Robust convergence of full waveform inversion by methods on based on physical intuition and geological knowledge: challenges ahead

Biondo Biondi

Stanford University, SEP

Going beyond first-order Born

- Making FWI more Rytov-like (phases vs. amplitudes)
  - Wave-equation tomography

    (Woodward, 1990; Luo and Schuster, 1991, …)
  - Correlation, adaptive waveform inversion, …
- Extend model space to improve linearization
  - Reflectivity-only extension => WEMVA

    (Biondi and Sava, 1999; Shen, Symes and Stolk , 2003)
  - Full extension (Symes, 2008; Biondi and Almomin, 2012, …)
- Add “geologic constraints”

Linearized τ extension $\tilde{\mathfrak{L}}(\tilde{\mathbf{v}}) = \mathfrak{L}(\mathbf{v}_0) + \tilde{\mathbf{L}}\delta\tilde{\mathbf{v}}^2(\tau)$

Full non-linear scattering

$$\left[\partial_{tt} - \mathbf{v}_0^2\nabla^2\right]\mathbf{P}_0 = \mathbf{f}$$

$$\left[\partial_{tt} - \mathbf{v}_0^2\nabla^2\right]\delta\mathbf{P} = \delta\mathbf{v}^2\nabla^2\left(\mathbf{P}_0 + \delta\mathbf{P}\right)$$

$\mathbf{v}_0$: Background velocity

$\mathbf{P}_0$: Background wavefield

$\delta\mathbf{v}$: Velocity perturbation

$\delta\mathbf{P}$: Scattered wavefield

$\mathbf{f}$: Source function

Linearized τ extension

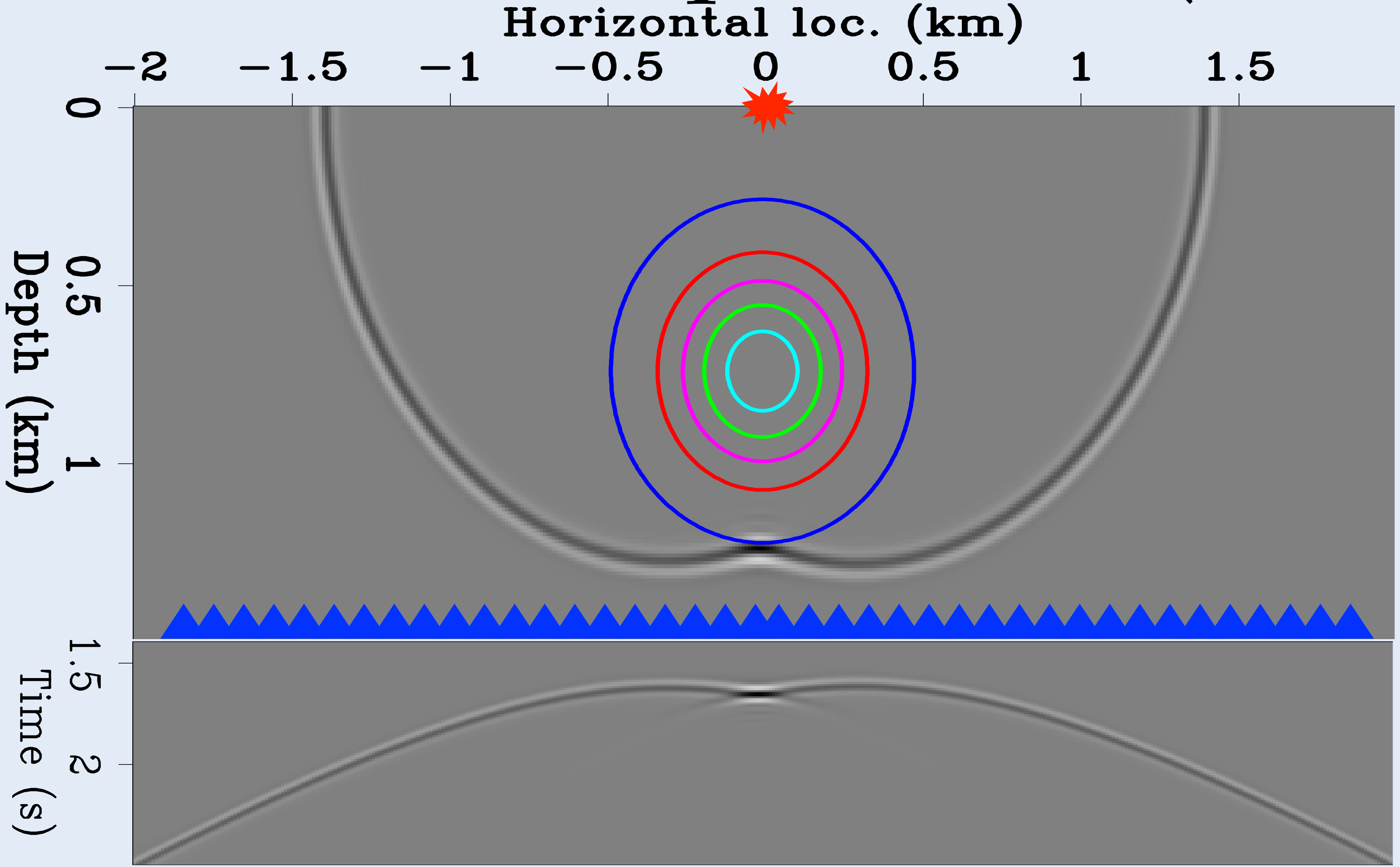
$$\left[\partial_{tt} - \mathbf{v}_0^2\nabla^2\right]\mathbf{P}_0 = \mathbf{f}$$

$$\left[\partial_{tt} - \mathbf{v}_0^2\nabla^2\right]\delta\tilde{\mathbf{P}} = \delta\tilde{\mathbf{v}}(\tau)^2 \overset{\tau}{*} \nabla^2\mathbf{P}_0$$

$\delta\tilde{\mathbf{v}}(\tau)$: Extended-velocity perturbation

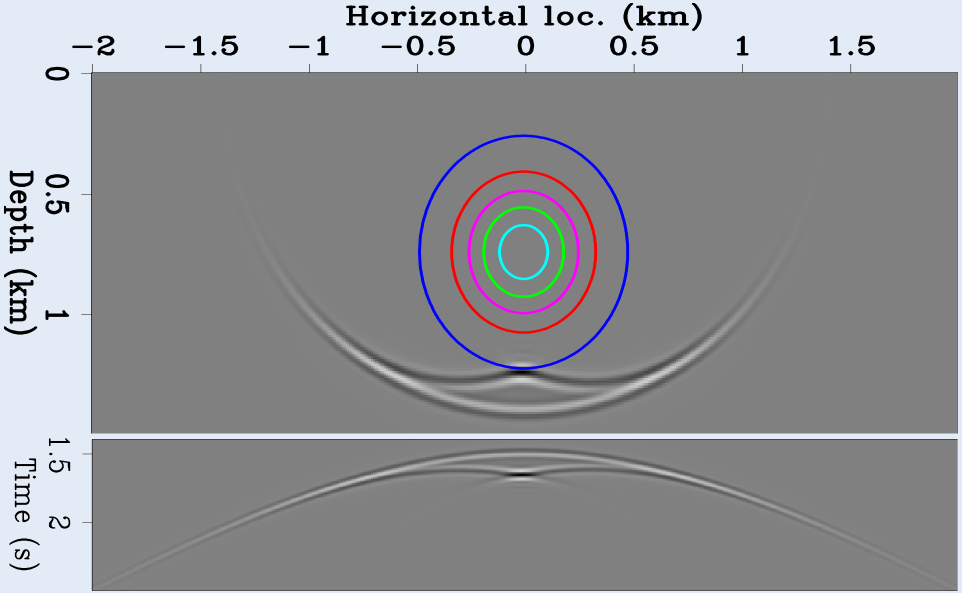
$\delta\tilde{\mathbf{P}}$: New scattered wavefield

Transmission experiment (t=1.4 s)

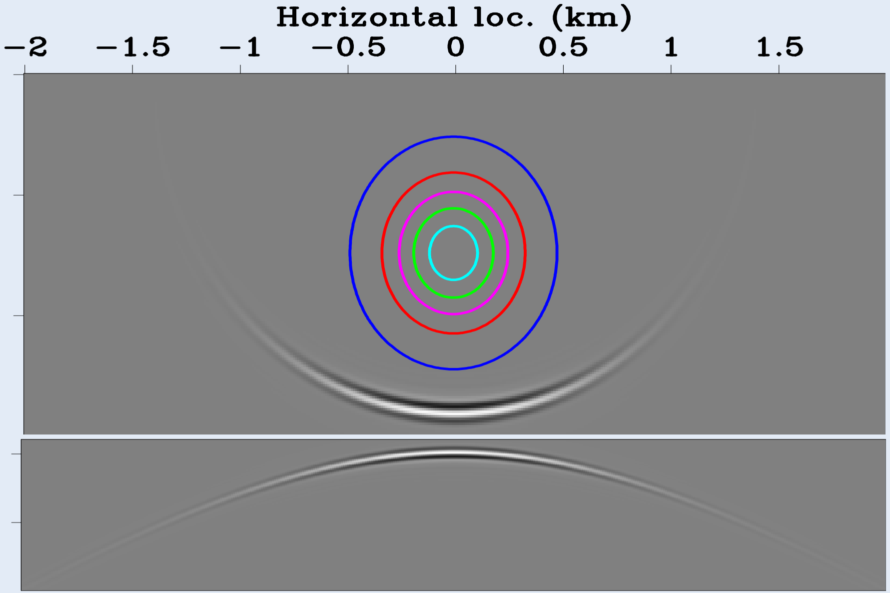


Limitations of Born linearization

Wavefield **and data residuals** by full non-linear scattering



Wavefield **and data residuals** by first-order Born scattering



Derivation of first-order Born scattering

Full non-linear scattering

$$\left[\boldsymbol{\partial}_{\mathbf{tt}} - \mathbf{v}_0^2\nabla^2\right]\mathbf{P}_0 = \mathbf{f}$$

$$\left[\boldsymbol{\partial}_{\mathbf{tt}} - \mathbf{v}_0^2\nabla^2\right]\delta\mathbf{P} = \delta\mathbf{v}^2\nabla^2\left(\mathbf{P}_0 + \delta\mathbf{P}\right)$$

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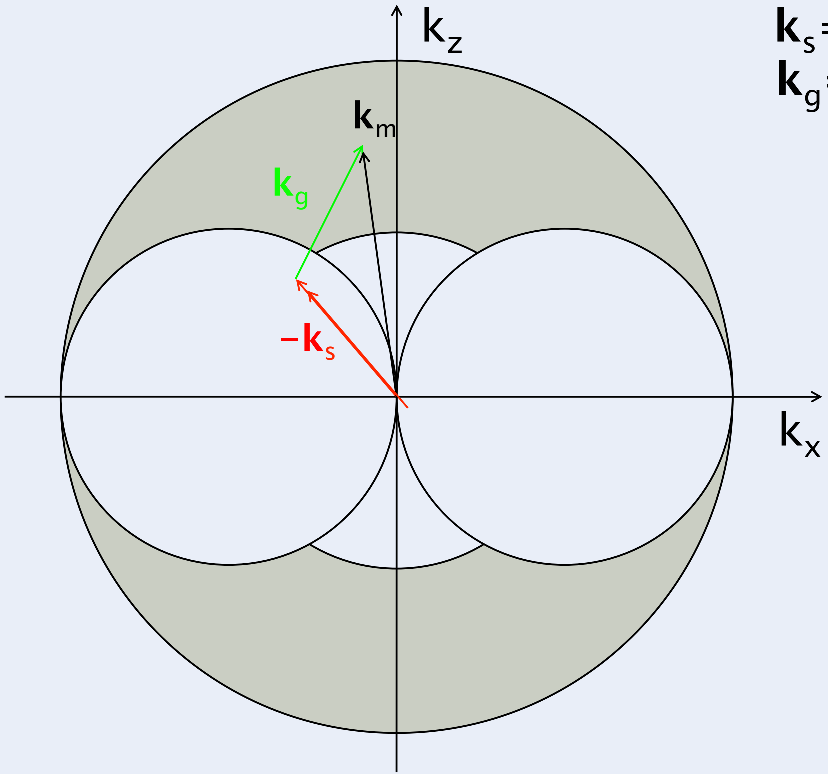
First-order Born scattering

$$\left[\boldsymbol{\partial}_{\mathbf{tt}} - \mathbf{v}_0^2\nabla^2\right]\mathbf{P}_0 = \mathbf{f}$$

$$\left[\boldsymbol{\partial}_{\mathbf{tt}} - \mathbf{v}_0^2\nabla^2\right]\delta\hat{\mathbf{P}} = \delta\mathbf{v}^2\nabla^2\mathbf{P}_0$$

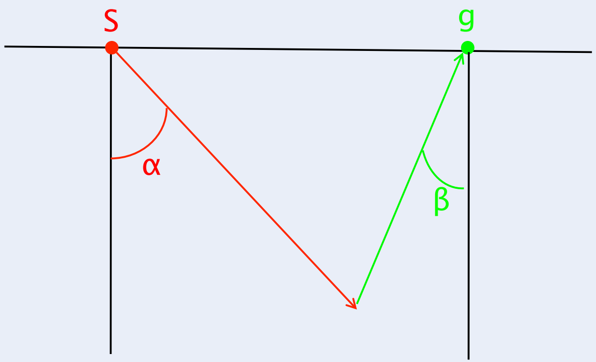
$\delta\hat{\mathbf{P}}$: Born scattered wavefield

Reflection (back scattered) data: One frequency ω



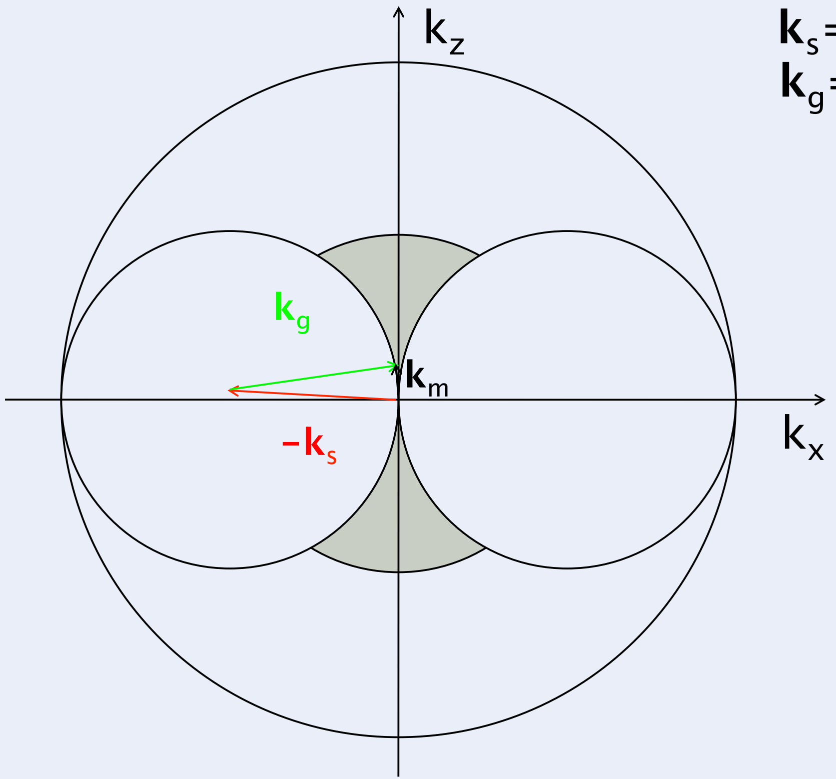
$\mathbf{k}_s = \omega/v \sin\alpha\, \mathbf{k}_x - \omega/v \cos\alpha\, \mathbf{k}_z$
$\mathbf{k}_g = \omega/v \sin\beta\, \mathbf{k}_x - \omega/v \cos b\, \mathbf{k}_z$

$\mathbf{K}_m = \mathbf{k}_g - \mathbf{k}_s$



biondo@stanford.edu

Refraction (forward scatt.) data: One frequency ω



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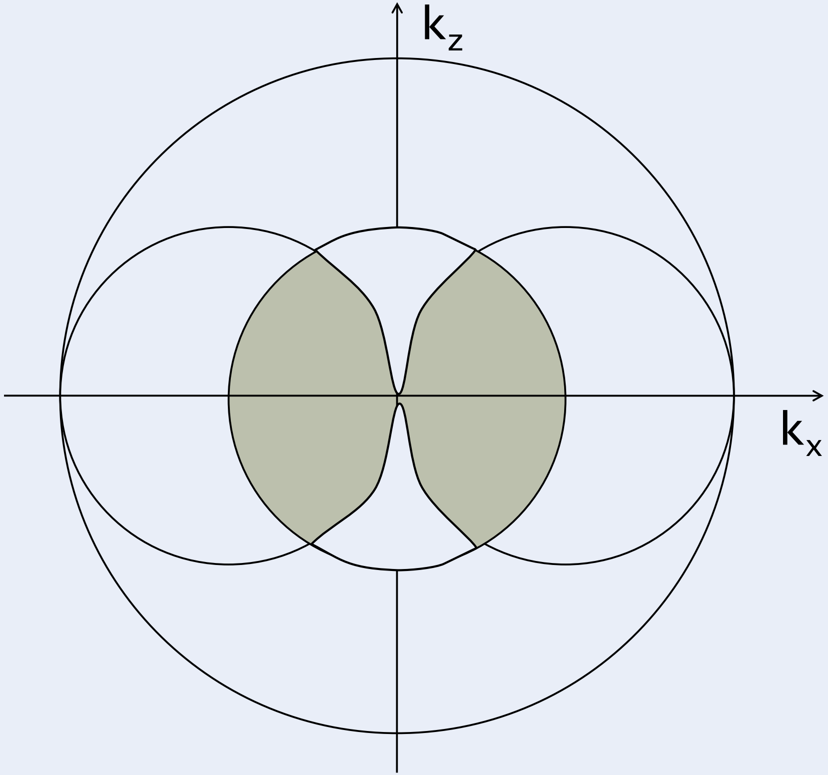
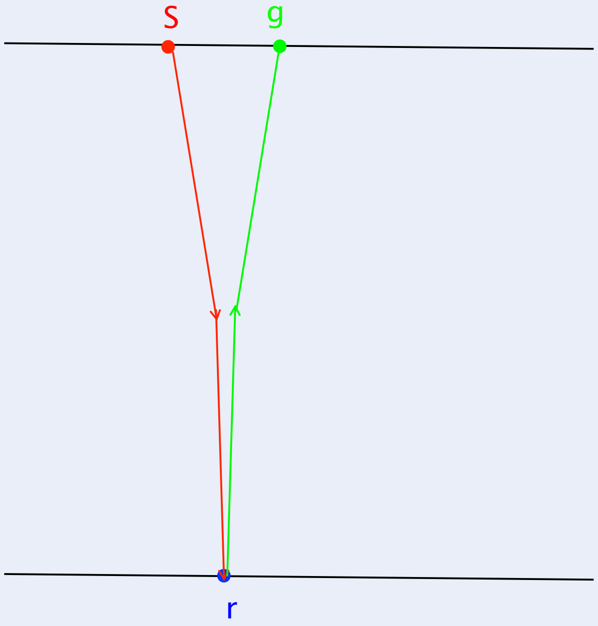
Transmission (forward scatt.) in reflection data



$K_{ms}=k_{s2}-k_{s1}$

$K_{mg}=k_{g2}-k_{g1}$



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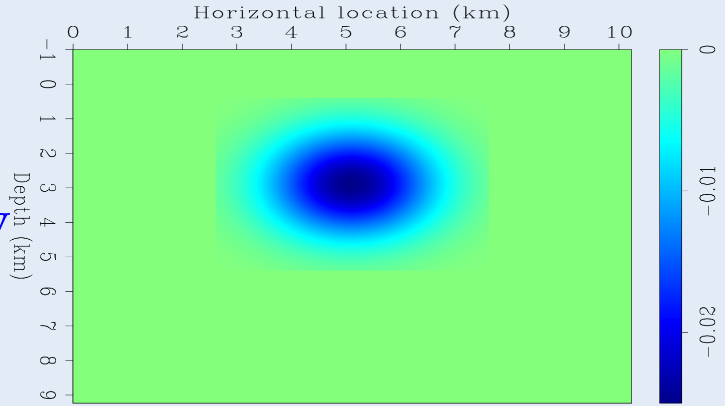
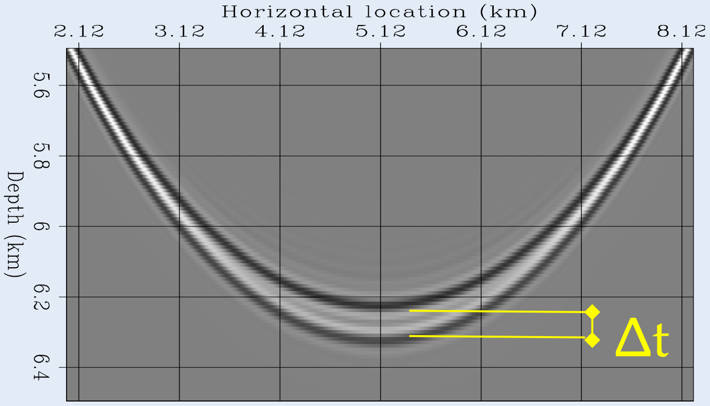
Data ← Non linear → Model

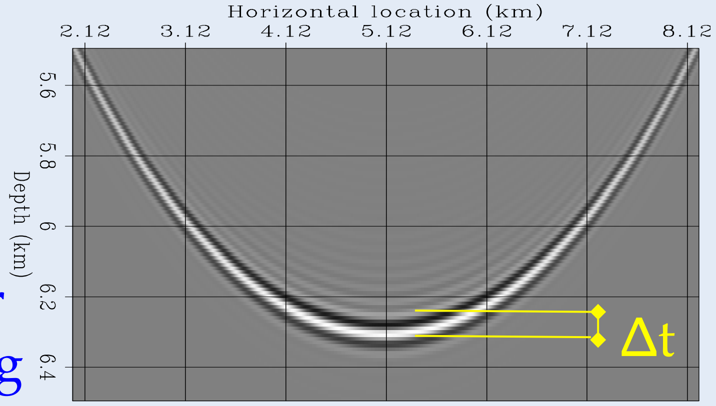
ΔVelocity

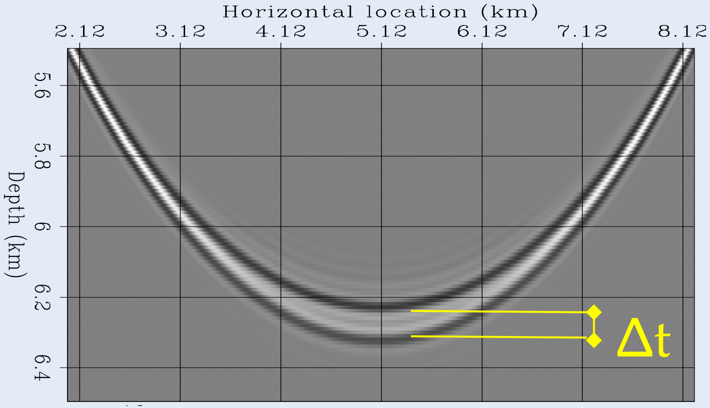




Δwave correct

Δwave with 1^{st} order scattering





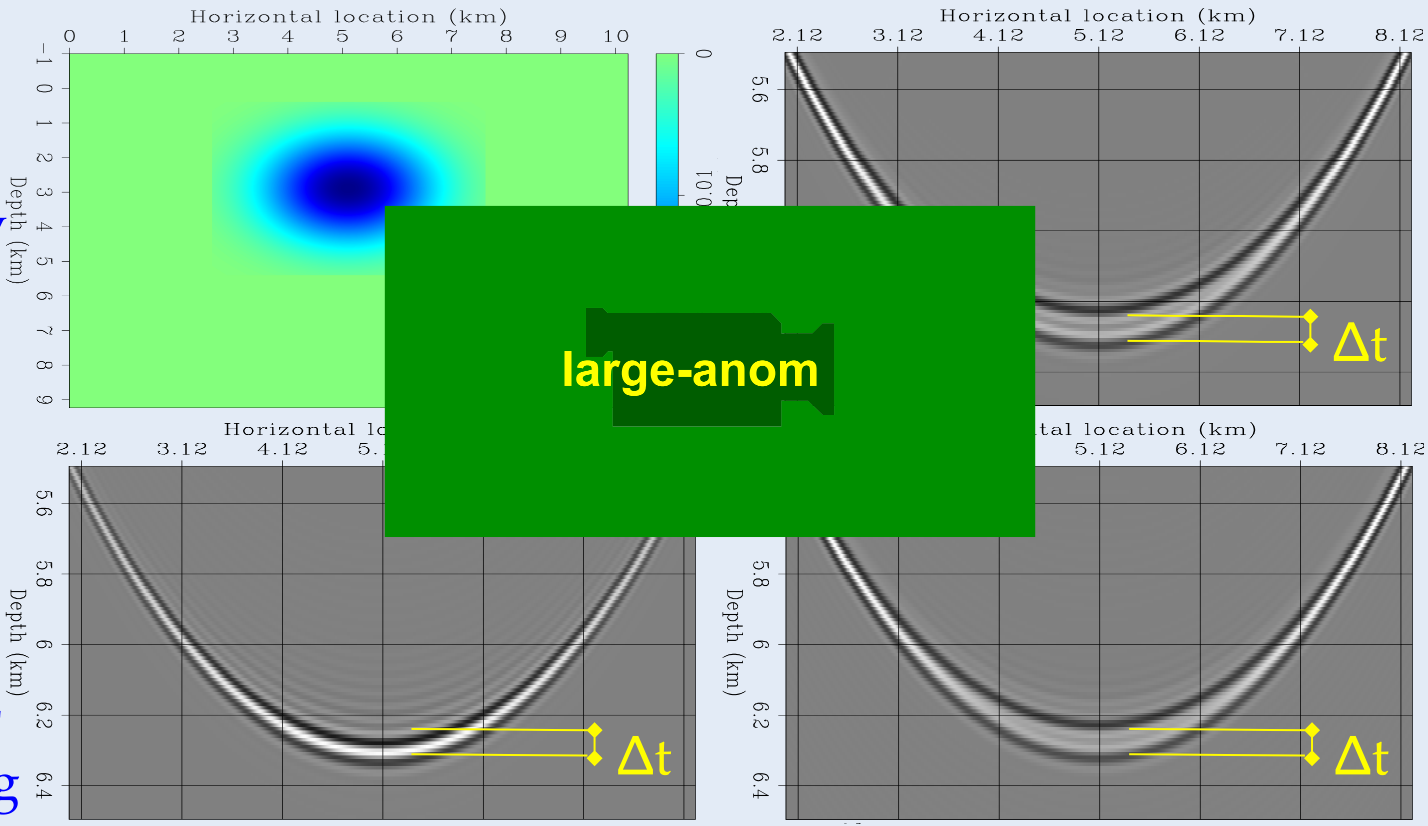
Δwave with 25^{th} order scattering

STANFORD
EXPLORATION PROJECT

Data ← Non linear → Model

ΔVelocity

Δwave correct



Δwave with 1st order scattering

Δwave with 25th order scattering

STANFORD EXPLORATION PROJECT

Linearized τ extension $\tilde{\mathfrak{L}}(\tilde{\mathbf{v}}) = \mathfrak{L}(\mathbf{v}_0) + \tilde{\mathbf{L}}\delta\tilde{\mathbf{v}}^2(\tau)$

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FWI vs. TFWI

FWI

$$J_{\text{FWI}}(\mathbf{v}) = \frac{1}{2}\left\|\mathfrak{L}(\mathbf{v}) - \mathbf{d}\right\|_2^2$$

J is the objective function to optimize,

$\mathfrak{L}$ is non-linear modeling operator,

$\mathbf{v}$ is velocity model,

$\mathbf{d}$ are data.

