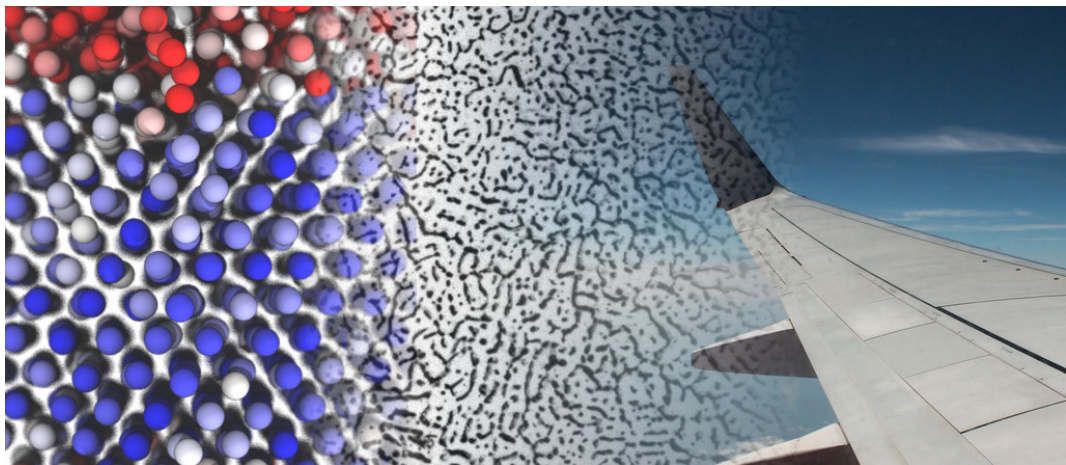


Machine learning for atomic-scale modeling: potentials and beyond

Michele Ceriotti
cosmo.epfl.ch

Mechanistic insights from atomic-scale modeling

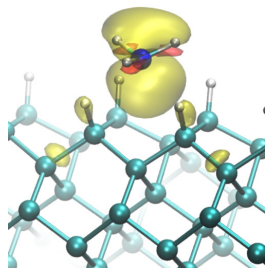
- Foundations for the predictive modeling of chemicals and materials
- Key challenge: accurate electronic properties + sampling of fluctuations/defects
- Machine-learning to the rescue, without losing the mechanistic insights



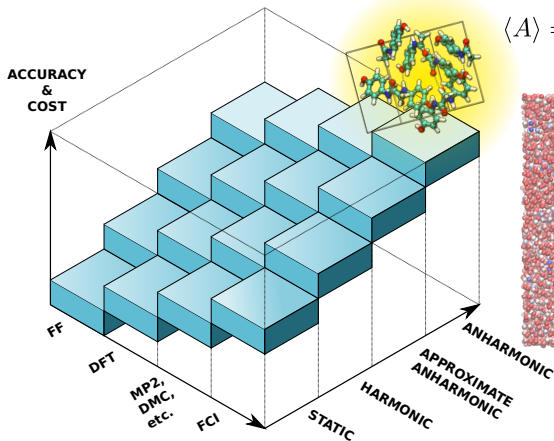
Mechanistic insights from atomic-scale modeling

- Foundations for the predictive modeling of chemicals and materials
- Key challenge: accurate electronic properties + sampling of fluctuations/defects
- Machine-learning to the rescue, without losing the mechanistic insights

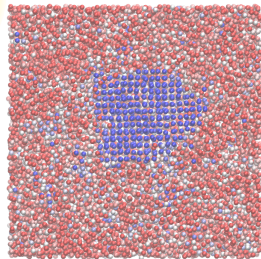
$$\hat{H}(\mathbf{q})|\Psi\rangle = V(\mathbf{q})|\Psi\rangle$$



**ACCURACY
OF
ENERGETICS**



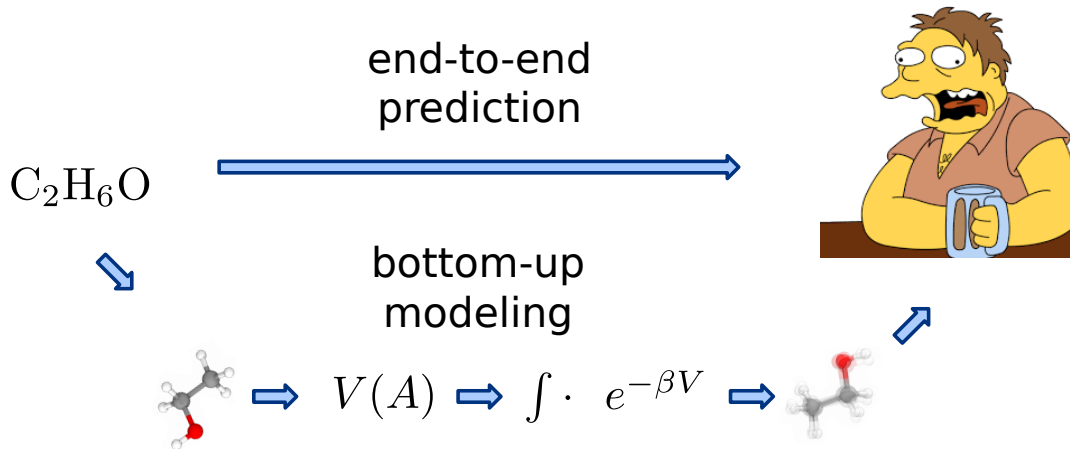
$$\langle A \rangle = \int d\mathbf{q} e^{-\beta V(\mathbf{q})} A(\mathbf{q})$$



**ACCURACY
OF
SAMPLING**

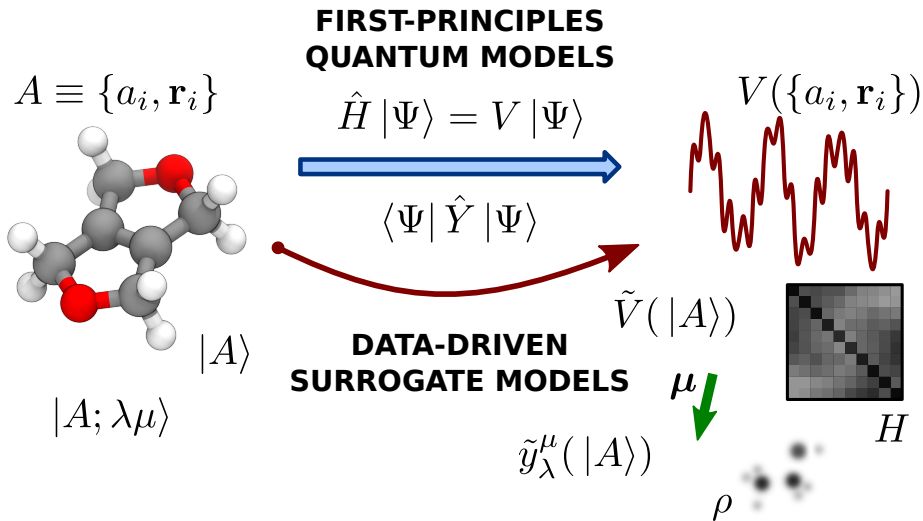
Mechanistic insights from atomic-scale modeling

- Foundations for the predictive modeling of chemicals and materials
- Key challenge: accurate electronic properties + sampling of fluctuations/defects
- Machine-learning to the rescue, without losing the mechanistic insights

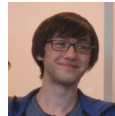


Surrogate models for quantum mechanics

- Electronic-structure calculations predict accurately materials properties
- Machine-learning models provide inexpensive approximations



A unified theory of atomic-scale machine learning



Machine learning à la carte

- Understanding the ingredients and the mixing rules to build custom ML frameworks for any type of atomistic modeling task



Machine learning à la carte

- Understanding the ingredients and the mixing rules to build custom ML frameworks for any type of atomistic modeling task



completeness



body-ordered
correlations



feature
engineering



model
nonlinearity



locality



message
passing



symmetry



equivariant
iterations

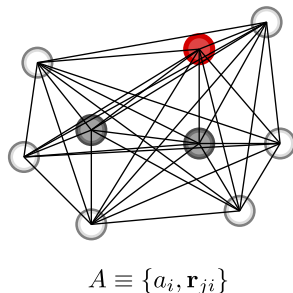
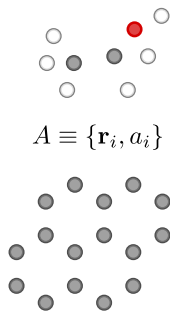
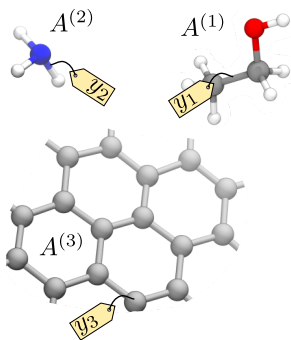
Machine learning à la carte

- Understanding the ingredients and the mixing rules to build custom ML frameworks for any type of atomistic modeling task



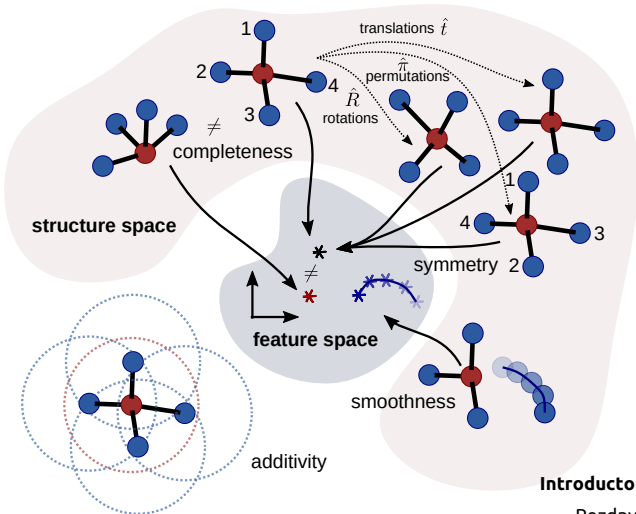
Molecules as atom clouds

- Molecules and materials can be treated as 3D point clouds, decorated by the chemical nature of the atoms
- An alternative view: each molecule is a fully-connected graph, with atoms as nodes and *separating vectors* as edges



Symmetry and physical priors

- Physical/mathematical requirements are imposed on the structure \rightarrow [representation] \rightarrow property mapping
- Additivity/locality + translation equivariance \rightarrow atom-centered formalism
- Roto-inversion ($O(3)$) and index permutation \rightarrow full equivariance

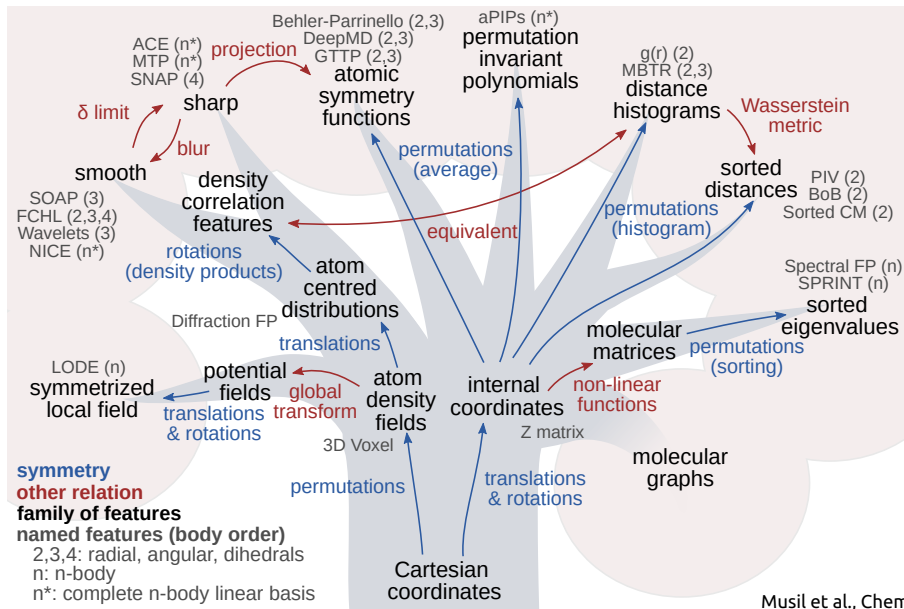


Introductory review: Musil et al., Chem. Rev. (2021)

Pozdnyakov et al. PRL (2020); arXiv: 2302.14770

A universal feature construction

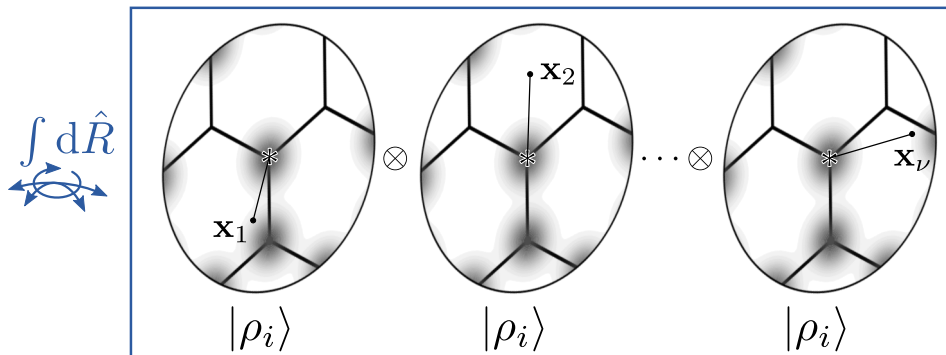
- Physical requirements constrain the design space of ML models



Musil et al., Chem. Rev. (2021)

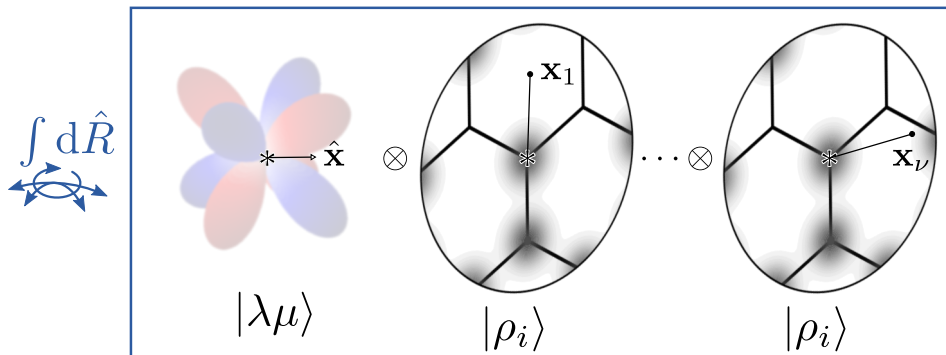
A universal feature construction

- Physical requirements constrain the design space of ML models
- Most frameworks can be expressed in terms of symmetrized n -body correlations of atom positions. Main difference - the choice of basis.
- Can be extended to encompass most ML architectures, including equivariance and message-passing



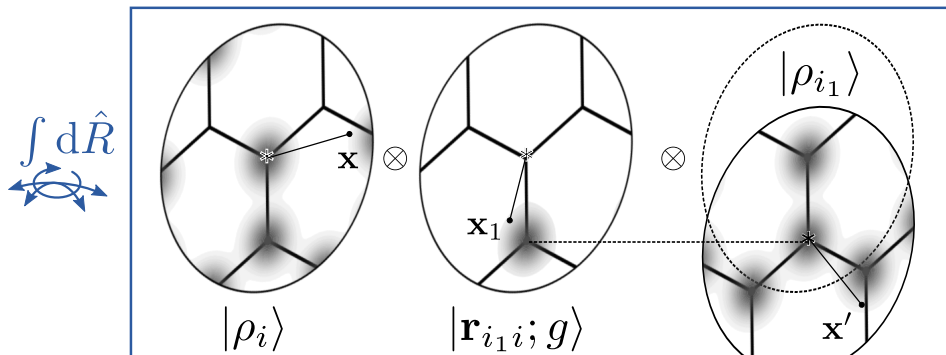
A universal feature construction

- Physical requirements constrain the design space of ML models
- Most frameworks can be expressed in terms of symmetrized n -body correlations of atom positions. Main difference - the choice of basis.
- Can be extended to encompass most ML architectures, including equivariance and message-passing



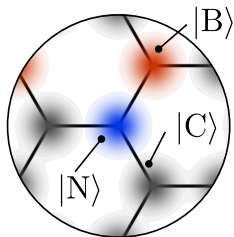
A universal feature construction

- Physical requirements constrain the design space of ML models
- Most frameworks can be expressed in terms of symmetrized n -body correlations of atom positions. Main difference - the choice of basis.
- Can be extended to encompass most ML architectures, including equivariance and message-passing



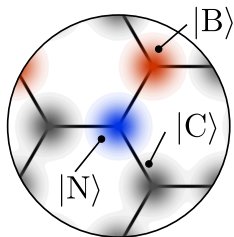
Machine-learning with pseudoelements

- How to learn with multiple species? Decorate atomic Gaussian with elemental kets $|C\rangle, |N\rangle, \dots$
- Tensor-product features lead to exponential scaling with the number of elements
- Expand each ket in a finite basis, $|b\rangle = \sum_a u_{ba} |a\rangle$. Optimize coefficients for “alchemical learning”



Machine-learning with pseudoelements

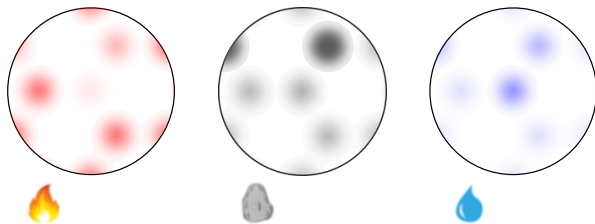
- How to learn with multiple species? Decorate atomic Gaussian with elemental kets $|C\rangle, |N\rangle, \dots$
- Tensor-product features lead to exponential scaling with the number of elements
- Expand each ket in a finite basis, $|b\rangle = \sum_a u_{ba} |a\rangle$. Optimize coefficients for “alchemical learning”



$$|C\rangle = 0.5 |\text{fire}\rangle + 0.1 |\text{stone}\rangle + 0.2 |\text{water}\rangle$$

$$|B\rangle = 0.2 |\text{fire}\rangle + 0.8 |\text{stone}\rangle + 0.3 |\text{water}\rangle$$

$$|N\rangle = 0.1 |\text{fire}\rangle + 0.1 |\text{stone}\rangle + 0.6 |\text{water}\rangle$$



Empedocles et al. (ca 360BC). Metaphor courtesy of Albert Bartók.
Willatt, Musil, MC, PCCP (2018)

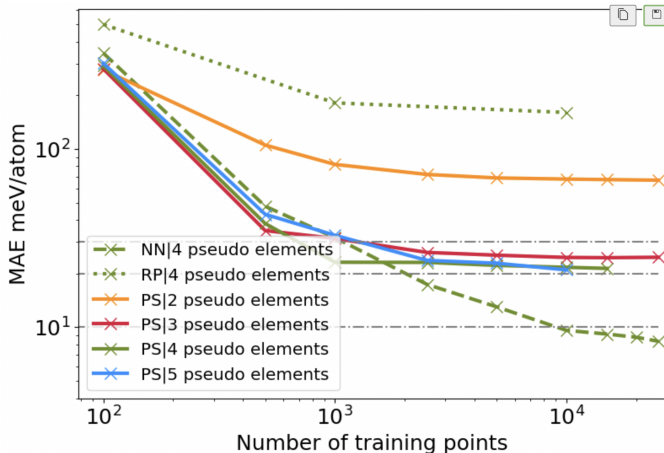
A potential for transition metal alloys

- 25 *d*-block elements that are used in high-entropy alloys.
- ~25'000 training structures with distorted *fcc* and *bcc* structures
- Alchemical compression with 3-body potential saturates at 3-4 pseudoelements.
- A non-linear layer brings error below 10meV MAE
- Interpretability: the alchemical weights reflect the ordering in the periodic table, with a twist

