IPAM Long Program
Computational Issues in Oil Field Applications - Tutorials
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Model-based optimization of oil and gas production

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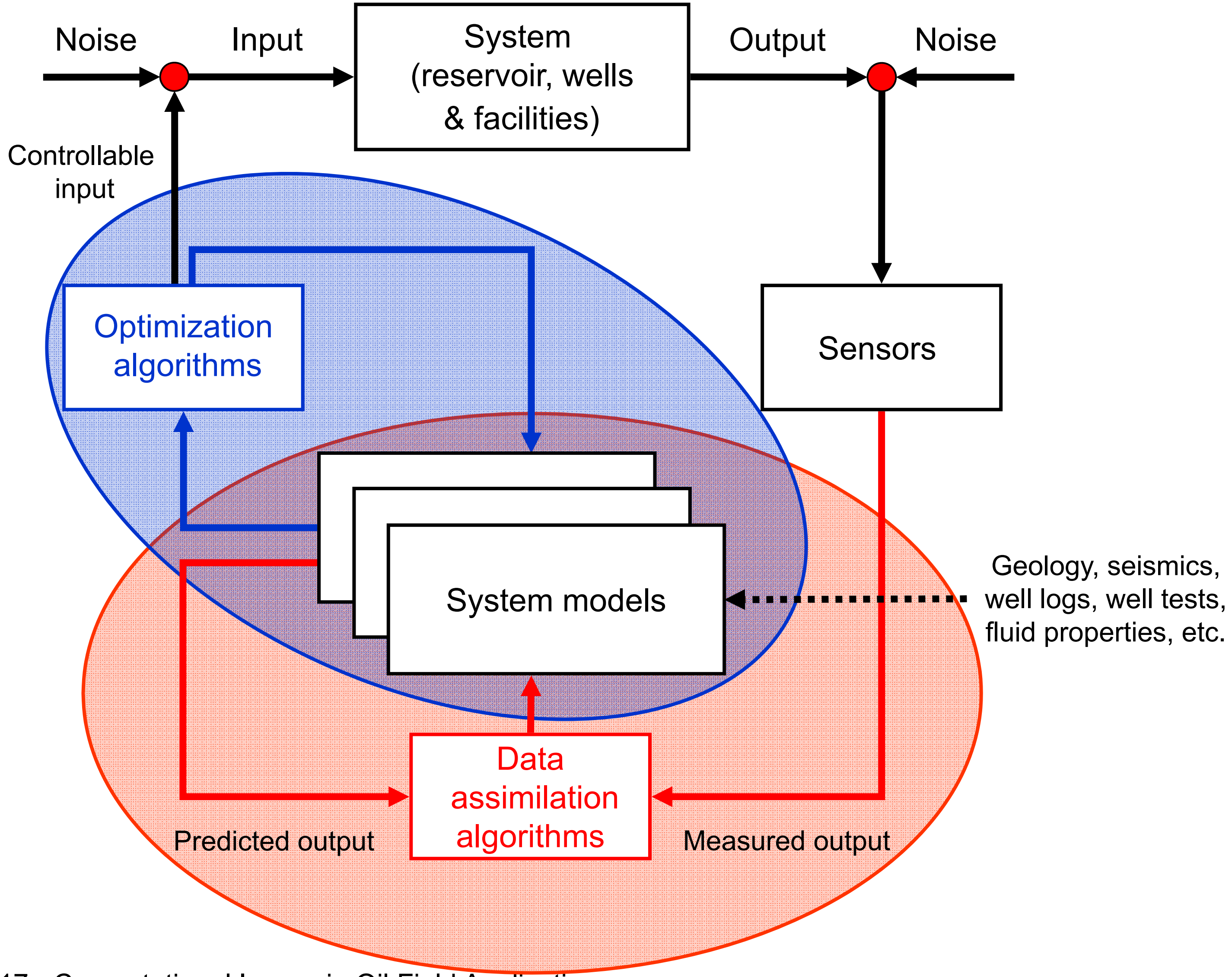
Closed-loop reservoir management



Notation: time-discretized equations

System eqs:	$\mathbf{g}_k\left(\mathbf{u}_k, \mathbf{x}_{k-1}, \mathbf{x}_k, \mathbf{m}\right) = \mathbf{0}$	
States:	$\mathbf{x} = \left[\mathbf{p}^T \quad \mathbf{s}^T\right]^T$	pressures, saturations
Parameters:	$\mathbf{m} = \left[\mathbf{k}^T \quad \boldsymbol{\varphi}^T\right]^T$	permeabilities, porosities
Inputs:	$\mathbf{u} = \left[\breve{\mathbf{p}}_{well}^T \quad \breve{\mathbf{q}}_{well}^T\right]^T$	well pressures/rates
Initial conditions:	$\mathbf{x}_0 = \breve{\mathbf{x}}_0$	
Time interval:	$k = 1, 2, \ldots, K$	

Production optimization: objective function

- Simple Net Present Value (NPV)
- N_{inj} injectors, N_{prod} producers

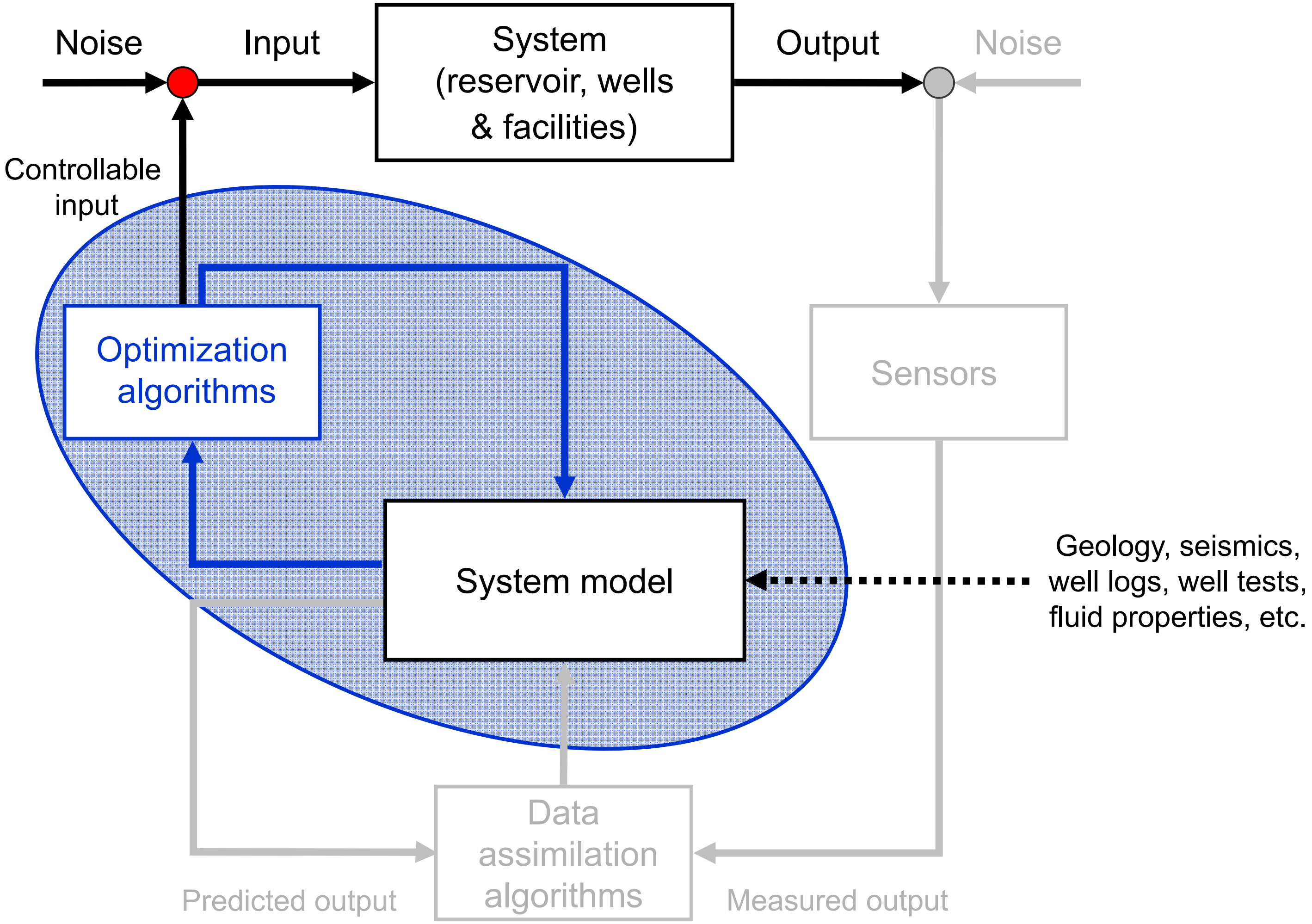
$$\mathcal{J} = \sum_{k=1}^{K} \left\{ \frac{\sum_{j=1}^{N_{prod}} \left[r_o \times \left(q_{o,j}\right)_k - r_{wp} \times \left(q_{wp,j}\right)_k \right] - \sum_{i=1}^{N_{inj}} r_{wi} \times \left(q_{wi,i}\right)_k}{\left(1+b\right)^{\frac{t_k}{\tau}}} \right\} \Delta t_k$$

- r = unit price or cost, b = discount factor, τ = 365 days
- Flow rates q_k functions of inputs u_k or outputs (states) x_k

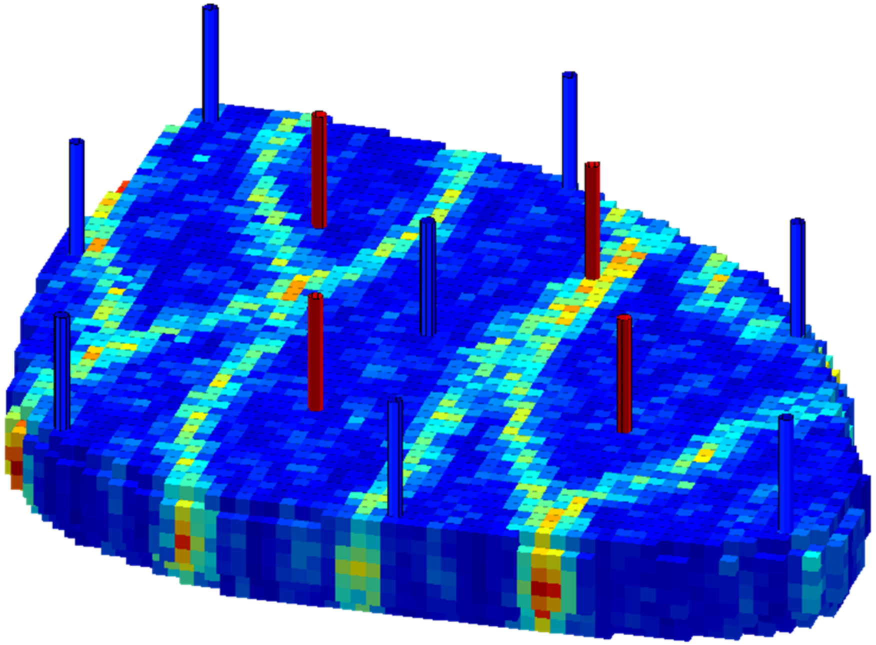
Production optimization: maximization problem

- Problem statement: $\max_{\mathbf{u}_{1:K}} \mathcal{J}\left(\mathbf{u}_{1:K}\right)$ subject to
- System equations: $\mathbf{g}_k\left(\mathbf{u}_k, \mathbf{x}_{k-1}, \mathbf{x}_k\right) = \mathbf{0}$
- Initial conditions: $\mathbf{x}_0 = \breve{\mathbf{x}}_0$
- Equality constraints: $\mathbf{c}_k\left(\mathbf{u}_k, \mathbf{x}_k\right) = \mathbf{0}$
- Inequality constraints: $\mathbf{d}_k\left(\mathbf{u}_k, \mathbf{x}_k\right) < \mathbf{0}$

1) “Open-loop” flooding optimization



12-well example (1)

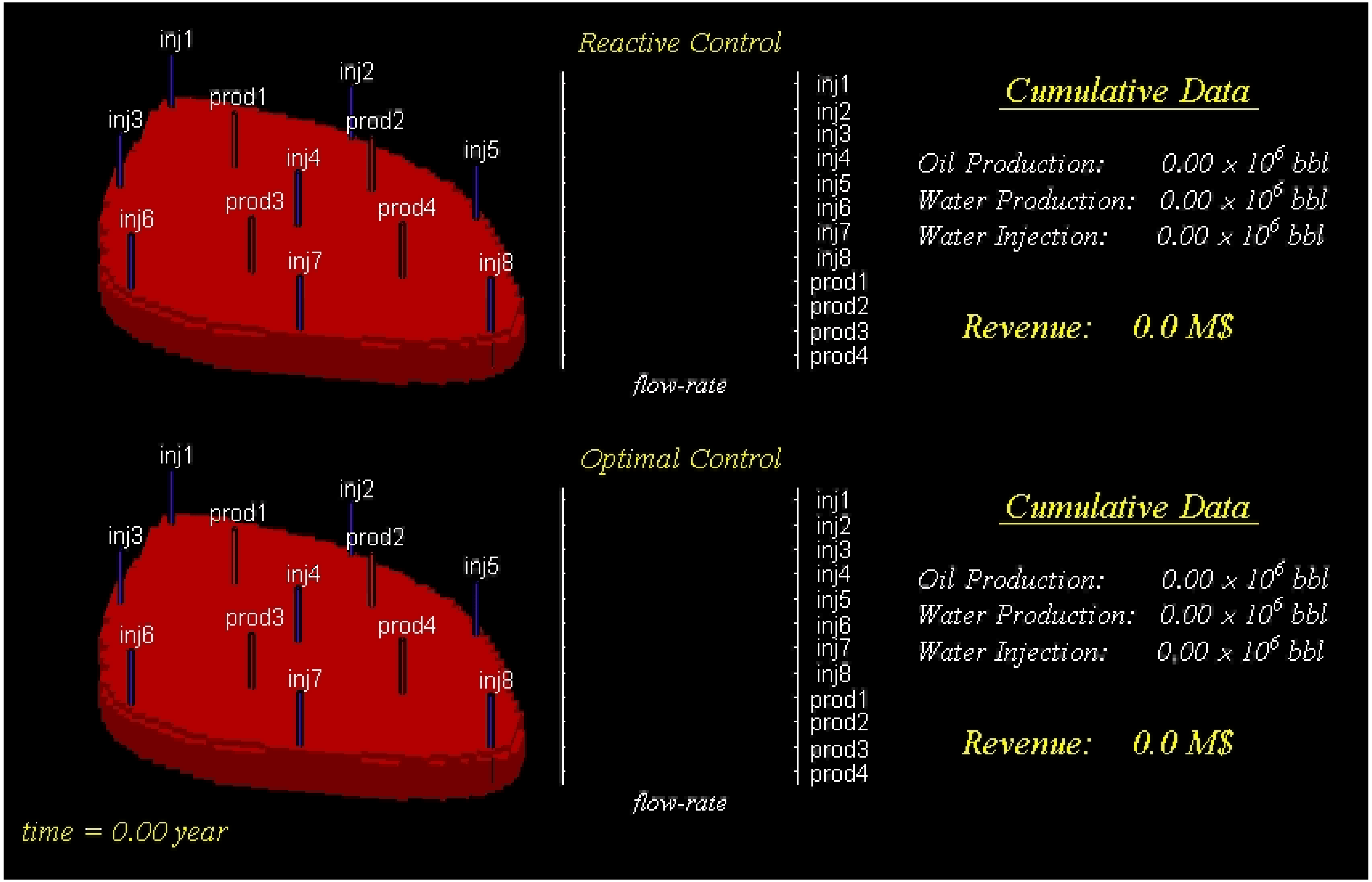


Van Essen et al., 2006

- 3D reservoir
- High-permeability channels
- 8 injectors, rate-controlled
- 4 producers, BHP-controlled
- Production period of 10 years
- 12 wells x 10 x 12 time steps

  => 1440 optimization parameters
- Bound constraints on controls
- Optimization of monetary value (oil revenues minus water costs)

12-well example (2)



Reactive Control

Cumulative Data

Oil Production: 0.00×10^6 bbl
Water Production: 0.00×10^6 bbl
Water Injection: 0.00×10^6 bbl

Revenue: 0.0 M$

Optimal Control

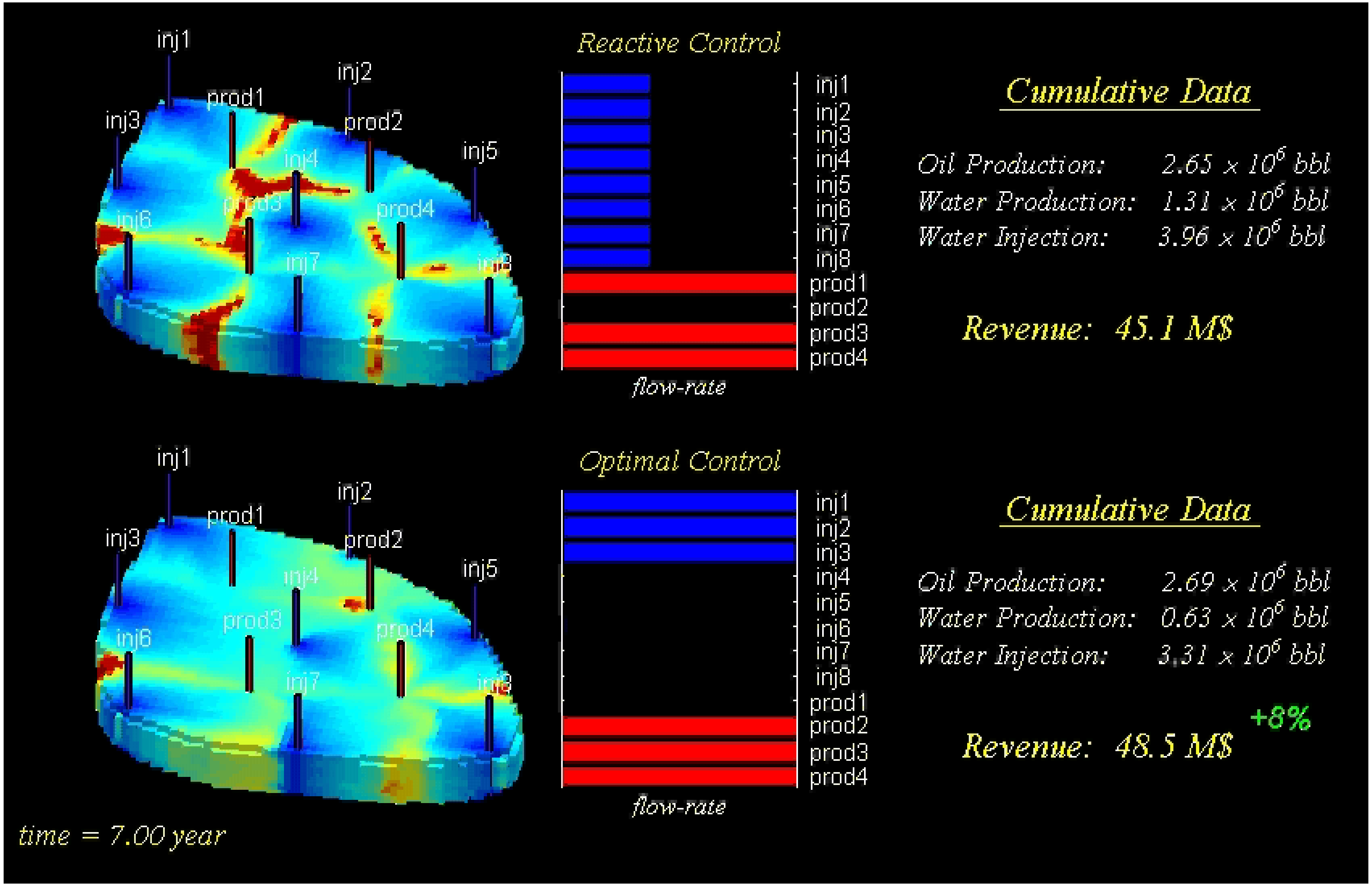
Cumulative Data

Oil Production: 0.00×10^6 bbl
Water Production: 0.00×10^6 bbl
Water Injection: 0.00×10^6 bbl

