Cosmological Structure Formation I

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References

- T. Padmanabhan, “Structure Formation in the Universe”, Cambridge (1994)
- S. D. M. White, “The Formation and Evolution of Galaxies”, Les Houches Lectures, August 1993, astro-ph/9410043

Outline

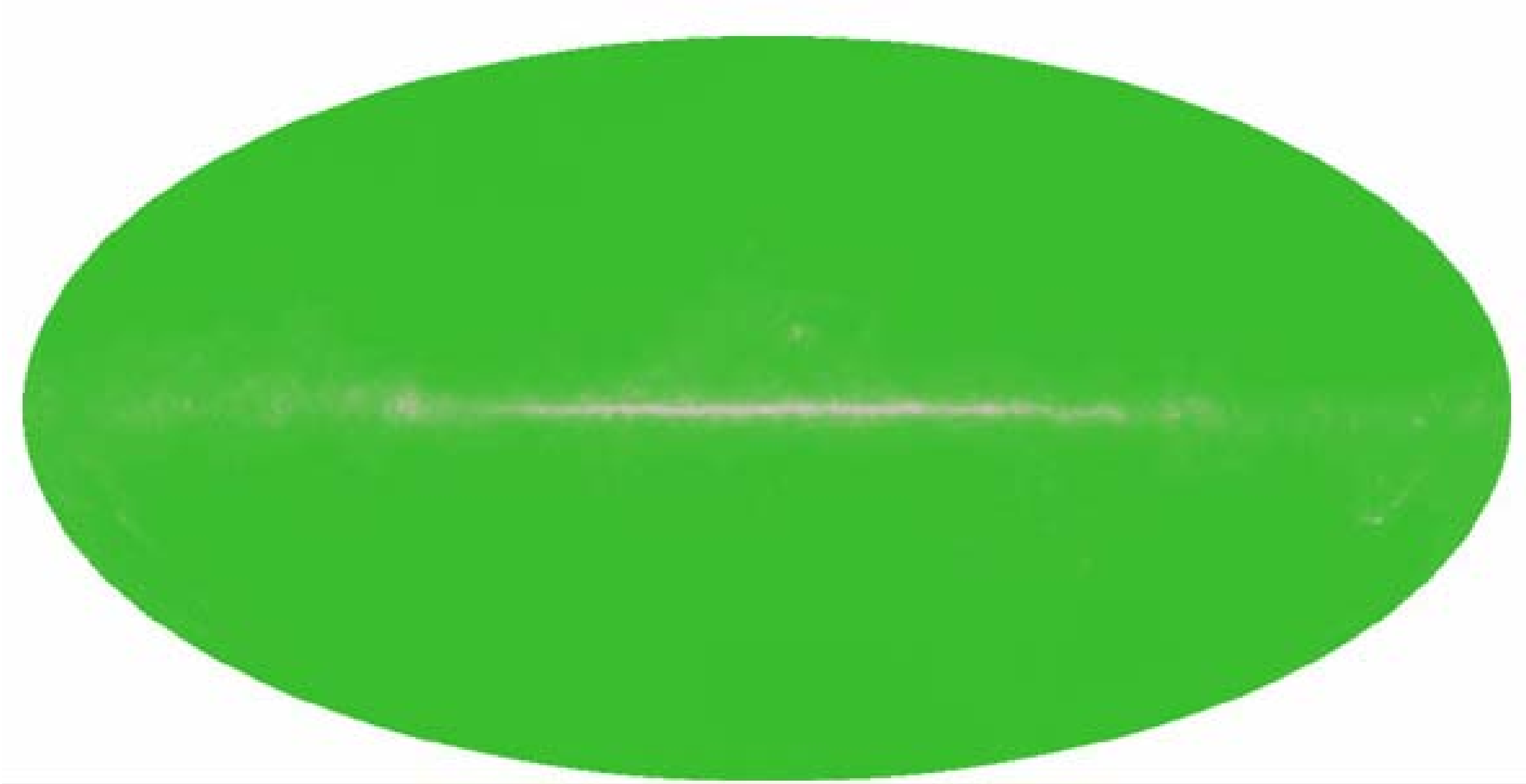
- Examples of cosmic structure
- Gravity: origin of cosmic structure
- Linear theory of perturbation growth
- Nonlinear simulations I. dark matter
  - hierarchical clustering
- Nonlinear theory of hierarchical clustering
- Nonlinear simulations II. baryons
  - hydrodynamic cosmology
  - the Enzo code

What is cosmic structure?

- Inhomogeneities in the distribution of matter in the universe at any epoch

$\delta = (\rho / \langle\rho\rangle) - 1$	Regime	Example
0	homogeneous	CMB
$O(\varepsilon)$	linear	CMB anisotropies
O(0.1)	quasi-linear	galaxy LSS
O(1-10)	nonlinear	Lyman alpha forest
O(>100)	virialized	galaxies, groups, clusters

Cosmic Microwave Background
Penzias & Wilson (1965)



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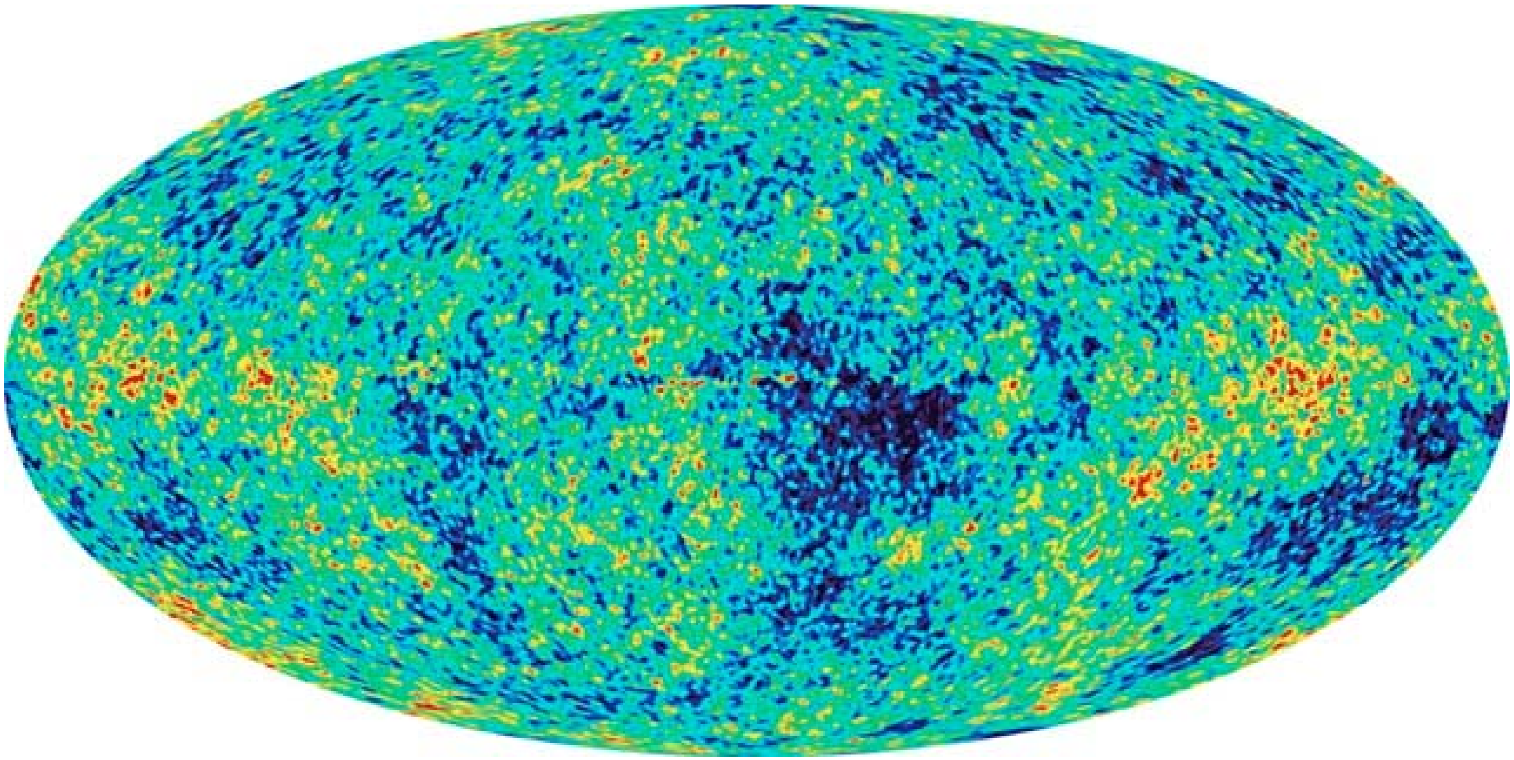
T=2.73 K

Cosmic Microwave Background
WMAP Year 1



$\Delta T/T \sim \delta \sim 10^{-4}$

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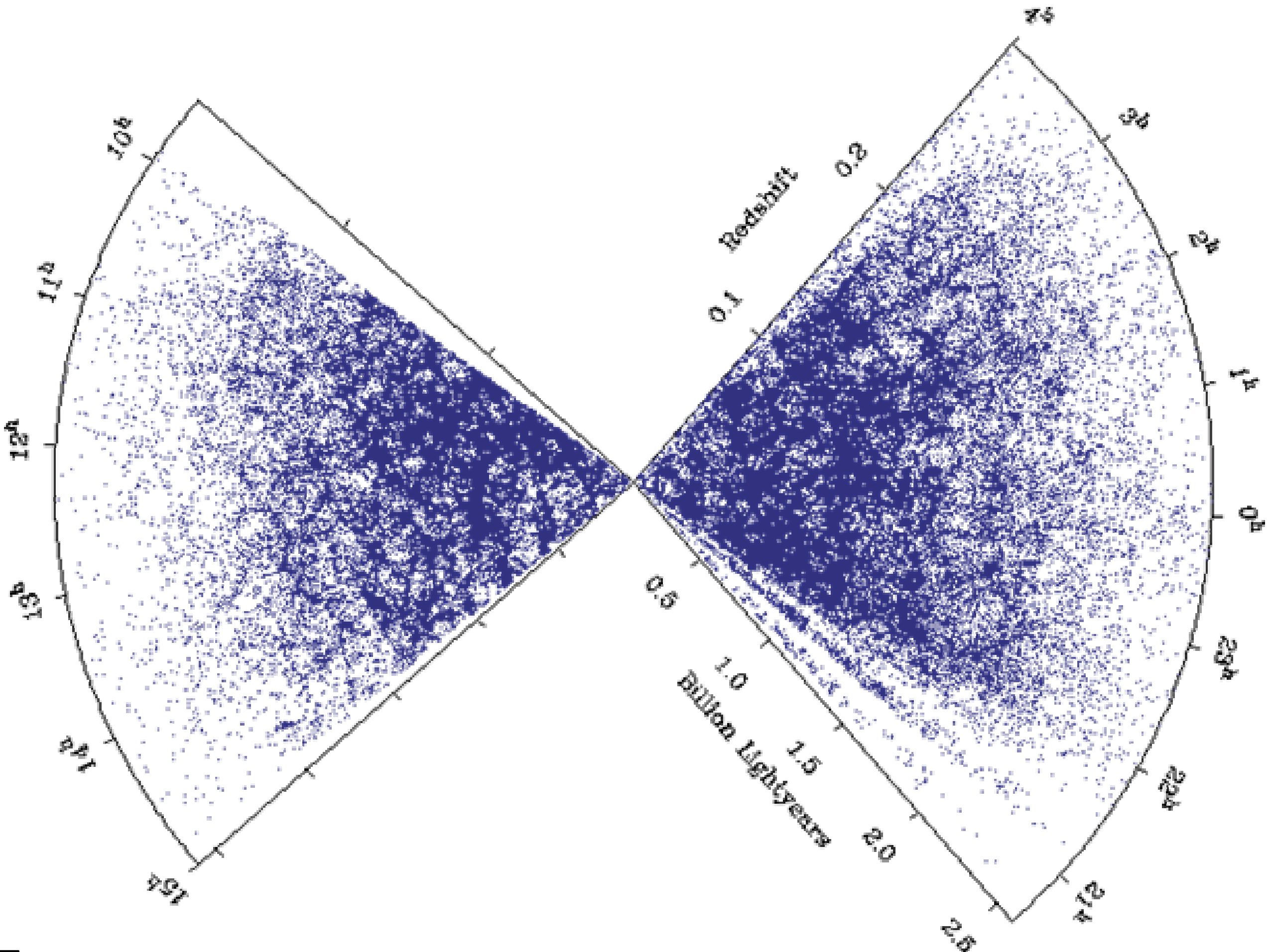
Galaxy Large Scale Structure: 2dF Galaxy Redshift Survey