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Period 1	H														
Period 2	Li	Be											B	C	N
Period 3	Na	Mg											Al	Si	P
Period 4	K	Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	Ga	Ge	As
Period 5	Rb	Sr	39 Y	40 Zr	41 Nb	42 Mo	Tc	44 Ru	45 Rh	46 Pd	47 Ag	Cd	In	Sn	Sb
Period 6	Cs	Ba	71 Lu	72 Hf	73 Ta	74 W	Re	Os	77 Ir	78 Pt	79 Au	Hg	Tl	Pb	Bi

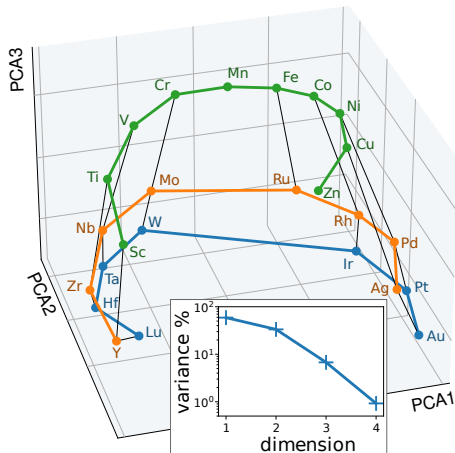
A potential for transition metal alloys

- 25 *d*-block elements that are used in high-entropy alloys.
- ~25'000 training structures with distorted *fcc* and *bcc* structures
- Alchemical compression with 3-body potential saturates at 3-4 pseudoelements.
- A non-linear layer brings error below 10meV MAE
- Interpretability: the alchemical weights reflect the ordering in the periodic table, with a twist



A potential for transition metal alloys

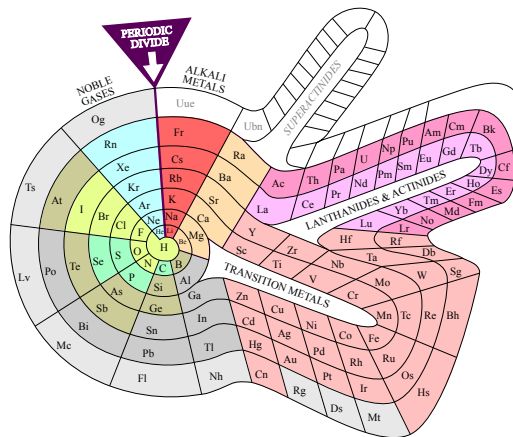
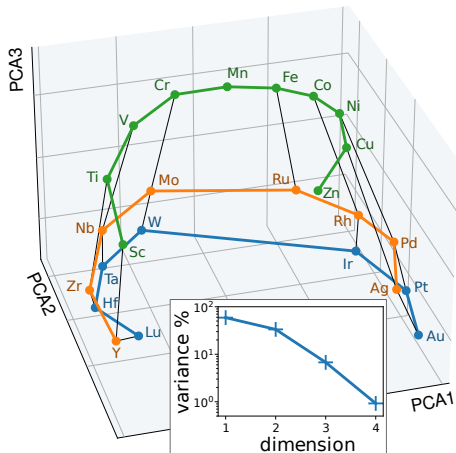
- 25 *d*-block elements that are used in high-entropy alloys.
- ~25'000 training structures with distorted *fcc* and *bcc* structures
- Alchemical compression with 3-body potential saturates at 3-4 pseudoelements.
- A non-linear layer brings error below 10meV MAE
- Interpretability: the alchemical weights reflect the ordering in the periodic table, with a twist



Lopnitsyna, Fraux, Springer, De, MC, arXiv: 2212.13254

A potential for transition metal alloys

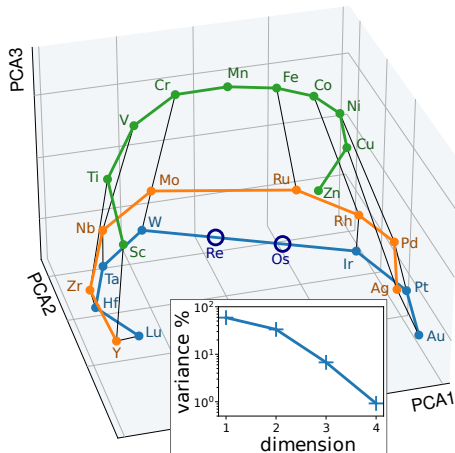
- 25 *d*-block elements that are used in high-entropy alloys.
- ~25'000 training structures with distorted *fcc* and *bcc* structures
- Alchemical compression with 3-body potential saturates at 3-4 pseudoelements.
- A non-linear layer brings error below 10meV MAE
- Interpretability: the alchemical weights reflect the ordering in the periodic table, with a twist



By DePiep - Own work, CC BY-SA 3.0, <https://commons.wikimedia.org/w/index.php?curid=27766488>

A potential for transition metal alloys

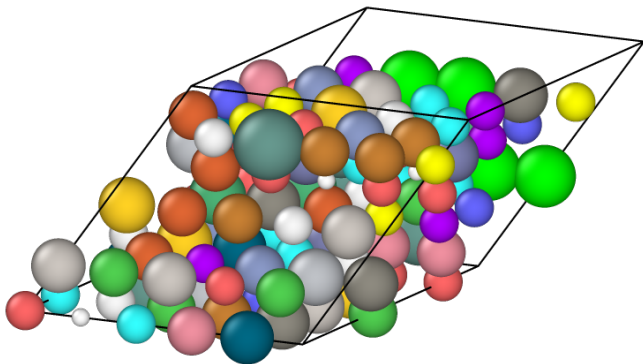
- 25 *d*-block elements that are used in high-entropy alloys.
- ~25'000 training structures with distorted *fcc* and *bcc* structures
- Alchemical compression with 3-body potential saturates at 3-4 pseudoelements.
- A non-linear layer brings error below 10meV MAE
- Interpretability: the alchemical weights reflect the ordering in the periodic table, with a twist



Lopnitsyna, Fraux, Springer, De, MC, arXiv: 2212.13254

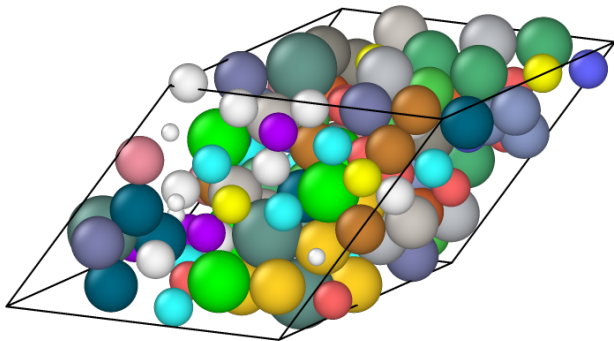
A computational Cantor alloy experiment

- Simulating a mixture of 25 elements at finite temperature, with replica exchange molecular dynamics and Monte Carlo particle exchanges
- Short-range order and segregation, with DFT accuracy



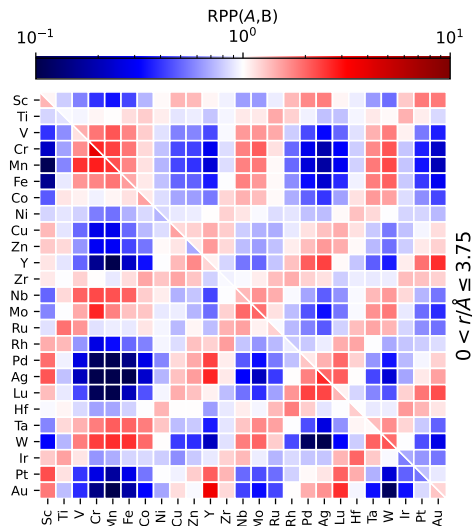
A computational Cantor alloy experiment

- Simulating a mixture of 25 elements at finite temperature, with replica exchange molecular dynamics and Monte Carlo particle exchanges
- Short-range order and segregation, with DFT accuracy



A computational Cantor alloy experiment

- Simulating a mixture of 25 elements at finite temperature, with replica exchange molecular dynamics and Monte Carlo particle exchanges
- Short-range order and segregation, with DFT accuracy

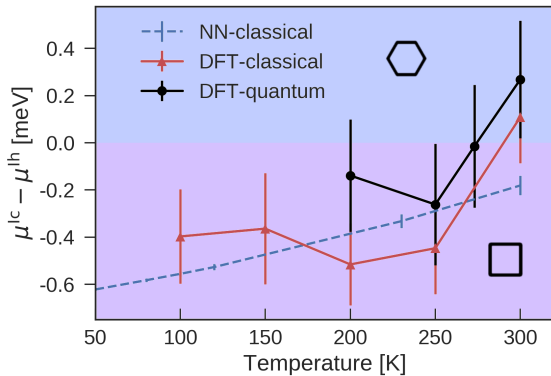
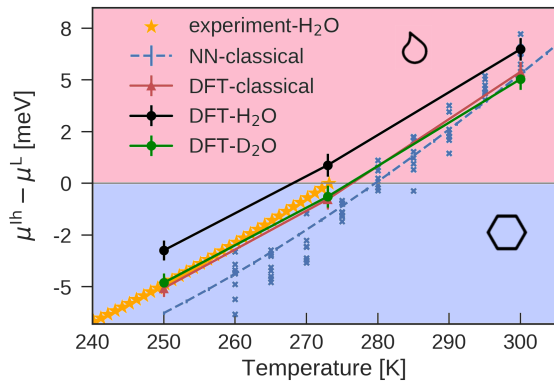


ML for atomistic modeling: thermodynamics, and beyond



Ab initio (thermo)dynamics made easy

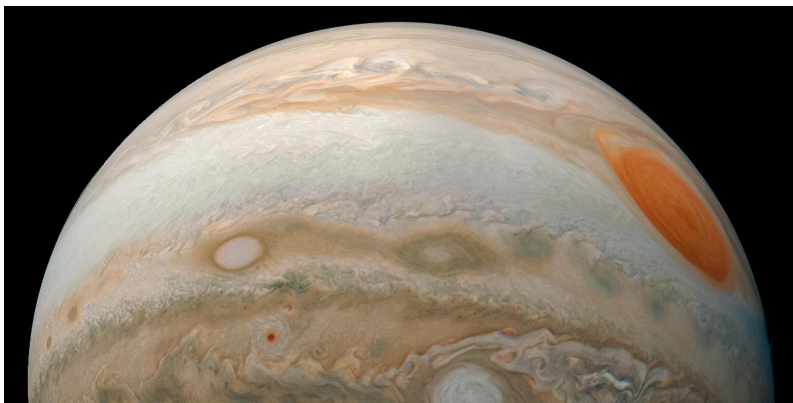
- Simulating matter at finite temperature, including quantum effects and dynamics is now much more affordable
- Accuracy (reference, long range physics, extrapolation) is still a concern: baselining, uncertainty quantification, free energy perturbation...



Cheng, Engel, Behler, Dellago, MC, PNAS (2019)

Ab initio (thermo)dynamics made easy

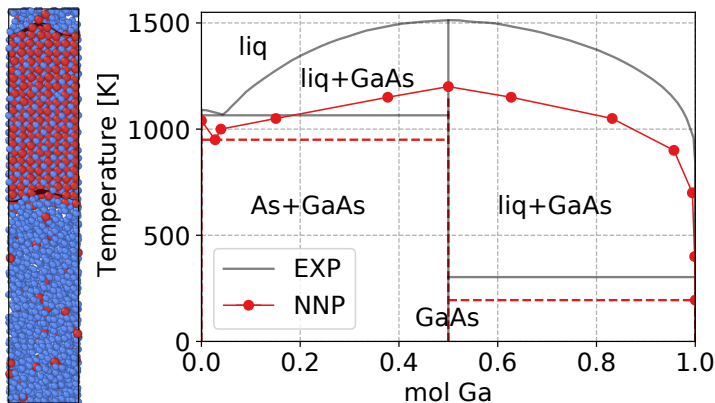
- Simulating matter at finite temperature, including quantum effects and dynamics is now much more affordable
- Accuracy (reference, long range physics, extrapolation) is still a concern: baselining, uncertainty quantification, free energy perturbation...



Cheng, Mazzola, Pickard, MC, Nature (2020)

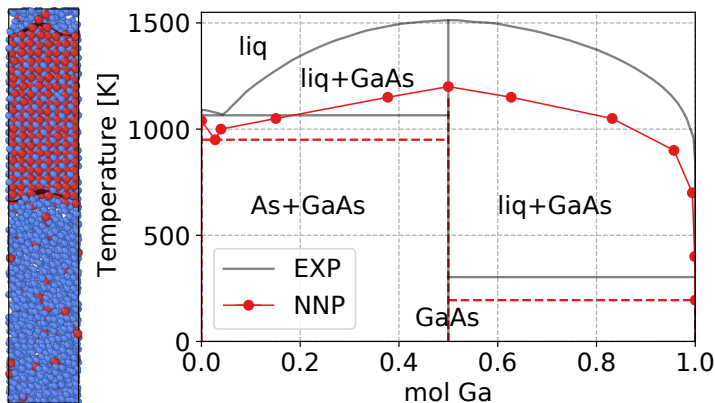
Ab initio (thermo)dynamics made easy

- Simulating matter at finite temperature, including quantum effects and dynamics is now much more affordable
- Accuracy (reference, long range physics, extrapolation) is still a concern: baselining, uncertainty quantification, free energy perturbation...



Ab initio (thermo)dynamics made easy

- Simulating matter at finite temperature, including quantum effects and dynamics is now much more affordable
- Accuracy (reference, long range physics, extrapolation) is still a concern: baselining, uncertainty quantification, free energy perturbation...



Calibrated committee models

- Ensemble of N_{RS} models, trained on subsets of the train set

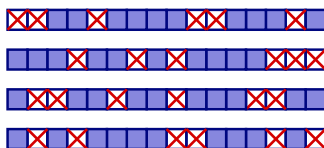
$$\bar{y}(A) = \frac{1}{N_{RS}} \sum_j \tilde{y}_j(A), \quad \sigma^2(A) = \frac{1}{N_{RS} - 1} \sum_i (\tilde{y}_i(A) - \bar{y}(A))^2$$

- Verify accuracy by the distribution of errors $P(|\bar{y}(A) - y_A(A)| | \sigma(A))$
- Adjust the spread by maximum likelihood $\tilde{y}_j \leftarrow \bar{y} + \alpha (\tilde{y}_j - \bar{y})$
- Calibrated model can be used for easy uncertainty propagation

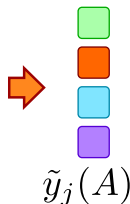
1. train set $\{(A, y_A)\}$



2. subselection



3. committee of models



Calibrated committee models

- Ensemble of N_{RS} models, trained on subsets of the train set

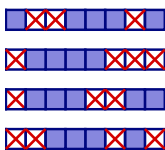
$$\bar{y}(A) = \frac{1}{N_{RS}} \sum_j \tilde{y}_j(A), \quad \sigma^2(A) = \frac{1}{N_{RS} - 1} \sum_i (\tilde{y}_j(A) - \bar{y}(A))^2$$

- Verify accuracy by the distribution of errors $P(|\bar{y}(A) - y_A(A)| | \sigma(A))$
- Adjust the spread by maximum likelihood $\tilde{y}_j \leftarrow \bar{y} + \alpha (\tilde{y}_j - \bar{y})$
- Calibrated model can be used for easy uncertainty propagation

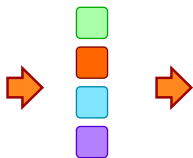
$\{(A, y_A)\}$



on

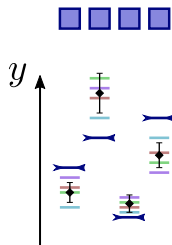


3. committee of models



$\tilde{y}_j(A)$

4. validation



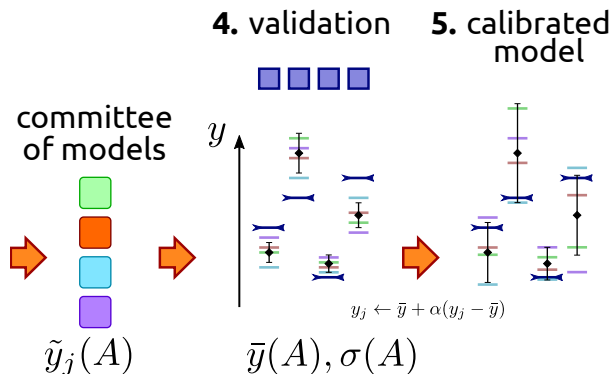
$\bar{y}(A), \sigma(A)$

Calibrated committee models

- Ensemble of N_{RS} models, trained on subsets of the train set

$$\bar{y}(A) = \frac{1}{N_{RS}} \sum_j \tilde{y}_j(A), \quad \sigma^2(A) = \frac{1}{N_{RS} - 1} \sum_i (\tilde{y}_j(A) - \bar{y}(A))^2$$

- Verify accuracy by the distribution of errors $P(|\bar{y}(A) - y_A(A)| | \sigma(A))$
- Adjust the spread by maximum likelihood $\tilde{y}_j \leftarrow \bar{y} + \alpha(\tilde{y}_j - \bar{y})$
- Calibrated model can be used for easy uncertainty propagation

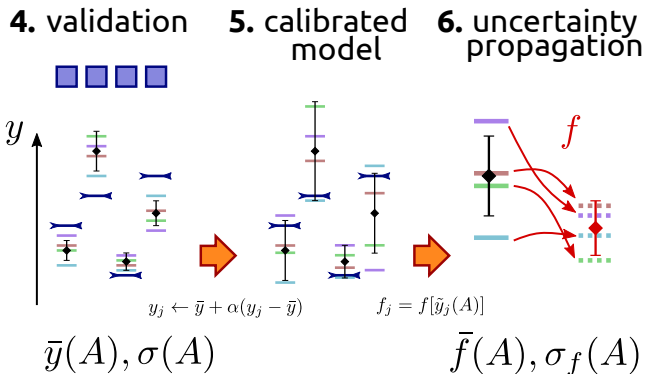


Calibrated committee models

- Ensemble of N_{RS} models, trained on subsets of the train set

$$\bar{y}(A) = \frac{1}{N_{RS}} \sum_j \tilde{y}_j(A), \quad \sigma^2(A) = \frac{1}{N_{RS} - 1} \sum_i (\tilde{y}_j(A) - \bar{y}(A))^2$$

- Verify accuracy by the distribution of errors $P(|\bar{y}(A) - y_A(A)| | \sigma(A))$
- Adjust the spread by maximum likelihood $\tilde{y}_j \leftarrow \bar{y} + \alpha(\tilde{y}_j - \bar{y})$
- Calibrated model can be used for easy uncertainty propagation



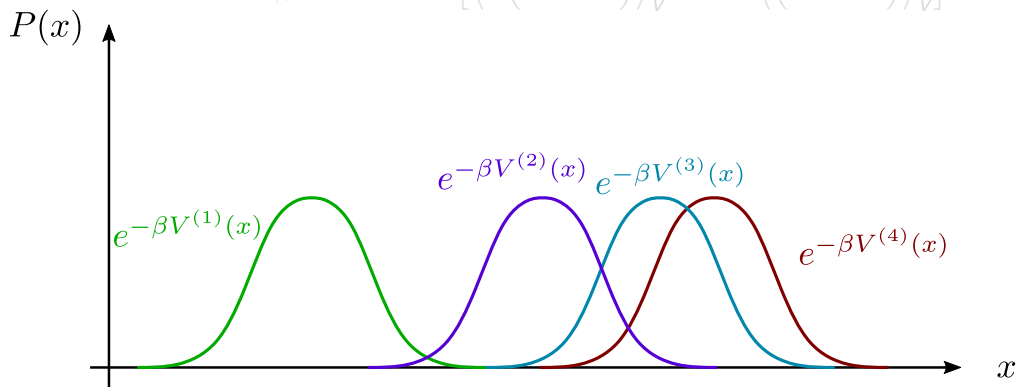
Uncertain sampling

- Errors in ML-based thermodynamic averages combine effects on the observable σ_a^2 and those from sampling σ_{aV}^2
- A committee of predictions can be obtained from a single trajectory!

$$\langle a \rangle_{V^{(i)}} = \left\langle a e^{\beta(\bar{V} - V^{(i)})} \right\rangle_{\bar{V}}$$

