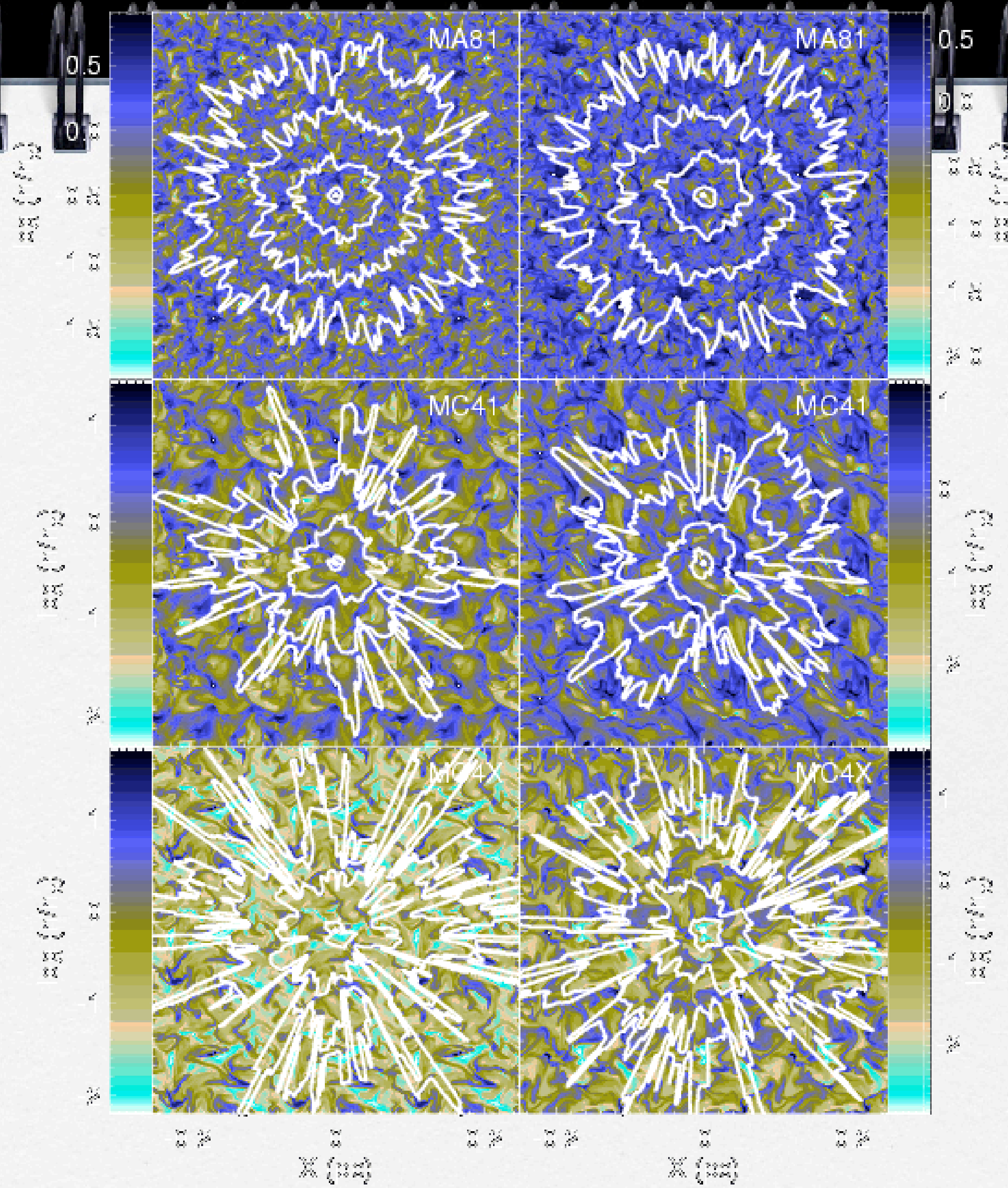
- Only OTVET so far has lead to publications with transfer and hydrodynamics coupled.





- Gnedin & Kravtsov in prep
- Implementation of approximative technique, OTVET, of Abel 1999 and Gnedin and Abel 2001

Techniques equally applicable for present day star formation



Li, MacLow & Abel 2004

Fig. 2.— Propagation of the I-front in a 384^3 density field simulated with different MHD models. *Left*: ionizing source is located at maximum density; *right*: ionizing source is located at minimum density. The contours give the position of the I-front from 0.1 to 100 years with the interval increasing evenly by a factor of 10. The size of box is 0.5 pc, $n_0 = 5 \times 10^3$ cm^{-3}.