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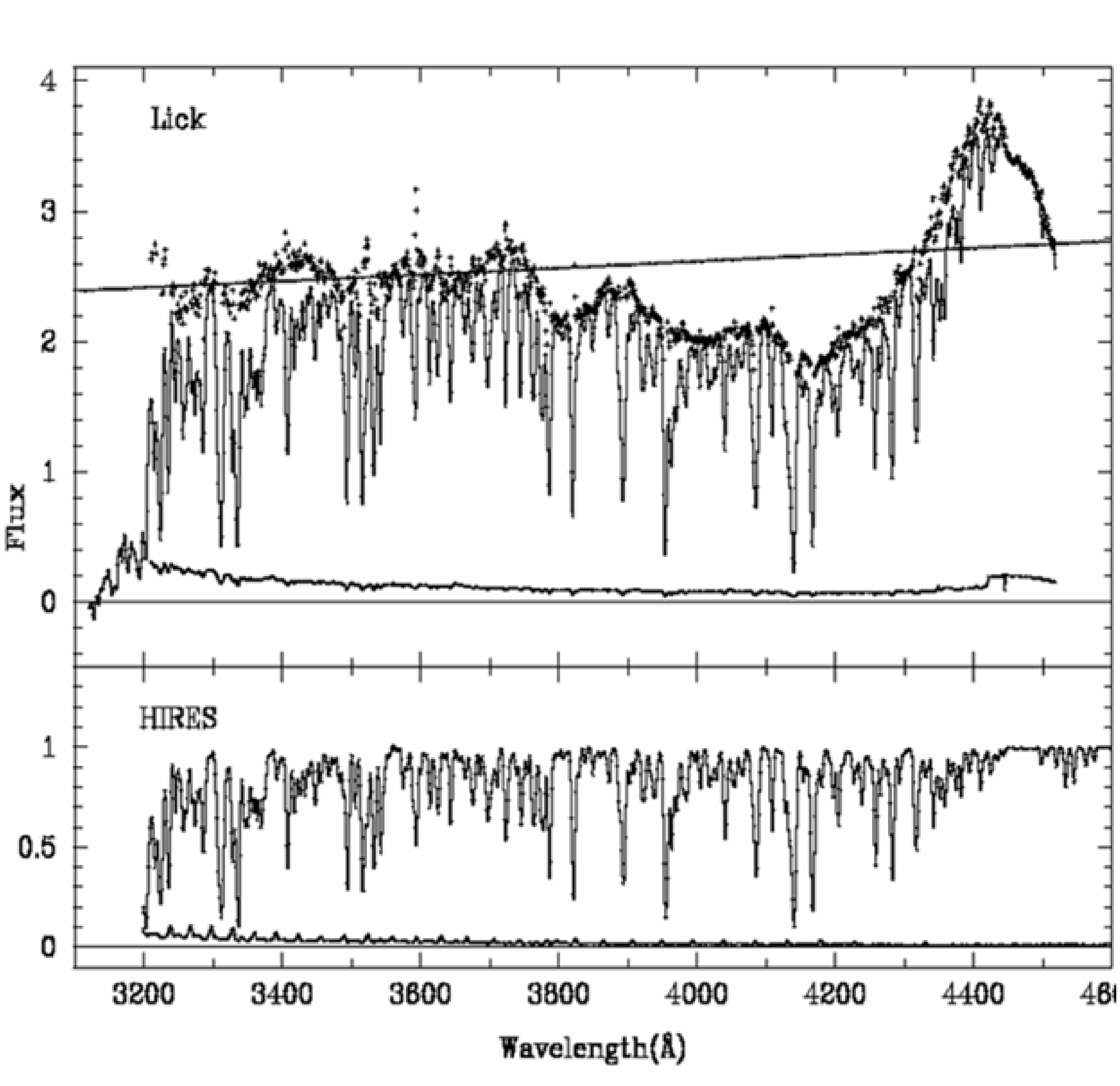
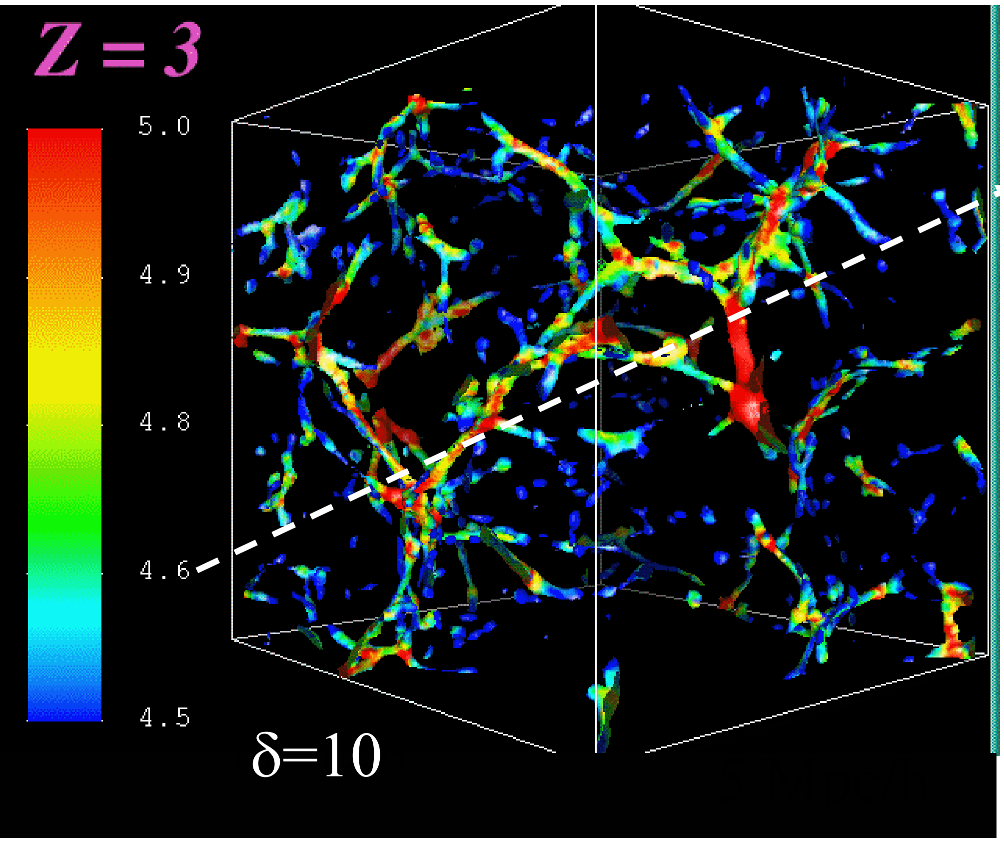
$$\langle \Delta n_g/n_g \rangle = b\langle \Delta\rho/\rho \rangle \sim 0.1$$

Lyman Alpha Forest:
HI absorption lines in quasar spectra

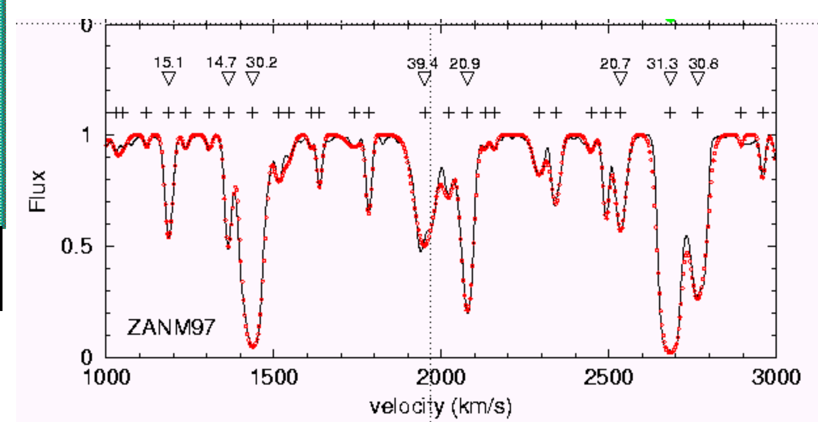


Physical Origin of the Lyman Alpha Forest

Zhang, Anninos, Norman (1995)



- intergalactic medium exhibits cosmic web structure at high z
- models explain observed hydrogen absorption spectra



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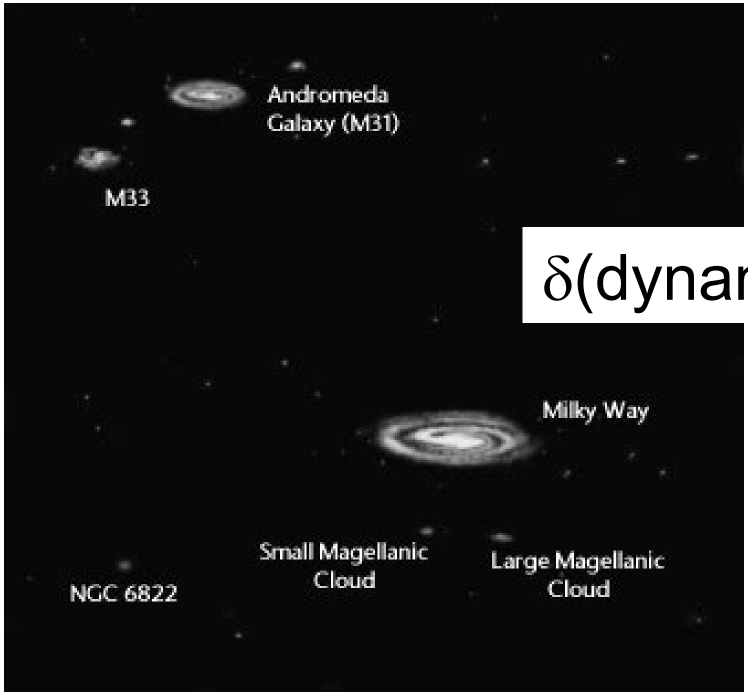
Galaxies, Groups & Clusters