TFWI

$$J_{\text{TFWI}}(\tilde{\mathbf{v}}) = \frac{1}{2}\left\|\tilde{\mathfrak{L}}(\tilde{\mathbf{v}}) - \mathbf{d}\right\|_2^2 \mp \varepsilon\left\|\mathfrak{F}(\tilde{\mathbf{v}})\right\|_2^2$$

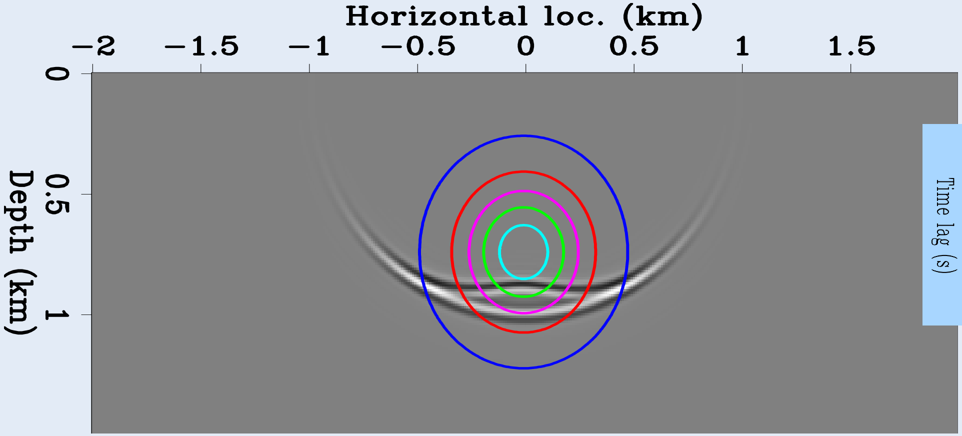
$\tilde{\mathfrak{L}}(\tilde{\mathbf{v}})$ is the extended modeling operator,

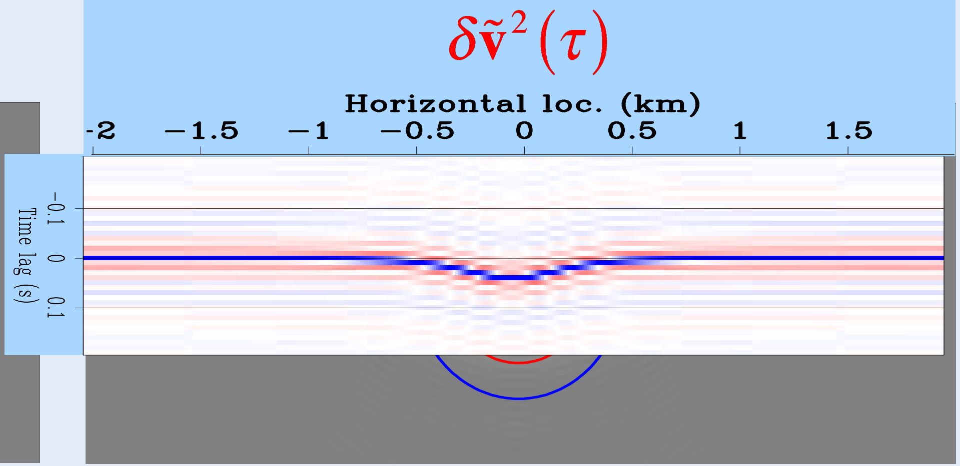
$\tilde{\mathbf{v}}$ is the extended velocity, e.g. $\tilde{\mathbf{v}}(\tau)$,

$\mathfrak{F}(\tilde{\mathbf{v}})$ measures focusing of $\tilde{\mathbf{v}}$, e.g. $\left\||\tau|\tilde{\mathbf{v}}\right\|_2^2$.

Beyond Born – Extended velocity

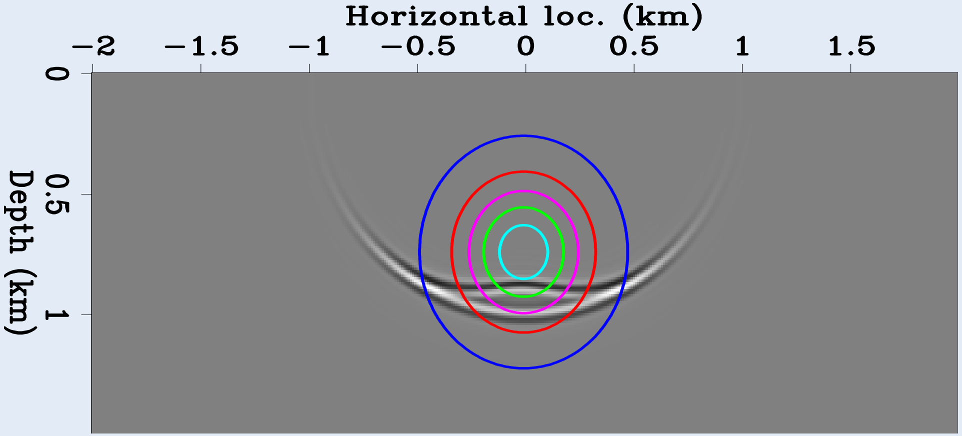
Horizontal section across anomaly

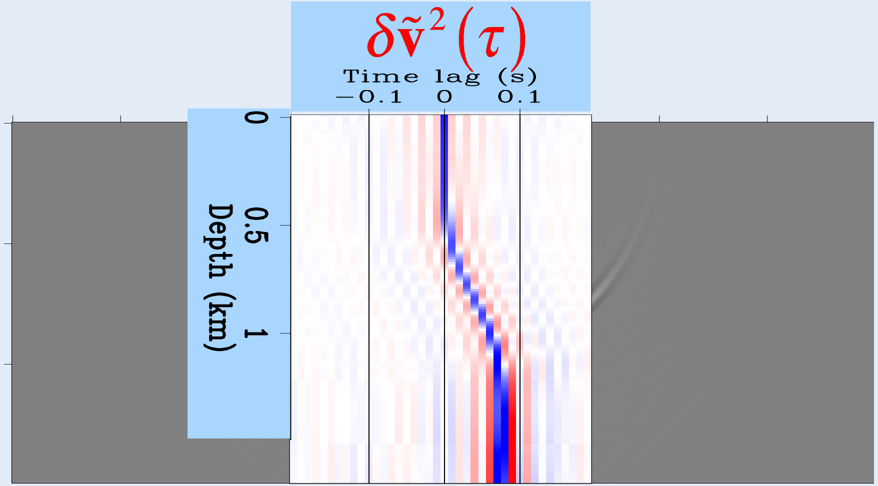




Beyond Born – Extended velocity

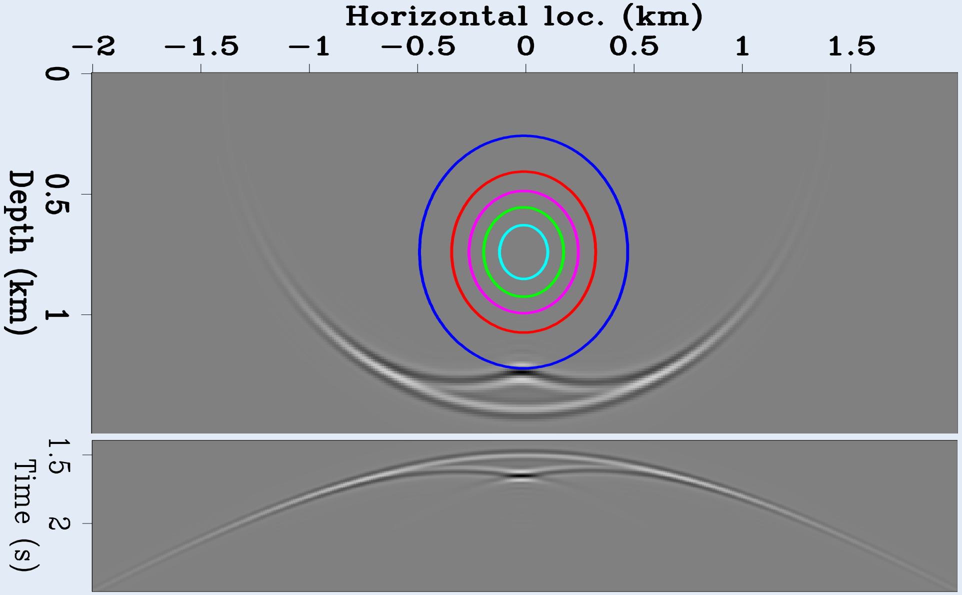
Vertical section across anomaly



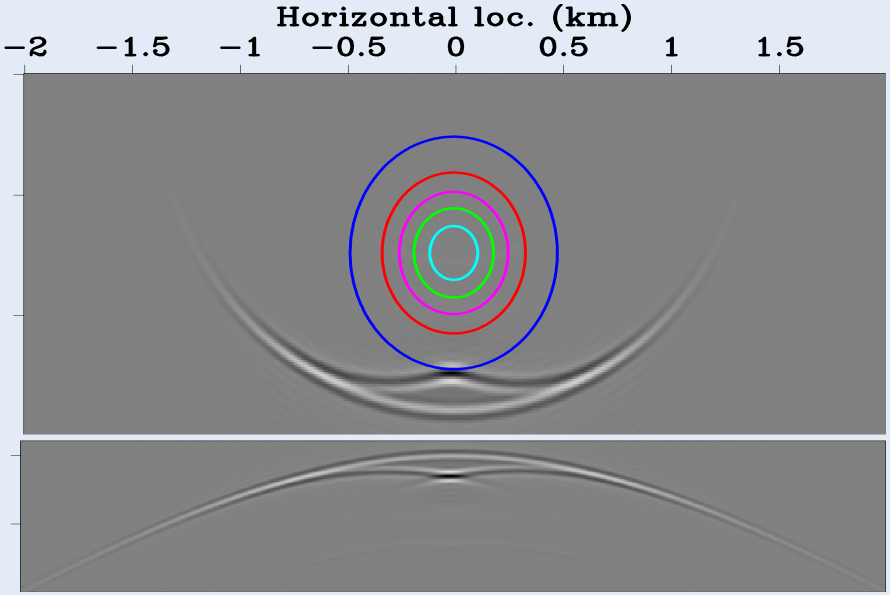


Beyond Born – Data residuals

Wavefield **and data residuals** by full non-linear scattering

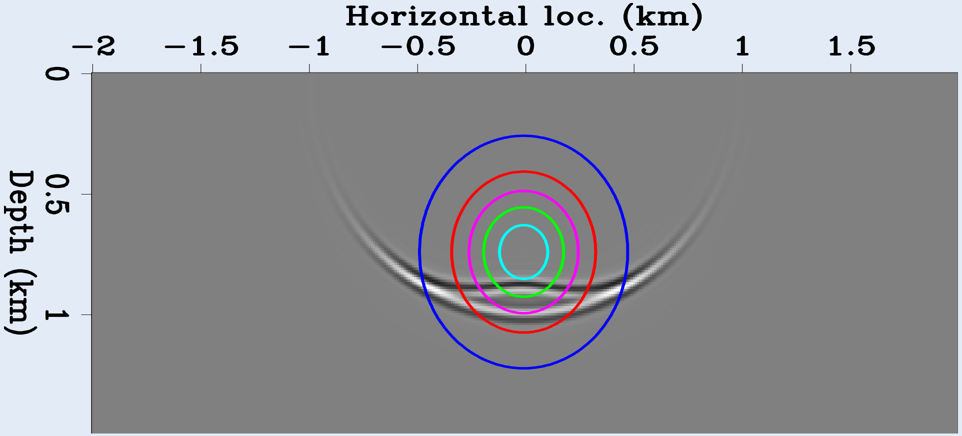


Wavefield **and data residuals** by linearized τ extension

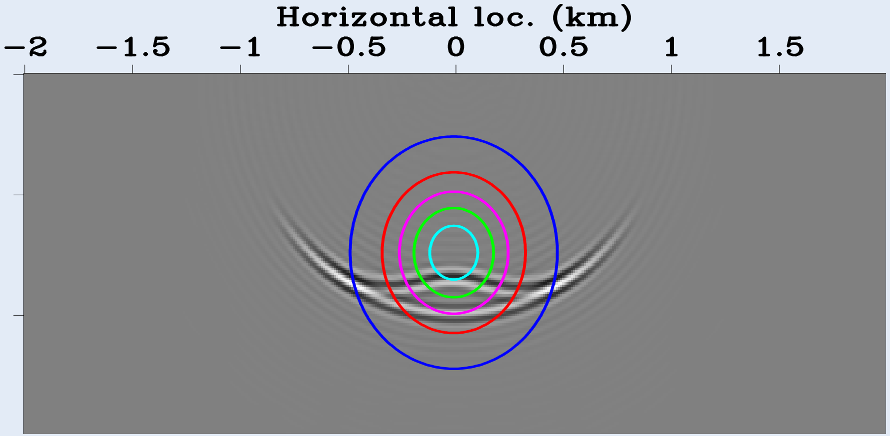


Beyond Born – $\Delta\tilde{\mathbf{v}}(\tau) = \tilde{\mathbf{L}}'\Delta\mathbf{d}$

Wavefield computed by full non-linear scattering



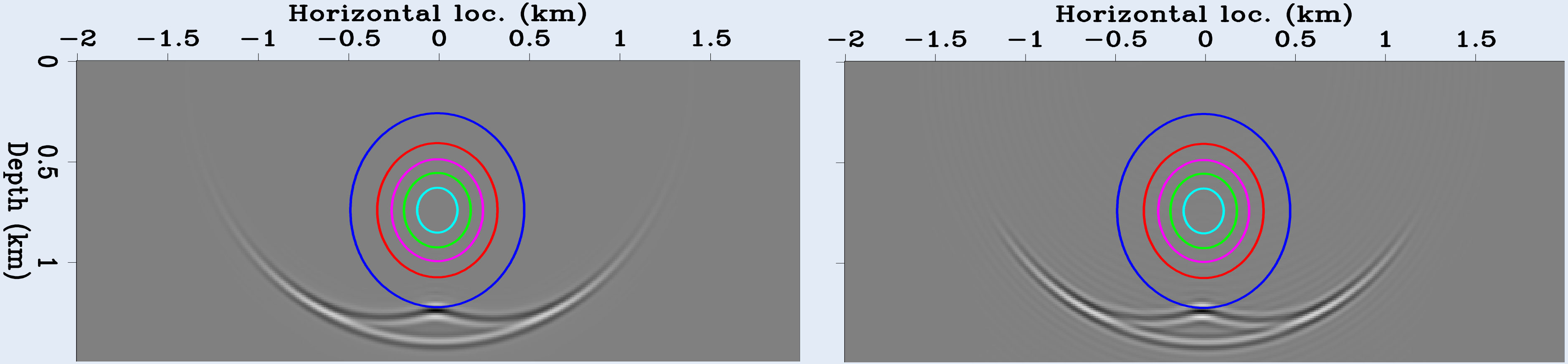
Wavefield by linearized τ extension with $\Delta\tilde{\mathbf{v}}(\tau) = \tilde{\mathbf{L}}'\Delta\mathbf{d}$



Beyond Born – $\Delta\tilde{\mathbf{v}}(\tau)=\tilde{\mathbf{L}}'\Delta\mathbf{d}$

Wavefield computed by full non-linear scattering

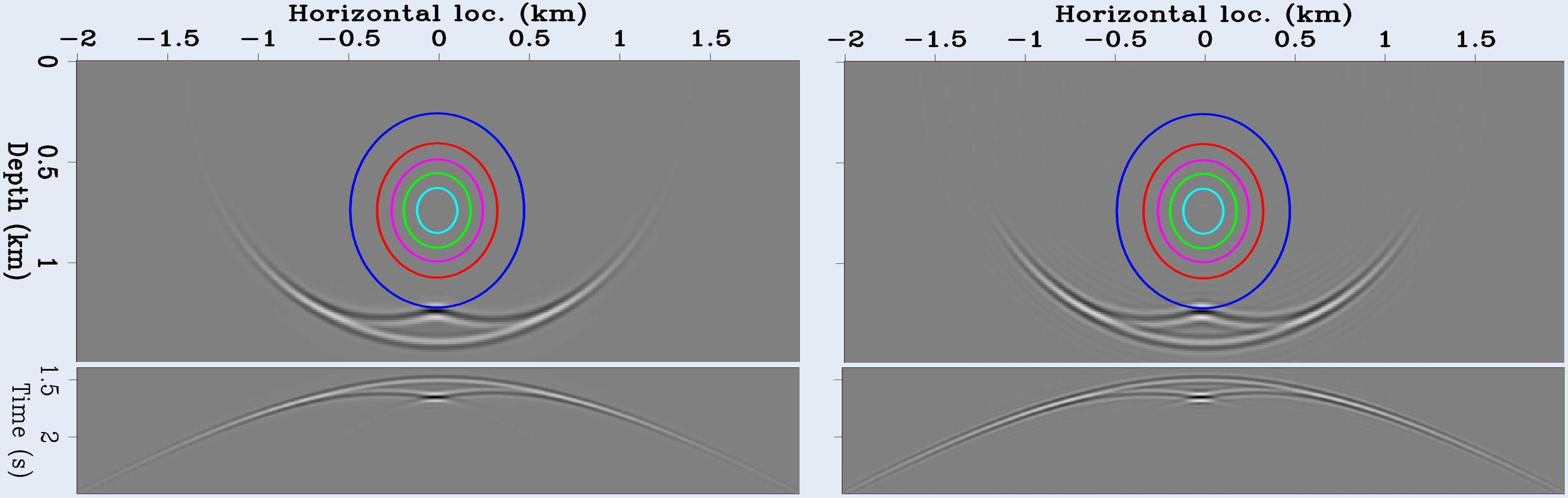
Wavefield by linearized τ extension with $\Delta\tilde{\mathbf{v}}(\tau)=\tilde{\mathbf{L}}'\Delta\mathbf{d}$



Beyond Born – $\Delta\tilde{\mathbf{v}}(\tau) = \tilde{\mathbf{L}}'\Delta\mathbf{d}$

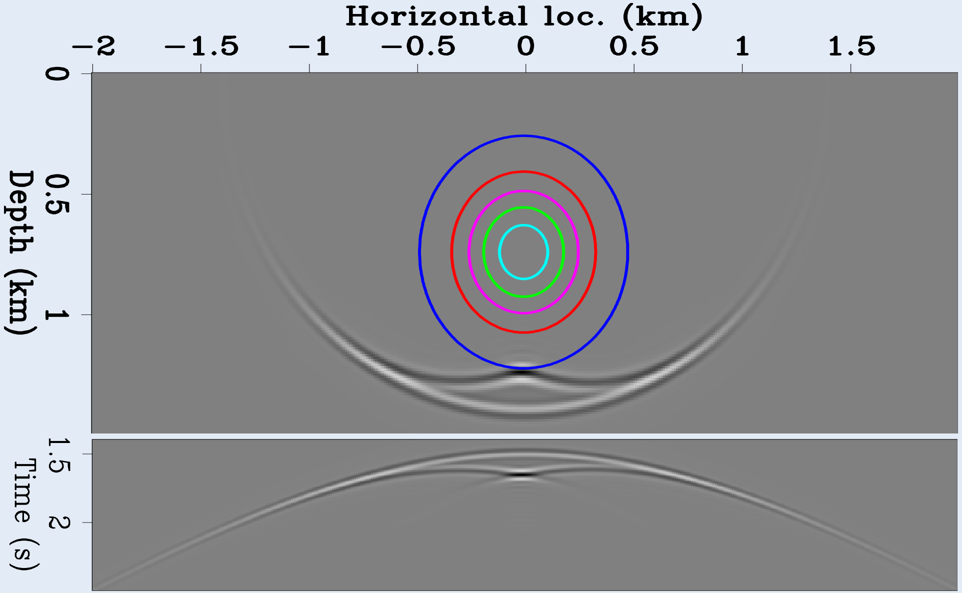
Data residuals computed by full non-linear scattering

Data residuals by linearized τ extension with $\Delta\tilde{\mathbf{v}}(\tau) = \tilde{\mathbf{L}}'\Delta\mathbf{d}$

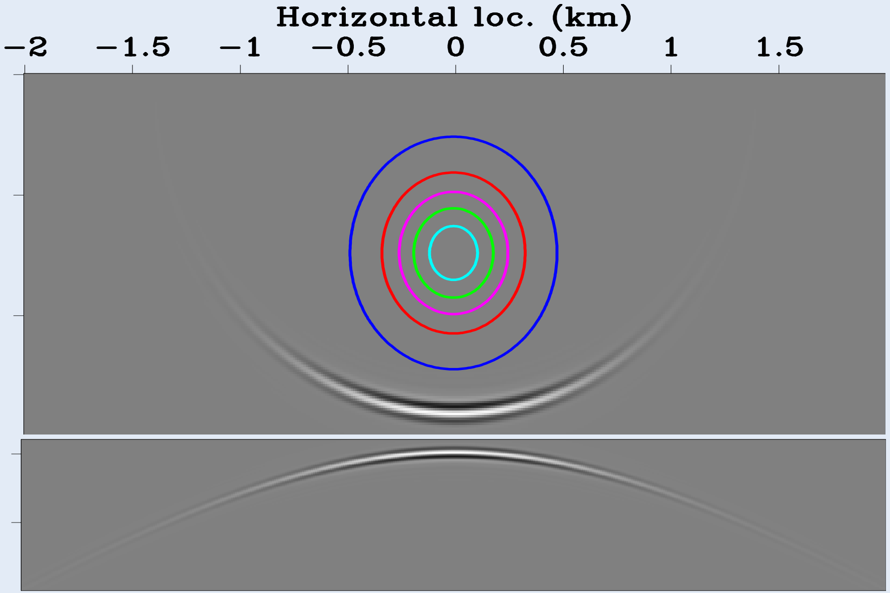


Limitations of Born linearization

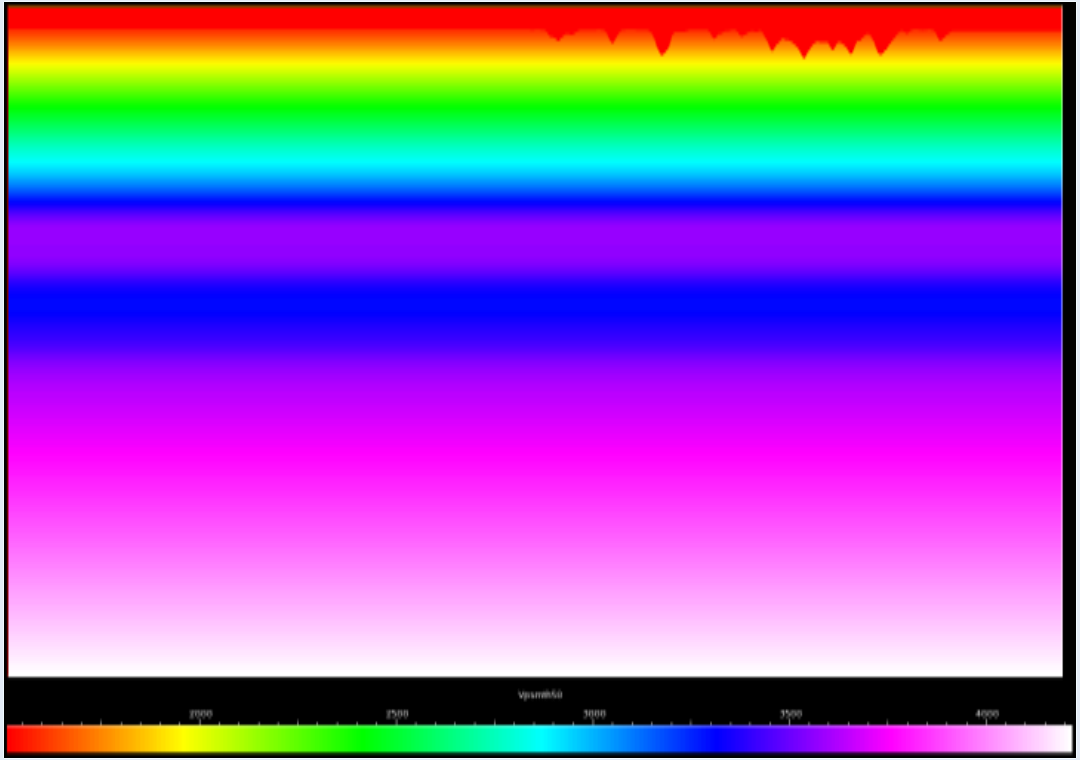
Wavefield **and data residuals** by full non-linear scattering

