Model-based production optimization and history matching – some (not so) recent developments

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Closed-loop reservoir management

Noise → Input → System (reservoir, wells & facilities) → Output → Noise

Controllable input → Optimization algorithms

Predicted output → Data assimilation algorithms → Measured output

System models

Sensors

Geology, seisms, well logs, well tests, fluid properties, etc.
1) “Robust” open-loop production optimization
12-well example (the “egg model”)

- 8 injectors, rate-controlled
- 4 producers, BHP-controlled
- Production period of 10 years
- 12 wells x 10 x 12 time steps
  
  => 1440 optimization parameters
- Bound constraints on controls
- Objective $J$: oil revenues minus water costs (‘NPV’)
- Forward model: fully implicit FV simulator (Dynamo MoReS, MRST)
- Optimizer: gradient- based (steepest ascent; line search with simple back tracking, gradients with adjoint formulation; projected constraints)
‘Robust’ optimization example (‘mean’ optimization)

- Number of realizations $N_r = 100$
- Optimize expectation of objective function $J$

$$\max_u \frac{1}{N_r} \sum_{i=1}^{N_r} J^i (u, m_i)$$

- $u$: inputs (well rates, pressures) for all optimization time steps
- $m$: parameters (permeabilities)

Van Essen et al., 2009
Robust optimization results

3 control strategies applied to set of 100 realizations:
reactive control, nominal optimization, robust optimization

Van Essen et al., 2009
Oil price uncertainty – time series

• Various complex models:
  – National Energy Modeling System (NEMS) (US DoE)
• We use: Auto-Regressive-Moving-Average model (ARMA) (Ljung, 1999)

\[ r_k = a_0 + \sum_{i=1}^{6} a_i r_{k-i} + \sum_{i=1}^{6} b_i e_{k-i} \]

• \( r_k \) = oil price
• \( e_k \) = white noise sequence
• \( a_0, a_i, b_i \) are constants
Oil price uncertainty – ensemble

- Base oil price 471 $/m³ = 75 $/bbl

Siraj et al. 2015

$n = 10$

$n = 100$
Mean optimization (MO)

\[ J_{MO} = \frac{1}{N_r} \sum_{i=1}^{N_r} J^i (u, m_i) \]
Mean-variance optimization (MVO)

\[ J_{\text{MVO}} = J_{\text{MO}} - \gamma J_{\text{V}} = \frac{1}{N_r} \sum_{i=1}^{N_r} J^i - \gamma \frac{1}{N_r - 1} \sum_{i=1}^{N_r} \left( J_{\text{MO}} - J^i \right)^2 \]

H. Markowitz (1952), Yeten et al. (2003), Bailey et al. (2005), Yasari et al. (2013), Capolei et al. (2015), Siraj et al. (2015), Liu and Reynolds (2016)

- Symmetric ‘risk measure’
- Penalizes the best cases
- Decision makers are mainly concerned with worst cases
Worst-case optimization (WCO)

\[
\max \min J(u, m_i) \quad \forall i
\]

- Min operator on discrete set is non-differentiable
- Reformulate with slack variable \( z \)

\[
\max z \quad \text{s.t.} \quad z \leq J(u, m_i) \quad \forall i
\]

- \( N_r \) inequality constraints
- Asymmetric ‘risk measure’
- Sensitive to outliers
- Usually very conservative
Optimizer KNITRO

- Large-scale non-linear constrained optimization
- Both interior-point (barrier) and active-set methods;
- Programmatic interfaces: C/C++, Fortran, Java, Python;
- Modeling language interfaces: AMPL ©, AIMMS ©, GAMS ©, MATLAB ©, MPL ©, Microsoft Excel Premium Solver ©;
Worst-case optimization (WCO) (geology)

- Worst-case increase: 3.60 %
- Average decrease: 1.54 %
MO, MVO and WCO (geology)

- MVO and WCO all reduce upside
MO, MVO and WCO (oil price)

- Note: WCO = single optimization with lowest oil price
- Same story: MVO and WCO all reduce upside
Mean worst-case optimization (MWCO)

\[ J_{\text{WCO}} = \max_u \min_{m_i} J(u, m_i) \]

- \( J_{\text{WCO}} \) is usually very conservative
- Can be controlled ad-hoc with weighted formulation:

\[ J_{\text{MWCO}} = J_{\text{MO}} - \lambda J_{\text{WCO}} \]

- Will not be pursued any further
Conditional value at risk (CVaR)

• Value at risk (VaR):

\[ \alpha_\beta (x) = \min \{ z \mid F_x(z) \leq \beta \} \]

• \( x \) is a random variable

• \( F_x(z) \) is the cdf \( P(x \leq z) \)

• \( \beta \in [0, 1[ \) is the confidence level

• In words: \( \beta \) fraction of objective function distribution

• Conditional Value at Risk (CVaR):

\[ \varphi_\beta (x) = E \{ x \mid x \leq \alpha_\beta \} \]
Worst case, VaR, and CVaR