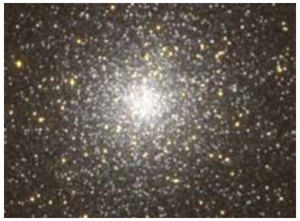


δ(dynamical) ~ 10^3

δ(disk) ~ 10^6

The Universe Exhibits a Hierarchy of Structures

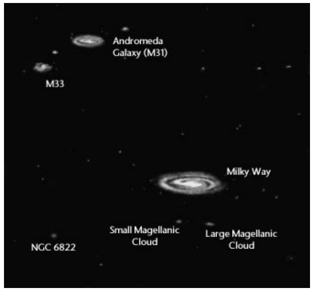
star clusters



dwarf galaxies

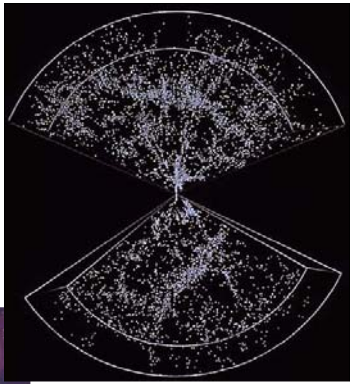
galaxies

galaxy groups



galaxy clusters

galaxy superclusters



10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9

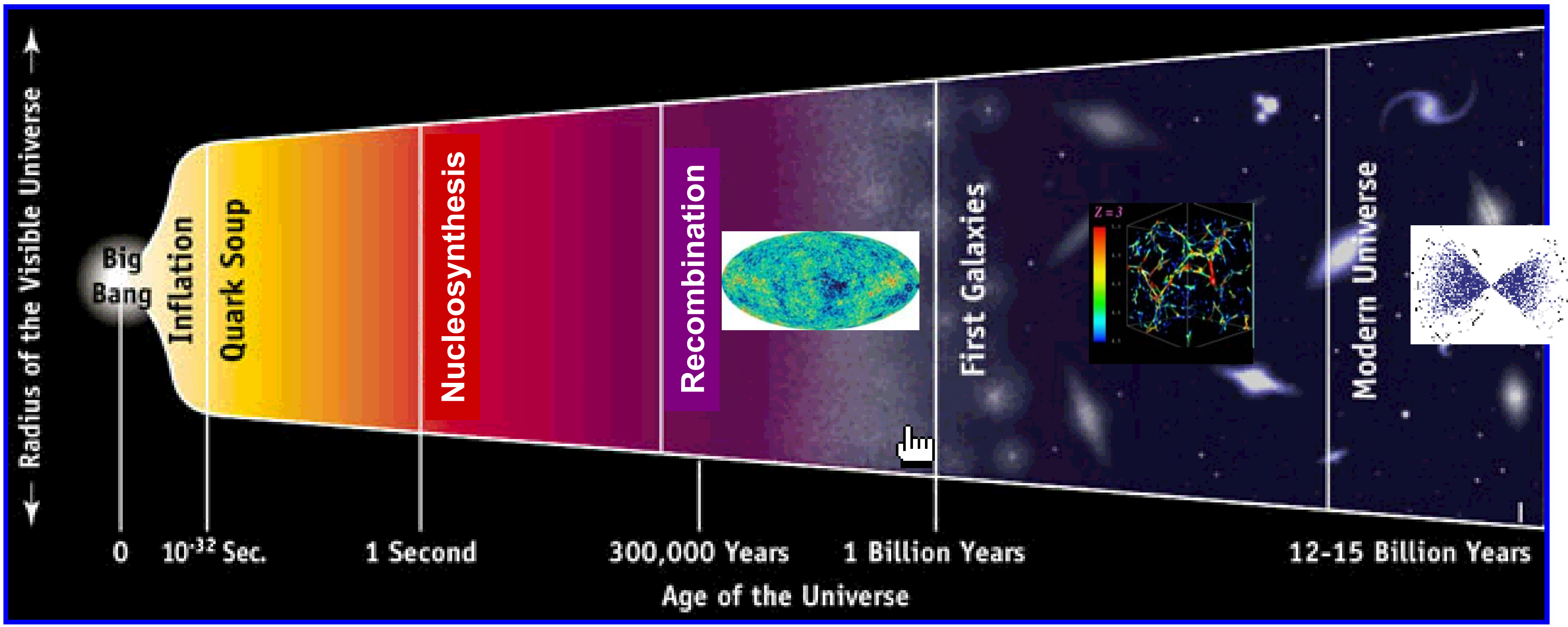
Light years

Structure Formation: Goals

- Understand
  - origin and evolution of cosmic structure from the Big Bang onward across all physical length, mass & time scales
  - Interplay between different mass constituents (dark matter, baryons, radiation), self-gravity, and cosmic expansion
  - Dependence on cosmological parameters
- Predict
  - Earliest generation of cosmic structures which have not yet been observed

History of the Universe

phase transitions gravitational instability

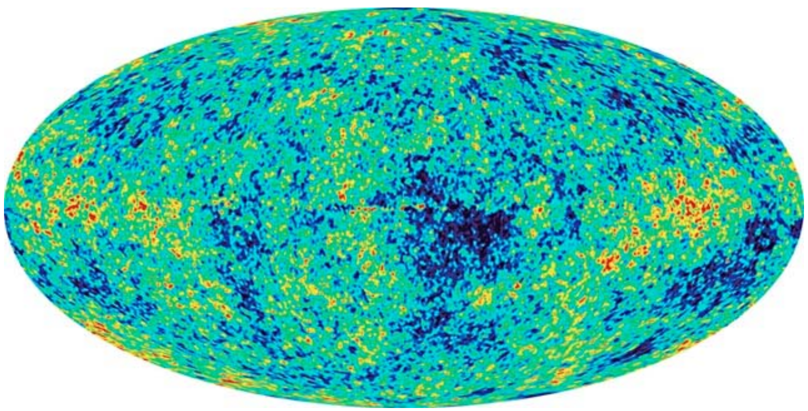


linear perturbation theory nonlinear simulations

Our universe then and now

Recombination (~380,000 yr)

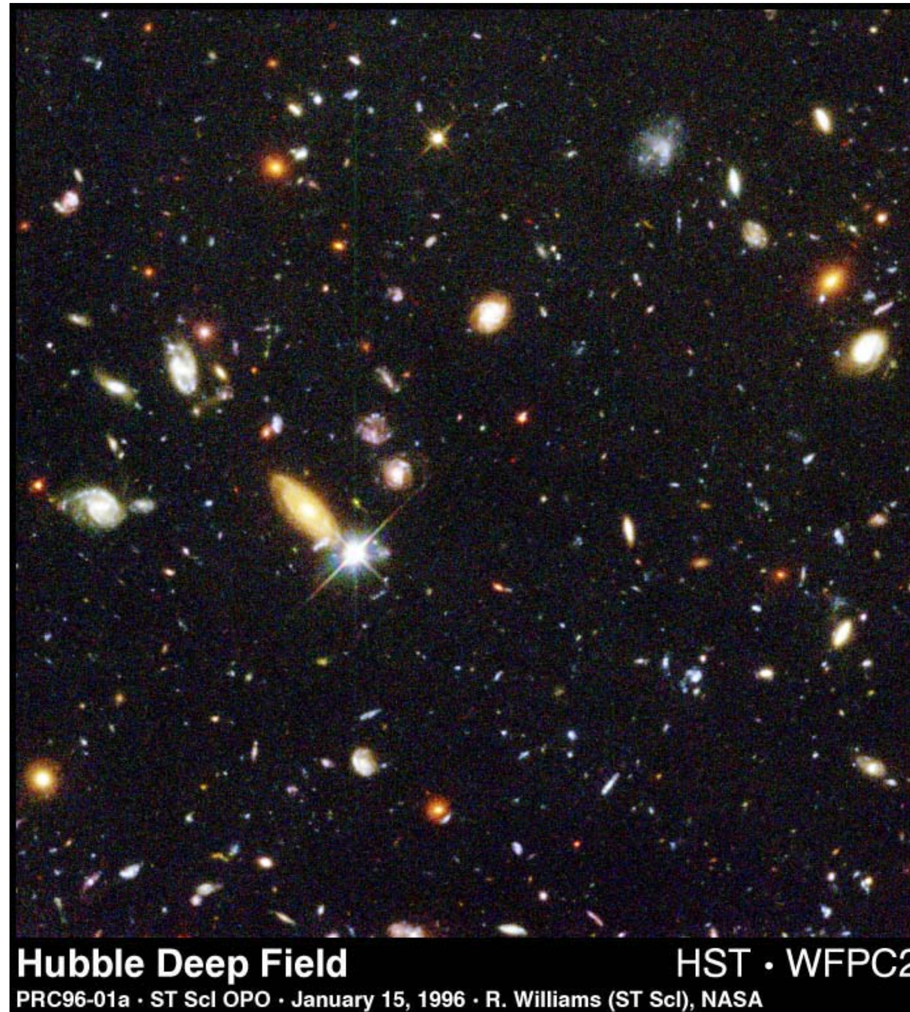
$\delta\rho/\langle\rho\rangle \sim 10^{-4}$



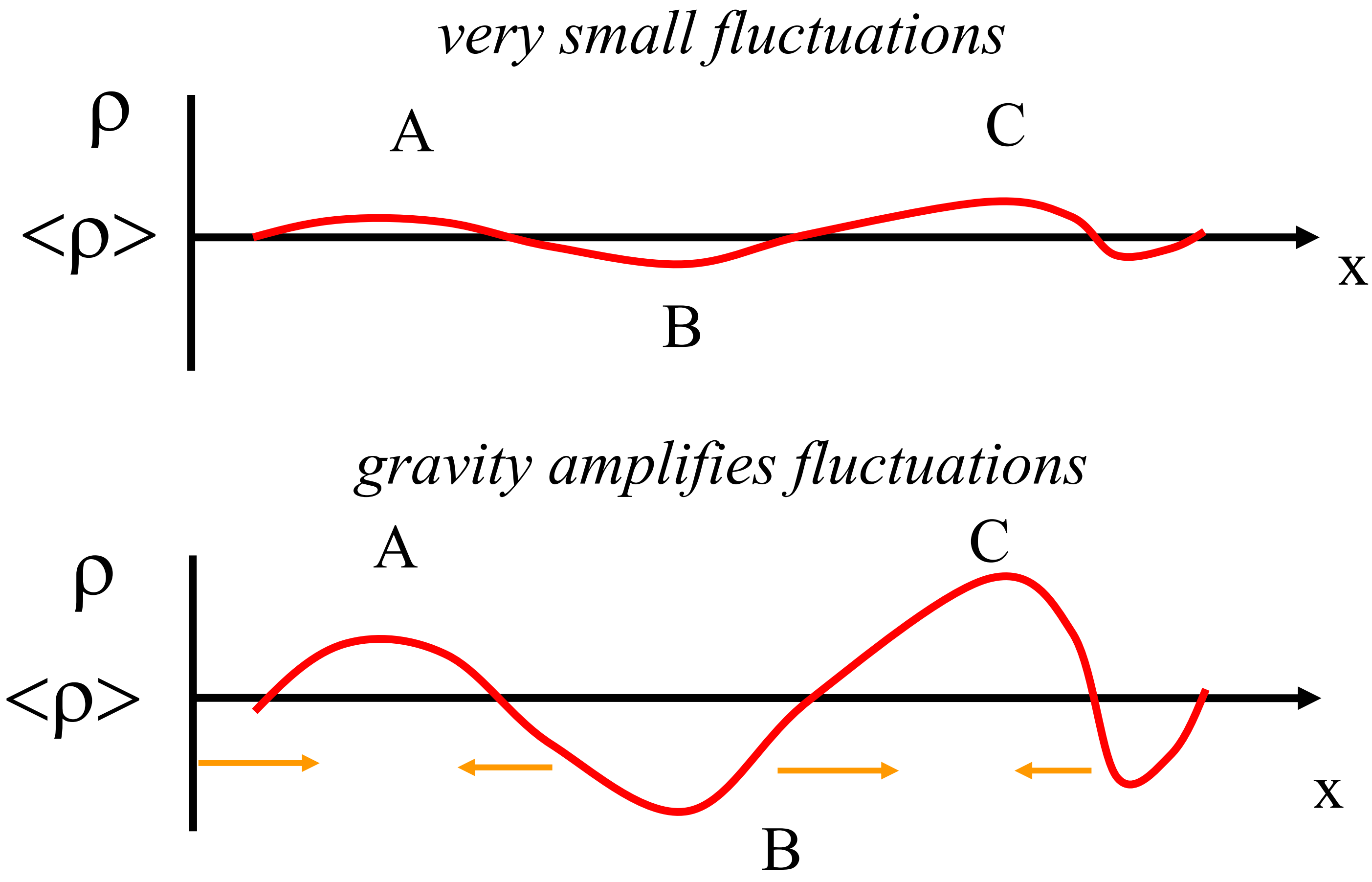
Wilkinson MAP (NASA)

Present ($\sim 14\times10^{9}$ yr)

$\delta\rho/\langle\rho\rangle \sim 10^{6}$

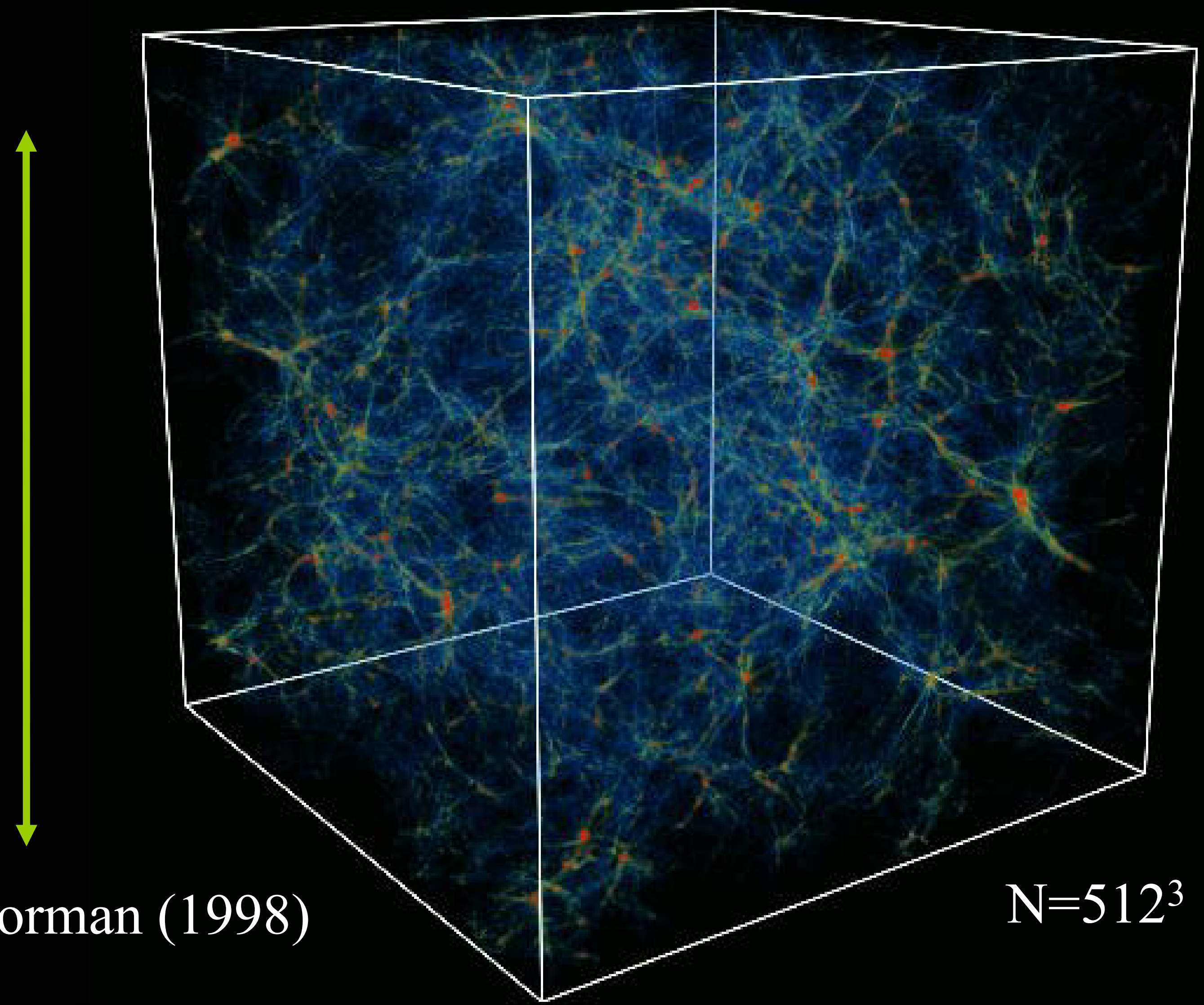


Gravitational Instability: Origin of Cosmic Structure



Gravitational Instability in 3-D: Origin of the "Cosmic Web"

