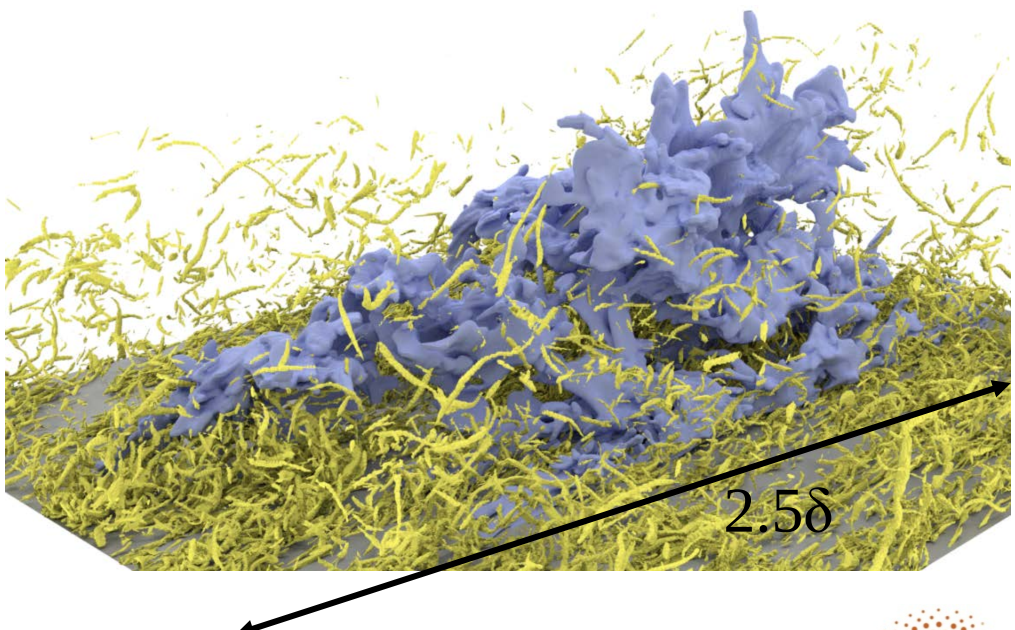
Big whorls have small whorls … ?

Javier Jiménez et al.

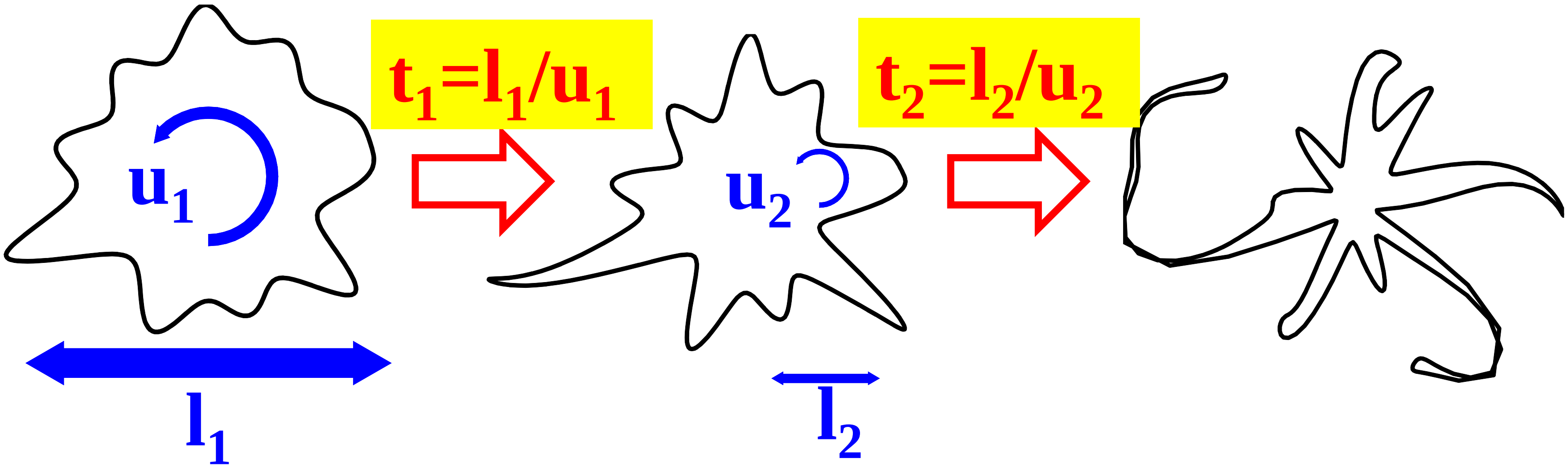
School of Aeronautics, Madrid



TBL: $Re_\tau=1800$, $u'^+=2$

J.A. Sillero

The temporal energy (flux) cascade

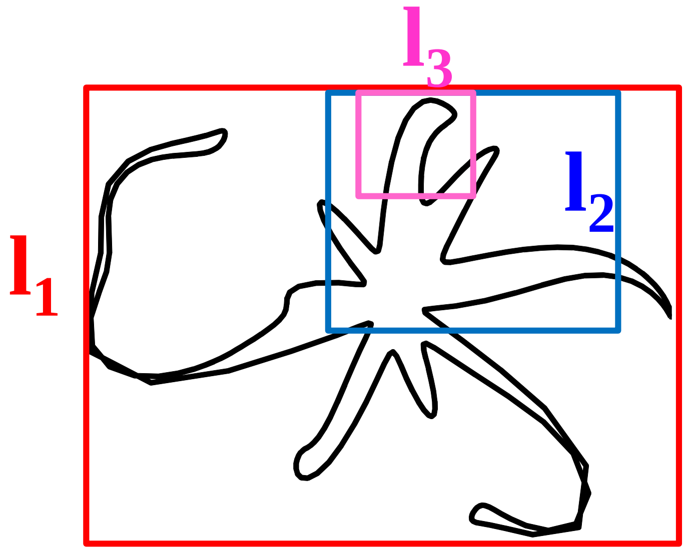


$$u_1^{\,2}/t_1=u_2^{\,2}/t_2=u_j^{\,3}/l_j=\varepsilon$$

$$\boxed{u=(\varepsilon\, l)^{1/3}}$$

Richardson 1922, Kolmogorov 1941, Obukhov 1941

The Kolmogorov energy cascade

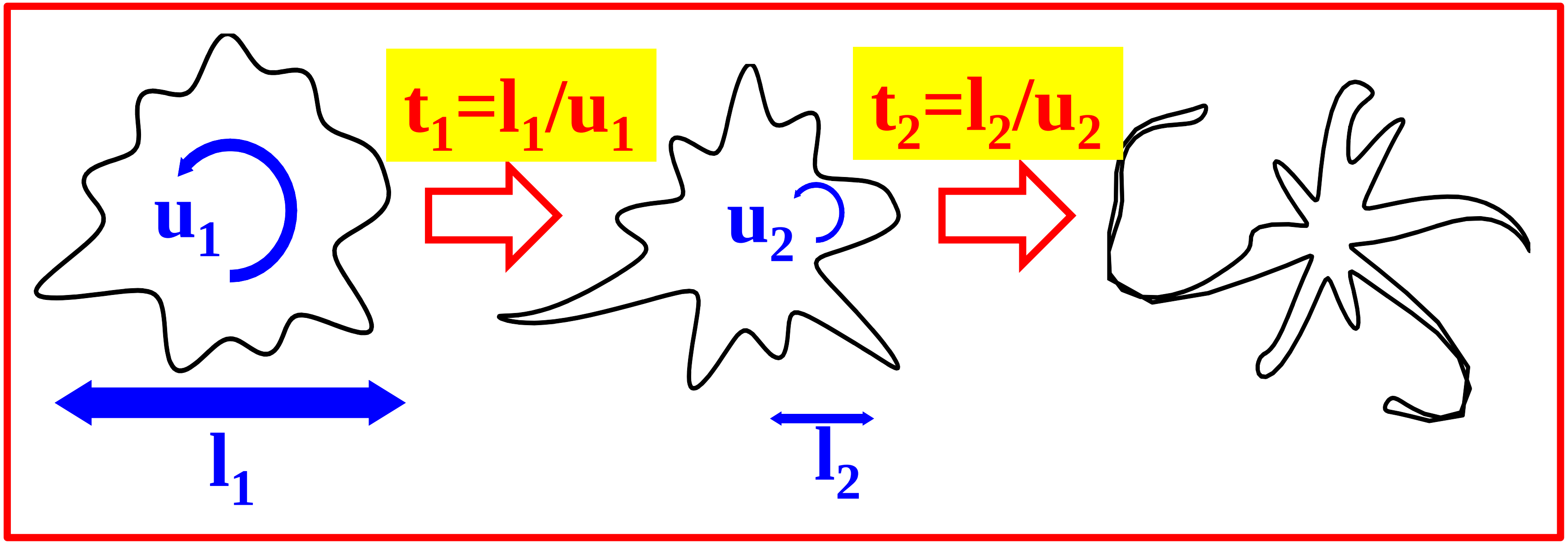


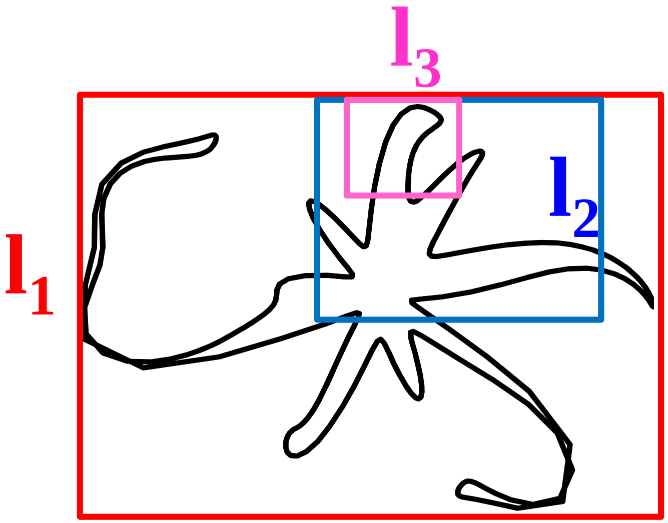
Big whorls have small whorls … (Richardson 1922)

