FLUID-STRUCTURE INTERACTION IN ARTERIAL BLOOD FLOW: MODELING ANALYSIS AND SIMULATIONS

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WITH STRONG INTERFACIAL EFFECTS

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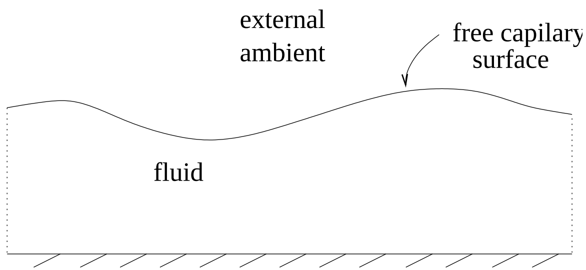
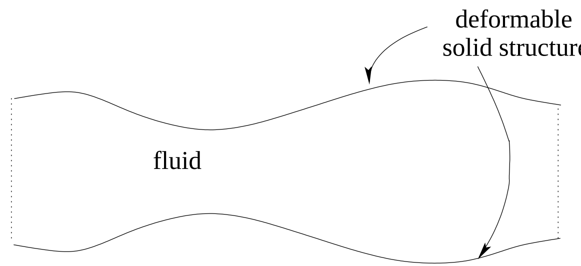
MODELING ANALYSIS AND SIMULATIONS

Free Boundary Flows

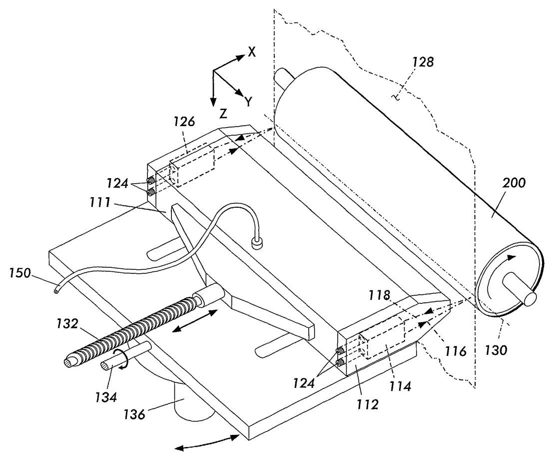
Free boundary flows involve deformable interfaces

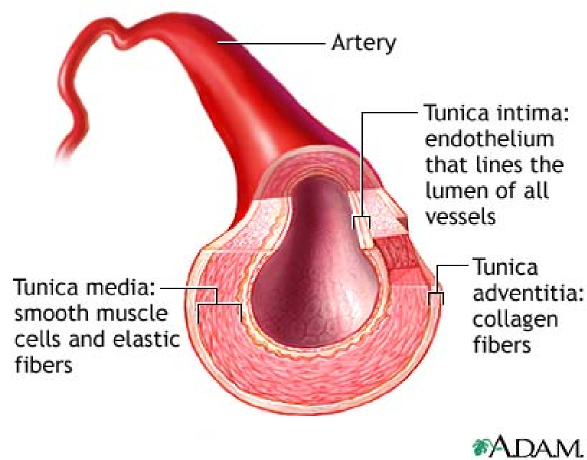




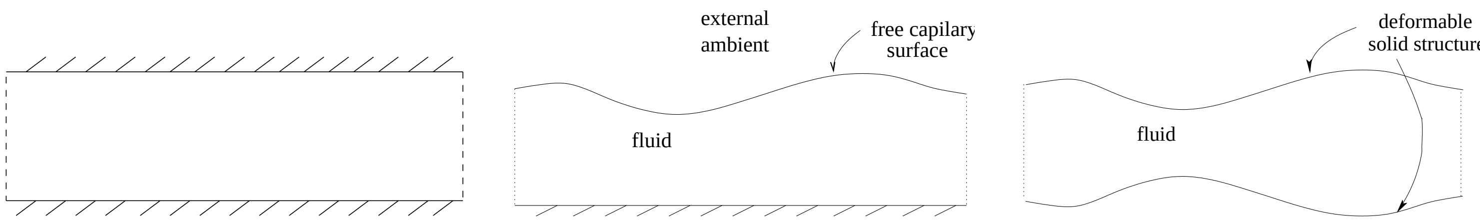
Coating Flows

Arterial Blood Flow





Free Boundary vs Fixed Domains



- ▷ Additional Unknowns. one has to solve not only for the fluid velocity and pressure, but simultaneously also for the location of the interface and its evolution in time:

$$\mathbf{u}(x, y; t),\ p(x, y; t),\ \eta(x; t) \tag{1}$$

Fluid-structure interaction: solve elasticity equations

- ▷ Additional Nonlinearities. The interface deformation is coupled to the fluid flow both kinematically (continuity of the velocities) and dynamically (balance of stresses), and the coupling is nonlinear.

Strong Interfacial Effects

Coating Flows

flow dominated
by capillary effects

$$Ca = \mu V/\sigma < 1$$

Ca: Capillary number
μ: fluid viscosity
V: characteristic velocity
σ: surface tension

Arterial Blood Flow

fluid and structure
of comparable densities

$$\varrho_s/\varrho_f \leq 1$$

ϱ_s: structure density
ϱ_f: fluid density

When interfacial effects are strong
the coupling at the deformable interface is higly non-linear
$\rightarrow$ numerical instabilities

Main Goal

STATE OF THE ART: Several commercial and non-commercial codes.
Free boundary flows with strong interfacial effects
are usually solved using strongly coupled schemes:
robust and stable
but with high computational costs and convergence issues

MAIN GOAL: Design schemes which combine
stability
low computational costs
modularity

STRATEGY: use operator splitting method for time-discretization.
Numerical instabilities can be controlled by treating carefully the
kinematic and the dynamic coupling conditions.

Plan of the talk

▷ Arterial Blood Flow

– Modeling: blood and vessel wall

– Full Problem vs Reduced Effective Models

– Numerical solutions of the Full Problem

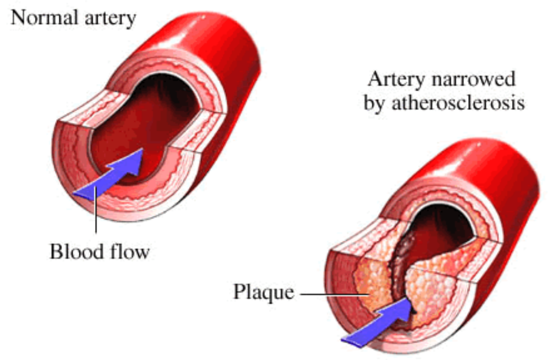
$\longrightarrow$ Kinematically-coupled algorithm

▷ Coating Flow

– Slot Coater

– Comparison with Monolothic Scheme by Pasquali (Rice U.)