Revenue: 0.0 M$

time = 0.00 year

12-well example (3)



Reactive Control

Cumulative Data

Oil Production: 2.65×10^6 bbl
Water Production: 1.31×10^6 bbl
Water Injection: 3.96×10^6 bbl

Revenue: 45.1 M$

Optimal Control

Cumulative Data

Oil Production: 2.69×10^6 bbl
Water Production: 0.63×10^6 bbl
Water Injection: 3.31×10^6 bbl

Revenue: 48.5 M$ +8%

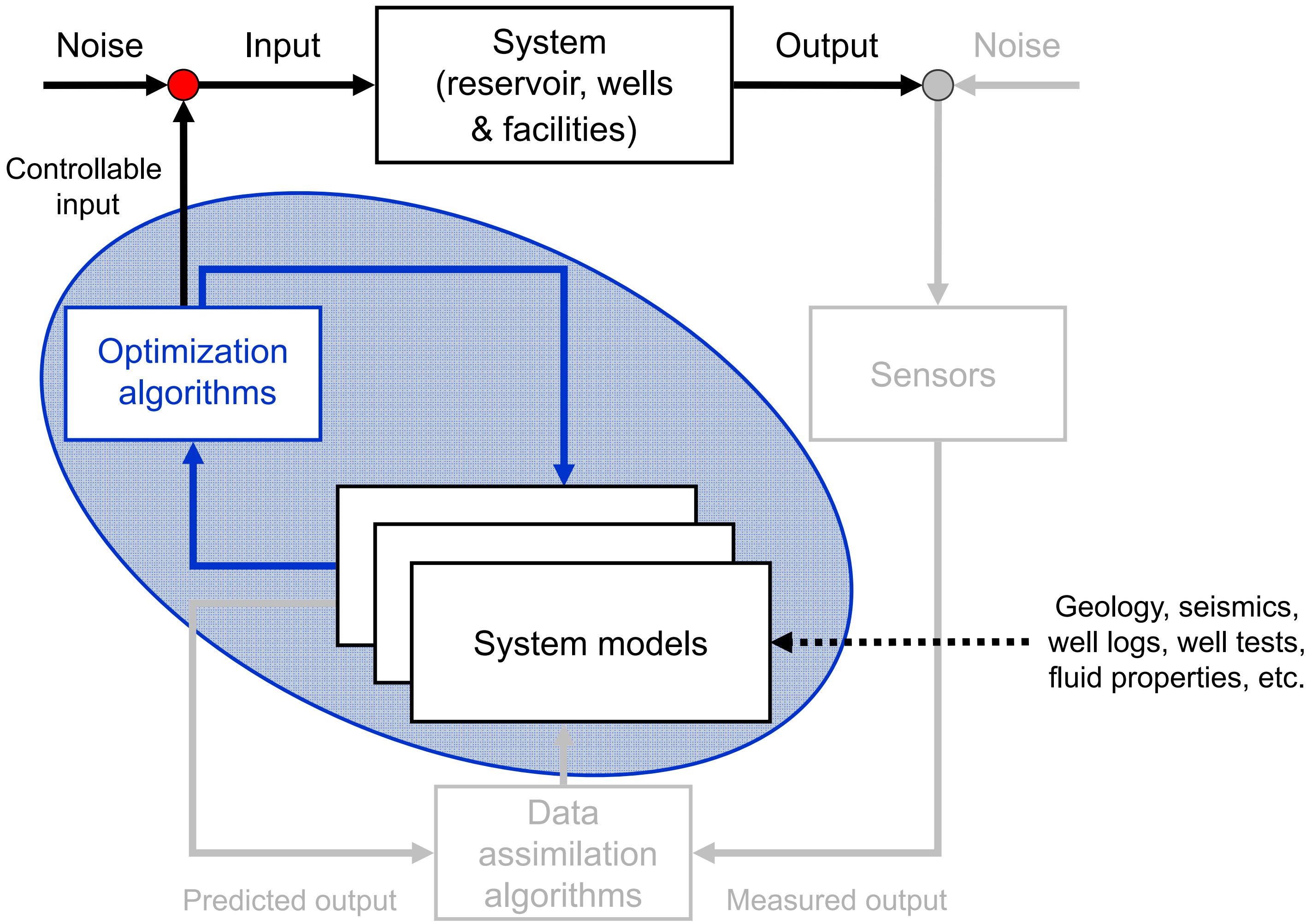
time = 7.00 year

Why this wouldn’t work

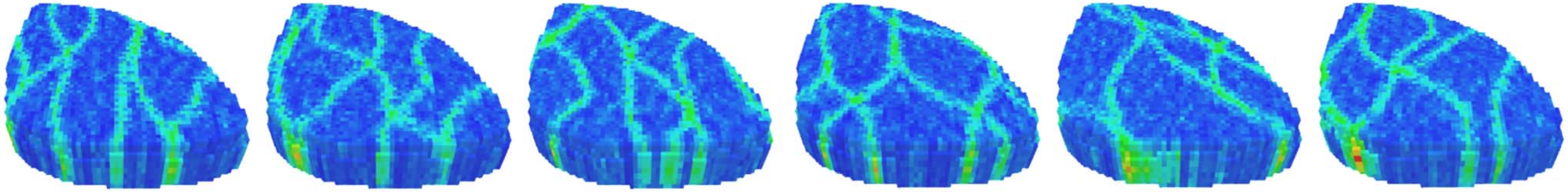
- Real wells are sparse and far apart
- Real wells have more complicated constraints
- Field management is usually production-focused
- Long-term optimization may jeopardize short-term profit
- Production engineers don’t trust reservoir models anyway

- We do not know the reservoir!

2) “Robust” open-loop flooding optimization



Robust optimization example



Van Essen et al., 2006

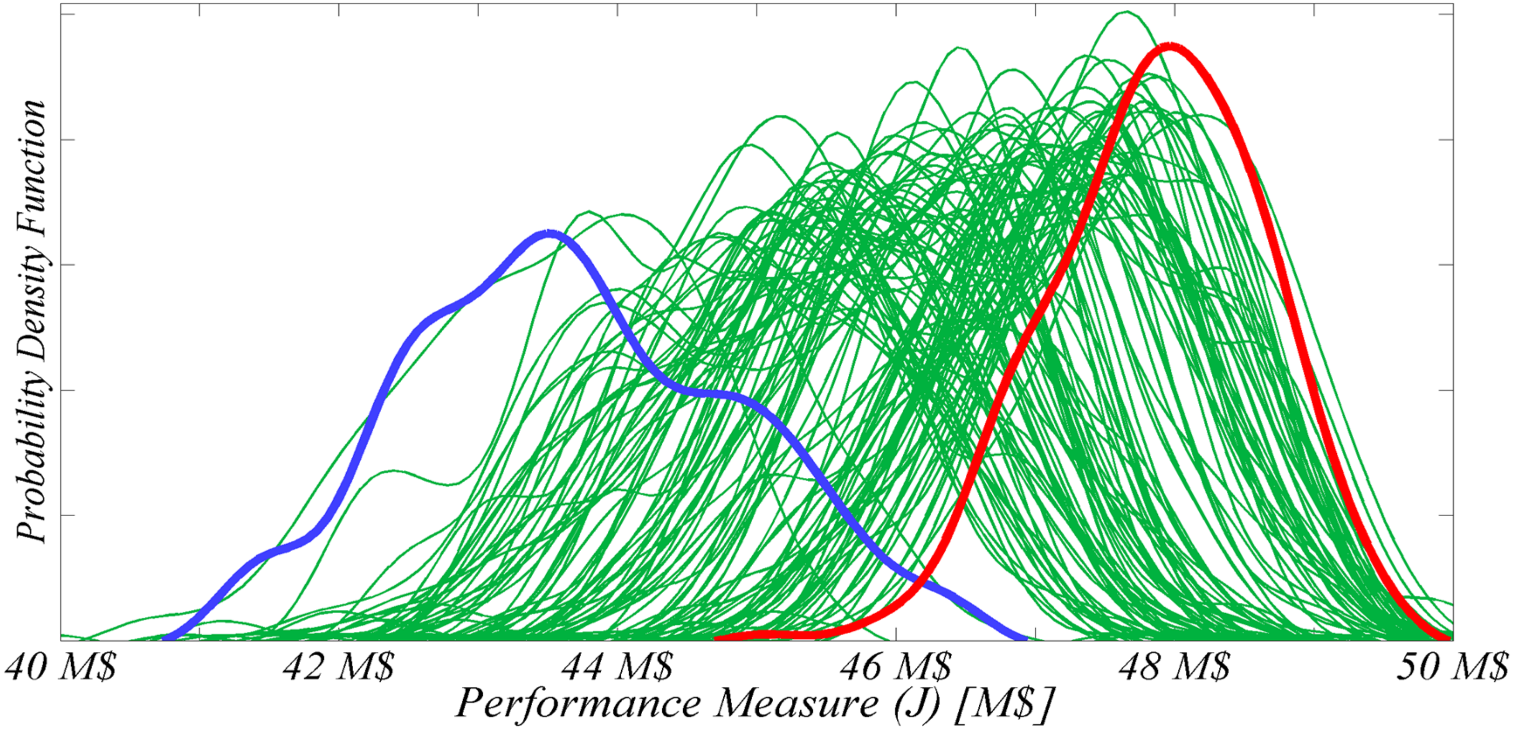
- 100 realizations
- Optimize expectation of objective function

$$\max_{\mathbf{u}_{1:K}} \frac{1}{N_r} \sum_{i=1}^{N_r} \mathcal{J}^i \left(\mathbf{u}_{1:K}, \mathbf{m}_i \right)$$

Robust optimization results

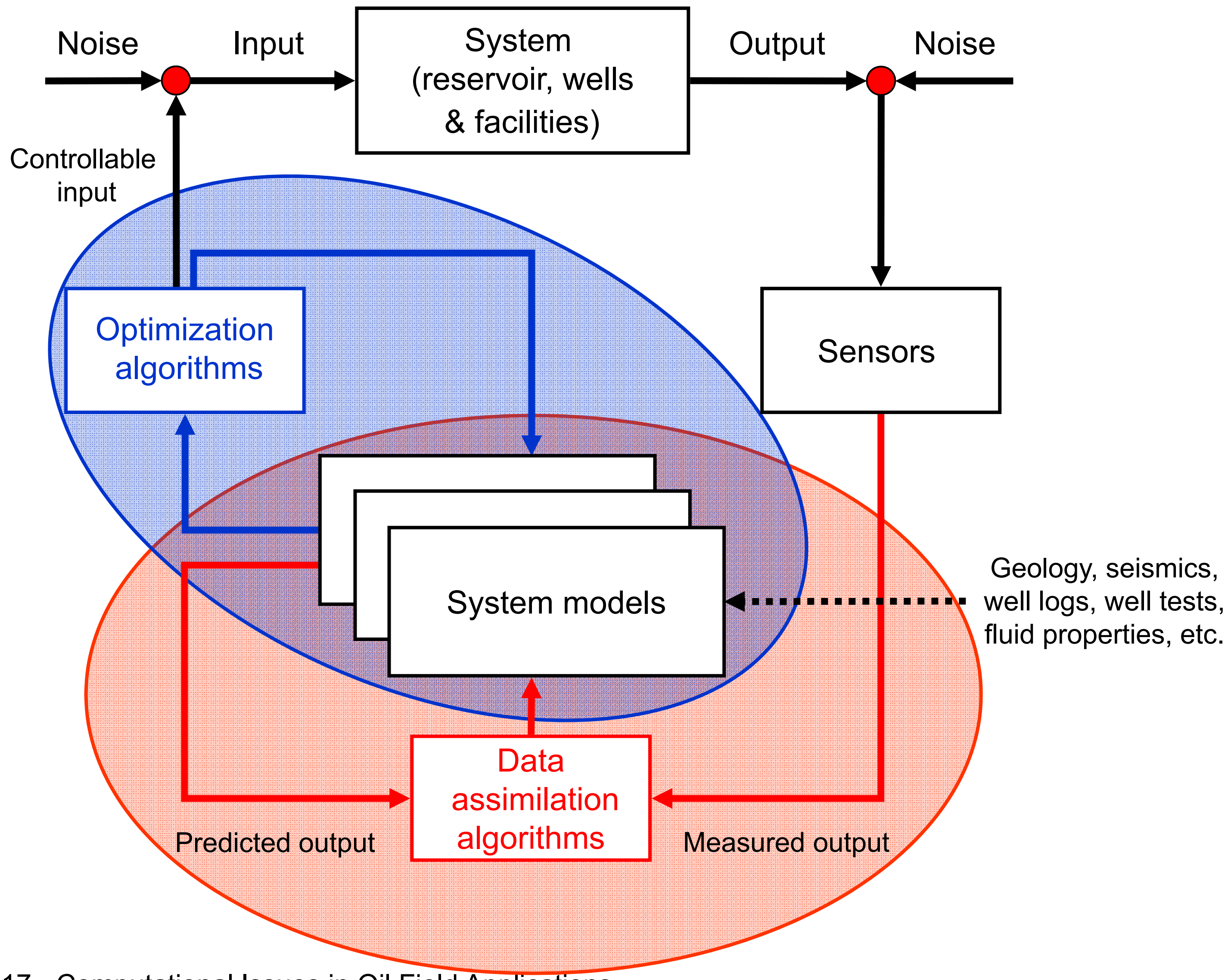
3 control strategies applied to set of 100 realizations:

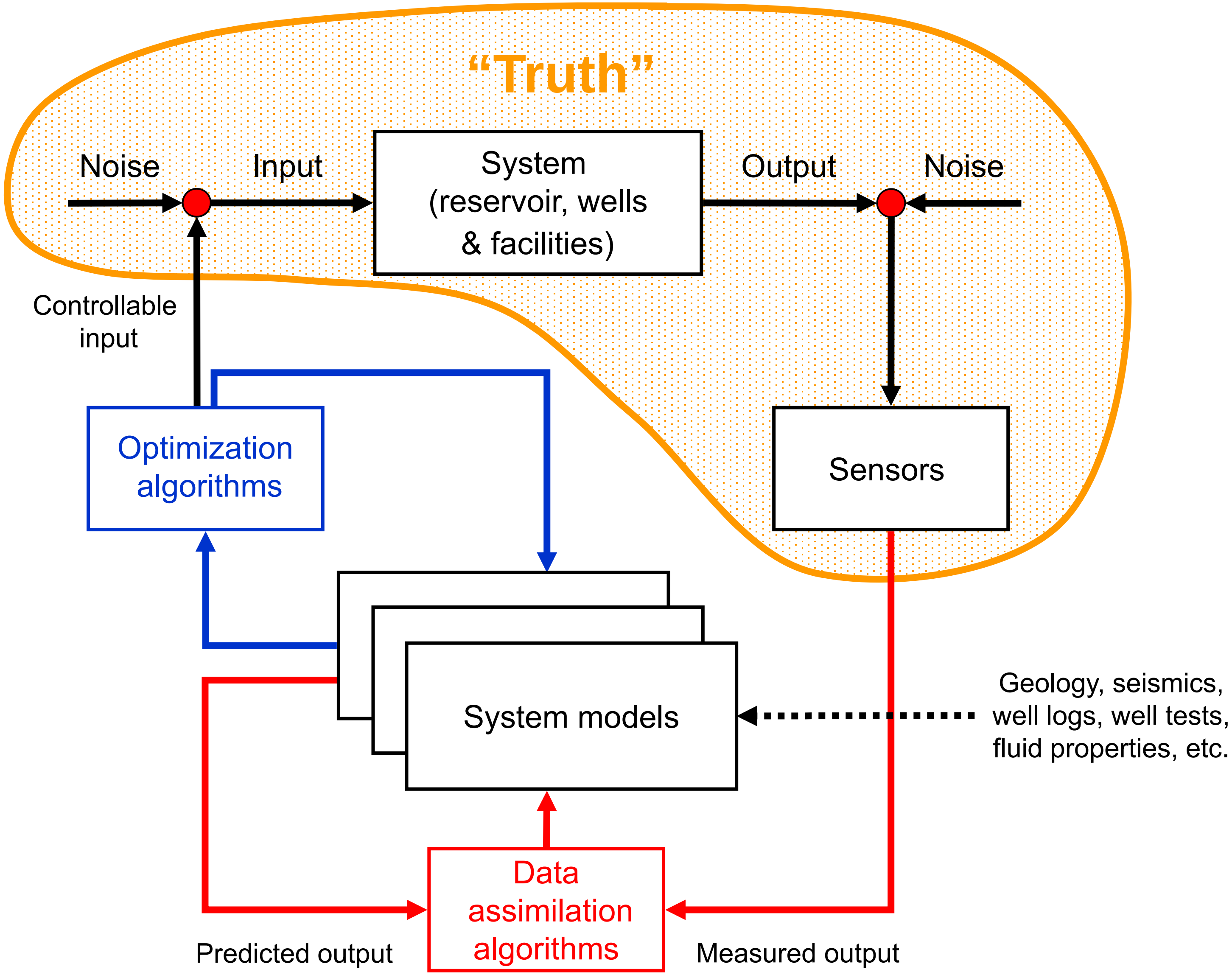
reactive control, nominal optimization, robust optimization



