Estimation of uncertainties of full moment tensors

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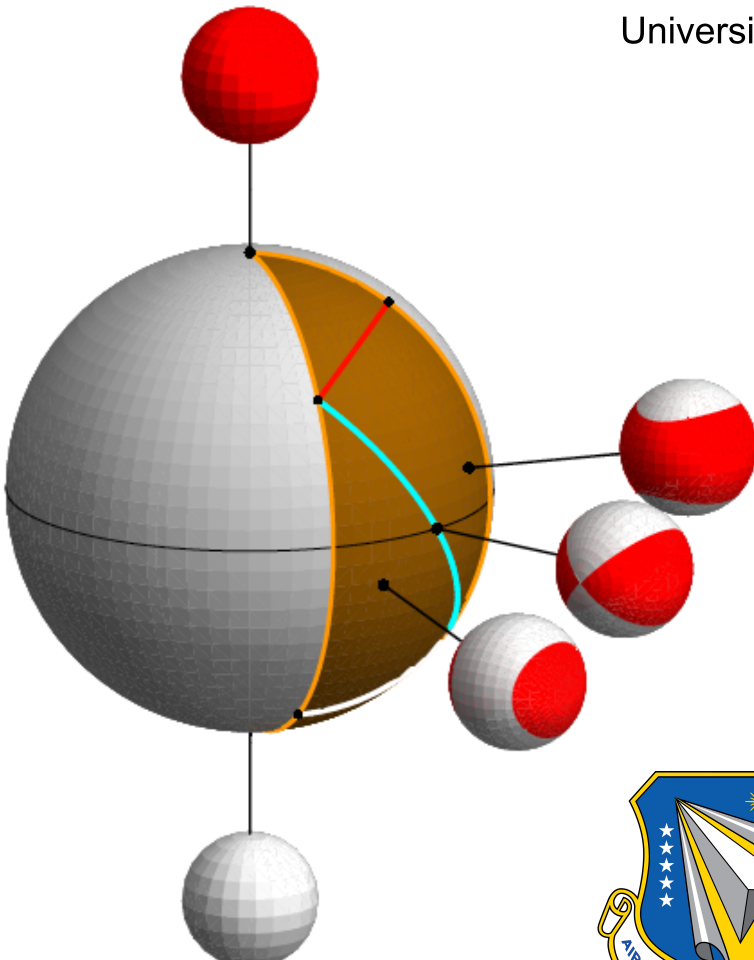
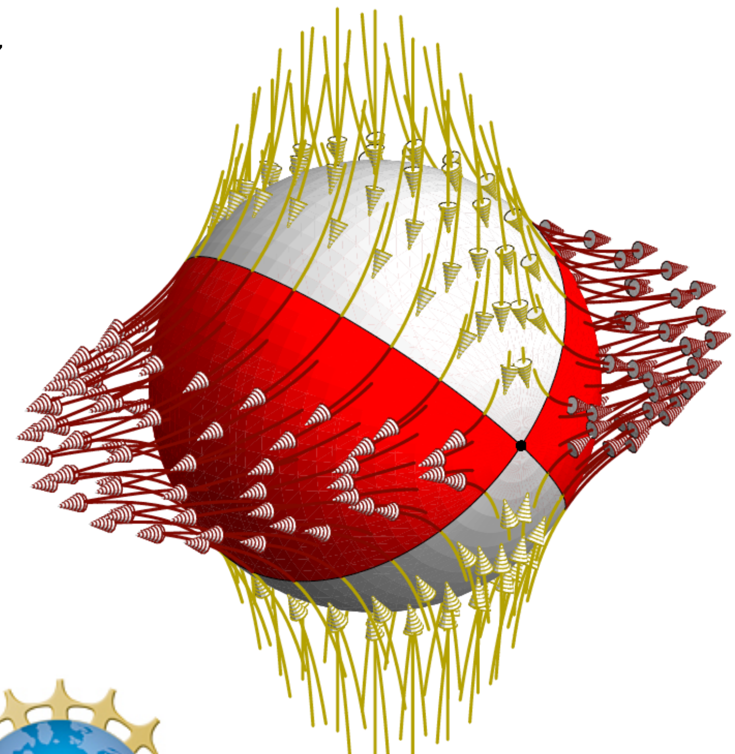


May 5, 2017

UCLA IPAM



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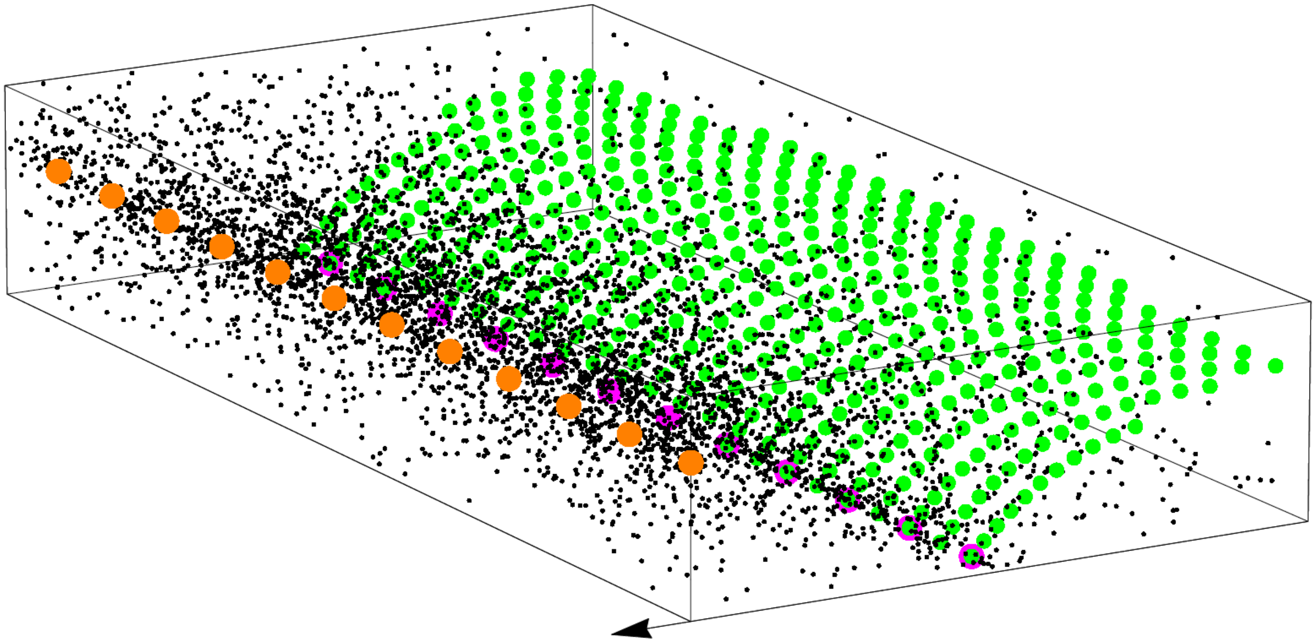
A moment tensor is a 3 x 3 symmetric matrix that characterizes a point-source mechanism for a source such as an earthquake.

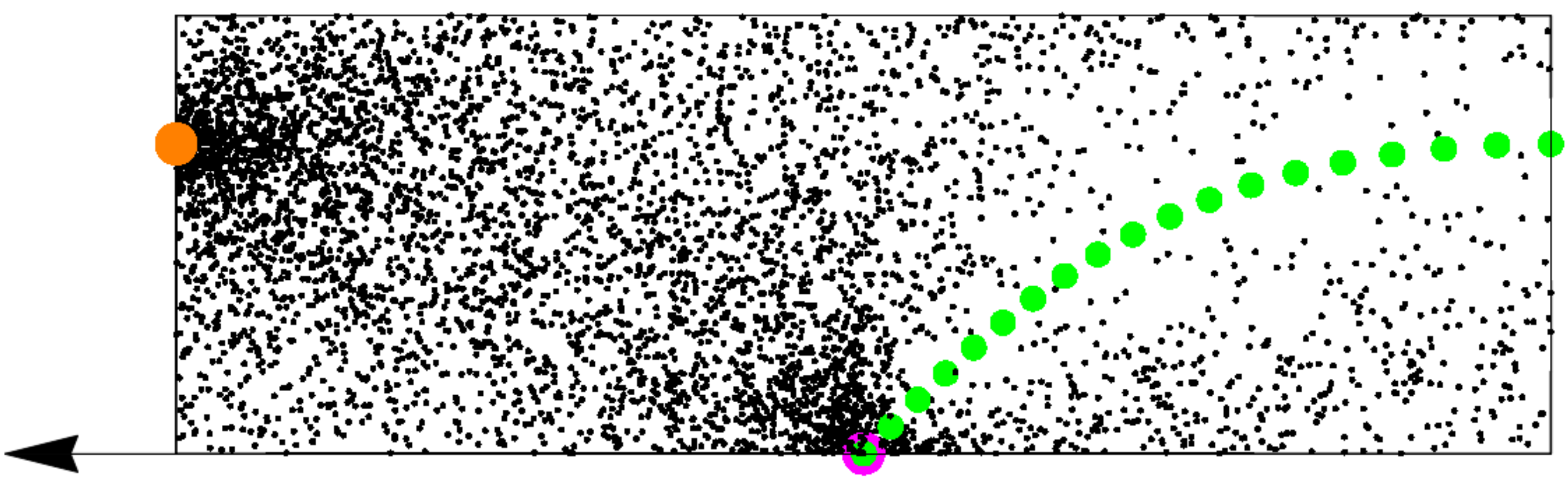
Moment tensors can be estimated from ground motion recorded by seismometers. (A model for Earth structure is needed.)

Why does moment tensor parameterization matter?

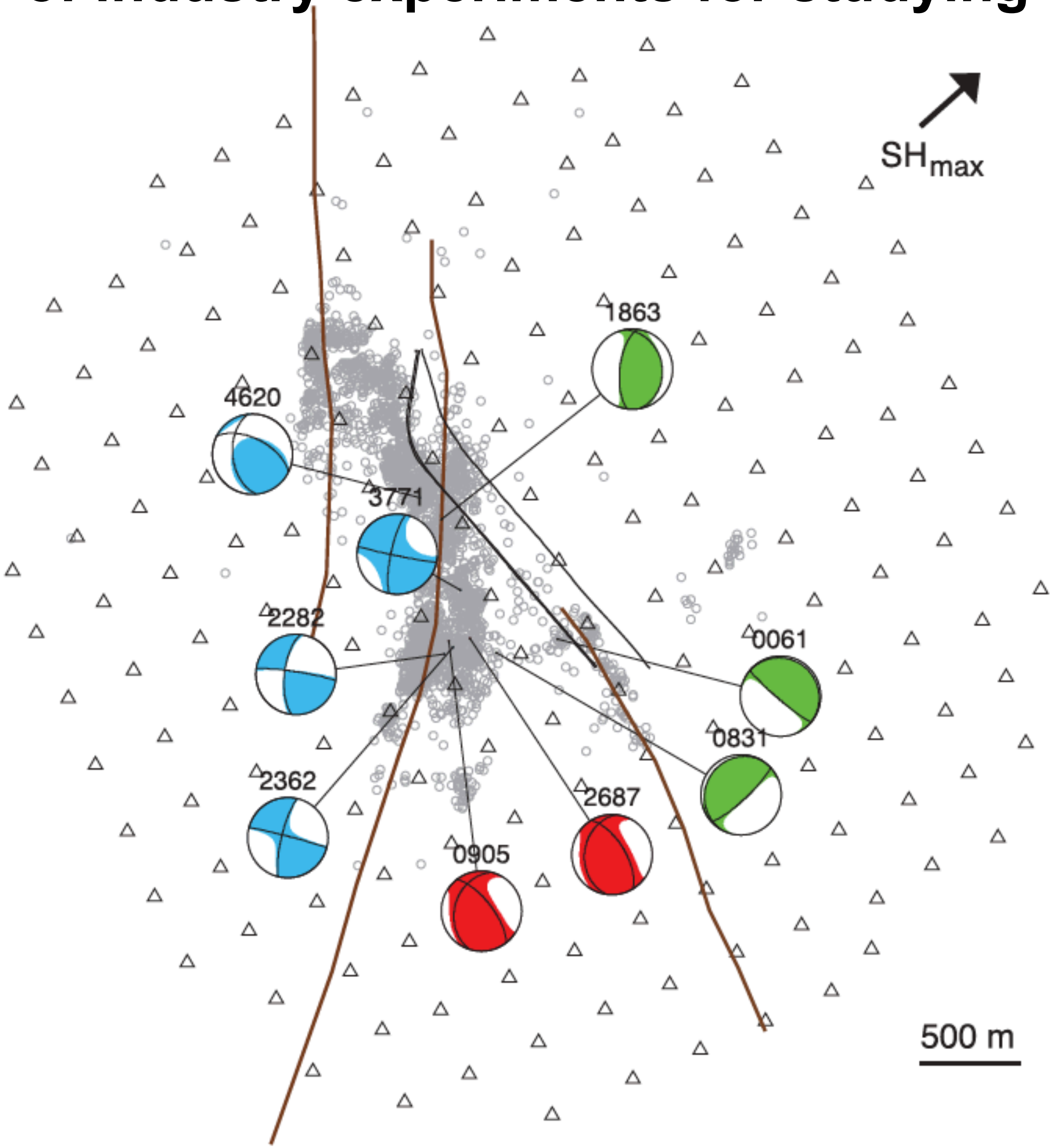
1. Allows for interpretation of the posterior probability density
2. Computational efficiency
3. Accurate numerical integration (dV increment needs to be known)
4. Provides a good framework for visualization



Representation of a recently published new parameterization for moment tensors. Its sampling of model parameter space turns out to be **highly non-uniform.**

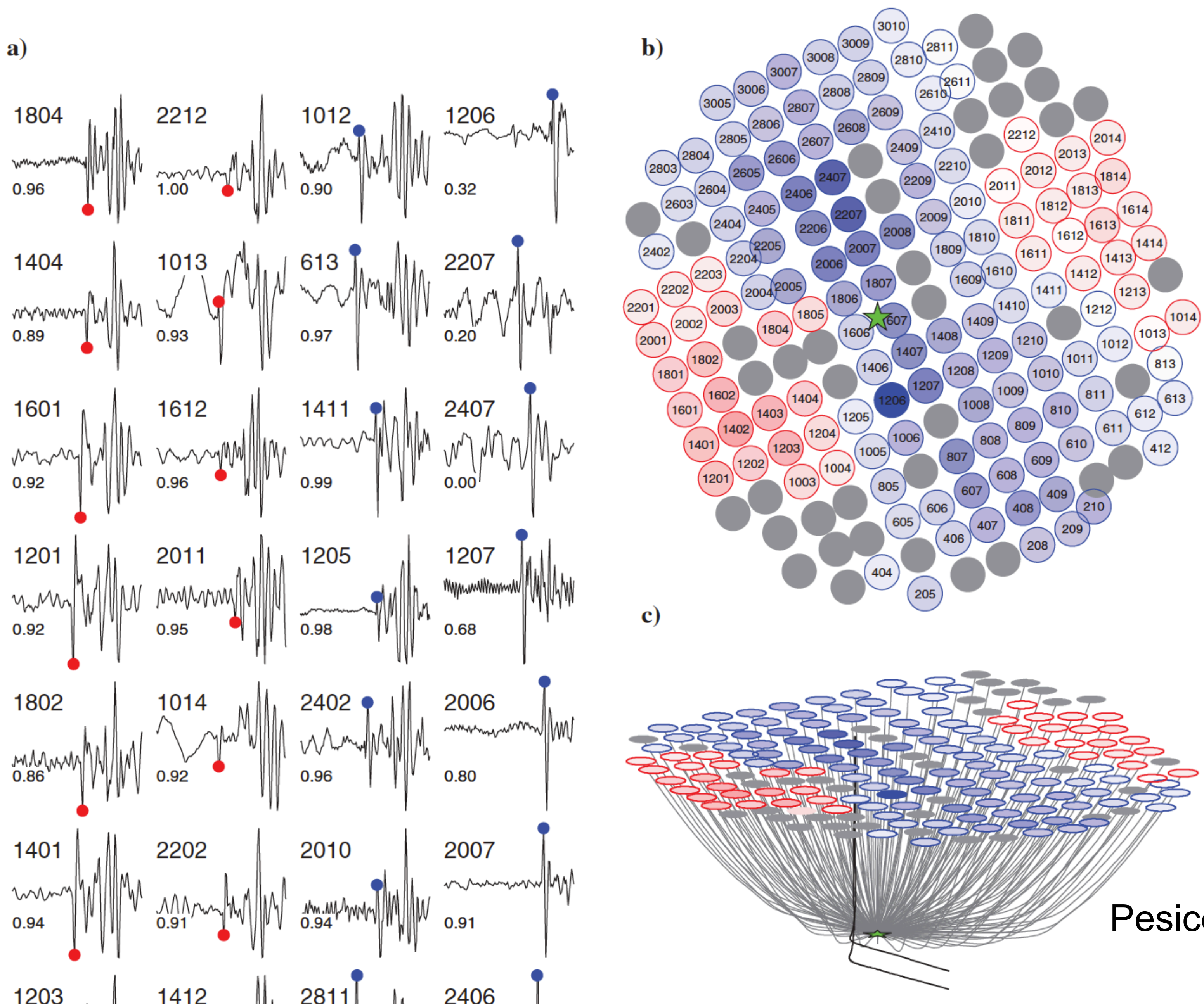


Examples of industry experiments for studying sources



Pesicek et al. (2016)

Examples of industry experiments for studying sources



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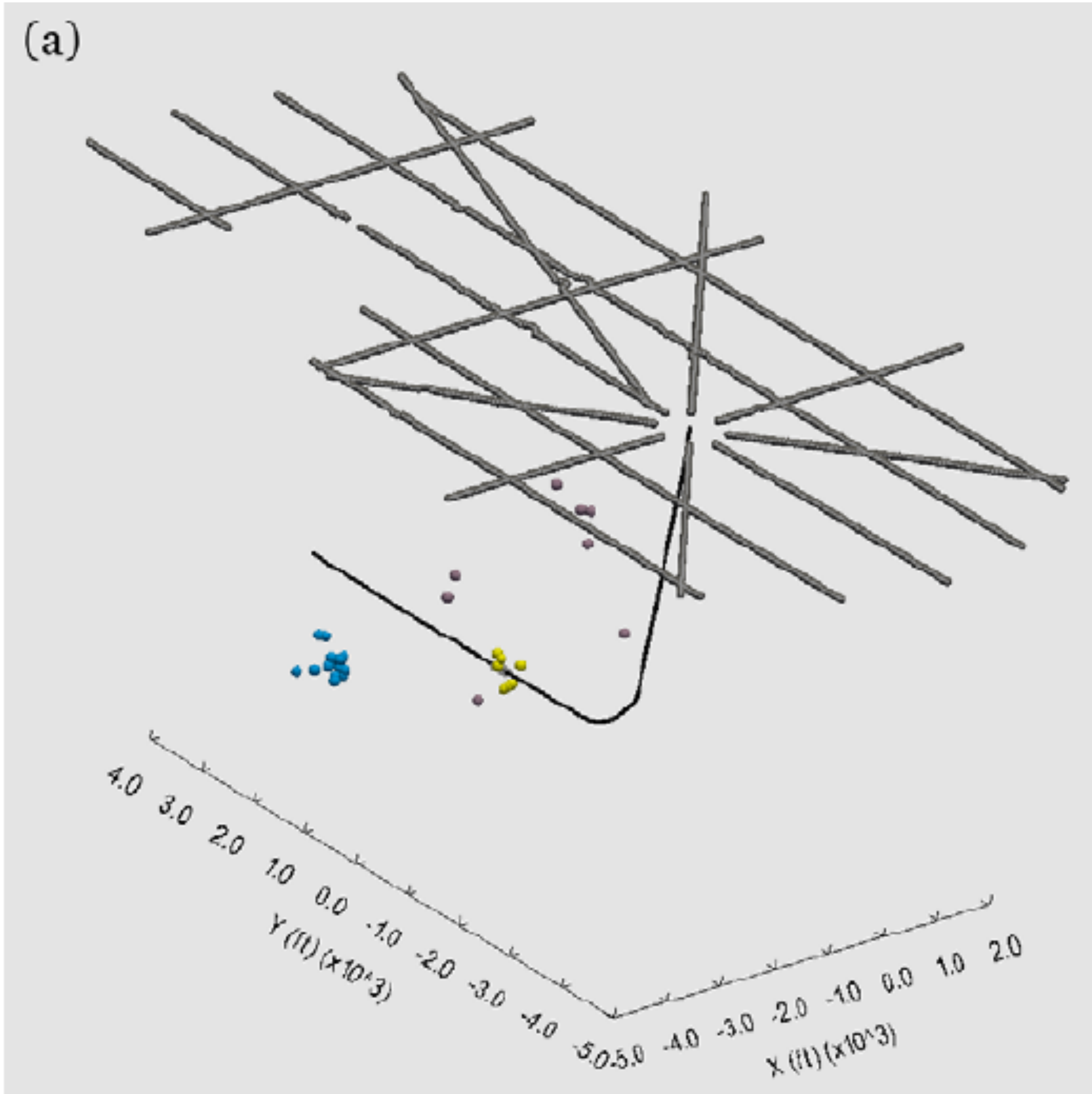
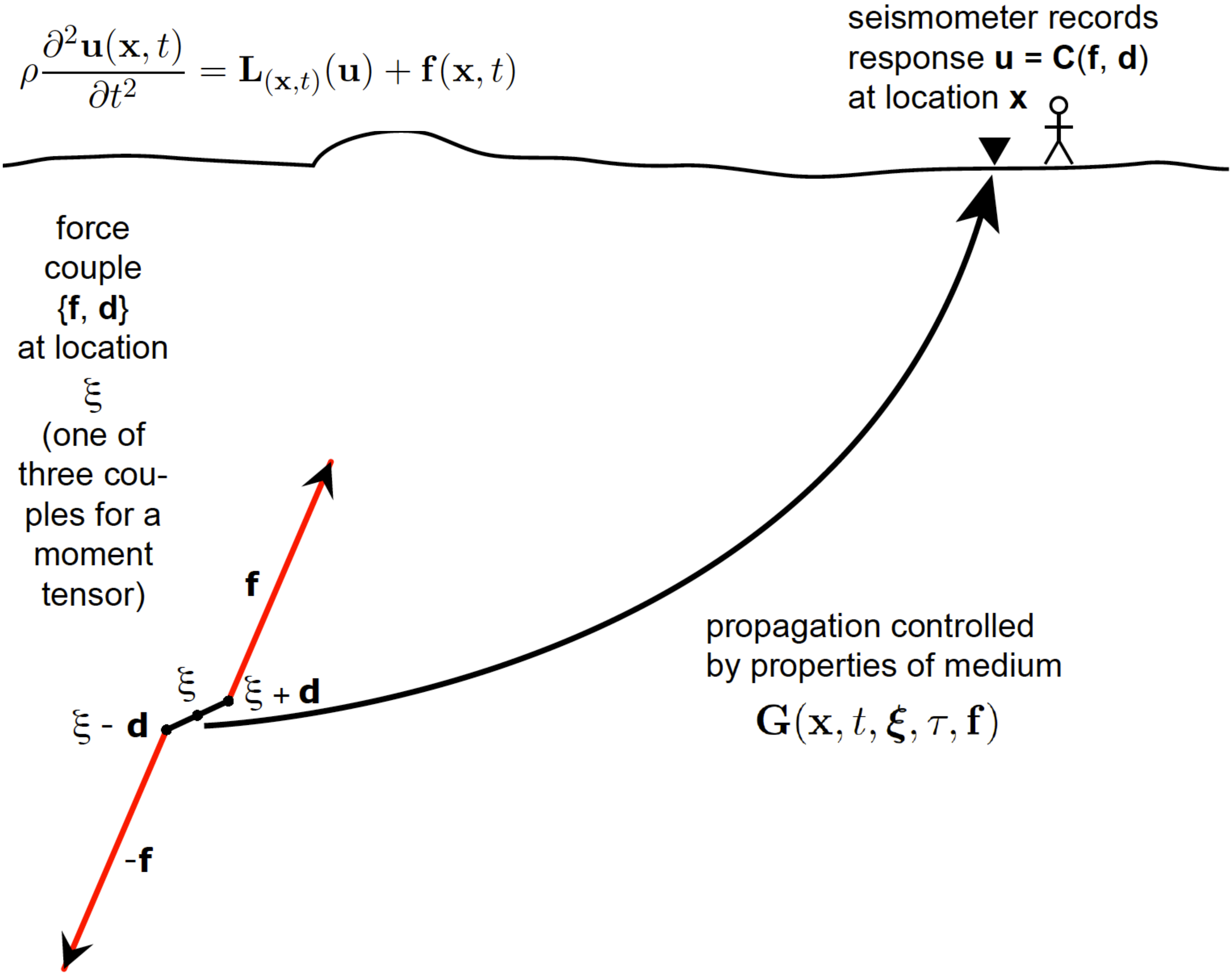
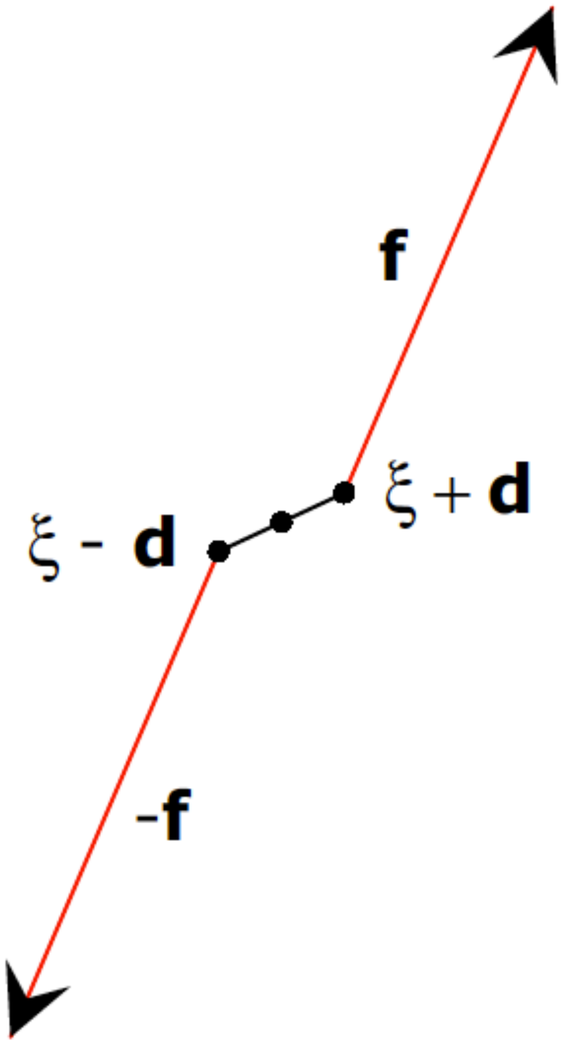
Examples of industry experiments for studying sources