- Statistically stable estimates with a Cumulant Expansion Approximation

$$\langle a \rangle_{V^{(i)}, \text{CEA}} \approx \langle a \rangle_{\bar{V}} - \beta \left[\left\langle a \left(V^{(i)} - \bar{V} \right) \right\rangle_{\bar{V}} - \langle a \rangle_{\bar{V}} \left\langle \left(V^{(i)} - \bar{V} \right) \right\rangle_{\bar{V}} \right]$$



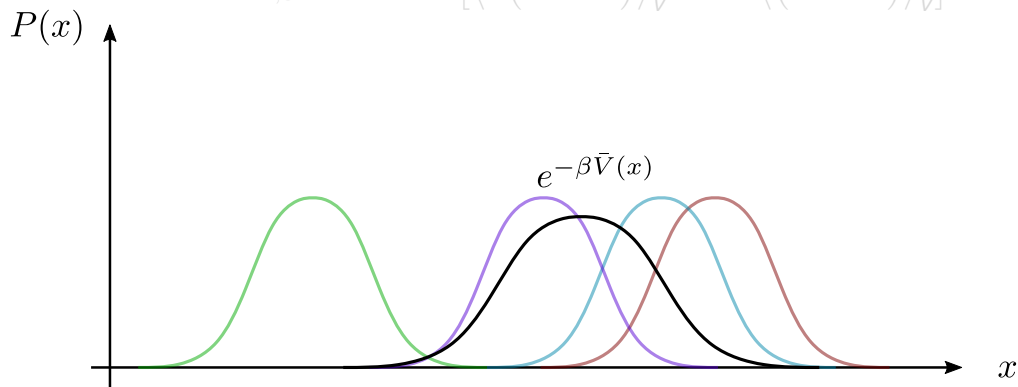
Uncertain sampling

- Errors in ML-based thermodynamic averages combine effects on the observable σ_a^2 and those from sampling σ_{aV}^2
- A committee of predictions can be obtained from a single trajectory!

$$\langle a \rangle_{V^{(i)}} = \left\langle a e^{\beta(\bar{V} - V^{(i)})} \right\rangle_{\bar{V}}$$

- Statistically stable estimates with a Cumulant Expansion Approximation

$$\langle a \rangle_{V^{(i)}, \text{CEA}} \approx \langle a \rangle_{\bar{V}} - \beta \left[\left\langle a \left(V^{(i)} - \bar{V} \right) \right\rangle_{\bar{V}} - \langle a \rangle_{\bar{V}} \left\langle \left(V^{(i)} - \bar{V} \right) \right\rangle_{\bar{V}} \right]$$



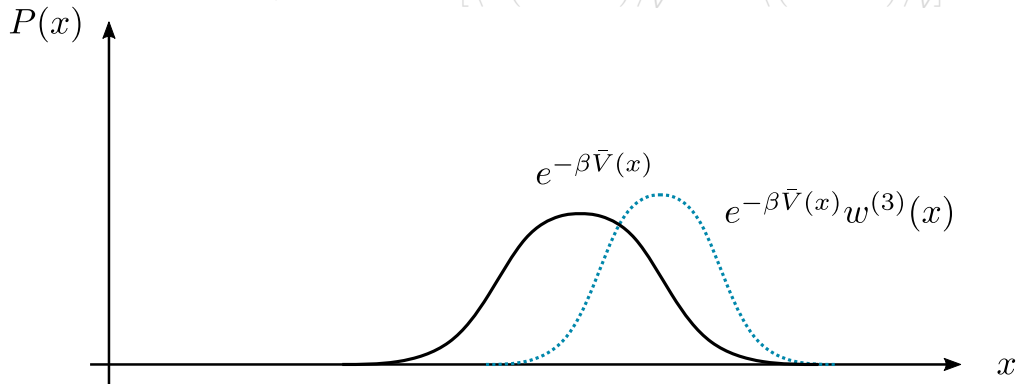
Uncertain sampling

- Errors in ML-based thermodynamic averages combine effects on the observable σ_a^2 and those from sampling σ_{aV}^2
- A committee of predictions can be obtained from a single trajectory!

$$\langle a \rangle_{V^{(i)}} = \left\langle a e^{\beta(\bar{V} - V^{(i)})} \right\rangle_{\bar{V}}$$

- Statistically stable estimates with a Cumulant Expansion Approximation

$$\langle a \rangle_{V^{(i)}, \text{CEA}} \approx \langle a \rangle_{\bar{V}} - \beta \left[\left\langle a \left(V^{(i)} - \bar{V} \right) \right\rangle_{\bar{V}} - \langle a \rangle_{\bar{V}} \left\langle \left(V^{(i)} - \bar{V} \right) \right\rangle_{\bar{V}} \right]$$



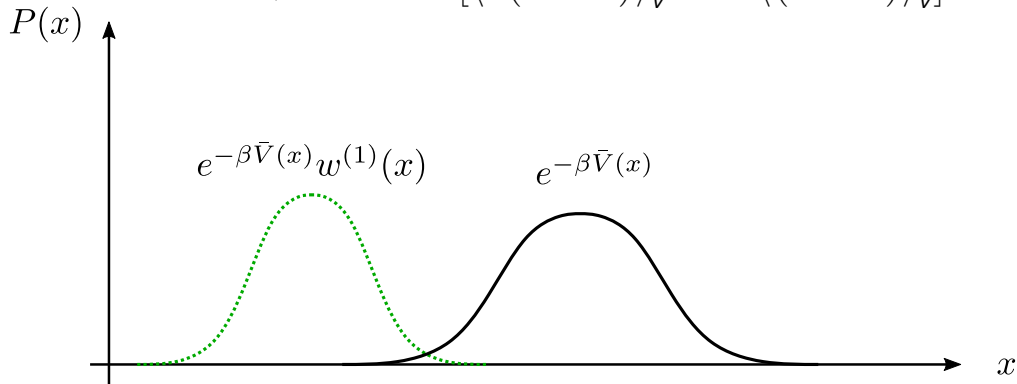
Uncertain sampling

- Errors in ML-based thermodynamic averages combine effects on the observable σ_a^2 and those from sampling σ_{aV}^2
- A committee of predictions can be obtained from a single trajectory!

$$\langle a \rangle_{V^{(i)}} = \left\langle a e^{\beta(\bar{V} - V^{(i)})} \right\rangle_{\bar{V}}$$

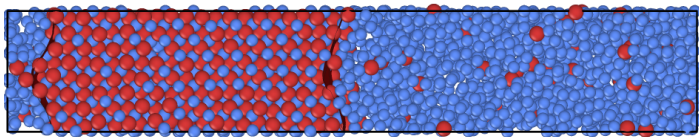
- Statistically stable estimates with a Cumulant Expansion Approximation

$$\langle a \rangle_{V^{(i)}, \text{CEA}} \approx \langle a \rangle_{\bar{V}} - \beta \left[\left\langle a \left(V^{(i)} - \bar{V} \right) \right\rangle_{\bar{V}} - \langle a \rangle_{\bar{V}} \left\langle \left(V^{(i)} - \bar{V} \right) \right\rangle_{\bar{V}} \right]$$



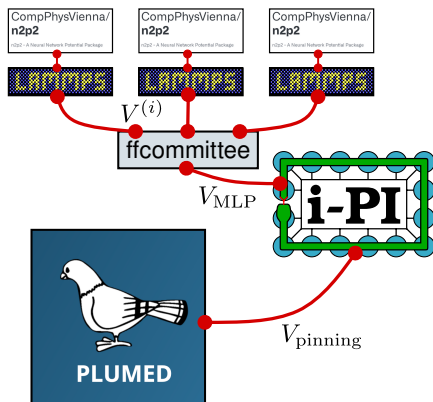
UQ for interface pinning

- Compute the phase diagram of $\text{Ga}_x\text{As}_{1-x}$: interface-pinning simulations for a 2-component system
 - DFT-accurate ML potential, i-PI+LAMMPS+PLUMED setup
 - Estimate uncertainty in melting points



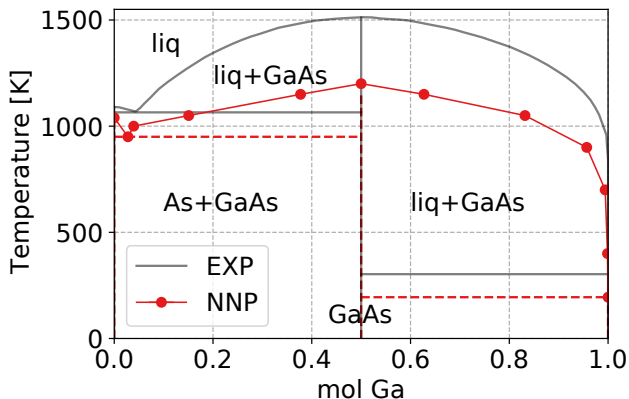
UQ for interface pinning

- Compute the phase diagram of $\text{Ga}_x\text{As}_{1-x}$: interface-pinning simulations for a 2-component system
- DFT-accurate ML potential, i-PI+LAMMPS+PLUMED setup
- Estimate uncertainty in melting points



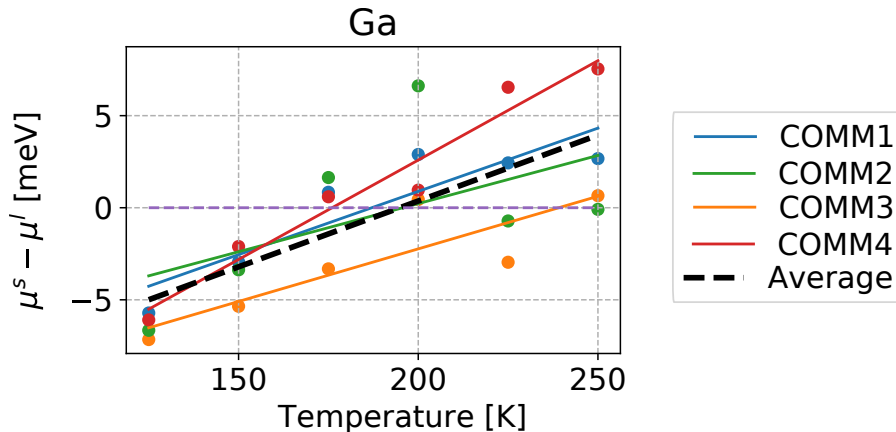
UQ for interface pinning

- Compute the phase diagram of $\text{Ga}_x\text{As}_{1-x}$: interface-pinning simulations for a 2-component system
- DFT-accurate ML potential, i-PI+LAMMPS+PLUMED setup
- Estimate uncertainty in melting points



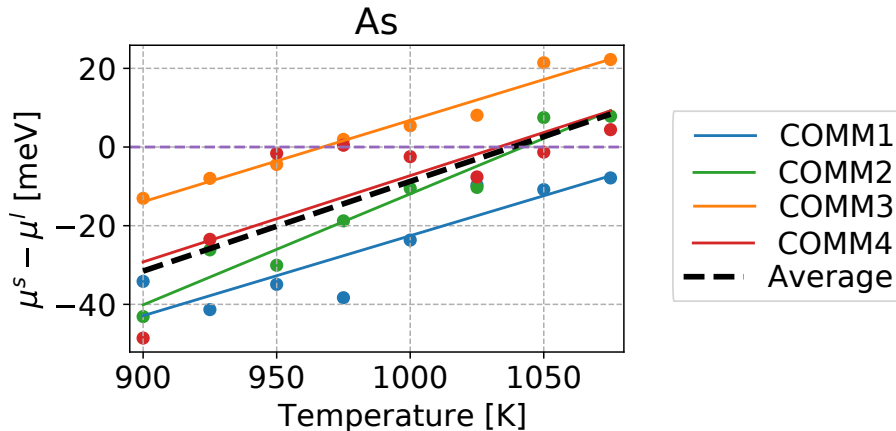
UQ for interface pinning

- Compute the phase diagram of $\text{Ga}_x\text{As}_{1-x}$: interface-pinning simulations for a 2-component system
- DFT-accurate ML potential, i-PI+LAMMPS+PLUMED setup
- Estimate uncertainty in melting points



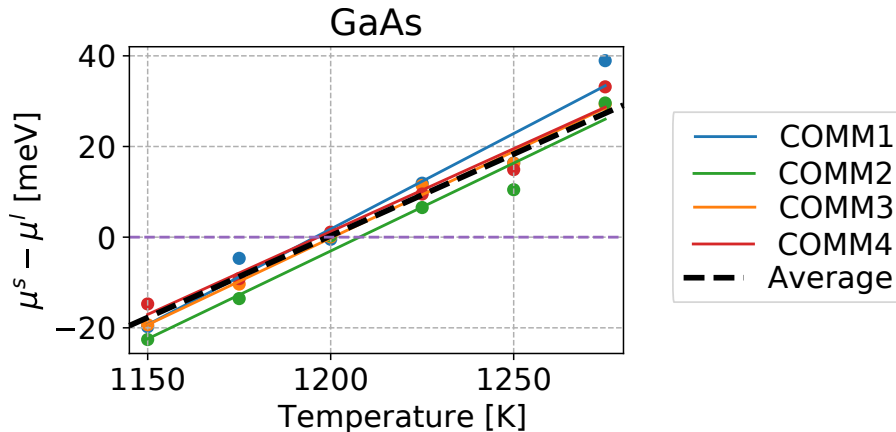
UQ for interface pinning

- Compute the phase diagram of $\text{Ga}_x\text{As}_{1-x}$: interface-pinning simulations for a 2-component system
- DFT-accurate ML potential, i-PI+LAMMPS+PLUMED setup
- Estimate uncertainty in melting points



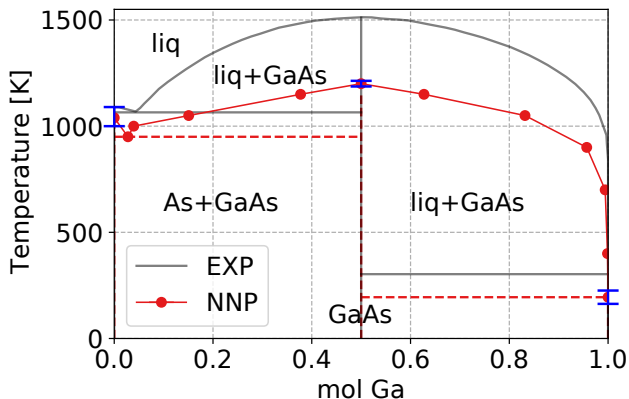
UQ for interface pinning

- Compute the phase diagram of $\text{Ga}_x\text{As}_{1-x}$: interface-pinning simulations for a 2-component system
- DFT-accurate ML potential, i-PI+LAMMPS+PLUMED setup
- Estimate uncertainty in melting points

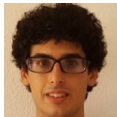


UQ for interface pinning

- Compute the phase diagram of $\text{Ga}_x\text{As}_{1-x}$: interface-pinning simulations for a 2-component system
- DFT-accurate ML potential, i-PI+LAMMPS+PLUMED setup
- Estimate uncertainty in melting points



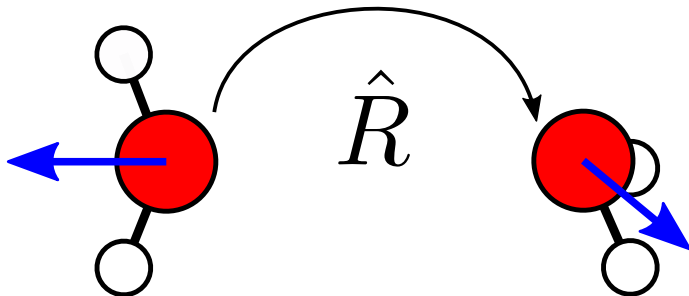
Integrated modeling with symmetry-adapted ML



Machine-learning for tensors

- Want to learn vectors or general tensors?
Need features that are *equivariant* to rotations

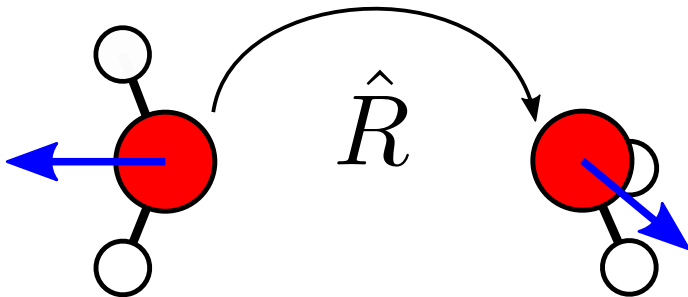
$$d_{\alpha}(\hat{R}A_i) = \sum_q \langle d|q\rangle \langle q|\hat{R}A_i; \overline{\rho_i^{\otimes \nu}; \alpha}\rangle$$



Machine-learning for tensors

- Want to learn vectors or general tensors?
Need features that are *equivariant* to rotations

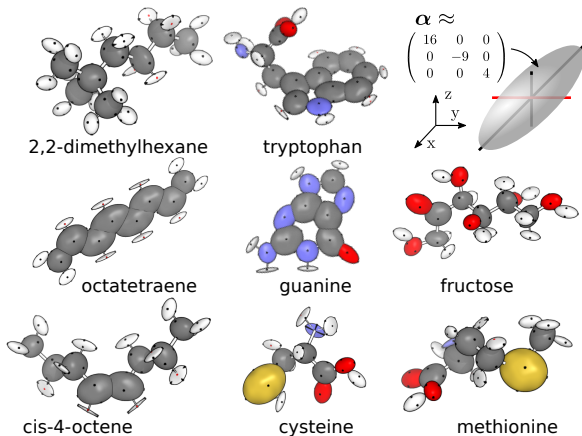
$$d_{\alpha}(\hat{R}A_i) = \sum_q \langle d|q\rangle \sum_{\alpha'} R_{\alpha\alpha'} \langle q|A_i; \overline{\rho_i^{\otimes \nu}; \alpha'}\rangle = \sum_{\alpha'} R_{\alpha\alpha'} d_{\alpha'}(A_i)$$



Machine-learning for tensors

- Want to learn vectors or general tensors?
Need features that are *equivariant* to rotations

$$y_{\mu}^{\lambda}(\hat{R}A_i) = \sum_q \langle d|q \rangle \sum_{\mu'} D_{\mu\mu'}^{\lambda}(\hat{R}) \langle q|A_i; \overline{\rho_i^{\otimes \nu}}; \lambda\mu \rangle$$

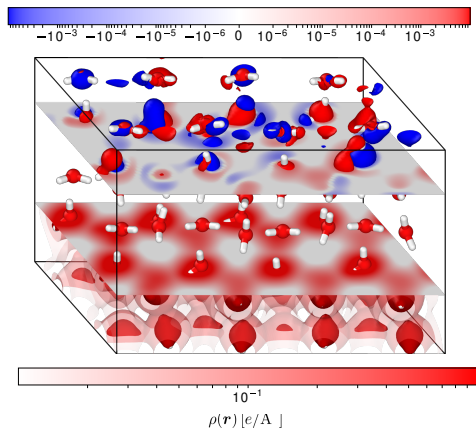


Wilkins, Grisafi, Yang, Lao, DiStasio, MC, PNAS (2019);

Machine-learning for tensors

- Want to learn vectors or general tensors?
Need features that are *equivariant* to rotations

$$y_{\mu}^{\lambda}(\hat{R}A_i) = \sum_q \langle d|q \rangle \sum_{\mu'} D_{\mu\mu'}^{\lambda}(\hat{R}) \langle q|A_i; \overline{\rho_i^{\otimes \nu}}; \lambda\mu \rangle$$

