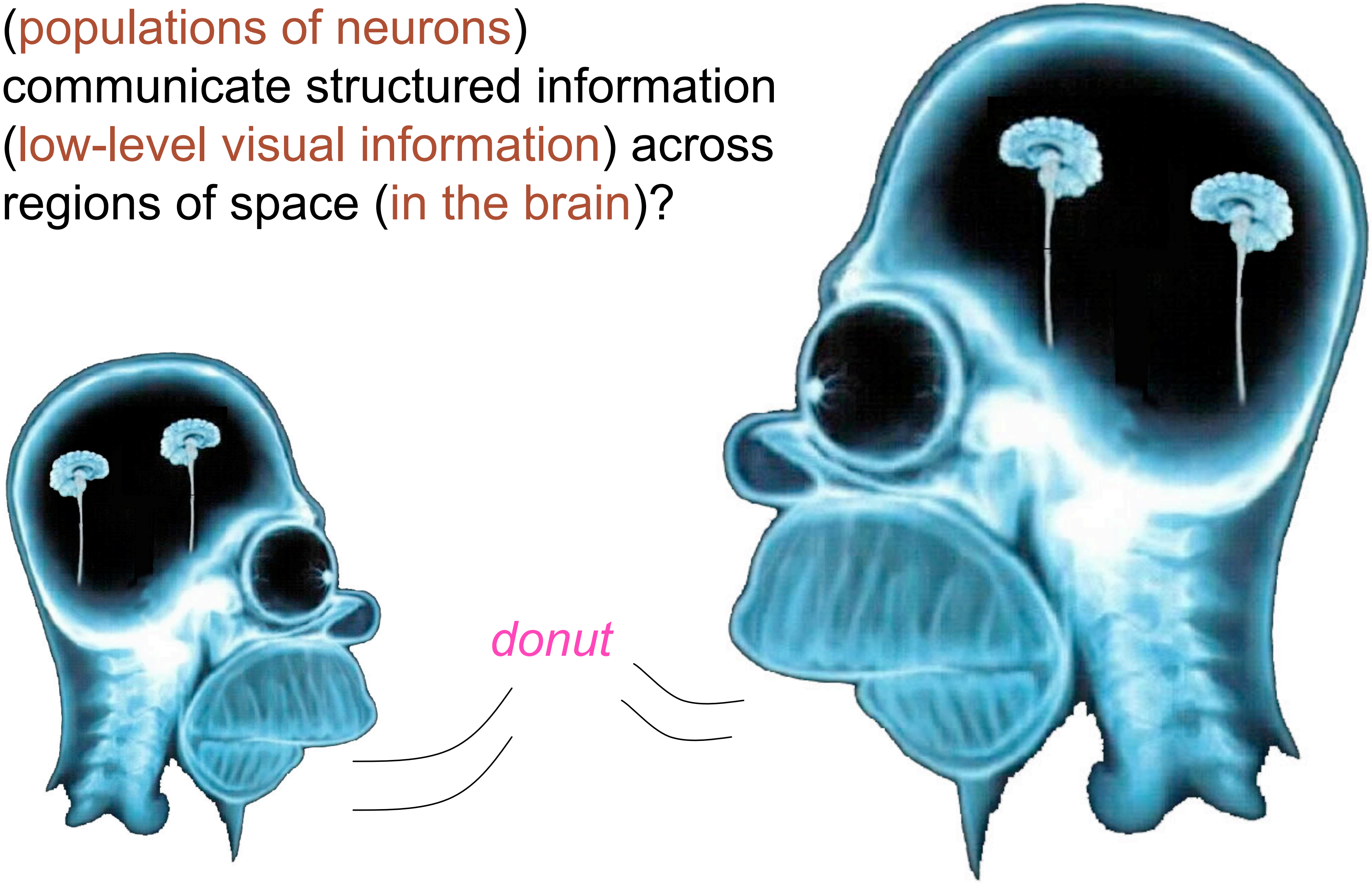
Adaptive Compressed Sensing

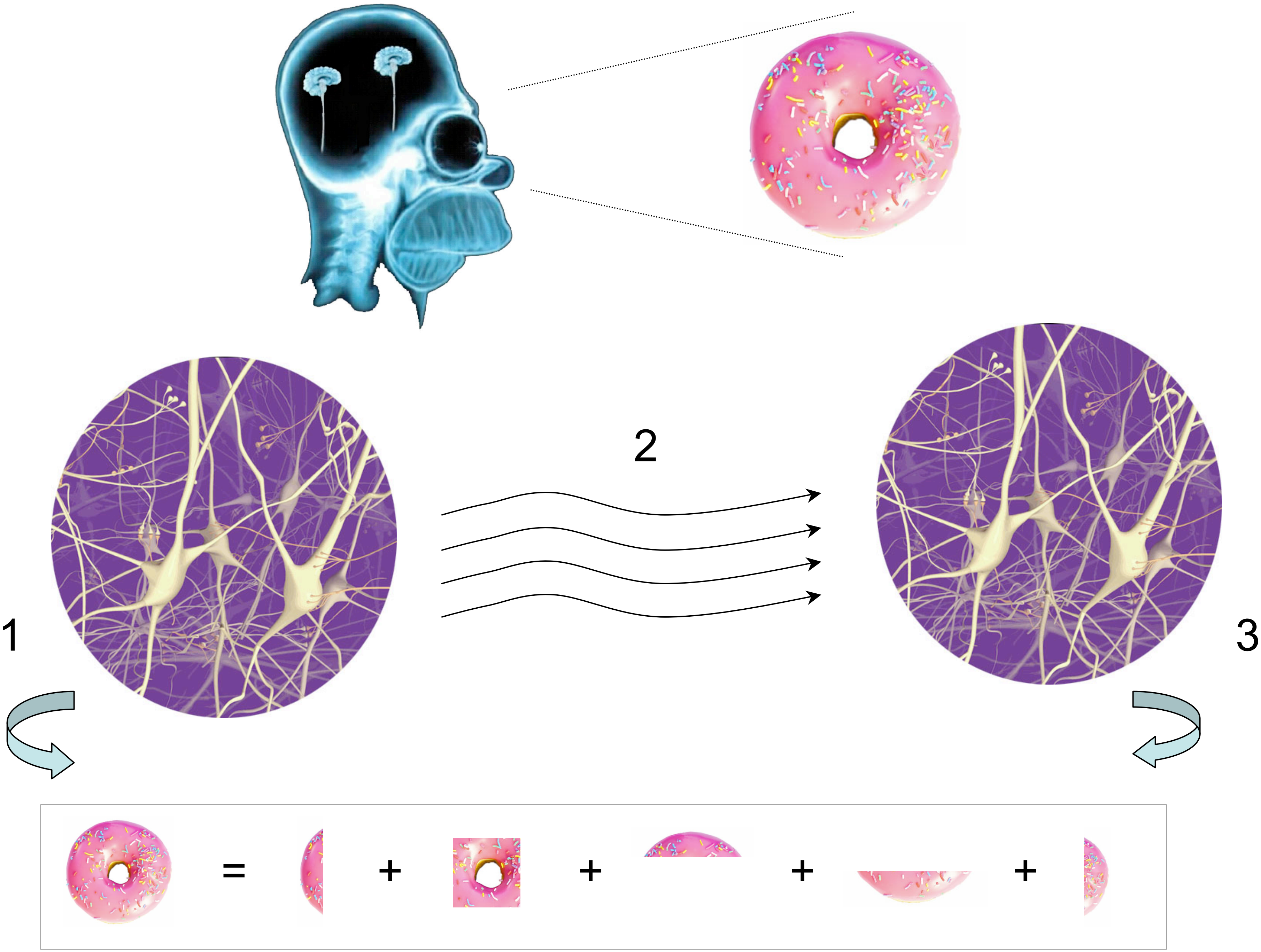
Chris Hillar

Redwood Center for Theoretical Neuroscience &
Mathematical Sciences Research Institute

(preliminary) work with W. Coulter, G. Isely, F. Sommer

Problem: How do agents (populations of neurons) communicate structured information (low-level visual information) across regions of space (in the brain)?





