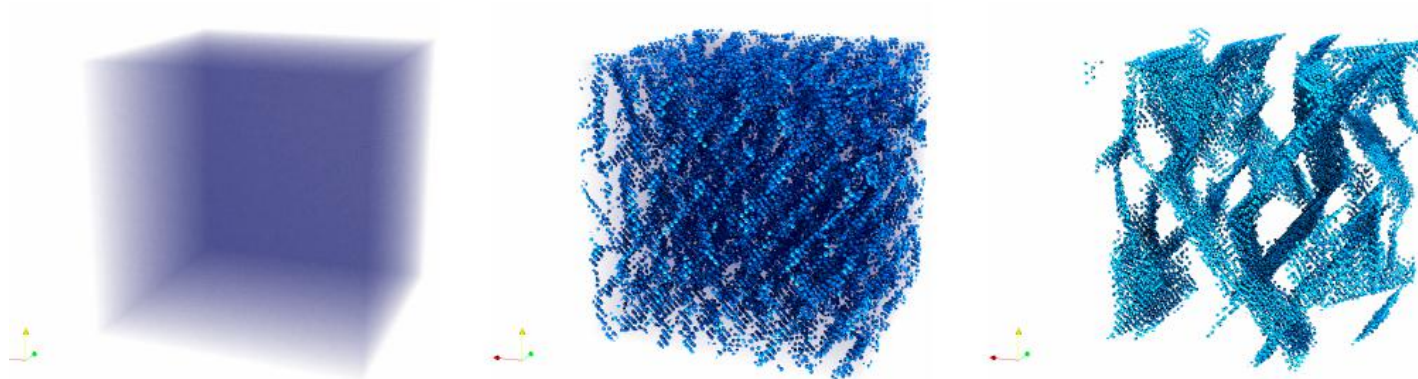


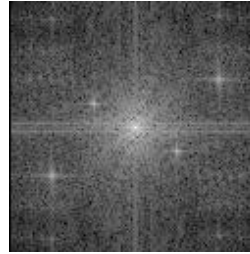
Atomistic modelling of phase transformation at large time and length scales: atomistic phase field approach

Helena Zapolsky,

*University of Rouen Normandie, GPM, UMR
6634, Saint-Etienne du Rouvray, France*



Acknowledgements

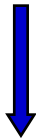
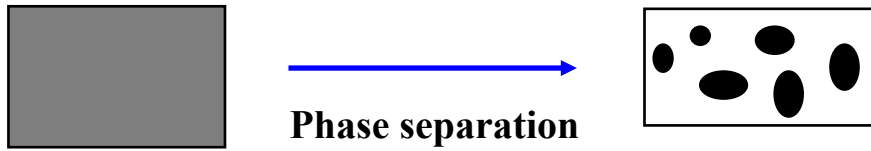


**G. Demange , M. Lavrskyi, O. Nakonechna, Y. Borges Gomes Lima,
A. Vaugeois, R. Patte**
GPM University of Rouen, France

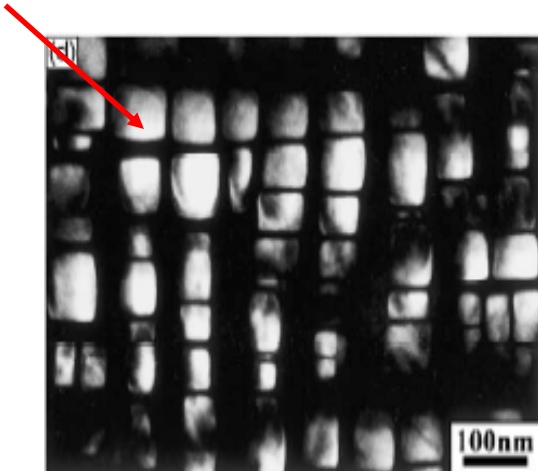
A.G. Khachaturyan
Rutgers University, Piscataway, NJ



Microstructure of alloys

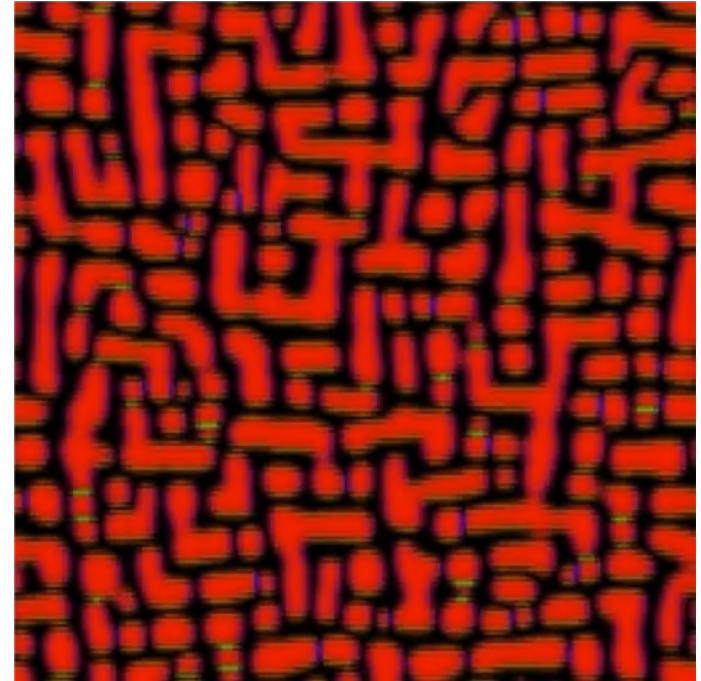


L12 ordered particles

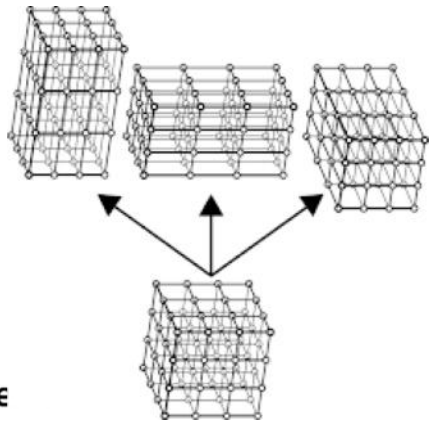


Ni-8%Al-6%Ti aged at 800°C during 45h

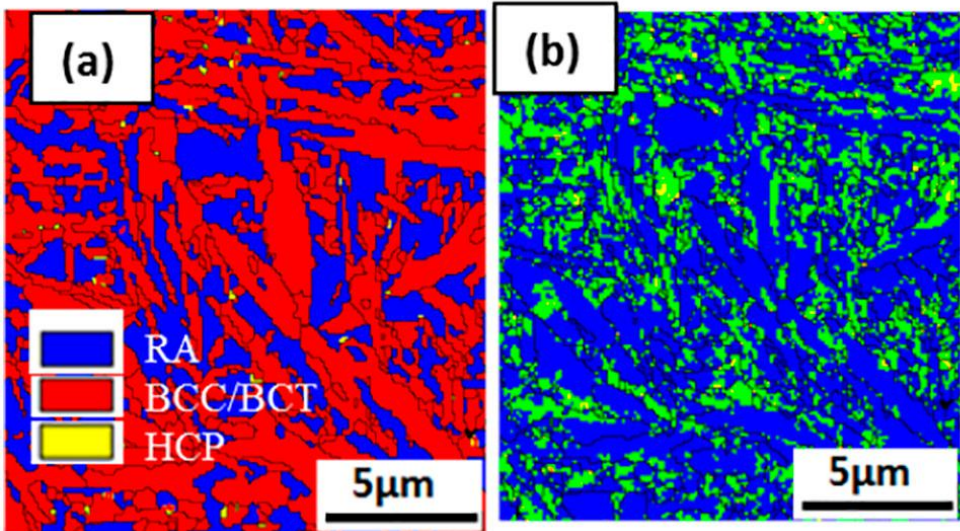
Phase Field modelling



Displacive transformations

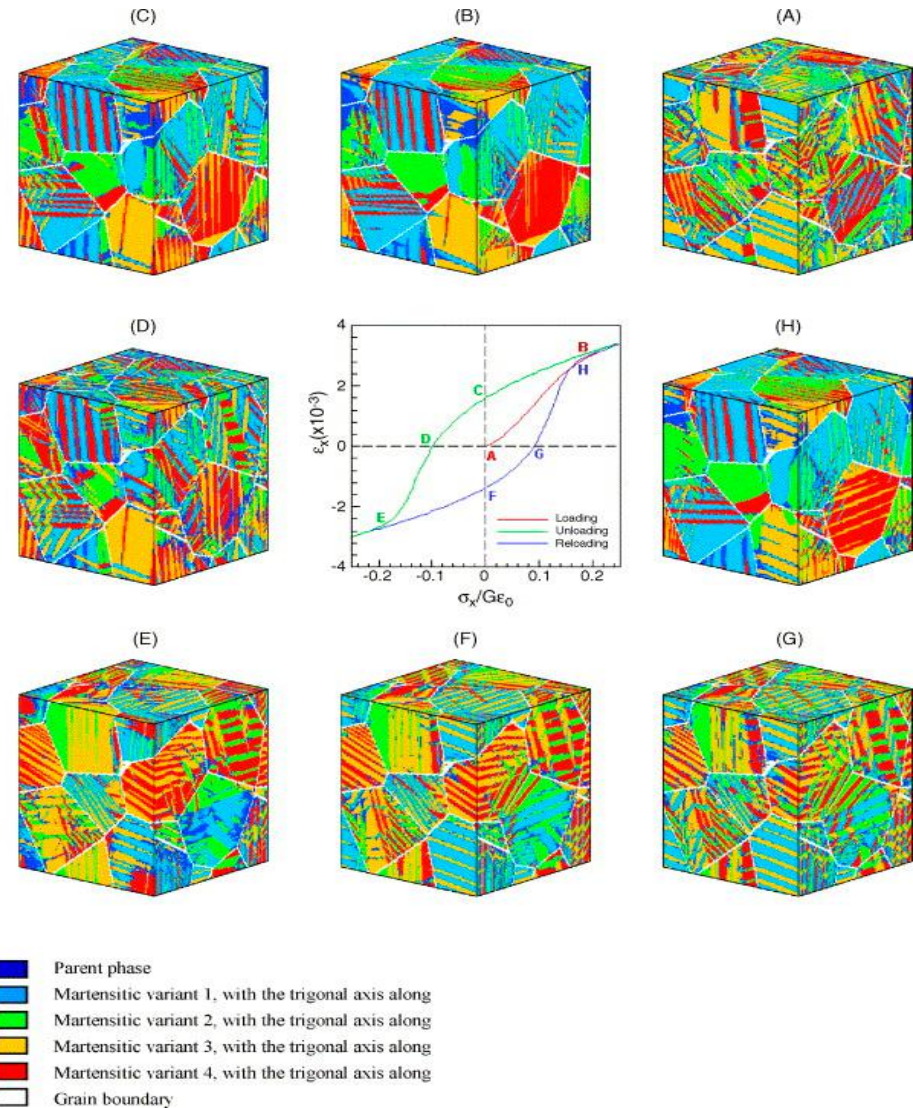


Sample



Microstructures of the steel

R. Hossain et al *Mat. Charact.* 149, 2019

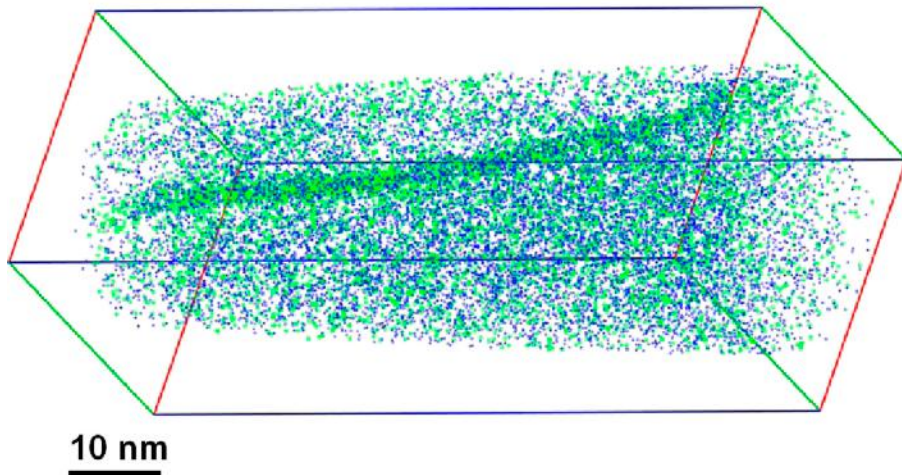


cubic \rightarrow trigonal martensitic transformation in a polycrystalline system

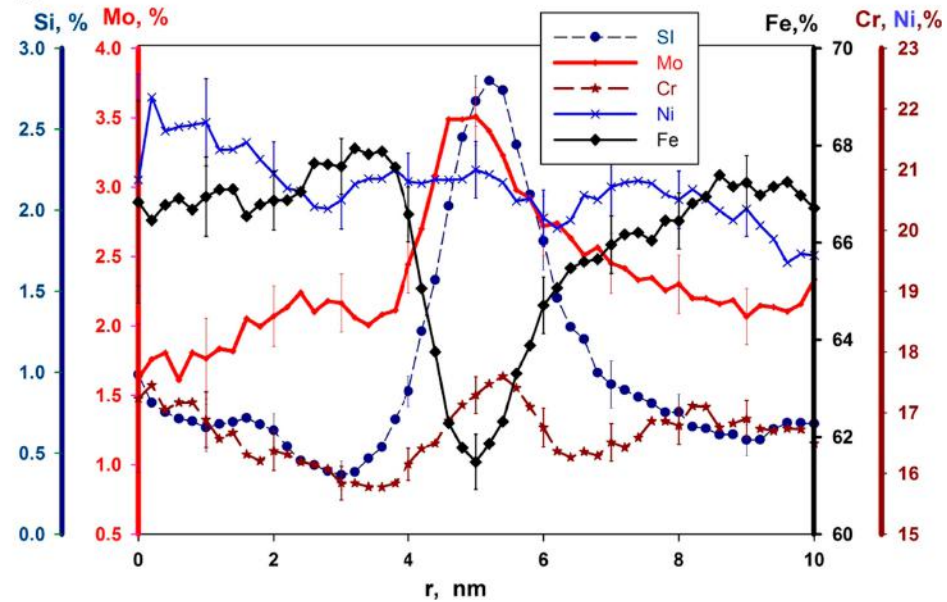
Y.M. Jin et al *Acta Mater.*, 49 (2001)

Grain boundary segregation induced strengthening of an ultrafine-grained austenitic stainless steel

a



b



An [APT](#) analysis of HPT 400 °C 316 steel reveals formation of Mo–Cr–Si rich segregation at a grain boundary—Mo and Si atoms are displayed.

Landau Khalatnikov kinetic equation (1954)



Landau-Khalatnikov relaxation equation

$$\frac{d\psi(\vec{r}, t)}{dt} = -\gamma \frac{\partial F}{\partial \psi(\vec{r}, t)}$$

Atomistic description (nm)

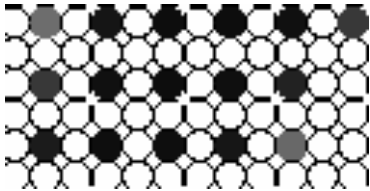
Continuos field description (μm)

Microscopique kinetic equation
Atomic density function theory

Time dependent Ginzburg-Landau equation

$$\frac{dP(\vec{r}, t)}{dt} = \sum_{\vec{r}'} L(\vec{r} - \vec{r}') \frac{\partial F}{\partial P(\vec{r}', t)}$$

$$\frac{\partial \eta(\vec{r}, t)}{\partial t} = -L \left(\frac{\delta F}{\delta \eta(\vec{r}, t)} \right) + \zeta_{\eta}(\vec{r}, t)$$



?

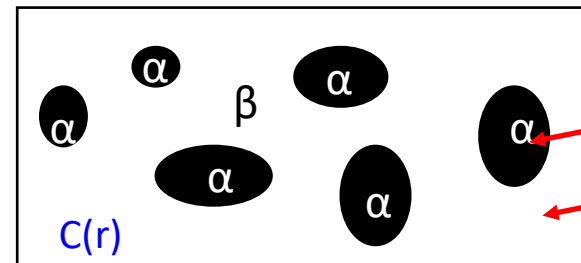


Cahn-Hilliard equation for concentrations

$$\frac{\partial c(\vec{r}, t)}{\partial t} = \vec{\nabla} M \vec{\nabla} \left(\frac{\delta F}{\delta c(\vec{r}, t)} \right) + \zeta_c(\vec{r}, t)$$

$P(r)$ is the probability to find atom in position r .

Phase field variables:
Concentration $c(r)$
Ordre parametre $\eta(r)$



$\eta=1$

$\eta=0$

Addressable Scales

Spatial scales:

Atomic scale

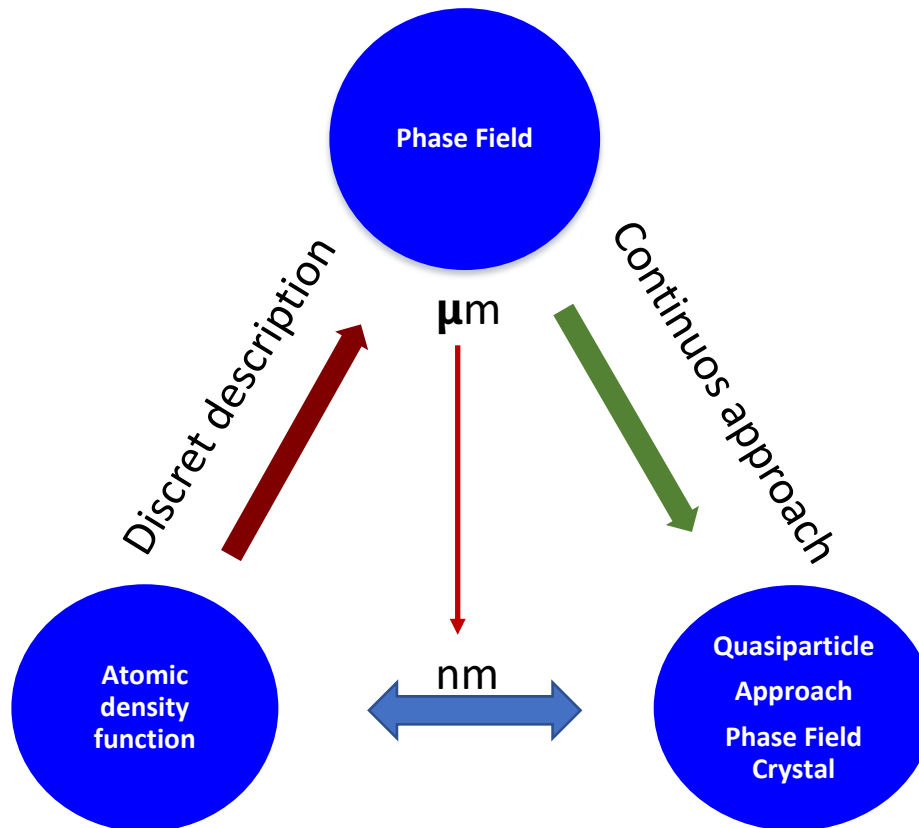
Nano-scale

Micron and sub-micron scale

Time scale:

sub-second - up to years

Phenomenological approaches



From Phase Field to Phase field Crystal Model

Stability of the system with respect to infinitesimal fluctuations

$$\delta c(r) = c(r) - \bar{c}$$

Free energy of heterogeneous system in continuous approximation

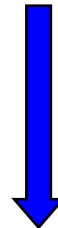
$$F = \int_V \left[f(c) + \frac{1}{2} \beta(c) (\nabla c)^2 \right] dV$$



Variation of free energy with respect to small fluctuations

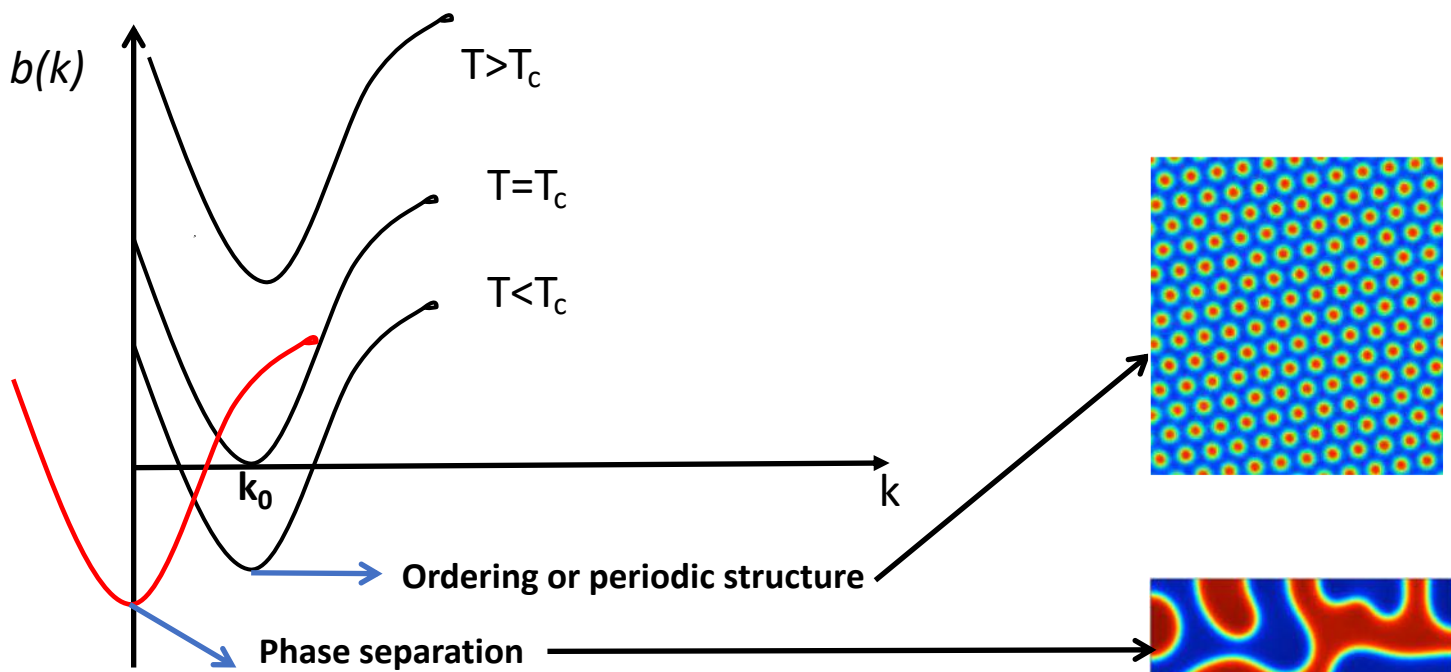
$$\delta c(r) = c(r) - \bar{c} = \frac{1}{N} \sum_k c(k) e^{ikr}$$

