Swarming: discrete models

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“Definition”

Aggregation of agents
of similar size and body type
generally moving in a coordinated way

Highly developed social organization:

Insects - Ants, Bees, Locusts, Termites

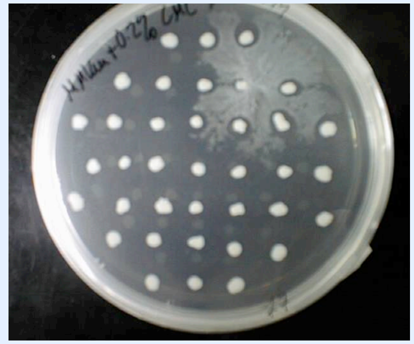
Animals - Fish, Birds, Wildebeast, Geese

Microorganisms - Bacteria,

Artificial Robots

Why?

barracuda



bacteria

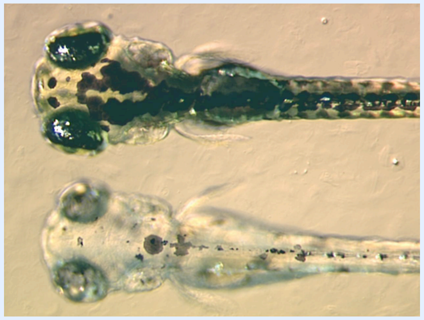
army ants

flocks

herds

jack, tuna

Fish:



Defense against predators - size deception
Hydrodynamic efficiency
Mating

Uniform schools
odd fish out will increase predator attack

Foraging - leaders may develop?

Imprinting - Fish joins school based upon
rearing similarities?

Ants, Termites and Bees:

Termite mating season

Resting bee swarm

Ant Colony:

Reproductive castes:
Queen, Males

Worker castes:
Sterile females

New colony:

Break-away

Swarming
Young males and queens
Genetic mixing

Environmental cues

Ants:

Dynamic phenomerone trails
Reinforced by successive passages
Dissipated after food source depleted
May attract predators

Modulations to signal
death, food sources, enemies

Can detect polarized light

Interactive learning
To lead naïve ant from nest to food

Beneficial to group

M. Moglich, Science 1974
N. Franks, T. Richardson, Nature 2006
Couzin, Nature 2006

"tandem running"

Leader ant waits for follower
Follower taps
Leader steps ahead
Acceleration/Deceleration

Approaches:

Behavioral ecology
Evolutionary biology

Parrish, Edelstein-Keshet Science 1999

Protect juvenile members
Deceive predators
Mating easier
Energetic benefits for motion

Compete for resources
Easier targets
Disease spread
Cannibalism

Just by chance – Aberrant behaviors?

AMERICAN MUSEUM NOVITATES

Published by
Number 1253 THE AMERICAN MUSEUM OF NATURAL HISTORY April 8, 1944
New York City

A UNIQUE CASE OF CIRCULAR MILLING IN ANTS, CONSIDERED IN RELATION TO TRAIL FOLLOWING AND THE GENERAL PROBLEM OF ORIENTATION[1]

BY T. C. SCHNEIRLA[2]

Ants died of exhaustion

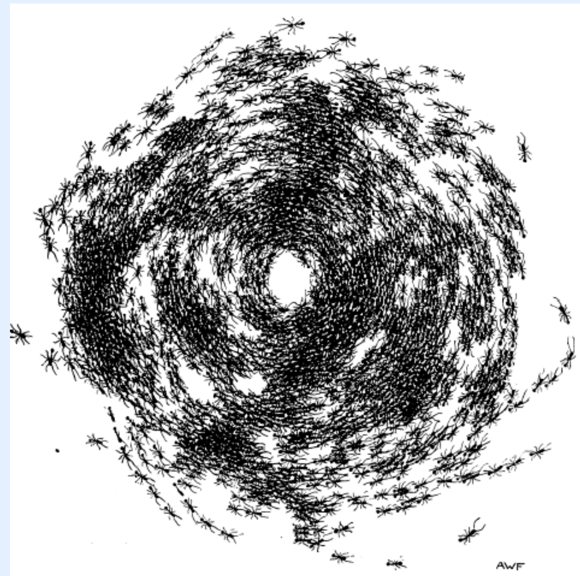


Fig. 1. The circular column of *Eciton praedator*, as drawn from a photograph taken shortly before 12:00 P.M. At that time the ring was approximately 14 cm. in diameter.

But also:

Unmanned Vehicle Operations

Exploration :

Space, Underwater

Dangerous missions:

Land-mine removal,
Earthquake recovery
Military missions

Individuals: limited capabilities

Teams: new, better properties without leaders

From Nature …

…to artificial systems?

Interactions:

Mediated by background:

Bacteria, plankton

Gradients of chemical or physical fields

food, light concentrations

temperature

electromagnetic fields

Direct information exchange between particles:

fish, birds

Nucleation agents:

External agents as triggers

tuna fish under floating objects

Interactions: design challenges

"I don't attribute emergent behaviors to amazing insights and interactions among the robots. I attribute them to me as the engineer not understanding the system.

One example of an emergent behavior that I was not anticipating: I was trying to get the robots to spread evenly throughout their environment, trying to have them move themselves so that there were robots everywhere in the whole room, leaving no empty spaces. And I made an error in the program; I flipped some signs in the equations. And when I ran the software, the robots formed into little clumps. Essentially they made polka dots on the floor, which was very entertaining after the fact."

James McLurkin, Nova-PBS December 2004

Approaches:

Discrete particle models: Equations of motion (coupled ODEs)

Albano PRL (1996), Shimoyama PRL (1996), Niwa JTB (1996), Levine PRE (2000), Mogilner JMB (2003), Gregoire PRL (2004), Birnir JSP (2007), Zhang PRE(2007)

Discrete particle models: Computer Rules

Viczek PRL (1995), Couzin Nature (2005), Franks JTB(2001)

Swarm Intelligence models

Ant Colony Optimization
(search for optimal paths - Dorigo 1992)

Particle Swarm Optimization
(optimizing fitness function on interacting particles - Kennedy 1995)

Stochastic Diffusion Search
(one to one random communication - Bishop 1989)

Continuum Fields (PDE-s)

Toner PRL (1995), Topaz JAM (2004), Grunbaum JMB (1994), Edelstein-Keshet (1998)

A First Study:

Vicsek algorithm CVA (PRL,1995):

Constant speed

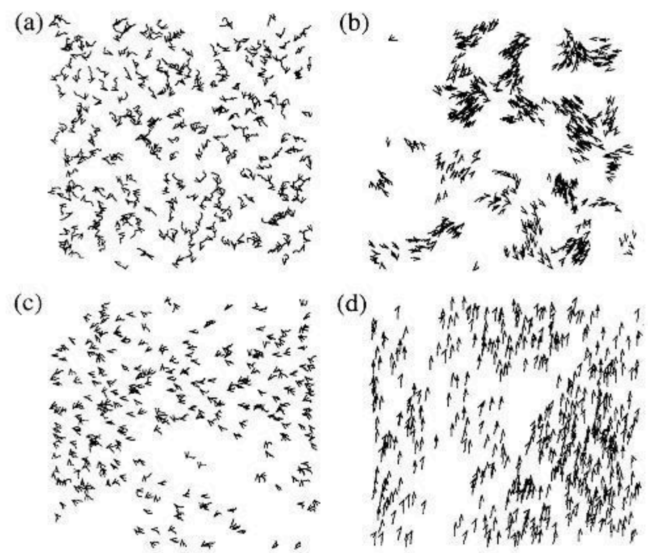
Velocity direction adjusts

according to neighbor directions

+ noise η

Phase transition to finite velocity

$|v| \sim (\eta_c - \eta)^{0.45}$



t=0 (a)

High density – high noise (c)

Low density – high noise (b)

High density – low noise (d)

Couzin et al J Theor Biol 2002
Buhl et al Science 2006: locusts
Toner et al PRL 1995
Gregoire et al PRL 2004

Starflag:

Fixed number of neighbors, no matter how far

Simple discrete model:

$$m_i \frac{\partial \vec{v}_i}{\partial t} = \boxed{\left(\alpha - \beta \left|\vec{v}_i\right|^2\right)\vec{v}_i} - \vec{\nabla}_i \sum_j U(\left|\vec{x}_i - \vec{x}_j\right|)$$

Rayleigh friction

Pumping - Self accelerating $\quad \alpha\, \vec{v}_i$

Dissipation - Self decelerating $\quad -\beta\, |\,\vec{v}_i\,|^2\, \vec{v}_i$

For $\quad \beta \left|\vec{v}_i\right|^2 = \alpha \quad$ the two terms balance and there is no pumping from or dissipating to the environment

Simple discrete model:

$$m_i \frac{\partial \vec{v}_i}{\partial t} = \left(\alpha - \beta \left|\vec{v}_i\right|^2\right)\vec{v}_i \boxed{- \vec{\nabla}_i \sum_j U\left(\left|\vec{x}_i - \vec{x}_j\right|\right)}$$

Morse potential

$$U\left(\left|\vec{x}_i - \vec{x}_j\right|\right) = -C_a e^{\frac{-\left|\vec{x}_i - \vec{x}_j\right|}{l_a}} + C_r e^{\frac{-\left|\vec{x}_i - \vec{x}_j\right|}{l_r}}$$

Simple discrete model:

$$m_i \frac{\partial \vec{v}_i}{\partial t} = \left(\alpha - \beta \left|\vec{v}_i\right|^2\right)\vec{v}_i - \vec{\nabla}_i \sum_j U\left(\left|\vec{x}_i - \vec{x}_j\right|\right)$$

Rayleigh friction $\beta \left|\vec{v}_i\right|^2 = \alpha$

$$U\left(\left|\vec{x}_i - \vec{x}_j\right|\right) = -C_a e^{\frac{-\left|\vec{x}_i - \vec{x}_j\right|}{l_a}} + C_r e^{\frac{-\left|\vec{x}_i - \vec{x}_j\right|}{l_r}}$$

Morse potential

Levine et al 2000

Schweitzer et al 2000

Mogilner et al 2003

Self propulsion:

Self-acceleration
+
Friction

↓

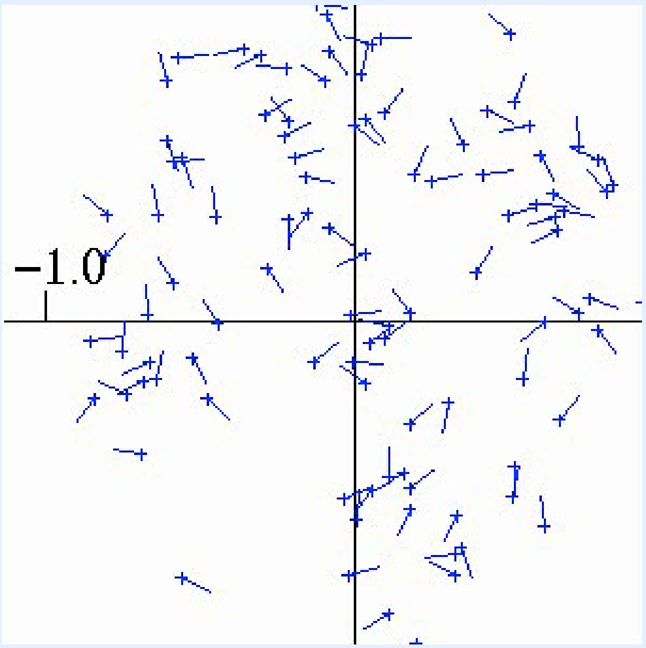
Optimal speed $\beta \left|\vec{v}_i\right|^2 = \alpha$

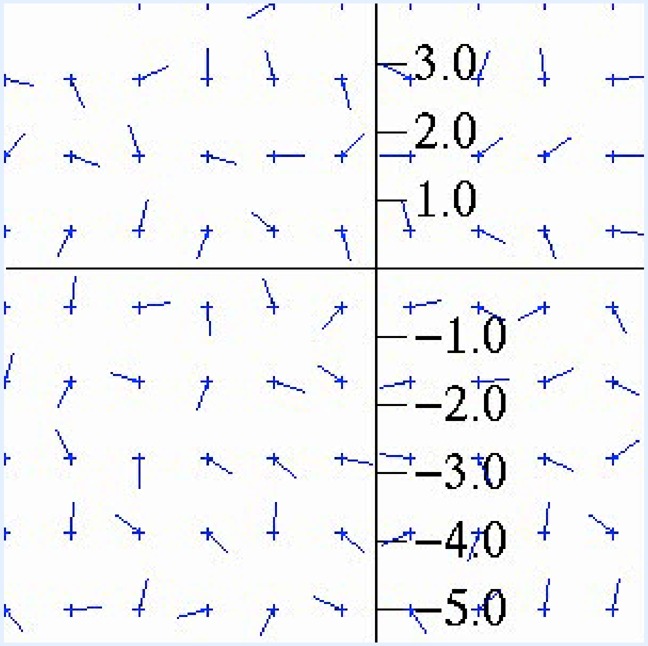
Attractive-Repulsive potential:

Ca, Cr, la, lr

Parameter choice

A few examples:





Example 1:

$\alpha = \beta = 0.5$

$C_a = 1.0,\ C_r = 40.0$

$l_a = 0.6, \quad l_r = 0.1$

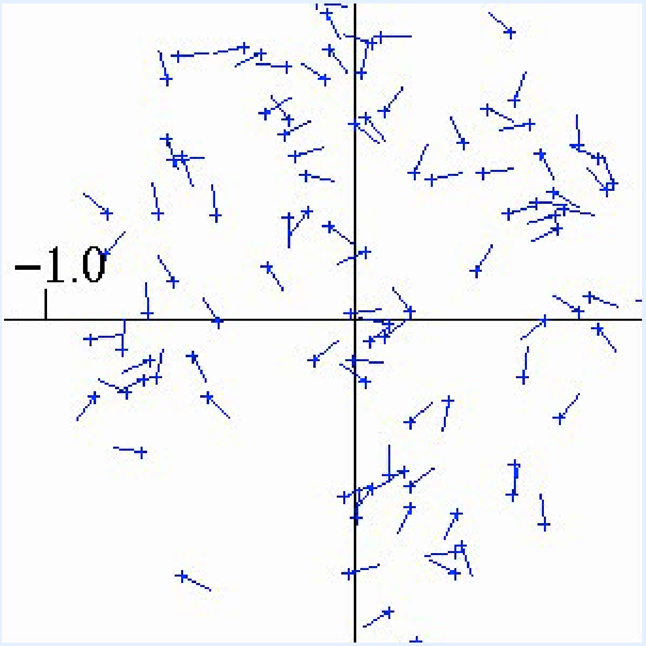
Example 2:

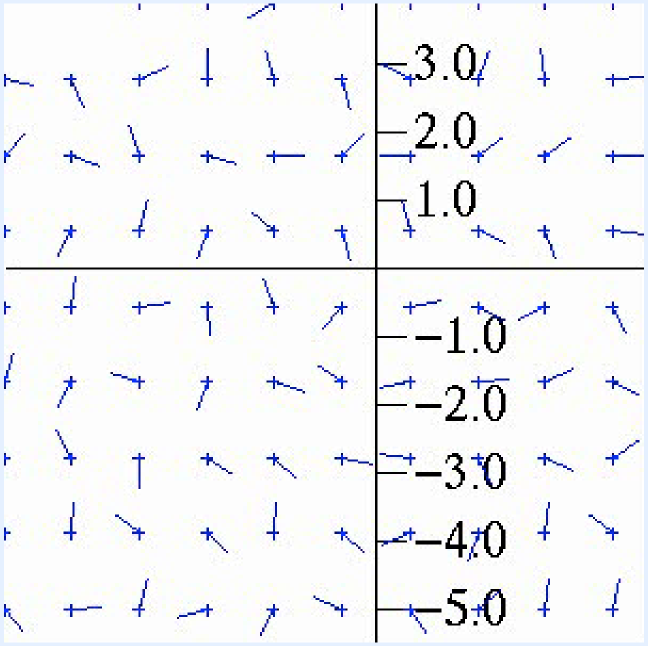
$\alpha = 0.8, \quad \beta = 0.5$

$C_a = 0.5,\ C_r = 1.0$

$l_a = 2.0, \quad l_r = 0.5$

A few examples:



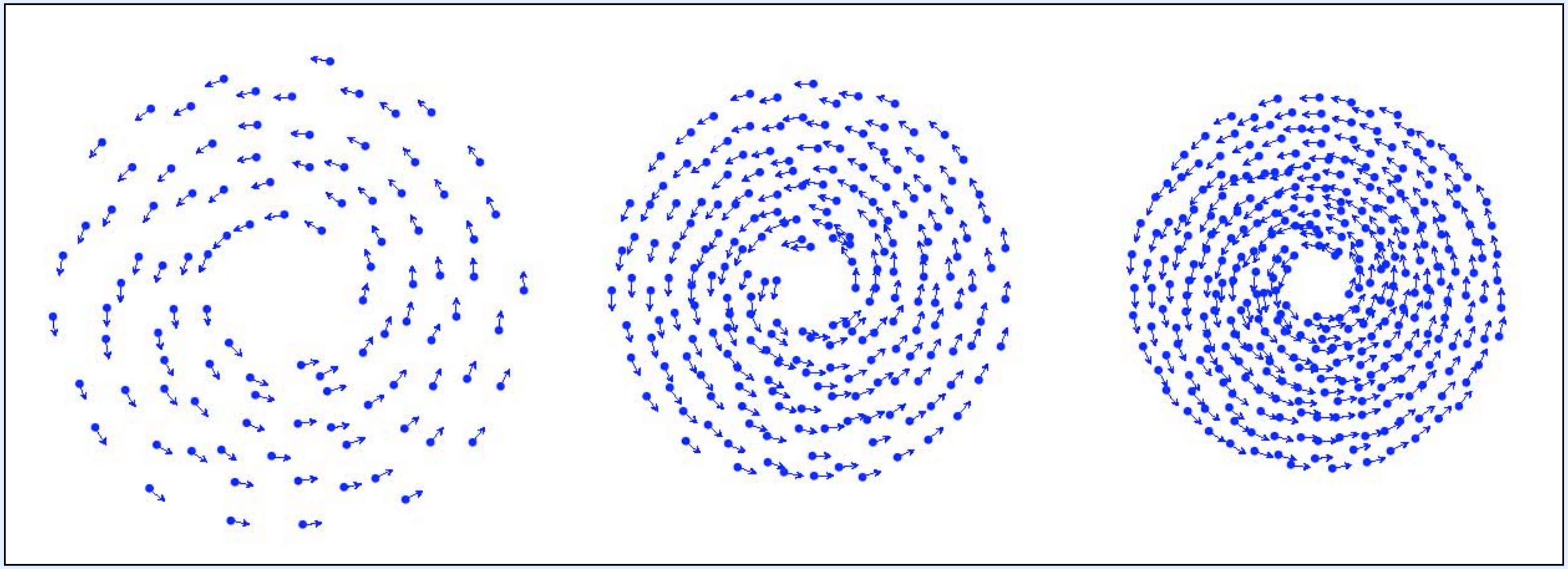


Why are they qualitatevely different?

What if we add more particles

Naive parameters:

N=100 N=200 N=300



Example 2

$\alpha = 0.8, \quad \beta = 0.5$

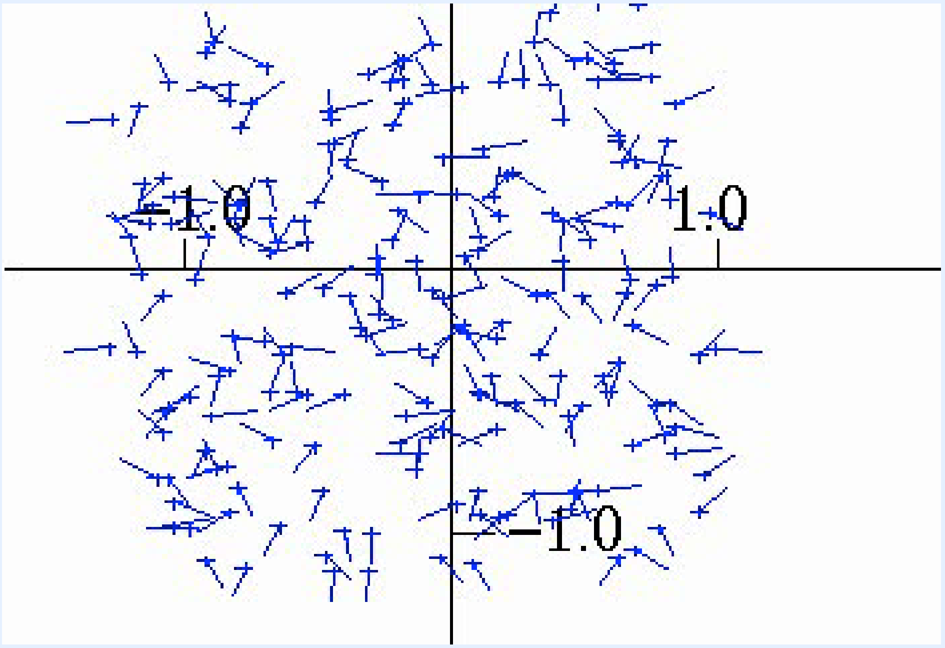
$C_a = 0.5, \; C_r = 1.0$

$l_a = 2.0, \quad l_r = 0.5$

The density is increasing!

Why is the system not extensive?

Another example:



$\alpha = 3, \quad \beta = 0.5$

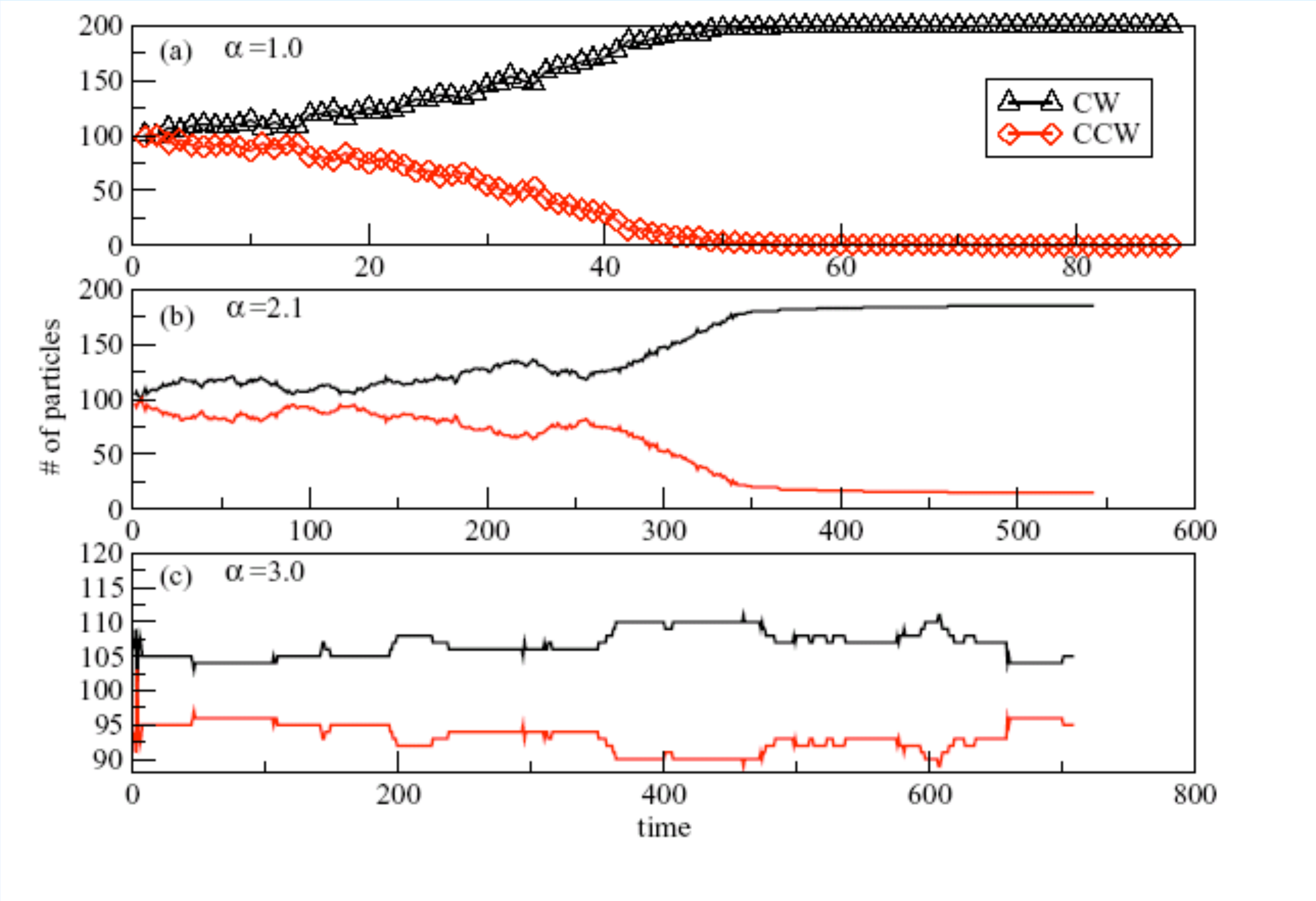
$C_a = 0.5, \; C_r = 1.0$

$l_a = 2.0, \quad l_r = 0.1$

Example 3

N=200

Double spirals



Persisting double spiral $\Longleftrightarrow$ Higher self propulsion

$$\beta \left|\vec{v}_i\right|^2 = \alpha$$

What is the role of the potential?

From Statistical Mechanics:

Given a many-body microscopic system

Is a ‘real’ macroscopic description possible?
i.e. thermodynamics

Interactions must obey ‘H-stability’ constraints

if not: CATASTROPHIC COLLAPSE!

