Introduction to Reservoir Simulation as Practiced in Industry

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Computational Issues in Oil Field Applications Tutorials
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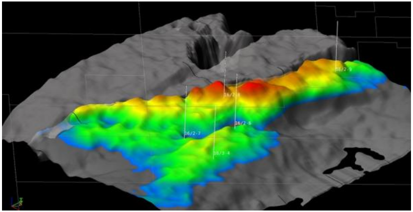
Petroleum reservoirs

Naturally occurring flammable liquid/gases found in geological formations

- Originating from organic sediments that have been compressed and 'cooked' to form hydrocarbons that migrated upward in sedimentary rocks until limited by a trapping structure
- Found in shallow reservoirs on land and deep under the seabed
- Only 30% of the reserves are 'conventional'; remaining 70% include *shale oil and gas*, heavy oil, extra heavy oil, and oil sands.

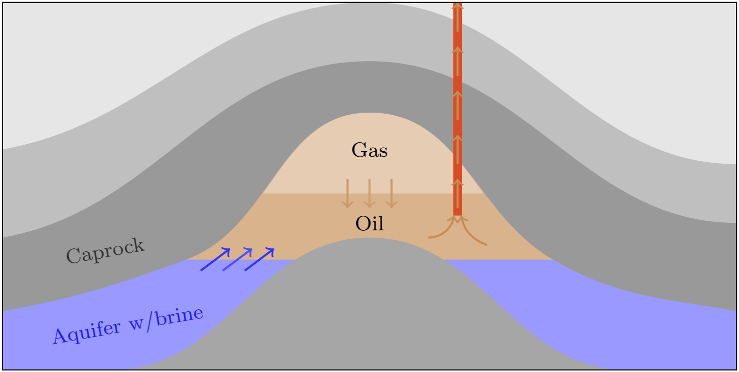
Uses of (refined) petroleum:

- Fuel (gas, liquid, solid)
- Alkenes manufactured into plastics and compounds
- Lubricants, wax, paraffin wax
- Pesticides and fertilizers for agriculture



Johan Sverdrup, new Norwegian 'elephant' discovery, 2011. Expected to be producing for the next 30+ years

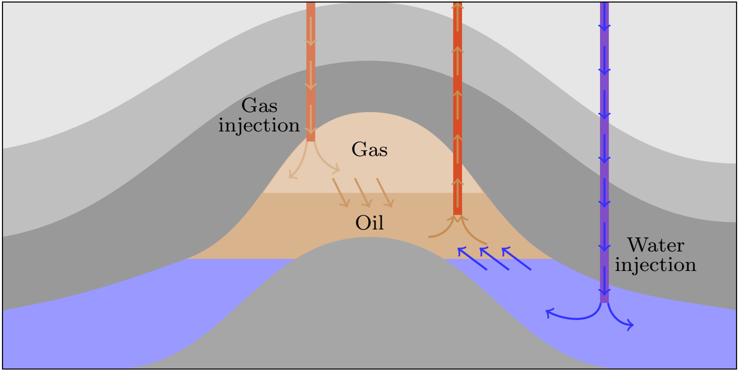
Production processes



Primary production – puncturing the 'balloon'

When the first well is drilled and opened for production, trapped hydrocarbon starts flowing toward the well because of over-pressure

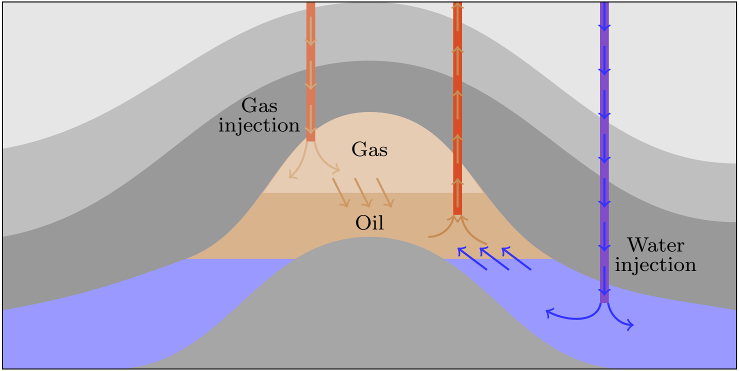
Production processes



Secondary production – maintaining reservoir flow

As pressure drops, less hydrocarbon is flowing. To maintain pressure and push more profitable hydrocarbons out, one starts injecting water or gas into the reservoir, possibly in an alternating fashion from the same well.

Production processes



Enhanced oil recovery

Even more crude oil can be extracted by gas injection (CO_2, natural gas, or nitrogen), chemical injection (foam, polymer, surfactants), microbial injection, or thermal recovery (cyclic steam, steam flooding, in-situ combustion), etc.

Why reservoir simulation?

To estimate reserves and support economic and operational decisions

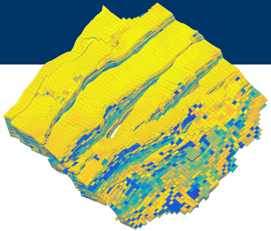
To this end, reservoir engineers need to:

- understand reservoir and fluid behavior
- quantify uncertainty
- test hypotheses and compare scenarios
- assimilate data
- optimize recovery processes

Reservoir models

Somewhat simplified, consist of three parts:

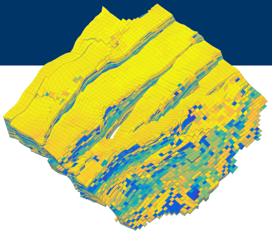
Reservoir models



Somewhat simplified, consist of three parts:

1. a **geological model** – volumetric grid with cell/face properties describing the porous rock formation

Reservoir models



Somewhat simplified, consist of three parts:

1. a **geological model** – volumetric grid with cell/face properties describing the porous rock formation

2. a **flow model** – describes how fluids flow in a porous medium (conservation laws + appropriate closure relations)

$$\partial_t(\phi b_w S_w) + \nabla\cdot(b_w \vec{u}_w) = b_w q_w$$

$$\partial_t[\phi(b_w S_o + b_g r_v S_g)] + \nabla\cdot(b_o \vec{u}_o + b_g r_v \vec{u}_g) = b_o q_o + b_g r_v q_g$$

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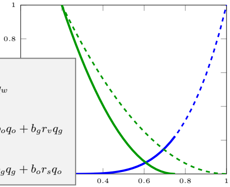

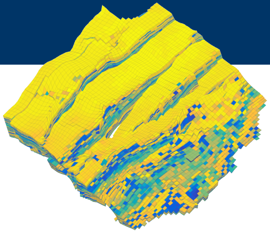

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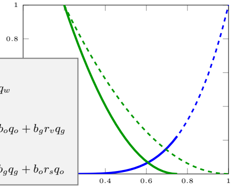

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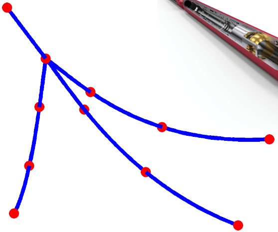
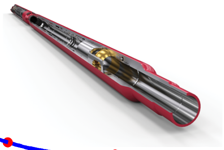
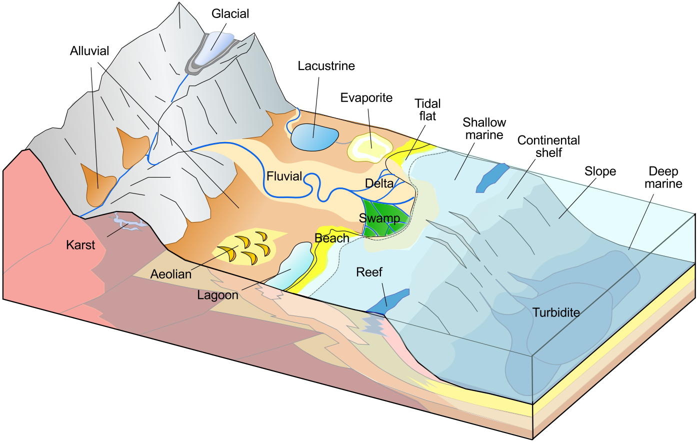
2. a **flow model** – describes how fluids flow in a porous medium (conservation laws + appropriate closure relations)

3. a **well model** – describes flow in and out of the reservoir, in the wellbore, flow control devices, surface facilities





Geologic model: sedimentary rocks



Mineral particles broken off by weathering and erosion

Transported by wind or water to a place where they settle and accumulate into a sediment, building up in lakes, rivers, sand deltas, lagoons, choral reefs, etc

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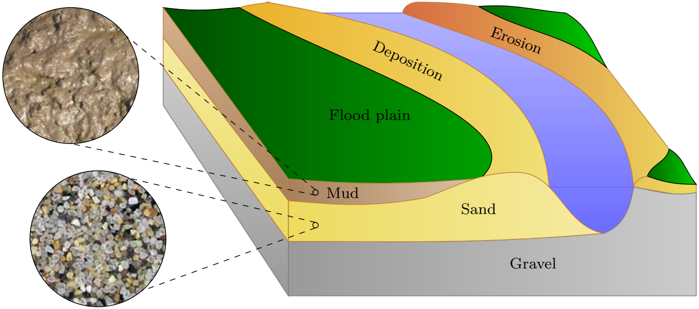

Layered structure with different mixtures of rock types with varying grain size, mineral type, and clay content

Thin beds that stretch hundreds or thousands of meters, typically horizontally or at a small angle. Gradually buried deeper and consolidated

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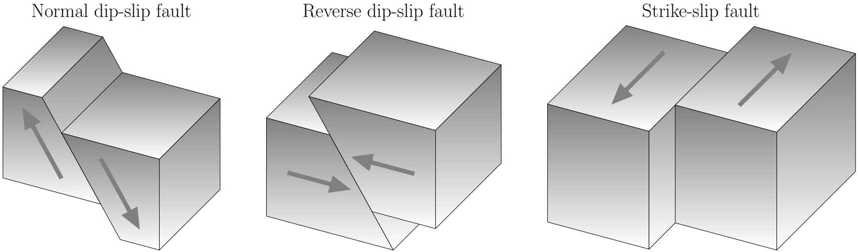
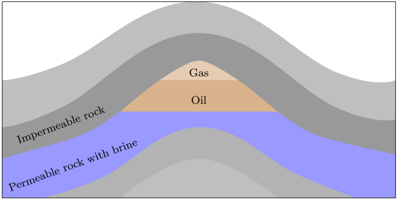


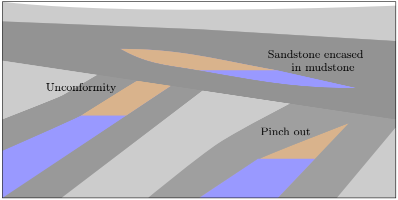
Geological activity will later fold, stretch, and fracture the consolidated rock

Geologic model: sedimentary rocks

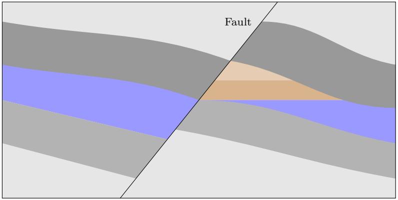
Structural trap: anticline



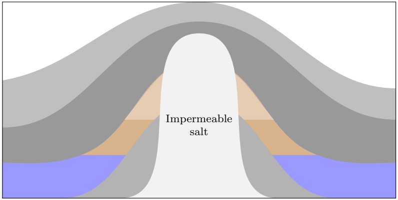
Stratigraphic traps



Fault trap



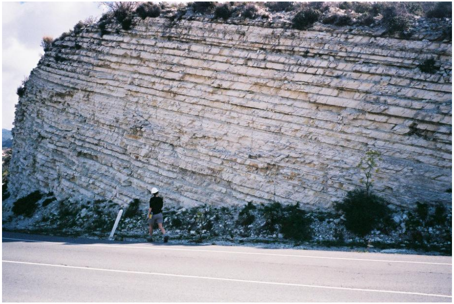
Salt dome



Geologic model: sedimentary rocks

Outcrops of sedimentary rocks from Svalbard, Norway. Length scale: ∼100 m

Geologic model: sedimentary rocks



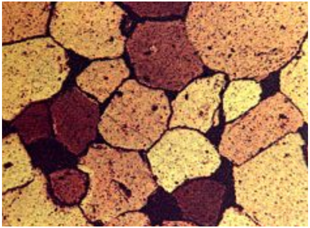
Layered geological structures typically occur on both large and small scales

Porous media flow – a multiscale problem

The scales that impact fluid flow in subsurface rocks range from

- the micrometer scale of pores and pore channels
- via dm-m scale of well bores and laminae sediments
- to sedimentary structures that stretch across entire reservoirs

Porous rocks are heterogeneous at all length scales (no scale separation)



