Robust convergence of full waveform inversion by methods on based on physical intuition and geological knowledge: challenges ahead

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Going beyond first-order Born

• Making FWI more Rylov-like (phases vs. amplitudes)
  • Wave-equation tomography
  • Correlation, adaptive waveform inversion, …
• Extend model space to improve linearization
  • Reflectivity-only extension => WEMVA
    (Biondi and Sava, 1999; Shen, Symes and Stolk, 2003)
  • Full extension (Symes, 2008; Biondi and Almomin, 2012, …)
• Add “geologic constraints”
**Linearized $\tau$ extension**

\[ \tilde{\mathcal{L}}(\tilde{v}) = \mathcal{L}(v_0) + \tilde{L} \delta \tilde{v}^2(\tau) \]

**Full non-linear scattering**

\[
\begin{align*}
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] P_0 &= f \\
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] \delta P &= \delta v^2 \nabla^2 (P_0 + \delta P)
\end{align*}
\]

$v_0$: Background velocity
$P_0$: Background wavefield
$\delta v$: Velocity perturbation
$\delta P$: Scattered wavefield
$f$: Source function

**Linearized $\tau$ extension**

\[
\begin{align*}
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] P_0 &= f \\
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] \delta \tilde{P} &= \delta \tilde{v}(\tau)^2 \ast \nabla^2 P_0 \\
\delta \tilde{v}(\tau)$: Extended-velocity perturbation
\delta \tilde{P}$: New scattered wavefield
Transmission experiment \((t=1.4 \text{ s})\)
Limitations of Born linearization

Wavefield and data residuals by full non-linear scattering

Wavefield and data residuals by first-order Born scattering
Derivation of first-order Born scattering

**Full non-linear scattering**

\[
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] P_0 = f \\
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] \delta P = \delta v^2 \nabla^2 (P_0 + \delta P)
\]

- \( v_0 \): Background velocity
- \( P_0 \): Background wavefield
- \( \delta v \): Velocity perturbation
- \( \delta P \): Scattered wavefield
- \( f \): Source function

**First-order Born scattering**

\[
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] \hat{P} = \delta v^2 \nabla^2 P_0
\]

- \( \hat{P} \): Born scattered wavefield
Reflection (back scattered) data: One frequency $\omega$

\[ k_s = \frac{\omega}{v} \sin \alpha \ k_x - \frac{\omega}{v} \cos \alpha \ k_z \]
\[ k_g = \frac{\omega}{v} \sin \beta \ k_x - \frac{\omega}{v} \cos \beta \ k_z \]

\[ K_m = k_g - k_s \]
Refraction (forward scatt.) data: One frequency $\omega$

\[ k_s = \omega/v \sin \alpha \, k_x - \omega/v \cos \alpha \, k_z \]
\[ k_g = \omega/v \sin \beta \, k_x - \omega/v \cos \beta \, k_z \]
\[ K_m = k_g - k_s \]
Transmission (forward scatt.) in reflection data

\[ K_{ms} = k_{s2} - k_{s1} \]
\[ K_{mg} = k_{g2} - k_{g1} \]
Data ↔ Non linear Model

ΔVelocity

Δwave correct

Δwave with 1st order scattering

Δt

Δwave with 25th order scattering

Data ç Non linear è Model

Δt

Δt

Δt
Data ↔ Non linear ↔ Model

ΔVelocity

Δwave correct

Δwave with 1st order scattering

Δt

Δwave with 25th order scattering
Linearized $\tau$ extension $\tilde{\mathcal{L}}(\tilde{v}) = \mathcal{L}(v_0) + \tilde{L}\delta\tilde{v}^2(\tau)$

**Full non-linear scattering**

$$\left[ \partial_{tt} - v_0^2 \nabla^2 \right] P_0 = f$$

$$\left[ \partial_{tt} - v_0^2 \nabla^2 \right] \delta P = \delta v^2 \nabla^2 (P_0 + \delta P)$$

$v_0$: Background velocity

$P_0$: Background wavefield

$\delta v$: Velocity perturbation

$\delta P$: Scattered wavefield

$f$: Source function

**Linearized $\tau$ extension**

$$\left[ \partial_{tt} - v_0^2 \nabla^2 \right] P_0 = f$$

$$\left[ \partial_{tt} - v_0^2 \nabla^2 \right] \delta \tilde{P} = \delta \tilde{v}(\tau)^2 \ast \nabla^2 P_0$$

$\delta \tilde{v}(\tau)$: Extended-velocity perturbation

$\delta \tilde{P}$: New scattered wavefield
**Linearized $\tau$ extension**

\[ \tilde{\mathcal{L}}(\tilde{\nu}) = \mathcal{L}\left(\nu_0\right) + \tilde{\mathcal{L}}\delta\tilde{\nu}^2(\tau) \]