Van Essen et al., 2006

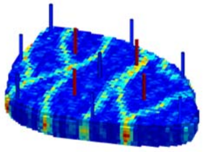
3) Closed-loop flooding optimization

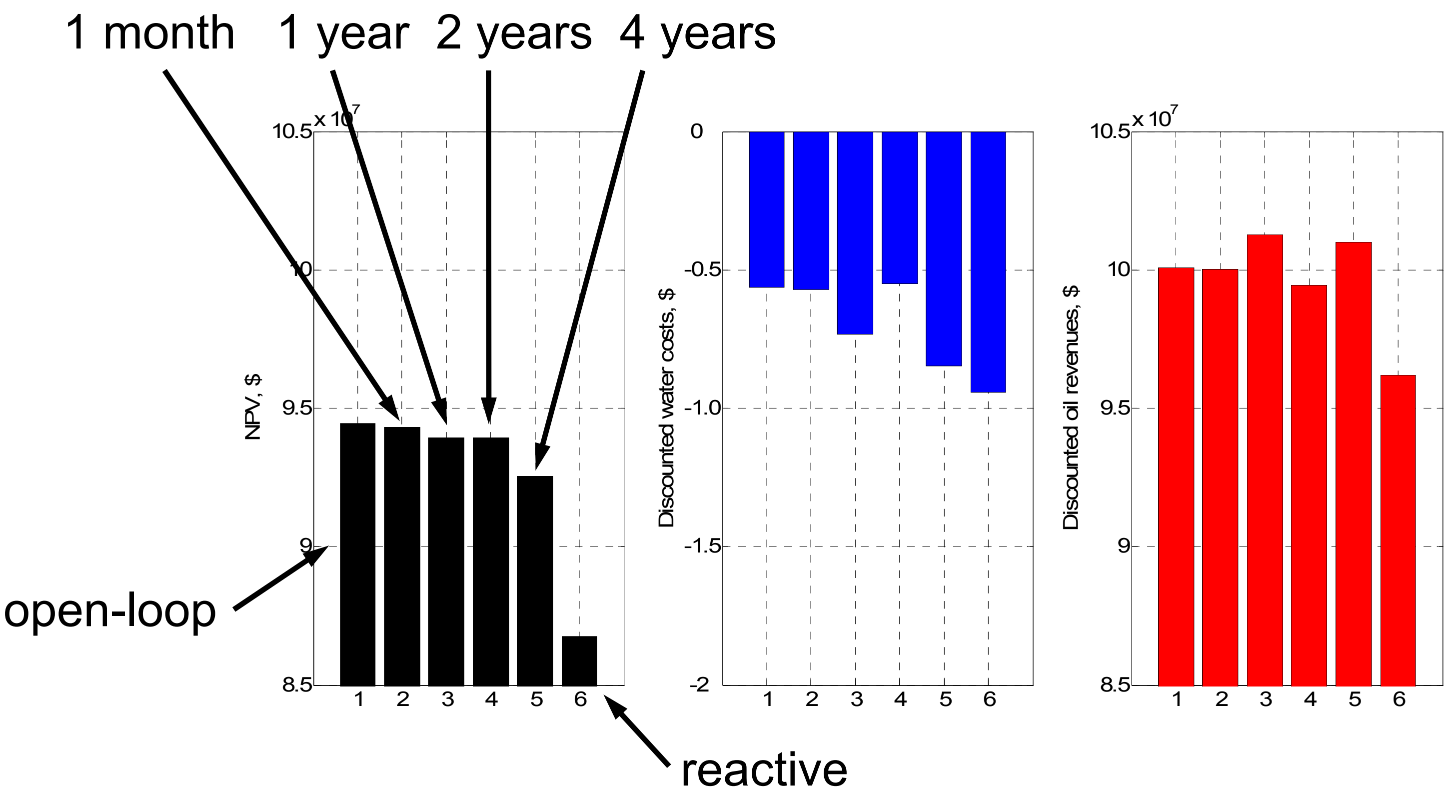



"Truth"
Noise
Input
System (reservoir, wells & facilities)
Output
Noise
Controllable input
Optimization algorithms
Sensors
System models
Geology, seismics, well logs, well tests, fluid properties, etc.
Predicted output
Data assimilation algorithms
Measured output

Closed-loop optimization

NPV and contributions from water & oil production





Optimization techniques

- Global versus local
- Gradient-based versus gradient-free
- Constrained versus non-constrained
- ‘Classical’ versus ‘non-classical’
  (simulated annealing, particle swarms, etc.)
- We use ‘optimal control theory’ or ‘adjoint-based’ optimization
- Has been proposed for history matching (Chen et al. 1974, Chavent et al. 1975, Li, Reynolds and Oliver 2003) and for flooding optimization (Ramirez 1987, Asheim 1988, Virnovski 1991, Zakirov et al. 1996, Sudaryanto and Yortsos, 2000, Brouwer and Jansen 2004, Sarma et al. 2004)

Optimal control theory, summary

- Gradient based optimization technique – local optimum
- Gradients of objective function with respect to controls obtained from ‘adjoint’ equation
- Gradients can be used with steepest ascent, quasi Newton, or trust-region methods
- Results in dynamic control strategy, i.e. controls change over time
- Computational effort independent of number of controls
- Output constraints not trivial; various techniques used
- Implementation is code-intrusive

Adjoint-Based Optimization

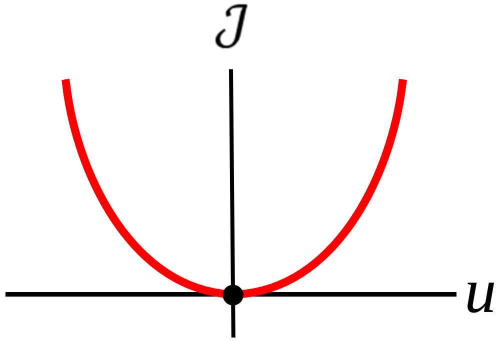
Part 1 - Theory

Unconstrained optimization (1D)

$$\mathcal{J}\left(u\right) = 2u^2$$

$$\frac{\partial \mathcal{J}}{\partial u} \equiv 4u = 0 \Rightarrow \begin{cases} u = 0 \\ \hdashline \mathcal{J} = 0 \end{cases}$$

$$\frac{\partial^2 \mathcal{J}}{\partial u^2} = 4 > 0 \Rightarrow \text{minimum}$$



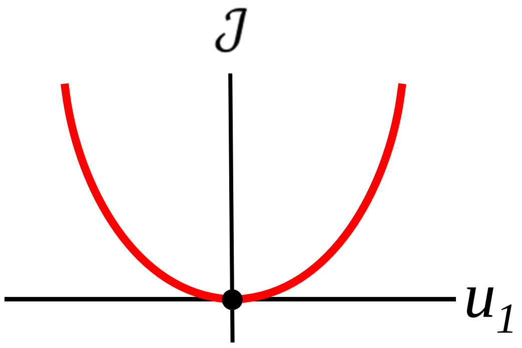
Objective function

Unconstrained optimization (2D)

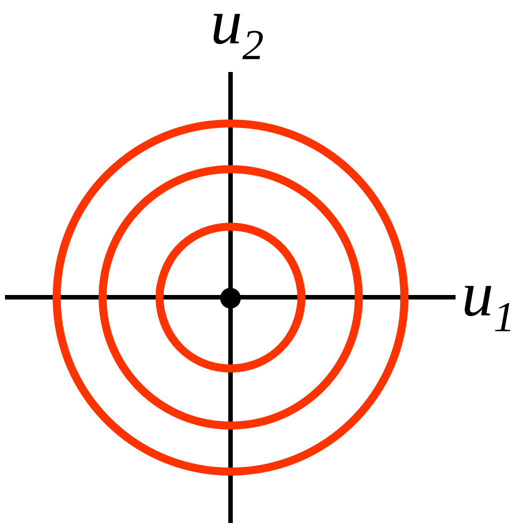
$$\mathcal{J}(\mathbf{u}) = 2\left(u_1^2 + u_2^2\right) \qquad \mathbf{u} = \left[u_1 \quad u_2\right]^T$$

$$\frac{\partial \mathcal{J}}{\partial \mathbf{u}} \equiv \left[4u_1 \quad 4u_2\right] = \mathbf{0}^T \Rightarrow \begin{cases} u_1 = 0 \\ u_2 = 0 \\ \hline \mathcal{J} = 0 \end{cases}$$

$$\frac{\partial^2 \mathcal{J}}{\partial \mathbf{u}^2} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} > 0 \Rightarrow \text{minimum}$$



Objective function



Contour lines

Constrained optimization (elimination)

$$\mathcal{J}(\mathbf{u}) = 2\left(u_1^2 + u_2^2\right) \quad \text{s.t.}$$

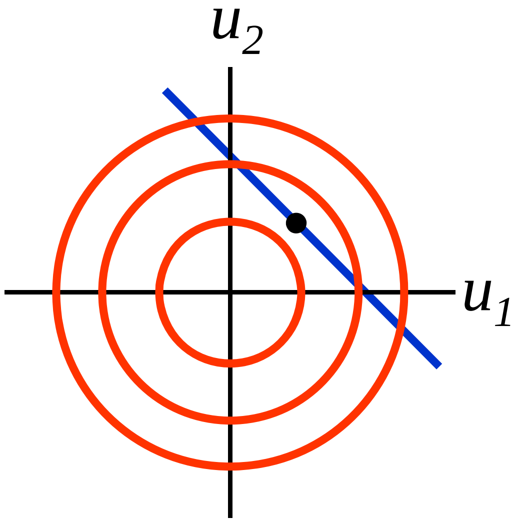
$$c(\mathbf{u}) \equiv u_1 + u_2 - 0.6 = 0$$

$$u_2 = 0.6 - u_1$$

$$\mathcal{J} = 4u_1^2 - 2.4u_1 + 0.72$$

$$\frac{\partial \mathcal{J}}{\partial u_1} \equiv 8u_1 - 2.4 = 0 \Rightarrow \begin{cases} u_1 = 0.3 \\ \hline u_2 = 0.3 \\ \mathcal{J} = 0.36 \end{cases}$$

$$\frac{\partial^2 \mathcal{J}}{\partial u_1^2} = 8 > 0 \Rightarrow \text{minimum}$$



Contour lines

Constrained optimization (Lagrange multipliers)

$$\mathcal{J}(\mathbf{u}) = 2\left(u_1^2 + u_2^2\right) \quad \text{s.t.}$$

$$c(\mathbf{u}) \equiv u_1 + u_2 - 0.6 = 0$$

$$\bar{\mathcal{J}}(\bar{\mathbf{u}}) \triangleq \mathcal{J}(\mathbf{u}) + \lambda c(\mathbf{u}) \qquad \bar{\mathbf{u}} = \begin{bmatrix} u_1 & u_2 & \lambda \end{bmatrix}^T$$

$$= 2\left(u_1^2 + u_2^2\right) + \lambda\left(u_1 + u_2 - 0.6\right)$$

$$\frac{\partial \bar{\mathcal{J}}}{\partial \bar{\mathbf{u}}} \equiv \begin{bmatrix} 4u_1 + \lambda & 4u_2 + \lambda & u_1 + u_2 - 0.6 \end{bmatrix} = \mathbf{0}^T \Rightarrow \begin{cases} u_1 = 0.3 \\ u_2 = 0.3 \\ \lambda = -1.2 \\ \hline \mathcal{J} = 0.36 \end{cases}$$

second-order conditions more complex

Lagrange multipliers – interpretation (a)

Recall elimination:

$$\mathcal{J}(\mathbf{u}) = 2\left(u_1^2 + u_2^2\right) \quad \text{s.t.} \qquad u_2 = 0.6 - u_1$$

$$c(\mathbf{u}) \equiv u_1 + u_2 - 0.6 = 0 \qquad \mathcal{J}(u_1) = 4u_1^2 - 2.4u_1 + 0.72$$

What if u_2 cannot be expressed in u_1 or v.v.?

Consider the total differential:

$$\frac{d\mathcal{J}}{du_1} = \left(\frac{\partial \mathcal{J}}{\partial u_1} + \frac{\partial \mathcal{J}}{\partial u_2} \frac{\partial u_2}{\partial u_1} \right)$$

But how do we compute $\partial u_2 / \partial u_1$?

Lagrange multipliers – interpretation (b)

Consider constraint $c\left(u_1, u_2\right) = 0$

Expressed in differential form:

$$\frac{\partial c}{\partial u_1}\partial u_1 + \frac{\partial c}{\partial u_2}\partial u_2 = 0$$

Can be rewritten as

$$\frac{\partial u_2}{\partial u_1} = -\left(\frac{\partial c}{\partial u_2}\right)^{-1}\frac{\partial c}{\partial u_1}$$

Implicit differentiation!

Lagrange multipliers – interpretation (c)

Given $\frac{d\mathcal{J}}{du_1}=\left(\frac{\partial\mathcal{J}}{\partial u_1}+\frac{\partial\mathcal{J}}{\partial u_2}\frac{\partial u_2}{\partial u_1}\right)$ and $\frac{\partial u_2}{\partial u_1}=-\left(\frac{\partial c}{\partial u_2}\right)^{-1}\frac{\partial c}{\partial u_1}$

we can now write

$$\frac{d\mathcal{J}}{du_1}=\frac{\partial\mathcal{J}}{\partial u_1}-\frac{\partial\mathcal{J}}{\partial u_2}\left(\frac{\partial c}{\partial u_2}\right)^{-1}\frac{\partial c}{\partial u_1}$$

which, in an optimum, can also be written as $\frac{d\mathcal{J}}{du_1}=0$

$$\Rightarrow \quad \frac{\partial\mathcal{J}}{\partial u_1}+\underbrace{-\frac{\partial\mathcal{J}}{\partial u_2}\left(\frac{\partial c}{\partial u_2}\right)^{-1}}_{\lambda_1}\frac{\partial c}{\partial u_1}=0$$

Lagrange multipliers – interpretation (d)

If we have $$\frac{\partial \mathcal{J}}{\partial u_1} + \underbrace{-\frac{\partial \mathcal{J}}{\partial u_2}\left(\frac{\partial c}{\partial u_2}\right)^{-1}}_{\lambda_1}\frac{\partial c}{\partial u_1} = 0$$

we can also derive that

$$\frac{\partial \mathcal{J}}{\partial u_2} + \underbrace{-\frac{\partial \mathcal{J}}{\partial u_1}\left(\frac{\partial c}{\partial u_1}\right)^{-1}}_{\lambda_2}\frac{\partial c}{\partial u_2} = 0$$

which, if $\lambda_1 = \lambda_2$, can be combined into $\frac{\partial \mathcal{J}}{\partial \mathbf{u}} + \lambda \frac{\partial c}{\partial \mathbf{u}} = \mathbf{0}^T$

$\lambda_1 = \lambda_2$ implies: $\frac{\partial \mathcal{J}}{\partial u_1}\left(\frac{\partial c}{\partial u_1}\right)^{-1} = \frac{\partial \mathcal{J}}{\partial u_2}\left(\frac{\partial c}{\partial u_2}\right)^{-1}$ OK in optimum

Use of Lagrange multipliers = implicit differentiation

Back to the real thing: Production optimization

- Problem statement: $\max\limits_{\mathbf{u}_{1:K}} \mathcal{J}\left(\mathbf{u}_{1:K}\right)$ subject to

- System equations: $\mathbf{g}_k\left(\mathbf{u}_k, \mathbf{x}_{k-1}, \mathbf{x}_k\right) = \mathbf{0}$
- Initial conditions: $\mathbf{x}_0 = \breve{\mathbf{x}}_0$
- Equality constraints: $\mathbf{c}_k\left(\mathbf{u}_k, \mathbf{x}_k\right) = \mathbf{0}$
- Inequality constraints: $\mathbf{d}_k\left(\mathbf{u}_k, \mathbf{x}_k\right) < \mathbf{0}$

- As a first step: disregard constraints $\mathbf{c}_k$ and $\mathbf{d}_k$

Gradient with implicit differentiation?