PLAT

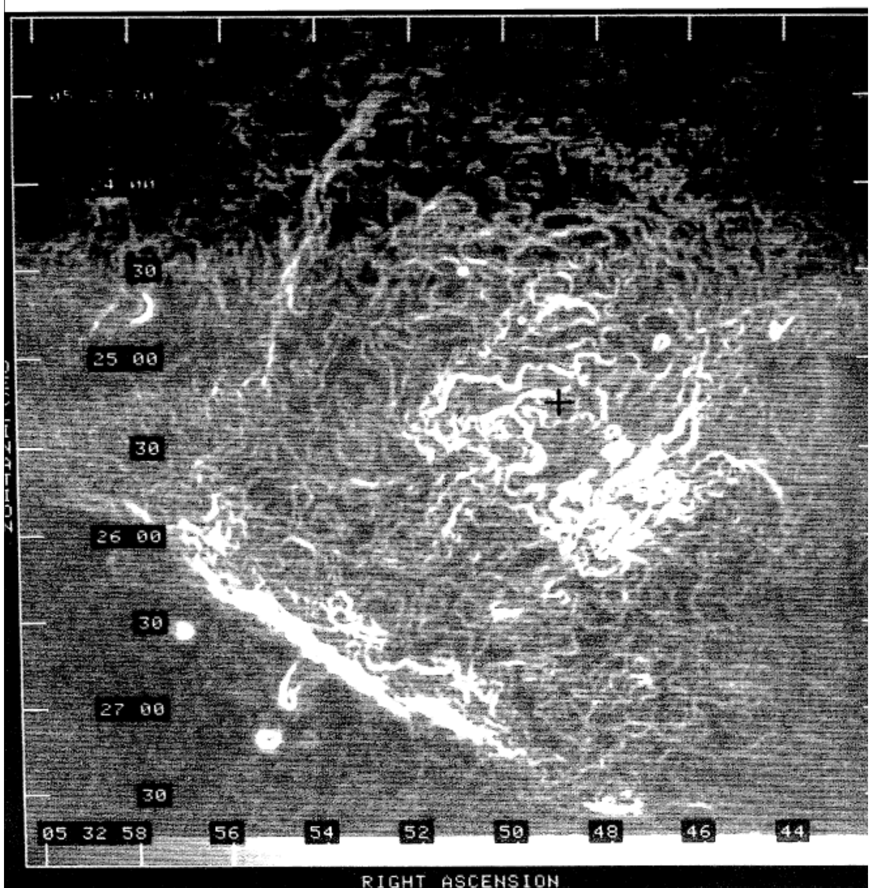
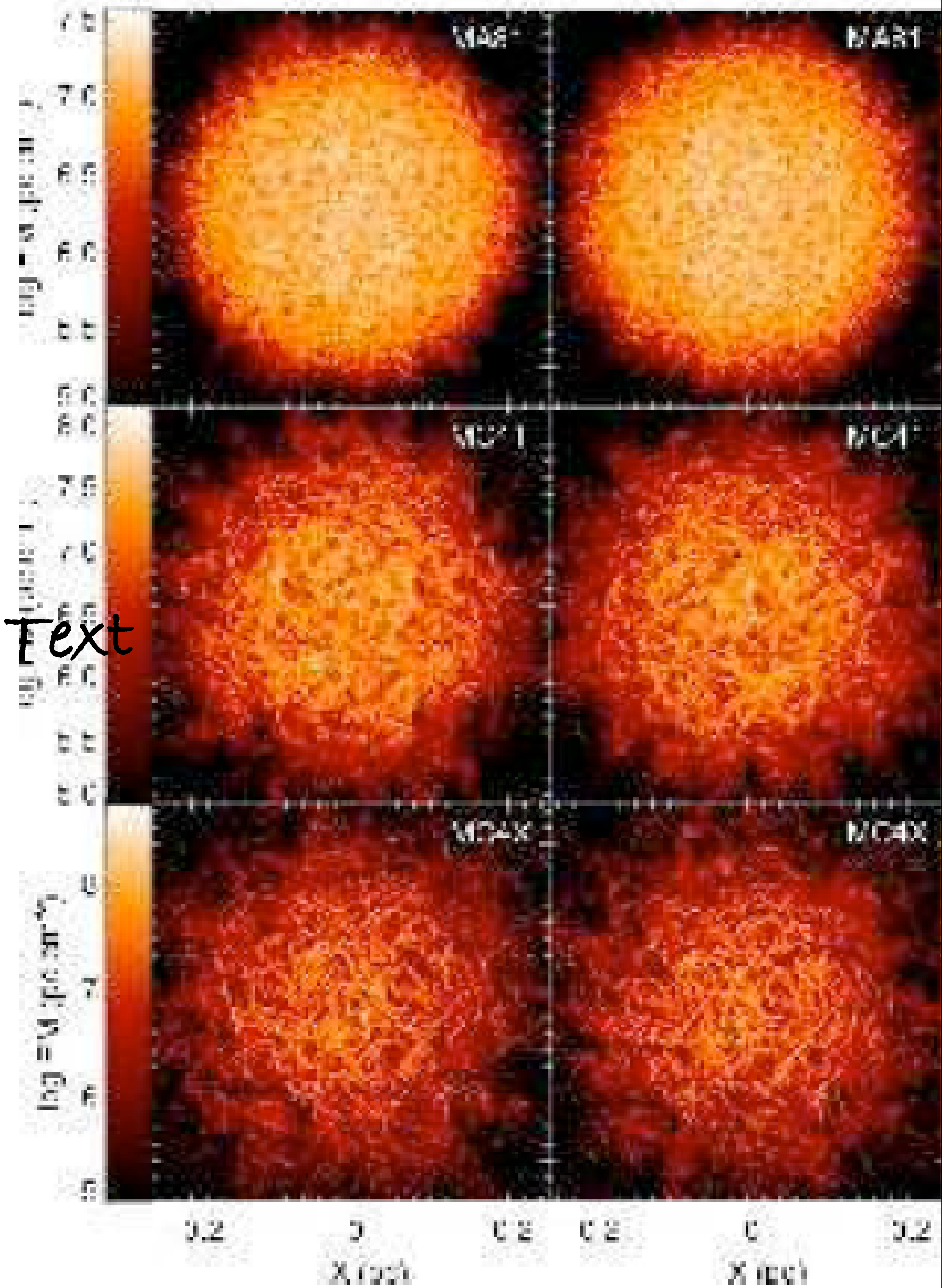