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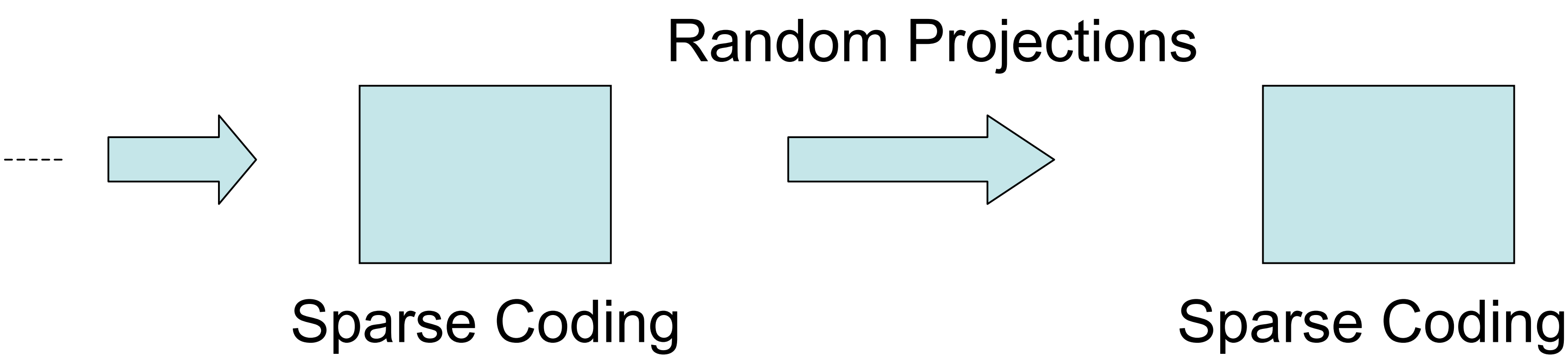
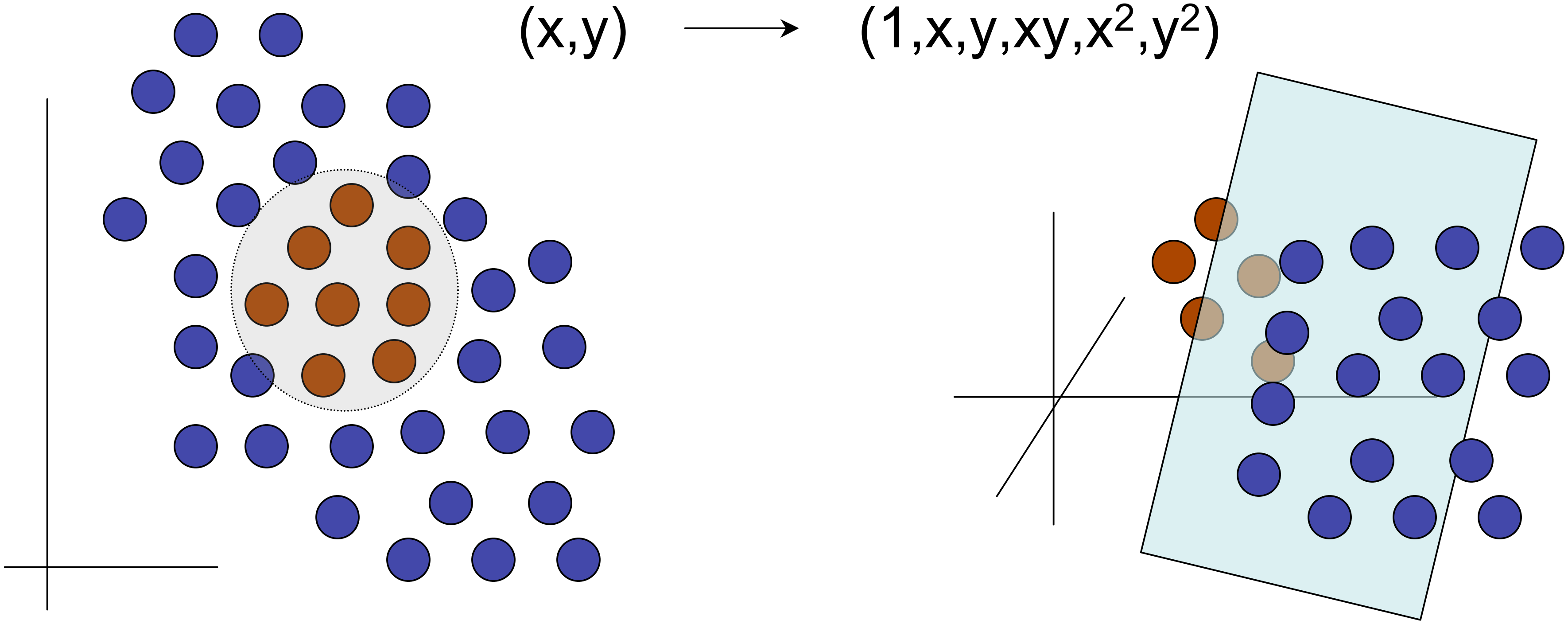
1. Population learning structure of signal
   - large number of neurons
2. Transmission & Compression
   - small bandwidth
3. Relearning structure
   - not knowing compression method



