CREWES

Elastic FWI in the P/S domain

sensitivities and rock physics parameterizations

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- P-, S- domain and standard multicomponent methods
- Elastic FWI formulation in the P-, S-domain

$$\begin{bmatrix} \delta s_\gamma(\vec{x}) \\ \delta s_\mu(\vec{x}) \\ \delta s_\rho(\vec{x}) \end{bmatrix} = \int d\vec{x}' \begin{bmatrix} H_{\gamma\gamma}(\vec{x},\vec{x}') & H_{\gamma\mu}(\vec{x},\vec{x}') & H_{\gamma\rho}(\vec{x},\vec{x}') \\ H_{\mu\gamma}(\vec{x},\vec{x}') & H_{\mu\mu}(\vec{x},\vec{x}') & H_{\mu\rho}(\vec{x},\vec{x}') \\ H_{\rho\gamma}(\vec{x},\vec{x}') & H_{\rho\mu}(\vec{x},\vec{x}') & H_{\rho\rho}(\vec{x},\vec{x}') \end{bmatrix}^{-1} \begin{bmatrix} g_\gamma(\vec{x}') \\ g_\mu(\vec{x}') \\ g_\rho(\vec{x}') \end{bmatrix}$$

- Features of (2D) PP, PS, and “joint” PP-PS FWI
- P/S FWI and AVO relationships
- Un-accounted for sensitivities
- Re-parameterizing for rock physics FWI

P / S wave domain

DICTUM

The fewer assumptions concerning wave propagation and scattering that are made in multicomponent processing, the less important P/S decomposition is.

P / S wave domain

DICTUM: The fewer assumptions concerning wave propagation and scattering that are made in multicomponent processing, the less important P/S decomposition is.

- while current model iterate is smooth (e.g., early iterations), $\phi / \vec{\psi}$ are simpler than $\vec{u}$
- practicalities (statics/near-surface, wavelet) may make separate treatment of PP, PS, etc preferable
- although in heterogeneous media P/S modes couple, field data supports interpretation of P/Sv/Sh propagating quasi-independently.
- Supports deeper understanding of FWI behaviour.

"Standard" multicomponent processing & interpretation

Stewart et al., 2002, Geophysics 67 1348–1363.

Hardage et al., 2013, Geophysical Reference Series 18, SEG.

Multicomponent reverse-time migration / mode decomposition

Chang and McMechan, 1987, Geophysics 52 1365–1375

Sun et al., 2001, Geophysics 66 1515–1518

Dellinger and Etgen, 1990, Geophysics 55 914–919

Vector reconstruction of multicomponent signals

Stanton and Sacchi, 2013, Geophysics 78 V131–V145

Multicomponent elastic FWI

Operto et al., 2013, The Leading Edge, 1040–1054

(... and refs within)

$$
\begin{bmatrix} \delta s_\gamma(\vec{x}) \\ \delta s_\mu(\vec{x}) \\ \delta s_\rho(\vec{x}) \end{bmatrix} = \int d\vec{x}' \begin{bmatrix} H_{\gamma\gamma}(\vec{x},\vec{x}') & H_{\gamma\mu}(\vec{x},\vec{x}') & H_{\gamma\rho}(\vec{x},\vec{x}') \\ H_{\mu\gamma}(\vec{x},\vec{x}') & H_{\mu\mu}(\vec{x},\vec{x}') & H_{\mu\rho}(\vec{x},\vec{x}') \\ H_{\rho\gamma}(\vec{x},\vec{x}') & H_{\rho\mu}(\vec{x},\vec{x}') & H_{\rho\rho}(\vec{x},\vec{x}') \end{bmatrix}^{-1} \begin{bmatrix} g_\gamma(\vec{x}') \\ g_\mu(\vec{x}') \\ g_\rho(\vec{x}') \end{bmatrix}
$$

$$
\delta P = \begin{bmatrix} \delta P_{pp}(xv) & \delta P_{ps}(xv) \\ \delta P_{sp}(xv) & \delta P_{ss}(xv) \end{bmatrix}
$$

measured $\alpha\beta$ data

predicted $\alpha\beta$ data

$$
\delta P_{\alpha\beta} = P_{\alpha\beta}(xv) - G_{\alpha\beta}(xv)
$$

$\alpha, \beta = p, s$

Minimize

$$
\Phi = \frac{1}{2} \sum_{xv} \mathrm{tr}\left\{ \delta P^H \delta P \right\}
$$

XV = experimental variables

(k_g, k_s, ω)

$(\theta_g, \theta_s, \omega)$

$\vdots$

$$
g_x(x,z) = -\sum_{xv} \mathrm{tr}\left\{ \overbrace{\begin{bmatrix} \dfrac{\partial G_{pp}(xv)}{\partial s_x(x,z)} & \dfrac{\partial G_{ps}(xv)}{\partial s_x(x,z)} \\[2ex] \dfrac{\partial G_{sp}(xv)}{\partial s_x(x,z)} & \dfrac{\partial G_{ss}(xv)}{\partial s_x(x,z)} \end{bmatrix}}^{\dfrac{\partial G(xv)}{\partial s_x(x,z)}}{}^{T} \begin{bmatrix} \delta P^{*}_{pp}(xv) & \delta P^{*}_{ps}(xv) \\[2ex] \delta P^{*}_{sp}(xv) & \delta P^{*}_{ss}(xv) \end{bmatrix} \right\}
$$

$$
H_{xy}(x,z,x',z') = \sum_{xv} \mathrm{tr}\left\{ \left(\frac{\partial G(xv)}{\partial s_x(x,z)} \right)^{H} \left(\frac{\partial G(xv)}{\partial s_y(x',z')} \right) \right\}
$$

for Gauss-Newton updates.