What we are looking for:

$$\frac{d\mathcal{J}}{d\mathbf{u}_k} = \frac{\partial \mathcal{J}_k}{\partial \mathbf{u}_k} + \sum_{j=k}^{K} \frac{\partial \mathcal{J}_j}{\partial \mathbf{x}_j} \frac{\partial \mathbf{x}_j}{\partial \mathbf{u}_k}$$

Effect of $\mathbf{u}_k$ on all subsequent time steps

Contributions from time steps $k \ldots K$

$$\frac{\partial \mathbf{x}_j}{\partial \mathbf{u}_k} = \frac{\partial \mathbf{x}_j}{\partial \mathbf{x}_{j-1}} \frac{\partial \mathbf{x}_{j-1}}{\partial \mathbf{x}_{j-2}} \cdots \frac{\partial \mathbf{x}_{k+2}}{\partial \mathbf{x}_{k+1}} \frac{\partial \mathbf{x}_{k+1}}{\partial \mathbf{x}_k} \frac{\partial \mathbf{x}_k}{\partial \mathbf{u}_k}$$

Requires a lot of implicit differentiation…

Gradient with Lagrange multipliers

- “Adjoin” constraints to objective function:

$$\bar{\mathcal{J}}\left(\mathbf{u}_{1:K},\mathbf{x}_{0:K},\boldsymbol{\lambda}_{0:K}\right)\triangleq\sum_{k=1}^{K}\left[\begin{array}{l}\mathcal{J}_k\left(\mathbf{u}_k,\mathbf{x}_k\right)\\+\boldsymbol{\lambda}_0^T\left(\mathbf{x}_0-\breve{\mathbf{x}}_0\right)\delta_{k-1}\\+\boldsymbol{\lambda}_k^T\mathbf{g}_k\left(\mathbf{u}_k,\mathbf{x}_{k-1},\mathbf{x}_k\right)\end{array}\right]$$

‘Modified objective function’

- Proceed as before: take first derivatives w.r.t. all independent variables and equate them to zero (i.e. force optimality conditions)

- Note that we can write: $\dfrac{\partial\mathbf{g}_k}{\partial\mathbf{x}_{k-1}}=\dfrac{\partial\mathbf{g}_{k+1}}{\partial\mathbf{x}_k}$ (index shift)

Optimality conditions (1)

$$\bar{\mathcal{J}}\left(\mathbf{u}_{1:K},\mathbf{x}_{0:K},\boldsymbol{\lambda}_{0:K}\right) \triangleq \sum_{k=1}^{K}\begin{bmatrix} \mathcal{J}_k\left(\mathbf{u}_k,\mathbf{y}_k\right) \\ +\boldsymbol{\lambda}_0^T\left(\mathbf{x}_0-\breve{\mathbf{x}}_0\right)\delta_{k-1} \\ +\boldsymbol{\lambda}_k^T\mathbf{g}_k\left(\mathbf{u}_k,\mathbf{x}_{k-1},\mathbf{x}_k\right) \end{bmatrix}$$

$$\frac{\partial \bar{\mathcal{J}}}{\partial \mathbf{u}_k} \equiv \frac{\partial \mathcal{J}_k}{\partial \mathbf{u}_k} + \boldsymbol{\lambda}_k^T \frac{\partial \mathbf{g}_k}{\partial \mathbf{u}_k} = \mathbf{0}^T \qquad k = 1, 2, \ldots, K$$

$$\frac{\partial \bar{\mathcal{J}}}{\partial \mathbf{x}_0} \equiv \boldsymbol{\lambda}_1^T \frac{\partial \mathbf{g}_1}{\partial \mathbf{x}_0} + \boldsymbol{\lambda}_0^T = \mathbf{0}^T$$

$$\frac{\partial \bar{\mathcal{J}}}{\partial \mathbf{x}_k} \equiv \frac{\partial \mathcal{J}_k}{\partial \mathbf{x}_k} + \boldsymbol{\lambda}_{k+1}^T \frac{\partial \mathbf{g}_{k+1}}{\partial \mathbf{x}_k} + \boldsymbol{\lambda}_k^T \frac{\partial \mathbf{g}_k}{\partial \mathbf{x}_k} = \mathbf{0}^T \qquad k = 1, 2, \ldots, K-1$$

$$\frac{\partial \bar{\mathcal{J}}}{\partial \mathbf{x}_K} \equiv \frac{\partial \mathcal{J}_K}{\partial \mathbf{x}_K} + \boldsymbol{\lambda}_K^T \frac{\partial \mathbf{g}_K}{\partial \mathbf{x}_K} = \mathbf{0}^T$$

Optimality conditions (2)

$$\bar{\mathcal{J}}\left(\mathbf{u}_{1:K},\mathbf{x}_{0:K},\boldsymbol{\lambda}_{0:K}\right) \triangleq \sum_{k=1}^{K}\left[\begin{array}{l}\mathcal{J}_k\left(\mathbf{u}_k,\mathbf{y}_k\right)\\ +\boldsymbol{\lambda}_0^T\left(\mathbf{x}_0-\breve{\mathbf{x}}_0\right)\delta_{k-1}\\ +\boldsymbol{\lambda}_k^T\mathbf{g}_k\left(\mathbf{u}_k,\mathbf{x}_{k-1},\mathbf{x}_k\right)\end{array}\right]$$

$$\frac{\partial\bar{\mathcal{J}}}{\partial\boldsymbol{\lambda}_0} \equiv \left(\mathbf{x}_0-\breve{\mathbf{x}}_0\right)^T = \mathbf{0}^T$$

$$\frac{\partial\bar{\mathcal{J}}}{\partial\boldsymbol{\lambda}_k} \equiv \mathbf{g}_k^T\left(\mathbf{u}_k,\mathbf{x}_{k-1},\mathbf{x}_k\right) = \mathbf{0}^T \qquad k = 1,2,\ldots,K$$

(Just recovers the initial conditions and system equations)

- The optimality conditions form a joint set of equations for the unknowns $\mathbf{u}_{1:K},\mathbf{x}_{0:K},\boldsymbol{\lambda}_{0:K}$
- Can in theory be solved simultaneously (Wathen et al.) but are usually treated sequentially.

Solving the resulting equations (1)

$$\left.\begin{array}{llll} \partial\bar{\mathcal{J}}/\partial\boldsymbol{\lambda}_0 & \left(\mathbf{x}_0-\breve{\mathbf{x}}_0\right)^T=\mathbf{0}^T & \Rightarrow & \mathbf{x}_0 \\ \partial\bar{\mathcal{J}}/\partial\boldsymbol{\lambda}_k & \mathbf{g}_k^T\left(\underset{\underset{\text{Initial guess!}}{\uparrow}}{\mathbf{u}_k},\mathbf{x}_{k-1},\mathbf{x}_k\right)=\mathbf{0}^T & \Rightarrow & \mathbf{x}_{1:K} \end{array}\right\}$$

Running
the simulator.
(Requires $\partial\mathbf{g}_k/\partial\mathbf{x}_k$)

Solving the resulting equations (1)

$$\left.\begin{array}{lll} \partial \bar{\mathcal{J}} / \partial \boldsymbol{\lambda}_0 & (\mathbf{x}_0 - \breve{\mathbf{x}}_0)^T = \mathbf{0}^T & \Rightarrow \quad \mathbf{x}_0 \\ \partial \bar{\mathcal{J}} / \partial \boldsymbol{\lambda}_k & \mathbf{g}_k^T(\mathbf{u}_k, \mathbf{x}_{k-1}, \mathbf{x}_k) = \mathbf{0}^T & \Rightarrow \quad \mathbf{x}_{1:K} \end{array}\right\}$$

Running
the simulator.
(Requires $\partial \mathbf{g}_k / \partial \mathbf{x}_k$)

$$\partial \bar{\mathcal{J}} / \partial \mathbf{x}_K \qquad \frac{\partial \mathcal{J}_K}{\partial \mathbf{x}_K} + \boldsymbol{\lambda}_K^T \frac{\partial \mathbf{g}_K}{\partial \mathbf{x}_K} = \mathbf{0}^T$$

Solving the resulting equations (1)

$\partial \bar{\mathcal{J}} / \partial \boldsymbol{\lambda}_0$ $$\left(\mathbf{x}_0 - \breve{\mathbf{x}}_0\right)^T = \mathbf{0}^T \quad \Rightarrow \quad \mathbf{x}_0$$

$\partial \bar{\mathcal{J}} / \partial \boldsymbol{\lambda}_k$ $$\mathbf{g}_k^T\left(\mathbf{u}_k, \mathbf{x}_{k-1}, \mathbf{x}_k\right) = \mathbf{0}^T \quad \Rightarrow \quad \mathbf{x}_{1:K}$$

Running
the simulator.
(Requires $\partial \mathbf{g}_k / \partial \mathbf{x}_k$)

$\partial \bar{\mathcal{J}} / \partial \mathbf{x}_K$ $$\left(\frac{\partial \mathbf{g}_K}{\partial \mathbf{x}_K}\right)^T \boldsymbol{\lambda}_K = -\left(\frac{\partial \mathcal{J}_K}{\partial \mathbf{x}_K}\right)^T \quad \Rightarrow \quad \boldsymbol{\lambda}_K$$

‘Final condition’

$\partial \bar{\mathcal{J}} / \partial \mathbf{x}_k$ $$\left(\frac{\partial \mathbf{g}_k}{\partial \mathbf{x}_k}\right)^T \boldsymbol{\lambda}_k = -\left(\frac{\partial \mathbf{g}_{k+1}}{\partial \mathbf{x}_k}\right)^T \boldsymbol{\lambda}_{k+1} \quad \Rightarrow \quad \boldsymbol{\lambda}_{K-1:1}$$

‘Backward’
integration
(linear)

$\partial \bar{\mathcal{J}} / \partial \mathbf{x}_0$ $$\boldsymbol{\lambda}_0 = \left(\frac{\partial \mathbf{g}_1}{\partial \mathbf{x}_0}\right)^T \boldsymbol{\lambda}_1 \quad \Rightarrow \quad \boldsymbol{\lambda}_0$$

Solving the resulting equations (2)

$\partial \bar{\mathcal{J}} / \partial \mathbf{u}_k$

$$\frac{\partial \mathcal{J}_k}{\partial \mathbf{u}_k} + \boldsymbol{\lambda}_k^T \frac{\partial \mathbf{g}_k}{\partial \mathbf{u}_k} \neq \mathbf{0}^T \quad ???$$

Usually not!

Solving the resulting equations (2)

$$\frac{\partial \bar{\mathcal{J}}}{\partial \mathbf{u}_k} = \frac{\partial \mathcal{J}_k}{\partial \mathbf{u}_k} + \boldsymbol{\lambda}_k^T \frac{\partial \mathbf{g}_k}{\partial \mathbf{u}_k}$$

Recall $$\frac{d\mathcal{J}}{d\mathbf{u}_k} = \frac{\partial \mathcal{J}_k}{\partial \mathbf{u}_k} + \sum_{j=k}^{K} \frac{\partial \mathcal{J}_j}{\partial \mathbf{y}_j} \frac{\partial \mathbf{x}_j}{\partial \mathbf{u}_k}$$

$$\frac{\partial \bar{\mathcal{J}}}{\partial \mathbf{u}_k} = \frac{d\mathcal{J}}{d\mathbf{u}_k} \quad !!!$$

Just what we need

Can now be used, e.g., in steepest ascent:

$$\mathbf{u}_k^{i+1} = \mathbf{u}_k^i + \alpha \left(\frac{d\mathcal{J}}{d\mathbf{u}_k^i} \right)^T$$

Summary adjoint-based optimization

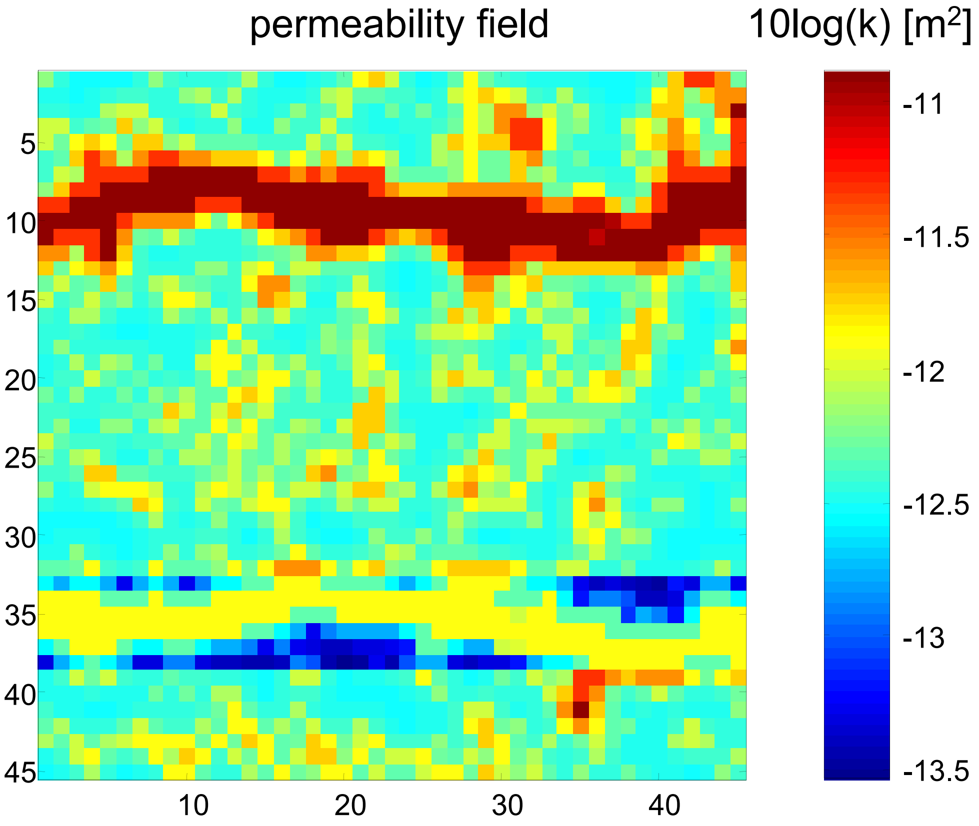
- Adjoint ~ implicit differentiation
- Computational effort independent of number of controls
- Gradient-based optimization – local optimum
- Constraint handling: GRG, lumping, SQP, augmented Lagrangian, … ; not trivial
- Beautiful, but code-intrusive and requires lots of programming => automatic differentiation
- Available in Eclipse (limited functionality), AD-GPRS, MRST, proprietary simulators
- Alternatives: ensemble methods (EnOpt, StoSAG), streamline-based methods, ‘non classical methods’ (particle swarm, etc.; often in combination with ‘proxies’ to reduce computational effort)

Adjoint-Based Optimization

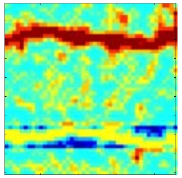
Part 2 - Examples

Classic example; smart horizontal wells

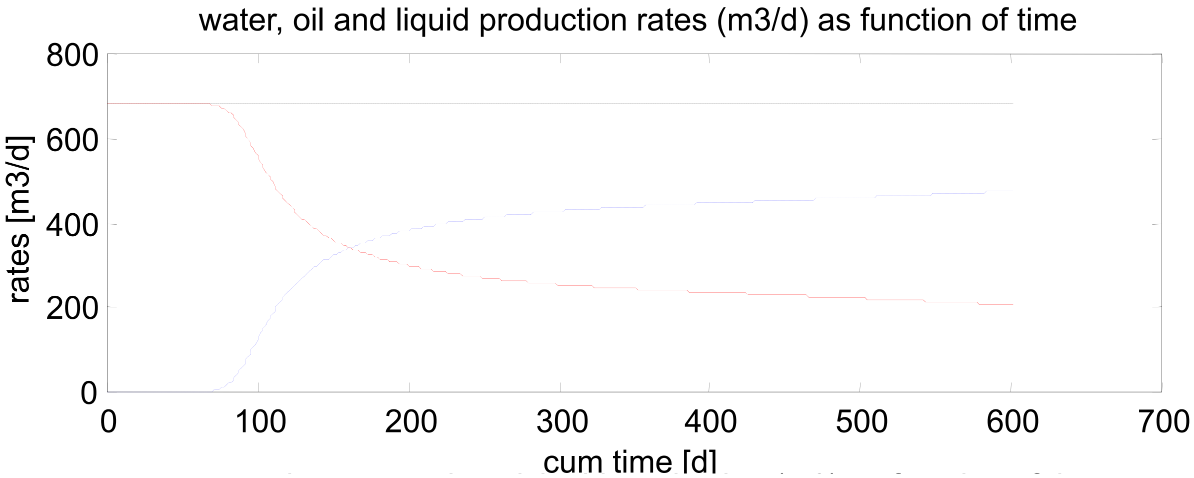
- 45 x 45 grid blocks
- 45 inj. & prod. segments
- p_{wf}, q_t at segments known
- 1 PV injected, $q_{inj} = q_{prod}$
- oil price r_o = 80 \$/m^3
- water costs r_w = 20 \$/m^3
- discount rate b = 0%

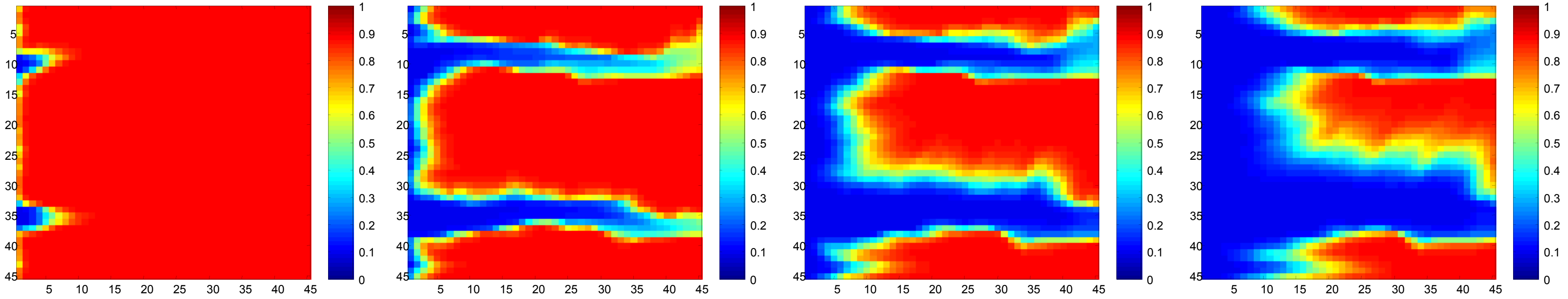


Brouwer and Jansen, 2004, SPEJ



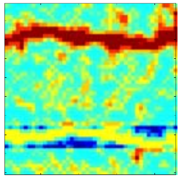
Results; conventional production

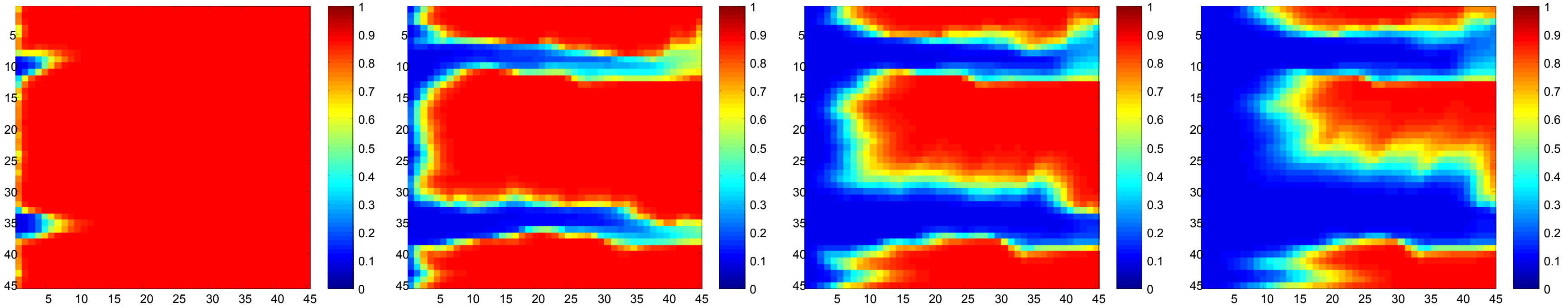




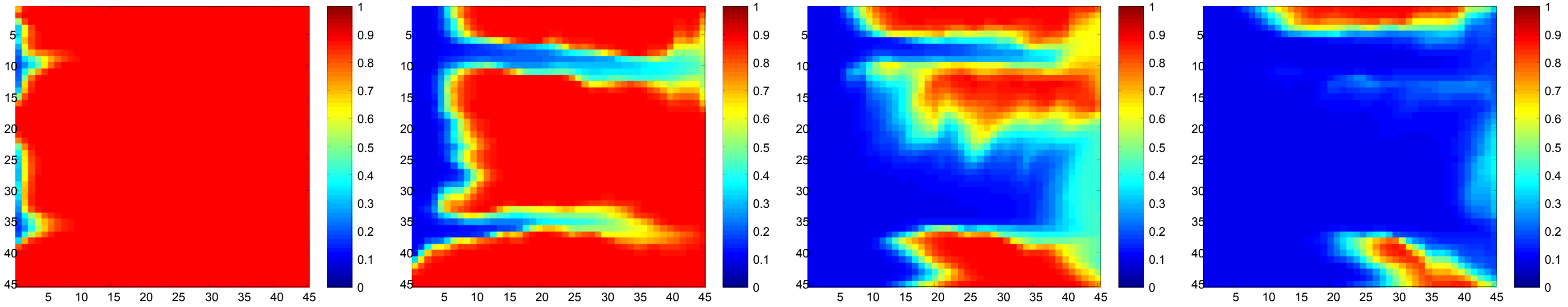
Equal pressures in all injector/producer segments