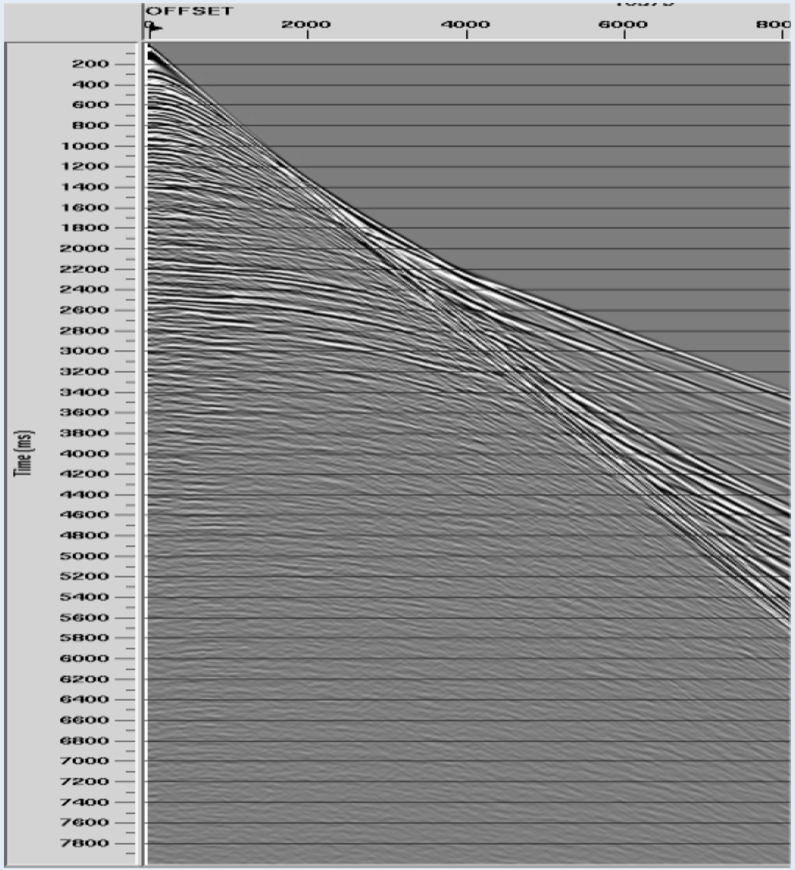


Wavefield **and data residuals** by first-order Born scattering



2014 SEG FWI Blind Test (3-35 Hz)





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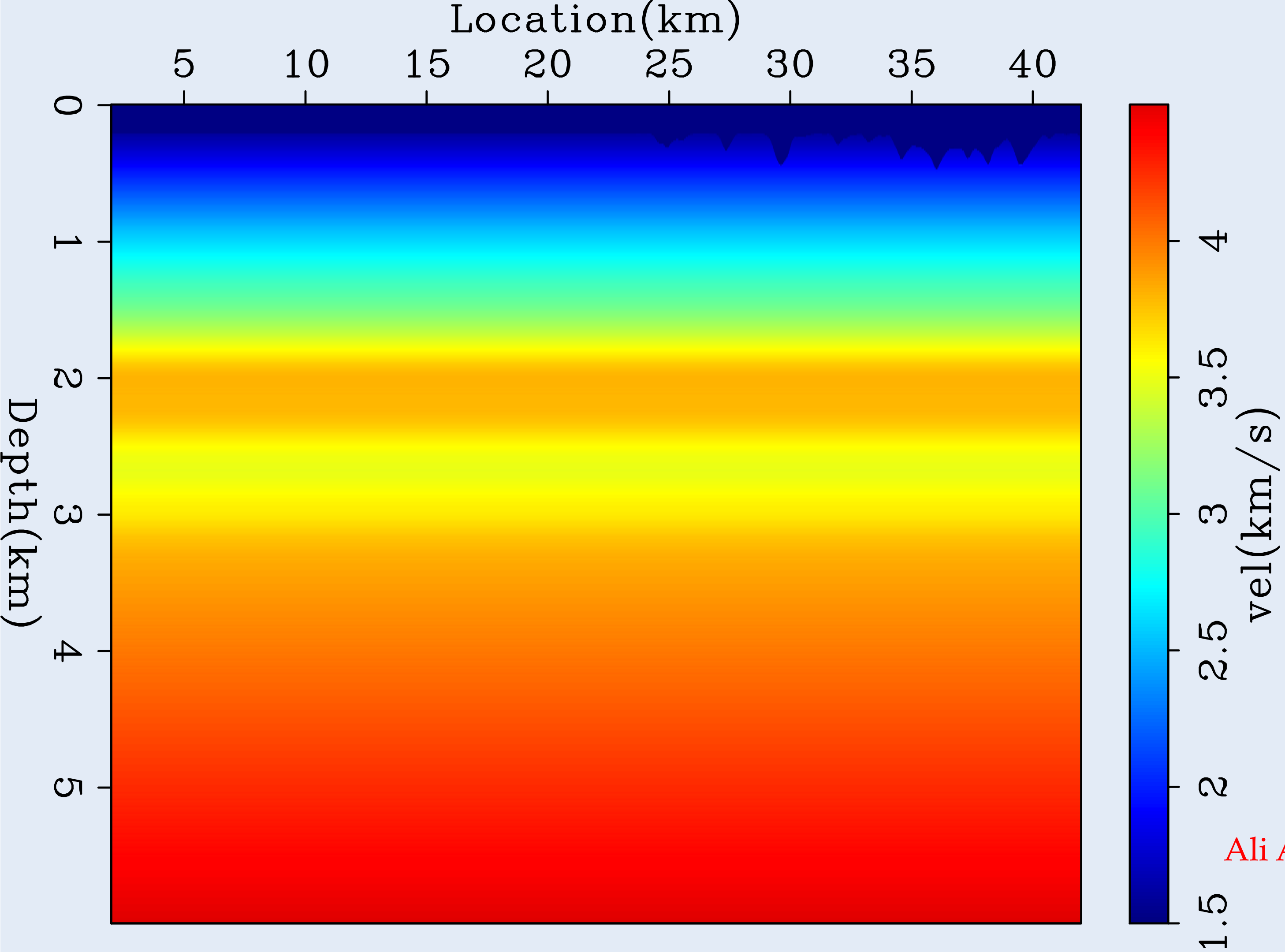

Initial velocity

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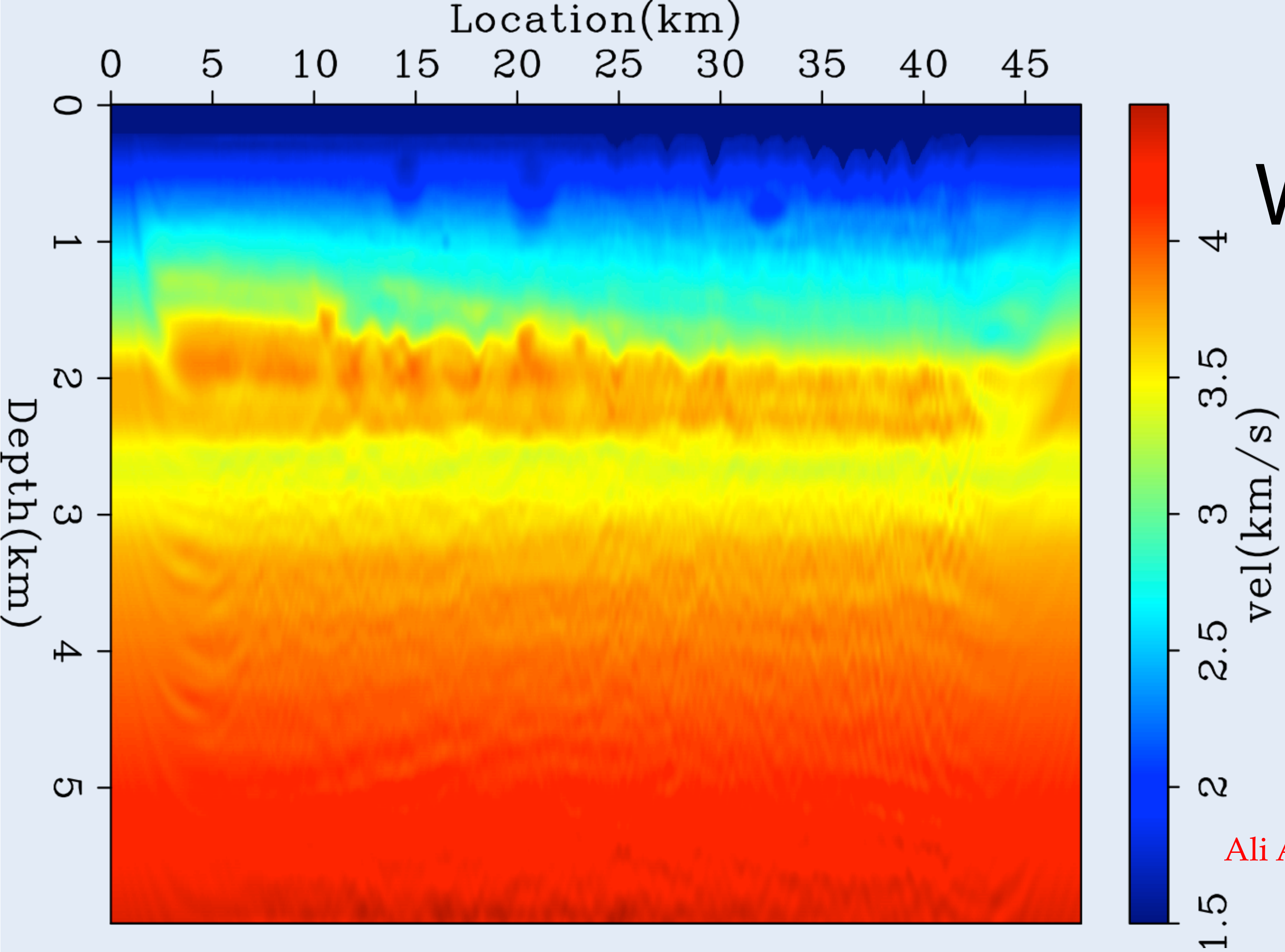

WET+FWI velocity

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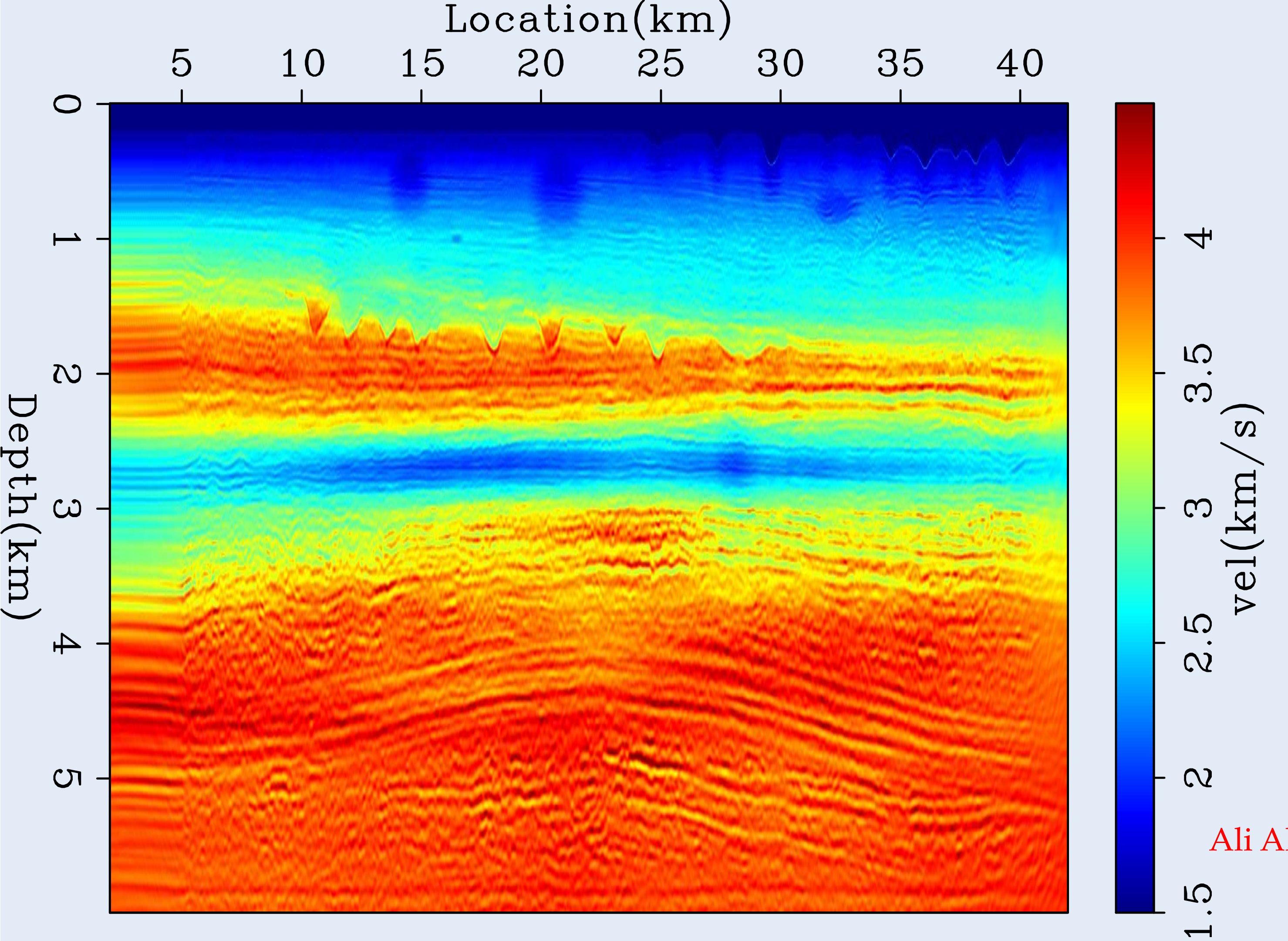

TFWI velocity

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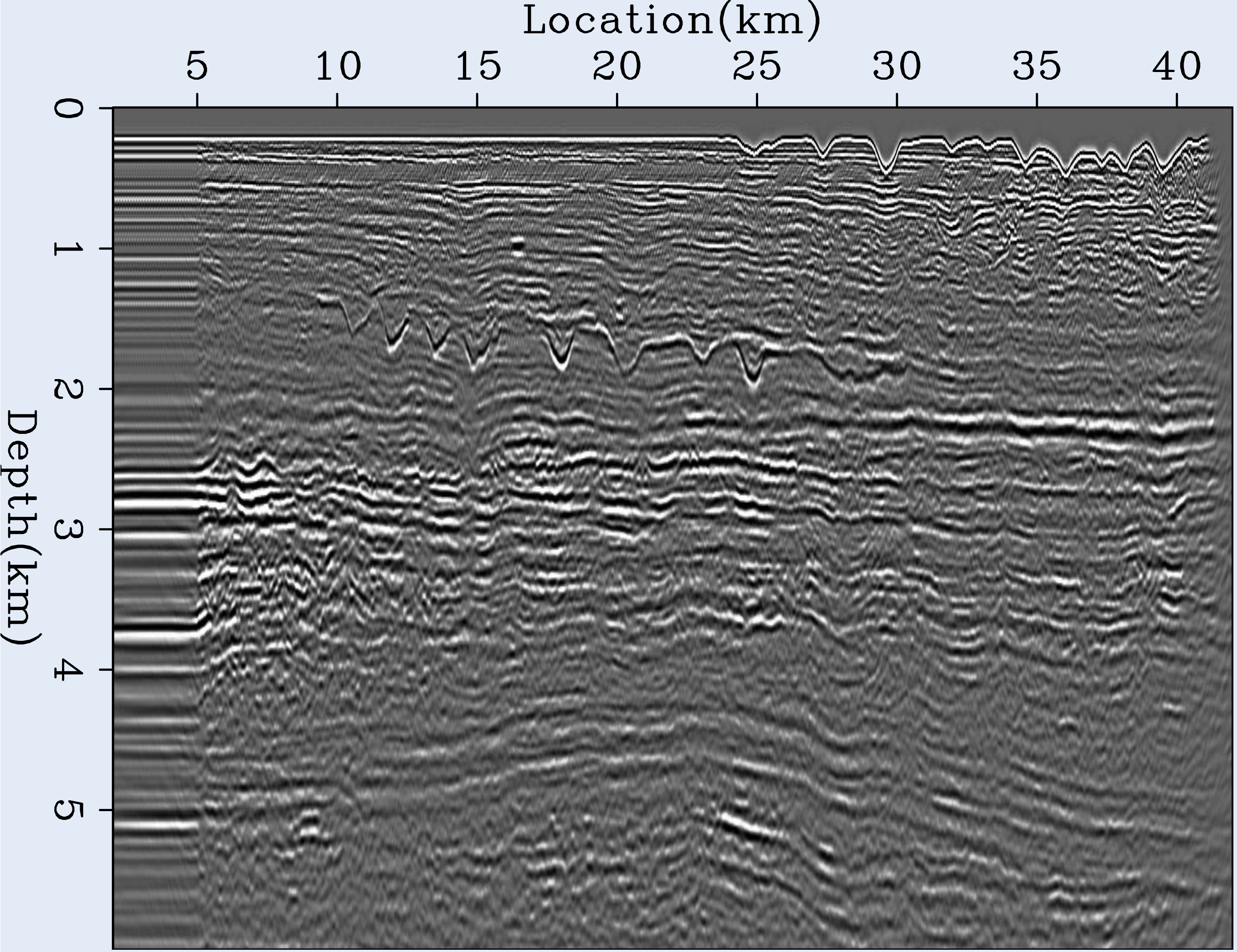

Image with initial velocity

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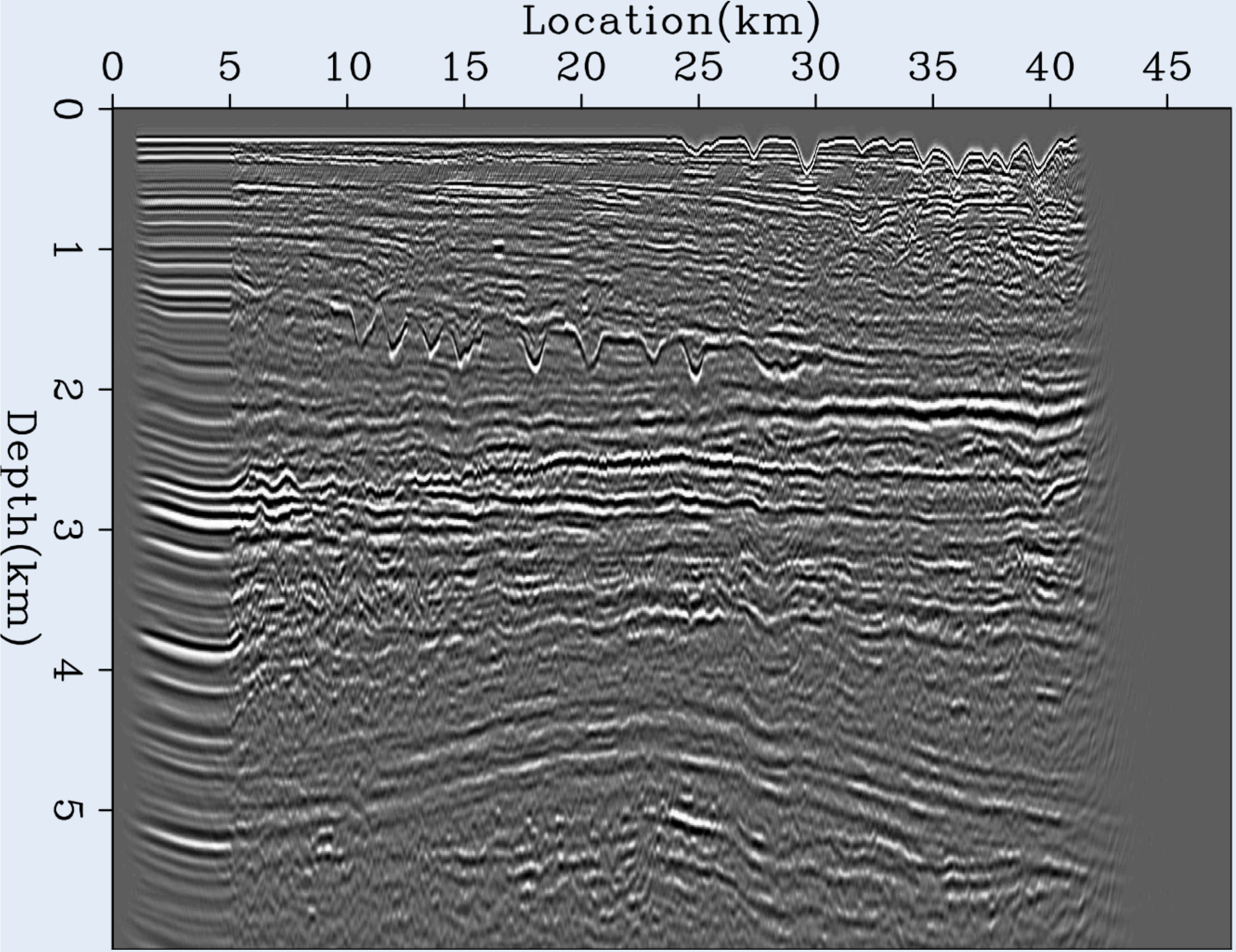

Image with WET+FWI velocity

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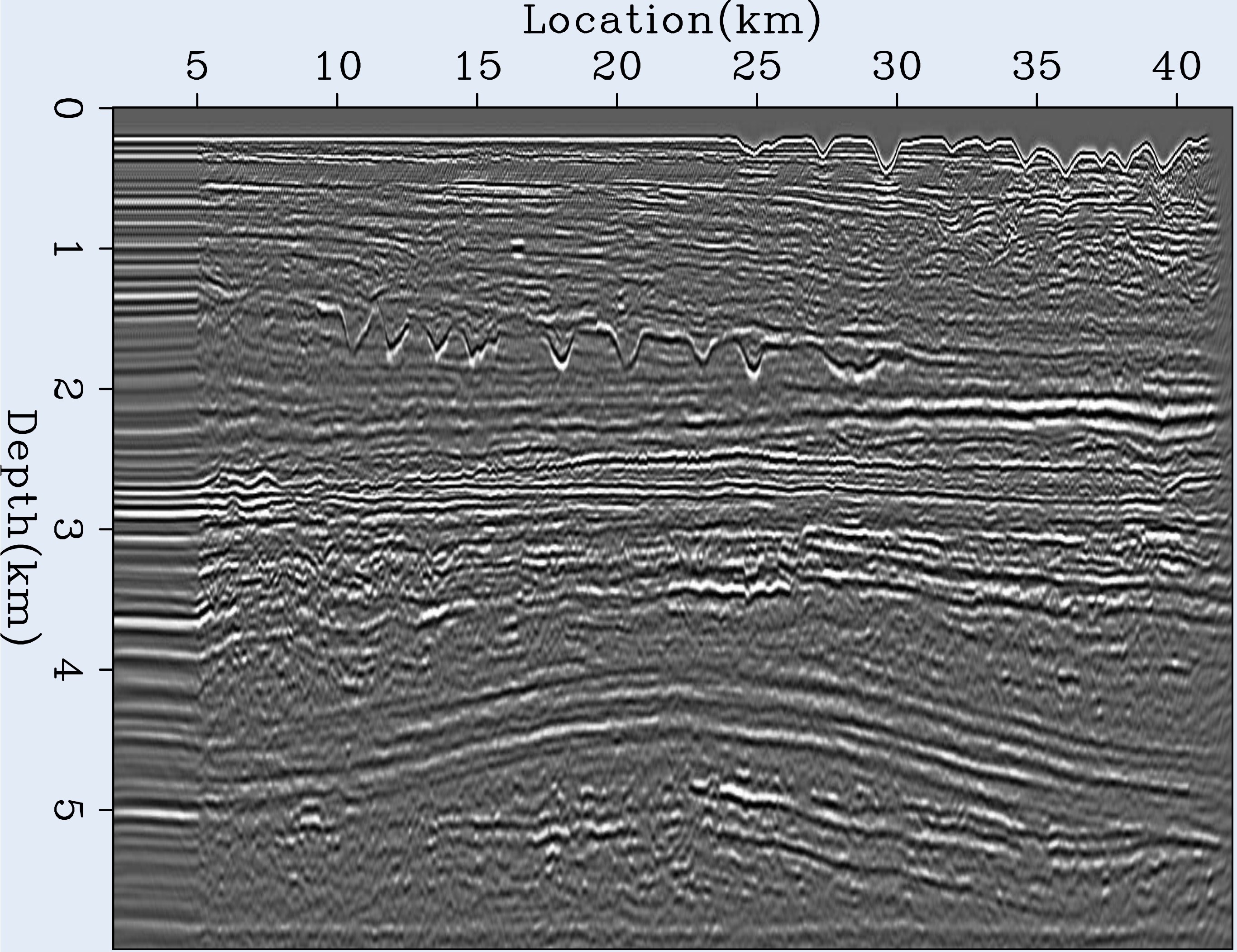