FT



$$\Delta F = \frac{1}{2} \int b(k) |c(k)|^2 \frac{d^3 k}{(2\pi)^3}$$

min F \longrightarrow min b(k)

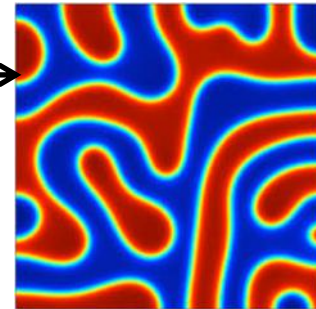
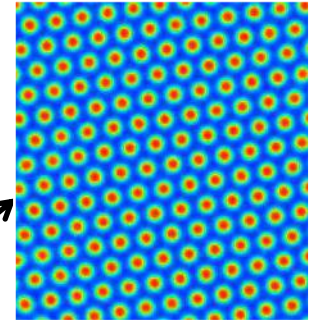


Two types of minima:

$$\frac{\partial b(k)}{\partial k} \Big|_{k=k_j} = 0$$

$k=0$ separation

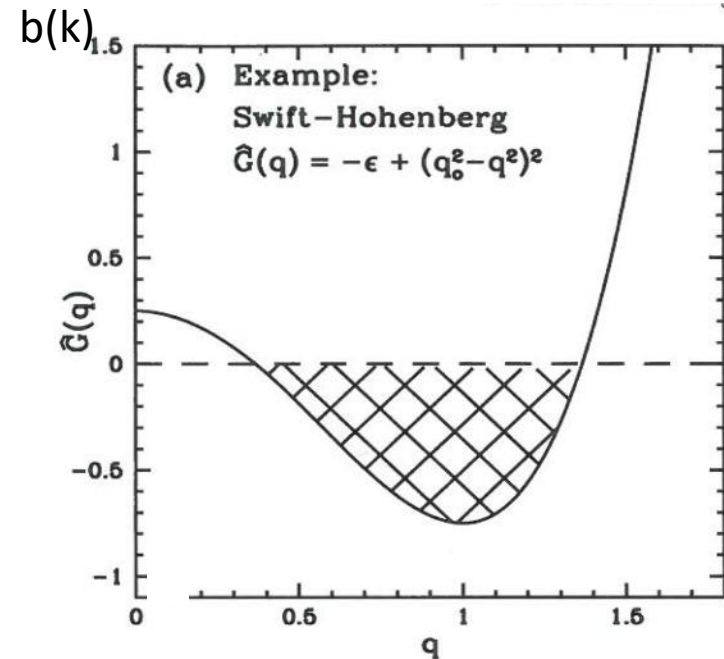
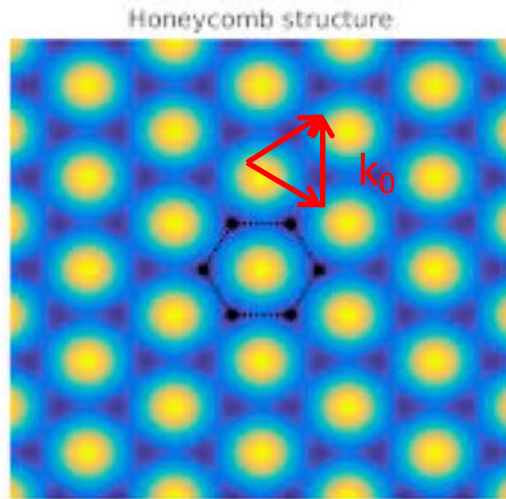
$k \neq 0$ periodic structures



Brazovskii model of crystallisation

The addition Ψ^3 in the free energy functional break \pm symmetry
(Brazovskii energy, 1975)

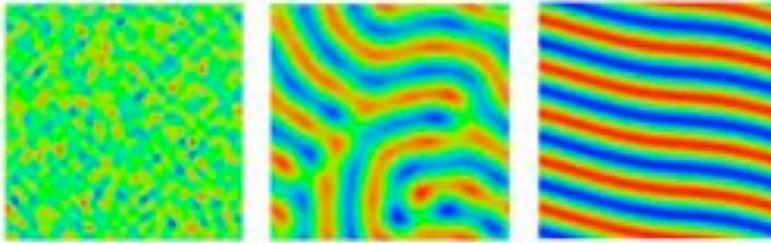
$$F = \int d\mathbf{r} \left[\frac{1}{2} \psi (-\epsilon + (k_0^2 + \nabla^2)^2) \psi + \alpha \frac{\psi^3}{3} + \frac{\psi^4}{4} \right]$$



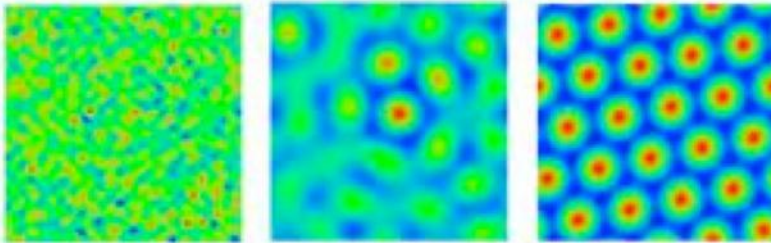
for $\epsilon=3/4$ and $k_0=1$

$$\frac{\partial \psi}{\partial t} = -L \frac{\delta F}{\delta \psi} = L(\varepsilon - (q_0^2 + \nabla^2)^2 \psi - \alpha \psi^2 - \psi^3)$$

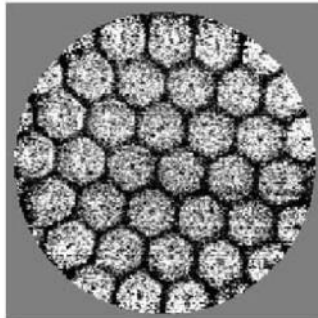
Swift-Hohenberg model of convection



stripes



honeycomb structure.



Rayleigh-Bénard convection

Phase Field Crystal method (K. Elder, 2002) -> zones with high concentration -> atoms

Limit transition to the Landau theory

$$F = \int \frac{1}{2} \sum_{\alpha\beta} V_{\alpha\beta}(k) \Phi_{\alpha}(k) \Phi_{\beta}^*(k) \frac{d^3k}{(2\pi)^3} + \int_V f(\{\rho_{\alpha}(r)\}) d^3r \quad (1)$$

Where $V_{\alpha\beta}(\mathbf{k})$ is the Fourier transforms of the effective potentials $W_{\alpha\beta}(\mathbf{r})$:

$$V(\mathbf{k}) = \int_V W(\mathbf{r}) \exp(-i\mathbf{k}\mathbf{r}) d^3r$$

Using Taylor expansion of $V_{\alpha\beta}(\mathbf{k})$ in k

$$V_{\alpha\beta}(k) = A_0^{\alpha\beta} + \frac{1}{2!} A_2^{\alpha\beta} k^2 + \frac{1}{4!} A_4^{\alpha\beta} k^4 + \dots + \frac{1}{n!} A_n^{\alpha\beta}$$

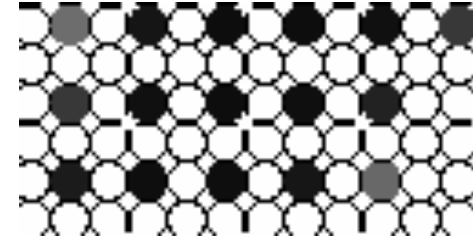
Eq.(1) is a generalized **Landau gradient expression**:

$$F = \int_V \sum_{\alpha\beta} \left(\frac{1}{2!} A_0^{\alpha\beta} \rho_{\alpha}(\mathbf{r}) \rho_{\beta}(\mathbf{r}) + \frac{1}{2!} A_2^{\alpha\beta} \nabla \rho_{\alpha}(\mathbf{r}) \nabla \rho_{\beta}(\mathbf{r}) + \frac{1}{4!} A_4^{\alpha\beta} \nabla^2 \rho_{\alpha}(\mathbf{r}) \nabla^2 \rho_{\beta}(\mathbf{r}) + \dots \right) d^3r + \int_V f(\{\rho(\mathbf{r})_{\alpha}\}) d^3r$$

The **Phase Field Crystal model** has used two first terms of the gradient expansion of the Landau theory.

From Atomic density function theory on constrained lattice to continuous approach

$$F = F_{chem} + E_{elast}$$

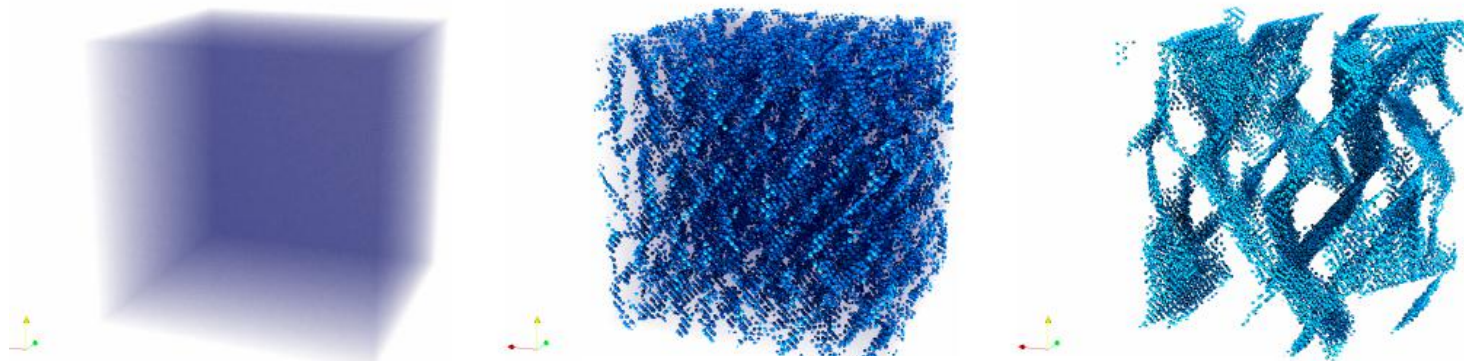


$$F_{chem} = \frac{1}{2} \sum_{r,r'} V(r-r') p(r) p(r') + k_B T \sum_{\bar{r}} \left\{ p(r) \text{Ln}(p(r) + [1-p(r)] \text{Ln}[1-(p(r)) \right\}$$

$$E_{elast} = \frac{1}{2} \sum_{pq} \int \frac{d^3 k}{(2\pi)^3} B_{pq}(\mathbf{n}) \theta_p(\mathbf{k}) \theta_q^*(\mathbf{k})$$

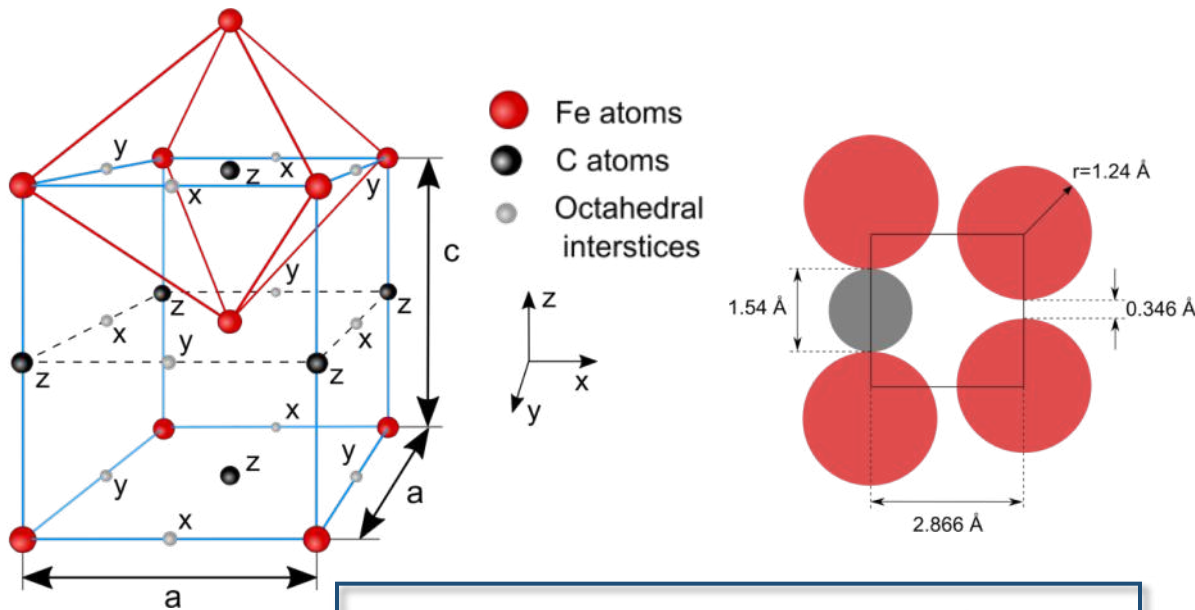
Microscopic kinetic Onsager equation

$$\frac{dP(r,t)}{dt} = \frac{1}{k_B T} \sum_{\alpha,\beta} \sum_{r'} L_{\alpha\beta}(r-r') c_\alpha c_\beta \frac{\partial F}{\partial P(r',t)}$$



Carbide formation in Fe-C system

Carbon atoms in the bcc lattice



Concentration expansion coefficients

$$u_{11} = \frac{a - a_0}{a_0 \bar{c}_c}; \quad u_{33} = \frac{c - a_0}{a_0 \bar{c}_c}$$

Deformation matrix

$$u_{ij}(1) = \begin{pmatrix} u_{33} & 0 & 0 \\ 0 & u_{11} & 0 \\ 0 & 0 & u_{11} \end{pmatrix}$$

$$u_{ij}(2) = \begin{pmatrix} u_{11} & 0 & 0 \\ 0 & u_{33} & 0 \\ 0 & 0 & u_{11} \end{pmatrix}$$

$$u_{ij}(3) = \begin{pmatrix} u_{11} & 0 & 0 \\ 0 & u_{11} & 0 \\ 0 & 0 & u_{33} \end{pmatrix}$$

Interaction potential consists of elastic and chemical parts:

$$\tilde{w}_{pq}(\mathbf{k}) = \tilde{w}_{pq}^{elas}(\mathbf{k}) + \tilde{w}_{pq}^{chem}(\mathbf{k})$$

$$p, q = 1, 2, 3$$

Elastic part of the interaction potential:

$$\tilde{W}_{pq}^{elast}(\mathbf{k}) = -F_i^{oct}(p, \mathbf{k}) G_{ij}(\mathbf{k}) F_i^{*oct}(q, \mathbf{k})$$

Kanzaki forces

$$F_x^{oct}(\mathbf{z}, \mathbf{k}) = -i a_0^2 \sigma_{11}^0 \sin\left(\frac{k_x a_0}{2}\right) \cos\left(\frac{k_y a_0}{2}\right)$$

$$F_y^{oct}(\mathbf{z}, \mathbf{k}) = -i a_0^2 \sigma_{11}^0 \cos\left(\frac{k_x a_0}{2}\right) \sin\left(\frac{k_y a_0}{2}\right)$$

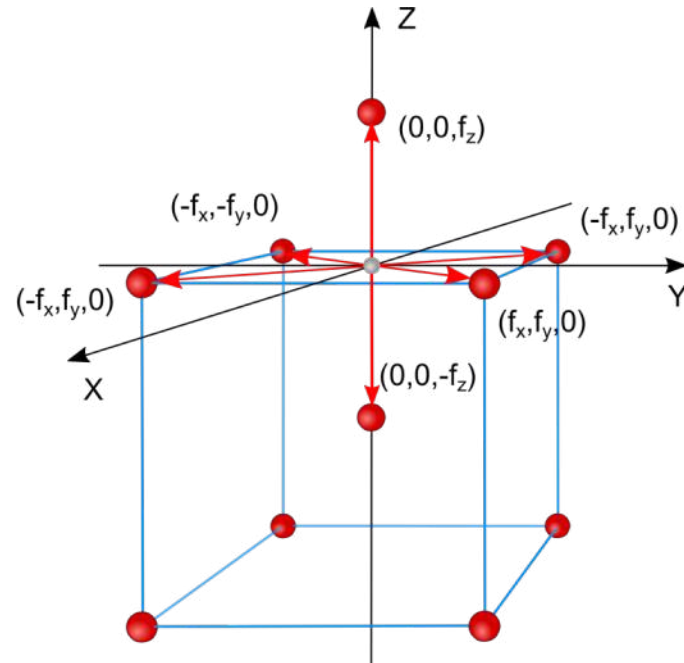
$$F_z^{oct}(\mathbf{z}, \mathbf{k}) = -i a_0^2 \sigma_{33}^0 \sin\left(\frac{k_z a_0}{2}\right)$$

$$\sigma_{11}^0 = (C_{11} + C_{12}) u_{11} + C_{12} u_{33}$$

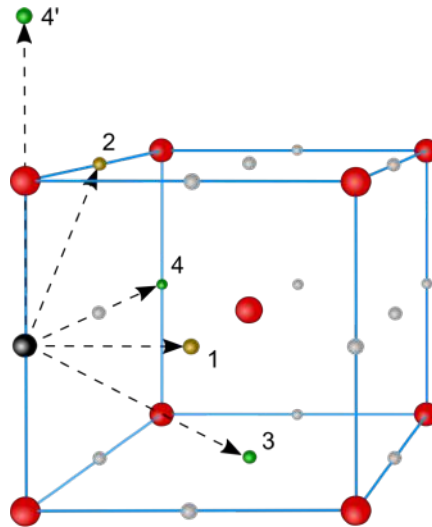
$$\sigma_{33}^0 = C_{11} u_{33} + 2C_{12} u_{11}$$

Green function

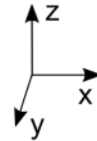
$$G_{ij}(\mathbf{k}) = [D_{ij}(\mathbf{k})]^{-1}$$



CHEMICAL PART OF INTERACTION POTENTIAL



- Metal atoms
- ● Octahedral interstices
- C atom



$$W_1^{chem} = 3.13$$

$$W_2^{chem} = 0.87$$

$$W_3^{chem} = 0.27$$

$$W_4^{chem} = 1.4$$

$$W_4^{chem*} = 0.23$$

$$\tilde{w}_{11}^{chem}(\mathbf{k}) = 8W_3 \cos\left(\frac{k_x a_0}{2}\right) \cos\left(\frac{k_y a_0}{2}\right) \cos\left(\frac{k_z a_0}{2}\right) + 2W_4 (\cos(k_y a_0) + \cos(k_z a_0)) + 2W_4^* \cos(k_x a_0)$$

$$\tilde{w}_{12}^{chem}(\mathbf{k}) = 2W_1 \cos\left(\frac{k_z a_0}{2}\right) + 4W_2 \cos\left(\frac{k_y a_0}{2}\right) \cos\left(\frac{k_x a_0}{2}\right)$$

Interaction potential

$$\tilde{w}_{pq}(\mathbf{k}) = -\frac{F_i^{oct}(p, \mathbf{k})G_{ij}(\mathbf{k})F_i^{*oct}(q, \mathbf{k})}{a_0^3 C_{44} u_{33}^2} + W_0 \frac{\tilde{w}_{pq}^{chem}(\mathbf{k})}{a_0^3 C_{44} u_{33}^2}$$