1. Population learning structure of signal

- large number of neurons

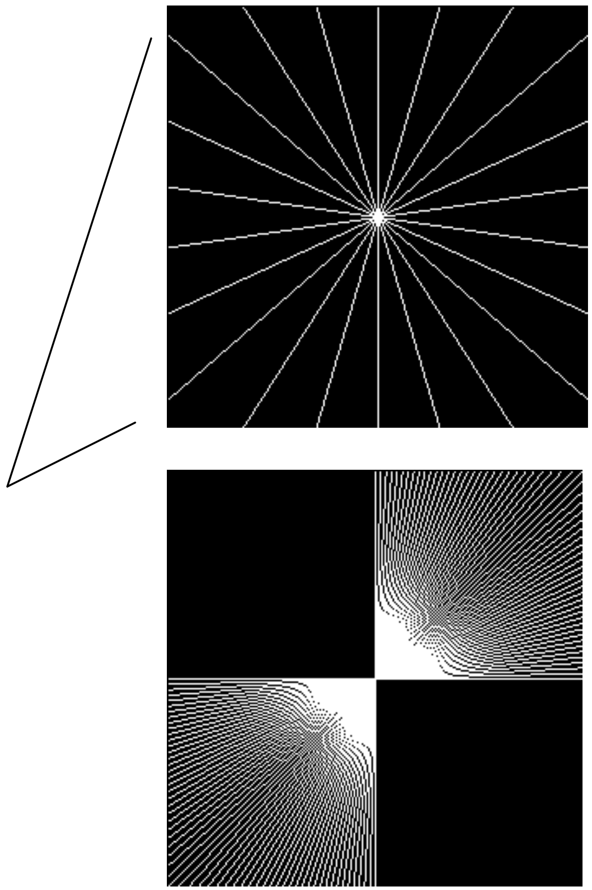


Question: How to discriminate blue / red in the plane?
Answer: (SVM) Map to high dimensional space, find separating hyperplanes! (ellipses are linear subspaces)