$$\frac{\partial G_{\alpha\beta}(xy)}{\partial s_x(x,z)} \quad ?$$

$$\sum_{j=1}^{2} \frac{\partial \sigma_{ij}}{\partial x_j} + f_i + \omega^2 \rho u_i = 0, \qquad \sigma_{ij} = \lambda D \delta_{ij} + 2\mu e_{ij}$$

$$\sum_{j=1}^{2} \frac{\partial \sigma_{ij}}{\partial x_j} + f_i + \omega^2 \rho u_i = 0, \qquad \sigma_{ij} = \lambda D \delta_{ij} + 2\mu e_{ij}$$

$$s_\gamma = s_\gamma(x,z) = \gamma(x,z), \qquad s_\mu = \mu(x,z), \qquad s_\rho = \rho(x,z)$$

$$\mathcal{L} \begin{bmatrix} u_x \\ u_z \end{bmatrix} = \begin{bmatrix} f_x \\ f_z \end{bmatrix}$$

$$\mathcal{L} = \left[\begin{pmatrix} \partial_x s_\gamma \partial_x + \partial_z s_\mu \partial_z & \partial_x (s_\gamma - 2 s_\mu) \partial_z + \partial_z s_\mu \partial_x \\ \partial_z (s_\gamma - 2 s_\mu) \partial_x + \partial_x s_\mu \partial_z & \partial_z s_\gamma \partial_z + \partial_x s_\mu \partial_x \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \omega^2 s_\rho \right]$$

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For constant $s_{\gamma 0}$, $s_{\mu 0}$, $s_{\rho 0}$ $\mathcal{L} = \mathcal{L}_0$ where

$$\mathcal{L}_0 = \begin{bmatrix} s_{\gamma 0} \partial_x^2 + s_{\mu 0} \partial_z^2 + \omega^2 s_{\rho 0} & (s_{\gamma 0} - 2 s_{\mu 0}) \partial_x \partial_z + s_{\mu 0} \partial_z \partial_x \\ (s_{\gamma 0} - 2 s_{\mu 0}) \partial_z \partial_x + s_{\mu 0} \partial_x \partial_z & s_{\gamma 0} \partial_z^2 + s_{\mu 0} \partial_x^2 + \omega^2 s_{\rho 0} \end{bmatrix}$$

If $G = \mathcal{L}^{-1}$, $G_0 = \mathcal{L}_0^{-1}$, then (defining $V = \mathcal{L}_0 - \mathcal{L}$)

$$\delta G = \underbrace{G - G_0 = G_0 V G}_{\text{Scattering / L.S. Eqn}} \sim G_0 V G_0$$

is a convenient step towards generating sensitivities

$$V = - \begin{bmatrix} \partial_x \delta s_\gamma \partial_x + \partial_z \delta s_\mu \partial_z + \omega^2 \delta s_\rho & \partial_x (\delta s_\gamma - 2 \delta s_\mu) \partial_z + \partial_z \delta s_\mu \partial_x \\ \partial_z (\delta s_\gamma - 2 \delta s_\mu) \partial_x + \partial_x \delta s_\mu \partial_z & \partial_z \delta s_\gamma \partial_z + \partial_x \delta s_\mu \partial_x + \omega^2 \delta s_\rho \end{bmatrix}$$

where $\delta s_\gamma = s_\gamma(x,z) - s_{\gamma 0}$, etc.

2D Helmholtz decomposition :

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \vec{\nabla}\phi + \vec{\nabla}\times\vec{\psi} = \begin{bmatrix} \partial_x \phi \\ \partial_y \phi \\ \partial_z \phi \end{bmatrix} + \begin{bmatrix} \partial_y \psi_z - \partial_z \psi_y \\ \partial_z \psi_x - \partial_x \psi_z \\ \partial_x \psi_y - \partial_y \psi_x \end{bmatrix}$$

2D Helmholtz decomposition :

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but with $\vec{\psi} = [0, \psi, 0]^T$ and $\partial_y(\;) = 0$:

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but with $\vec{\psi} = [0, \psi, 0]^T$ and $\partial_y(\ ) = 0$:

$$\begin{bmatrix} u_x \\ 0 \\ u_z \end{bmatrix} = \begin{bmatrix} \partial_x \phi \\ 0 \\ \partial_z \phi \end{bmatrix} + \begin{bmatrix} -\partial_z \psi \\ 0 \\ \partial_x \psi \end{bmatrix} \rightarrow \begin{bmatrix} u_x \\ u_z \end{bmatrix} = \begin{bmatrix} \partial_x & -\partial_z \\ \partial_z & \partial_x \end{bmatrix} \begin{bmatrix} \phi \\ \psi \end{bmatrix}$$

$$\begin{bmatrix} u_x \\ u_z \end{bmatrix} = \Pi^T \begin{bmatrix} \phi \\ \psi \end{bmatrix}, \quad \Pi = \begin{bmatrix} \partial_x & \partial_z \\ -\partial_z & \partial_x \end{bmatrix}$$

e.g., Stolt & Weglein 2012

The Π operator diagonalizes $\mathcal{L}_0$:

$$\Gamma_0 L_0 = \Pi \mathcal{L}_0 \Pi^{-1}$$

where $\Gamma_0 = \begin{bmatrix} \gamma_0 & 0 \\ 0 & \mu_0 \end{bmatrix}$ and $L_0 = \begin{bmatrix} \frac{\omega^2}{V_{p0}^2} + \nabla^2 & 0 \\ 0 & \frac{\omega^2}{V_{s0}^2} + \nabla^2 \end{bmatrix}$

and $V_{p0}^2 = \gamma_0/\rho_0$, $V_{s0}^2 = \mu_0/\rho_0$.

The inverses of the elements of L_0 involve Green's functions satisfying

$$\left[\nabla^2 + \frac{\omega^2}{V_{p0}^2}\right] G_{p0}(x,z,x',z') = \delta(x-x')\,\delta(z-z')$$

$$\left[\nabla^2 + \frac{\omega^2}{V_{s0}^2}\right] G_{s0}(x,z,x',z') = \delta(x-x')\,\delta(z-z')$$

In this domain the scattering equation is

$$\delta G = \underbrace{\begin{bmatrix} \delta G_{pp}(xv) & \delta G_{ps}(xv) \\ \delta G_{sp}(xv) & \delta G_{ss}(xv) \end{bmatrix}}_{\text{variation will be w.r.t. mode-preserved and mode-converted fields}} = \iint dx'dz' \underbrace{\begin{bmatrix} G_{p0}(xv, x', z') & 0 \\ 0 & G_{s0}(xv, x', z') \end{bmatrix}}_{\text{convenient, mode-decomposed}} V(x', z') \underbrace{\begin{bmatrix} G_{p0}(x' z' xv) & 0 \\ 0 & G_{s0}(x' z' xv) \end{bmatrix}}_{\text{on source and receiver side}} + \ldots$$

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Still to be done: form of V!