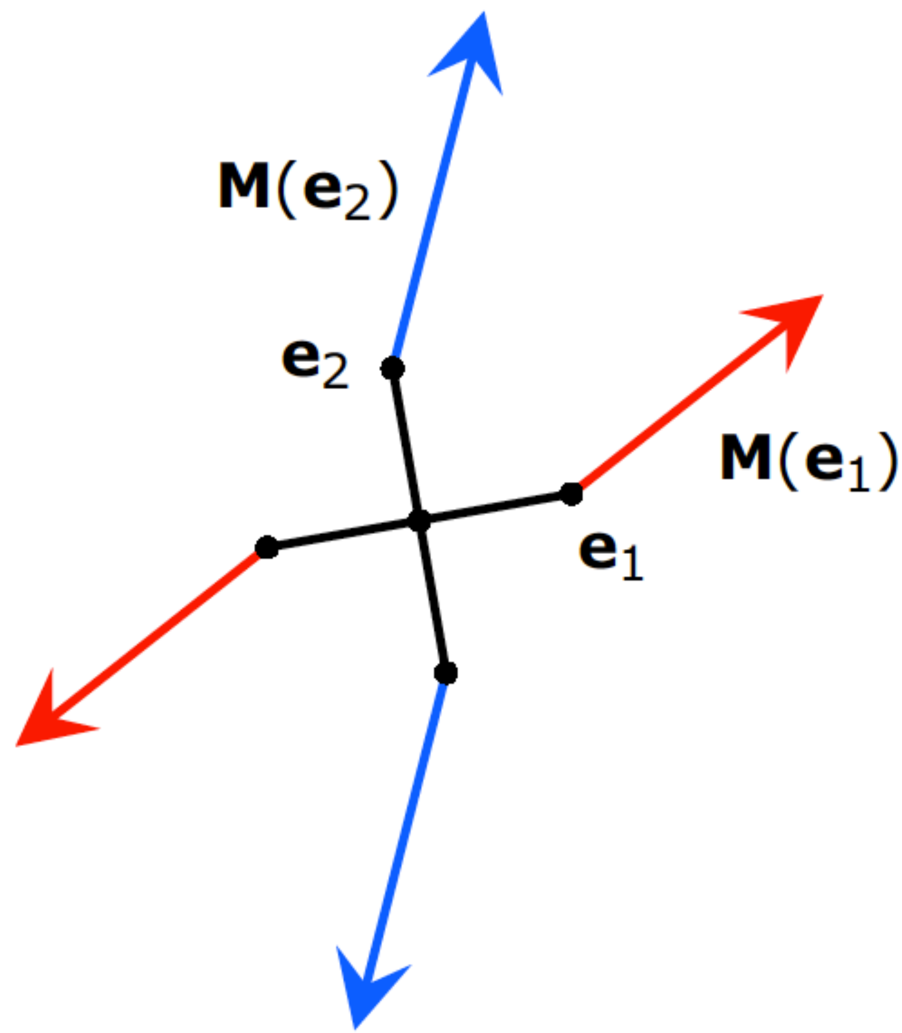
Clarke et al. (2013)


$\rho \frac{\partial^2 \mathbf{u}(\mathbf{x},t)}{\partial t^2} = \mathbf{L}_{(\mathbf{x},t)}(\mathbf{u}) + \mathbf{f}(\mathbf{x},t)$
seismometer records response **u = C(f, d)** at location **x**
force couple **{f, d}** at location ξ (one of three couples for a moment tensor)
f
-f
ξ
ξ - **d**
ξ + **d**
propagation controlled by properties of medium
$\mathbf{G}(\mathbf{x}, t, \boldsymbol{\xi}, \tau, \mathbf{f})$

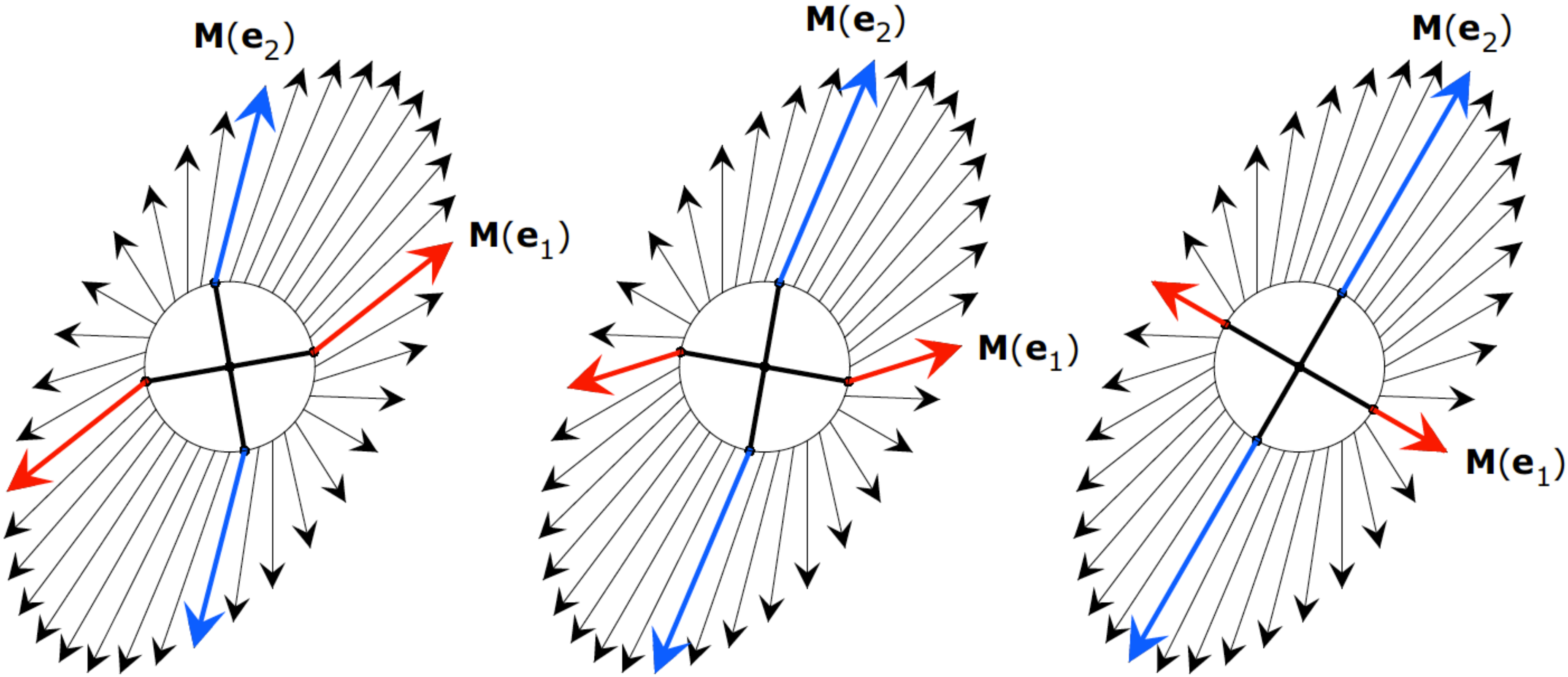
A single couple $\{\mathbf{f}, \mathbf{d}\}$



2D moment tensor
= two force couples
$\{\mathbf{e_1}, M(\mathbf{e_1})\}$ and $\{\mathbf{e_2}, M(\mathbf{e_2})\}$



Choose any orthonormal basis to get entries of moment tensor matrix.



Notation

$$[\mathbf{\Lambda}] = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix},$$

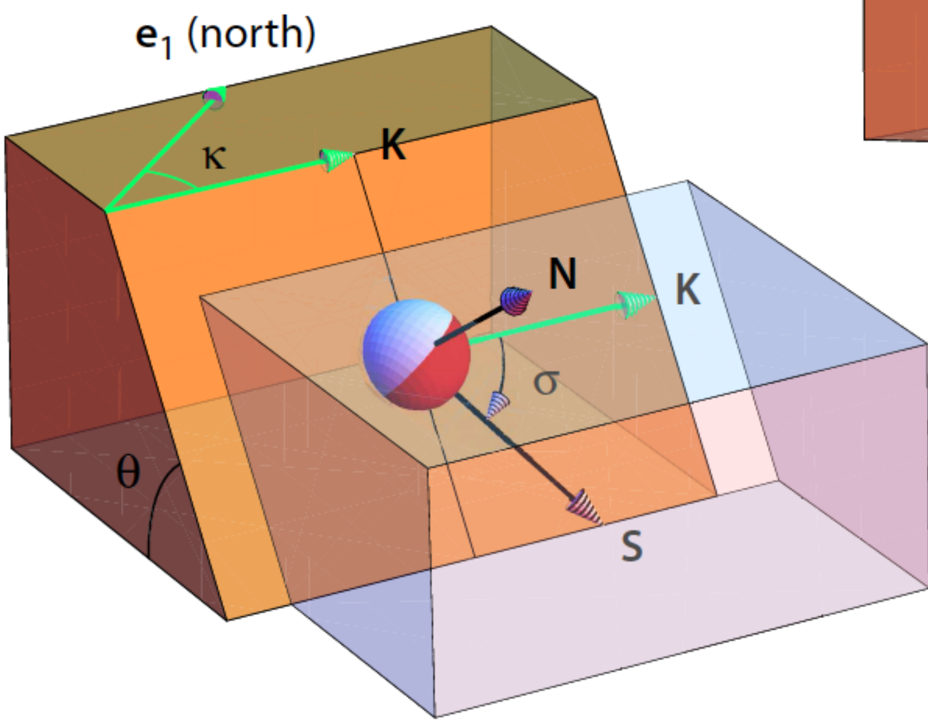
$$[\mathbf{\Lambda}]_U = U\,[\mathbf{\Lambda}]U^{-1}.$$

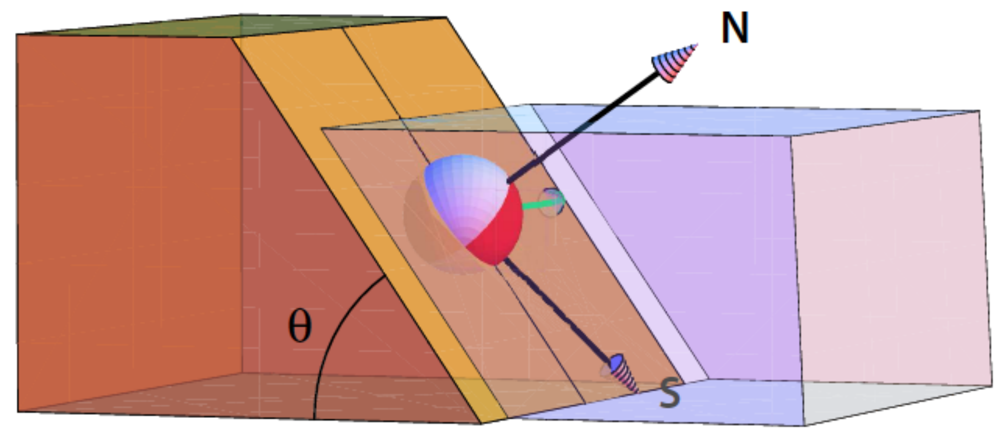
Notation

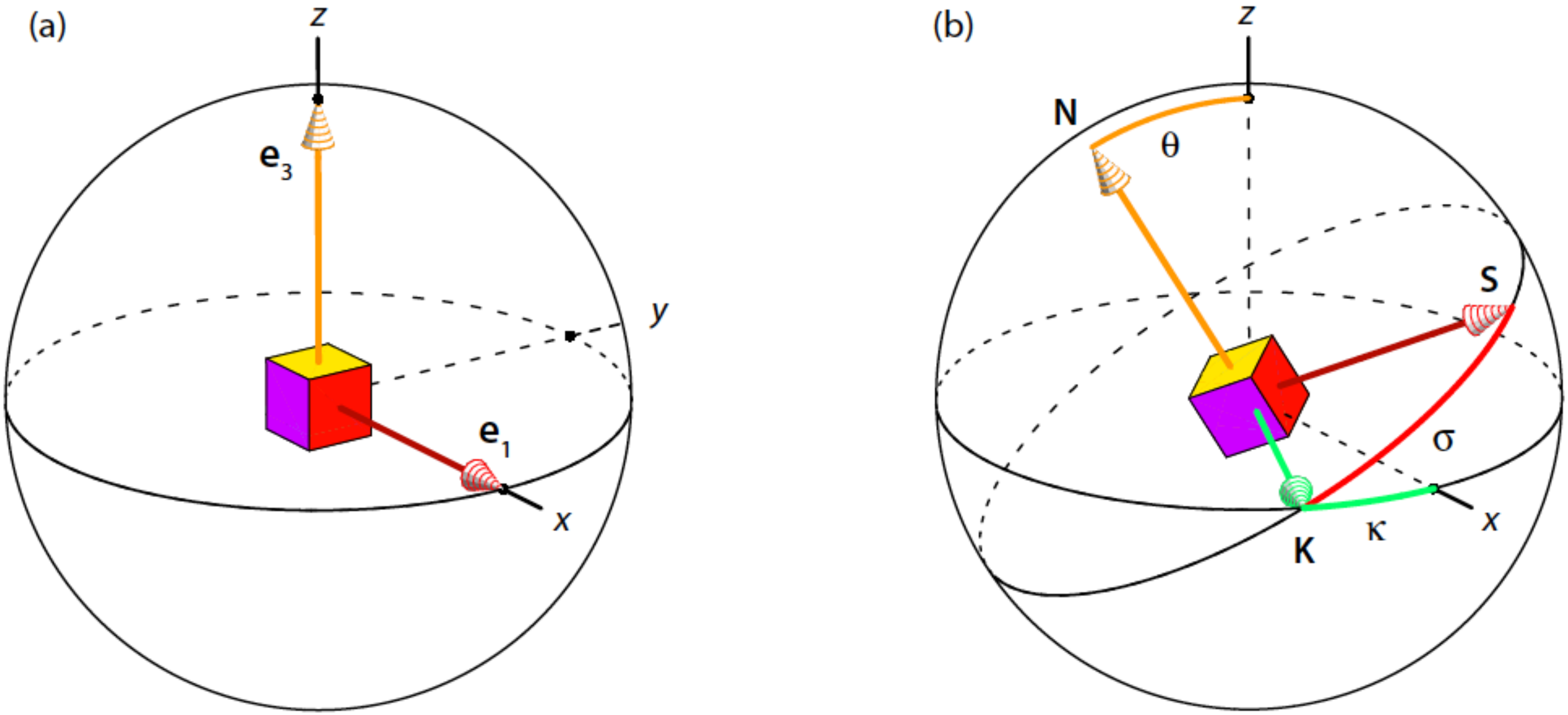
The symbols X_ξ, Y_ξ and Z_ξ denote the rotations through angle ξ about the coordinate axes. Thus

$$Z_\xi = \begin{pmatrix} \cos\xi & -\sin\xi & 0 \\ \sin\xi & \cos\xi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad Y_\xi = \begin{pmatrix} \cos\xi & 0 & \sin\xi \\ 0 & 1 & 0 \\ -\sin\xi & 0 & \cos\xi \end{pmatrix}, \quad \text{etc.} \tag{3}$$

The moment tensor as slip on a fault







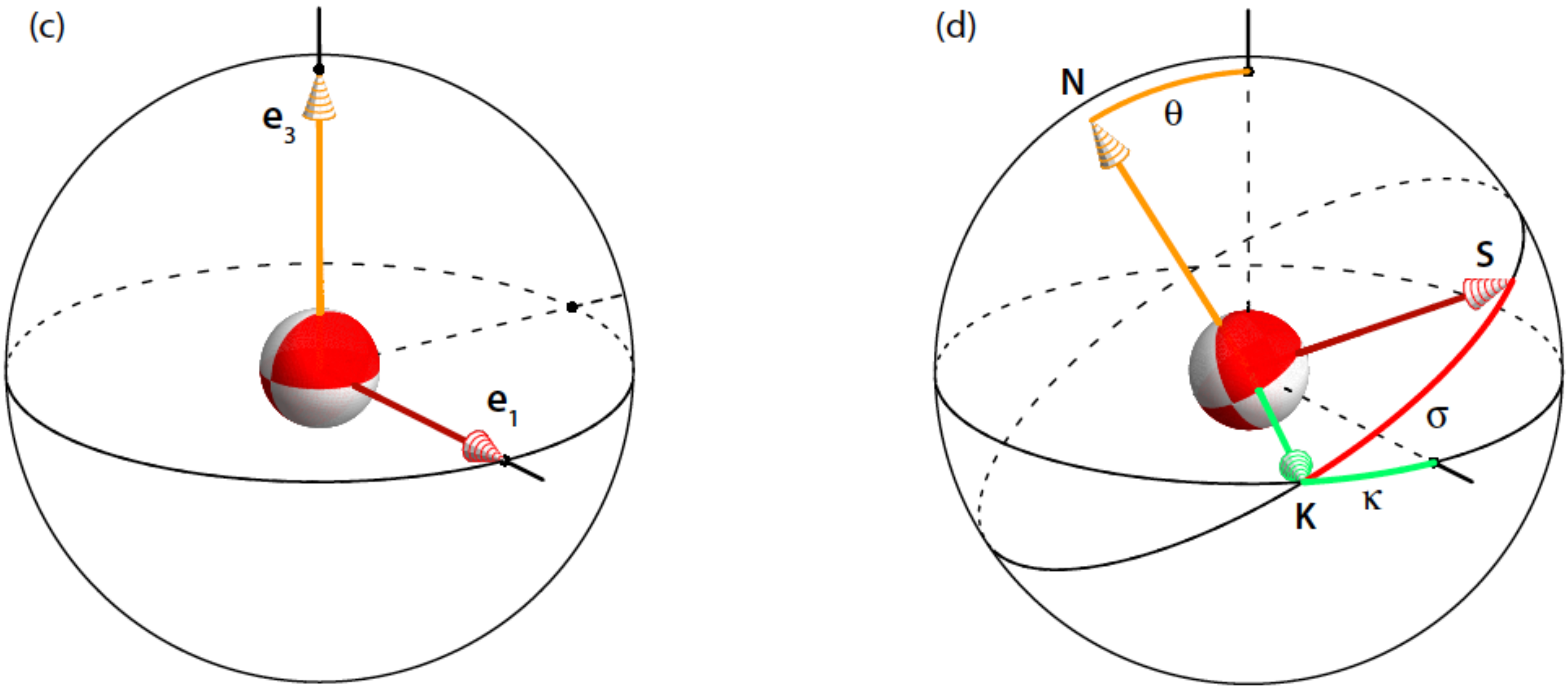
$$\mathbf{K} = R_\phi(\mathbf{e}_3) \cdot \mathbf{e}_1 = Z_\phi \cdot \mathbf{e}_1 \qquad \text{(strike vector)}$$

$$\mathbf{N} = R_\theta(\mathbf{K}) \cdot \mathbf{e}_3 \qquad \text{(normal vector)}$$

$$\mathbf{S} = R_\sigma(\mathbf{N}) \cdot \mathbf{K} \qquad \text{(slip vector),}$$