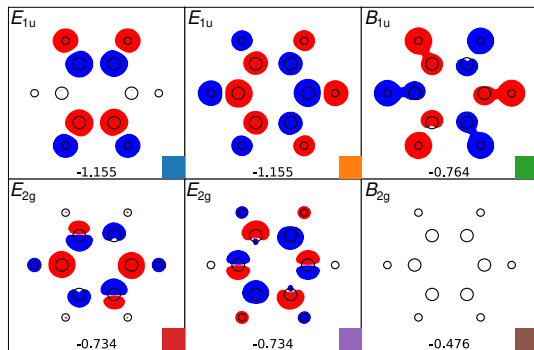
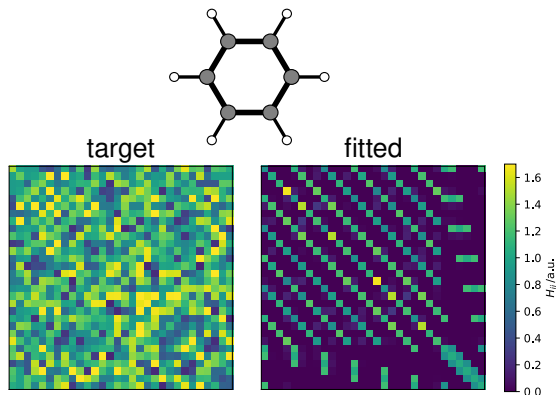


Grisafi et al., ACS Cent. Sci. (2019); Lewis, Grisafi, MC, Rossi, JCTC (2021)

Machine-learning for tensors

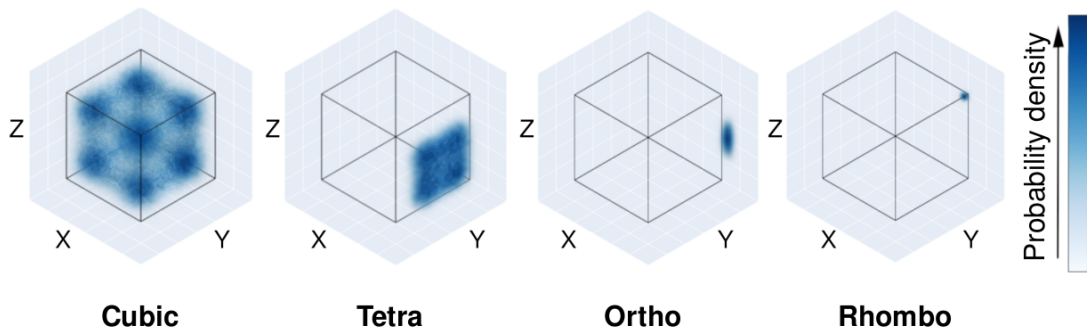
- Want to learn vectors or general tensors?
Need features that are *equivariant* to rotations

$$y_{\mu}^{\lambda}(\hat{R}A_i) = \sum_q \langle d|q \rangle \sum_{\mu'} D_{\mu\mu'}^{\lambda}(\hat{R}) \langle q|A_i; \overline{\rho_i^{\otimes \nu}}; \lambda\mu \rangle$$



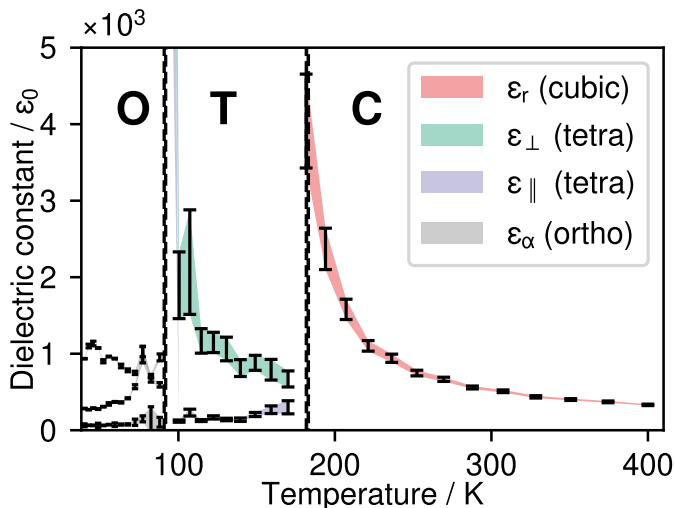
Structural and functional properties, combined

- Predicting *any* property accessible to quantum calculations
- Realistic time and size scales, with first-principles accuracy *and* mapping of structural and functional properties



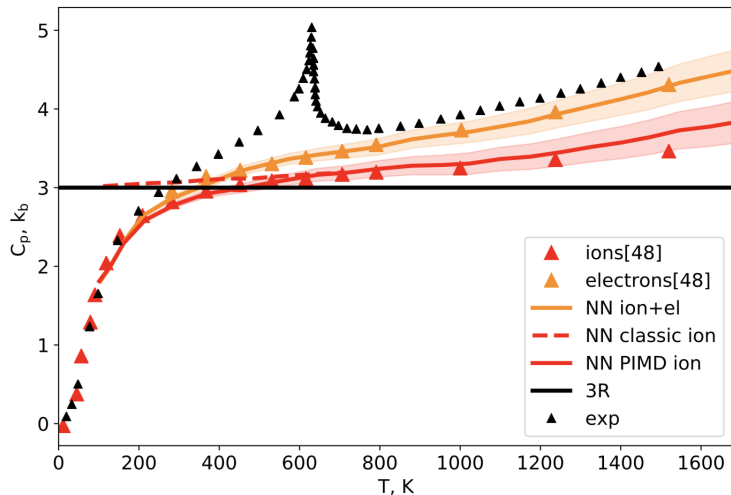
Structural and functional properties, combined

- Predicting *any* property accessible to quantum calculations
- Realistic time and size scales, with first-principles accuracy *and* mapping of structural and functional properties



Structural and functional properties, combined

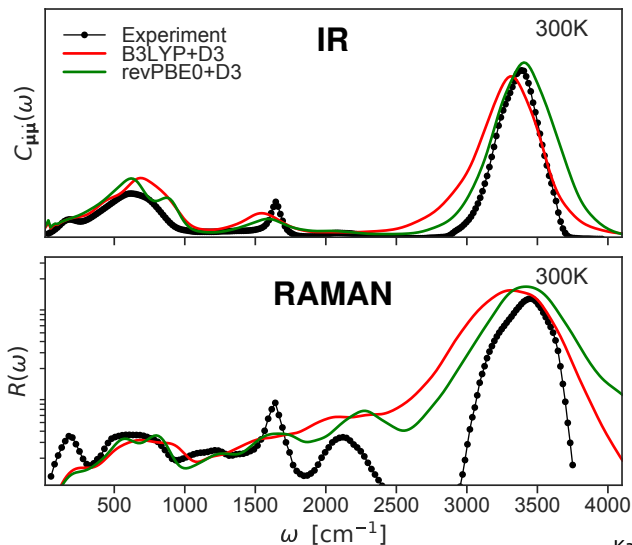
- Predicting *any* property accessible to quantum calculations
- Realistic time and size scales, with first-principles accuracy *and* mapping of structural and functional properties



N. Lopanitsyna, C. Ben Mahmoud, MC, Phys. Rev. Mater. (2021)

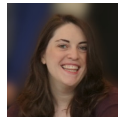
Structural and functional properties, combined

- Predicting *any* property accessible to quantum calculations
- Realistic time and size scales, with first-principles accuracy *and* mapping of structural and functional properties



Kapil, Wilkins, Lan, MC, JCP (2020)

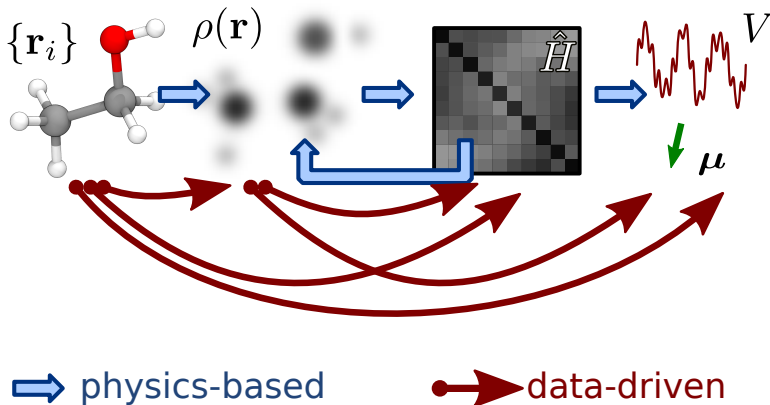
Beyond ML



Blurring the lines between ML and QM

- Interoperable quantum mechanical / machine-learning calculations: mix & match physics and data
- Example: finite- T electron free energies from ground state energy and electronic DOS

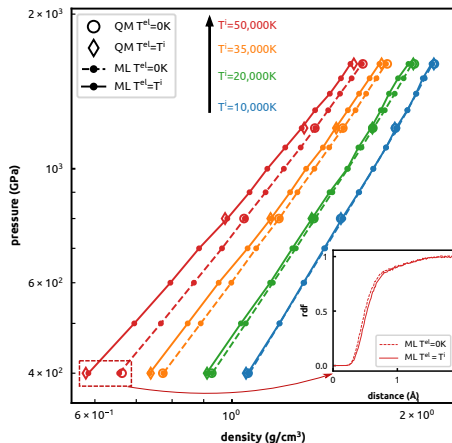
$$A(T_{\text{el}}) \approx E(0) + \int \epsilon g^0(\epsilon) [f^{T_{\text{el}}}(\epsilon) - f^0(\epsilon)] d\epsilon - T_{\text{el}} \int g^0(\epsilon) s^{T_{\text{el}}}(\epsilon) d\epsilon$$



Blurring the lines between ML and QM

- Interoperable quantum mechanical / machine-learning calculations: mix & match physics and data
- Example: finite- T electron free energies from ground state energy and electronic DOS

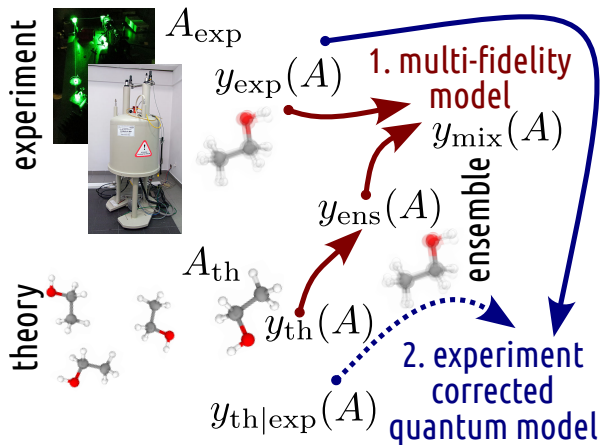
$$A(T_{\text{el}}) \approx E(0) + \int \epsilon g^0(\epsilon) [f^{T_{\text{el}}}(\epsilon) - f^0(\epsilon)] d\epsilon - T_{\text{el}} \int g^0(\epsilon) s^{T_{\text{el}}}(\epsilon) d\epsilon$$



Ben Mahmoud, Grasselli, MC, PRB (2022)

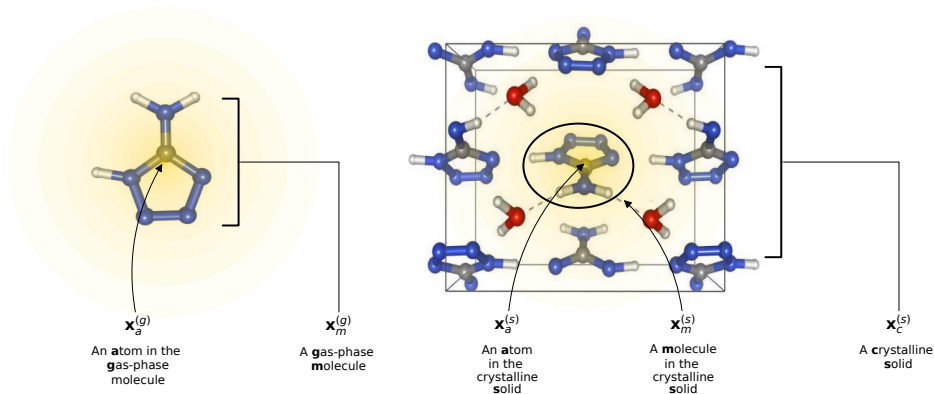
Blurring the lines between theory and experiments

- Multi-fidelity models to combine electronic-structure calculations and experimental constraints
- Conceptual challenge: reconciling what theory and experiments measures



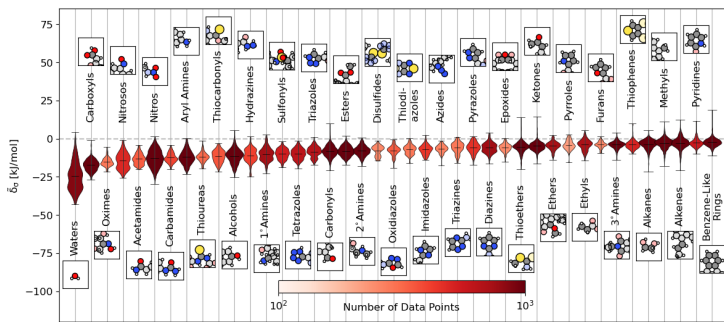
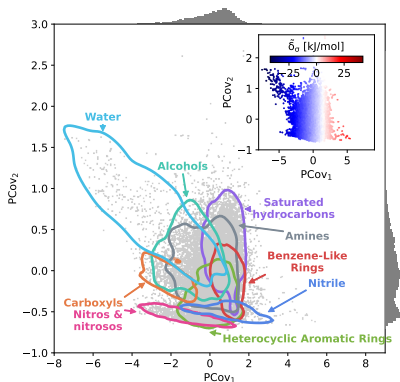
Between predictions and understanding

- Using ML to interpret the outcome of simulations, and to analyze datasets
- ML “introspection”: use knock-out models to identify key structure-property relations
- Integrating end-to-end and bottom-up modeling



Between predictions and understanding

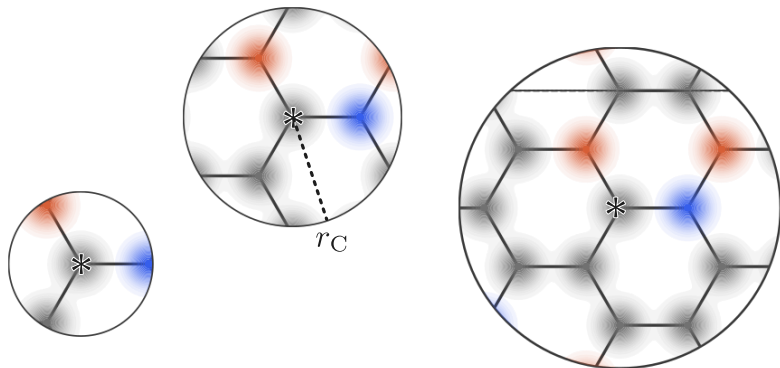
- Using ML to interpret the outcome of simulations, and to analyze datasets
 - ML “introspection”: use knock-out models to identify key structure-property relations
 - Integrating end-to-end and bottom-up modeling



Cersonsky, Pakhnova, Engel, MC, Chem. Sci. (2023); <https://molmotifs.matcloud.xyz/>

Between predictions and understanding

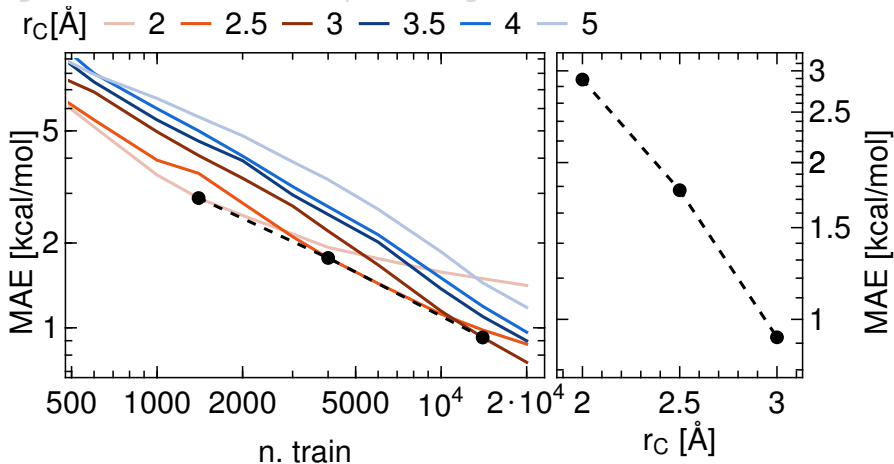
- Using ML to interpret the outcome of simulations, and to analyze datasets
- ML “introspection”: use knock-out models to identify key structure-property relations
- Integrating end-to-end and bottom-up modeling



Bartók, De, Poelking, Kermode, Bernstein, Csányi, **MC**, Science Advances (2017)

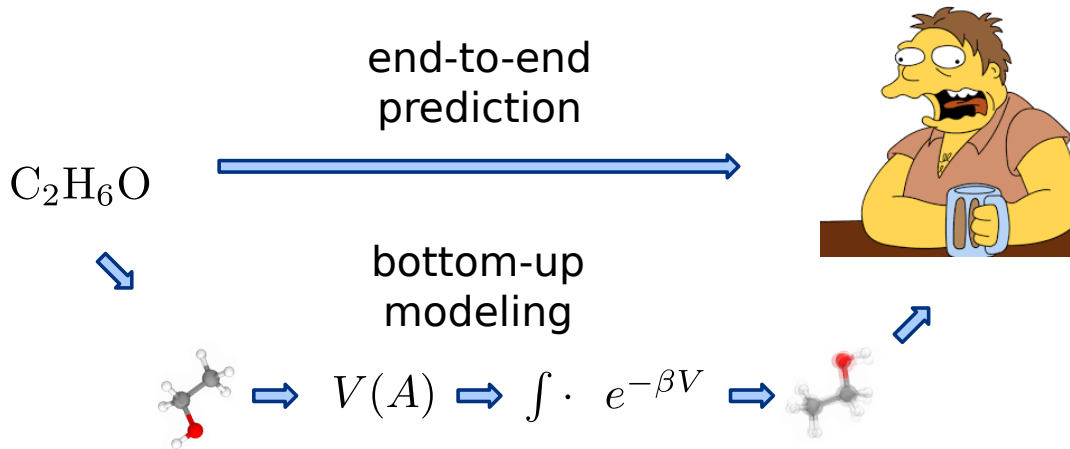
Between predictions and understanding

- Using ML to interpret the outcome of simulations, and to analyze datasets
- ML “introspection”: use knock-out models to identify key structure-property relations
- Integrating end-to-end and bottom-up modeling



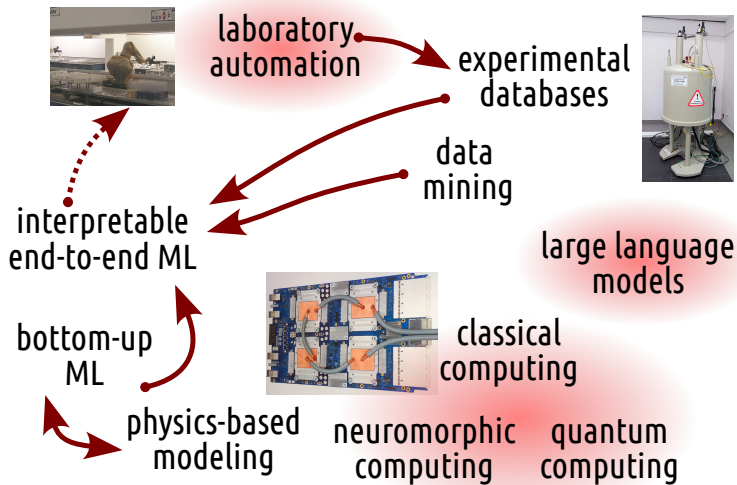
Between predictions and understanding

- Using ML to interpret the outcome of simulations, and to analyze datasets
- ML “introspection”: use knock-out models to identify key structure-property relations
- Integrating end-to-end and bottom-up modeling



