IPAM – Turbulence - 2014

Fundamental concepts in turbulent boundary layer flows and their use in engineering applications

(or: using LES to develop better wind farms)

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- Dr. Richard Stevens (JHU, postdoc) – LES + CWBL
- Dr. Michael Wilczek (JHU postdoc, soon MPI Göttingen)
  - spatio-temporal spectra + LES
- Prof. Dennice Gayme (JHU) CWBL + power grid modeling
- Prof. Johan Meyers (Univ. Leuven) – LES
- Prof. Marc Calaf (formerly JHU, EPFL & now U. Utah) – LES
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WHEN WIND FARMS GROW UP:
The windturbine-array boundary layer

Horns Rev 1 owned by Vattenfall.
Photographer Christian Steiness

Arrays are getting bigger: when L > 10 H (H: height of ABL),
approach "fully developed" state

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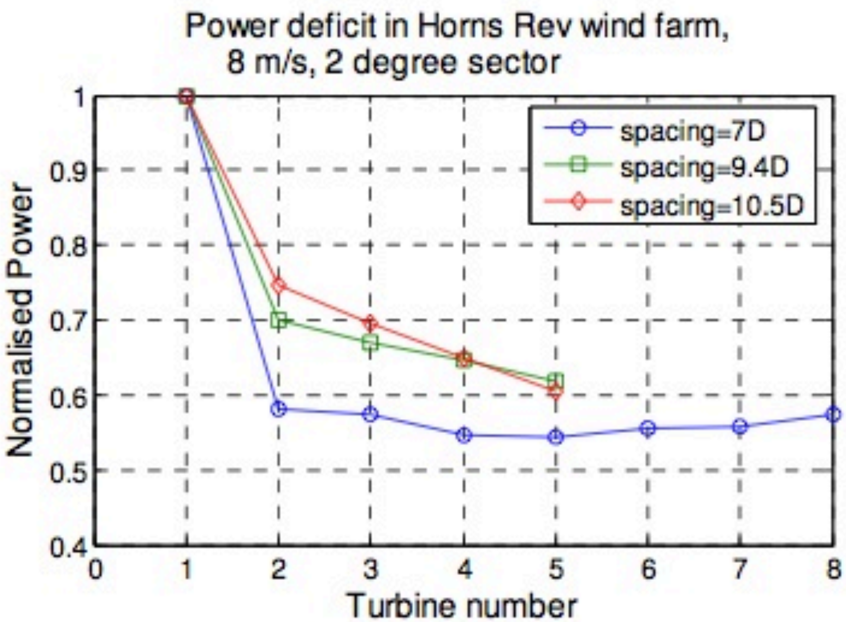
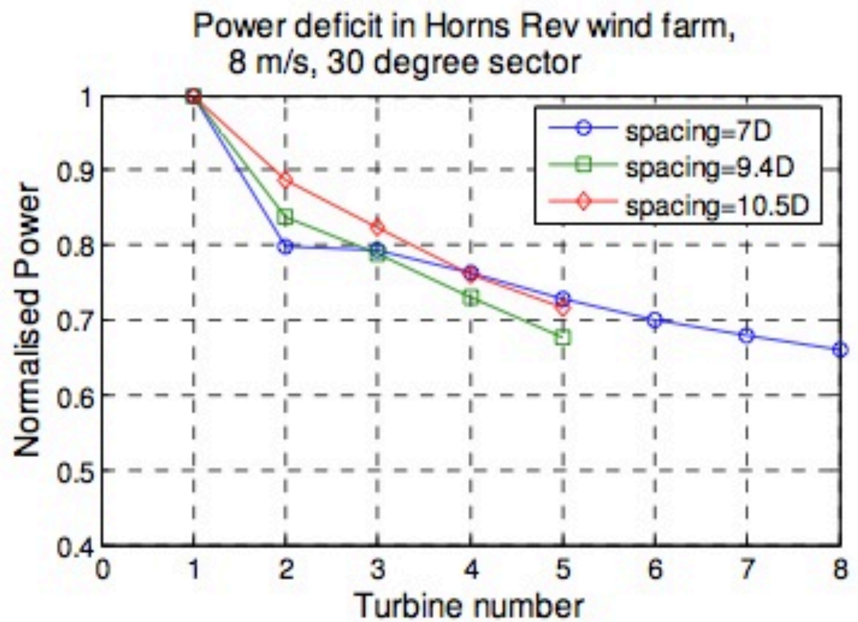
Wind farm power degradation: effects of wakes





"Modelling and measurements of wakes in large wind farms" Barthelemie, Rathmann, Frandsen, Hansen et al… J. Physics Conf. Series **75** (2007), 012049

Engineering models (wake superposition) have trouble coupling with atmospheric boundary
Layer (ABL) structure

Some basic concepts and their use in "engineering better wind farms"

- Logarithmic behavior of mean velocity profile
  (Prandtl, von Karman ...)

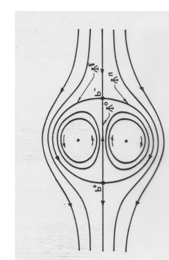
$$\frac{u(z)}{u_*} = \frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right)$$

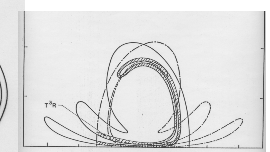
- Logarithmic scaling of variance
  (Townsend, Perry, Smith, Marusic, Hultmark ..)

$$\left\langle u_x^2 \right\rangle = u_*^2\left[B - A\log\left(\frac{z}{H}\right)\right]$$

- Random sweeping hypothesis
  (Kraichnan, Lumley, ...)

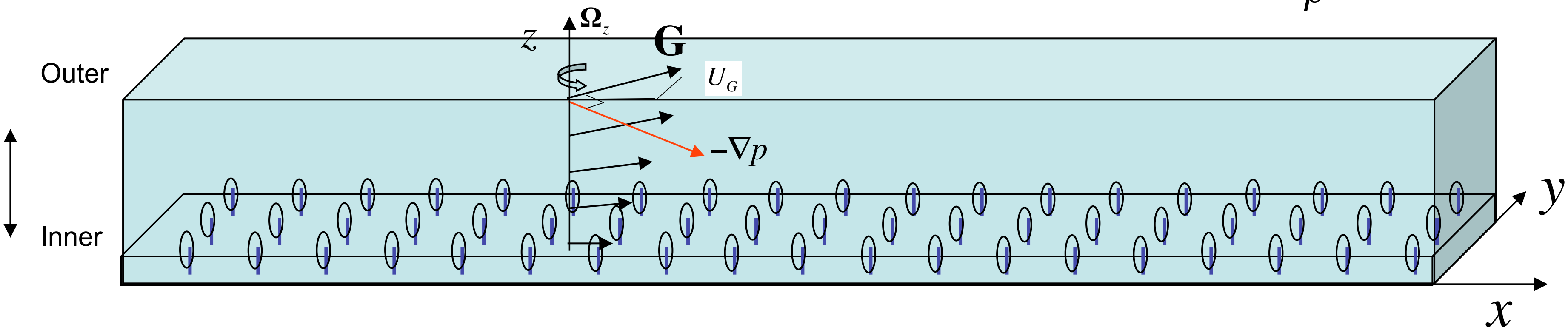
- Topological Fluid Dynamics, Lagrangian Chaos
  (Aref, Ottino, Wiggins ...)





WTABL: Forcing by geostrophic wind

Above ABL (in mid-latitudes): geostrophic balance $$2\boldsymbol{\Omega} \times \mathbf{G} - \frac{1}{\rho}\nabla P \approx 0$$



Coupled through a stress $(u_*)^2$:

Outer length-scale: $H = \frac{u_*}{f}$ $f = 2\Omega \sin\phi \approx 10^{-4} s^{-1}$ (mid-latitudes)

Inner length-scale: z_0

Inner-outer matching: $$\frac{u(z)}{u_*} = \frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right) \qquad \frac{G}{u_*} = \sqrt{A^2 + \left[\frac{1}{\kappa}\ln\left(\frac{u_*}{f z_0}\right) - C\right]^2}$$

Given G and z_0 -> find u_* and H – and then $u(z_h)$, velocity at WT

+ effects of thermal stratification (will not be focused upon in this talk)

Analogy:

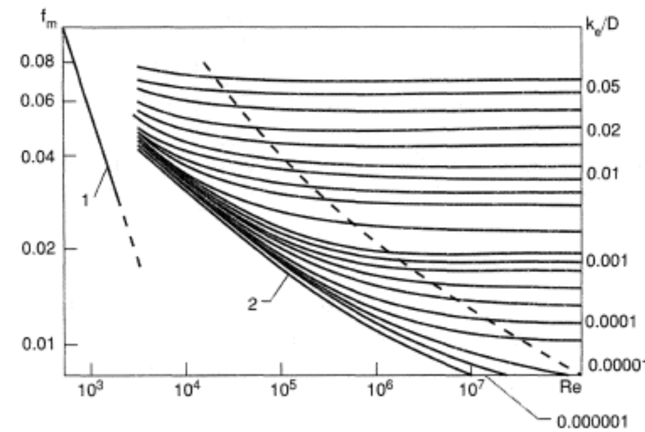
Atmospheric boundary layer as fully developed, half-channel flow

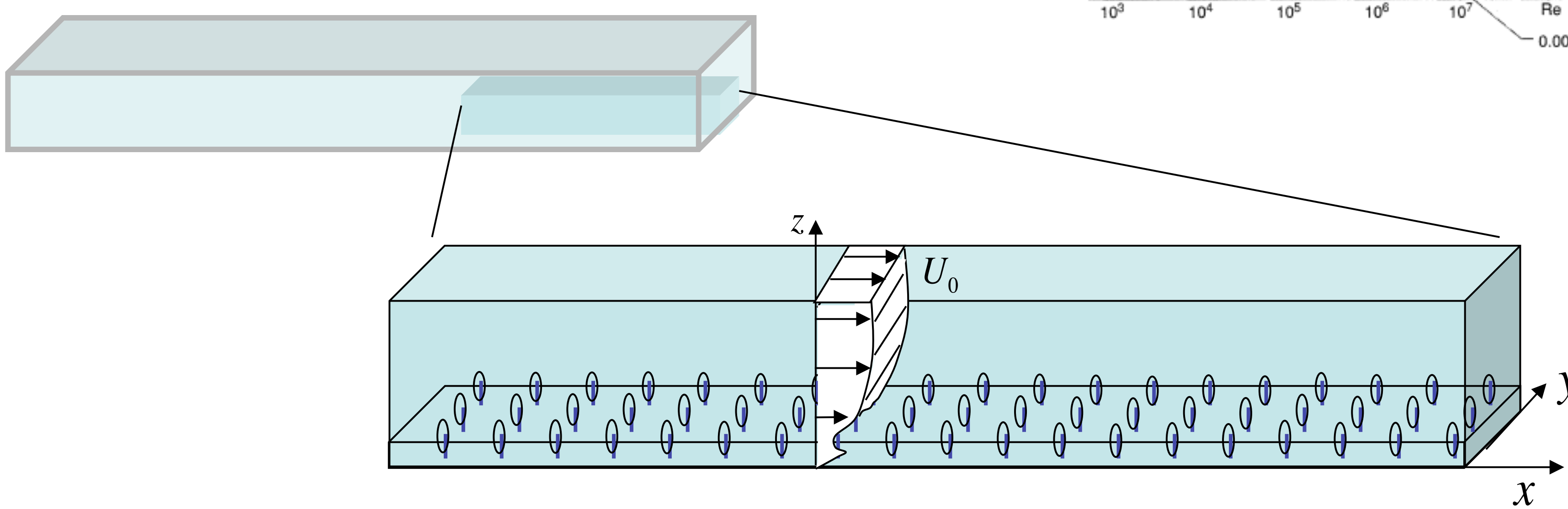
$$\langle u \rangle = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right) \quad \rightarrow \quad \frac{\tau_w}{\rho} = u_*^2 = \left(\frac{\kappa}{\ln(z_{ref}/z_0)}\right)^2 \langle u \rangle^2 (z_{ref})$$

f/2 – friction factor

V = RI, we need R

Moody diagram

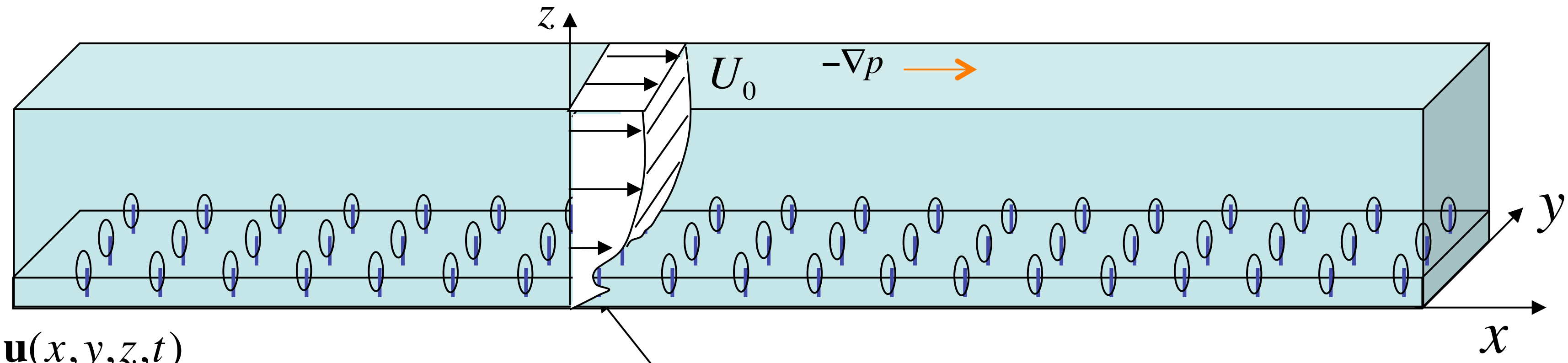




The "fully developed" pressure-grad-driven WTABL:

What is the generic structure of this specific type of boundary layer?