D. Ruelle,
Statistical Mechanics,
Rigorous results

From Statistical Mechanics:

MICRO	→	MACRO
x_i, v_i for N particles		Volume, density, pressure
Many variables		Few variables
H-STABILITY		Extensive behavior (more particles occupy more volume)
IF NOT H-STABLE		Catastrophic collapse (all particles converge to a small volume)

Easy Recipe:

Take all configurations of the system
for fixed agent number N,
that is all possible positions, all possible velocities

Calculate the energy
kinetic and potential

Sum over all contributions of a “likelihood” function

Higher energy means less likely

This sum is called the partition function
And contains ALL relevant macroscopic information
that are derived via elementary math operations

H-Stability means that the partition function is mathematically well defined

D Ruelle, Statistical Mechanics Rigorous Results

H-stability:

A system of N >> 1 interacting agents
is H-stable if a non-negative
constant B exists such that:

$$\sum_{i>j}^{N} U(|x_i - x_j|) \geq -BN$$

where the l.h.s. is the total potential

Pairwise interactions:
H-stable constraints on the two-body potential

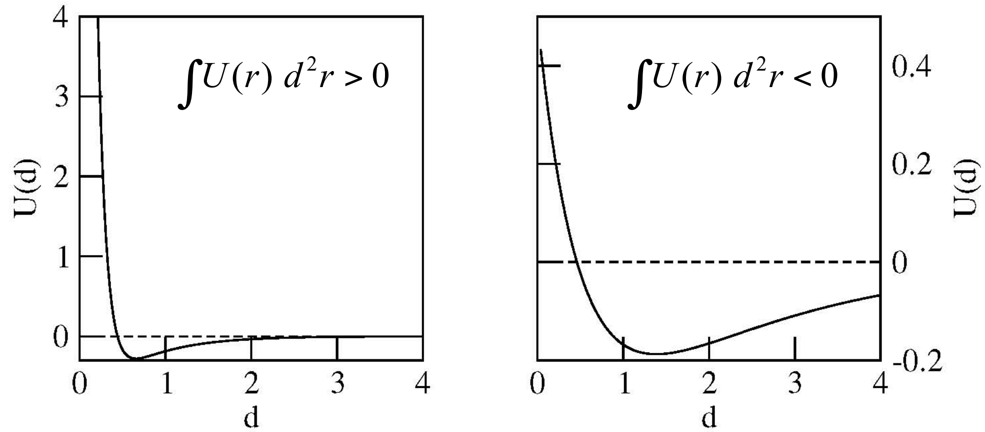
An H-Stable condition:

Pair-wise potential

$$\int U(r)\, d^n r < 0$$

Catastrophic !

Pair-wise potential:



Qualitatively similar

Soft-core, exponentially decaying, minimum exists

TWO particles will find a minimum, optimal distance

in BOTH cases

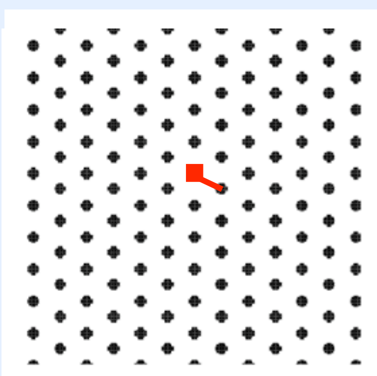
An H-Stable condition:

Pair-wise potential

$$\int U(r)\, d^n r < 0$$

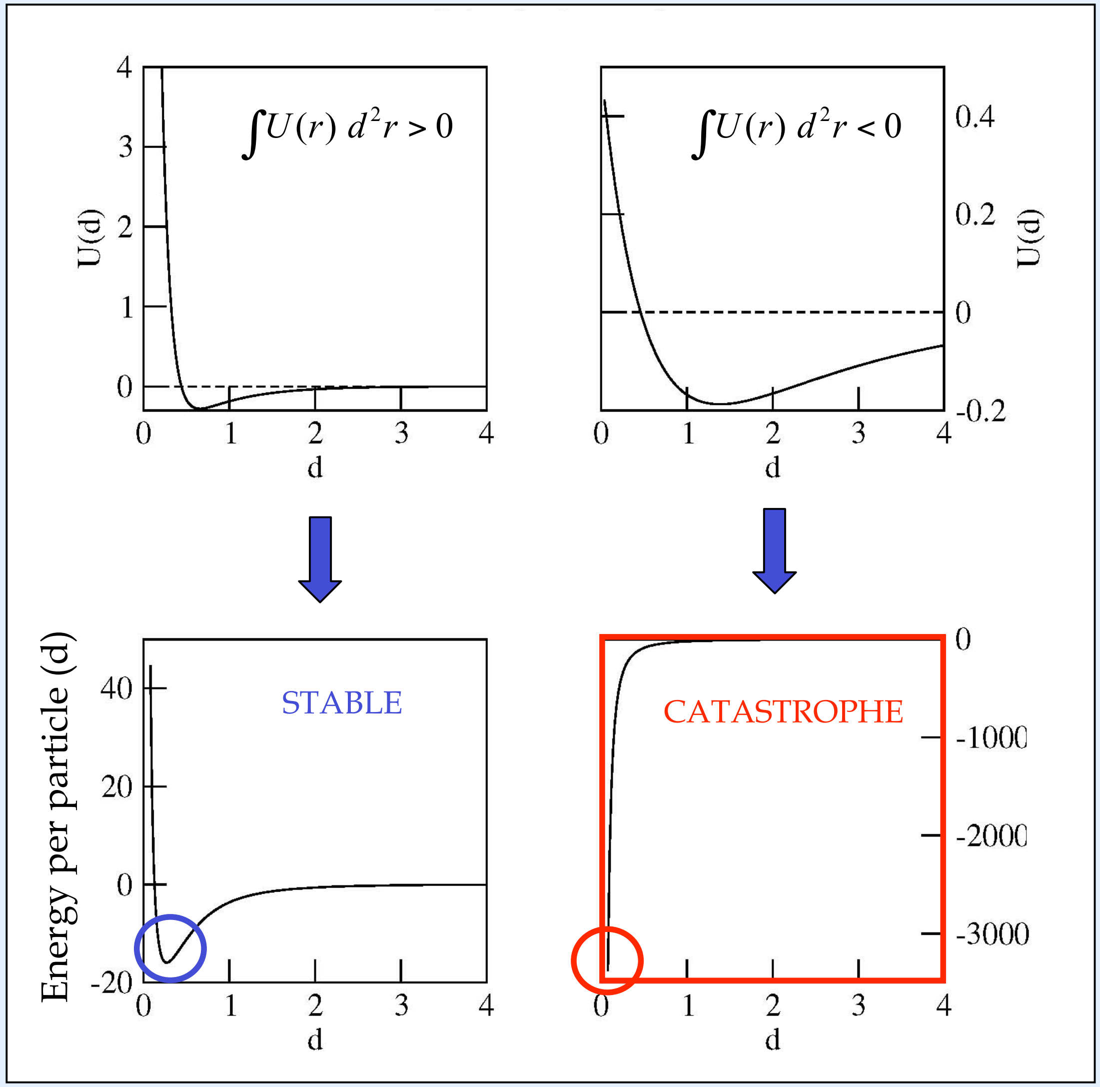
Catastrophic !

Example: particles on lattice



d

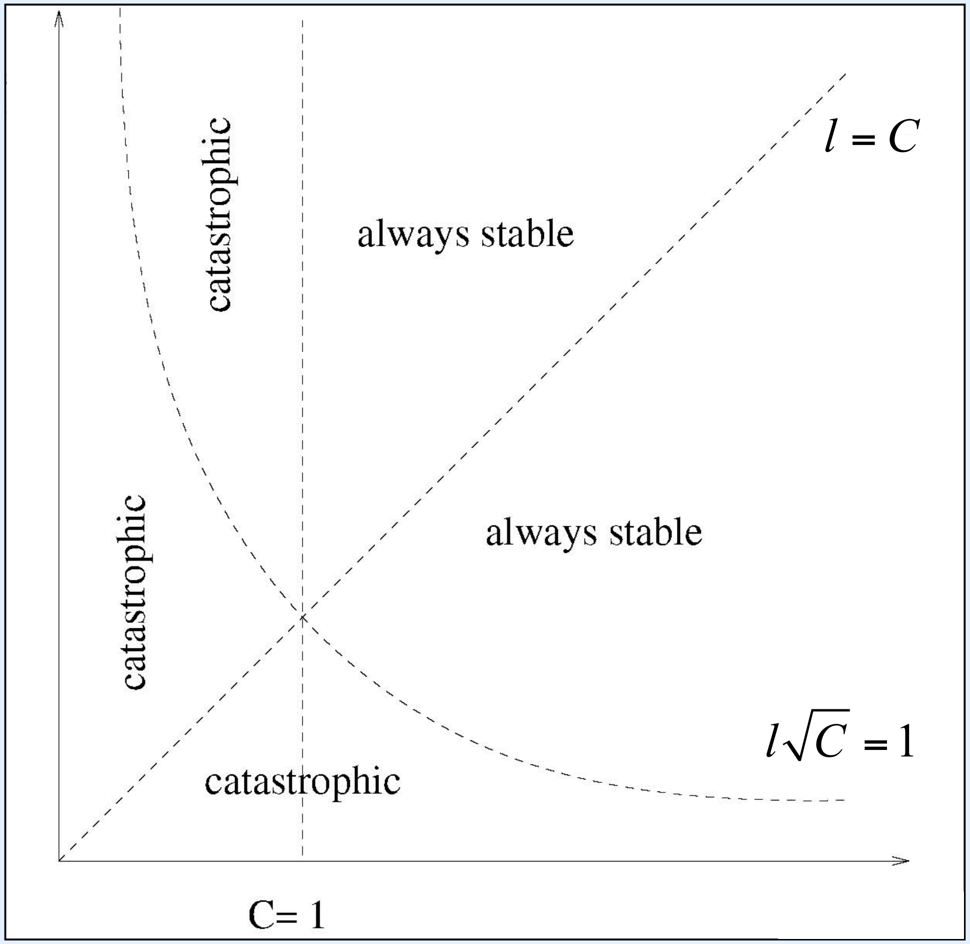
Pair-wise potential:



H-stability: Guiding interaction criteria

Morse Potential and H-stability:

$\boxed{l \equiv l_r / l_a}$



$\boxed{C \equiv C_r / C_a}$

$\beta |\vec{v}_i|^2 = \alpha$

$\vec{v}_i$ in rotation

Catastrophic:

particles collapse as

$N \to \infty$

MRD, Chuang, Bertozzi, Chayes PRL 2006

Stable:

volume occupied as

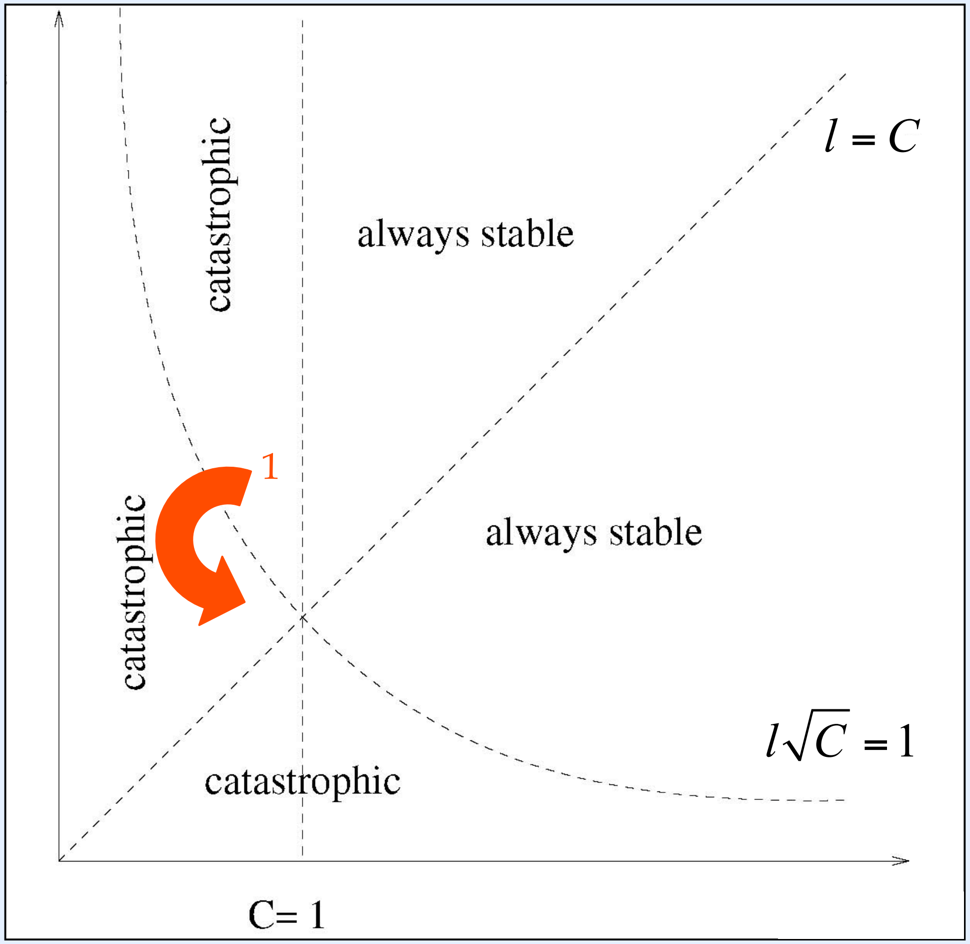
$N \to \infty$

Morse Potential and H-stability:

$l \equiv l_r / l_a$

$\beta |\vec{v}_i|^2 = \alpha$

$\vec{v}_i$ in rotation



$C \equiv C_r / C_a$

Catastrophic:

particles collapse as

$N \to \infty$

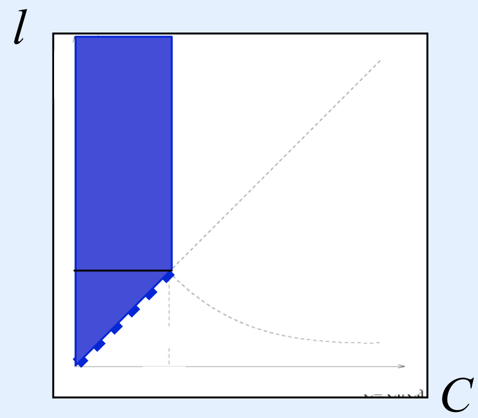
MRD, Chuang, Bertozzi, Chayes PRL 2006

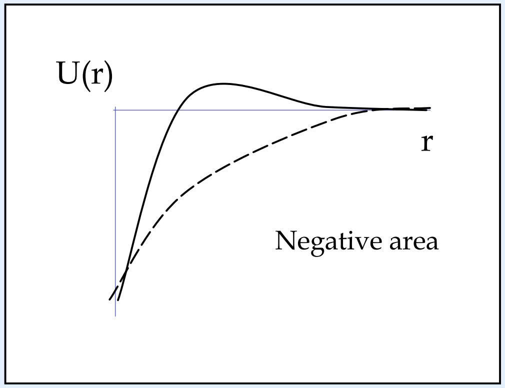
Stable:

volume occupied as

$N \to \infty$

Catastrophic features and patterns:

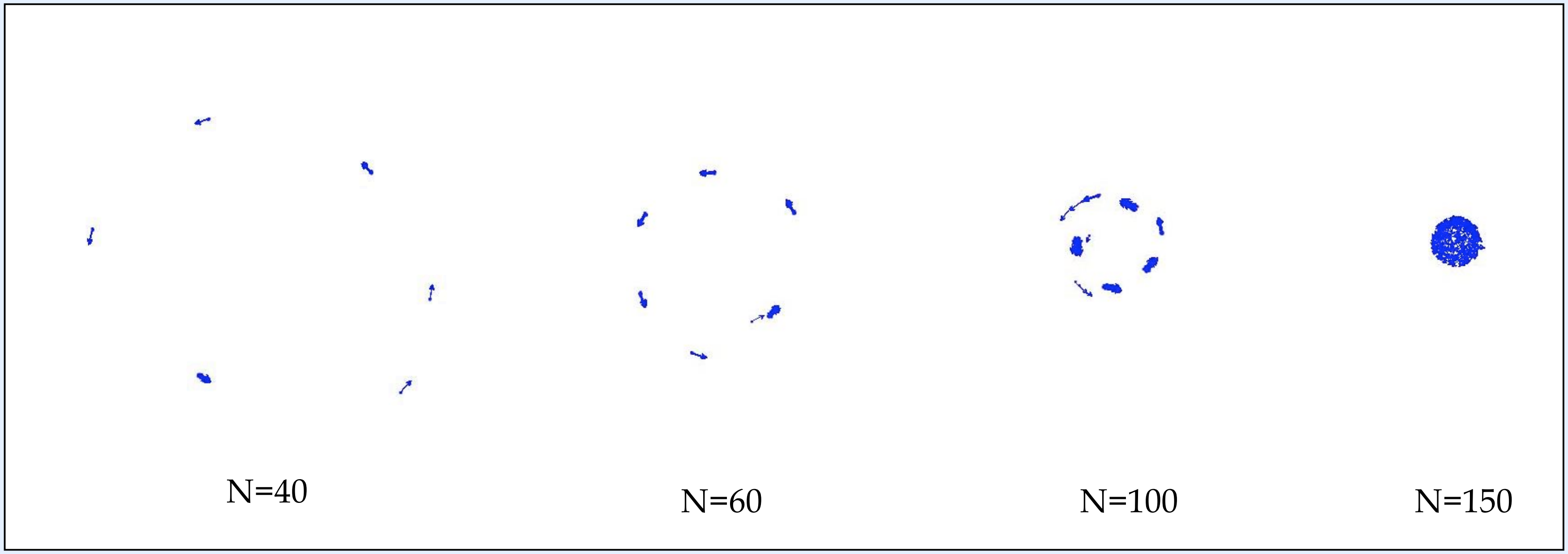




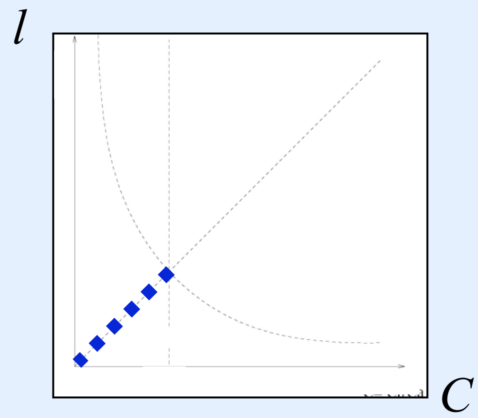
No intrinsic separation

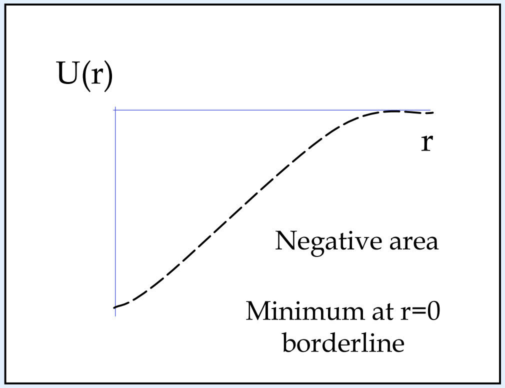
Self-propelling speed

Random initial conditions



Catastrophic features and patterns:

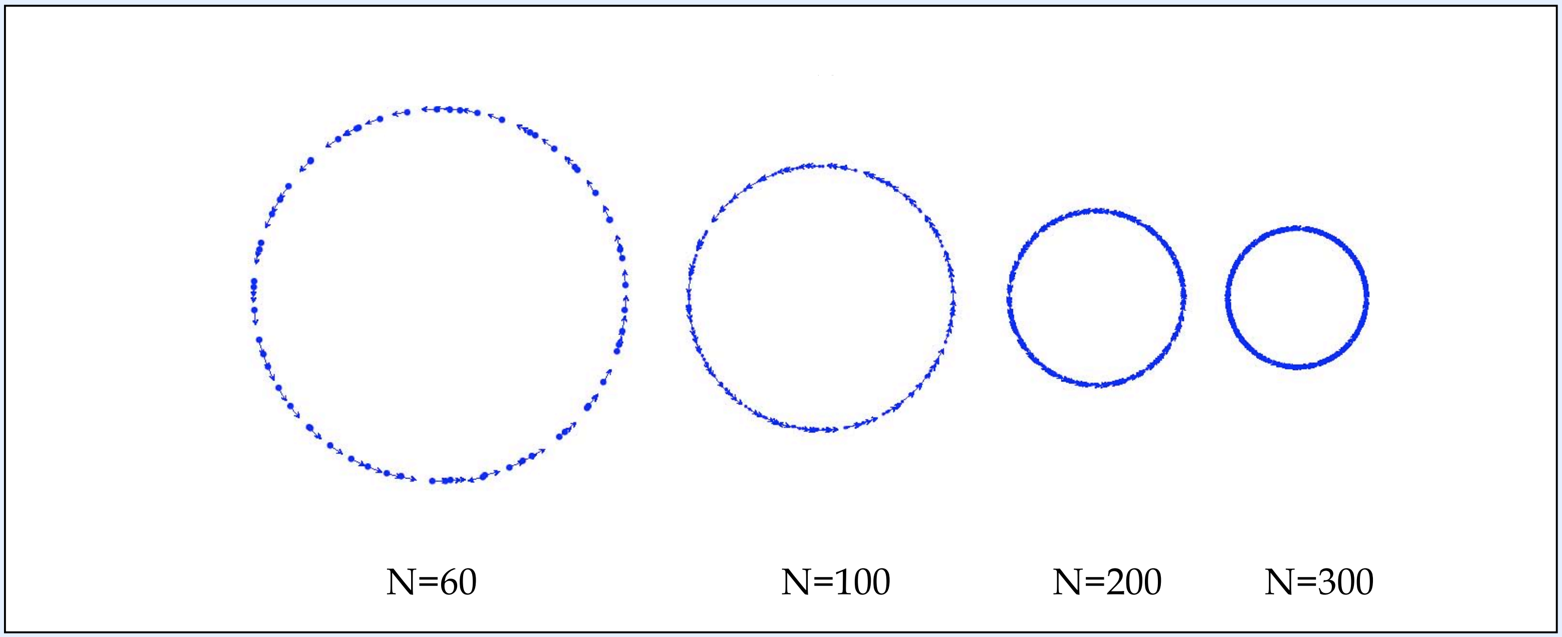




Intrinsic length-scale=0

Self-propelling speed

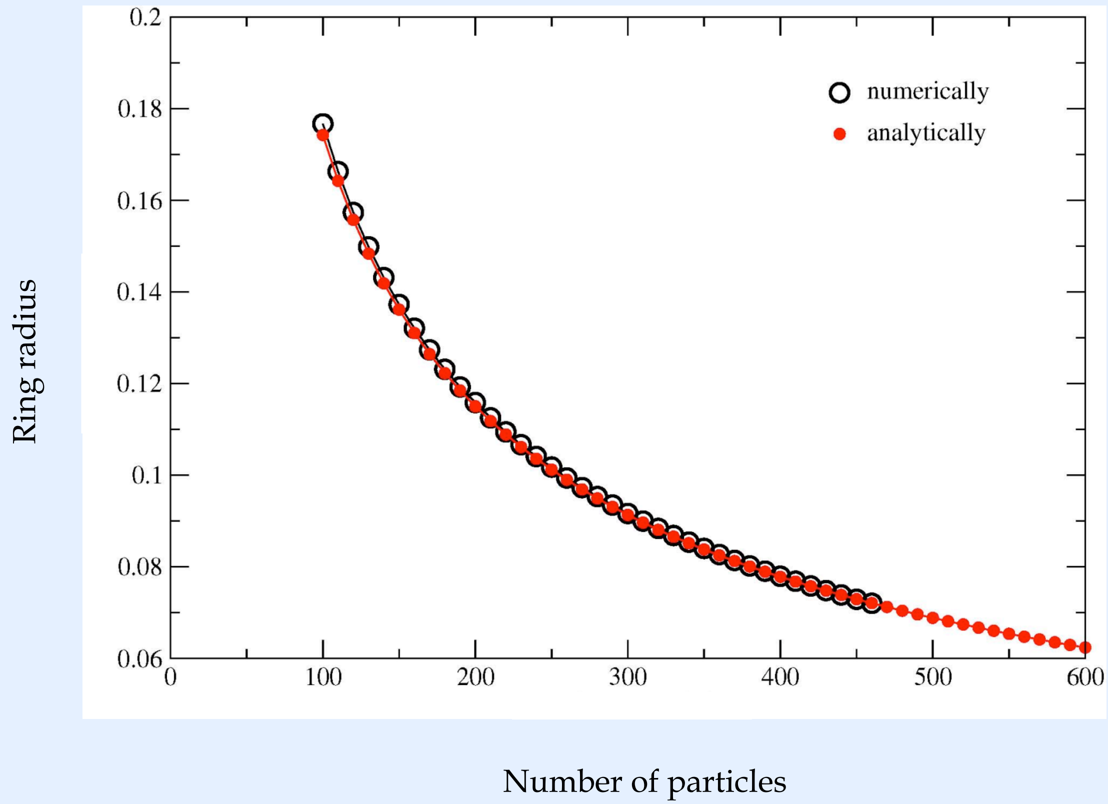
Random initial conditions



Ring Formation:

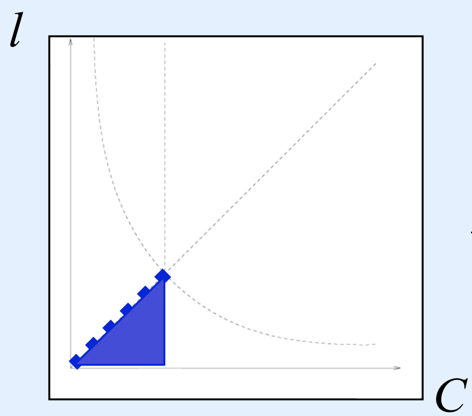
Implicit formula:

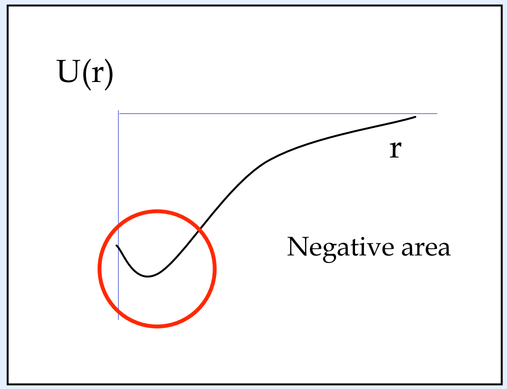
$$\frac{\alpha}{2\beta r} = \sum_{n=1}^{N/2} \left[\frac{C_a}{l_a} e^{-2r\sin(2\pi/N)/l_a} - \frac{C_r}{l_r} e^{-2r\sin(2\pi/N)/l_r} \right] \sin\frac{\pi n}{N}$$



Excellent agreement!