Arterial Blood Flow - Motivation

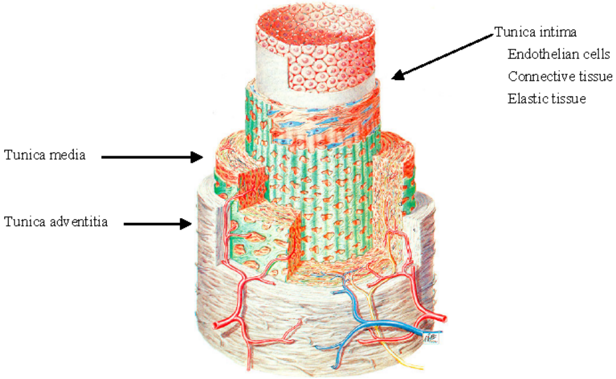


- cardiovascular diseases $\leftrightarrow$ changes in blood flow and wall deformation (e.g. atherosclerosis, aneurysms,...)
- long-term success of clinical treatments $\leftrightarrow$ changes induced on blood flow and body reaction
- fluid-structure interaction are mathematically very challenging

Proper resolution of fluid-structure interaction is one of the core problems!

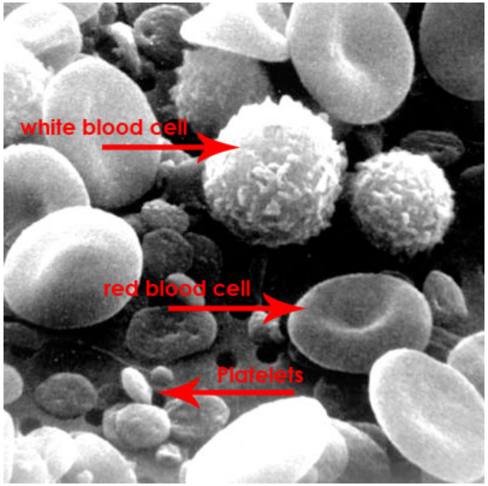
Arterial Blood Flow - Modeling

Arterial wall



heterogeneous and anisotropic
nonlinear and viscoelastic
pre-stressed

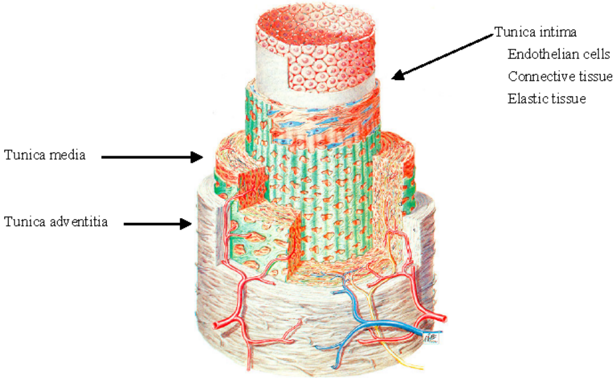
Blood



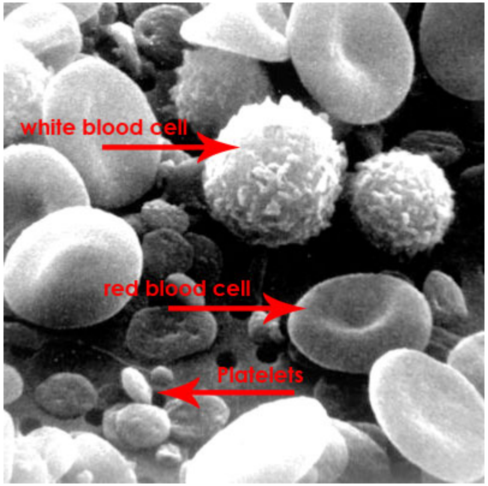
suspension
cells and plasma
complex rehology

Arterial Blood Flow - Modeling

Arterial wall



Blood



Models for vessel wall (structure) and blood (fluid) which is:

- complicated enough to catch the interesting phenomena

- simple enough to be solved in a reasonable way

$\longrightarrow$ Reduced Effective Models helps!

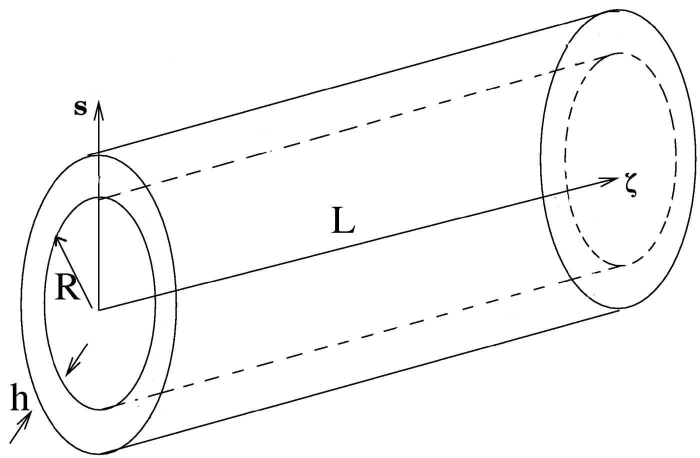
Reduced Effective models

- based multiscale analysis (e.g. $R/L = \varepsilon$). Huge literature

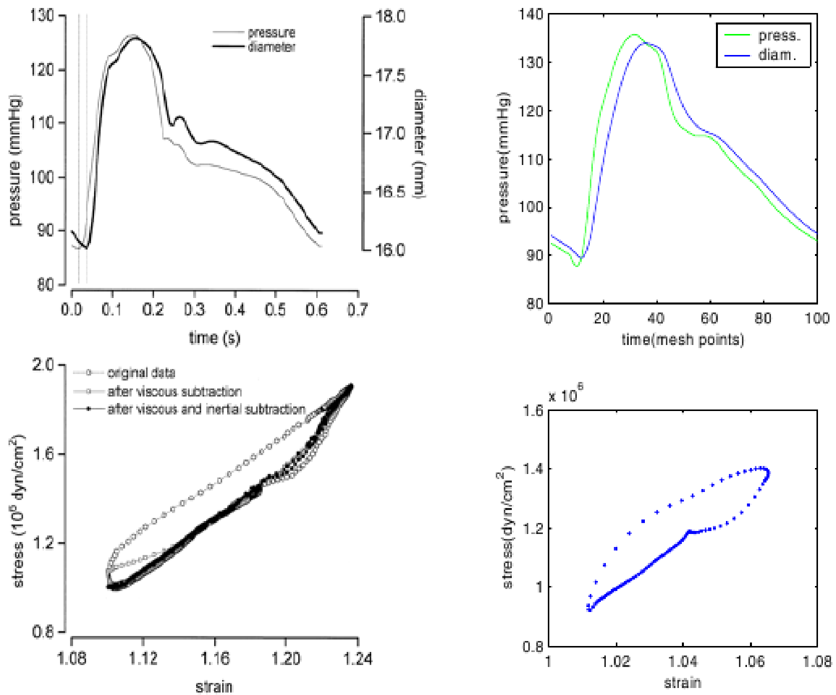
  Barnard et al. 1964, Noseda 1974, Quarteroni et al. 2000, Olufsen et al. 2000, Formaggia et al. 2002, Smith et al. 2002

- avaraged equations on cross section $\longrightarrow$ ad hoc closure

- homogenization theory (Canic and Mikelic 2002) $\longrightarrow$ no ad hoc closure



Wall viscoelasticity: mathematical or physiological?