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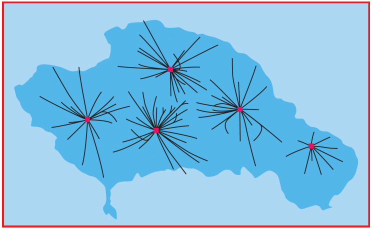

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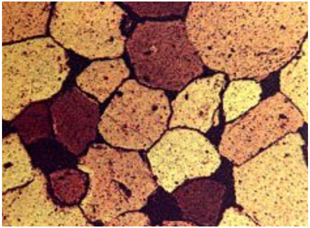
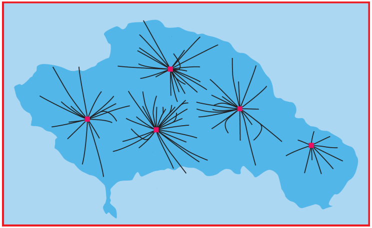
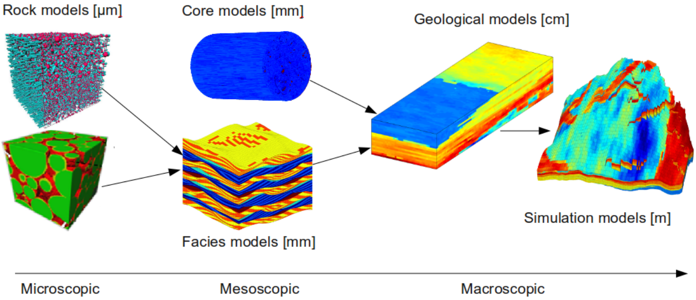
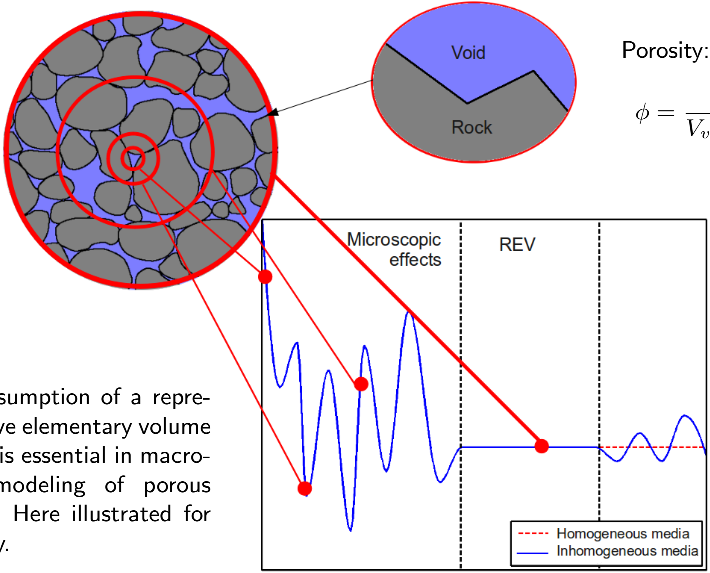






Flow model: representative elementary volume



Porosity:

$$\phi = \frac{V_v}{V_v + V_r}$$

The assumption of a representative elementary volume (REV) is essential in macroscale modeling of porous media. Here illustrated for porosity.

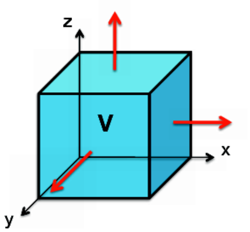
Governing equations for fluid flow

In its simplest form – two main principles

- Conservation of mass

$$\frac{\partial}{\partial t}\int_V m\,dx + \oint_{\partial V} \vec{F}\cdot\vec{n}\,ds = \int_V r\,dx$$

m=mass, $\vec{F}$=flow rate, r=fluid sources



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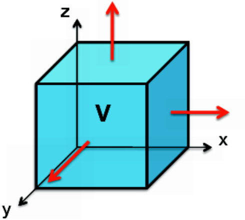
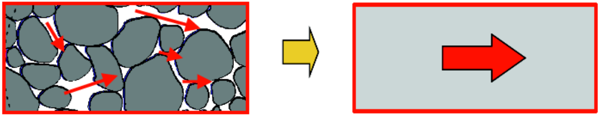


- Darcy's law:

$$\vec{u} = -K(\nabla p - \rho g \nabla z)$$

empirical law for describing processes on an unresolved scale.



Similar to Fourier's law (heat) [1822], Ohm's law (electric current) [1827], Fick's law (concentration) [1855], except that we now have *two driving forces*

Darcy's law and permeability

In reservoir engineering:

$$\vec{u} = -\frac{\mathbf{K}}{\mu}\big(\nabla p - \rho g \nabla z\big)$$

Intrinsic permeability **K** measures ability to transmit fluids

Anisotropic and diagonal by nature, full tensor due to averaging.

Reported in units Darcy: $1 \text{ d} = 9.869233 \cdot 10^{-13} \text{ m}^2$

Fluid velocity:

Darcy's law is formulated for *volumetric flux*, i.e., volume of fluid per total area per time. The *fluid velocity* is volume per area occupied by fluid per time, i.e., $\vec{v} = \frac{\vec{u}}{\phi}$.

Theoretical basis (M. K. Hubbert, 1956):

Darcy's law derived from the Navier–Stokes equations by averaging, neglecting intertial and viscous effects

Single-phase, incompressible flow

Model equations for single-phase flow:

$$\frac{\partial(\phi\rho)}{\partial t} + \nabla \cdot \big(\rho\vec{u}\big) = q, \qquad \vec{u} = -\frac{\mathbf{K}}{\mu}\big(\nabla p - \rho g \nabla z\big)$$

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Assume constant density ρ, unit fluid viscosity μ, and neglect gravity g $\longrightarrow$ flow equation on mixed form

$$\nabla \cdot \vec{u} = q, \qquad \vec{u} = -\mathbf{K}\nabla p$$

or as a Poisson equation with variable coefficients

$$-\nabla\big(\mathbf{K}\nabla p\big) = q$$

Single-phase, slightly compressible flow

Introduce compressibilities for rock and fluid

$$\frac{d\phi}{dp} = c_r\phi, \qquad \frac{d\rho}{dp} = c_f\rho$$

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Insert into conservation equation

$$\frac{\partial(\phi\rho)}{\partial t} = \nabla \cdot \left(\rho \frac{\mathbf{K}}{\mu} \nabla p \right)$$

$$\left[(c_r + c_f)\phi\rho \right] \frac{\partial p}{\partial t} = \frac{c_f \rho}{\mu} \nabla p \cdot \mathbf{K} \nabla p + \frac{\rho}{\mu} \nabla \cdot (\mathbf{K} \nabla p)$$

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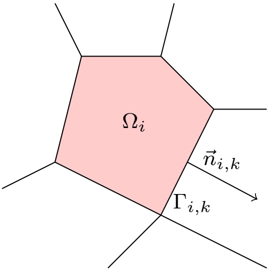
If c_f is sufficiently small, so that $c_f\nabla p\cdot\mathbf{K}\nabla p \ll \nabla\cdot(\mathbf{K}\nabla p)$, we get

$$\frac{\partial p}{\partial t} = \frac{1}{\mu\phi c}\nabla\cdot\big(\mathbf{K}\nabla p\big), \qquad c = c_r + c_f$$

Numerical discretization

Assumption: a grid $\mathcal{G}$ consisting of a collection of polyhedral cells $\{\Omega_i\}$



Numerical discretization

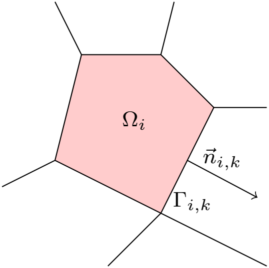
Assumption: a grid $\mathcal{G}$ consisting of a collection of polyhedral cells $\{\Omega_i\}$

Mass conservation per grid cell:

$$\int_{\Omega_i} \nabla \cdot \vec{u}\, dx = \oint_{\partial\Omega_i} \vec{u} \cdot \vec{n}\, ds = \int_{\Omega_i} q\, dx$$

$$\sum_k u_{i,k} = q_i$$

Pressure is cell-wise constant, flux is continuous across cell interfaces



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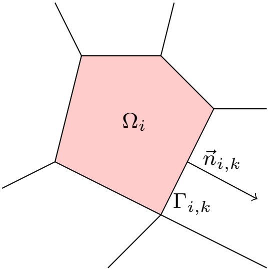
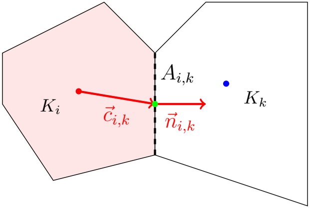


Assume **K** is constant within each cell

$$u_{i,k} = -\int_{\Gamma_{ik}} \mathbf{K}\nabla p \cdot \vec{n}_{ik}\, ds$$
$$\approx A_k\, \mathbf{K}\, \frac{(p_i - \pi_{i,k})\vec{c}_{ik}}{|\vec{c}_{ik}|^2} \cdot \vec{n}_{ik}$$
$$= T_{i,k}\big(p_i - \pi_{i,k}\big)$$



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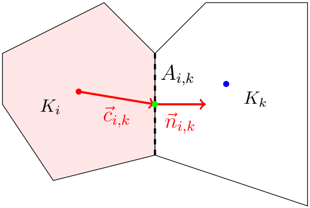

Next, we use continuity of flux and pressure to eliminate the interface pressures

$$u_{i,k} = T_{ik}\big(p_i - p_k\big)$$

Numerical discretization

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$$
\begin{aligned}
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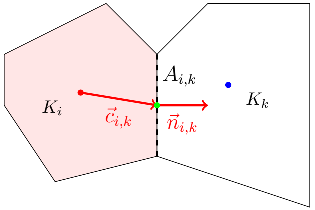
&\approx A_k\, \mathbf{K}\, \frac{(p_i - \pi_{i,k})\vec{c}_{ik}}{|\vec{c}_{ik}|^2} \cdot \vec{n}_{ik} \\
&= T_{i,k}\big(p_i - \pi_{i,k}\big)
\end{aligned}
$$



Next, we use continuity of flux and pressure to eliminate the interface pressures

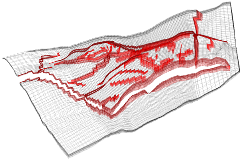
$$u_{i,k} = T_{ik}\big(p_i - p_k\big)$$

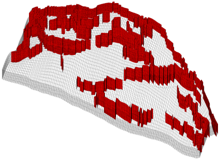
Mass conservation $q_i = \sum_k u_{i,k}$ gives a linear system

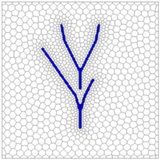
$$\mathbf{A}\boldsymbol{p} = \boldsymbol{q}, \qquad \text{where } A_{ij} = \begin{cases} \sum_j T_{ij}, & k = i, \\ -T_{ik}, & k \neq i. \end{cases}$$

Grids: volumetric representation of the reservoir

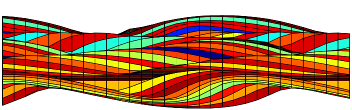
The structure of the reservoir (geological surfaces, faults, etc) + well paths

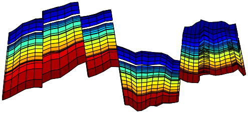




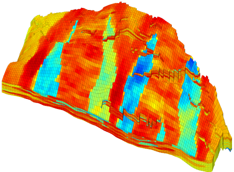


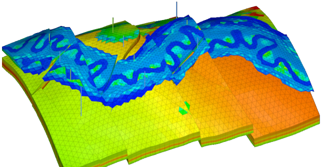
The stratigraphy of the reservoir (sedimentary structures)



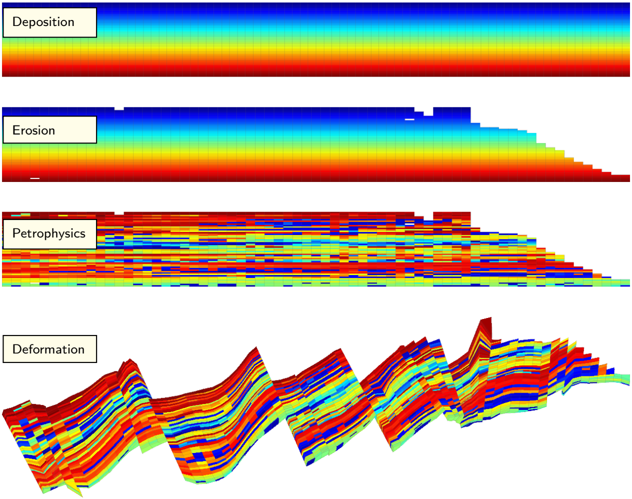


Petrophysical parameters (permeability, porosity, net-to-gross, . . .)