Pressure-gradient driven (half-channel flow)



$\mathbf{u}(x,y,z,t)$ DNS

$\tilde{\mathbf{u}}(x,y,z,t)$ LES

$\overline{\mathbf{u}}(x,y,z)$ RANS

$$U(z)=\langle \overline{u}(x,y,z)\rangle_{xy}$$

horizontal (canopy) average

<u>**Outline**: **3 parts**</u>

Part I: What is the "averaged" velocity distribution? $U(z)=\langle \overline{u}(x,y,z)\rangle_{xy}$

engineering model for wind farm optimization (CWBL model)

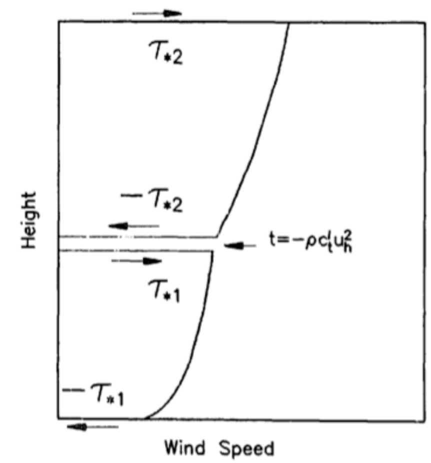
Part II: Fluctuations: spatio-temporal spectra (in boundary layers first)

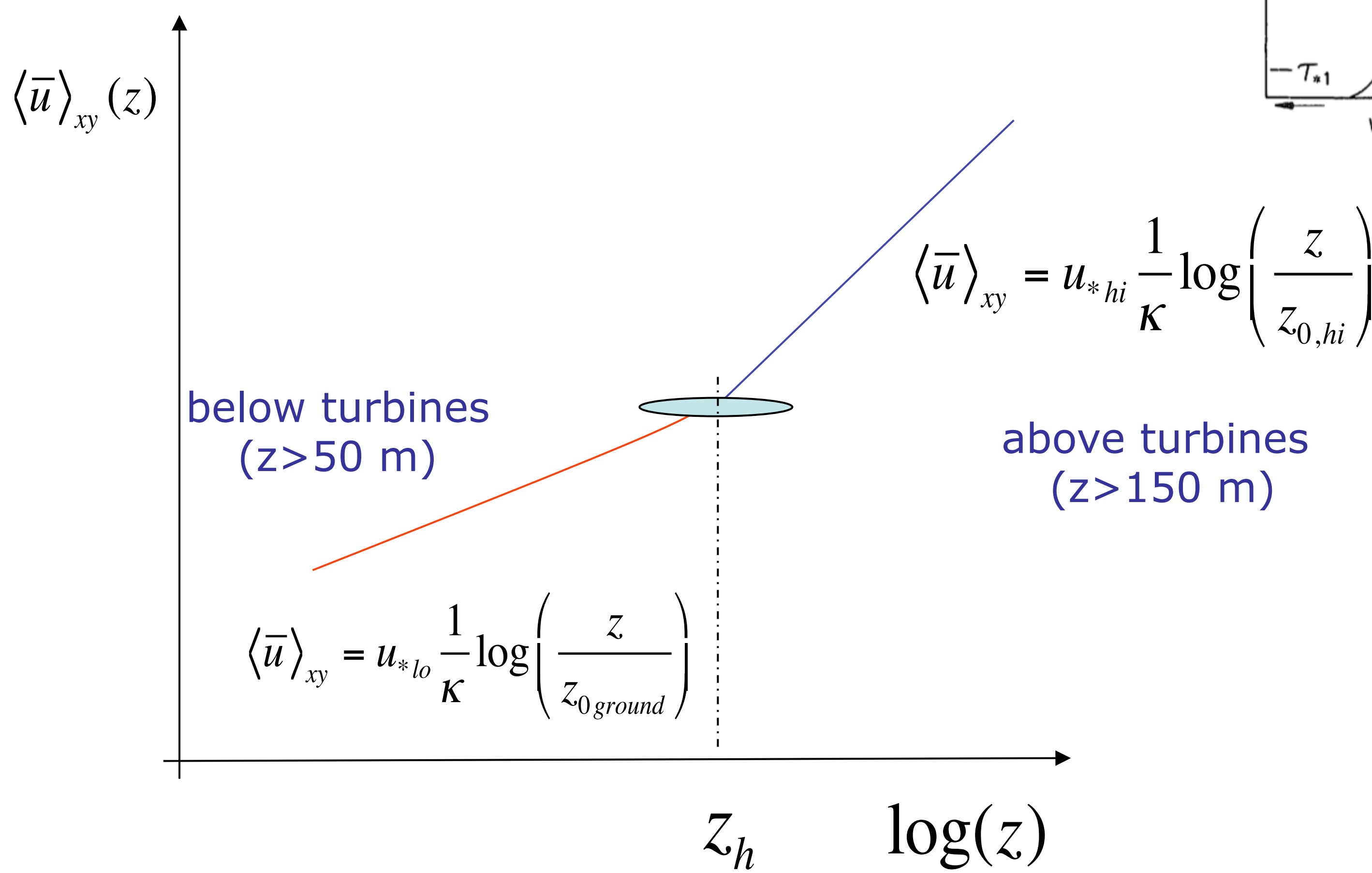
Part III: Visualizing where the energy comes from.

Part I: The fully developed WTABL & momentum theory

$$u_{*\,hi}^{2} \approx u_{*\,lo}^{2} + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_h^2$$

Frandsen 1992 (also Newman 1977)
postulated the existence of 2 log laws:





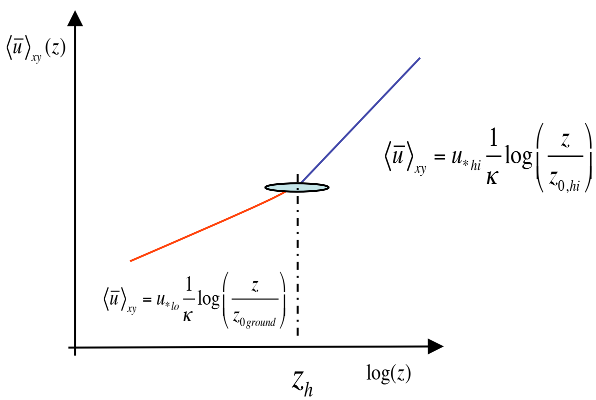
Review: The fully developed WTABL & momentum theory

S. Frandsen 1992, Frandsen et al. 2006:

Knowns: $u_{*hi}, z_{0,ground}, C_T, s_x, s_y$

3 unknowns: $z_{0,hi}, U_h, u_{*lo}$

$$\begin{cases} u_{*hi}^2 \approx u_{*lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_h^2 \\ U_h = u_{*hi} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,hi}}\right) \\ u_{*hi} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,hi}}\right) = u_{*lo} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,ground}}\right) \end{cases}$$



Solve for effective roughness:

$$z_{0,hi} = z_h \exp\left(-\kappa \left[\frac{\pi C_T}{8 s_x s_y} + \left(\frac{\kappa}{\ln(z_h / z_{0,ground})} \right) \right]^{-1/2} \right)$$

$$\frac{\tau_w}{\rho} = u_*^2 = \left(\frac{\kappa}{\ln(z_{ref} / z_{0,hi})} \right)^2 \langle u \rangle^2 (z_{ref})$$

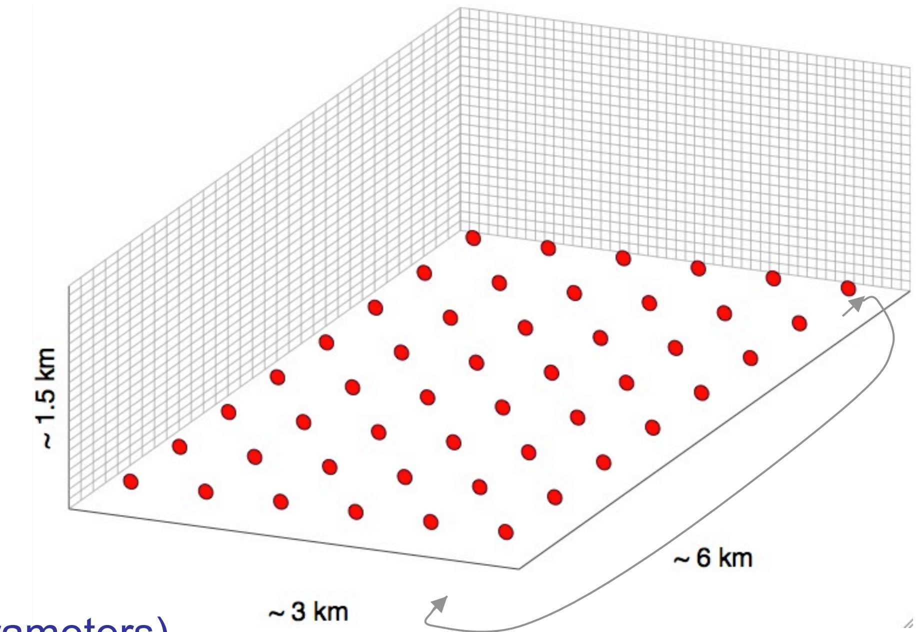
f/2 – friction factor

Next: perform Large Eddy Simulations (LES) of WTABL
Simulations setup:

- LES code: horizontal pseudo-spectral (periodic B.C.), vertical: centered 2nd order FD (Moeng 1984, Albertson & Parlange 1999, Porté-Agel et al. 2000, Bou-Zeid et al. 2005)

$$H = 1000 - 1500m, \quad L_x = \pi H - 2\pi H, \quad L_y = \pi H$$

$$(N_x \times N_y \times N_z) = 128 \times 128 \times 128$$



- Horizontal periodic boundary conditions (only good for FD-WTABL)

- Top surface: zero stress, zero w

- Bottom surface B.C.: Zero w + Wall stress: Standard wall function relating wall stress to first grid-point velocity

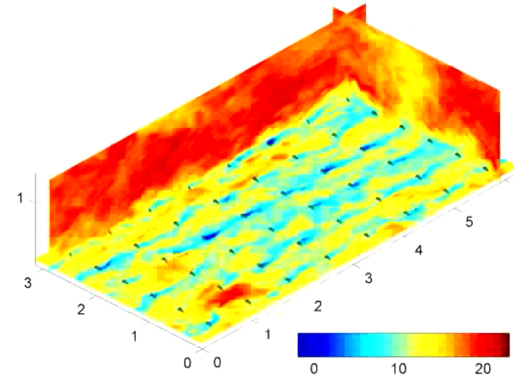
- Scale-dependent dynamic Lagrangian model - eddy-viscosity closure but (*no* adjustable parameters)

- Actuator disk modeling for wind turbines $f_{Tx} = -\frac{1}{2} C_T U_{ref}^2 \frac{\delta A_{yz}}{\delta V}, \quad C_T = 0.75$

- More details: Calaf, Meneveau & Meyers, "Large eddy simulation study of fully developed wind-turbine array boundary layers" Phys. Fluids. **22** (2010) 015110

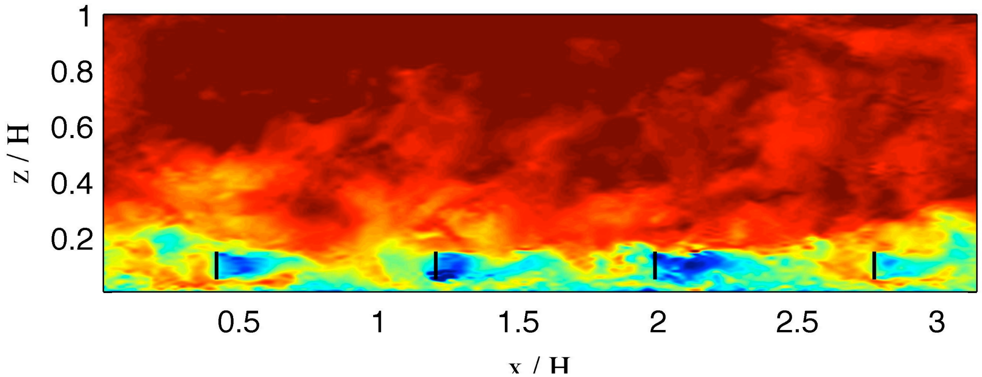
Sample realizations

Instantaneous stream-wise velocity contours:

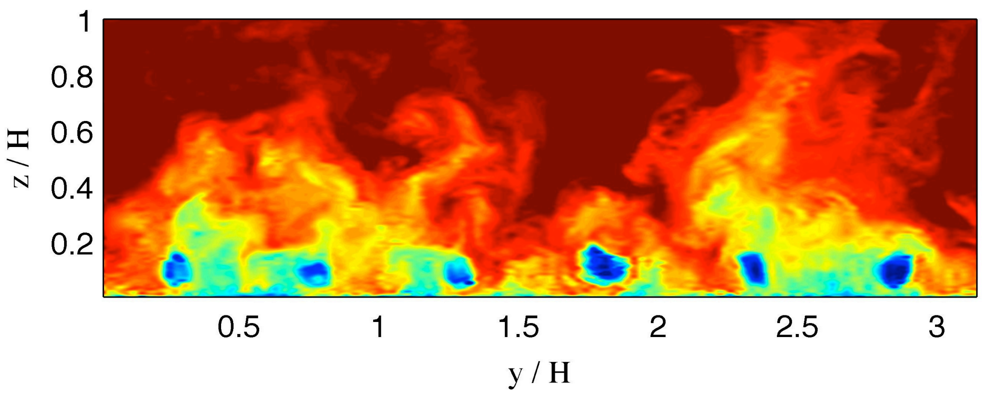


u/u*

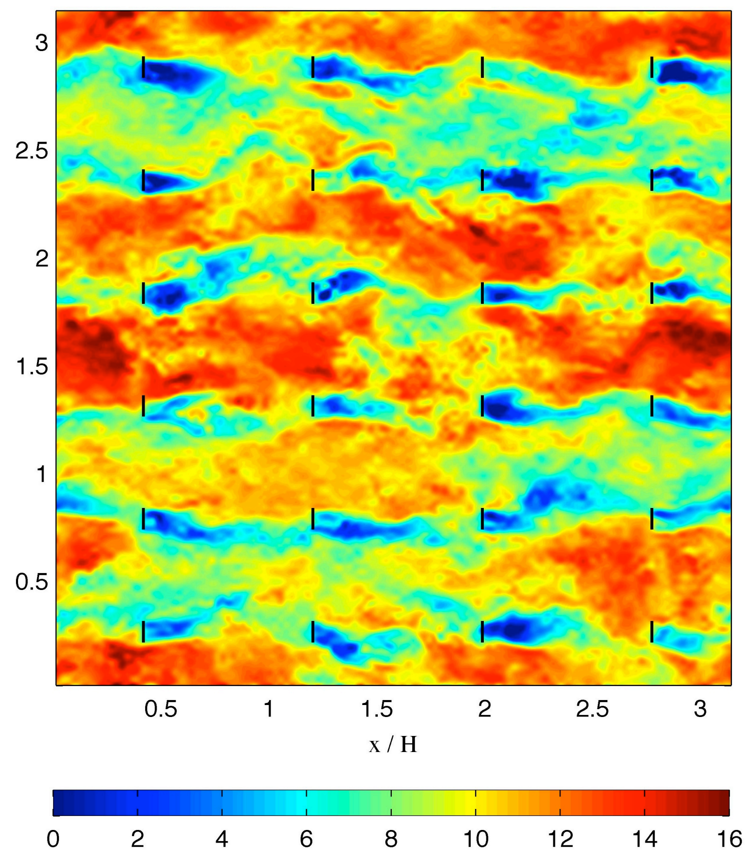
side-view



front-view



top-view



u/u*

$H = 1000 - 1500m, \quad L_x = \pi H - 2\pi H, \quad L_y = \pi H$

$(N_x \times N_y \times N_z) = 128 \times 128 \times 128$

In-silico wind farm (Large-Eddy-Simulation) JHU-LES code
Visualization courtesy of D. Brock (Extended Services XSEDE)