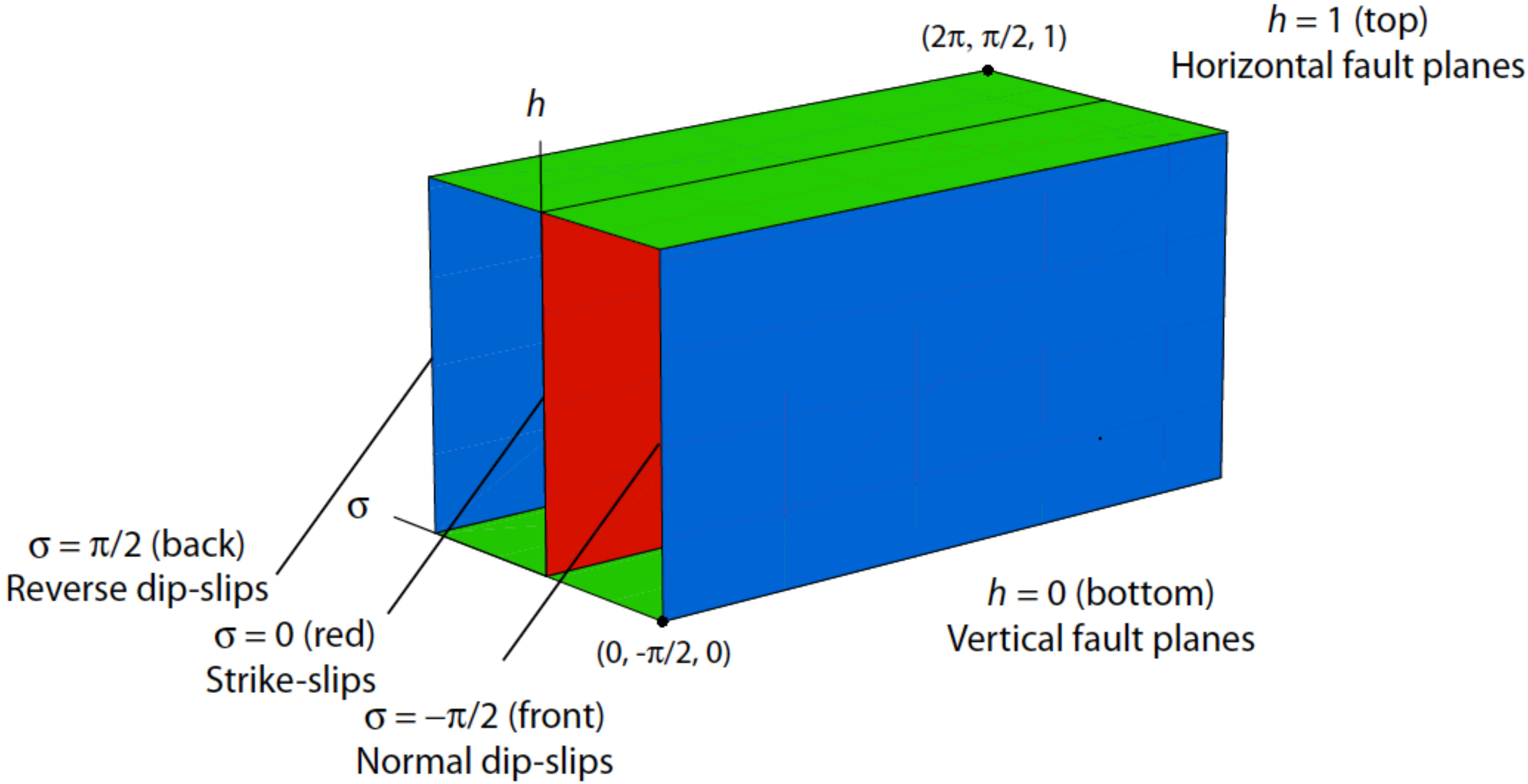
$$M = [\mathbf{\Lambda}]_{V\,Y_{-\pi/4}}$$

$$V(\mathbf{S},\ \mathbf{N}) = (\mathbf{S},\ \mathbf{N} \times \mathbf{S},\ \mathbf{N}),$$



(c)
e3
e1
(d)
N
θ
S
σ
K
κ

The "orientation block" (3 out of 6 moment tensor parameters)

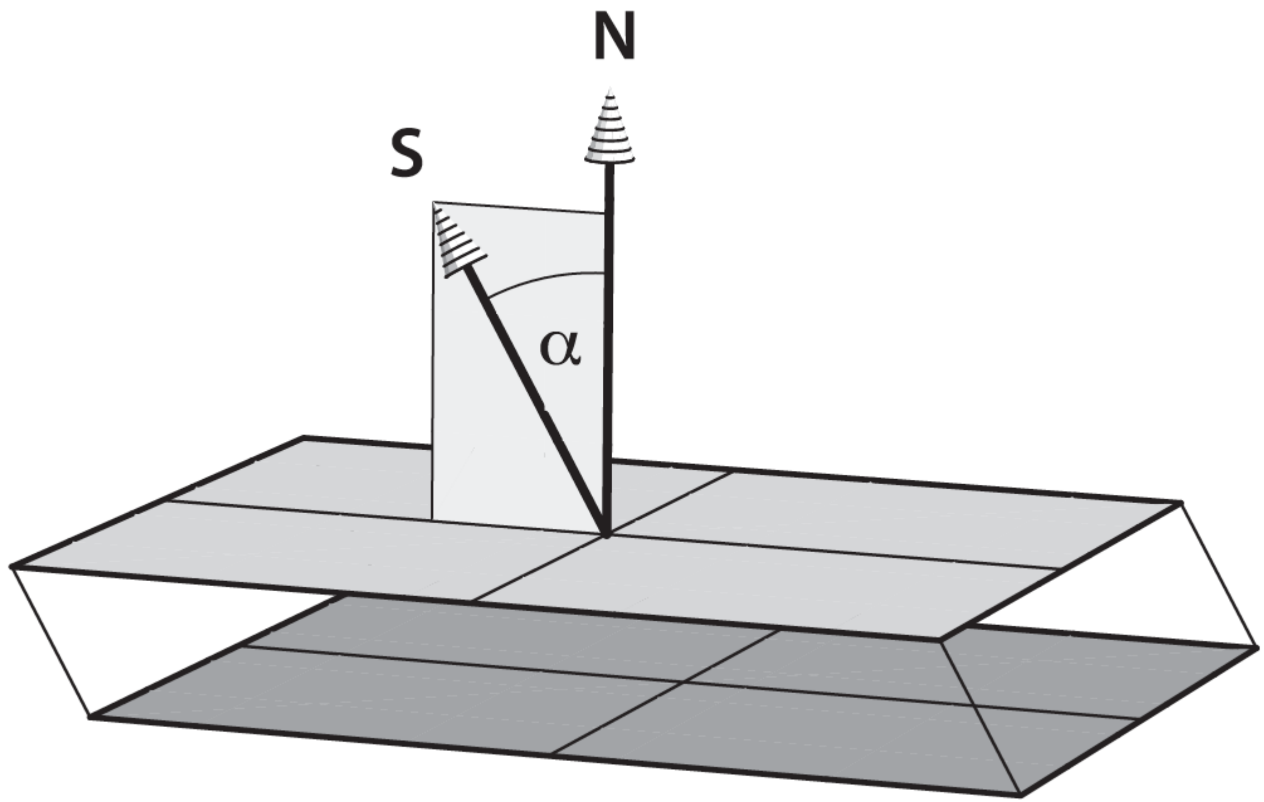


This will provide all the moment tensor orientations (but half the faults).

Note that the strike, dip, and rake are for the closest double couple of a moment tensor.

Full moment tensors allow for opening (or closing) across a fault.

(Aki and Richards, 1980)



α is the angle between slip vector and normal vector:

α = 90 for a double couple

α = 0 for a tensional crack

α = 180 for a compressional crack

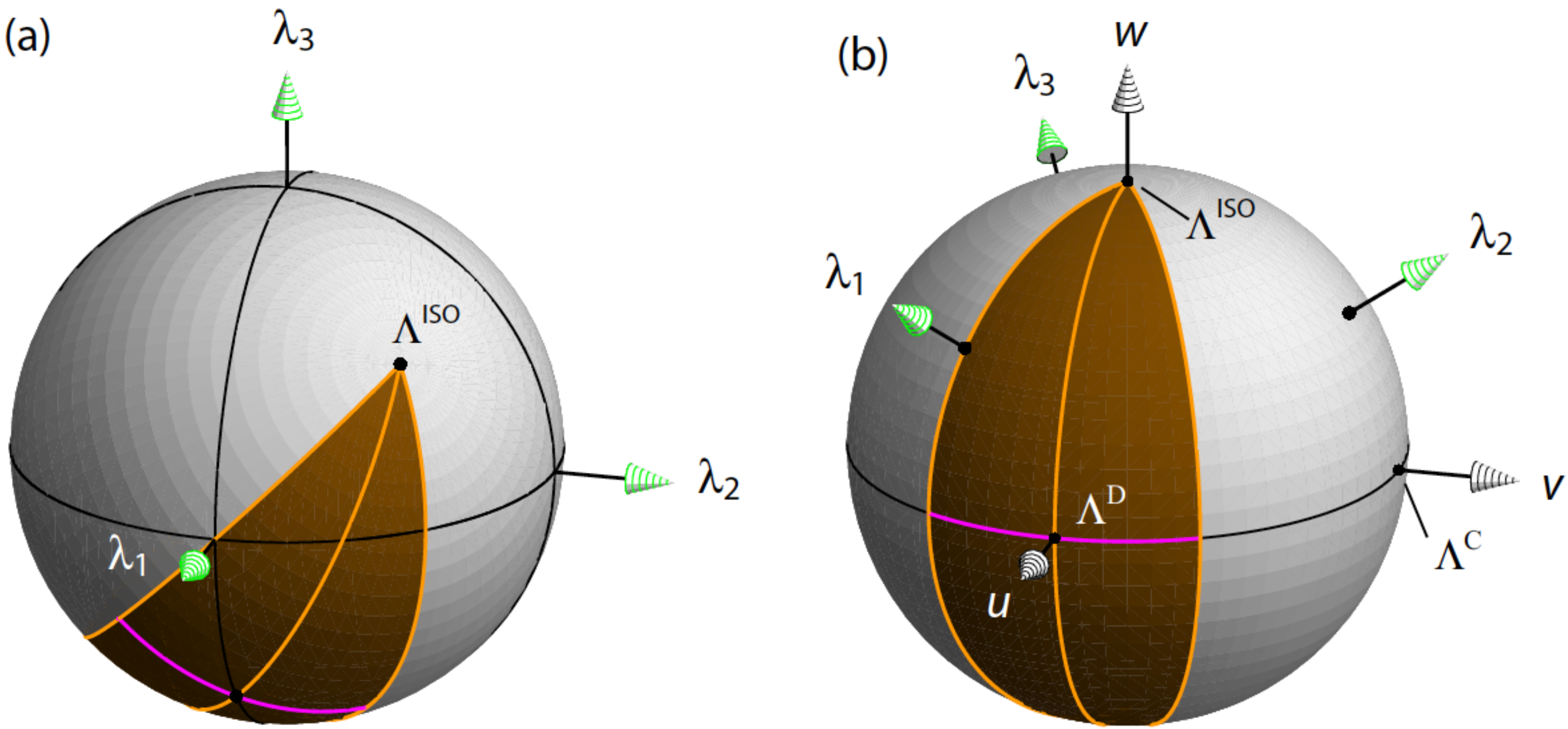
$$[\mathbf{\Lambda}] = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$$[\mathbf{\Lambda}]_U = U\,[\mathbf{\Lambda}]U^{-1}$$

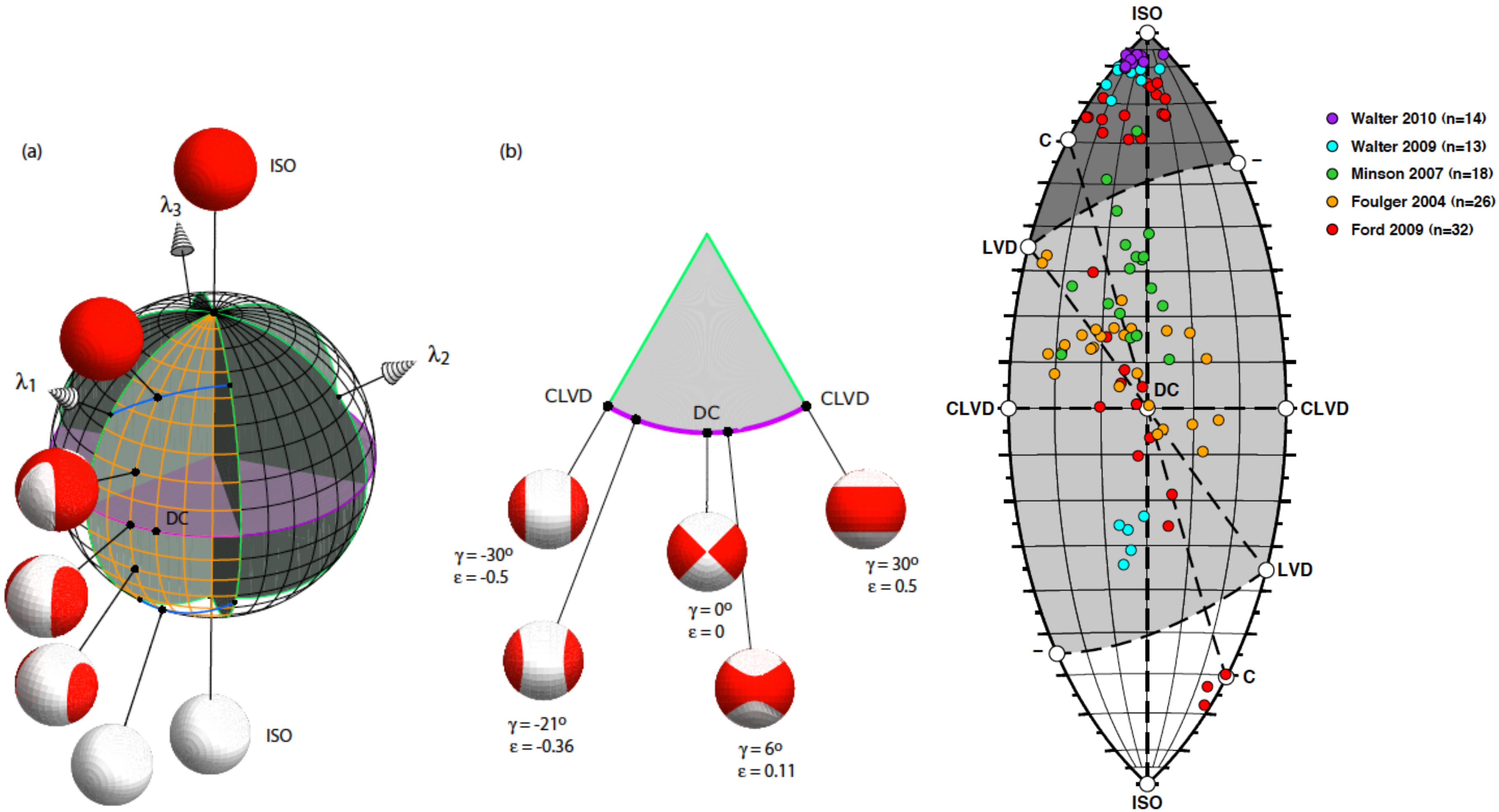
basis U
= moment tensor orientation

eigenvalue triple (λ_1, λ_2, λ_3)
= moment tensor source type

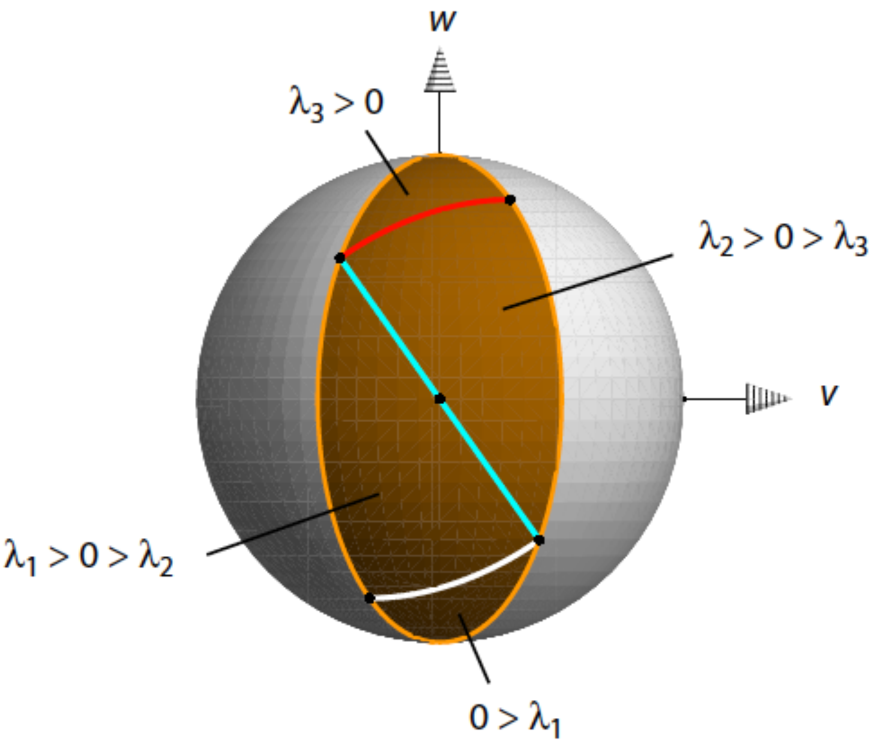
The lune is the set of ordered and normalized eigenvalues.

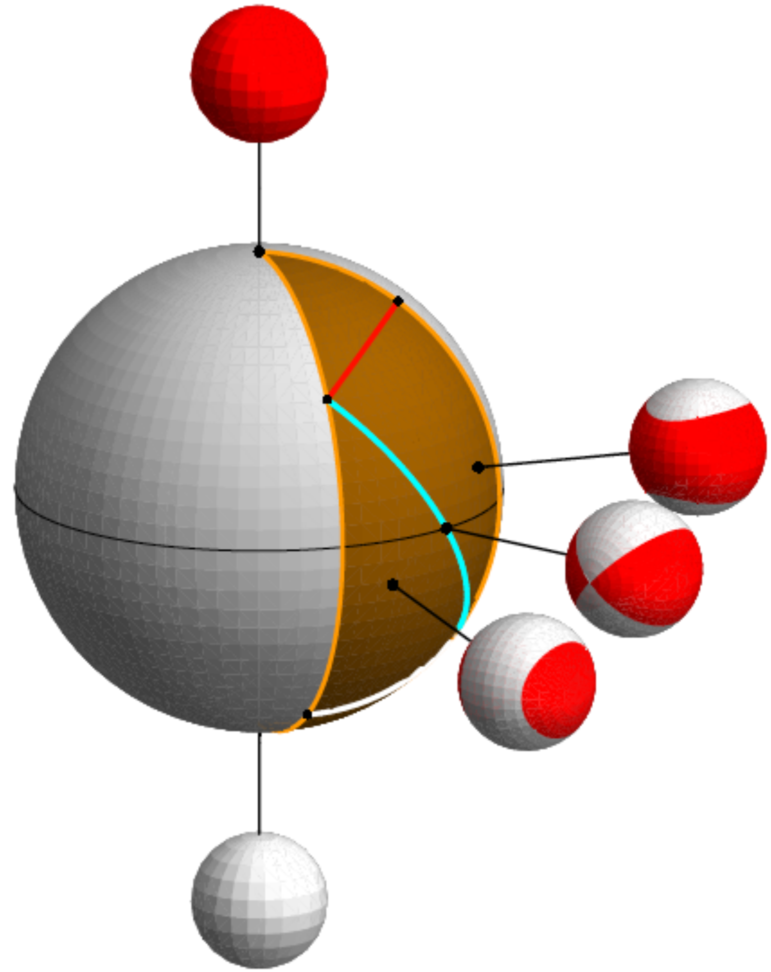


Moment tensor source types

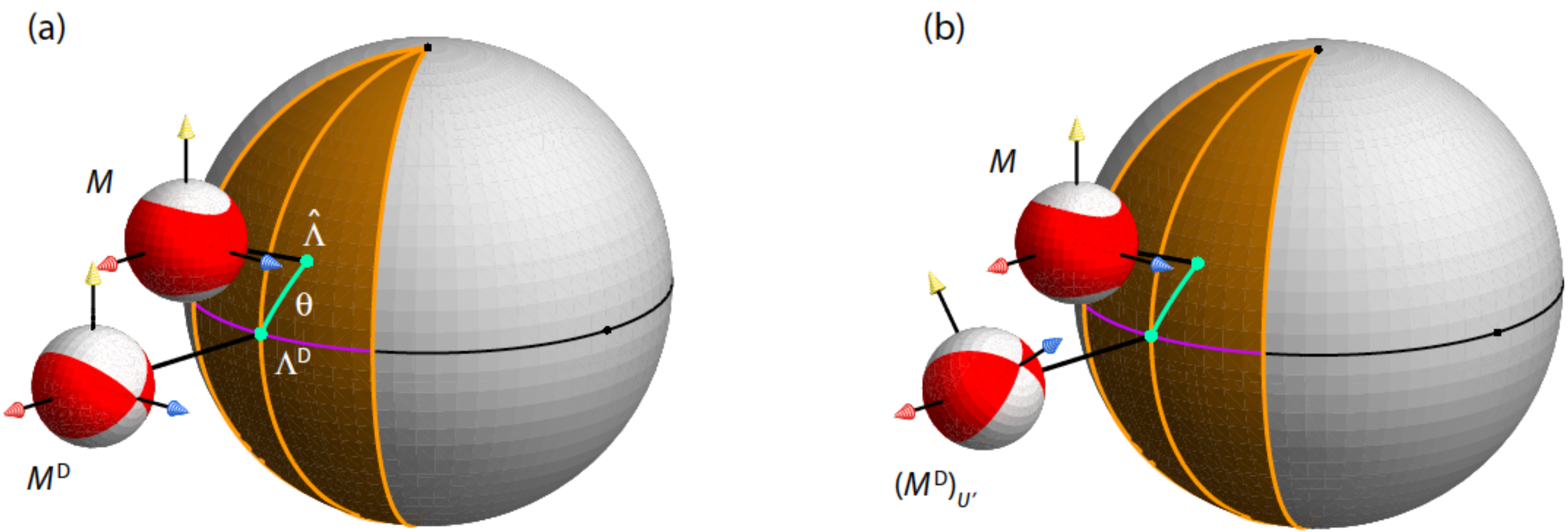


Moment tensor source types

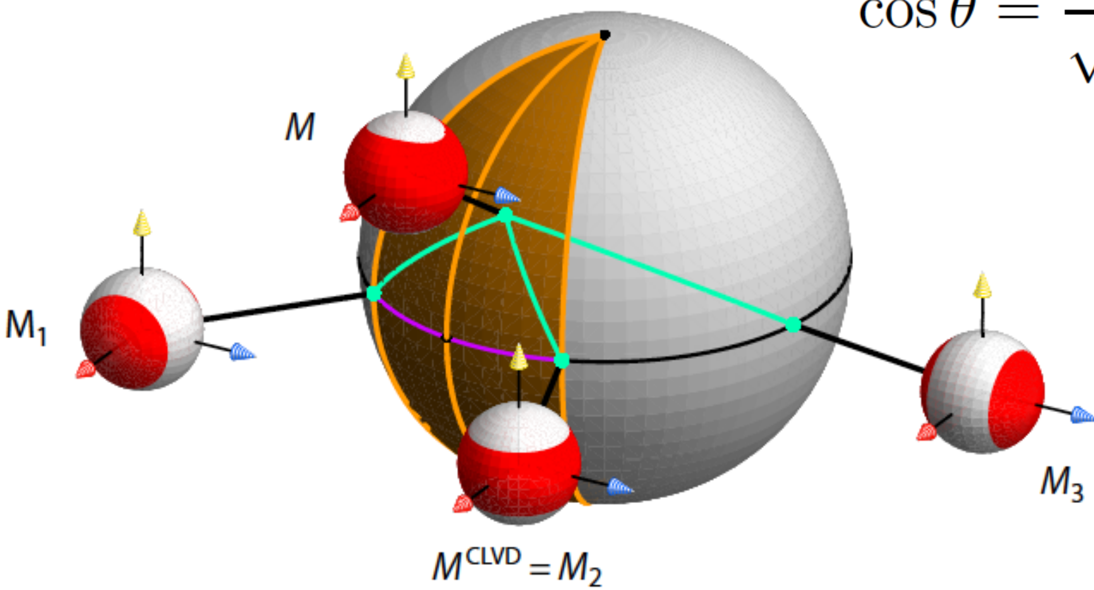




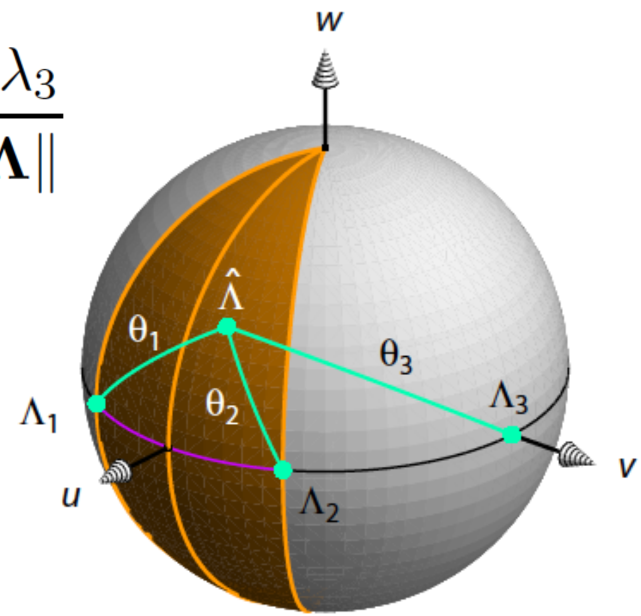
The arc distance on the lune is the minimum angle between moment tensors in matrix space.