Results; rate-constrained (1)

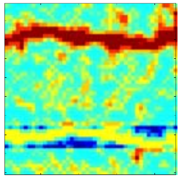




Conventional (equal pressure in all segments, no control)



Best possible (identical total rates, no pressure constraints)



Results; rate-constrained (2)

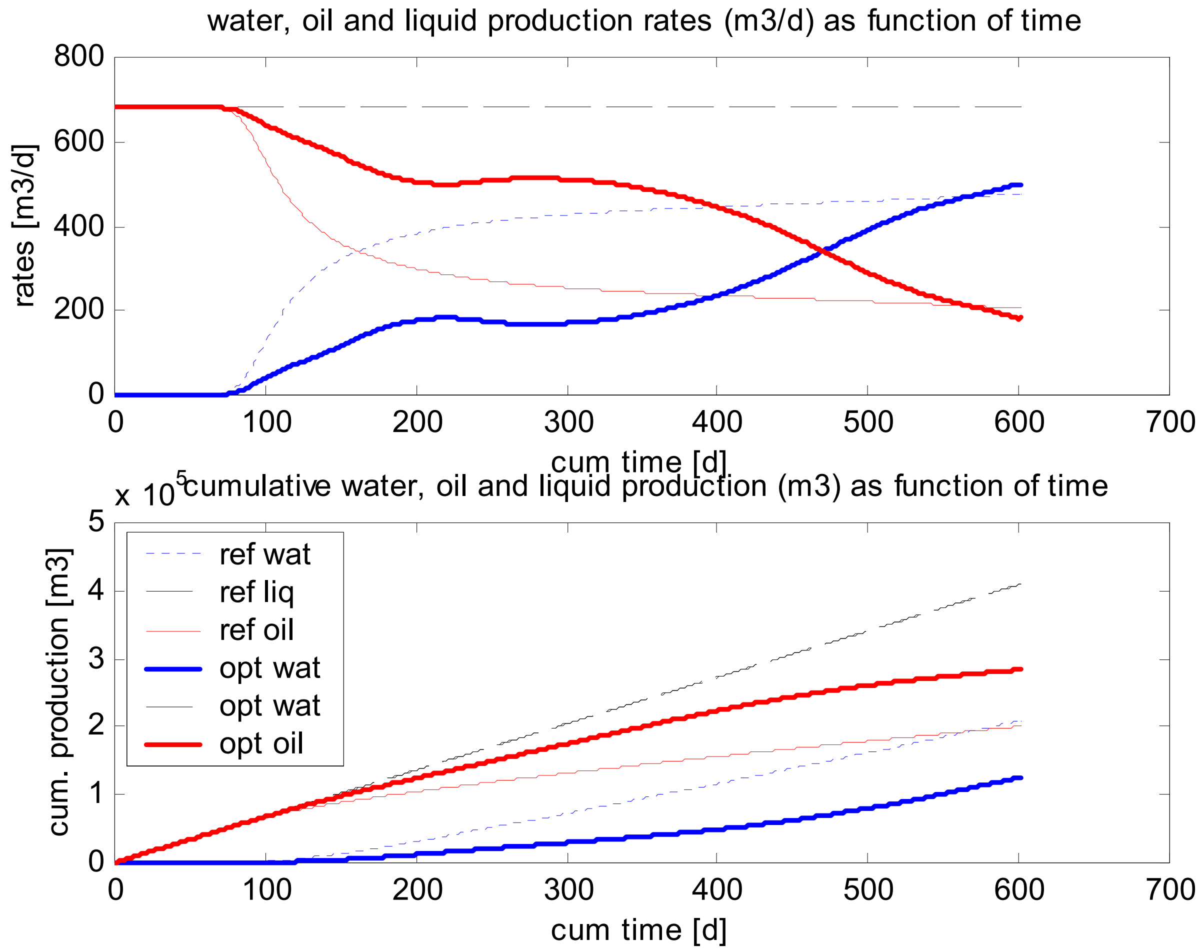
NPV

+60%

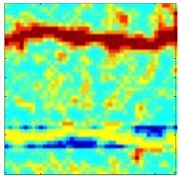
Production

+ 41% cum oil

- 45% cum wat

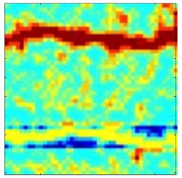


Pressure-constrained operation



- Limited energy available
- Total injection/production rate dependent on number of active wells

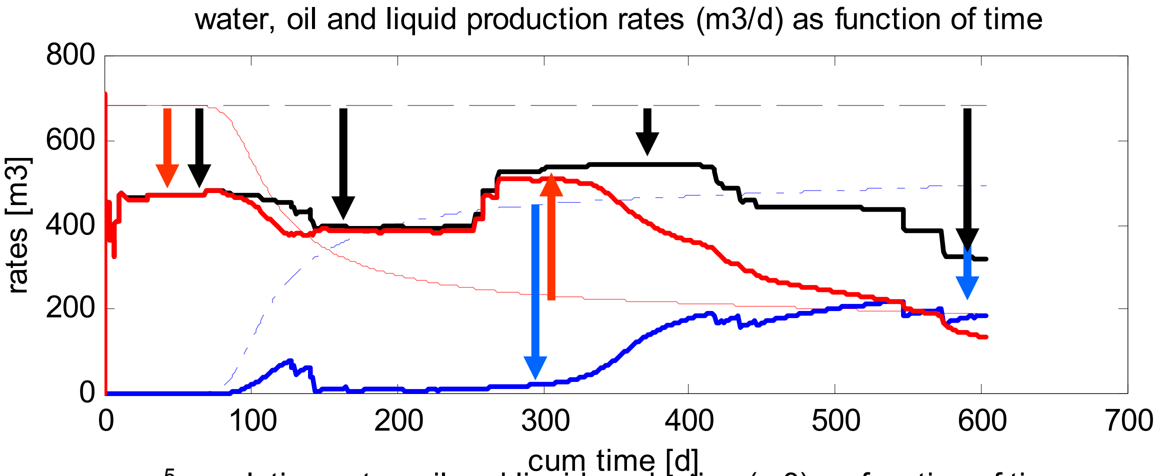
Results: pressure-constrained

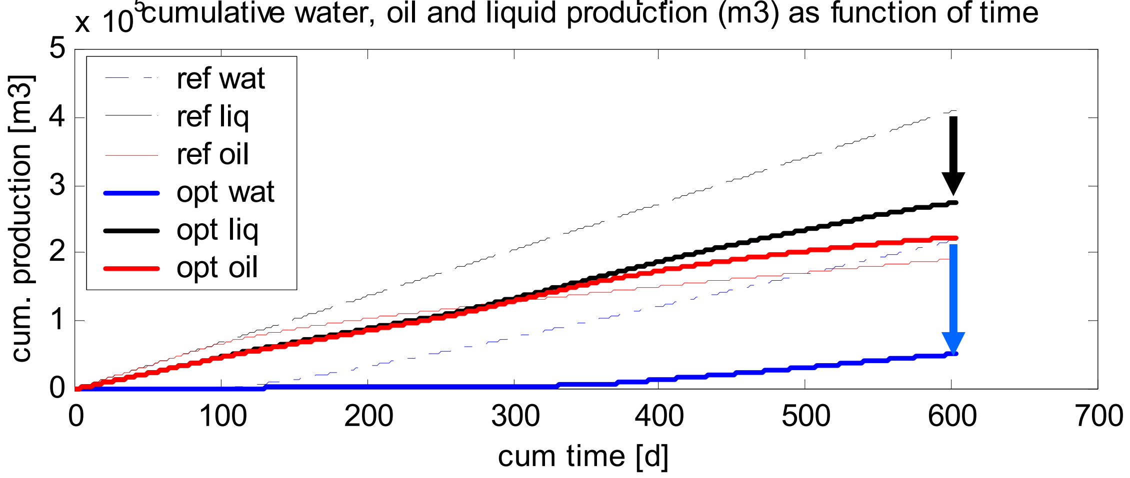


Improvement
in NPV
+53%

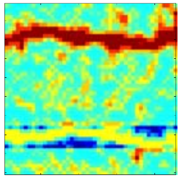
Production
+16% cum oil
-77% cum water

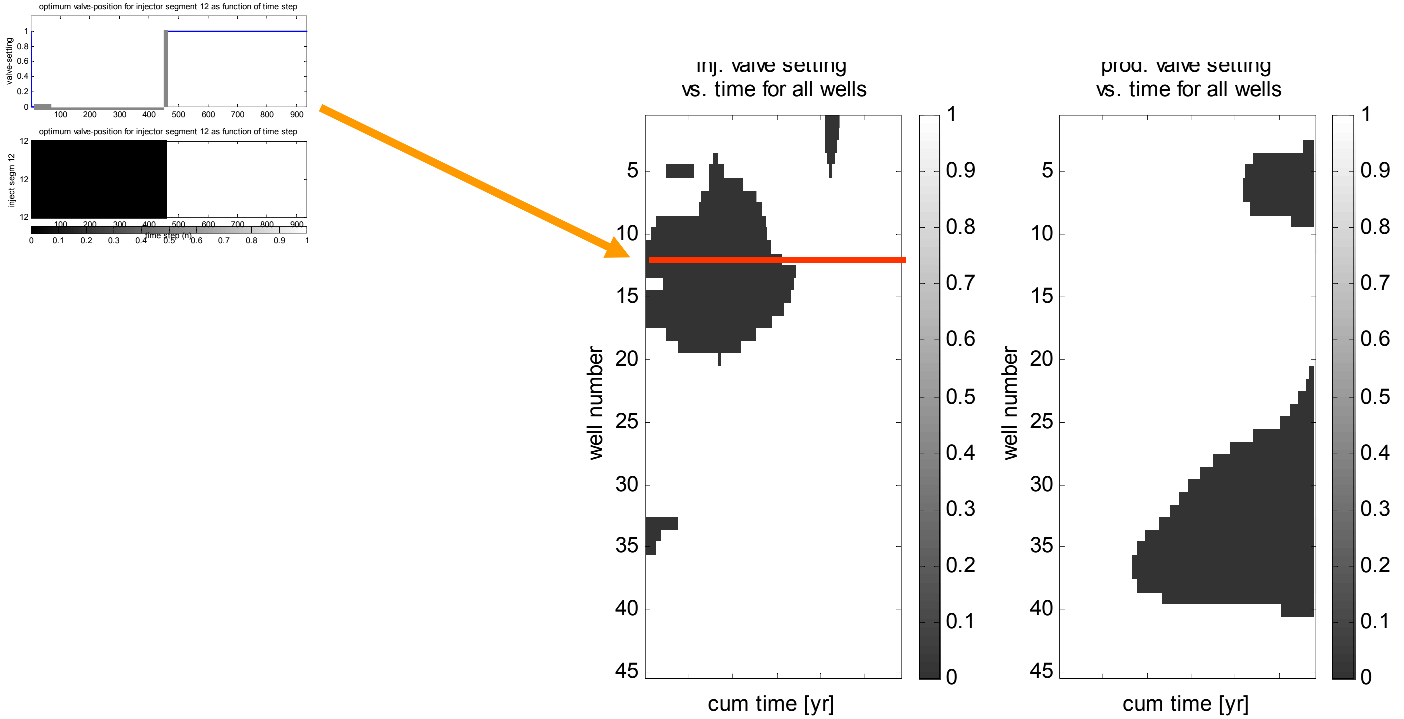
Injection
-32% cum water


water, oil and liquid production rates (m3/d) as function of time


cumulative water, oil and liquid production (m3) as function of time

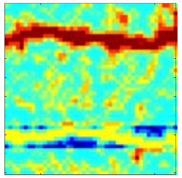
Optimum valve-settings (1)

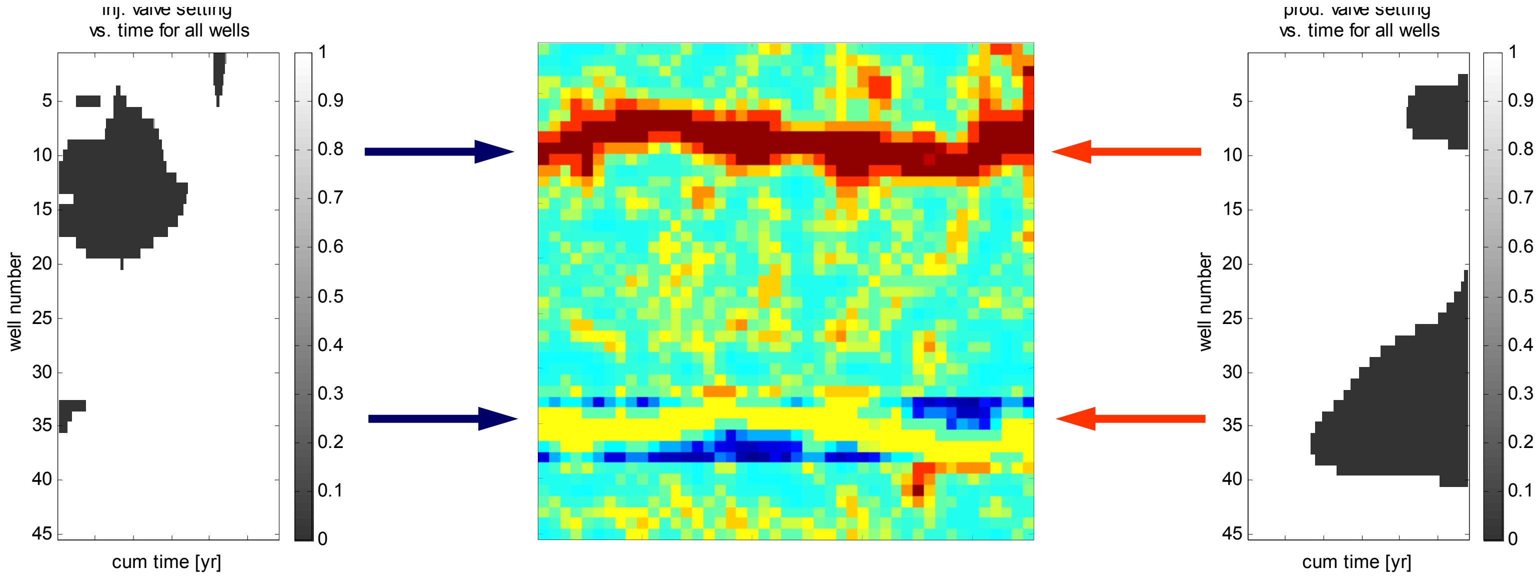




- Bang-bang (on-off) solution
- Necessary condition: linear controls, linear constraints

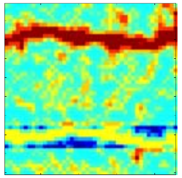
Optimum valve-settings (2)