1 billion light years



Bryan & Norman (1998)

$N=512^3$

Evolution of Cosmic Structure: Key Issues

- origin and character of primordial density fluctuations
  - Inflation: scale-free, gaussian random field
- linear evolution of the power spectrum $P(k) \propto |\delta^*(k)|^2$ with time (redshift) for each mass component before recombination
- nonlinear evolution of density fluctuations due to gravitational and internal forces after recombination
- above in an expanding universe with known cosmological parameters (ΛCDM)

Matter Power Spectrum

- Fourier transform of linear density field

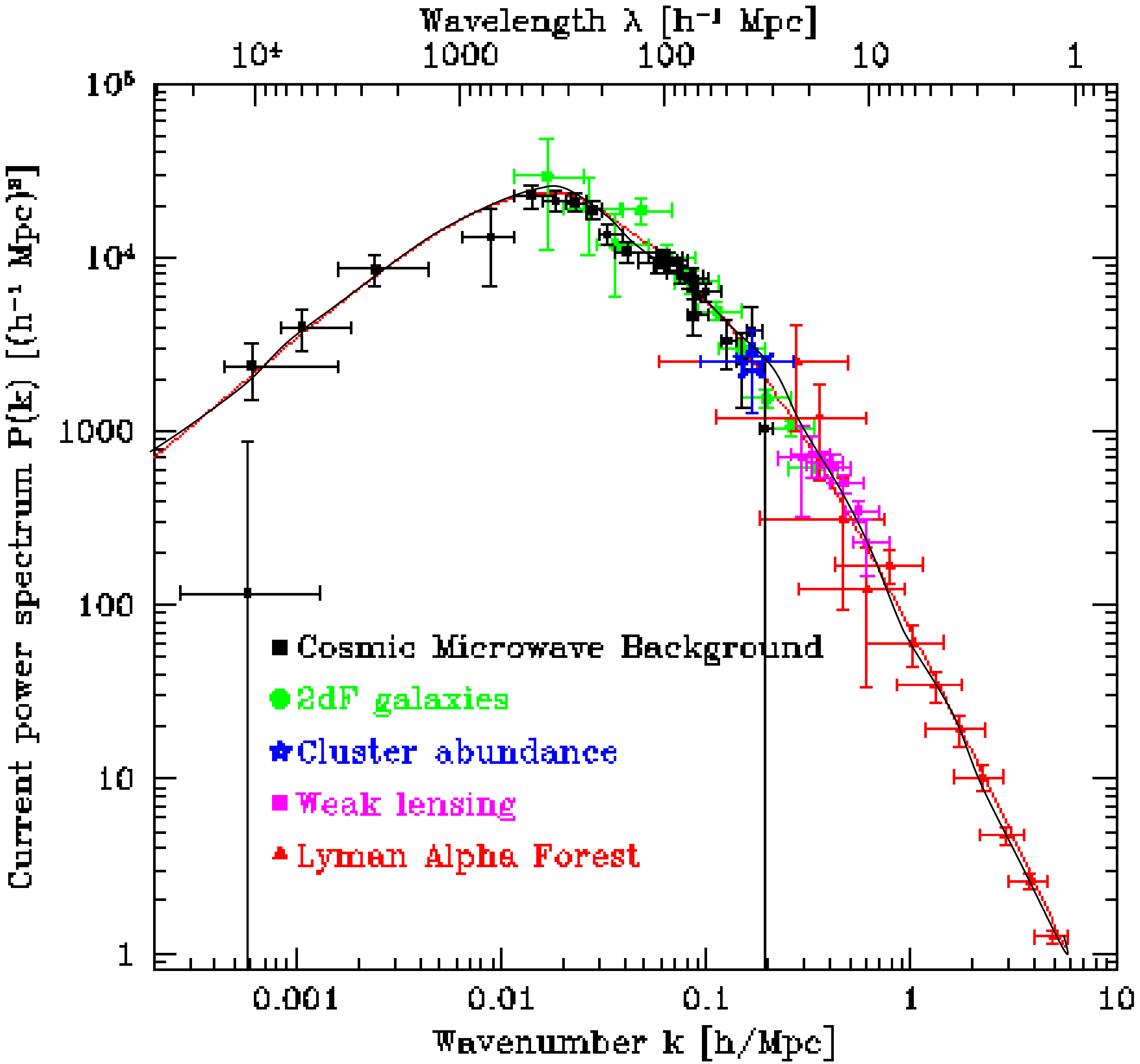
$$\delta(\mathbf{x}) = \frac{(\rho(\mathbf{x}) - \bar{\rho})}{\bar{\rho}}$$

$$\delta_{\mathbf{k}} = \frac{1}{V}\int d^3x\,\delta(\mathbf{x})e^{i\mathbf{k}\cdot\mathbf{x}}$$

- Power spectrum is defined as:

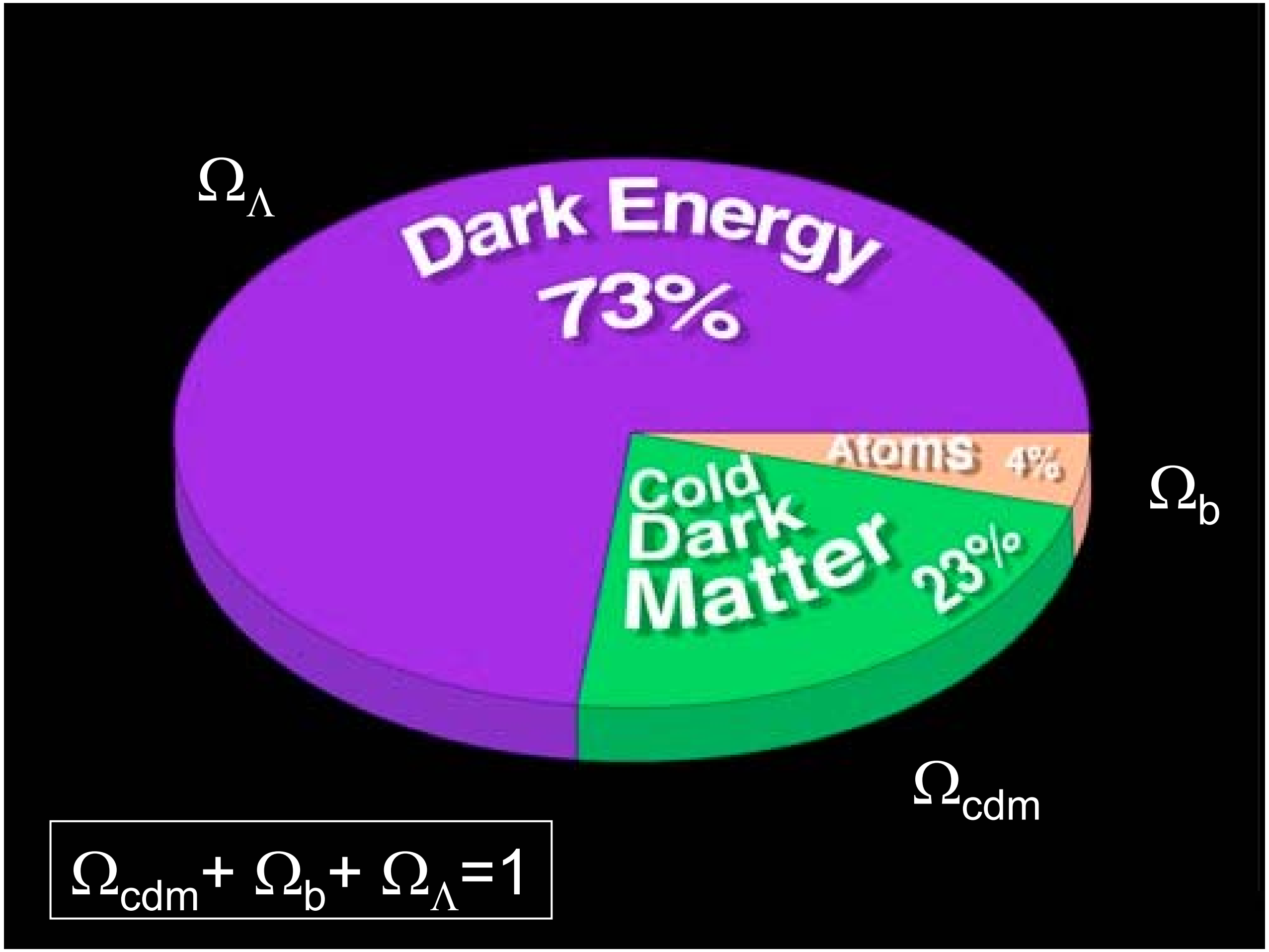
$$P(k) = \left|\delta_{\mathbf{k}}\right|^2 \text{ for all } |\mathbf{k}| \text{ between } k \text{ and } k + dk$$

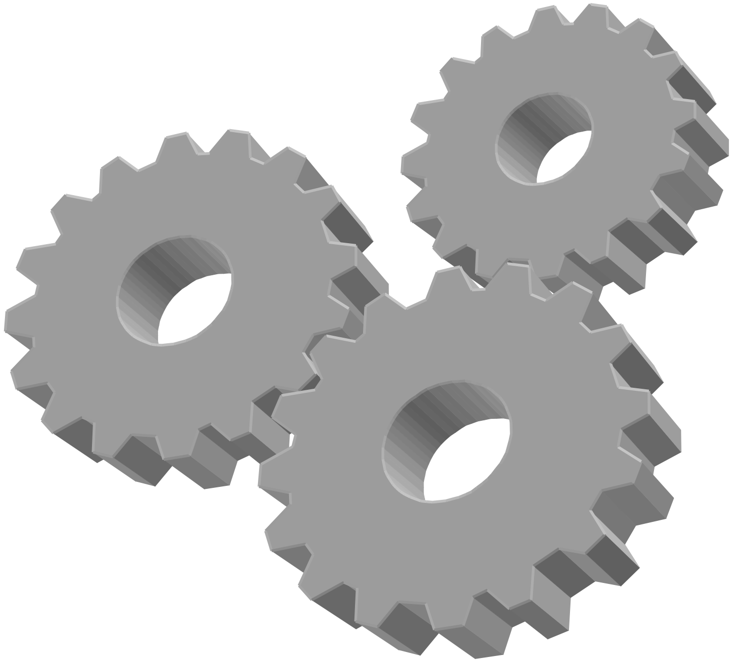
ΛCDM Matter Power Spectrum



http://www.hep.upenn.edu/~max

Mass-Energy Budget of the Universe (WMAP)