$$\partial_r(\Delta u^3) = -4\varepsilon/5$$ (Kolmogorov 1941)

$$u = (\varepsilon\, l)^{1/3}$$

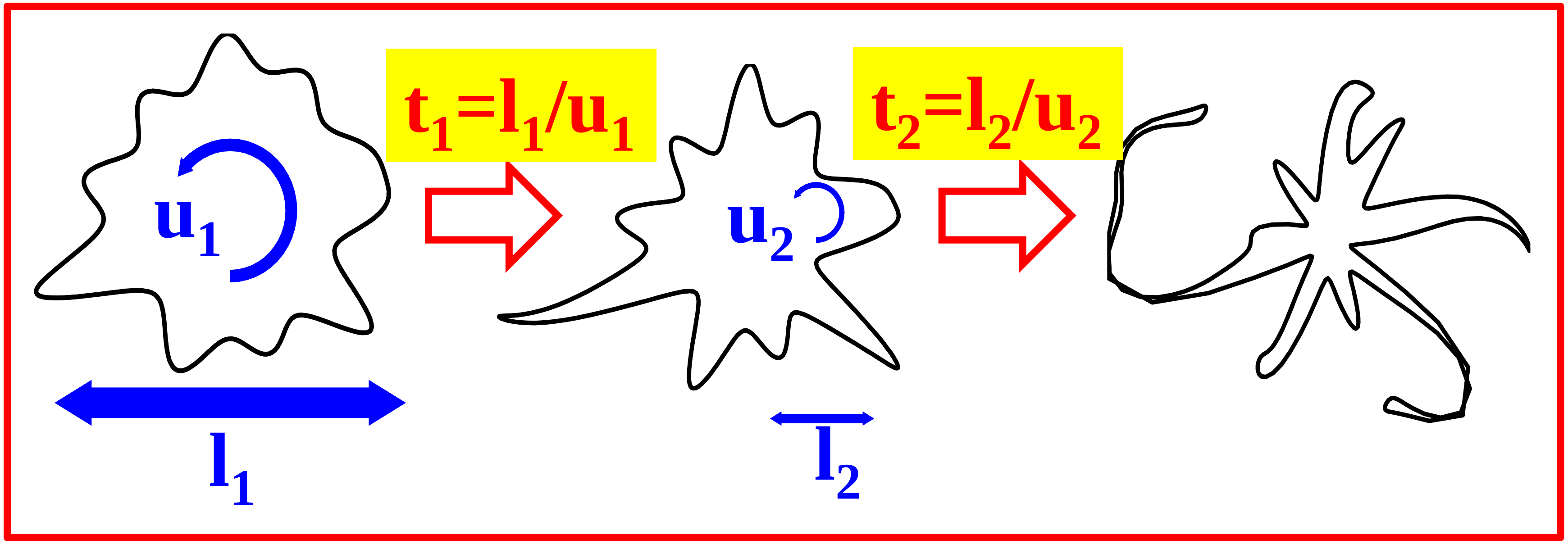
Which is the real cascade?

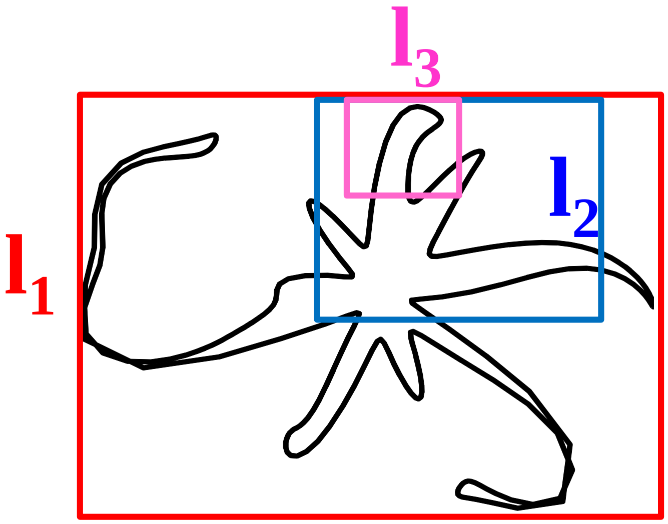




?

Why should we care?



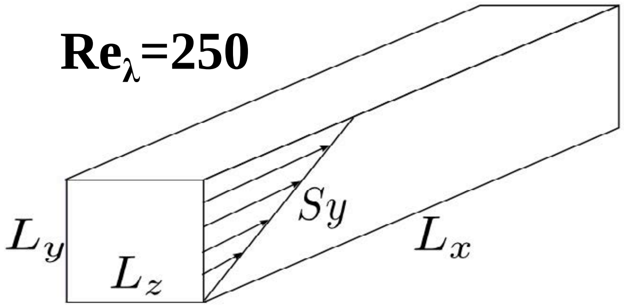


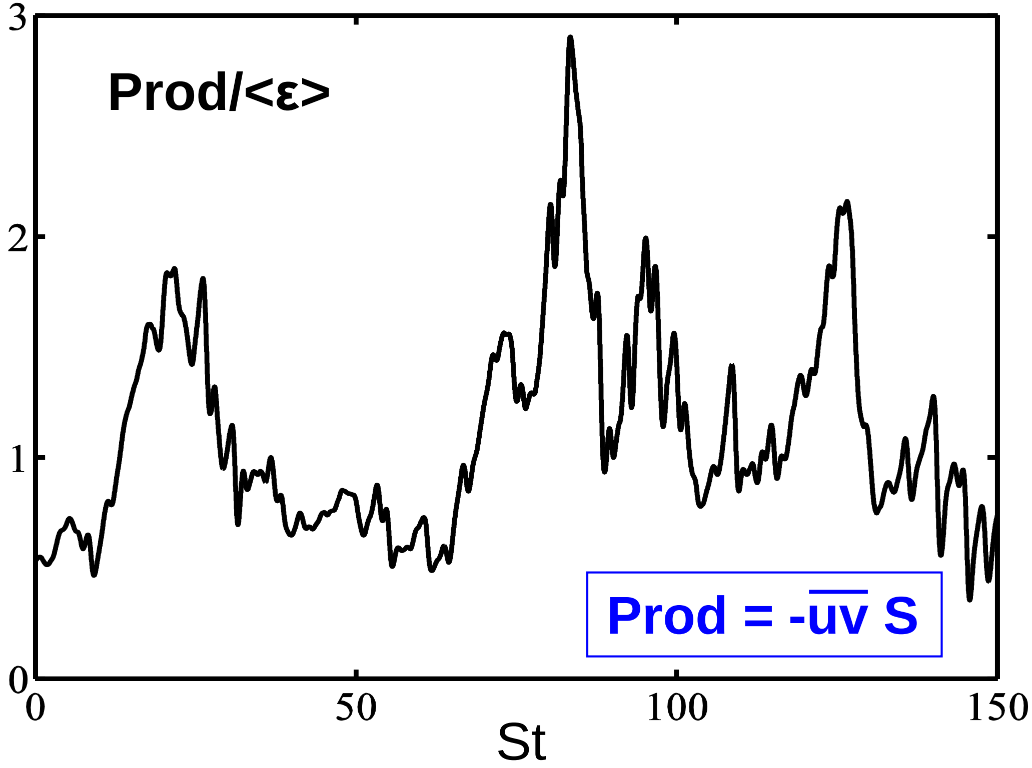
LES, Control, etc.

