Agent-based modeling for animal migration and gang behavior

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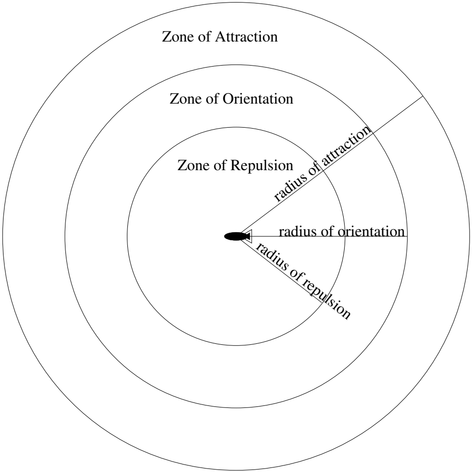
Interacting particle models and their associated challenges

- Interacting particle models: models with discrete particles which interact among themselves with a set of rules of interaction
- It is expensive to simulate a suitably large number of particles!
- Naively coded versions of most discrete interacting particle model scale as N^2, where N is the total number of fish
  - This limits the number of particles used in simulations
- Simulating many particles is important:
  - Local information becomes global information via local interaction with sufficiently many particles

Interactions in this model



Fish, Oceans, and the World!

Our model consists of three main classes [1]:

- The *Fish* class stores coordinate and velocity data for a particle,
- The *Ocean* class is meant to represent a single body of water,
- The *World* class is a bigger body of water composed of several connected oceans.

Each Fish stores an x and y coordinate for its location in the world. A Fish stores its velocity as the cosine and sine of its direction angle together with a non-negative speed. The Ocean class has a member variable "fish" which is an array of Fish living in that Ocean.

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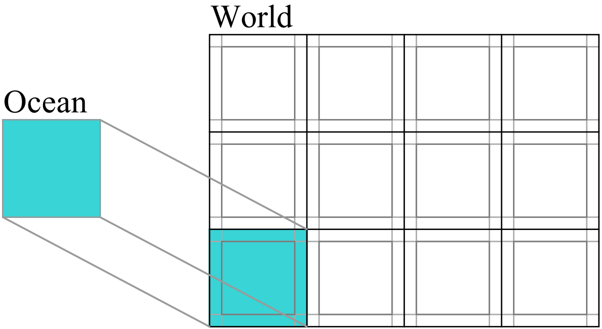


Figure: Oceans are connected in a rectangular grid which resides in a World.

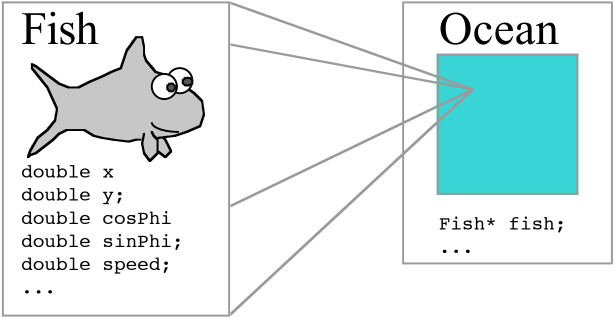


Figure: The Fish class stores coordinate and velocity data for a particle. The Ocean class keeps an array of Fish located in that ocean.

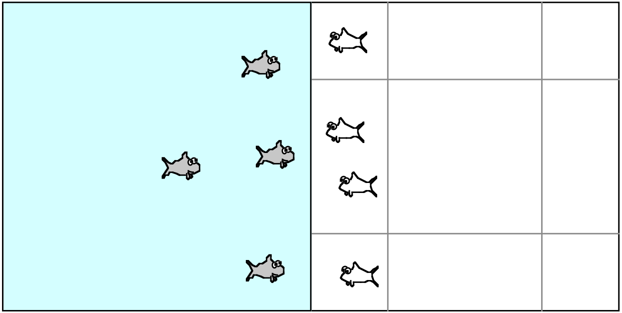


Figure: Fish in an ocean need to interact with sufficiently nearby fish on other oceans. We add fish near the boundary of a neighboring ocean as “ghost fish” to the current ocean.

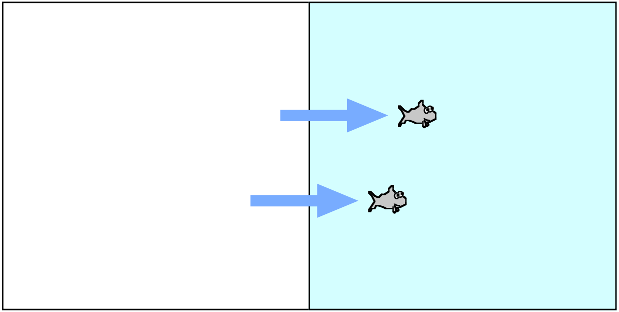


Figure: After the fish move, some fish might have crossed over into the domain of a neighboring ocean. When this happens, those fish need to get copied into that neighboring ocean and removed from the current ocean.