If you want to know "how close" a moment tensor is to being a DC (or CLVD or ISO), then measure the arc distance to it.



$$\cos\theta = \frac{\lambda_1 - \lambda_3}{\sqrt{2}\,\|\mathbf{\Lambda}\|}$$



Note: here all beachballs have the same frame.

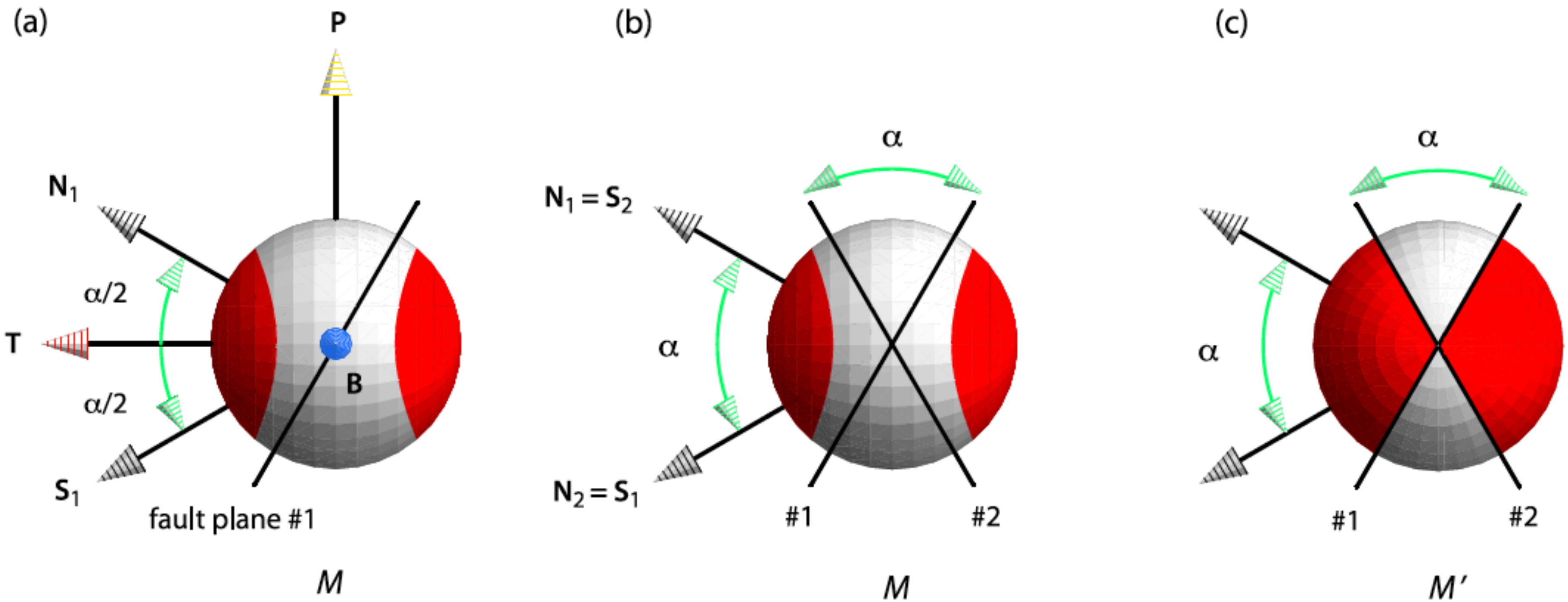
Formula for DC "percentage" from the literature:

$$c^{\mathrm{DC}} = 1 - \frac{2\,|\lambda_1 - 2\lambda_2 + \lambda_3|\,\Big|\,|\lambda_1 + \lambda_2 + \lambda_3| - 3\max\left(|\lambda_1|, |\lambda_3|\right)\Big|}{3\max\left(|\lambda_1|, |\lambda_3|\right)\,\max\left(\,|\lambda_1 + \lambda_2 - 2\lambda_3|, |2\lambda_1 - \lambda_2 - \lambda_3|\right)} - \frac{|\lambda_1 + \lambda_2 + \lambda_3|}{3\max\left(|\lambda_1|, |\lambda_3|\right)}$$

Moment tensor showing the fault plane in the classical model

$$M_{pq} = \mu\,(n_p s_q + n_q s_p) + \lambda\,\delta_{pq}\,\mathbf{S}\cdot\mathbf{N}$$

Aki and Richards (1980)

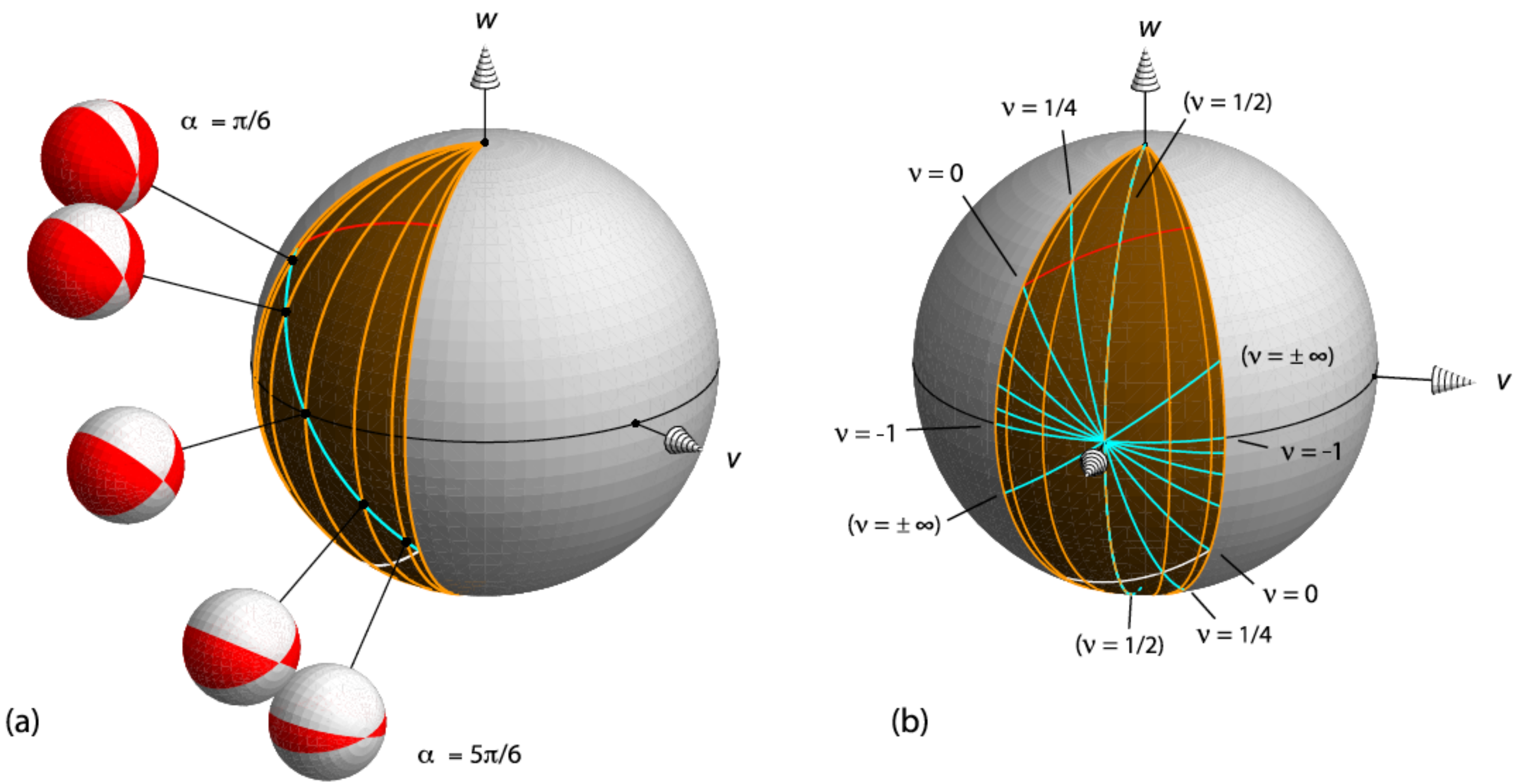


$$\mathbf{N} = Y_{-\alpha/2}\,\mathbf{e}_1 = \big(\cos(\alpha/2),\ 0,\ \sin(\alpha/2)\big)$$

$$\mathbf{S} = Y_{+\alpha/2}\,\mathbf{e}_1 = \big(\cos(\alpha/2),\ 0,\ -\sin(\alpha/2)\big)$$

N and S are fixed for a fixed lune longitude.

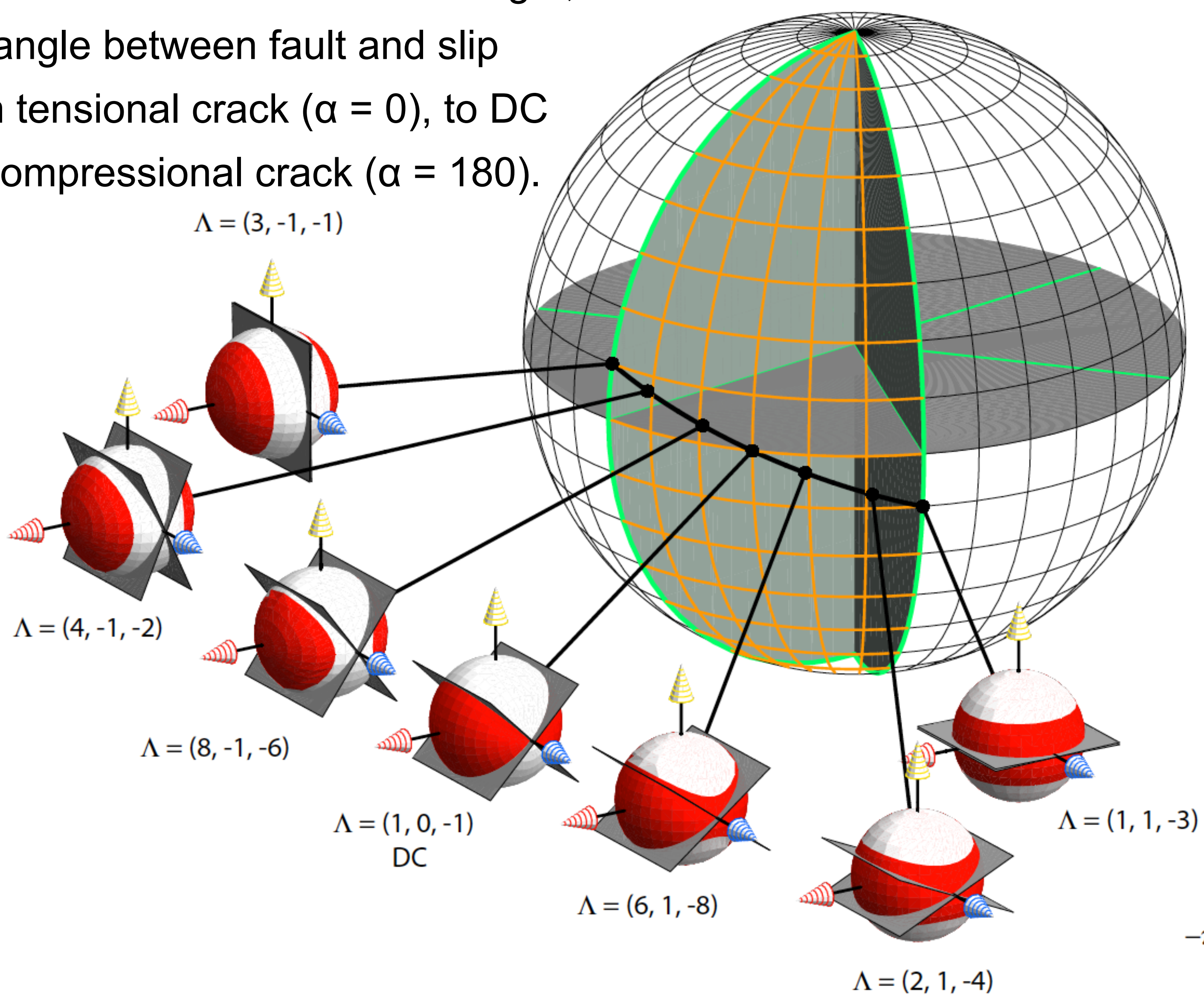
The classical model of moment tensors



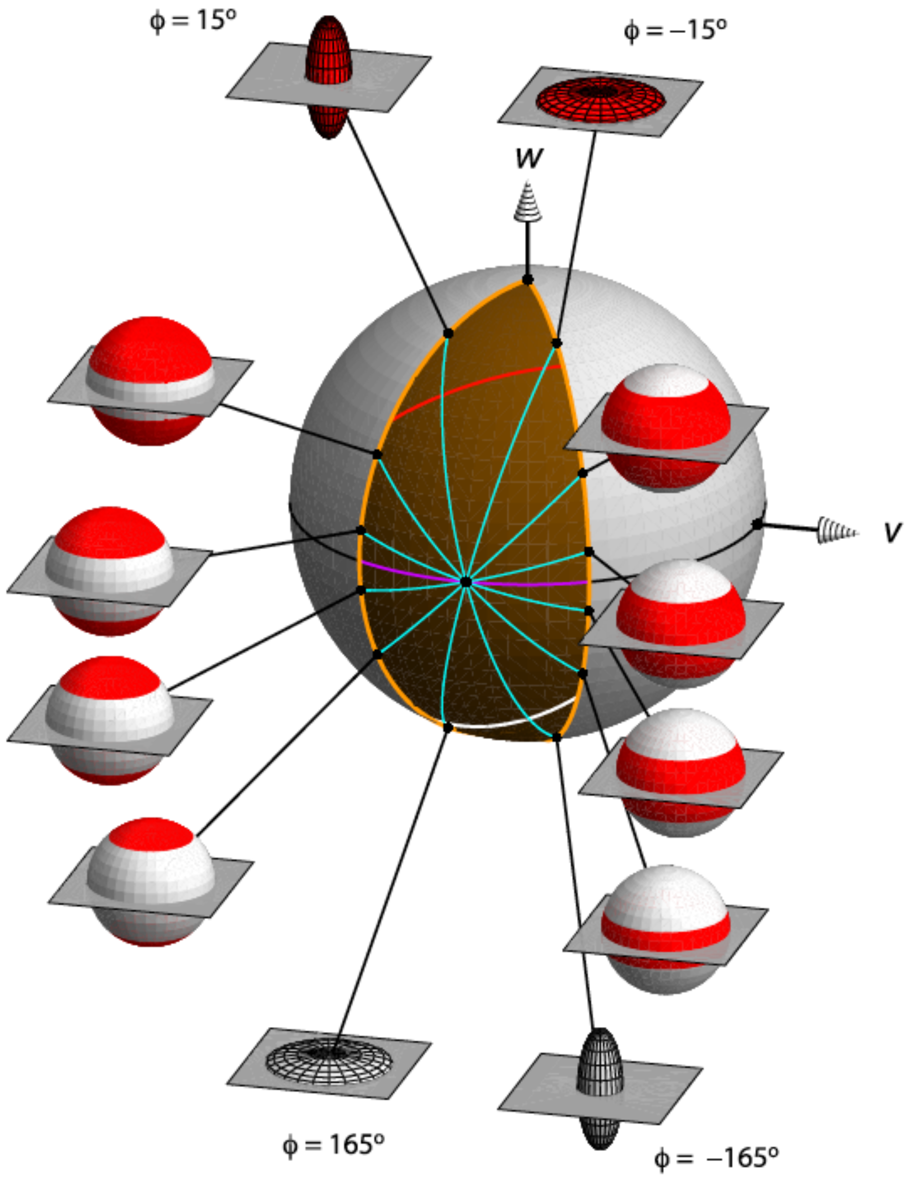
$$\widehat{\mathbf{\Lambda}}(\alpha,\, \nu) = \frac{1}{\|\ \|}\left(\frac{\cos\alpha}{1-2\nu}+1,\, \frac{2\nu\cos\alpha}{1-2\nu},\, \frac{\cos\alpha}{1-2\nu}-1\right)$$

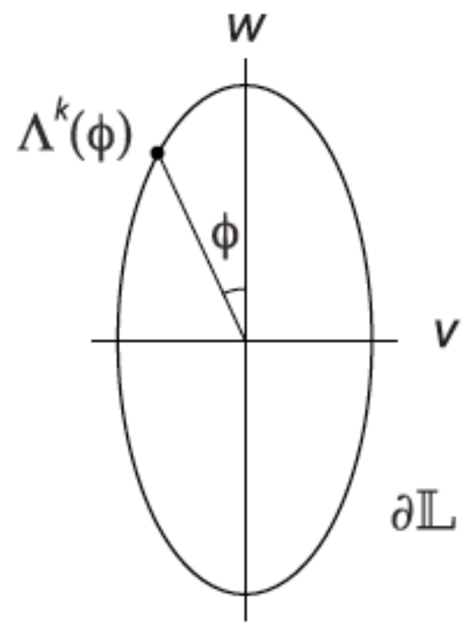
Subset of moment tensors with fixed Poisson ratio

As lune longitude increases from left to right, so does the angle between fault and slip vectors, from tensional crack (α = 0), to DC (α = 90), to compressional crack (α = 180).

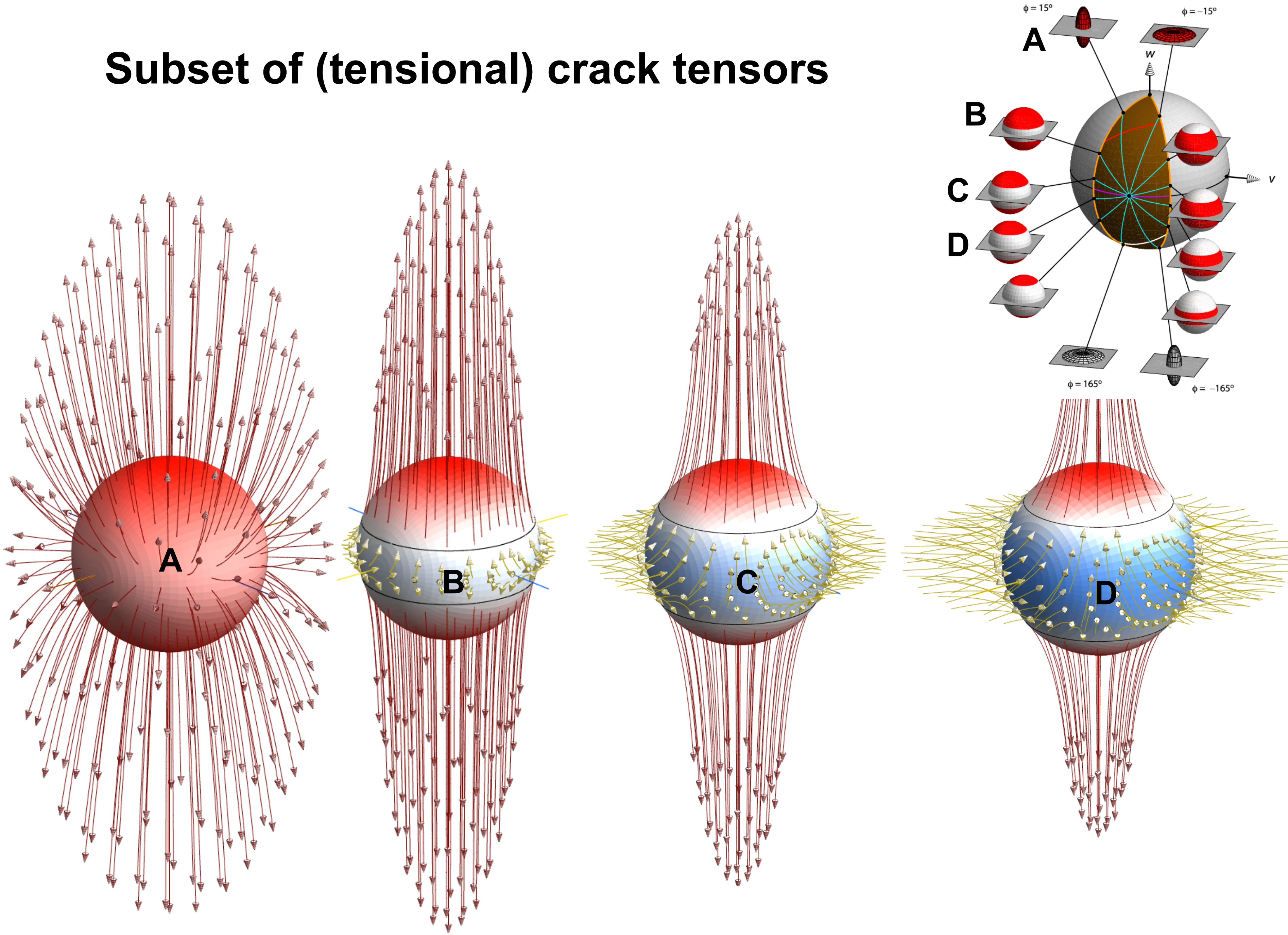


Subset of crack tensors

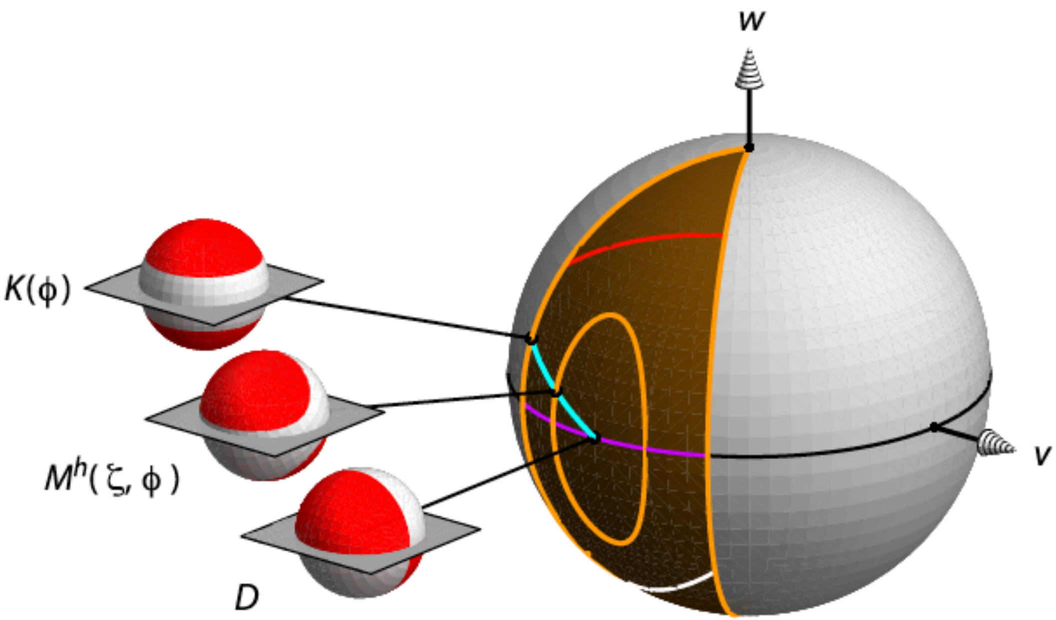


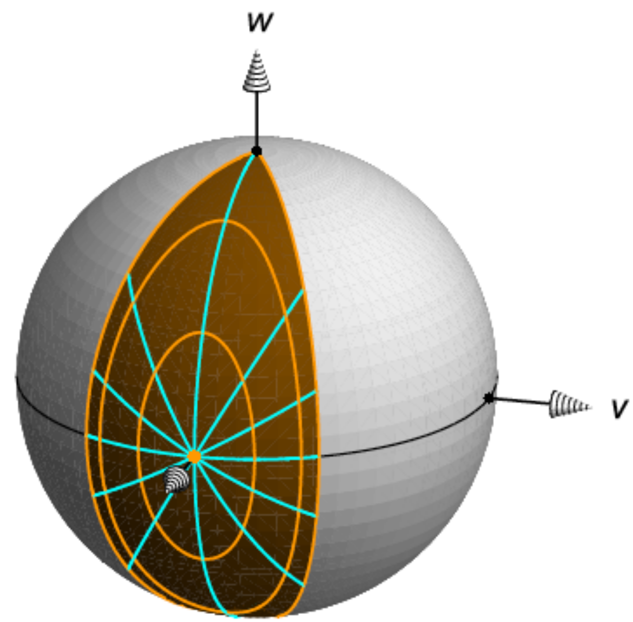


Subset of (tensional) crack tensors

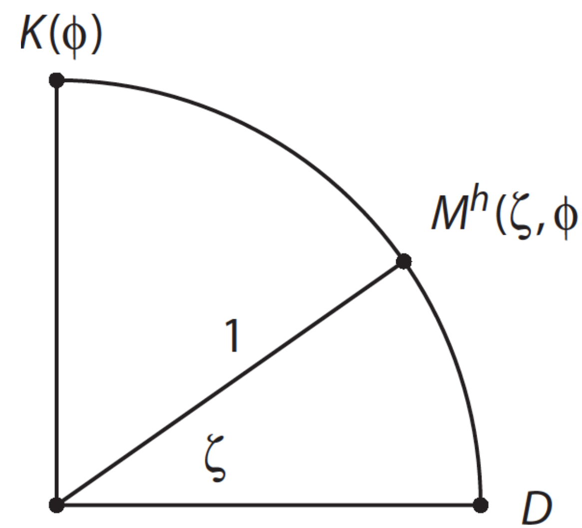


From the basic crack-plus-double-couple model (Minson et al., 2007), we can derive an orthogonal measure of the "crack fraction" in any moment tensor.