Linear Theory

Linear Perturbations

define $H(\tau) \equiv \dot{a}/a, \quad \Omega \equiv 3\mathrm{H}^2/8\pi G\overline{\rho}$

then, pressure - free perturbation obeys

$$\ddot{\delta} + \frac{\dot{a}}{a}\dot{\delta} - \frac{3}{2}\Omega\left(\frac{\dot{a}}{a}\right)^2 \delta = 0$$

for EdS universe $(\Omega = 1)$, above becomes

$$\ddot{\delta} + 2\dot{\delta}/\tau - 6^2\delta/\tau^2 = 0$$

growing mode : $\delta = D(\tau)\delta_0 \propto \tau^2 \propto t^{2/3} \propto a$

decaying mode : $\delta \propto \tau^{-3} \propto 1/t \propto a^{-3/2}$

Saturated Growth in Low Ω_m Universe

– Ω is the driving term in growth of δ
- Makes a sudden transition in Λ model
- Growth like EdS for $a<a_c$, saturates thereafter

$$H^2 = \frac{8\pi G\bar{\rho}}{3} - \frac{\kappa}{a^2} + \frac{\Lambda}{3}. \quad \kappa = 0, \pm 1; \ \Lambda \text{ cosmological constant}$$

$$\rightarrow H_0^2 \Omega_0 (\Omega^{-1} - 1) a_0^3 / a^3 = -\kappa / a^2 + \Lambda / 3$$

$$\text{open universe}: (\kappa < 0, \Lambda = 0)$$

$$\Omega^{-1} - 1 \propto a \rightarrow \Omega = 1/(1 + a/a_c)$$

$$\text{flat universe}: (\kappa = 0, \Lambda > 0)$$

$$\Omega^{-1} - 1 \propto a^3 \rightarrow \Omega = 1/(1 + a^3/a_c^3)$$

$$a_c = \text{value of expansion factor when } \Omega = 0.5$$

Mass Fluctuations in Spheres

- Approximate P(k) locally with power-law

$$|\delta_k|^2 \propto D^2(\tau) k^n$$

- Mean square mass fluctuation (variance)

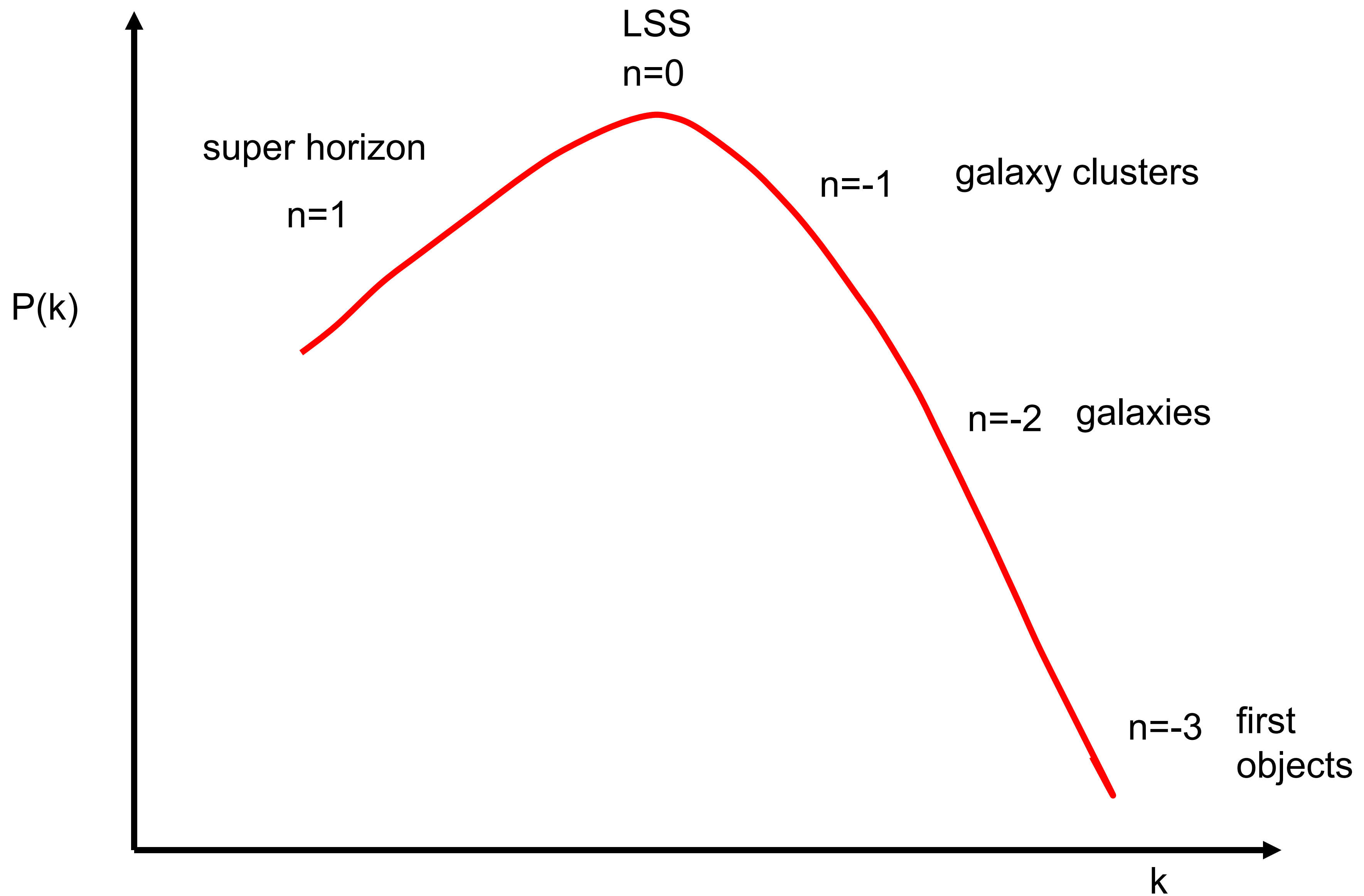
$$\left\langle (\delta M / M)^2 \right\rangle = \int d^3k \, W_T^2(kR) |\delta_k|^2 \propto D^2 R^{-(3+n)} \propto D^2 M^{-(3+n)/3}$$

$W_T(kR)$ is Fourier transform of top-hat window function

$$\mathrm{W}(\mathbf{x}) = \begin{cases} 3/4\pi R^3, & |\mathbf{x}| < R \\ 0, & |\mathbf{x}| \geq R \end{cases}$$

$$\rightarrow W_T(kR) = 3\left[\sin(kR)/kR - \cos(kR)\right]/(kR)^2$$

CDM Power Spectrum



Mass Fluctuations, cont'd

- RMS mass fluctuations

$$\sigma(M) \equiv \left\langle (\delta M / M)^2 \right\rangle^{1/2} \propto DM^{-(3+n)/6}$$

For n>-3, RMS fluctuations a decreasing function of M

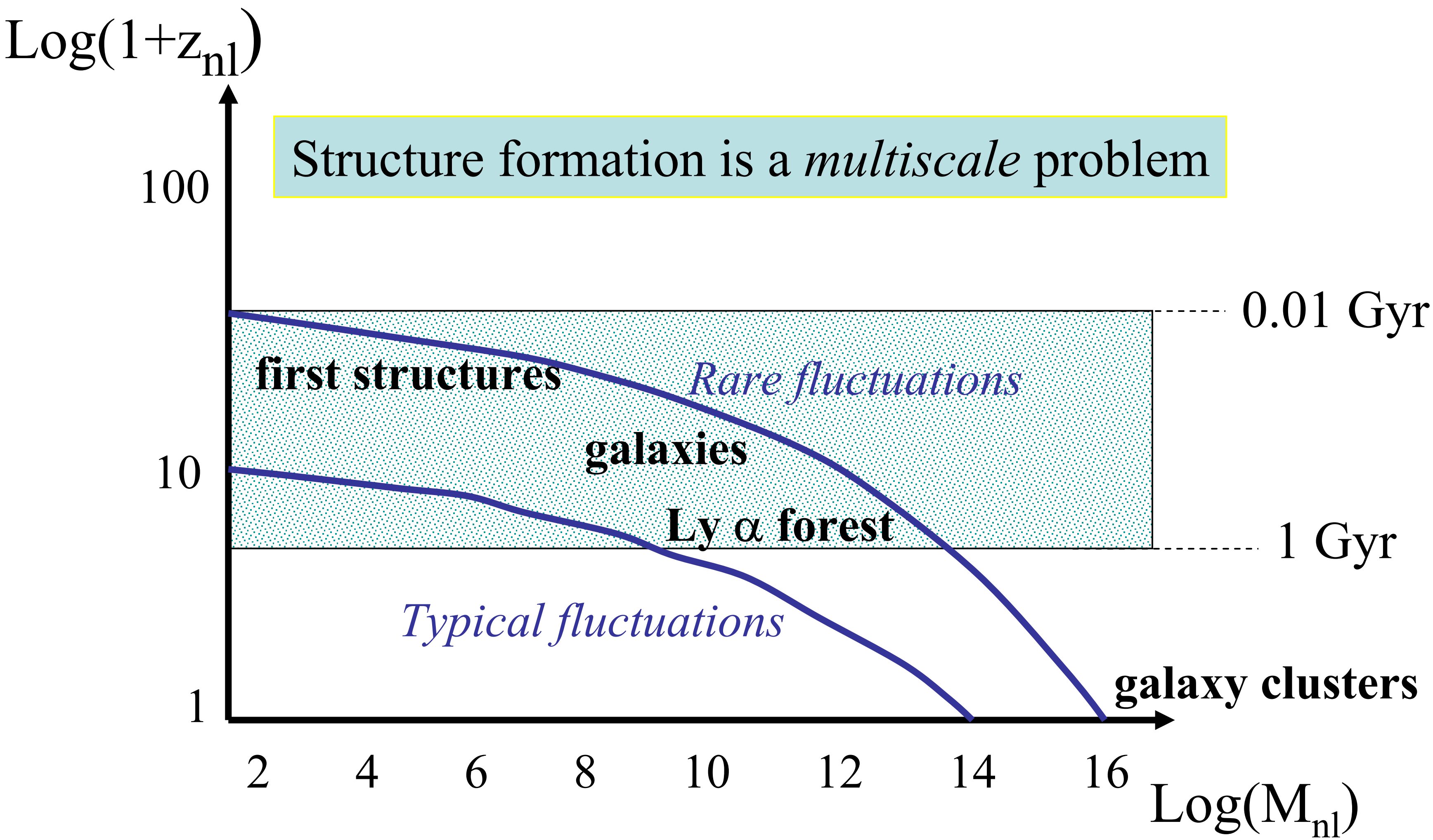
- Nonlinear mass scale

$$\text{setting} \quad \sigma(\mathrm{M}_{\mathrm{nl}}) = 1$$

$$\text{get } M_{nl}(\tau) \propto D(\tau)^{6/(3+n)} \quad (\propto (1+z)^{-6/(3+n)} \text{ for EdS})$$

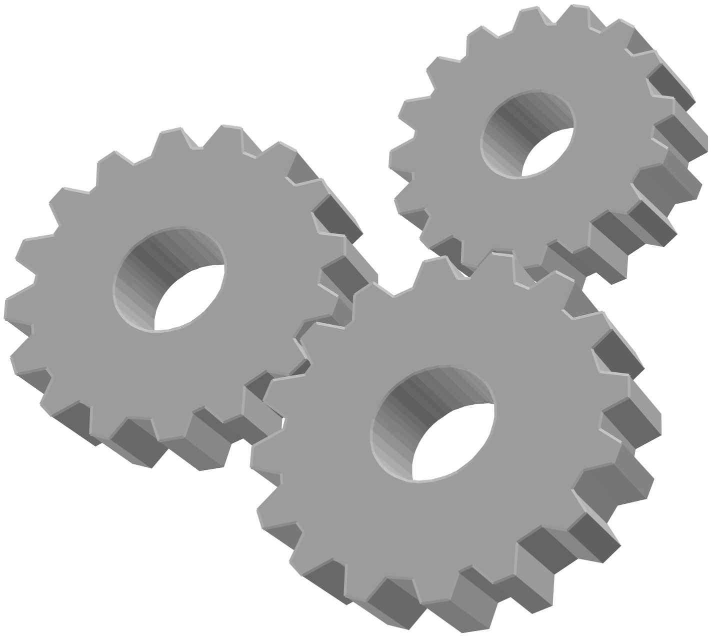
For n>-3, smallest mass scales become nonlinear first

Evolution of nonlinear mass scale with redshift in ΛCDM



Numerical Cosmology Goals

- Simulate the processes governing the formation and evolution of the observable structures in the universe
  - *galaxies, quasars, clusters, superclusters*
- Find the "best fit" cosmological model through detailed observational comparison
- Make quantitative predictions for new era of high redshift observations