In this domain the scattering equation is

$$\delta G = \begin{bmatrix} \delta G_{PP}(xv) & \delta G_{PS}(xv) \\ \delta G_{SP}(xv) & \delta G_{SS}(xv) \end{bmatrix} = \iint dx'dz' \begin{bmatrix} G_{P0}(xv, x',z') & 0 \\ 0 & G_{S0}(xv, x',z') \end{bmatrix} V(x',z') \begin{bmatrix} G_{P0}(x'z' xv) & 0 \\ 0 & G_{S0}(x'z' xv) \end{bmatrix} + \ldots$$

variation will be w.r.t. mode-preserved and mode-converted fields

convenient, mode-decomposed

on source and receiver side

Still to be done: form of V!

We have in hand the displacement ϑ, and so

$$V(x,z) = \Pi \underbrace{\vartheta(x,z)}_{} \Pi^T \Gamma_0^{-1} (\partial_x^2 + \partial_z^2)^{-1}$$

But, be guided in next steps by the convenience of ϑ for, e.g., re-parameterization

Integrating by parts to move the operators off $\mathcal{V}$

$$\delta G(xv) = \iint dx'dz' \left[\Pi^T \begin{pmatrix} G_{p0}(xv, x', z') & 0 \\ 0 & G_{s0}(xv, x', z') \end{pmatrix} \right]^T \mathcal{V}(x', z')\, \Pi^T \begin{bmatrix} \frac{V_{p0}^2}{\gamma_0 \omega^2} G_{p0}(x', z', xv) & 0 \\ 0 & \frac{V_{s0}^2}{\mu_0 \omega^2} G_{s0}(x', z', xv) \end{bmatrix} + \ldots$$

we can then use

$$\mathcal{V}(x,z) = \begin{bmatrix} \omega^2 \delta s_\rho(x,z) + \partial_x \delta s_\gamma(x,z)\partial_x + \partial_z \delta s_\mu(x,z)\partial_z & \partial_x[\delta s_\gamma(x,z) - 2\delta s_\mu(x,z)]\partial_z + \partial_z \delta s_\mu(x,z)\partial_x \\ \partial_z[\delta s_\gamma(x,z) - 2\delta s_\mu(x,z)]\partial_x + \partial_x \delta s_\mu(x,z)\partial_z & \omega^2 \delta s_\rho(x,z) + \partial_z \delta s_\gamma(x,z)\partial_z + \partial_x \delta s_\mu(x,z)\partial_x \end{bmatrix}$$

Not yet an expression for sensitivities, because:

- perturbations in all 3 parameters are simultaneously accommodated;
- perturbations are distributed in space.

To get at real sensitivities, procedure

- set δs's for 2 properties to zero
- set current $\delta s_x(x', z')$ to $\delta s_x(x,z)\,\delta(x-x')\,\delta(z-z')$

Then find the 1^{st} order coefficient for $\delta s_x(x,z)$.

Interim results ($xv = k_g, k_s, \omega$).

L, R matrices containing weights, derivatives acting on G_{P0}, G_{S0}.

$$\frac{\partial G(k_g,k_s,\omega)}{\partial s_\gamma(x,z)} = -G_L^\gamma \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} G_R^\gamma$$

$$\frac{\partial G(k_g,k_s,\omega)}{\partial s_\mu(x,z)} = -G_{L1}^\mu \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} G_{R1}^\mu - G_{L2}^\mu \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix} G_{R2}^\mu$$

$$\frac{\partial G(k_g,k_s,\omega)}{\partial s_\rho(x,z)} = -G_L^\rho \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} G_R^\rho$$

$$G_R^\rho = \begin{bmatrix} i\omega & 0 \\ 0 & i\omega \end{bmatrix} \begin{bmatrix} \frac{V_{P0}^2}{\gamma_0 \omega^2}\partial_x G_{P0} & \frac{-V_{S0}^2}{\mu_0 \omega^2}\partial_z G_{S0} \\ \frac{V_{P0}^2}{\gamma_0 \omega^2}\partial_z G_{P0} & \frac{V_{S0}^2}{\mu_0 \omega^2}\partial_x G_{S0} \end{bmatrix}$$

$$G_L^\rho = \begin{bmatrix} i\omega & 0 \\ 0 & i\omega \end{bmatrix} \begin{bmatrix} \partial_x G_{P0} & \partial_z G_{P0} \\ -\partial_z G_{S0} & \partial_x G_{S0} \end{bmatrix}$$

In a homogeneous initial medium,

$$G_{p0}(k_g, 0, x, z, \omega) = \frac{\rho_0}{2i\nu_g} e^{i\vec{k}_p^g \cdot \vec{x}}$$

$$G_{p0}(x, z, k_s, 0, \omega) = \frac{\rho_0}{2i\nu_s} e^{i\vec{k}_p^s \cdot \vec{x}}$$

$$G_{s0}(k_g, 0, x, z, \omega) = \frac{\rho_0}{2i\eta_g} e^{i\vec{k}_s^g \cdot \vec{x}}$$

$$G_{s0}(x, z, k_s, 0, \omega) = \frac{\rho_0}{2i\eta_s} e^{i\vec{k}_s^s \cdot \vec{x}}$$

$$\nu_g = \frac{\omega}{V_{p0}} \sqrt{1 - \frac{V_{p0}^2 k_g^2}{\omega^2}}$$

$$\nu_s = \frac{\omega}{V_{p0}} \sqrt{1 - \frac{V_{p0}^2 k_s^2}{\omega^2}}$$

$$\eta_g = \frac{\omega}{V_{s0}} \sqrt{1 - \frac{V_{s0}^2 k_g^2}{\omega^2}}$$

$$\eta_s = \frac{\omega}{V_{s0}} \sqrt{1 - \frac{V_{s0}^2 k_s^2}{\omega^2}}$$

$$\vec{k}_p^g = \begin{bmatrix} -k_g \\ \nu_g \end{bmatrix} \qquad \vec{k}_p^s = \begin{bmatrix} k_s \\ \nu_s \end{bmatrix}$$

$$\vec{k}_s^g = \begin{bmatrix} -k_g \\ \eta_g \end{bmatrix} \qquad \vec{k}_s^s = \begin{bmatrix} k_s \\ \eta_s \end{bmatrix}$$