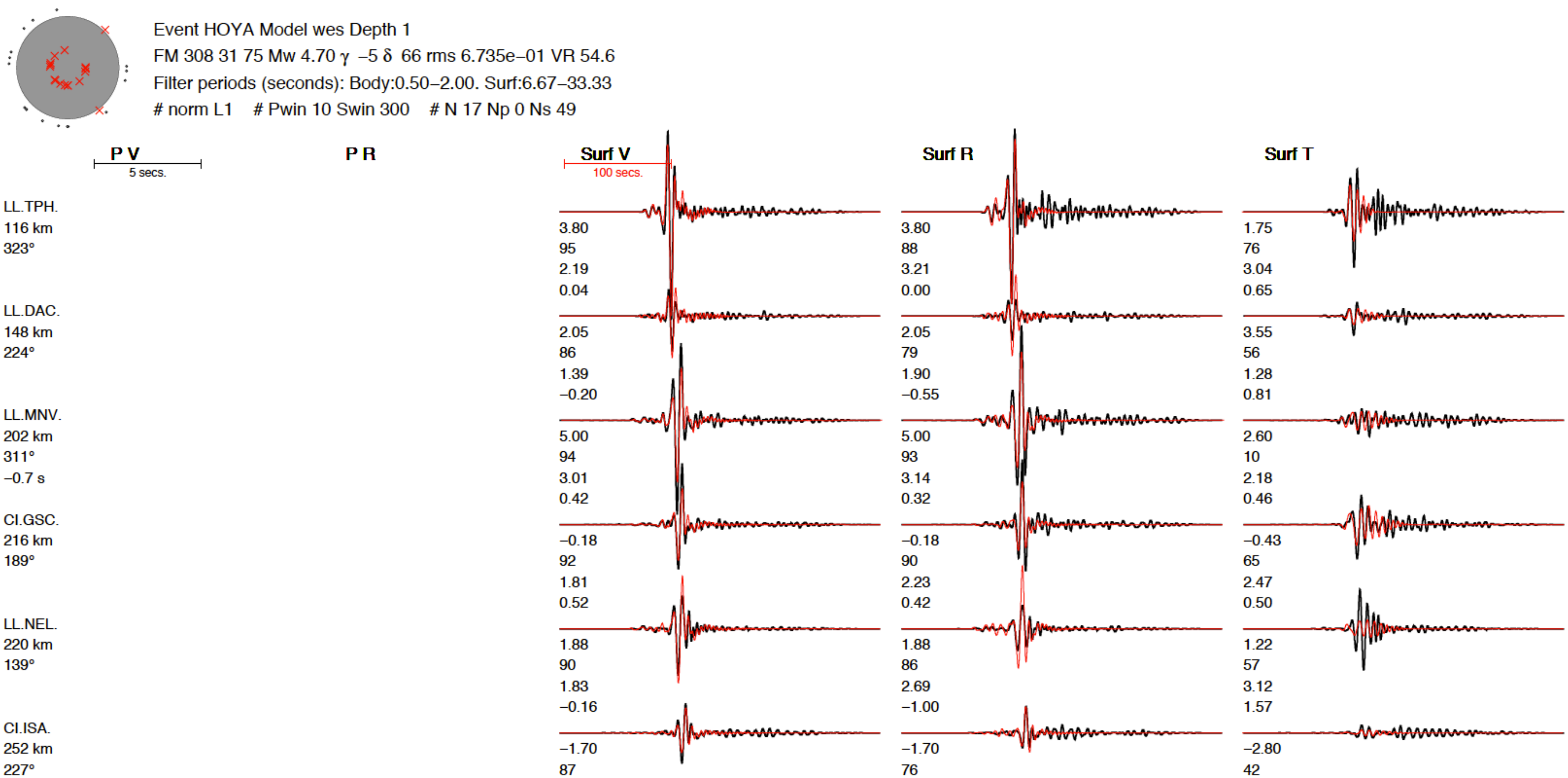
$$M^h(\zeta,\ \phi) = (\cos\zeta)\,D + (\sin\zeta)\,K(\phi),$$



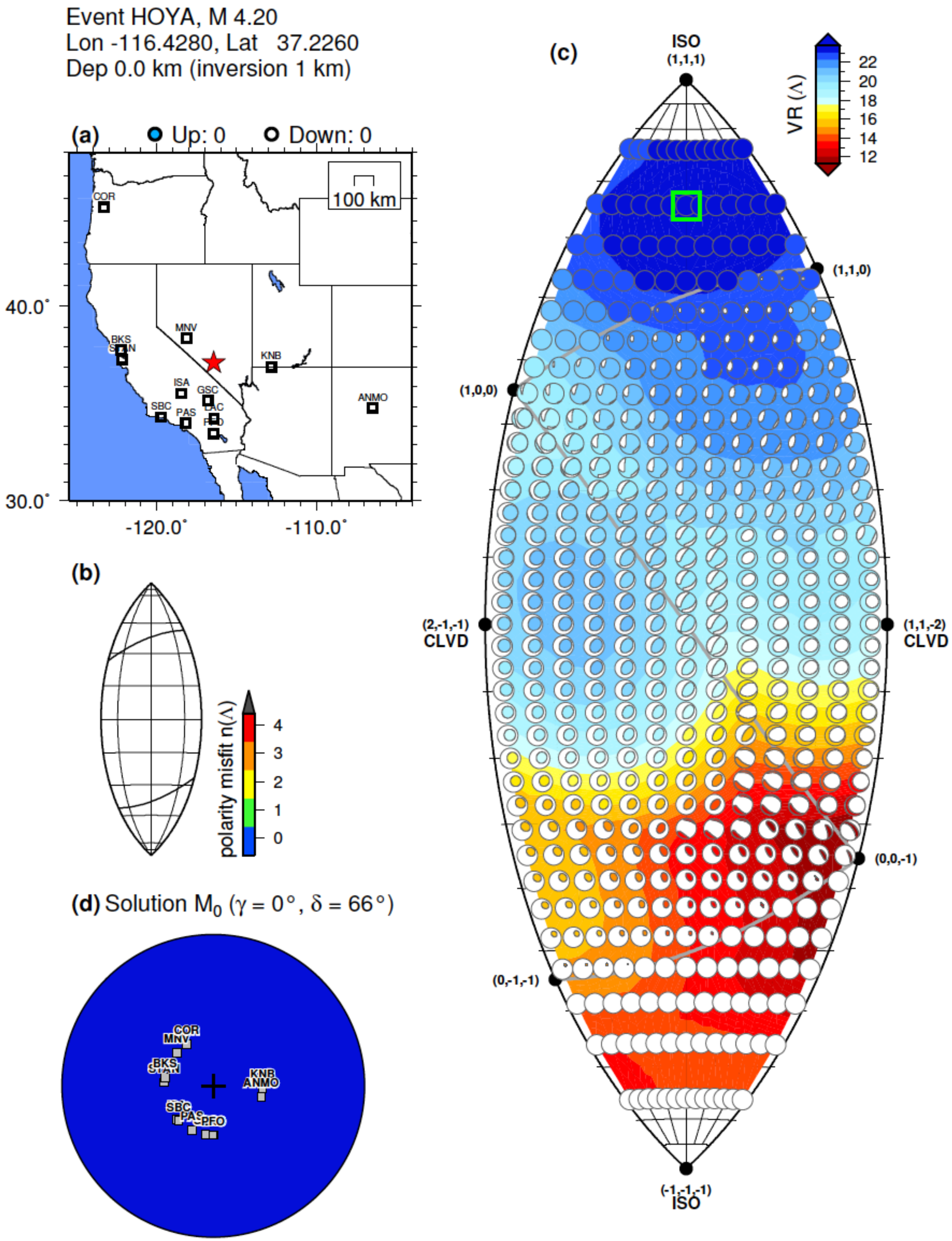
A key point is that *D* and *K* do not have the same frame.

Estimating a moment tensor involves fitting time-shifted synthetic seismograms to observed seismograms.

Example waveform fits for HOYA nuclear explosion
periods 6 – 33 s (red = synthetics, black = data)



Celso Alvizuri



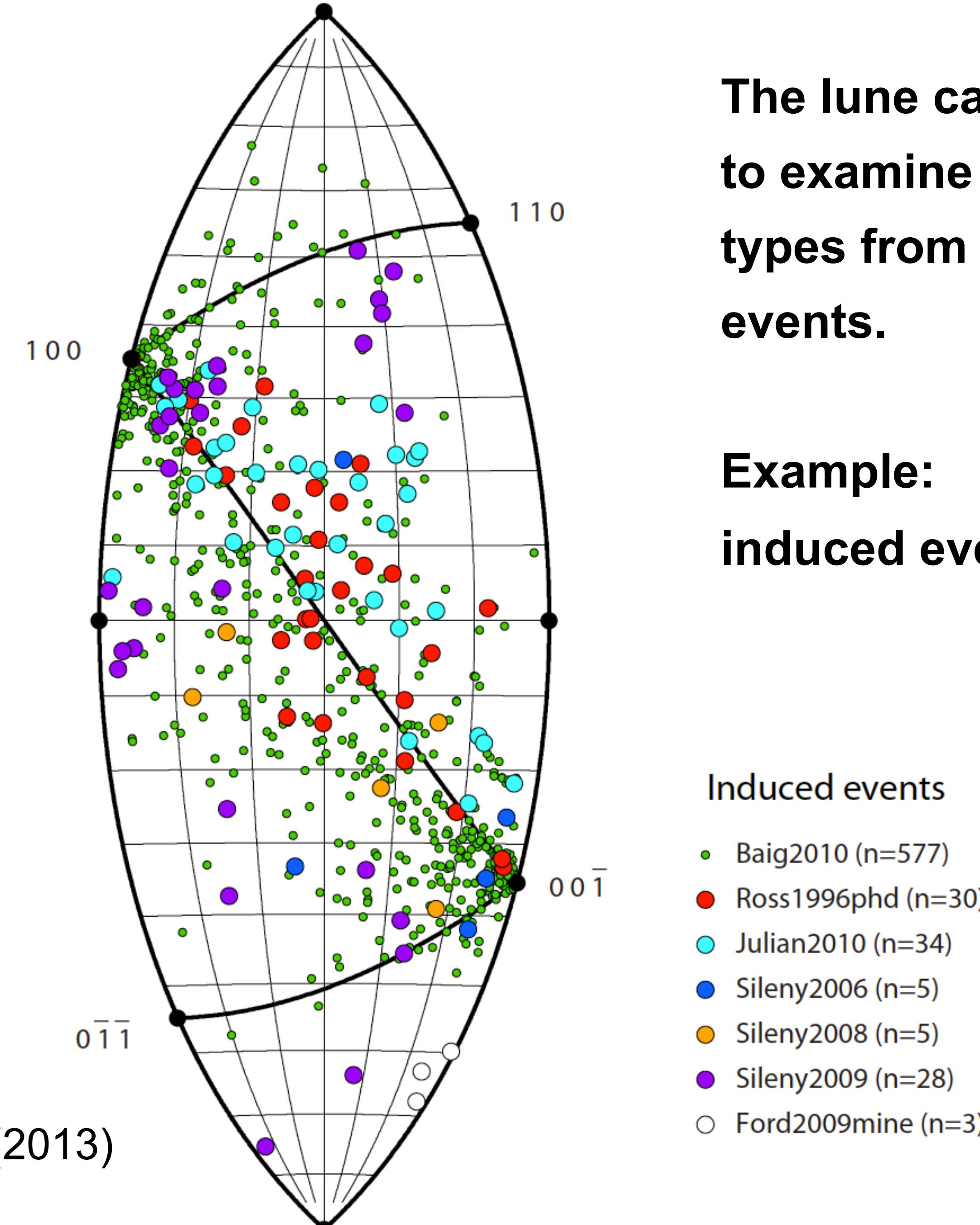
The lune can be used to examine the misfit function in the space of model parameters.

Example: HOYA nuclear explosion

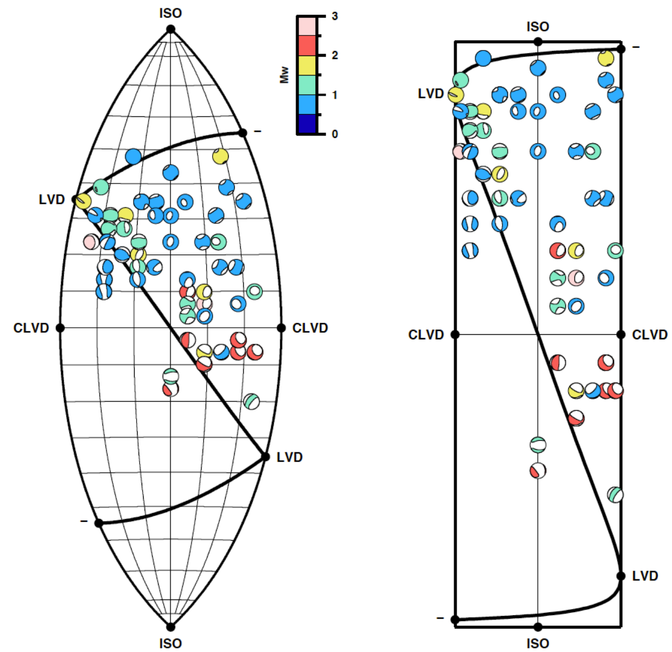
Celso Alvizuri

The lune can be used to examine source types from different events.

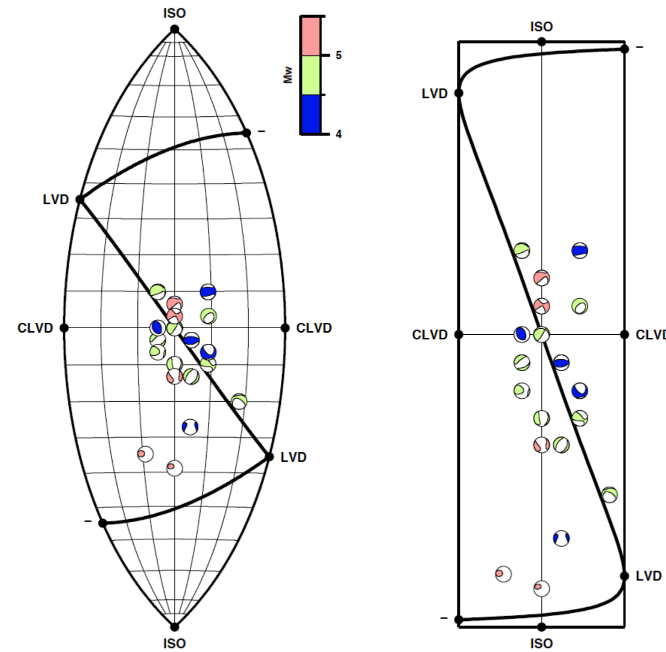
Example:
induced events



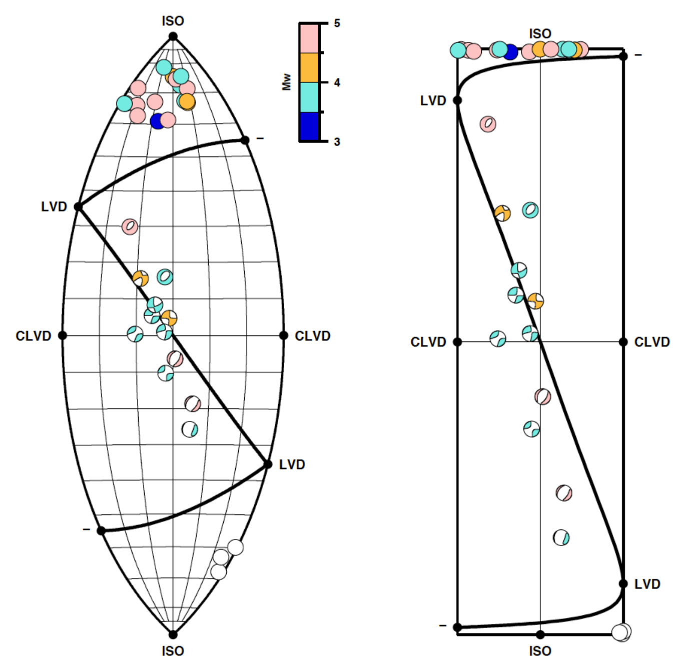
Tape and Tape (2013)
Figure S14



(a)

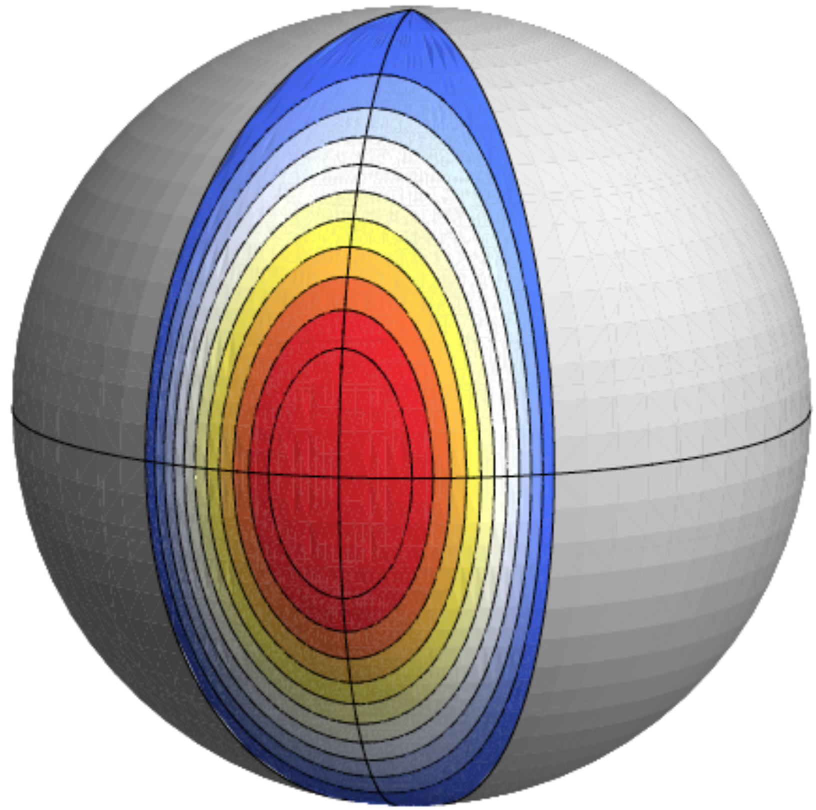


(b)



Homogeneous probability density for full moment tensors

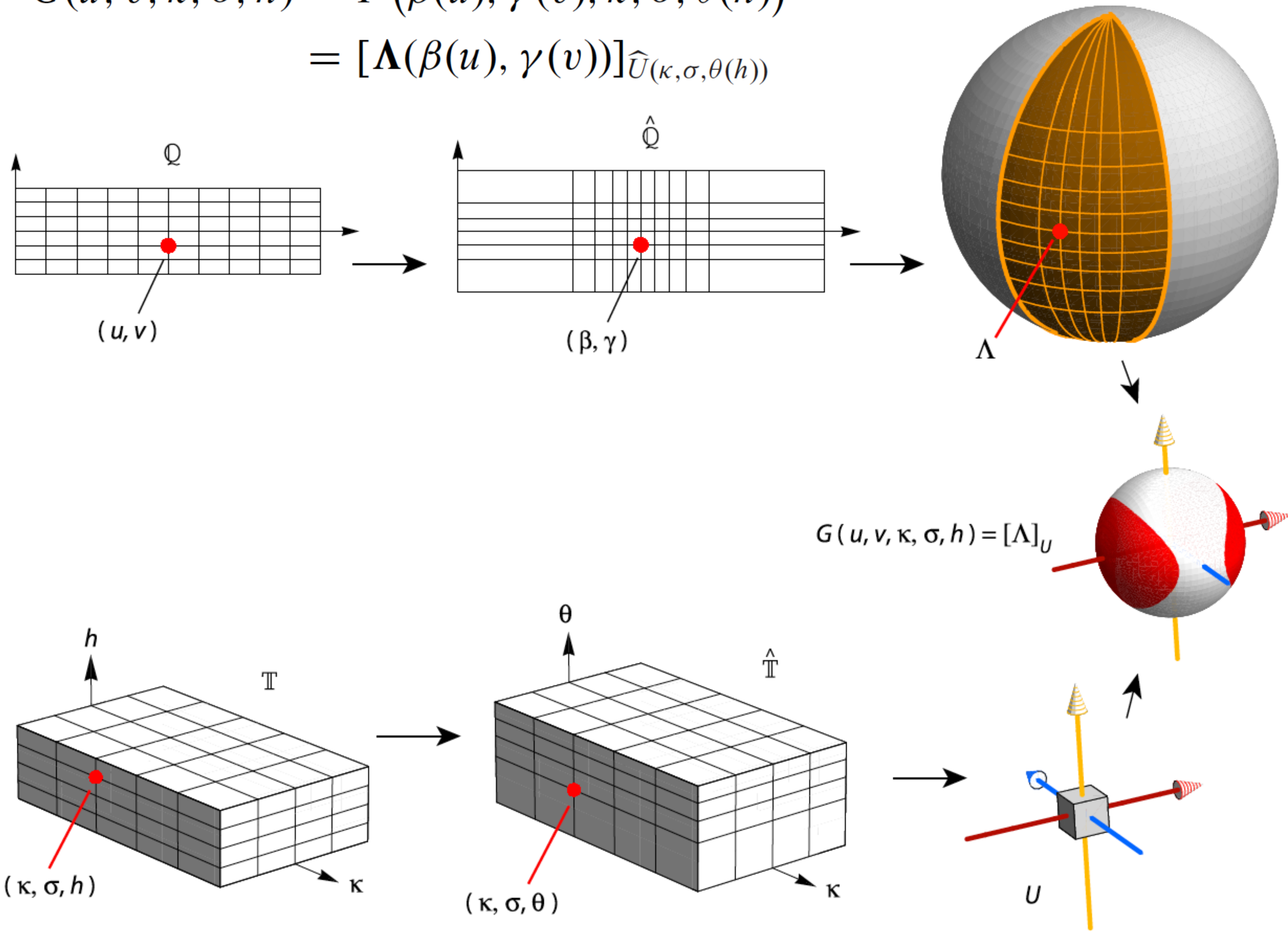
Ford et al. (2010):
“However, as one moves away from the center of the source-type plot (location of a DC mechanism), source orientation becomes less important to the seismic radiation so that the top and bottom of the plot are uniquely represented by an explosion or implosion, respectively (Hudson et al., 1989). A more appropriate population of synthetic sources would take into account the variable degrees of freedom represented in the source-type plot so that source types near the edges of the plot would not have the same density of orientations as those in the center.“