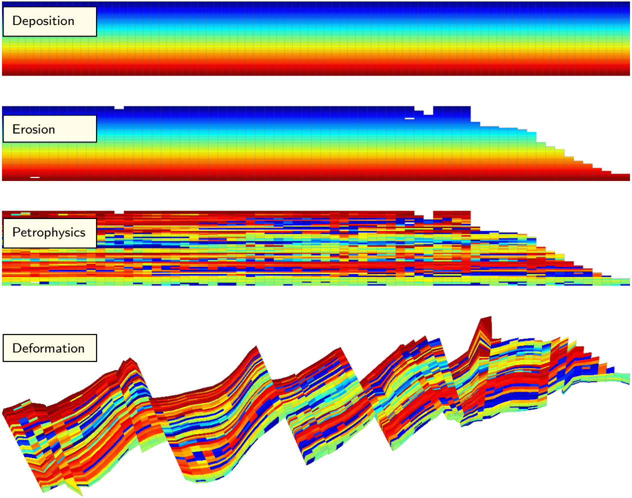


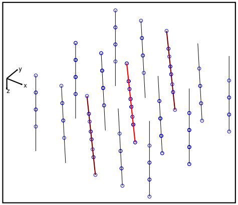


Grids: mimicking geological processes



Grids: mimicking geological processes





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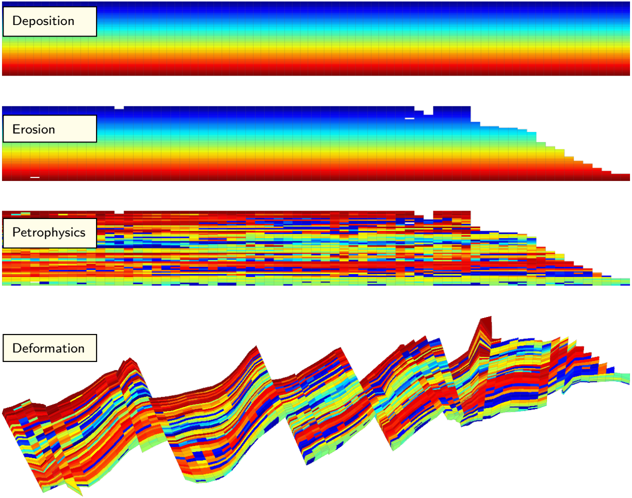

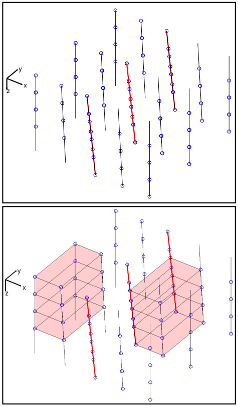

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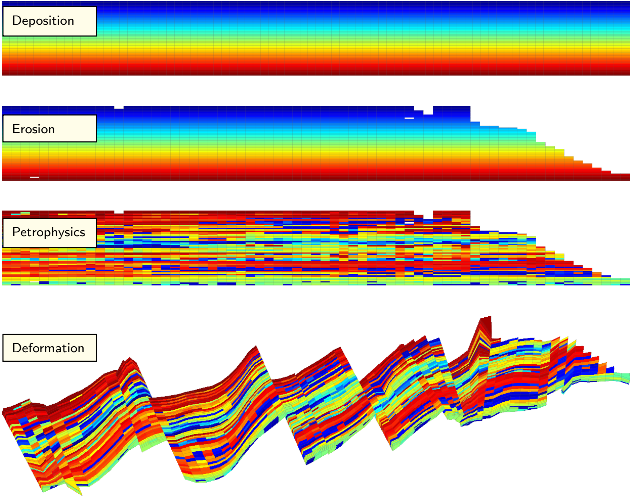
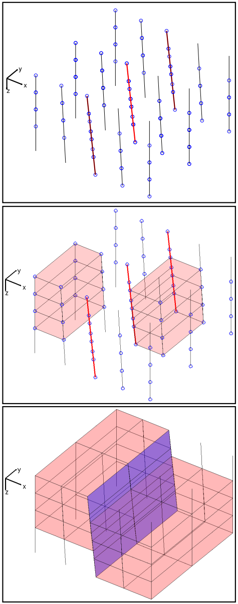
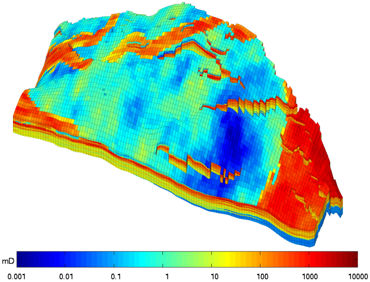
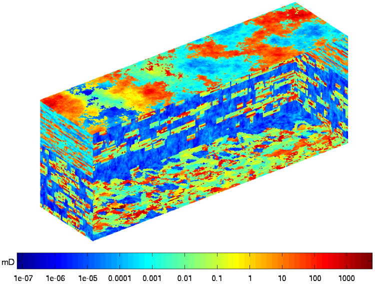
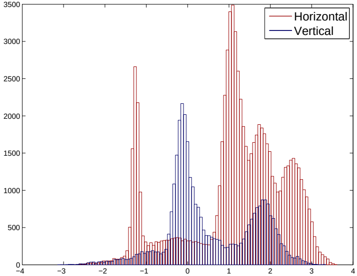


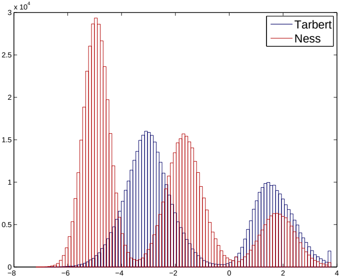


Petrophysical parameters









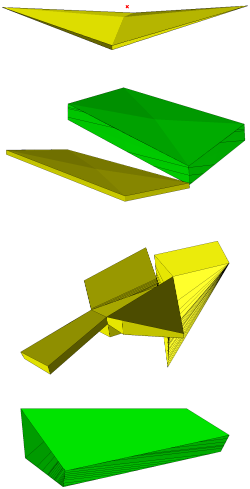
Research challenge: numerical robustness

Complex, unstructured grids with many obscure challenges

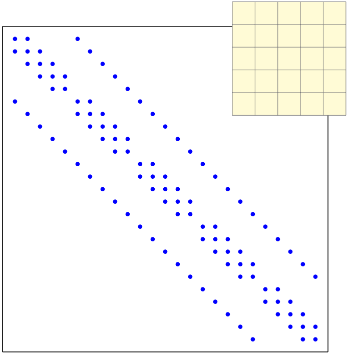
- Grid dictated by geology, not chosen freely to maximize accuracy of numerical discretization
- Topology is generally unstructured, non-neighboring connections
- Cells deviate strongly from box shape, high aspect ratios, many faces/neighbors, small faces, ...
- Potential inconsistencies: bilinear vs tetrahedral surfaces

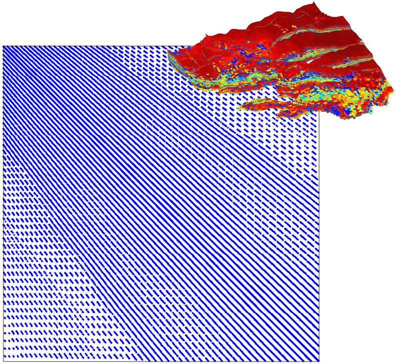
Petrophysics:

- Many orders of magnitude variations
- Strong discontinuities
- No clear scale separation (long and short correlations)



Research challenge: efficient solvers

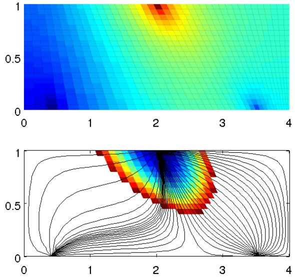




Large coefficient variations, complex sparsity patterns, etc. Call for efficient iterative solvers and preconditioning methods $\longrightarrow$ good test problems for multigrid methods

Research challenge: consistent discretizations

Problem: standard finite-volume methods are not consistent unless the grid is **K** orthogonal



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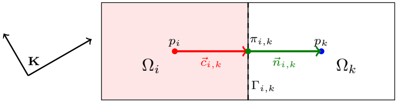
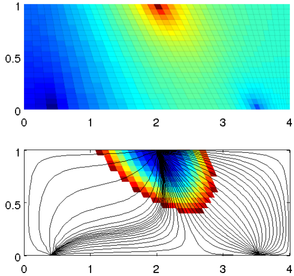


$$u_{ik} = -\int_{\Gamma_{ik}} \left(K_{xx}\partial_x p + K_{xy}\partial_y p + K_{xz}\partial_z p\right) ds$$

Here, $\partial_y p$ and $\partial_z p$ cannot be estimated from p_i and p_k $\longrightarrow$ transverse flux $K_{xy}p_y$ and $K_{xz}p_z$ neglected $\longrightarrow$ inconsistent scheme



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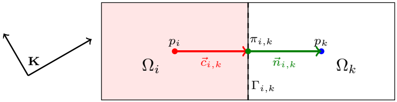

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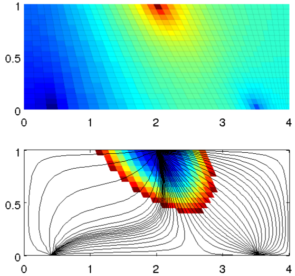
Many methods developed to amend this

- (mortar) mixed finite elements
- multipoint flux approximation (MFPA)
- mimetic finite difference
- vertex approximate gradient (VAG)
- *nonlinear TPFA*



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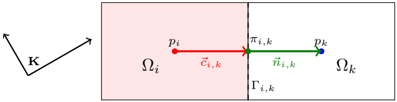

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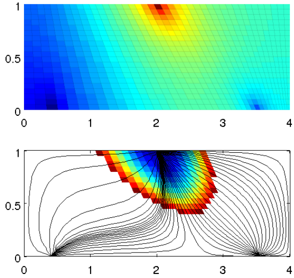
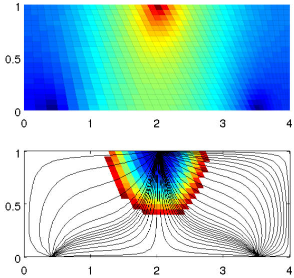
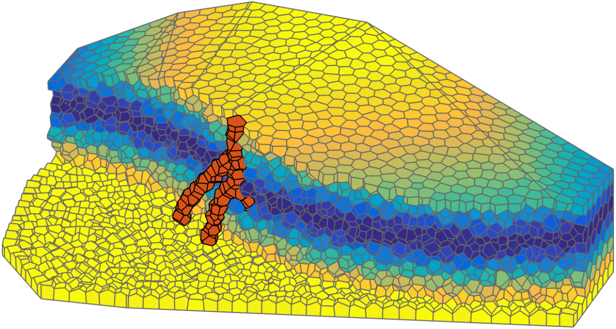




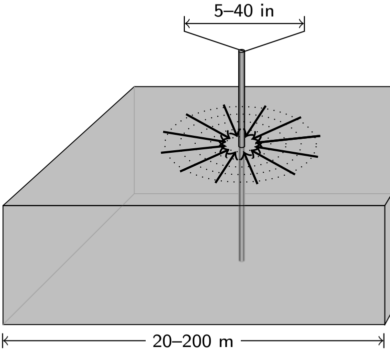
Example: comparison of consistent methods

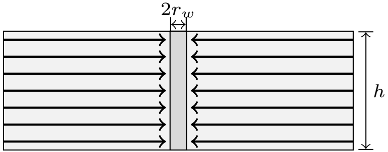
Example: 3D Voronoi grid adapting to branching well. Anisotropic and spatially varying permeability



Method	dof	nnz	ratio	cond
TPFA	9026	126002	13.96	9.64e+02
NTPFA	11920	280703	23.55	2.92e+07
MFD	60321	1538305	25.50	9.37e+15
VEM1	52350	3404977	65.04	4.91e+11
VEM2	227459	38770593	170.45	—

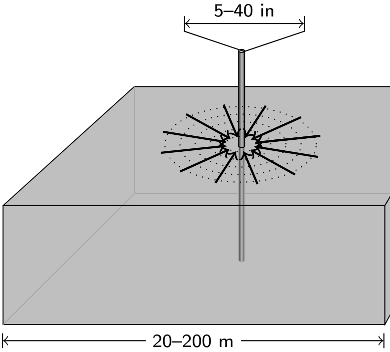
Wells: flow in and out of the reservoir

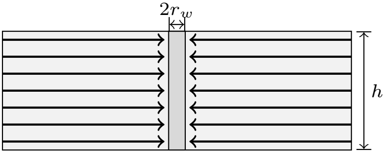




Inflow and outflow take place on a *subgrid* scale, with large variations in pressure over short distances.