Catastrophic Features:

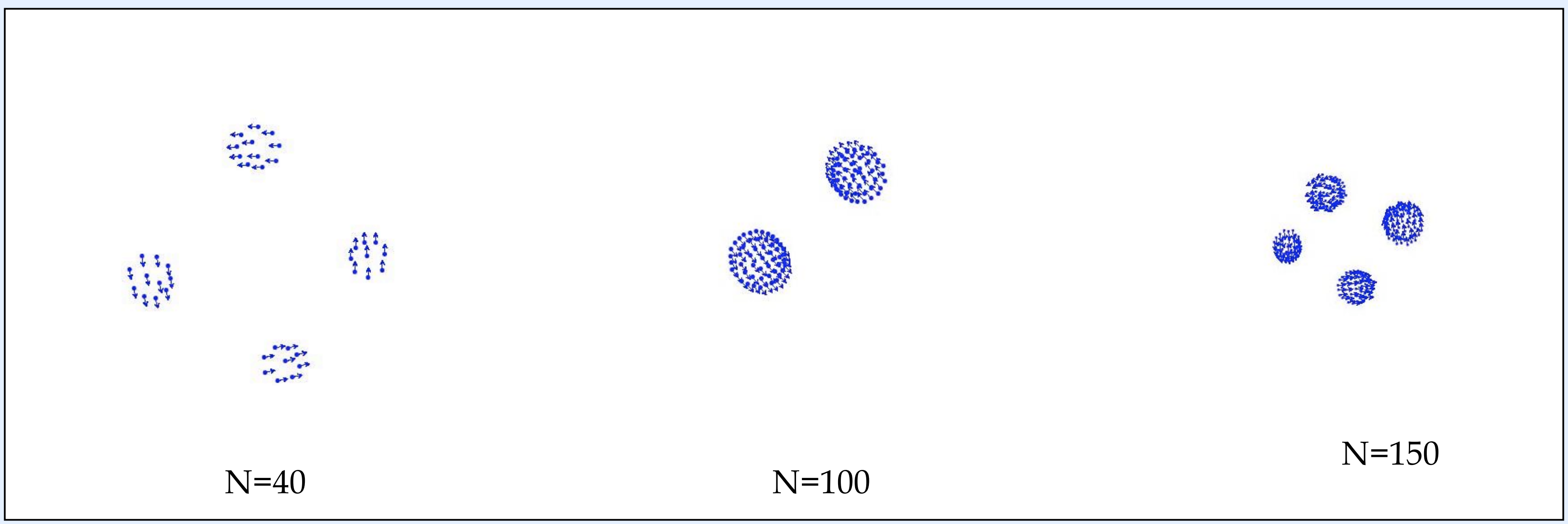




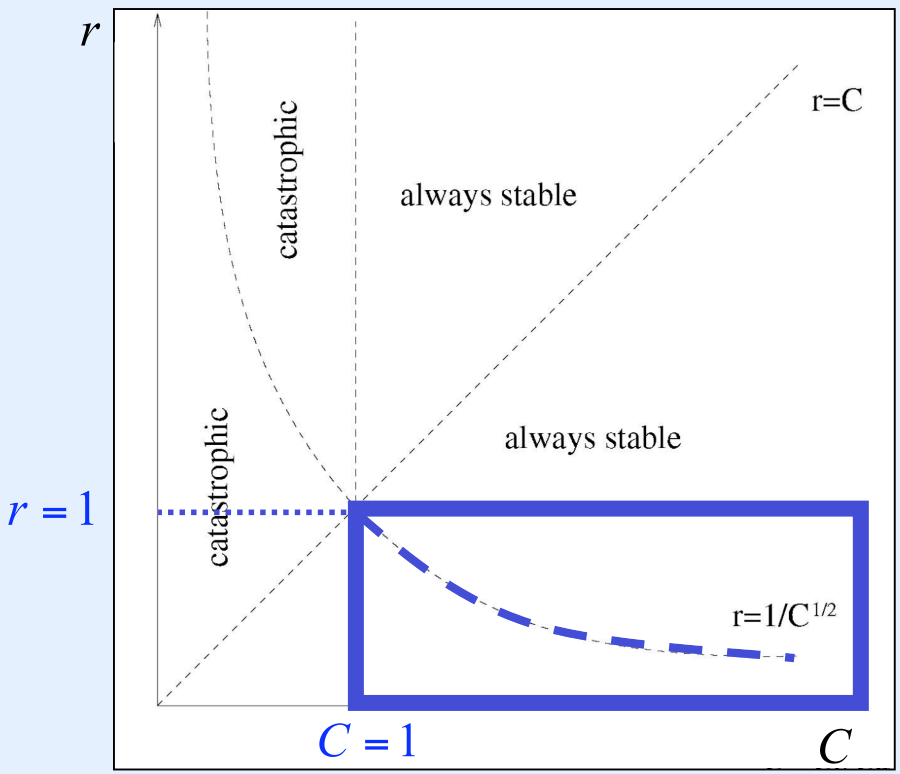
Finite intrinsic separation

Self-propelling speed

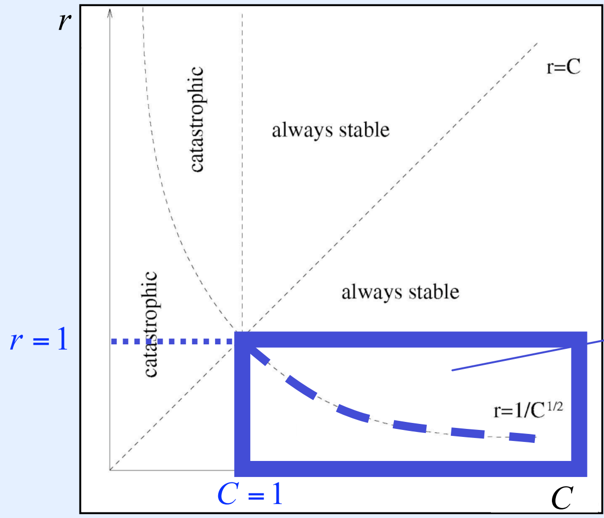
Random initial conditions

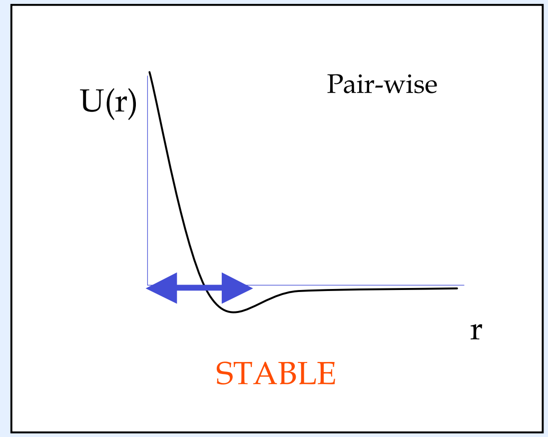


Potential features and patterns:



Potential Features:





Optimal spacing

`Crystalline'

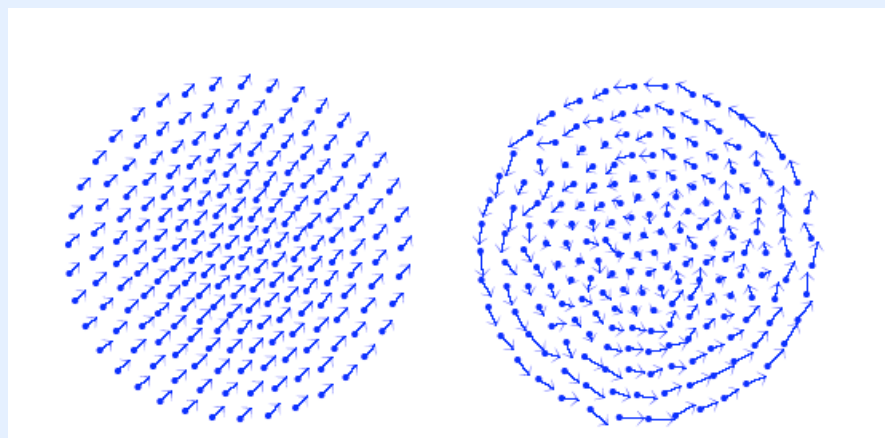
Small values α/β

Example 1

Different random
initial conditions,
speed

In both cases:
inter-particle spacing
constant

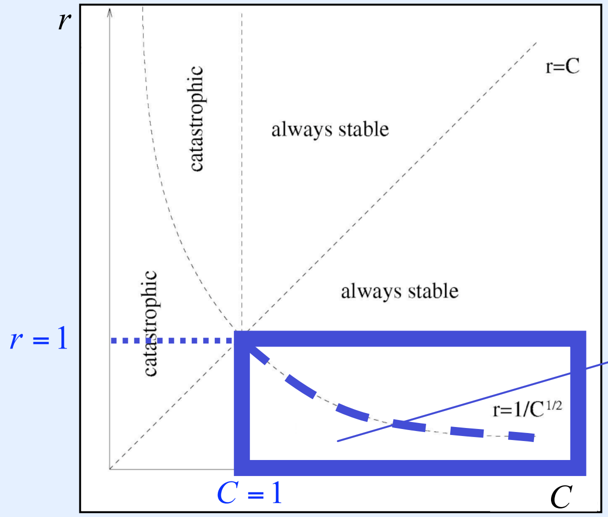
H-STABLE

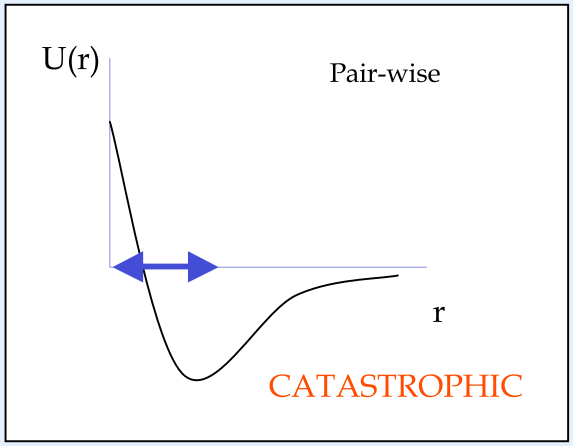


Flock
$v^2 = \alpha/\beta$

Rigid disk
Same as example 1
$v=\omega\, r$

Potential Features:



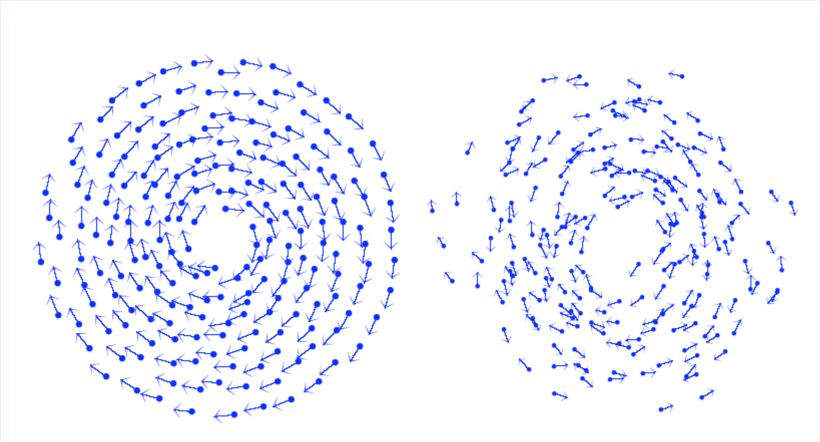


Optimal spacing

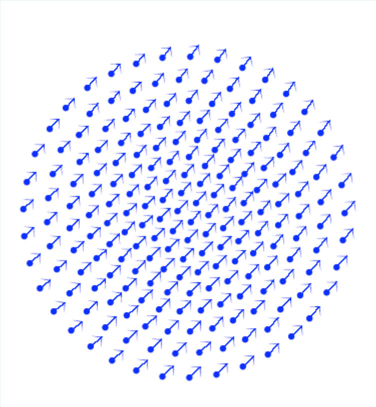
Collapse at large N

Larger values α/β

Example 2



Core free, same as example 2



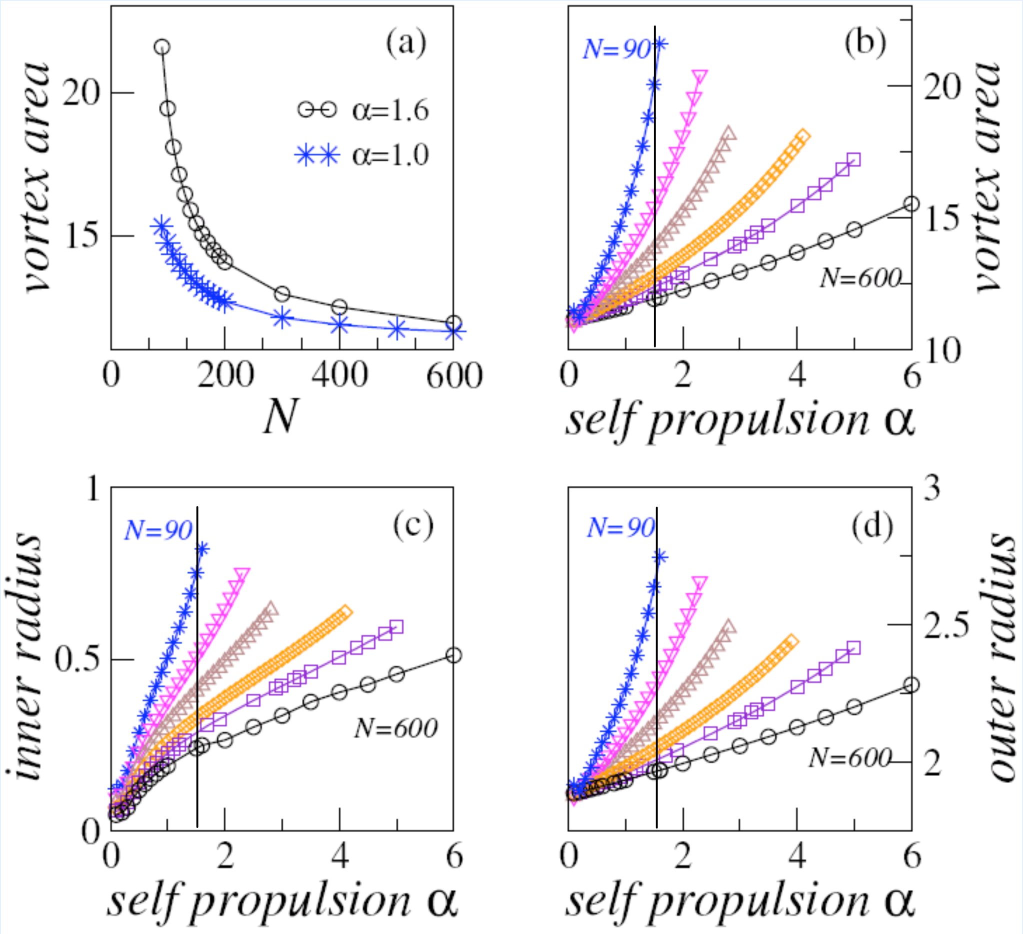
Flock

In both cases:

$v^2 \sim \alpha/\beta$

CATASTROPHIC

Catastrophic Vortices:



Area decreases with N!