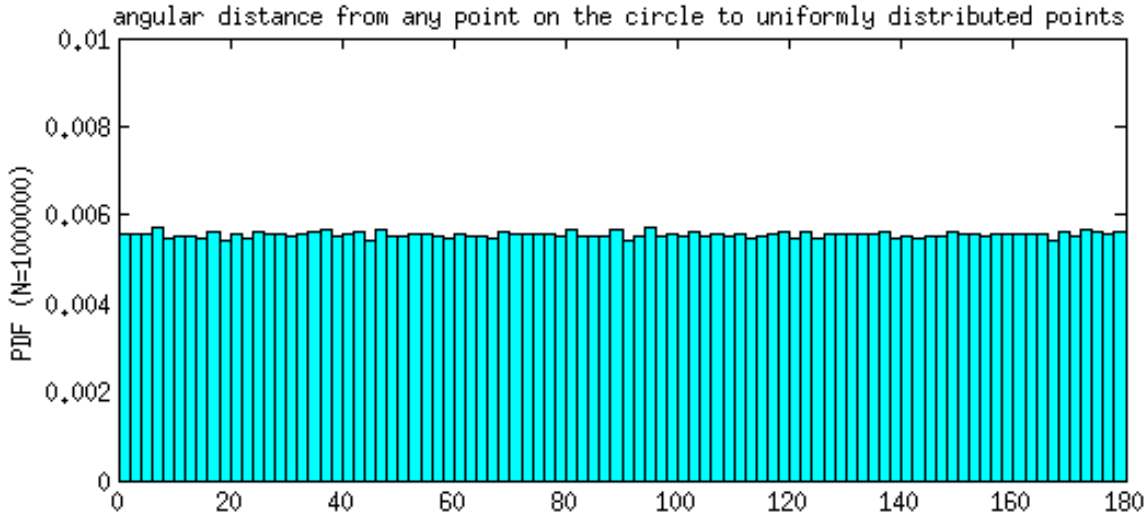
Analytical function of the homogeneous probability density plotted on the lune

Tape and Tape (2015)

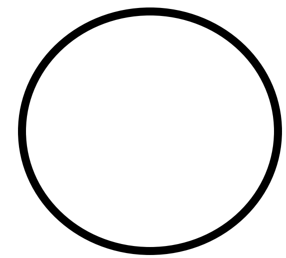
$$
\begin{aligned}
G(u, v, \kappa, \sigma, h) &= F\big(\beta(u), \gamma(v), \kappa, \sigma, \theta(h)\big) \\
&= [\boldsymbol{\Lambda}(\beta(u), \gamma(v))]_{\widehat{U}(\kappa, \sigma, \theta(h))}
\end{aligned}
$$

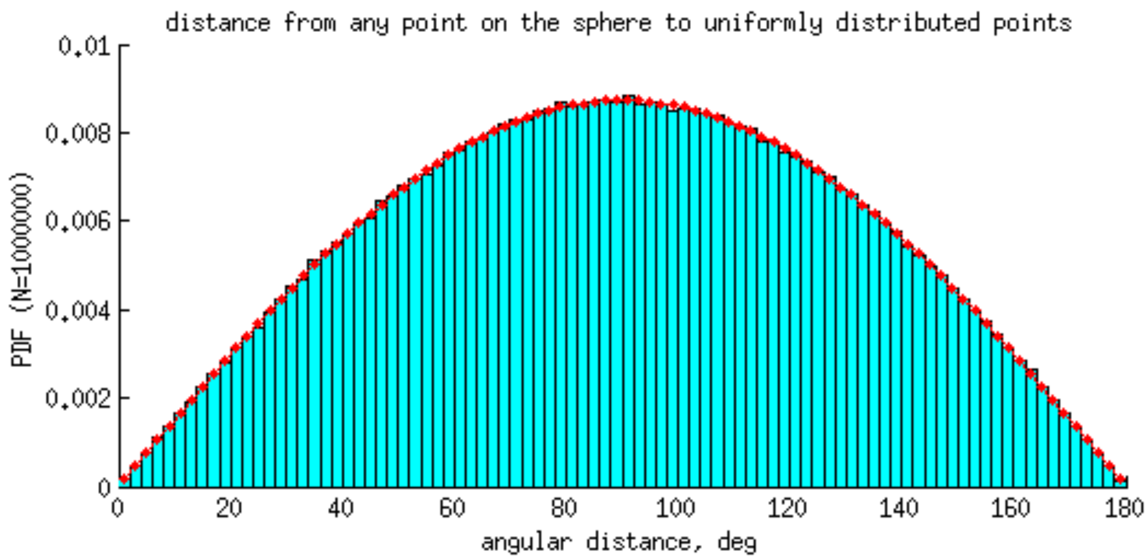


Uniform distribution for 1D (circle) and 2D (sphere) analogs

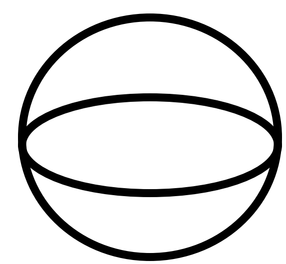


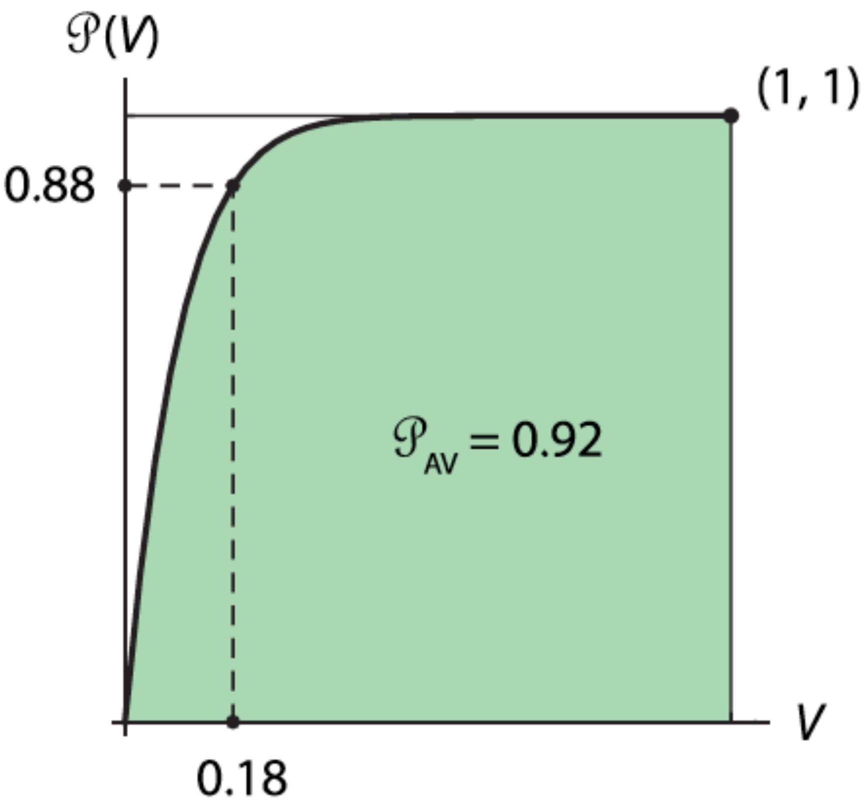
S^1

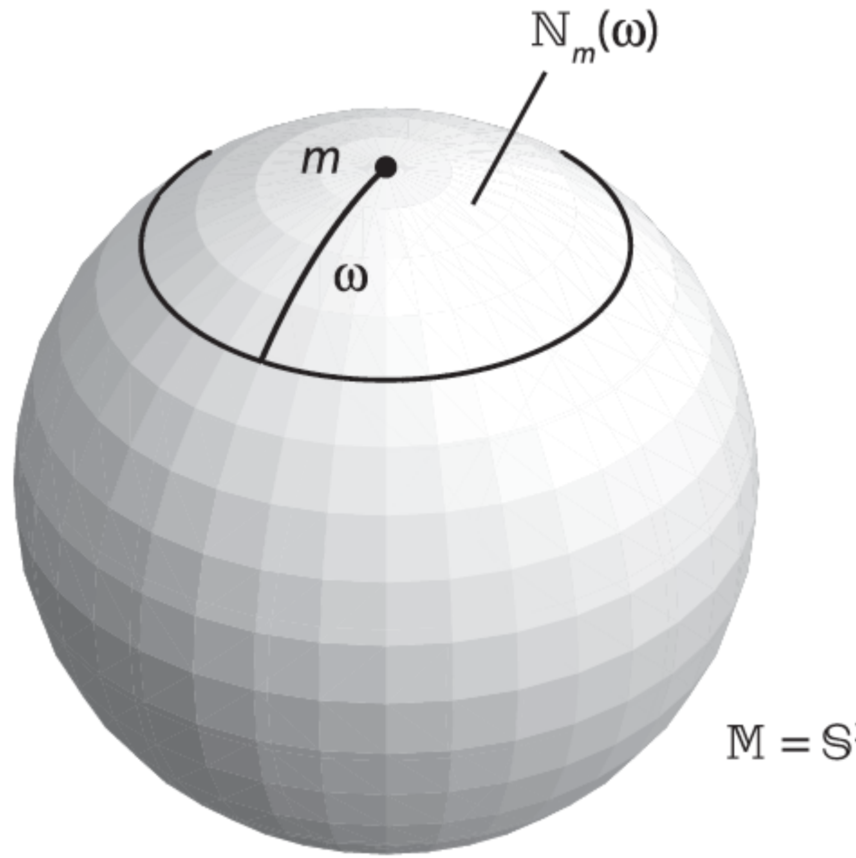




S^2



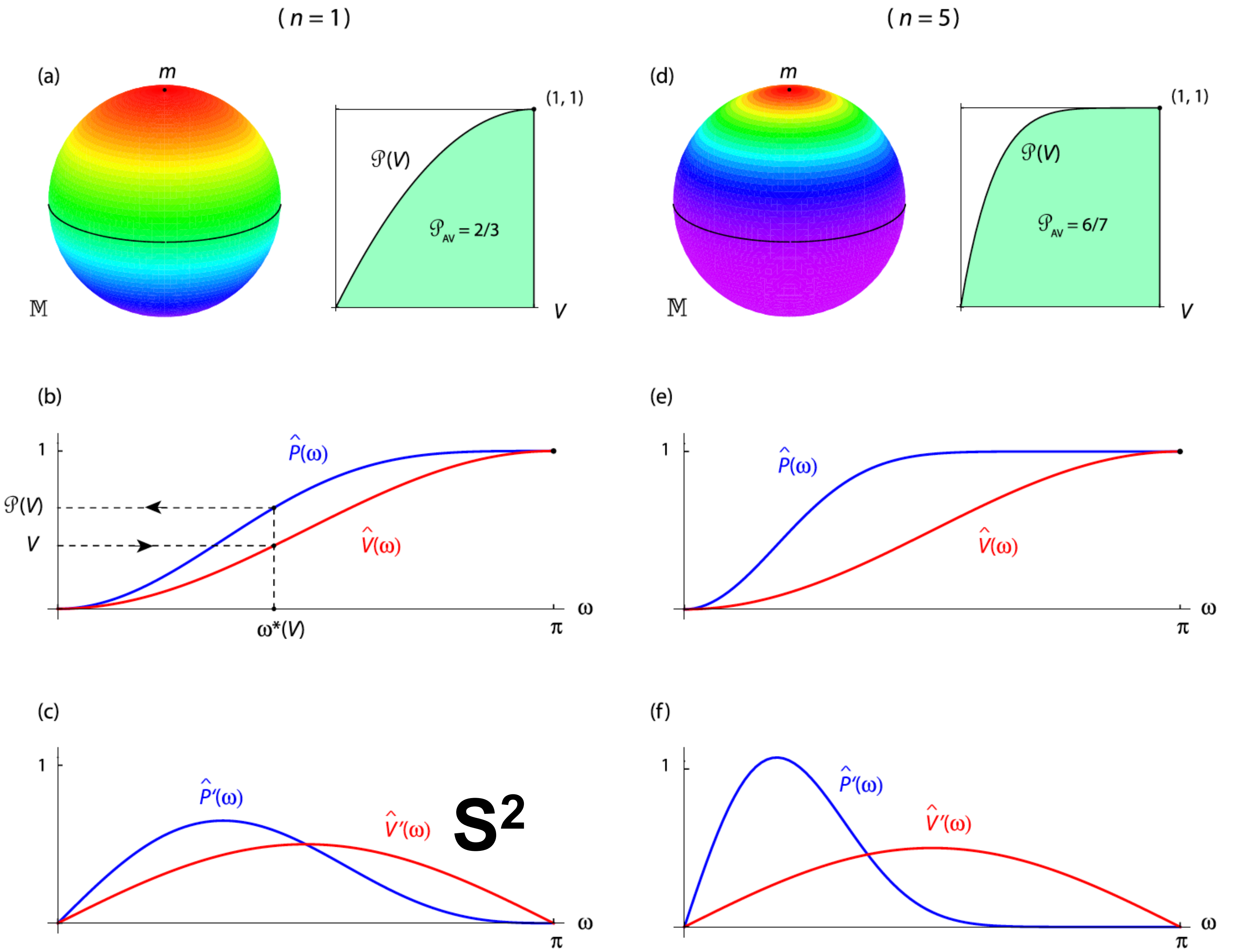


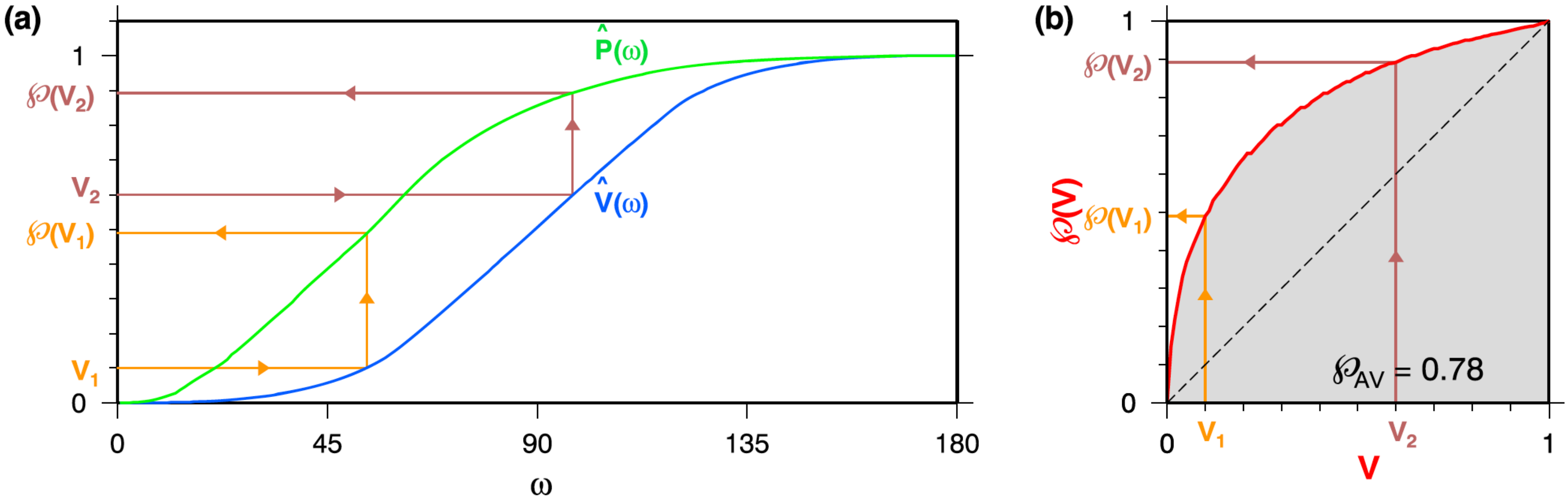


S^2

Tape and Tape (2016)

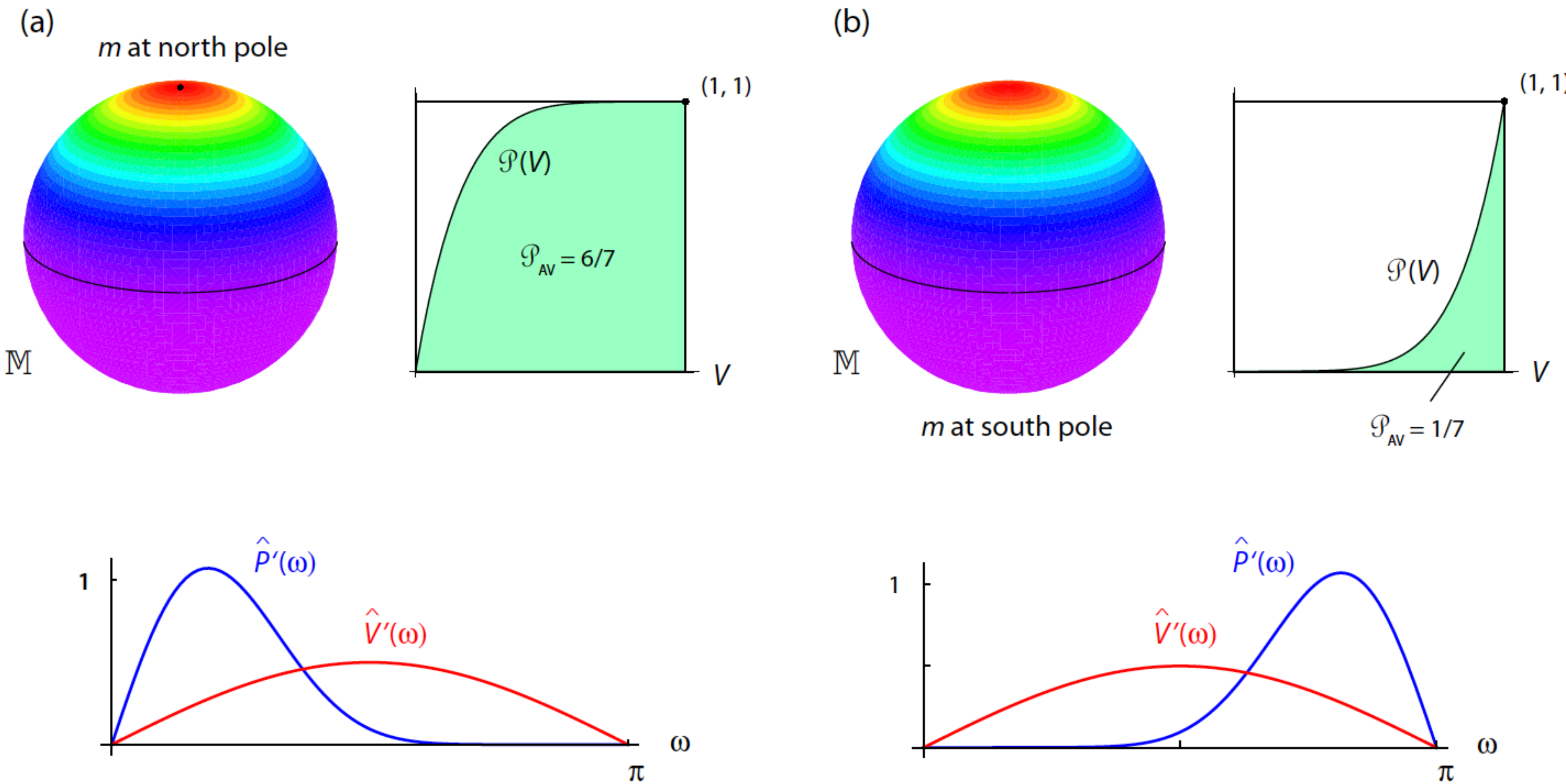
EXAMPLE: A favorable and a more favorable confidence curve



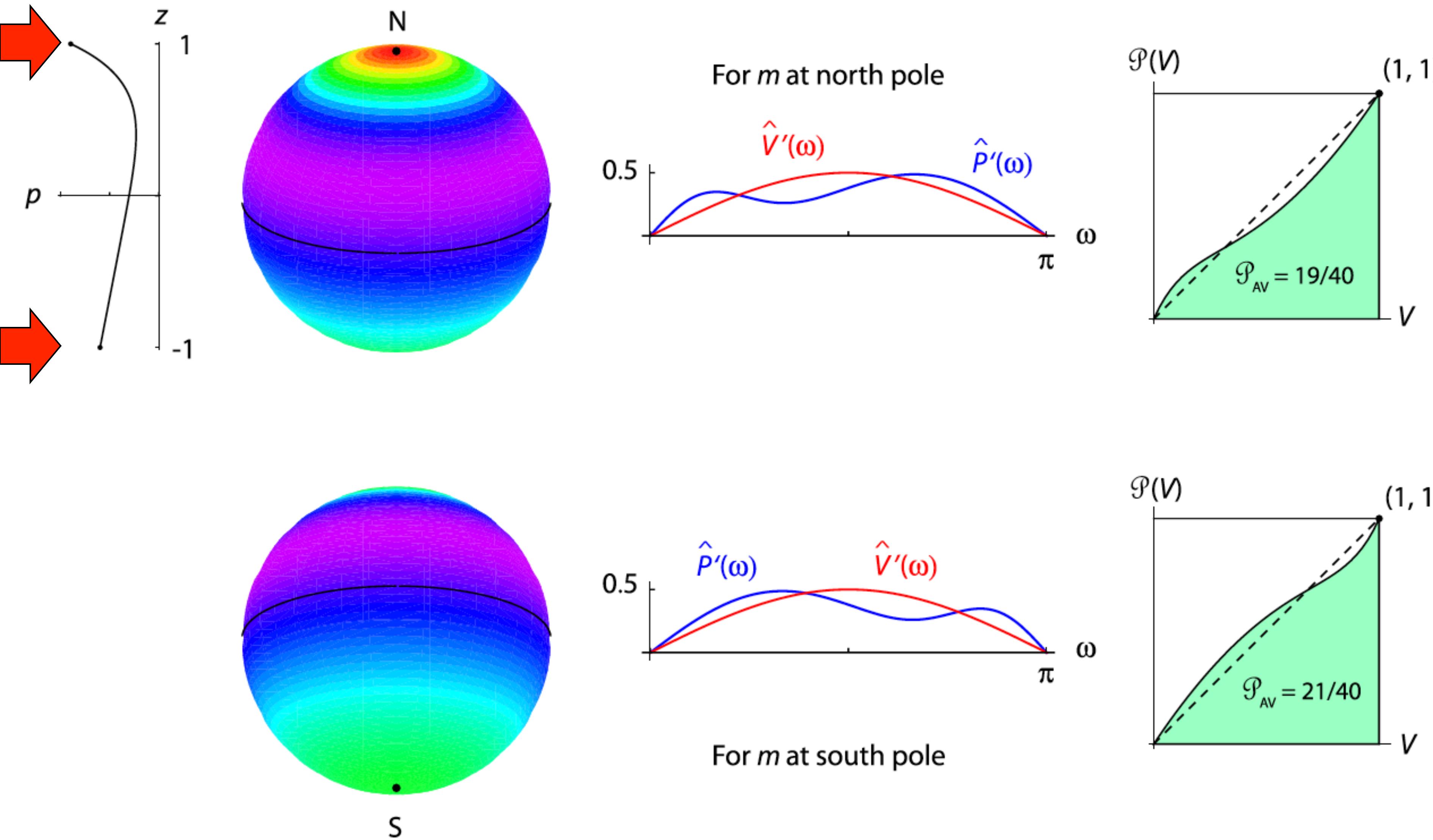


Silwal and Tape (2016)

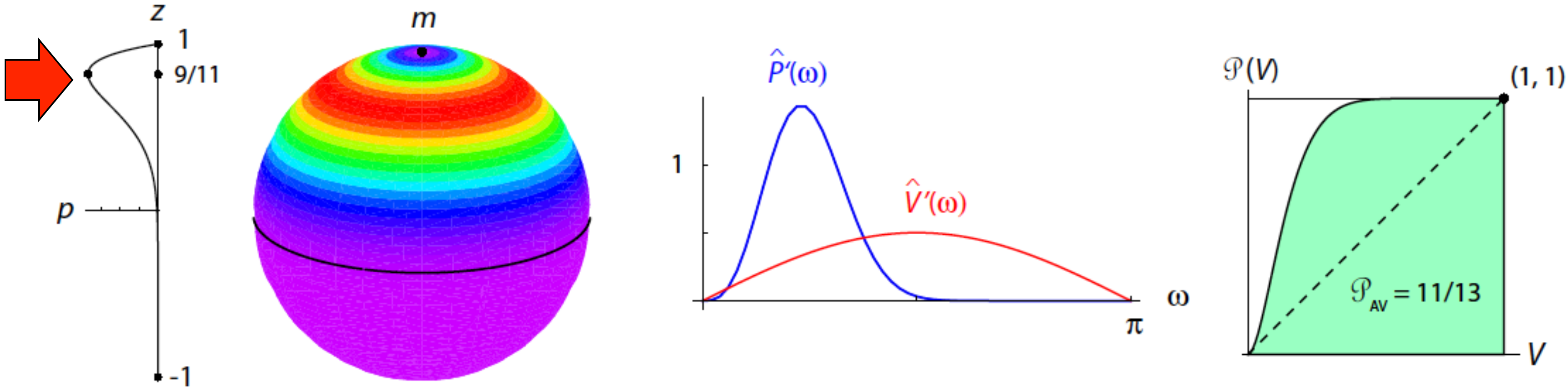
EXAMPLE: Reference tensor at M = max(p) and -M



EXAMPLE: Multi-modal probability density



EXAMPLE: Reference tensor has low probability density, but confidence is favorable.