Applying the derivatives & weights.

$$\frac{\partial G(k_g, k_s, \omega)}{\partial s_\gamma(x,z)} = -G_r^\gamma \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} G_R^\gamma$$

$$= \begin{bmatrix} k_g^2 G_{po} & \nu_g^2 G_{po} \\ k_g \eta_g G_{so} & -k_g \eta_g G_{so} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -\frac{V_{po}^2}{\gamma_o \omega^2} k_s^2 G_{po} & \frac{V_{so}^2}{\mu_o \omega^2} k_s \eta_s G_{so} \\ -\frac{V_{po}^2}{\gamma_o \omega^2} \nu_s^2 G_{po} & -\frac{V_{so}^2}{\mu_o \omega^2} k_s \eta_s G_{so} \end{bmatrix}$$

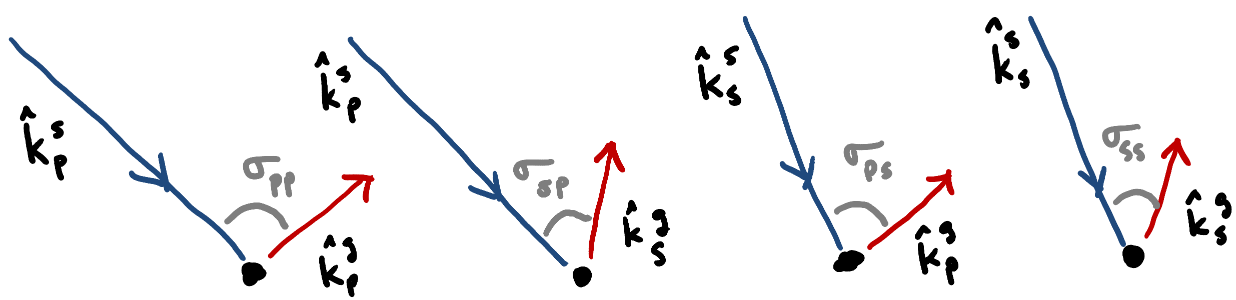
$$= \begin{bmatrix} G_{po}(k_g\ 0\ x\ z) & 0 \\ 0 & G_{so}(k_g\ 0\ x\ z) \end{bmatrix} \begin{bmatrix} K_{pp}^\gamma & K_{ps}^\gamma \\ K_{sp}^\gamma & K_{ss}^\gamma \end{bmatrix} \begin{bmatrix} G_{po}(x\ z\ k_s\ 0) & 0 \\ 0 & G_{so}(x\ z\ k_s\ 0) \end{bmatrix}$$

which though shown for the homogeneous case, holds equally well for smoothly-varying heterogeneous media.

$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_\gamma(x,z)} = \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \begin{bmatrix} K_{pp}^{\gamma 0} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix}$$

$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_\mu(x,z)} = \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \begin{bmatrix} K_{pp}^{\mu 0}(\hat{k}_p^g \times \hat{k}_p^s)^2 & K_{ps}^{\mu 0}(\hat{k}_p^g \cdot \hat{k}_s^s)(\hat{k}_p^g \times \hat{k}_s^s) \\ K_{sp}^{\mu 0}(\hat{k}_s^g \cdot \hat{k}_p^s)(\hat{k}_s^g \times \hat{k}_p^s) & K_{ss}^{\mu 0}\left[(\hat{k}_s^g \cdot \hat{k}_s^s)^2 - (\hat{k}_s^g \times \hat{k}_s^s)^2\right] \end{bmatrix} \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s} \end{bmatrix}$$

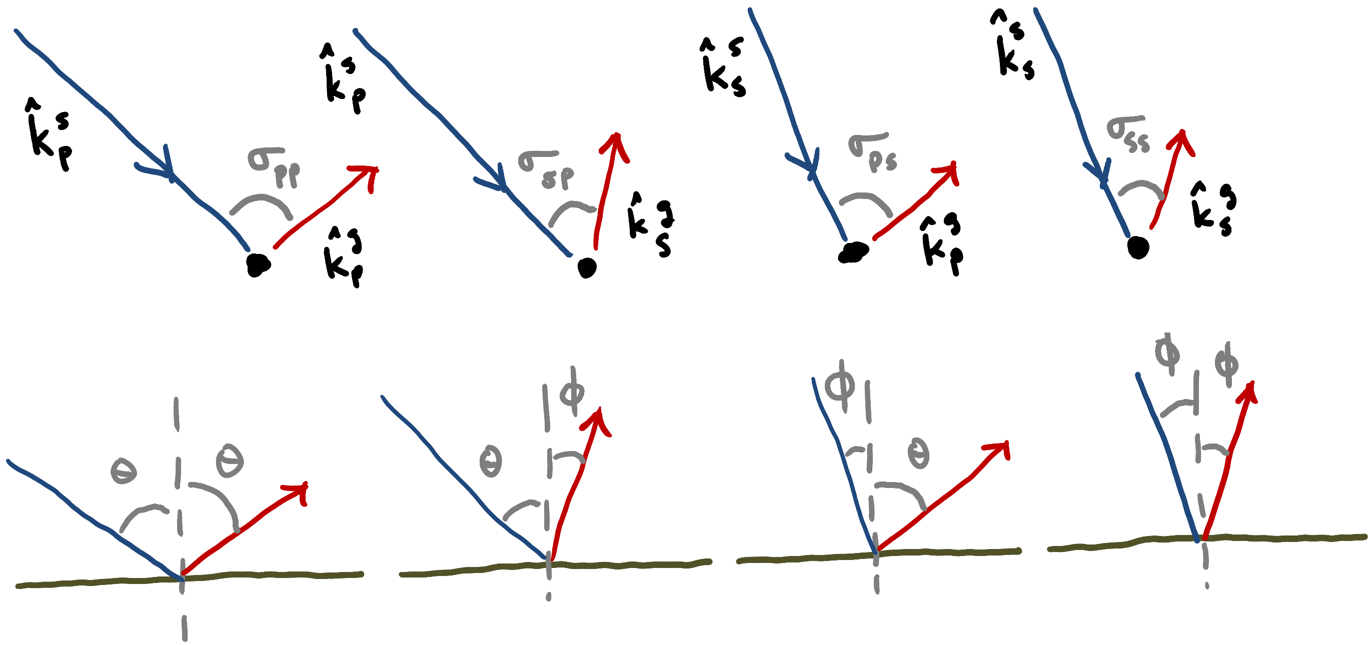
$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_\rho(x,z)} = \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \begin{bmatrix} K_{pp}^{\rho 0}(\hat{k}_p^g \cdot \hat{k}_p^s) & K_{ps}^{\rho 0}(\hat{k}_p^g \times \hat{k}_s^s) \\ K_{sp}^{\rho 0}(\hat{k}_s^g \times \hat{k}_p^s) & K_{ss}^{\rho 0}(\hat{k}_s^g \cdot \hat{k}_s^s) \end{bmatrix} \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix}$$



