1

The Hénon approach: Fast simulations of collisional self-gravitating systems

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$IBFTC CARL06   NODECK
C
C   EVOLUTION [illegible] SYSTEME SPHERIQUE PAR LA METHODE DE MONTE-CARLO.
C   CALCUL EXACT DU [illegible]
C
C   INDICES DE COMMANDE
C
C   IC(2) = 1 , IMPRESSION DETAILLEE.
C   IC(3) = 1 , REPRISE SUR CARTES D'UN [illegible]
C               DES MASSES.
C   IC(5) = 0 , CONDITIONS INITIALES = MODELE HOMOGENE, DISTRIBUTION MAXWELLIENNE
C               DES VITESSES (NON STATIONNAIRE).
C           1 , POLYTROPE N = 1 , VITESSES ISOTROPES.
C           2 , MODELE HOMOGENE, DISTRIBUTION HOMO[illegible]
C               SPHERE (VON HOERNER) (NON STATIONNAIRE).
C           3 , POLYTROPE N = 5 , VITESSES ISOTROPES.
C           4 , MODELE HOMOGENE, ORBITES CIRCULAIRES.
C           5 , DENSITE EN 1/R**2, ORBITES RADIALES.
C   IC(6) = 0 , AMAS [illegible]
C           1 , AMAS [illegible]
C           2 , AMAS NON ISOLE. EVASION [illegible] SUPERIEUR A RE (POUR COMPARAISON
C               AVEC LARSON).
C   IC(8) = 0 , MASSES EGALES.
C           1 , DISTRIBUTION DE MASSES DE WIELEN.
C           2 , DISTRIBUTION DE MASSES DE AARSETH.
C           3 , DISTRIBUTION CONTINUE EN M**-2.
C   IC(11) = 1 , ENREGISTREMENT DES RESULTATS SUR BANDE MAGNETIQUE ET SUR CARTES.
C                SI DE PLUS NPINIT EST POSITIF, LE CALCUL EST REPRIS A PARTIR DU
C                PAS NPINIT LU SUR BANDE. SI NPINIT = -1 , LE CALCUL EST REPRIS
C                A PARTIR DE VALEURS LUES SUR CARTES.
C   IC(12) = 1 , ON PREND DT = B ET C = 1000.
C   IC(13) = 0 , LE PREMIER NOMBRE ALEATOIRE N0 EST LU SUR CARTE,
C            1 , LE PREMIER NOMBRE ALEATOIRE N0 EST PRIS EGAL AU DERNIER DU
C                CALCUL PRECEDENT.
C
C   VARIABLES DE COMMON.
C
C   A(P) = MOMENT ANGULAIRE (PAR UNITE DE MASSE) DE LA SUPER-ETOILE P.
C   AA = MOMENT ANGULAIRE.
C   AM(P) = MASSE DE LA SUPER-ETOILE P.
C   AMA = MASSE.
C   AMMIN , AMMAX = MASSES PARTIELLES A GAUCHE DES RAYONS MINIMAL ET MAXIMAL.
C   AMT = MASSE TOTALE.
C   B = VALEUR IMPOSEE DE DV2/V2 PRES DU CENTRE.
C   C = CONSTANTE DANS LA FONCTION DE PONDERATION.
C   CK = COEFFICIENT INTERVENANT DANS LE TIRAGE AU SORT D'UNE NOUVELLE VALEUR DE
C        R (VOIR CARL89).
C   DAM(P) = MASSE PARTIELLE = MASSE DU SOUS-ARBRE DE GAUCHE DE P, PLUS LA MASSE
C            DE P.
C   DAMR(P) = SOMME PARTIELLE DES RAPPORTS MASSE/RAYON = SOMME POUR LE SOUS-ARBRE
C             DE DROITE DE P, PLUS LA VALEUR POUR P.
C   DELT = PAS INDIVIDUEL (POUR UN COUPLE).
C   DEMIA2 = DEMI-CARRE DU MOMENT ANGULAIRE.
C   DIVERS = TABLE POUR COMMUNICATION PROVISOIRE DE NOMBRES PAR LE COMMON.
C   DT = PAS GLOBAL = TEMPS CORRESPONDANT A UN GRAND CYCLE.
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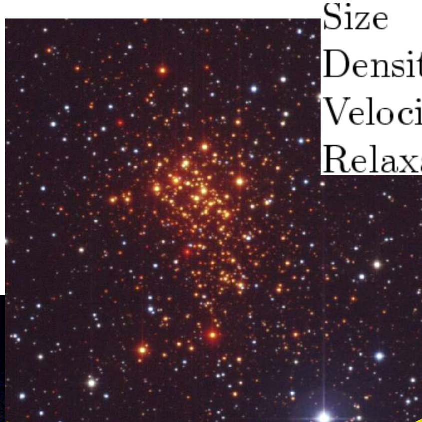
The stage: collisional stellar clusters

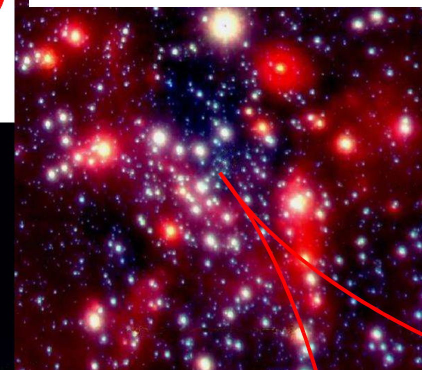
Stellar cluster

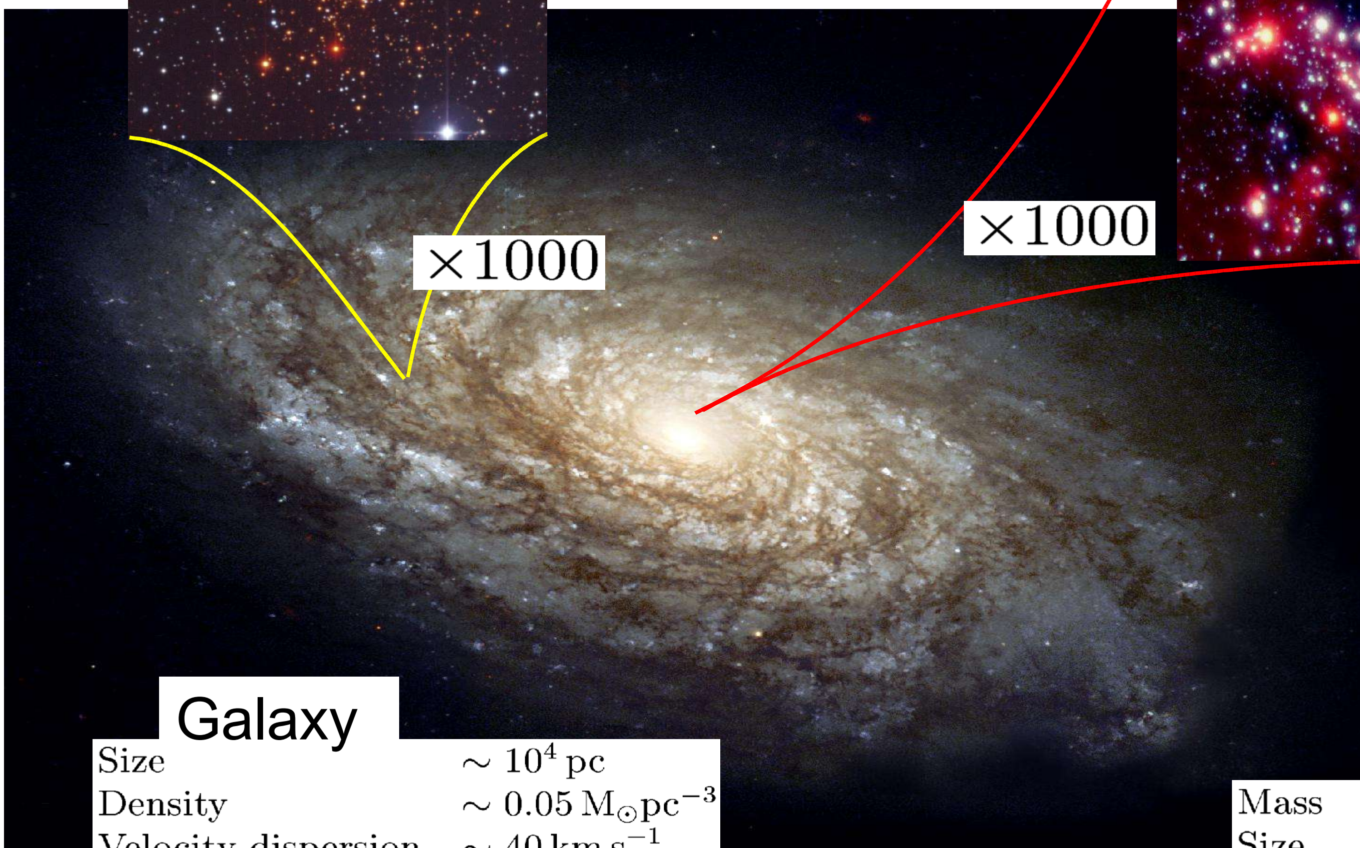
Size	$\sim 1-10\,\mathrm{pc}$
Density	$\sim 10^{3-6}\,\mathrm{M_\odot pc^{-3}}$
Velocity dispersion	$\sim 10-20\,\mathrm{km\,s^{-1}}$
Relaxation time	$\sim 10^{7-9}$ years

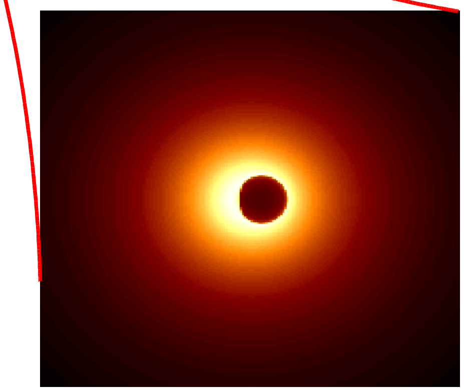


Galactic nucleus

Size	$\sim 1-10\,\mathrm{pc}$
Density	$\sim 10^{7}\,\mathrm{M_\odot pc^{-3}}$
Velocity dispersion	$\sim 100-1000\,\mathrm{km\,s^{-1}}$
Relaxation time	$\sim 10^{8-9}$ years







Galaxy

Size	$\sim 10^{4}\,\mathrm{pc}$
Density	$\sim 0.05\,\mathrm{M_\odot pc^{-3}}$
Velocity dispersion	$\sim 40\,\mathrm{km\,s^{-1}}$
Relaxation time	$\sim 10^{15}$ years

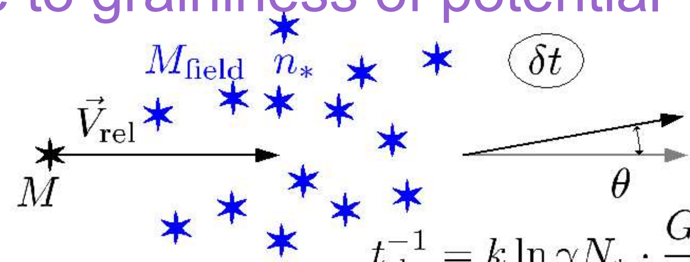
Massive Black Hole

Mass	$10^{6-9}\mathrm{M_\odot}$
Size	$R_\mathrm{S} = 2GM_\mathrm{BH}/c^2 = 10^{-7} - 10^{-4}\,\mathrm{pc}$
Rotation	??