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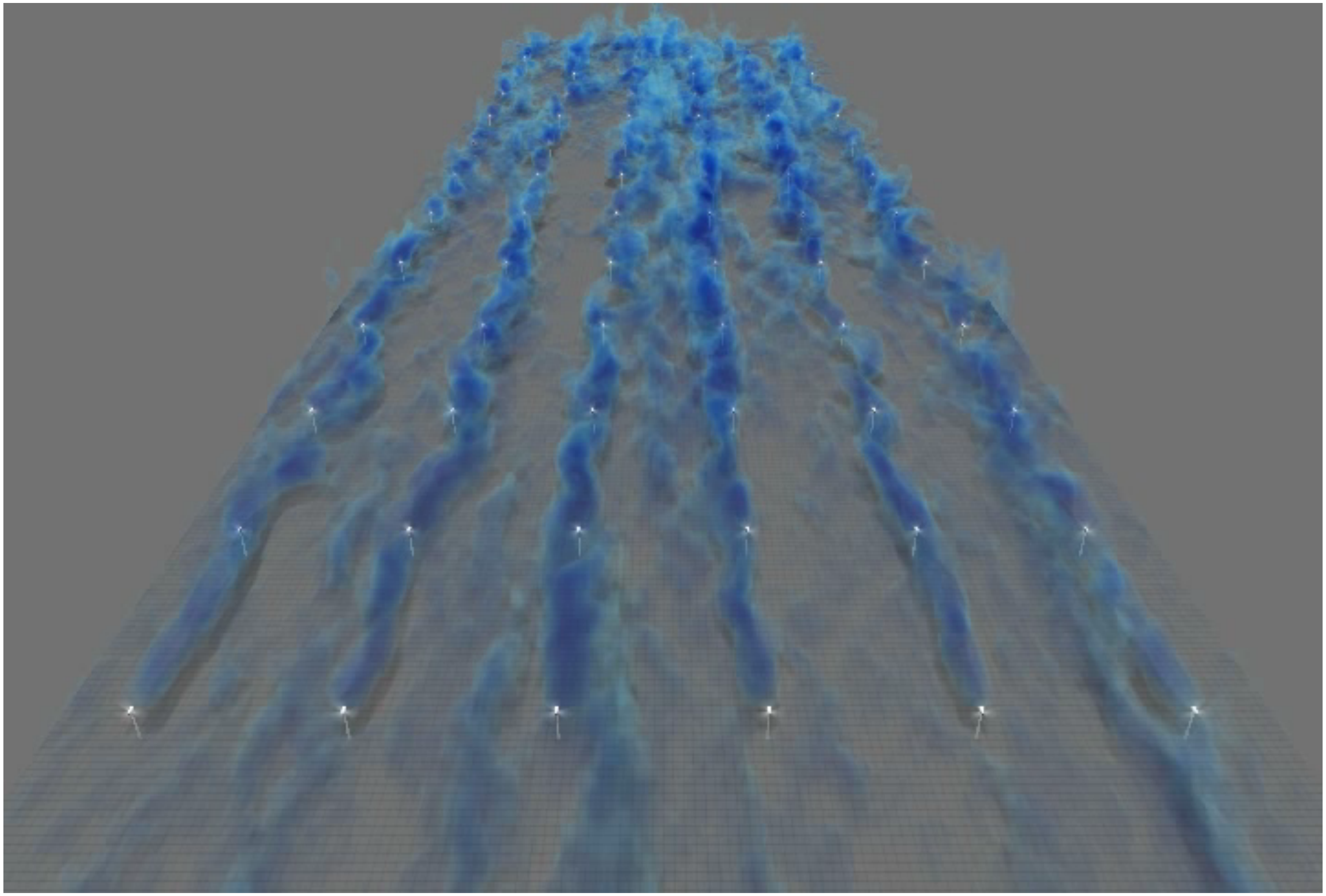
$H = 1500m, \quad L_x = 8\pi H, \quad L_y = 3\pi H$

$(N_x \times N_y \times N_z) = 1024 \times 512 \times 512$

Volume rendering of “low u” + fluid particles (white)

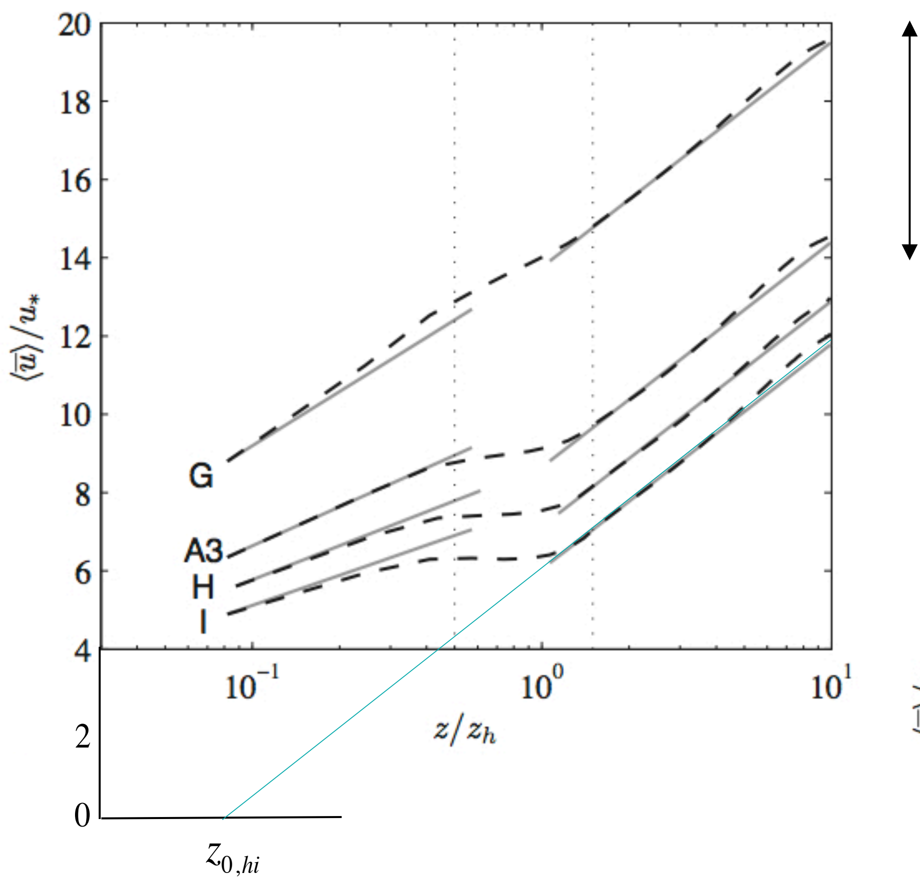


Suite of LES cases

TABLE I. Summarizing parameters of the various LES cases. Between brackets is indicated which code is used: “L” refers to the KULeuven code and “J” refers to the JHU-LES code.

	s_x/s_y	s_x	$4s_xs_y/\pi$	N_t	$L_x \times L_y \times H$	$N_x \times N_y \times N_z$	$z_{0,\mathrm{lo}}$	C_T'	c_{ft}'
A1 (L)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	1.33	0.025
A2 (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	1.33	0.025
A3 (L)	1.5	7.85	52.36	8×6	$2\pi\times\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.025
A4 (L)	1.5	7.85	52.36	8×6	$2\pi\times\pi\times1.5$	$128\times192\times92$	10^{-4}	1.33	0.025
B (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	2.00	0.038
C (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-4}	0.60	0.012
D (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-3}	1.33	0.025
E (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-5}	1.33	0.025
F (J)	1.5	7.85	52.36	4×6	$\pi\times\pi\times1$	128^3	10^{-6}	1.33	0.025
G (L)	1.5	15.7	209.4	4×3	$2\pi\times\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.0064
H (L)	1.5	6.28	33.51	10×8	$2\pi\times1.07\pi\times1$	$128\times192\times57$	10^{-4}	1.33	0.040
I (L)	1.5	5.24	23.27	12×9	$2\pi\times\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.057
J (L)	2	9.07	52.36	7×7	$2.02\pi\times1.01\pi\times1$	$128\times192\times61$	10^{-4}	1.33	0.025
K (L)	1	6.41	52.36	10×5	$2.04\pi\times1.02\pi\times1$	$128\times192\times60$	10^{-4}	1.33	0.025

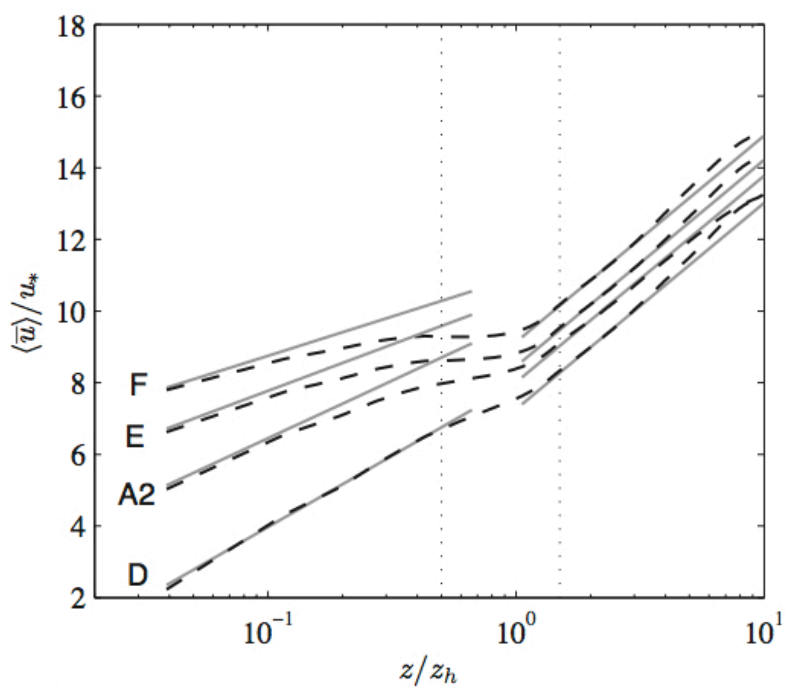
Measuring $z_{0,hi}$ from LES (horizontally averaged) mean velocities



measure $z_{0,hi}$ from intercept

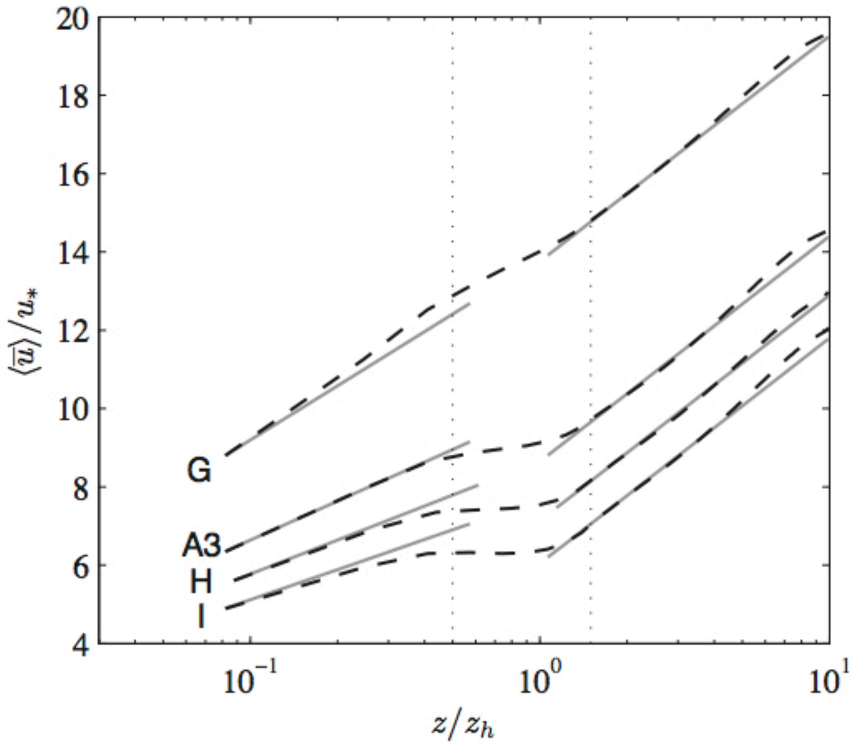
$$\langle \overline{u} \rangle_{xy} = u_{*hi} \frac{1}{\kappa} \log\left(\frac{z}{z_{0,hi}} \right)$$

(essentially the "Clauser plot" method)



Double log-law confirmed, but predicted $z_{0,hi}$ does not match Frandsen model

"Wake upgrade" to Frandsen's top-down model



$$\left(\kappa u_* z_h + \nu_w\right)\frac{\partial \langle \overline{u} \rangle}{\partial z} = u_*^2$$

In wake, reduced slope:

$$\frac{\partial \langle \overline{u} \rangle}{\partial z} = \frac{1}{\kappa u_* z_h + \nu_w} u_*^2$$

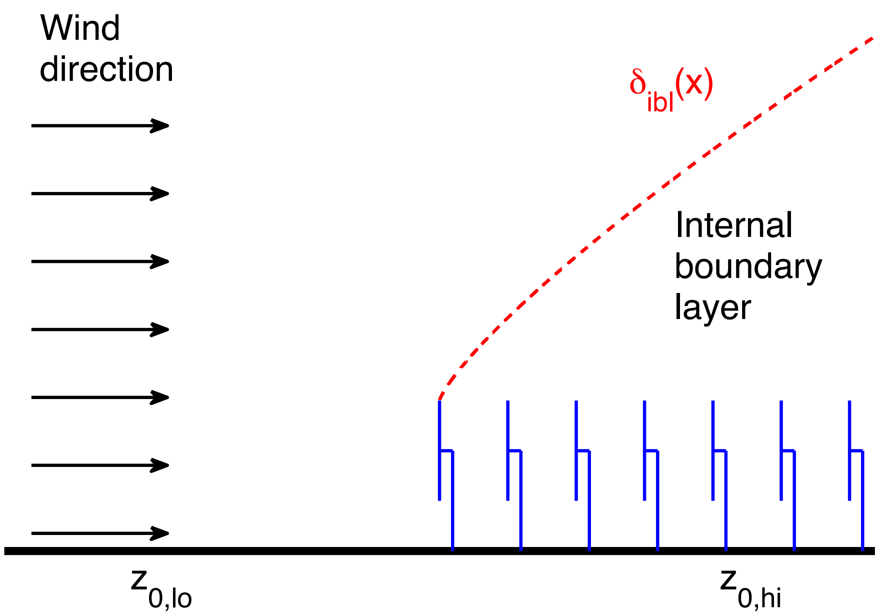
$$\nu_w = \sqrt{\tfrac{1}{2} c_{ft}} \langle \overline{u} \rangle D$$

$$\nu_w^* = \frac{\sqrt{\tfrac{1}{2} c_{ft}} \langle \overline{u}(z_h) \rangle D}{\kappa u_* z_h} \approx 28 \sqrt{\tfrac{1}{2} c_{ft}}$$

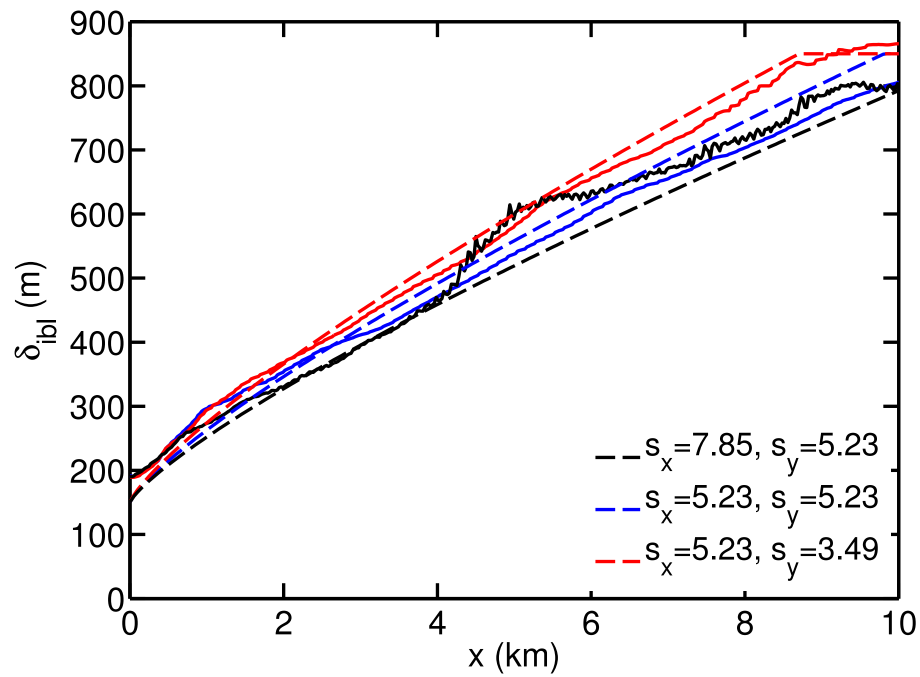
$$z_{0,hi} = z_h \left(1 + \frac{D}{2z_h}\right)^{\beta} \exp\left(-\left[\frac{\pi C_T}{8\kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1 - \frac{D}{2z_h}\right)^{\beta}\right]\right)^{-2}\right]^{-1/2}\right)$$

$$\text{where} \quad \beta = \frac{28\sqrt{\tfrac{1}{2} c_{ft}}}{1 + 28\sqrt{\tfrac{1}{2} c_{ft}}},$$