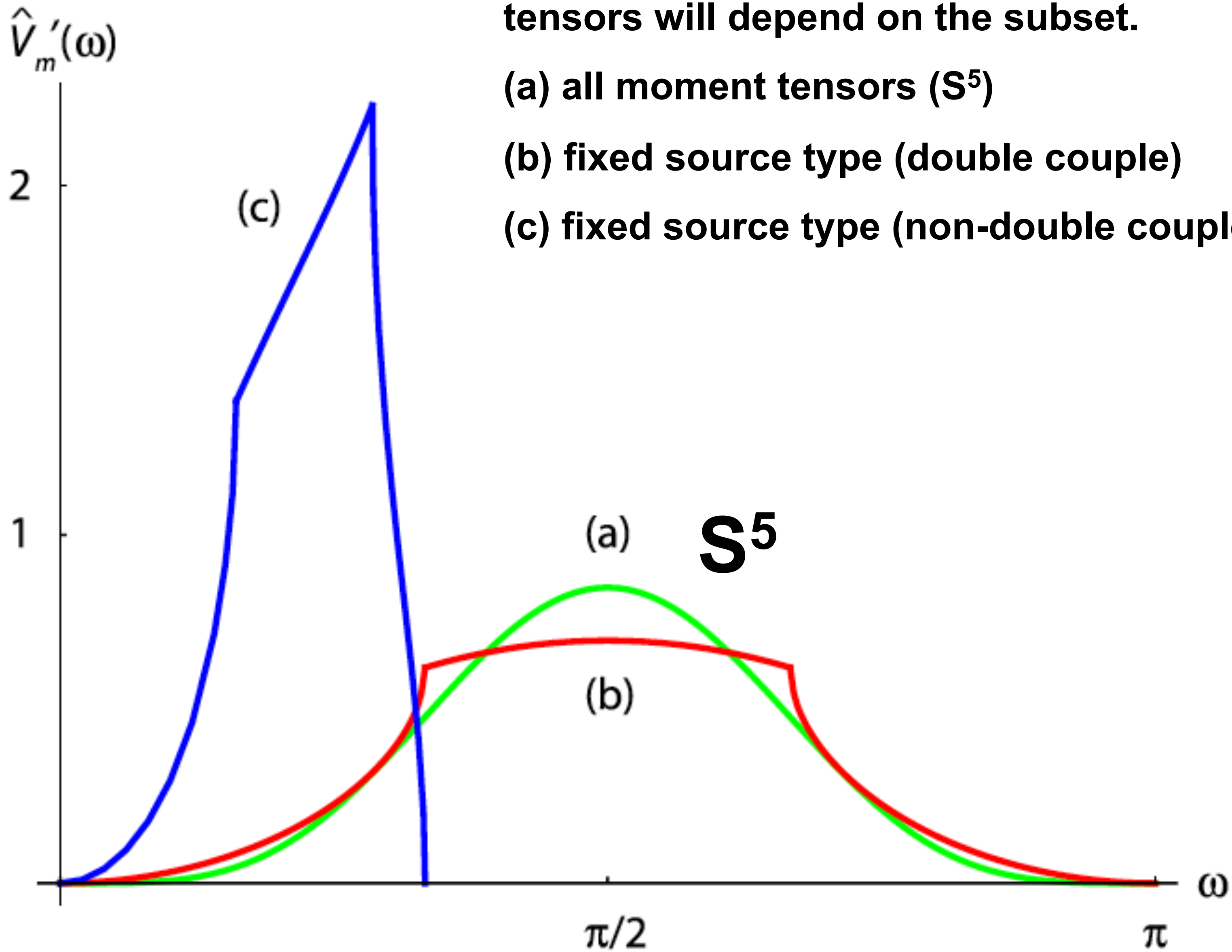


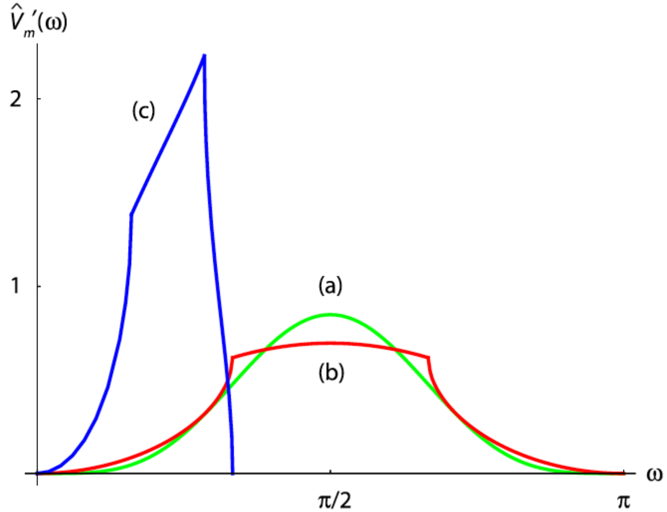
The uniform distribution of moment tensors will depend on the subset.

(a) all moment tensors (S^5)

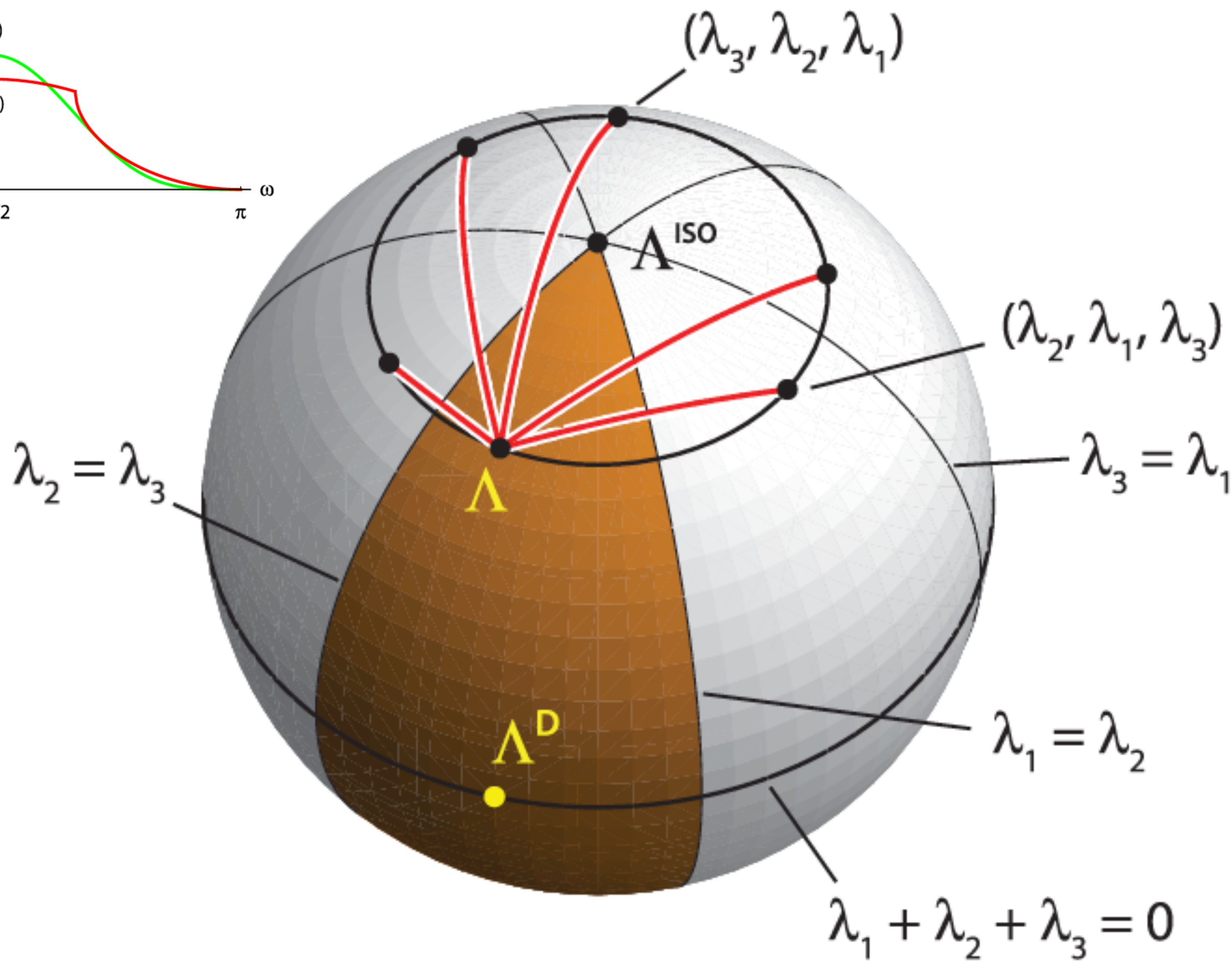
(b) fixed source type (double couple)

(c) fixed source type (non-double couple)

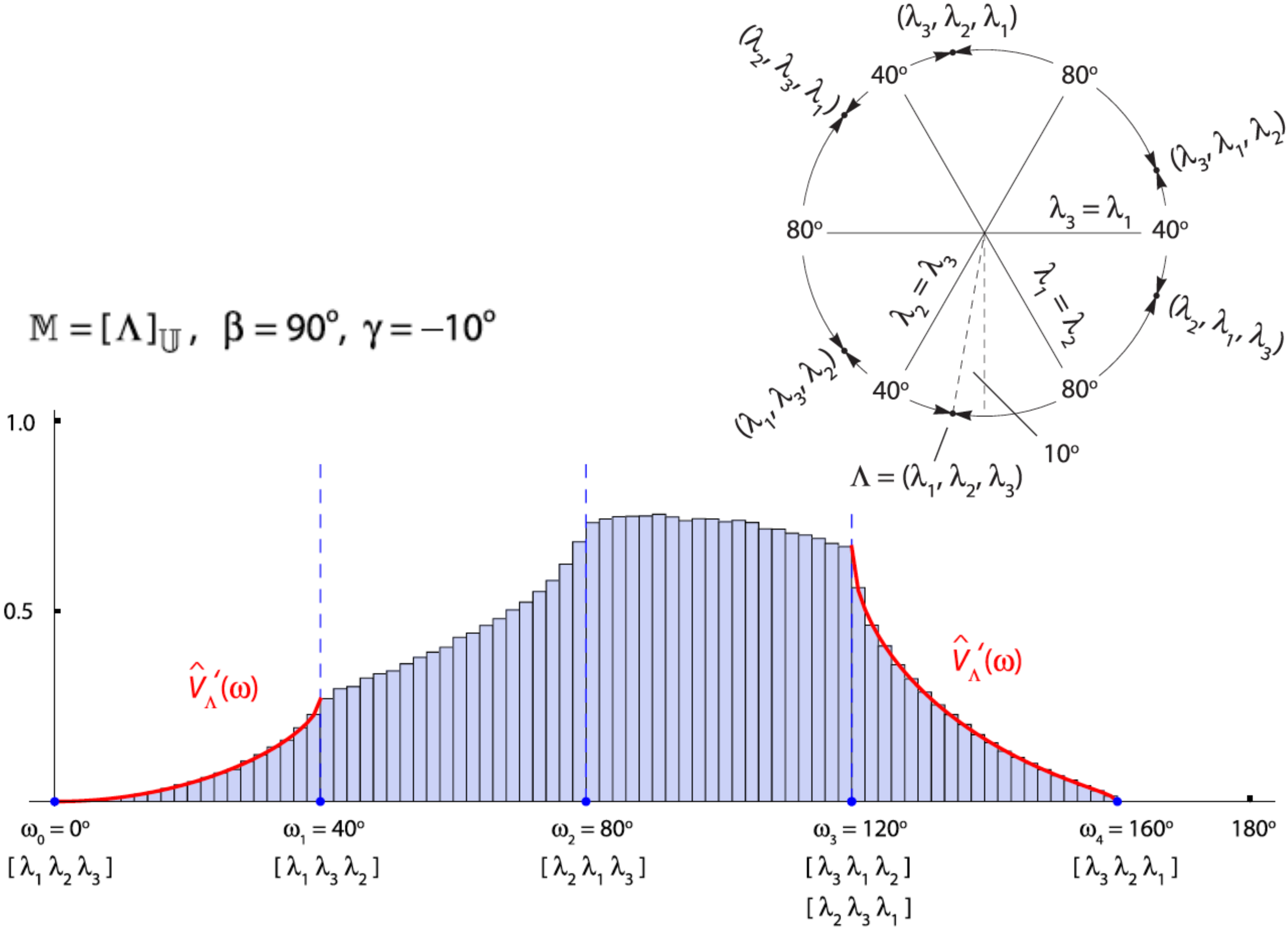


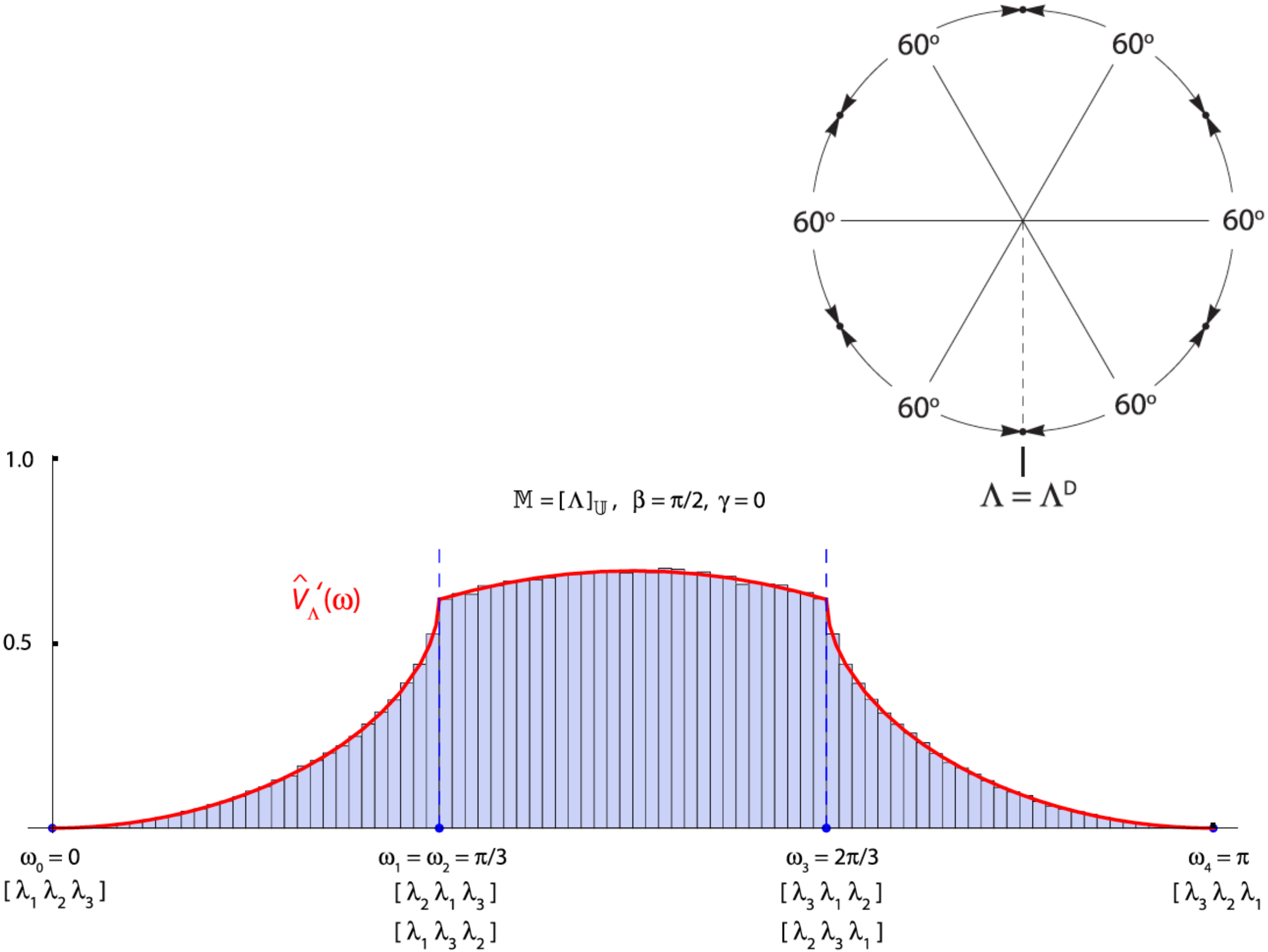


$\hat{V}_m'(\omega)$
2
1
(c)
(a)
(b)
ω
$\pi/2$
π



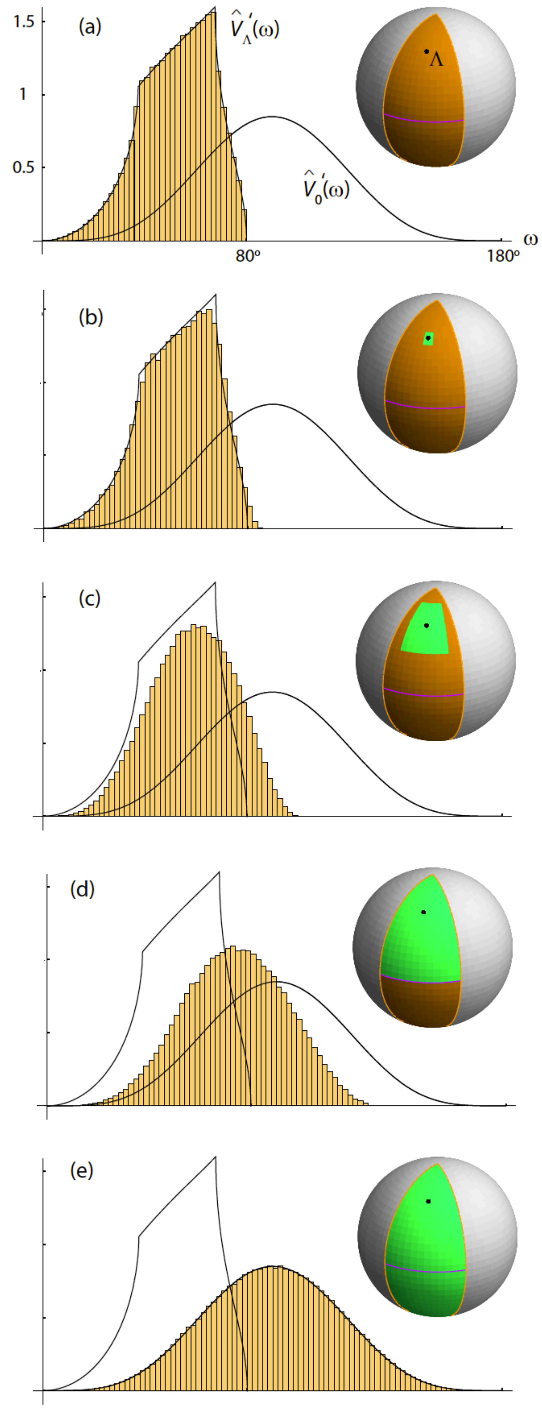
$(\lambda_3, \lambda_2, \lambda_1)$
Λ^{ISO}
$(\lambda_2, \lambda_1, \lambda_3)$
$\lambda_3 = \lambda_1$
$\lambda_2 = \lambda_3$
Λ
$\lambda_1 = \lambda_2$
Λ^D
$\lambda_1 + \lambda_2 + \lambda_3 = 0$


$\mathbb{M} = [\Lambda]_{\mathbb{U}}, \ \beta = 90^o, \ \gamma = -10^o$
$(\lambda_3, \lambda_2, \lambda_1)$
$(\lambda_2, \lambda_3, \lambda_1)$
$(\lambda_3, \lambda_1, \lambda_2)$
$(\lambda_2, \lambda_1, \lambda_3)$
$(\lambda_1, \lambda_3, \lambda_2)$
$\Lambda = (\lambda_1, \lambda_2, \lambda_3)$
$\lambda_3 = \lambda_1$
$\lambda_2 = \lambda_3$
$\lambda_1 = \lambda_2$
40°
80°
10°
1.0
0.5
$\hat{V}_\Lambda'(\omega)$
$\hat{V}_\Lambda'(\omega)$
$\omega_0 = 0^o$
$[\lambda_1 \lambda_2 \lambda_3]$
$\omega_1 = 40^o$
$[\lambda_1 \lambda_3 \lambda_2]$
$\omega_2 = 80^o$
$[\lambda_2 \lambda_1 \lambda_3]$
$\omega_3 = 120^o$
$[\lambda_3 \lambda_1 \lambda_2]$
$[\lambda_2 \lambda_3 \lambda_1]$
$\omega_4 = 160^o$
$[\lambda_3 \lambda_2 \lambda_1]$
180°