Dimensionless
parameter

$$a_0^3 C_{44} u_{33}^2 = 12.17 \text{ eV}$$

Concentration expansion
coefficients [1]

$$u_{11} = -0.09$$

$$u_{33} = 0.86$$

$$t_1 = u_{11}/u_{33} = -0.1$$

Fe_α

$$a_0 = 2,865 \text{ \AA}$$

$$C_{11} = 242 \text{ GPa}$$

$$C_{12} = 146.5 \text{ GPa}$$

$$C_{44} = 112 \text{ GPa}$$

Carbon diffusion in martensite phase at room temperature

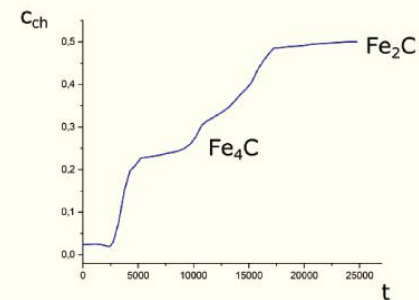
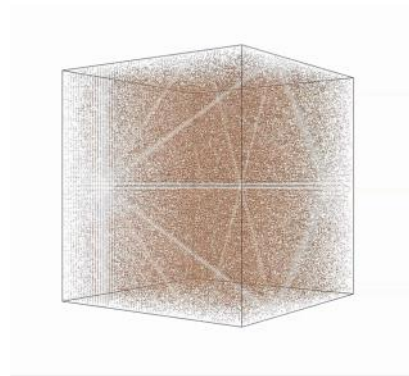
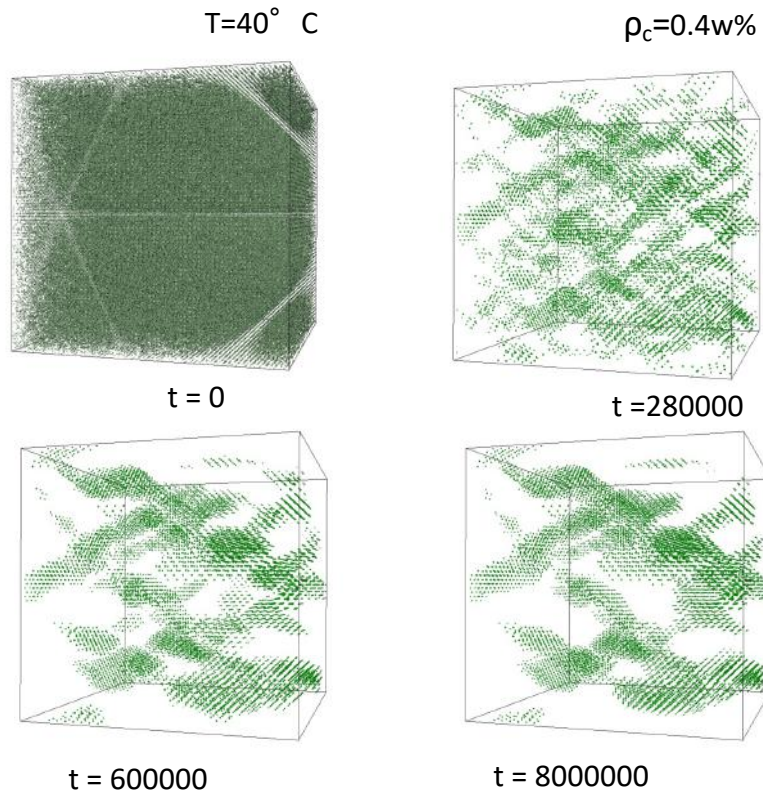
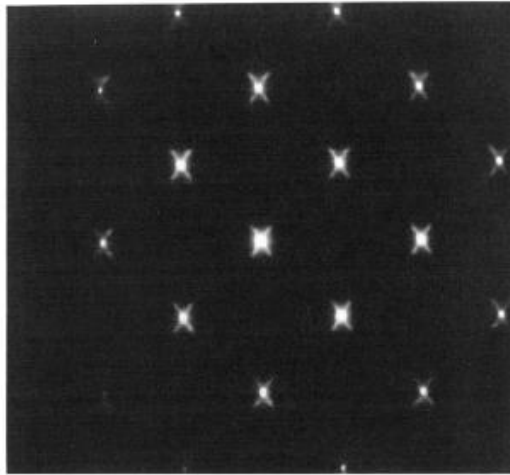
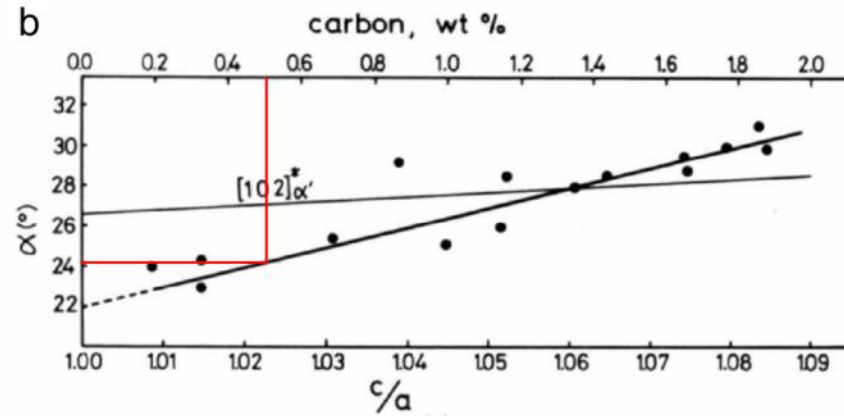
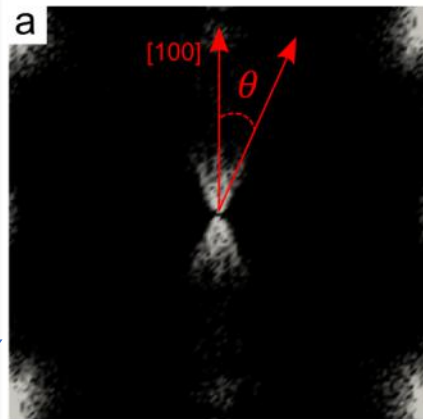


FIGURE 5.12: Dependence of the concentration $c_{ch}(\mathbf{r}, t)$ on time at the point $\mathbf{r}(x, y, z)$ with coordinates (52, 31, 61). The final stoichiometric formula for the blue curve is Fe_2C with an intermediate Fe_4C concentration. The time units are in t .

Diffraction of carbon reach zones observed after aging at room temperature



diffraction pattern obtained from Fe-15Ni-1C martensite after aging at room temperature for 26 min (K.A. Taylor et al. Met.Trans.A V.20A, 1989)

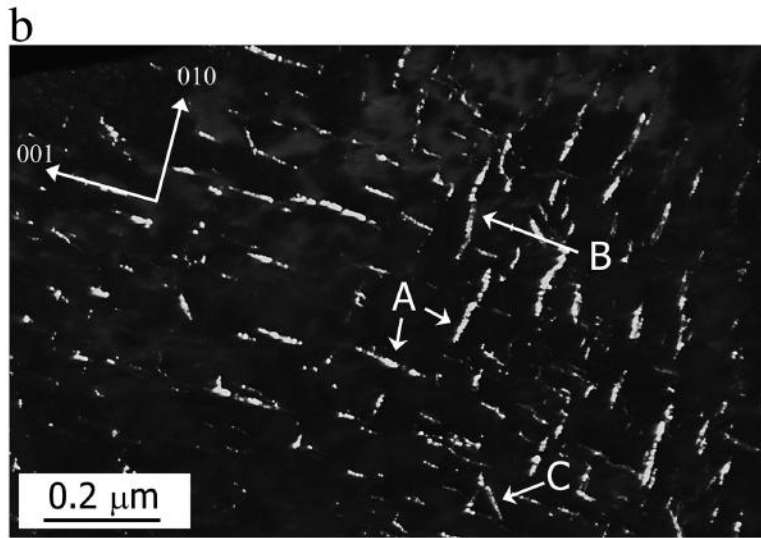
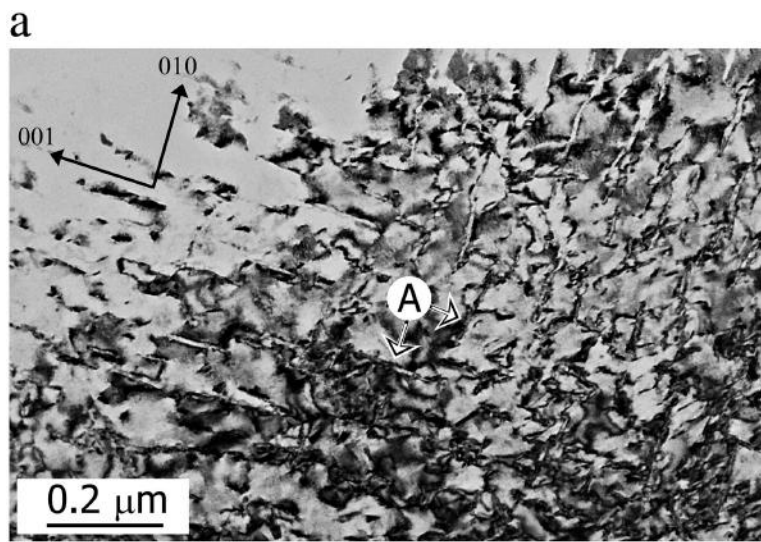


Diffraction pattern from simulation

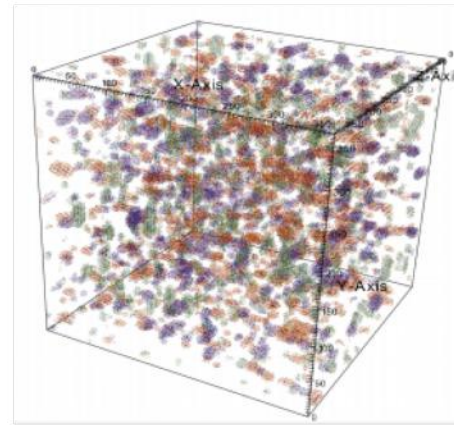
$$\bar{c}_C = 0.51 \text{ wt.}\%$$

$$\theta_{\text{obtained}} \approx 23.4^\circ \rightarrow \sim [2\bar{1}0]$$

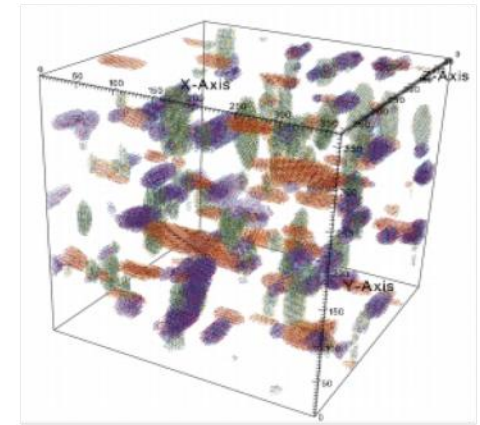
$$\theta_{\text{exp}} \approx 24^\circ$$



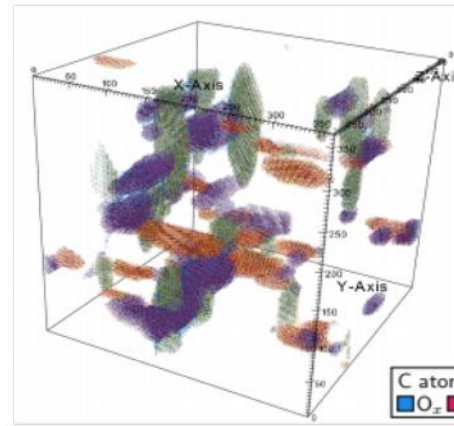
η -Fe₂C carbide



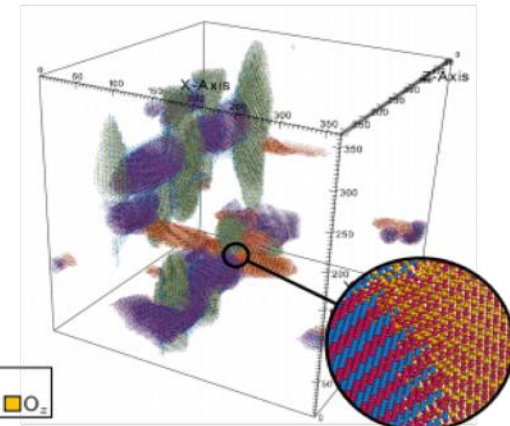
(a) $t^* = 1$



(b) $t^* = 10$



(c) $t^* = 50$



(d) $t^* = 110$

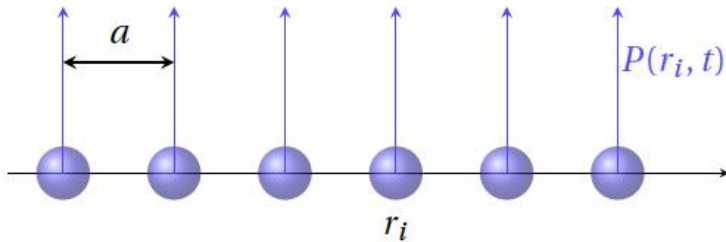
Transition-iron-carbide precipitates in a lath of tempered martensite. (a) Bright-field and (b) centered-dark-field TEM images.

S.W.Thompson Mater. Charact. V.106, 2015

Schwab et al npj Comput Mat under revision

From discrete to continuum ADF model

Rigide Ising lattice



(a) Probability $P(r_i, t)$ to find atom on site r_i

- $\mathbb{P}(r)$: probability to find atom at position r
- 1 atome \Leftrightarrow 1 site

$$\sum_r \mathbb{P}(r) = N_{atoms}$$

Continuous model

Meaning of $\mathbb{P}(r)$?

$$\sum_r \mathbb{P}(r) \neq N_{atoms}$$

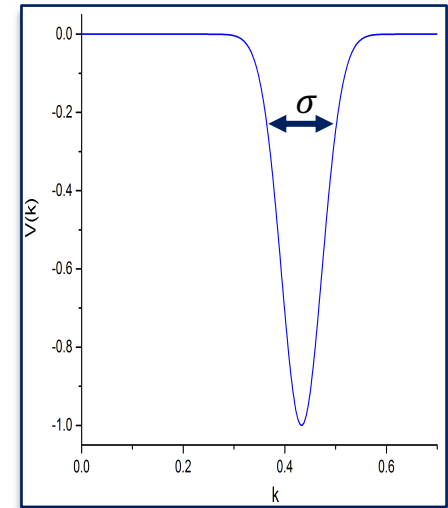
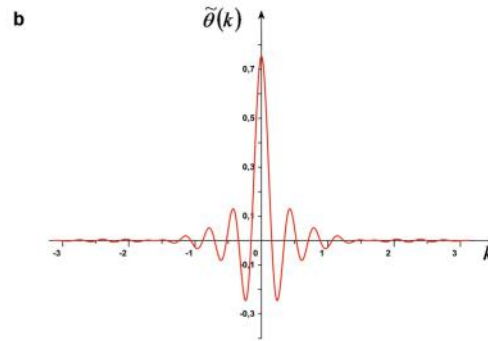
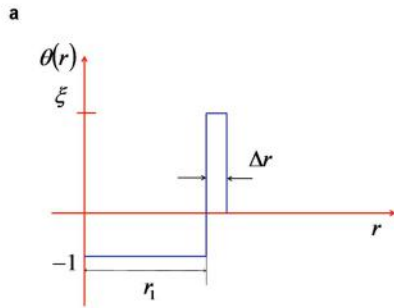
(M. Lavrskiy et al., npj computational materials 2017)

$$\sum_r \mathbb{P}(\mathbf{r}) = N_{fraton}$$

The Fourier Transform representation of the model Hamiltonian:

$$H = \frac{1}{2N} \sum_{\mathbf{k}} \sum_{\alpha=1}^{\alpha=m} \sum_{\beta=1}^{\beta=m} \tilde{w}_{\alpha\beta}(\mathbf{k}) \tilde{\rho}_{\alpha}(\mathbf{k}) \tilde{\rho}_{\beta}(\mathbf{k})^*$$

Model Potential
$$W_{\alpha\beta}(\mathbf{r} - \mathbf{r}') = \underbrace{\theta_{\alpha}(\mathbf{r} - \mathbf{r}') \delta_{\alpha\beta}}_{\text{Short range}} + \underbrace{\lambda_{\alpha\beta} W_{\alpha\beta}^{\text{LR}}(\mathbf{r} - \mathbf{r}')}_{\text{Long range}}.$$



$$\theta_{\alpha}(\mathbf{r}) = \begin{cases} -1 & \text{if } r \leq R_{\alpha} \\ \xi & \text{if } R_{\alpha} < r \leq R_{\alpha} + \Delta R_{\alpha} \\ 0 & \text{otherwise.} \end{cases}$$

Exemple bcc structure

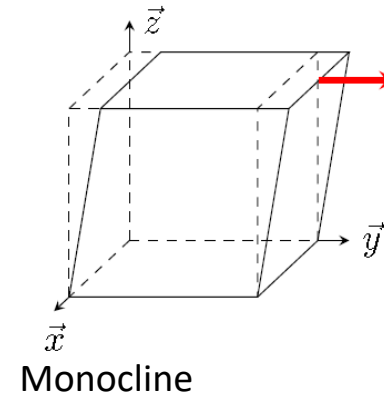
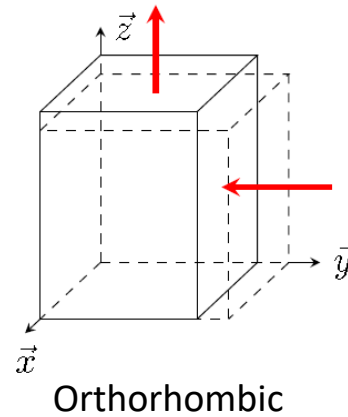
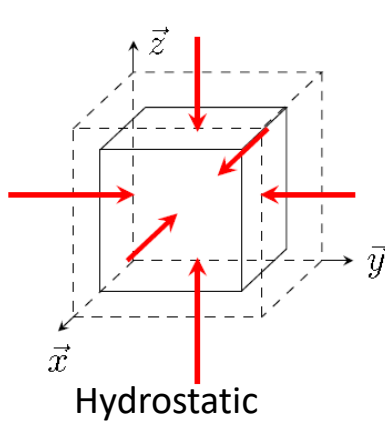
$$\hat{W}^{\text{LR}}(k) = -\exp\left(-\frac{(k - k^{\text{bcc}})^2}{2(\sigma^{\text{bcc}})^2}\right)$$

Principal loading conditions – calculation of elastic constants

1) hydrostatic, $(x, y, z) \rightarrow (1 - \varepsilon)x, (1 - \varepsilon)y, (1 - \varepsilon)z,$

2) orthorhombic, $(x, y, z) \rightarrow (1 + \varepsilon)x, (1 - \varepsilon)y, z$

3) monoclinique $(x, y, z) \rightarrow x + \varepsilon y, y, z$



$$F_{cubic.}(\varepsilon) = F_0 + V_0 \frac{3}{2} (C_{11} + 2C_{12}) \varepsilon^2$$

$$F_{orthor.}(\varepsilon) = F_0 + V_0 (C_{11} - C_{12}) \varepsilon^2$$

$$F_{monoc.}(\varepsilon) = F_0 + V \frac{C_{44}}{2} \varepsilon^2$$

Kinetic equation (n-components alloy)

$$\frac{d \rho_{\alpha}(\mathbf{r}, t)}{dt} = \sum_{\mathbf{r}'} L_{\alpha\beta}(\mathbf{r}, \mathbf{r}') \frac{\delta F}{\delta \rho_{\beta}(\mathbf{r}', t)}$$

Free energy functional is:

$$F_{chem} = \frac{1}{2} \sum_{\alpha, \beta} \sum_{r, r'} V_{\alpha\beta}(r - r') p_{\alpha}(r) p_{\beta}(r') + k_B T \sum_{\alpha} \sum_{\vec{r}} \{ p_{\alpha}(r) \text{Ln}[p_{\alpha}(r)] + [1 - p_{\alpha}(r)] \text{Ln}[1 - (p_{\alpha}(r))] \}$$

Reduced variables:

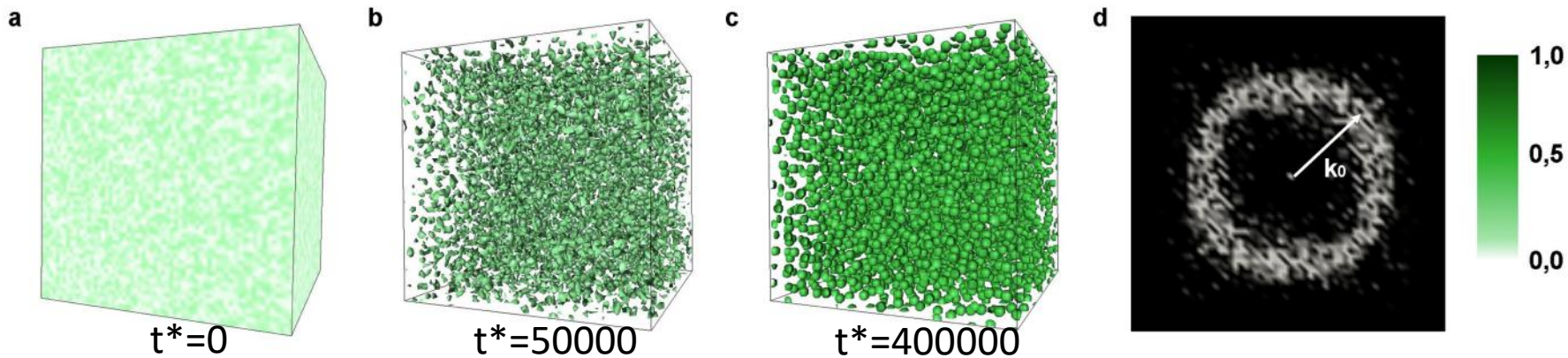
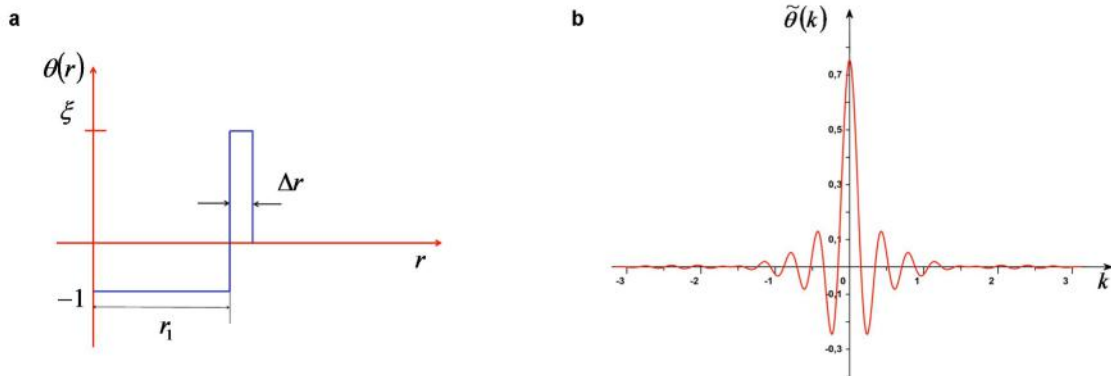
time is measured in units of typical atomic migration time, τ_0

energy is measured in units of $k_B T_0$, where T_0 is the solidification temperature

the grid lattice increment, l , (the spacing of the underlying Ising lattice), is defined as a fraction of the atomic radius