The physics of dense clusters

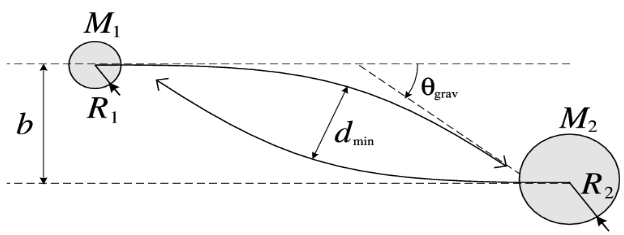
- "Collisional" = 2-body relaxation

  E, J drift and diffusion due to graininess of potential
  - Core collapse
  - Mass segregation



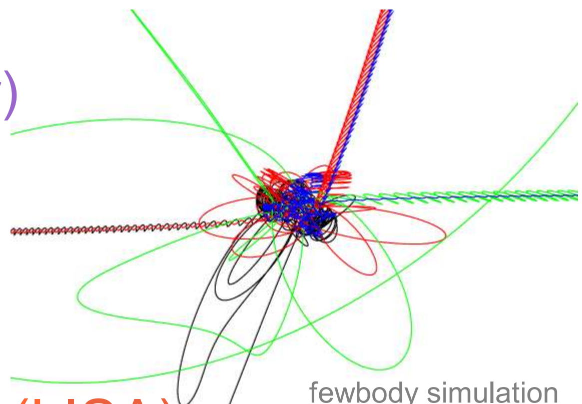
$$\langle\theta\rangle = 0$$
$$\langle\theta^2\rangle = \left(\frac{\pi}{2}\right)^2 \frac{\delta t}{t_{\rm rlx}}$$
$$t_{\rm rlx}^{-1} = k\ln\gamma N_* \cdot \frac{GMM_{\rm field}n_*}{V_{\rm rel}^3} \approx \frac{\ln\gamma N_*}{N_*} t_{\rm orb}^{-1}$$

- Collisional = Direct collisions between stars



$$t_{\rm coll}^{-1} = S_{\rm coll}V_{\rm rel}n_* \quad S_{\rm coll} = \pi(R+R_{\rm field})^2\left(1+\frac{2G(M+M_{\rm field})}{(R+R_{\rm field})V_{\rm rel}^2}\right)$$
$$t_{\rm coll} \simeq 2.1\times10^{12}\,{\rm yr}\,\frac{10^6\,{\rm pc}^{-3}}{n_*}\,\frac{\sigma_v}{30\,{\rm km\,s}^{-1}}\,\frac{{\rm R}_\odot}{R_*}\,\frac{{\rm M}_\odot}{M_*}$$

- Binary interactions
  - Heat source stopping collapse (Hard gets harder)
  - Ejections
- Interactions with a central (I)MBH
  - Tidal disruptions UV/X Flares
  - Plunges through horizon
  - Captures by emission of gravitational waves (LISA)
- Stellar evolution



fewbody simulation by J. Fregeau

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WANTED: Million-star cluster dynamics method

How to follow the evolution of systems containing 10^6 to 10^8 stars over millions to billions of years, when collisional effects are important?

- Direct approach: *N*-body
  - Newtonian gravity without approximation. "Just" solve Newton equations for *N* particles
    $$\frac{d^2\vec{X}_i}{dt} = -G\sum_{j\neq i} m_j \frac{\vec{X}_i - \vec{X}_j}{\left|\vec{X}_i - \vec{X}_j\right|^3}$$
  - No spatial symmetry or dynamical equilibrium assumed
  - Very time consuming $T_{\rm CPU}/t_{\rm rlx} \propto N^3 \Rightarrow N < 10^6$
- Continuum approaches: Fokker-Planck & Gas codes
  - Follow evolution of DF or "fluid" of stars
  - Very fast
  - Very approximate (difficulties with mass spectrum, stellar evolution, collisions...)
- The best of both worlds?

The **Hénon Monte Carlo** scheme...

The Monte Carlo stellar dynamics method

Hénon 71a,b, 73, 75; Stodołkiewicz 82, 85, 86; Giersz 98, 01
Joshi et al. 00, 01; Fregeau et al. 03; Gürkan et al. 04; Freitag & Benz 01, 02

- Follow evolution *of* orbits not evolution *on* orbits
- Uses 3 assumptions:
  - **Spherical symmetry**
  - **Dynamical equilibrium**
  - **Diffusive 2-body relaxation** (Chandrasekhar; Fokker-Planck)

$$dn_* = f(\vec{X},\vec{V})d^3X d^3V = 8\pi^2 F(E,J) R^2 dR V_\mathrm{t} dV_\mathrm{t} dV_\mathrm{r}$$

$$E = \frac{1}{2}\vec{V}^2 + \Phi(R) \quad J = R\cdot V_\mathrm{t}$$

- Represents the cluster with particles
  - 1 particle = 1 spherical shell (given orbital and stellar prop.)
  - 1 particle = many stars (possibly) ⇒ No limit on N_*
  - Local time steps $\delta t \leq f_{\delta t}\cdot \min(T_\mathrm{rlx}, T_\mathrm{coll}, \ldots)$
- Allows rich physics
  - Cluster (+central object) self-gravity; *V*-anisotropy; Any *M*-spectrum
  - 2-body relaxation; Stellar collisions (use SPH data); Stellar evolution
  - “Loss-cone processes”: Tidal disruptions; Plunges; GW-captures
- Fast $T_\mathrm{CPU}/t_\mathrm{rlx} \propto N\ln N \Rightarrow N \approx 10^4 - 10^7$

How does the MC code work?

- Initialization
  - Realization of cluster with *N* particles according to DF *F(E)*
    E_i, J_i, R_i
  - Attribution of masses M_i according to IMF
- Main loop (modifies 2 or all particles per step)

1) Selection of pair of neighboring particles $P_{\rm selec} \propto \delta t(R)^{-1}$
2) Test for collisions: rand() < $P_{\rm coll}$; modify $M_{1,2}$ & $V_{1,2}$ if needed

   Similarly: Test for binary interaction; Test for interaction with central BH
3) Relaxation simulated by "Super-encounter"

$$\theta_{\rm SE} = \frac{\pi}{2}\sqrt{\frac{\delta t}{t_{\rm rlx}}}$$

4) New orbital parameters $E_{1,2}$ & $J_{1,2}$ computed
5) For each particle, new position R_i picked at random on (E_i,J_i)-orbit