Wells: flow in and out of the reservoir





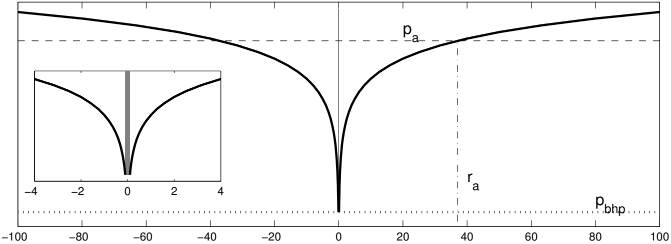
Inflow and outflow take place on a *subgrid* scale, with large variations in pressure over short distances.

Solution: use a linear inflow-performance relation

$$q = J\big(p_R - p_{bh}\big)$$

Here, p_{bh} is flowing pressure in wellbore and p_R average pressure in cell

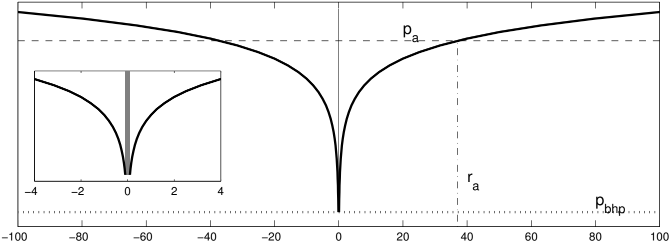
Wells: analytic subscale model



Pseudo-steady, radial flow. Mass conservation in cylinder coordinates

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} = 0 \qquad \longrightarrow \qquad u = C/r.$$

Wells: analytic subscale model



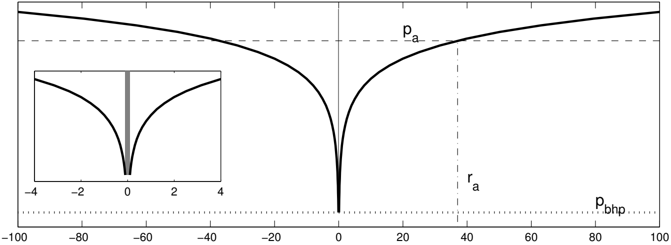
Pseudo-steady, radial flow. Mass conservation in cylinder coordinates

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} = 0 \qquad \longrightarrow \qquad u = C/r.$$

Integrating around a small cylinder surrounding the well,

$$q = \oint \vec{u} \cdot \vec{n}\, ds = -2\pi h C$$

Wells: analytic subscale model



Insert into Darcy's law and integrate from wellbore radius r_w to drainage radius r_d at which $p = p_d$ is constant:

$$u = -\frac{q}{2\pi r h} = -\frac{K}{\mu}\frac{dp}{dr} \quad \longrightarrow \quad 2\pi K h \int_{p_{bh}}^{p_d} \frac{dp}{q\mu} = \int_{r_w}^{r_d} \frac{dr}{r}$$

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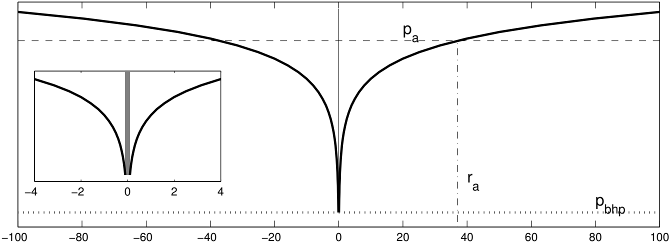

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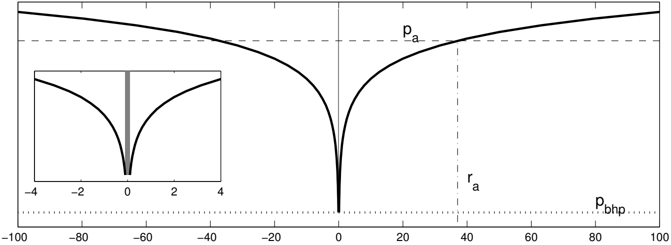
Solution

$$q = \frac{2\pi K h}{\mu \ln(r_d/r_w)} \big(p_d - p_{bh}\big)$$

Wells: analytic subscale model



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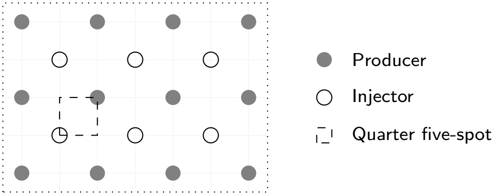
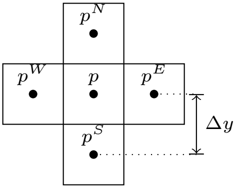
Solution (volumetric average pressure $p = p_a$ at $r_a = 0.472 r_d$)

$$q = \frac{2\pi K h}{\mu \ln(r_d/r_w)}\big(p_d - p_{bh}\big) = \frac{2\pi K h}{\mu\big(\ln(r_d/r_w) - 0.75\big)}\big(p_a - p_{bh}\big)$$

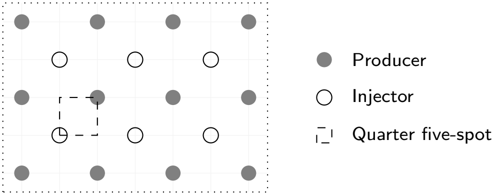
Wells: analytic subscale model

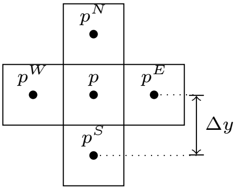




Repeated five-spot $\longrightarrow$ symmetric solution. Discretize Poisson's equation:

$$-\frac{Kh}{\mu}\left[4p - p^W - p^N - p^W - p^S\right] = q \qquad \longrightarrow \qquad p = p^E - \frac{q\mu}{4Kh}$$





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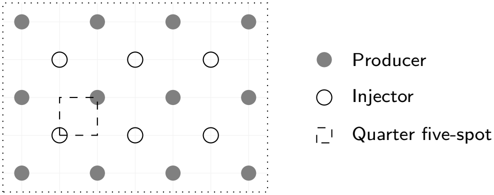
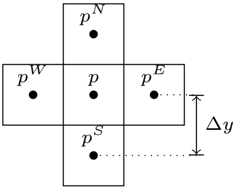
Analytic model valid in neighboring blocks

$$p = p_{bh} + \frac{q\mu}{2\pi Kh}\ln\left(\Delta x / r_w\right) - \frac{q\mu}{4Kh}$$

Wells: analytic subscale model





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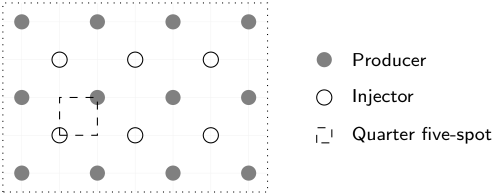

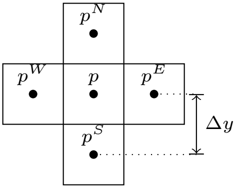

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Peaceman's formula:

$$q = \frac{2\pi Kh}{\mu \ln\left(r_e / r_w\right)}\left(p - p_{bh}\right), \qquad r_e = e^{-\frac{\pi}{2}}\sqrt{\Delta x \Delta y} \approx 0.20788\sqrt{\Delta x \Delta y}$$

Wells: many complications

There are several known extensions to Peaceman's well model:

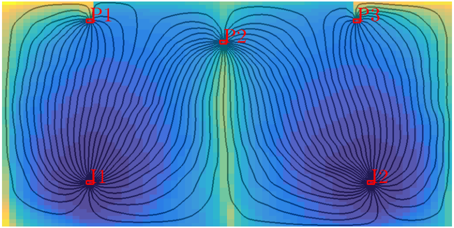
- Diagonal permeability tensor $K \to \sqrt{K_x K_y}$
- Rectangular grid cells (more complex formula for r_e)
- Horizontal wells, off-centered wells, multiple wells, ...
- Near-well effects (permeability increase/reduction)
- Other grid types and discretization schemes

Despite obvious limiting assumptions, Peaceman's model is used rather uncritically in industry. Need for more accurate/robust/versatile models...

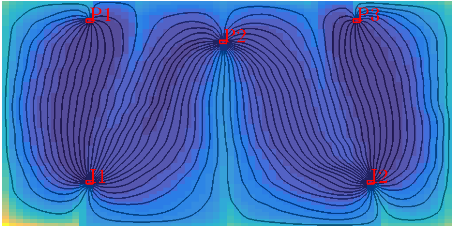
In general: need to describe flow within wellbore and annulus, downhole equipment, surface facilities, control strategies (choking, reinjection) involving complex logic, ...

What can you do with single-phase flow?

Run basic diagnostics of your model to establish basic timelines, volumetric connections, measures of dynamic heterogeneity, etc



Forward time of flight



Residence time

Can be computed by tracing streamline or by finite-volume methods solving steady transport equations $\vec{u} \cdot \nabla h = f(x)$

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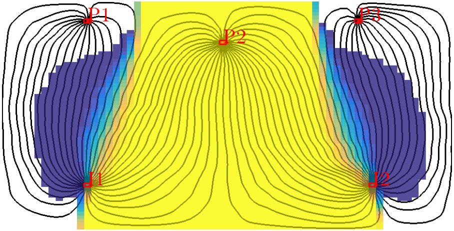
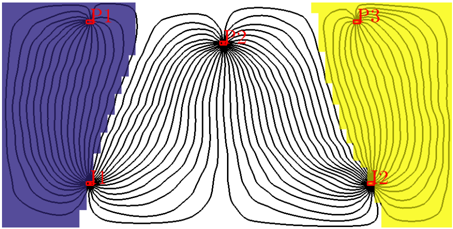


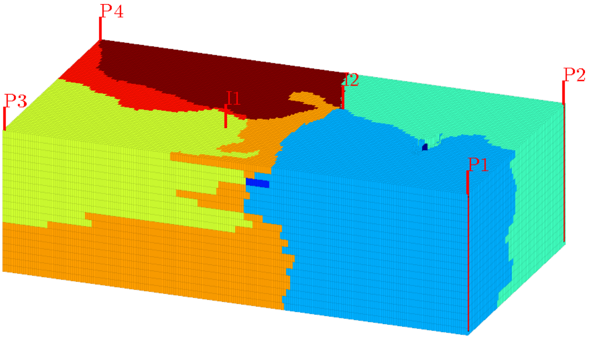
Influence regions

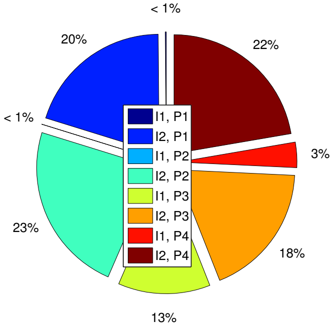


Well-pair regions

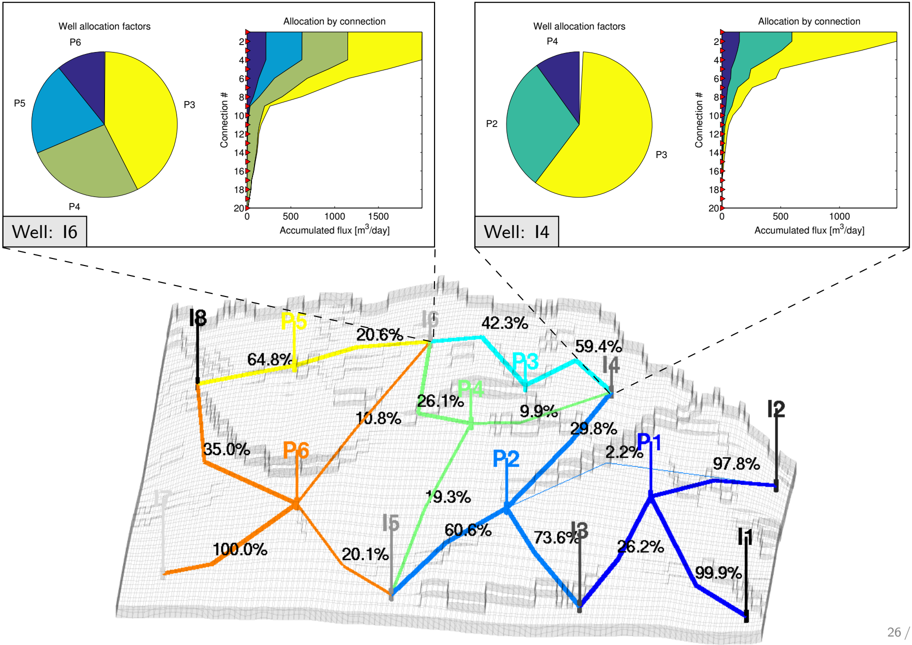
Can be computed by tracing streamline or by finite-volume methods solving steady transport equations $\vec{u} \cdot \nabla h = f(x)$