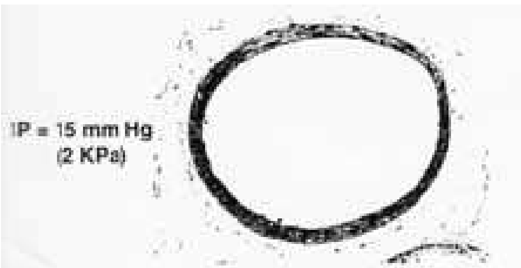


Left: In Vivo. Armentano et al. 1995

Right: Reduced Effective Model. Canic, Tambaca, Guidoboni, Mikelic, Hartley, Rosenstrauch 2006

(large-to-medium arteries, linear elasticity, thin shell, Kelvin-Voigt viscoelasticity)

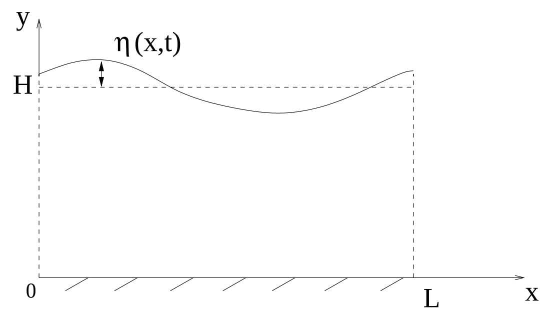
Other Modeling Issues



- Prestress: present but not very important (Mikelic, Guidoboni, Canic 2007)
- Wall Displacement: radial >> longitudinal → consider only radial
- Wall Thickness: we will consider thin walls
- Blood: large arteries: small non-Newtonian effects → Navier-Stokes
- Well-posedness: open field!!!

Now we will see a specific model as a benchmark
to present and test our novel numerical approach

Benchmark Problem: Mathematical Model



Fluid: Navier-Stokes eqs

$$\varrho_f(\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u}) = -\nabla p + \mu\Delta\mathbf{u}, \quad \nabla\cdot\mathbf{u} = 0, \quad \text{in } \Omega\times(0,T),$$

$\mathbf{u} = (u_1, u_2)$: fluid velocity; p: pressure, ϱ_f: fluid density, μ: fluid viscosity.

Dynamic and Kinematic Interfacial Coupling:

$$\varrho_s h_s \partial_t^2 \eta + a\eta - b\partial_x^2\eta - \gamma\partial_t\partial_x^2\eta = p|_{y=\eta(x,t)} \quad \text{on } (0,L)\times(0,T),$$

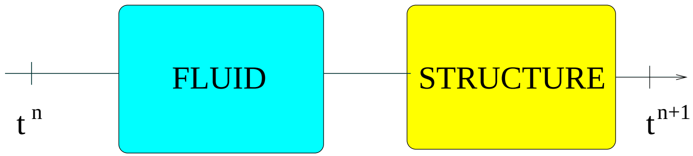
$$\partial_t\eta = u_2|_{y=\eta(x,t)}, \quad u_1|_{y=\eta(x,t)} = 0$$

η: transverse displacement; ϱ_s: structure density; h_s: structure thickness; a, b, γ: elastic constants. (Causin, Gerbeau, Nobile 2005)

Traditional Partitioned Schemes (Loosely Coupled)

Farhat et al. 1998, Zhao et al. 1998, Quarteroni et al. 2000

1. Given η and $\partial_t \eta$ at t^n
2. Use $\partial_t \eta$ at t^n as Dirichlet condition for the fluid $\longrightarrow$ compute $\mathbf{u}$, p
   fluid solver with given b.c.
3. Use it to force the structure $\longrightarrow$ compute η and $\partial_t \eta$
   structure solver with given load
4. go to t^{n+1} and Step1



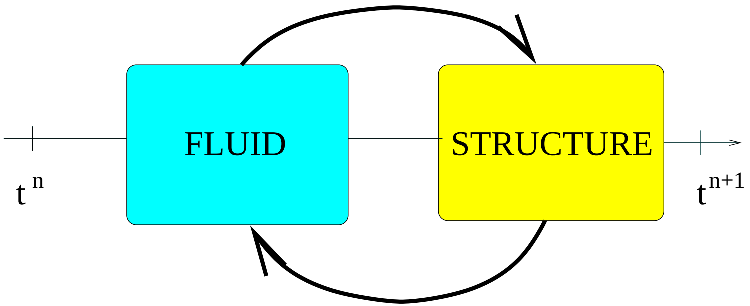
cheap and modular

stable in aeroelasticity, unstable in blood flow

ADDED MASS EFFECT Causin, Gerbeau, Nobile 2005

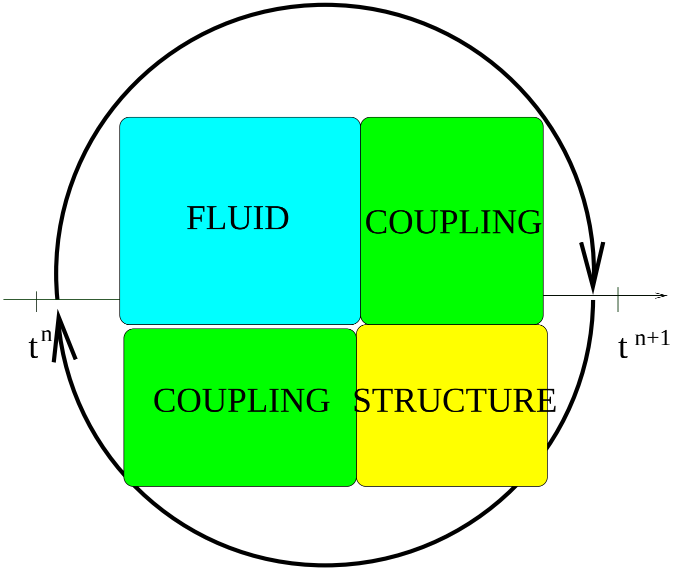
Strongly Coupled Schemes

- Implicit: iterate between fluid and structure till the coupling conditions are satisfied to a certain tolerance. Quarteroni, Nobile, Formaggia



Strongly Coupled Schemes

- Implicit: iterate between fluid and structure till the coupling conditions are satisfied to a certain tolerance. Quarteroni, Nobile, Formaggia
- Monolithic: linearize the problem, write a large system involving all the unknwowns, use iterative techniques to solve the nonlinear problem. Hughes, Taylor



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- Quasi-Monolithic: a thin structure is incorporated into the fluid equations via a Robin-like boundary condition. Nobile, Vergara

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- Semi-Implicit: strong coupling between fluid pressure and structure displacement, while the fluid velocity is decoupled. Gerbeau, Grandmont, Quaini, Quarteroni

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Our Goal: combine stability of strongly coupled schemes
with low computational cost and modularity of loosely coupled schemes

Kinematically-coupled scheme: Main Ideas

Guidoboni, Glowinski, Cavallini, Canic, Lapin (AML)

→ Time-discretization via operator splitting.
Splitting at differential level: freedom to use different time steps and/or space approximations in different substeps

→ Cardinal role of kinematic condition. (Kinematically-coupled scheme)
(1) first-order formulation → operator-splitting theory
(2) link velocities of fluid & structure → Added-Mass Effect

