Advanced Gaussian Process Function Approximation for Uncertainty Quantification and Autonomous Experimentation

Focus: Kernel Designs and Scalability

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Motivation: Stochastic Modeling and Autonomous Experimentation

Preliminaries: The Basics of Gaussian-Processes and Autonomous Data Acquisition

> **Challenges:** Approximation Accuracy, UQ, Domain Awareness, and Scalability



Advancements: Flexible Non-Stationary and Compactly Supported Kernel Designs



Synergy: Community and Software



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The **Gaussian stochastic process** is a popular framework for function approximation from noisy data.







Raster scanning is collecting lots of redundant data and only works in 2d.

Random data collection might work in higher dims but is not optimal.

Intuitive experiment control is labor intensive and often suboptimal.



We need a way to choose optimal measurements independent of dimensionality.

Depends on instrument parameters The Autonomous Experiment Loop Automated Sample **Measurements** Depends on synthesis, processing In Situ/Ex Situ Sample Prep, 3D Printing, Robotics, Remote Access Detector image and environmental parameters gp Automated Data Intelligent Analysis/Dim. Decision Making Communication A function over the Reduction Stochastic Processes, Optimization PCA, NMF, NNs, Num. Infrastructure parameter space Integrations, Peak Detection Active By File, ZMQ, S3, Cloud Services Learning

Instrument



Autonomous SAXS Exploration of Nanoscale Ordering in a Blade-Coated Polymer-Grafted Nanorod Film

Facility: AFRL and NSLS II | **Technique:** SAXS | **Collabs:** Strait, Vaia, Fukuto, Yager, Li | **Achievement:** 15% of data required, higher resolution in areas of interest







Center for Functional Nanomaterials Brookhaven National Laboratory







Defect Identification through Autonomous Scanning Tunneling Spectroscopy

Facility: Molecular Foundry @ LBNL | **Technique:** STS Microscopy | **Collabs:** Thomas, Rossi | **Achievement:** ~4% of data required, ~35 hrs vs ~1 mo acq. Time



Autonomous Steering of ARPES Data Acquisition Facilities: ALS @ LBNL | Technique: ARPES | Collabs: Melton, Rotenberg, Zwart, Hexemer | Achievement: 12% of data required







K-Means-Driven Gaussian Process Data Collection for **Angle-Resolved Photoemission Spectroscopy**, Charles N. Melton, Marcus M. Noack, Taisuke Ohta, Thomas E. Beechem, Jeremy Robinson, Xiaotian Zhang, Aaron Bostwick, Chris Jozwiak, Roland J. Koch, Petrus H. Zwart, Alexander Hexemer, and Eli Rotenberg

Autonomous Control of Synchrotron Infrared Spectroscopy

Facility: ALS @ LBNL | **Technique:** IR Spec. Micr. | **Collabs:** Holman, Zwart, Chen, Lee | **Achievement:** ~5% of data required, collected in ~10% of the time, materials targeted



Spectral Map

400 sample points

9.8K sample points

1 hour

9 hours













Estimated Spectral Accuracy



Other applied-science fields benefit from stochastic function approximations and UQ ...

3

Battery-lifetime prediction can be formulated as a stochastic process

Institution: ETA @LBNL | Collabs: Harris, Battaglia, Bakhtian | Achievement/Objective: Early prediction of battery lifetime



Stochastic Climate Modeling via GPs Institution: CASCADE @LBNL | Collabs: Mark Risser, Bill Collins | Achievement/Objective: Large-Scale Stochastic Climate Models





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Gaussian Processes get their name from a normal distribution over functions

$$p(\mathbf{f}) = \frac{1}{\sqrt{(2\pi)^{\dim}|\mathbf{K}|}} \exp\left[-\frac{1}{2}(\mathbf{f} - \mathbf{m})^T \mathbf{K}^{-1}(\mathbf{f} - \mathbf{m})\right]$$
$$\mathbf{K}_{ij} = k(\mathbf{x}_i, \mathbf{x}_j)$$
$$f(\mathbf{x}) = \sum \alpha_i k(\mathbf{x}_i, \mathbf{s}; h)$$
$$m(\mathbf{x}_0) = \boldsymbol{\mu} + \mathbf{k}^T (\mathbf{K} + \mathbf{I}_e)^{-1} (\mathbf{y} - \boldsymbol{\mu})$$
$$\sigma^2(\mathbf{x}_0) = k(\mathbf{x}_0, \mathbf{x}_0) - \mathbf{k}^T (\mathbf{K} + \mathbf{I}_e)^{-1} \mathbf{k}$$

Defining the Covariance, a.k.a. the Kernel Trick

















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uncertainty

ground truth







(2) Standard GPs don't adhere to physical constraints.





(3) Standard GPs might not have (optimal) solutions that satisfy optimization constraints.



(4) GPs don't scale well.