The numerical solution -> the semi-implicit Fourier spectral method

Self-assembly of fractons with only short range interaction

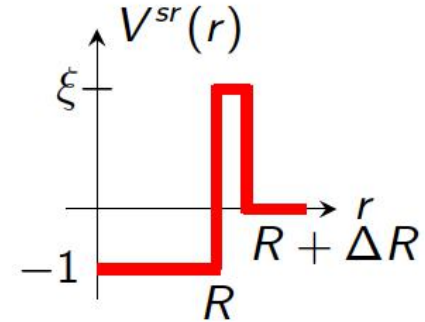


$$\hat{\rho} = 0.1, \hat{l} = 0.5, \Delta \hat{r} = 0.17, \text{ and } \hat{T} = 0.63 \quad \hat{\lambda}_1 = 45.23, \xi = 4$$

α-Fe polycrystal

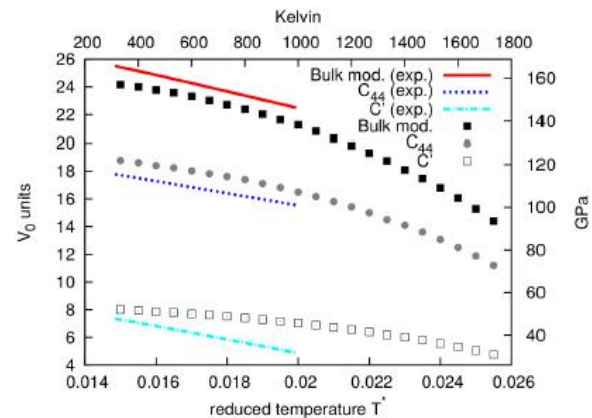
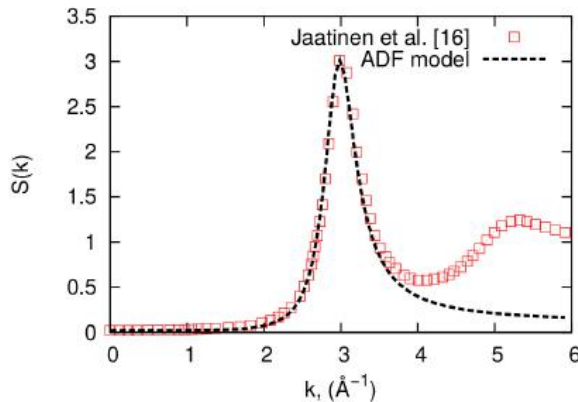
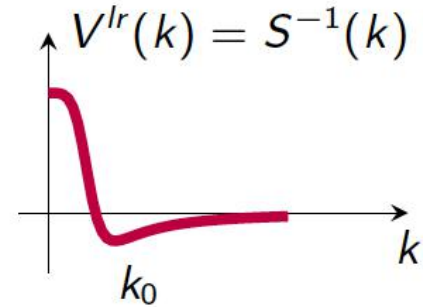
- Short range potential :

$$V^{sr}(r) = \begin{cases} -1 & \text{if } r < R \\ \xi & \text{if } R \leq r \leq R + \Delta R \\ 0 & \text{if } r > R + \Delta R \end{cases}$$



- Long range potential¹ :

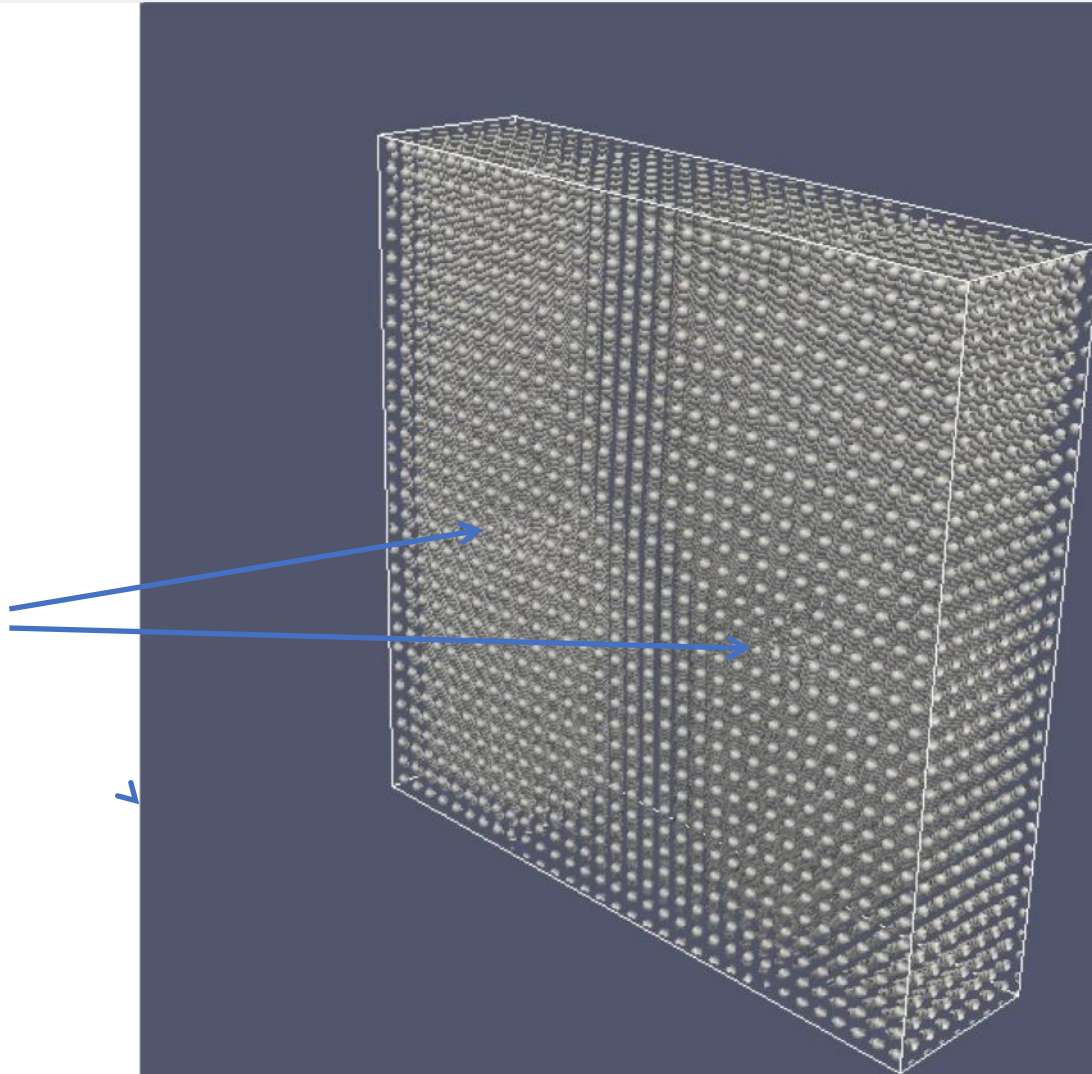
$$V^{lr}(k) = 1 - \frac{k^4}{(k^2 - k_1^2)^2 + k_2^4}$$



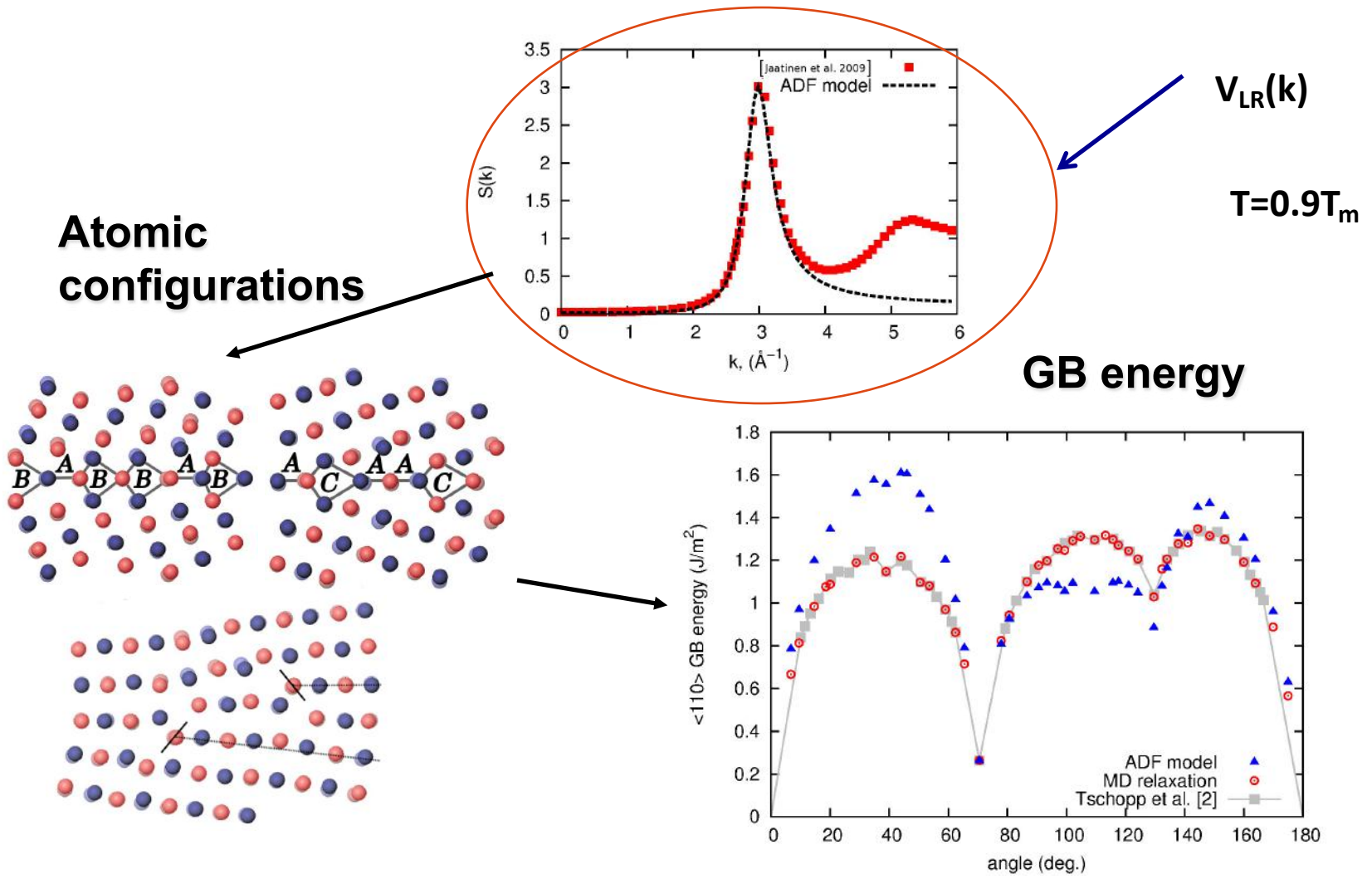
¹ O. Kapikranian, H. Zapolsky & al. Phys. Rev. B 00, 004100 (2015)
 [16] Jaatinen et al. Phys. Rev. E 80 (2009) 031602

GB growth (tilt angle $\theta = 3.58^\circ$)

Dislocations
at GB



The atomic density function (ADF) model of tilt grain boundaries in α -Fe

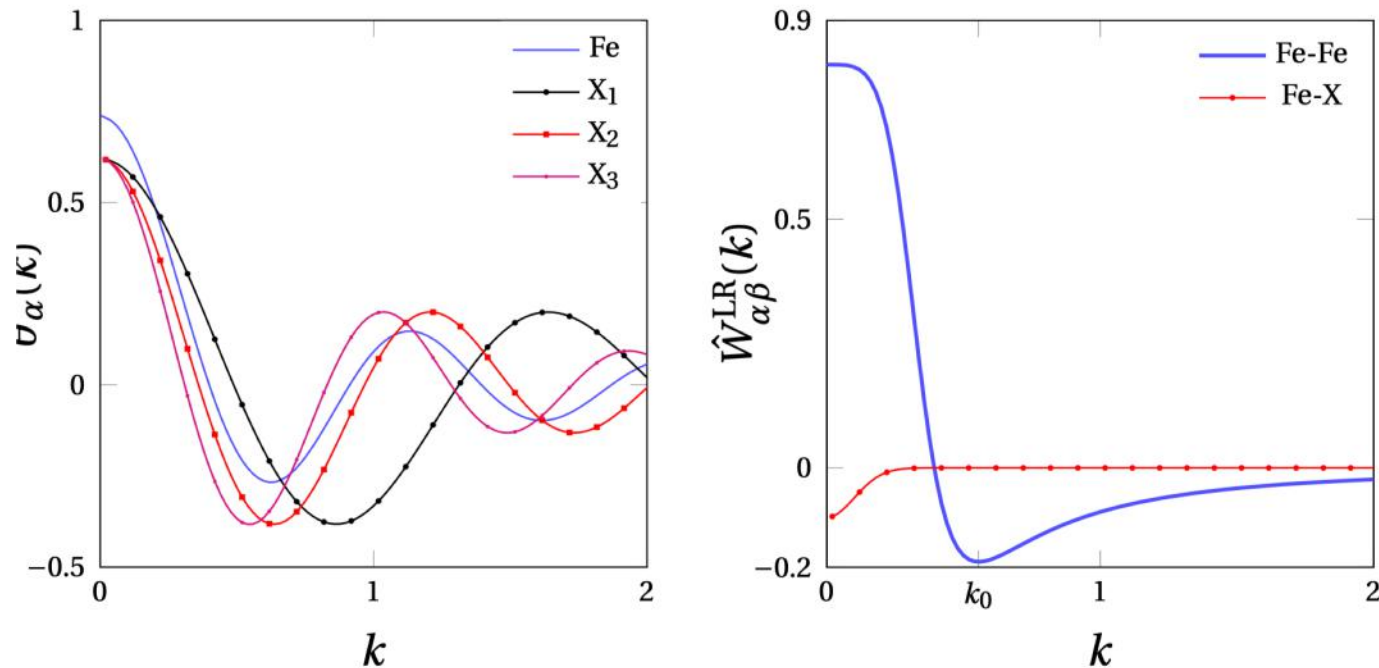


Size-Dependent Solute Segregation at Symmetric Tilt $\langle 100 \rangle$ Grain Boundaries in α -Fe

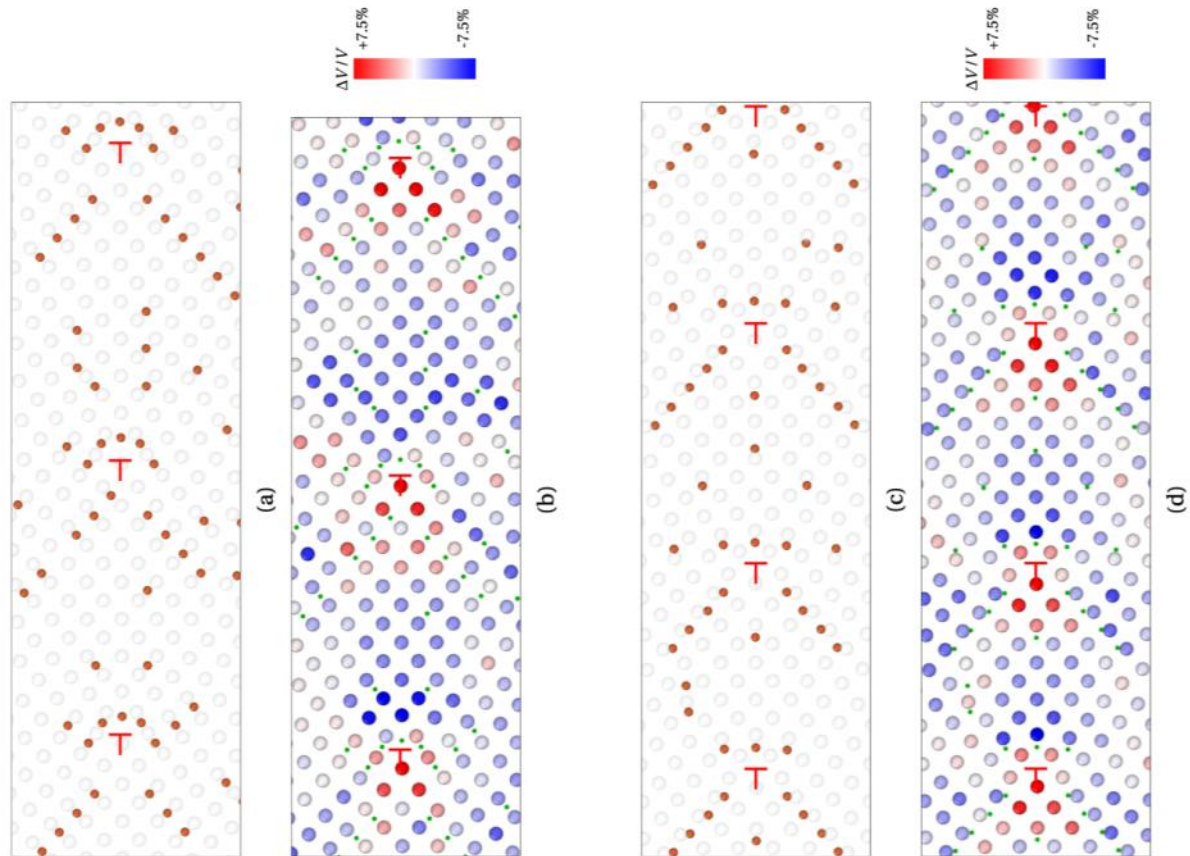
Three types of solute atoms X_1 , X_2 and X_3 with atomic radii $R_1 < R_{Fe}$, $R_2 = R_{Fe}$ and $R_3 > R_{Fe}$ corresponding to phosphorus (P), antimony (Sb) and tin (Sn)

Low angle GBs, with $\theta = 7.15^\circ$ and $\theta = 9.53^\circ$, and two high angle GBs, $\Sigma 5$ (310) ($\theta = 36.95^\circ$) and $\Sigma 29$ (730) ($\theta = 46.40^\circ$).

Interaction potentials



Formation of Cottrell atmospheres around dislocations



Formation of Cottrell atmospheres at $\langle 100 \rangle$ edge dislocations (red \perp marks) after segregation of solute atoms at $\langle 100 \rangle$ symmetric tilt LAGBs. **(a,b)** X_1 (small) atoms segregation for a misorientation angle $\theta = 7.15^\circ$. **(c,d)** X_2 (larger) atoms for $\theta = 9.53^\circ$. **(a,c)** solute atoms distribution (orange) and Fe atoms (transparent). **(b,d)** Volume per atom variation $\Delta V/V$ (Voronoi analysis). Red—dilatation, blue—compression. X atoms are spotted by green dots in **(b,d)**.

HAGB in α -Fe

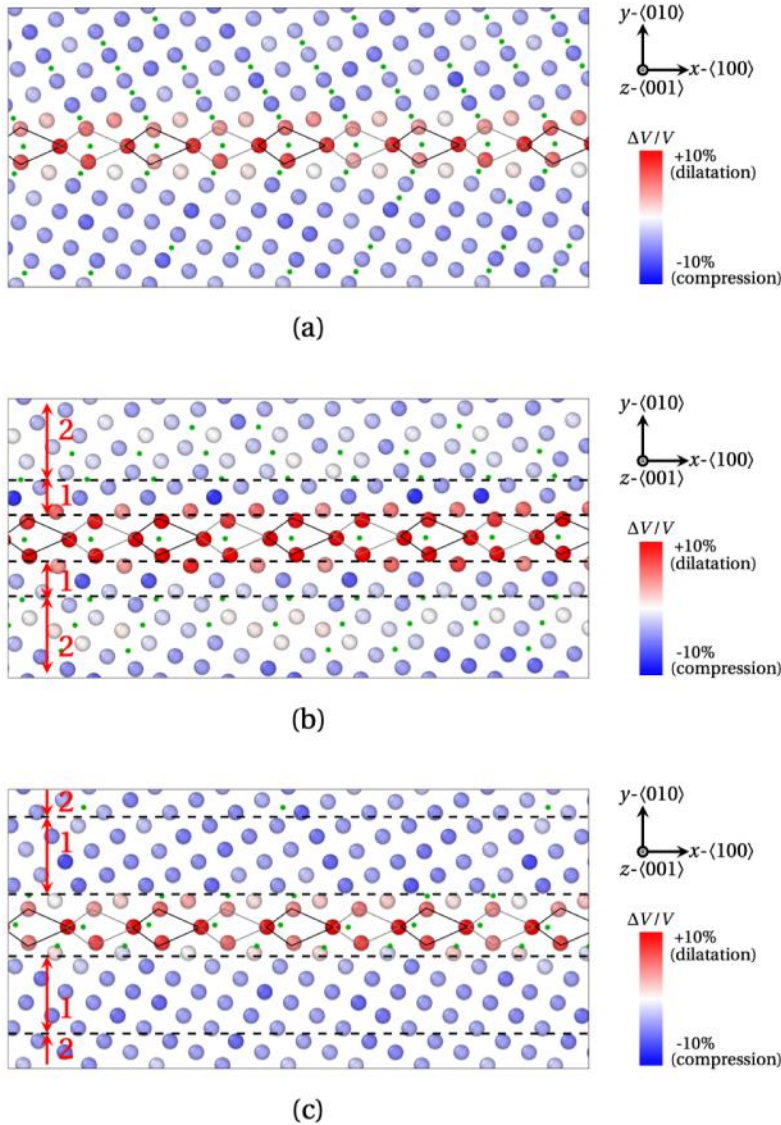
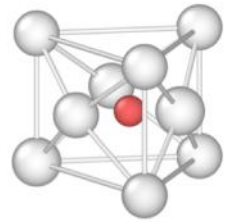
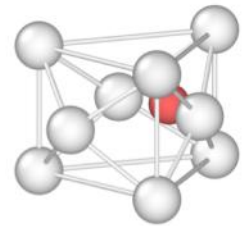


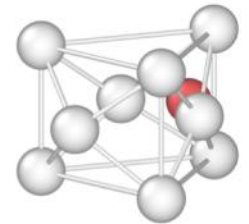
Figure 8. Influence of the volume per atom (Voronoi analysis) on the segregation of X_1 (a) X_2 (b) and X_3 (c) solute atoms at $\Sigma 5$ (310) ($\theta = 36.87^\circ$). X_1 atoms are spotted by green dots. The depleted (over-compressed) and segregated (less compressed) areas are delineated by dashed black lines, and referred to as zones 1 and 2 in (b,c).