Image with TFWI velocity

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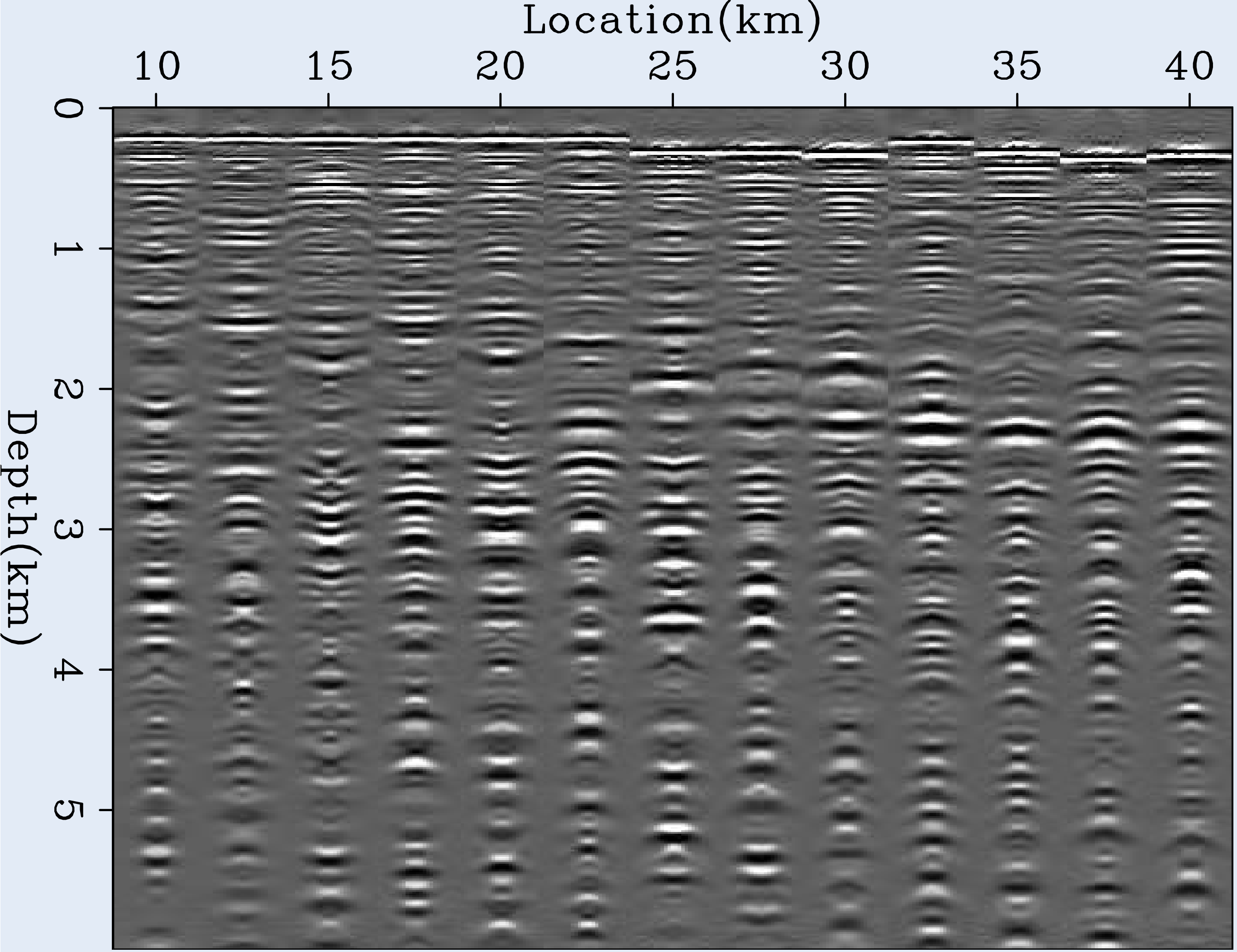

CIGs with initial velocity

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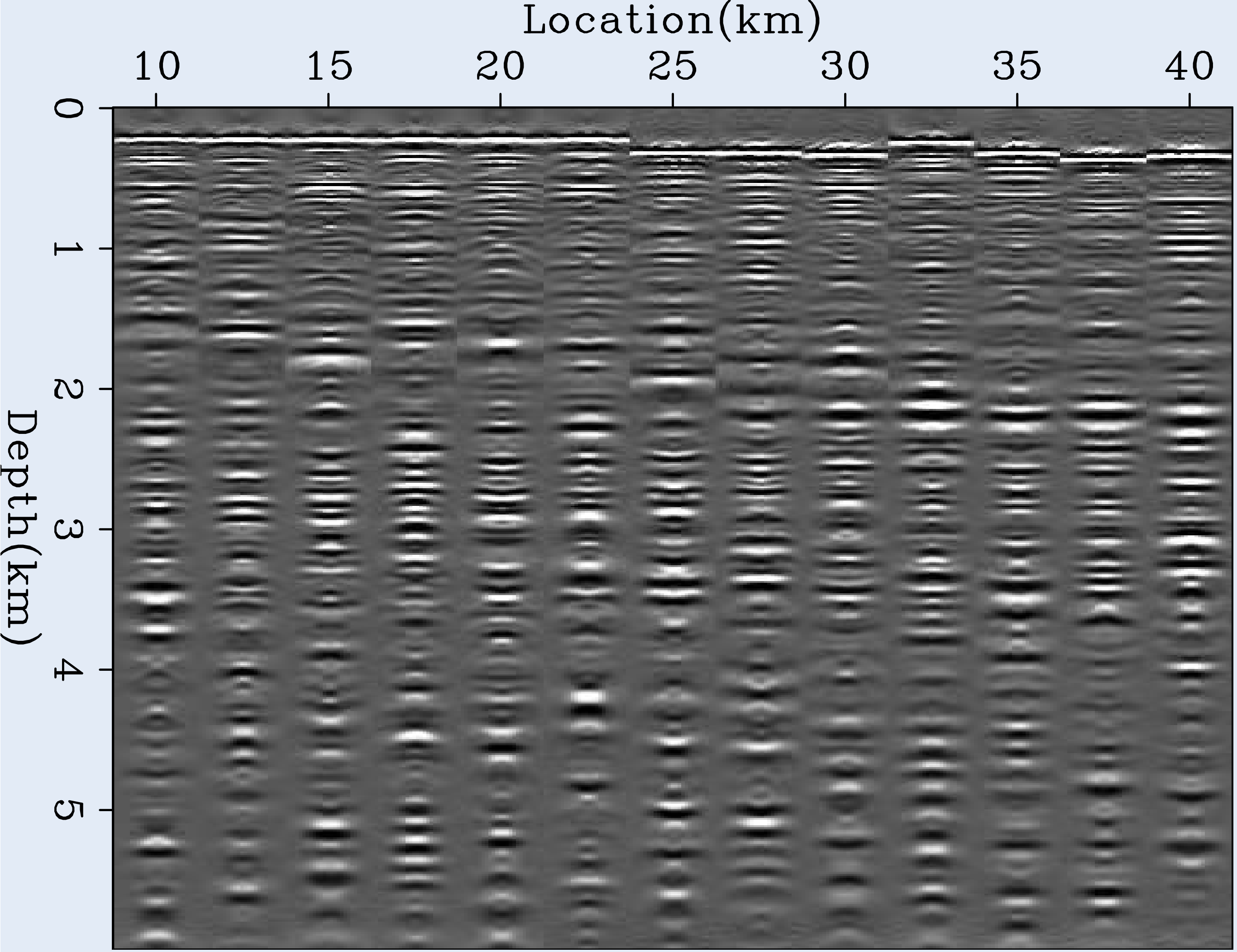

CIGs with TFWI velocity

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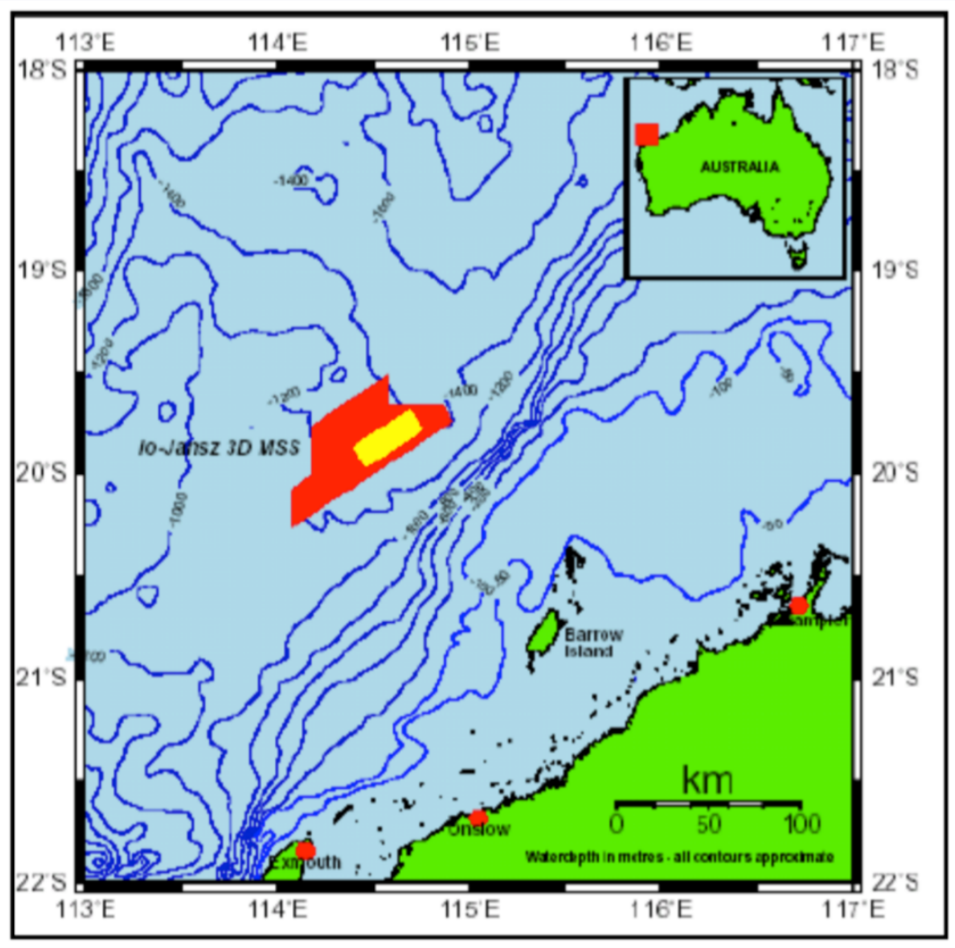

IO-Jansz

NW Australia

Conventional streamer data

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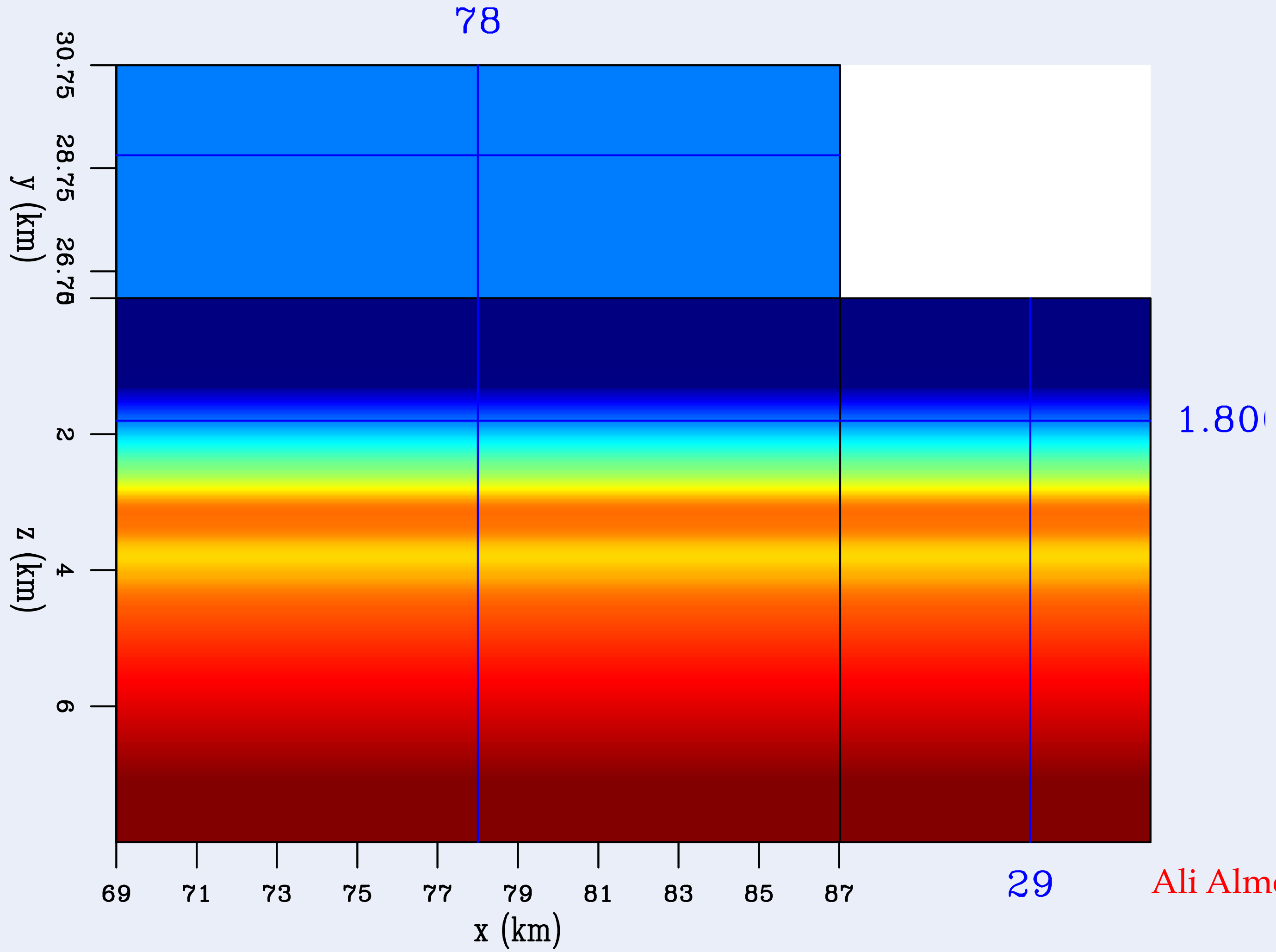
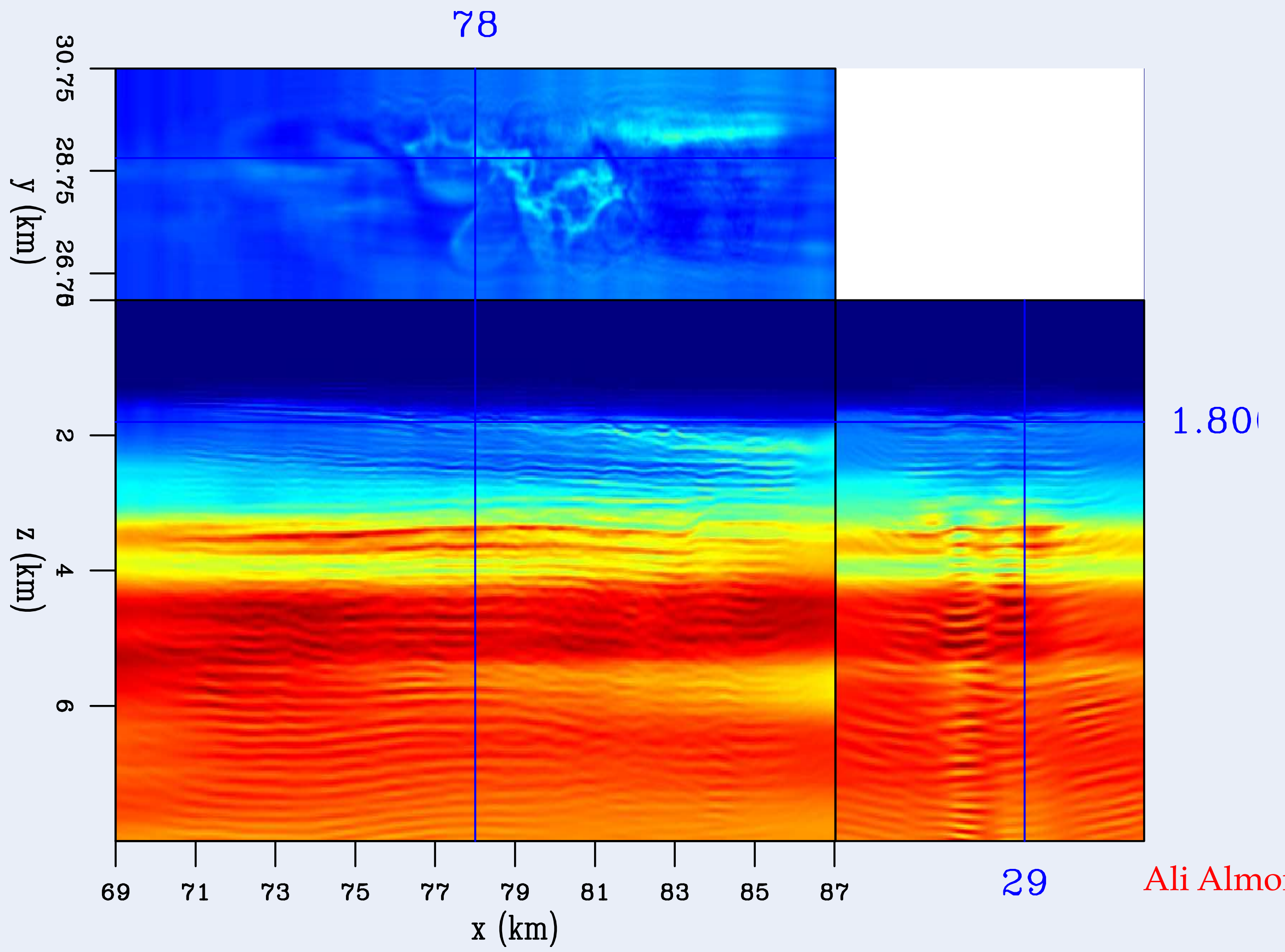


Initial velocity



TFWI velocity

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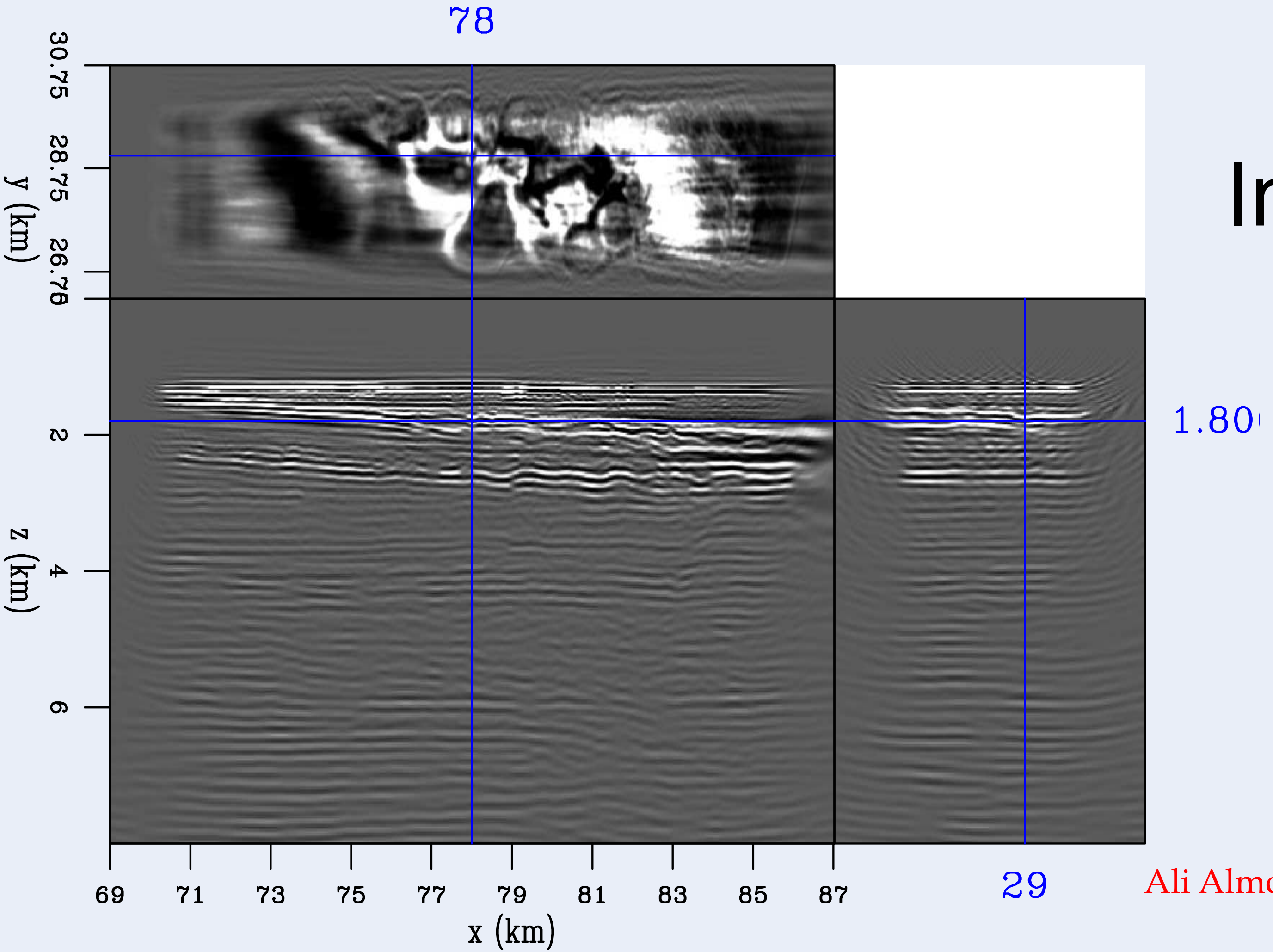

Image with initial velocity

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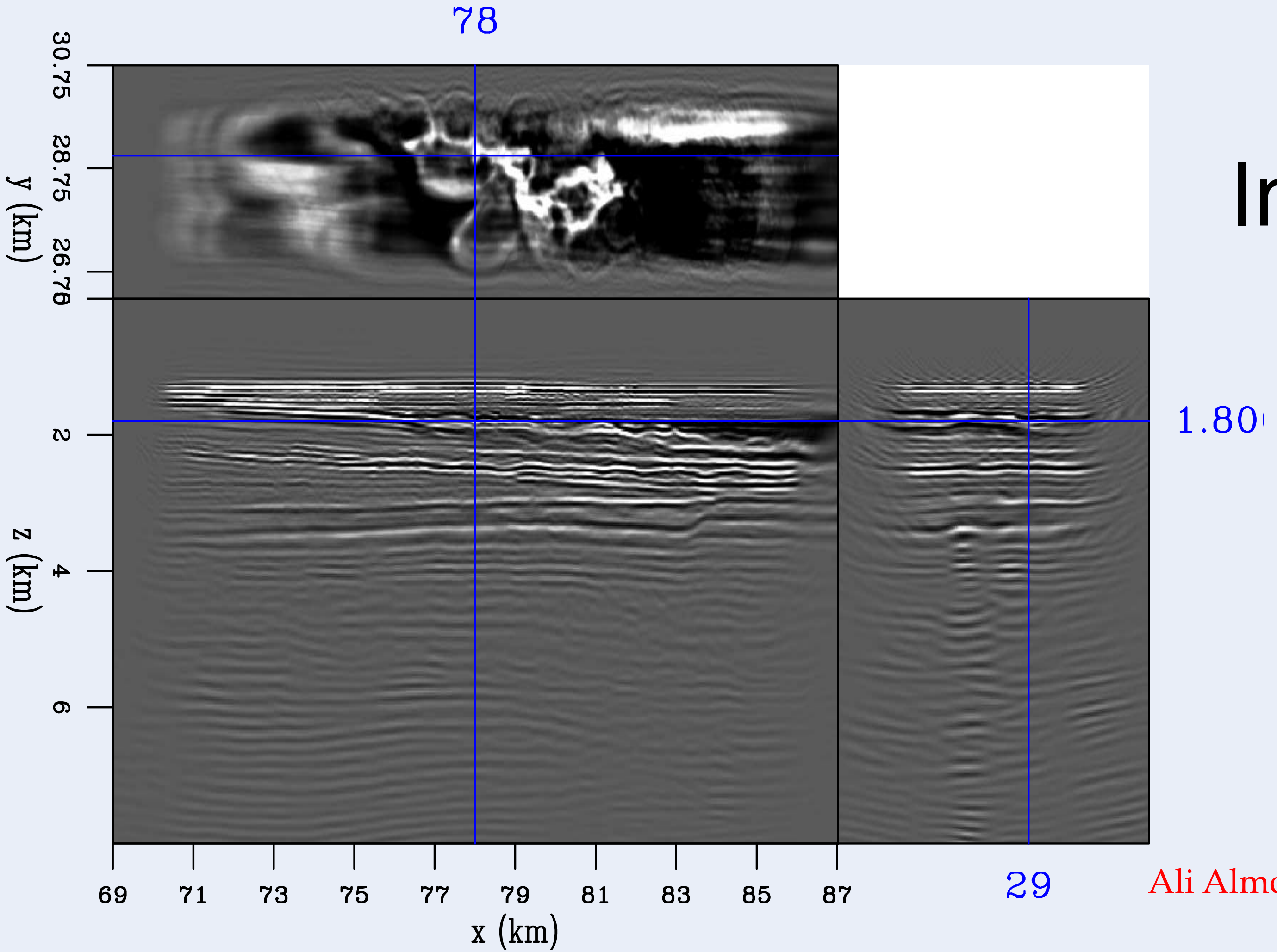
Image with TFWI velocity



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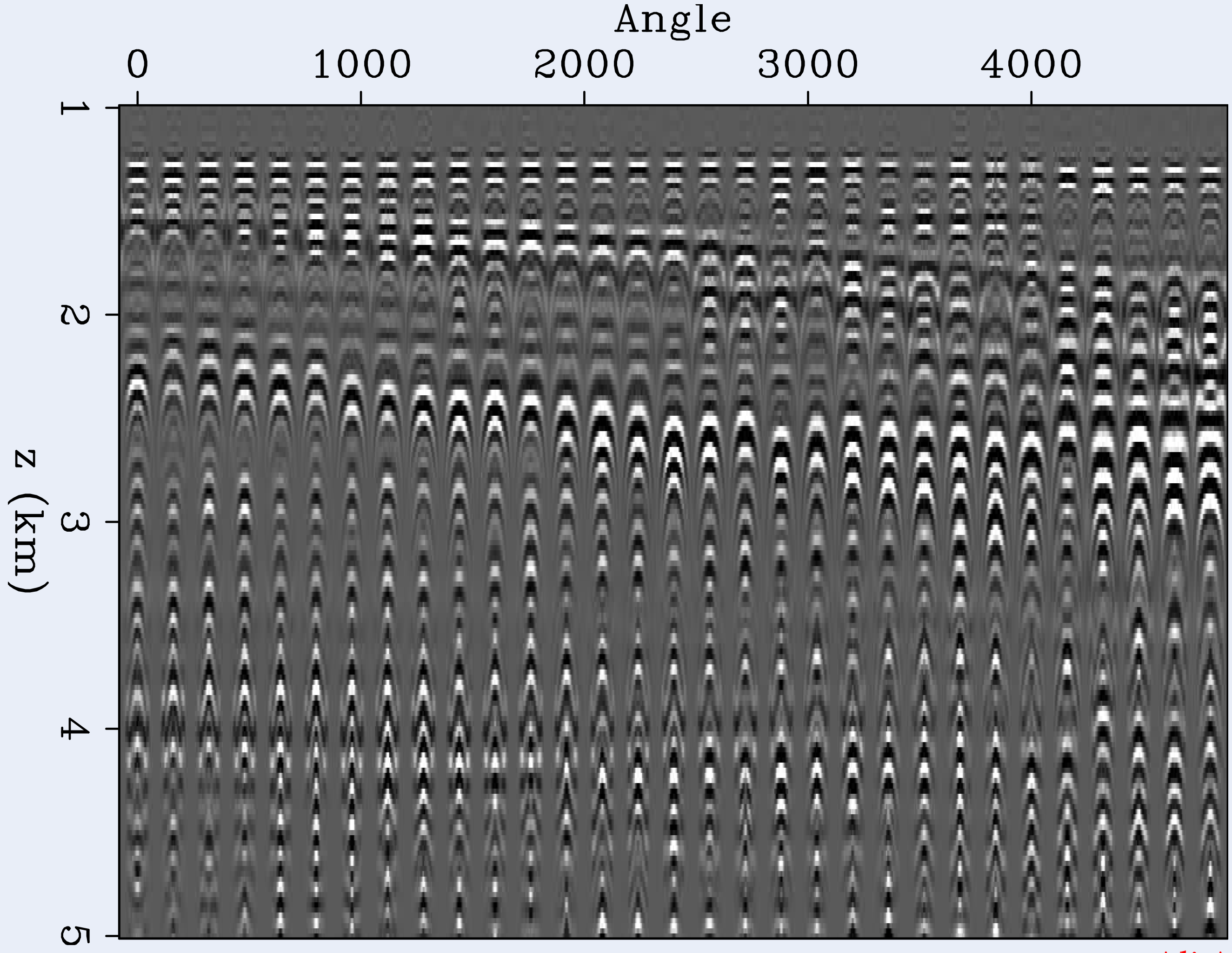