- P-, S- domain and standard multicomponent methods ✓
- Elastic FWI formulation in the P-, S-domain ✓

$$\begin{bmatrix} \delta s_\gamma(\vec{x}) \\ \delta s_\mu(\vec{x}) \\ \delta s_\rho(\vec{x}) \end{bmatrix} = \int d\vec{x}' \begin{bmatrix} H_{\gamma\gamma}(\vec{x},\vec{x}') & H_{\gamma\mu}(\vec{x},\vec{x}') & H_{\gamma\rho}(\vec{x},\vec{x}') \\ H_{\mu\gamma}(\vec{x},\vec{x}') & H_{\mu\mu}(\vec{x},\vec{x}') & H_{\mu\rho}(\vec{x},\vec{x}') \\ H_{\rho\gamma}(\vec{x},\vec{x}') & H_{\rho\mu}(\vec{x},\vec{x}') & H_{\rho\rho}(\vec{x},\vec{x}') \end{bmatrix}^{-1} \begin{bmatrix} g_\gamma(\vec{x}') \\ g_\mu(\vec{x}') \\ g_\rho(\vec{x}') \end{bmatrix}$$

- Features of (2D) PP, PS, and “joint” PP-PS FWI
- P/S FWI and AVO relationships
- Un-accounted for sensitivities
- Re-parameterizing for rock physics FWI



$$K_{pp}^{\gamma} = (\gamma_0 v_{p0}^2)^{-1}, \quad K_{ps}^{\gamma} = K_{sp}^{\gamma} = K_{ss}^{\gamma} = 0$$

$$K_{pp}^{\mu} = -8(\gamma_0 v_{p0}^2)^{-1}\sin^2\theta, \quad K_{ps}^{\mu} = -2(\mu_0 v_{p0}^2)^{-1}\left(1 - \frac{v_{p0}}{v_{s0}}\right)\sin\theta$$

$$K_{sp}^{\mu} = -2(\gamma_0 v_{s0}^2)^{-1}\left(1 + \frac{v_{p0}}{v_{s0}}\right)\sin\theta, \quad K_{ss}^{\mu} = (\mu_0 v_{s0}^2)^{-1}\left[1 - 8\sin^2\phi\right]$$

$$K_{pp}^{\ell} = \gamma_0^{-1}(1 - 2\sin^2\theta), \quad K_{ps}^{\ell} = \mu_0^{-1}\left(1 + \frac{v_{s0}}{v_{p0}}\right)\sin\theta$$

$$K_{sp}^{\ell} = \gamma_0^{-1}\left(\frac{v_{p0}}{v_{s0}}\right)\left(1 + \frac{v_{p0}}{v_{s0}}\right)\sin\theta, \quad K_{ss}^{\ell} = \mu_0^{-1}(1 - 2\sin^2\phi)$$

$$-\sum_{k_g,\omega} \frac{\partial G_{\alpha\beta}(k_g,k_s,\omega)}{\partial s_x(x,z)}\, \delta P^{*}_{\alpha\beta}(k_g,k_s,\omega)$$

$$-\sum_{k_g,\omega} \frac{\partial G_{\alpha\beta}(k_g,k_s,\omega)}{\partial s_x(x,z)} \, \delta P^*_{\alpha\beta}(k_g,k_s,\omega)$$

$$S(z) = \begin{cases} 0 & z<0 \\ 1 & z>1 \end{cases}$$

$$= R_{\alpha\beta} \, K^x_{\alpha\beta} \; W(\alpha,\beta) \; S(z-z_1)$$

$\sim (\hat{k}^g_p \cdot \hat{k}^s_s),\ (\hat{k}^g_s \times \hat{k}^s_p)$, etc.

now with $g=s$

$$\frac{s_{\rho_0}^3 \pi}{4} \; \frac{V_{\alpha 0} V_{\beta 0}^2}{\cos\overset{\theta}{\phi} \; \cos^2\overset{\theta}{\phi}}$$

$$-\sum_{k_g,\omega} \frac{\partial G_{\alpha\beta}(k_g,k_s,\omega)}{\partial s_x(x,z)} \delta P^*_{\alpha\beta}(k_g,k_s,\omega)$$

$$S(z) = \begin{cases} 0 & z<0 \\ 1 & z>1 \end{cases}$$

$$= R_{\alpha\beta} K^x_{\alpha\beta} \; W(\alpha,\beta) \; S(z-z_1)$$

$\sim (\hat{k}^g_p \cdot \hat{k}^s_s), (\hat{k}^g_s \times \hat{k}^s_p)$, etc.

now with $g=s$

$$\frac{s_{\rho_0}^3 \pi}{4} \frac{V_{\alpha 0} V_{\beta 0}^2}{\cos\theta \cos^2\theta}$$