The energy cascade

Homogeneous Shear Flow

Re_λ=250



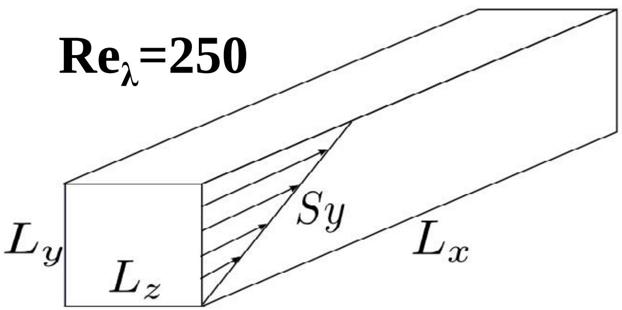


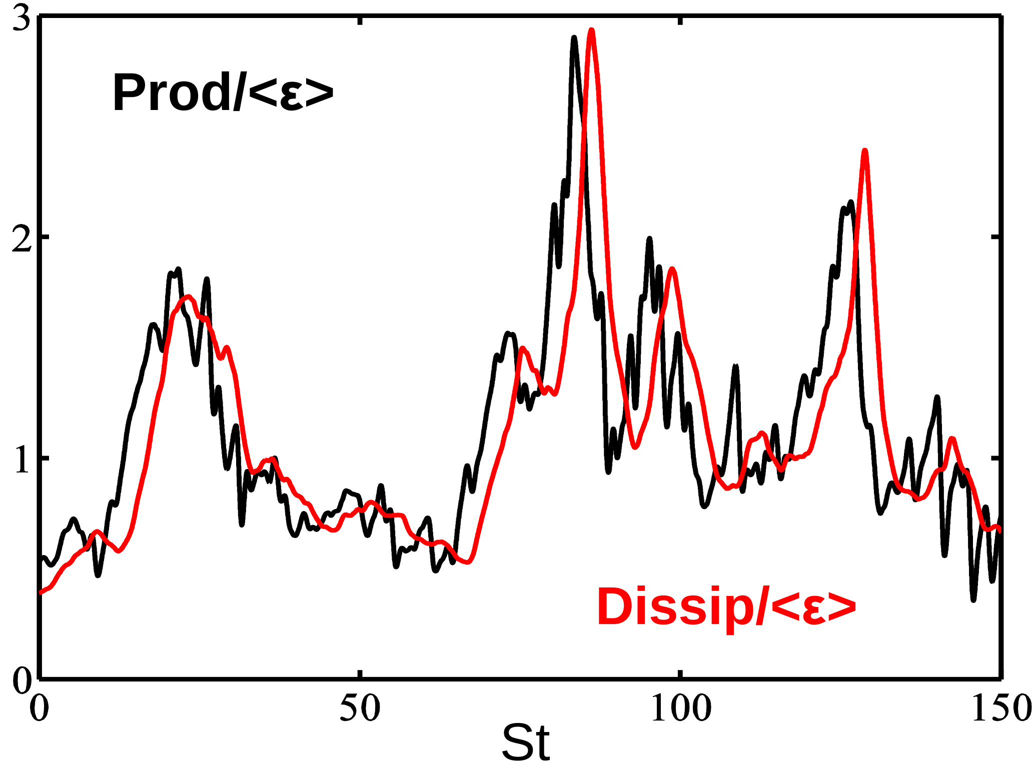
Dong, Sekimoto & J, 2012

The energy cascade takes time
Large to Small

Homogeneous Shear Flow

Re_λ=250





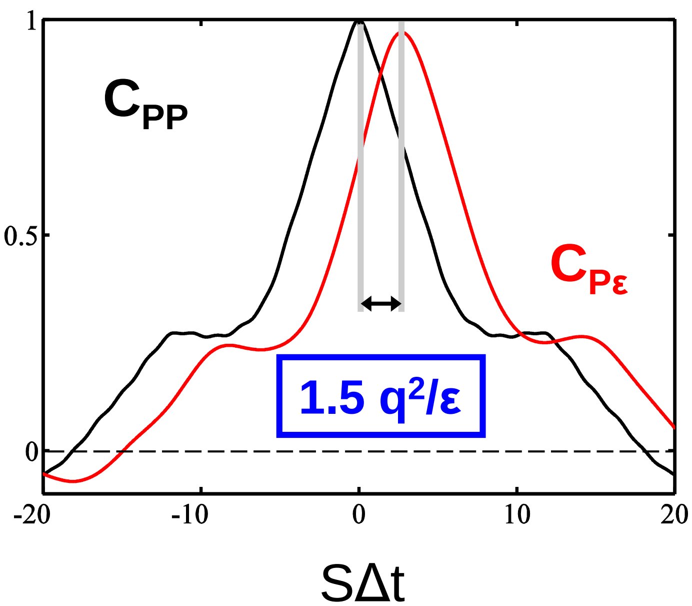
Dong, Sekimoto & J, 2012

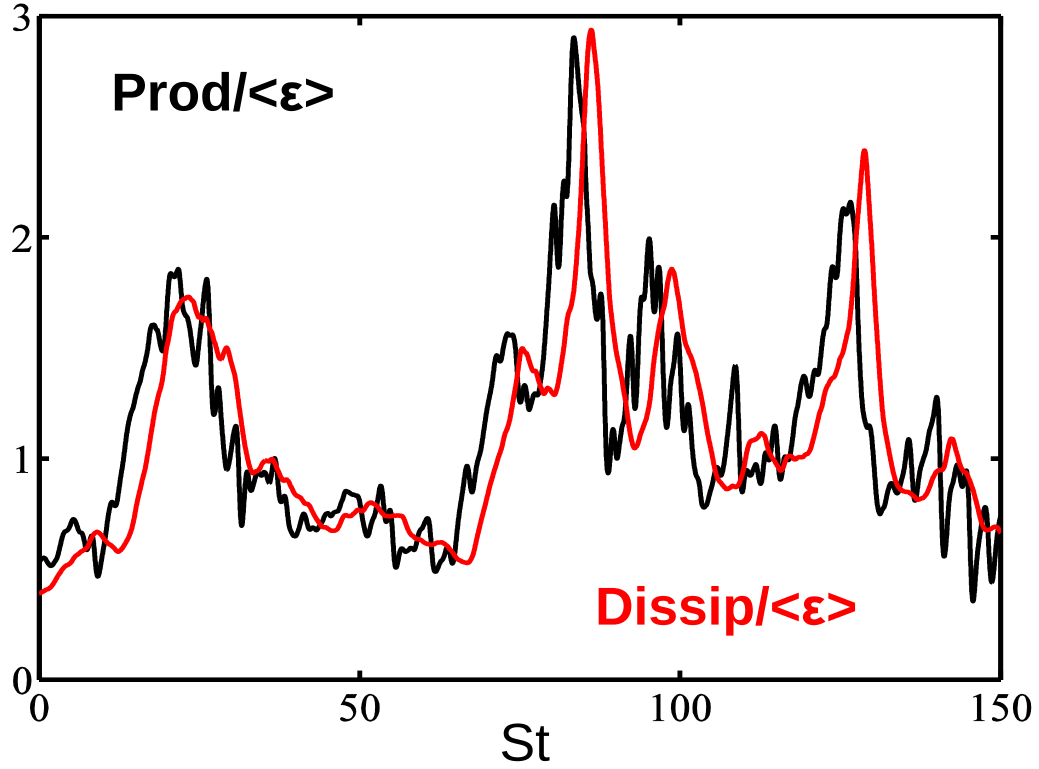
The energy cascade takes time

Large to Small

Homogeneous Shear Flow

Temporal cross-correl.





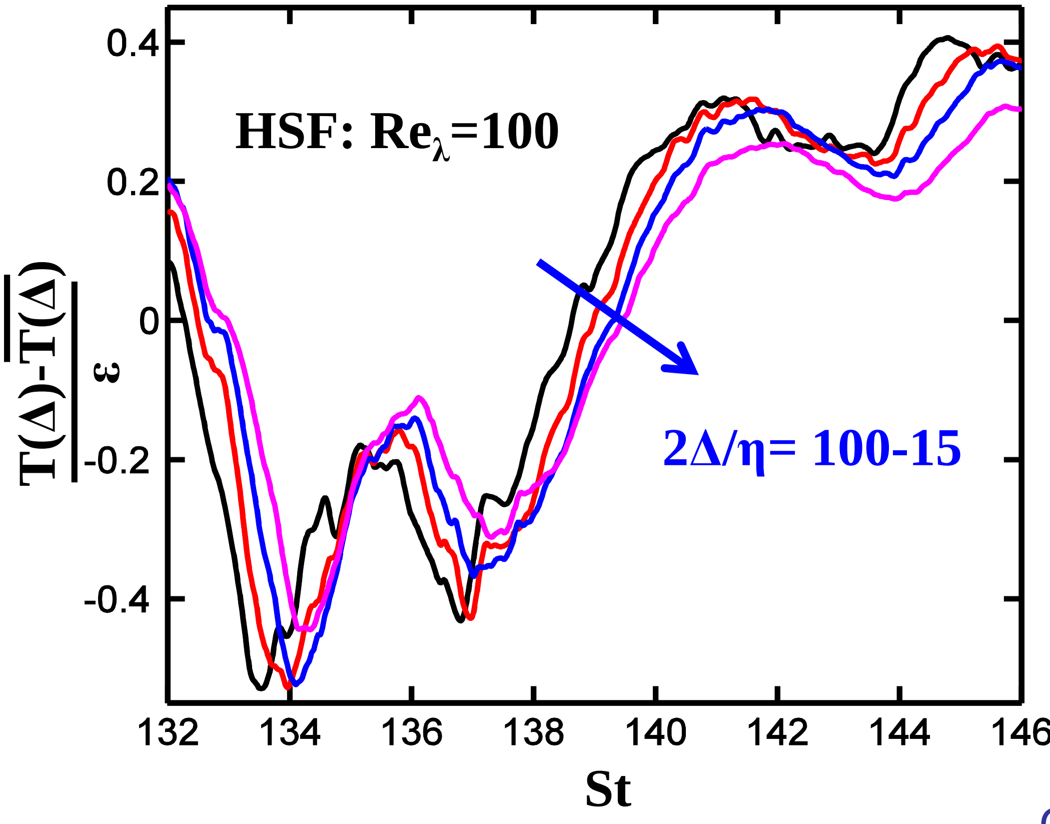
Dong, Sekimoto & J, 2012

The cross-scale energy flux

$u = \bar{u} + u'$; $\bar{u} = G * u$; $G \sim \exp[-(x_1^2+x_2^2+x_3^2)/\Delta^2]$