**Full non-linear scattering**

\[
\begin{aligned}
\left[ \partial_{tt} - \nu_0^2 \nabla^2 \right] P_0 &= f \\
\left[ \partial_{tt} - \nu_0^2 \nabla^2 \right] \delta P &= \delta\nu^2 \nabla^2 \left( P_0 + \delta P \right)
\end{aligned}
\]

$v_0$: Background velocity

$P_0$: Background wavefield

$\delta v$: Velocity perturbation

$\delta P$: Scattered wavefield

$f$: Source function

**Linearized $\tau$ extension**

\[
\begin{aligned}
\left[ \partial_{tt} - \nu_0^2 \nabla^2 \right] P_0 &= f \\
\left[ \partial_{tt} - \nu_0^2 \nabla^2 \right] \delta \tilde{P} &= \delta\tilde{\nu}(\tau)^2 \star \nabla^2 P_0
\end{aligned}
\]

$\delta\tilde{\nu}(\tau)$: Extended-velocity perturbation

$\delta\tilde{P}$: New scattered wavefield
FWI vs. TFWI

**FWI**

\[
J_{\text{FWI}}(v) = \frac{1}{2} \| \mathcal{L}(v) - d \|^2_2
\]

**TFWI**

\[
J_{\text{TFWI}}(\tilde{v}) = \frac{1}{2} \| \tilde{\mathcal{L}}(\tilde{v}) - d \|^2_2 \pm \varepsilon \| \mathcal{F}(\tilde{v}) \|^2_2
\]

\( J \) is the objective function to optimize, \( \mathcal{L} \) is non-linear modeling operator, \( v \) is velocity model, \( d \) are data.

\( \tilde{\mathcal{L}}(\tilde{v}) \) is the extended modeling operator, \( \tilde{v} \) is the extended velocity, e.g. \( \tilde{v}(\tau) \), \( \mathcal{F}(\tilde{v}) \) measures focusing of \( \tilde{v} \), e.g. \( \| \tau|\tilde{v}| \|^2_2 \).
Beyond Born – Extended velocity

Horizontal section across anomaly

\[ \delta \tilde{v}^2(\tau) \]
Beyond Born – Extended velocity

Vertical section across anomaly

$\delta \tilde{V}^2(\tau)$

Time lag (s)

Depth (km)

Horizontal loc. (km)
Beyond Born – Data residuals

Wavefield and data residuals by full non-linear scattering

Wavefield and data residuals by linearized \( \tau \) extension
Beyond Born – \[ \Delta \tilde{v}(\tau) = \tilde{L}' \Delta d \]

Wavefield computed by full non-linear scattering

Wavefield by linearized \( \tau \) extension with \( \Delta \tilde{v}(\tau) = \tilde{L}' \Delta d \)
Beyond Born —  $\Delta \tilde{\nu}(\tau) = \tilde{L}'\Delta d$

Wavefield computed by full non-linear scattering

Wavefield by linearized $\tau$ extension with $\Delta \tilde{\nu}(\tau) = \tilde{L}'\Delta d$
**Beyond Born** $- \Delta \tilde{v}(\tau) = \tilde{L}' \Delta d$

Data residuals computed by full non-linear scattering

Data residuals by linearized $\tau$ extension with $\Delta \tilde{v}(\tau) = \tilde{L}' \Delta d$
Limitations of Born linearization

Wavefield and data residuals by full non-linear scattering

Wavefield and data residuals by first-order Born scattering
2014 SEG FWI Blind Test (3-35 Hz)

Ali Almomin’s thesis – SEP 164
Initial velocity

Ali Almomin’s thesis – SEP 164
WET+FWI velocity

Ali Almomin’s thesis – SEP 164
Image with initial velocity

Ali Almomin’s thesis – SEP 164
Image with WET+FWI velocity

Ali Almomin’s thesis – SEP 164
CIGs with initial velocity
CIGs with TFWI velocity

Ali Almomin’s thesis – SEP 164
IO-Jansz

NW Australia

Conventional streamer data

Ali Almomin’s thesis – SEP 164
Image with initial velocity
Image with TFWI velocity
CIGs with initial velocity
CIGs with TFWI velocity

Ali Almomin’s thesis – SEP 164
What’s the matter with salt bodies?
They are everywhere!
They are important!

BP says 1 billion additional barrels 'possible' in Gulf of Mexico hubs

BP finds trove of oil in Gulf of Mexico using new subsea imaging

The algorithm that allowed BP to see under salt? Designed by whiz-kid @stanford grad Xukai Shen via @HoustonChron 7:30 AM - 27 Apr 2017

BP finds hidden trove of oil in Gulf of Mexico via new subsea imaging British oil major BP has discovered 200 million barrels of oil in a hidden cache in the Gulf of Mexico, thanks to a technological houstonchronicle.com
They are complicated!