$\phi \quad \phi$

$$g_x(x,z) = \frac{s_{\rho_0}^3 \pi}{4} \sum_{k_s} \left[R_{pp} K^x_{pp} \frac{V_{p_0}^3}{\cos^3\theta} + R_{ps} K^x_{ps} \frac{V_{p_0}^2 V_{s_0}}{\cos^2\theta \cos\phi} + R_{sp} K^x_{sp} \frac{V_{p_0} V_{s_0}^2}{\cos\theta \cos^2\phi} + R_{ss} K^x_{ss} \frac{V_{s_0}^3}{\cos^3\phi} \right] S(z-z_1)$$

$$\underbrace{\begin{bmatrix} (\mathrm{WTS}, \mathrm{DATA})\, S(z-z_1) \\ (\mathrm{WTS}, \mathrm{DATA})\, S(z-z_1) \\ (\mathrm{WTS}, \mathrm{DATA})\, S(z-z_1) \end{bmatrix}}_{\vec{g}} = -\int dz' \underbrace{\begin{bmatrix} \mathrm{WTS}\ \delta(z-z') & \cdots & (\mathrm{WTS})\ \delta(z-z') \\ \vdots & \ddots & \\ (\mathrm{WTS})\ \delta(z-z') & & (\mathrm{WTS})\ \delta(z-z') \end{bmatrix}}_{H(z, z')} \underbrace{\begin{bmatrix} \delta s_\gamma\ S(z'-z_1) \\ \delta s_\mu\ S(z'-z_1) \\ \delta s_e\ S(z'-z_1) \end{bmatrix}}_{\vec{\delta s}}$$

Case 1: PP data only

$$
\begin{bmatrix} \sum_{\theta=1}^{N} R_{pp} K_{pp}^{\gamma} \\ \sum_{\theta=1}^{N} R_{pp} K_{pp}^{\mu} \\ \sum_{\theta=1}^{N} R_{pp} K_{pp}^{\rho} \end{bmatrix}
=
\underbrace{\begin{bmatrix}
\sum_{\theta=1}^{N} {K_{pp}^{\gamma}}^{2} & \sum_{\theta=1}^{N} K_{pp}^{\gamma} K_{pp}^{\mu} & \sum_{\theta=1}^{N} K_{pp}^{\gamma} K_{pp}^{\rho} \\
\sum_{\theta=1}^{N} K_{pp}^{\mu} K_{pp}^{\gamma} & \sum_{\theta=1}^{N} {K_{pp}^{\mu}}^{2} & \sum_{\theta=1}^{N} K_{pp}^{\mu} K_{pp}^{\rho} \\
\sum_{\theta=1}^{N} K_{pp}^{\rho} K_{pp}^{\gamma} & \sum_{\theta=1}^{N} K_{pp}^{\rho} K_{pp}^{\mu} & \sum_{\theta=1}^{N} {K_{pp}^{\rho}}^{2}
\end{bmatrix}}_{L_{pp}(N)}
\begin{bmatrix} \delta s_{\gamma} \\ \delta s_{\mu} \\ \delta s_{\rho} \end{bmatrix}
$$

$\det L_{pp}(1) = 0$

$\det L_{pp}(2) = 0$

$\det L_{pp}(3) \neq 0$

Case 2: "Joint" PP-PS data

$$
\begin{bmatrix}
\sum_{\theta=1}^{N} R_{pp} K_{pp}^{\gamma} \\
\sum_{\theta=1}^{N} R_{pp} K_{pp}^{\mu} + R_{sp} K_{sp}^{\mu} \\
\sum_{\theta=1}^{N} R_{pp} K_{pp}^{\rho} + R_{sp} K_{sp}^{\rho}
\end{bmatrix}
=
\underbrace{\begin{bmatrix}
\sum_{\theta=1}^{N} K_{pp}^{\gamma\,2} & \sum_{\theta=1}^{N} K_{pp}^{\gamma} K_{pp}^{\mu} & \sum_{\theta=1}^{N} K_{pp}^{\gamma} K_{pp}^{\rho} \\
\sum_{\theta=1}^{N} K_{pp}^{\gamma} K_{pp}^{\mu} & \sum_{\theta=1}^{N} \left(K_{pp}^{\mu\,2} + K_{sp}^{\mu\,2}\right) & \sum_{\theta=1}^{N} \left(K_{pp}^{\mu} K_{pp}^{\rho} + K_{sp}^{\mu} K_{sp}^{\rho}\right) \\
\sum_{\theta=1}^{N} K_{pp}^{\rho} K_{pp}^{\gamma} & \sum_{\theta=1}^{N} \left(K_{pp}^{\rho} K_{pp}^{\mu} + K_{sp}^{\rho} K_{sp}^{\mu}\right) & \sum_{\theta=1}^{N} \left(K_{pp}^{\rho\,2} + K_{sp}^{\rho\,2}\right)
\end{bmatrix}}_{L_{pp\text{-}ps}(N)}
\begin{bmatrix}
\delta s_{\gamma} \\
\delta s_{\mu} \\
\delta s_{\rho}
\end{bmatrix}
$$

$\det L_{pp\text{-}ps}(1) = 0$

$\det L_{pp\text{-}ps}(2) \neq 0$

$\det L_{pp\text{-}ps}(3) \neq 0$

Case 3: PS data only

$$\begin{bmatrix} 0 \\ \begin{bmatrix} \sum_{\theta=1}^{N} R_{sp} K_{sp}^{\mu} \\ \sum_{\theta=1}^{N} R_{sp} K_{sp}^{\rho} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sum_{\theta=1}^{N} K_{sp}^{\mu^2} & \sum_{\theta=1}^{N} K_{sp}^{\mu} K_{sp}^{\rho} \\ 0 & \sum_{\theta=1}^{N} K_{sp}^{\rho} K_{sp}^{\mu} & \sum_{\theta=1}^{N} K_{sp}^{\rho^2} \end{bmatrix} \begin{bmatrix} \delta s_{\gamma} \\ \begin{bmatrix} \delta s_{\mu} \\ \delta s_{\rho} \end{bmatrix} \end{bmatrix}$$

Comes about because $K_{sp}^{\gamma} = K_{ps}^{\gamma} = K_{ss}^{\gamma} = 0$,

i.e., a perturbation in γ has no influence

on the P-to-S, S-to-P, and S-to-S modes.