![Diagram showing worst case, CVaR, and VaR under curve.](image-url)
Semi variance

\[ \text{Var}_+ (x) = E \{ \max \left[ x - E(x), 0 \right] \}^2 \]

\[ \text{Var}_- (x) = E \{ \max \left[ E(x) - x, 0 \right] \}^2 \]
MCVaR (geology)

\[ J_{MCVaR} = J_{MO} - \omega J_{VaR} \]

- Computationally tedious (integration)
MCVaR (oil price)

\[ J_{\text{MCVaR}} = J_{\text{MO}} - \omega J_{\text{VaR}} \]

- Not convincingly successful
Conclusions ‘risk measures’

• MVO (symmetric) leads to strong reduction in upside
• Asymmetric risk measures (WCO, CVaR, SV and their ‘mean’ varieties) improve the situation somewhat
• MCVaR seems to perform best, but is computationally demanding and requires choice of weighting parameter
• Improvements under oil price uncertainty lower than expected
• Joint geological - oil price scenarios not yet tested
2) Computer-assisted history matching

System (reservoir, wells & facilities)

Noise Input Output Noise

Controllable input

Optimization algorithms

Sensors

System models

Data assimilation algorithms

Geology, seismics, well logs, well tests, fluid properties, etc.

Predicted output Measured output
Upper/lower economic bounds

Idea:
• Explicitly search for HM-models that provide upper and lower bounds of economic forecasts (for a given production strategy)
• Proposed solution: hierarchical optimization
• Motivation: after obtaining a history match there is still a lot of room in the parameter space to optimize a second objective

• Van Essen et al., *Computational Geosciences* (2016); ECMOR (2010)
Hierarchical optimization

• Order objectives according to importance
  1. Good history-match ($V$)
  2. Maximize/minimize (economic) forecasts ($J$)

• Optimize objectives sequentially
• Optimality of upper objective constrains optimization of lower one
• Use redundant degrees of freedom (DOF) in decision variables, after meeting primary objective (take a walk in the null space)
Null space wandering in 3D
Hierarchical optimization

\[ V_{\text{min}} := \min_m V(\bar{u}, m) \]

s.t. \[ g_k(\bar{u}_{k-1}, x_k, m) = 0, \quad k = 1, \ldots, K, \quad x_0 = \bar{x}_0 \]

\[ \max_m J(\bar{u}, m) / \min_m J(\bar{u}, m) \]

s.t. \[ g_k(\bar{u}_{k-1}, x_k, m) = 0, \quad k = 1, \ldots, K, \quad x_0 = \bar{x}_0 \]

\[ V(m) - V_{\text{min}} \leq \varepsilon \]

- primary optimization problem
- secondary optimization problem
- relaxation of constraint
Formal method: Null-space approach

Idea: find ‘free’ directions and use these to optimize second objective function

1. Find optimal match \( m \) for primary objective \( V \)

2. Determine null-space \( N \) of input parameter space \( S_m \) around \( m \). (\( N \) relates to those directions in \( S_m \) to which \( V \) is insensitive)

3. Find improving direction \( d \) for secondary objective \( J \)

4. Project \( d \) onto basis of \( N \) to get projected direction \( d^* \) \( (d^* \) is improving direction for \( J \) but does not affect \( V \))

5. Update \( m \) using projected direction \( d^* \)

6. Perform steps 2 – 5 until convergence
Alternative: switching method

Idea: alternate unconstrained step to optimize $J$ with correction step to return to $V_{\text{min}}$

- New objective function
  \[ W = \Omega_1(V) \cdot V + \Omega_2(V) \cdot J, \]

- \[
  \Omega_1(V) = \begin{cases} 
  1 & \text{if } V - V_{\text{min}} > \varepsilon \\
  0 & \text{if } V - V_{\text{min}} \leq \varepsilon 
\end{cases} ,
  \quad \Omega_2(V) = \begin{cases} 
  0 & \text{if } V - V_{\text{min}} > \varepsilon \\
  1 & \text{if } V - V_{\text{min}} \leq \varepsilon 
\end{cases}
\]

where $\Omega_1$ and $\Omega_2$ are ‘switching’ functions

- Gradients of $W$ with respect to the model parameters
  \[
  \frac{\partial W}{\partial m} = \Omega_1(V) \cdot \frac{\partial V}{\partial m} + \Omega_2(V) \cdot \frac{\partial J}{\partial m}
\]
Switching method
Modified switching method