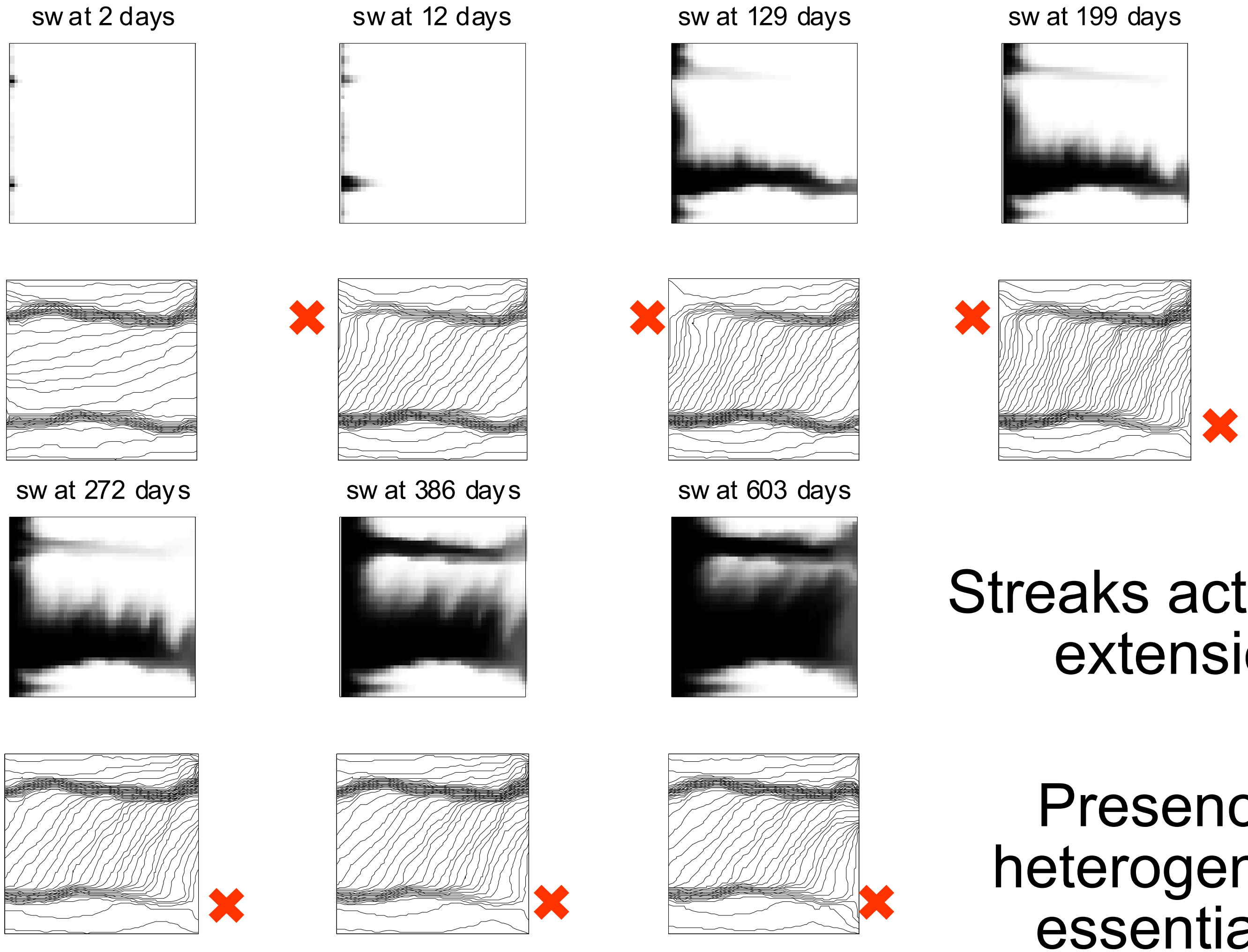




All the action is around the heterogeneities

Optimum valve settings (3)





sw at 2 days

sw at 12 days

sw at 129 days

sw at 199 days

sw at 272 days

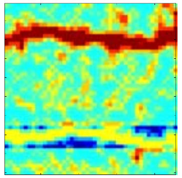
sw at 386 days

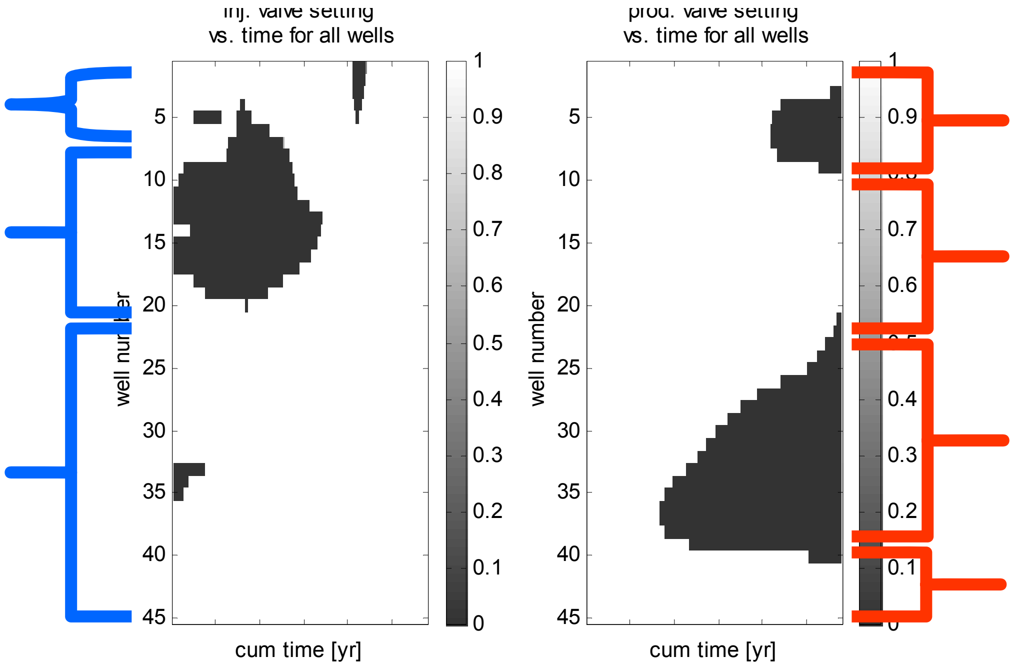
sw at 603 days

Streaks act as well extensions

Presence of heterogeneities essential for optimization

Optimum valve-settings (4)





3 valves
in injector

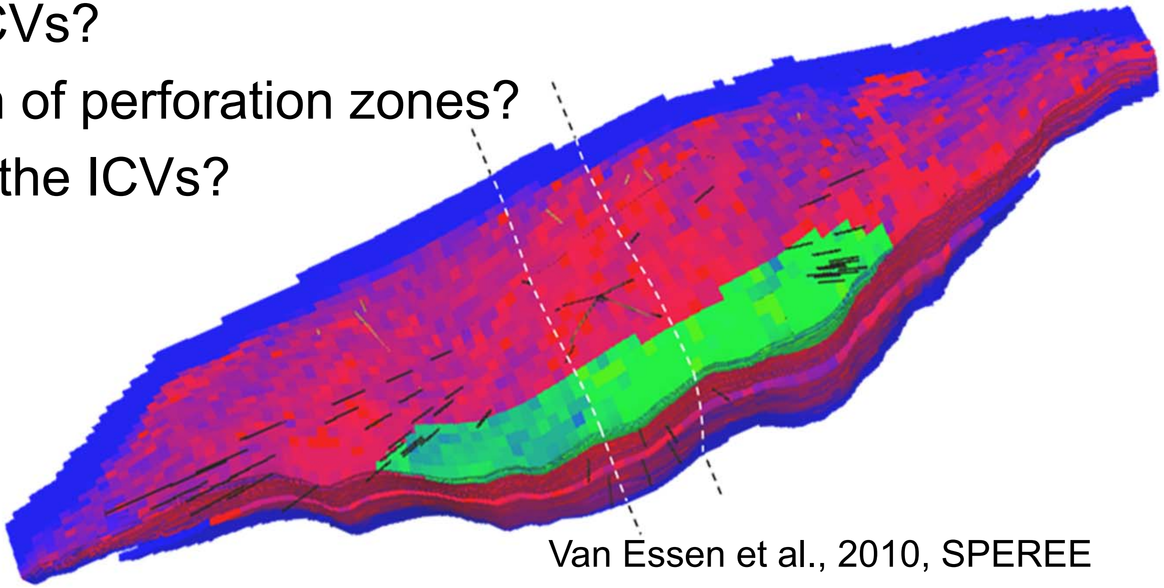
4 valves in
producer

No need for 45 segments per well

St. Joseph field re-development case

Objective: to determine the value of down-hole control in planned water injectors, in terms of incremental cumulative oil production

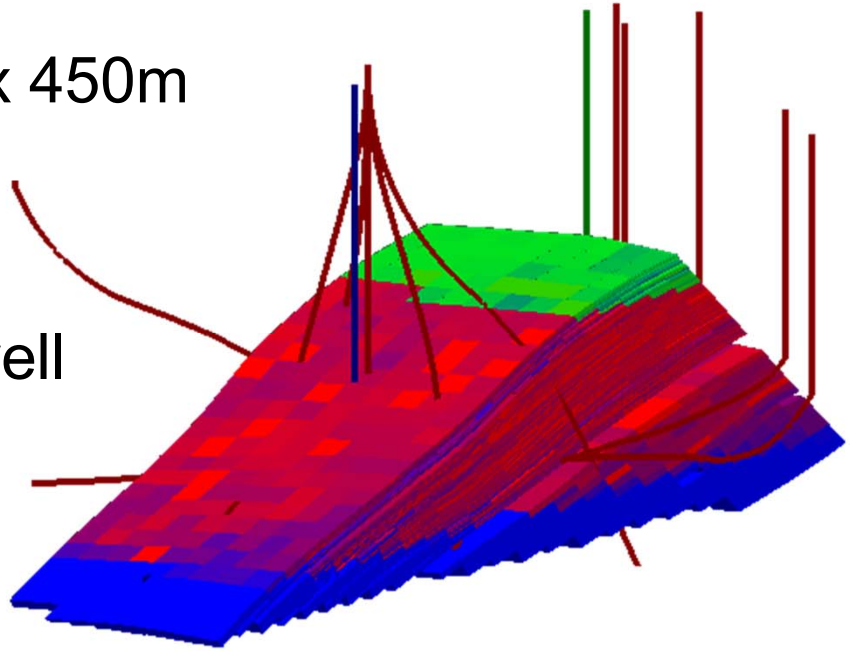
- Maximum number of ICVs: 5
- Water injection rate: 10,000 bbl/d per well
- Trajectory of water injector fixed
- Optimum number of ICVs?
- Optimum configuration of perforation zones?
- Optimum operation of the ICVs?



Van Essen et al., 2010, SPEREE

Pilot study on sector model

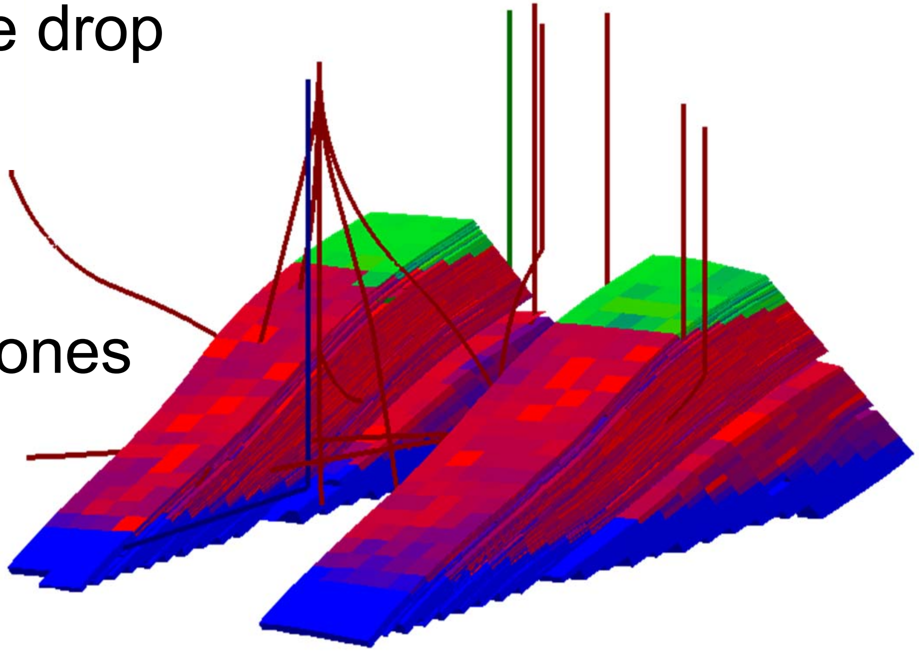
- Strongly layered structure
- Very limited vertical communication
- Dips approximately 20º
- 21,909 active grid blocks
- Dimensions 1600m x 500m x 450m
- No aquifer support
- 1 gas injection well
- 1 (planned) water injection well
- 7 production wells in sector



Smart water injection well

Properties

- Fixed flow rate of 10,000 bbl/d
- Fixed location and trajectory
- Horizontal section perforated
- Lift table captures pressure drop



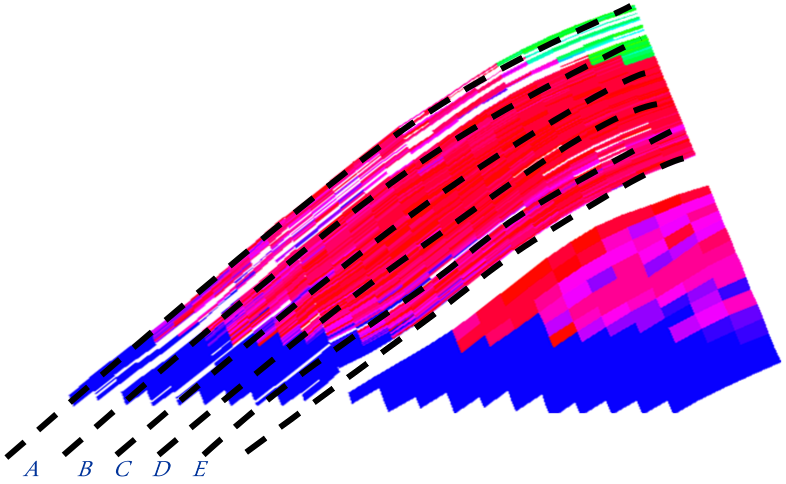
Variables

- Number of ICVs
- Length of the perforation zones
- Operation of ICVs

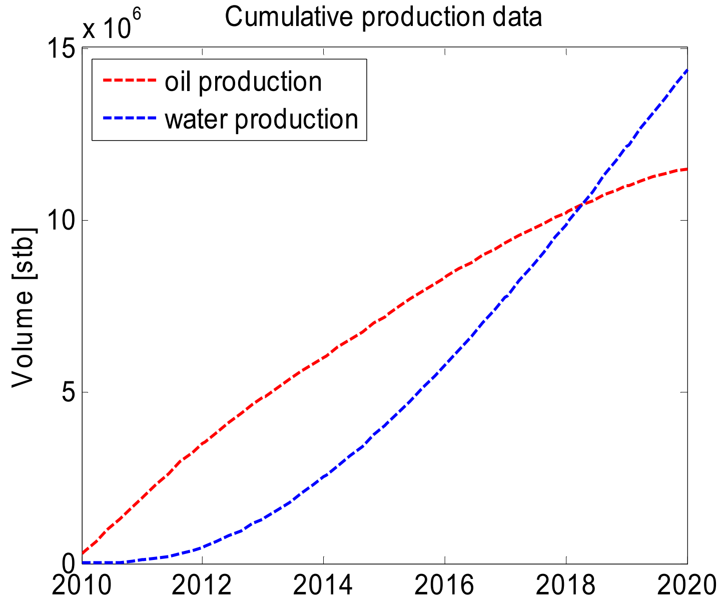
- Controls: kdh multipliers

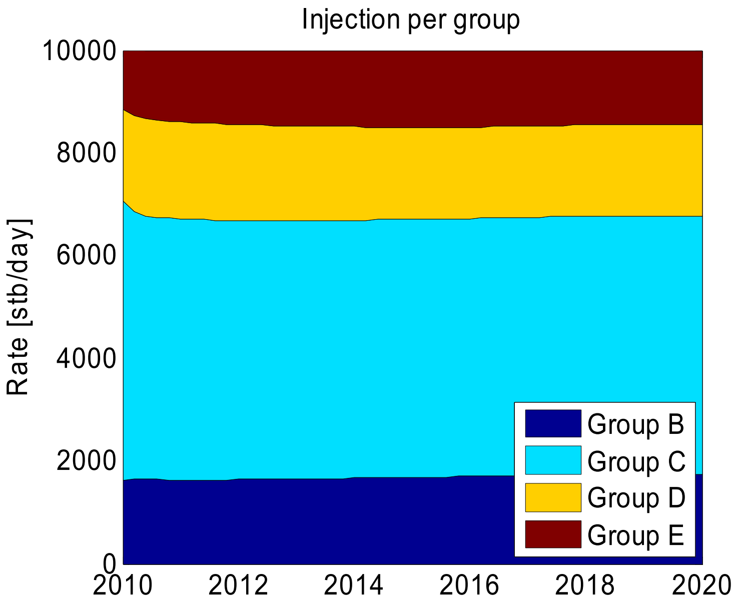
Base case

- No control
  - All kdh multipliers in 102 layers equal to 1
- Water injection into each layer result of permeability, pressure difference, etc.
  - Performance quantified in terms of cumulative oil production
- Also water injection rate into each zone is determined
  - Zones B, C, D and E
  - No injection in A



Base case results

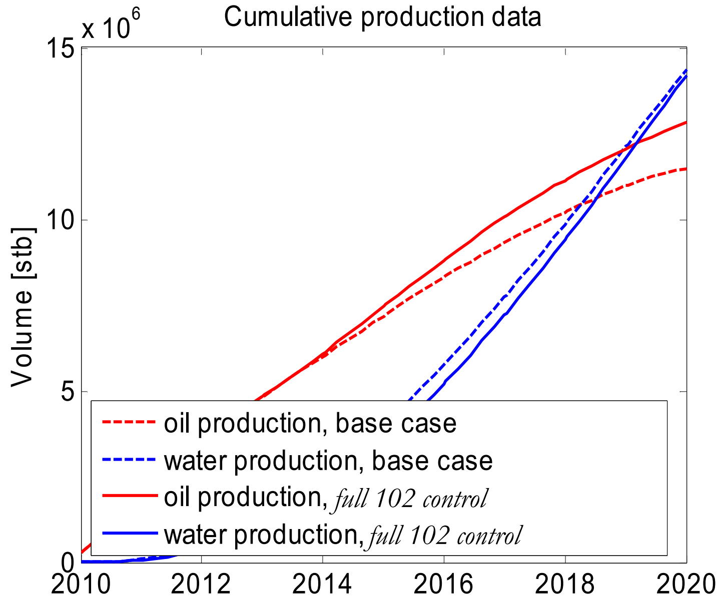


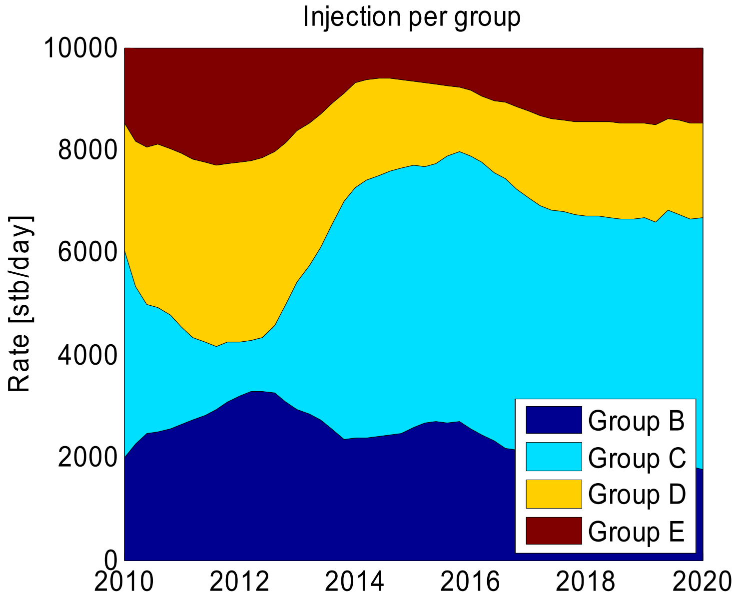


Van Essen et al., 2010, SPEREE

- Cumulative oil production: 11.47 MMstb

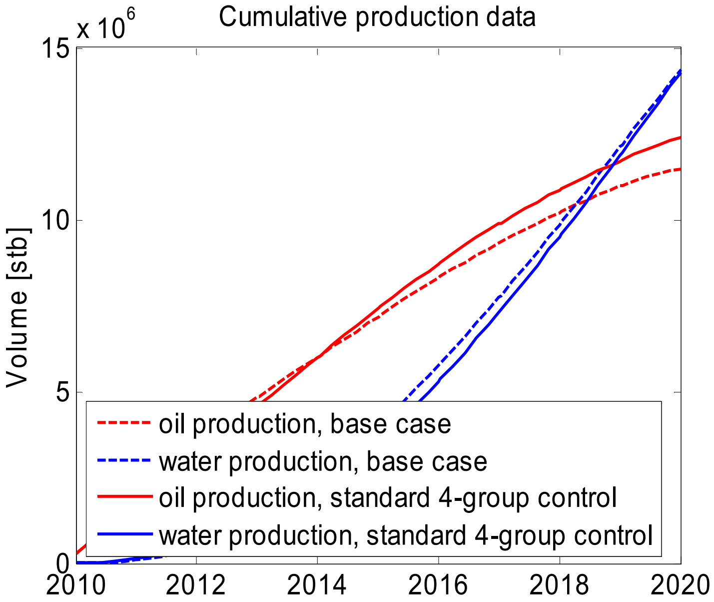
Full 102 zone control (‘technical limit’)