Flow diagnostics





Flow diagnostics



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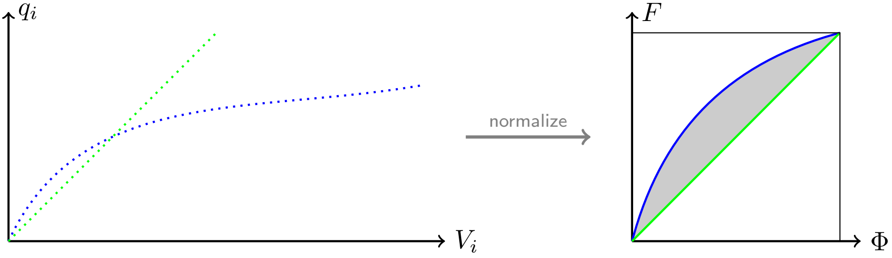
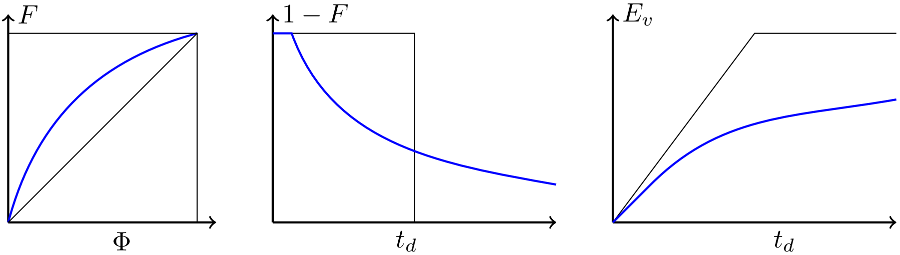


Flow diagnostics



F-Φ diagram

Fractional recovery

Sweep efficiency

Model reduction: flow-based upscaling

$$-\nabla \cdot (\mathbf{K}\nabla p) = f, \qquad \text{in } \Omega$$

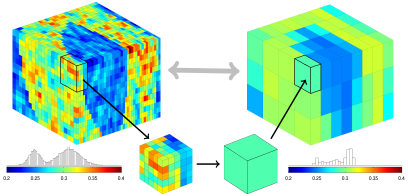
Subdivide grid into coarse blocks. For each block B, we seek a tensor $\mathbf{K}^*$ such that

$$\int_B \mathbf{K}\nabla p \, dx = \mathbf{K}^* \int_B \nabla p \, dx,$$

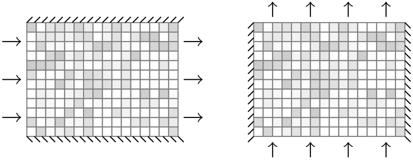
That is, we use Darcy's law on the coarse scale

$$\bar{u} = -\mathbf{K}^* \overline{\nabla p}$$

to relate the net flow rate $\bar{u}$ through B to the average pressure gradient $\overline{\nabla p}$ inside B.



Typical approach: flow-based upscaling



Many alternatives, few are sufficiently accurate and robust

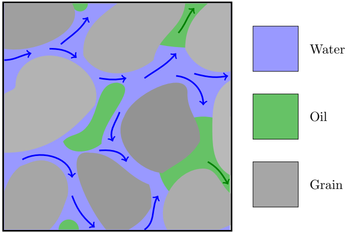
See talks by Y. Efendiev and H. Tchelepi

Multiphase flow

Hydrocarbon typically consists of different chemical species like methane, ethane, propane, etc. Common modelling practice to group fluid components into **phases**, i.e., a mixture of components having similar flow properties.

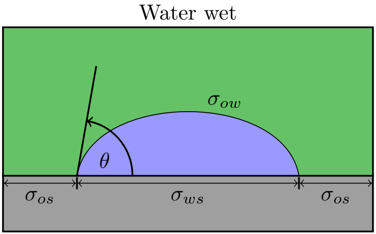
Most common phases:

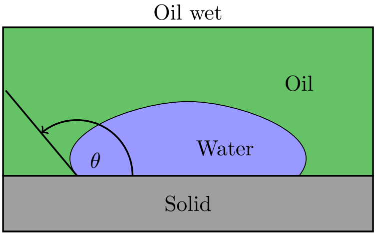
aqueous, liquid, and vapor



Fundamental physics: wettability

Immiscible phases separated by a infinitely thin surface having associated surface tension





Contact angle θ: determined by balance of adhesive and cohesive forces

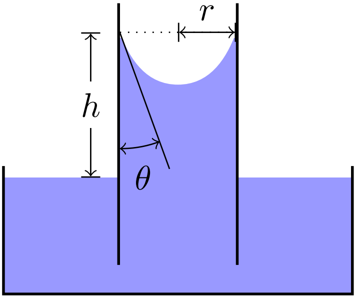
Young's equation (energy balance): $\sigma_{ow} \cos\theta = \sigma_{os} - \sigma_{ws}$

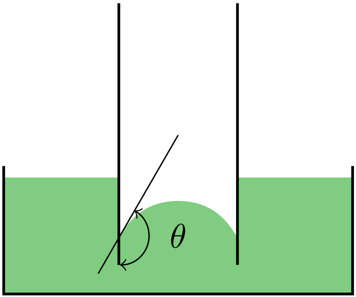
Water generally shows greater affinity than oil to stick to the rock surface $\longrightarrow$ reservoirs are predominantly water-wet systems

Fundamental physics: capillary pressure

Different equilibrium pressure in two phases separated by curved interface:

$$p_c = p_n - p_w = \underbrace{\frac{2\pi r\,\sigma\cos\theta}{\pi r^2}}_{\text{upward force}} = \frac{2\sigma\cos\theta}{r} = \underbrace{\frac{\pi r^2\, gh(\rho_l - \rho_a)}{\pi r^2}}_{\text{downward force}} = \Delta\rho g h$$

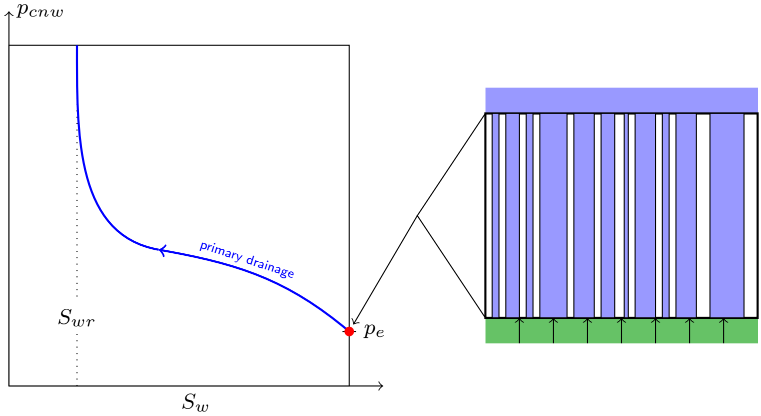




Fundamental physics: drainage (primary migration)

Saturation: fraction of pore volume filled by a given fluid phase

Drainage: non-wetting fluid displacing wetting fluid, controlled by widest non-invaded pore throat



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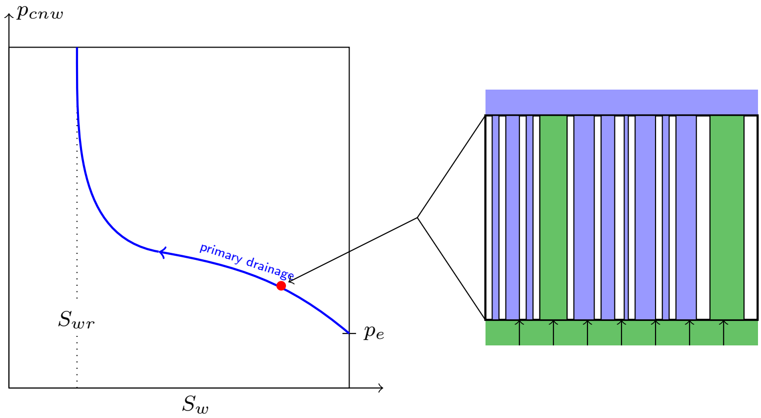

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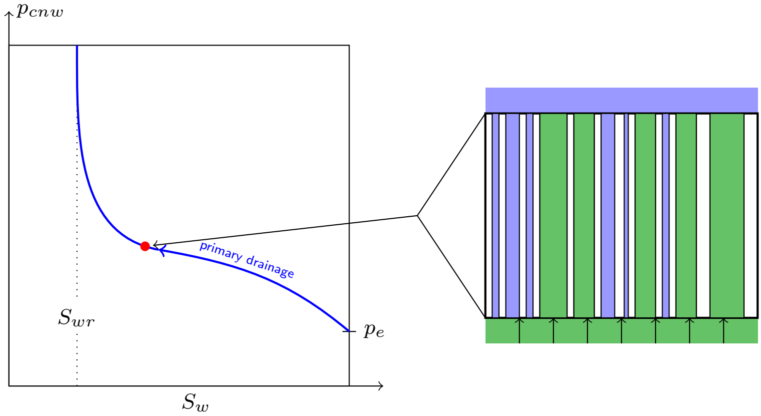

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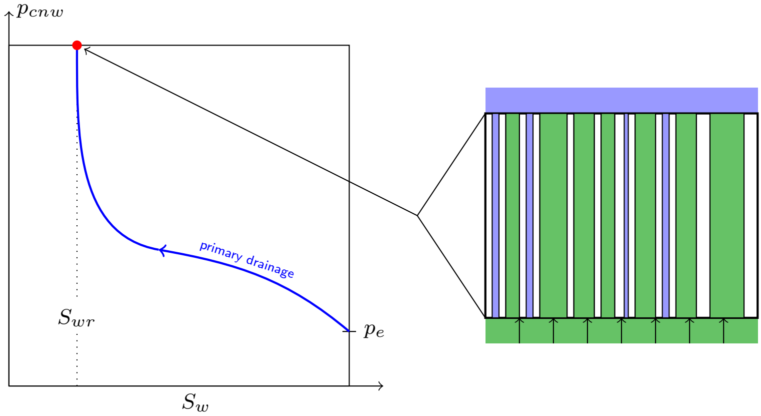
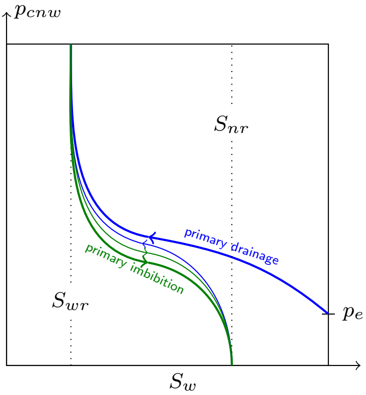
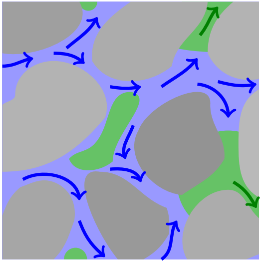


Fundamental physics: imbibition (hydrocarbon recovery)

Imbibition: wetting fluid displaces non-wetting fluid, controlled by the size of the *narrowest* non-invaded pore.

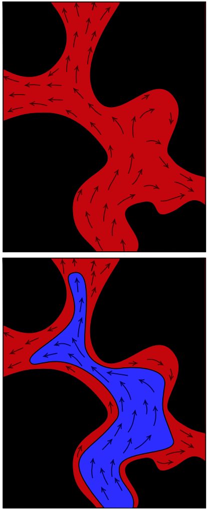
Will not follow the same capillary curve $\longrightarrow$ **hysteresis** (cause: trapped oil droplets, different wetting angle for advancing and receding interfaces)





EOR: inject substances to alter wetting properties to mobilize immobile oil, $S_{or} \to 1$

Extensions of model equations to multiphase flow



Three-phase Darcy velocities (Muskat, 1936):

$$\vec{u}_\alpha = -\frac{\mathbf{K}_\alpha(S_\alpha)}{\mu_\alpha}\left(\nabla p_\alpha - \rho_\alpha g \nabla z\right)$$

Assuming each phase consists of only one component, the mass-balance equations for each phase read (Muskat, 1945):

$$\frac{\partial(\phi \rho_\alpha S_\alpha)}{\partial t} + \nabla \cdot \left(\rho_\alpha \vec{u}_\alpha\right) = q_\alpha$$