(b)

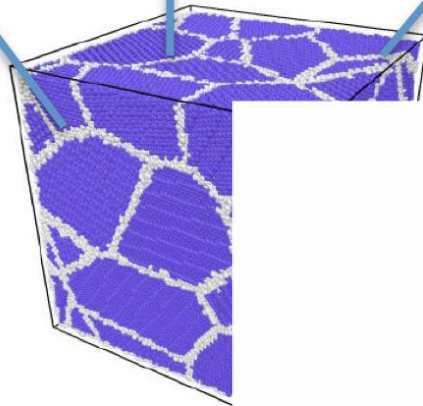
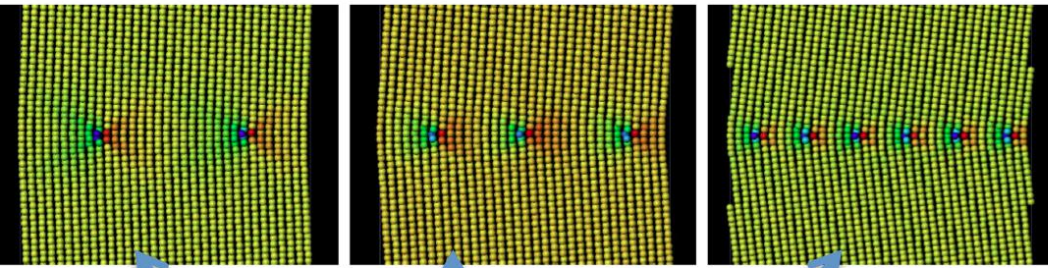


(e)

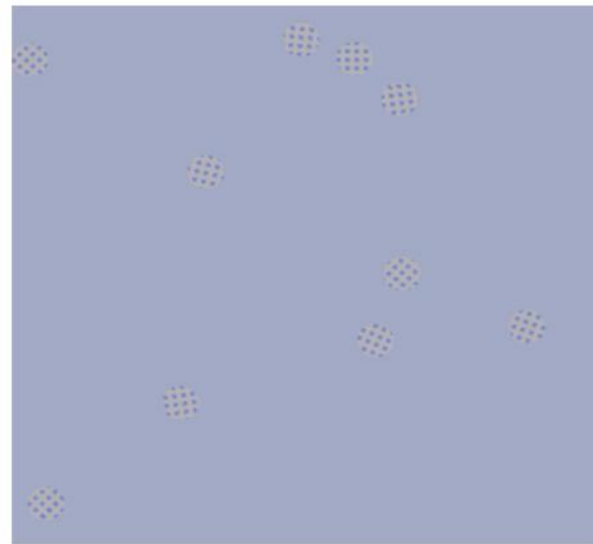


- Solute atoms are located inside of the capped trigonal prism in dilatation region.
- With increasing of atomic size the solute atoms are concentrated more and more near GB, in dilatation region

Polycrystal in α -Fe

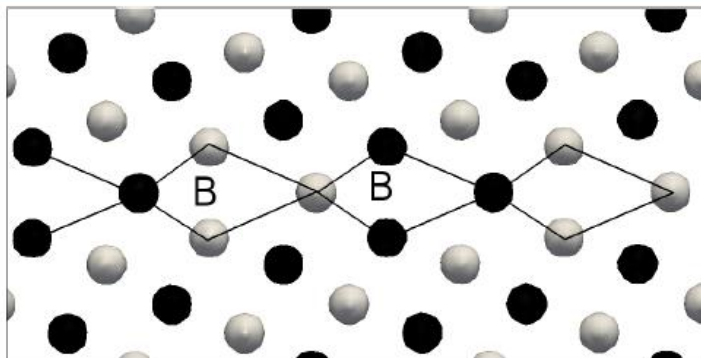


$$V=70^3nr$$

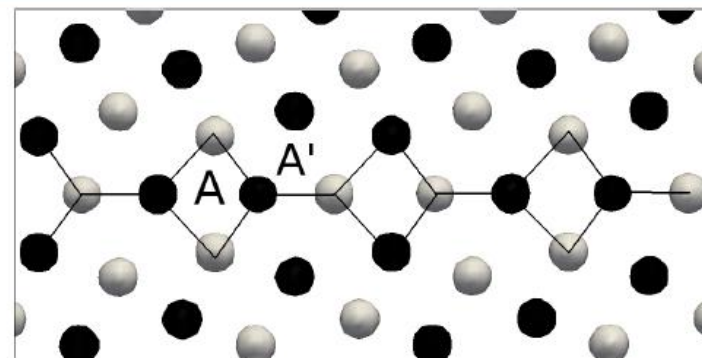


GRAIN BOUNDARIES STRUCTURE IN α -IRON UNDER APPLIED STRESS: $\epsilon=0.1$

$\Sigma 5(310)$, $\theta = 36.87$

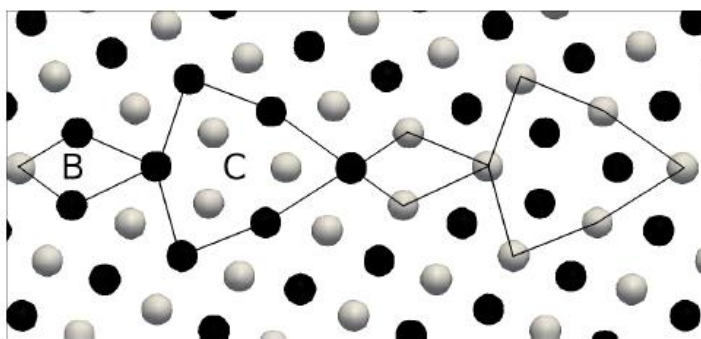


Equilibrium

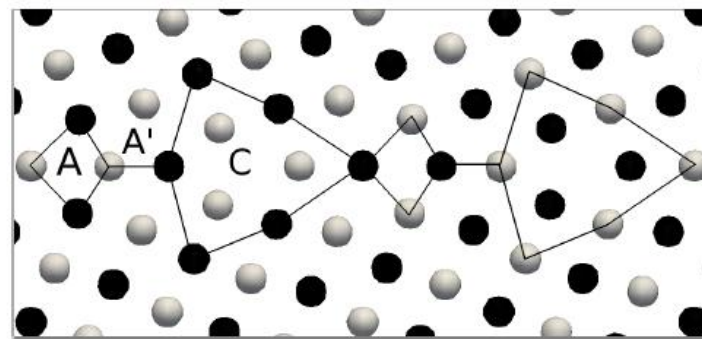


Under constrain

$\Sigma 29(730)$, $\theta = 46.40$



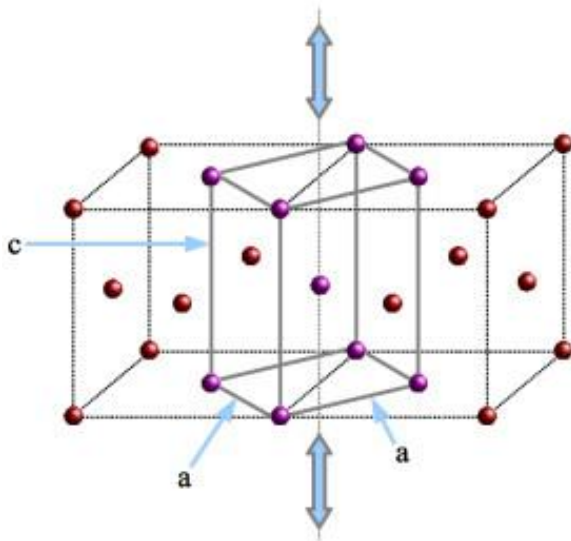
Equilibrium



Under constrain

MARTENSITIC TRANSFORMATIONS

Bain OR: $\{010\}_\gamma \parallel \{001\}_\alpha$
 $\{110\}_\gamma \parallel \{100\}_\alpha$



The Bain model of the martensitic transformation

KS OR: $\{111\}_\gamma \parallel \{110\}_\alpha$
 $\{101\}_\gamma \parallel \{111\}_\alpha$

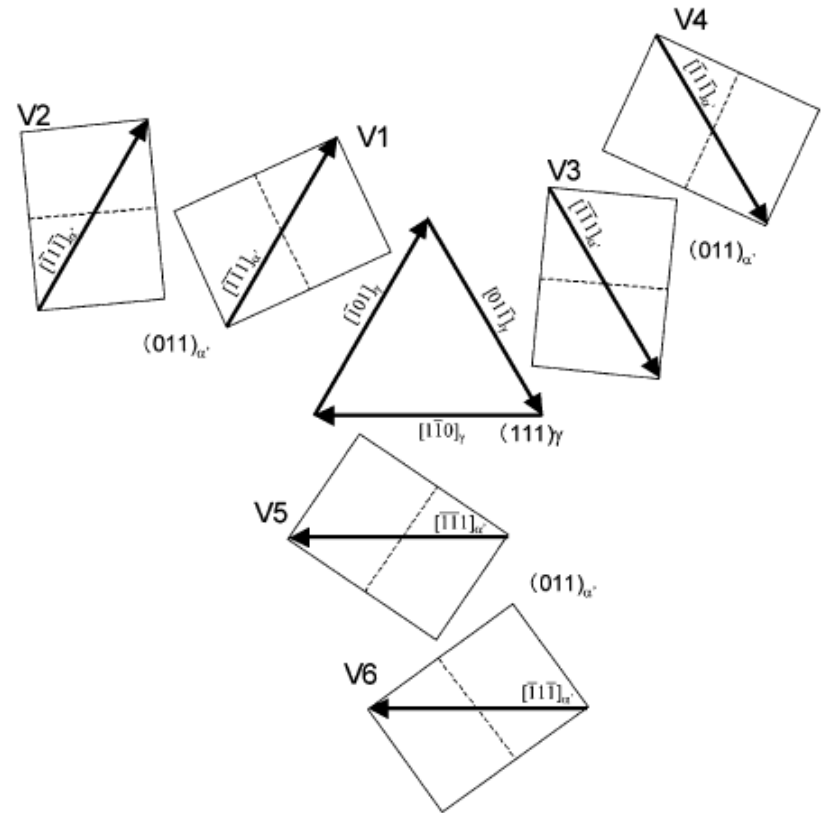


Fig. 1. Six crystallographic variants (V1–V6) for the K–S orientation relationship that evolves on a (111) austenite plane. The triangle and rectangles indicate the (111) plane of austenite (γ : fcc) and the (011) plane of martensite (α' : bcc), respectively.

MARTENSITIC TRANSFORMATIONS

The 24 crystallographic variants for the K-S orientation relationship

Variant	Plane parallel	Direction parallel
V1	$(111)_\gamma // (011)_{\alpha'}$	$[\bar{1}01]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V2	$(111)_\gamma // (011)_{\alpha'}$	$[\bar{1}01]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$
V3	$(111)_\gamma // (011)_{\alpha'}$	$[01\bar{1}]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V4	$(111)_\gamma // (011)_{\alpha'}$	$[01\bar{1}]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$
V5	$(111)_\gamma // (011)_{\alpha'}$	$[1\bar{1}0]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V6	$(111)_\gamma // (011)_{\alpha'}$	$[1\bar{1}0]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$
V7	$(1\bar{1}1)_\gamma // (011)_{\alpha'}$	$[10\bar{1}]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V8	$(1\bar{1}1)_\gamma // (011)_{\alpha'}$	$[10\bar{1}]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$
V9	$(1\bar{1}1)_\gamma // (011)_{\alpha'}$	$[\bar{1}\bar{1}0]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V10	$(1\bar{1}1)_\gamma // (011)_{\alpha'}$	$[\bar{1}\bar{1}0]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$
V11	$(1\bar{1}1)_\gamma // (011)_{\alpha'}$	$[011]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V12	$(1\bar{1}1)_\gamma // (011)_{\alpha'}$	$[011]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$
V13	$(\bar{1}11)_\gamma // (011)_{\alpha'}$	$[0\bar{1}1]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V14	$(\bar{1}11)_\gamma // (011)_{\alpha'}$	$[0\bar{1}1]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$
V15	$(\bar{1}11)_\gamma // (011)_{\alpha'}$	$[\bar{1}0\bar{1}]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V16	$(\bar{1}11)_\gamma // (011)_{\alpha'}$	$[\bar{1}0\bar{1}]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$
V17	$(\bar{1}11)_\gamma // (011)_{\alpha'}$	$[110]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V18	$(\bar{1}11)_\gamma // (011)_{\alpha'}$	$[110]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$
V19	$(11\bar{1})_\gamma // (011)_{\alpha'}$	$[\bar{1}10]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V20	$(11\bar{1})_\gamma // (011)_{\alpha'}$	$[\bar{1}10]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$
V21	$(11\bar{1})_\gamma // (011)_{\alpha'}$	$[0\bar{1}\bar{1}]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V22	$(11\bar{1})_\gamma // (011)_{\alpha'}$	$[0\bar{1}\bar{1}]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$
V23	$(11\bar{1})_\gamma // (011)_{\alpha'}$	$[10\bar{1}]_\gamma // [\bar{1}\bar{1}1]_{\alpha'}$
V24	$(11\bar{1})_\gamma // (011)_{\alpha'}$	$[10\bar{1}]_\gamma // [\bar{1}1\bar{1}]_{\alpha'}$

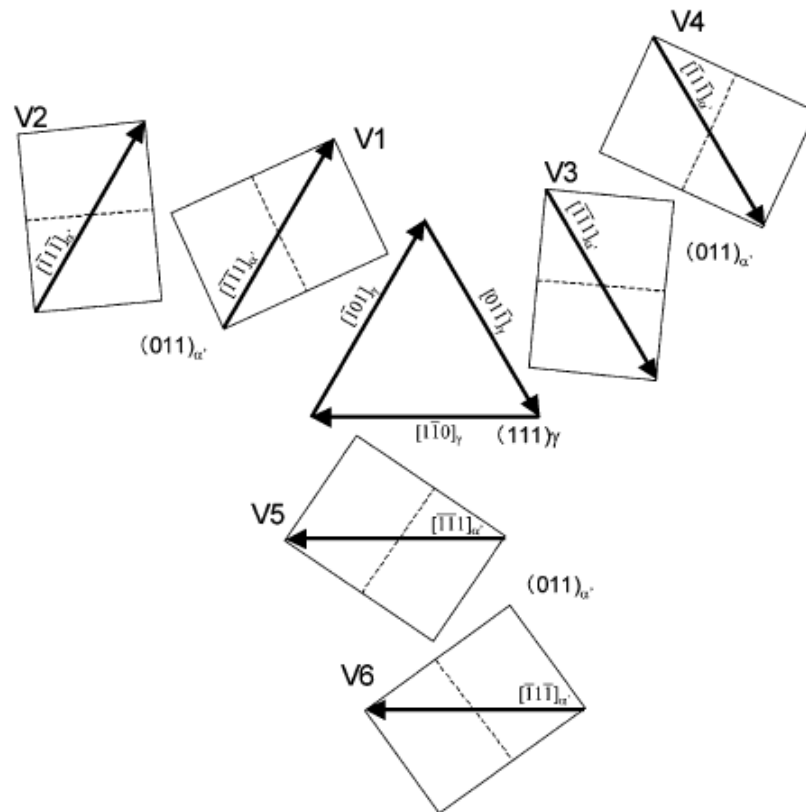
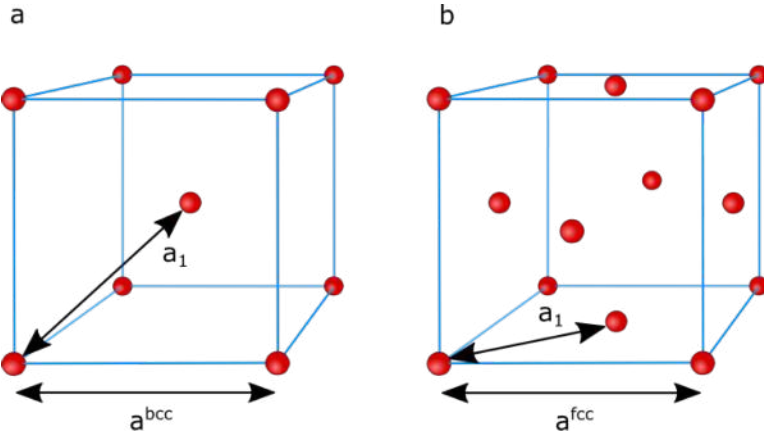


Fig. 1. Six crystallographic variants (V1-V6) for the K-S orientation relationship that evolves on a (111) austenite plane. The triangle and rectangles indicate the (111) plane of austenite (γ : fcc) and the (011) plane of martensite (α' : bcc), respectively.

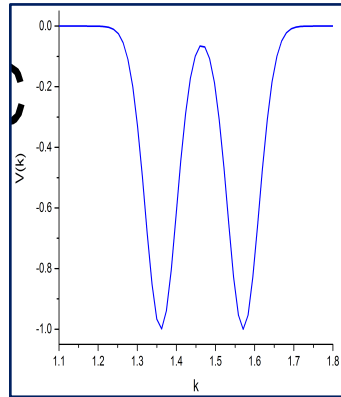
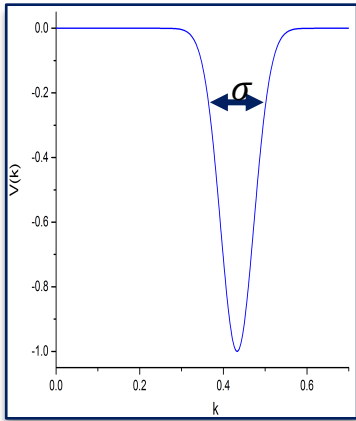
FCC-BCC transition



$$\frac{a_0^{fcc}}{a_0^{bcc}} = \sqrt{\frac{3}{2}} \approx 1.225$$

$$a_0^{fcc} = 8\Delta x$$

$$\sigma = 0.05$$



Elastic constants of the given potential

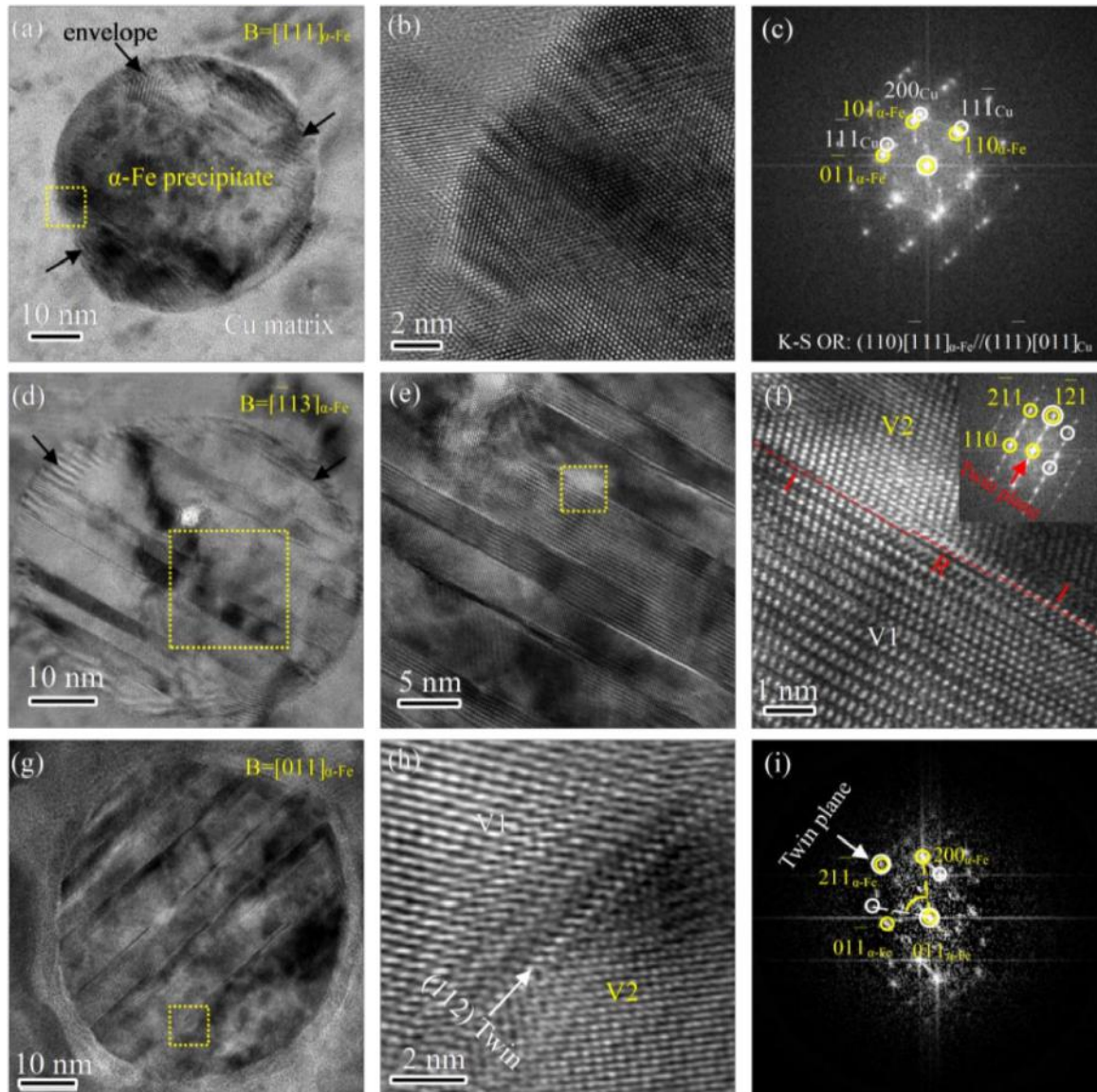
$$\frac{C_{11}^{bcc}}{C_{11}^{fcc}} \approx 1.68, \quad \frac{C_{12}^{bcc}}{C_{12}^{fcc}} \approx 1.25, \quad \frac{C_{44}^{bcc}}{C_{44}^{fcc}} \approx 1.3$$