- P-, S- domain and standard multicomponent methods ✓
- Elastic FWI formulation in the P-, S-domain ✓

$$\begin{bmatrix} \delta s_\gamma(\vec{x}) \\ \delta s_\mu(\vec{x}) \\ \delta s_\rho(\vec{x}) \end{bmatrix} = \int d\vec{x}' \begin{bmatrix} H_{\gamma\gamma}(\vec{x},\vec{x}') & H_{\gamma\mu}(\vec{x},\vec{x}') & H_{\gamma\rho}(\vec{x},\vec{x}') \\ H_{\mu\gamma}(\vec{x},\vec{x}') & H_{\mu\mu}(\vec{x},\vec{x}') & H_{\mu\rho}(\vec{x},\vec{x}') \\ H_{\rho\gamma}(\vec{x},\vec{x}') & H_{\rho\mu}(\vec{x},\vec{x}') & H_{\rho\rho}(\vec{x},\vec{x}') \end{bmatrix}^{-1} \begin{bmatrix} g_\gamma(\vec{x}') \\ g_\mu(\vec{x}') \\ g_\rho(\vec{x}') \end{bmatrix}$$

- Features of (2D) PP, PS, and “joint” PP-PS FWI ✓
- P/S FWI and AVO relationships ✓
- Un-accounted for sensitivities
- Re-parameterizing for rock physics FWI

Case 3: PS data only

$$\begin{bmatrix} 0 \\ \sum_{\theta=1}^{N} R_{sp} K_{sp}^{\mu} \\ \sum_{\theta=1}^{N} R_{sp} K_{sp}^{\rho} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sum_{\theta=1}^{N} K_{sp}^{\mu^2} & \sum_{\theta=1}^{N} K_{sp}^{\mu} K_{sp}^{\rho} \\ 0 & \sum_{\theta=1}^{N} K_{sp}^{\rho} K_{sp}^{\mu} & \sum_{\theta=1}^{N} K_{sp}^{\rho^2} \end{bmatrix} \begin{bmatrix} \delta s_{\gamma} \\ \delta s_{\mu} \\ \delta s_{\rho} \end{bmatrix}$$

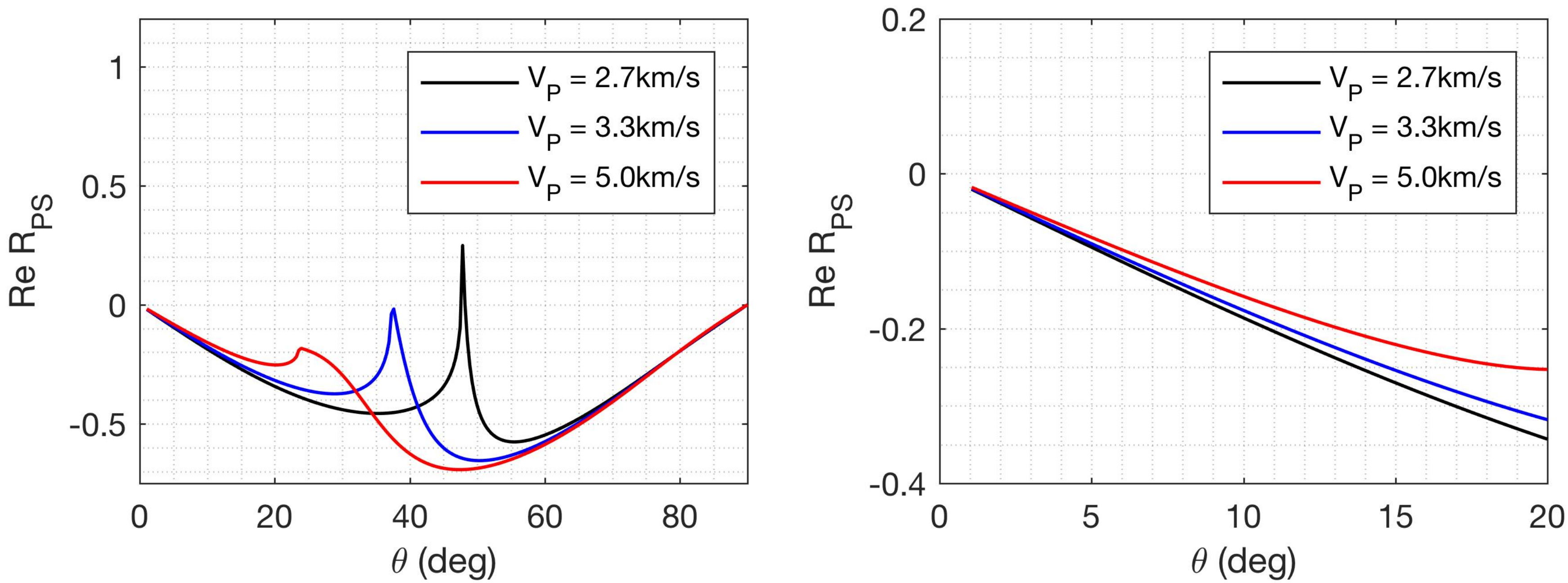
Comes about because $K_{sp}^{\gamma} = K_{ps}^{\gamma} = K_{ss}^{\gamma} = 0$,

i.e., a perturbation in γ has no influence

on the P-to-S, S-to-P, and S-to-S modes.

c.f. Aki-Richards approximation for R_{sp} :

$$R_{sp}(\theta) \sim C_s(\theta) \frac{\Delta V_s}{V_s} + C_\rho(\theta) \frac{\Delta \rho}{\rho}$$



Intrinsic feature of FWI sensitivities – based on a localized, infinitesimal variation. Anything more requires an altered formulation. (see Geng, 2017)

- P-, S- domain and standard multicomponent methods ✓
- Elastic FWI formulation in the P-, S-domain ✓

$$\begin{bmatrix} \delta s_\gamma(\vec{x}) \\ \delta s_\mu(\vec{x}) \\ \delta s_\rho(\vec{x}) \end{bmatrix} = \int d\vec{x}' \begin{bmatrix} H_{\gamma\gamma}(\vec{x},\vec{x}') & H_{\gamma\mu}(\vec{x},\vec{x}') & H_{\gamma\rho}(\vec{x},\vec{x}') \\ H_{\mu\gamma}(\vec{x},\vec{x}') & H_{\mu\mu}(\vec{x},\vec{x}') & H_{\mu\rho}(\vec{x},\vec{x}') \\ H_{\rho\gamma}(\vec{x},\vec{x}') & H_{\rho\mu}(\vec{x},\vec{x}') & H_{\rho\rho}(\vec{x},\vec{x}') \end{bmatrix}^{-1} \begin{bmatrix} g_\gamma(\vec{x}') \\ g_\mu(\vec{x}') \\ g_\rho(\vec{x}') \end{bmatrix}$$