Fly apart α increases with N:

Centrifugal force mv^2/r vs. interactions

$m\alpha/(\beta r)$ force vs. N-dense system

$\alpha_{max} \sim N$

β fixed, catastrophic vortex regime

Other potentials?

Lennard-Jones
Hard disks

Always stable

Power law divergences

$$U\left(\left|\vec{x}_i - \vec{x}_j\right|\right) = \frac{-C_a e^{\frac{-\left|\vec{x}_i - \vec{x}_j\right|}{l_a}} + C_r e^{\frac{-\left|\vec{x}_i - \vec{x}_j\right|}{l_r}}}{\left|\vec{x}_i - \vec{x}_j\right|^p}$$

$p >= 2$

Always stable

$p < 2$

Stable vs. catastrophic

Separatrix $\ell\ C^{(2-p)}=1$

Other potentials: Heynes J Phys C (2007)

Locusts:

From Disorder to Order in Marching Locusts - J. Buhl et al. Science 2006

Recent models from theoretical physics have predicted that mass-migrating animal groups may share group-level properties, irrespective of the type of animals in the group. One key prediction is that as the density of animals in the group increases, a rapid transition occurs from disordered movement of individuals within the group to highly aligned collective motion. Understanding such a transition is crucial to the control of mobile swarming insect pests such as the desert locust. We confirmed the prediction of a rapid transition from disordered to ordered movement and identified a critical density for the onset of coordinated marching in locust nymphs. We also demonstrated a dynamic instability in motion at densities typical of locusts in the field, in which groups can switch direction without external perturbation, potentially facilitating the rapid transfer of directional information.

How to go from discrete to continuum?

Irving Kirkwood:

$$f(R_1,\ldots,R_N,p_1,\ldots,p_N,t)$$

f: Probability distribution function in phase space

$$\int f\ \mathrm{d}R_1\ldots dR_N dp_1\ldots dp_N = 1$$

$$\frac{\partial f}{\partial t} = \sum_{k=1}^{N}\left[-\frac{p_k}{m_k}\cdot\nabla_{R_k} f + \nabla_{R_k} U\cdot\nabla_{p_k} f\right]$$

Hamiltonian equations of motion, U potential

Liouville equation for conserved system

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Hamiltonian equations of motion, U potential

Liouville equation for conserved system

$$\frac{\partial R_k}{\partial t} = v_k = \frac{p_k}{m_k};$$

$$\frac{\partial p_k}{\partial t} = F_k = -\nabla_{R_k} U$$

Irving Kirkwood:

$$f(R_1,\ldots,R_N,p_1,\ldots,p_N,t)$$

f: Probability distribution function in phase space

$$\int f\ \mathrm{d}R_1\ldots dR_N dp_1\ldots dp_N = 1$$

$$\frac{\partial f}{\partial t} = \sum_{k=1}^{N}\left[-\frac{p_k}{m_k}\cdot\nabla_{R_k} f + \nabla_{R_k} U\cdot\nabla_{p_k} f\right]$$

Hamiltonian equations of motion, U potential

Liouville equation for conserved system

$$a(R_1,\ldots,R_N,p_1,\ldots,p_N)$$

a: Dynamic variable

$$\left\langle a, f\right\rangle = \int a\, f\ dR_1\ldots dR_N dp_1\ldots dp_N$$

expectation value = Macroscopic value of a

Use Liouville equation to find dynamics of variable <a,f>

Hydrodynamics equations, JCP 1950

Irving Kirkwood 2:

$$\rho(r,t) = \sum_{k=1}^{N} m_k \left\langle \delta(R_k - r), f \right\rangle$$

MACROSCOPIC DENSITY

$$\rho(r,t)\, u(r,t) = \sum_{k=1}^{N} \left\langle p_k \delta(R_k - r), f \right\rangle$$

MEAN FIELD VELOCITY

$$E_k(r,t) = \sum_{k=1}^{N} \left\langle \frac{p_k^2}{2m_k} \delta(R_k - r), f \right\rangle$$

KINETIC ENERGY DENSITY

Continuity equation, momentum transport, energy transport

Non-Hamiltonian systems?

But: These Liouville equations are valid for conserved systems!

CAN PROVE existence of Liouville's equation for NON Hamiltonian systems

CAN generalize Irving Kirkwood continuum limit!

Our simple model becomes:

Continuum:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = \alpha \vec{v} - \beta |\vec{v}|^2 \vec{v} - \vec{\nabla} \int U(\vec{r} - \vec{r}\,')\, \rho(\vec{r}\,', t) dr'$$

Euler

Irving-Kirkwood

$$\rho(r,t) = \sum_{k=1}^{N} m_k \left\langle \delta(R_k - r), f \right\rangle$$

average in phase space

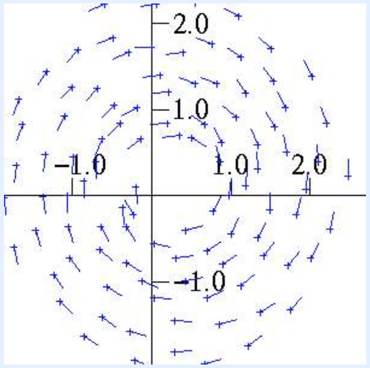
Continuum swarms

set rotational velocities

$$\vec{v} = \sqrt{\frac{\alpha}{\beta}}(-\sin\theta, \cos\theta)$$

$$\int_0^\infty \rho(R)\, U(r-R)\, dR = D - \frac{\alpha}{\beta}\ln r$$

Density implicitly defined

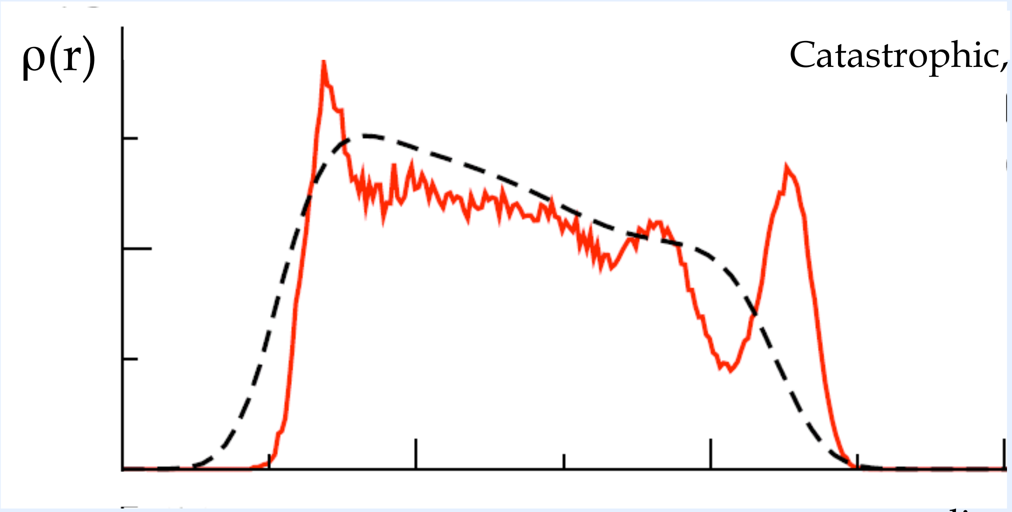


Constant speed,
Catastrophic, discrete

$\alpha = 1.0, \quad \beta = 0.5$

$C_a = 0.5, \; C_r = 1.0$

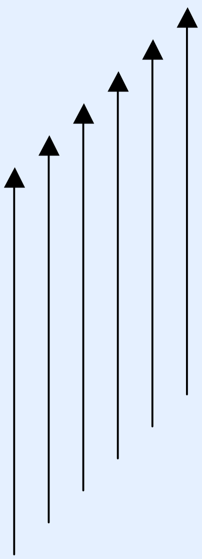
$l_a = 2.0, \quad l_r = 0.5$



radius

Continuum equations:

TRANSLATIONAL MOTION:



Uniform density,
velocity

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = \alpha \vec{v} - \beta |\vec{v}|^2 \vec{v} - \vec{\nabla} \int U(\vec{r} - \vec{r}\,')\, \rho(\vec{r}\,', t) dr'$$

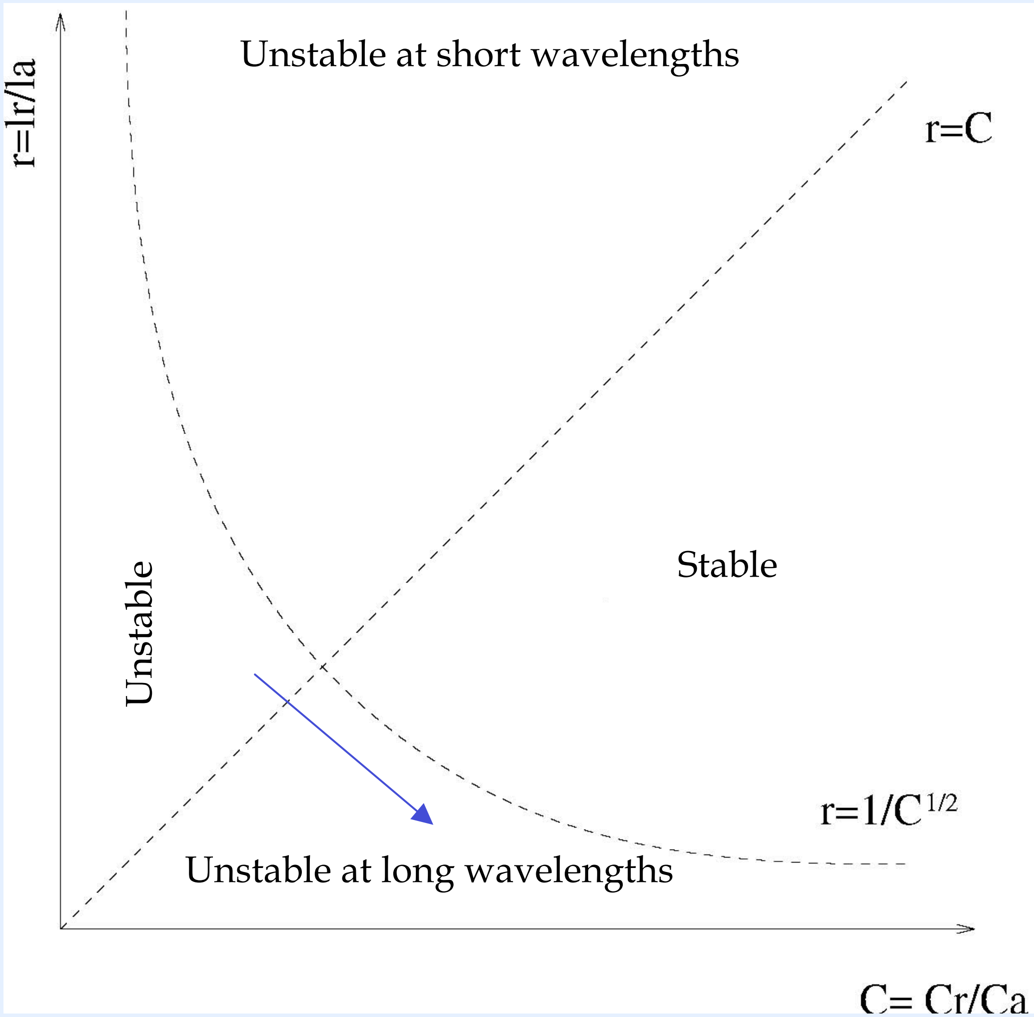
Linear stability analysis around

$$\vec{v} = \sqrt{\frac{\alpha}{\beta}} \hat{y}$$

$$\rho = \rho_0$$

$C_r, C_a, l_a, l_r \rightarrow$ Predict instabilities, most unstable wavelengths