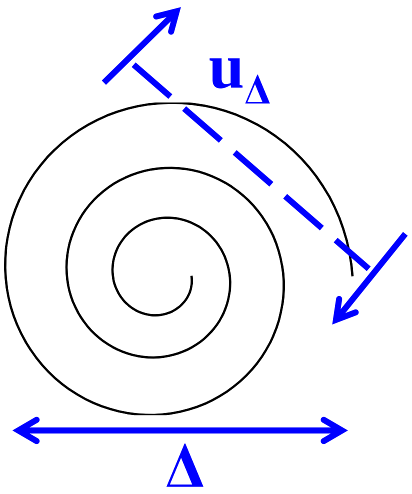
$$(\partial_t + \overline{u}_j \partial_j)\, \frac{1}{2}\overline{u}_i \overline{u}_i = -\partial_j \left(\overline{u}_j \overline{p} + \overline{u}_i \tau_{ij} - 2\nu \overline{u}_i \overline{S}_{ij} \right) - 2\nu \overline{S}_{ij} \overline{S}_{ij} + \overline{S}_{ij} \tau_{ij}$$

T



Cardesa & J, 2014

The velocity of the cascade

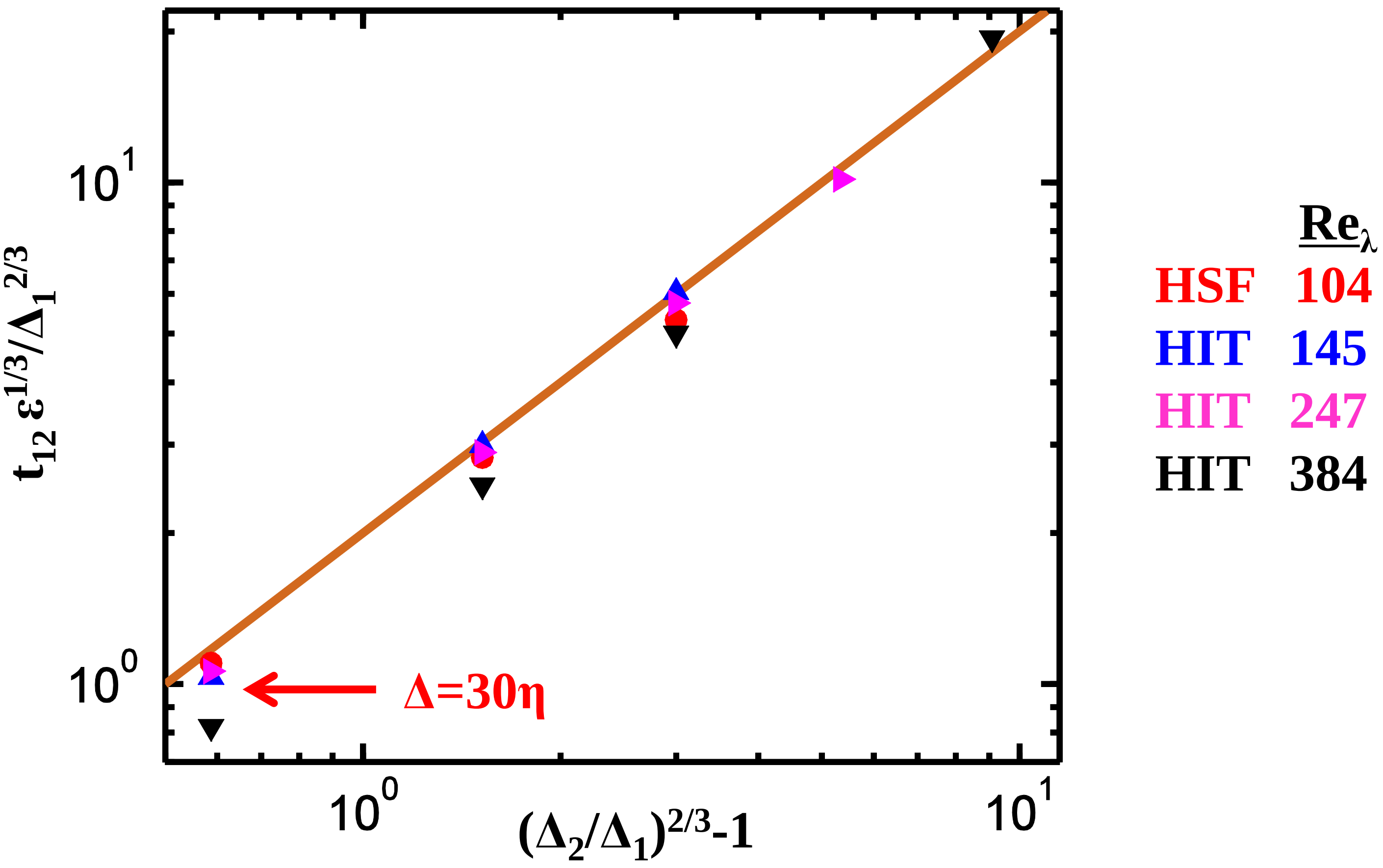


$$t(\Delta) \sim \Delta/u_\Delta \sim \Delta/(\varepsilon\Delta)^{1/3} = \Delta^{2/3}/\varepsilon^{1/3}$$

$$t_{12} = t(\Delta_1) - t(\Delta_2) = \Delta_1^{2/3}/\varepsilon^{1/3} \, [(\Delta_2/\Delta_1)^{2/3} - 1]$$