CIGs with initial velocity

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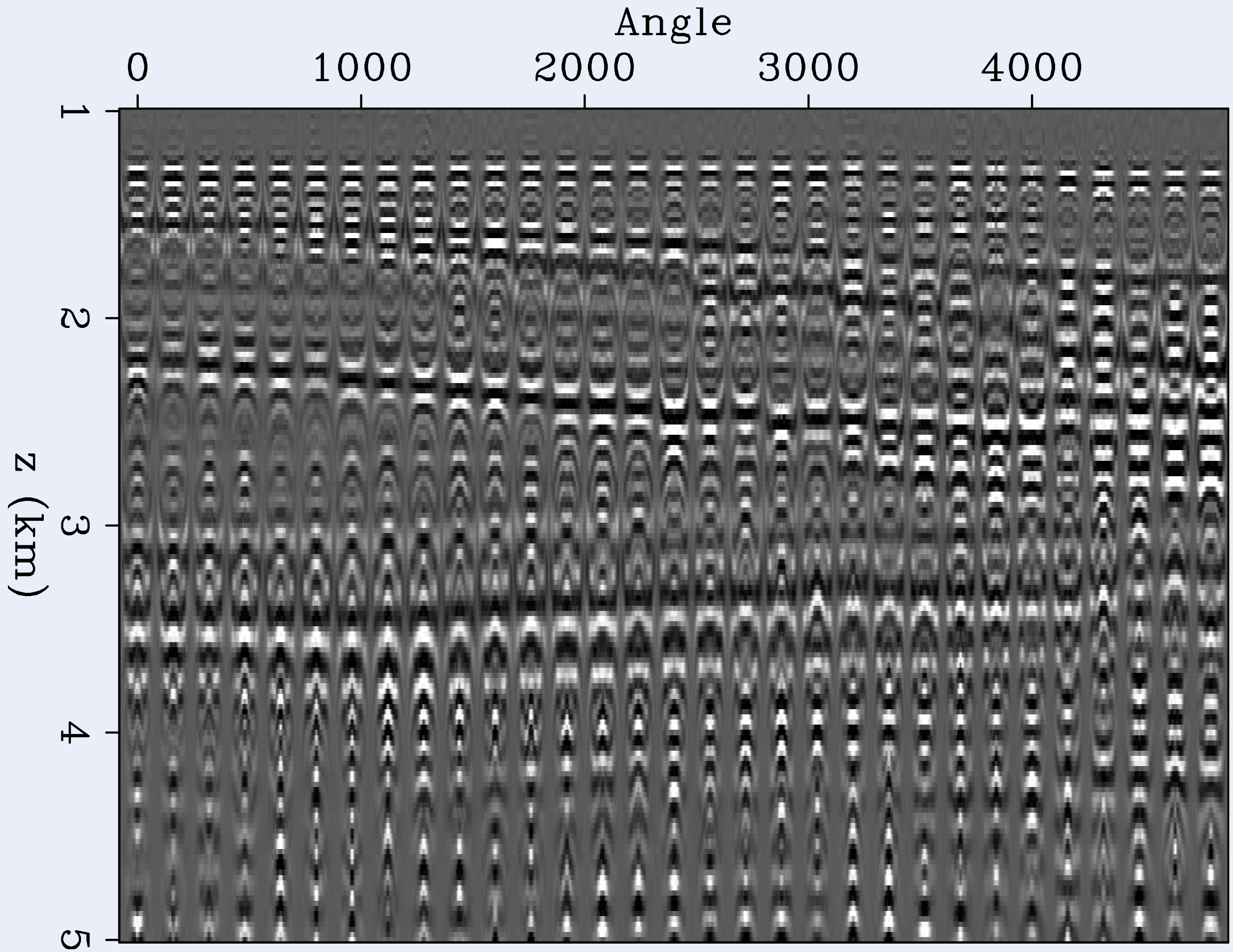

CIGs with TFWI velocity

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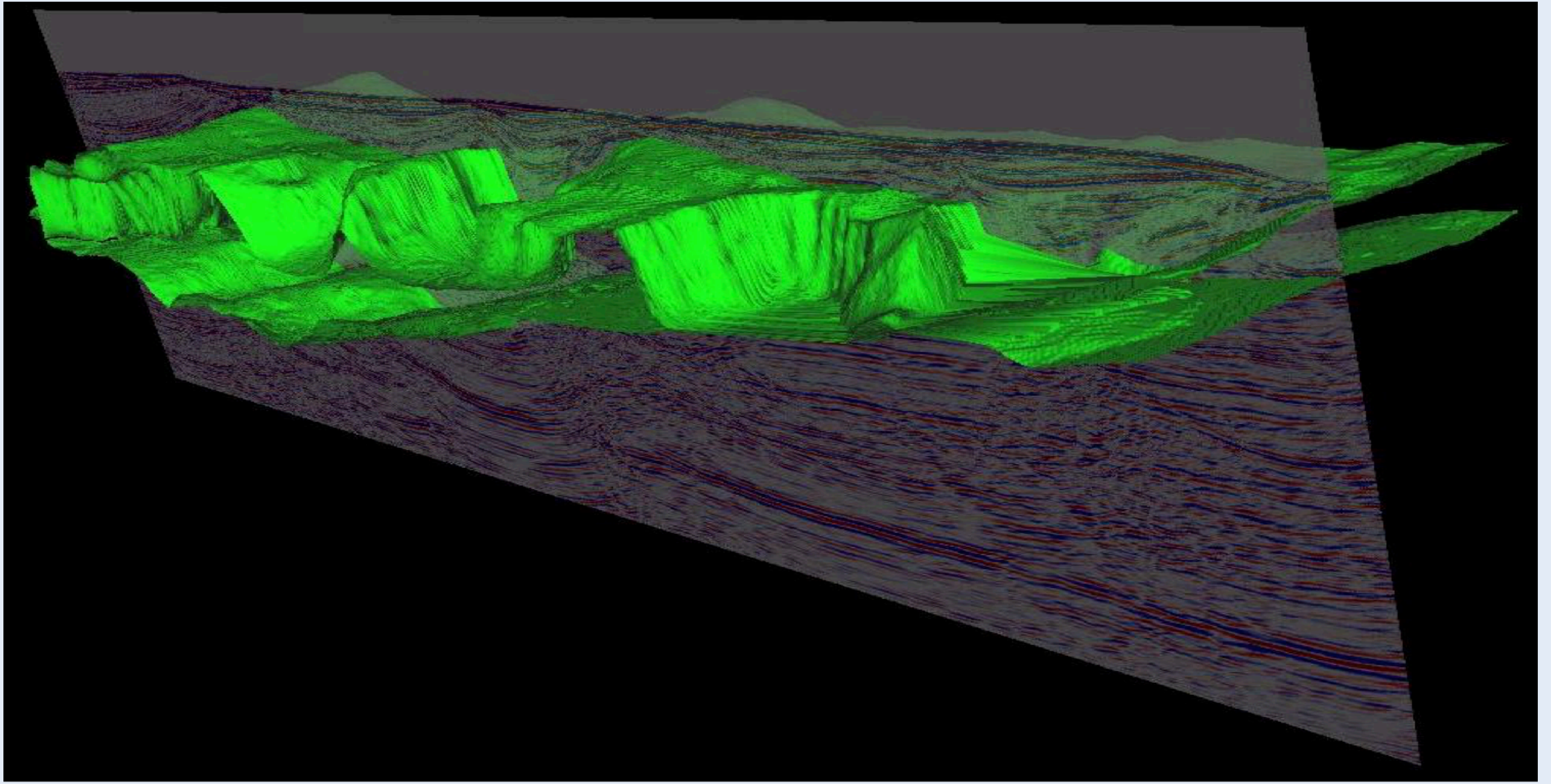
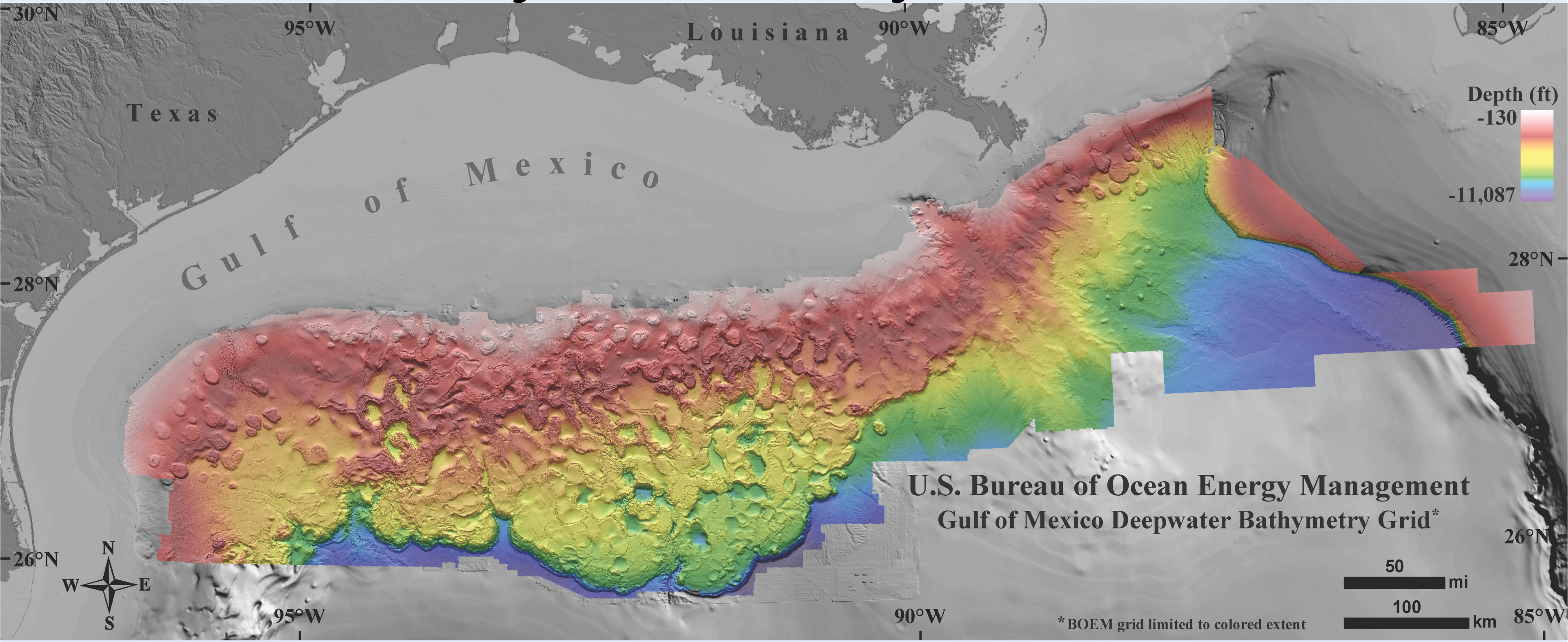
What's the matter with salt bodies?



They are everywhere!



They are important!

BUSINESS NEWS | Mon May 1, 2017 | 6:50pm BST

BP says 1 billion additional barrels 'possible' in Gulf of Mexico hubs

BP finds trove of oil in Gulf of Mexico using new subsea imaging

By David Hunn | April 27, 2017 | Updated: April 27, 2017 3:37pm

Photo: ©2007 BP PLC

IMAGE 1 OF 2
The view from the deck of a supply ship, looking at the Atlantis platform in the Gulf of Mexico, USA.

David Hunn

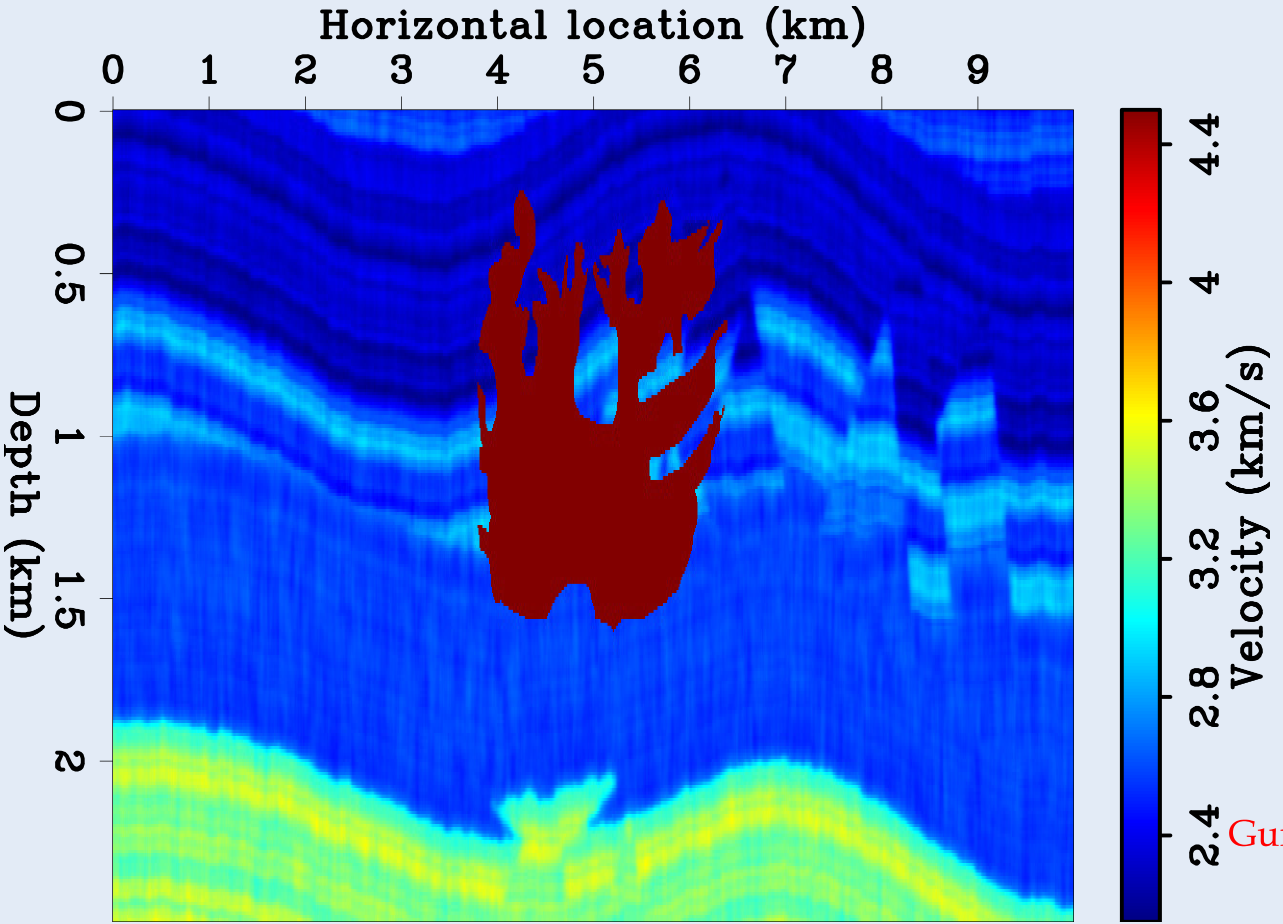
Follow

The algorithm that allowed BP to see under salt? Designed by whiz-kid @stanford grad Xukai Shen
…tonchronicle.com/business/energ… via @HoustonC…

7:30 AM - 27 Apr 2017

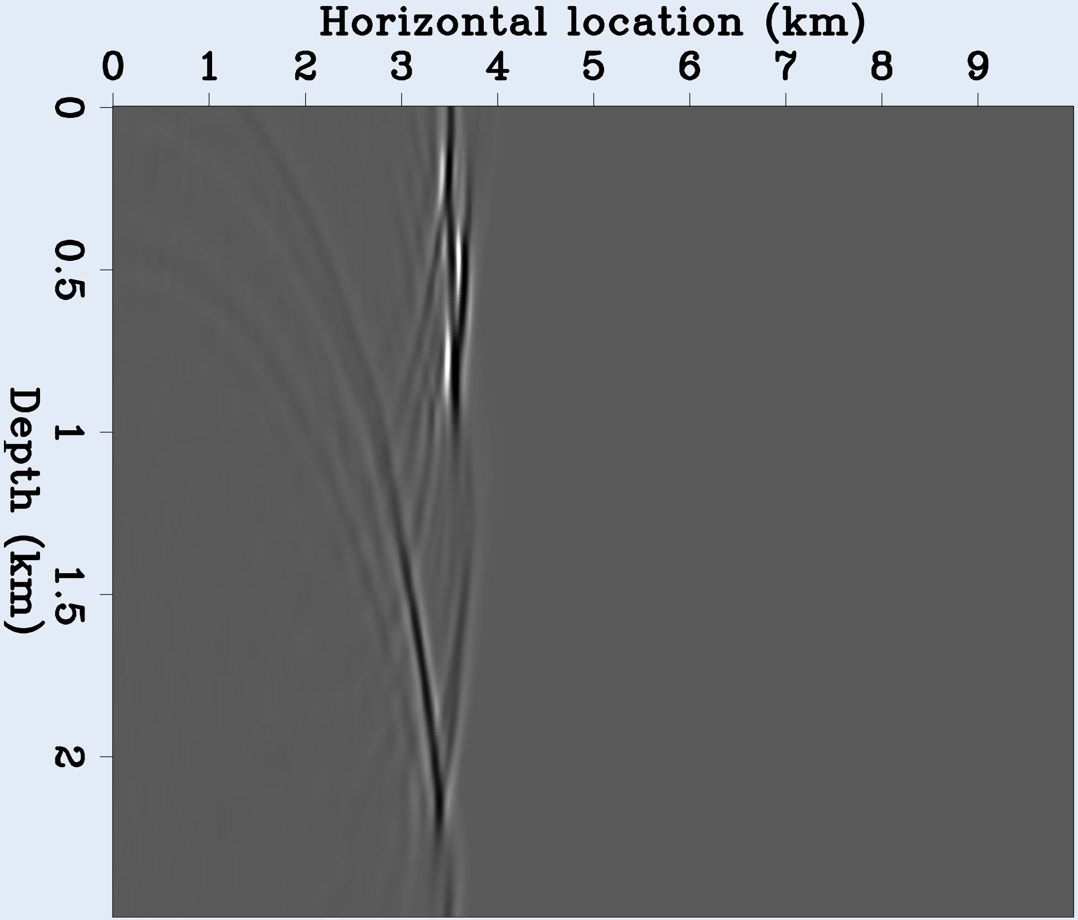
BP finds hidden trove of oil in Gulf of Mexico via new subsea i…
British oil major BP has discovered 200 million barrels of oil in a hidden cache in the Gulf of Mexico, thanks to a technological
houstonchronicle.com

They are complicated!



Courtesy of
Guillaume Barnier
(SEP)

and they make a mess of the wavefield



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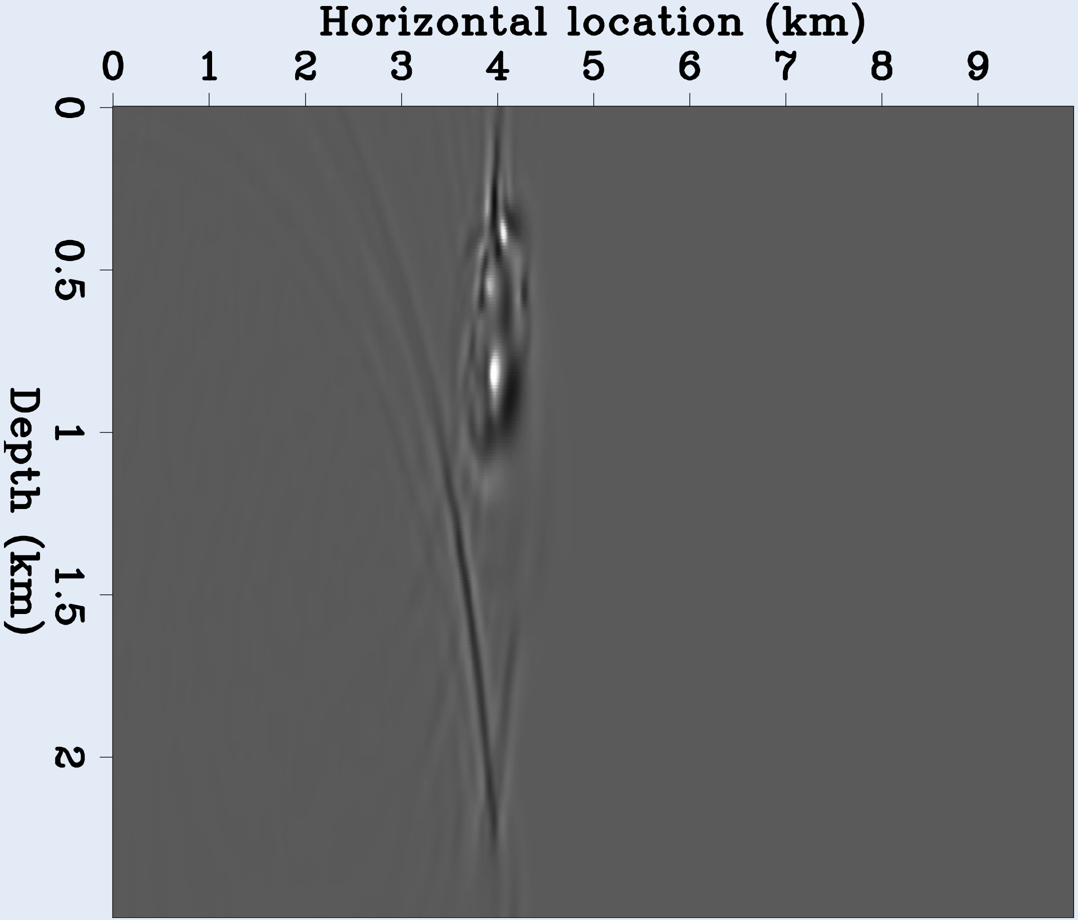