Linear stability analysis

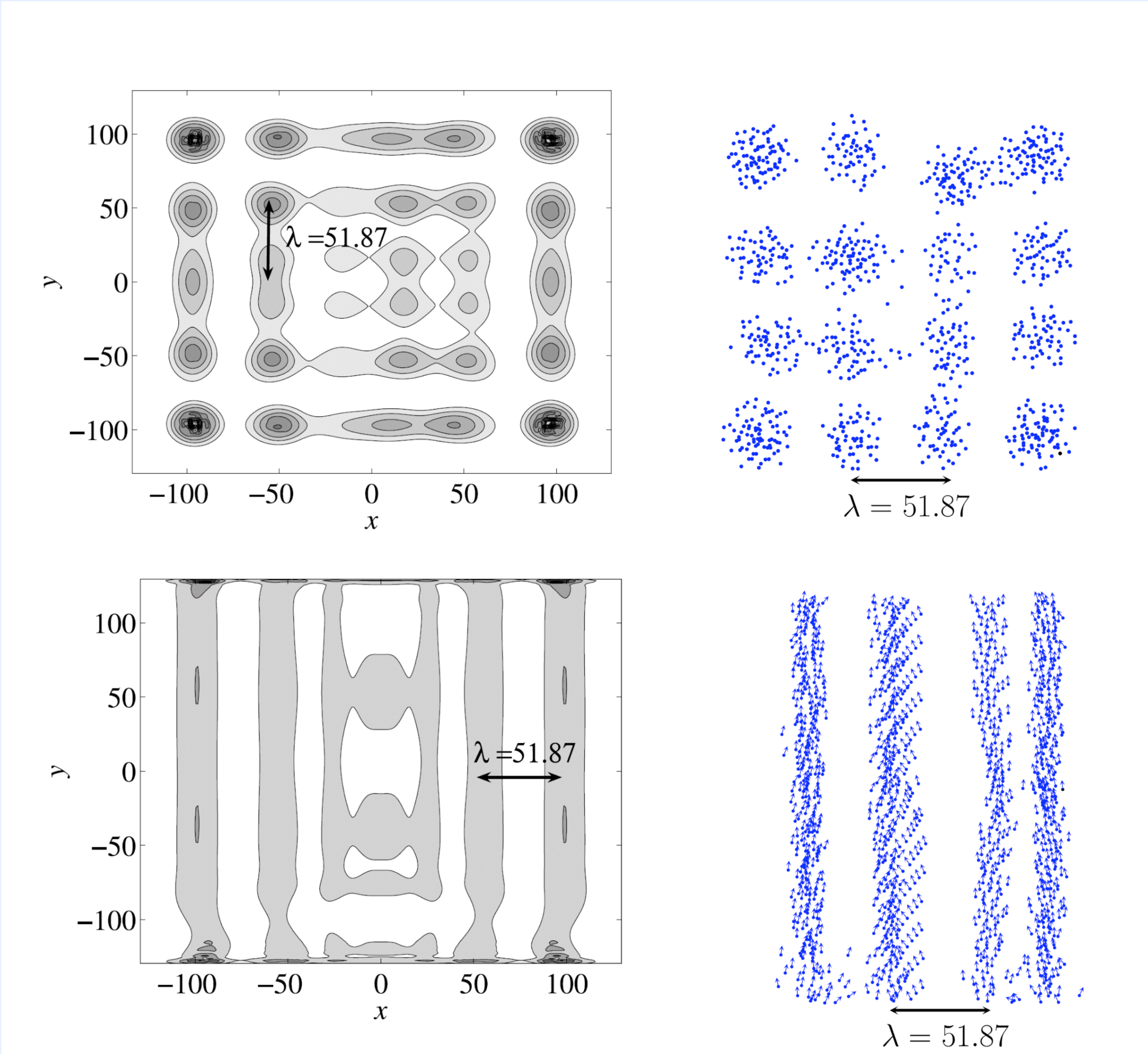


At t=0 translational motion

Patterns

α=0

α=1



Continuum

Discrete

$\beta = 0.5$

$C_a = 0.5, \; C_r = 1.0$

$l_a = 2.0, \quad l_r = 1.35$

At t=0
parallel velocities

LSA estimate:
largest λ = 51.87

Noise?

$$m_i \frac{\partial \vec{v}_i}{\partial t} = \left(\alpha - \beta \left|\vec{v}_i\right|^2\right)\vec{v}_i - \vec{\nabla}_i \sum_j U\left(\left|\vec{x}_i - \vec{x}_j\right|\right) + \eta(t)$$

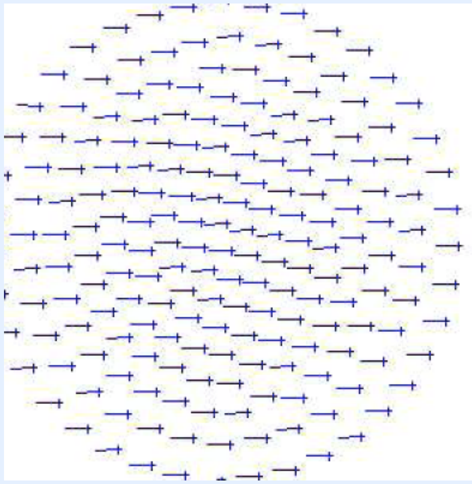
$$U\left(\left|\vec{x}_i - \vec{x}_j\right|\right) = -C_a e^{\frac{-\left|\vec{x}_i - \vec{x}_j\right|}{l_a}} + C_r e^{\frac{-\left|\vec{x}_i - \vec{x}_j\right|}{l_r}}$$

$$\boxed{\begin{gathered} \langle \eta(t)\, \eta(t') \rangle = \sigma^2\, \delta(t-t') \\ \langle \eta(t) \rangle = 0 \end{gathered}}$$

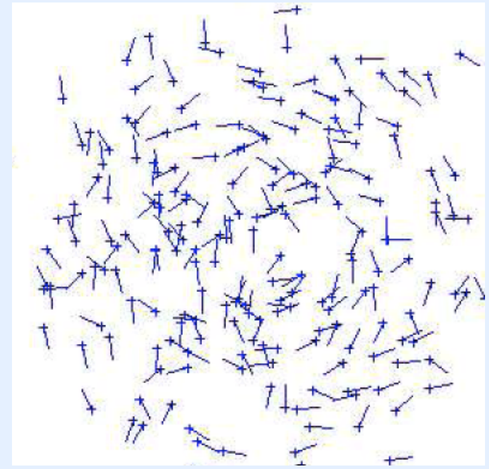
white noise

Transition from flock to vortex
For large noise values

Noise?



noise



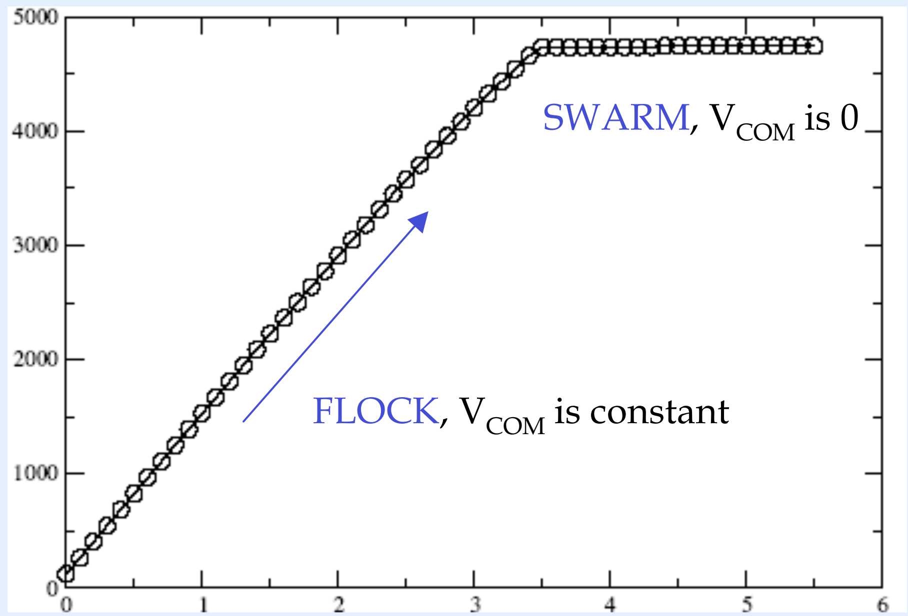
$\eta(t)=0$

COM is moving

$\eta(t) > \eta_{\text{critical}}$

COM is fixed

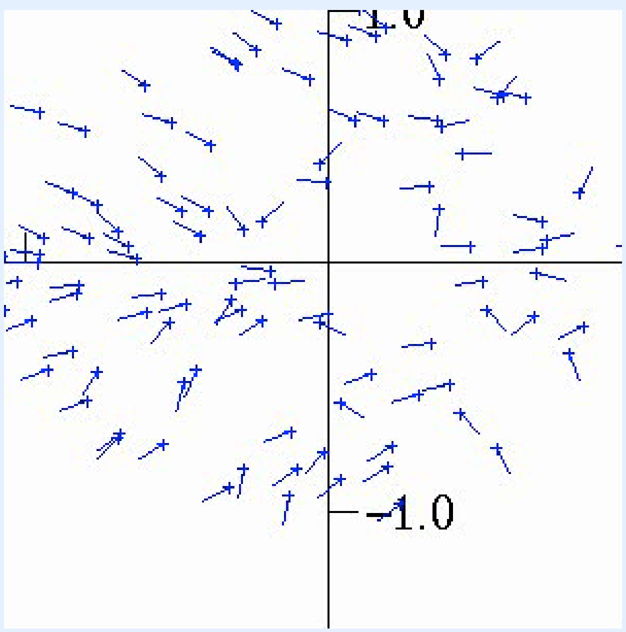
x of center of mass

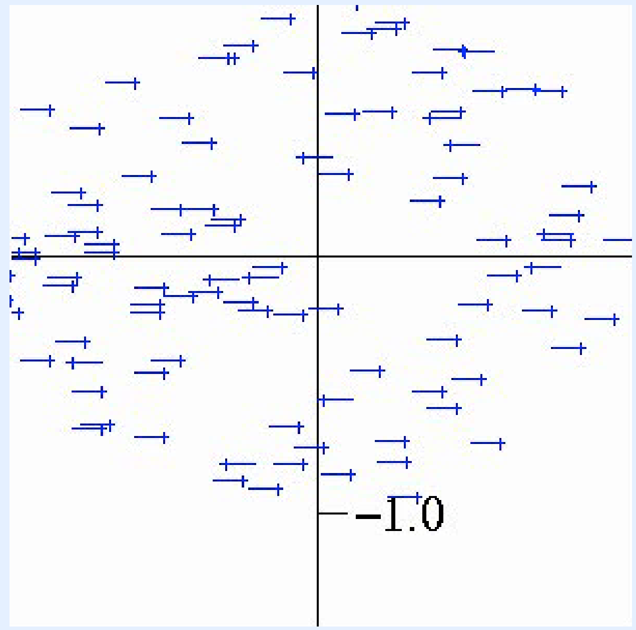


σ noise (regular time increases)

Magnetic field:

H < 0

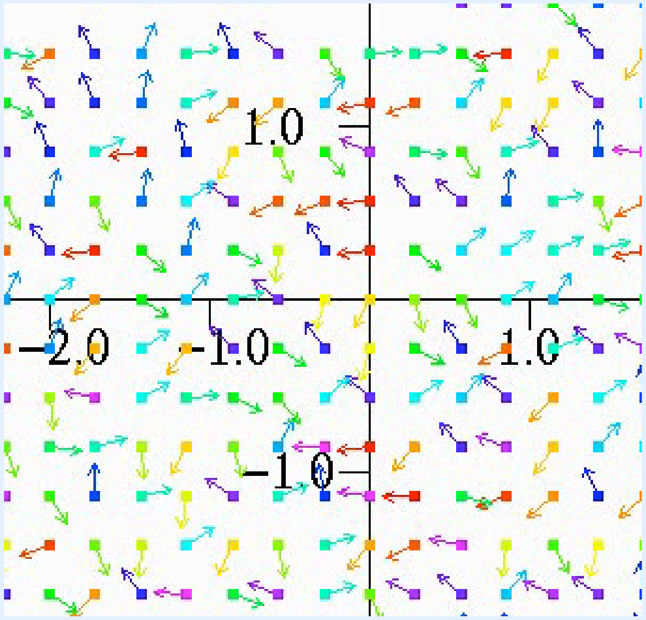




H > 0

$$m_i \frac{\partial \vec{v}_i}{\partial t} = \left(\alpha - \beta \left|\vec{v}_i\right|^2\right)\vec{v}_i - \vec{\nabla}_i \sum_j U\left(\left|\vec{x}_i - \vec{x}_j\right|\right) \boxed{+ v_i \times H}$$

Variable masses:



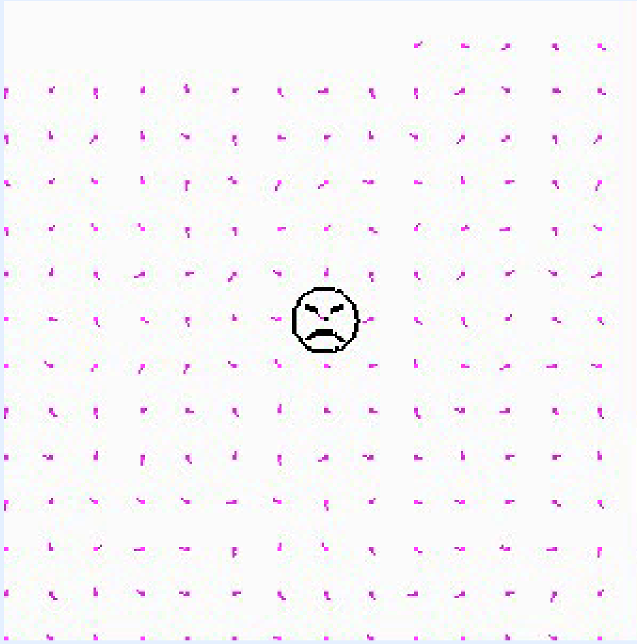
Variable masses
Vortex

Segregation

$m_i\ \alpha/\beta\ r_i$= Interactions

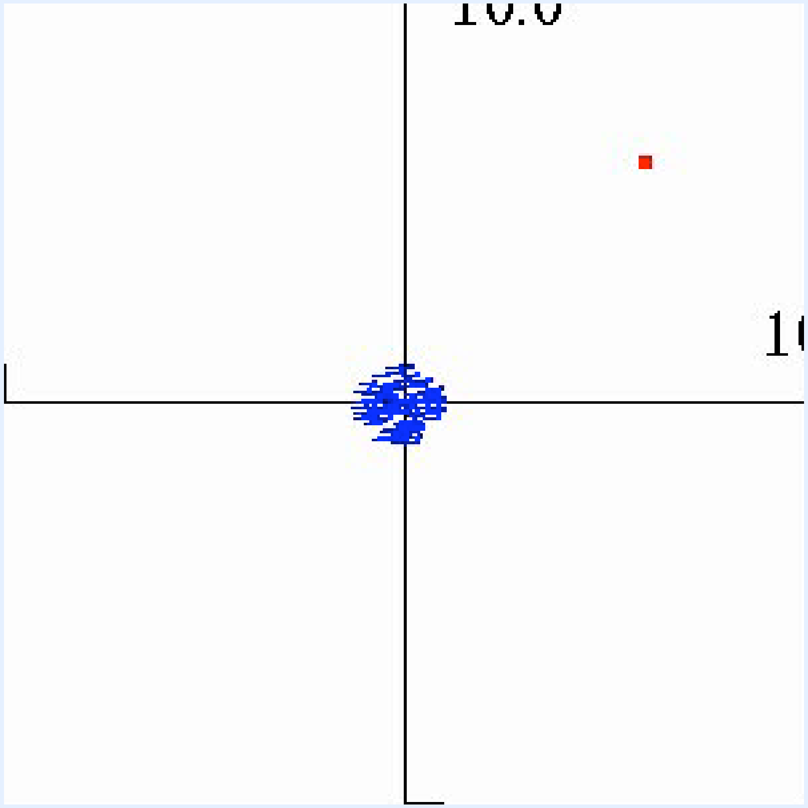
Same segregation behavior
for variable α_i-s

Site avoidance:



Split Patterns

Site convergence:



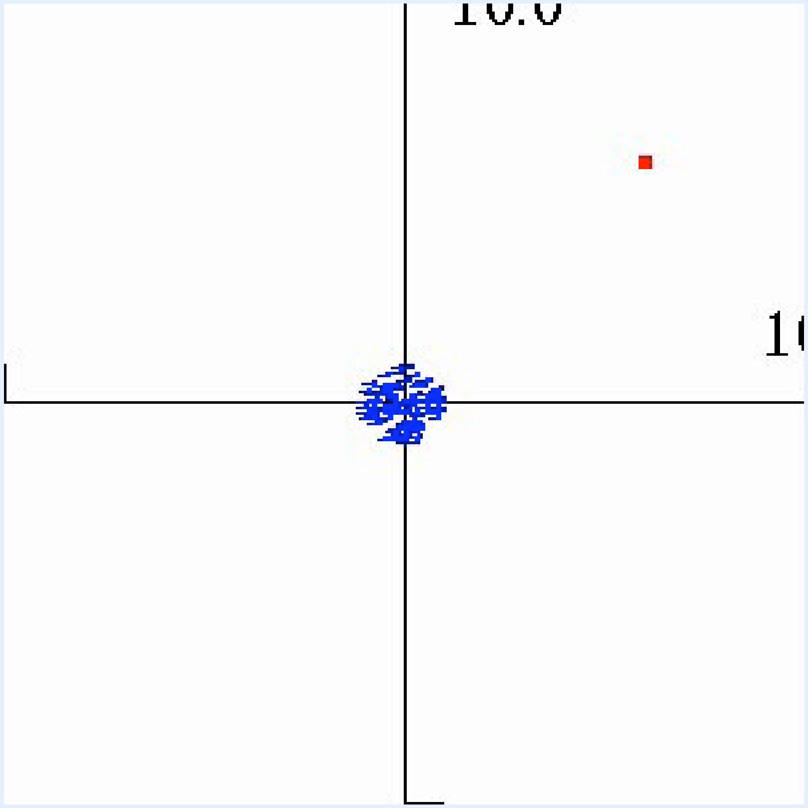
T=0 Flock

Medium attraction to target

Wait a little bit

T=T_{final} Swarm

Site convergence:



T=0 Flock

Medium attraction to target

When center of mass
is close to target

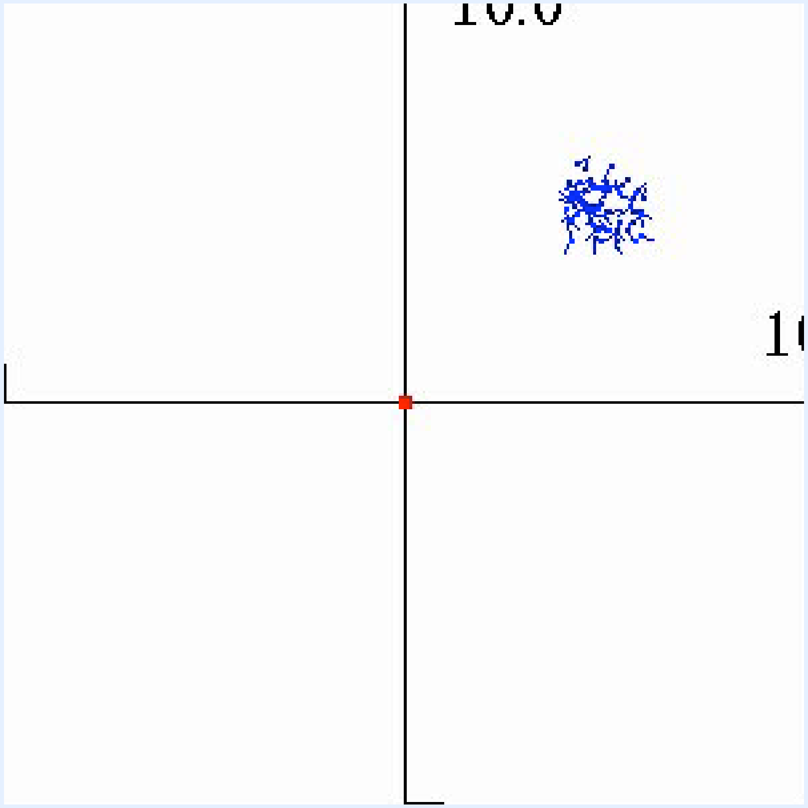
Turn on noise

Randomize

Turn off noise

$T=T_{final}$ Swarm

Chemotaxis:



Diffusing chemical at origin

Point Source, Decay

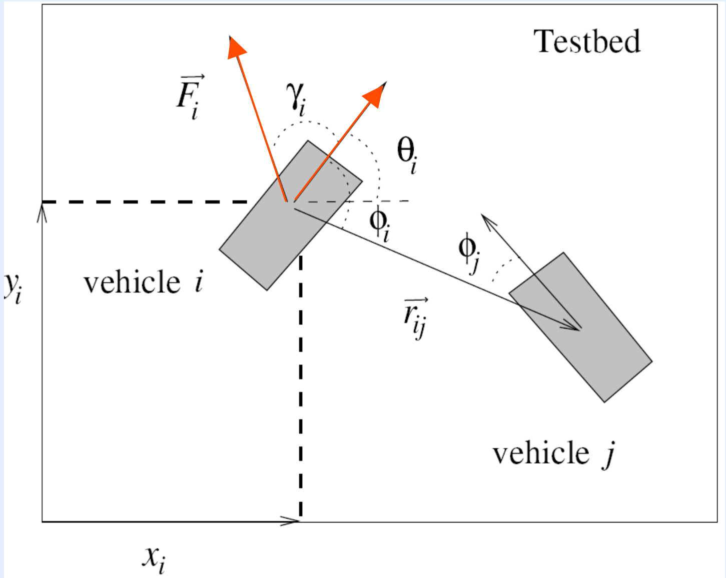
Particle gradient over length

Application to robots?

Cars have intrinsic speed v

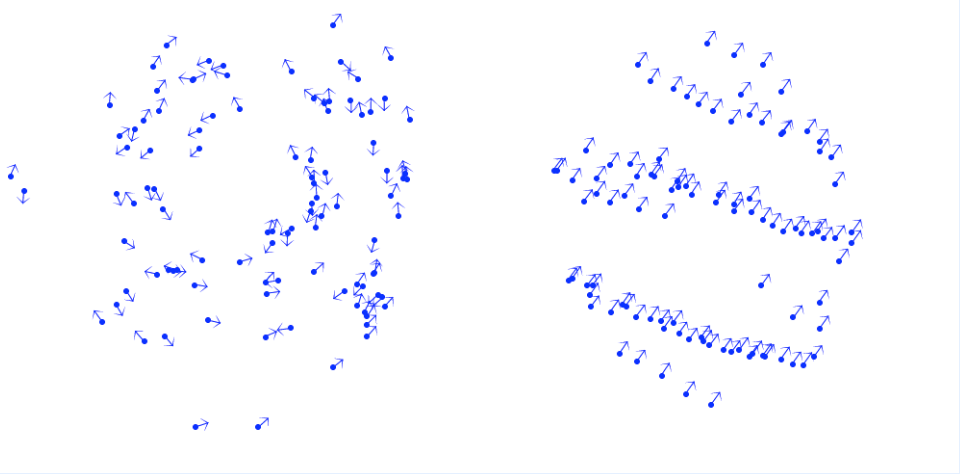
Pairwise potentials – Morse type

Steer in direction of the total force γ_ι



$$\dot{x}_i = v\cos\theta_i$$
$$\dot{y}_i = v\sin\theta_i$$

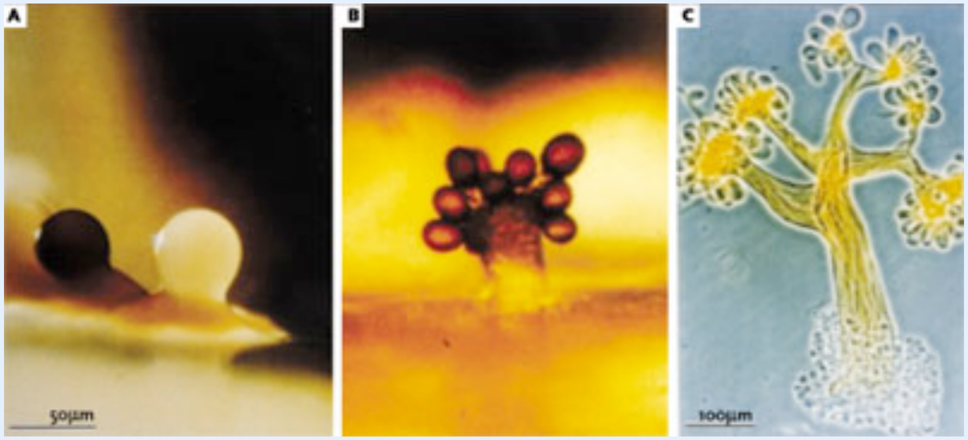
$$\dot{\theta}_i = \begin{cases} \omega & \text{if } \gamma_i > threshold \\ -\omega & \text{if } \gamma_i < -threshold \\ 0 & \text{otherwise} \end{cases}$$



R. Huang et al, ICRA 2007

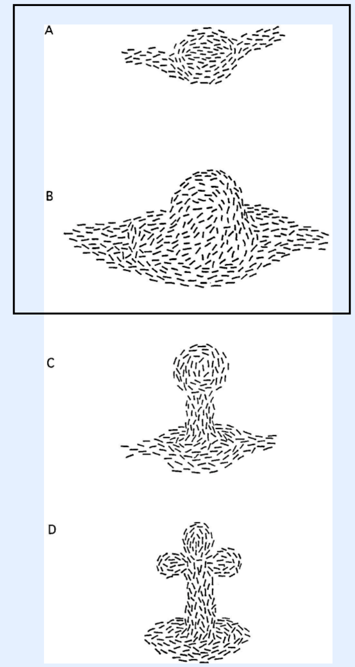
Mostly simulated vehicles

Applications to biology?



Myxococcus xanthus

Stigmatella aurantiaca

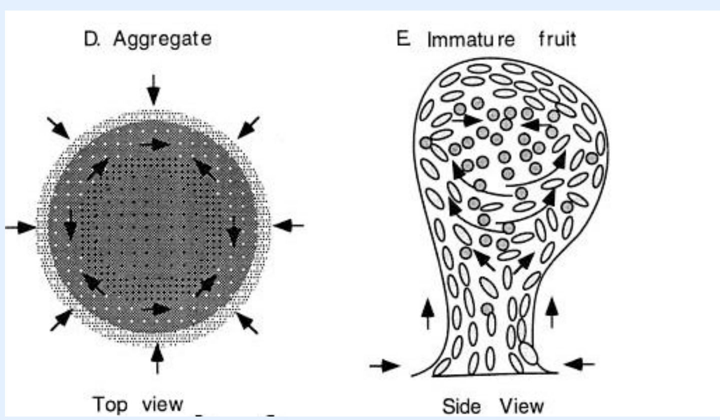


t

Under starving conditions the bacteria will aggregate

2D double spirals collapse into 3D aggregates

Direct interactions



Maybe!

Conclusions:

Potential determines stability of structures in large agent limit

H-stability

statistical mechanics – biology – device control

can apply to other potentials

can tune cross-over from stable-dispersive

to catastrophic-site convergent

natural systems: ‘movement ecology?’