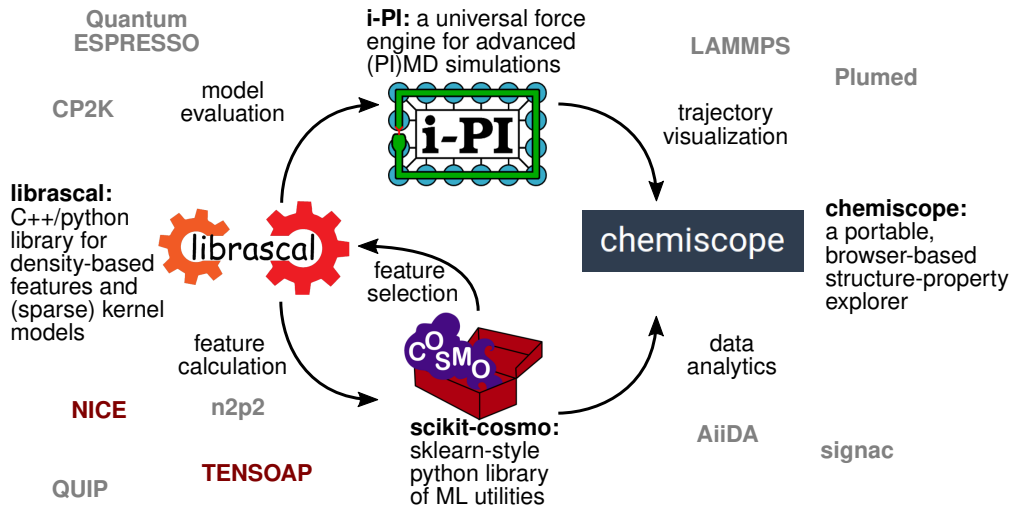
Everything, everywhere, all at once

- Modular paradigm of chemical knowledge. Machine-learning as “information glue”
- Modular software design as the technological infrastructure



Everything, everywhere, all at once

- Modular paradigm of chemical knowledge. Machine-learning as “information glue”
- Modular software design as the technological infrastructure



<https://github.com/lab-cosmo/>

Questions?

cosmo.epfl.ch

Follow @lab_COSMO



Swiss National
Science Foundation

Questions?

cosmo.epfl.ch

Follow @lab_COSMO

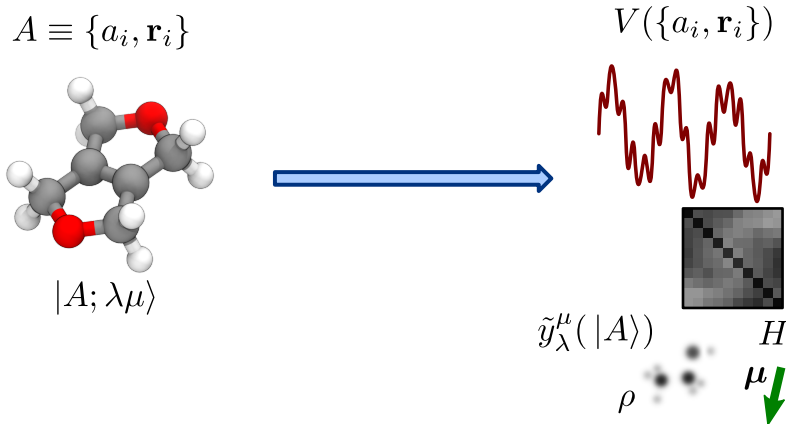


Code → github.com/lab-cosmo
Representations → Musil et al., ChemRev (2021)
Integrated models → MC, MRS Bulletin (2022)

More details

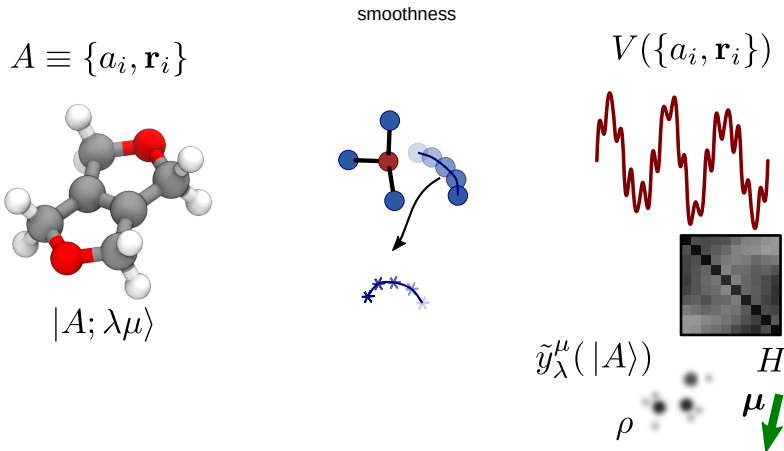
From atoms to properties

- Physical/mathematical requirements are imposed on the structure \rightarrow [representation] \rightarrow property mapping



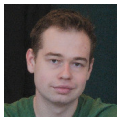
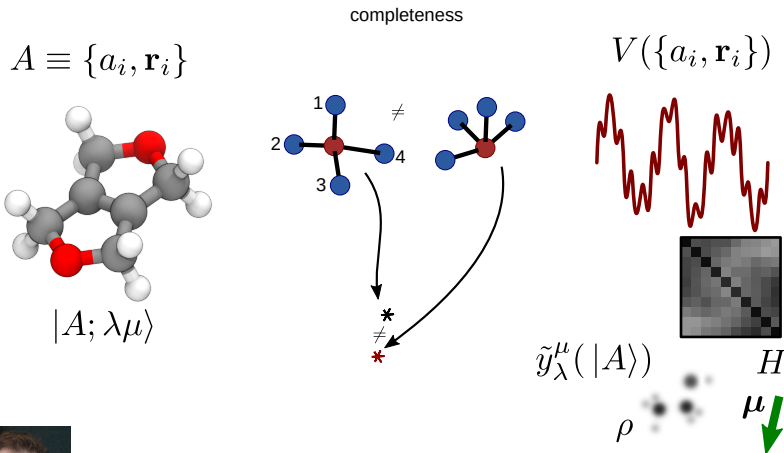
From atoms to properties

- Physical/mathematical requirements are imposed on the structure \rightarrow [representation] \rightarrow property mapping



From atoms to properties

- Physical/mathematical requirements are imposed on the structure \rightarrow [representation] \rightarrow property mapping

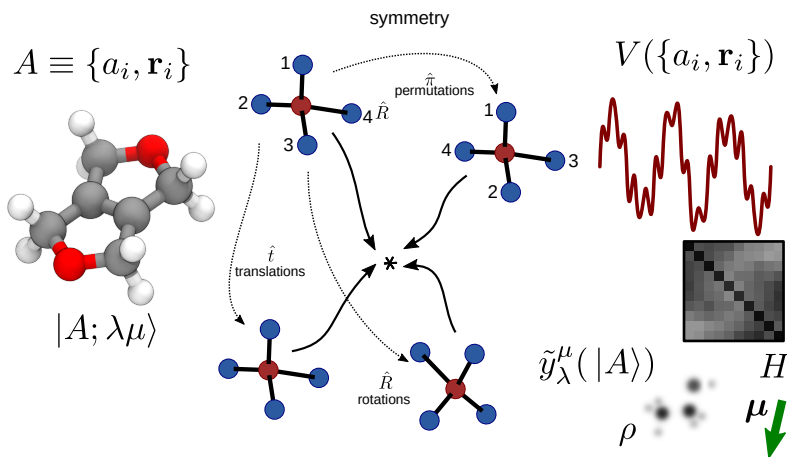


Pozdnyakov et al. PRL (2020)
Pozdnyakov & MC, MLST (2022)

Introductory review: Musil et al., Chem. Rev. (2021)

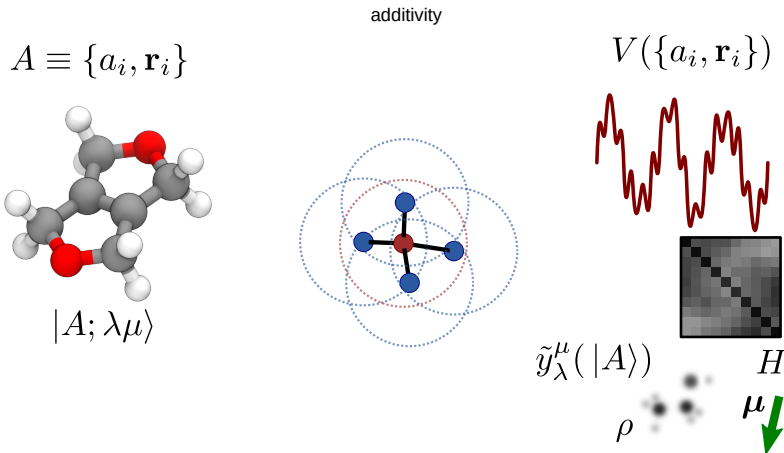
From atoms to properties

- Physical/mathematical requirements are imposed on the structure \rightarrow [representation] \rightarrow property mapping



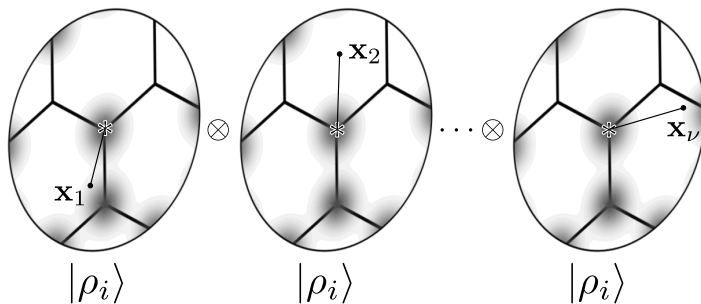
From atoms to properties

- Physical/mathematical requirements are imposed on the structure \rightarrow [representation] \rightarrow property mapping



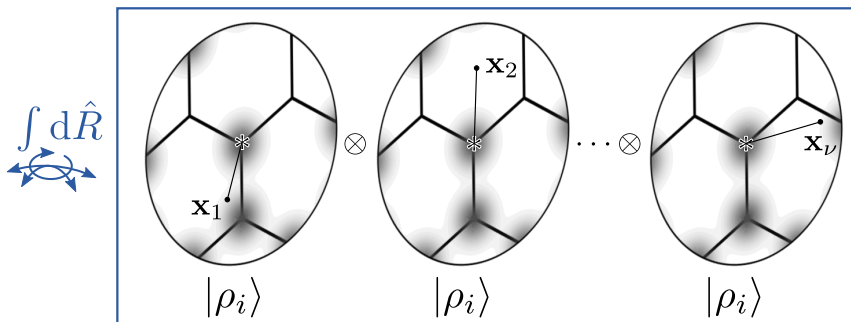
A universal feature construction

- Physical requirements constrain the design space of ML models
- Most frameworks can be expressed in terms of (symmetrized) n -body correlations of atom positions. Main difference - the choice of basis
- Easy extension in many directions:
 - fully equivariant framework (NICE)
 - features to describe long-range interactions (LODE)
 - message-passing, N -center features (MP-ACDC)



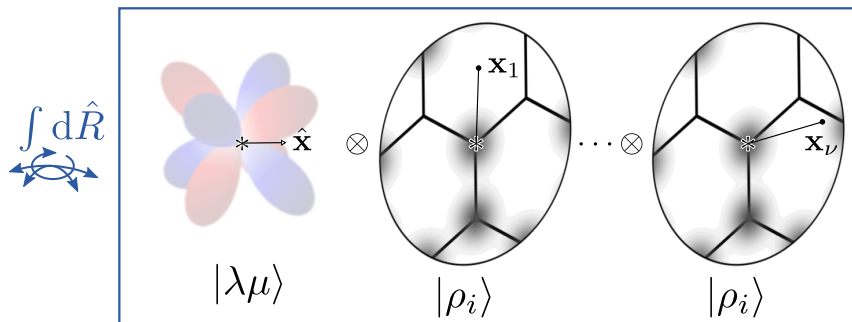
A universal feature construction

- Physical requirements constrain the design space of ML models
- Most frameworks can be expressed in terms of (symmetrized) n -body correlations of atom positions. Main difference - the choice of basis
- Easy extension in many directions:
 - fully equivariant framework (NICE)
 - features to describe long-range interactions (LODE)
 - message-passing, N -center features (MP-ACDC)



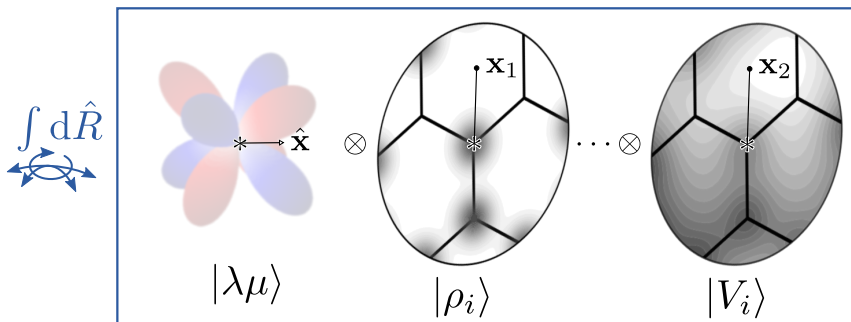
A universal feature construction

- Physical requirements constrain the design space of ML models
- Most frameworks can be expressed in terms of (symmetrized) n -body correlations of atom positions. Main difference - the choice of basis
- Easy extension in many directions:
 - fully equivariant framework (NICE)
 - features to describe long-range interactions (LODE)
 - message-passing, N -center features (MP-ACDC)



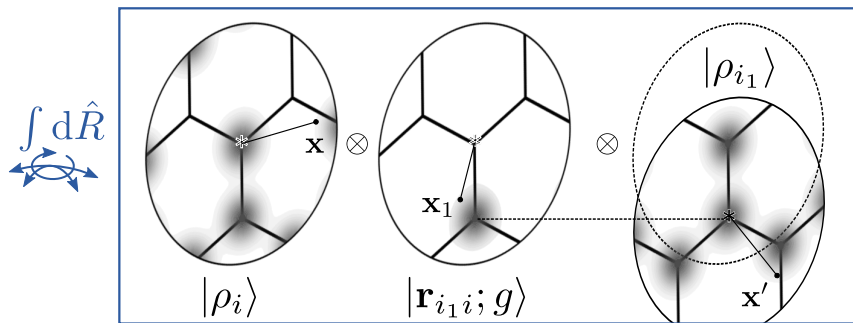
A universal feature construction

- Physical requirements constrain the design space of ML models
- Most frameworks can be expressed in terms of (symmetrized) n -body correlations of atom positions. Main difference - the choice of basis
- Easy extension in many directions:
 - fully equivariant framework (NICE)
 - features to describe long-range interactions (LODE)
 - message-passing, N -center features (MP-ACDC)



A universal feature construction

- Physical requirements constrain the design space of ML models
- Most frameworks can be expressed in terms of (symmetrized) n -body correlations of atom positions. Main difference - the choice of basis
- Easy extension in many directions:
 - fully equivariant framework (NICE)
 - features to describe long-range interactions (LODE)
 - message-passing, N -center features (MP-ACDC)



Atom-centered correlations

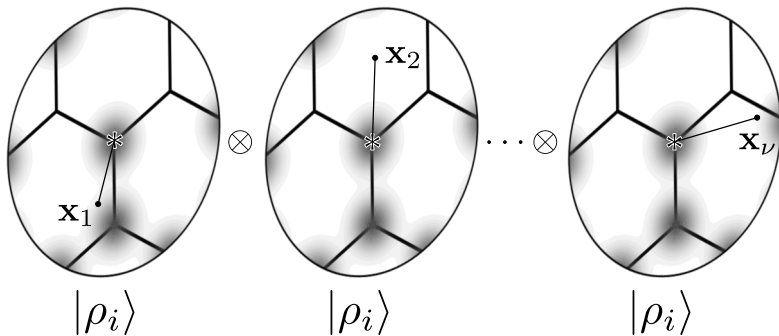
- A permutation-invariant function of neighbor positions can be linearly expanded in terms of tensor products of the neighbor density
- Symmetrized correlations \leftrightarrow invariant models
- Extension to a fully equivariant framework

$$\begin{aligned}y(A_i) &\approx \sum_{i_1 i_2 \in A_i} \tilde{y}(\mathbf{r}_{i i_1}, \mathbf{r}_{i i_2}) = \sum_{i_1 i_2 \in A_i} \int d\mathbf{x}_1 d\mathbf{x}_2 \tilde{y}(\mathbf{x}_1, \mathbf{x}_2) \delta(\mathbf{x}_1 - \mathbf{r}_{i i_1}) \delta(\mathbf{x}_2 - \mathbf{r}_{i i_2}) \\ &= \int d\mathbf{x}_1 d\mathbf{x}_2 \tilde{y}(\mathbf{x}_1, \mathbf{x}_2) \rho_i(\mathbf{x}_1) \rho_i(\mathbf{x}_2) = \int d\mathbf{x}_1 d\mathbf{x}_2 \tilde{y}(\mathbf{x}_1, \mathbf{x}_2) \langle \mathbf{x}_1 | A_i \rangle \langle \mathbf{x}_2 | A_i \rangle\end{aligned}$$

$$y(A_i) \approx \sum_{q_1 q_2} y_{q_1 q_2} \langle q_1 | A_i \rangle \langle q_2 | A_i \rangle$$

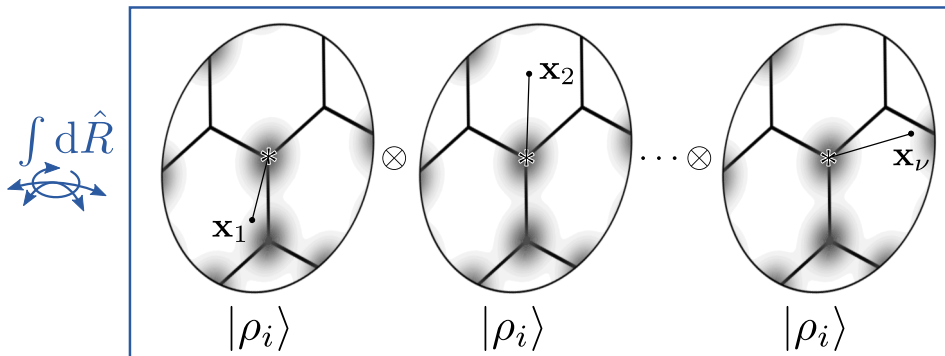
Atom-centered correlations

- A permutation-invariant function of neighbor positions can be linearly expanded in terms of tensor products of the neighbor density
- Symmetrized correlations \leftrightarrow invariant models
- Extension to a fully equivariant framework



Atom-centered correlations

- A permutation-invariant function of neighbor positions can be linearly expanded in terms of tensor products of the neighbor density
- Symmetrized correlations \leftrightarrow invariant models
- Extension to a fully equivariant framework



Atom-centered correlations

- A permutation-invariant function of neighbor positions can be linearly expanded in terms of tensor products of the neighbor density
- Symmetrized correlations \leftrightarrow invariant models
- Extension to a fully equivariant framework

$$y(A_i) \approx \sum_{i_1 i_2 \in A_i} \int d\hat{R} \tilde{y}(\hat{R} \mathbf{r}_{i_1}, \hat{R} \mathbf{r}_{i_2}) = \int d\mathbf{x}_1 d\mathbf{x}_2 \tilde{y}(\mathbf{x}_1, \mathbf{x}_2) \int d\hat{R} \langle \mathbf{x}_1 | \hat{R} A_i \rangle \langle \mathbf{x}_2 | \hat{R} A_i \rangle$$

$$y(A_i) \approx \sum_{nn'l} y_{nn'l} \sum_m \langle nlm | A_i \rangle \langle n' lm | A_i \rangle$$