Generalizing to developing case:



Internal boundary layer finite size wind-farm



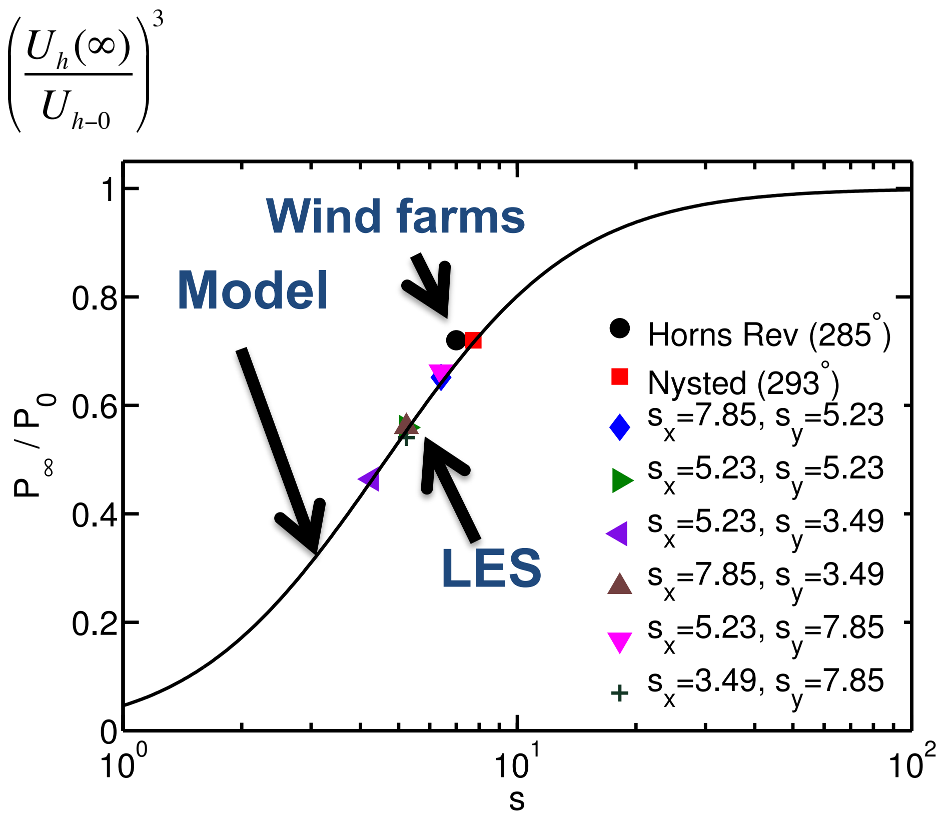
Comparison LES with model

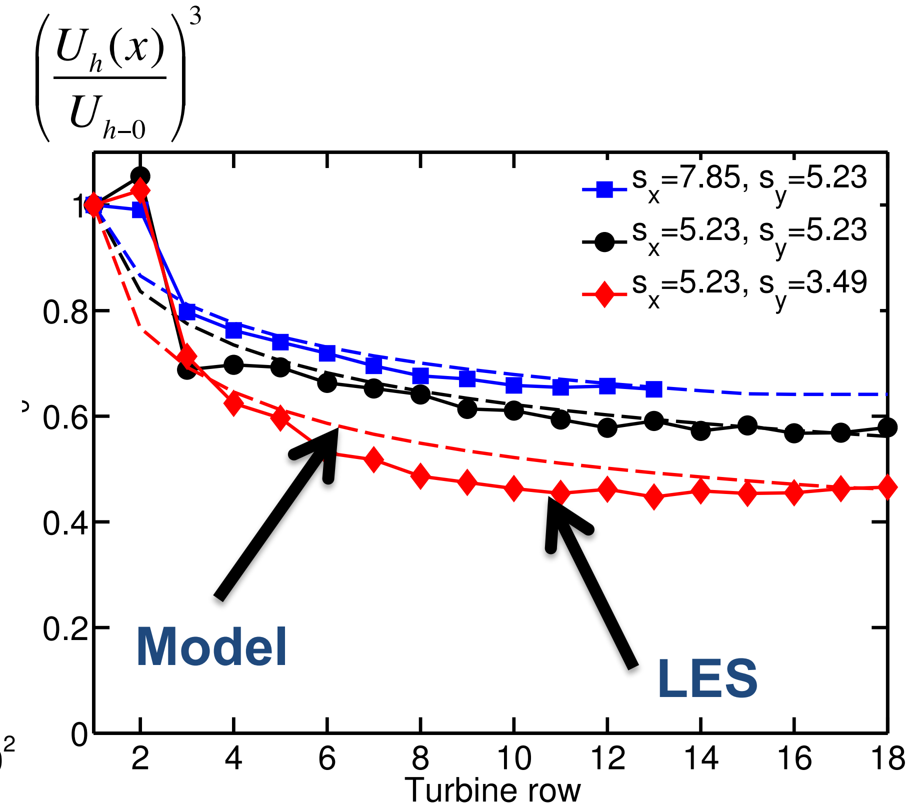
$$\frac{\delta_{\mathrm{IBL}}(x)}{z_{0,\mathrm{hi}}} = \frac{\delta_{\mathrm{IBL}}(0)}{z_{0,\mathrm{hi}}} + C_1 \left(\frac{x}{z_{0,\mathrm{hi}}} \right)^{4/5}$$

with C_1=0.33

***Meneveau, J. Turbulence (2012); Stevens**, submitted to Wind Energy (2014)*

Comparing model with LES & (staggered) wind farm data





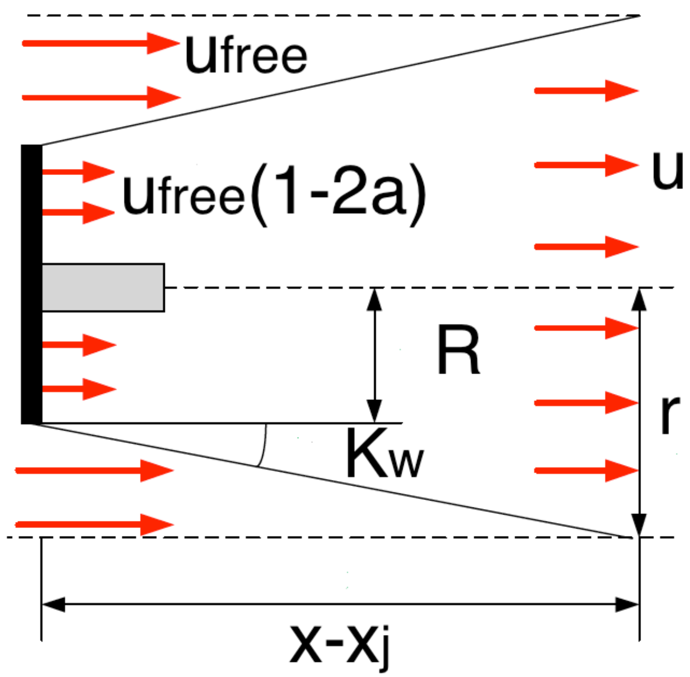
Relative power output fully developed regime

Downstream power development **staggered wind-farms**

Stevens, submitted to Wind Energy (2014)

Coupled wake boundary layer model (CWBL)

(R. Stevens, D. Gayme, CM)

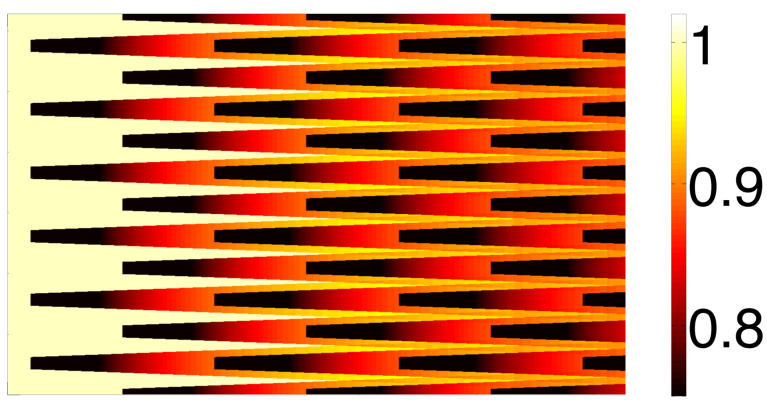


PART I: Momentum wake model gives velocity deficit

$$\delta u(\mathbf{x};j) = u_{\text{free}} - u(\mathbf{x};j) = \frac{2\,a\,u_{\text{free}}}{[1 + k_w(x - x_j)/R]^2}$$

Lissaman (1979) / Jensen (1984)

Wake expansion models



Wake interaction with squared velocity deficits

$$u(\mathbf{x}) = u_{\text{free}} - \sqrt{\sum_{j \in J_A} \delta u^2(\mathbf{x};j)},$$

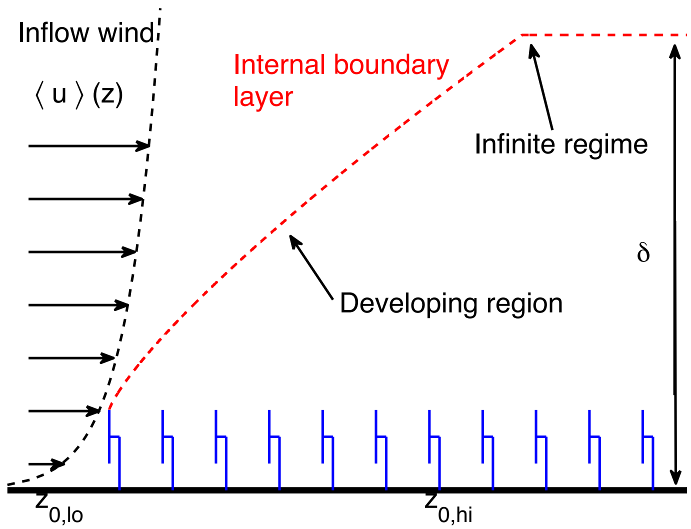
Lissaman (1979) / Katic et al. (1986)

Coupled wake boundary layer model (CWBL)

(R. Stevens, D. Gayme, CM)

Effective maximum span-wise spacing, s_{ye}

Topdown model



$$z_{0,\mathrm{hi}} = z_{\mathrm{h}} \left(1 + \frac{D}{4 z_{\mathrm{h}}}\right)^{\beta} \exp\left(- \left[\frac{C_T}{8 s_x s_{ye} \kappa^2} + \left(\ln\left[\frac{z_{\mathrm{h}}}{z_{0,\mathrm{lo}}} \left(1 - \frac{D}{2 z_{\mathrm{h}}}\right)^{\beta} \right] \right)^{-2} \right]^{-1/2} \right)$$

Wake expansion superposition model

Effective wake expansion coefficient, k_w

Two way coupling leads to improved results!

Stevens, Gayme, Meneveau, submitted to JRSE (2014), arXiv:1408.1730

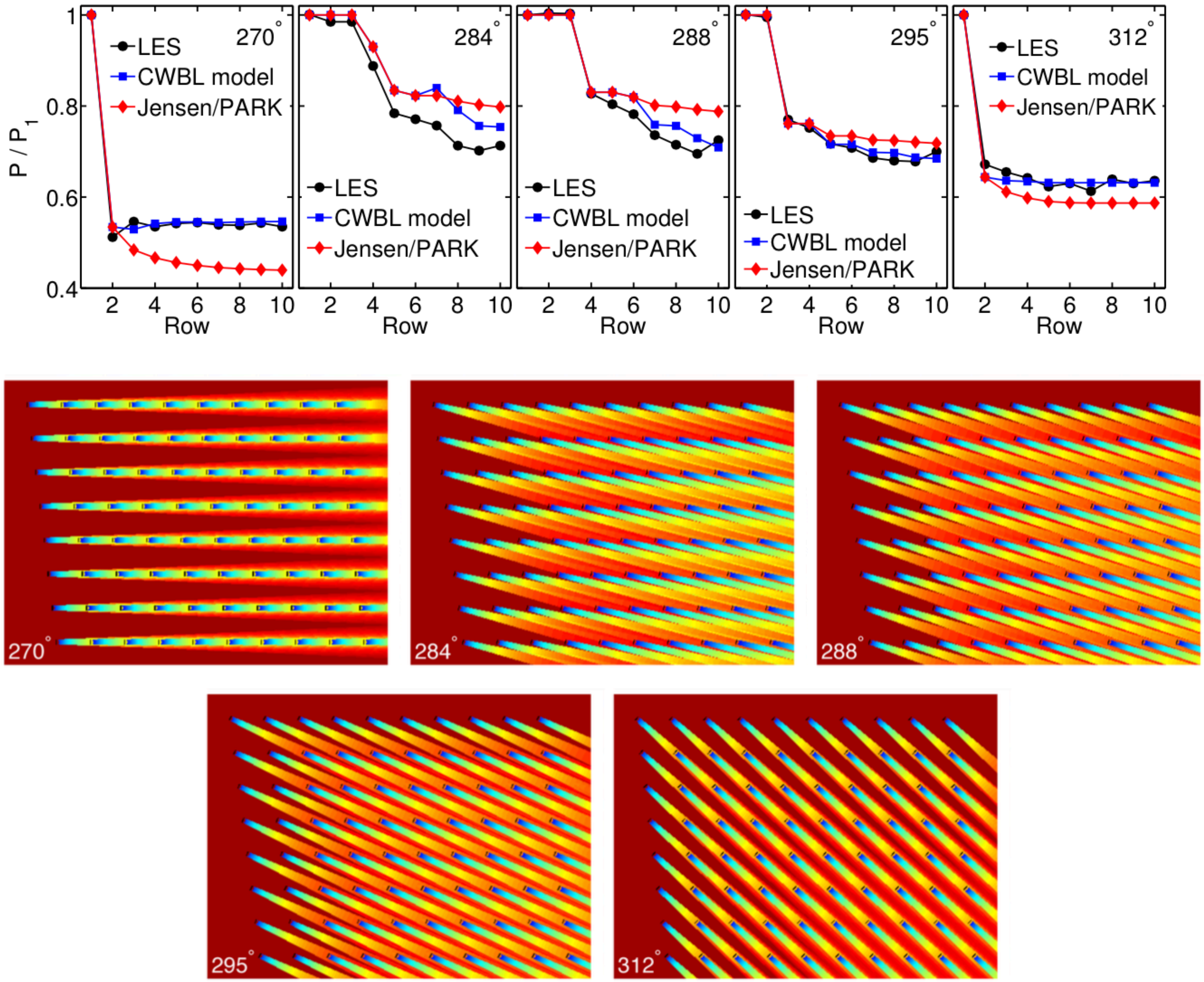
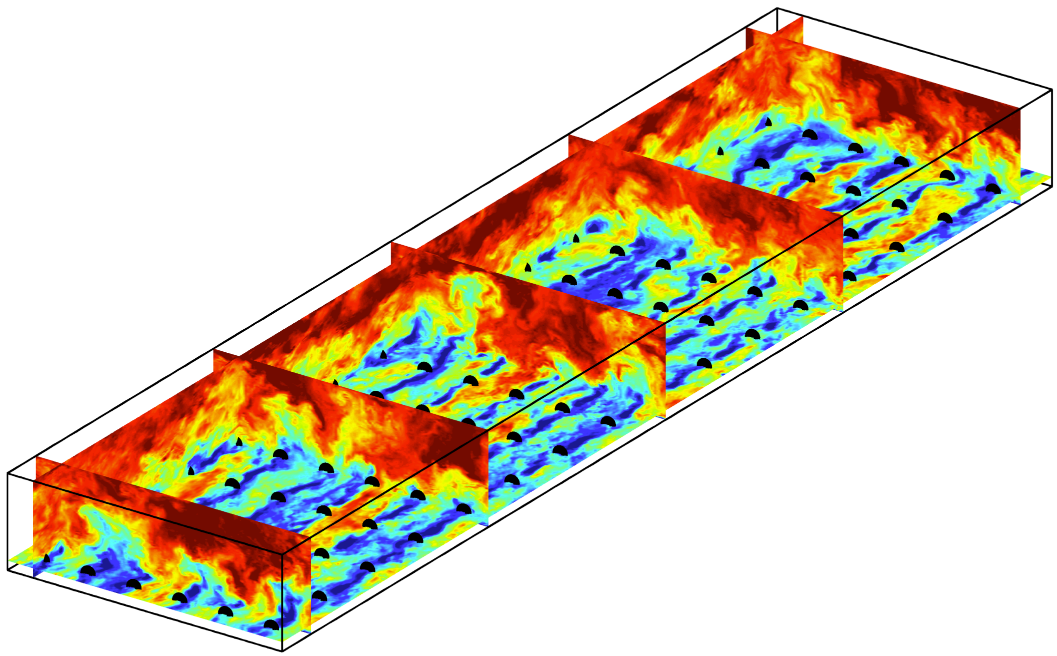


Figure 6. Comparison between the LES results from Porté-Agel [2] (black circles), the CWBL model (blue squares) and the Jensen/PARK model (red diamonds) for Horns Rev. The top panels indicate the results for the wind-directions 270°, 284°, 288°, 295°, and 312°. The lower panels show a visualization of the normalized velocity field at hub-height obtained from the CWBL model for these cases.