Courtesy of
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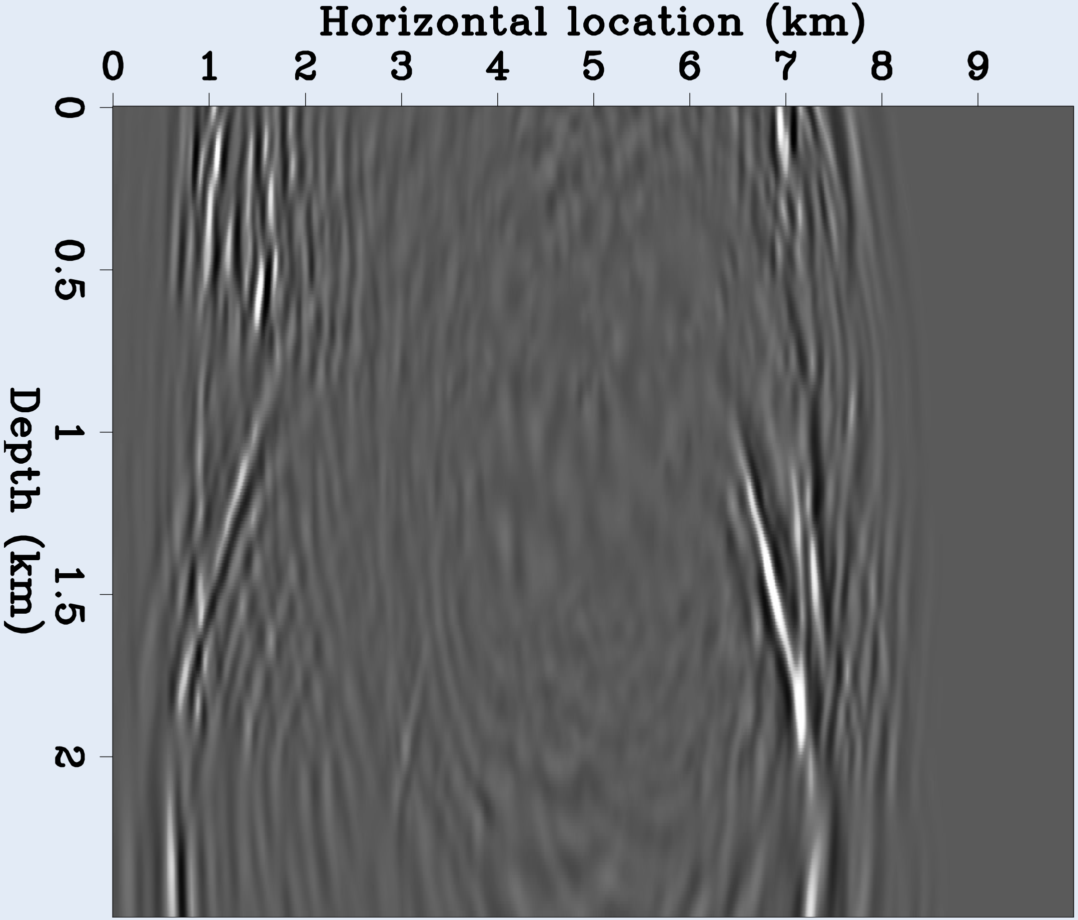
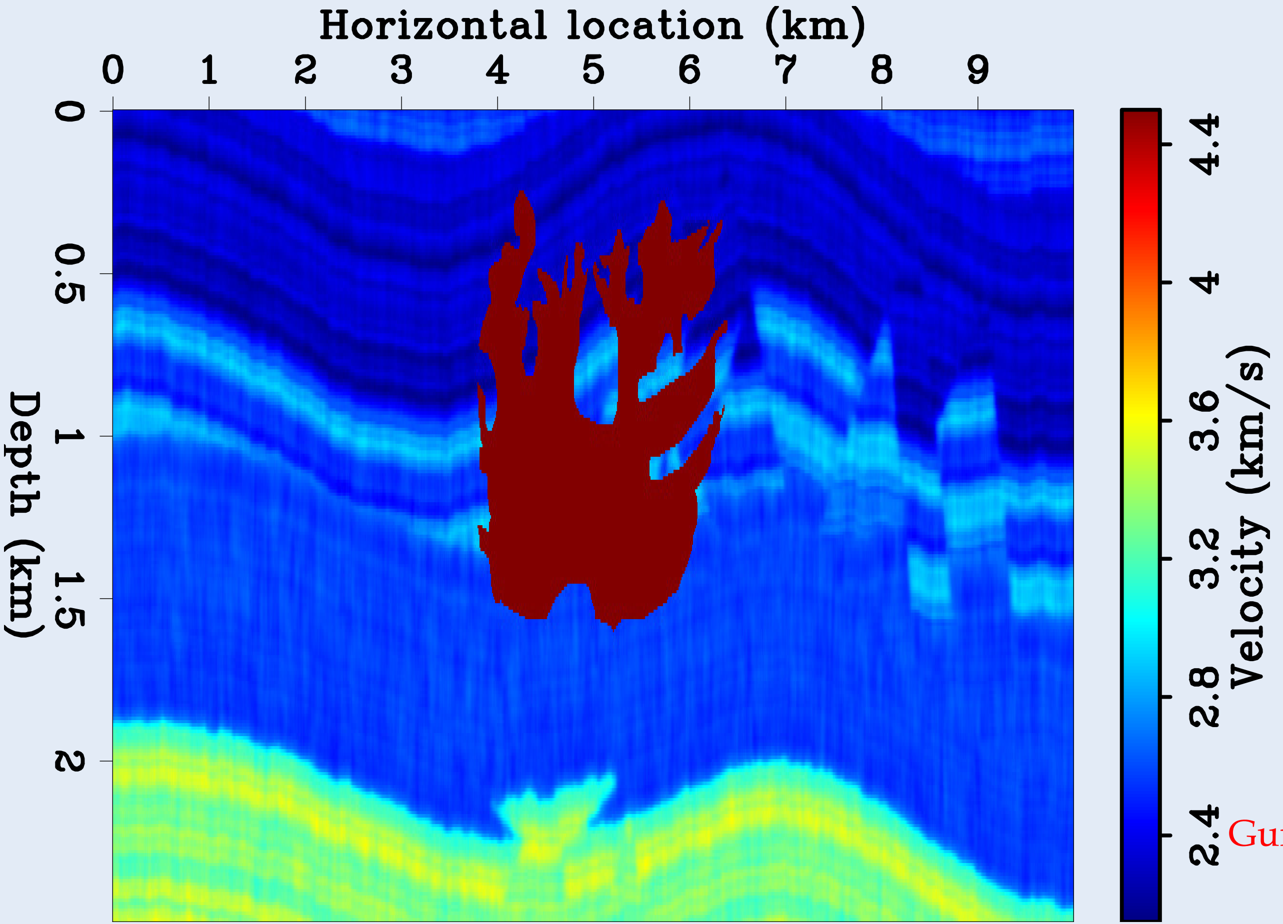
(SEP)

and they make a mess of the wavefield



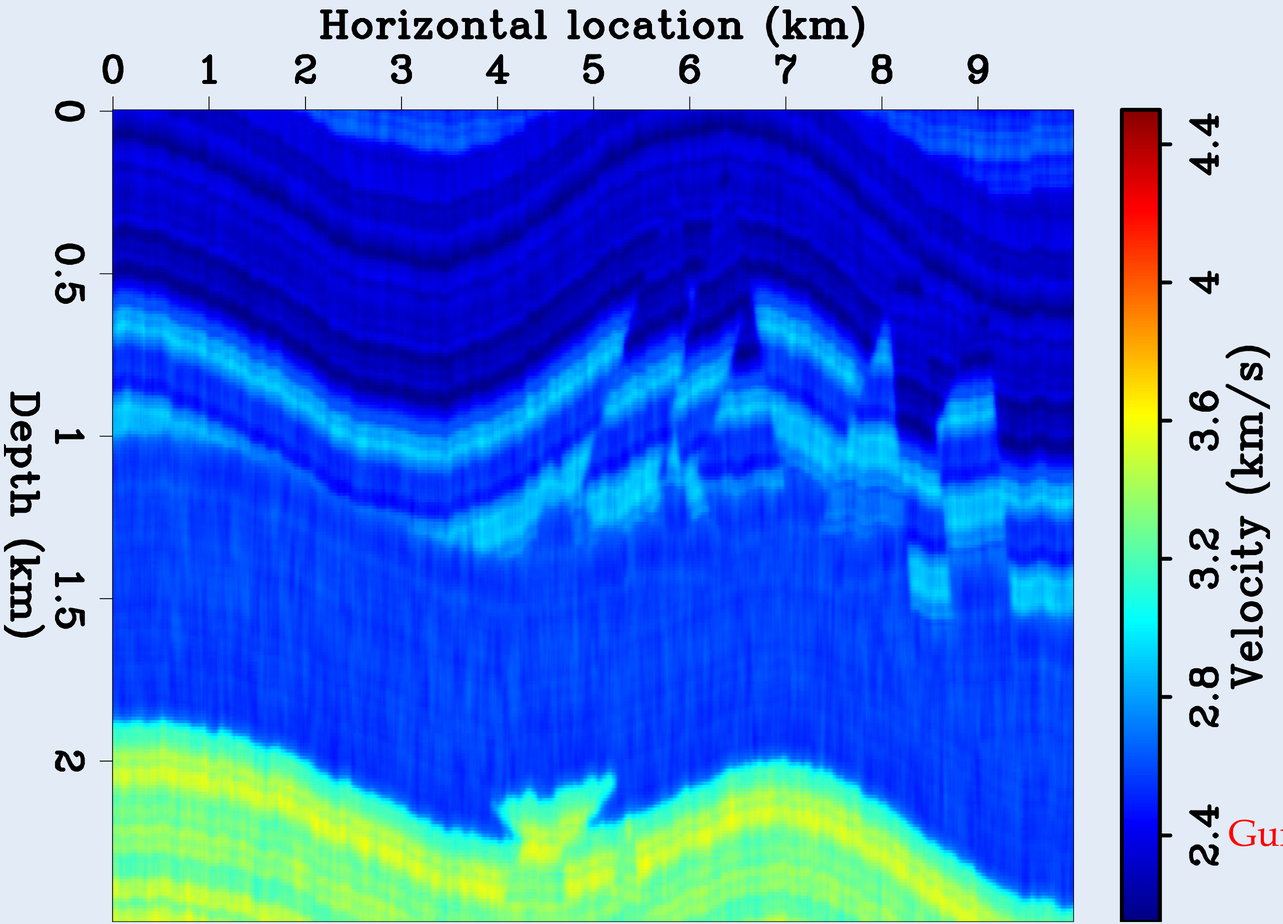
Courtesy of
Guillaume Barnier
(SEP)

True model



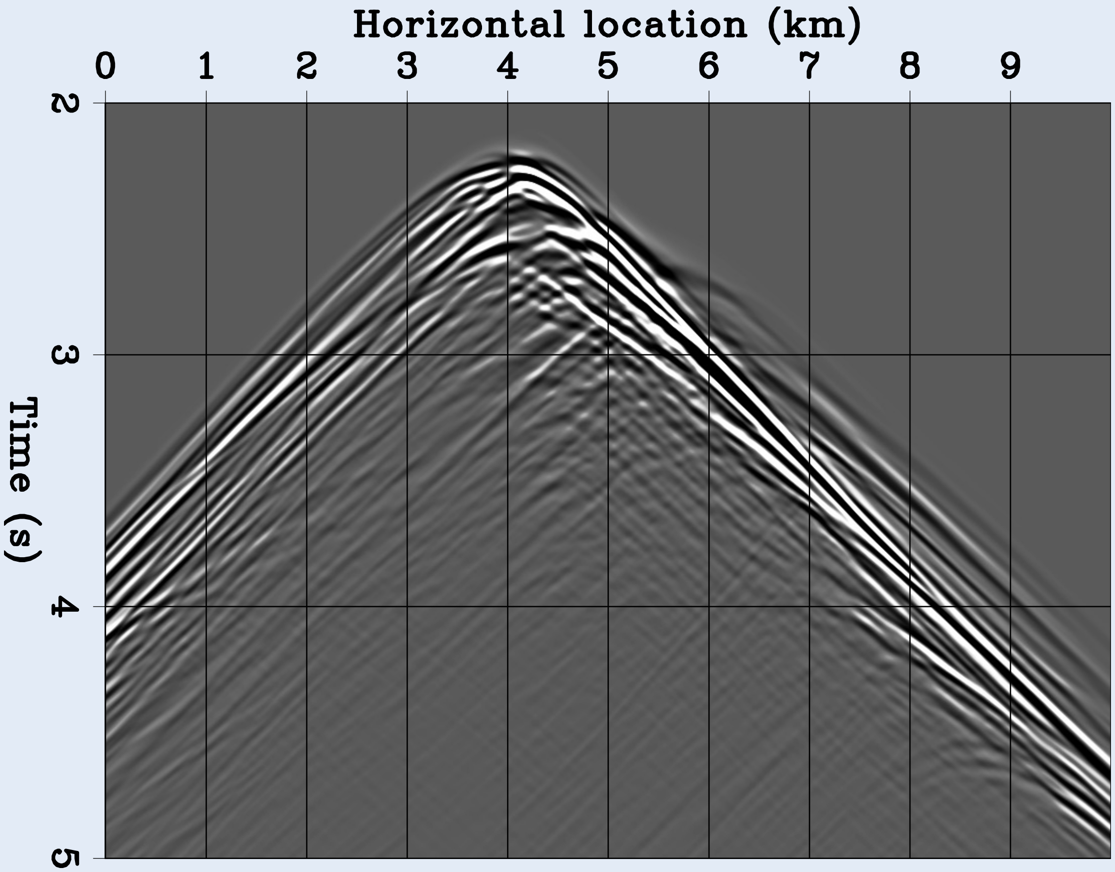
Courtesy of
Guillaume Barnier
(SEP)

Starting model



Courtesy of
Guillaume Barnier
(SEP)

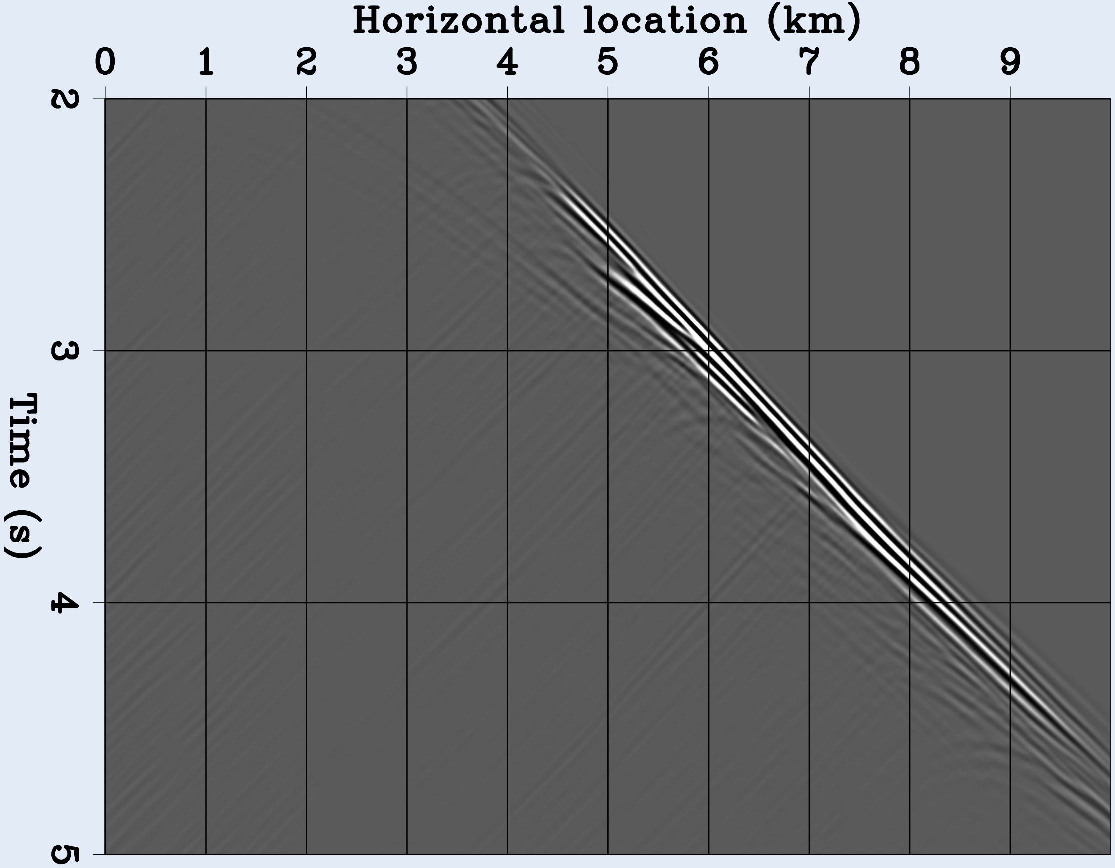
Initial data residuals



Courtesy of Guillaume Barnier (SEP)

Predicted residuals - No extension

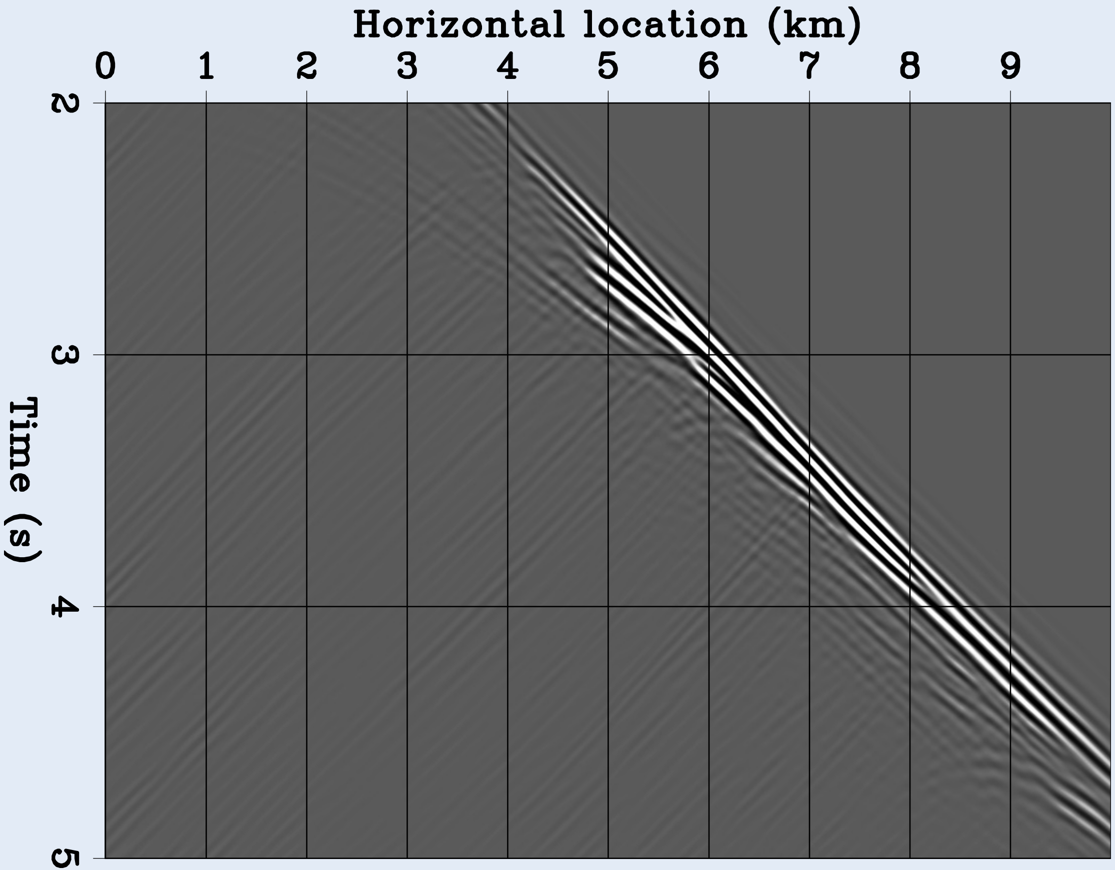
$$\overline{\Delta \mathbf{d}} = \mathbf{L}\mathbf{L}'\Delta \mathbf{d}$$



Courtesy of Guillaume Barnier (SEP)

Predicted residuals - τ extension

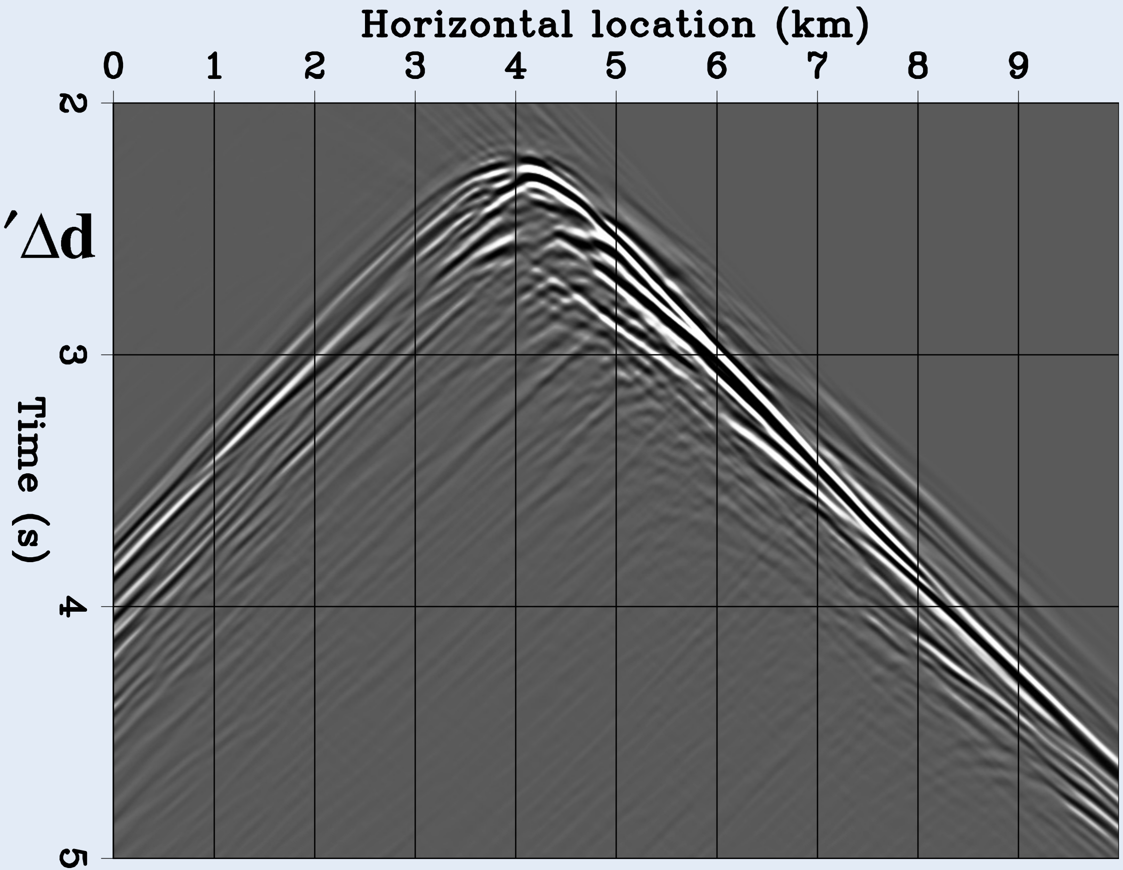
$$\overline{\Delta \mathbf{d}} = \tilde{\mathbf{L}}\tilde{\mathbf{L}}'\Delta \mathbf{d}$$



Courtesy of Guillaume Barnier (SEP)

Predicted residuals - τ extension

$$\widetilde{\Delta \mathbf{d}} = \tilde{\mathbf{L}}\left(\tilde{\mathbf{L}}'\tilde{\mathbf{L}}\right)^{\dagger}\tilde{\mathbf{L}}'\Delta\mathbf{d}$$

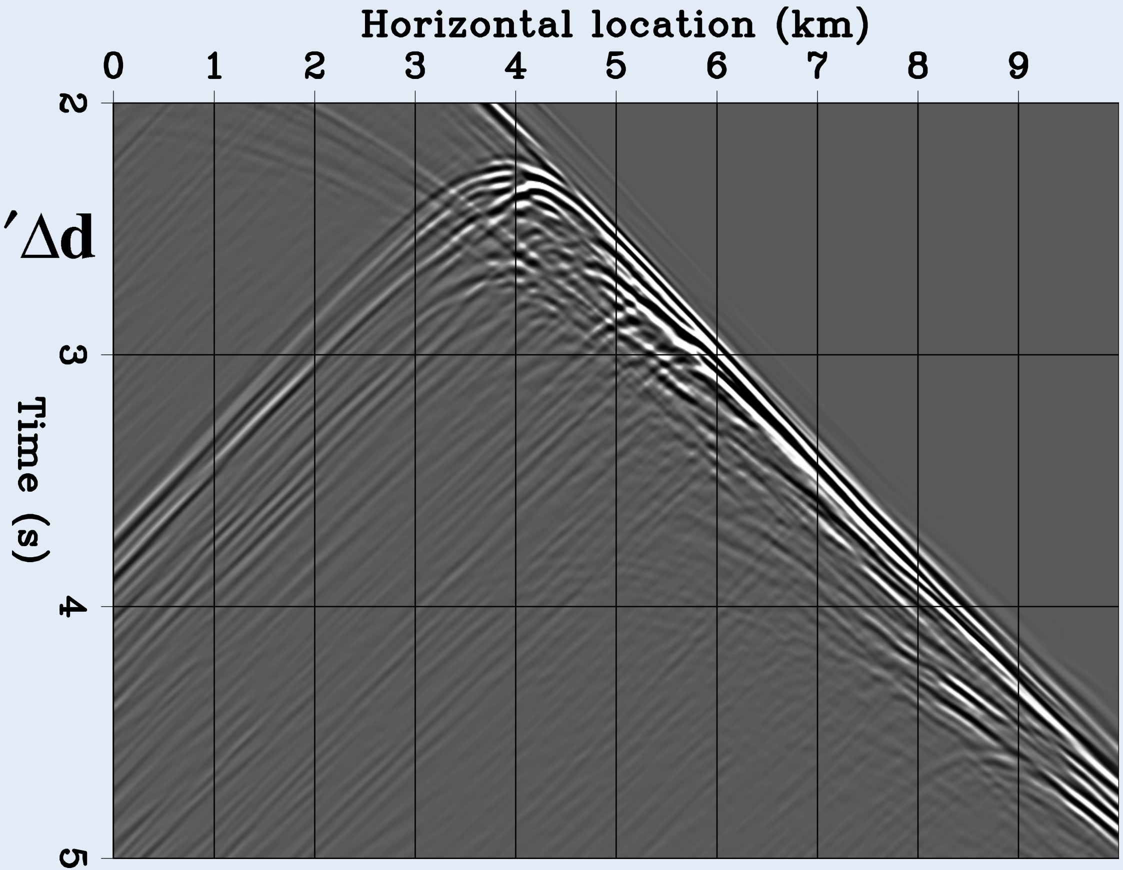


28^{th} iteration

Courtesy of Guillaume Barnier (SEP)

Predicted residuals - No extension

$$\widetilde{\Delta\mathbf{d}} = \mathbf{L}\left(\mathbf{L}'\mathbf{L}\right)^{\dagger}\mathbf{L}'\Delta\mathbf{d}$$

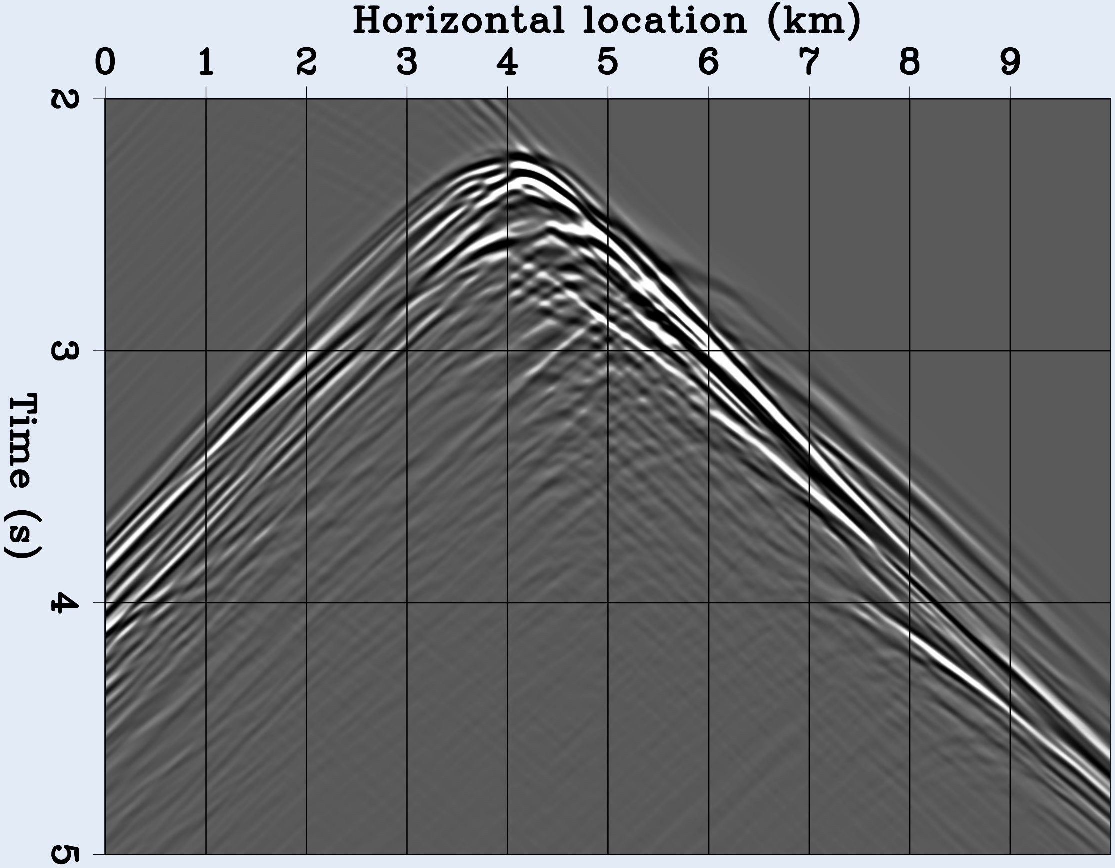


28th iteration

Courtesy of Guillaume Barnier (SEP)