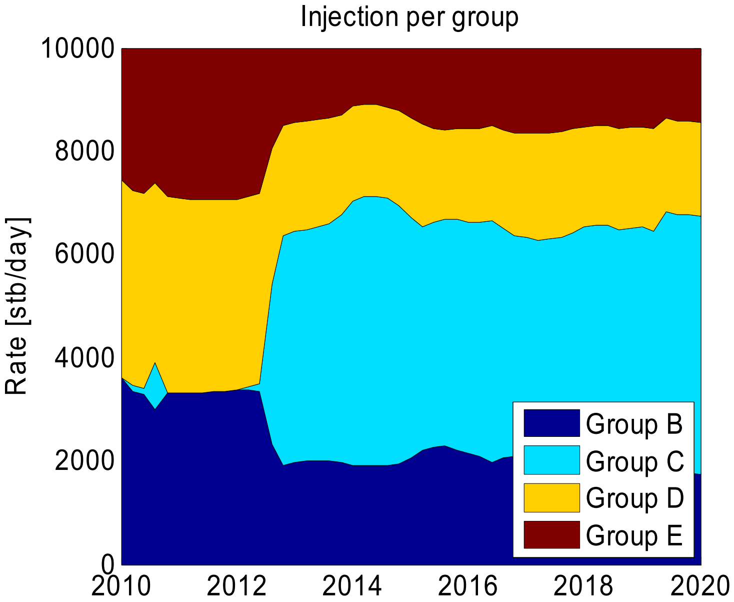




- Cumulative oil production: 12.82 MMstb
- Increase of 11.7% (1.35 MMstb)

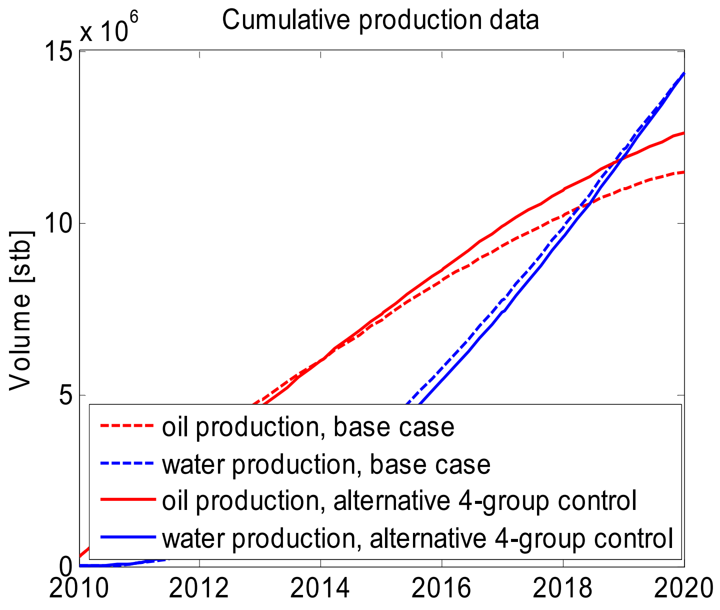
Standard 4-group control (geological insight)

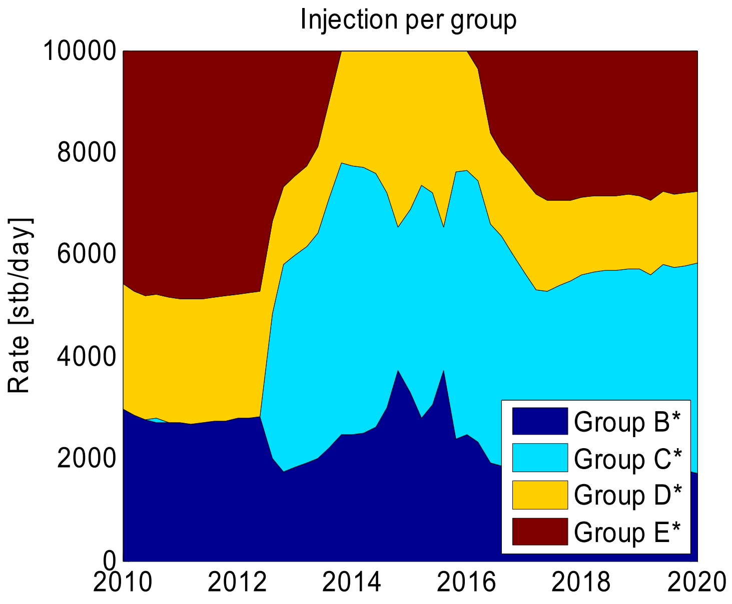

Cumulative production data


Injection per group

- Cumulative oil production: 12.40 MMstb
- Increase of 8.1% (0.93 MMstb)

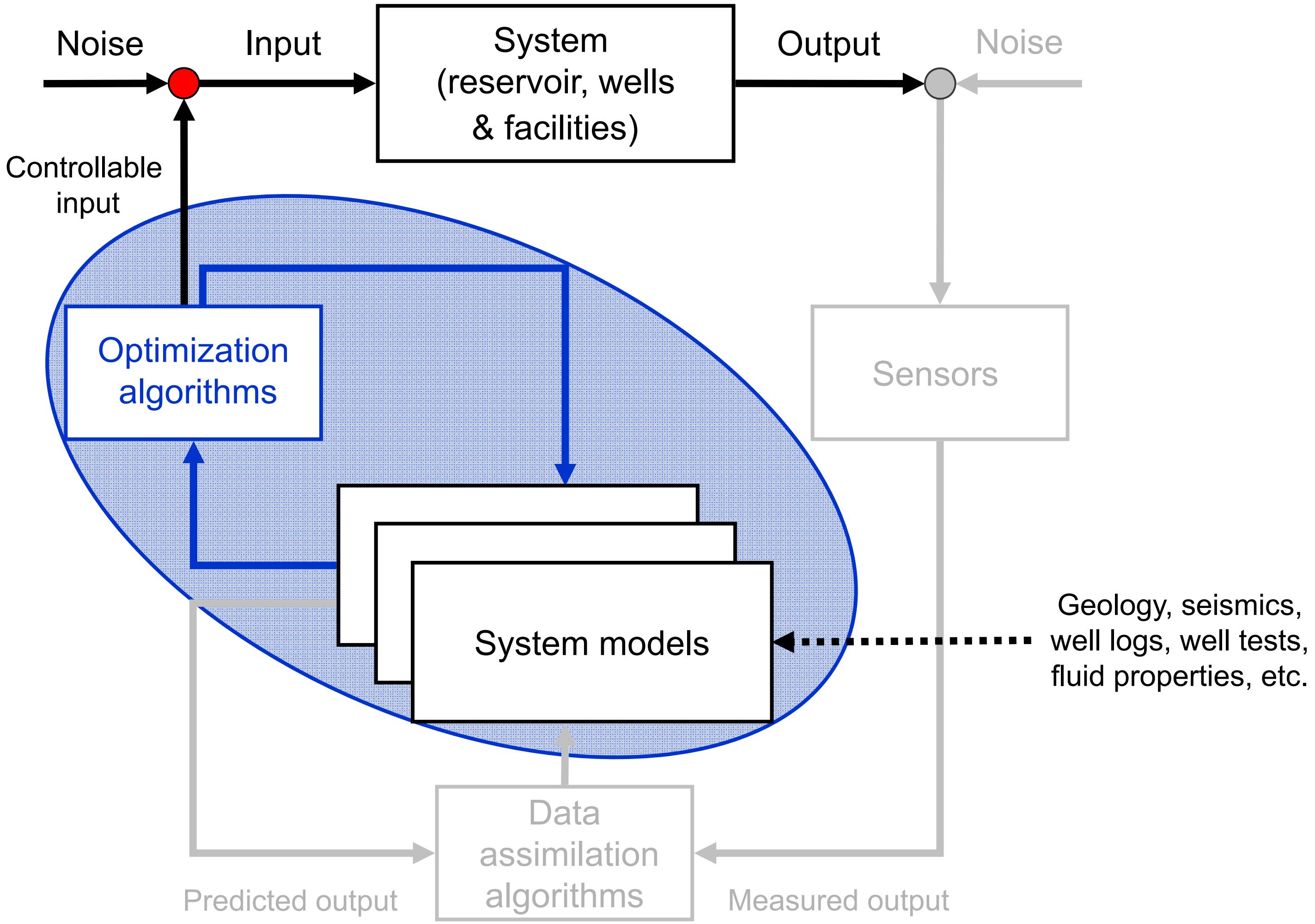
Alternative 4-group control (optimal grouping)



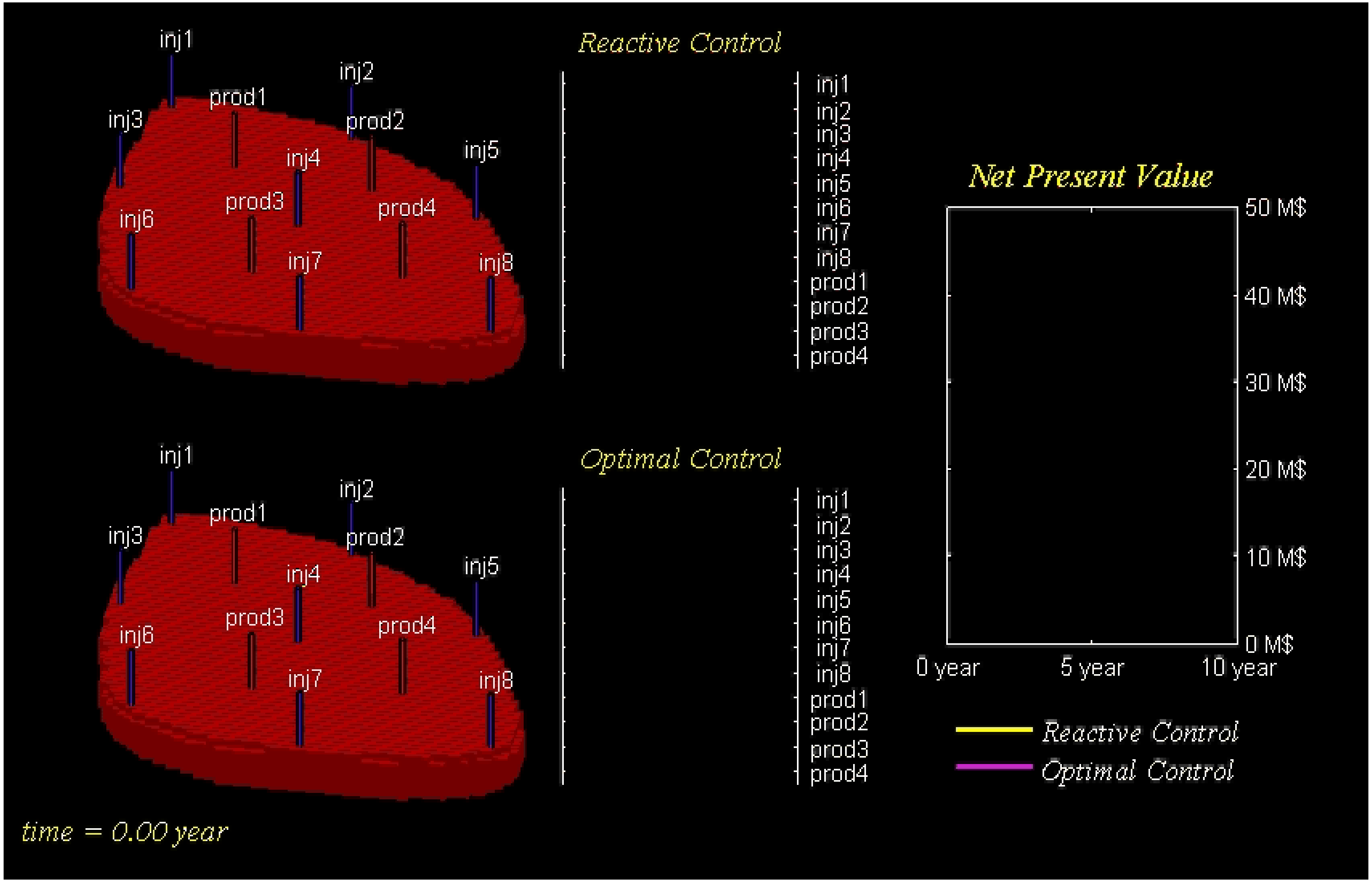


- Cumulative oil production: 12.62 MMstb
- Increase of 10.0% (1.15 MMstb)

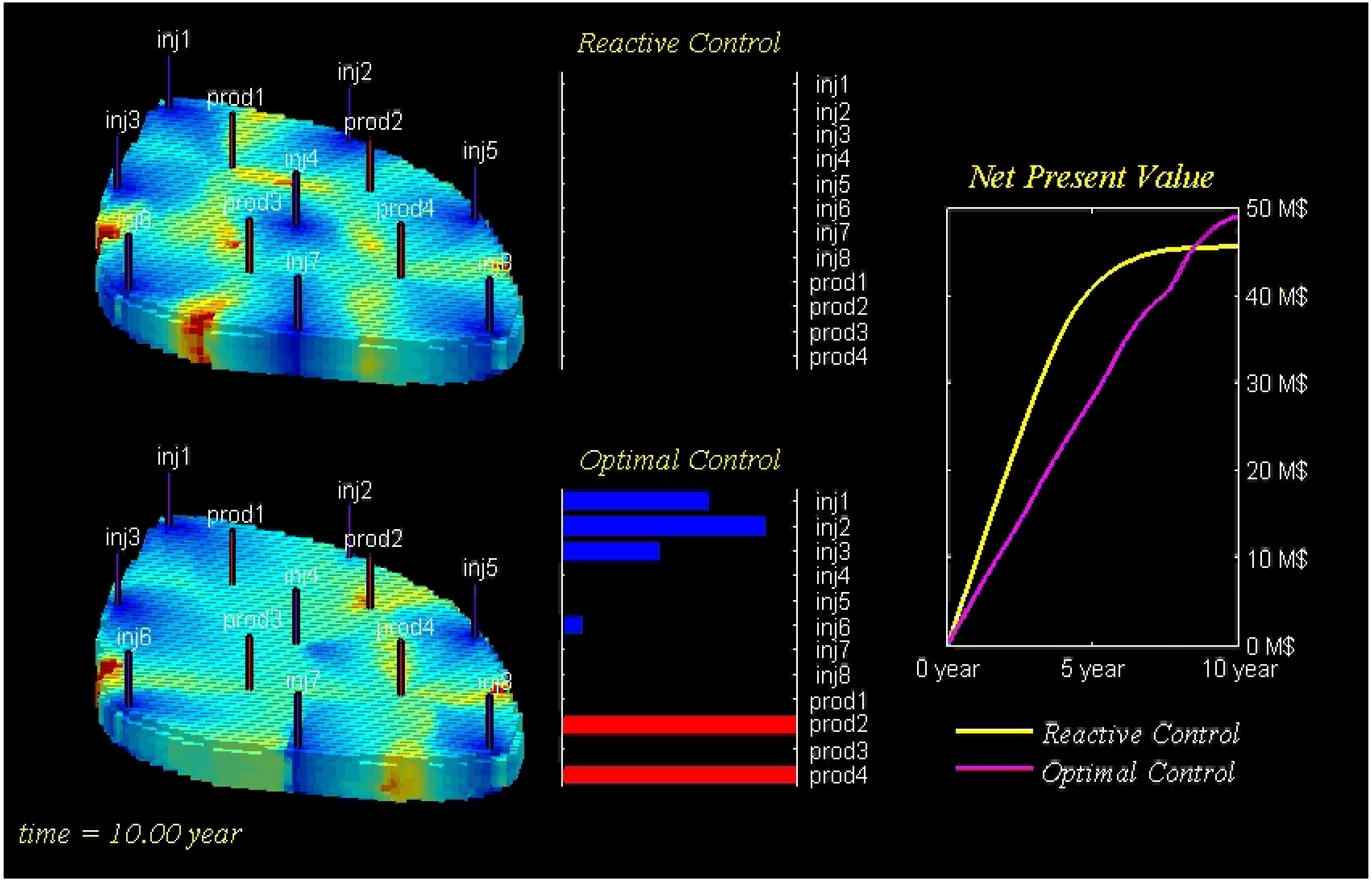
Link with short-term optimization



Life-cycle optimization vs. reactive control (1)