Part II: What can we say about fluctuations?

- Power fluctuations *P(t)*

- Unsteady loading on turbines *F(t)*



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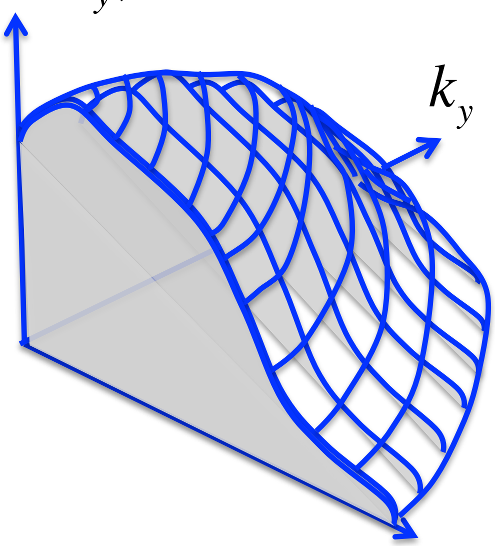
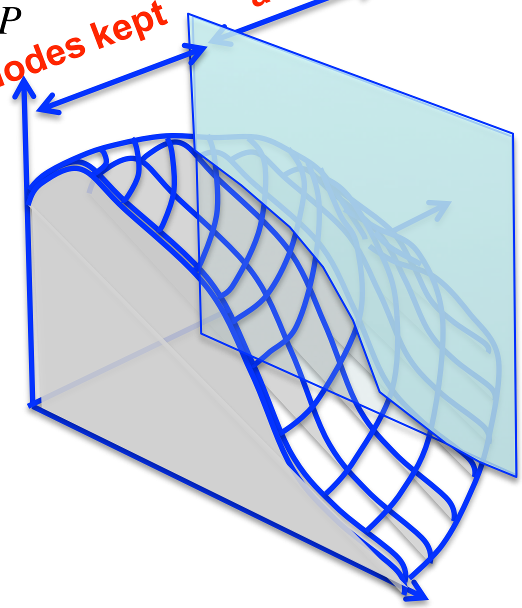
Space-time structure of fluctuations:

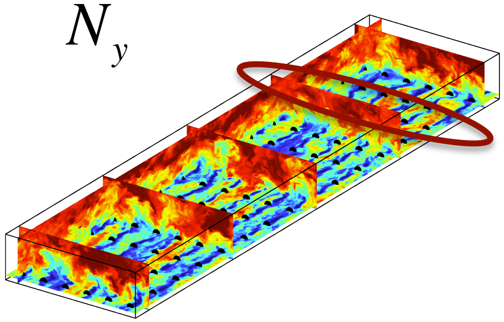
We would like wavenumber-frequency spectrum $E_P(k_x, k_y, \omega)$





Spectrum of spatial aggregates:

$\frac{1}{N_y}\Sigma P$



$$E_{\Sigma P}(\omega) = \sum_{k_x}\sum_{k_y} E_P(k_y, \omega) \times \left(\frac{\sin(k_y L)}{k_y L}\right)^2$$

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- We would like to have a model for wavenumber-frequency spectrum of power in farm boundary layers $E_P(k_x, k_y, \omega)$

- But first we need a model for the wavenumber-frequency spectrum of stream-wise velocity in ABL turbulence as function of height

$$E_u(k_x, k_y, \omega; z)$$

- This needs to be expressible in practical fashion (i.e. analytically “manipulatable” expression)

- Nothing in textbooks, unfortunately

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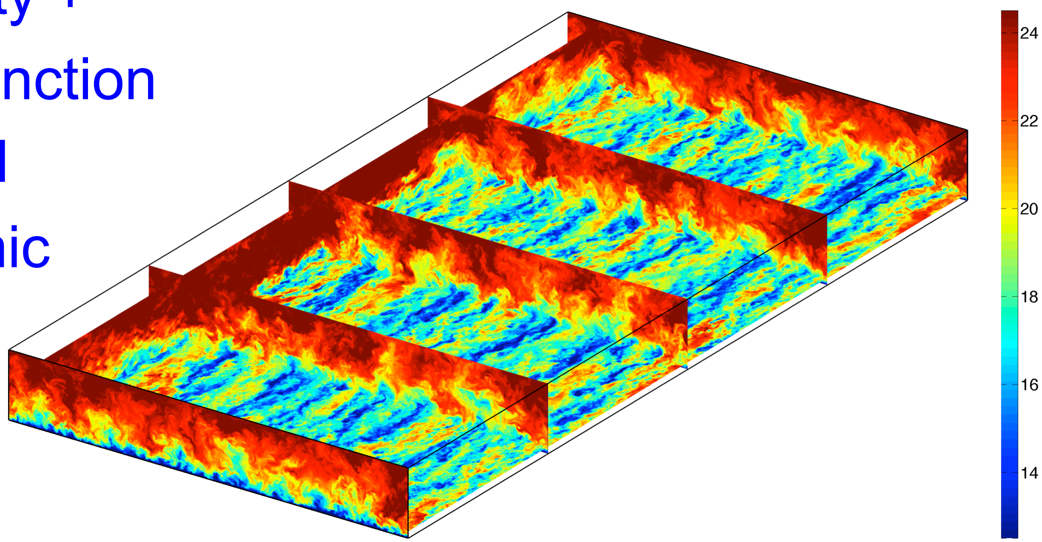
Data from Large Eddy Simulation

- LESGO code: horizontal pseudo-spectral,
  vertical: centered 2nd order finite difference
- Horizontal periodic boundary condition (bc)
- 512x256x128 (4π x 2π x H)
- Top surface: zero stress,
  No vertical velocity
- Bottom bc: No vertical velocity +
  Wall stress: Standard wall function
  with prescribed $z_0=10^{-4}$H
- Lagrangian scale-dep dynamic SGS model

1024x512x256 (4πH x 2πH x H)



Stevens, Wilczek & CM
(2014, JFM 2014)

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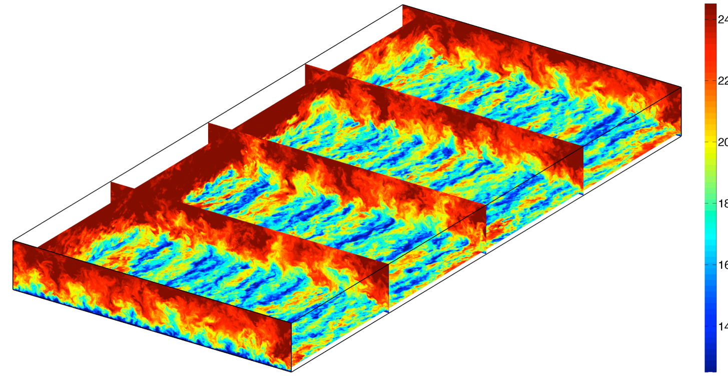
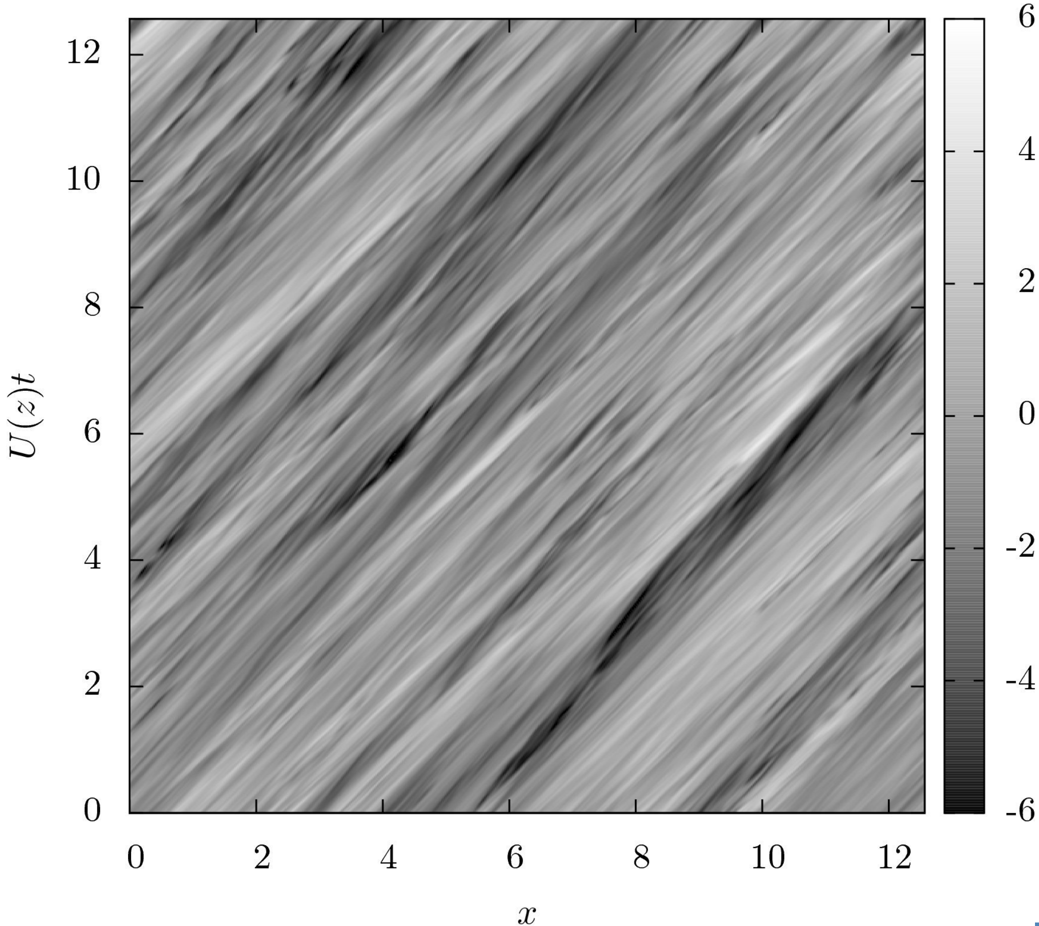
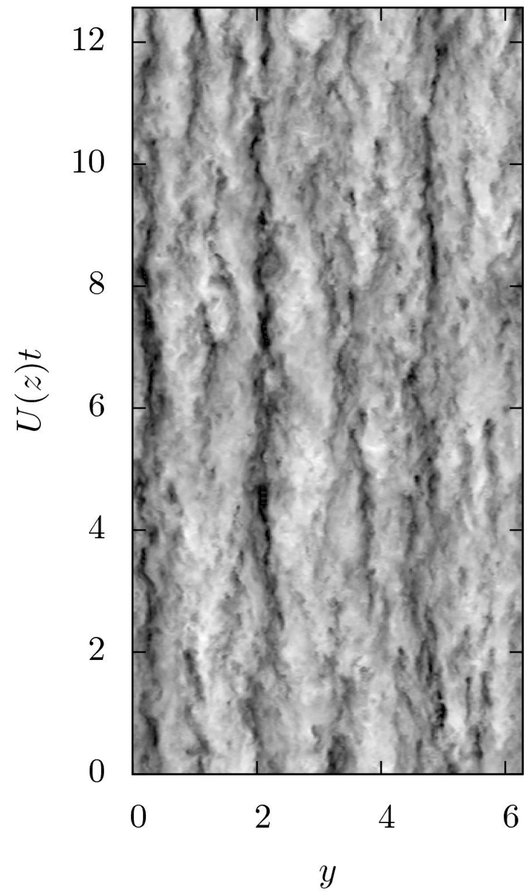
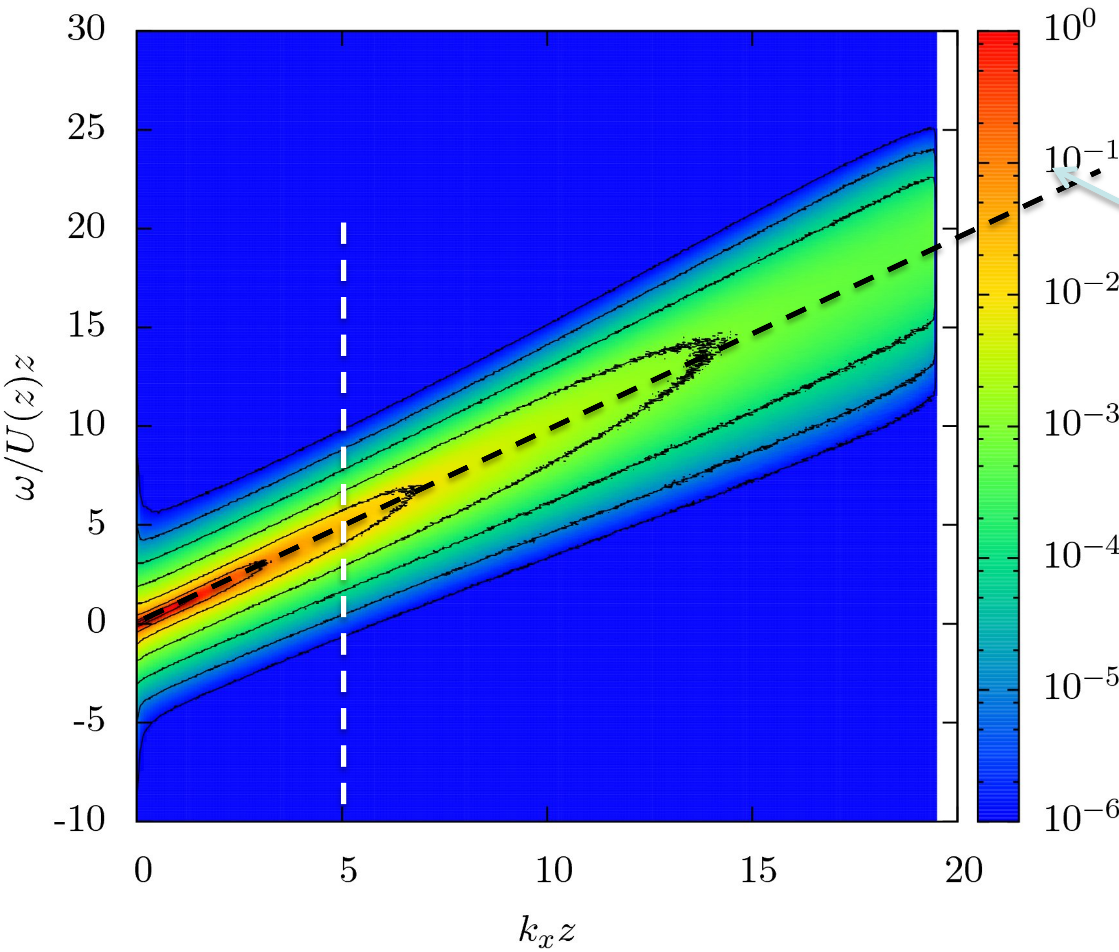
Data from LES
at z/H = 0.15 – in log-layer:
space-time contours