FIG. 5a

. 5.—These images are similar to Fig. 4 except that extended features have been filtered out. Fig. 5b, which includes M43 to its northeast, is displaye
st level which is different from Fig. 5a.

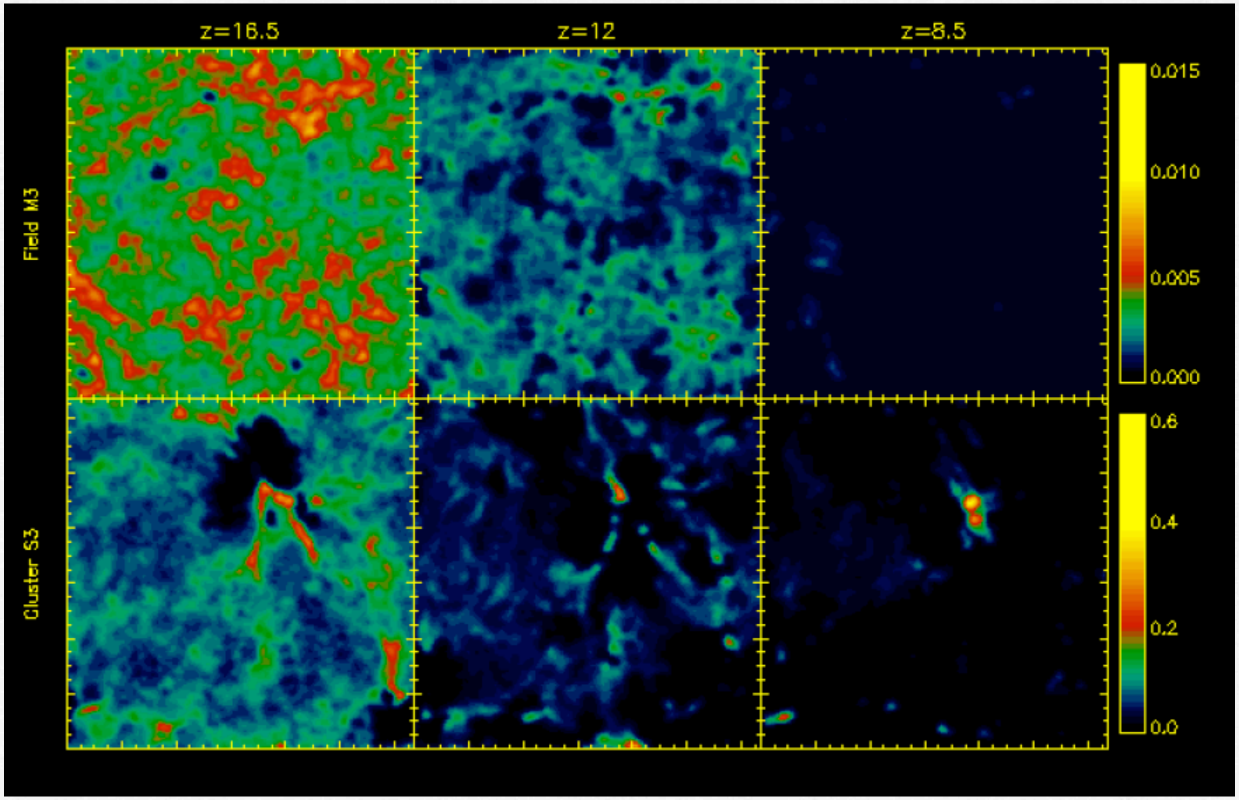
ZADEH (*see* **361**, L20)

Yusef-Zadeh et al.



Monte Carlo

Being applied successfully!



Ciardi & Ferrara 2000-

Conclusions

- There are many novel applications for 3D transport
- Early stages of doing three-D transport in cosmological hydrodynamics
- Lots of problems to be solved
- A few implementations available already