→ Split hyperbolic & parabolic parts.
Traditional schemes follow multi-physics: fluid vs structure
Our scheme follows multi-math: hyperbolic vs parabolic
Deeply related to our analytical studies on well-posedness
Kim, Canic, Guidoboni, Mikelic

Mathematical Model: New notation

Assume small displacement $\longrightarrow$ fix fluid domain

$$\varrho_f \partial_t \mathbf{u} = \Phi(\mathbf{u}, p), \quad \nabla \cdot \mathbf{u} = 0, \quad \text{in } \Omega \times (0,T),$$
$$\varrho_s h_s \partial_t^2 \eta = \Psi(\eta) + \Pi(\partial_t \eta) + \Upsilon(\mathbf{u}, p) \quad \text{on } (0,L) \times (0,T),$$
$$\partial_t \eta = u_2|_{y=H} \quad \text{on } (0,L) \times (0,T),$$

with:

$$\Phi(\mathbf{u}, p) = -\varrho_f \mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \mu \Delta \mathbf{u}$$
$$\Psi(\eta) = -a\eta + b\partial_x^2 \eta \longrightarrow \text{elasticity}$$
$$\Pi(\partial_t \eta) = \gamma \partial_x^2 \partial_t \eta \longrightarrow \text{viscoelasticity}$$
$$\Upsilon(\mathbf{u}, p) = p|_{y=H} \longrightarrow \text{interfacial hydrodynamic load}$$

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Use the kinematic condition $\partial_t \eta = u_2|_{y=H}$
to obtain a first-order formulation

Mathematical Model: New formulation

$$\varrho_f \partial_t \mathbf{u} = \Phi(\mathbf{u}, p), \quad \nabla \cdot \mathbf{u} = 0, \quad \text{in } \Omega \times (0,T),$$
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becomes:

$$\varrho_f \partial_t \mathbf{u} = \Phi(\mathbf{u}, p), \quad \nabla \cdot \mathbf{u} = 0, \quad \text{in } \Omega \times (0,T)\ ,$$
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Mathematical Model: New formulation

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$$\partial_t \eta = u_2|_{y=H} \quad \text{on } (0,L) \times (0,T)\ .$$

Now we can properly apply the operator splitting technique for the time discretization

Operator Splitting Method

Consider the initial value problem:

$$\partial_t \varphi + A(\varphi, t) = 0 \text{ in } (0, T), \quad \varphi(0) = \varphi_0 \tag{2}$$

Assume $A = A_1 + A_2$.

Let $\varphi^n = \varphi(t^n)$ be given.

1. Solve first

$$\partial_t \varphi + A_1(\varphi, t) = 0 \text{ in } (t^n, t^{n+1}), \quad \varphi(t^n) = \varphi^n \tag{3}$$

   and then set $\varphi(t^{n+1}) = \varphi^{n+1/2}$.

2. Then solve

$$\partial_t \varphi + A_2(\varphi, t) = 0 \text{ in } (t^n, t^{n+1}), \quad \varphi(t^n) = \varphi^{n+1/2} \tag{4}$$

   and then set $\varphi(t^{n+1}) = \varphi^{n+1}$.

(see e.g. Yanenko (1971), Marchuk(1975,1990), Glowinski (2003))

Operator Splitting Method - Remarks

- the decomposition is not unique
- the communication between sub-steps is through the initial conditions
  $\longrightarrow$ modules of black boxes
- freedom of using different time steps and/or spatial approx. for the same variable in different substeps
- stability may be achieved with different choices for the decomposition of A
- splitting error may compromise accuracy
  $\longrightarrow$ symmetrization

Kinematically-coupled scheme

Step1:

$$\varrho_f \partial_t \mathbf{u} = \Phi(\mathbf{u}, p), \quad \nabla \cdot \mathbf{u} = 0, \quad \text{in } \Omega \times (0,T) \ ,$$

$$\varrho_s h_s \partial_t u_2|_{y=H} = \Psi(\eta) + \Pi(u_2|_{y=H}) + \Upsilon(\mathbf{u}, p) \quad \text{on } (0,L) \times (0,T)$$

$$\partial_t \eta = u_2|_{y=H} \quad \text{on } (0,L) \times (0,T) \ .$$

Step2:

$$\varrho_f \partial_t \mathbf{u} = \Phi(\mathbf{u}, p), \quad \nabla \cdot \mathbf{u} = 0, \quad \text{in } \Omega \times (0,T) \ ,$$

$$\varrho_s h_s \partial_t u_2|_{y=H} = \Psi(\eta) + \Pi(u_2|_{y=H}) + \Upsilon(\mathbf{u}, p) \quad \text{on } (0,L) \times (0,T)$$

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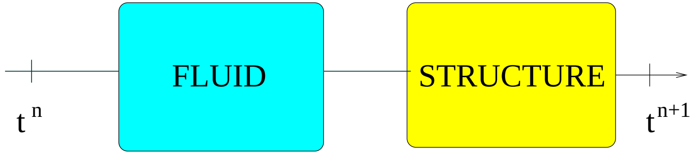


Kinematically-coupled scheme: Step1

Given $\mathbf{u}(t^n) = \mathbf{u}^n$, $\eta(t^n) = \eta^n$, and $\partial_t \eta(t^n) = g^n$, solve:

$$\varrho_f \partial_t \mathbf{u} = \Phi(\mathbf{u}, p), \quad \nabla \cdot \mathbf{u} = 0, \quad \text{in } \Omega \times (t^n, t^{n+1}) \, ,$$

$$\varrho_s h_s \partial_t u_2|_{y=H} = \Pi(u_2|_{y=H}) + \Upsilon(\mathbf{u}, p) \, , \quad \text{on } (0, L) \times (t^n, t^{n+1}) \, ,$$

and then set $\mathbf{u}(t^{n+1}) = \mathbf{u}^{n+1/2}$ and $p(t^{n+1}) = p^{n+1}$.

MAIN IDEA:

The hydrodynamic part of the structure equation

$\Pi(u_2|_{y=H}) + \Upsilon(\mathbf{u}, p)$

(viscoelasticity and fluid stress on the interface)

are treated together with the fluid equations

$\longrightarrow$ more inertia on the interface avoiding Added-Mass effect

Kinematically-coupled scheme: Step2

Given $\eta(t^n) = \eta^n$, and $\partial_t \eta(t^n) = u_2|_{y=H}^{n+1/2}$, solve:

$$\partial_t \eta = u_2|_{y=H}$$

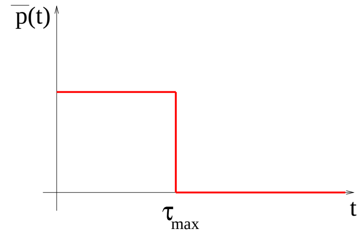
$$\varrho_s h_s \partial_t u_2|_{y=H} = \Psi(\eta)$$

on $(0, L) \times (t^n, t^{n+1})$, and then set $\eta(t^{n+1}) = \eta^{n+1}$ and $\partial_t \eta(t^{n+1}) = u_2|_{y=H}^{n+1}$.