Data from LES (at z/H = 0.15 – in log-layer): streamwise wavenumber-frequency spectrum



Taylor's hypothesis would predict delta function "line" …

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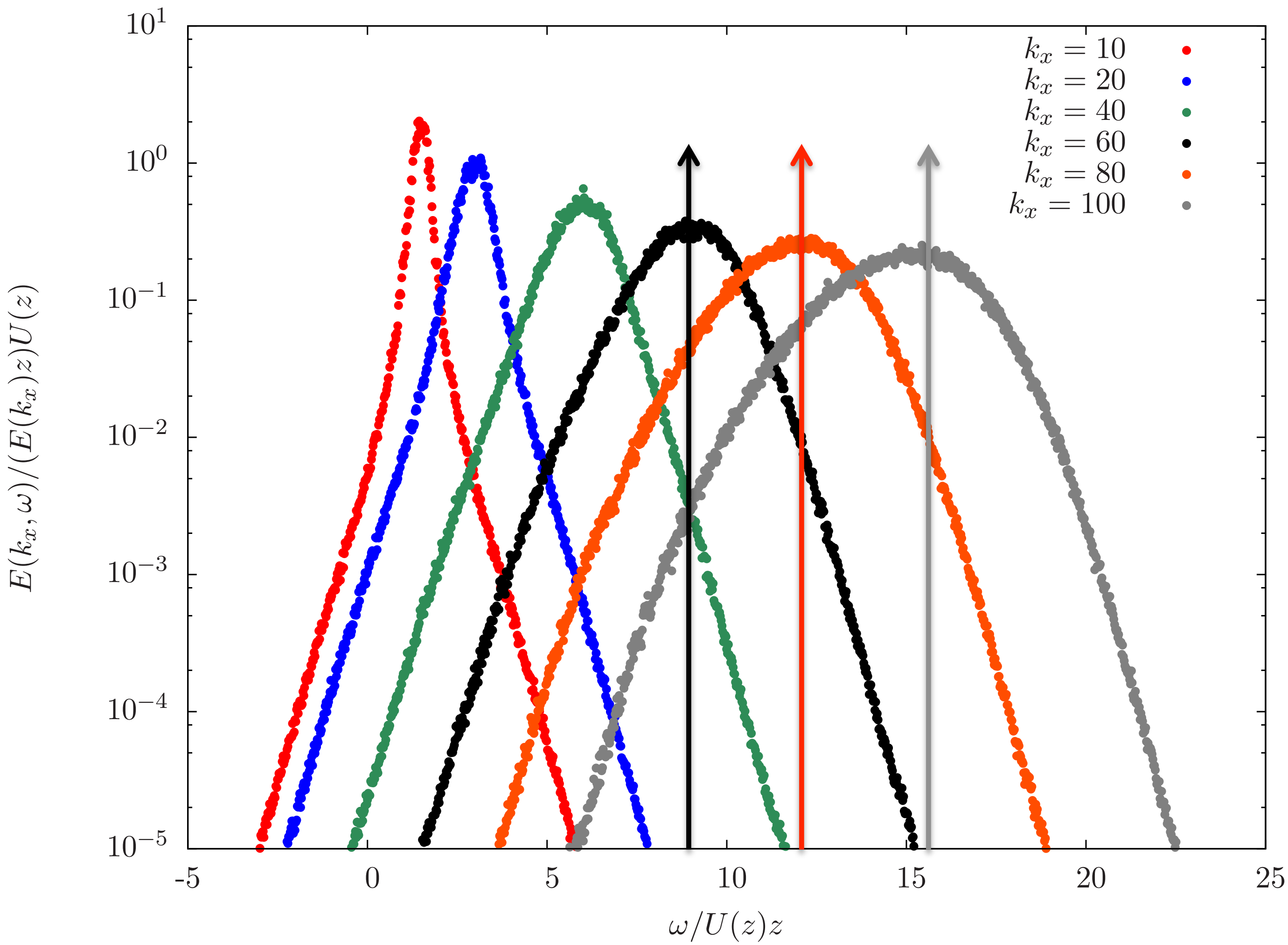
Data from LES (at z/H = 0.15 – in log-layer):
streamwise wavenumber-frequency spectrum



For these spectra, Taylor's hypothesis would predict delta functions at

$$\frac{\omega}{U/z} = k_x z$$

Model based on Kraichnan's random sweeping hypothesis

Model by Wilczek & Narita (2012, PRE **86**, 066308), applied here to 2D (in wall-parallel xy plane):

Consider *u'* small-scale fluctuation of passive turbulence being advected by a mean velocity [U(z)] and a random **large-scale, slow** Gaussian turbulence field **v**, independent of *u'*.

$$\frac{\partial}{\partial t} u'(x,y,z,t) + (\mathbf{U}+\mathbf{v}) \cdot \nabla u'(x,y,z,t) = 0$$

$$\frac{\partial}{\partial t} \hat{u}(\mathbf{k},z,t) + \mathrm{i}(\mathbf{U}+\mathbf{v}) \cdot \mathbf{k}\, \hat{u}(\mathbf{k},z,t) = 0 \qquad \mathbf{k} = (k_x, k_y)^T$$

Analytical solution:

$$\hat{u}(\mathbf{k},z,t) = \hat{u}(\mathbf{k},z,0) \exp\left[-\mathrm{i}(\mathbf{U}+\mathbf{v}) \cdot \mathbf{k}\, t\right]$$

Note: only advection. This does not yet account for shear (RDT), as done by J. Mann 1994 for spatial spectra

Relevant prior works:

Kraichnan (1964, Phys. Fluids **7**)
Lumley (Phys Fluids **8**, 1965)
Tennekes (1975, JFM **67**)
Clifford & Wyngaard (1977)
Wyngaard (2010)
George et al (1989)
Chen & Kraichnan (1989),
Del Alamo & Jimenez (2009)
G. He & Zhang (2009, elliptic model) …

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Model based on Kraichnan's random sweeping hypothesis

Model by Wilczek & Narita (2012, PRE **86**, 066308), applied here to 2D (in wall-parallel xy plane):

$$\hat{u}(\mathbf{k},z,t) = \hat{u}(\mathbf{k},z,0)\exp\left[-\mathrm{i}(\mathbf{U}+\mathbf{v})\cdot\mathbf{k}\,t\right] \qquad \mathbf{k} = (k_x,k_y)^T$$

Multiply by *u*-hat at different k and time and average, *u* uncorrelated from **v**:

$$\left\langle \hat{u}(\mathbf{k},z,t)\hat{u}(\mathbf{k}',z,t+\tau)\right\rangle = \left\langle \hat{u}(\mathbf{k},z,0)\hat{u}(\mathbf{k}',z,0)\right\rangle \left\langle \exp\left[-\mathrm{i}(\mathbf{U}+\mathbf{v})\cdot\mathbf{k}'\tau\right]\right\rangle$$

Key step:

Since **v** is Gaussian, average of exponential can be evaluated:

$$\left\langle \exp\left[-\mathrm{i}(\mathbf{U}+\mathbf{v})\cdot\mathbf{k}\,\tau\right]\right\rangle = \exp\left[-\mathrm{i}\mathbf{U}\cdot\mathbf{k}\,\tau\right]\exp\left[-\frac{1}{2}\left\langle(\mathbf{v}\cdot\mathbf{k})^2\right\rangle\tau^2\right]$$

$$\langle \hat{u}(\mathbf{k},z,t)\hat{u}(\mathbf{k}',z,t+\tau)\rangle = \langle \hat{u}(\mathbf{k},z,0)\hat{u}(\mathbf{k}',z,0)\rangle \exp[-i\mathbf{U}\cdot\mathbf{k}\,\tau]\exp\left[-\frac{1}{2}\langle(\mathbf{v}\cdot\mathbf{k})^2\rangle\tau^2\right]$$

$$\mathbf{k} = (k_x, k_y)^T$$

Additional Fourier transform in time:

Doppler shift

$$E_{11}(\mathbf{k},\omega;z) = E_{11}(\mathbf{k};z)\left[2\pi\langle(\mathbf{v}\cdot\mathbf{k})^2\rangle\right]^{-1/2}\exp\left[-\frac{(\omega-\mathbf{k}\cdot\mathbf{U})^2}{2\langle(\mathbf{v}\cdot\mathbf{k})^2\rangle}\right].$$

Spatial k_x, k_y spectrum
as function of z

Doppler broadening
if v→0, → delta - function
(Taylor's hypothesis)

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Engineering parameterization of spatial k_x, k_y spectra:

Standard streamwise 1-D spectrum for streamwise velocity fluctuations ..

$$E_{11}(k_x;z)=\begin{cases} \dfrac{C_1}{\kappa^{2/3}}u_*^2 H & k_x<1/H \\ \dfrac{C_1}{\kappa^{2/3}}u_*^2 k_x^{-1} & 1/H<k_x<1/z \\ C_1\left(\dfrac{u_*^3}{\kappa z}\right)^{2/3} k_x^{-5/3} & k_x>1/z. \end{cases}$$

In 2D: high wavenumber isotropic turbulence ..

$$\Phi_{ij}\left(\tilde{\mathbf{k}}\right)=\frac{E\left(\tilde{k}\right)}{4\pi\tilde{k}^2}\left(\delta_{ij}-\frac{\tilde{k}_i\tilde{k}_j}{\tilde{k}^2}\right), \qquad \tilde{\mathbf{k}}=(k_x,k_y,k_z)^T$$

$$E_{11}^{>}(\mathbf{k})=2\int \mathrm{d}k_z\,\Phi_{11}\left(\tilde{\mathbf{k}}\right)=\int \mathrm{d}k_z\,\frac{E\left(\tilde{k}\right)}{2\pi\tilde{k}^2}\left(1-\frac{k_x^2}{\tilde{k}^2}\right) = \frac{\Gamma\left(\frac{1}{3}\right)}{5\sqrt{\pi}\,\Gamma\left(\frac{5}{6}\right)}C_K\varepsilon^{2/3}\left[1-\frac{8}{11}\frac{k_x^2}{k^2}\right]k^{-8/3}$$

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Engineering parameterization of spatial k_x, k_y spectra:

k^{-1} in k_x direction ..

$$E_{11}^{<}(\mathbf{k}) = A\left[\left(\frac{1}{H}\right)^4 + k_x^4\right]^{-1/4}$$

Blending with cross-over at k=1/z:

$$E_{11}(\mathbf{k};z) = \left[1-\theta_\alpha\left(k-\frac{1}{z}\right)\right] A\left[\left(\frac{1}{H}\right)^4 + k_x^4\right]^{-1/4} + \theta_\alpha\left(k-\frac{1}{z}\right)\frac{\Gamma\left(\frac{1}{3}\right)}{5\sqrt{\pi}\,\Gamma\left(\frac{5}{6}\right)} C_K \varepsilon^{2/3}\left[1-\frac{8}{11}\frac{k_x^2}{k^2}\right]k^{-8/3}$$

where $\theta_\alpha(k-k_0) = \frac{1}{2}\left(\tanh[\alpha(k-k_0)]+1\right)$

$$\varepsilon = \frac{u_*^3}{\kappa z}$$

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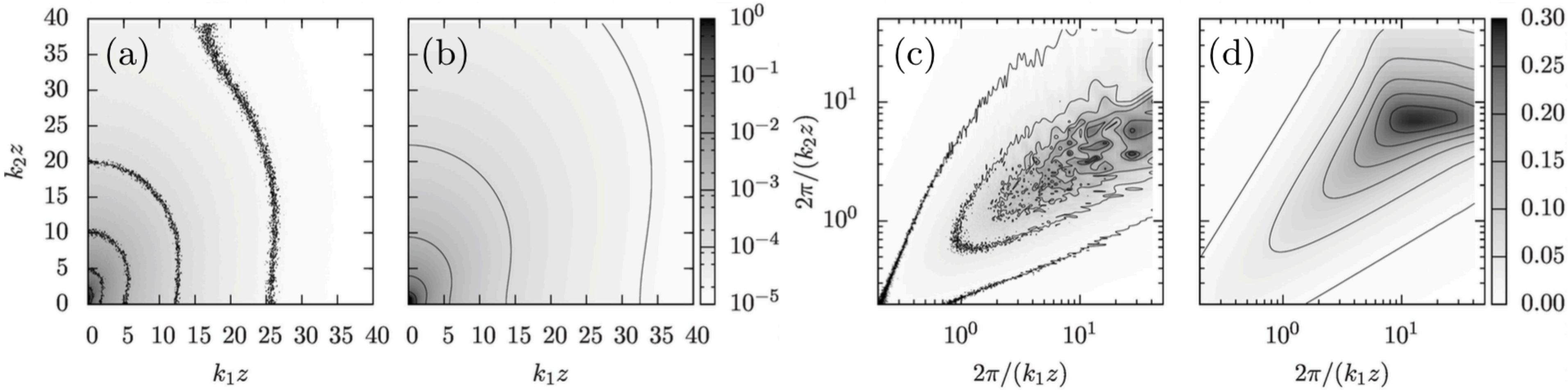
Tests compared to LES (at z/H=0.15):

$$E_{11}(\mathbf{k};z) = \left[1-\theta_\alpha\right] A \left[\left(\frac{1}{H}\right)^4 + k_x^4\right]^{-1/4} + \theta_\alpha \frac{\Gamma\left(\frac{1}{3}\right)}{5\sqrt{\pi}\,\Gamma\left(\frac{5}{6}\right)} C_K \varepsilon^{2/3} \left[1 - \frac{8}{11}\frac{k_x^2}{k^2}\right] k^{-8/3}$$

LES data **Blended model** **LES data** **Blended model**



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Full engineering model for spatio-temporal spectrum:

Tests at z/H=0.15:

$$E_{11}(k_x,k_y,\omega;z)=\left\{[1-\theta_\alpha]A\left[\left(\frac{1}{H}\right)^4+k_x^4\right]^{-1/4}+\theta_\alpha\frac{\Gamma\left(\frac{1}{3}\right)}{5\sqrt{\pi}\,\Gamma\left(\frac{5}{6}\right)}C_K\varepsilon^{2/3}\left[1-\frac{8}{11}\frac{k_x^2}{k^2}\right]k^{-8/3}\right\}\left[2\pi\sigma^2(z)\right]^{-1/2}\exp\left[-\frac{(\omega-\mathbf{k}\cdot\mathbf{U})^2}{2\sigma^2(z)}\right]$$