Kolmogorov, 1941

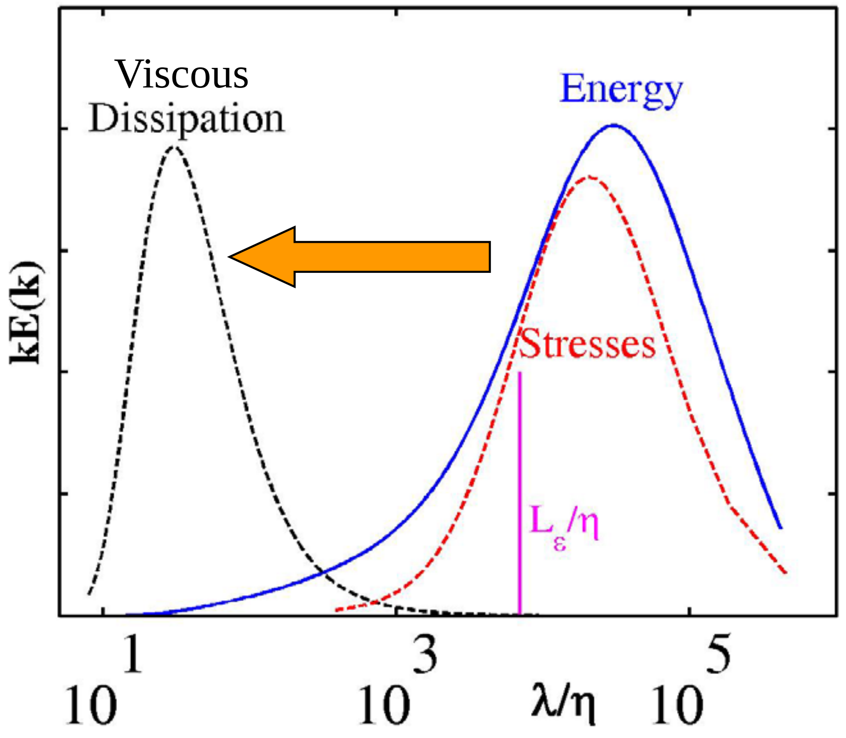
The velocity of the cascade



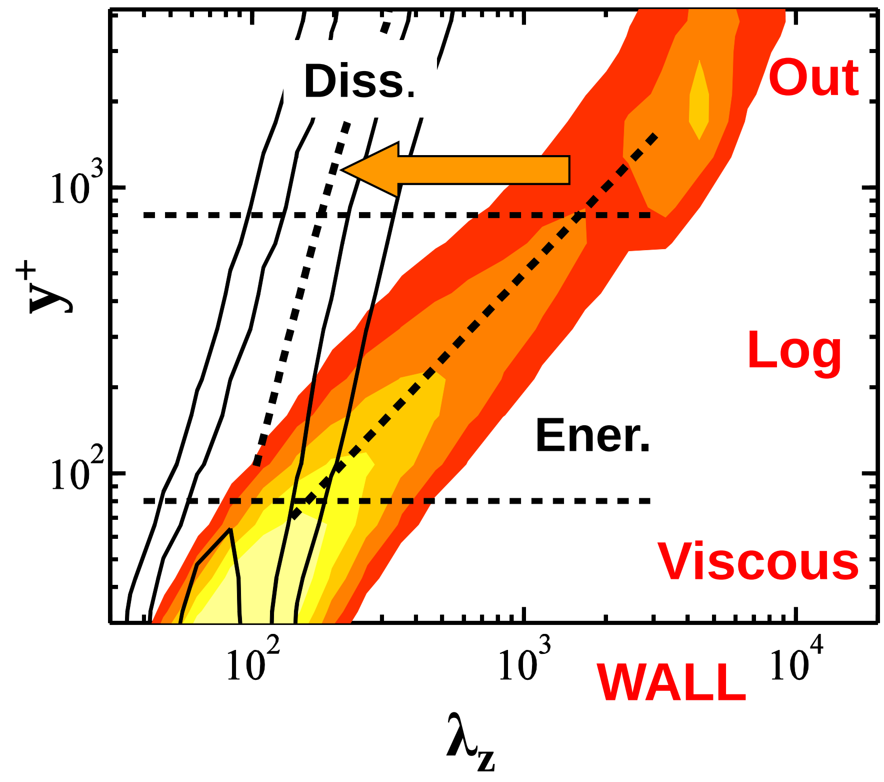
Cardesa & J, 2014

Wall-bounded Flows

Homogeneous Turbulence

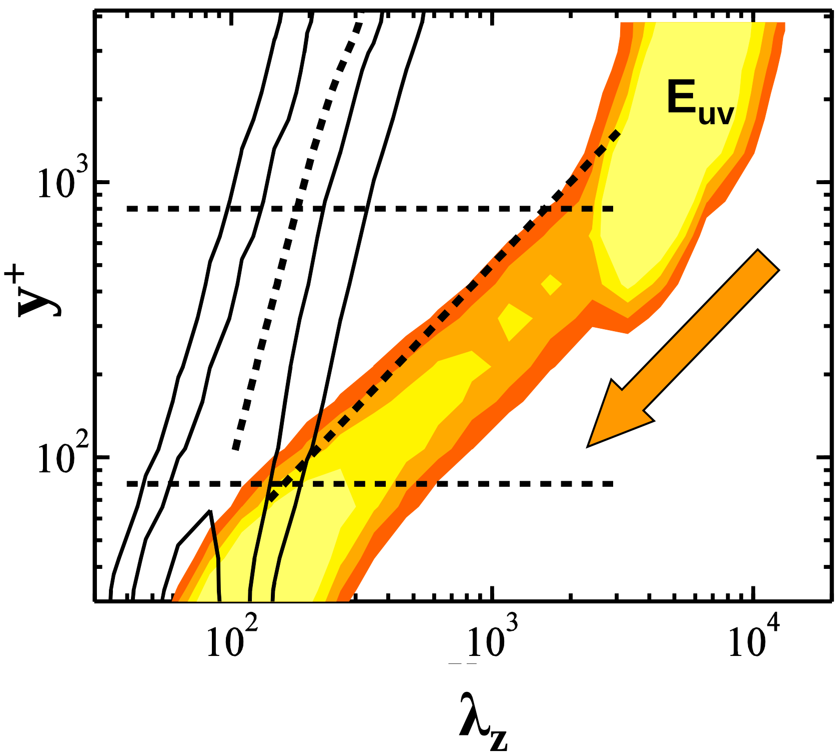


Wall Turbulence (Channel)

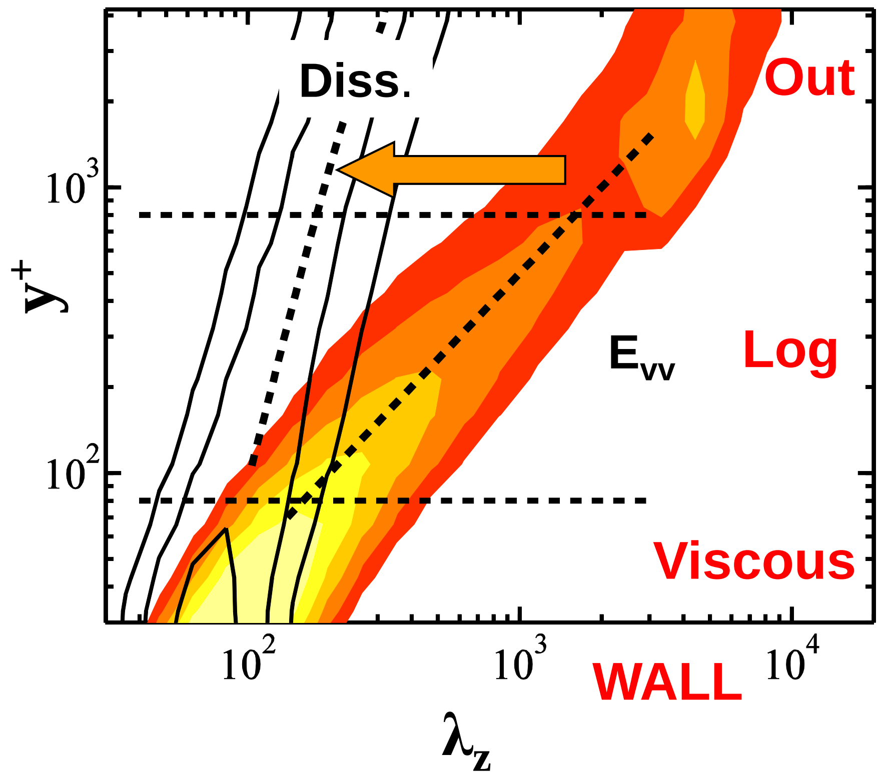


Momentum Flux Cascade

Reynolds stress (Channel)

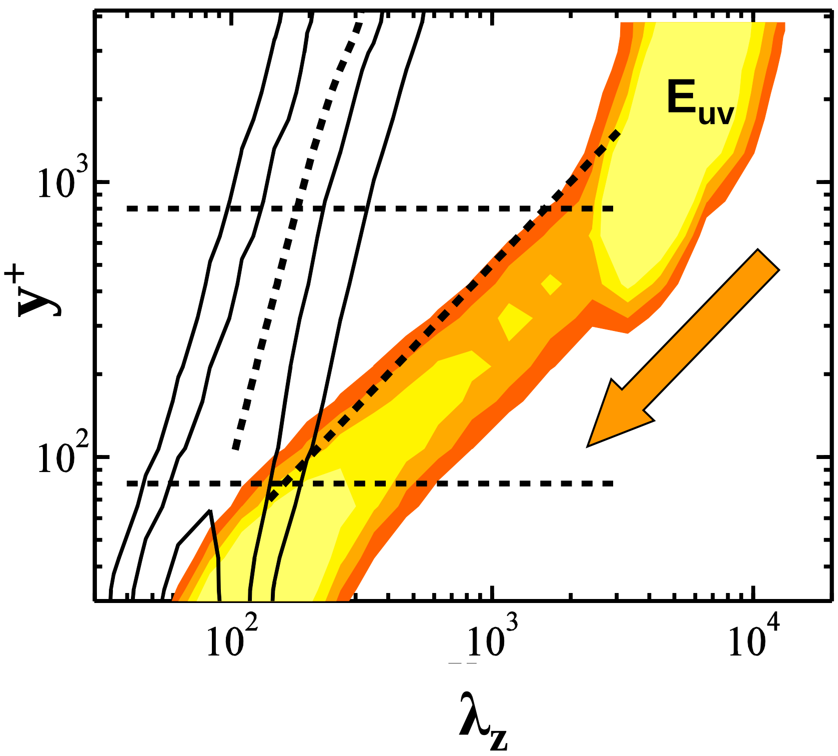


Energy (Channel)

