Upscaling of Compositional Models for Single-Porosity and Discrete-Fracture-Matrix Systems

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Collaborators

• Fractured systems:
  • Mohammad Karimi-Fard (Stanford)
  • Robin Hui (Chevron)
  • Brad Mallison (Chevron)

• Single-porosity systems:
  • Hangyu Li (now at Shell)
General Motivation

• Current reservoir engineering applications often require very large numbers of (automated) runs

• Key examples include field development / well control optimization and uncertainty quantification

• Compositional flow simulation, which is expensive, required for EOR and shale oil operations

• Performing $O(10^3-10^5)$ detailed compositional runs is computationally prohibitive (and not necessary)
Outline

• Upscaling discrete-fracture-matrix (DFM) systems

• Global compositional upscaling for single-porosity models

• Ensemble level upscaling to generate many (approximate) upscaled models for use in UQ
Classification of Numerical Upscaling Methods

- Methods based on coarse gridding and single-phase flow computations
  - Local, global, border-region, local-global procedures
  - Provide $k^*$, $T^*$, $Wl^*$, $\phi^*$ (not $k_{rj}^*$ etc)

- Methods based on compositional flow computations
  - Flow domains as above
  - Provide $k^*$, $T^*$, $Wl^*$, $\phi^*$ and $k_{rj}^*$ etc
  - Expensive, so reuse is important

- Ensemble level procedures use statistical assignment for majority of upscaled models
DFM Grid Generation

- Provides discrete-fracture-matrix model (discretization as per Brad’s description)
- Could also use EDFM (Li & Lee, 2008) or a combination for fine-scale simulations
Aggregation of Fine Cells

- Coarse “control volumes” constructed via aggregation of fine cells; based on geometry, geology & flow information.

Model has a dual-continuum character; similarities with GMsFEM models (Efendiev, Chung, …)
Coarse-scale Properties (M-M & F-F)

$\nabla \cdot (k \nabla p) = 0$

$p_L = 1, \quad p_R = 0 \quad (\text{flow in } x)$

$T_{IJ}^* = \frac{\Sigma q}{\langle p \rangle_I - \langle p \rangle_J}$

- coarse cell $I$
- coarse cell $J$
Coarse-scale Properties (F-M)

fracture-matrix flow

pseudo-steady-state solution

\[ \nabla \cdot (k \nabla p) = \phi F \]

\[ p = 0 \text{ in } \Gamma_f \]

\[ \partial p / \partial n = 0 \text{ on } \Gamma_b \]

\[ T^*_{IJ} = \frac{\Sigma q}{\langle p \rangle_I - \langle p \rangle_J} \]

(slightly different from Brad Mallison’s treatment)
DFM Model – Problem Setup

- 663 discrete fractures in two orthogonal sets, 5° dip, \( k_f = 100,000 \text{ mD} \), \( k_m = 0.1 \text{ mD} \)
- \( \sim650,000 \) matrix and \( \sim24,000 \) fracture control volumes
- Multiple coarse models from one set of global 1-p runs
- 6 components, 3 phases (IX runs for all models)
Gas Production Results

- Injectors: $q = 30,000$ bbl/day with maximum BHP; injected gas ($0.23$ H$_2$S, $0.47$ CH$_4$, ...) is miscible with reservoir oil
- Producers: $q = 30,000$ bbl/day with minimum BHP
**H$_2$S Mole Fraction & Errors in Cumulatives**

- Fine IX simulation required 62 hours on 64 CPUs

<table>
<thead>
<tr>
<th>Resolution</th>
<th>Error oil (%)</th>
<th>Error gas (%)</th>
<th>Error inj (%)</th>
<th>Speedup factor</th>
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<tr>
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<td>3.4</td>
<td>1.5</td>
<td>2.8</td>
<td>634</td>
</tr>
</tbody>
</table>

- After 2 years

**fine (100%)**

**coarse (10%; 70x)**
Global Compositional Upscaling

- **Mass conservation on fine scale:**

\[
\nabla \cdot \left( \sum_{j=1}^{n_p} x_{ij} \rho_j \lambda_j k \cdot \nabla p \right) - \sum_{j=1}^{n_p} x_{ij} \rho_j q_j^w = \frac{\partial}{\partial t} \left( \phi \sum_{j=1}^{n_p} x_{ij} \rho_j S_j \right), \quad i = 1, n_c
\]

- **Mass conservation on coarse scale:**

\[
\nabla \cdot \left( \sum_{j=1}^{n_p} x_{ij}^c \alpha_{ij} \rho_j c \lambda_j^* k^* \cdot \nabla p^c \right) - \sum_{j=1}^{n_p} x_{ij}^c \alpha_{ij} \rho_j c q_j^{w,c} = \frac{\partial}{\partial t} \left( \phi^* \sum_{j=1}^{n_p} x_{ij}^c \rho_j c S_j^c \right)
\]