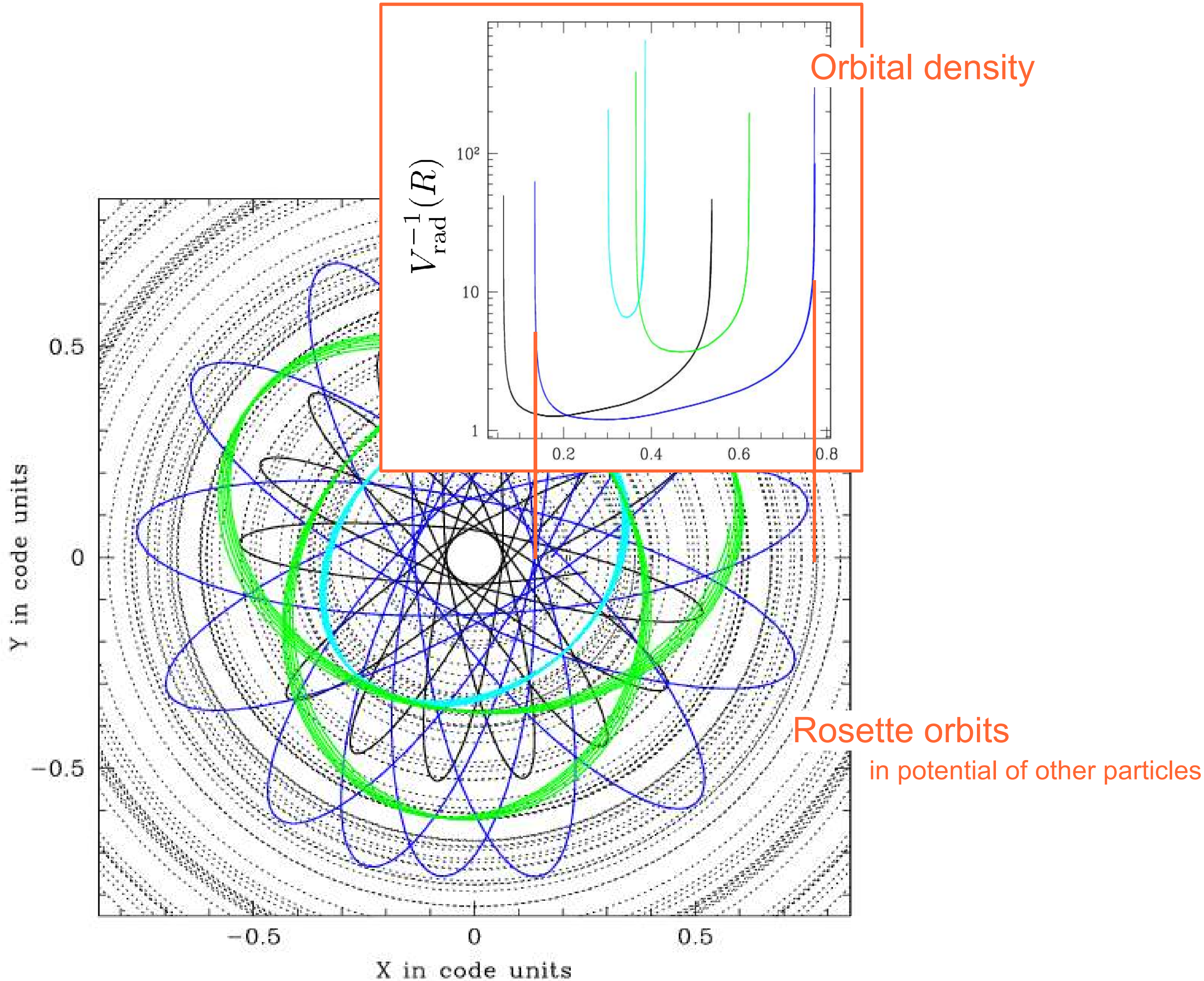
   Cluster's potential updated

$$\frac{dP}{dR}(R) \propto \frac{1}{V_{\rm rad}(R)}$$

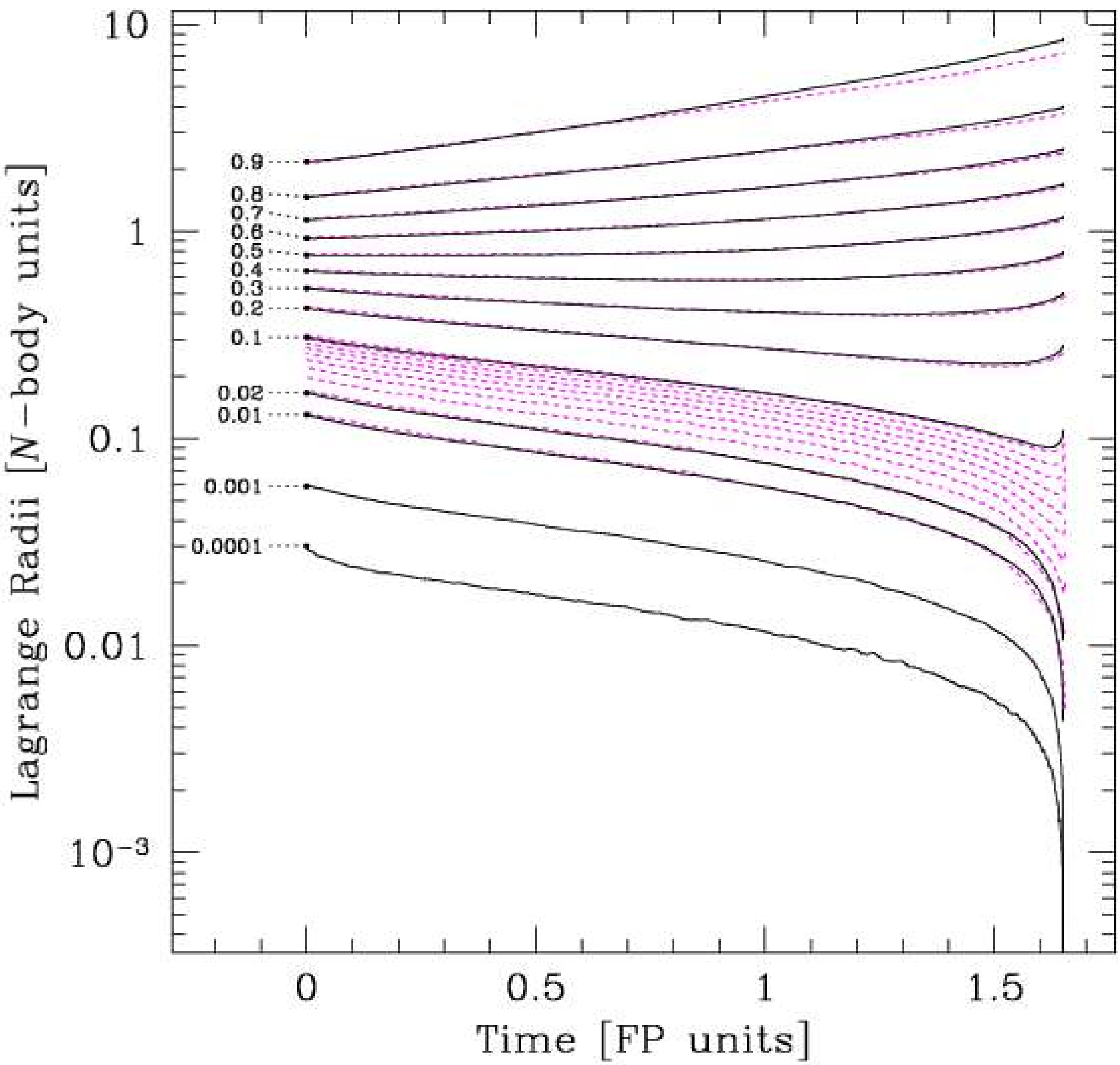
Go back to 1

And add many complications!...

Selection of particle position in MC code



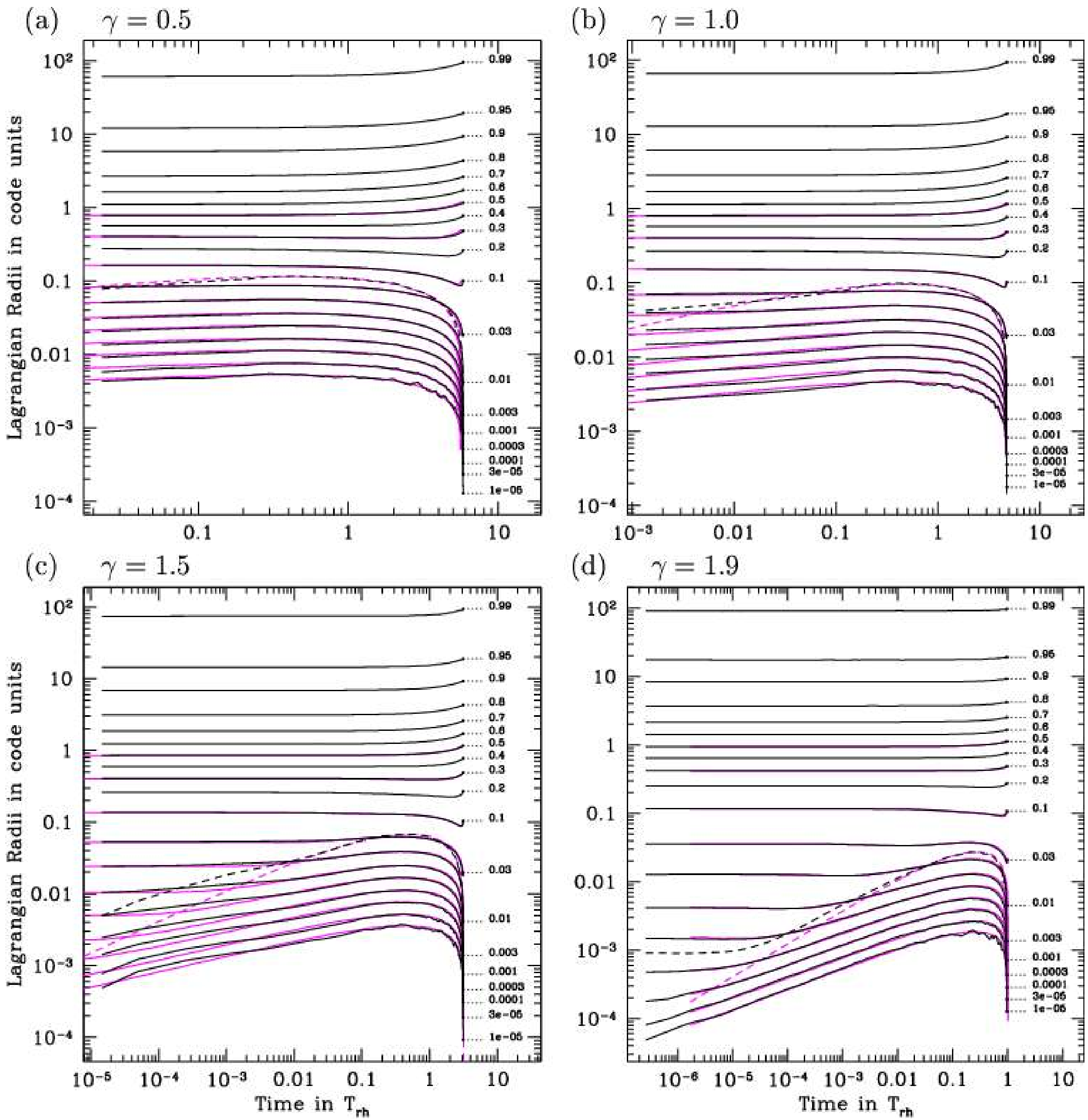
Testing the MC code...



Gravothermal collapse
of single-mass Plummer model

Comparison with direct *N*-body
(Baumgardt)

Testing the MC code...