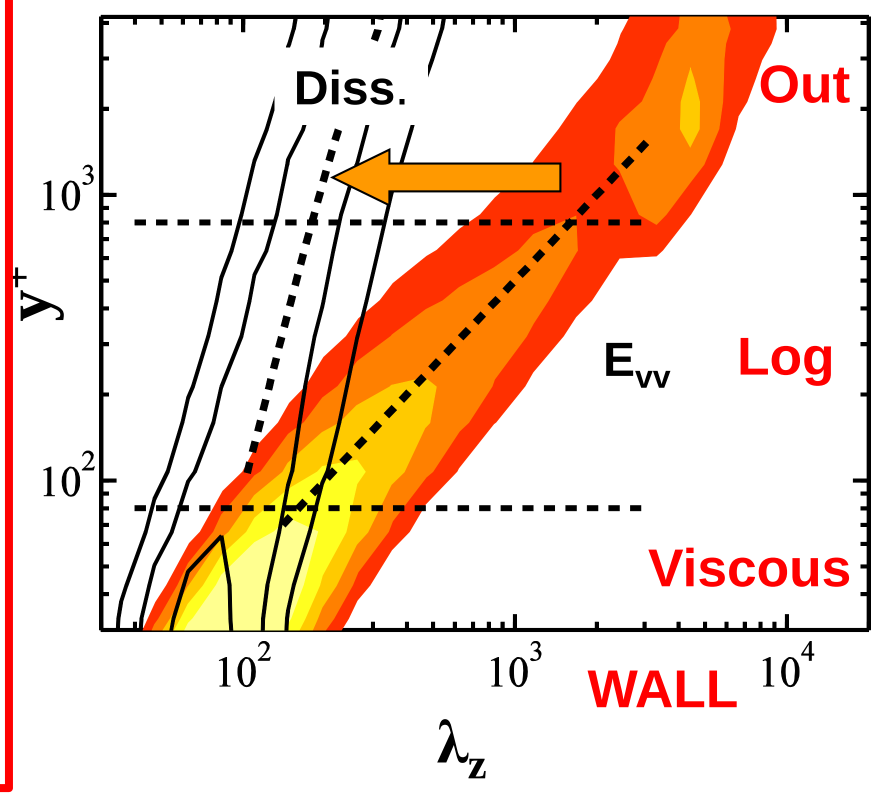


Momentum Flux Cascade

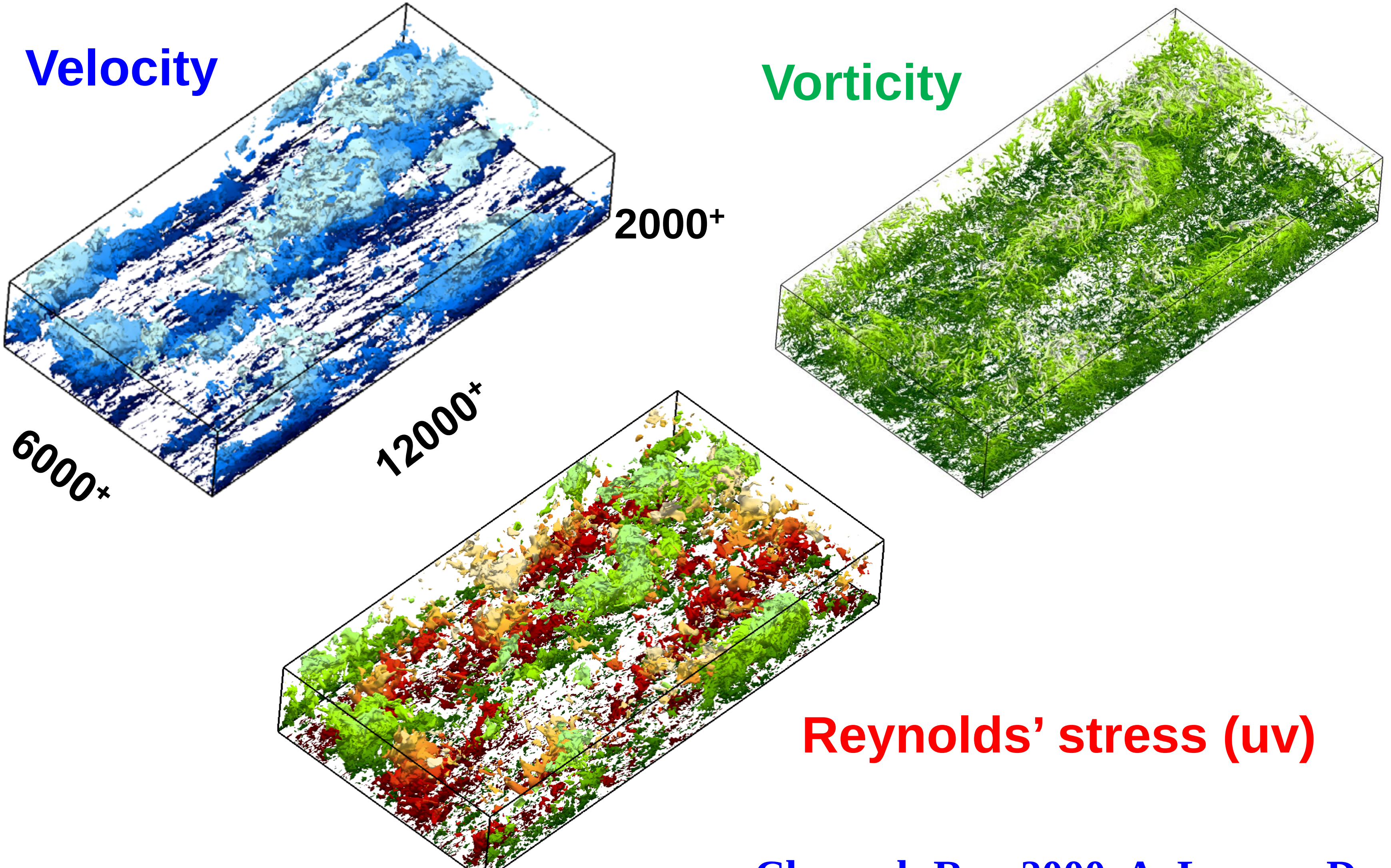
Reynolds stress (Channel)



Energy (Channel)