 $\sim\!5000$ climate stations x 10 000 days

2.5 * 10¹⁵ floats

- $2 * 10^{16}$ bytes = $2 * 10^{7}$ Gbytes RAM
- ~ 625 000 desktop computers



Advanced GPs lead to tough optimization problems. (5)

$$\arg \max_{\phi} \left(\log(L(D,\phi)) = -\frac{1}{2} (\mathbf{y} - \boldsymbol{\mu}(\phi)) (\mathbf{K}(\phi) + \mathbf{V})^{-1} (\mathbf{y} - \boldsymbol{\mu}(\phi)) - \frac{1}{2} \log(|\mathbf{K}(\phi) + \mathbf{V}|) - \frac{\dim(\mathbf{y})}{2} \log(2\pi) \right)$$



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(2)

(3)

(4)

Kernels can be utilized to increase the flexibility of the function approximation and to inject domain-awareness into the model



Non-stationary kernels can make UQ much more realistic.

Stationary



d...Euclidean distance *l*... length scale σ_s^2 ... signal variance

Parametric Non-Stationary



$f(\mathbf{x}_1, \mathbf{x}_2) = f(\mathbf{x}_1) f(\mathbf{x}_2) \ k_{stat}(\mathbf{x}_1, \mathbf{x}_2)$ $f(x) = \sum_i^N \alpha_i \beta(\mathbf{x}_i, \mathbf{x}; w)$ $\beta(\mathbf{x}_i, \mathbf{x}; w) = \exp\left[-||\mathbf{x}_i - \mathbf{x}|| \ w^2\right]$

 $w\ldots$ width parameter

Deep Kernel



 $k_{deep} = k_{stat}(||g(\mathbf{x}_1) - g(\mathbf{x}_2)||)$





Non-stationary kernels can make UQ much more realistic.

$$k_{stat}(\mathbf{x}_1, \mathbf{x}_2) = \sigma_s^2 \left(1 - \frac{\sqrt{3d}}{l}\right) \exp\left[-\frac{\sqrt{3d}}{l}\right]$$

d...Euclidean distance *l*... length scale σ_s^2 ... signal variance

$$k_{non}(\mathbf{x}_1, \mathbf{x}_2) = f(\mathbf{x}_1) f(\mathbf{x}_2) \ k_{stat}(\mathbf{x}_1, \mathbf{x}_2)$$
$$f(x) = \sum_i^N \alpha_i \beta(\mathbf{x}_i, \mathbf{x}; w)$$
$$\beta(\mathbf{x}_i, \mathbf{x}; w) = \exp[-||\mathbf{x}_i - \mathbf{x}|| \ w^2]$$

w... width parameter

$$(101) mod \\ (101) mod \\ (10$$

posterior variance





ground truth







Accurate treatment of noise is vital for a successful AE.

$$p(\mathbf{y}|\mathbf{f}) = \frac{1}{\sqrt{(2\pi)^{\dim}|\mathbf{V}|}} \exp\left[-\frac{1}{2}(\mathbf{y}-\mathbf{f})^T \mathbf{V}^{-1}(\mathbf{y}-\mathbf{f})\right] \stackrel{\text{g}}{\approx}$$



 $y \in [0, 1]$

N = 50

N = 200



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Kernels can constrain the function space (RKHS) to only allow for

physically-viable solutions Facilities: Thales @ ILL, Grenoble, France | Technique: Neutron Scattering | Collabs: Boehm, Mutti, Weber | Achievement: Constraining the RKHS to 6-fold symmetric functions

$$k(\mathbf{x}_i, \mathbf{x}_j) = \frac{1}{36} \sum_{\phi} \sum_{\theta} \tilde{k}(\mathcal{R}_{\phi} \mathbf{x}_i, \mathcal{R}_{\theta} \mathbf{x}_j)$$











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BRA

NATIONAL LABORATORY

NATIONAL SYNCHROTRON LIGHT SOURCE I

Physics Knowledge in the Form of Periodicity for X-Ray Scattering

Facility: NIST, CFN, NSLS II | **Technique:** SAXS, GISAXS | **Collabs**: Yager, Fukuto, Seppala | **Achievement:** Use of non-stationary kernels to learn and exploit local characteristics





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Kernels can do even more...



Autonomous exploration of multidimensional material state-spaces underlying self-assembly of copolymer mixtures

Facility: CFN & NSLS II at BNL | **Technique:** small angle x-ray scattering | **Collabs:** Russel, Fukuto, Yager | **Achievement:** Optimal mapping of a 5-dim. material state-space



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What do we know about the physics:

- 1. Space distances are isotropic in 2 directions and anisotropic in time direction.
- 2. The model function is in [0,1].
- 3. The sum across all 8 tasks at any position in the input space is 1.
- 4. 1st order differentiability in all directions





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gp2Scale: Exact Gaussian Processes on Millions of Data Points

— let kernels discover, not induce, sparsity

Building Block 1: Ultra-Flexible, Compactly Supported and Non-Stationary Kernels



 $k:\mathcal{X} imes\mathcal{X} o\mathbb{R}$



Building Block 3: MCMC Constrained Training $argmax_h \ln(L)$ subject to s < sparsity requirement $argmax_{h,s} \left(\ln(L) + (1-s) \ln(L)
ight)$