2. Transmission & Compression

- small bandwidth

sparse signal:
sum of small
number of causes

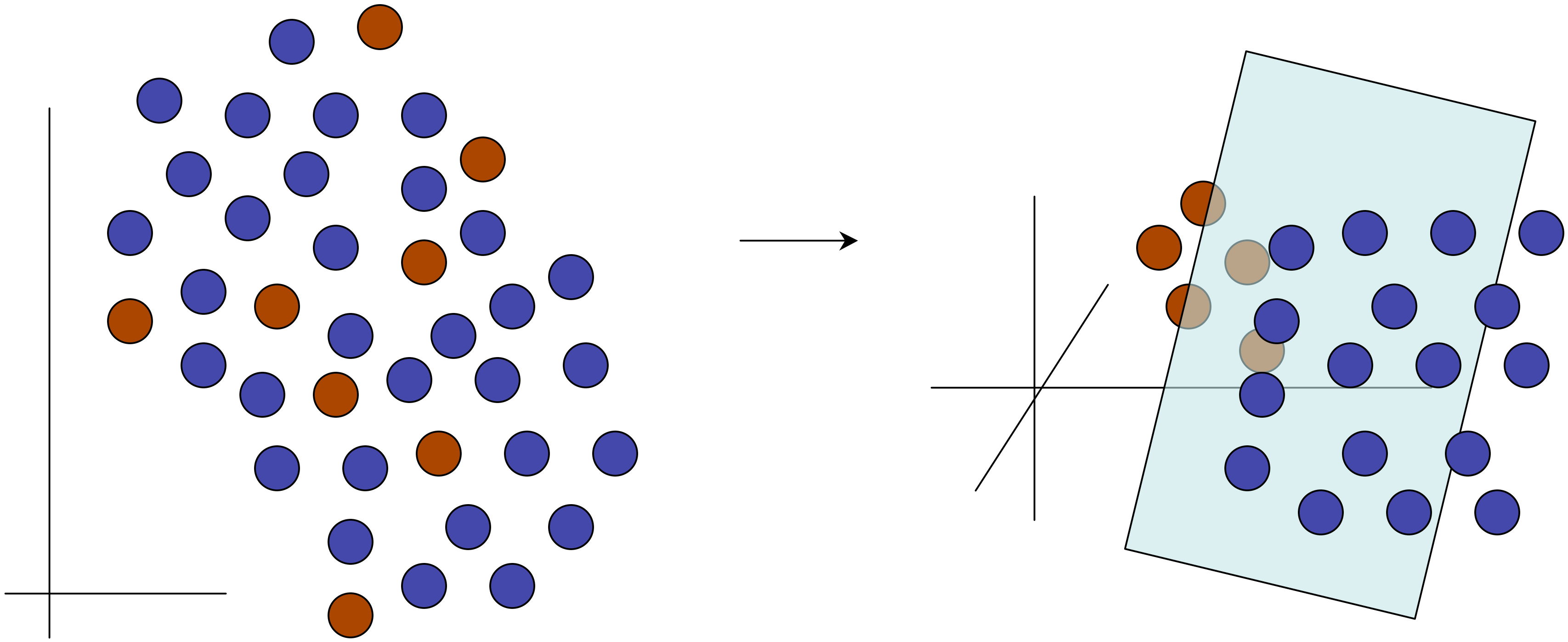


projections

Danielyan, et al ‘08
(MRI) Lustig, et al ‘07

3. Relearning structure

- not knowing compression method



Challenge: Tease out structure after (unknown) compression

Sparse coding of natural images

- Natural images have a structure that is sparse

$$x = \sum_{i=1}^{m} a_i \varphi_i + \nu$$

image patch coefficients features noise

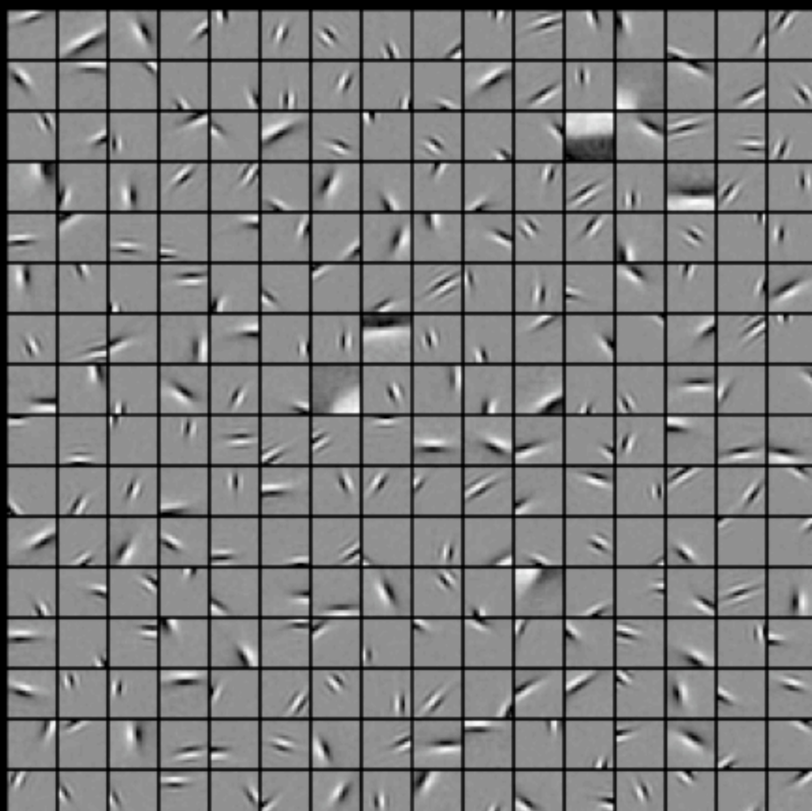
- Inferring the sparse representation is non-linear

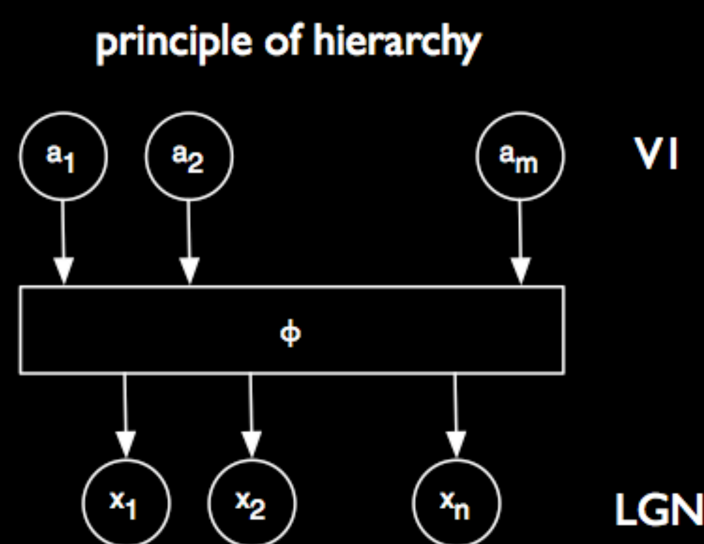
$$\min_a \frac{1}{2}\|x - \Phi a\|_2^2 + \lambda C(a)$$

reconstruction tradeoff sparsity

Learning basis functions

- Goal: learn the optimal set of basis functions such that images have a sparse representation
- Learned basis functions resemble receptive fields of neurons in primary visual cortex





(Olshausen & Field, Nature, '96)