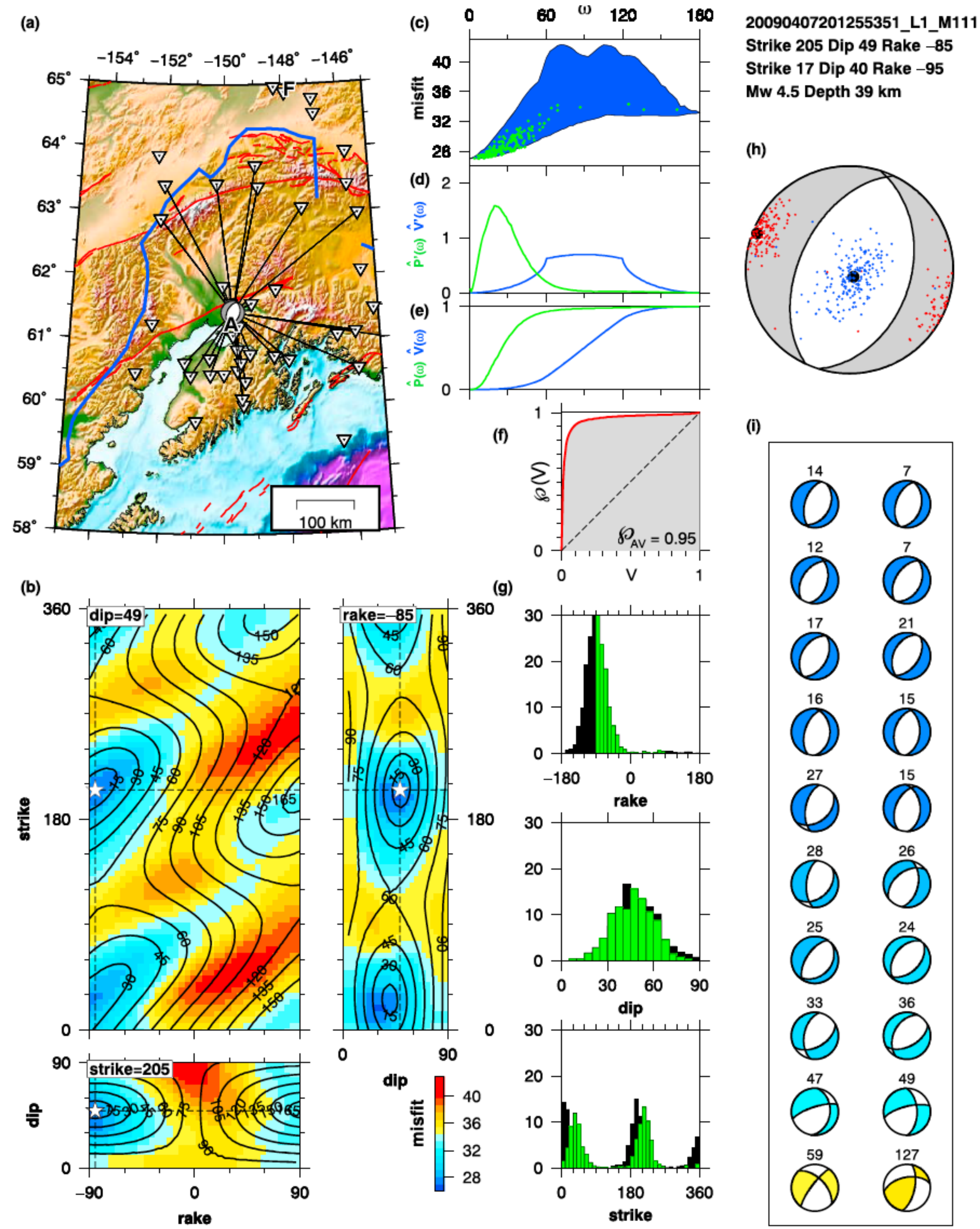


60°
60°
60°
60°
60°
60°
Λ = Λ^D
1.0
0.5
𝕄 = [Λ]_𝕌 , β = π/2, γ = 0
V̂′_Λ(ω)
ω_0 = 0
[λ_1 λ_2 λ_3]
ω_1 = ω_2 = π/3
[λ_2 λ_1 λ_3]
[λ_1 λ_3 λ_2]
ω_3 = 2π/3
[λ_3 λ_1 λ_2]
[λ_2 λ_3 λ_1]
ω_4 = π
[λ_3 λ_2 λ_1]

Uniform distribution of moment tensors: evolution from fixed source type (top) to all source types (bottom)

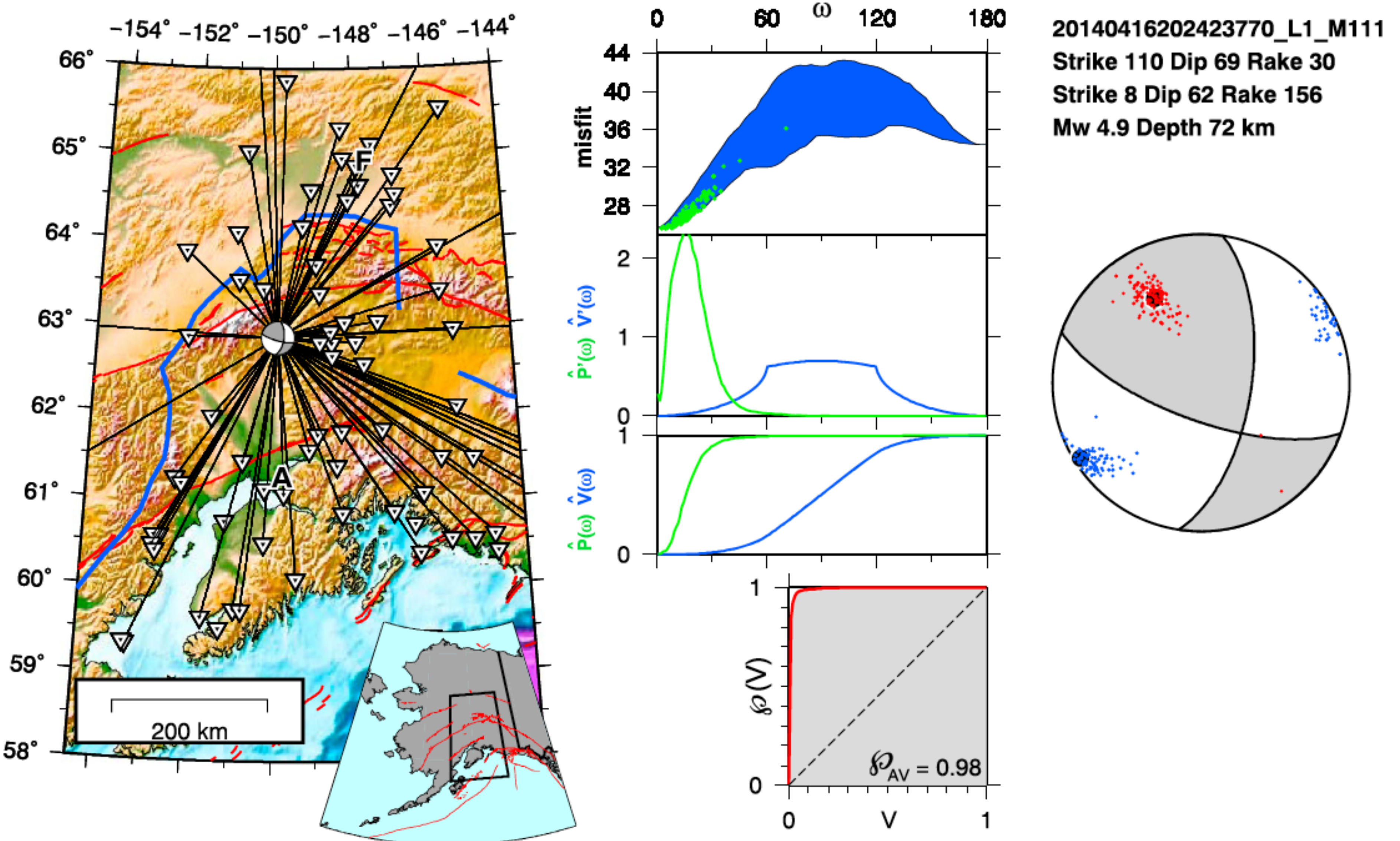


Uncertainty analysis for the case of an assumed double couple moment tensor



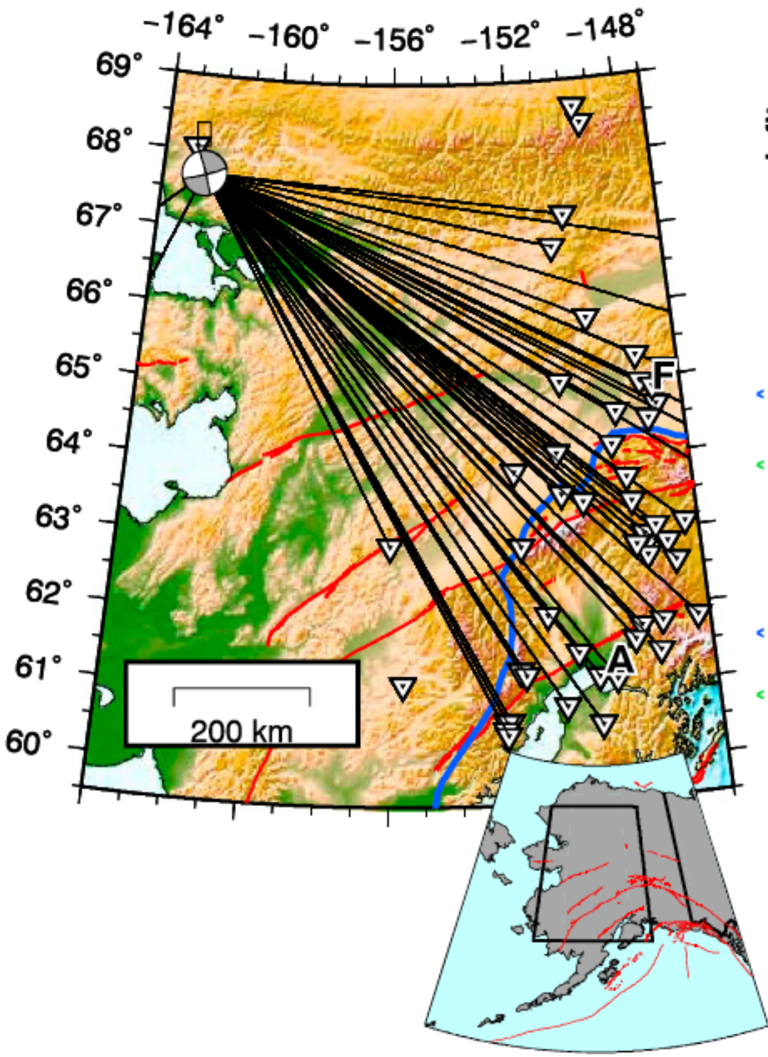
Silwal and Tape (2016)

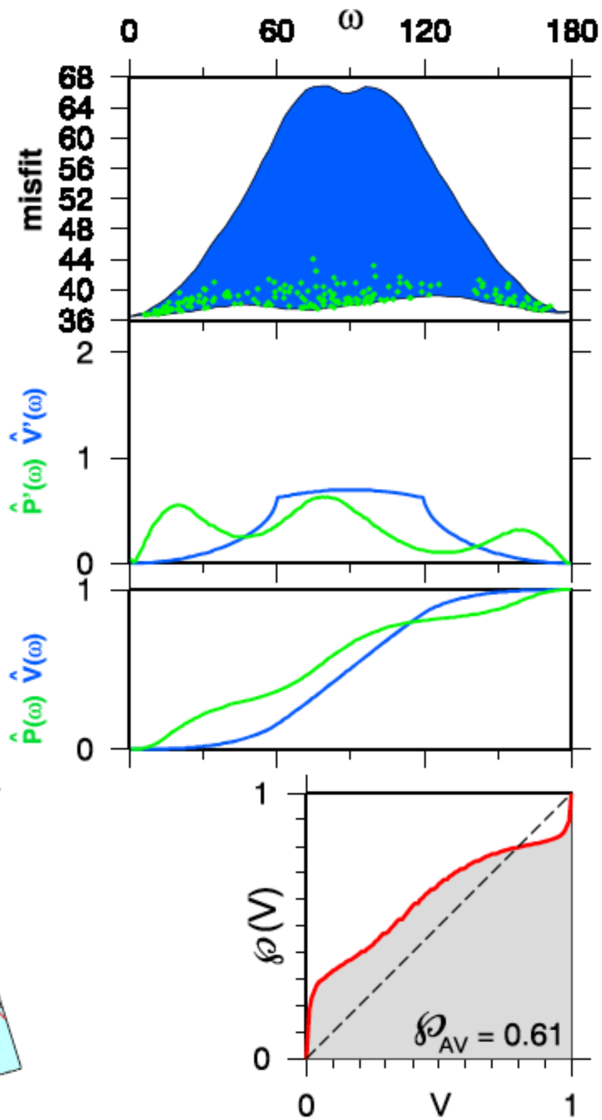
High Confidence



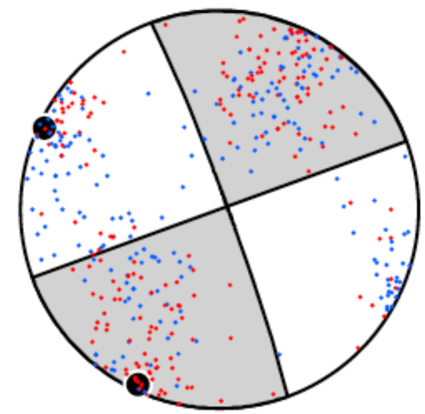
Silwal and Tape (2016)

Low Confidence



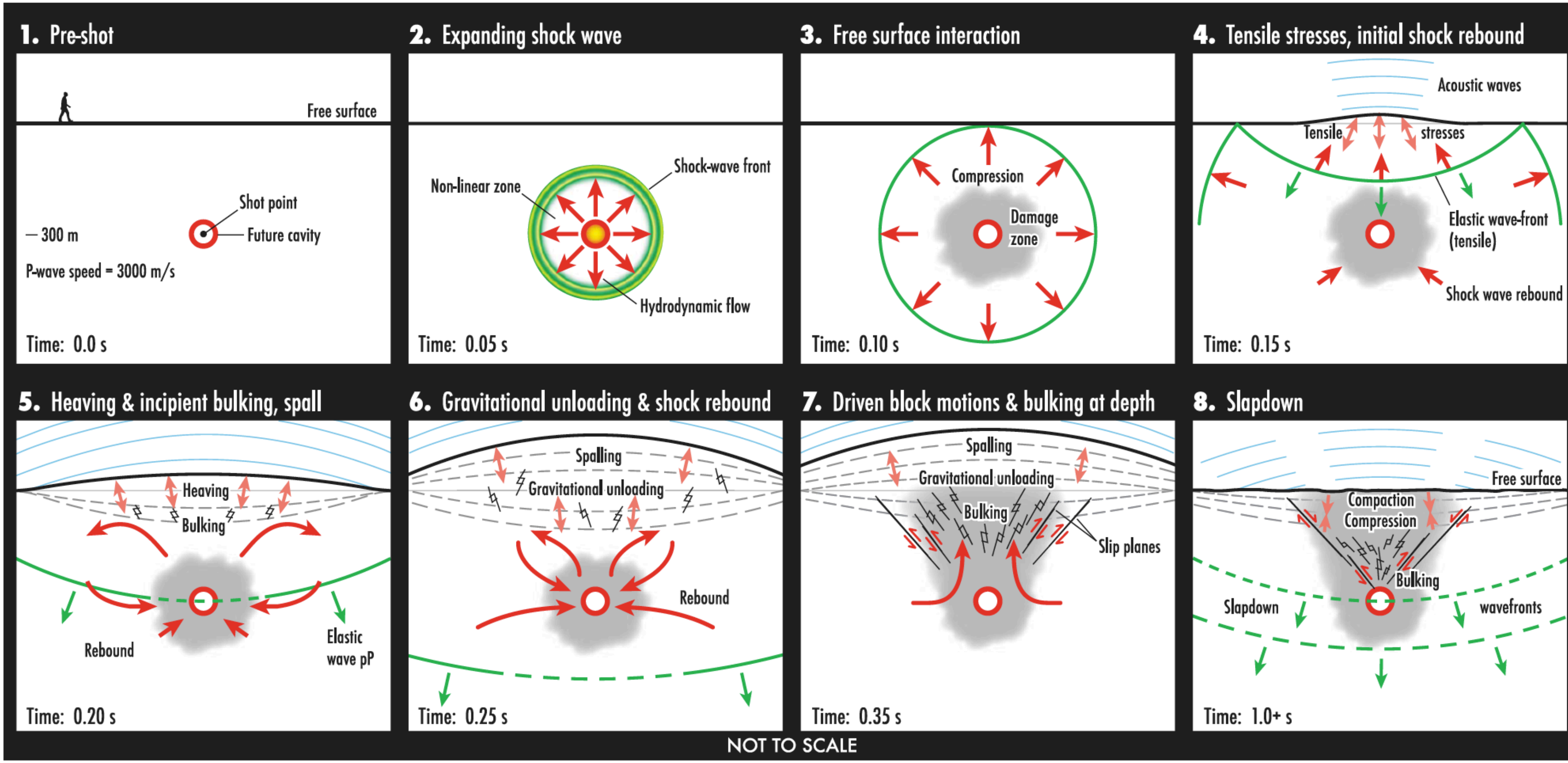


20140418184418152_L1_M111
Strike 340 Dip 86 Rake 0
Strike 250 Dip 90 Rake 176
Mw 5.2 Depth 20 km



Silwal and Tape (2016)

How is source complexity of an explosion imprinted onto the moment tensor?

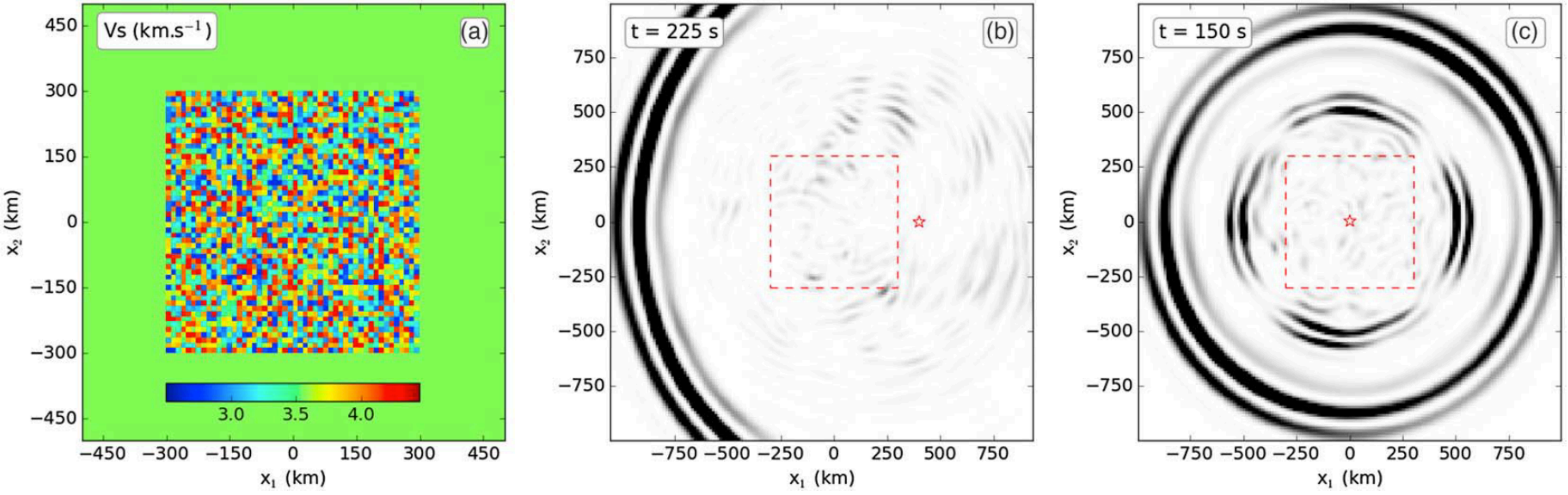


Patton and Taylor (2011)

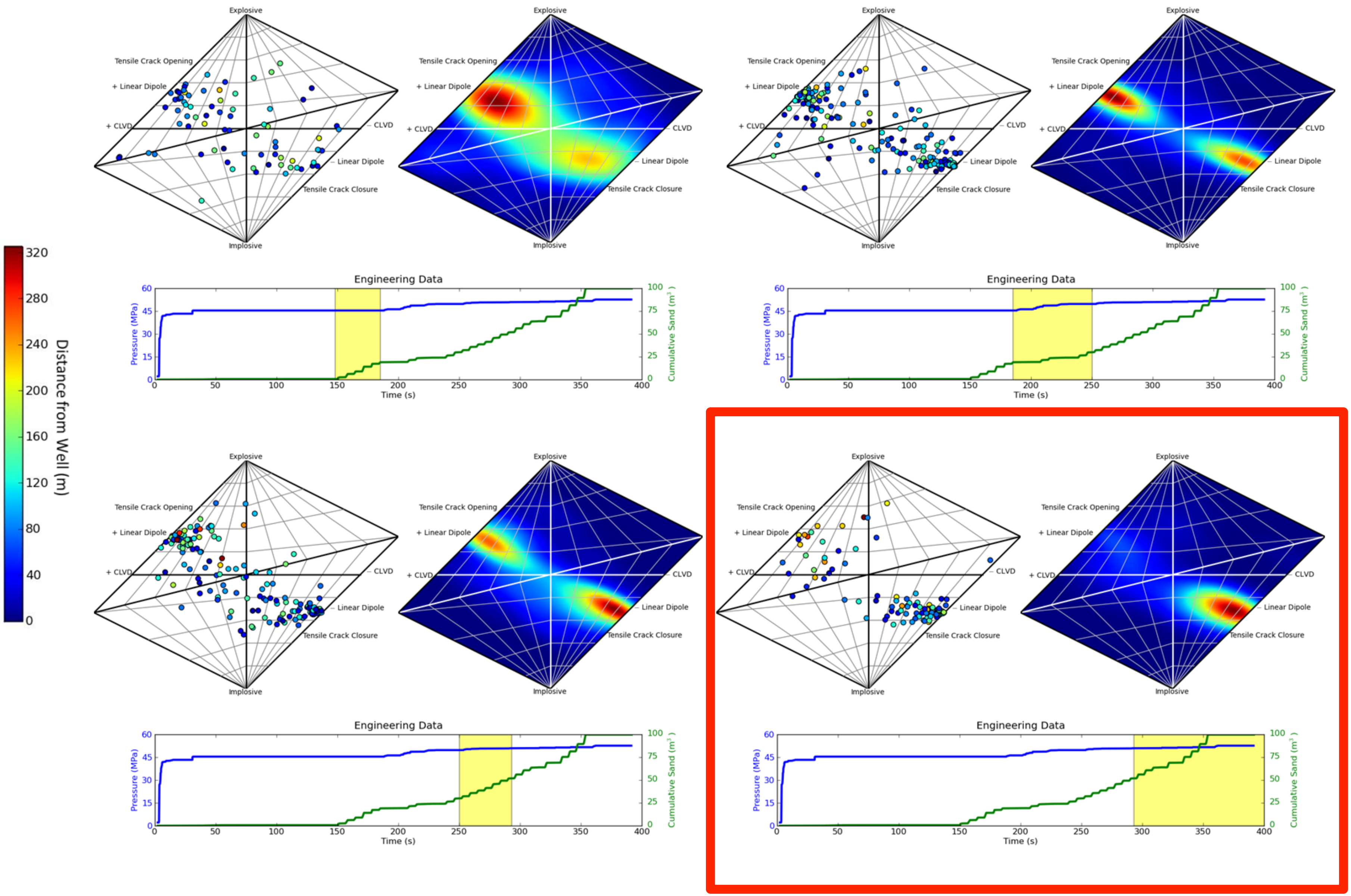
How is structural complexity imprinted onto the moment tensor?

Homogenized moment tensor and the effect of near-field heterogeneities on nonisotropic radiation in nuclear explosion

Gaël Burgos[1,2], Yann Capdeville[3], and Laurent Guillot[1]



Source mechanisms change as well pressure changes



Baig and Urbancic (2010)

Final remarks

1. The eigenvalue space of moment tensors (the lune) provides a useful framework for understanding and interpreting full moment tensors and for characterizing uncertainties.
2. Choices in the misfit function are extremely important (bandpass, time windows, allowable time shifts, etc).
3. Fundamental challenge: to interpret moment tensor in physical terms (e.g., oblique opening crack, basic crack-plus-double-couple, two-process model, etc)

download moment tensor scripts on github (feedback welcome):
https://github.com/carltape/compearth

ctape@alaska.edu

THANK YOU FOR YOUR ATTENTION!