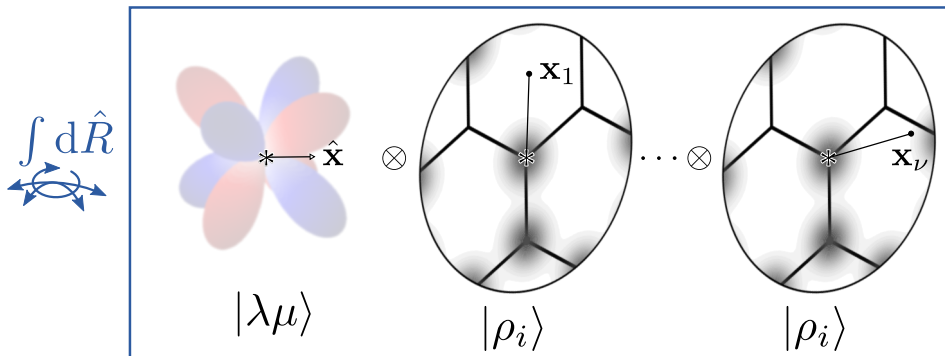
Atom-centered correlations

- A permutation-invariant function of neighbor positions can be linearly expanded in terms of tensor products of the neighbor density
- Symmetrized correlations \leftrightarrow invariant models
- Extension to a fully equivariant framework

$$\begin{aligned} y_{\lambda}^{\mu}(A_i) &\approx \sum_{i_1 i_2 \in A_i} \int d\hat{\mathbf{x}} d\mathbf{x}_1 d\mathbf{x}_2 \tilde{y}(\hat{\mathbf{x}}, \mathbf{x}_1, \mathbf{x}_2) Y_{\lambda}^{\mu}(\hat{\mathbf{x}}) \delta(\mathbf{x}_1 - \mathbf{r}_{i_1}) \delta(\mathbf{x}_2 - \mathbf{r}_{i_2}) \\ &= \int d\hat{\mathbf{x}} d\mathbf{x}_1 d\mathbf{x}_2 \tilde{y}(\hat{\mathbf{x}}, \mathbf{x}_1, \mathbf{x}_2) \langle \hat{\mathbf{x}} | \lambda \mu \rangle \langle \mathbf{x}_1 | A_i \rangle \langle \mathbf{x}_2 | A_i \rangle \end{aligned}$$

Atom-centered correlations

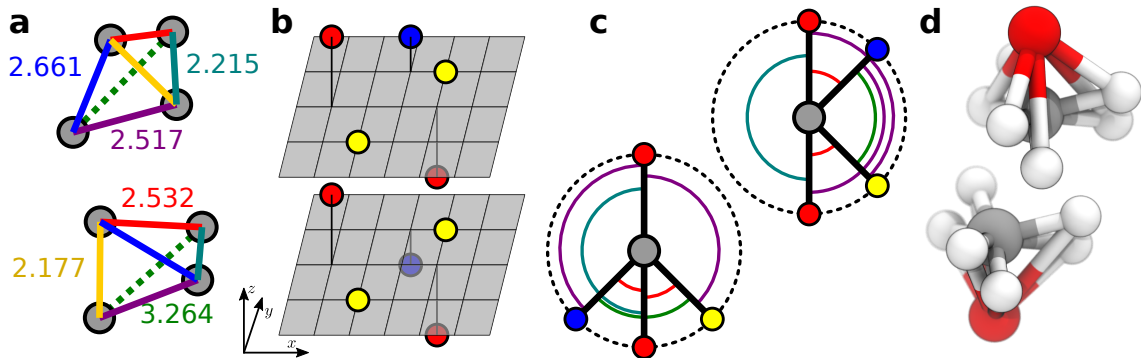
- A permutation-invariant function of neighbor positions can be linearly expanded in terms of tensor products of the neighbor density
- Symmetrized correlations \leftrightarrow invariant models
- Extension to a fully equivariant framework



Glielmo, Sollich, De Vita, PRB (2017); Grisafi, Wilkins, Csányi, & MC, PRL (2018);

Fundamental open questions

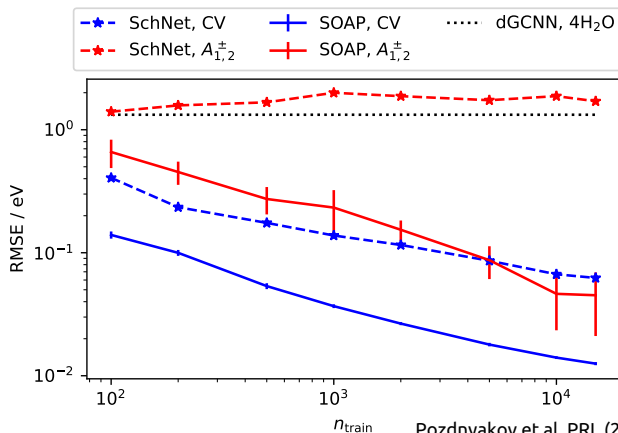
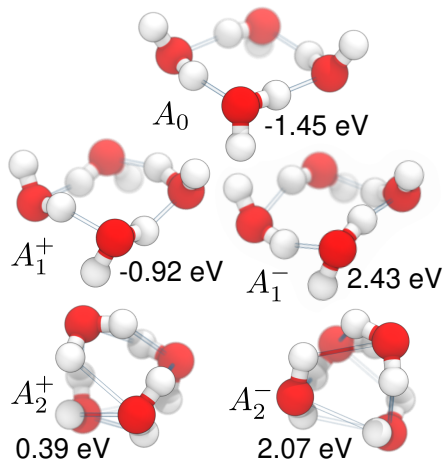
- “Completeness” of representations: what does it take to make a universal approximator?
- Relation with “deep learning” models: fundamental and practical relevance



Pozdnyakov et al. PRL (2020);
Pozdnyakov, MC, MLST (2022);
Nigam, Fraux, Pozdnyakov, MC, JCP (2022)

Fundamental open questions

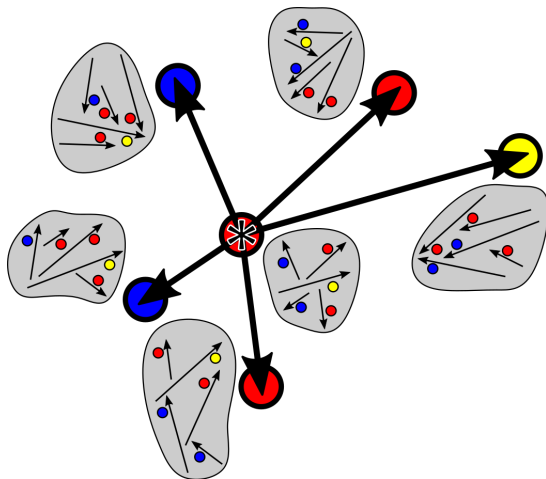
- “Completeness” of representations: what does it take to make a universal approximator?
- Relation with “deep learning” models: fundamental and practical relevance



Pozdnyakov et al. PRL (2020);
Pozdnyakov, MC, MLST (2022);
Nigam, Fraux, Pozdnyakov, MC, JCP (2022)

Fundamental open questions

- “Completeness” of representations: what does it take to make a universal approximator?
- Relation with “deep learning” models: fundamental and practical relevance

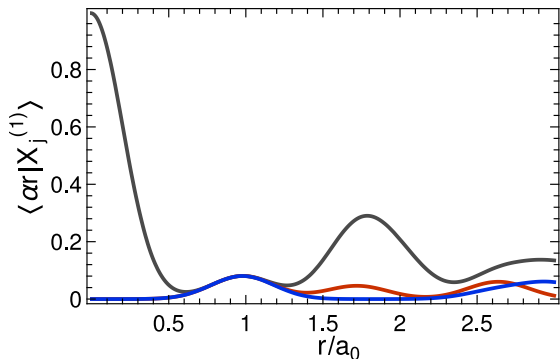
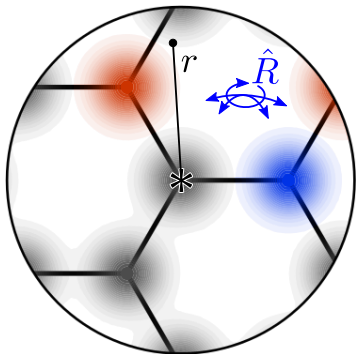


Pozdnyakov et al. PRL (2020);
Pozdnyakov, MC, MLST (2022);
Nigam, Fraux, Pozdnyakov, MC, JCP (2022)

A hierarchy of body-ordered features

- Rotationally-averaged representations are essentially the same n -body correlations that are used in statistical theories of liquids
- Linear models built on $|\overline{\rho_i^{\otimes \nu}}; g \rightarrow \delta\rangle$ yield $(\nu + 1)$ -body potential expansion

$$V(A_i) = \sum_{ij} V^{(2)}(r_{ij}) + \sum_{ij} V^{(3)}(r_{ij}, r_{ik}, \omega_{ijk}) \dots$$

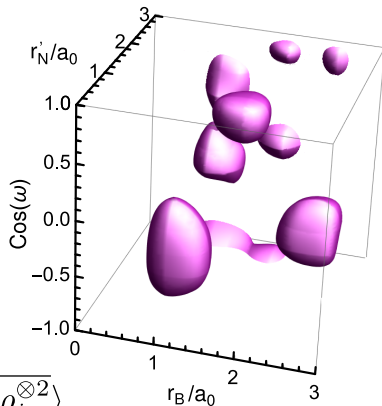
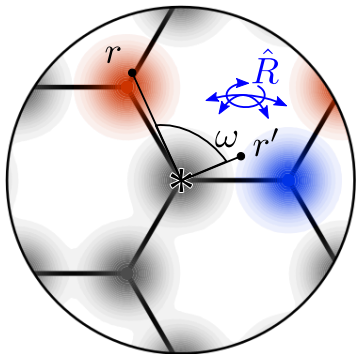


$$\langle ax | \overline{\rho_i^{\otimes 1}} \rangle = \int d\hat{R} \langle ax \hat{\mathbf{x}} | \hat{R} | \rho_i \rangle$$

A hierarchy of body-ordered features

- Rotationally-averaged representations are essentially the same n -body correlations that are used in statistical theories of liquids
- Linear models built on $|\overline{\rho_i^{\otimes \nu}}; g \rightarrow \delta\rangle$ yield $(\nu + 1)$ -body potential expansion

$$V(A_i) = \sum_{ij} V^{(2)}(r_{ij}) + \sum_{ij} V^{(3)}(r_{ij}, r_{ik}, \omega_{ijk}) \dots$$

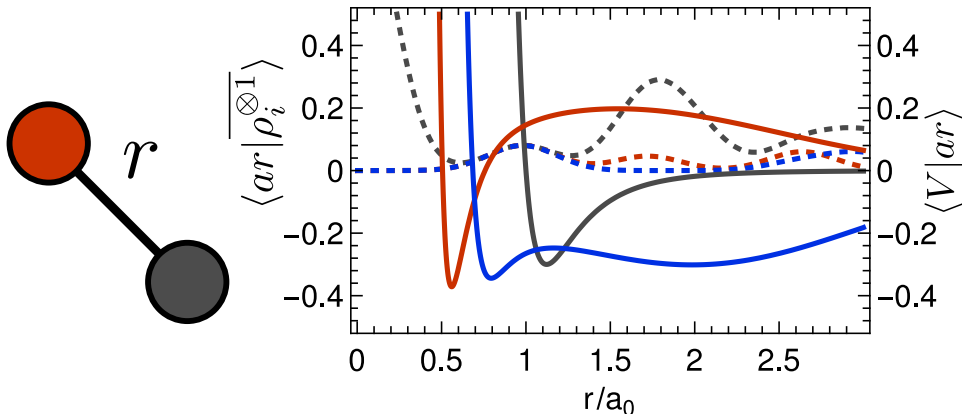


$$\begin{aligned} & \langle a_1 x_1; a_2 x_2; \omega | \overline{\rho_i^{\otimes 2}} \rangle \\ &= \int d\hat{R} \langle a_1 x_1 \hat{\mathbf{x}} | \hat{R} | \rho_i \rangle \langle a_2 x_2 \hat{\mathbf{x}}'(\omega) | \hat{R} | \rho_i \rangle \end{aligned}$$

A hierarchy of body-ordered features

- Rotationally-averaged representations are essentially the same n -body correlations that are used in statistical theories of liquids
- Linear models built on $\overline{|\rho_i^{\otimes \nu}; \mathbf{g} \rightarrow \delta\rangle}$ yield $(\nu + 1)$ -body potential expansion

$$V(A_i) = \sum_{ij} V^{(2)}(r_{ij}) + \sum_{ij} V^{(3)}(r_{ij}, r_{ik}, \omega_{ijk}) \dots$$



$$V(A_i) = \int dx \langle V | ax \rangle \langle ax | \overline{|\rho_i^{\otimes 1} \rangle} \approx \sum_j V_a(r_{ij})$$

Willatt, Musil, MC, JCP (2019); Drautz, PRB (2019); Glielmo, Zeni, De Vita, PRB (2018)

A hierarchy of equivariant features

- Equivariant N -body features transform like angular momenta

$$|\hat{R}\mathbf{A}; \overline{\rho_i^{\otimes \nu}; \lambda \mu}\rangle \sim \sum_{\mu'} D_{\mu \mu'}^\lambda(R) |\mathbf{A}; \overline{\rho_i^{\otimes \nu}; \lambda \mu'}\rangle$$

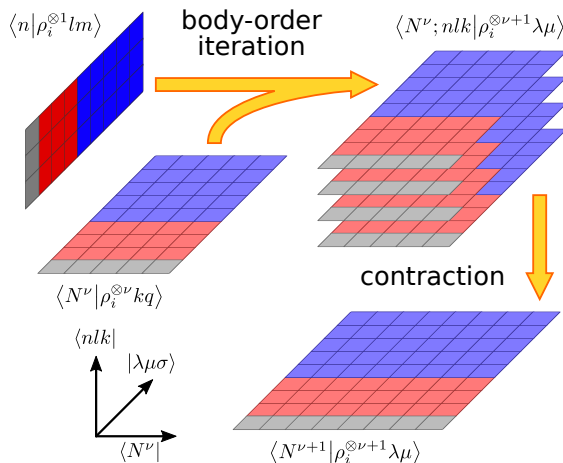
- Recursive construction based on sums of angular momenta and an expansion of the atom density \rightarrow *Clebsch-Gordan iteration*

$$\begin{aligned} \langle n_1 | \overline{\rho_i^{\otimes 1}; \lambda \mu} \rangle &\equiv \langle n_1 \lambda \mu | \rho_i \rangle \\ \langle \dots; n_\nu l_\nu k_\nu; n l k | \overline{\rho_i^{\otimes (\nu+1)}; \lambda \mu} \rangle &= \sum_{qm} \langle n | \overline{\rho_i^{\otimes 1}; l m} \rangle \langle \dots; n_\nu l_\nu k_\nu | \overline{\rho_i^{\otimes \nu}; k q} \rangle \langle l m; k q | \lambda \mu \rangle \end{aligned}$$

- Can be used to compute efficiently *invariant* features $|\overline{\rho_i^{\otimes \nu}; 00}\rangle$
 \rightarrow **a complete linear basis of invariant polynomials**

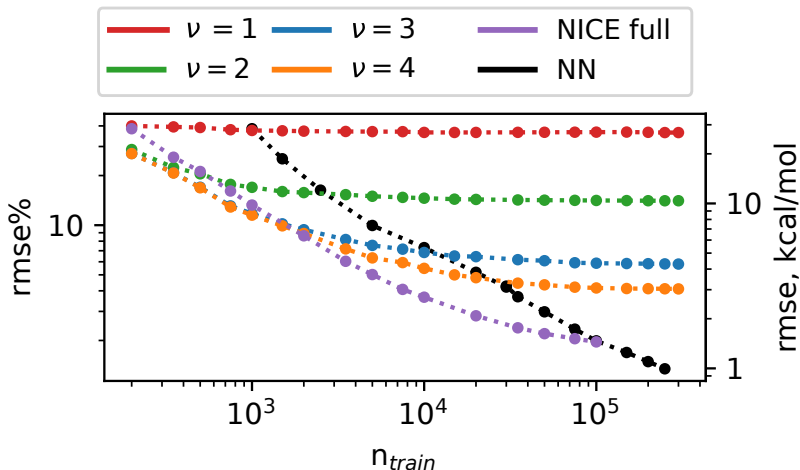
NICE features for ML

- Problem: number of features grows exponentially with ν
- Solution: N -body iterative contraction of equivariants (NICE)
 - After each body order increase, most relevant features are selected for the next iteration
 - Systematic convergence with ν and contraction truncation



NICE features for ML

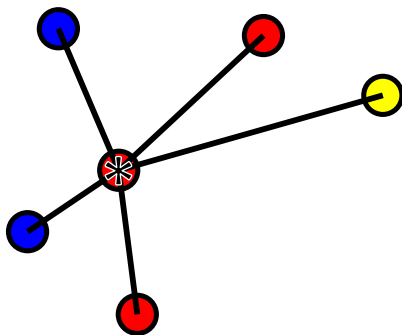
- Problem: number of features grows exponentially with ν
- Solution: N -body iterative contraction of equivariants (NICE)
 - After each body order increase, most relevant features are selected for the next iteration
 - Systematic convergence with ν and contraction truncation



Nigam, Pozdnyakov, MC, JCP (2020); <https://github.com/cosmo-epfl/nice>

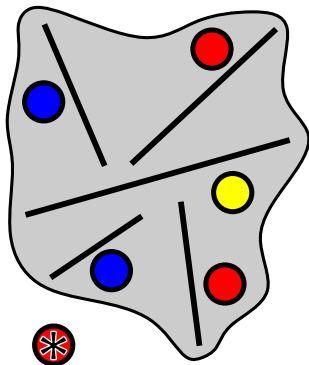
How about graph convolution?

- Atoms are nodes in a fully-connected network. Edges are decorated by (functions of) interatomic distances r_{ij}
- Each node is decorated by the nature of its neighbors and their distance $h(A_i) = (a_i, \{(a_j, r_{ij})\})$
- The multiset of neighbors and edges is hashed, and used as a label to describe the nodes. The process can be iterated



How about graph convolution?

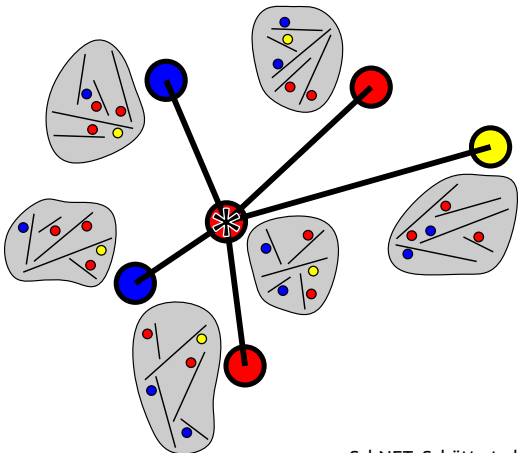
- Atoms are nodes in a fully-connected network. Edges are decorated by (functions of) interatomic distances r_{ij}
- Each node is decorated by the nature of its neighbors and their distance $h(A_i) = (a_i, \{(a_j, r_{ij})\})$
- The multiset of neighbors and edges is hashed, and used as a label to describe the nodes. The process can be iterated



SchNET: Schütt et al., JCP (2018); Gilmer *et al.*, ICML (2017)

How about graph convolution?