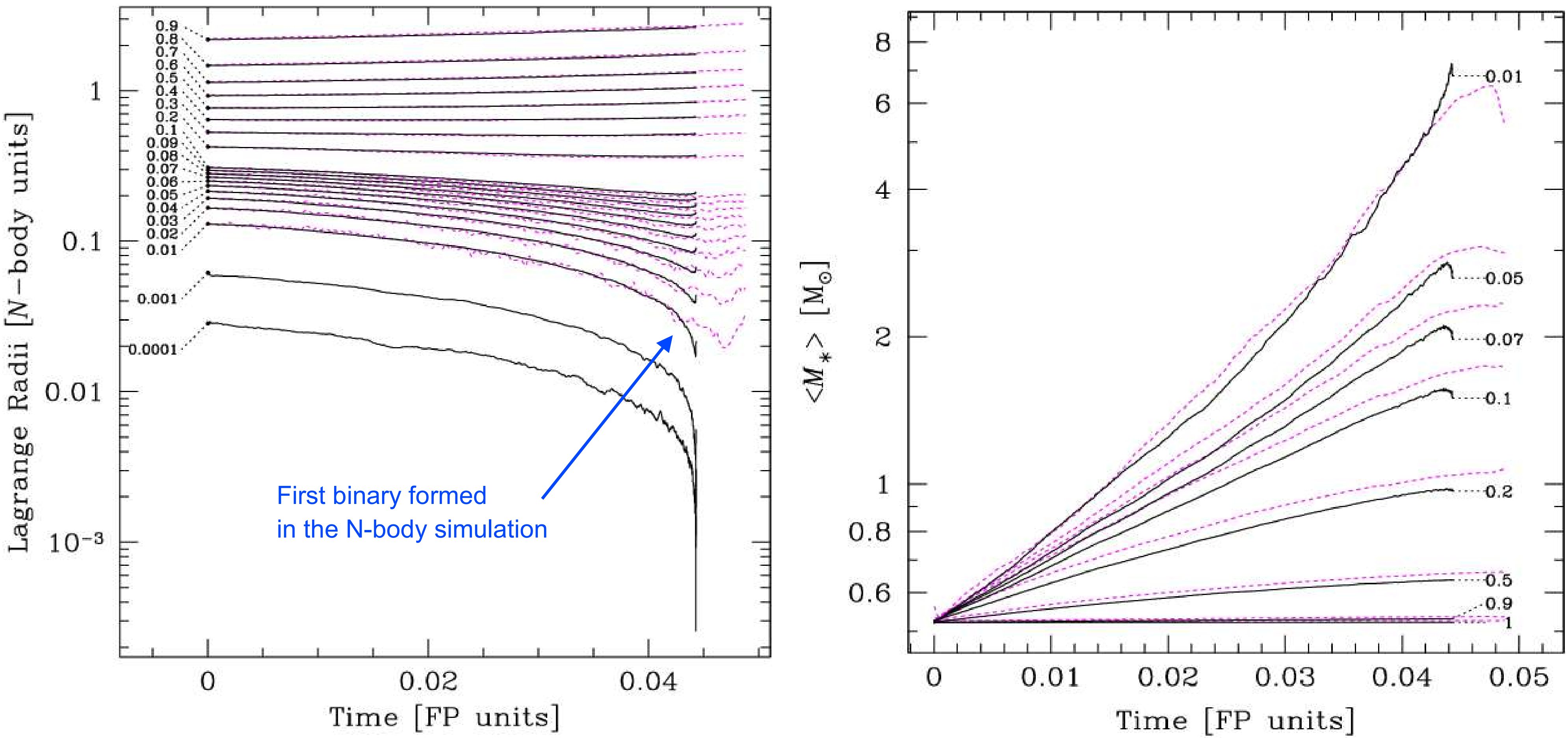
Gravothermal expansion and collapse of single-mass Dehnen models

Comparison with Quinlan 96 (FP code)

Testing the MC code...

Core collapse driven by mass segregation (Plummer, Salpeter IMF 0.2-10 M_{sun})

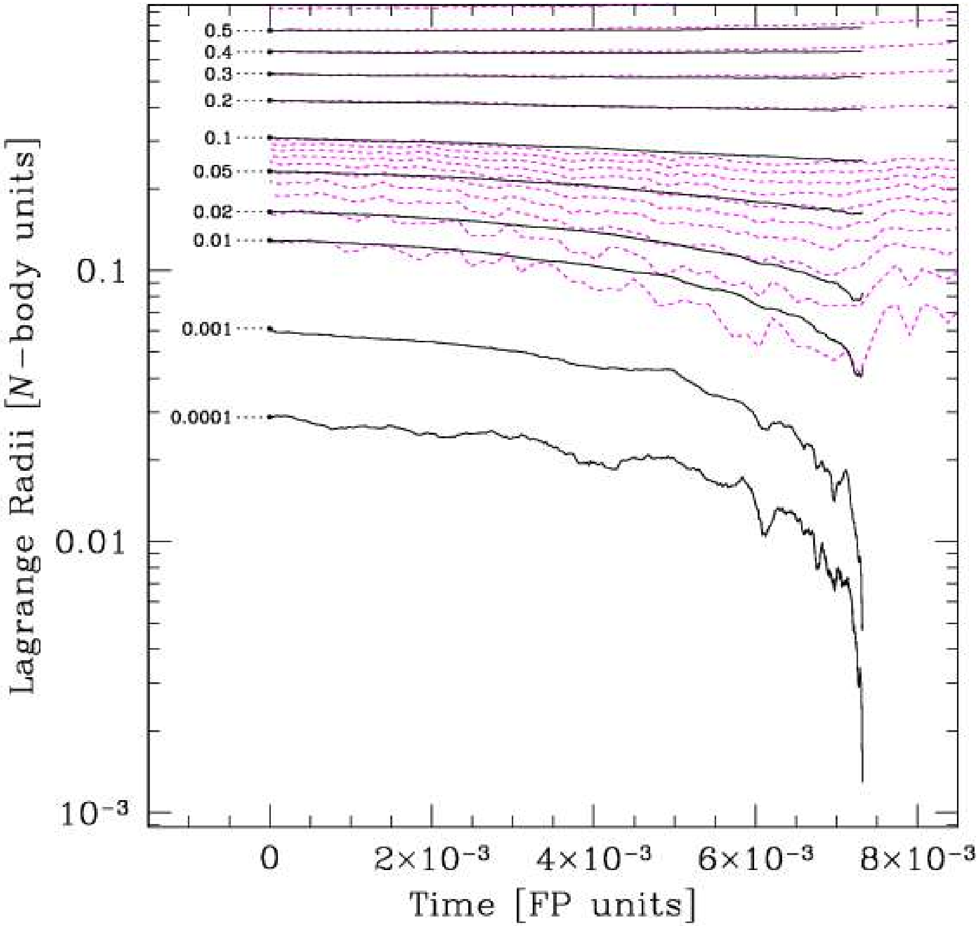
Comparison with direct *N*-body (Baumgardt)

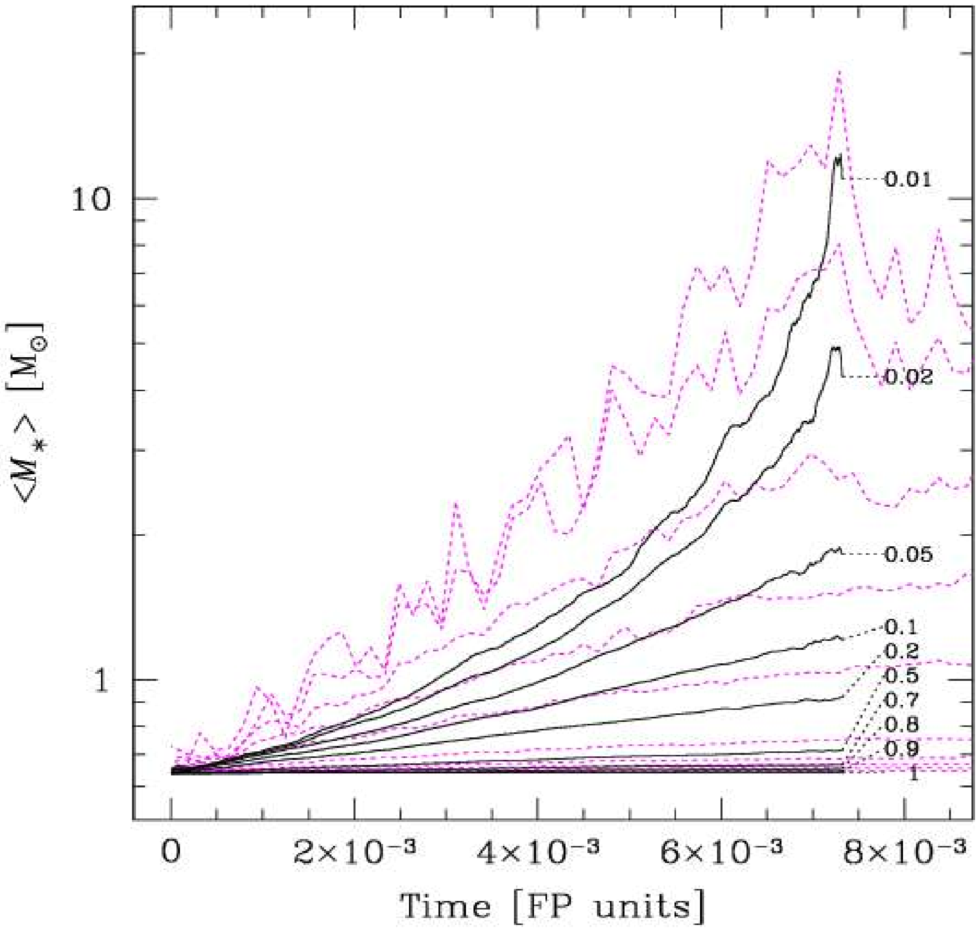


Testing the MC code...

Core collapse driven by mass segregation (Plummer, Salpeter IMF 0.2-100 M_{sun})

Comparison with direct *N*-body (Baumgardt)... Some issues!