Experimental elastic constants for iron

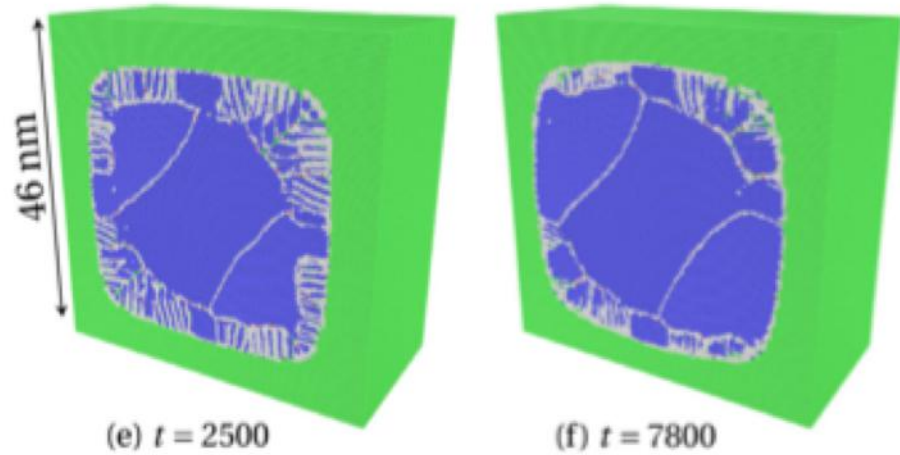
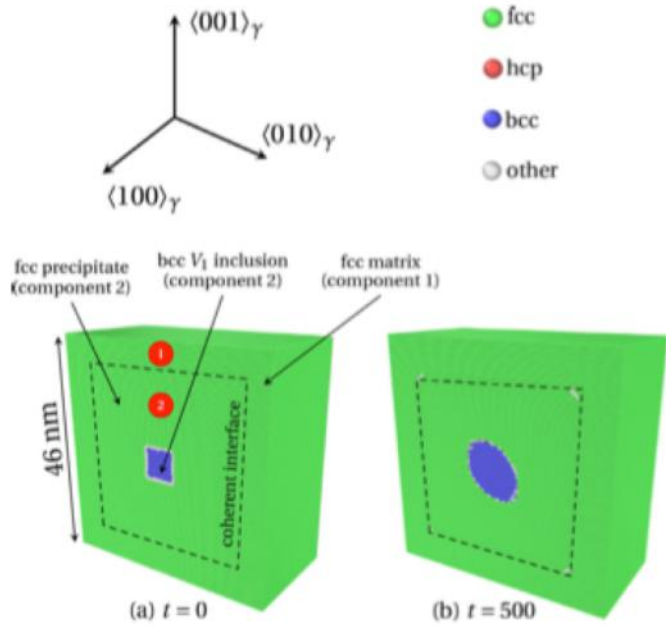
$$\frac{C_{11}^{bcc}}{C_{11}^{fcc}} \approx 1.60, \quad \frac{C_{12}^{bcc}}{C_{12}^{fcc}} \approx 1.15, \quad \frac{C_{44}^{bcc}}{C_{44}^{fcc}} \approx 1.51$$

$$\tilde{w}_{LR}(\mathbf{k}) = \exp\left(-\frac{(k-k_{01})^2}{2\sigma^2}\right) + 0.1 \exp\left(-\frac{(k-k_{02})^2}{2\sigma^2}\right)$$

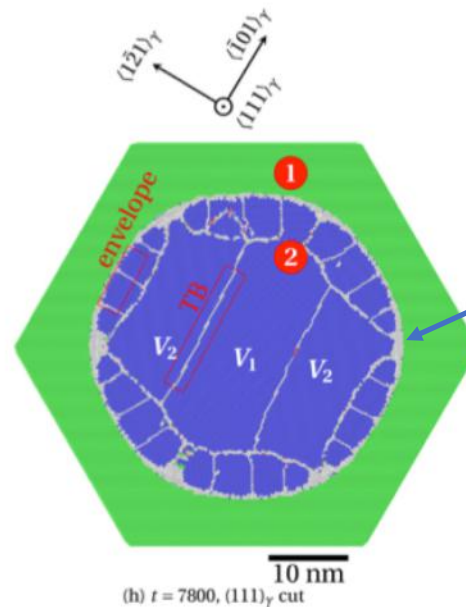
Twinning structure of α -Fe (bcc) precipitate in Cu (fcc) matrix in the Cu-2.0Fe-0.5Co wt%



Growth of α -Fe particle in the Cu matrix

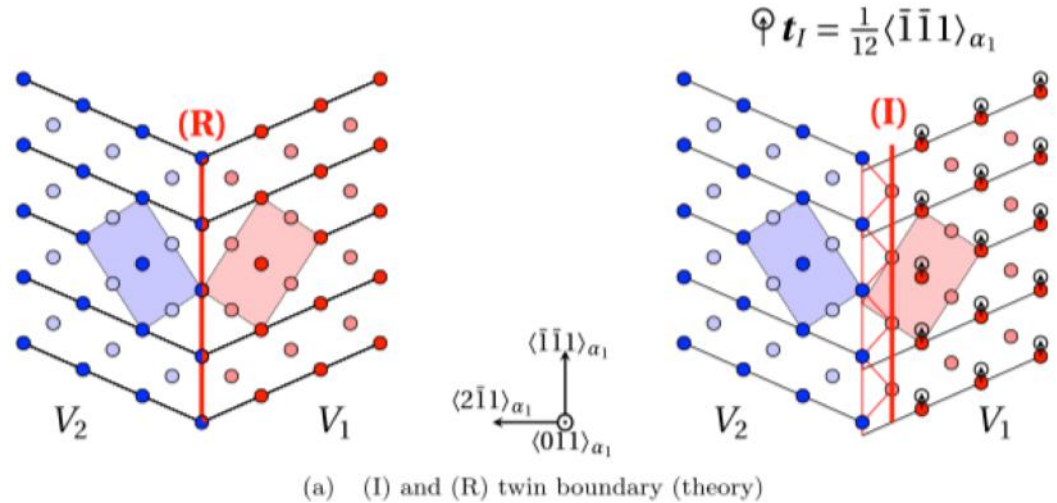


Size of the simulation box 1024^3
 46^3 nm

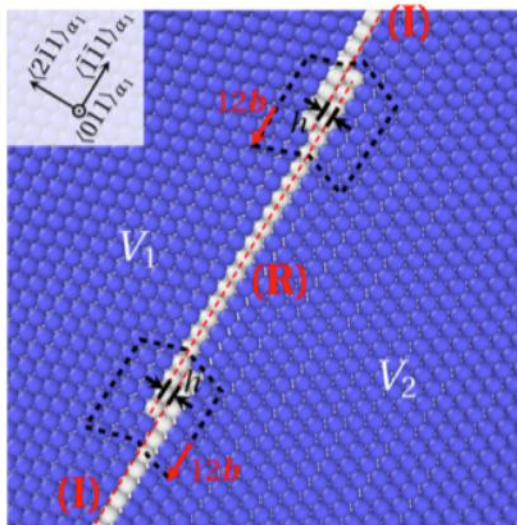


Two-variants structure with
 Kurdjumov–Sachs OR

Twin boundaries



Reflection (R) and isocel(I) twin boundaries structures

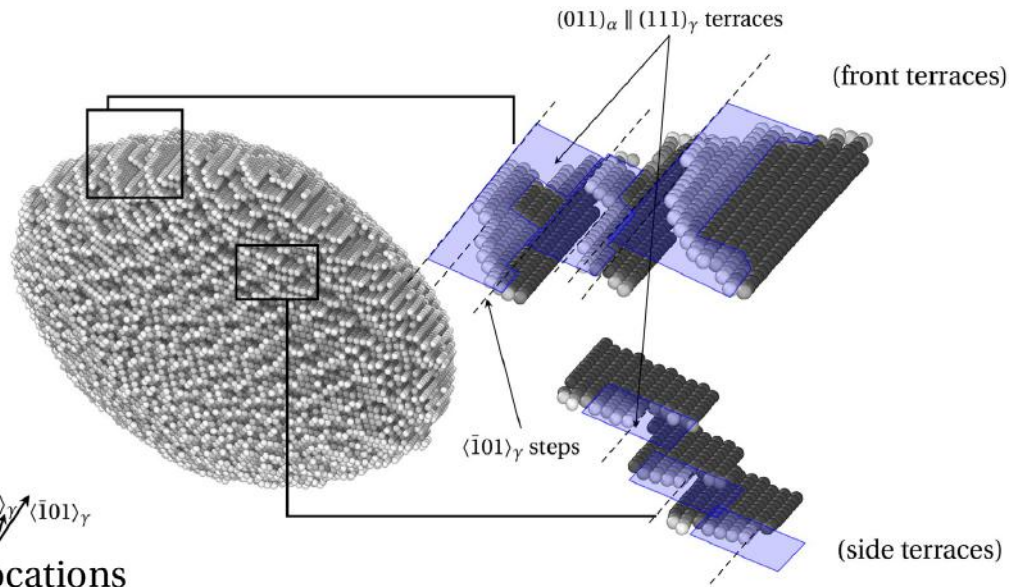
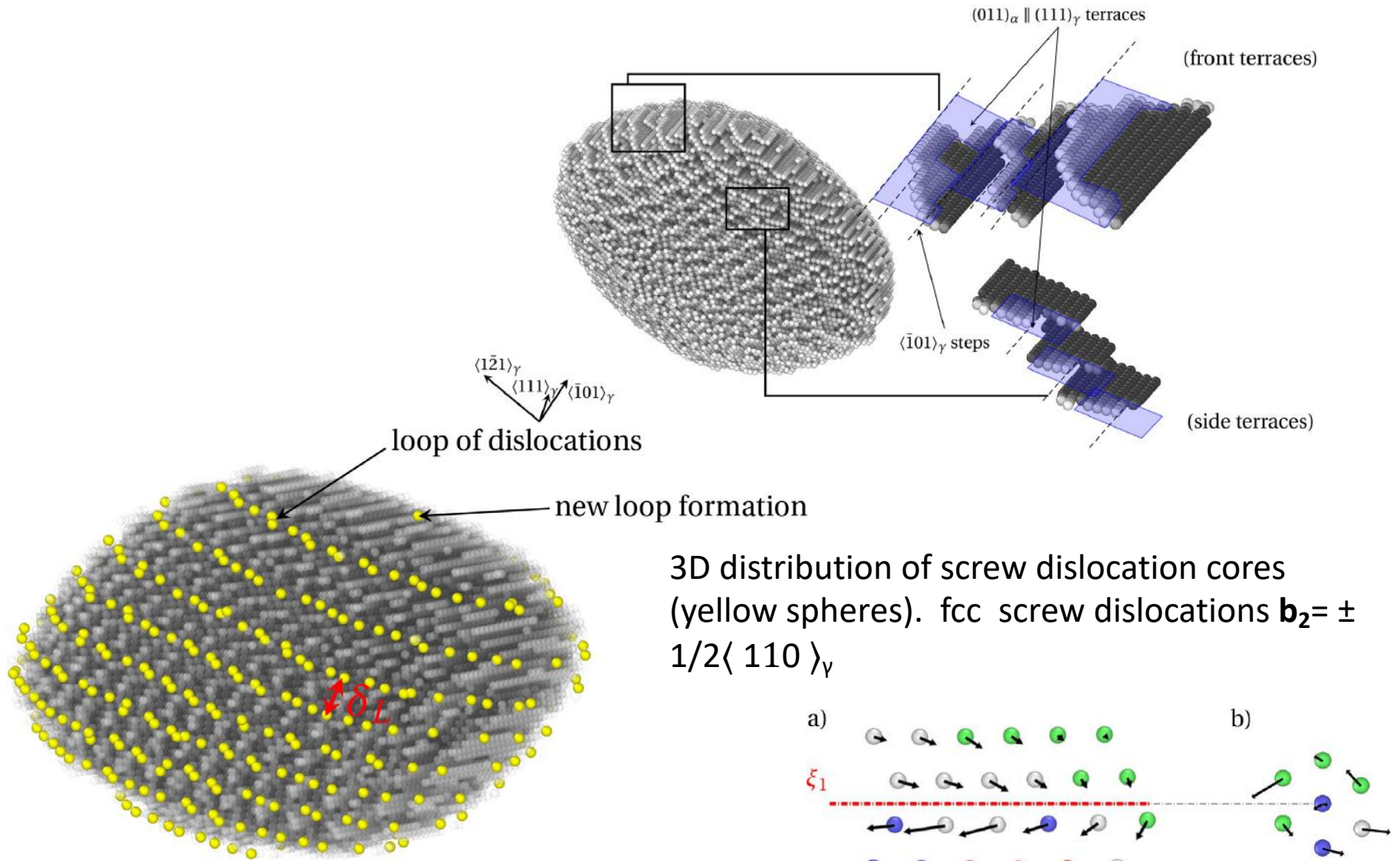


(a) IR+RI couple

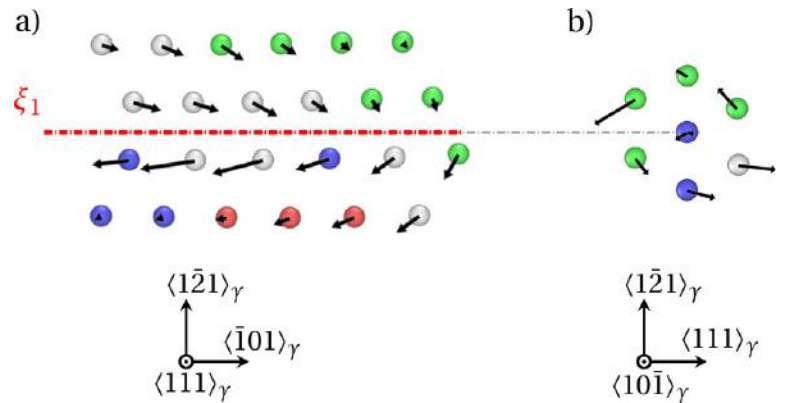
During the propagation of TB, the transitions (I) \rightarrow (R) and (R) \rightarrow (I)– is accompanied by the stacking fault in $\langle \bar{2}11 \rangle_{\alpha_1}$ planes, which results in the shift of the interface along the direction $\langle \bar{2}11 \rangle_{\alpha_1}$ perpendicular to the twinning plane.

Partial twin dislocation couple (IR+RI) with step
 $h = a_2(2\sqrt{6})$, Burgers vector $\mathbf{b} = \frac{1}{12} \langle \bar{1}\bar{1}1 \rangle_{\alpha_1}$

Fcc/bcc interface



3D distribution of screw dislocation cores (yellow spheres). fcc screw dislocations $\mathbf{b}_2 = \pm 1/2 \langle 110 \rangle_\gamma$



Role of transformation interface

Solid-state phase transformations during steel processing



Microstructure future

- phases volume fraction
- topology and morphology of phases
- grain size distribution



Mechanical properties of the steel



moving transformation interface

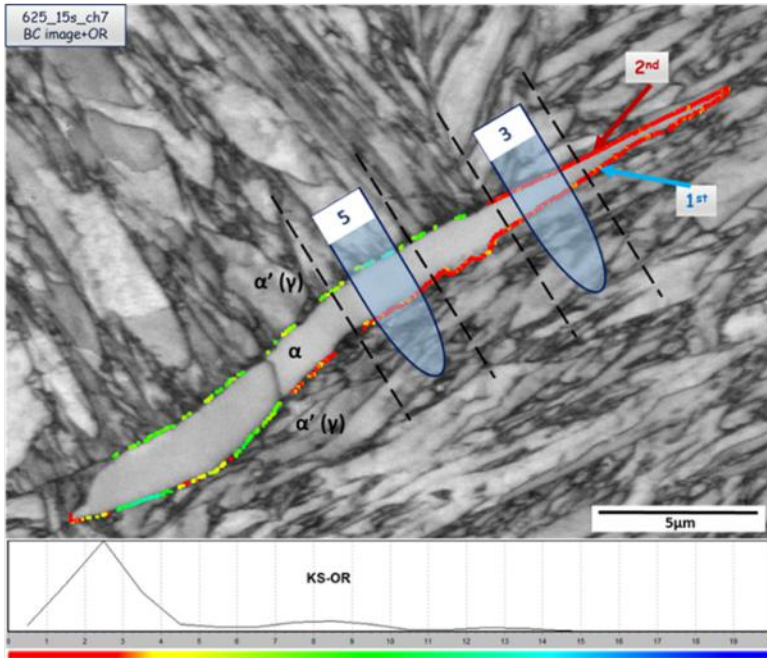


- Interface interaction with the alloying elements
- Orientation relationship (ORs) between two phases
- Interface properties (coherency, thickness and shape)
- Atomic structure of the interface
- Ability to dissipate energy: interfacial friction



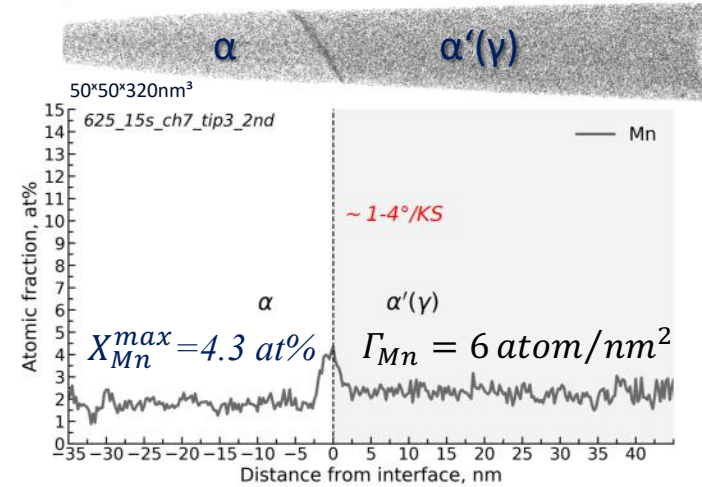
Experimental investigation of the transformation interface at the nanoscale and modeling

15s at 625 °C: interface #7

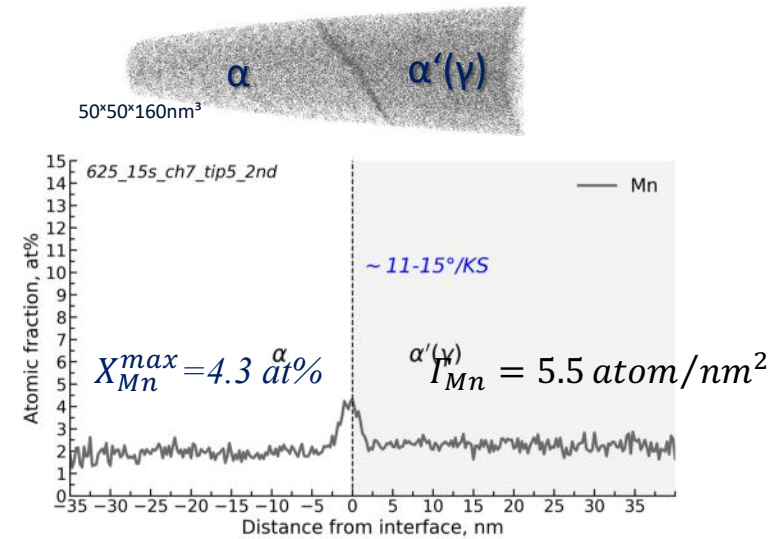


p 4

tip3_2nd

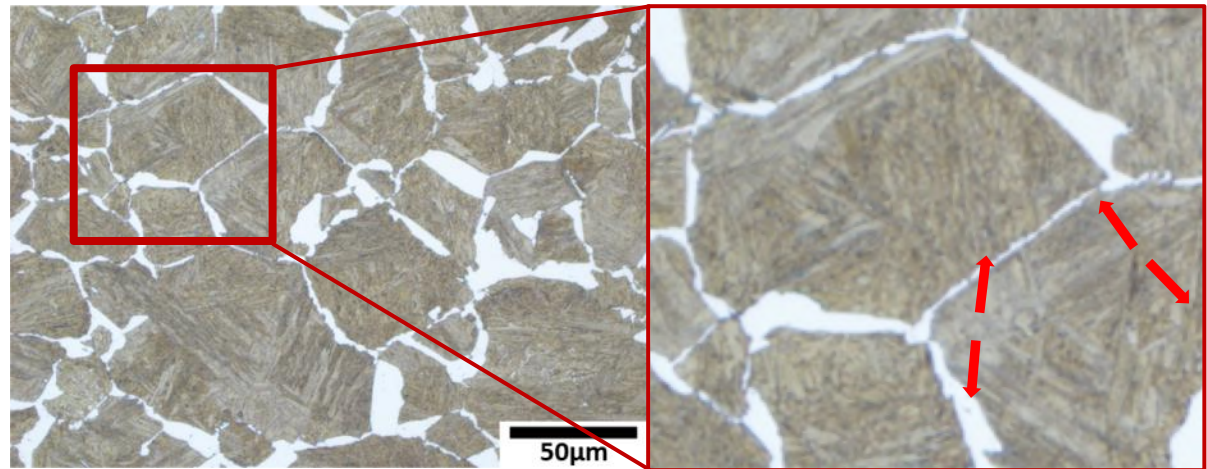


tip5_2nd

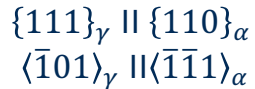


Austenite-to-ferrite phase transformation

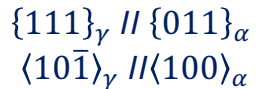
- ❑ The change in the crystal structure (from fcc to bcc).
- ❑ Diffusional process: the redistribution of interstitial and substitutional elements.