(\(\alpha\)-factor model after Barker and Fayers, 1994; also Iranshahr, Chen & Voskov, 2014; Salehi, Voskov & Tchelepi, 2017)
Global Upscaling Procedure (1)

- Single-phase parameters $T^*$  
  (solve $\nabla \cdot (k \cdot \nabla p) = q^w$)

  \[ \sum q^f = \sum T^f \Delta p^f \quad \longleftrightarrow \quad q^c = T^* \Delta p^c \]

- Transmissibility:
  \[ T^* = \frac{\sum q^f}{\langle p^f \rangle_n - \langle p^f \rangle_{n+1}} \]
Global Upscaling Procedure (2)

- Two-phase functions $k_{rj}^*$ (solve full compositional model)

$$\sum q_j^f = \sum T^f \Delta p^f \frac{k_{rj}^f}{\mu_j^f} \rho_j^f \quad \leftrightarrow \quad q_j^c = T^* \Delta p^c \frac{k_{rj}^*}{\mu_j^c} \rho_j^c, \quad j = o, g$$

- Relative permeability: $k_{rj}^*(S_g^c) = \frac{\langle \mu_j^f \rangle \sum q_j^f}{T^* \langle \rho_j^f \rangle \{\langle p^f \rangle_n - \langle p^f \rangle_{n+1}\}}$
Global Upscaling Procedure (3)

Compositional functions $\alpha_{ij}$ (same global solution)

$$\sum q_{ij}^f = \sum T^f \Delta p^f \frac{k_{rj}^f}{\mu_j^f} \rho_j^f x_{ij}^f$$

$$q_{ij}^c = T^* \Delta p^c \frac{k_{rj}^c}{\mu_j^c} \rho_j^c \alpha_{ij} x_{ij}^c$$

$\alpha$-factor:

$$\alpha_{ij}(S_g^c) = \frac{\langle \mu_j^f \rangle \sum q_j^f}{T^* k_{rj}^* \langle x_{ij}^f \rho_j^f \rangle \langle p^f \rangle_n - \langle p^f \rangle_{n+1}}$$
Global Upscaling Procedure (4)

- Upscaled functions for well terms also computed
- Use of $T^*$, $k^*_{rj}$ & $\alpha_{ij}$ does not assure that $q^c_{ij} = \sum q^f_{ij}$
- Iterate on $\alpha_{ij}$ to minimize $\left| q^c_{ij} - \sum q^f_{ij} \right|$ 

(after Y. Chen, Mallison and Durlofsky, 2008)
Flowchart of Global Upscaling

\[ T^*, k^*_{rj} \text{ and } \alpha_{ij} \text{ from global fine-scale simulation (save } \sum q_{ij}^f) \]

At each time step, run global coarse model with \( T^*, k^*_{rj} \text{ and } \alpha_{ij} \)

Coarse-scale component flow rate: \( q_{ij}^c \)

Update \( \alpha_{ij} \):

\[
\alpha_{ij}^{v+1} = d \alpha_{ij}^v + (1 - d) (\sum q_{ij}^f / q_{ij}^c) \alpha_{ij}^v
\]

\( d \): damping factor
Eight-Component Near-Miscible System

- **Initial conditions:**
  - 1150 psi, 373 K
  - \{C_1(10\%), CO_2(1\%), C_2(1\%), C_3(1\%), C_4(10\%), C_6(10\%), C_8(20\%), C_{15}(47\%)\}

- Permeability using sGsim
  - \sigma_{lnk} = 1.6, \ l_x = 0.3, \ l_y = 0.3
  - 55 x 55 \rightarrow 11 x 11 \text{ (uniform grid)}

- Inject \{C_1(20\%), CO_2(80\%)\}
  - at 1700 psi

- Produce at 750 psi
Oil and Gas Field Production Rates

Oil rate

Standard upscaling
\((T^* \text{ \& } WI^* \text{ only})\)

Fine-scale
Oil and Gas Field Production Rates

Oil rate

- Standard upscaling ($T^*$ & $WI^*$ only)
- Global upscaling
- Fine-scale
Oil and Gas Field Production Rates

Oil rate

Gas rate

Standard upscaling
($T^* \& WI^*$ only)

Global upscaling

Fine-scale
$C_1$ and $CO_2$ Field Production Rates

$C_1$ rate

Fine-scale

Global upscaling

Standard upscaling

$CO_2$ rate

Time (days)
Impact of $\alpha_{ij}$ and Iteration

- Different (4-component) system, $C_1$ injected
- Five-spot, $99 \times 99 \rightarrow 11 \times 11$, $\sigma_{\text{lnk}} = 1.8$