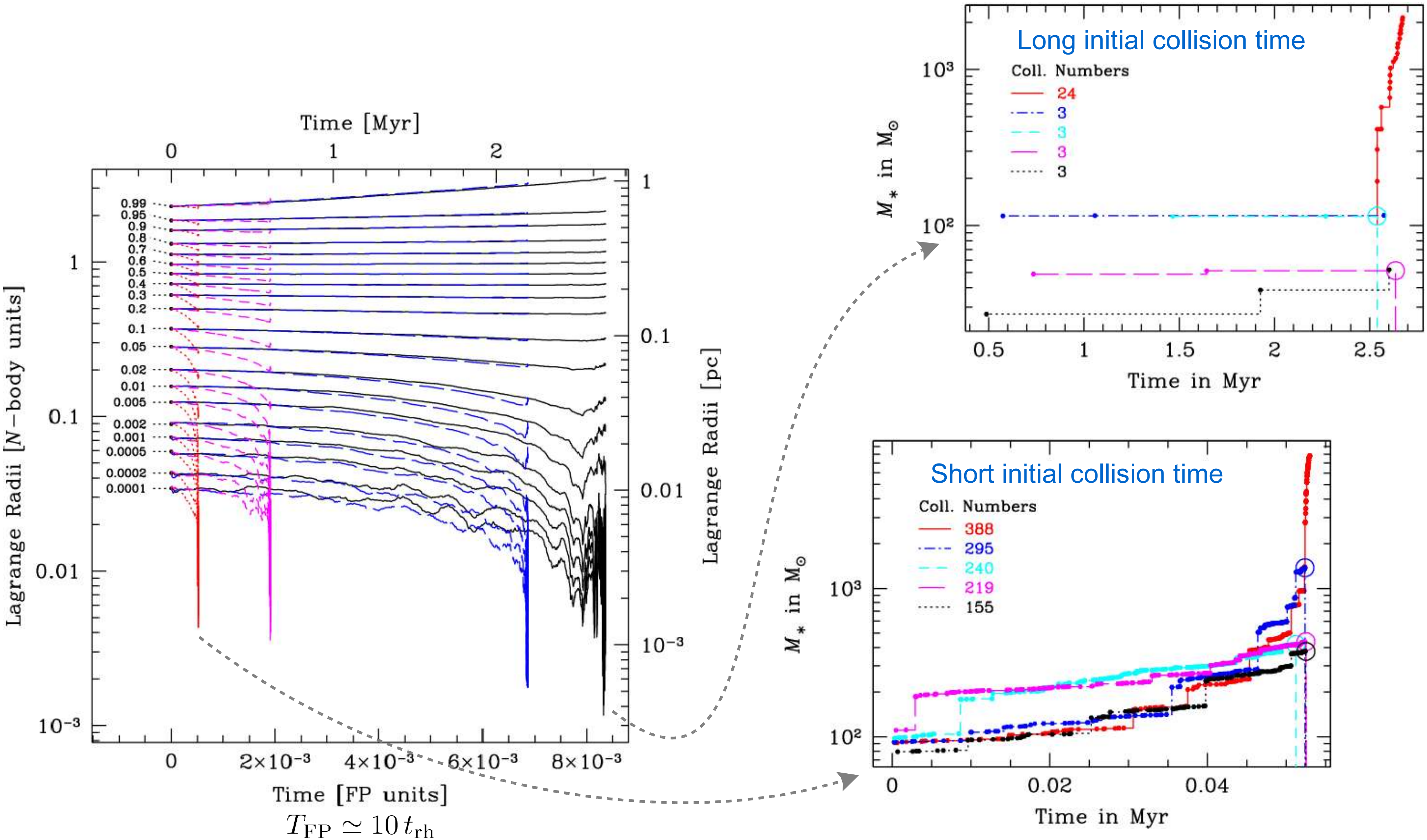




Full collapse in less than 200 dynamical times (*N*-body)!

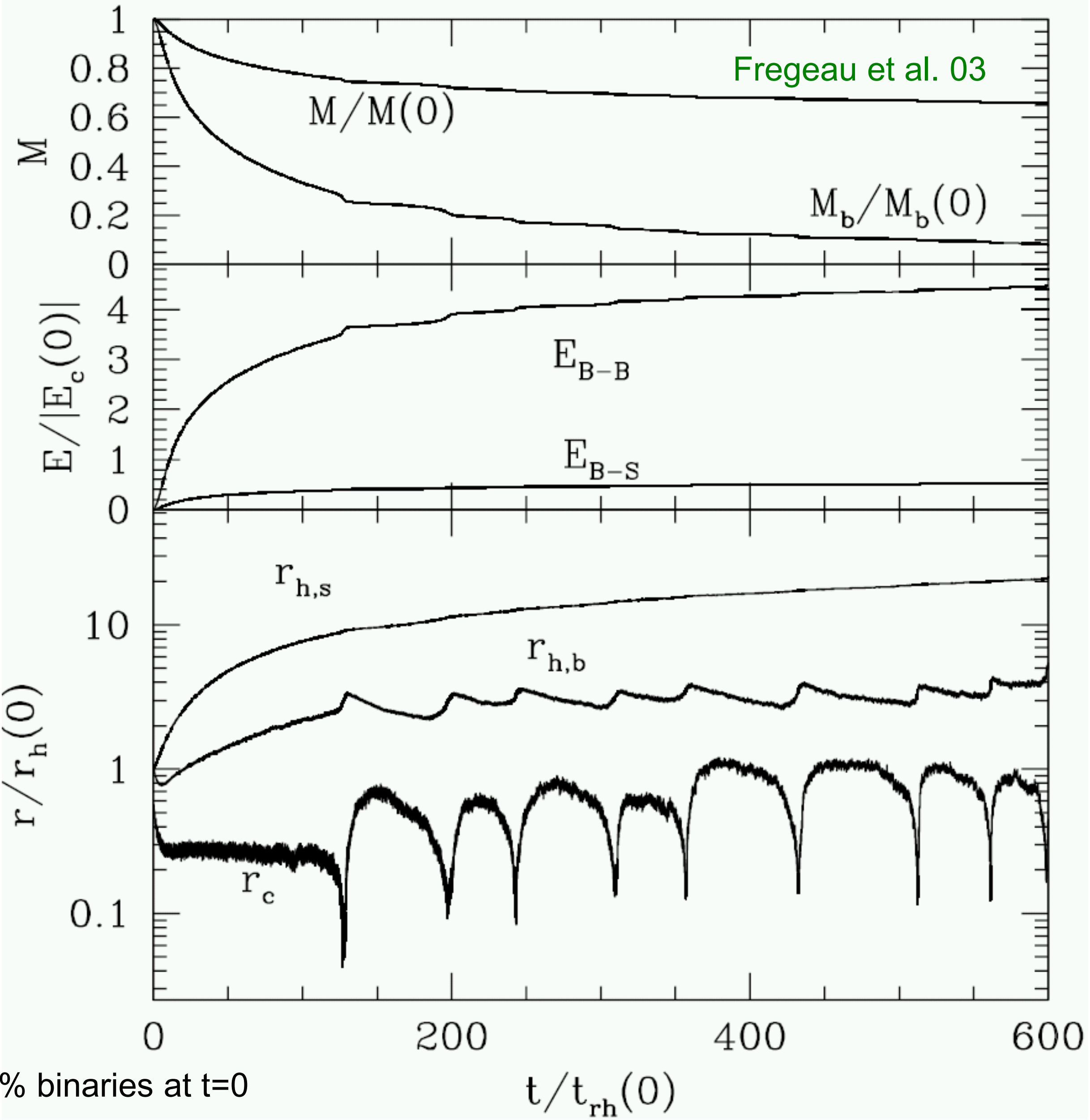
Application: Collisional runaway

Gürkan, Freitag & Rasio 2004; Freitag, Rasio & Baumgardt 2005; Freitag, Gürkan & Rasio 2005



Application: Dynamical role of binaries

- Some particles represent binaries
- Binary interactions through "recipes" or explicit 3/4-body integrations
- Complication: evolution of binary stars...
- See also hybrid MC-Gas code by Giersz & Spurzem