MAIN IDEA:
The elastic part of the structure equation
$\Psi(\eta)$
is non-dissipateive and it is treated in a separate step
$\rightarrow$ non-dissipative solver

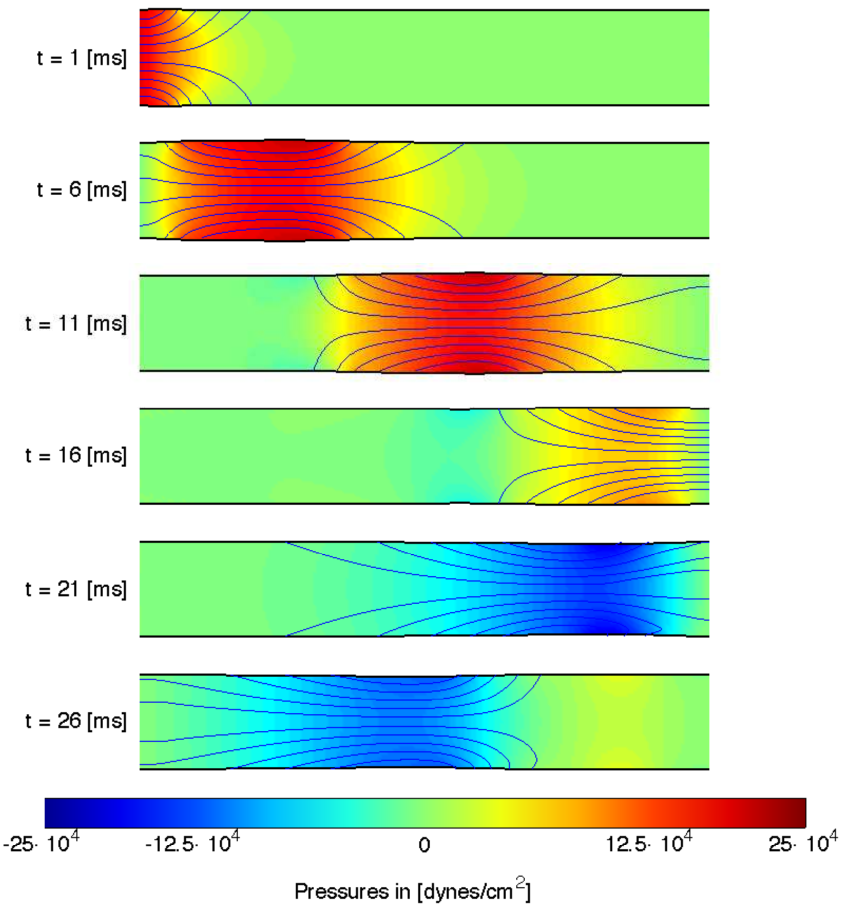
Blood Flow - Numerical Test

Benchmark Test: Formaggia et al. 2001. SHOW MOVIE

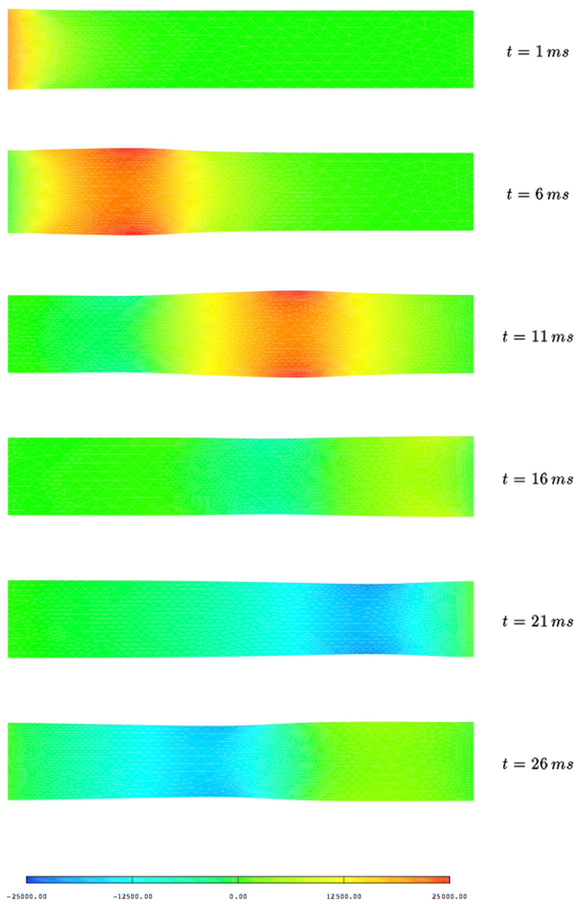


viscosity	μ	0.035	poise
fluid density	ρ_f	1	g/cm^3
young modulus	E	$0.75\ 10^6$	$dynes/cm^3$
poisson coefficient	σ	0.5	[1]
structure density	ρ_s	1.1	g/cm^2
structure thickness	h_s	0.1	cm
shear modulus	G	$\frac{E\ hs}{2(1+\sigma)}$	dynes/cm
structure viscoelasticity	γ	0.01	poise · cm
inlet pressure	$\bar{p}_{max}$	$2\ 10^4$	$dynes/cm^2$
inlet pressure duration	τ_{max}	5	ms

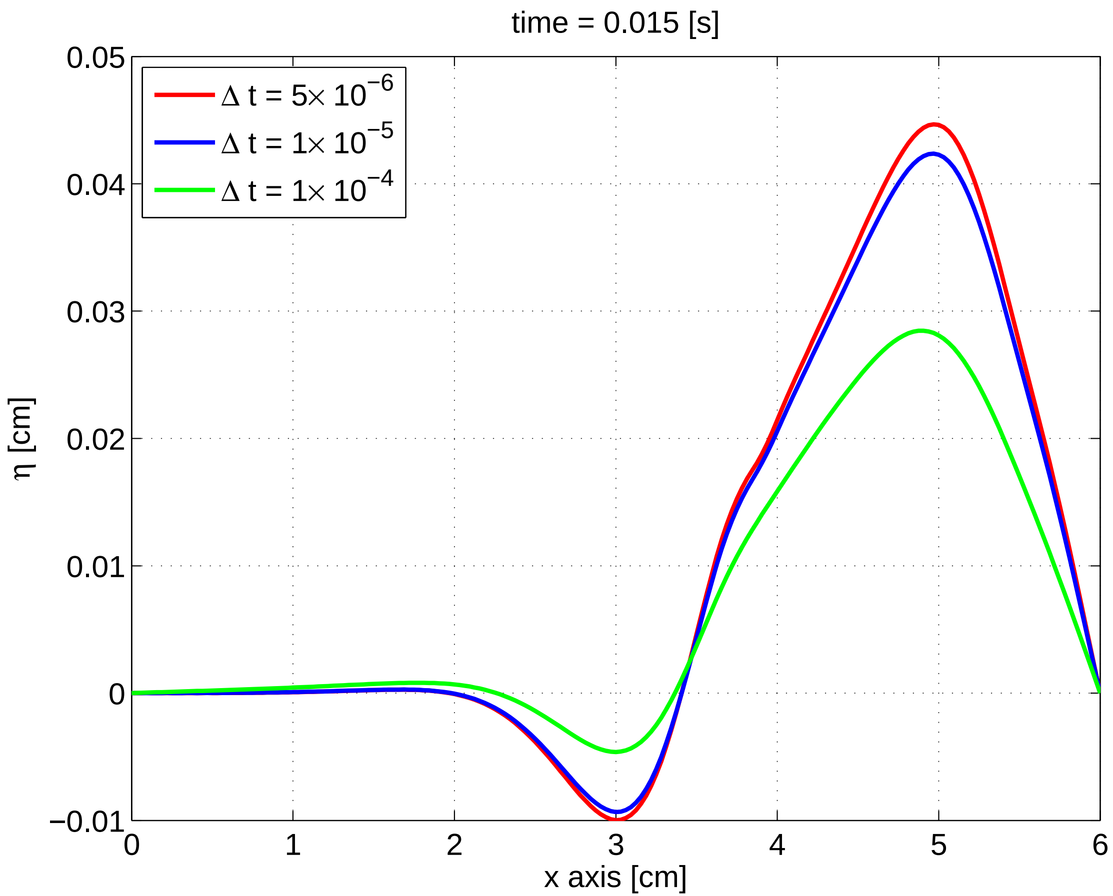
Results with the kinematically-coupled scheme