- Features of (2D) PP, PS, and "joint" PP-PS FWI ✓
- P/S FWI and AVO relationships ✓
- Un-accounted for sensitivities ✓
- Re-parameterizing for rock physics FWI

Petrophysical properties

- LMR (Goodway 1997)
- poroelastic (Russell & Gray 2011)
- "Geogain" (Gidlow et al., 1992)

⋮

rigidity, compressibility are more sensitive to presence of pore fluids, lithology, etc., than are properties organized to reflect seismic velocities

"L" "M"

LMR : $\lambda\rho$, $\mu\rho$ (+ ρ)

robustly determined

challenging

$$\delta s_\gamma \to I_\lambda \delta s_L - (J_\lambda + 2 J_\mu) \delta s_\rho + 2 I_\mu \delta s_M$$

$$\delta s_\mu \to I_\mu \delta s_M - J_\mu \delta s_\rho$$

$$\delta s_\rho \to \delta s_\rho$$

$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_\gamma(x,z)} = \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \begin{bmatrix} K_{pp}^{\gamma 0} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix}$$

$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_\mu(x,z)} = \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \begin{bmatrix} K_{pp}^{\mu 0}(\hat{k}_p^g \times \hat{k}_p^s)^2 & K_{ps}^{\mu 0}(\hat{k}_p^g \cdot \hat{k}_s^s)(\hat{k}_p^g \times \hat{k}_s^s) \\ K_{sp}^{\mu 0}(\hat{k}_s^g \cdot \hat{k}_p^s)(\hat{k}_s^g \times \hat{k}_p^s) & K_{ss}^{\mu 0}\left[(\hat{k}_s^g \cdot \hat{k}_s^s)^2 - (\hat{k}_s^g \times \hat{k}_s^s)^2\right] \end{bmatrix} \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s} \end{bmatrix}$$

$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_\rho(x,z)} = \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \begin{bmatrix} K_{pp}^{\rho 0}(\hat{k}_p^g \cdot \hat{k}_p^s) & K_{ps}^{\rho 0}(\hat{k}_p^g \times \hat{k}_s^s) \\ K_{sp}^{\rho 0}(\hat{k}_s^g \times \hat{k}_p^s) & K_{ss}^{\rho 0}(\hat{k}_s^g \cdot \hat{k}_s^s) \end{bmatrix} \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix}$$

$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_L(x,z)} = K_L \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \begin{bmatrix} K_{pp}^{\delta 0} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix}$$

$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_M(x,z)} = K_M \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \begin{bmatrix} K_{pp}^{\mu 0}(\hat{k}_p^g \times \hat{k}_p^s)^2 & K_{ps}^{\mu 0}(\hat{k}_p^g \cdot \hat{k}_s^s)(\hat{k}_p^g \times \hat{k}_s^s) \\ K_{sp}^{\mu 0}(\hat{k}_s^g \cdot \hat{k}_p^s)(\hat{k}_s^g \times \hat{k}_p^s) & K_{ss}^{\mu 0}\left[(\hat{k}_s^g \cdot \hat{k}_s^s)^2 - (\hat{k}_s^g \times \hat{k}_s^s)^2\right] \end{bmatrix} \begin{bmatrix} G_{p0} & 0 \\ 0 & G_s \end{bmatrix}$$

$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_\rho(x,z)} = \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \quad ? \quad \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix}$$

$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_L(x,z)} = K_L \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \begin{bmatrix} K_{pp}^{\delta 0} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix}$$

$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_M(x,z)} = K_M \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \begin{bmatrix} K_{pp}^{\mu 0}(\hat{k}_p^g \times \hat{k}_p^s)^2 & K_{ps}^{\mu 0}(\hat{k}_p^g \cdot \hat{k}_s^s)(\hat{k}_p^g \times \hat{k}_s^s) \\ K_{sp}^{\mu 0}(\hat{k}_s^g \cdot \hat{k}_p^s)(\hat{k}_s^g \times \hat{k}_p^s) & K_{ss}^{\mu 0}\left[(\hat{k}_s^g \cdot \hat{k}_s^s)^2 - (\hat{k}_s^g \times \hat{k}_s^s)^2\right] \end{bmatrix} \begin{bmatrix} G_{p0} & 0 \\ 0 & G_s \end{bmatrix}$$

$$\frac{\partial G(k_g\, k_s\, \omega)}{\partial s_\rho(x,z)} = \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix} \begin{bmatrix} K_{pp}^{\rho 0}(\hat{k}_p^g \cdot \hat{k}_p^s) + K_{pp}^{\rho 1}(\hat{k}_p^g \cdot \hat{k}_p^s)^2 + K_{pp}^{\rho 3} & K_{ps}^{\rho 0}(\hat{k}_p^g \times \hat{k}_s^g) + K_{ps}^{\rho 1}(\hat{k}_p^g \cdot \hat{k}_s^s)(\hat{k}_p^g \times \hat{k}_p^s) \\ K_{sp}^{\rho 0}(\hat{k}_s^g \times \hat{k}_p^s) + K_{sp}^{\rho 2}(\hat{k}_s^g \cdot \hat{k}_p^s)(\hat{k}_s^g \times \hat{k}_p^s) & K_{ss}^{\rho 0}(\hat{k}_s^g \cdot \hat{k}_s^s) + K_{ss}^{\rho 1}\left[(\hat{k}_s^g \cdot \hat{k}_s^s)^2 - (\hat{k}_s^g \times \hat{k}_s^s)^2\right] \end{bmatrix} \begin{bmatrix} G_{p0} & 0 \\ 0 & G_{s0} \end{bmatrix}$$

P/S-domain FWI

- sensitivities in terms of $\hat{k}_{in}^{P,S}$, $\hat{k}_{out}^{P,S}$, weights

- mismatch between pointwise sensitivity and gross sensitivities due to specular scattering

- geophysicists staying industry-relevant! Can we learn from quantitative AVO and frame rock physics FWI?

- Hessian operator critical to this and to questions of appraisal and data sufficiency.

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