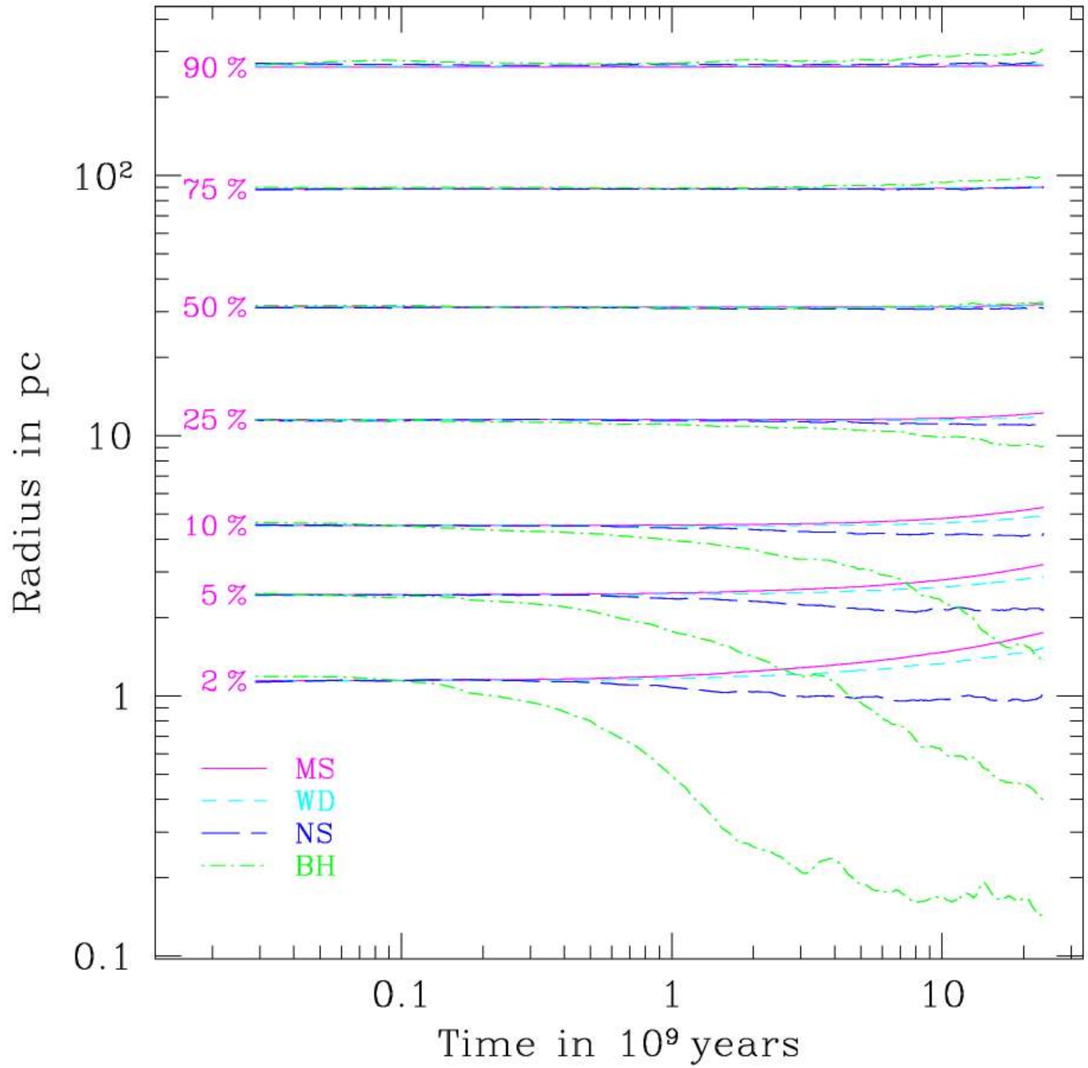


Plummer model with 20% binaries at t=0

Application: Simulation of SgrA*-type nucleus

Freitag 2003
Freitag & Kalogera, in progress

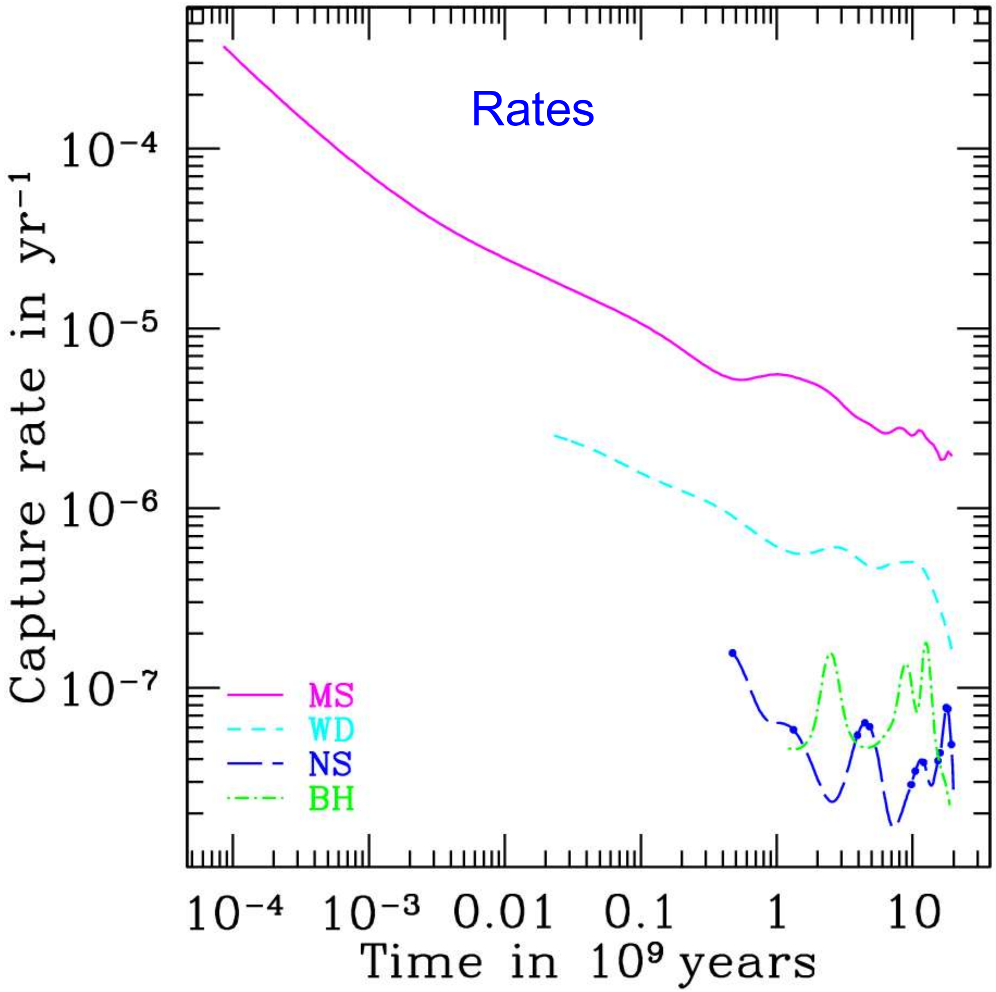
Overall structure evolution
Segregation of stellar BHs to the center

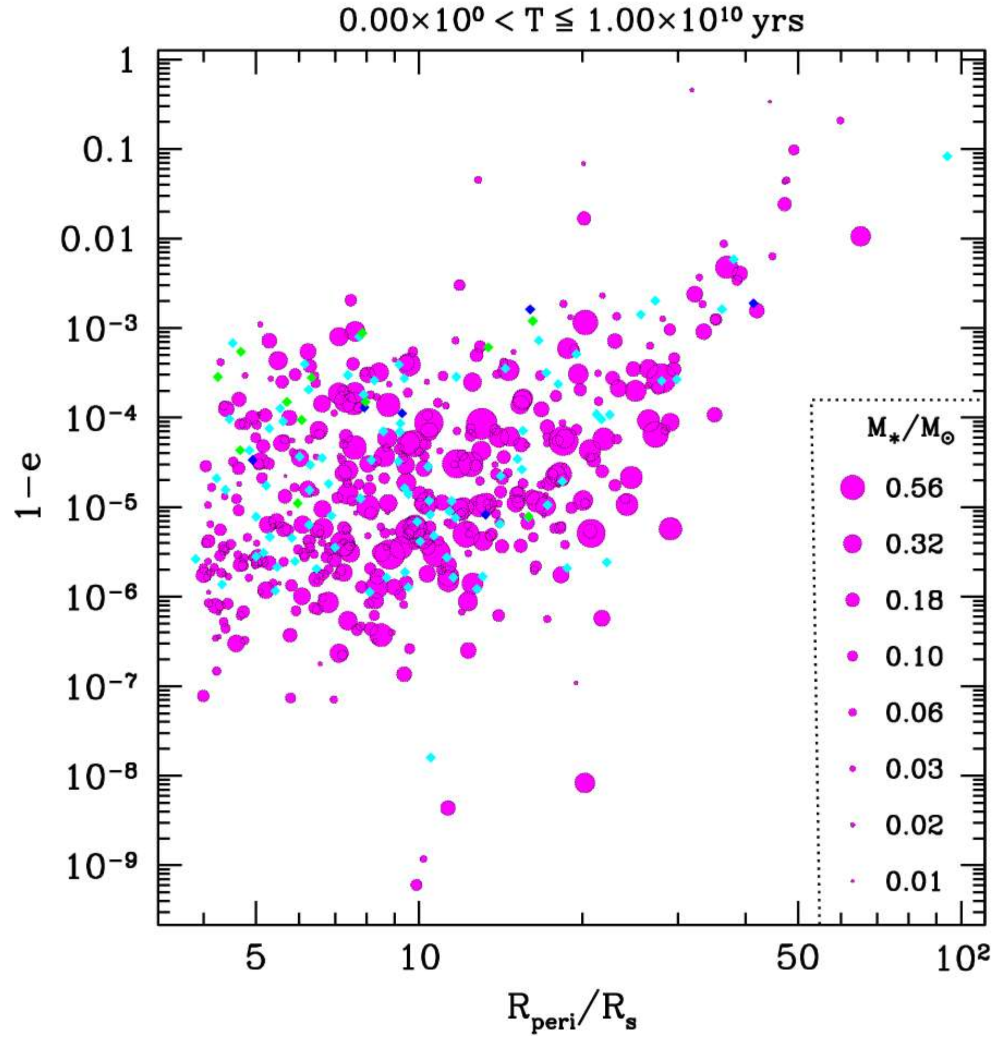


Simulation of SgrA-type nucleus*

Formation of extreme-mass ratio binaries

GW sources for LISA





The future of MC cluster dynamics codes

- Non-spherical geometries
  - Thin disks (Nyffenegger & Benz for planetesimal disks, in progress)
  - Hybrid *N*-body – MC approach
    - "Collisionless" N-body to compute orbits
    - MC to introduce collisional effects in well controlled fashion

Implemented by Spitzer for spherical systems.
Already suggested by Hénon for non-spherical case!
Similar to SCF+FP relaxation by Sigurdsson et al.

- Specialized implementations
  - Dynamics of cusp stars around MBH
    - Fixed potential
    - Individual time steps to resolve "loss-cone" processes (tidal interactions, captures and orbit evolution by GW emission...)
- Better understanding of the algorithm? :-]

Back to the future...

From Hénon 1971, Ap&SS 13, 284:

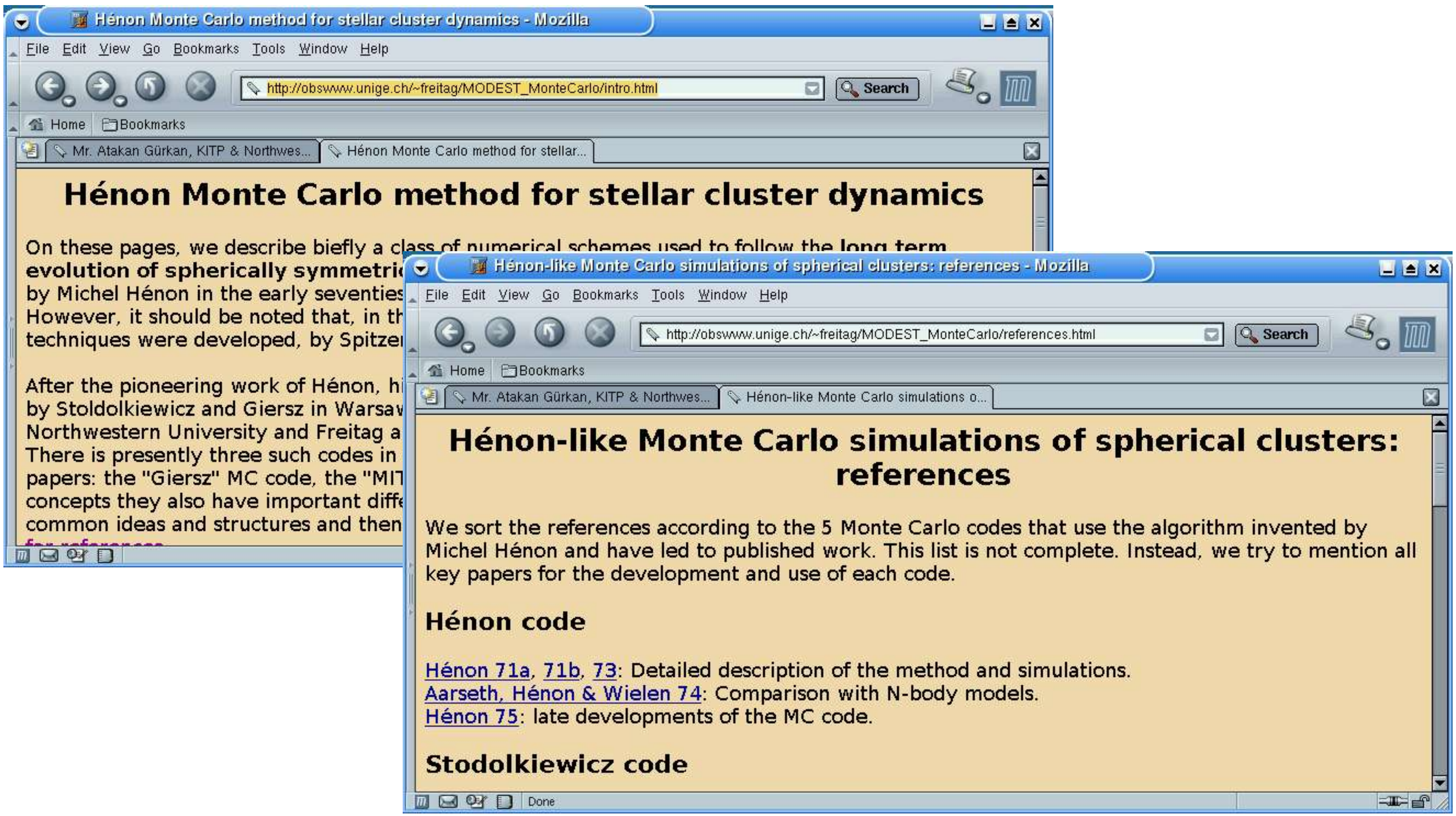
4. Possible Extensions

It would be interesting to try to extend the Monte Carlo method to other cases than the spherical symmetry, and in particular to the case of axial symmetry. However, before one can apply the method as presented here, the characteristics of the motion of a star in the mean field must be known; more specifically, in order to be able to select at random a new position of the star, one must know precisely which part of space is visited by the orbit, and what fraction of time the star spends in each volume element. Mathematically, this means that one must know explicitly all the isolating integrals of the motion. In the case of axial symmetry, therefore, serious difficulties would arise from the ill-defined nature of the third integral (Hénon and Heiles, 1964).

This problem can be eliminated by sacrificing the first half of the Monte Carlo method: that is, one integrates numerically the motion of the stars in the mean field, with a time step small compared to t_c. The second half of the method could be preserved: one would still compute the effect of encounters by selecting at random one field star. The computing time would be proportional to nN. The value of N would have to be specified, since both time scales t_c and t_r would appear in the computation.

Hénon-type Monte Carlo on the web

- “MODEST” page
  - Start at http://www.manybody.org/modest/
  - Goto “Working Groups”, select “WG3; Stellar Dynamics”
  - Look for “Monte Carlo” in “Methods”



The End

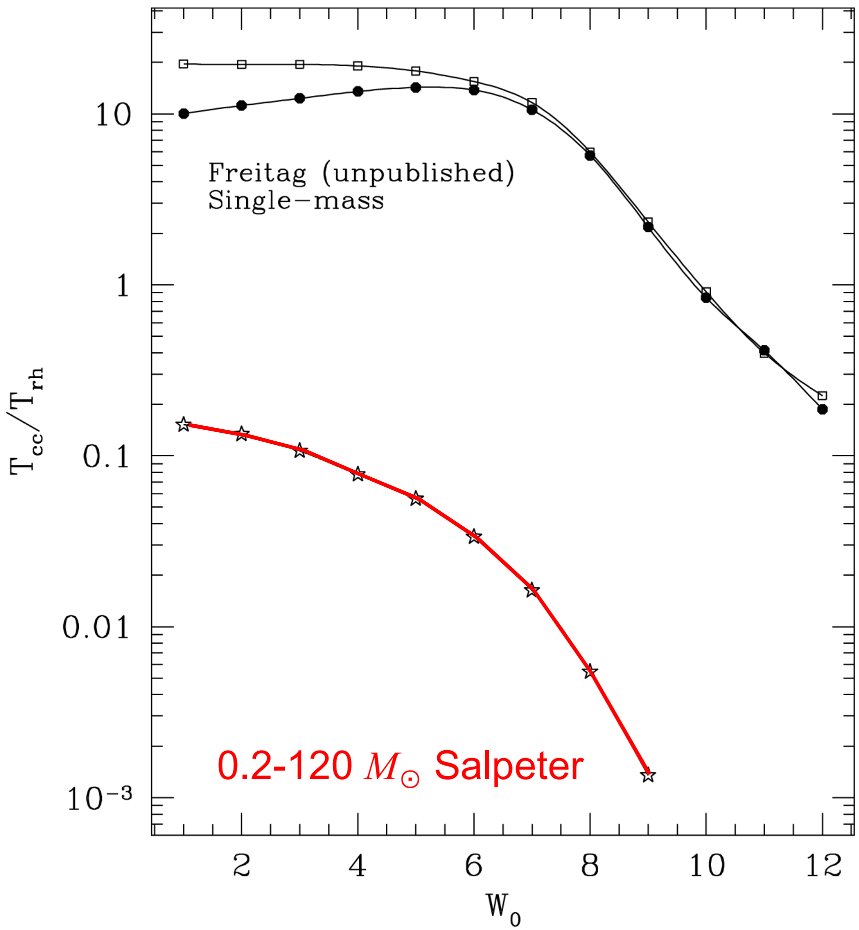
Thank you.

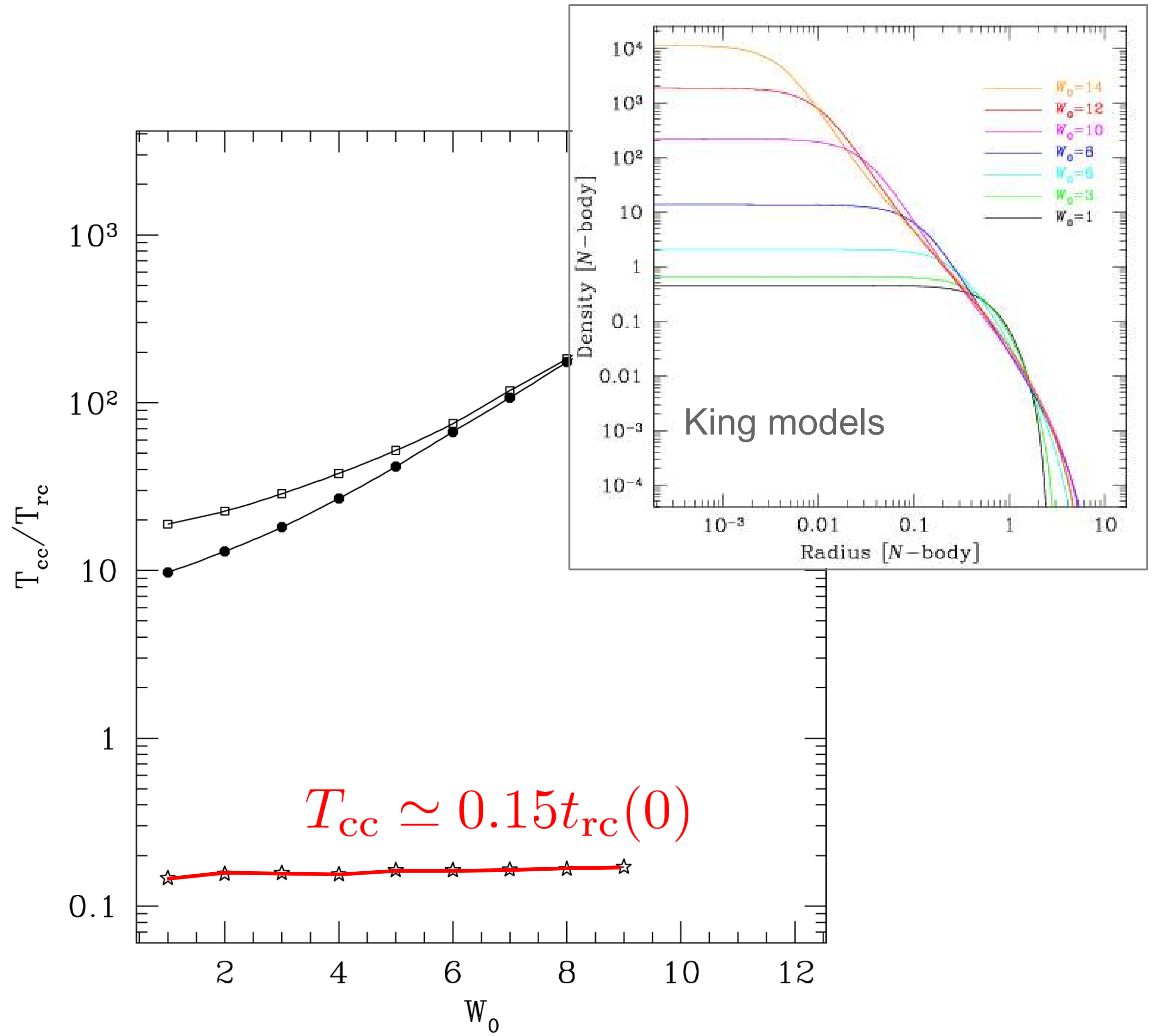
General motivations

- Massive black holes are present at the center of dense stellar clusters
  - SMBH in galactic nuclei (certainly!)
  - IMBH in smaller nuclei (maybe...)
- How do these (I)MBHs (or their seeds) form?
  - Gaseous processes?
  - Stellar dynamical processes?
- How do they grow?
  - Gaseous processes?
  - Stellar dynamical processes?
- How do they reveal themselves?
  - Direct gas accretion
  - Structure of cluster (velocity, photometric profiles; peculiar stars...)
  - Accretion of stellar gas
    - Stellar winds, Stellar collisions, Tidal disruptions
  - Emission of GWs by “captured” stars

How fast is core collapse?