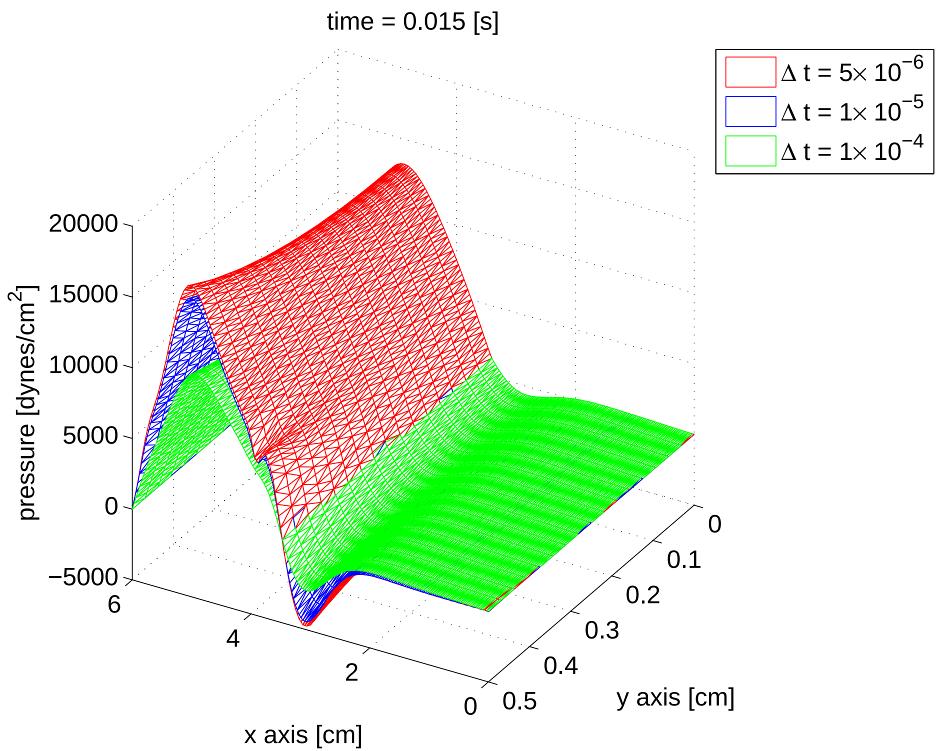
Results with implicit scheme by Formaggia et al. 2001