Nonlinear Simulations I: Cold Dark Matter

Cold Dark Matter

- Dominant mass constituent: Ω_{cdm}~0.23
- Only interacts gravitationally with ordinary matter (baryons)
- Collisionless dynamics governed by Vlasov-Poisson equation

$$\partial_t f(\vec{x},\vec{\upsilon},t) + \vec{\upsilon}\cdot\nabla_x f - \nabla\phi\cdot\nabla_\upsilon f = 0$$

$$\nabla^2\phi = 4\pi G\int d^3\vec{\upsilon}\; f(\vec{x},\vec{\upsilon},t)$$

- Solved numerically using N-body methods

The Universe is an IVP suitable for computation

- *Globally*, the universe evolves according to the Friedmann equation

$$H(t)^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

Hubble parameter

scale factor a(t)

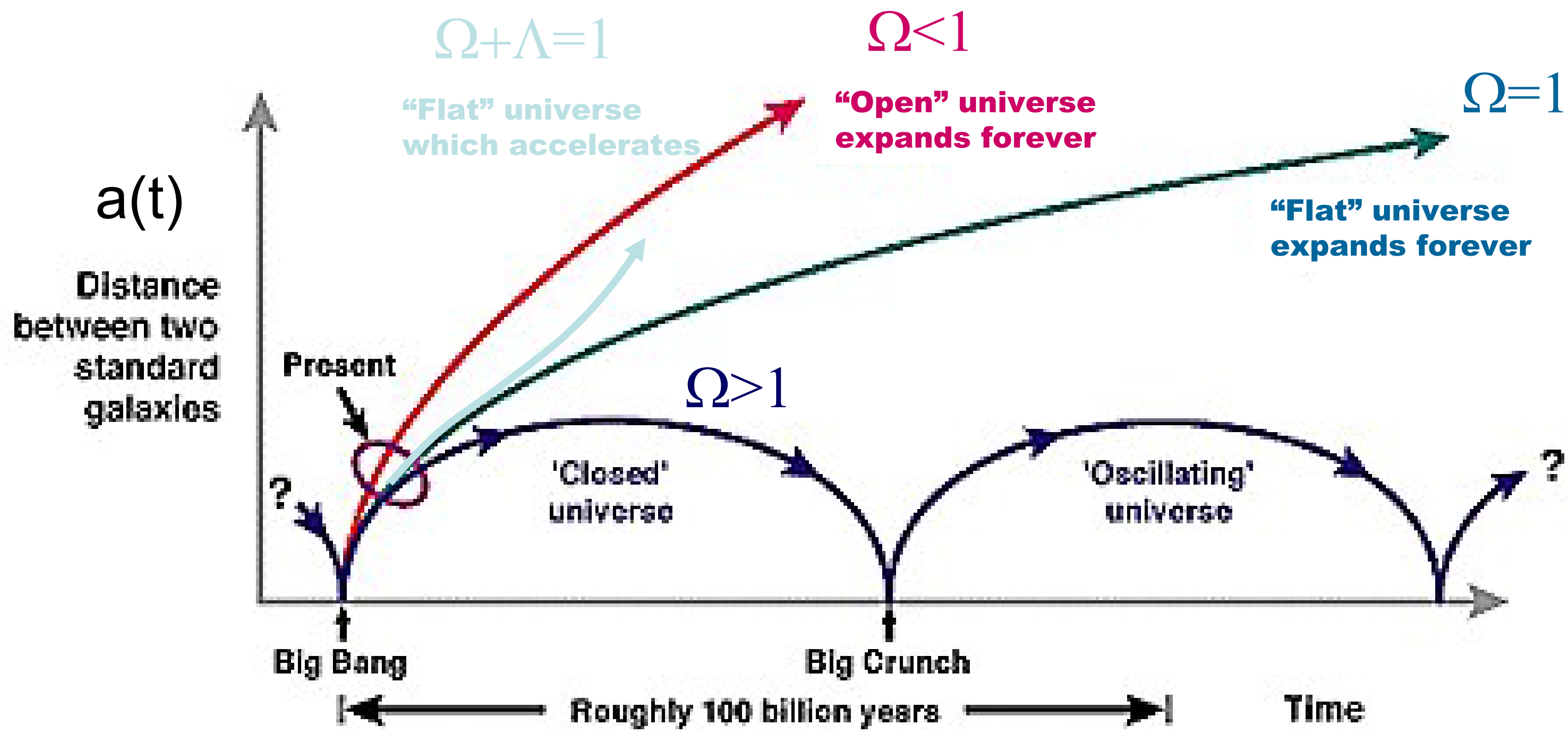
mass-energy density

spacetime curvature

cosmological constant

Friedmann Models: The Omega Factor

$\Omega = \langle\rho\rangle / \rho_{crit}$

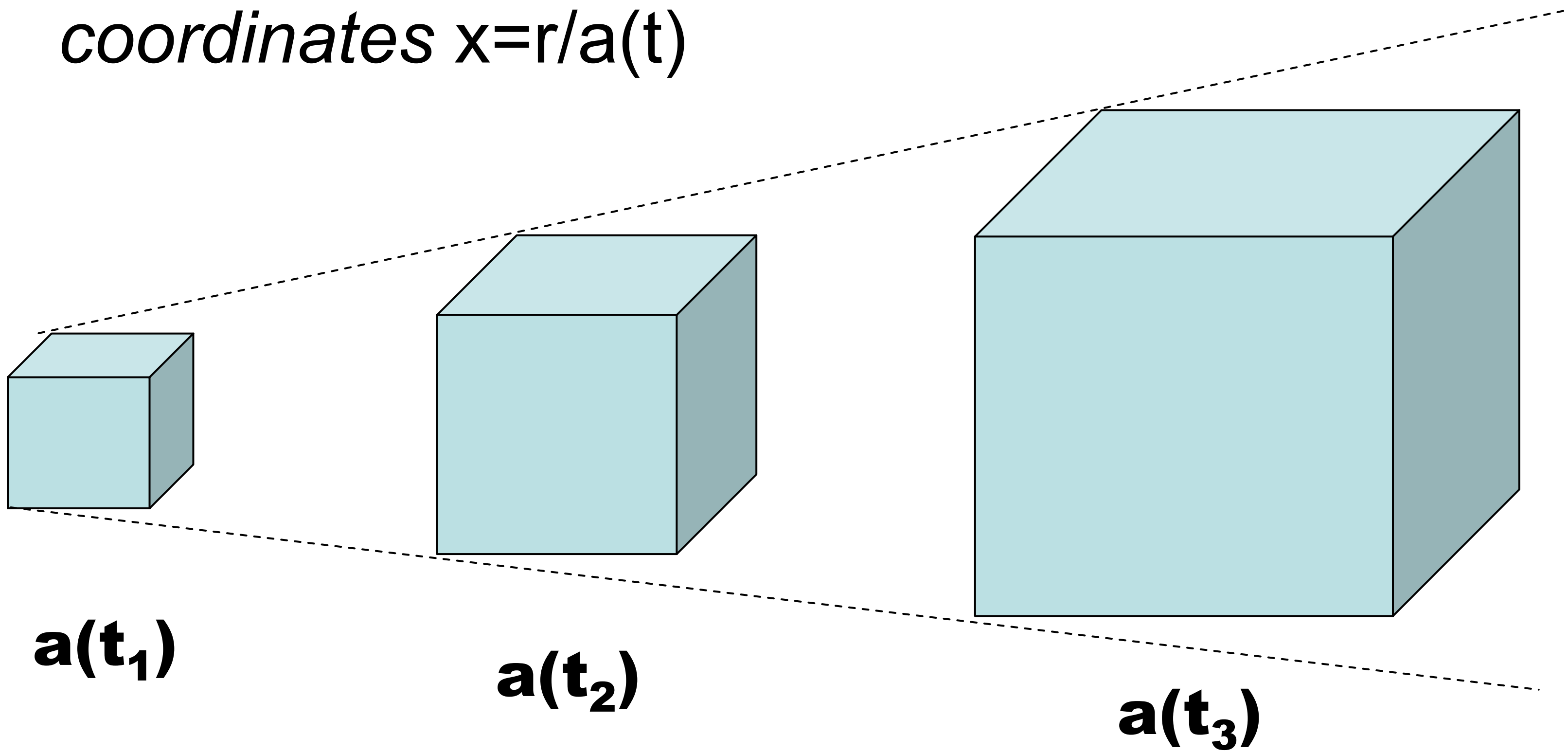


The Universe is an IVP...

- *Locally*, its contents obey:
  - Newton's laws of gravitational N-body dynamics for stars and collisionless dark matter (CDM)
  - Euler or MHD equations for baryonic gas/plasma
  - Atomic, molecular, and radiative processes important for the condensation of stars and galaxies from diffuse gas

Gridding the Universe

- Transformation to *comoving coordinates* $x = r/a(t)$
- Triply-periodic boundary conditions



Dark Matter Dynamics in an Expanding Universe

$$\frac{dx_{dm}}{dt} = v_{dm}$$

$$\frac{dv_{dm}}{dt} = -2\frac{\dot{a}}{a}v_{dm} - \frac{1}{a^2}\nabla\phi$$

Newton's laws

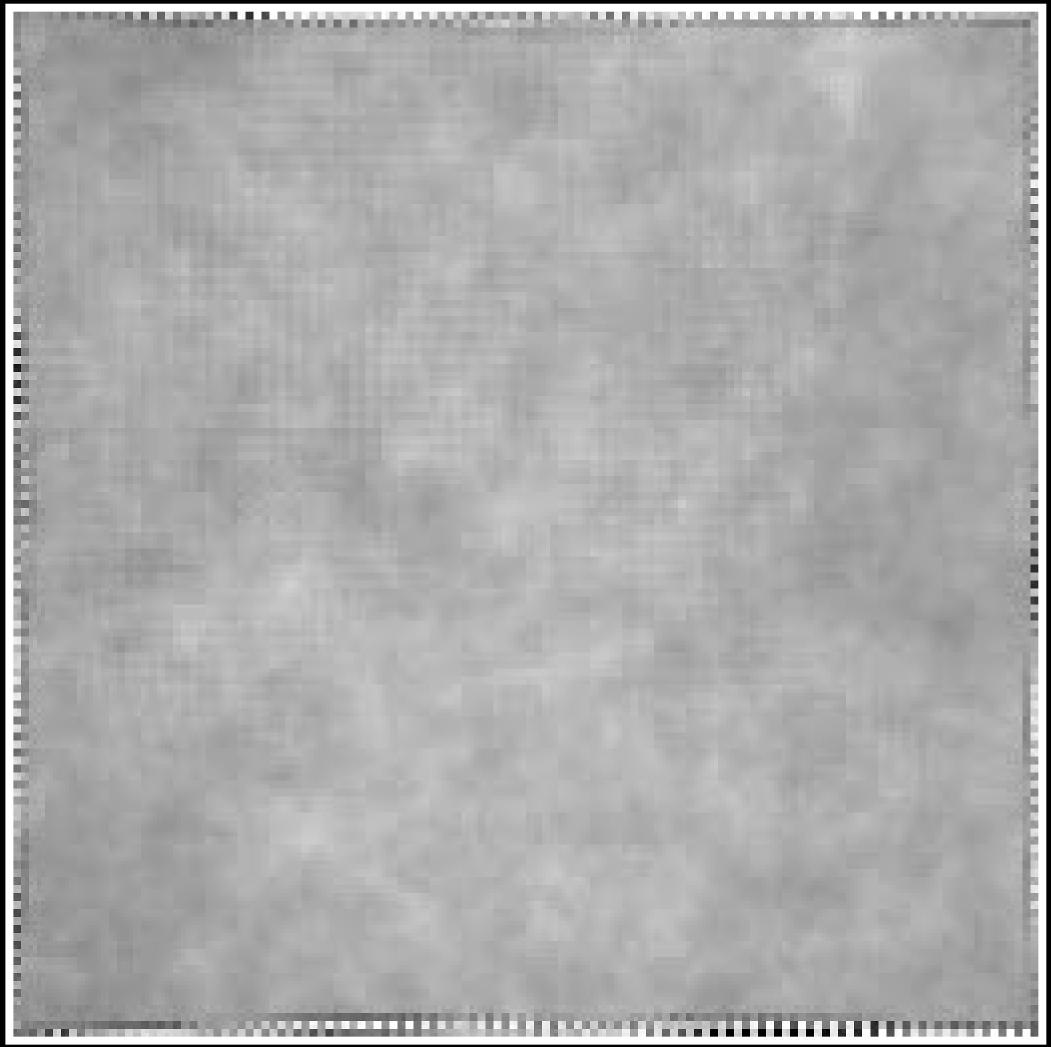
$$\nabla^2\phi = 4\pi G a^2(\rho - \overline{\rho})$$

Poisson equation

$$\frac{da}{dt} = H_0\left[\Omega_m\left(\frac{1}{a} - 1\right) + \Omega_\Lambda(a^2 - 1) + 1\right]^{1/2}$$