- Atoms are nodes in a fully-connected network. Edges are decorated by (functions of) interatomic distances r_{ij}
- Each node is decorated by the nature of its neighbors and their distance $h(A_i) = (a_i, \{(a_j, r_{ij})\})$
- The multiset of neighbors and edges is hashed, and used as a label to describe the nodes. The process can be iterated



SchNET: Schütt et al., JCP (2018); Gilmer *et al.*, ICML (2017)

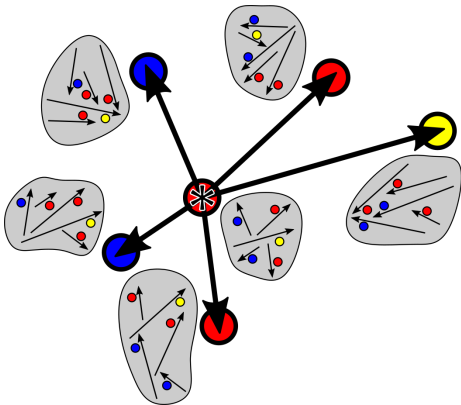
Equivariant graph convolution and ACDC

- Equivariant MP schemes can be understood as carrying around information on the directionality of the edges
- The construction of N -centers correlations can include features centered on multiple atoms, and message-passing-like contractions

$$|\rho_i^{\otimes[\nu \leftarrow \nu_1]}\rangle = \sum_{i_1} |\rho_{i_1}^{\otimes \nu}\rangle \otimes |\mathbf{r}_{i_1 i}\rangle \otimes |\rho_{i_1}^{\otimes \nu_1}\rangle$$

- Symmetry-adapted versions can be obtained with CG iterations

$$\langle q_1 l_1; q_2 l_2 | \lambda \mu \rangle = \sum_{m_1 m_2} \langle q_1 | l_1 m_1 \rangle \langle q_2 | l_2 m_2 \rangle \langle l_1 m_1; l_2 m_2 | \lambda \mu \rangle$$



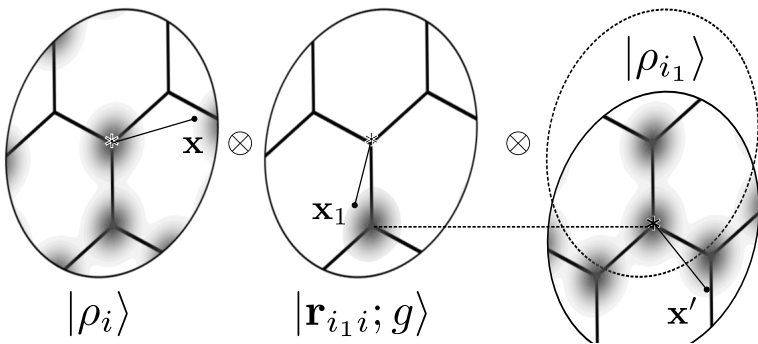
Equivariant graph convolution and ACDC

- Equivariant MP schemes can be understood as carrying around information on the directionality of the edges
- The construction of N -centers correlations can include features centered on multiple atoms, and message-passing-like contractions

$$|\rho_i^{\otimes[\nu \leftarrow \nu_1]}\rangle = \sum_{i_1} |\rho_{i_1}^{\otimes \nu}\rangle \otimes |\mathbf{r}_{i_1 i}\rangle \otimes |\rho_{i_1}^{\otimes \nu_1}\rangle$$

- Symmetry-adapted versions can be obtained with CG iterations

$$\langle q_1 l_1; q_2 l_2 | \lambda \mu \rangle = \sum_{m_1 m_2} \langle q_1 | l_1 m_1 \rangle \langle q_2 | l_2 m_2 \rangle \langle l_1 m_1; l_2 m_2 | \lambda \mu \rangle$$



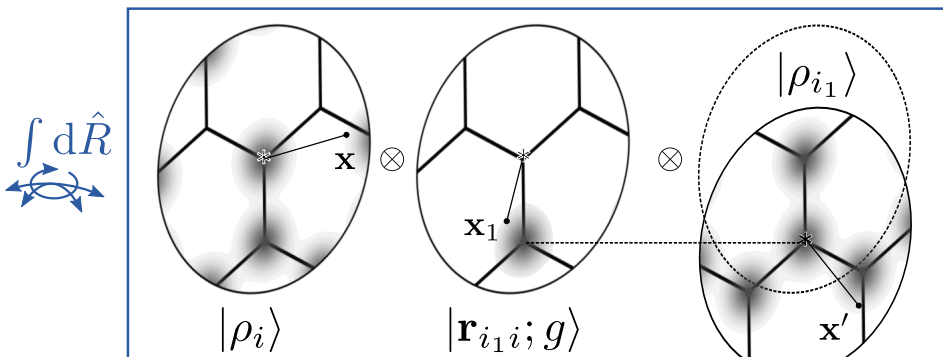
Equivariant graph convolution and ACDC

- Equivariant MP schemes can be understood as carrying around information on the directionality of the edges
- The construction of N -centers correlations can include features centered on multiple atoms, and message-passing-like contractions

$$|\rho_i^{\otimes[\nu \leftarrow \nu_1]}\rangle = \sum_{i_1} |\rho_i^{\otimes \nu}\rangle \otimes |\mathbf{r}_{i_1 i}\rangle \otimes |\rho_{i_1}^{\otimes \nu_1}\rangle$$

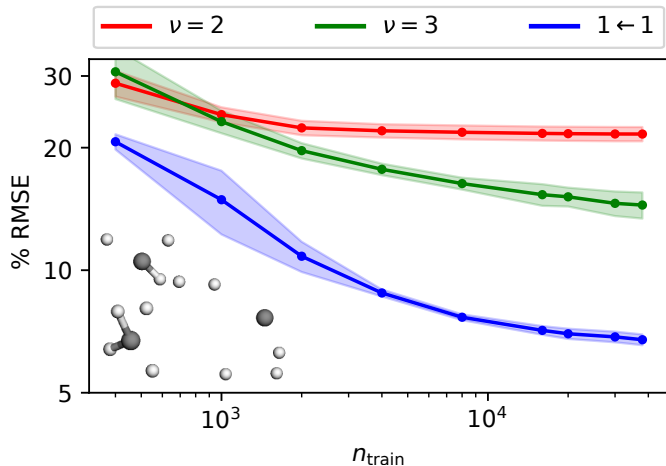
- Symmetry-adapted versions can be obtained with CG iterations

$$\langle q_1 l_1; q_2 l_2 | \lambda \mu \rangle = \sum_{m_1 m_2} \langle q_1 | l_1 m_1 \rangle \langle q_2 | l_2 m_2 \rangle \langle l_1 m_1; l_2 m_2 | \lambda \mu \rangle$$



Details matter: resolution and range

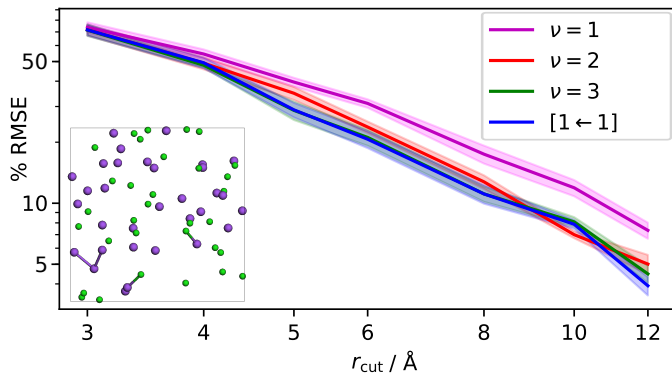
- Empirical tests of the role of MP constructs
- Much better discretization convergence for body-ordered expansions
- . . . but very little impact on long-range interactions



Nigam, Pozdnyakov, Fraux, MC, JCP (2022); Batatia et al. arxiv:2205.06643

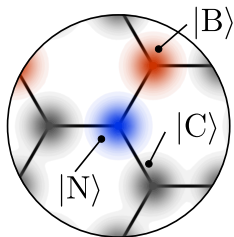
Details matter: resolution and range

- Empirical tests of the role of MP constructs
- Much better discretization convergence for body-ordered expansions
- . . . but very little impact on long-range interactions



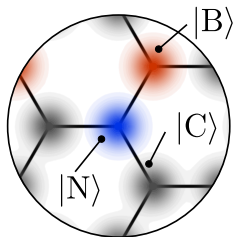
Machine-learning with pseudoelements

- How to learn with multiple species? Decorate atomic Gaussian with elemental kets $|C\rangle, |N\rangle, \dots$
- Tensor-product features lead to exponential scaling with the number of elements
- Expand each ket in a finite basis, $|b\rangle = \sum_a u_{ba} |a\rangle$. Optimize coefficients for “alchemical learning”



Machine-learning with pseudoelements

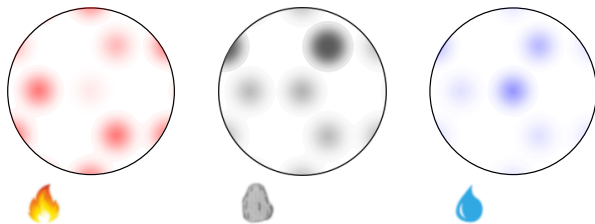
- How to learn with multiple species? Decorate atomic Gaussian with elemental kets $|C\rangle, |N\rangle, \dots$
- Tensor-product features lead to exponential scaling with the number of elements
- Expand each ket in a finite basis, $|b\rangle = \sum_a u_{ba} |a\rangle$. Optimize coefficients for “alchemical learning”



$$|C\rangle = 0.5 |\text{fire}\rangle + 0.1 |\text{stone}\rangle + 0.2 |\text{water}\rangle$$

$$|B\rangle = 0.2 |\text{fire}\rangle + 0.8 |\text{stone}\rangle + 0.3 |\text{water}\rangle$$

$$|N\rangle = 0.1 |\text{fire}\rangle + 0.1 |\text{stone}\rangle + 0.6 |\text{water}\rangle$$



Epedocles et al. (ca 360BC). Metaphor courtesy of Albert Bartók.
Willatt, Musil, MC, PCCP (2018)

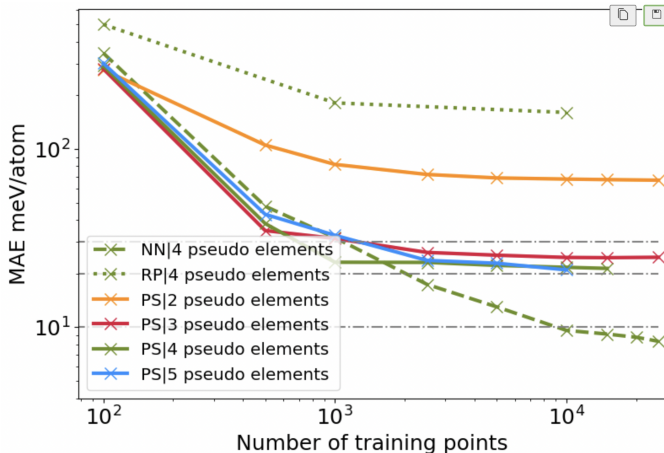
A potential for transition metal alloys

- 25 *d*-block elements that are used in high-entropy alloys.
- ~25'000 training structures with distorted *fcc* and *bcc* structures
- Alchemical compression with 3-body potential saturate at 3-4 pseudoelements.
- A non-linear layer brings error below 10meV MAE
- Interpretability: the alchemical weights reflect the ordering in the periodic table, with a twist

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Period 1	H														
2	Li	Be											B	C	N
3	Na	Mg											Al	Si	P
4	K	Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	Ga	Ge	As
5	Rb	Sr	39 Y	40 Zr	41 Nb	42 Mo	Tc	44 Ru	45 Rh	46 Pd	47 Ag	Cd	In	Sn	Sb
6	Cs	Ba	71 Lu	72 Hf	73 Ta	74 W	Re	Os	77 Ir	78 Pt	79 Au	Hg	Tl	Pb	Bi

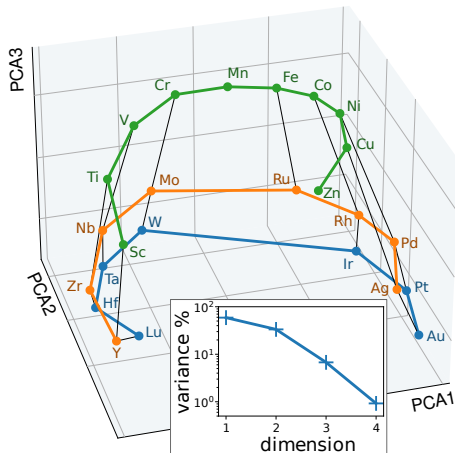
A potential for transition metal alloys

- 25 *d*-block elements that are used in high-entropy alloys.
- ~25'000 training structures with distorted *fcc* and *bcc* structures
- Alchemical compression with 3-body potential saturate at 3-4 pseudoelements.
- A non-linear layer brings error below 10meV MAE
- Interpretability: the alchemical weights reflect the ordering in the periodic table, with a twist



A potential for transition metal alloys

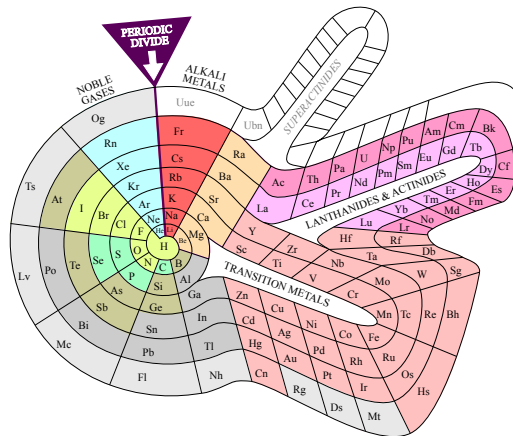
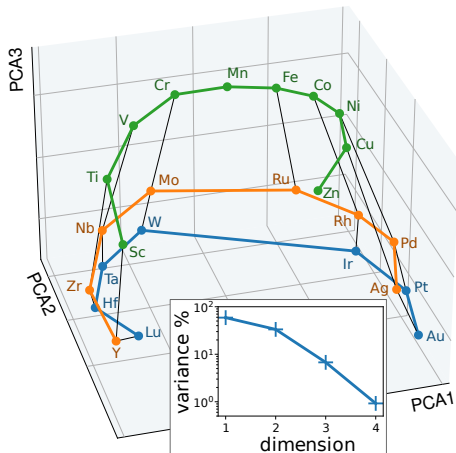
- 25 *d*-block elements that are used in high-entropy alloys.
- ~25'000 training structures with distorted *fcc* and *bcc* structures
- Alchemical compression with 3-body potential saturate at 3-4 pseudoelements.
- A non-linear layer brings error below 10meV MAE
- Interpretability: the alchemical weights reflect the ordering in the periodic table, with a twist



Lopatinysna, Fraux, Springer De, MC, in preparation

A potential for transition metal alloys

- 25 *d*-block elements that are used in high-entropy alloys.
- ~25'000 training structures with distorted *fcc* and *bcc* structures
- Alchemical compression with 3-body potential saturate at 3-4 pseudoelements.
- A non-linear layer brings error below 10meV MAE
- Interpretability: the alchemical weights reflect the ordering in the periodic table, with a twist

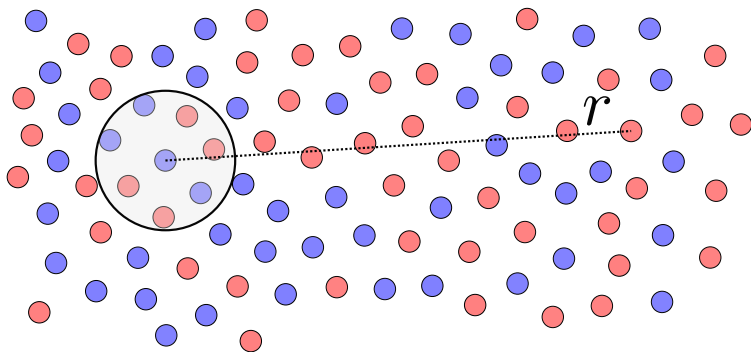


By DePiep - Own work, CC BY-SA 3.0, <https://commons.wikimedia.org/w/index.php?curid=27766488>

Long-distance equivariant representation

- Idea: *local* representation that reflects long-range *asymptotics*

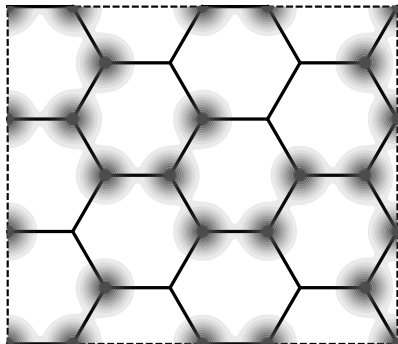
- 1 Atom-density potential $\langle ar|V \rangle = \int \langle ar'|\rho \rangle / |\mathbf{r}' - \mathbf{r}| d\mathbf{r}'$
- 2 Efficient evaluation in reciprocal space
- 3 Usual gig: symmetrize, decompose locally, learn!



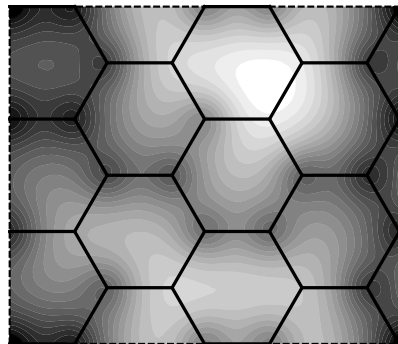
Long-distance equivariant representation

- Idea: *local* representation that reflects long-range *asymptotics*

- 1 Atom-density potential $\langle \mathbf{a}\mathbf{r} | V \rangle = \int \langle \mathbf{a}\mathbf{r}' | \rho \rangle / |\mathbf{r}' - \mathbf{r}| d\mathbf{r}'$
- 2 Efficient evaluation in reciprocal space
- 3 Usual gig: symmetrize, decompose locally, learn!



$$\langle \mathbf{a}\mathbf{r} | \rho \rangle = \sum_i \delta_{\mathbf{a}\mathbf{a}_i} g(\mathbf{r} - \mathbf{r}_i)$$

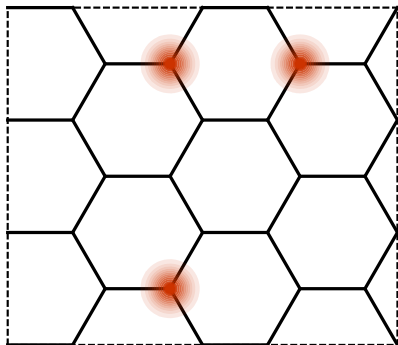


$$\langle \mathbf{a}\mathbf{r} | V \rangle = \int \langle \mathbf{a}\mathbf{r}' | \rho \rangle / |\mathbf{r}' - \mathbf{r}| d\mathbf{r}'$$

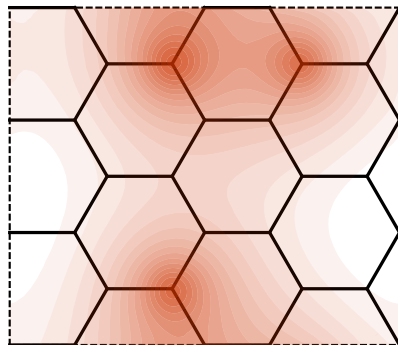
Long-distance equivariant representation

- Idea: *local* representation that reflects long-range *asymptotics*

- Atom-density potential $\langle \mathbf{a}\mathbf{r} | V \rangle = \int \langle \mathbf{a}\mathbf{r}' | \rho \rangle / |\mathbf{r}' - \mathbf{r}| d\mathbf{r}'$
- Efficient evaluation in reciprocal space
- Usual gig: symmetrize, decompose locally, learn!



$$\langle \mathbf{a}\mathbf{r} | \rho \rangle = \sum_i \delta_{\mathbf{a}\mathbf{a}_i} g(\mathbf{r} - \mathbf{r}_i)$$

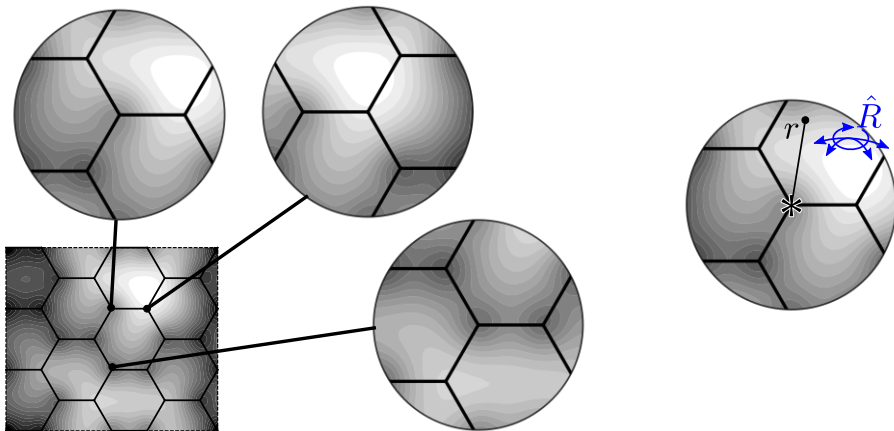


$$\langle \mathbf{a}\mathbf{r} | V \rangle = \int \langle \mathbf{a}\mathbf{r}' | \rho \rangle / |\mathbf{r}' - \mathbf{r}| d\mathbf{r}'$$