Kurdjumov-Sachs (KS):



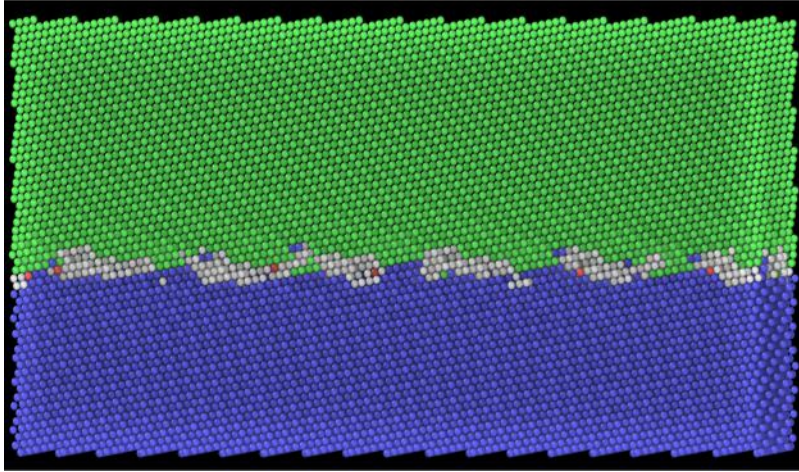
Nishiyama-Wassermann (NW):



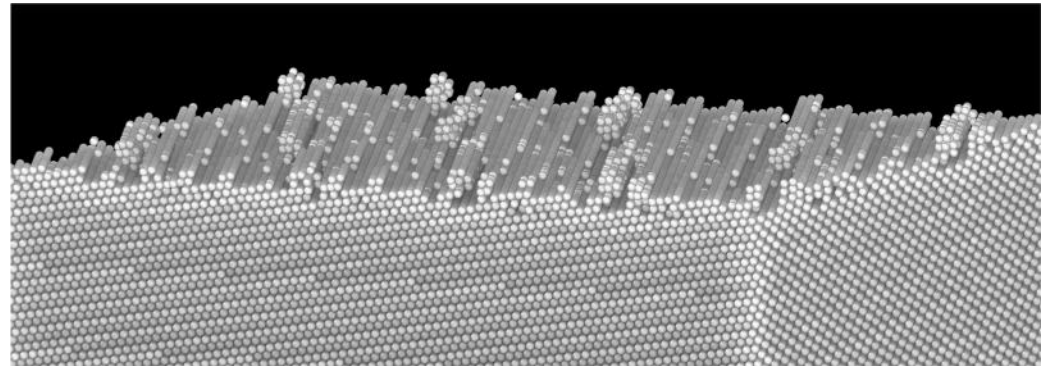
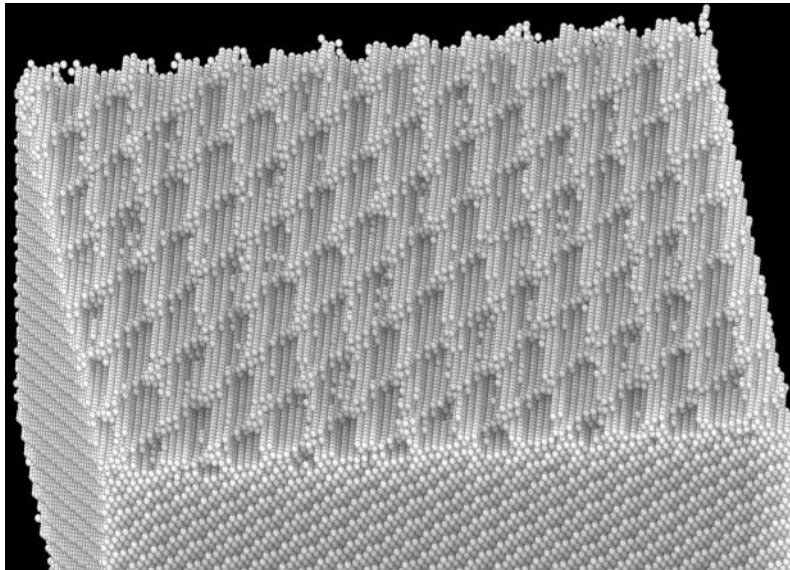
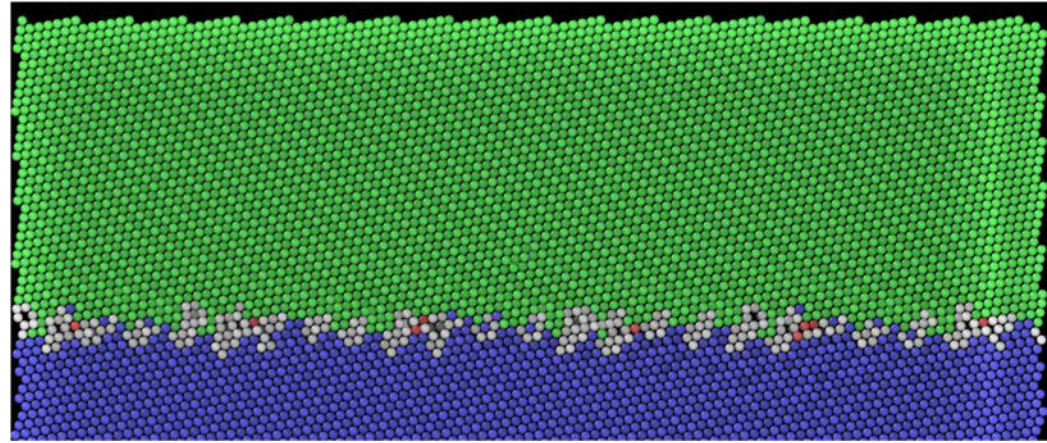
At higher T: disordered interfaces migrate faster
At lower T: coherent interfaces migrate faster ¹

FCC-BCC plane interface

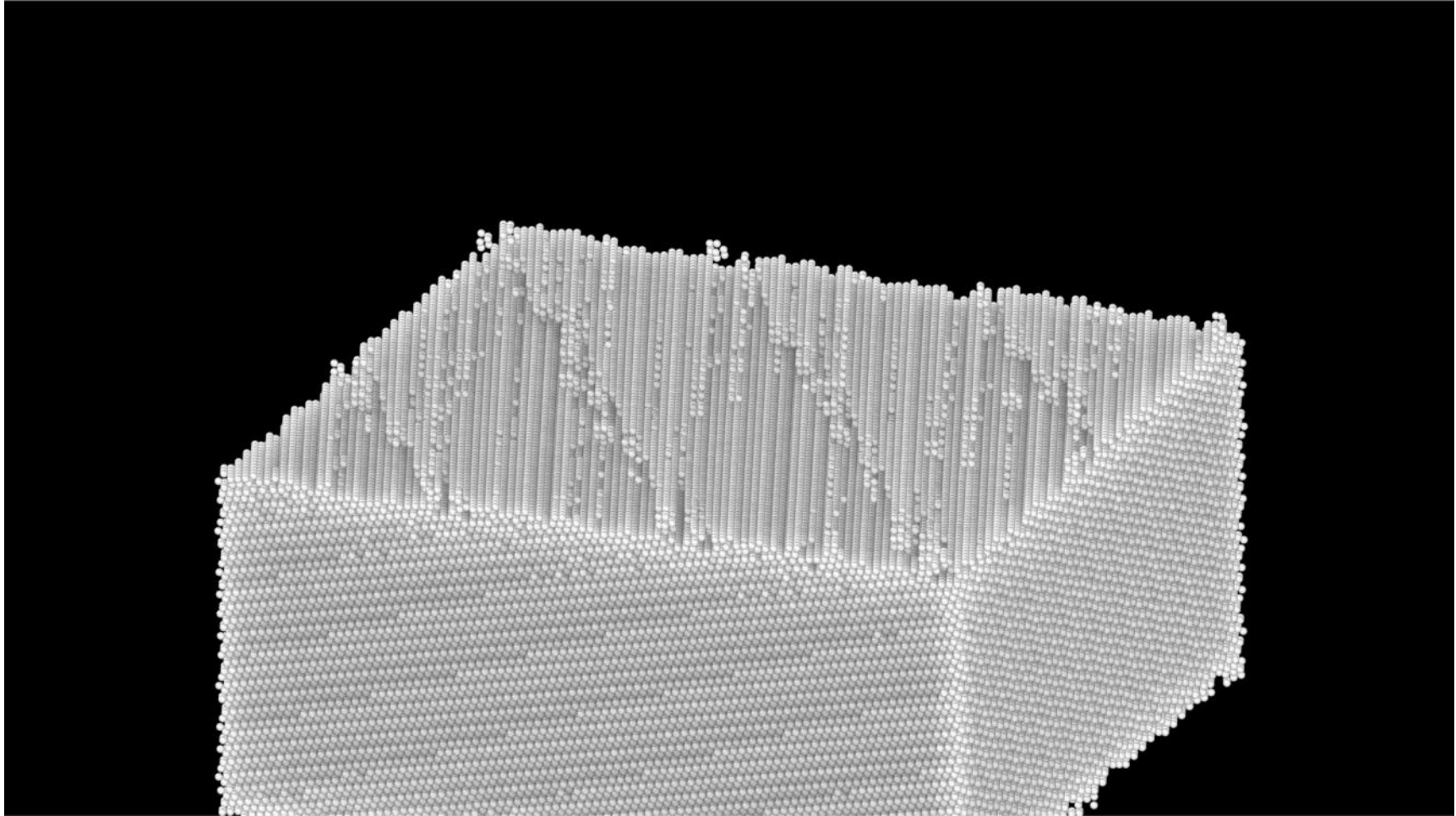
Greninger-Troiano OR



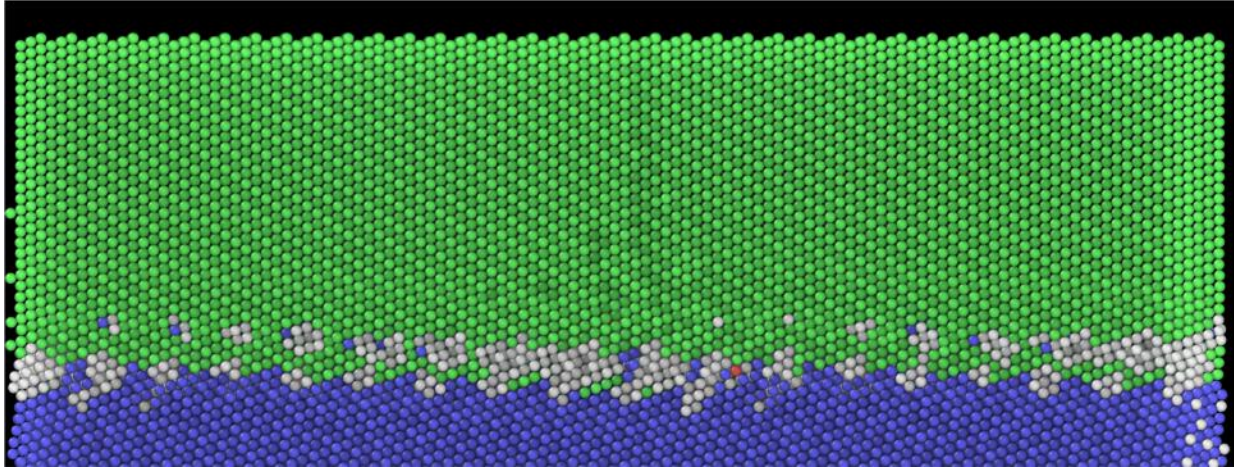
Kurdjumov-Sachs OR



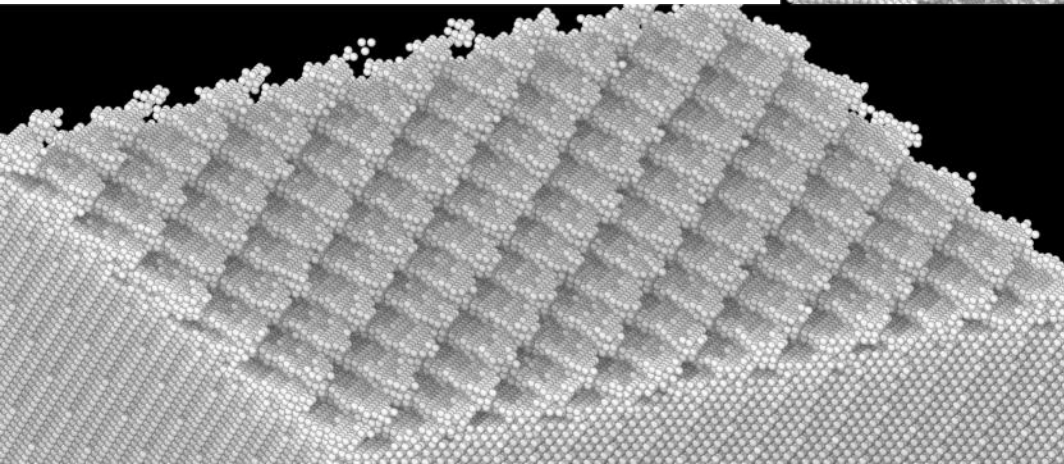
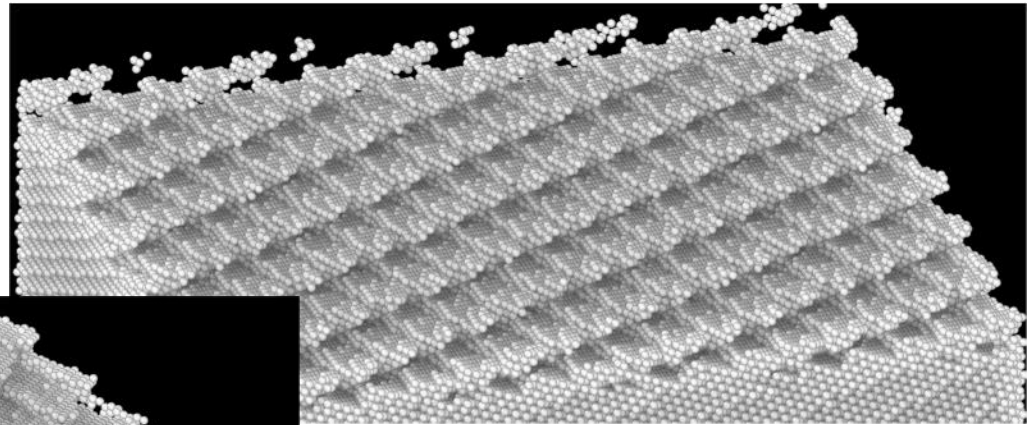
Propagation of (575) interface with KS OR



Nishiyama Wasserman Orientation Relationship

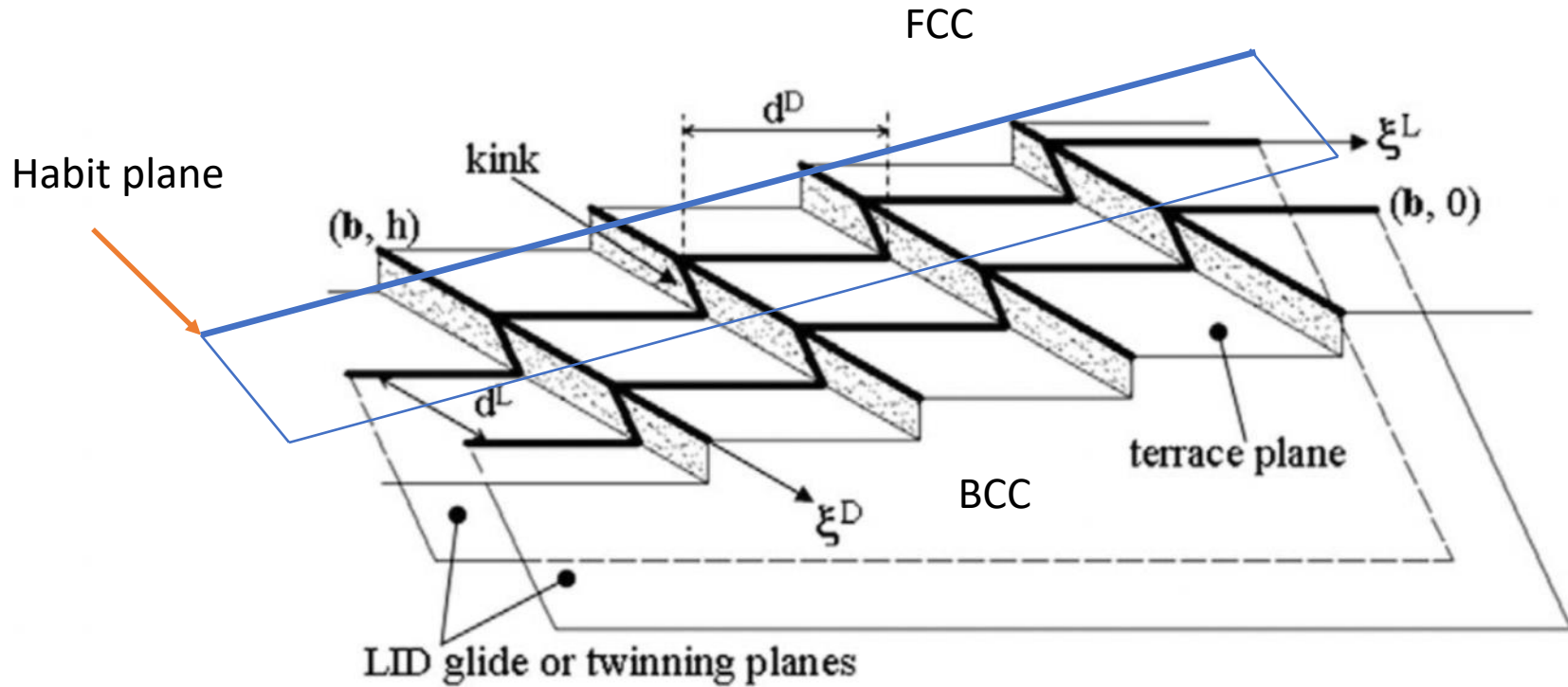


(121) Habit plane



(575) Habit plane

FCC→BCC phase transformation in Fe

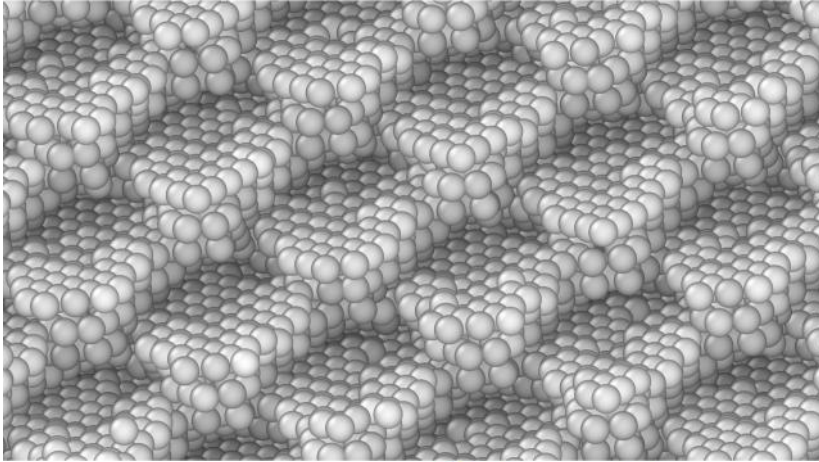


To identify the crystal defects

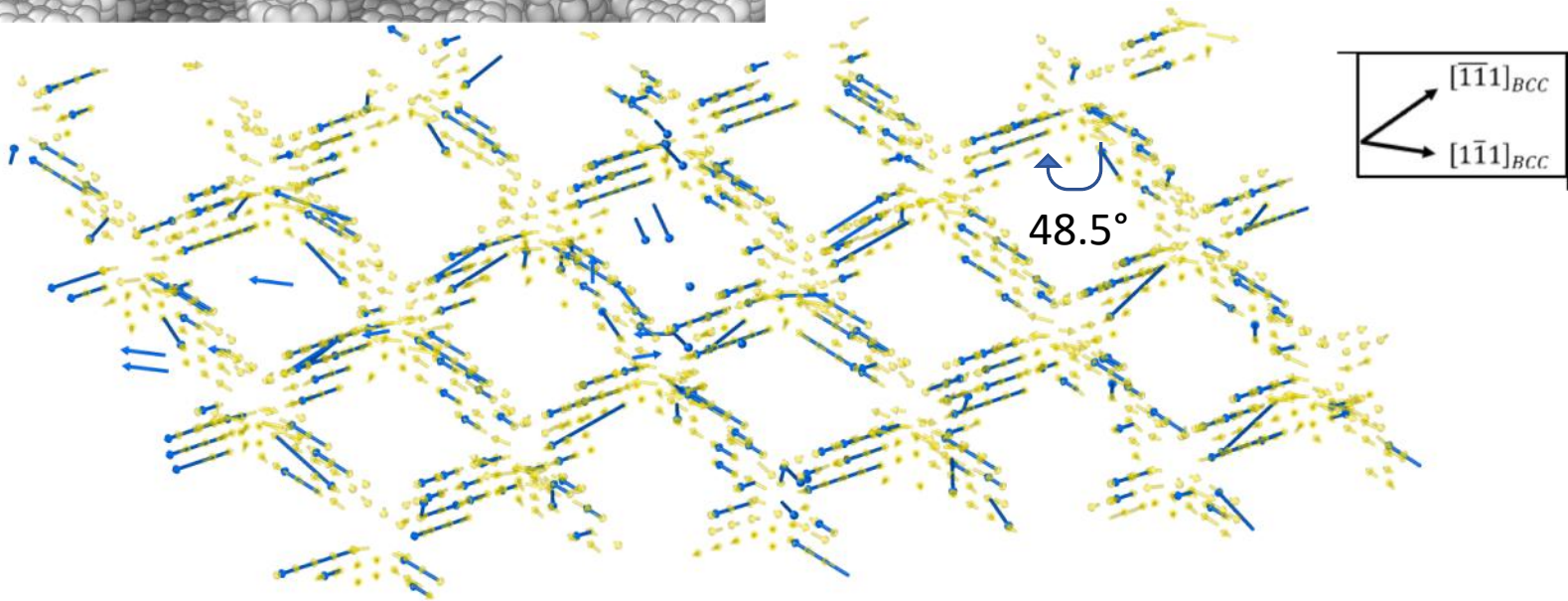


Slip Vector Analysis (SVA)