Sparsity inducing penalties

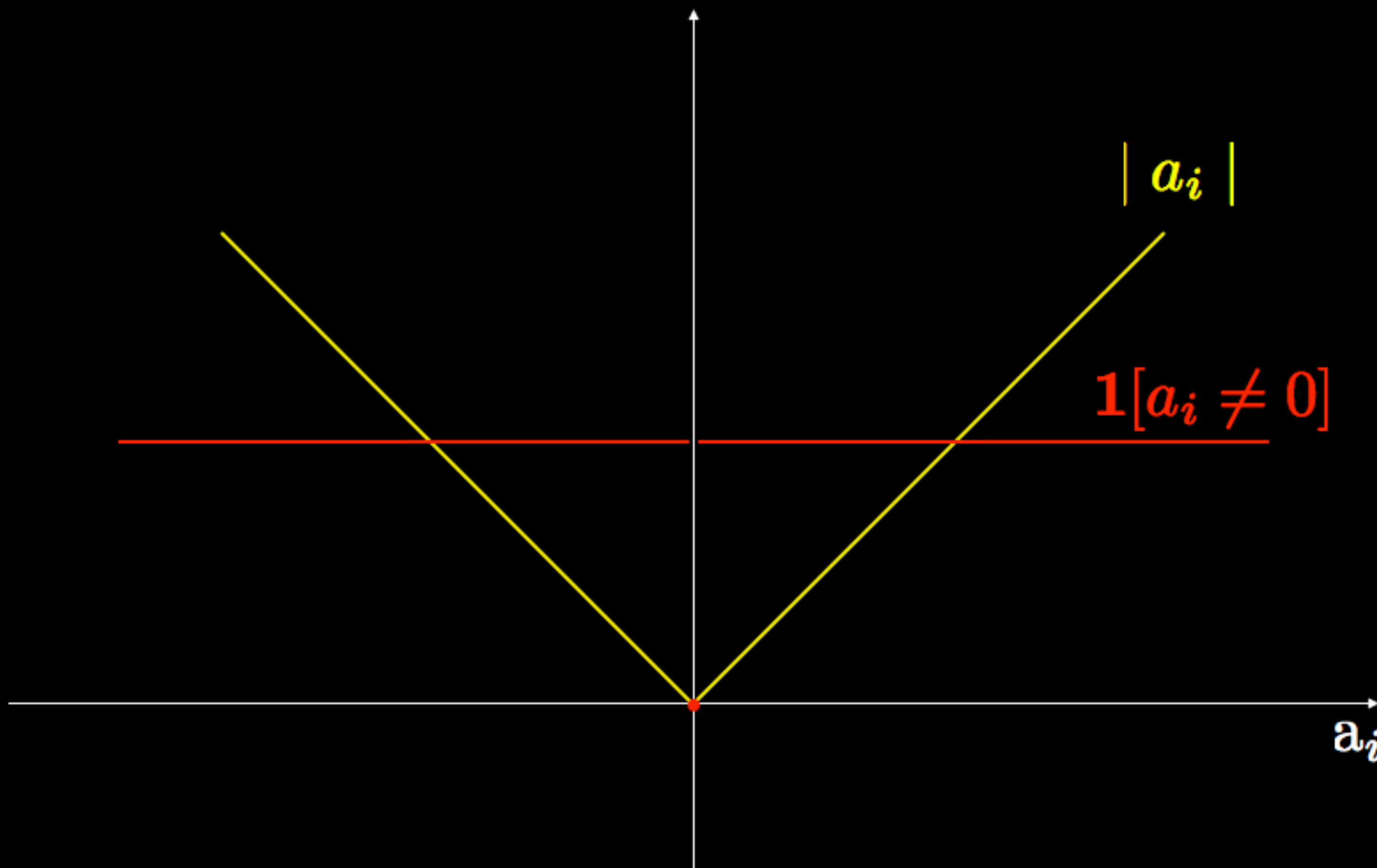
- The correct measure of sparsity is the 0-norm which is the number of non-zero coefficients
- Ideally we'd like to solve:

$$\min_a \underbrace{\tfrac{1}{2}\|x - \Phi a\|_2^2}_{\text{reconstruction}} + \lambda \underbrace{\|a\|_0}_{\text{sparsity}}$$