Macro-scale capillarity concept (Leverett, 1941):

$$p_c(S_w) = J\sqrt{\frac{\phi}{K}}\sigma\cos\theta$$

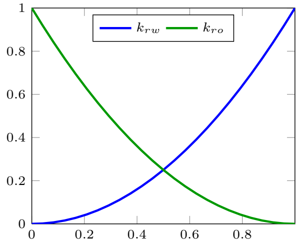
Relative permeability

Effective permeability experienced by one phase is reduced by the presence of other phases. Relative permeabilities

$$k_{r\alpha} = k_{r\alpha}(S_{\alpha_1}, \ldots, S_{\alpha_m}),$$

are nonlinear functions that attempt to account for this effect. Notice that

$$\sum_{\alpha} k_{r\alpha} < 1$$



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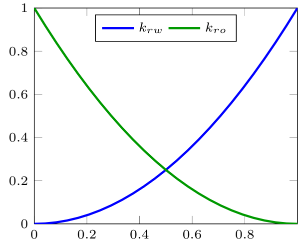
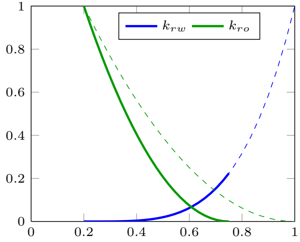
This gives Darcy's law on the form

$$\begin{aligned}\vec{u}_\alpha &= -\frac{\mathbf{K} k_{r\alpha}}{\mu_\alpha}\big(\nabla p_\alpha - \rho_\alpha g \nabla z\big) \\ &= -\mathbf{K}\lambda_\alpha\big(\nabla p_\alpha - \rho_\alpha g \nabla z\big)\end{aligned}$$





General flow equations for two-phase flow

Gathering the equations, we have

$$\frac{\partial(\phi\rho_\alpha S_\alpha)}{\partial t} + \nabla \cdot \big(\rho_\alpha \vec{u}_\alpha\big) = q_\alpha, \qquad \alpha = \{w, n\}$$

$$\vec{u}_\alpha = -\frac{\mathbf{K} k_{r\alpha}}{\mu_\alpha}\big(\nabla p_\alpha - \rho_\alpha g \nabla z\big)$$

$$p_c = p_n - p_w, \qquad S_w + S_n = 1$$

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Commercial reservoir simulators: insert functional relationships $p_c = P_c(S_w)$ and ρ_α and ϕ as function of p_α, and discretize with backward Euler in time and the two-point scheme in space

In academia: common practice to rewrite the equations to better reveal their mathematical nature

Fractional flow formulation

Choose S_w and p_n as primary unknowns, consider incompressible flow (i.e., ρ is constant and can be divided out)

$$\phi \frac{\partial S_\alpha}{\partial t} + \nabla \cdot \vec{u}_\alpha = q_\alpha .$$

Fractional flow formulation

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Sum mass-conservation equations:

$$\phi \frac{\partial}{\partial t}(S_n + S_w) + \nabla \cdot (\vec{u}_n + \vec{u}_w) = q_n + q_w$$

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Sum Darcy equations

$$\vec{u} = \vec{u}_n + \vec{u}_w = -(\boldsymbol{\lambda}_n + \boldsymbol{\lambda}_w)\nabla p_n + \boldsymbol{\lambda}_w \nabla p_c + (\boldsymbol{\lambda}_n \rho_n + \boldsymbol{\lambda}_w \rho_w) g \nabla z$$

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Inserted into $\nabla \cdot \vec{u} = q$ gives pressure equation

$$-\nabla \cdot (\lambda \mathbf{K} \nabla p_n) = q - \nabla \big[\boldsymbol{\lambda}_w \nabla p_c + (\boldsymbol{\lambda}_n \rho_n + \boldsymbol{\lambda}_w \rho_w) g \nabla z \big]$$

Fractional flow formulation

Choose S_w and p_n as primary unknowns, consider incompressible flow (i.e., ρ is constant and can be divided out)

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Sum Darcy equations

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$$\underbrace{-\nabla \cdot (\lambda \mathbf{K} \nabla p_n) = q}_{\text{Poisson}} - \underbrace{\nabla \big[\boldsymbol{\lambda}_w \nabla p_c + (\boldsymbol{\lambda}_n \rho_n + \boldsymbol{\lambda}_w \rho_w) g \nabla z\big]}_{\text{only function of } S_w}$$

Fractional flow formulation

Multiply phase velocity by mobility of other phase and subtract

$$\lambda_n \vec{u}_w - \lambda_w \vec{u}_n = \lambda_w \lambda_n \mathbf{K} \left[\nabla p_c + (\rho_w - \rho_n) g \nabla z \right]$$

Fractional flow formulation

Multiply phase velocity by mobility of other phase and subtract

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Solve for $\vec{u}_w$ and insert into conservation equation

$$\phi \frac{\partial S_w}{\partial t} + \nabla \cdot \big[f_w \big(\vec{u} + \boldsymbol{\lambda}_n \Delta\rho g \nabla z \big) \big] = q_w - \nabla \cdot \big(f_w \boldsymbol{\lambda}_n P_c' \nabla S_w \big)$$

Fractional flow formulation

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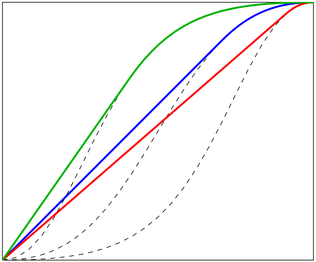
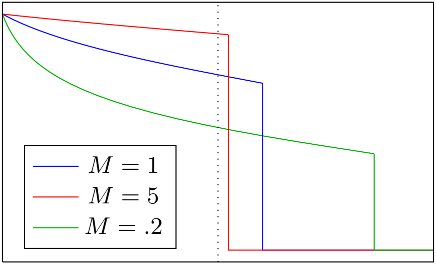
Setting $P_c \equiv 0$ and $g \equiv 0$ for simplicity

$$-\nabla\big(\mathbf{K}\lambda(S)\nabla p\big) = q, \qquad \vec{u} = -\mathbf{K}\lambda(S)\nabla p,$$
$$\phi \partial_t S + \nabla \cdot (\vec{u} f(S)) = 0$$

System of one **elliptic** pressure equation and one **hyperbolic** saturation equation. Typically: solved sequentially with specialized methods

Buckley–Leverett solution for 1D displacement

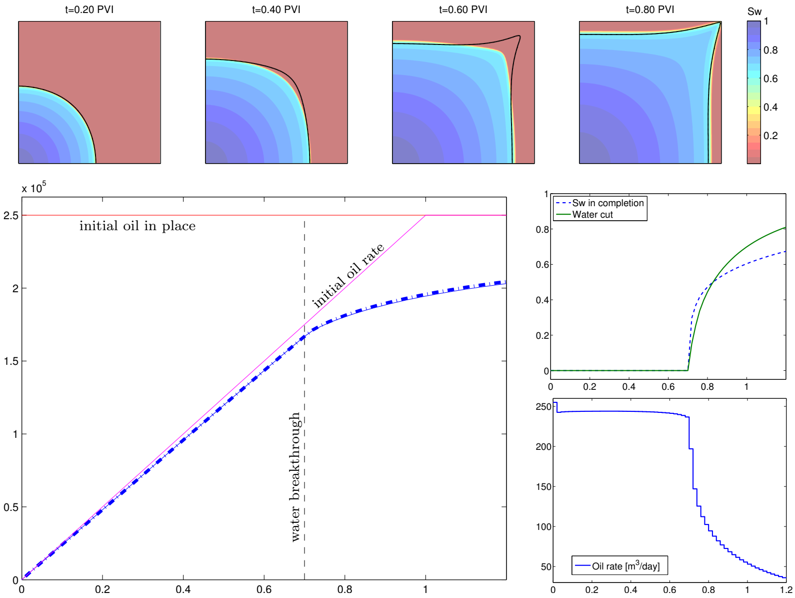




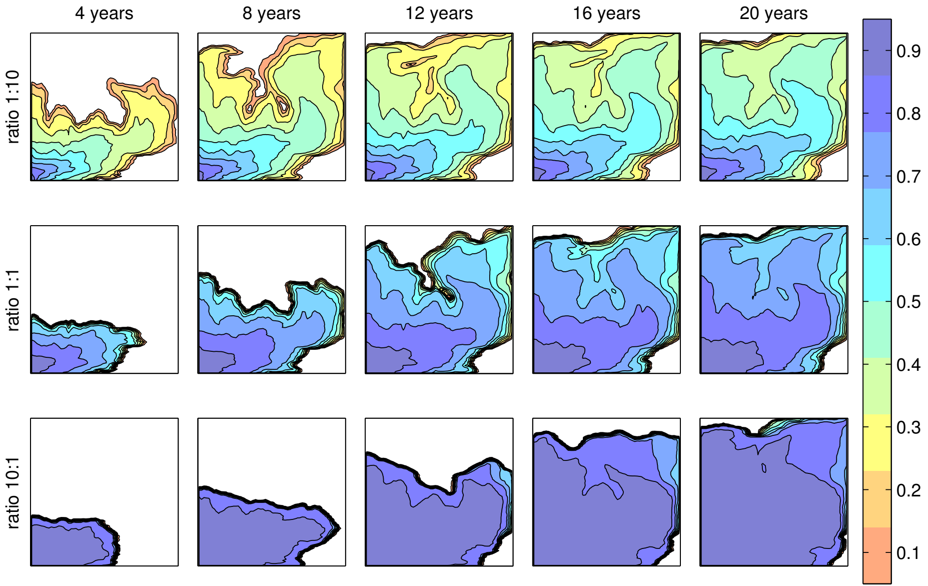
$$S_t + f(S)_x = q, \qquad f(S) = \frac{S^2}{S^2 + M(1-S)^2}, \quad M = \mu_w/\mu_n$$

Here, $M = .2$ gives poor local displacement efficiency, $M = 5$ gives very good

Simulation examples: quarter-five spot



Simulation examples: quarter-five spot



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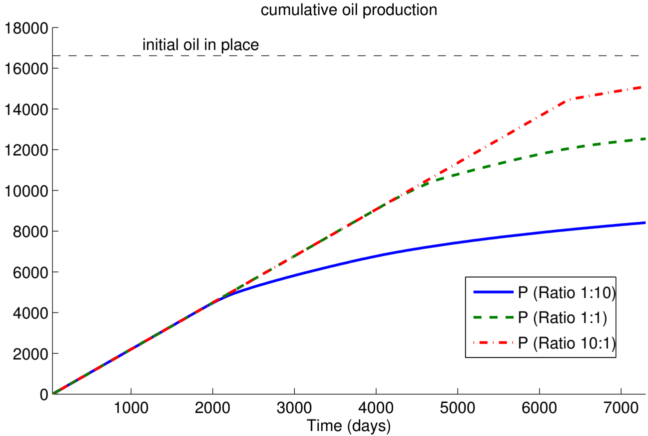

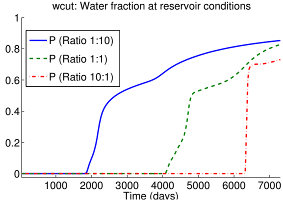

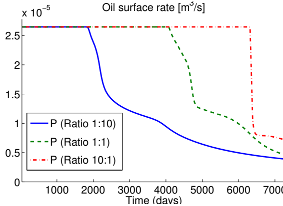

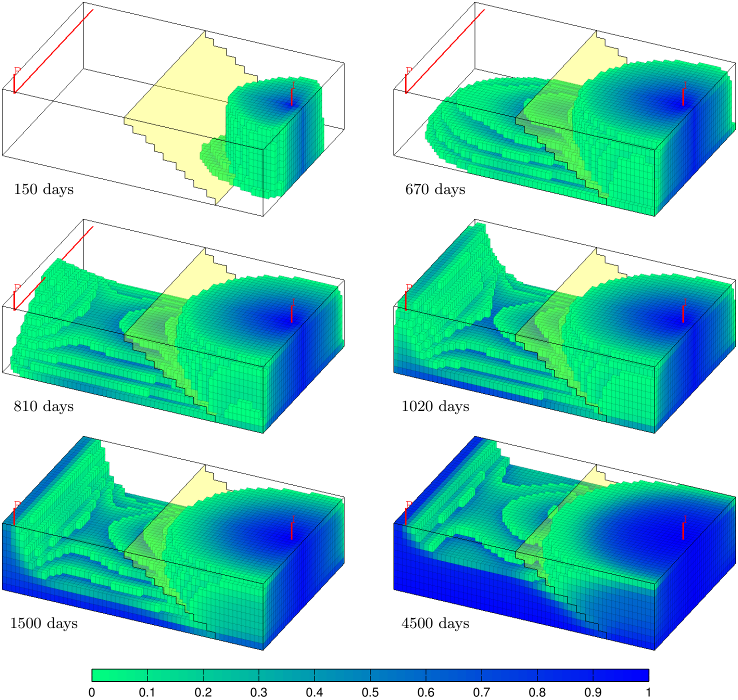
150 days
670 days
810 days
1020 days
1500 days
4500 days
0
0.1
0.2
0.3
0.4
0.5
0.6
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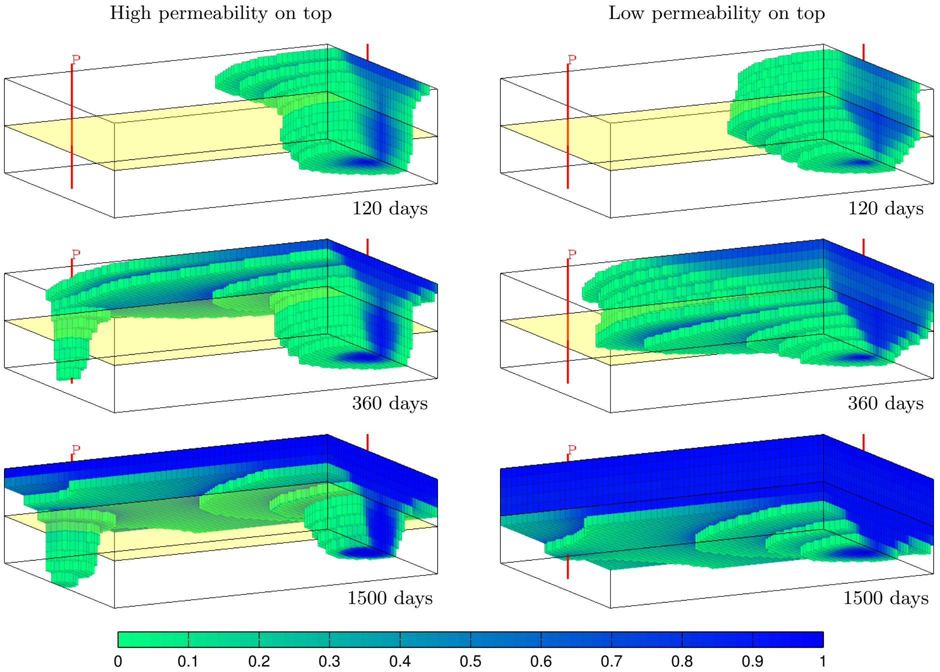
0.8
0.9
1


High permeability on top
Low permeability on top
P
120 days
360 days
1500 days
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Multicomponent flows

System with N phases and M components

Notation: c_α^ℓ mass fraction of component ℓ in phase α

Multicomponent flows

System with N phases and M components

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Equations: conservation for phases or components?