Field oil rate

(Li, 2014)
Robustness Test (8-comp system)

• “Training” case: $BHP_{inj} = 1700$ psi, $BHP_{prod} = 750$ psi

• “Test” case: vary BHPs by ±20% every 200 days (to mimic optimization run) – use upscaled functions from training run
Oil and Gas Field Production Rates

Oil rate

Gas rate

Fine-scale

Training

Global upscaling

Oil rate (bbl/day)

Gas rate (mscf/day)

Time (days)
Multiple Test Cases

- Training case: $BHP_{inj} = 1700 \text{ psi}$, $BHP_{prod} = 750 \text{ psi}$
- 100 test cases with time-varying BHPs

\[\begin{align*}
BHP_{inj} & = 1800 \\
& = 1700 \\
& = 1500
\end{align*}\]
Multiple Test Cases

- Training case: $BHP_{\text{inj}} = 1700$ psi, $BHP_{\text{prod}} = 750$ psi
- 100 test cases with time-varying BHPs
  
  \[
  BHP_{\text{inj}}
  \]

\begin{align*}
\vdots & \quad 1800 \\
\vdots & \quad 1700 \\
\vdots & \quad 1500
\end{align*}
Multiple Test Cases

- Training case: $\text{BHP}_{\text{inj}} = 1700 \text{ psi, BHP}_{\text{prod}} = 750 \text{ psi}$
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Multiple Test Cases

- Training case: $\text{BHP}_{\text{inj}} = 1700 \text{ psi}$, $\text{BHP}_{\text{prod}} = 750 \text{ psi}$
- 100 test cases with time-varying BHPs
Cumulative Production at 500 days

- **Oil**
  - Global upscaling
  - Standard upscaling

- **Gas**
  - Training results
3D Model (5-spot pattern)

- Initial conditions:
  - 1150 psi, 334 K
  - \{C_1(0.2), \text{CO}_2(0.01), C_4(0.29), C_{10}(0.5)\}

- Permeability using sGsim
  - \(l_x = 0.3, \ l_y = 0.3, \ l_z = 0.1\)
  - 55 x 55 x 15 \(\rightarrow\) 11 x 11 x 5

- Inject \(\{\text{C}_1(10\%), \ \text{CO}_2(90\%)\}\) at 1500 psi

- Produce at 750 psi
Oil and Gas Field Production Rates

Oil rate

Gas rate

Fine-scale Standard

Global upscaling
Oil and Gas Production Rates (Producer 4)

**Oil rate**

- **Fine-scale**
- **Standard**

**Gas rate**

- **Global upscaling**
Gas Saturation Distribution
(300 days, coarse layer 4)
Pressure Distribution
(300 days, coarse layer 4)
Ensemble Level Upscaling for UQ

1. Generate multiple fine-scale geological realizations

2. Apply full flow-based global upscaling to a small fraction of realizations

3. Calibrate the numerically upscaled $k_{rg}^*, k_{ro}^*$, $\alpha_{ij}$ to coarse-scale features

4. Statistically assign $k_{rg}^*, k_{ro}^*$ & $\alpha_{ij}$ for new realizations

(figure after Y. Chen)
Ensemble Level Upscaling for UQ

1. Generate multiple fine-scale geological realizations

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(figure after Y. Chen)
Parameterize $k_{rg}^*$

- Parameterize by the difference between $k_{rg}^*$ and $k_{rg}^f$

$$\delta k_{rg} = \frac{1}{N} \sum (k_{rg}^* - k_{rg}^f)$$
Calibrate ($k_{rg}^*$) for Upscaled Realizations

Features:
- $\sigma(\ln k)$
- $\sigma(u_{1p})/\overline{u_{1p}}$
- $\sum q^f$

Numerically upscaled function ($k_{rg}^*$):
- $\delta k_{rg}(f_1, f_2, f_3)$
- $k_{rg}^* \rightarrow \delta k_{rg}$
\( \delta k_{rg} \) from Features for New Realizations

Features:

- \( \sigma(\ln k) \)
- \( \sigma(u_{1p})/\bar{u}_{1p} \)
- \( \Sigma q^f \)

\( \delta k_{rg} \) from features:

\( \delta k_{rg}(f_1, f_2, f_3) \)
Statistically Assign $\delta k_{rg}$ for New Realizations

$\delta k_{rg}$ from features

Variogram from numerically upscaled $\delta k_{rg}$

Co-sGsim

$\delta k_{rg}$ for new realization
Multiple Realizations (4-component system)

- Permeability using sGsim
  - Initial oil: \{C_1(0.1), CO_2(0.18), C_4(0.38), C_{10}(0.34)\}