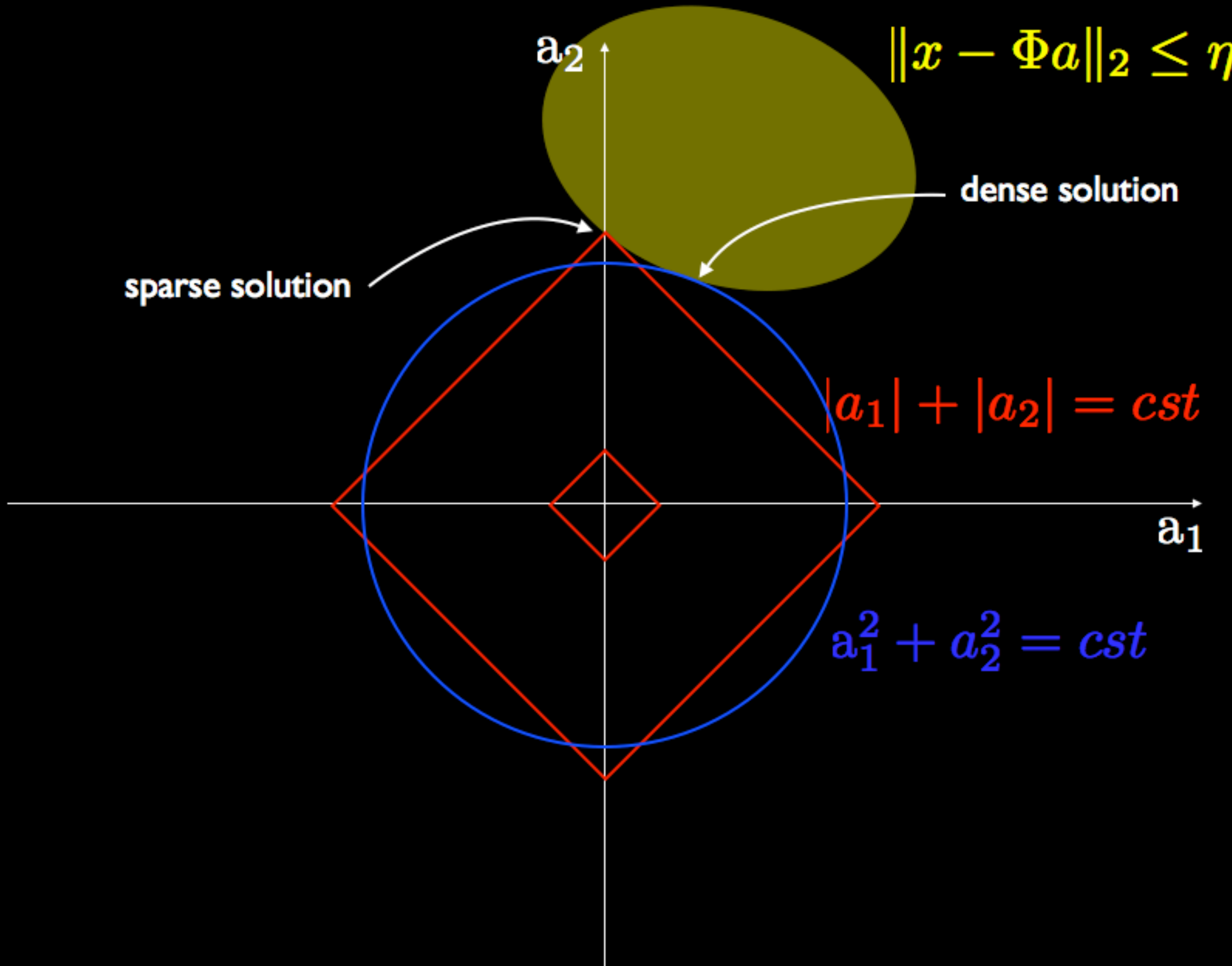
- This problem is NP-hard!
- However it has been shown that the problem with 1-norm is a good approximation

$$\min_a \tfrac{1}{2}\|x - \Phi a\|_2^2 + \lambda \underbrace{\|a\|_1}_{\text{convex!}}$$

Sparsity penalty



Geometrical intuition



Signal Processing

- Sparse coding ideas are useful when working with signals that have a sparse structure
- Examples of sparse signals
  - Natural images
  - Natural sounds
  - EEG data
  - geophysical data
  - hyperspectral data
  - biosensing (DNA microarrays)
  - Astronomical data

Compression of sparse signals

- We compress n-dimensional signals that are k-sparse
- Standard approach known as transform coding:

$$\min_a \frac{1}{2}\|x - \Phi a\|_2^2 + \lambda \|a\|_1$$

- Problems with this approach
  - full n samples of the signals are acquired (often k << n)
  - need to compute the coefficients
  - need to encode the location of the non-zero coefficients

An alternative: compressed sensing

$$\longrightarrow \boxed{\text{random measurements}} \xrightarrow[\text{m}]{y} \boxed{\text{decode}} \longrightarrow$$

$$\begin{aligned} y_1 &= x^T w_1 \\ y_2 &= x^T w_2 \\ &\vdots \\ y_m &= x^T w_m \end{aligned}$$