Multicomponent flows

System with N phases and M components

Notation: c_α^ℓ mass fraction of component ℓ in phase α

Equations: conservation for phases or components?

Choose components to avoid source terms for mass transfer

$$\frac{\partial}{\partial t}\Big(\phi \sum_\alpha c_\alpha^\ell \rho_\alpha S_\alpha\Big) + \nabla \cdot \Big(\sum_\alpha c_\alpha^\ell \rho_\alpha \vec{u}_\alpha + \vec{J}_\alpha^\ell\Big) = \sum_\alpha c_\alpha^\ell \rho_\alpha q_\alpha,$$

Here, J is diffusion, e.g., Fickian

$$\vec{J}_\alpha^\ell = -\rho_\alpha S_\alpha \mathbf{D}_\alpha^\ell \nabla c_\alpha^\ell,$$

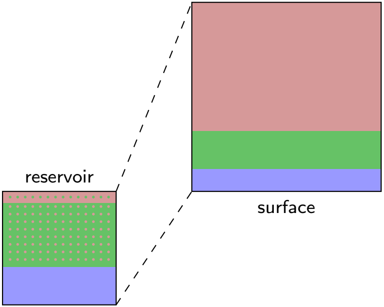
More in talk by K. Jessen

Black-oil equations

Hydrocarbon components lumped together to a light 'gas' and a heavier 'oil' pseudocomponent at surface conditions

	W	O	G
A	X		
L		X	X
V		X	X

Simple PVT: formation-volume factors, $B_\alpha = V_\alpha / V_\alpha^s = \rho_\alpha^s / \rho_\alpha$ or $b_\alpha = 1/B_\alpha$



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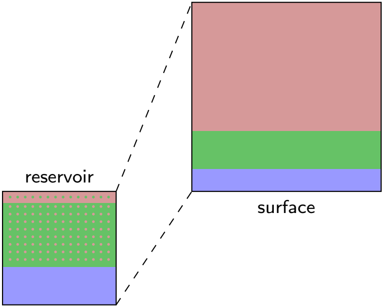


Simple PVT: formation-volume factors,
$B_\alpha = V_\alpha / V_\alpha^s = \rho_\alpha^s / \rho_\alpha$ or $b_\alpha = 1/B_\alpha$

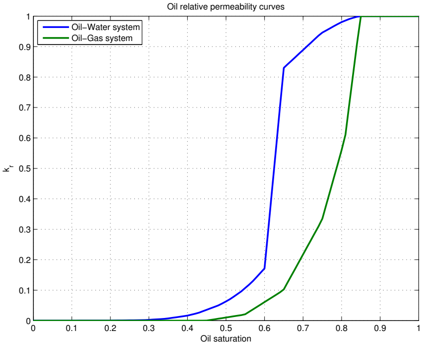
Conservation equations:

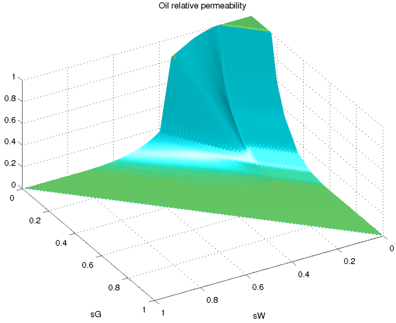
$$\partial_t \big(\phi b_o S_o\big) + \nabla \cdot \big(b_o \vec{u}_o\big) = b_o q_o$$
$$\partial_t \big(\phi b_w S_w\big) + \nabla \cdot \big(b_w \vec{u}_w\big) = b_w q_w$$
$$\partial_t \big[\phi \big(b_g S_g + b_o r_{so} S_o\big)\big] + \nabla \cdot \big(b_g \vec{u}_g + b_o r_{so} \vec{u}_o\big) = b_g q_g + b_o r_{so} q_o$$

Dissolved gas in oil: $r_{so} = V_g^s / V_o^s$. Similarly: oil vaporized in gas r_{sg}

Example: fluid model from SPE9

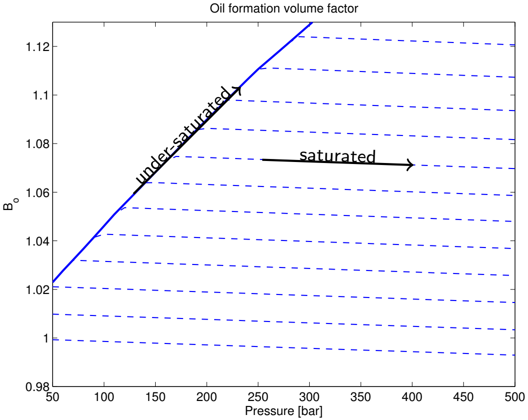
Relative permeability for oil

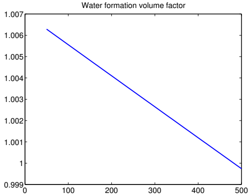


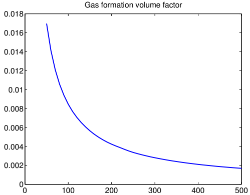


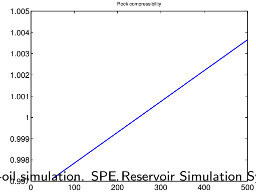
J. E. Killough (1995). Ninth SPE comparative solution project: A reexamination of black-oil simulation. SPE Reservoir Simulation Symposium

Formation-volume factors (inverse densities):









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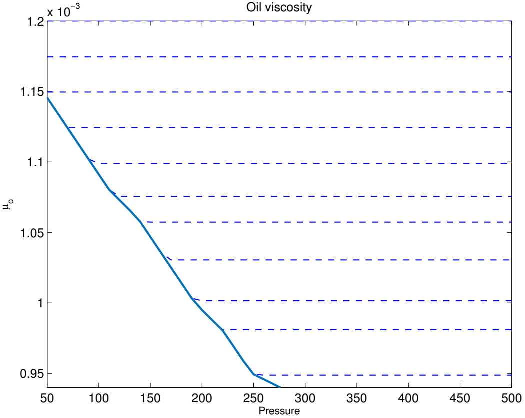
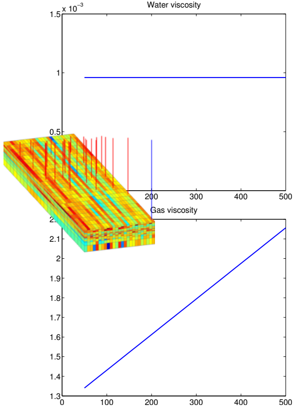
Example: fluid model from SPE9

Viscosities:

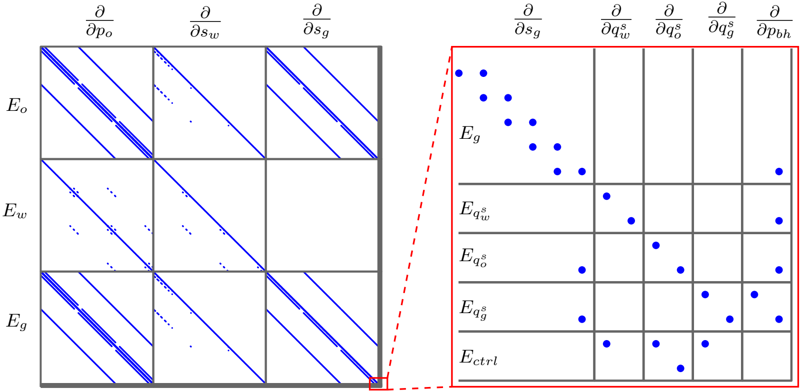




Black-oil: discretization and linearization

Discretization: backward Euler in time, two-point flux-approximation with upstream mobility in space.

Newton's method for nonlinear equation: $\dfrac{\partial E}{\partial x^i}\delta x^i = E(x^i)$



Black-oil: solution strategies

Solution procedure

1. Eliminate well variables q_o^s, q_w^s, q_g^s, and p_{bh}
2. Set first block-row equal to sum of block-rows, leave out rows that may harm diagonal dominance in block (1,1)
3. Set up two-stage preconditioner:
   - $\mathbf{M}_1^{-1}$: solves pressure subsystem
   - $\mathbf{M}_2^{-1}$: ILU0 decomposition of the full system
4. Solve full system with GMRES using preconditioner $\mathbf{M}_2^{-1}\mathbf{M}_1^{-1}$
5. Recover remaining variables

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For larger models, pressure subsystem should be solved with algebraic multigrid

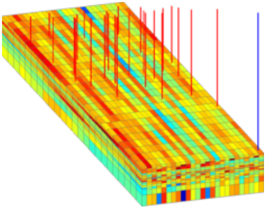
Time-step control

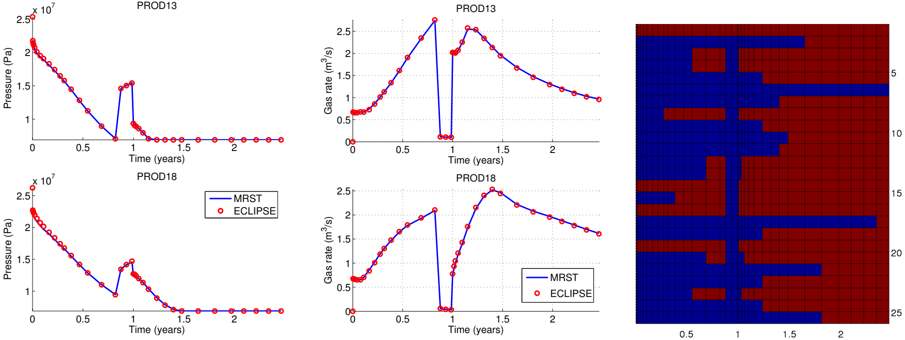
- chop if too large changes in variables
- chop if convergence failure
- more advanced logic to maintain targeted iteration count

Elaborate logic for well control and surface facilities

Example: SPE 9 benchmark

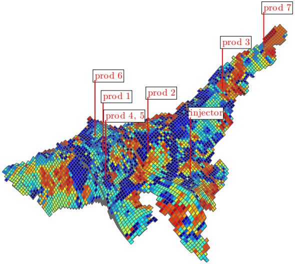
- Grid with 9000 cells
- 1 water injector, rate controlled, switches to bhp
- 25 producers, oil-rate controlled, most switch to bhp
- Appearance of free gas due to pressure drop
- Production rates lowered to 1/15 between days 300 and 360