Prediction error - τ extension

$$\widetilde{\Delta \mathbf{d}} - \Delta \mathbf{d}$$

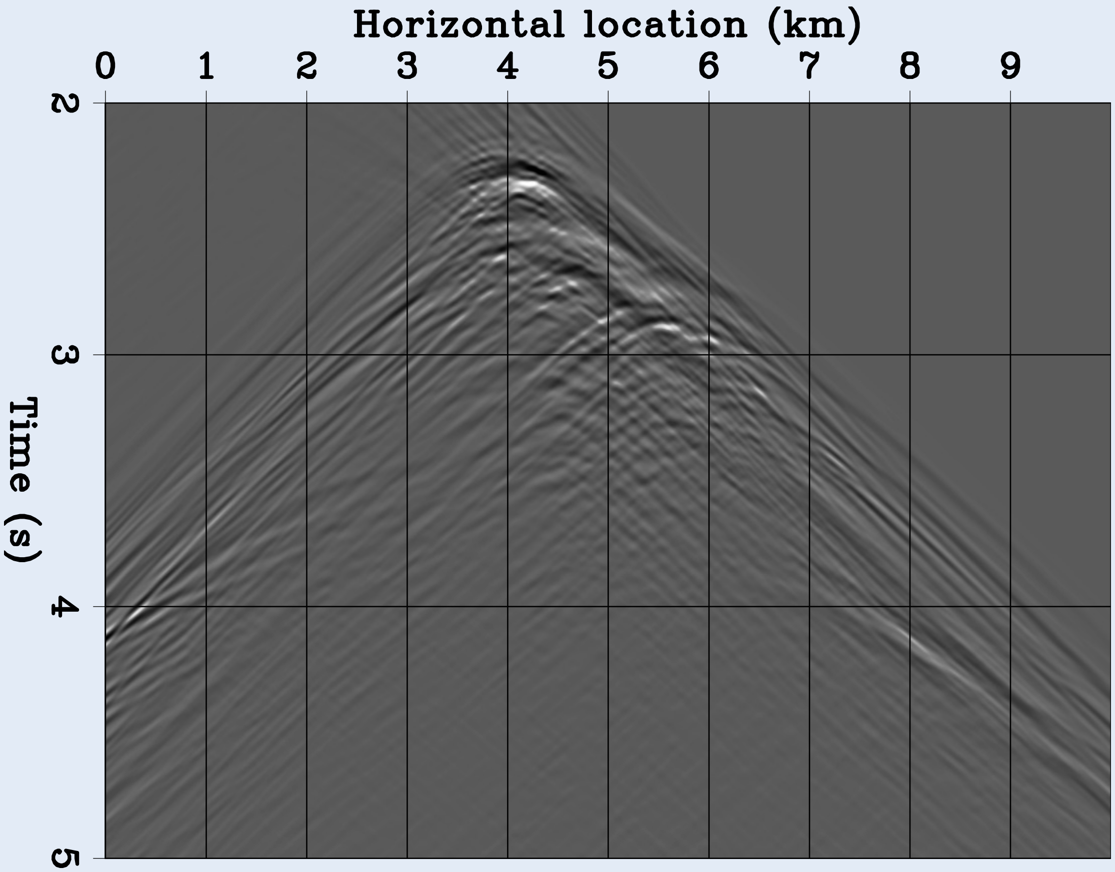


1st iteration

Courtesy of Guillaume Barnier (SEP)

Prediction error - τ extension

$$\widetilde{\Delta \mathbf{d}} - \Delta \mathbf{d}$$

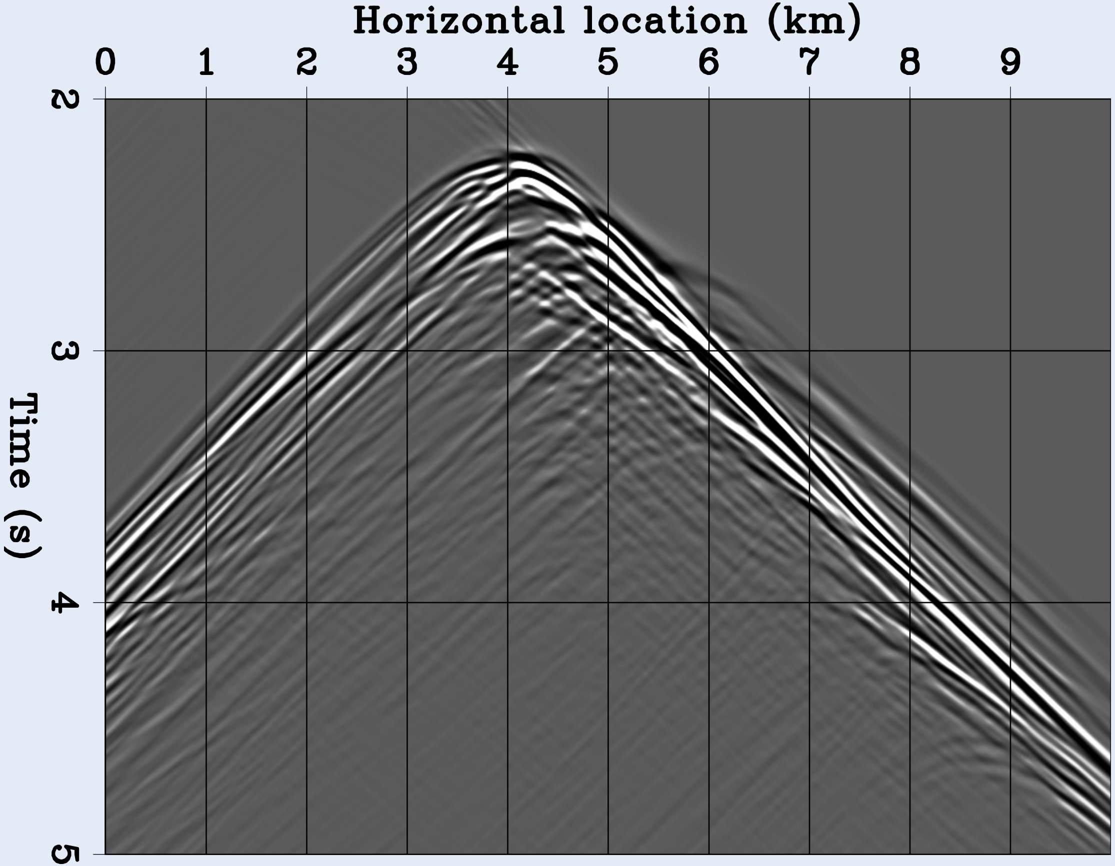


28th iteration

Courtesy of Guillaume Barnier (SEP)

Prediction error - No extension

$$\widetilde{\Delta \mathbf{d}} - \Delta \mathbf{d}$$

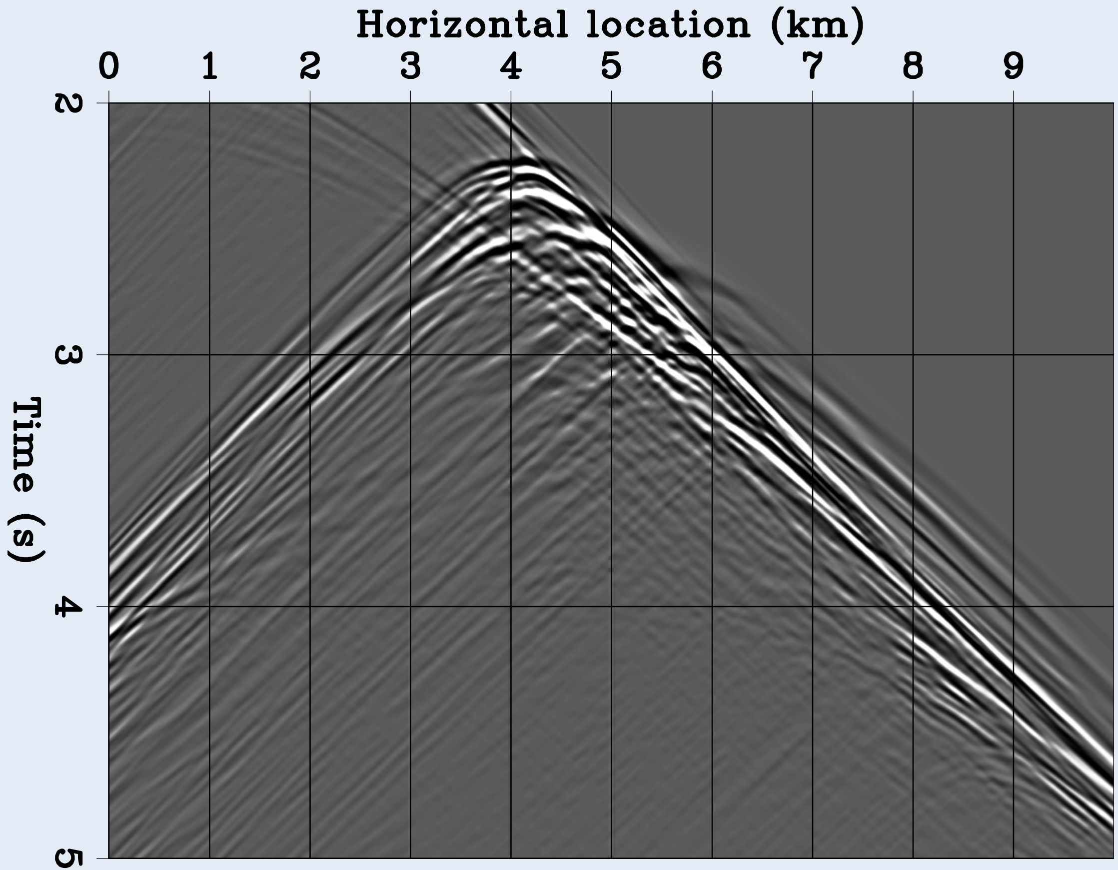


1st iteration

Courtesy of Guillaume Barnier (SEP)

Prediction error - No extension

$$\widetilde{\Delta \mathbf{d}} - \Delta \mathbf{d}$$



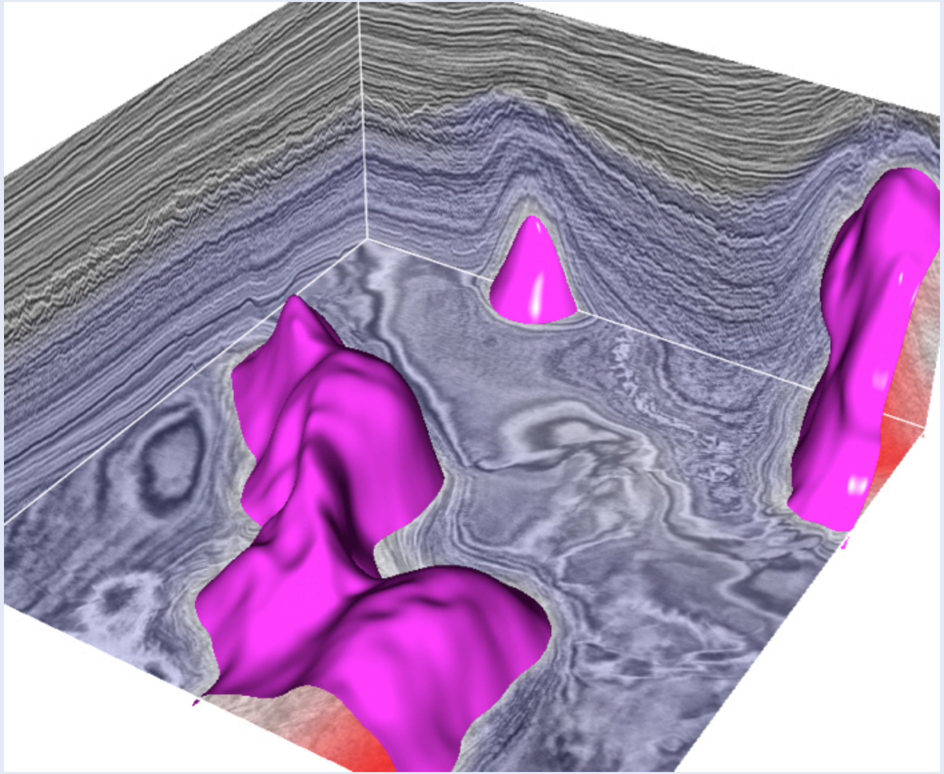
28th iteration

Courtesy of Guillaume Barnier (SEP)

Why use level sets?

Lewis et al. (EAGE 2012)
Guo and de Hoop (SEG 2013)

- Salt bodies can often be approximated as homogeneous.
- Level sets define boundaries of homogeneous bodies
- This makes them a useful tool for salt modeling



Wu, X., 2016, Methods to compute salt likelihoods and extract salt boundaries from 3D seismic images. **Geophysics**, 81(6)

Courtesy of Taylor Dhalke (SEP)

Derivation: Objective function

Typical FWI:

$$\psi(m) = \| F(m) - d_{obs} \|_2^2$$

Level set FWI:

$$\psi(m(\phi, b)) = \| F(m(\phi, b)) - d_{obs} \|_2^2$$

Courtesy of Taylor Dhalke (SEP)

Derivation: New model space

$$m(\phi,b)=H(\phi)(c_s-b)+b$$

Courtesy of Taylor Dhalke (SEP)

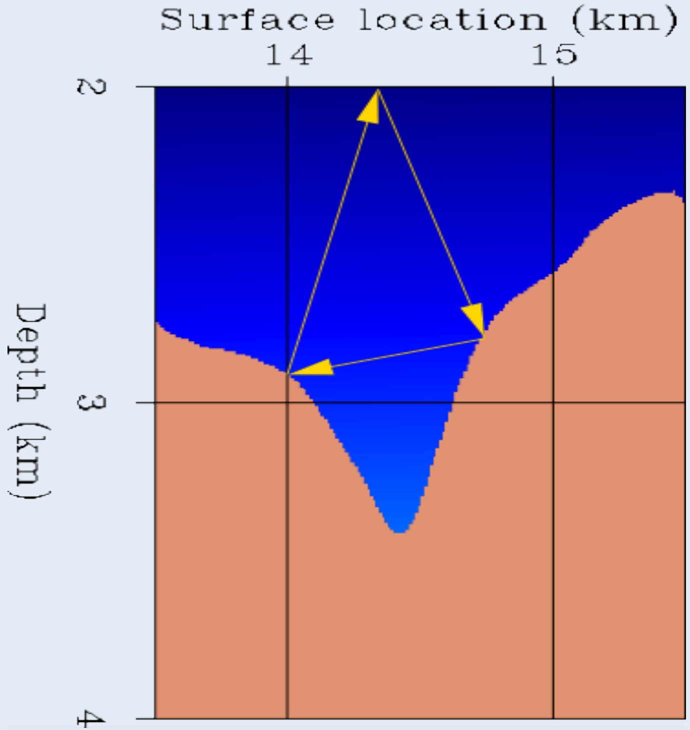
Derivation: New model space

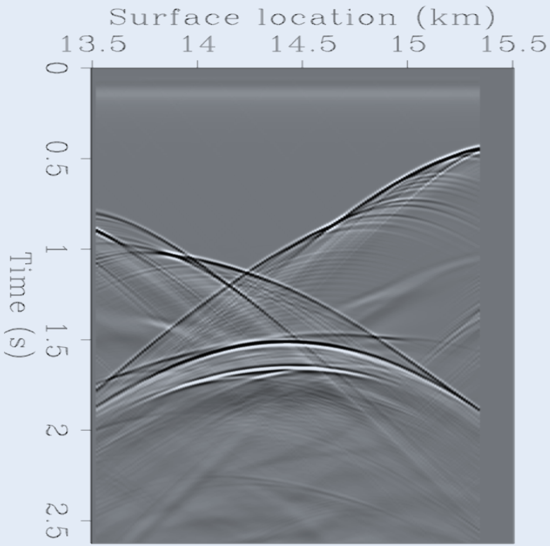
$$m(\phi,b)=H(\phi)(c_s-b)+b$$

Heaviside function **Salt velocity** **Background velocity**

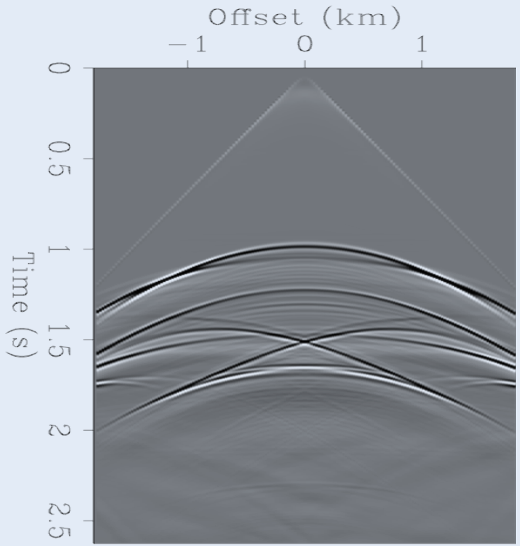
Courtesy of Taylor Dhalke (SEP)

Prismatic waves caused by salt





Zero-offset section



CMP gather

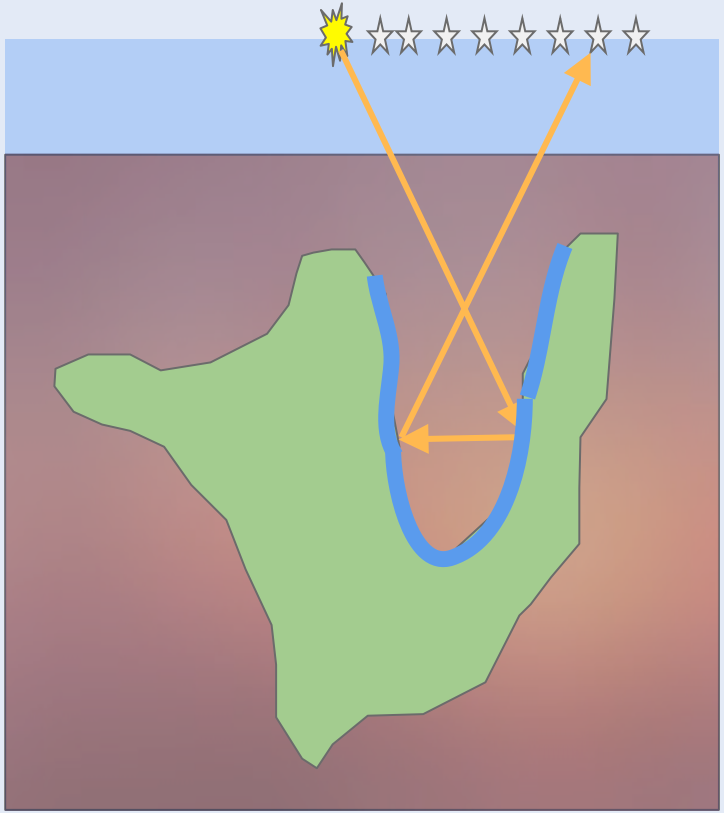
Figure 3.16 & 3.17

biondo@stanford.edu

Motivation: Why use the Hessian?

What about the interaction of salt **boundaries inside canyons**?

What about the interaction between the **salt position** and the **background velocity**?



Courtesy of Taylor Dhalke (SEP)

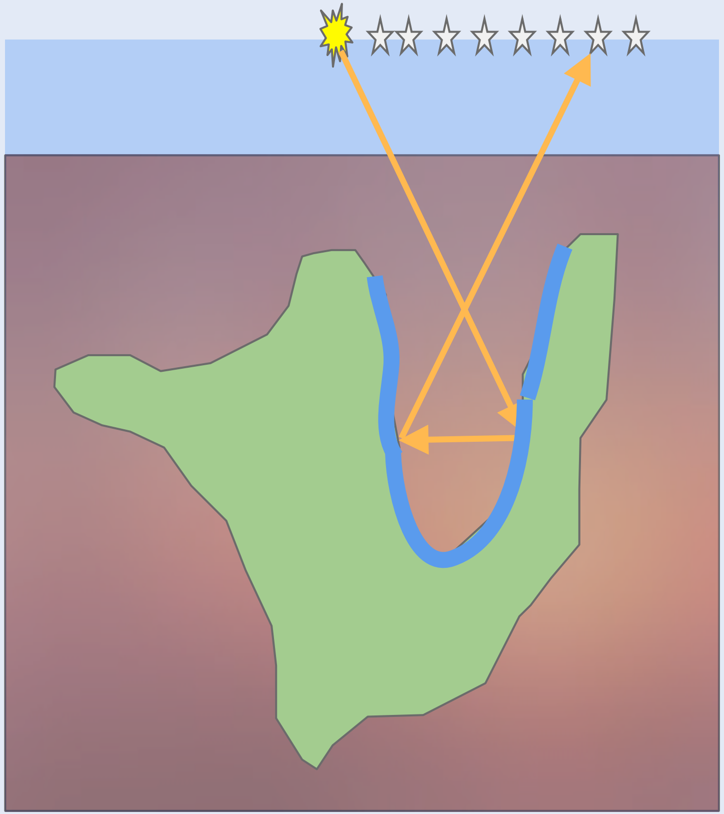
Motivation: Why use the Hessian?

What about the interaction of salt **boundaries inside canyons**?

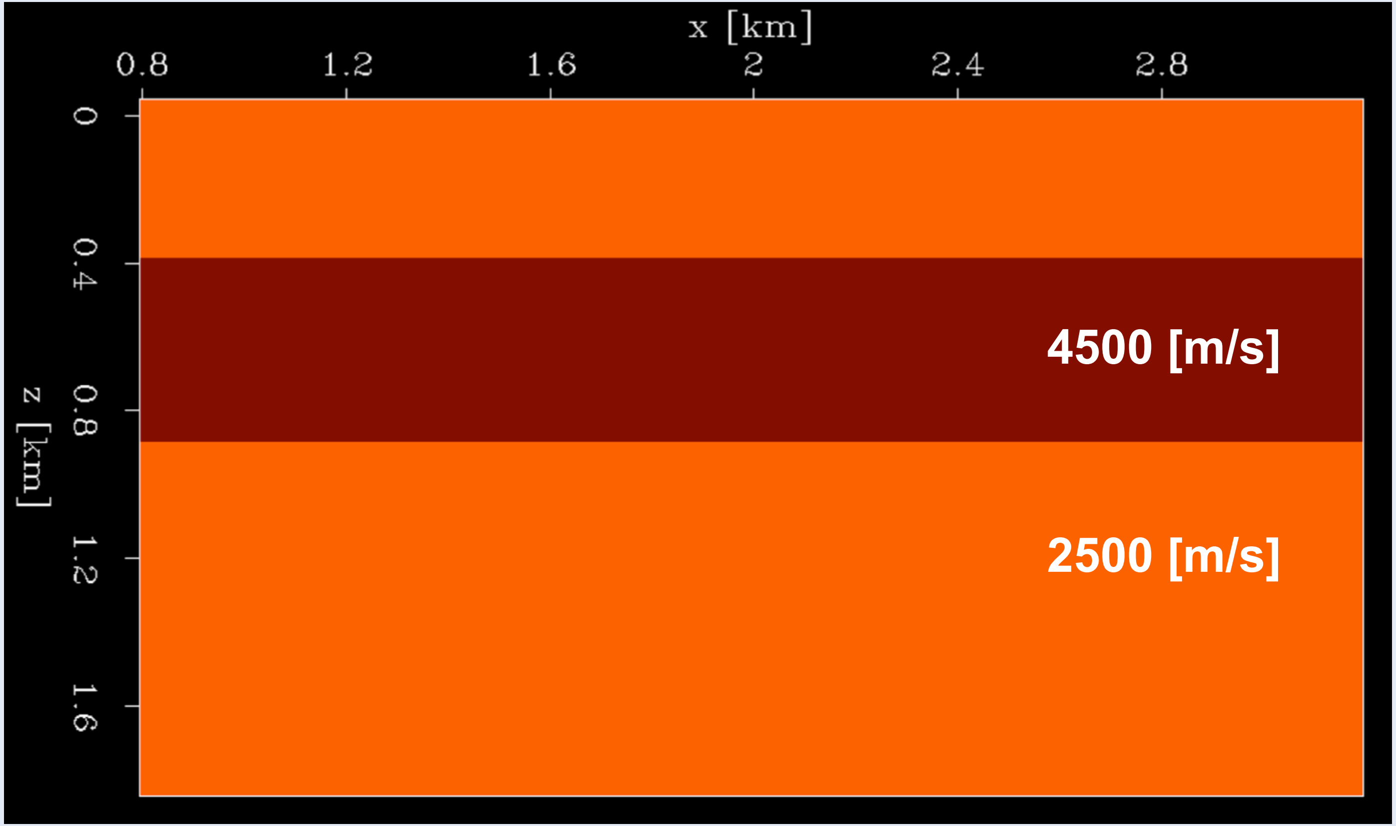
What about the interaction between the **salt position** and the **background velocity**?

Can we formulate the Hessian for our objective function that helps account for these interactions?