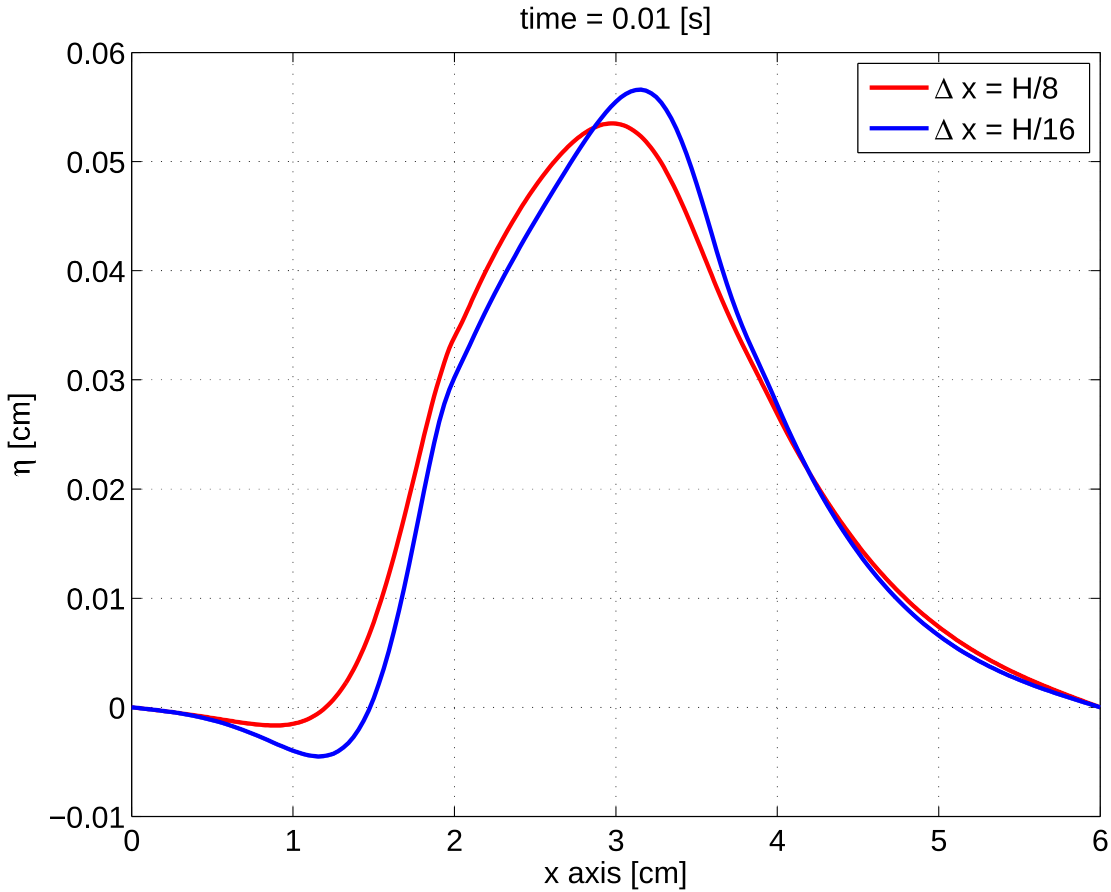
Different time steps: displacement



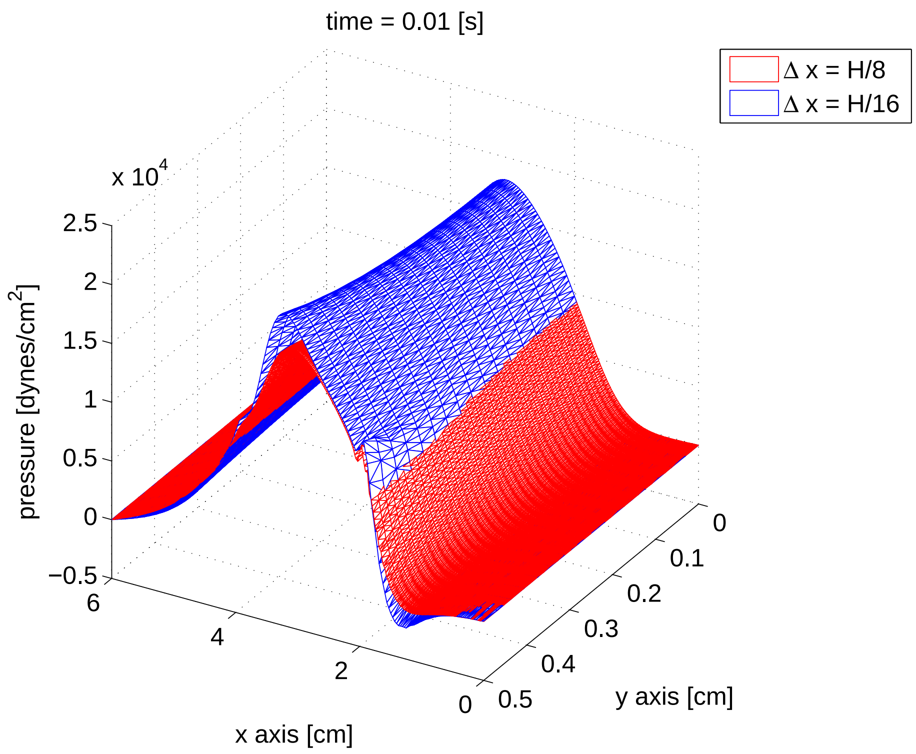
Different time steps: pressure



Different time steps and mesh size: displacement



Different time steps and mesh size: pressure



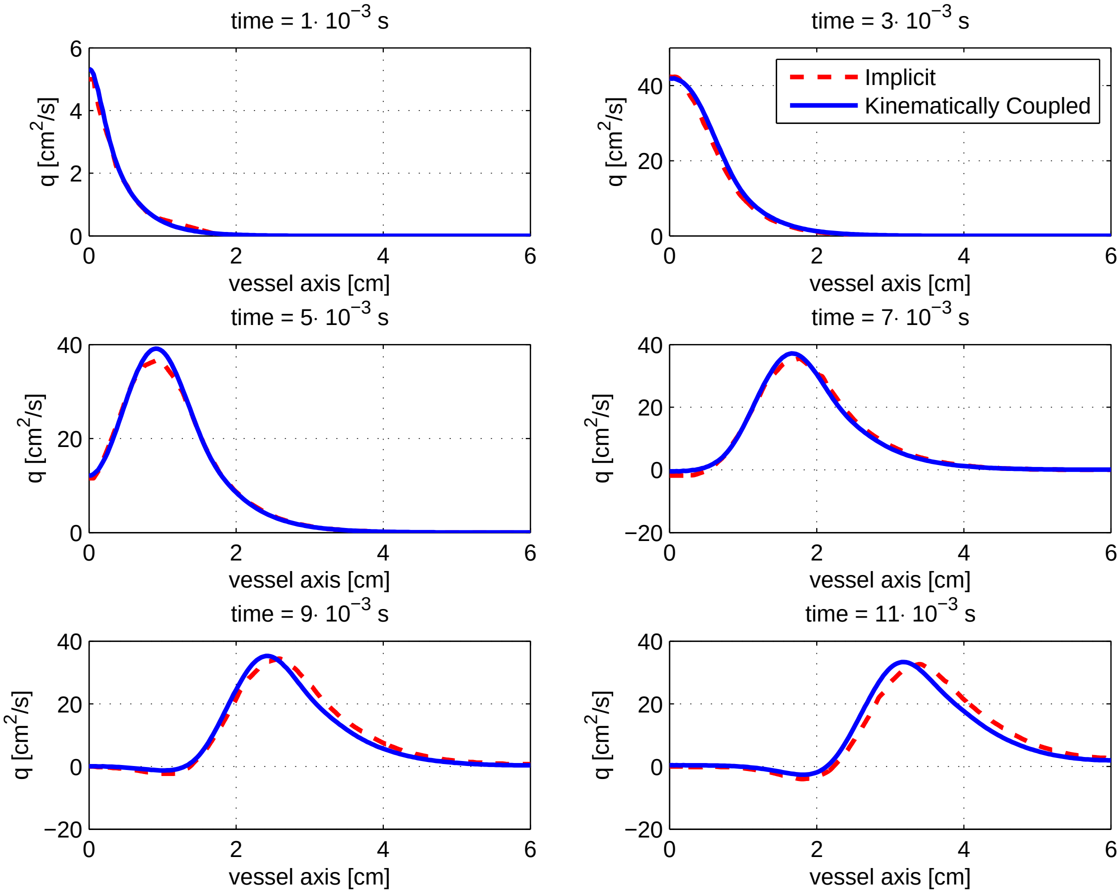
Blood Flow - Numerical Test

Benchmark Test: density ratio beyond critical limit

viscosity	μ	0.035	poise
fluid density	ρ_f	10	g/cm^3
young modulus	E	$0.75\ 10^6$	$dynes/cm^3$
poisson coefficient	σ	0.5	[1]
membrane density	ρ_s	1.1	g/cm^2
membrane thickness	h_s	0.1	cm
shear modulus	G	$\frac{E\ hs}{2(1+\sigma)}$	dynes/cm
membrane viscoelasticity	γ	0.01	poise · cm
inlet pressure	$\bar{p}_{max}$	$2\ 10^4$	$dynes/cm^2$
inlet pressure duration	τ_{max}	5	ms

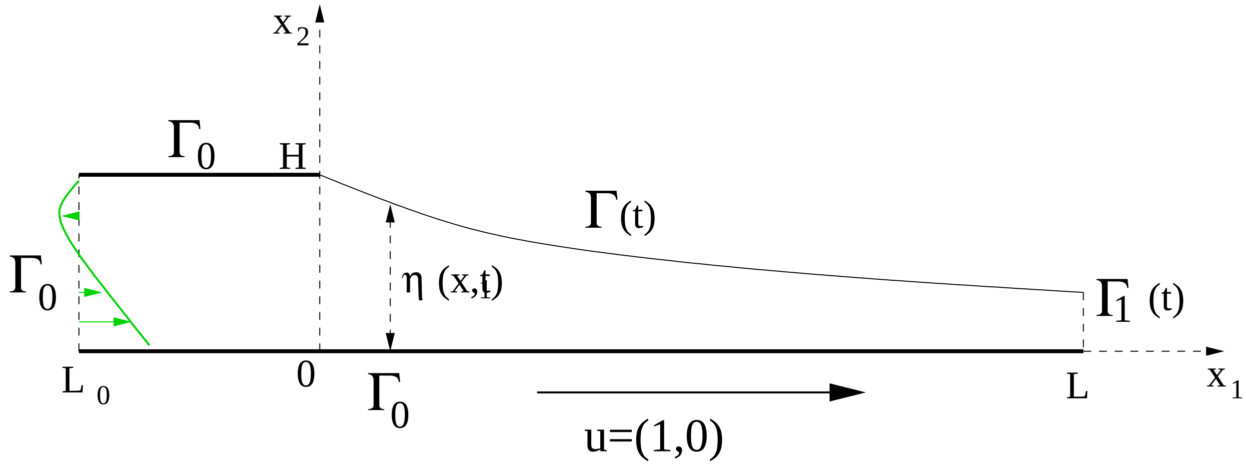
SHOW MOVIE

Comparison with implicit schemes - Flow rate



(Implict scheme results from Nobile PhD Thesis)