Courtesy of Guillaume Barnier (SEP)
and they make a mess of the wavefield
and they make a mess of the wavefield

Courtesy of Guillaume Barnier (SEP)
and they make a mess of the wavefield
True model

Horizontal location (km)

Depth (km)

Velocity (km/s)

Courtesy of Guillaume Barnier (SEP)
Starting model

Horizontal location (km)

Depth (km)

Velocity (km/s)

Courtesy of Guillaume Barnier (SEP)
Initial data residuals

Courtesy of Guillaume Barnier (SEP)
Predicted residuals - No extension

\[ \Delta d = LL' \Delta d \]
Predicted residuals - $\tau$ extension

$\overline{\Delta d} = \tilde{L}\tilde{L}'\Delta d$

Courtesy of Guillaume Barnier (SEP)
Predicted residuals - $\tau$ extension

\[ \tilde{\Delta d} = \tilde{L}(\tilde{L}'\tilde{L})^\dagger \tilde{L}'\Delta d \]

Horizontal location (km)

Time (s)

28th iteration

Courtesy of Guillaume Barnier (SEP)
\[ \tilde{\Delta}d = L(L'\mathbf{L})^+ L'\Delta d \]

Predicted residuals - No extension

28\textsuperscript{th} iteration

Courtesy of Guillaume Barnier (SEP)
Prediction error - $\tau$ extension

$\Delta d - \Delta \tilde{d}$

$1^{st}$ iteration

Courtesy of Guillaume Barnier (SEP)
Prediction error - $\tau$ extension

$\widetilde{\Delta d} - \Delta d$

Horizontal location (km)

28th iteration

Courtesy of Guillaume Barnier (SEP)
Prediction error - No extension

\[ \Delta d - \tilde{\Delta d} \]

1st iteration

Horizontal location (km)

Time (s)

Courtesy of Guillaume Barnier (SEP)
Prediction error - No extension

$\Delta d - \Delta d$

28\textsuperscript{th} iteration

horizontal location (km)

Time (s)

Courtesy of Guillaume Barnier (SEP)
Why use level sets?

Salt bodies can often be approximated as homogeneous.

Level sets define boundaries of homogeneous bodies

This makes them a useful tool for salt modeling

Lewis et al. (EAGE 2012)
Guo and de Hoop (SEG 2013)

Wu, X., 2016, Methods to compute salt likelihoods and extract salt boundaries from 3D seismic images. *Geophysics*, 81(6)
Derivation: Objective function

Typical FWI:

$$\psi(m) = \| F(m) - d_{obs} \|^2_2$$

Level set FWI:

$$\psi(m(\phi, b)) = \| F(m(\phi, b)) - d_{obs} \|^2_2$$

Courtesy of Taylor Dhalke (SEP)
Derivation: New model space

\[ m(\phi, b) = H(\phi)(c_s - b) + b \]
Derivation: New model space

\[ m(\phi, b) = H(\phi)(c_s - b) + b \]

- Heaviside function
- Salt velocity
- Background velocity

Courtesy of Taylor Dhalke (SEP)
Prismatic waves caused by salt

Figure 3.16 & 3.17
What about the interaction of salt boundaries inside canyons?

What about the interaction between the salt position and the background velocity?

Motivation: Why use the Hessian?

Courtesy of Taylor Dhalke (SEP)
Motivation: Why use the Hessian?

What about the interaction of salt boundaries inside canyons?

What about the interaction between the salt position and the background velocity?

Can we formulate the Hessian for our objective function that helps account for these interactions?

Courtesy of Taylor Dhalke (SEP)
True model

- 40 shots
- 235 receivers
- 7 Hz source wavelet

Courtesy of Taylor Dhalke (SEP)
True model - Initial model difference

- Initial salt base is too deep

Courtesy of Taylor Dhalke (SEP)
Adjoint Born Image

Courtesy of Taylor Dhalke (SEP)
Phi search direction (steepest descent)

Courtesy of Taylor Dhalke (SEP)
Phi search direction (truncated full Newton)

Courtesy of Taylor Dhalke (SEP)
Initial Phi

Courtesy of Taylor Dhalke (SEP)
Updated Phi (steepest descent)

Courtesy of Taylor Dhalke (SEP)
Updated Phi (truncated full Newton)

Courtesy of Taylor Dhalke (SEP)
Conclusions

• We (as a community) are making progress towards making FWI to converge more robustly.

• Highly scattering geobodies (e.g. salt bodies) are still a challenge.

• It is unlikely that there is a silver bullet; different geologic settings call for different solutions.
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