random vectors

$$y = Wx$$

$$m = O\left(k \log \frac{n}{k}\right)$$

number of such measurements

we know $x = \Phi a$ sparse

hence $y = W\Phi a$

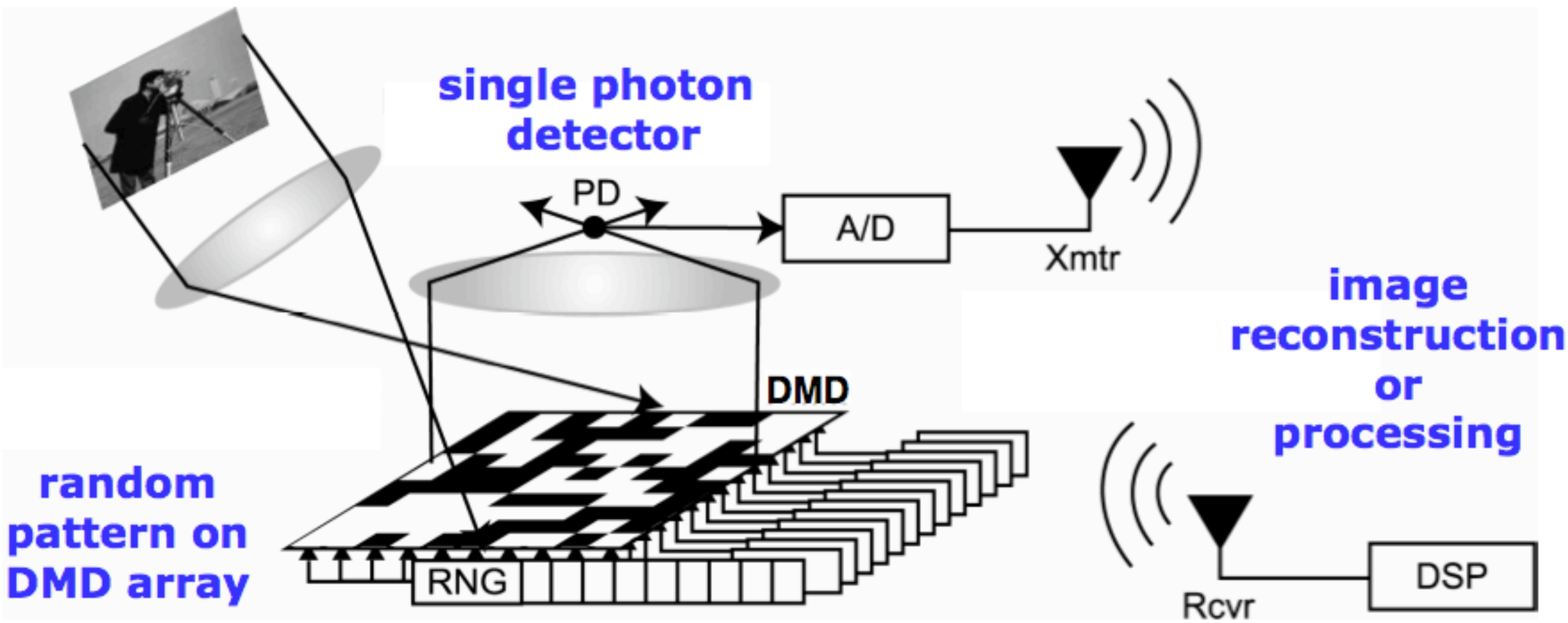
therefore we solve

$$\min_a \frac{1}{2}\|y - W\Phi a\|_2^2 + \lambda\|a\|_1$$

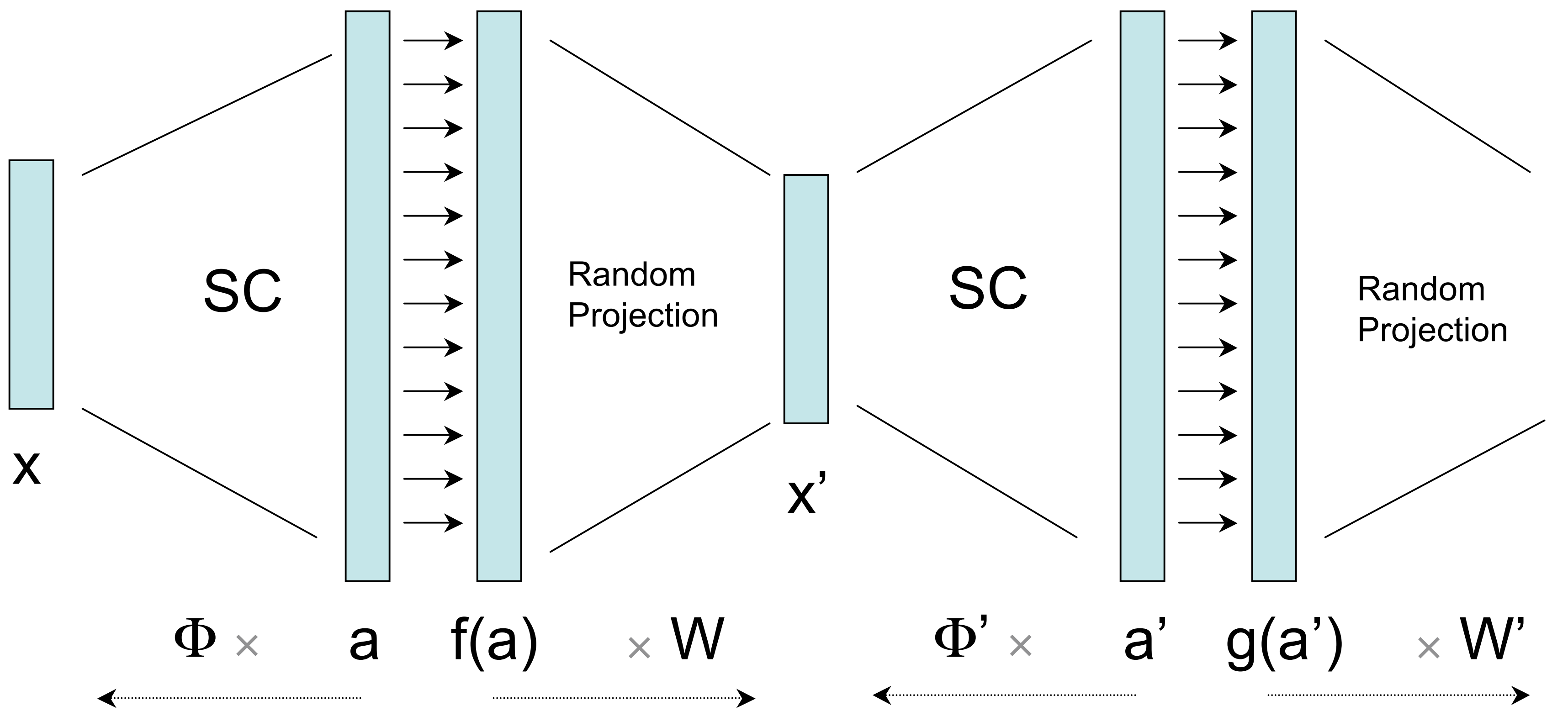
$$\hat{x} = \Phi a^*$$

(Candes, Donoho, Tao, etc..)

Rice Single-Pixel CS Camera



Communication between Bandwidth-limited Regions



Adaptive Compressed Sensing (ACS)

Receptive Fields

$$E(\mathbf{x}, \mathbf{a}, \mathbf{\Psi}) = \frac{1}{2}||\mathbf{x} - \mathbf{\Psi}\mathbf{a}||^2 + S(\mathbf{a})$$