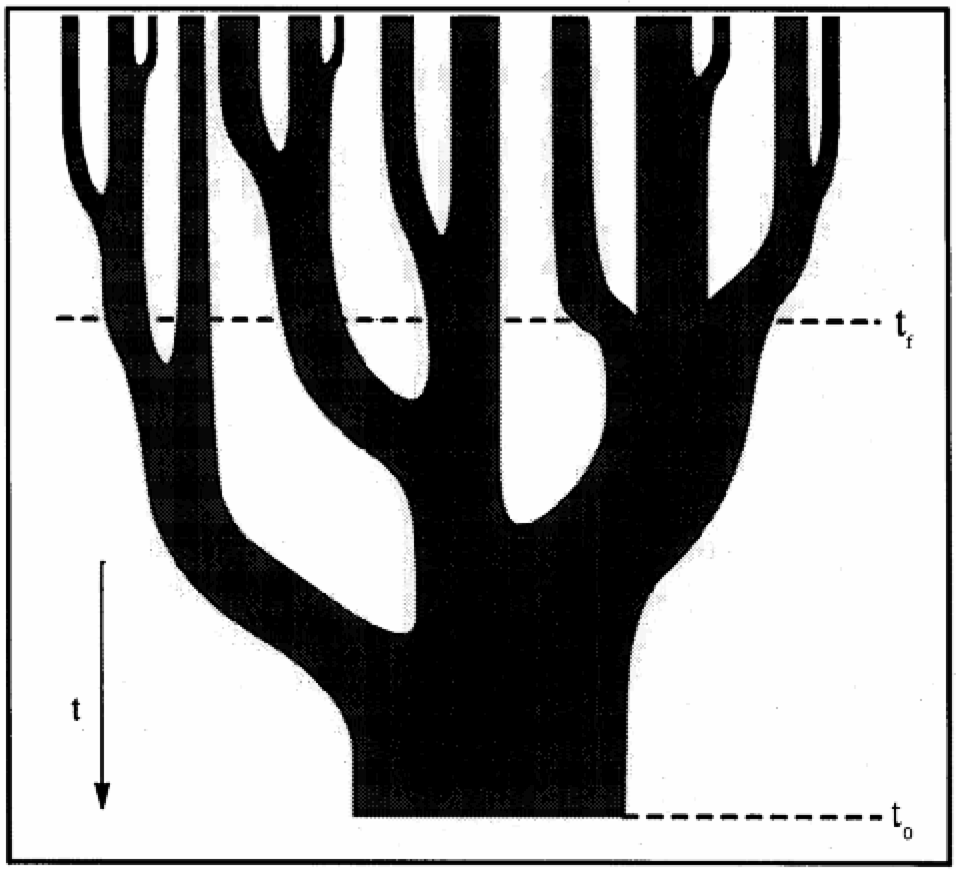
Friedmann equation

Hierarchical Clustering of Cold Dark Matter



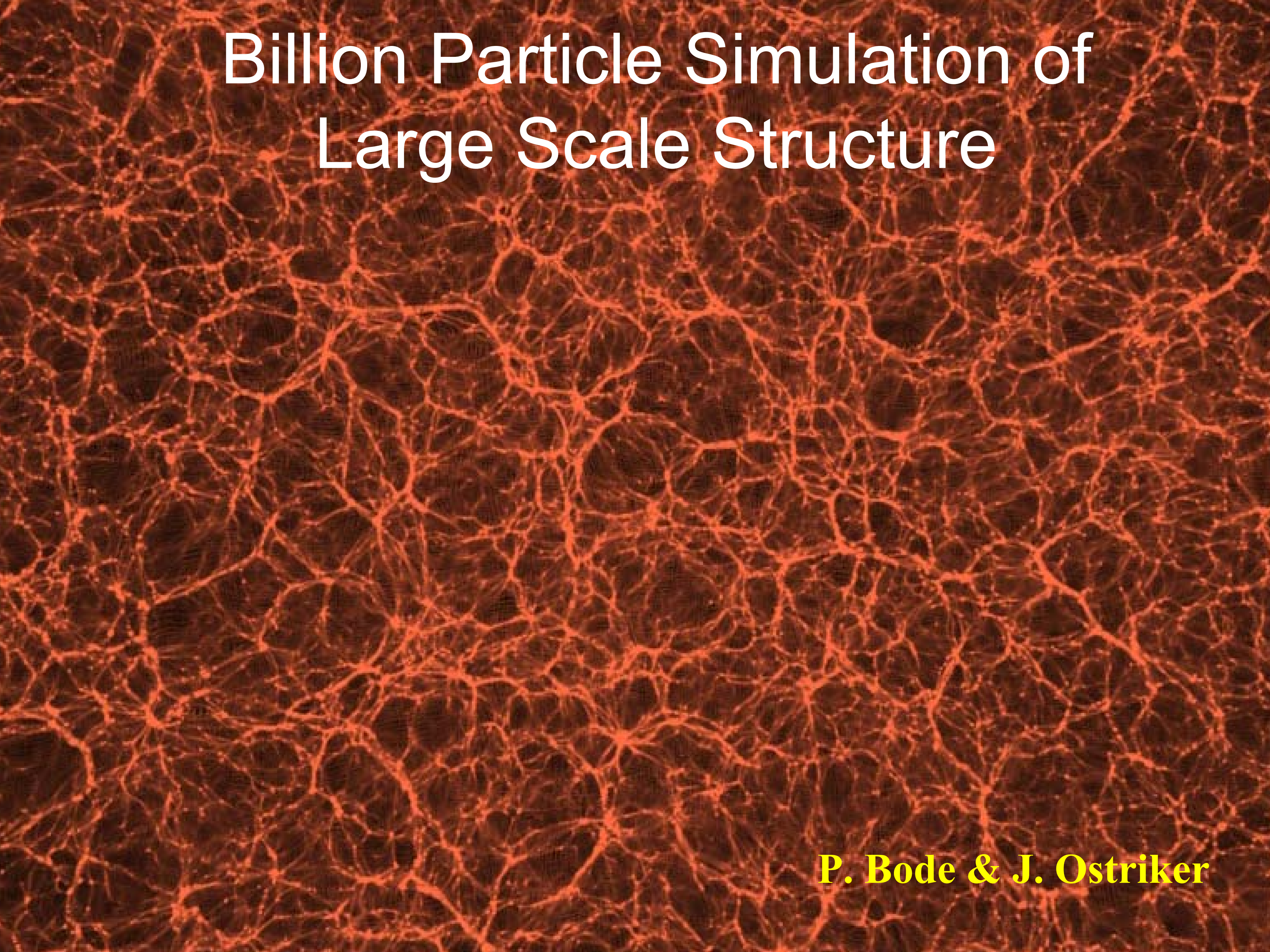
Hierarchical Structure Formation: Like a River

- large galaxies form from mergers of sub-galactic units
- Galaxy groups form from merger of galaxies
- Galaxy clusters form from merger of groups
- Where did it begin, and where will it end?

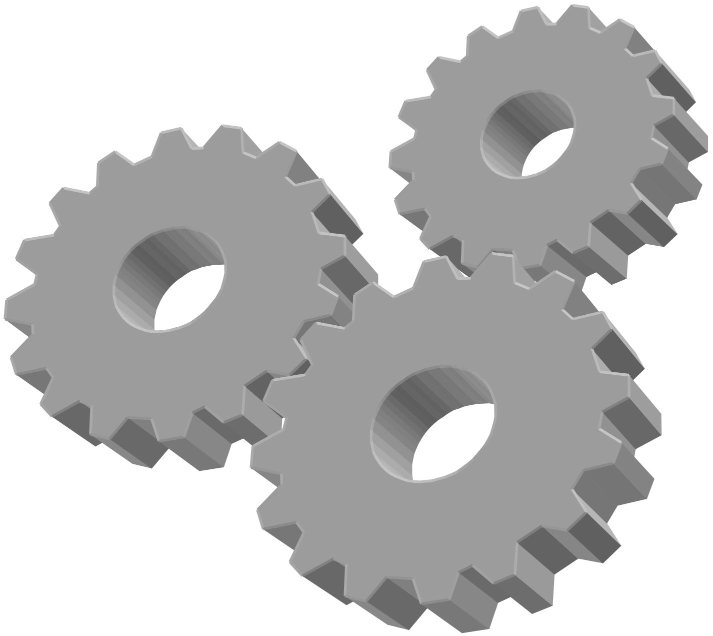


Lacey & Cole (1993)

Billion Particle Simulation of Large Scale Structure



P. Bode & J. Ostriker



Nonlinear Theory

Dark Matter Halo Mass Function

- Principal quantity of interest is the globally averaged number density of collapsed objects of mass M as a function of redshift=z

$$< n(M,z) >$$

- Dark matter halos define the gravitational potential wells galaxies, galaxy groups, and clusters of galaxies form in
- Sensitive function of cosmological parameters

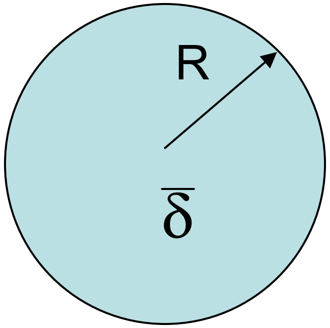
Spherical Top-Hat Model

- Simplest analytic model of nonlinear evolution of a discrete perturbation

consider spherical region with uniform overdensity $\bar{\delta}$ and radius R

by Birkhoff's theorem (GR), EOM is:

$$\frac{d^2R}{dt^2} = -\frac{GM}{R^2} = -\frac{4\pi G}{3}\bar{\rho}(1+\bar{\delta})R$$



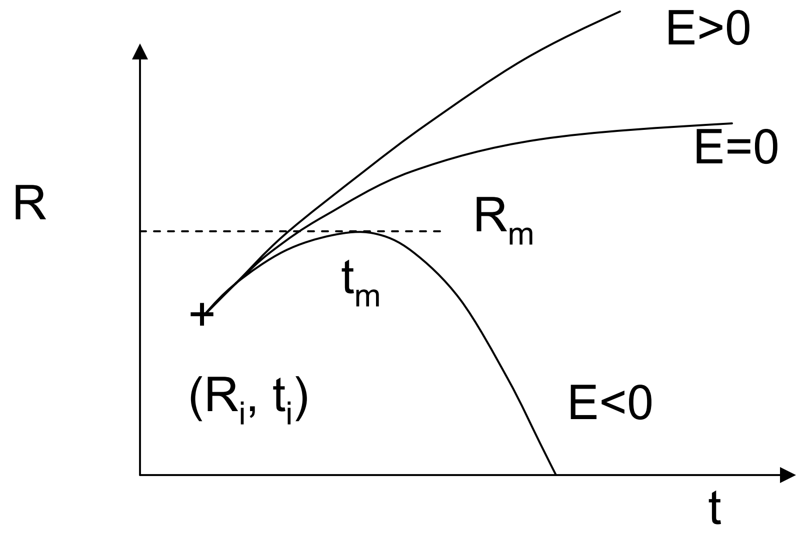
whereas the Friedmann equation is:

$$\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\bar{\rho}a$$

$\therefore$ perturbation evolves like a universe of different density,

but the same initial time and expansion rate

Spherical Top-Hat, cont'd



first integral of motion :

$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 - \frac{GM}{R} = E$$

if E < 0, perturbation is bound, and obeys

$$R/R_m = \frac{1}{2}(1-\cos\Theta), \quad t/t_m = (\Theta - \sin\Theta)/\pi$$

where R_m and t_m are radius and time at "turnaround"

Turnaround and Collapse

mean *linear* overdensity with respect to EdS universe:

$$\bar{\delta}_{lin} = \frac{3}{20}(6\pi t / t_m)^{2/3} \propto a_{EdS} \quad (\bar{\delta} << 1)$$

$$\bar{\delta}_{lin}(t_m) = 1.063$$

nonlinear overdensity at turnaround:

$$\bar{\delta}_{nl}(t_m) = 4.6$$

Collapse: $\mathrm{R} \to 0$ as $\mathrm{t} \to 2\mathrm{t}_\mathrm{m}$

$$\therefore \delta_{collapse} = \bar{\delta}_{lin}(2t_m) = \frac{3}{20}(12\pi)^{2/3} = 1.686$$

Virialization

collapse halted when virial equilibrium established : $|U| = 2K$

assume total energy $E = K\text{-}|U|$ is conserved

at turnaround : $K = 0,\ E = \text{-}|U| = -\frac{3}{5}\frac{GM^2}{R_m}$ (homogeneous sphere)