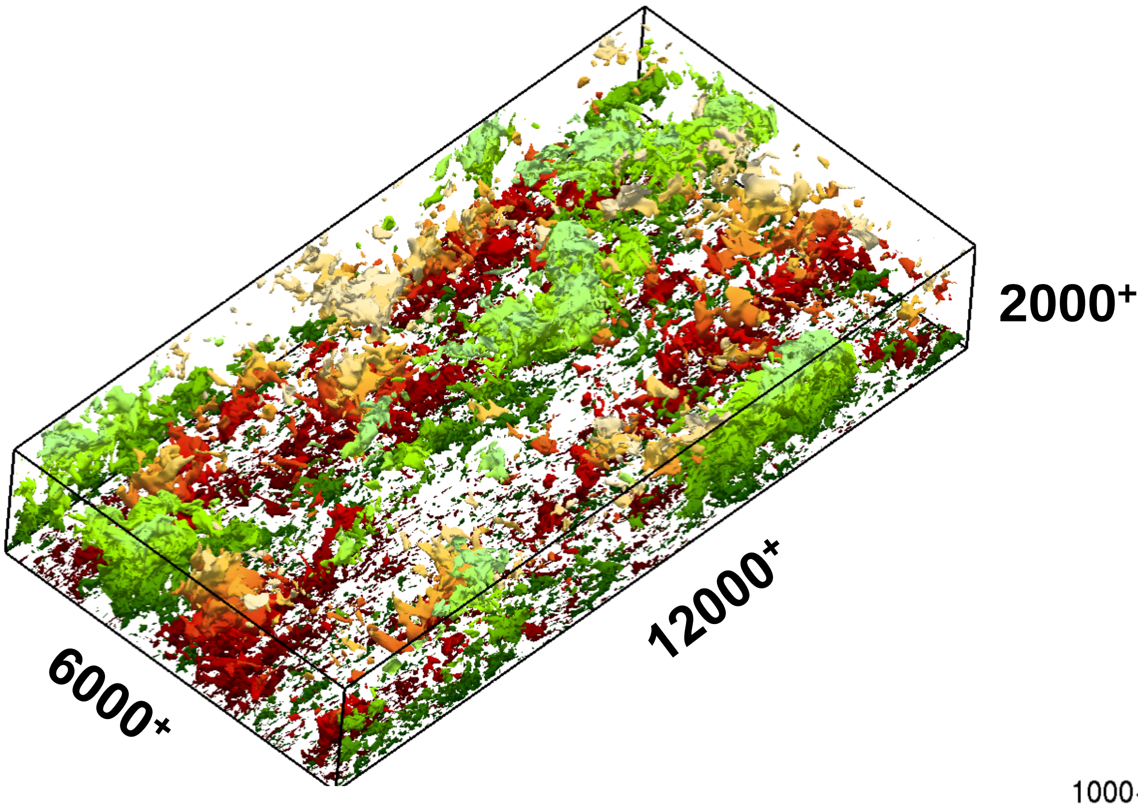


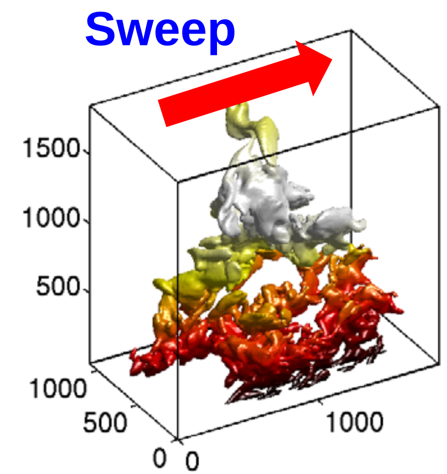
Flow fields of the Logarithmic layer

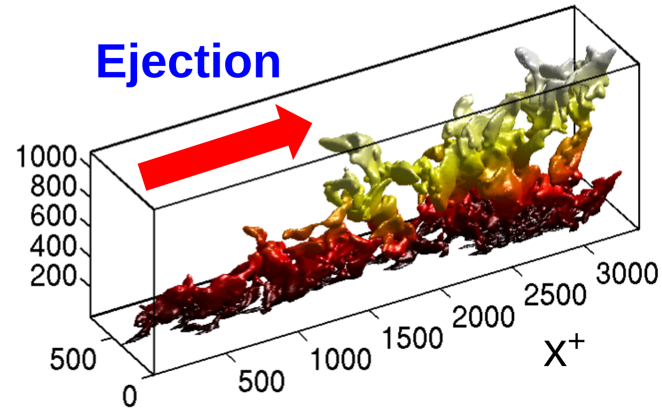


Channel: Re_τ=2000. A. Lozano-Durán

Momentum Structures of the Logarithmic layer



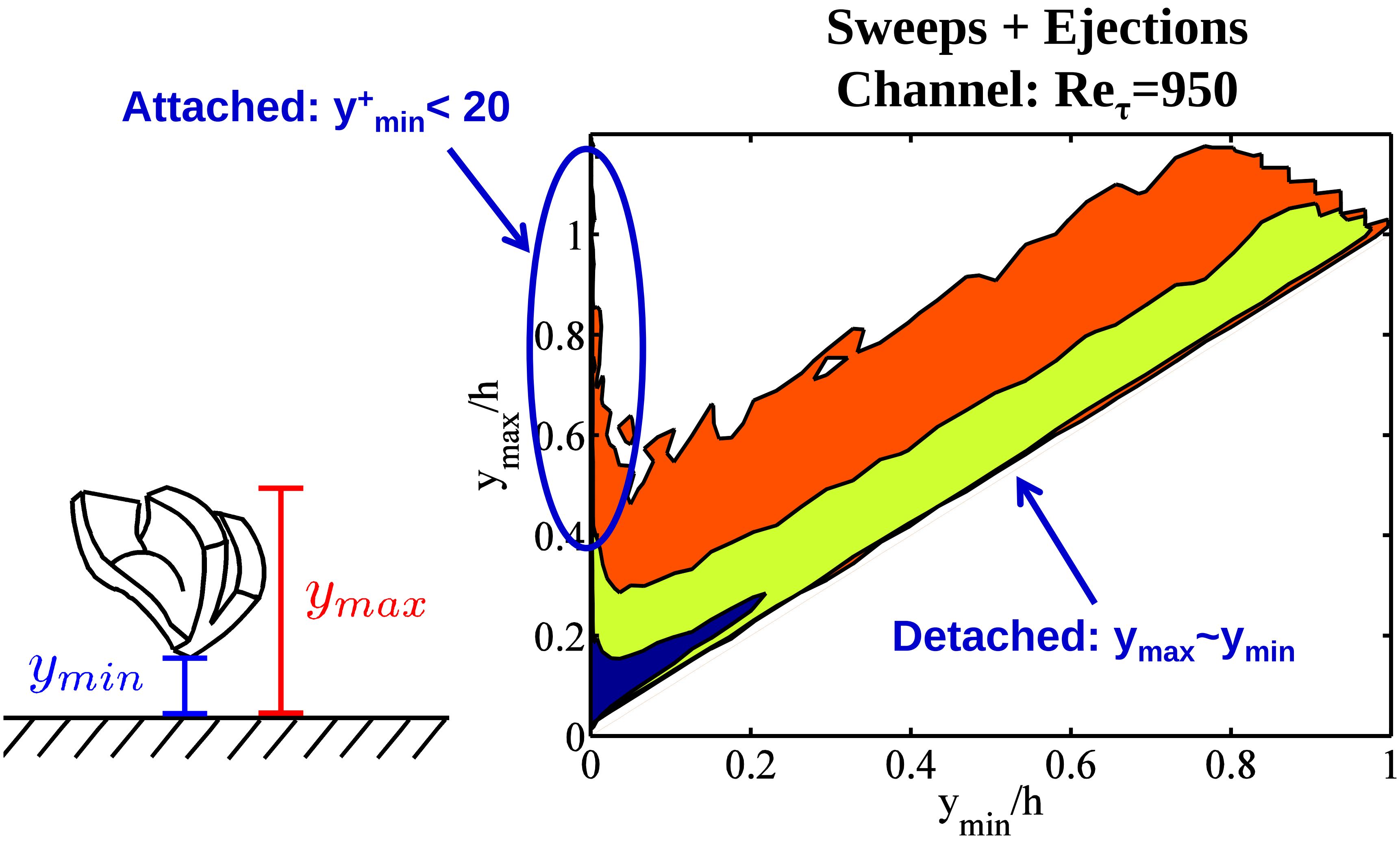




-uv > 1.75 u'v'

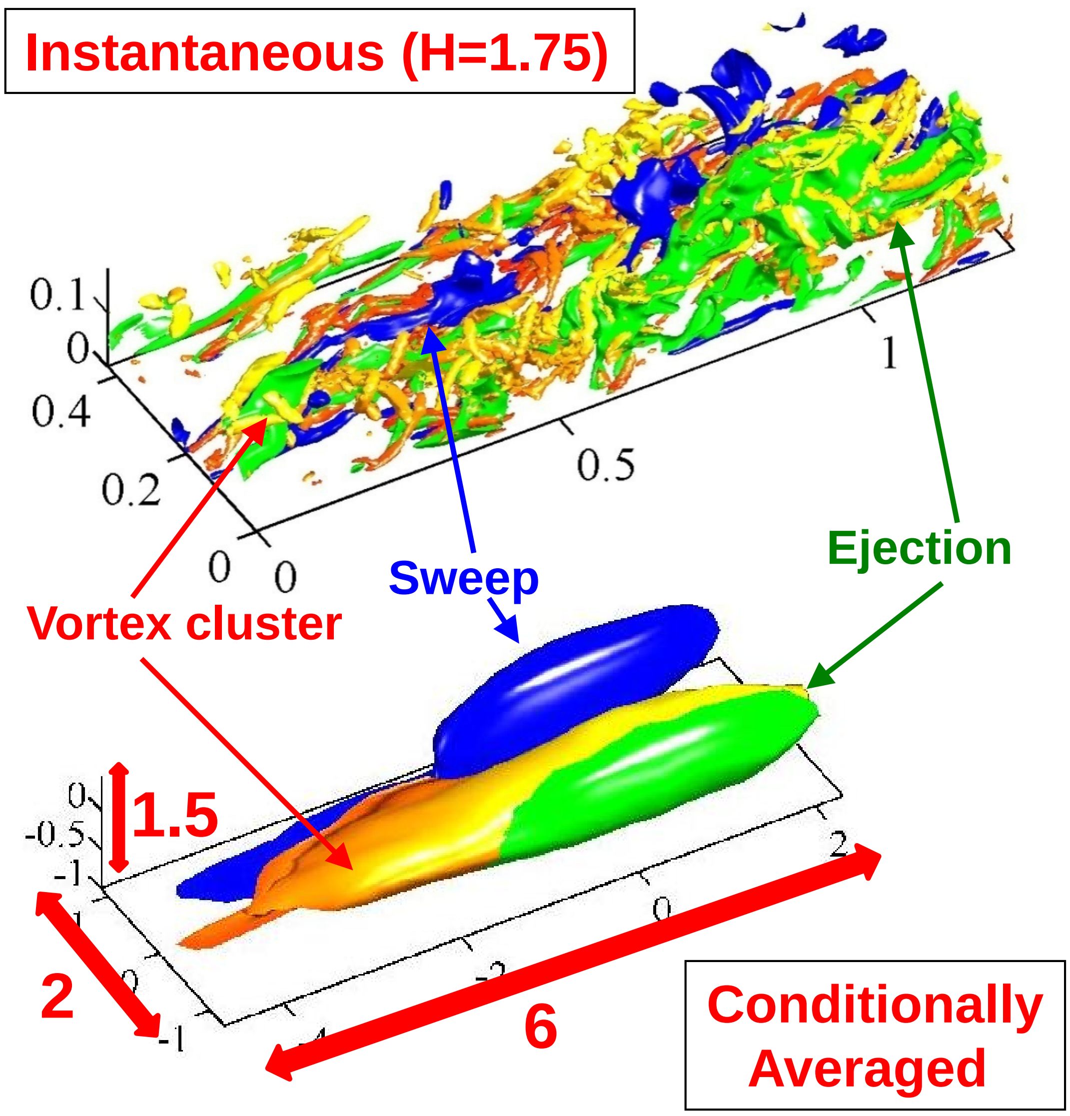
Channel: Re_τ=2000. A. Lozano-Durán

"Attached" and "Detached" Eddies

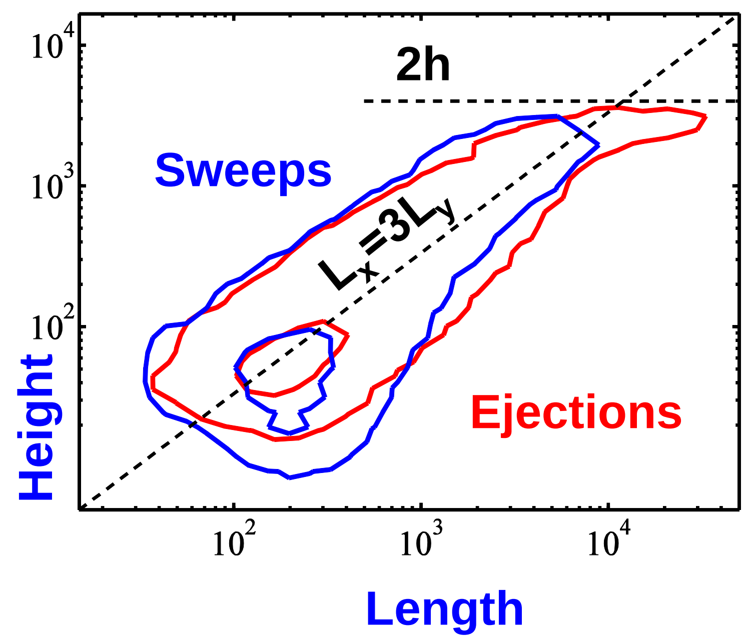


Del Alamo et al. (2006); Lozano-Duran, Flores & J (2012)