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0.26436384, 0.21300795, 0.19834864,, 0.20183792, 0.82454492, 0.81746336,
0.68064155, 0.66793227, 0.02274104,, 0.37098925, 0.66477699, 0.71282514,
0.13523289, 0.85643443, 0.43357488,, 0.71829634, 0.98986933, 0.60671197,
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0.10520203, 0.50040440, 0.40001400,, 0.11025004, 0.50500533,0.00011151

The Traditional Training/Optimization Workflow needs a Large Number of Function Evaluations and Blocks the Main Thread





0.26436384, 0.21300795, 0.19834864, ..., 0.20183792, 0.82454492, 0.81746336 0.68064155, 0.66793227, 0.02274104, ..., 0.37098925, 0.66477699, 0.71282514, 0.13523289, 0.85643443, 0.43357488, ..., 0.71829634, 0.98986933, 0.60671197, 0.43558082, 0.95638452, 0.99928695, ..., 0.63067478, 0.38601846, 0.52014594 0.09019633, 0.03045269, 0.55291218, ..., 0.66801905, 0.75265345, 1.64352323 0.68064155, 0.66793227, 0.02274104, ..., 0.37098925, 0.66477699, 0.71282514, 0.13523289, 0.85643443, 0.43357488, ..., 0.71829634, 0.98986933, 0.60671197 0.43558082, 0.95638452, 0.99928695, ..., 0.63067478, 0.38601846, 0.5201459 0.68064155, 0.66793227, 0.02274104, ..., 0.37098925, 0.66477699, 0.71282514, 0.13523289, 0.85643443, 0.43357488, ..., 0.71829634, 0.98986933, 0.60671197 0.43558082, 0.95638452, 0.99928695, ..., 0.63067478, 0.38601846, 0.52014594 0.43558082, 0.95638452, 0.99928695, ..., 0.63067478, 0.38601846, 0.52014594 0.09019633, 0.03045269, 0.55291218, ..., 0.66801905, 0.75265345, 1.64352323 0.68064155, 0.66793227, 0.02274104, ..., 0.37098925, 0.66477699, 0.71282514 0.13523289, 0.85643443, 0.43357488, ..., 0.71829634, 0.98986933, 0.60671197 0.43558082, 0.95638452, 0.99928695, ... 0.63067478, 0.38601846,0.52014594 0.68064155, 0.66793227, 0.02274104, ..., 0.37098925, 0.66477699, 0.71282514 0.13523289, 0.85643443, 0.43357488, ..., 0.71829634, 0.98986933, 0.60671197 0.43558082, 0.95638452, 0.99928695, ..., 0.63067478, 0.38601846, 0.52014594 0.09019633, 0.03045269, 0.55291218, ..., 0.66801905, 0.75265345, 1.64352323 0.68064155, 0.66793227, 0.02274104, ..., 0.37098925, 0.66477699, 0.71282514, 0.13523289, 0.85643443, 0.43357488, ..., 0.71829634, 0.98986933, 0.60671197 0.43558082, 0.95638452, 0.99928695, ..., 0.63067478, 0.38601846, 0.52014594 0.68064155, 0.66793227, 0.02274104, ..., 0.37098925, 0.66477699, 0.71282514, 0.13523289, 0.85643443, 0.43357488, ..., 0.71829634, 0.98986933, 0.60671197

Minimizing Number of Function Evaluations: Asynchronous Distributed Training



Optimization of the Log-Likelihood with HGDL

4

Using High Performance Asynchronous Distributed Optimization for Robust and Efficient GP Training.

$$\begin{split} \log(L(\mathcal{D};\phi,\mu)) &= \\ &-\frac{1}{2}(\mathbf{y}-\boldsymbol{\mu}(\mathbf{x}))^T(\mathbf{K}(\phi)+\mathbf{I}_e)^{-1}(\mathbf{y}-\boldsymbol{\mu}(\mathbf{x}))-\frac{1}{2}\log(|\mathbf{K}(\phi)+\mathbf{I}_e|) \end{split}$$

Optimization Challenges: Ill-Posedness, Non-Uniqueness, Costly Func. Evals + High-Dim., Blocking Execution





HGDL yields:

- 1. a set of unique solutions
- HPC readiness of training and prediction
- 3. asynchronous training
- 4. Measure for uniqueness of solutions



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gpCAM is a CAMERA project that is the result of a broad collaboration between many institutions and facilities.





CASE: A Community for Autonomous Scientific Experimentation



Established in 2021 217+ members

https://autonomous-discovery.lbl.gov/



The product is three APIs



init hyperparameters = np.array([1,1,1]) hyperparameter bounds = np.array([[0.01,100],[0.01,100.0],[0.01,100]]) my ae = AutonomousExperimenterGP (parameters, instrument, init hyperparameters, hyperparameter bounds, init dataset size=10)

entry["value"] = np.sin(np.linalg.norm(entry["position"]))

gpcam.lbl.gov

62000 downloads 21 facilities and counting A GUI, called Tsuchinoko, is being developed by Ron Pandolfi pip install tsuchinoko





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