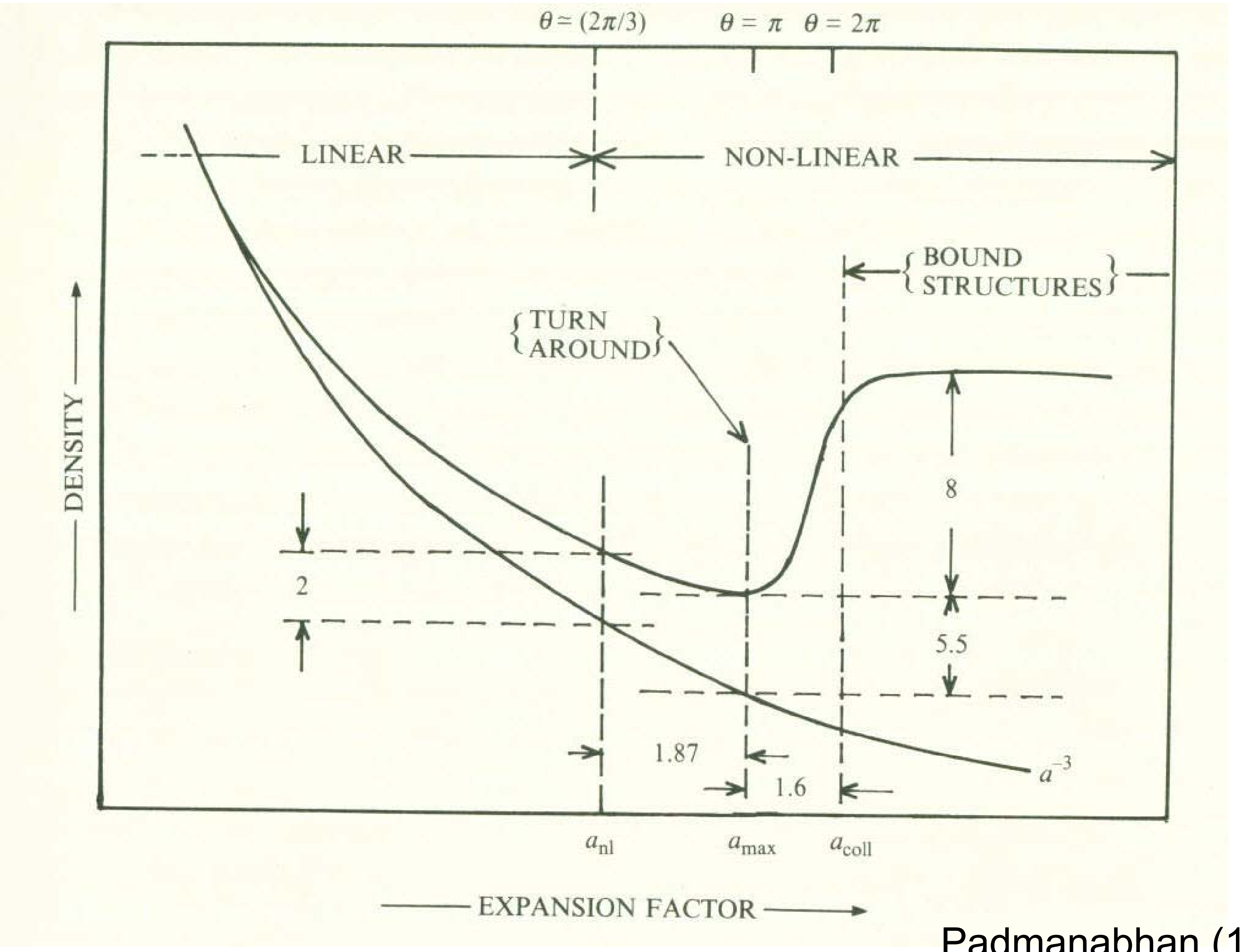
at virialization : $E = K\text{-}2K = \text{-}K$

$$\therefore K = -E = \frac{1}{2}|U| = \frac{1}{2}\cdot\frac{3}{5}\frac{GM^2}{R_{vir}}$$

equating

$$\frac{1}{2}\cdot\frac{3}{5}\frac{GM^2}{R_{vir}} = \frac{3}{5}\frac{GM^2}{R_m} \rightarrow R_{vir} = \frac{1}{2}R_m$$

Evolution of Top-hat Perturbation



Padmanabhan (1994)

Statistics of Hierarchical Clustering

- Press & Schechter (1974) derived a simple, yet accurate formula for estimating the number of virialized objects of mass M as a function of τ (or equivalently, z)
- Basic idea is to smooth density field on a scale R at such that mass scale of interest satisfies

$$M = \frac{4\pi}{3}\bar{\rho}(\tau)R^3$$

Press-Schechter Theory

- Because the density field (both smoothed and unsmoothed) is a Gaussian random field, probability that mean overdensity in spheres of radius R exceeds a critical value δ_c is

$$p(R,\tau) = \int_{\delta_c}^{\infty} d\delta \frac{1}{\sqrt{2\pi}\sigma(R,\tau)} \exp\left(-\frac{\delta_c^2}{2\sigma^2(R,\tau)} \right)$$

where

$$\sigma^2(R,\tau) = \int d^3k \, W_T^2(kR) P(k,\tau)$$

P-S Theory, cont'd

- P-S suggested this fraction be identified with the fraction of particles which are part of a nonlinear lump with mass exceeding M if we take δ_c=1.686

$$\text{i.e.} \quad n(>M,\tau) = 2\frac{\bar{\rho}}{M} p(R,\tau) = \int_M^\infty dM' \frac{dn}{dM'}$$

$$\therefore \frac{dn}{dM} = -2\frac{\bar{\rho}}{M}\frac{\partial p}{\partial M} = -\sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M}\frac{\delta_c}{\sigma^2}\frac{d\sigma}{dM}\exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)$$

n.b. factor of 2 is a fudge factor so that every particle belongs to a halo of some mass

P-S theory, cont'd

$$\sigma(R,\tau) = D(\tau)\sigma_0(R) \propto DR^{-\frac{n+3}{2}} \propto DM^{-\frac{n+3}{6}}$$

explicitly, we find

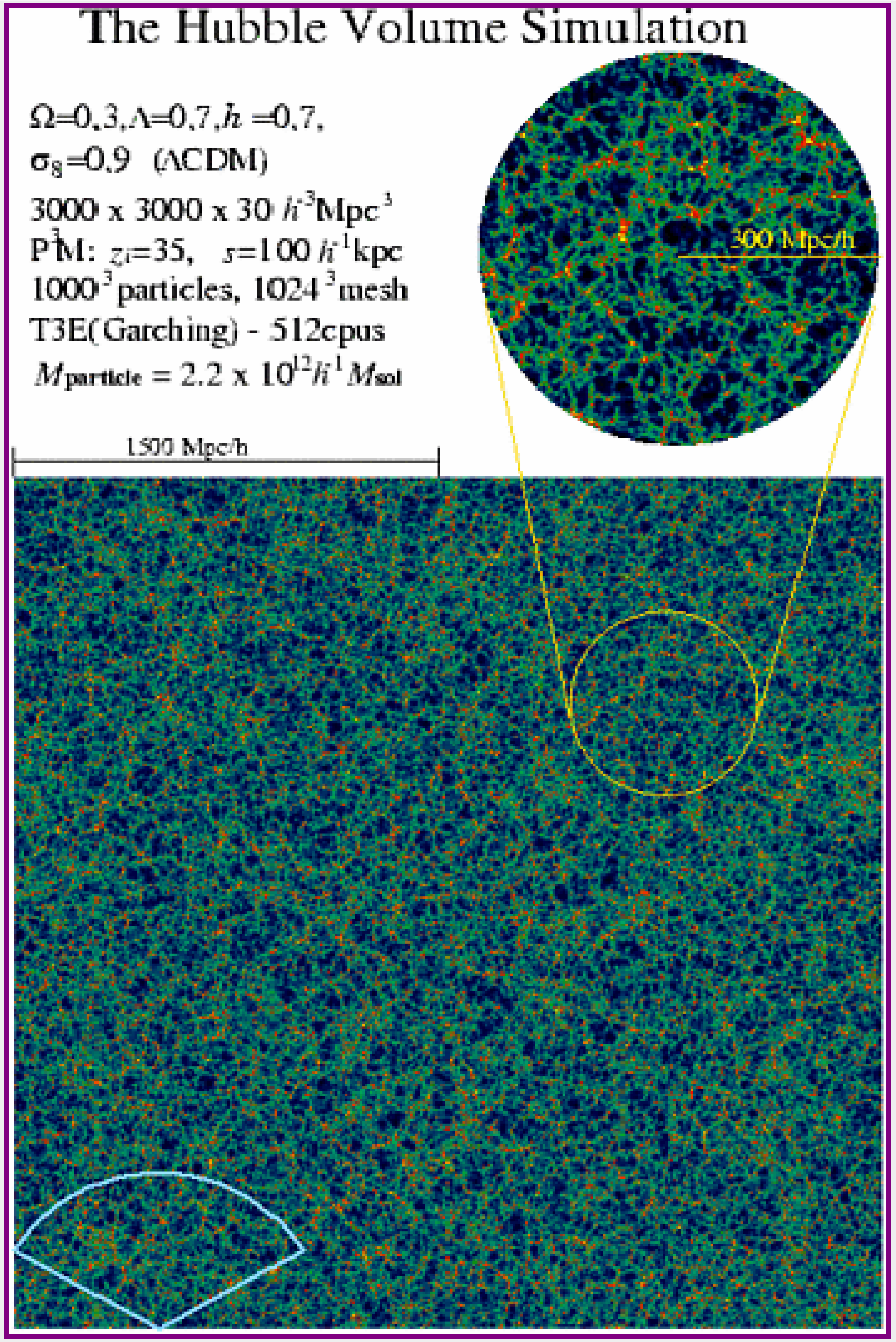
$$\frac{dn}{dM}dM = \left(\frac{2}{\pi}\right)^{1/2} \frac{\bar{\rho}}{M^2}\left(1+\frac{n}{3}\right)\left[\frac{M}{M_{nl}(\tau)}\right]^{\frac{n-3}{6}} \exp\left[-\left(\frac{M}{M_{nl}(\tau)}\right)^{\frac{3+n}{3}}/2\right]dM$$

where now M_{nl} satisfies

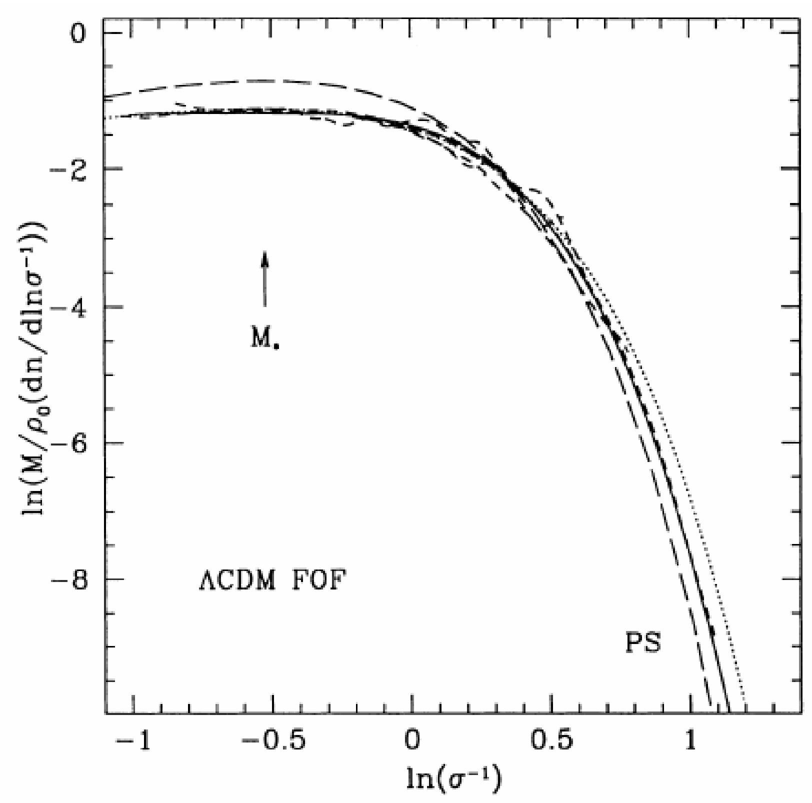
$$\sigma(M_{nl},\tau) = \delta_c$$

Mass function is a power-law for $M<M_{nl}$ and an exponential for $M>M_{nl}$

Comparison with N-body Simulations



- PS under-predicts most massive objects and over-predicts "typical" objects



Jenkins et al. (1998)