The problem of superindividuals

- In the real migrations which we are trying to accurately capture, it is safe to assume there are around $5 \cdot 10^{10}$ fish
- In our simulations, we use roughly $5 \cdot 10^4$ particles
- This means each particle represents 10^6 fish
- Each particle must therefore be thought of as a superindividual
- With these superindividuals, we captured the migration

The goal

- One fish per particle
- Then we could more confidently justify our behavioral rules, since they are based on data obtained from interactions among individual fish

So this leads to a question: how does the system change as we change the number of particles?

Assumptions

- We assume uniform density of particles and fish in the schools
- The interaction length of the particles should be much less than the size of the school
- We further assume the velocities of the particles are equal

Propagation of information through the school

- If sufficiently dense, local interactions between particles allows information to propagate through a school
  - Temperature information
  - Information about predators
  - Information about food
- Our scaling should preserve the speed at which this information propagates through the school

Relating time and spatial scales

- Let q_k denote the position and u_k denote the unit vector in the direction of the velocity of the k^{th} particle
- $\Delta q_k = \Delta t \cdot v \cdot u_k \Rightarrow \Delta q_k \propto \Delta t$
- To ensure particle k does not move beyond its range of vision in one timestep, we need a scaling between the radius of interaction, r, and the timestep
  - $r \propto \Delta t$
- We can think of Δq as defining a spatial scale

Constant density of actual fish

- In the actual migration, there are a given number of fish within a given area
- Each simulation needs to relate back to this real situation
- Schematic:
  - $\frac{fish}{region} = (\frac{particles}{interaction \cdot zone})(\frac{fish}{particle})(\frac{interaction \cdot zone}{region})$
- Let N denote the total number of particles in a simulation, F denote the number of fish in the migration, and A_w denote the total area of the region
- Let M denote the number of particles per interaction zone
  - Constant across interaction zones due to constant density assumption
  - For computational intensity, need M is constant across different simulations (so the number of neighbors for each particle remains constant)

Relating time and space to the number of particles

- Then for a given simulation indexed by i, $\frac{F}{A_w} = (M)(\frac{F}{N_i})(\frac{A_w}{\pi r_i^2}) \Rightarrow \frac{1}{A_w^2} = \frac{M}{(\pi r_i^2)N_i}$
- For two different simulations:
  - $\frac{M}{(\pi r_0^2)N_0} = \frac{M}{(\pi r_1^2)N_1} \Rightarrow (\frac{r_1}{r_0})^2 = \frac{N_0}{N_1}$
- $r_1 = r_0\sqrt{\frac{N_0}{N_1}}$
- Considering r_0 and N_0 to have come from a reference simulation:
  - $\Delta t \propto r \propto \sqrt{\frac{1}{N}}$

Our parameters for the migrations

- $\Delta t = 0.05$ days
- Initial speed $v_k \simeq 4 - 8$ km/day
- $r_r = 0.01$ or about ~ 120 m
- $r_o = r_a = 0.1$ or about ~ 1.2 km
- Number of particles is roughly $5 \cdot 10^4$

Scaling down to an individual level

How do the particles scale as we take N^s to 1? A rough estimate for the total number of fish in a migration is $F \simeq 5 \cdot 10^{10}$.

- $N_0 \simeq 5 \cdot 10^4$ and $N_1 \simeq 5 \cdot 10^{10}$
- $\frac{\Delta q_0}{\frac{1}{\sqrt{N_0}}} = \frac{\Delta q_1}{\frac{1}{\sqrt{N_1}}}$ and $\Delta q_0 \simeq 1.2$ km $\Rightarrow \Delta q_1 \simeq 1.2$ meters
- $\Delta t_0 = 0.05$ days and $\frac{\Delta t_0}{\Delta q_0} = \frac{\Delta t_1}{\Delta q_1} \Rightarrow \Delta t_1 = 4.32$ seconds
- Radii scale with Δq, so
    - $r_{r_0} \simeq 120$ meters $\Rightarrow r_{r_1} \simeq 12$cm
    - $r_{o_0} = r_{a_0} \simeq 1.2$ km $\Rightarrow r_{o_1} = r_{a_1} \simeq 1.2$m

These are all biologically reasonable!

Continuing work on the fish model

- DEB (as discussed by Birnir)
- Scaling
- Hydrodynamic description

Crime group at UCLA and UCI

Much work has been done, as seen this morning and in the poster session, including:

- Burglary model: hot spots
- Policing
- REU work on gangs: violence and networks

Routine activities

Modeling territoriality using gang graffiti

We will focus on mobile offenders and stationary “targets” in this territorial model:

- Gang members move around in space
- Gang members are *tagging* stationary points
- Other gang members move around based on the tag field

Background for territorial development

- Lewis, Murray. *Modelling territoriality and wolf-deer interactions*. Nature (1993) 366, 738–740)
- Lewis, White, Murray. *Analysis of a model for wolf territories*. J. Math. Bio. (1997) 35 : 749–774
- Moorcroft, Lewis, Crabtree. *Home range analysis using a mechanistic home range model*. Ecology (1999) 80(5), pp. 1656–1665
- Briscoe, Lewis, Parrish. *Home range formation in wolves due to scent marking*. Bulletin of Mathematical Biology (2002) 64, 261–284
- Moorcroft, Lewis, Crabtree. *Mechanistic home range models capture spatial patterns and dynamics of coyote territories in Yellowstone*. Proc. R. Soc. B (2006) 273, 1651–1659