Capillary Surface - Slot Coater



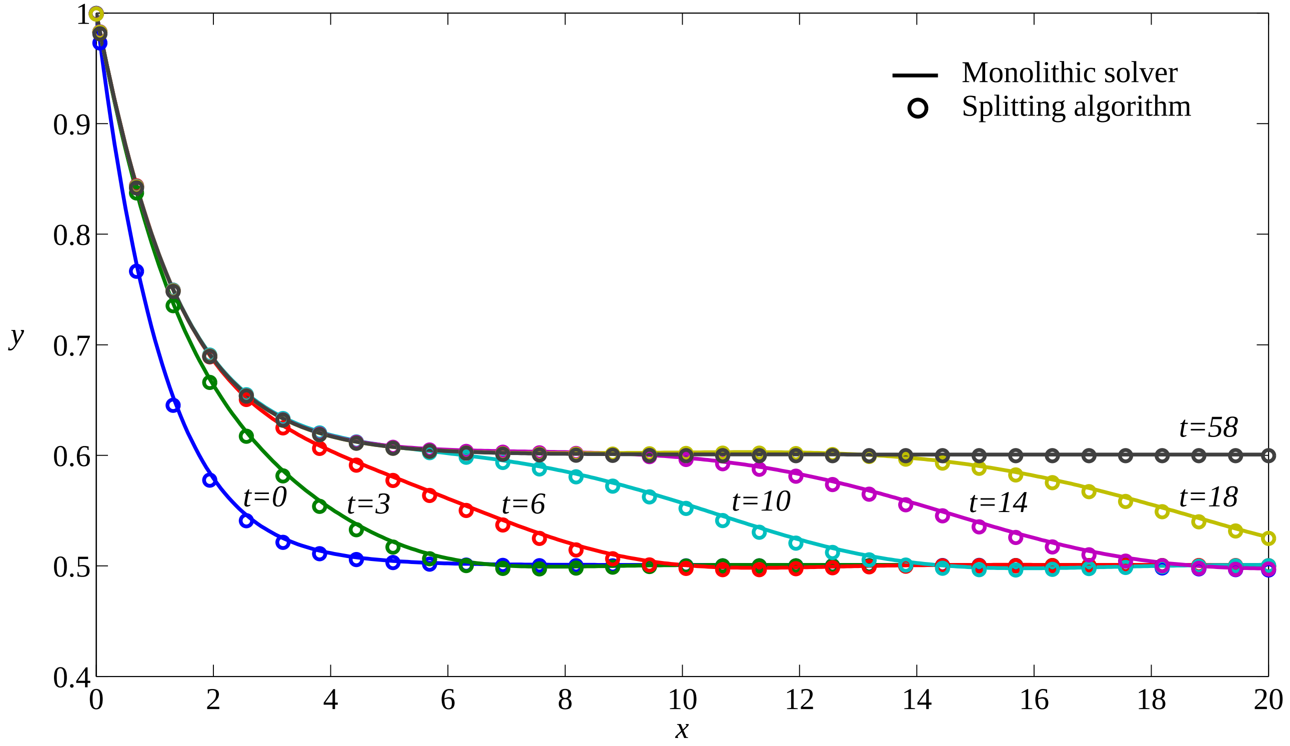
Milestone work: Silliman (1979)

Monolithic Algorithm: Pasquali and Scriven (2004).

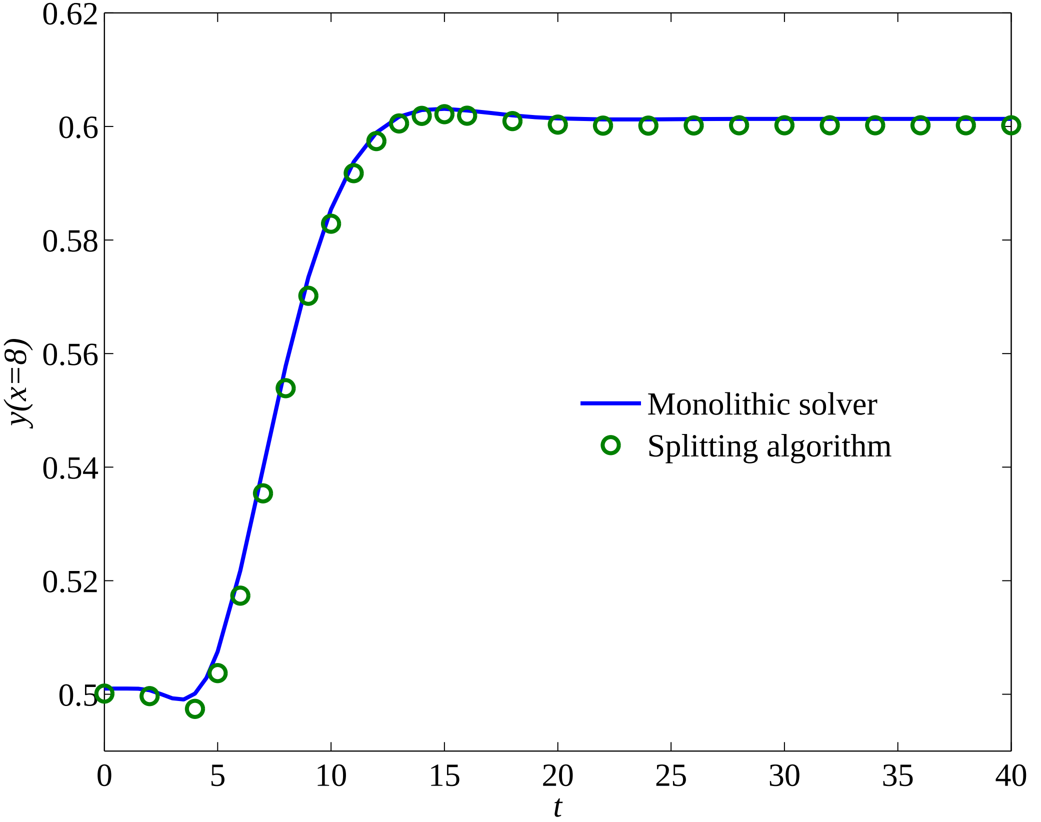
Comparison splitting/monolithic:

G. Guidoboni, R. Glowinski, M. Pasquali. Submitted to JCAM.

Time evolution of the free surface profile



Time evolution of the height of the free surface at $x = 8$



FUTURE DIRECTIONS

- Thick structure: work in progress
- Accuracy: first-order. Symmetrization?
- 3-D
- Clinically relevant problems

Thank you!

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GRANTS

NSF/NIH NIGMS, ARP Texas Higher Education Board, UH