- Goal is to keep $V$ close to $V_{\text{min}}$ with update in $J$-direction
- Projection of the gradients $J$ onto the first-order approximation of the null-space of $V$:

$$\frac{\partial \tilde{J}}{\partial \mathbf{m}} := \frac{\partial J}{\partial \mathbf{m}} \cdot \left[ \mathbf{I} - \frac{\partial V^T}{\partial \mathbf{m}} \cdot \frac{\partial V}{\partial \mathbf{m}} \right],$$

which gives an alternative switching search direction $\mathbf{d}$

$$\mathbf{d} = \Omega_1(V) \cdot \frac{\partial V}{\partial \mathbf{m}} + \Omega_2(V) \cdot \frac{\partial J}{\partial \mathbf{m}} \cdot \left[ \mathbf{I} - \left| \frac{\partial V}{\partial \mathbf{m}} \right|^T \cdot \frac{\partial V}{\partial \mathbf{m}} \right]$$
Example 1: egg model

As before, except:

- Production history of 1.5 years (monthly measurements)
- Forecasts for next 4.5 years
Example 1: optimization method

- In-house reservoir simulator (fully-implicit black oil)
- Minimization with adjoint-based gradients, steepest-descent and line search
- Twin approach: ‘truth’ to generate synthetic; uniform model (correct mean) as prior for history match
- History match objective (first optimization):

\[
V = \sum_{k=1}^{K} (d_k - y_k)^T P^{-1}_{d_k} (d_k - y_k)
\]

where \(d\) are measured data and \(y\) predicted data

- Economic objective (second optimization):

\[
J = \sum_{k=1}^{K} \left\{ \sum_{i=1}^{N_{inj}} r_{wi} \cdot (u_{wi,i})_k + \sum_{j=1}^{N_{prod}} \left[ r_{wp} \cdot (y_{wp,j})_k + r_o \cdot (y_{o,j})_k \right] \cdot \Delta t_k \right\}
\]
**Example 1: hierarchical optimization**

<table>
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<th>Secondary optimization problem</th>
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<td>History-matching</td>
<td>Bounds on economic forecast</td>
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<td>0 – 1.5 years</td>
<td>1.5 – 6 years</td>
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</table>

- Simulation run by prescribing:
  - injection rates (from history)
  - BPHs producers (from history)
- Minimize $V$ (mismatch between measured & simulated data)
- Data (288 points):
  - BHPs of injectors
  - Oil/water flow rates producers
- Controls: grid block perms

- Simulation run by prescribing:
  - injection rates (constant)
  - BHPs producers (constant)
- Maximize/minimize $J$ (NPV over 4.5 years)
- $r_o = 9 \$/bbl, r_w = -1 \$/bbl, 0 disc.
- Weakly constrained by minimum primary objective $V_{min}$
- Controls: grid block perms
Example 1: HM results - pressures

pressures upper bound model

pressures lower bound model

measurement points
Example 1: HM results – flow rates

[Graphs showing flow rates for producers 1 to 4 over time, with oil flow rates and water flow rates depicted.]
Example 1: incremental permeability fields

“Lower bound”
model

“Upper bound”
model
Example 1: HM & forecast – pressures

pressures upper bound model
pressures lower bound model
measurement points
Example 1: HM & forecast – flow rates

![Graphs showing flow rates over time for producers 1 to 4.]

- Oil flow rates upper bound model
- Oil flow rates lower bound model
- Water flow rates upper bound model
- Water flow rates lower bound model

- Measurement points
- Current time
Example 1: forecast range in NPV

Historic & Predicted Economic Performance

Future

Past

Net Present Value [M$]

0 5 10 15 20 25

0 1 2 3 4 5 6

time [years]

Future

Net Present Value [M$]

0 5 10 15

2 3 4 5 6

time [years]

average prediction

current time

Prediction range

current time

+63%

-63%

predicted economic performance

average prediction

current time

+63%

-63%
Example 2: Brugge field

- 60,048 cells
- Own-generated synthetic truth
- 10 yrs ‘production data’ + ‘interpreted 4D’; 10% error
- Starting model for HM randomly selected out of ensemble
- 11 producers, BHP-controlled with bounds; reactive
- 20 injectors, fixed rate-controlled
Example 2: HM results (prod. only) – water rates

- 0.5% deviation allowed in objective function value
- 19.5 % difference in NPV
Example 2: Updated permeability fields

Differences in permeabilities in 9 layers

natural log mD
Example 2: HM results – effect of ‘data type’
Example 2: HM results – effect of ‘data type’
Example 2: HM results – effect of threshold value (1)
Example 2: HM results – effect of threshold value (2)
Conclusions ‘upper and lower bounds’

• Method can be used to gain more insight in the possible economic consequences of the lack of information in the data
  – NPV, total production, ultimate recovery, or other.
  – Economic impact alternative data sources, e.g. 4D seismic data

• No guaranteed lower/upper bounds, due to local optima

• Considerable number of iterations required until convergence
  – May be improved using more efficient optimization scheme (Quasi-Newton, conjugate gradient method, …)

• Wandering in the null space can be useful after all
References

• Robust optimization

• Upper/lower bounds