Architecture of my code:

Code uses the object-oriented properties of C++. The objects are:

- LatticePoint: keeps track of things happening at the level of a grid node
- City: keeps track of things happening at the level of a city, e.g. connectivity of LatticePoints
- Environment: keeps track of info necessary for drawing the City and draws it

Original dynamics for territorial development

All rates are probabilistic

- Suppose two gangs, red and blue
- Agents are conserved
- Red gang puts down red tag, blue gang puts down blue tag
- Red gang moves preferentially away from blue tag, and blue gang moves preferentially away from red tag
- Tags decay in time

Coarsening in the original model

We found that in some parameter regimes:

- Uniform initial distribution
- Coarsening into red and blue territories!
- We would like to be able to prove that such coarsening occurs

Analyzing the emergence of such coarsening using a spin system

Reframe the problem in terms of a spin system:

- Spin at each lattice site consists of (η, g)
- $\eta \in \{-1, 0, 1\}$
- $g \in \mathbb{R}$

Think of something similar to the Ising model, but now with both agents and tags

The Hamiltonian

Consider

$$-\mathcal{H} = J \sum_{<i,j>} \eta_i g_j + K \sum_i \eta_i g_i + y \sum_i \eta_i^2 - \lambda \sum_i g_i^2$$

- Hamiltonian captures the essential dynamics of the original model
- Using this, we can prove similar dynamics to the original model

Proving a phase transition

- Peierls argument: does not work here
  - Unlike Ising model, no agent-to-agent coupling, which makes defining a “shoreline” difficult
- Reflection positivity does apply here
  - Basic idea: Use a checkerboard estimate to bound the probability of a “bad” pair of lattice points

A discussion of detailed balance

- The question of graffiti decay adds some extra interest to the problem
- The resolution of this question could be quite interesting

Gang violence project

We now consider mobile offenders and mobile targets

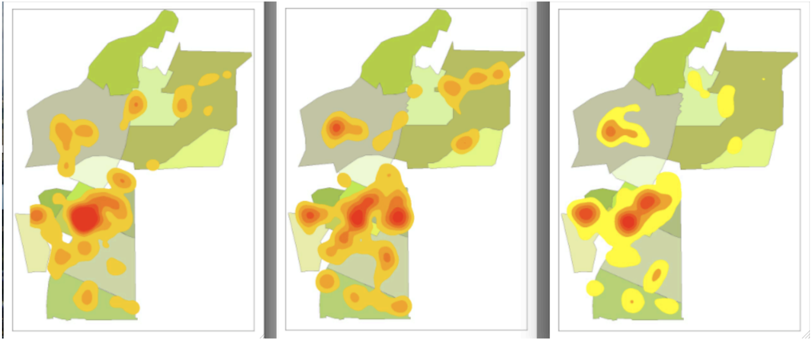
- Some work was done on this by REU students this summer, with promising results
- Would like to consider a model with rules directly from criminologists and gang specialists
- Eventual goal: better understand what causes gang violence

Motivation for an agent-based model in particular

Want to take a bottom-up approach

- More accessible to criminologists
- More flexible for exploring different behavioral rules
- Allows for implementation of more heuristic rules

The data



Anecdotal rules

- Each gang has a central gathering space, called a *set space*
- Gangs form dynamically evolving rivalries with one another
- After major gang violence, there is a period of low violence before retaliation
- Group size affects probability of fights
- Avoidance of certain established territories
- Most violence occurs outside own territory

Code architecture

Again making use of the object-oriented nature of C++

- The City contains an array of gang members
- Each gang member can have many attributes, such as:
  - Location
  - Speed
  - Aggressiveness
  - Status in the gang
  - Gang affiliation
- The City also has a rivalry matrix of intergang rivalry strengths
- The rivalry matrix is updated as gang violence occurs

What comes next

- Currently working with criminologists at UCI and the LAPD to determine reasonable rules for agent behavior
- Once we have a good idea of the set of possible motivations for both movement and gang violence, implement these in the model to see which ones are necessary to produce patterns of violence similar to data

Ongoing and future work on crime

- Finish verifying the phase transition in the spin system model for territorial development
- Further discuss reasonable behavioral rules for the gang violence project
- Compare to violence data
- Consider continuum Fokker-Planck-type description of such a model
- One possible extensions of the gang violence model to include rivalry network formation
- Once the model is working, possible to analyze how the rivalry networks influence the dynamics by altering the rivalries but not the behavioral rules

Thank you to the ONR for supporting this conference, and thanks very much to all the speakers and poster presenters!

Lamia Youseff, Alethea Barbaro, Peterson Trethewey, Björn Birnir, and John Gilbert.
Parallel modeling of fish interaction.
IEEE 11th International Conference on Computational Science and Engineering, pages 234–241, 2008.