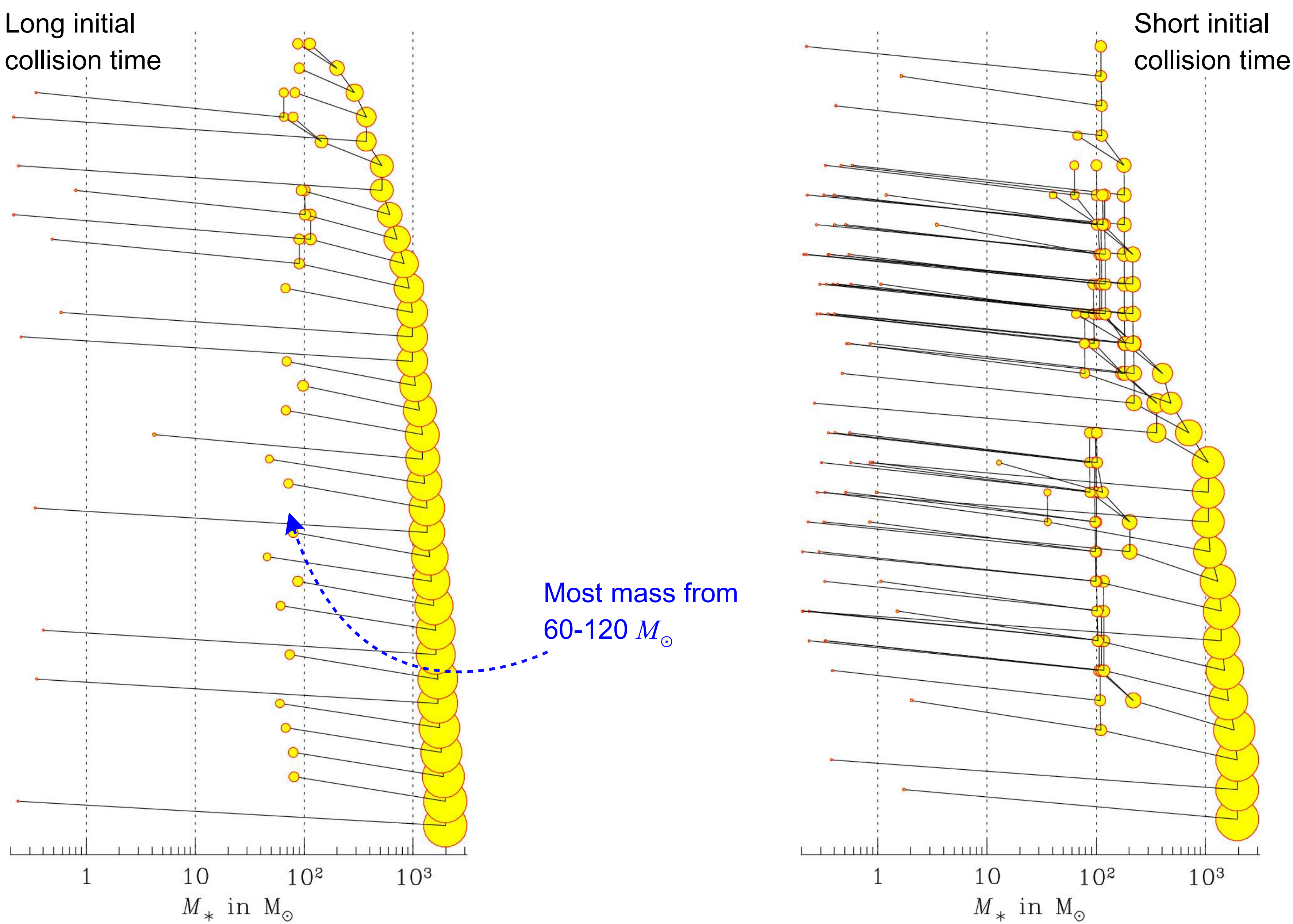
MC runs with various cluster structures





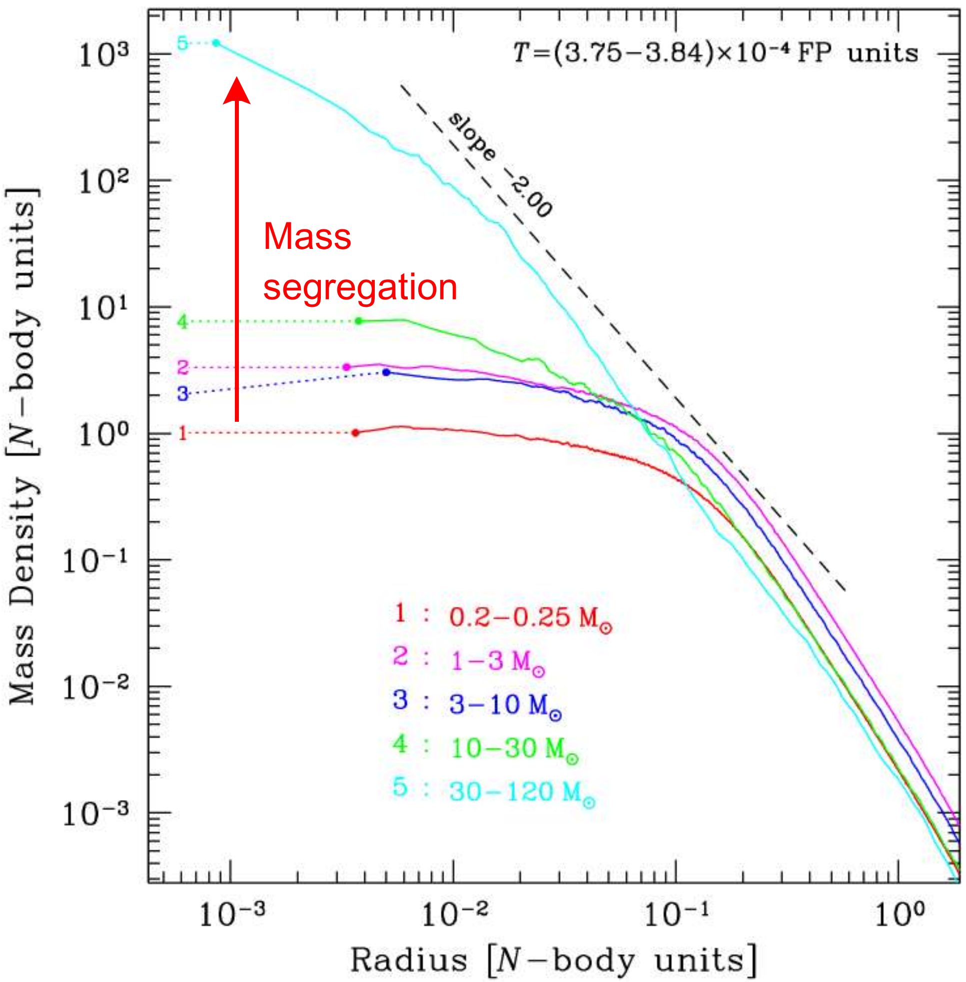
Gürkan, Freitag & Rasio 2004

Runaway merger trees

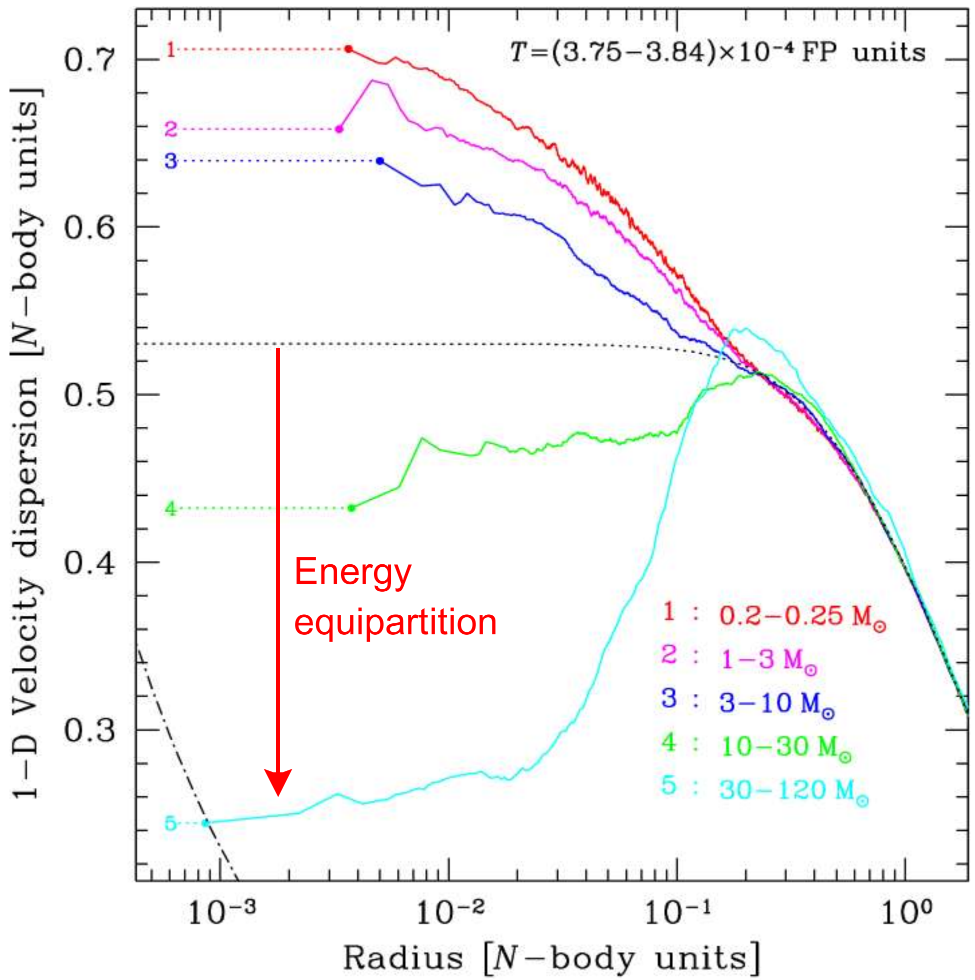


Cluster structure during runaway

Density of stars



Velocity dispersions



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