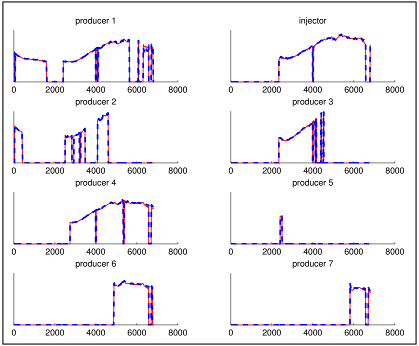




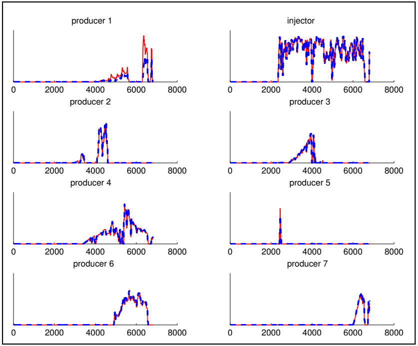
Example: the Voador field

- South wing of the reservoir (Petrobras)
- Gradients obtained through adjoint simulations
- Validate: open-source / commercial simulator:
  - 20 years of historic data
  - virtually identical results
  - main challenge: needed to reverse-engineer description of wells. . .





bottom-hole pressure

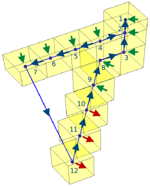


water rate

Multisegment wells

More accurate modelling:

- Network models
- Represent annulus
- Flow inside wellbore
- (Autonomous) inflow control devices
- Artificial lift, etc

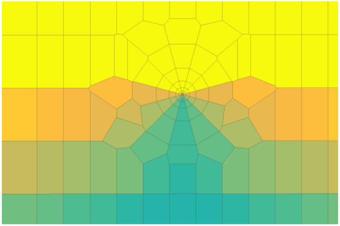


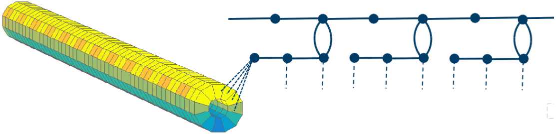
In nodes:

- pressure p
- mass fractions x_w^m, x_o^m, x_g^m

In segments:

- mass rates v^m





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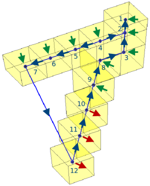


In nodes:

- pressure p
- mass fractions x_w^m, x_o^m, x_g^m

In segments:

- mass rates v^m

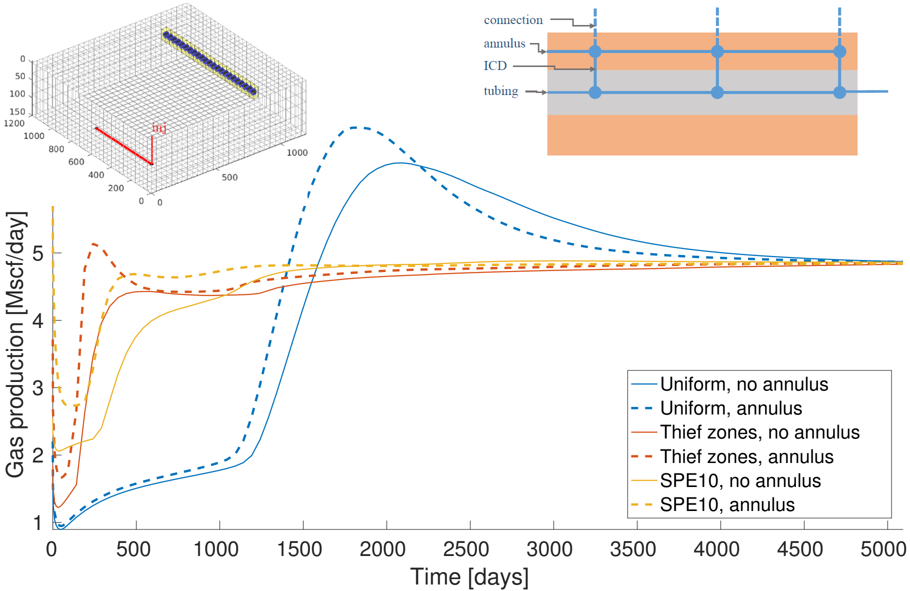
Discrete mass conservation in nodes:

$$\underbrace{\frac{V}{\Delta t}\left(x_c\rho - x_c^0\rho\right)}_{\text{accumulation}} + \underbrace{\operatorname{div}\left(v_c^m\right)}_{\text{in- and outflux to neighboring nodes}} - \underbrace{q_c^m}_{\text{source term (well control, connections)}} = 0$$

Discrete pressure drop equations in segments:

$$\operatorname{grad}(p) - \underbrace{g\operatorname{avg}(\rho)\operatorname{grad}(z)}_{\text{gravity term}} - \underbrace{h\big(v^m, \operatorname{uw}(\rho), \operatorname{uw}(\mu)\big)}_{\text{heuristic pressure drop term}} = 0$$

Example: effect of modeling annulus



Enhanced Oil Recovery: polymer flooding

Simple model: introduce extra immiscible component and mixture law

Polymer transported in water:

$$\vec{u}_w = -\frac{k_{rw}(S)}{\mu_{w,\mathsf{eff}}(c)\,R_k(c)}\mathbf{K}\big(\nabla p_w - \rho_w g \nabla z\big)$$

$$\vec{u}_p = -\frac{k_{rw}(S)}{\mu_{p,\mathsf{eff}}(c) R_k(c)}\mathbf{K}\big(\nabla p_w - \rho_w g \nabla z\big)$$

Conservation of polymer component:

$$\partial_t\big[\phi(1-S_{ipv})cb_w S + \rho_r\, C^a(1-\phi)\big] + \nabla\cdot\big(cb_w\vec{u}_p\big) = q_p$$

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viscosity enhancement

Tod

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Todd–Longstaff mixing:

$$\frac{1}{\mu_{w,\text{eff}}} = \frac{1 - c/c_m}{\mu_m(c)^{\omega} \mu_w^{1-\omega}} + \frac{c/c_m}{\mu_m(c)^{\omega} \mu_p^{1-\omega}}$$

nt

permeability reduction

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nt

ce

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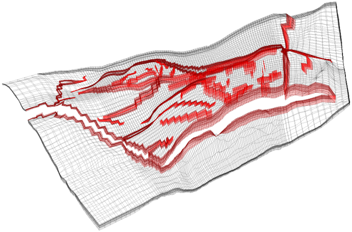
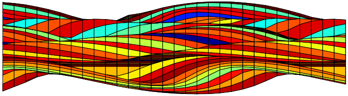
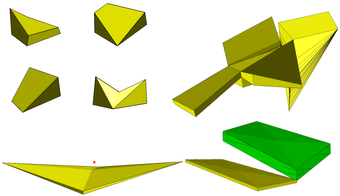
permeability reduction

adsorption

Summary





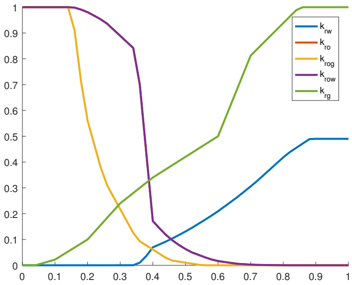


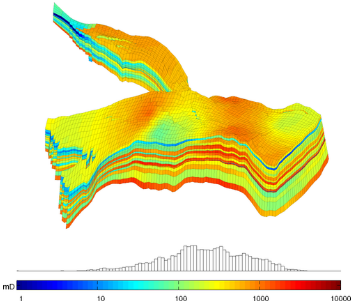
- Geological models: complex unstructured grids having many obscure challenges
- Flow models: system of highly nonlinear parabolic PDEs with elliptic and hyperbolic sub-character
- Well models: subscale models, complex logic, strong impact on flow
- Validation and availability in software

Challenges:

- Main point of grid: describe stratigraphy and structural architecture, i.e., not chosen freely to maximize accuracy of numerical discretization
- Industry standard: corner-point / stratigraphic grids
- Grid topology is generally unstructured, with non-neighboring connections
- Geometry: deviates (strongly) from box shape, high aspect ratios, many faces/neighbors, small faces, ...
- Potential inconsistencies since faces are bilinear or tetrahedral surfaces

Summary





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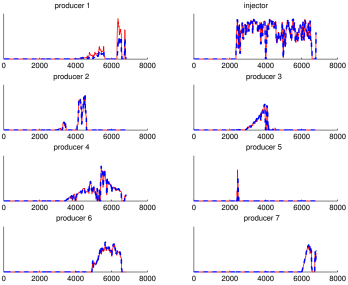
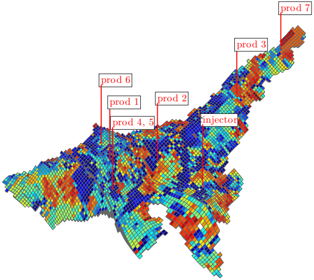
Challenges:

- Delicate balances: viscous forces, gravity, capillary, ...
- Strong coupling between 'elliptic' and 'hyperbolic' variables (small scale: capillary, large scale: gravity)
- Large variation in time constants and coupling
- Orders-of-magnitude variations in permeability
- Parameters with discontinuous derivatives
- Path-dependence: hysteretic parameters
- Sensitive to subtle changes in interpolation of tabulated physical data
- Monotonicity and mass conservation

Summary





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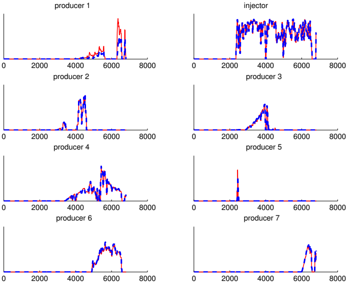
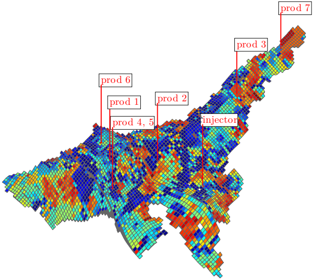
Challenges:

- Near singular radial flow in near-well zone (much larger flow than inside reservoir)
- Induce nonlocal connections
- Completely different multiphase flow inside wellbore
- Coupling to surface facilities
- Abrupt changes in driving forces
- Control strategies with intricate logic which is highly sensitive to state values

⋮

Summary





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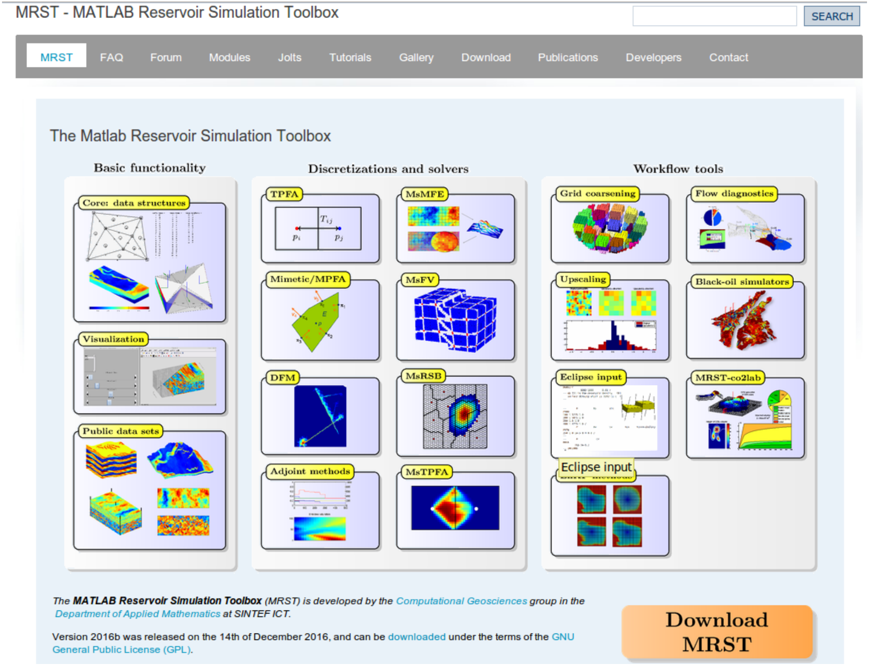
Challenges:

- New methods tend to be immature and too simplified
- Researchers: incompressible flow and explicit methods. Industry: implicit methods for compressible flow
- Industry relies on a few software providers and has strong faith in software with (undocumented) safeguards and algorithmic choices
- Oil companies seldom give away data
- Realistic models involve a large number of intricate details (Eclipse has 2–3000 keywords. . .)



http://www.sintef.no/MRST

MRST - MATLAB Reservoir Simulation Toolbox

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Originally:

- developed to support research on multiscale methods and mimetic discretizations
- first public release as open source, April 2009

Today:

- general toolbox for rapid prototyping and verification of new computational methods
- wide range of applications
- two releases per year
- each release has from 900 (R2013a) to 2100 (R2015a) unique downloads

Users:

- academic institutions, oil and service companies
- large user base in USA, Norway, China, Brazil, United Kingdom, Iran, Germany, Netherlands, France, Canada, . . .

Publications:

- used in 24 PhD theses and 59 master theses
- used in more than 100 scientific papers by people outside of SINTEF

The MATLAB Reservoir Simulation Toolbox (MRST) is developed by the Computational Geosciences group in the Department of Applied Mathematics at SINTEF ICT.

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