- 100 realizations:
  - 99 x 99 → 11 x 11

- Inject pure C_1 at 1500 psi, produce at 500 psi

- 10 realizations upscaled globally, EnLU applied for other 90 realizations
Flow Statistics of Gas Production Rates

Standard upscaling
($T^* \& W^I$ only)
Flow Statistics of Gas Production Rates

- Standard upscaling
  \((T^* \text{ & } WI^* \text{ only})\)
- Full global compositional upscaling (ref)

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**Graphs:**
- **Fine Gas Rate:**
  - P10
  - P50
  - P90

- **Full global compositional upscaling:**
  - P10
  - P50
  - P90
Flow Statistics of Gas Production Rates

Standard upscaling
($(T^* \& WI^* \text{ only})$

Full global compositional upscaling (ref)

EnLU compositional upscaling

Flow Statistics of Gas Production Rates:

- **Fine** upscaling

  - $P_{10}$
  - $P_{50}$
  - $P_{90}$

- **Full global** compositional upscaling (ref)

  - $P_{10}$
  - $P_{50}$
  - $P_{90}$

- **EnLU** compositional upscaling

  - $P_{10}$
  - $P_{50}$
  - $P_{90}$

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Gas rate (mscf/day)

Time (days)
Flow Statistics of Oil Production Rates

Standard upscaling
($T^* \& WI^*$ only)

Full global compositional upscaling (ref)

EnLU compositional upscaling

![Graphs showing oil production rates for different scaling methods.]

- **P10**, **P50**, and **P90** represent the 10th, 50th, and 90th percentiles of oil production rates, respectively.
- **Fine** scale diagram highlights differences in production rates compared to coarser scales.
- **Time (days)** and **Oil rate (bbl/day)** are on the x and y axes, respectively.
Flow Statistics of C$_1$ Production Rates

Standard upscaling
($T^*$ & $WI^*$ only)

Full global compositional upscaling (ref)

EnLU compositional upscaling

<table>
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<tr>
<th>Time (days)</th>
<th>C$_1$ Rate (lb-mol/day)</th>
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<tr>
<td>0</td>
<td>0</td>
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<tr>
<td>1000</td>
<td>500</td>
</tr>
<tr>
<td>2000</td>
<td>1000</td>
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<td>3000</td>
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<table>
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<tr>
<th>Fine</th>
<th>P10</th>
<th>P50</th>
<th>P90</th>
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<td>0</td>
<td>0</td>
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<td>1000</td>
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<td>3000</td>
<td>1500</td>
<td>1500</td>
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<table>
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<tr>
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<th>P10</th>
<th>P50</th>
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<tr>
<td>3000</td>
<td>2000</td>
<td>2000</td>
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Note: The diagrams show the flow statistics for different scenarios, with lines indicating the 10th, 50th, and 90th percentiles (P10, P50, P90) over time (days) for C$_1$ production rates in lb-mol/day.
Realization-by-Realization Error (Gas Rate)

\[ E_j = \frac{\int_t (q_j^c - q_j^f) \, dt}{\int_t q_j^f \, dt} \]

- standard coarse
- comp upsc (iter)
- EnLU
Realization-by-Realization Error (Gas Rate)

Median errors
- std coarse: 34%
- comp upsc: 3.3%
- EnLU: 9.3%
Flow Statistics of Gas Production Rates
(5 realizations upscaled)

Standard upscaling
\((T^* & WI^* \text{ only})\)

Full global compositional upscaling (ref)

EnLU compositional upscaling
Channel Realizations (4 components)

- Problem setup
  - Initial oil: \{C_1(0.2), CO_2(0.01), C_4(0.29), C_{10}(0.5)\}
  - 100 realizations: 55 x 55 → 11 x 11

- 10 realizations upscaled globally, EnLU applied for other 90 realizations
- Inject CO_2 at 1500 psi, produce at 500 psi
Flow Statistics – Oil Rates (channel model)

Standard upscaling

Median (realization) rate errors
std coarse: 20.5%
comp upsc: 1.3%
EnLU: 5.7%

EnLU
Summary

• Described single-phase (for DFM) and compositional upscaling approaches

• Aggregation-based method can efficiently generate a range of coarsened models

• Global compositional upscaling is expensive but provides high accuracy and acceptable robustness

• Ensemble level upscaling provides reasonable flow statistics with 5-10% of models run on fine-scale
Current and Future Work

- Combine aggregation-based treatments with multilevel Monte Carlo for UQ
- Apply/extend compositional upscaling to shale oil simulations
- Further develop error models to correct upscaled models and guide fine-scale simulation
References for Results Presented


Alternative DFM Gridding Framework

- Local fracture distribution
- Matrix permeability distribution
- Coarse model