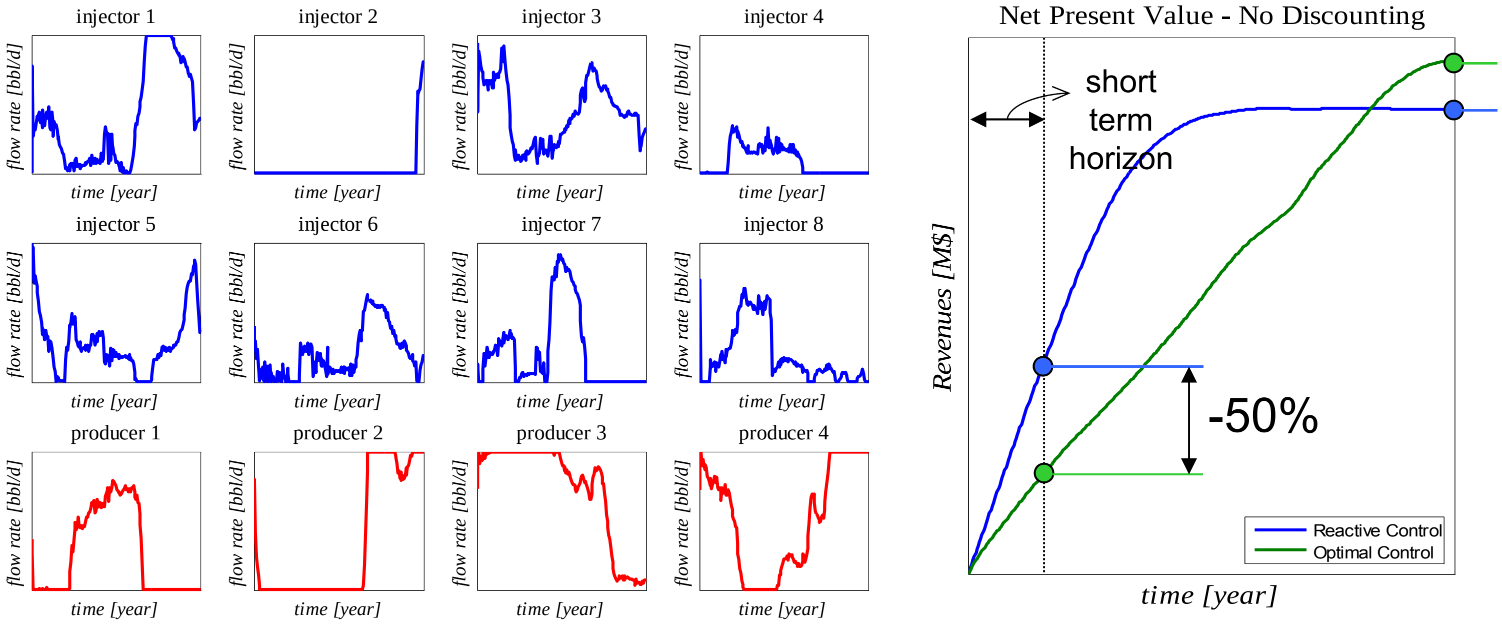


Life-cycle optimization vs. reactive control (2)



Life-cycle optimization vs. reactive control (3)

- Life-cycle optimization attractive for reservoir engineers
  - Increased NPV due to improved sweep efficiency



Van Essen et al., 2011, SPEJ

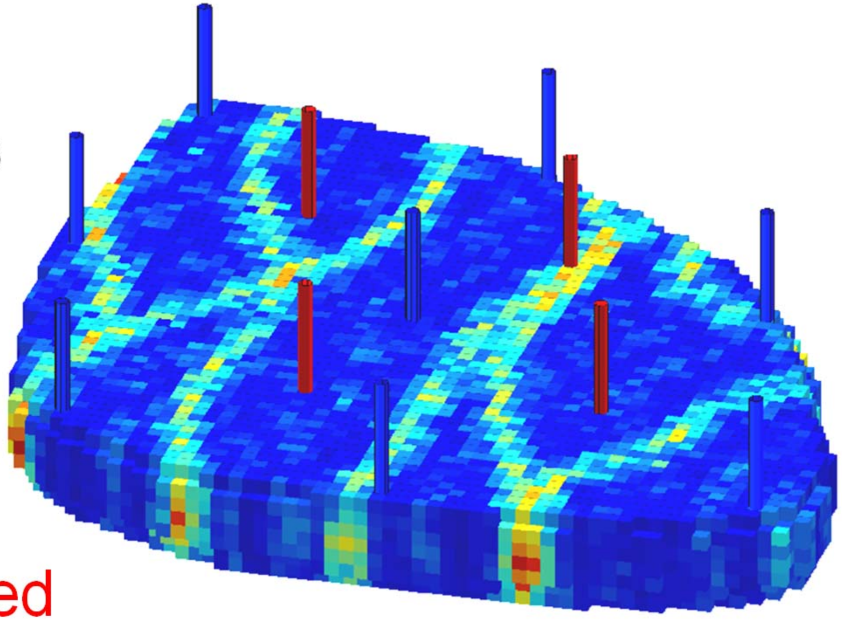
- Not so attractive from production engineering point of view
  - Decreased short term production
  - Erratic behavior of optimal operational strategy

Hierarchical optimization

- Take production objectives into account by incorporating them as additional optimization criteria:
- Formal solution:
  - Order objectives according to importance
  - Optimize objectives sequentially
  - Optimality of upper objective constrains optimization of lower one
- Only possible if there are redundant degrees of freedom in input parameters after meeting primary objective

Objective function with ridges

Example: Hierarchical optimization using null-space approach (1)

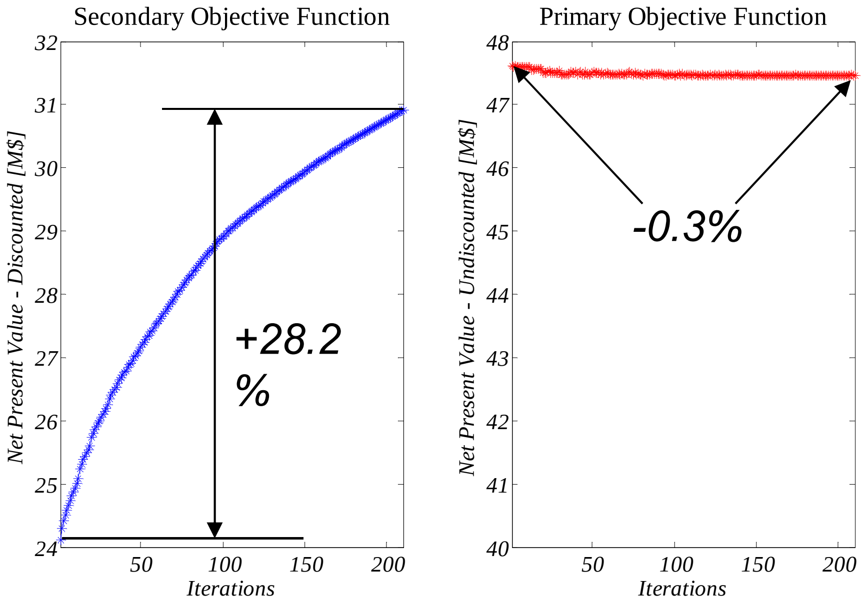


- 3D reservoir
- 8 injection / 4 production wells
- Period of 10 years
- Producers at constant BHP
- Rates in injectors optimized

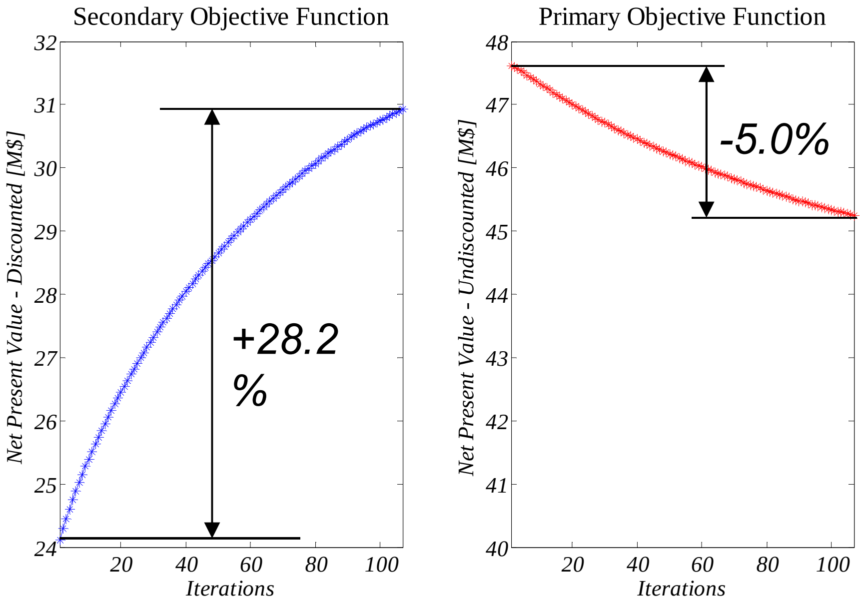
- *Primary objective*: undiscounted NPV over the life of the field
- *Secondary objective*: NPV with very high discount factor (25%) to emphasize importance of short term production

Example: Hierarchical optimization using null-space approach (2)

Optimization of secondary objective function - constrained to null-space of primary objective

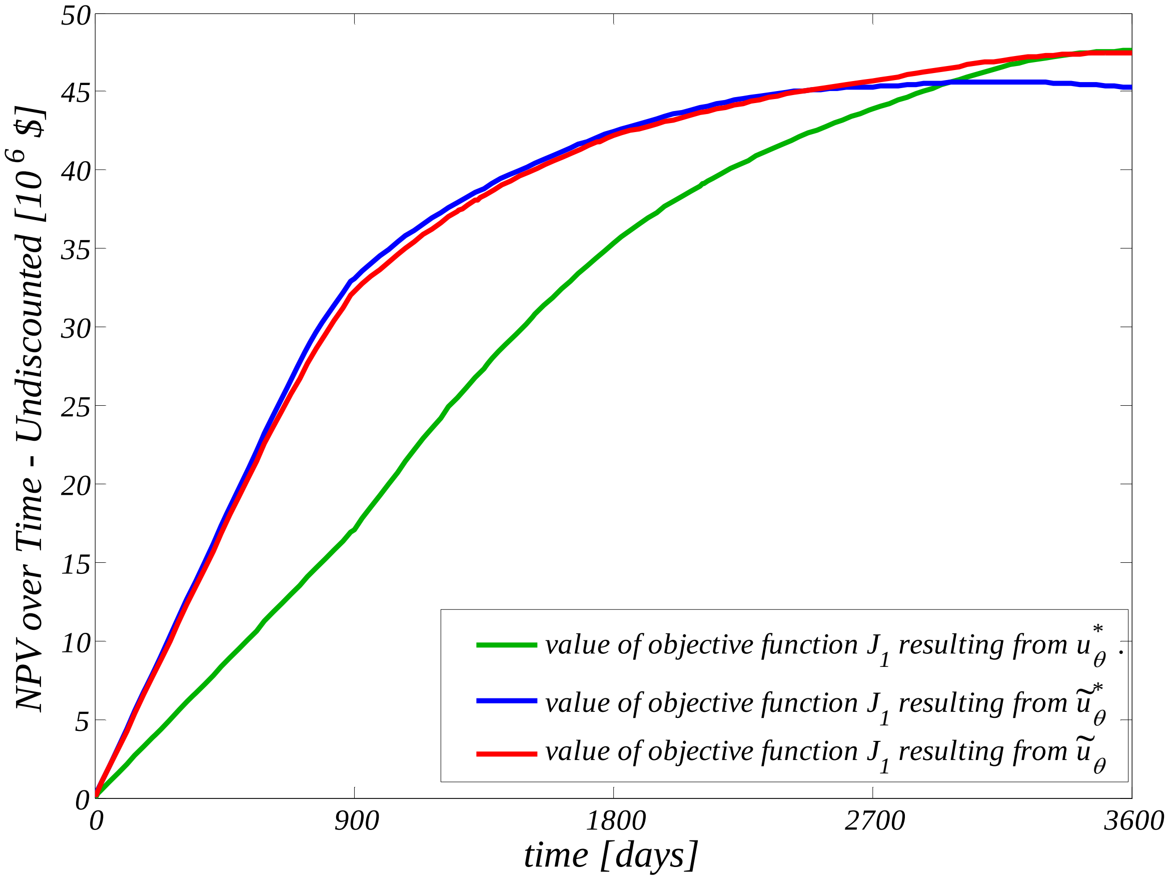


Optimization of secondary objective function - unconstrained

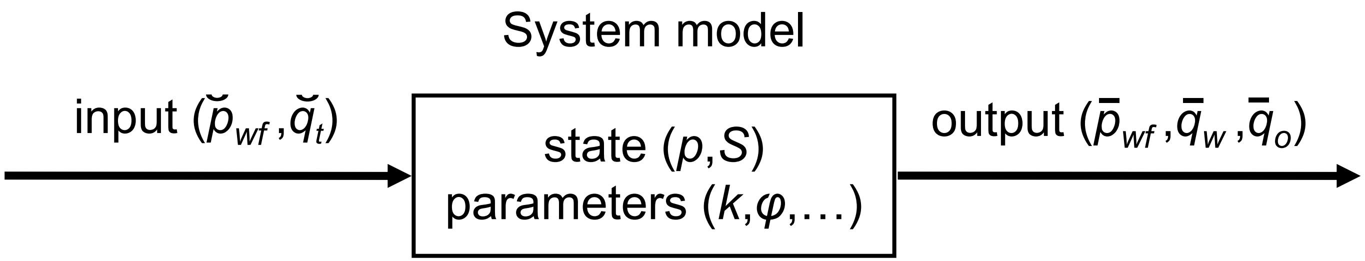


Van Essen et al., 2011, SPEJ

Example: Hierarchical optimization using null-space approach (3)



Observability, controlability, identifiability



Controlability of a dynamic system is the ability to influence the states through manipulation of the inputs.

Observability of a dynamic system is the ability to determine the states through observation of the outputs.

Identifiability of a dynamic system is the ability to determine the parameters from the input-output behavior.

All very limited for reservoir simulation models!

Zandvliet, M. et al., 2008: *Computational Geosciences* **12** (4) 808-822.

Van Doren, J.F.M., et al. 2013: *Computational Geosciences* **17** (5) 773-788.

Model based optimization – conclusions

‘Well control’ optimization :

- Adjoint-based techniques work well; constraints, regularization, storage, efficiency, still to be improved
- Alternatives: gradient-free, particle swarms, EnOpt, StoSAG
- Controllability very limited. Increased by heterogeneities

Well location optimization (not discussed):

- Gradient-free seems to work best
- Combination with well control optimization

Field implementation:

- Well control optimization: none reported
- Acceptance will require combi with short-term optimization
- Computer-assisted history matching: thriving!
- Well location/trajectory optimization: up and coming!
- Advisory mode – tools for discussion

References adjoint-based optimization (1)

Review paper (with additional references)

Jansen, J.D., 2011: Adjoint-based optimization of multiphase flow through porous media – a review. *Computers and Fluids* **46** (1) 40-51. DOI: 10.1016/j.compfluid.2010.09.039.

Early use in history matching

Chavent, G., Dupuy, M. and Lemonnier, P., 1975: History matching by use of optimal theory. *SPE Journal* **15** (1) 74-86. DOI: 10.2118/4627-PA.

Chen, W.H., Gavalas, G.R. and Wasserman, M.L., 1974: A new algorithm for automatic history matching. *SPE Journal* **14** (6) 593-608. DOI: 10.2118/4545-PA.

Li, R., Reynolds, A.C., and Oliver, D.S., 2003: History matching of three-phase flow production data. *SPE Journal* **8** (4): 328-340. DOI: 10.2118/87336-PA.

Early use in flooding optimization

Ramirez, W.F., 1987: *Application of optimal control theory to enhanced oil recovery*, Elsevier, Amsterdam.

Asheim, H., 1988: Maximization of water sweep efficiency by controlling production and injection rates. Paper SPE 18365 presented at the SPE European Petroleum Conference, London, UK, October 16-18. DOI: 10.2118/18365-MS.

Virnovski, G.A., 1991: Water flooding strategy design using optimal control theory, *Proc.* 6th European Symposium on IOR, Stavanger, Norway, 437-446.

References adjoint-based optimization (2)

Zakirov, I.S., Aanonsen, S.I., Zakirov, E.S., and Palatnik, B.M., 1996: Optimization of reservoir performance by automatic allocation of well rates. *Proc.* 5th European Conference on the Mathematics of Oil Recovery (ECMOR V), Leoben, Austria.

Sudaryanto, B. and Yortsos, Y.C., 2000: Optimization of fluid front dynamics in porous media using rate control. *Physics of Fluids* **12** (7) 1656-1670. DOI: 10.1063/1.870417.

TU Delft series

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Van Doren, J.F.M., Markovinović, R. and Jansen, J.D., 2006: Reduced-order optimal control of waterflooding using POD. *Computational Geosciences* **10** (1) 137-158. DOI: 10.1007/s10596-005-9014-2.

Zandvliet, M.J., Bosgra, O.H., Van den Hof, P.M.J., Jansen, J.D. and Kraaijevanger, J.F.B.M., 2007: Bang-bang control and singular arcs in reservoir flooding. *Journal of Petroleum Science and Engineering* **58**, 186-200. DOI: 10.1016/j.petrol.2006.12.008.

Lien, M., Brouwer, D.R., Manseth, T. and Jansen, J.D., 2008: Multiscale regularization of flooding optimization for smart field management. *SPE Journal* **13** (2) 195-204. DOI: 10.2118/99728-PA.

Zandvliet, M.J., Handels, M., Van Essen, G.M., Brouwer, D.R. and Jansen, J.D., 2008: Adjoint-based well placement optimization under production constraints. *SPE Journal* **13** (4) 392-399. DOI: 10.2118/105797-PA.

References adjoint-based optimization (3)

Van Essen, G.M., Zandvliet, M.J., Van den Hof, P.M.J., Bosgra, O.H. and Jansen, J.D., 2009: Robust waterflooding optimization of multiple geological scenarios. *SPE Journal* **14** (1) 202-210. DOI: 10.2118/102913-PA.

Van Essen, G.M., Jansen, J.D., Brouwer, D.R. Douma, S.G., Zandvliet, M.J., Rollett, K.I. and Harris, D.P., 2010: Optimization of smart wells in the St. Joseph field. *SPE Reservoir Evaluation and Engineering* **13** (4) 588-595. DOI: 10.2118/123563-PA.

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Farshbaf Zinati, F., Jansen, J.D. and Luthi, S.M., 2012: Estimating the specific productivity index in horizontal wells from distributed pressure measurements using an adjoint-based minimization algorithm. *SPE Journal* **17** (3) 742-751. DOI: 10.2118/135223-PA.

Namdar Zanganeh, M., Kraaijevanger, J.F.B.M., Buurman, H.W., Jansen, J.D., Rossen, W.R., 2014: Challenges in adjoint-based optimization of a foam EOR process. *Computational Geosciences* **18** (3-4) 563–577. DOI: 10.1007/s10596-014-9412-4.

de Moraes R.J., Rodrigues, J.R.P., Hajibeygi, H. and Jansen, J.D., 2017: Multiscale gradient computation for subsurface flow models. *Journal of Computational Physics*. Published online. DOI: 10.1016/j.jcp.2017.02.024.

References adjoint-based optimization (4)

Computational aspects

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Han, C., Wallis, J., Sarma, P. et al., 2013: Adaptation of the CPR preconditioner for efficient solution of the adjoint equation. *SPE Journal* **18** (2) 207-213. DOI: org/10.2118/141300-PA.

Algebraic formulation

Rodrigues, J.R.P., 2006: Calculating derivatives for automatic history matching. *Computational Geosciences* **10** (1) 119-136. DOI: 10.1007/s10596-005-9013-3.

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