Long-distance equivariant representation

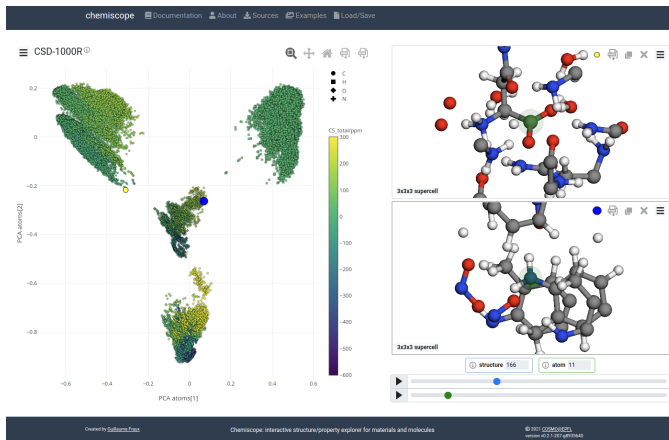
- Idea: *local* representation that reflects long-range *asymptotics*

- 1 Atom-density potential $\langle a\mathbf{r}|V\rangle = \int \langle a\mathbf{r}'|\rho\rangle / |\mathbf{r}' - \mathbf{r}| d\mathbf{r}'$
- 2 Efficient evaluation in reciprocal space
- 3 Usual gig: symmetrize, decompose locally, learn!



chemscope - a modular structure-property explorer

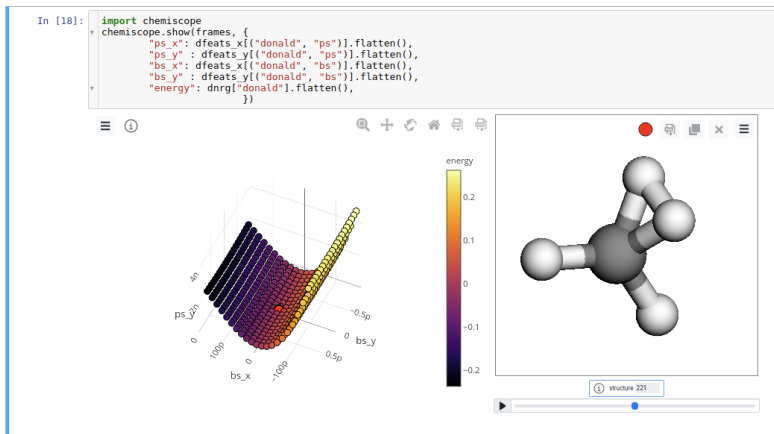
- Interactive visualization of materials datasets
- chemscope.org is a web-based visualizer with a modular design
 - online viewer, opens .json files created from python or command line
 - self-contained offline HTML for archiving/SI
 - jupyter widget to use during data analysis



Fraux, Cersonski, MC, JOSS (2020); <https://chemscope.org>

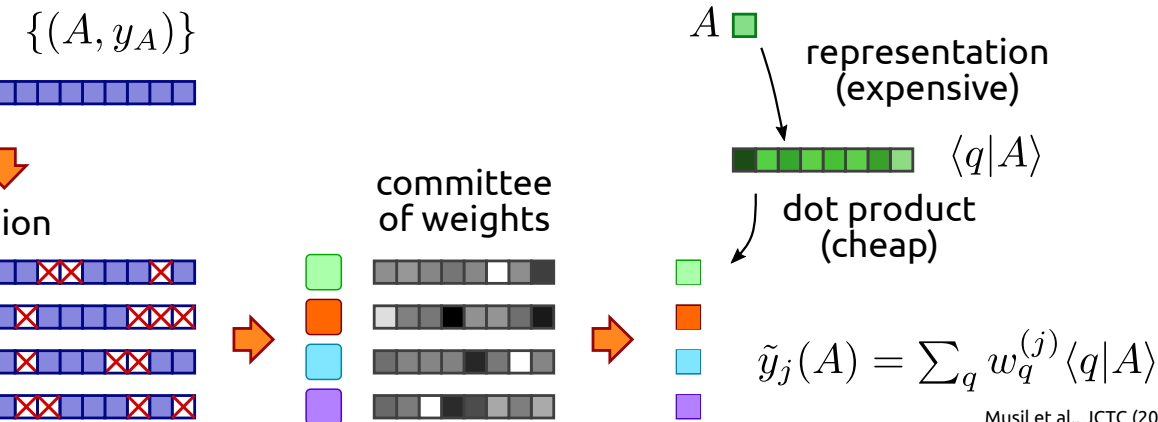
chemscope - a modular structure-property explorer

- Interactive visualization of materials datasets
- chemscope.org is a web-based visualizer with a modular design
 - online viewer, opens .json files created from python or command line
 - self-contained offline HTML for archiving/SI
 - jupyter widget to use during data analysis



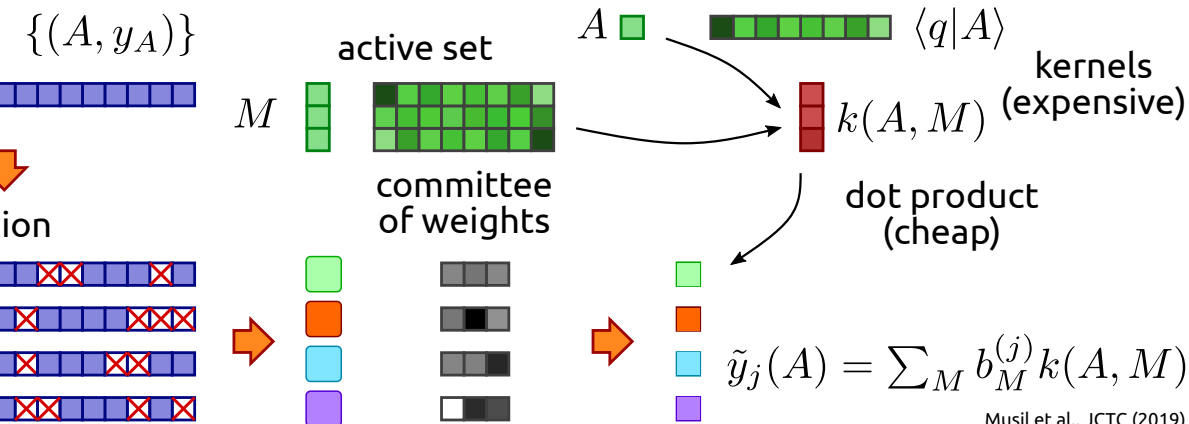
Uncertainty estimation on a shoestring

- Most expensive step is usually features/kernels evaluation. Generating committee by resampling allows inexpensive error estimation
- Same idea applies to linear & (sparse) kernel models
- Extension to NNs by only re-training output layer



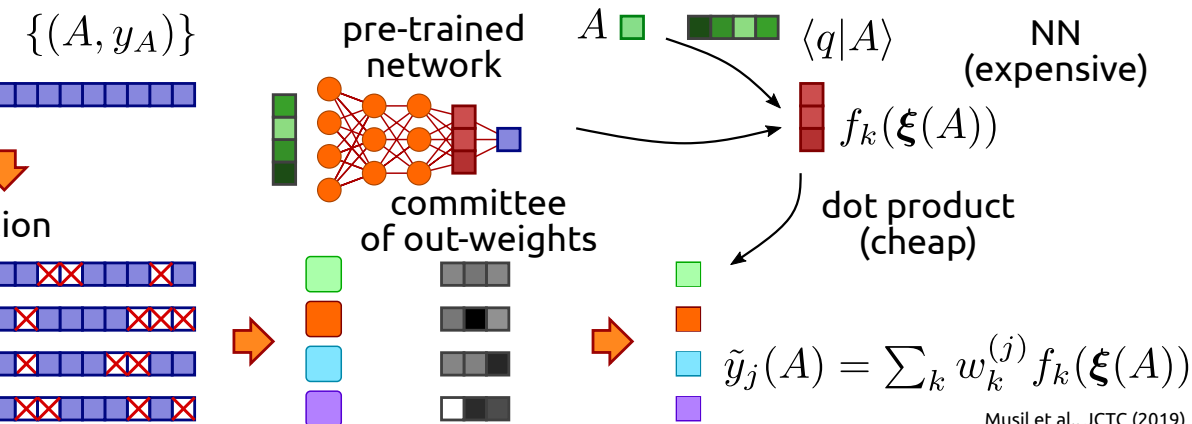
Uncertainty estimation on a shoestring

- Most expensive step is usually features/kernels evaluation. Generating committee by resampling allows inexpensive error estimation
- Same idea applies to linear & (sparse) kernel models
- Extension to NNs by only re-training output layer



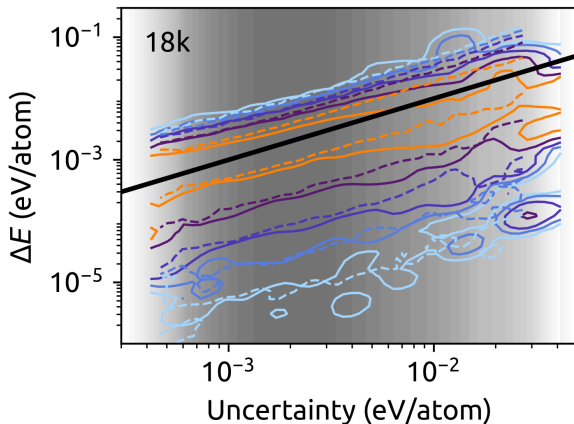
Uncertainty estimation on a shoestring

- Most expensive step is usually features/kernels evaluation. Generating committee by resampling allows inexpensive error estimation
- Same idea applies to linear & (sparse) kernel models
- Extension to NNs by only re-training output layer



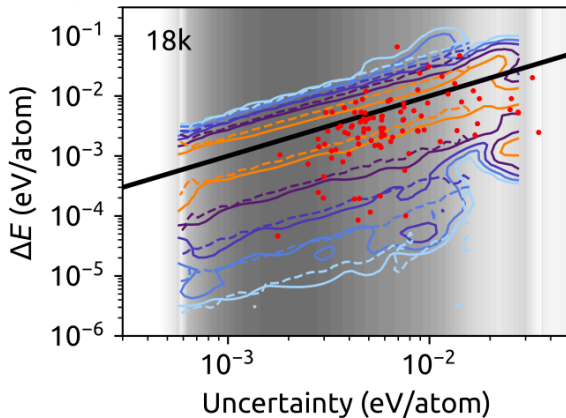
Uncertainty estimation in action

- Error vs uncertainty for the atomization energies of QM9
 - Uncertainty \neq error! Spread of predictions is what matters
 - “Rejected” QM9 structures are highly uncertain
- Raman spectra with uncertainty propagation



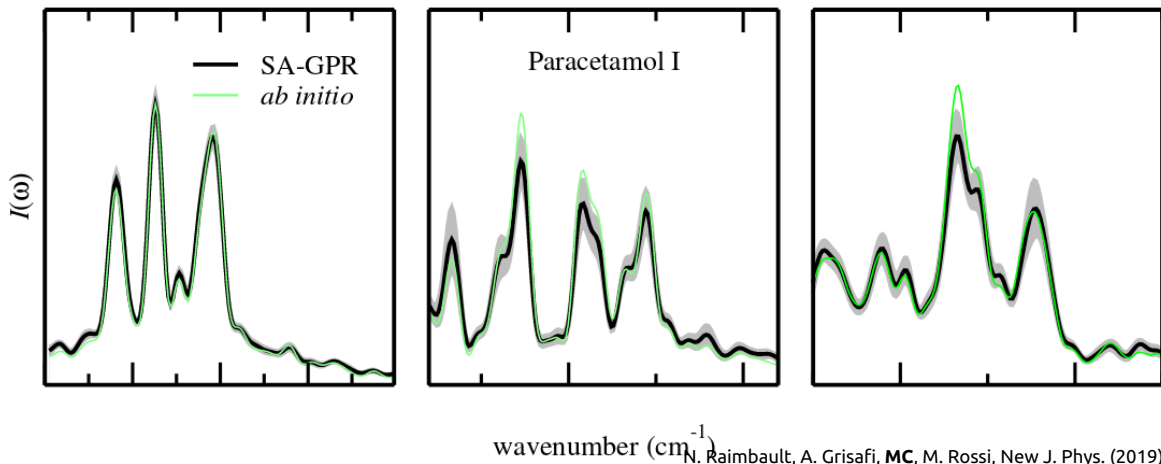
Uncertainty estimation in action

- Error vs uncertainty for the atomization energies of QM9
 - Uncertainty \neq error! Spread of predictions is what matters
 - "Rejected" QM9 structures are highly uncertain
- Raman spectra with uncertainty propagation



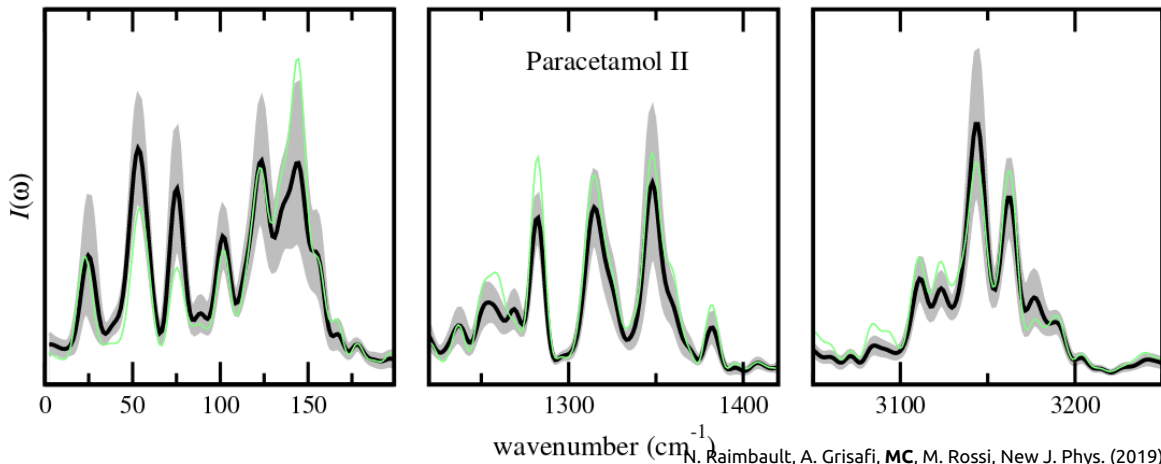
Uncertainty estimation in action

- Error vs uncertainty for the atomization energies of QM9
 - Uncertainty \neq error! Spread of predictions is what matters
 - “Rejected” QM9 structures are highly uncertain
- Raman spectra with uncertainty propagation



Uncertainty estimation in action

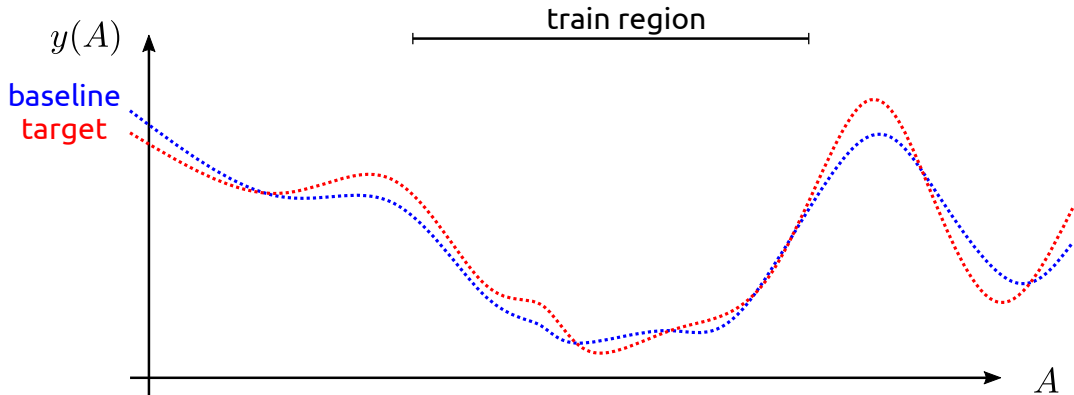
- Error vs uncertainty for the atomization energies of QM9
 - Uncertainty \neq error! Spread of predictions is what matters
 - "Rejected" QM9 structures are highly uncertain
- Raman spectra with uncertainty propagation



Weighted baseline models

- ML potential as a correction to a (semi)empirical baseline
- Use baseline error (constant σ_b) and uncertainty (structure-dependent $\sigma(A)$) to get a weighted-baseline model that avoids instability

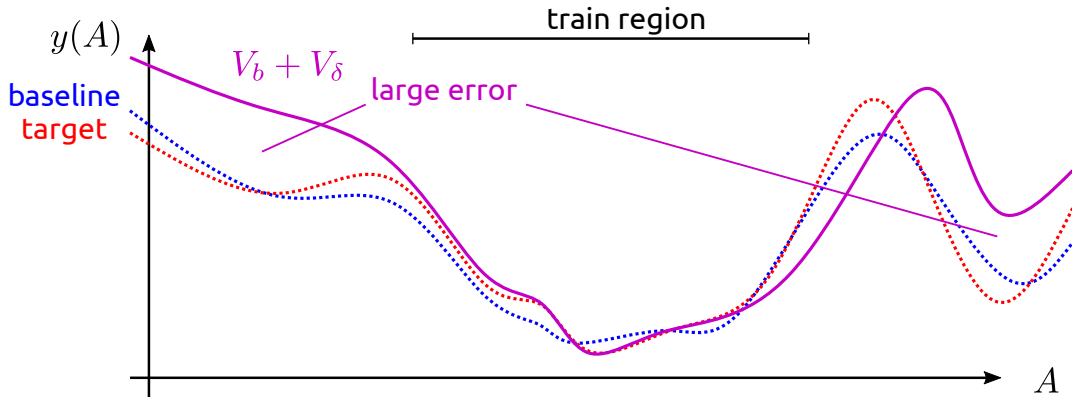
$$V(A) = V_b(A) + \bar{V}_\delta(A)$$



Weighted baseline models

- ML potential as a correction to a (semi)empirical baseline
- Use baseline error (constant σ_b) and uncertainty (structure-dependent $\sigma(A)$) to get a weighted-baseline model that avoids instability

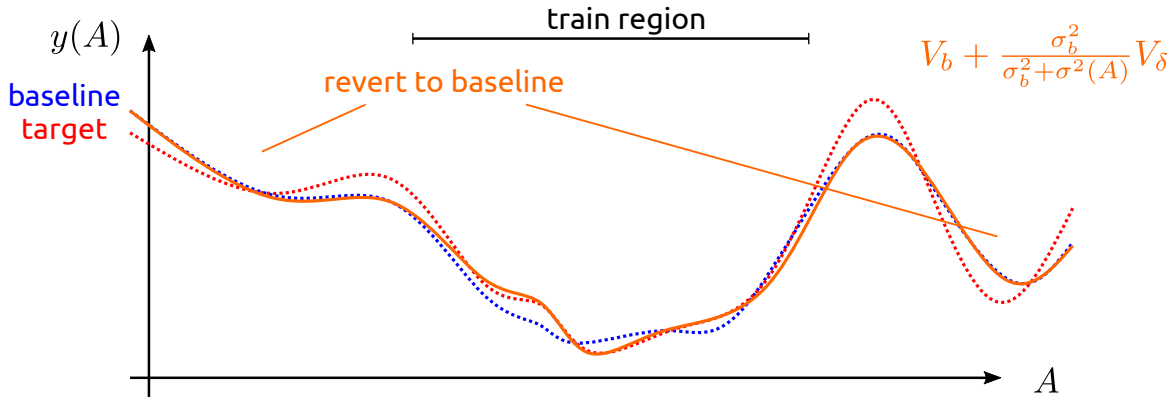
$$V(A) = V_b(A) + \bar{V}_\delta(A)$$



Weighted baseline models

- ML potential as a correction to a (semi)empirical baseline
- Use baseline error (constant σ_b) and uncertainty (structure-dependent $\sigma(A)$) to get a weighted-baseline model that avoids instability

$$U(A) = \left[\frac{1}{\sigma_b^2} + \frac{1}{\sigma^2(A)} \right]^{-1} \left[\frac{1}{\sigma_b^2} V_b(A) + \frac{1}{\sigma^2(A)} [V_b(A) + \bar{V}_\delta(A)] \right]$$



Weighted baseline models

- ML potential as a correction to a (semi)empirical baseline
- Use baseline error (constant σ_b) and uncertainty (structure-dependent $\sigma(A)$) to get a weighted-baseline model that avoids instability