Courtesy of Taylor Dhalke (SEP)



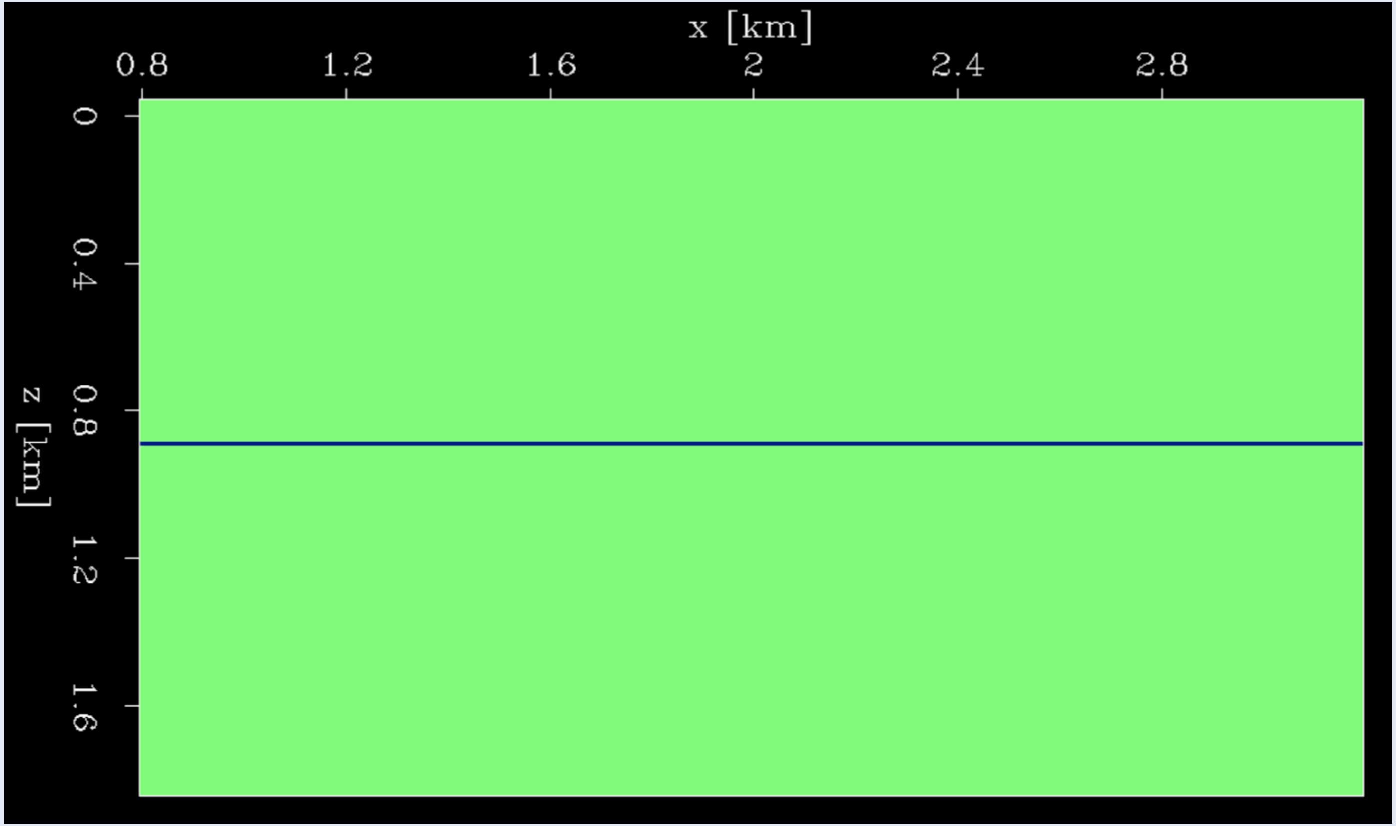
True model



Courtesy of Taylor Dhalke (SEP)

- 40 shots
- 235 receivers
- 7 Hz source wavelet

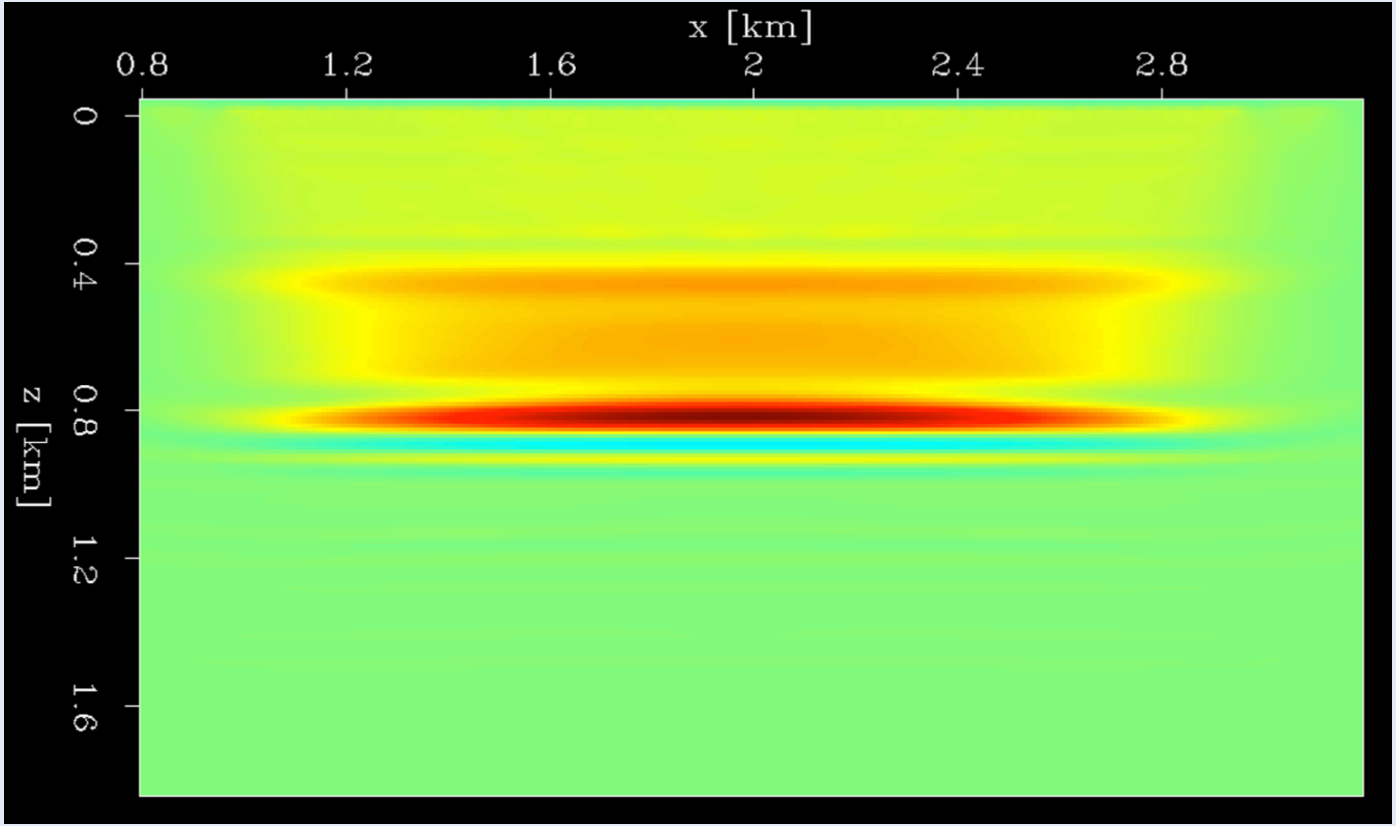
True model - Initial model difference



Courtesy of Taylor Dhalke (SEP)

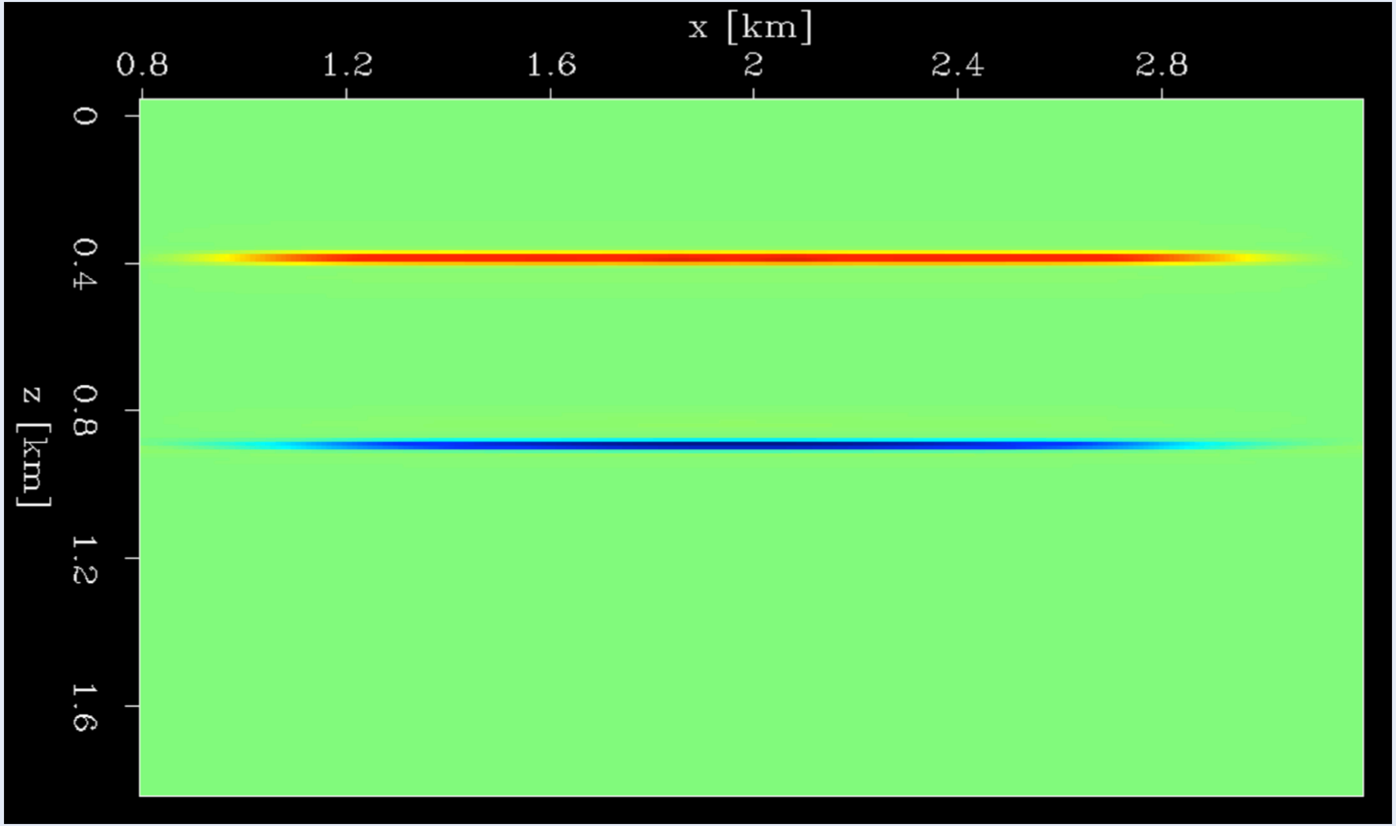
- **Initial salt base is too deep**

Adjoint Born Image



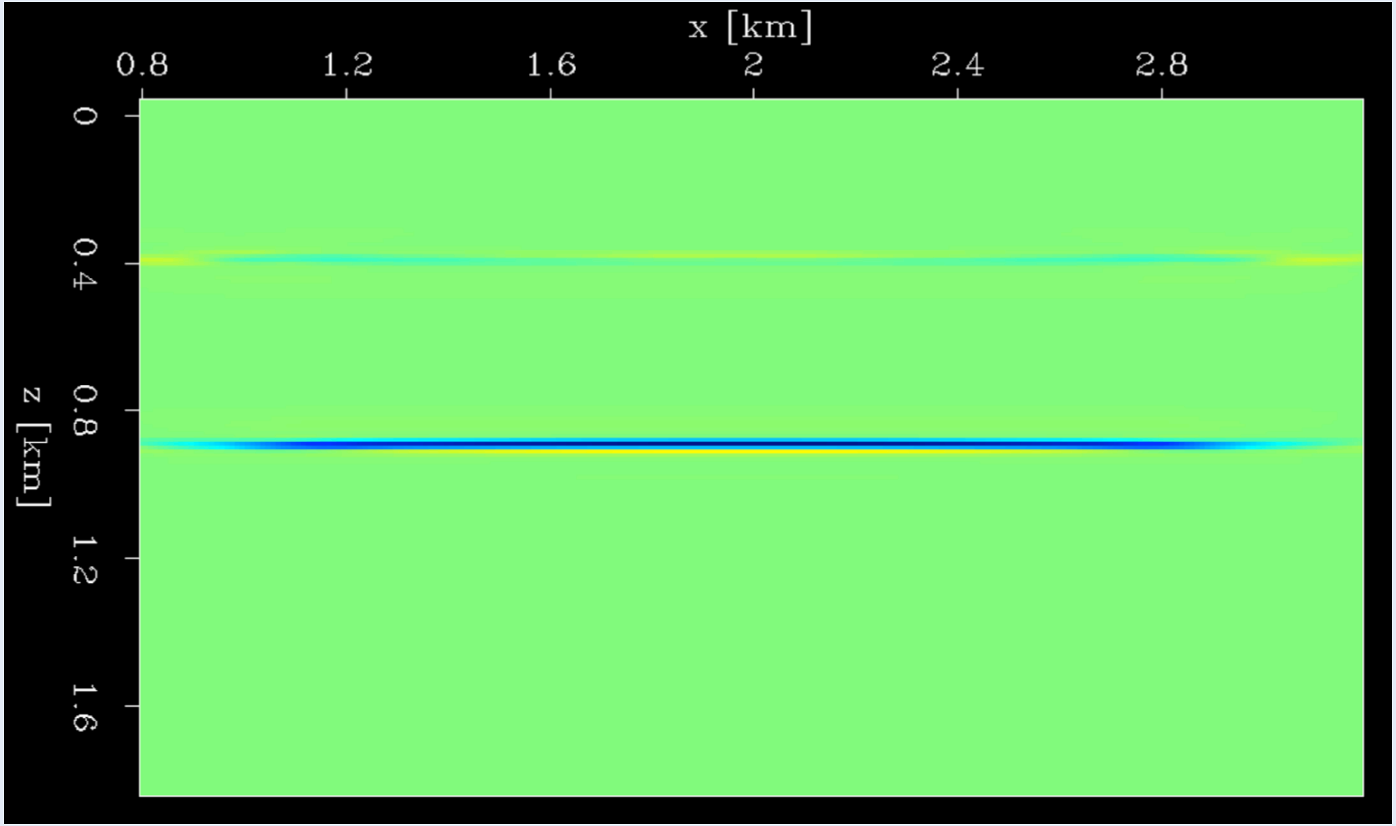
Courtesy of Taylor Dhalke (SEP)

Phi search direction (steepest descent)



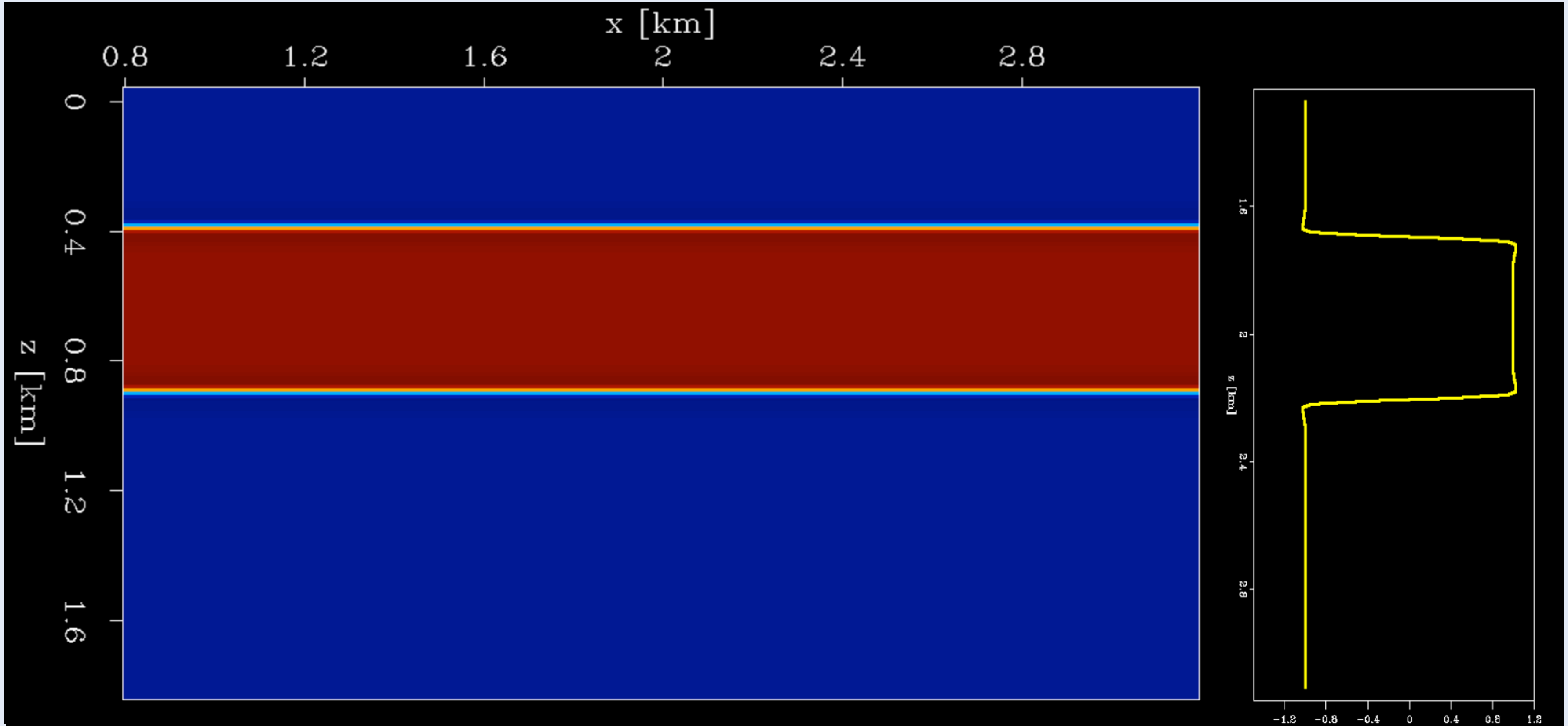
Courtesy of Taylor Dhalke (SEP)

Phi search direction (truncated full Newton)



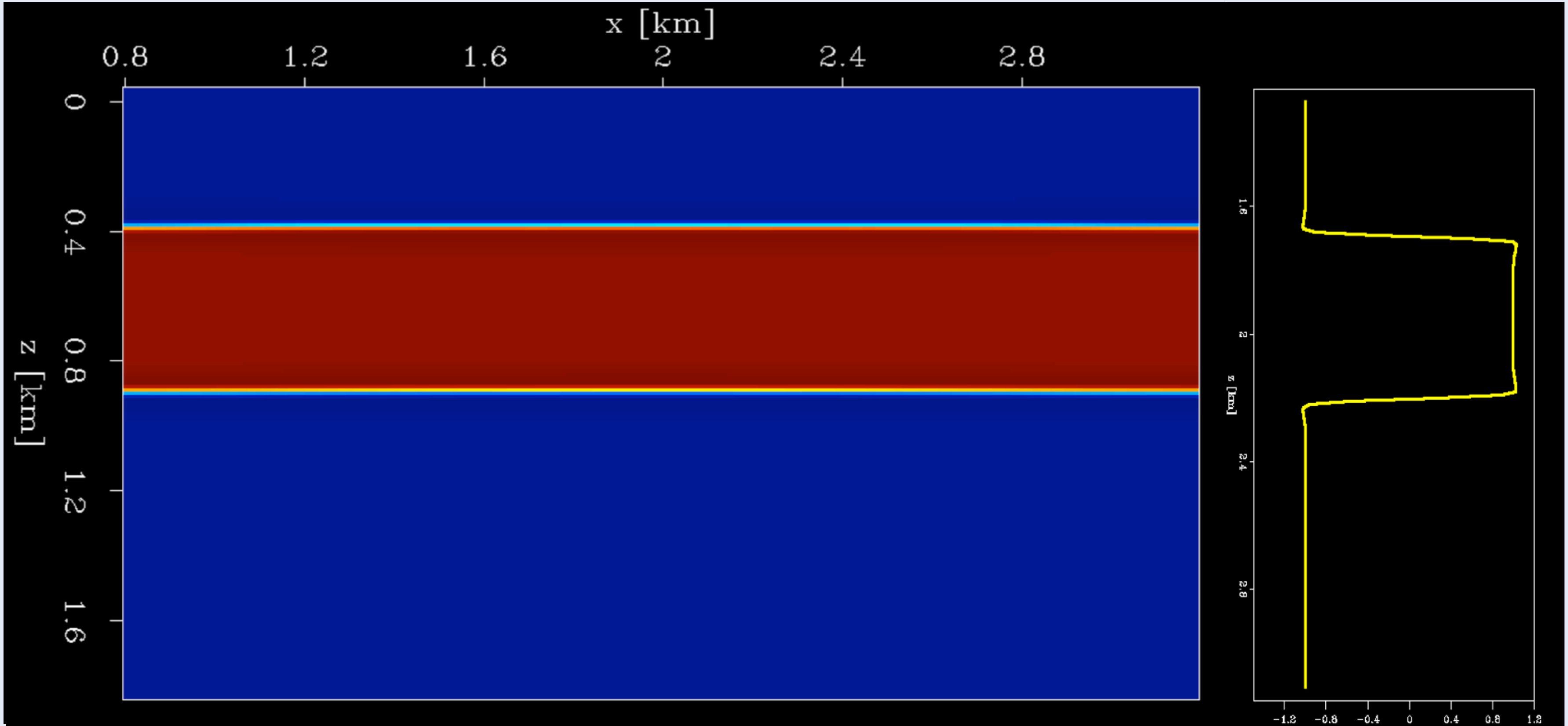
Courtesy of Taylor Dhalke (SEP)

Initial Phi



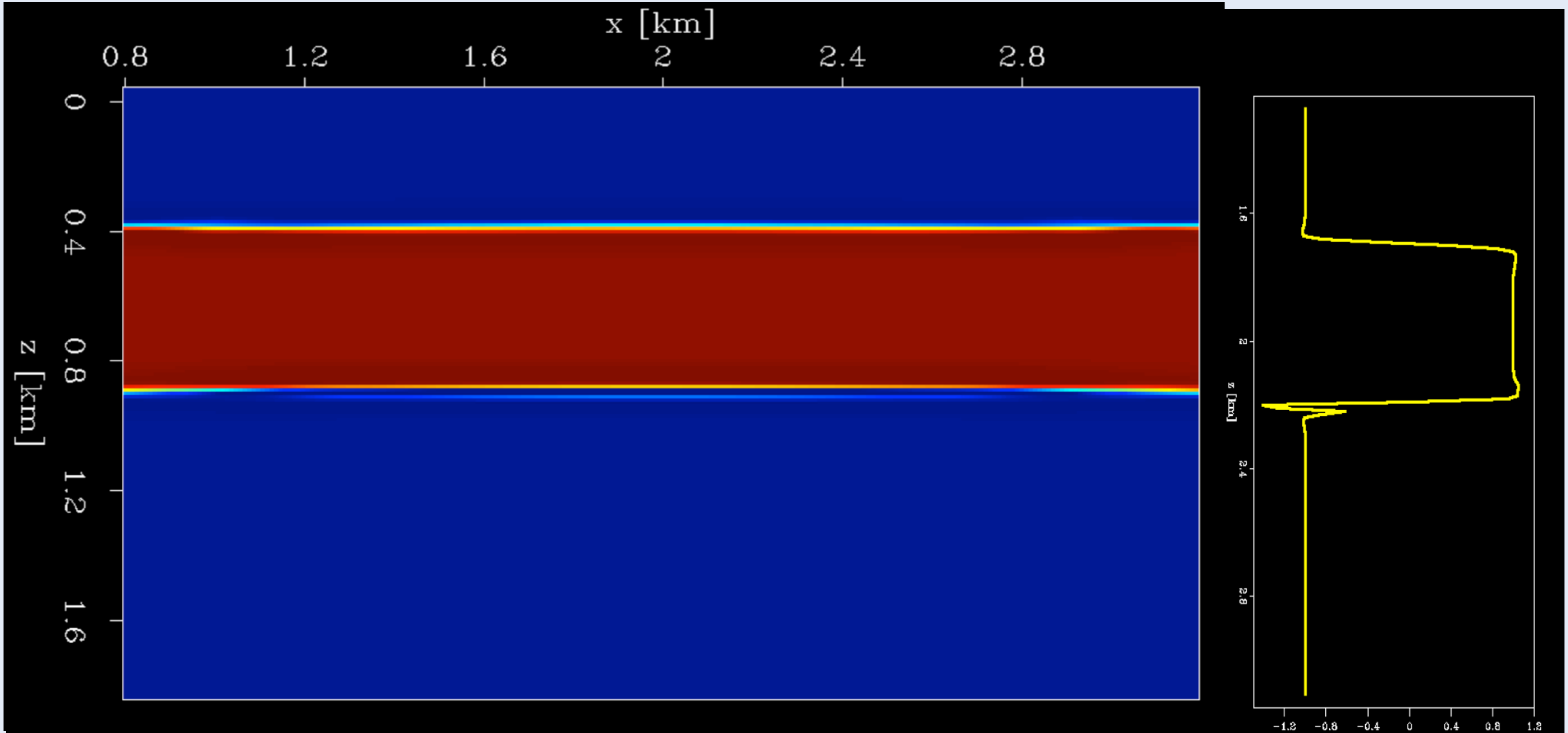
Courtesy of Taylor Dhalke (SEP)

Updated Phi (steepest descent)



Courtesy of Taylor Dhalke (SEP)

Updated Phi (truncated full Newton)



Courtesy of Taylor Dhalke (SEP)

Conclusions

- We (as a community) are making progress towards making FWI to converge more robustly.
- Highly scattering geobodies (e.g. salt bodies) are still a challenge.
- It is unlikely that there is a silver bullet; different geologic settings call for different solutions.

Acknowledgments

- Ali Almomin, Guillaume Barnier and Taylor Dhalke for doing the work, and providing images and slides.
- IPAM.
- Chevron for IO-Jansz data.
- Stanford Exploration Project affiliate members for financial support
- Chevron for financial support of Stanford Center of Research Excellence.
- Stanford Center for Computational Earth and Environmental Science for computational support.