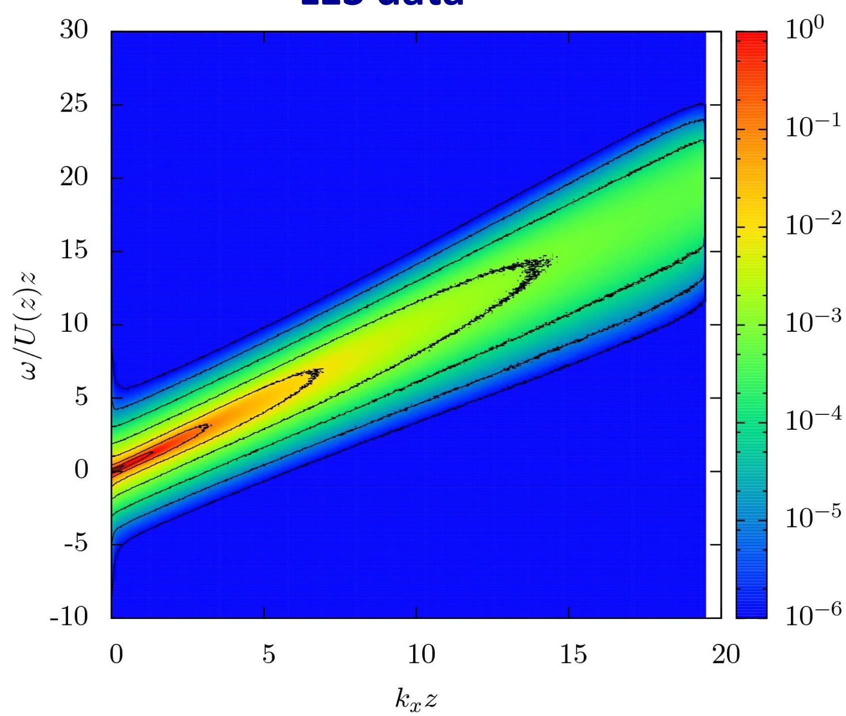
$$U(z)=\frac{u_*}{\kappa}\log\left(\frac{z}{z_0}\right)$$

$$\left\langle v_x^2\right\rangle=u_*^2\left[B-A\log\left(\frac{z}{H}\right)\right]$$

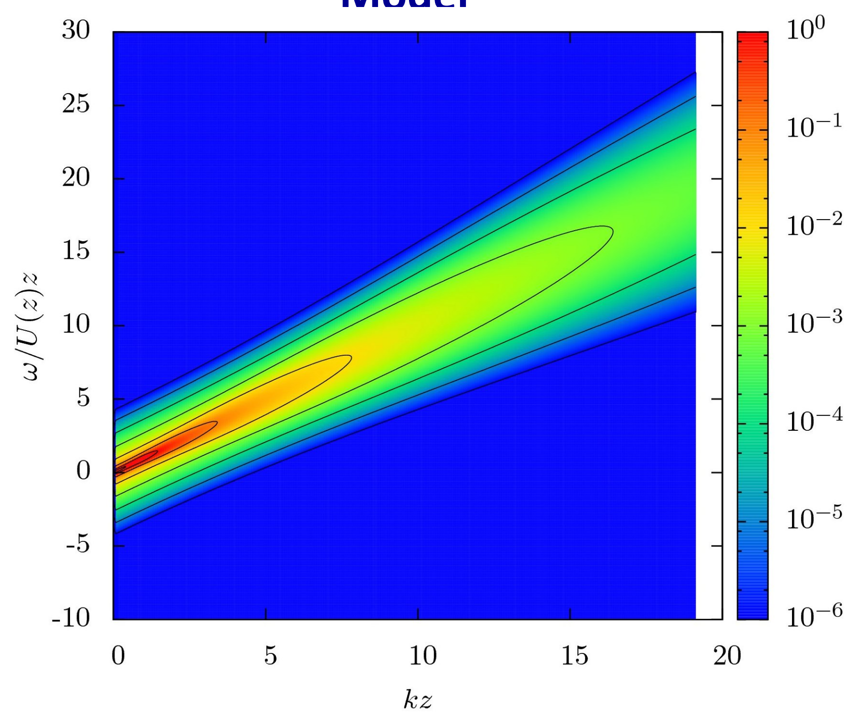
$$\sigma^2(z)=\left\langle(\mathbf{v}\cdot\mathbf{k})^2\right\rangle=\left\langle v_x^2\right\rangle k_x^2+\left\langle v_y^2\right\rangle k_y^2$$

$$\sigma^2(z)=\left\langle v_x^2\right\rangle\left[k_x^2+Ck_y^2\right],\quad \mathrm{A}=0.96,\quad \mathrm{B}=2.41,\quad C=0.33,\quad \kappa=0.4$$

LES data



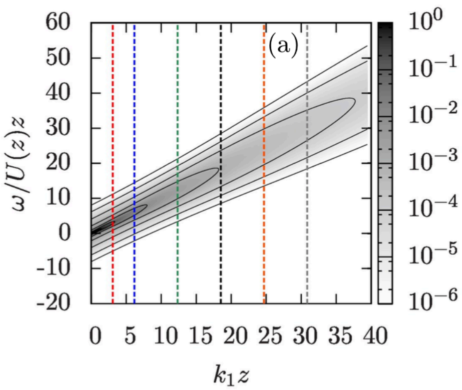
Model

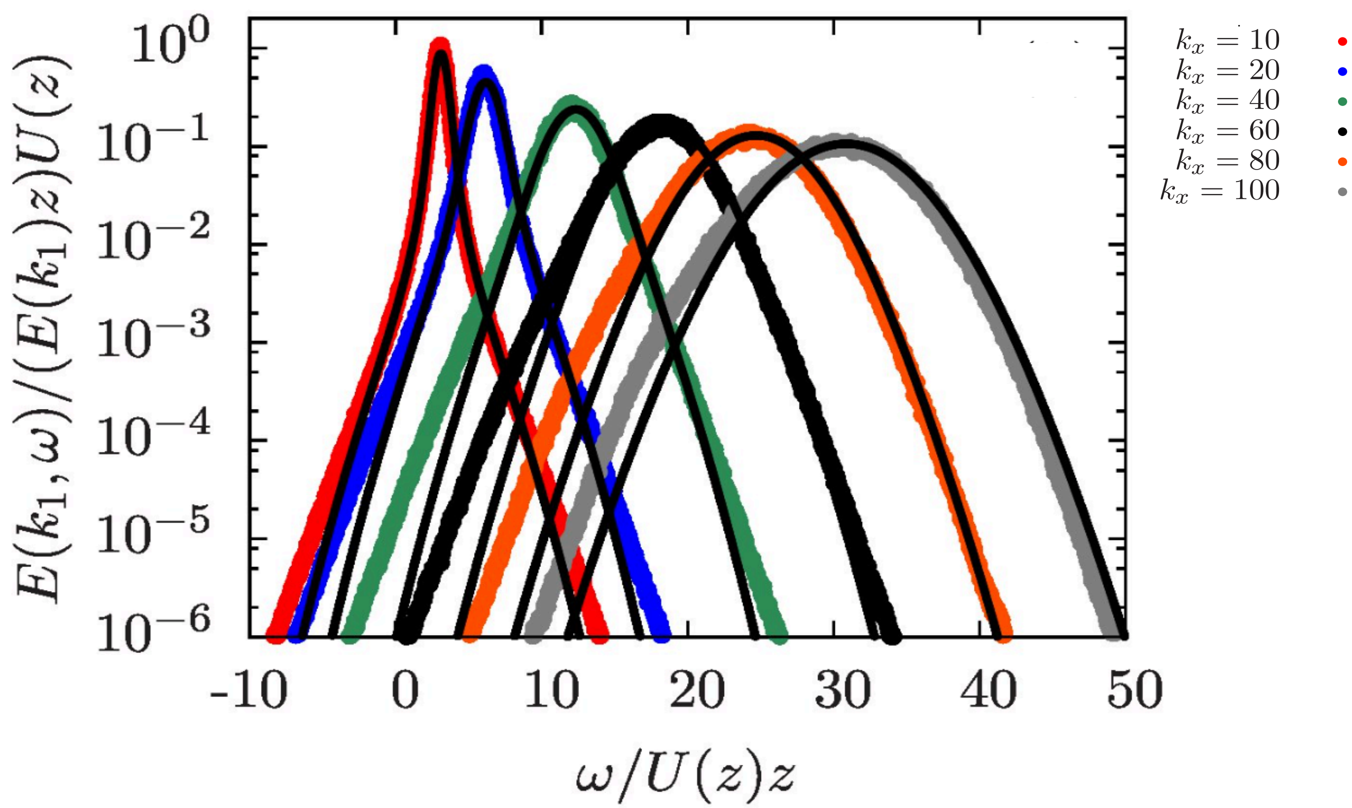


Full engineering model for spatio-temporal spectrum:

Tests at z/H=0.15:

$$E_{11}(k_x,k_y,\omega;z)=\left\{\left[1-\theta_\alpha\right]A\left[\left(\frac{1}{H}\right)^4+k_x^4\right]^{-1/4}+\theta_\alpha\frac{\Gamma\left(\frac{1}{3}\right)}{5\sqrt{\pi}\,\Gamma\left(\frac{5}{6}\right)}C_K\varepsilon^{2/3}\left[1-\frac{8}{11}\frac{k_x^2}{k^2}\right]k^{-8/3}\right\}\left[2\pi\sigma^2(z)\right]^{-1/2}\exp\left[-\frac{(\omega-\mathbf{k}\cdot\mathbf{U})^2}{2\sigma^2(z)}\right]$$



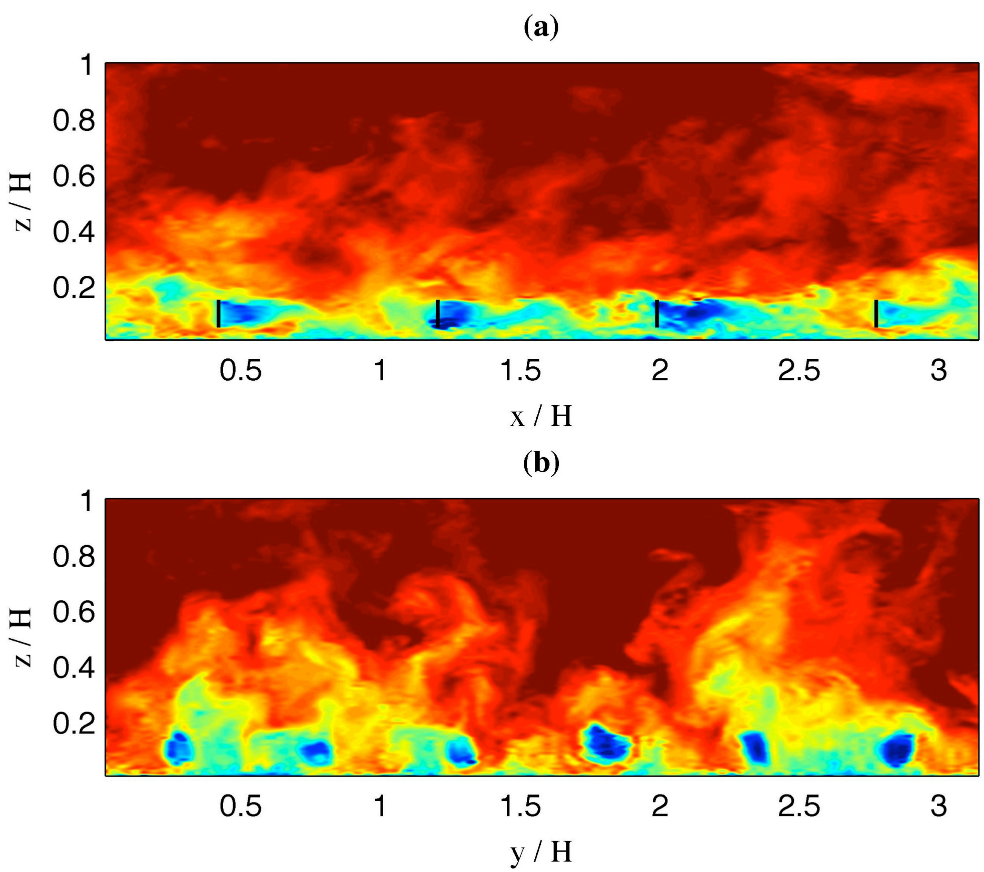


Symbols: LES

Lines: Model

Very good agreement

Part III: Where does the kinetic energy (mean velocity) at wind turbines come from?



Consider total mechanical energy transport of mean flow, in statistically steady turbulent flow

$$E = \frac{1}{2}\overline{u}_i\overline{u}_i + \frac{1}{\rho}\hat{p}$$

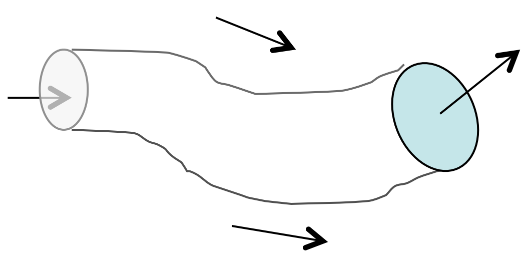
$$\overline{F}_{E,j} = E\ \overline{u}_j + \overline{u_i'u_j'}\ \overline{u}_i - 2\nu\overline{S}_{ij}\ \overline{u}_i$$

$$\frac{\partial}{\partial x_j}(\rho\overline{F}_{E,j}) = -\rho\overline{u_i'u_j'}\frac{\partial\overline{u}_i}{\partial x_j} - 2\mu\overline{S}_{ij}\overline{S}_{ij} + \rho\overline{u}_i\overline{f}_i$$