Nishiyama Wasserman Orientation Relationship

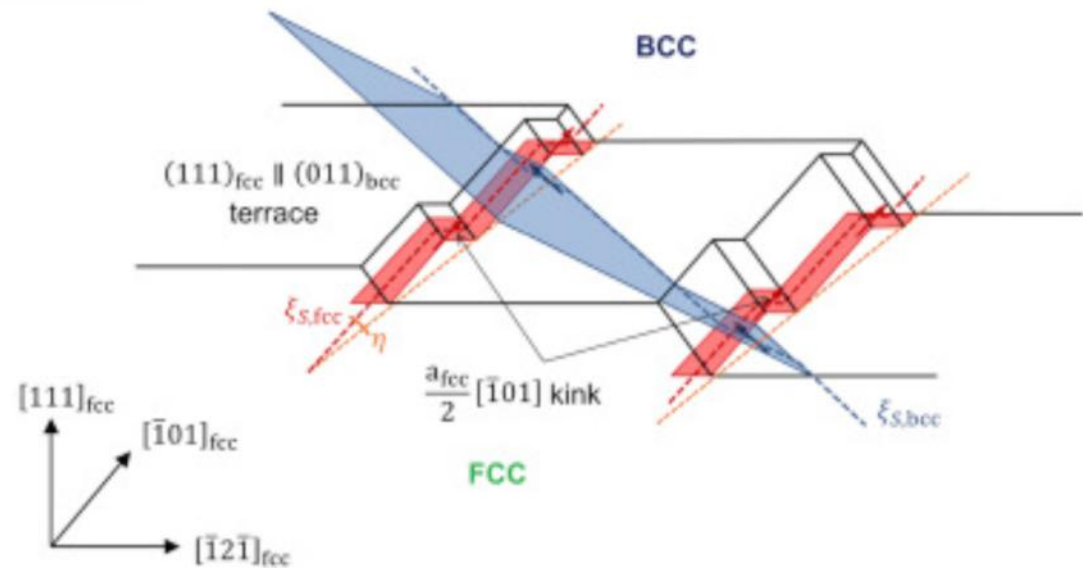
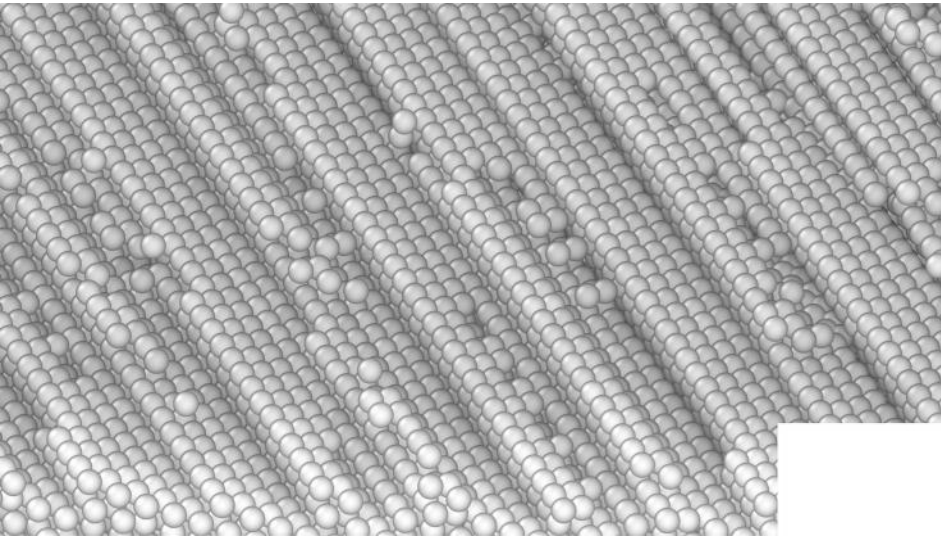


- Average step height : $h = a_{BCC}\sqrt{3} \approx 0,50 \text{ nm}$.
- Average terrace length : $1,0 \text{ nm} \leq l \leq 1,5 \text{ nm}$.



Slip Vector Analysis of NW (575)_{FCC} interface with a BCC reference. Blue vectors are detected dislocations mean line direction.

Kurdjumov-Sachs OR



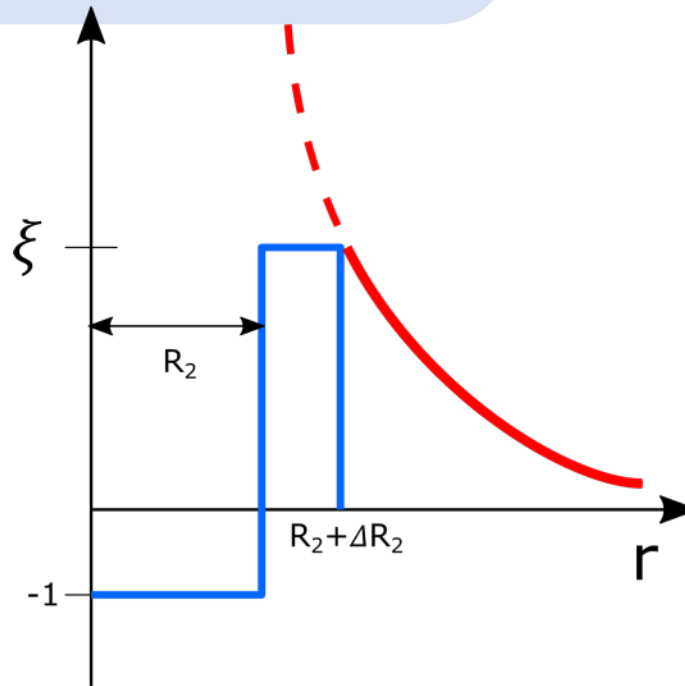
screw dislocations exist with Burgers vector $a_{\text{fcc}}/2[\bar{1}01]$ lying next to the steps along $[101]_{\text{fcc}}$, with the stacking fault between partials lying on the $(111)_{\text{fcc}}$ terrace.

Interaction of the Fcc/BCC interface with Carbon atoms

Long-range potential to model the C-C repulsion:

$$w_{CC}^{LR}(\mathbf{r}) = \begin{cases} 0, & \text{if } r < (R_C + \Delta R_C) \\ e^{-\alpha r}, & \text{if } r \geq (R_C + \Delta R_C) \end{cases}$$

$$\alpha = 0.34 \text{ [2]}$$

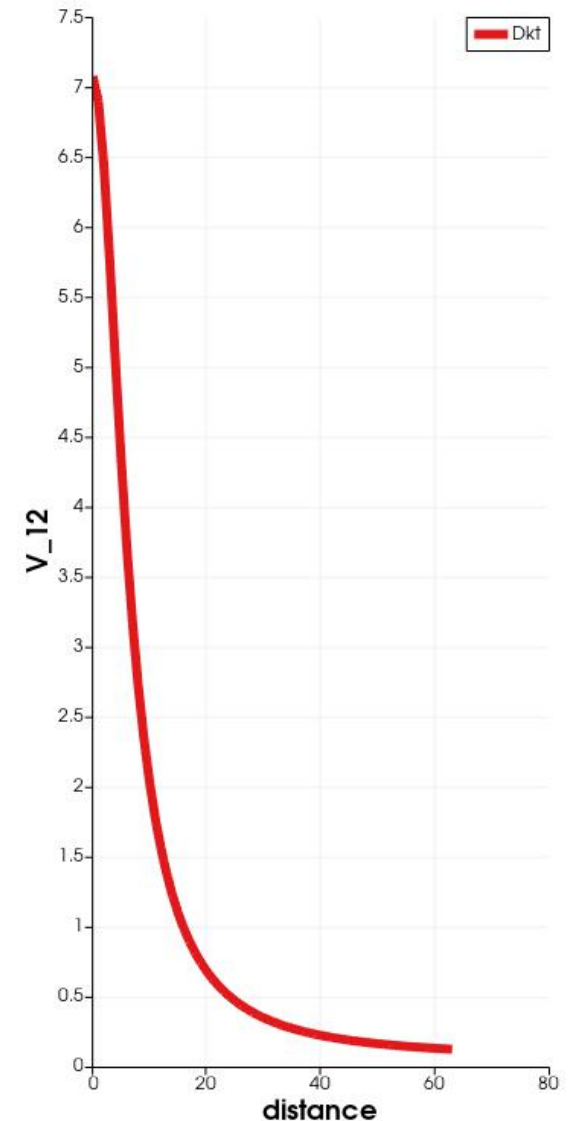


Long-range potential to model the Fe-C repulsion
(Screened Coulomb potential):

$$w_{FeC}(\mathbf{r}) = \frac{e^2}{r} \exp\left(-\frac{r^2}{r_D^2}\right)$$

$r_D = 0.2036 \cdot a_0$ [3], where r_D - Debye radius

$$w_{FeC}(\mathbf{k}) = \lambda_2 \frac{1}{(1/r_D)^2 + k^2}$$



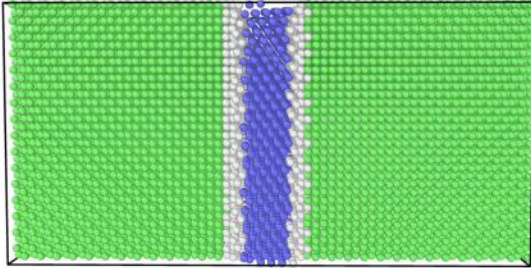
$$\hat{L}^0 = \begin{pmatrix} L_{FeFe}^0 & L_{FeC}^0 \\ L_{FeC}^0 & L_{CC}^0 \end{pmatrix}$$

➤ $L_{FeC}^0 = 0$ - since C atoms are in interstitial positions, there is no Fe/C exchange

➤ $L_{CC}^0 / L_{FeFe}^0 > 1$ - Carbon atoms are more mobile than iron atoms

$$\hat{L}^0 = \begin{pmatrix} 0.67 & 0 \\ 0 & 1.5 \end{pmatrix}$$

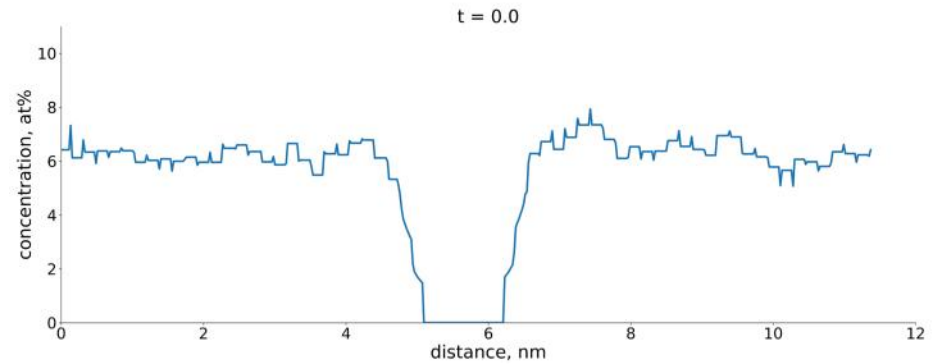
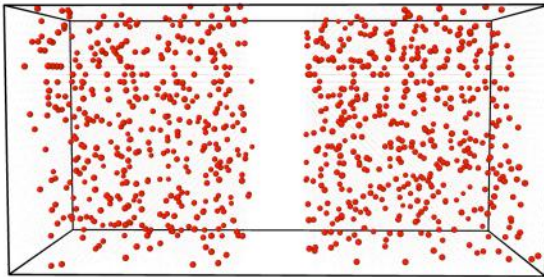
Segregation of C atoms at non-KS interfaces



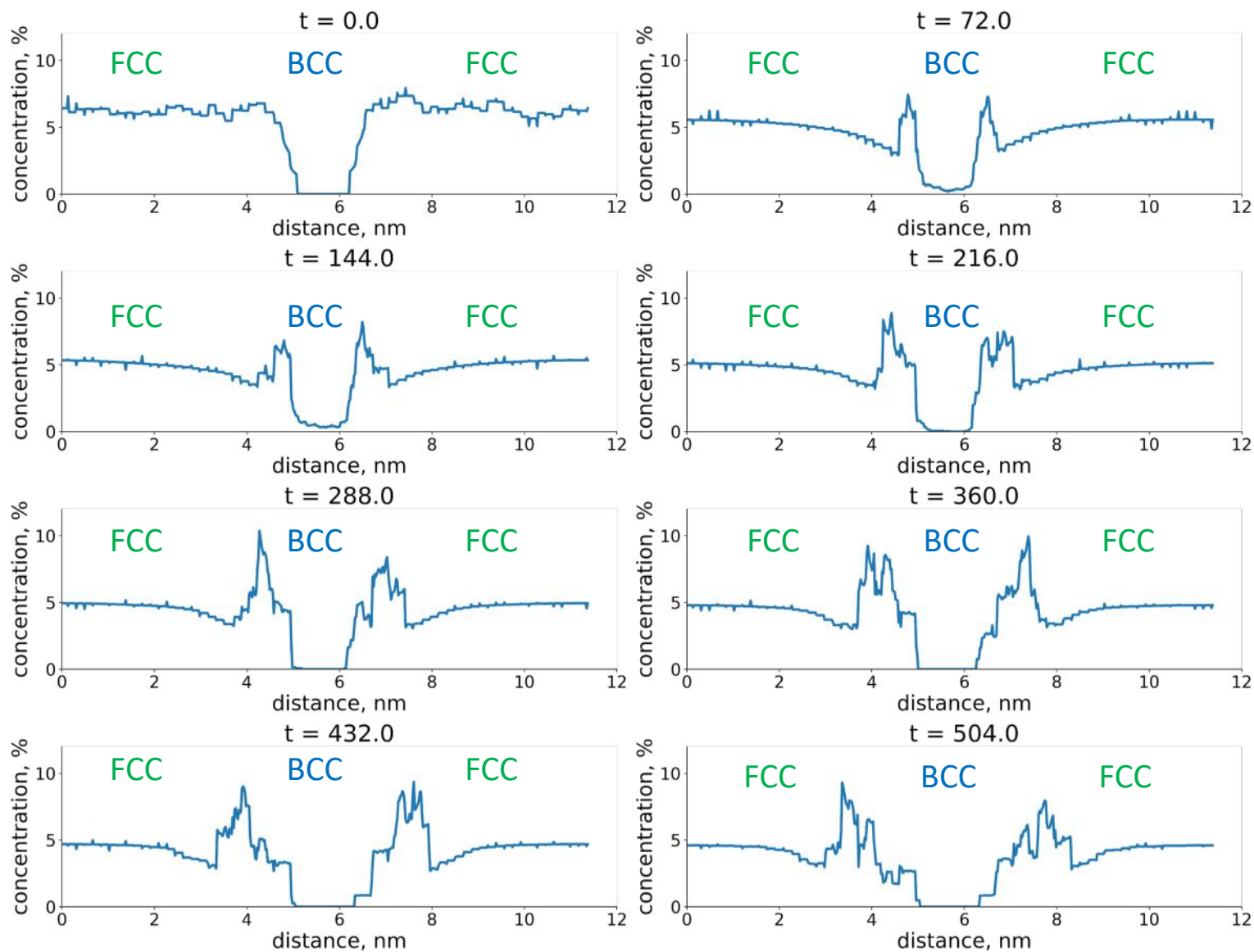
OVITO software: Common Neighbor Analysis (CAN)

- BCC;
- FCC;
- “other”;
- Carbon

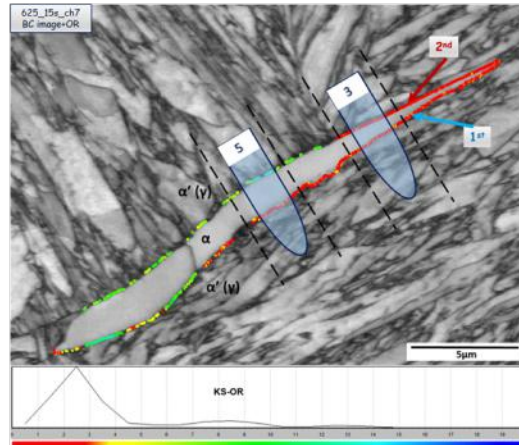
$$V = 256 \times 256 \times 512$$



Segregation of C atoms at non-KS interfaces

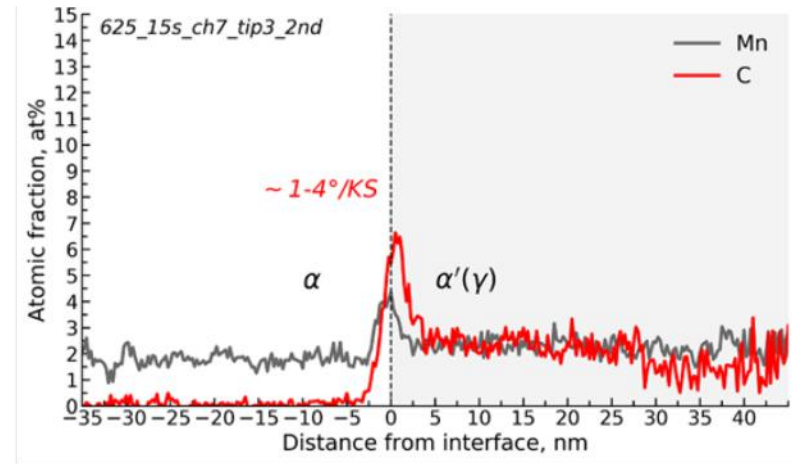
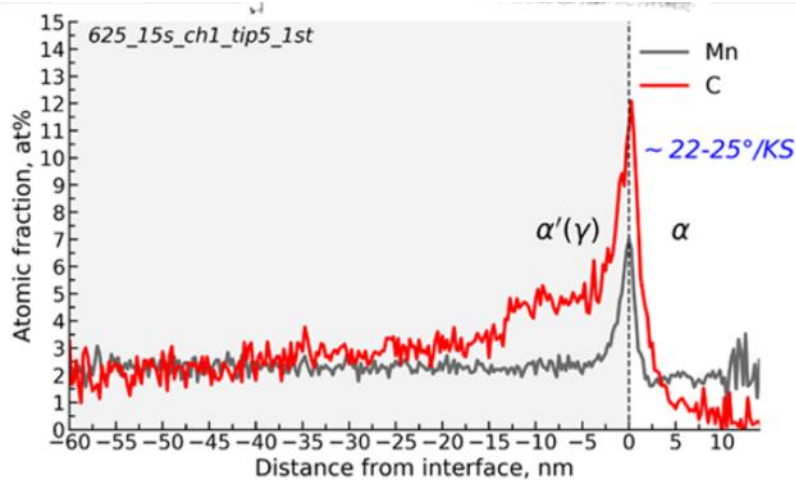


APT data Fe-0,787At%C-2At%Mn



(a)

(b)



3D reconstructions and concentration profiles of C and Mn atoms across
(a) the 1st interface of tip5 (isothermal holding at 625°C during 15s),
(b) the 2nd interface of tip3 (isothermal holding at 625°C during 15s).

Segregation of C atoms at fcc/bcc moving interface

FCC/BCC interface with C atoms

- ❖ Interface with Non-KS OR propagates faster than KS
- ❖ Concentration of C at non-KS OR interface is around 12%
- ❖ Concentration of C at KS OR interface is around 5-6 %

Versatility of Phase Field Atomic and Nanoscale Modeling

$$\frac{\partial c(\mathbf{r}, t)}{\partial t} = (\nabla M \nabla) \frac{\delta F}{\delta c}$$

$$\frac{\partial \eta(\mathbf{r}, t)}{\partial t} = -L \frac{\delta F}{\delta \eta}$$

Different types of fields:

- atomic density, $\rho(r)$ –solidification, atomic ordering
- composition $c(r)$, decomposition
- atomic LRO parameters, $\eta(r)_i$, (atomic ordering)
- transformation strain, $\epsilon(r)_{ij}$, (martensitic transformation)
- polarization, P , (ferroelectrics)
- magnetization, $M(r)$, (ferromagnets)
- Burgers vector, b , (multi-dislocations systems)
- crack opening, h (multi-crack systems)