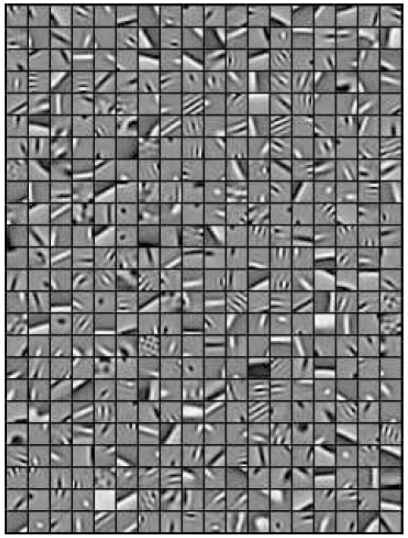
$$\mathbf{a}(\mathbf{x}) := \arg\min_{a} E(\mathbf{x}, \mathbf{a}, \mathbf{\Psi})$$

$$RF := \frac{1}{|I|}\sum_{\mathbf{x}\in I} \mathbf{x} \cdot \mathbf{a}(\mathbf{x})^{\top} \quad = \quad \text{Cov(x,a(x))}$$

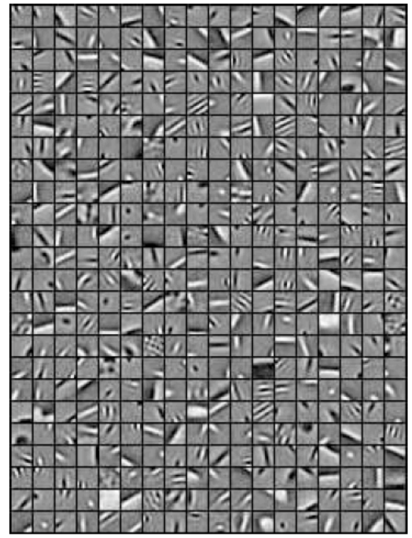
Receptive field of a neuron = What images it responds to

Learned ψ and Receptive Fields

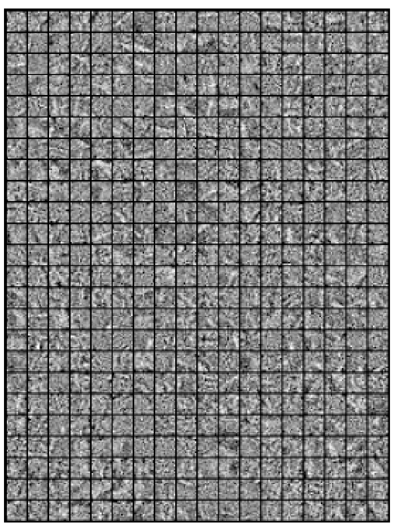
SC Ψ



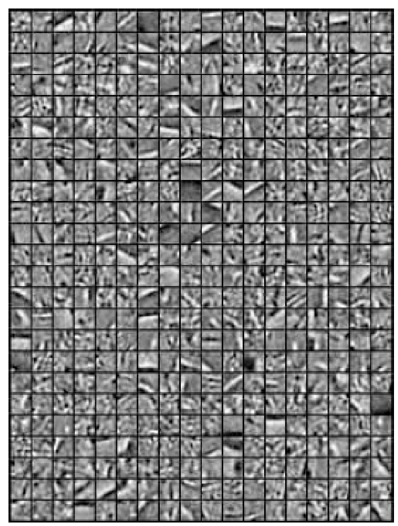
SC RF



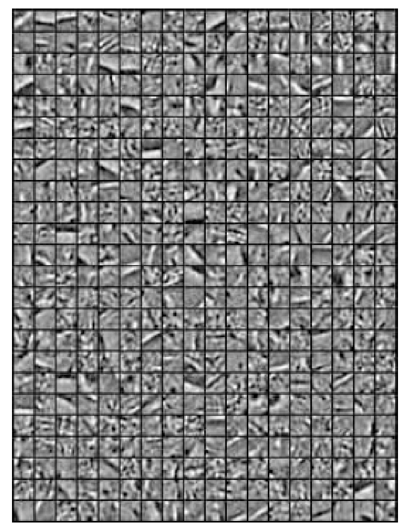
ACS FF



ACS RF



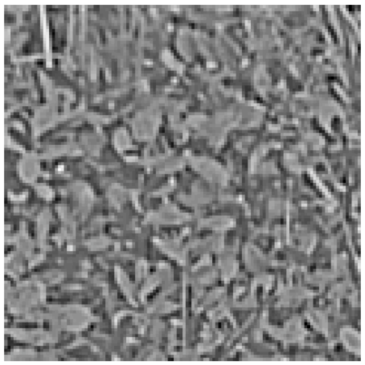
ACS (2^{nd} stage) RF



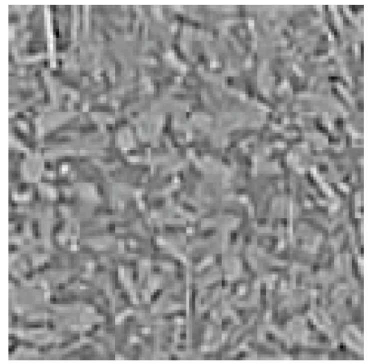
Reconstructions

Difficulty: how to verify if structure of signal passed on to second stage?

Original

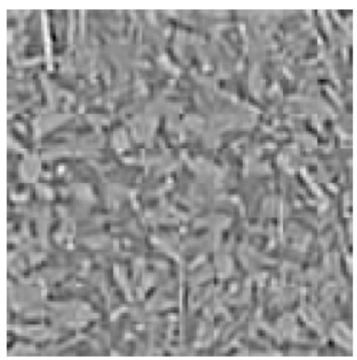


Sparse Coding

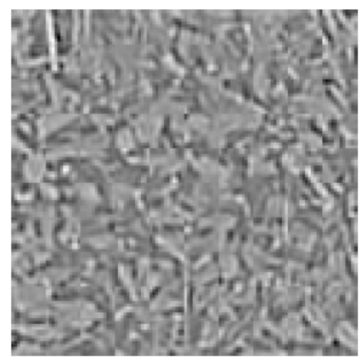


Using Learned ψ

ACS



ACS (2nd stage)



Using Receptive Fields

Thanks!