Mean-flow total energy transport vector field

Tangent lines – bundles – tubes

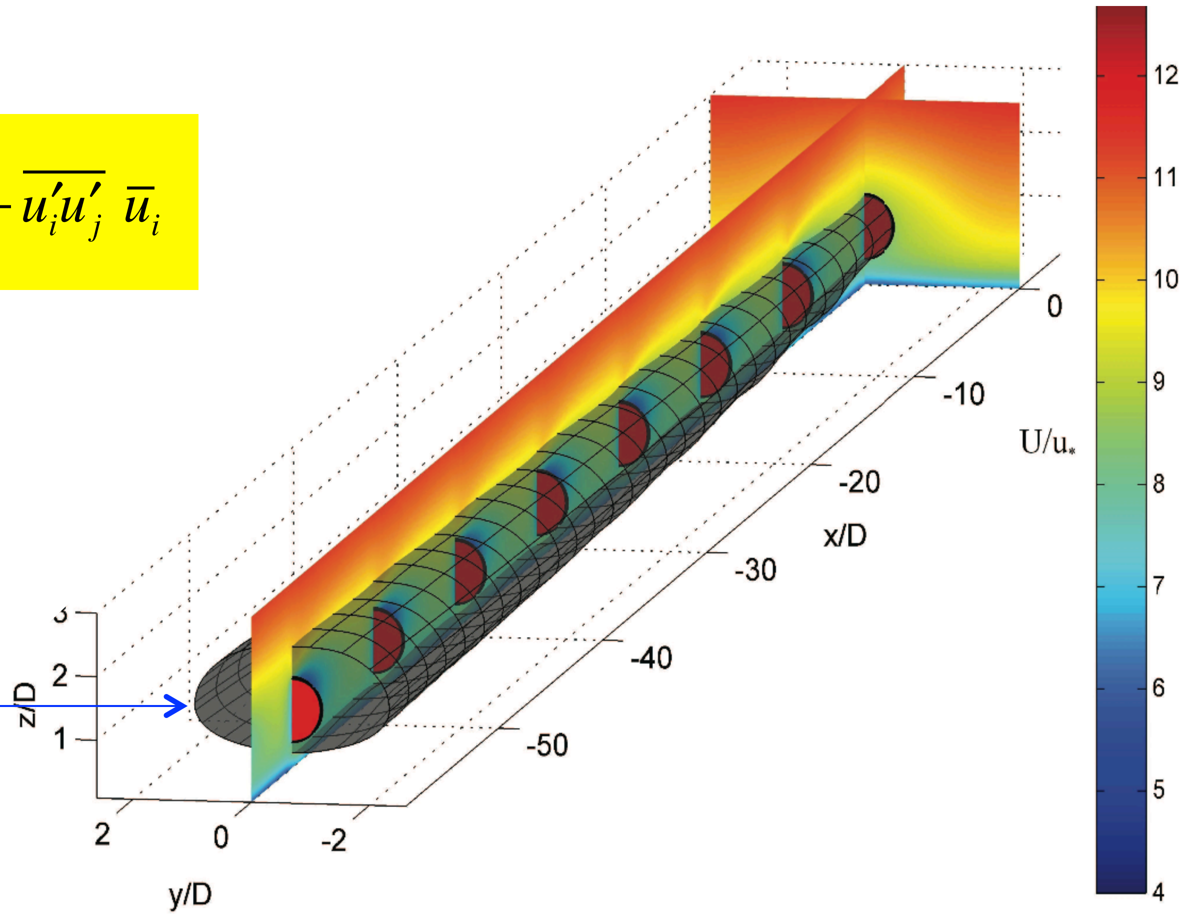
$$\iint_{A_2}\rho\overline{F}_{E,j}n_j\,\mathrm{d}\mathbf{x} + \iint_{A_1}\rho\overline{F}_{E,j}n_j\,\mathrm{d}\mathbf{x} = -\iiint_{\Omega}\left(2\mu\overline{S}_{ij}\overline{S}_{ij} - \rho\overline{u_i'u_j'}\frac{\partial u_i}{\partial x_j}\right)\mathrm{d}\mathbf{x} + \iiint_{\Omega}\rho\overline{u}_i(\overline{f}_i + f_{i,\infty})\mathrm{d}\mathbf{x}$$



Energy transport tubes in wind farms:

$$\overline{F}_{E,j} = \left(\frac{1}{2}\overline{u}_i\overline{u}_i + \frac{1}{\rho}\hat{p} \right) \overline{u}_j + \overline{u_i'u_j'}\ \overline{u}_i$$

Total energy transport tube passing through last wind turbine



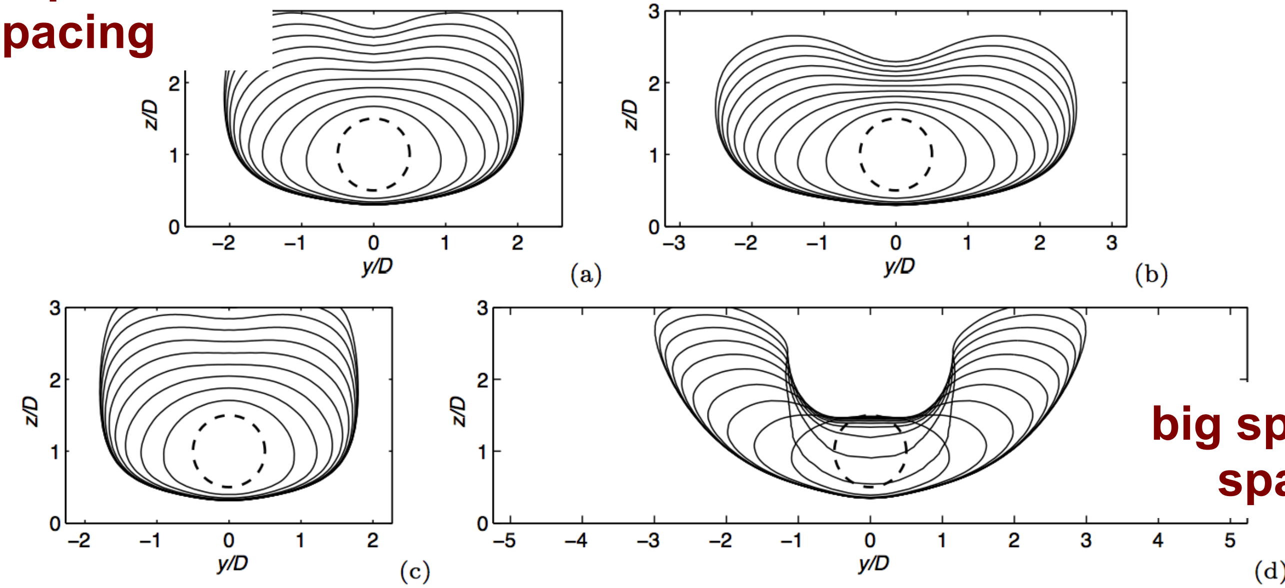
"Flow visualization using momentum and energy transport tubes and applications to turbulent flow in wind farms"
J. Meyers & C.M. 2013 (JFM)

Effects of various wind farm layouts on transport tube geometry

"Poincaré sections"

In-line

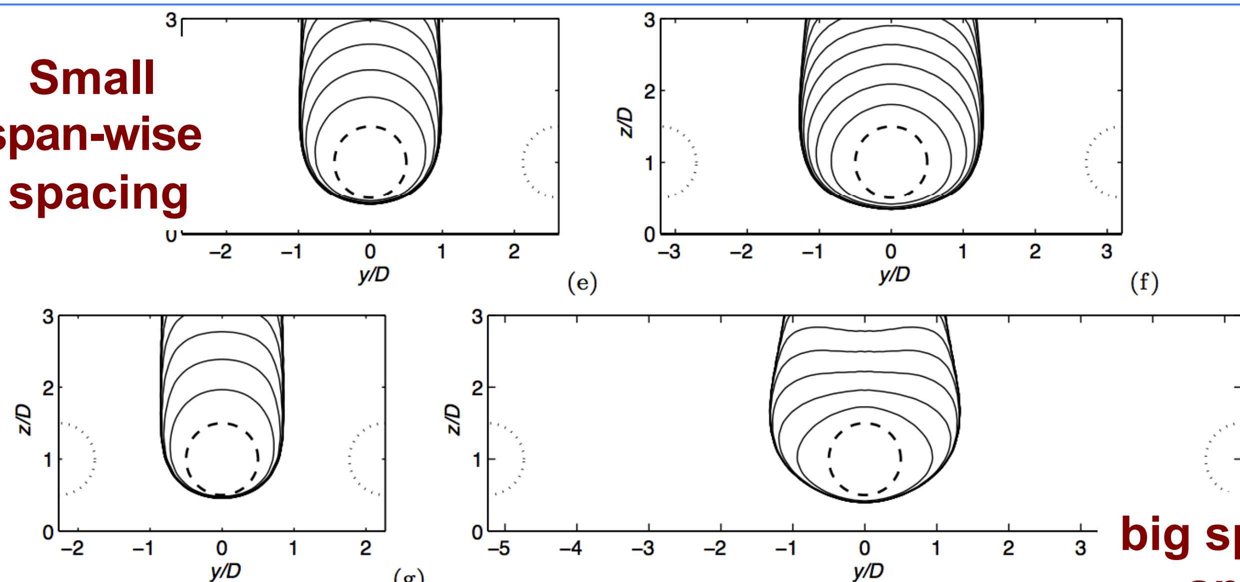
Small span-wise spacing



big span-wise spacing

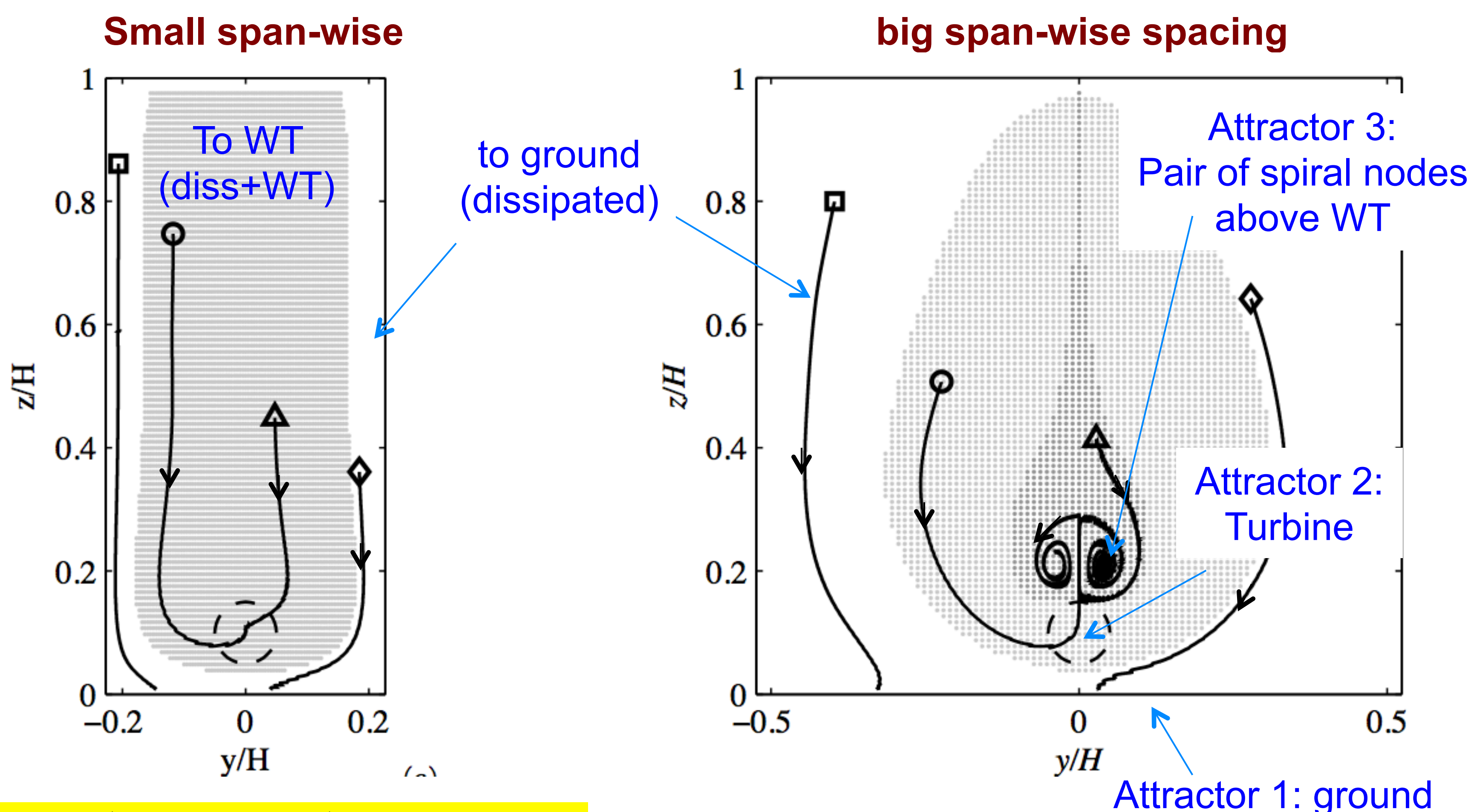
staggered

Small span-wise spacing



big span-wise spacing

Energy transport lines: non-Hamiltonian attractors & basins of attraction



$$\overline{F}_{E,j} = \left(\frac{1}{2} \overline{u}_i \overline{u}_i + \frac{1}{\rho} \hat{p} \right) \overline{u}_j + \overline{u_i' u_j'}\ \overline{u}_i$$

"Flow visualization using momentum and energy transport tubes and applications to turbulent flow in wind farms"
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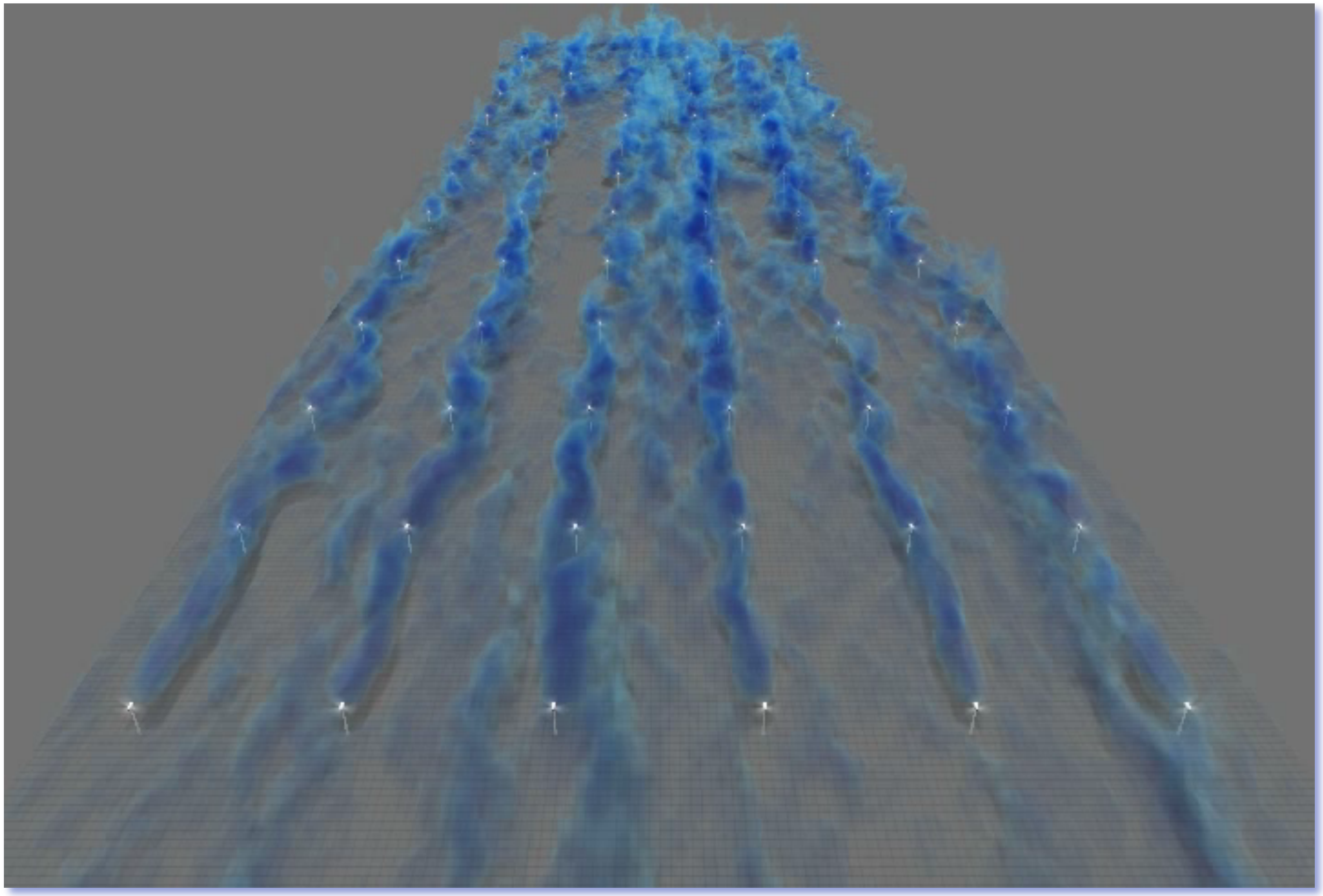
Thank you !   Questions ?



In-silico wind farm (Large-Eddy-Simulation) JHU-LES code
Visualization courtesy of D. Brock (Extended Services XSEDE)

JOHNS HOPKINS
Center for Environmental
& Applied Fluid Mechanics

JHU Mechanical Engineering

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