Attached Sweeps and Ejections

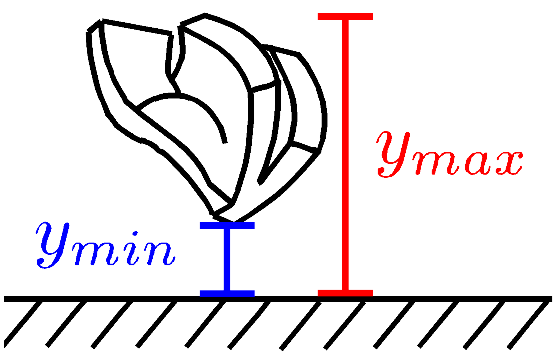


Momentum Transfer is self-similar

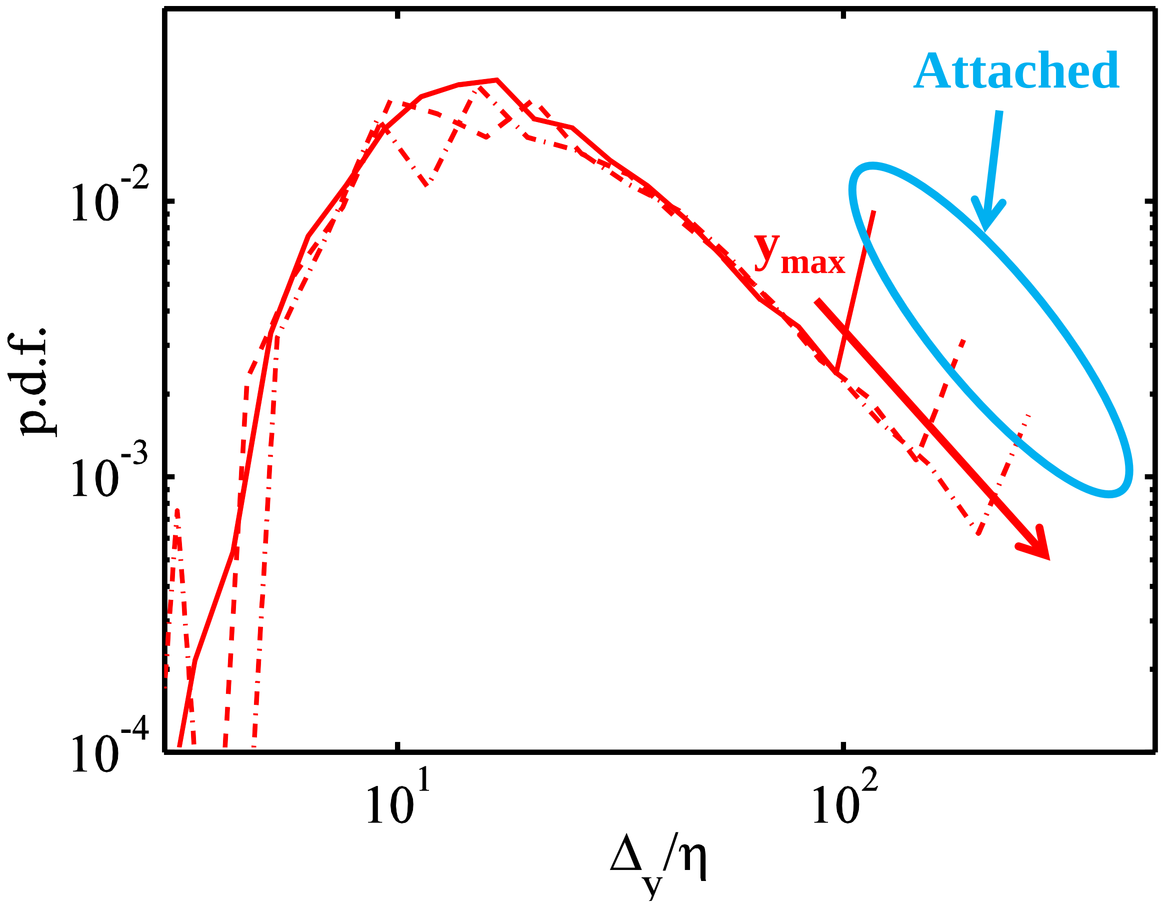


Lozano-Duran, Flores & J (2012)

"Attached" and "Detached" Eddies



Sweeps + Ejections
Channel: Re_τ=2000



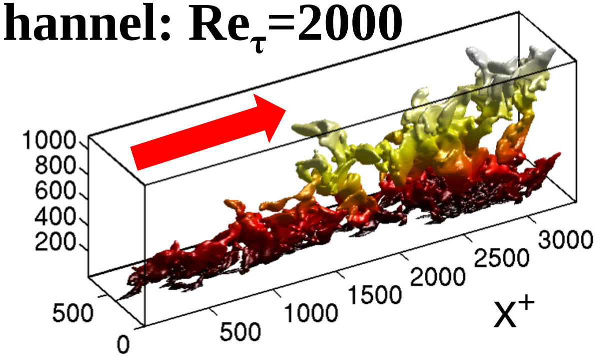
Lozano-Duran, Flores & J (2012)

"Large" and "Small" Eddies

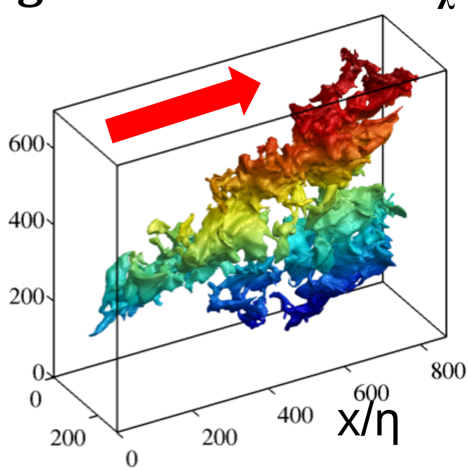
Homogeneous Shear Turbulence

Ejections

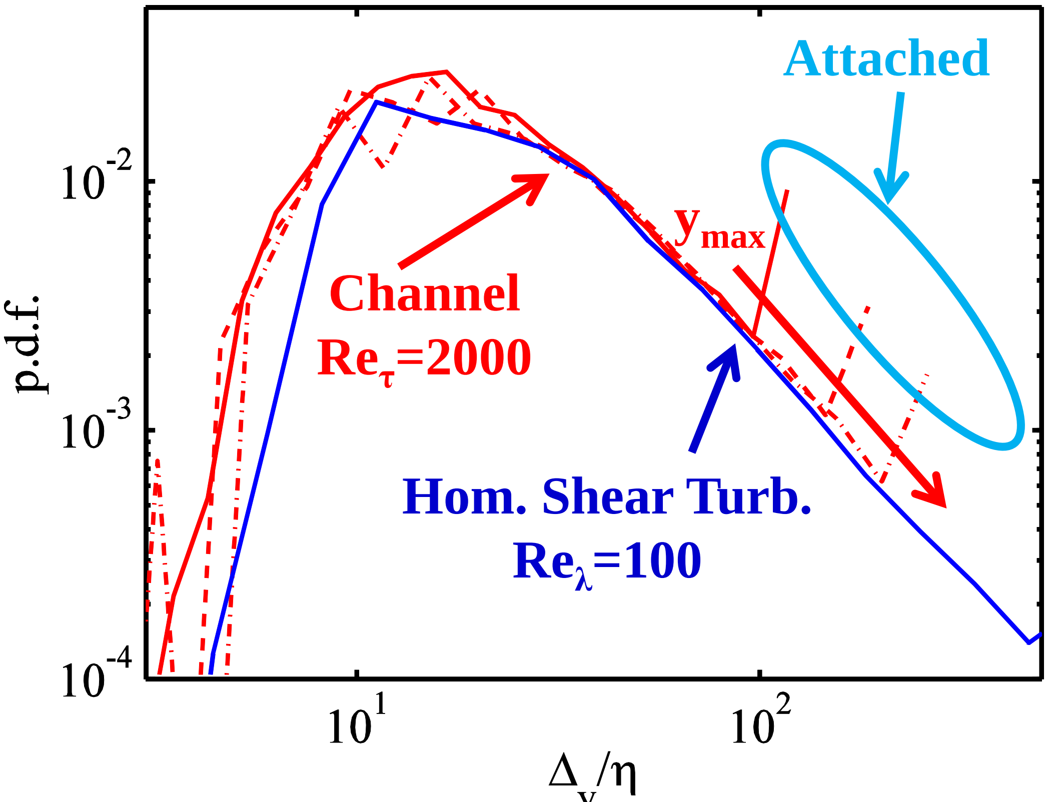
Channel: Re_τ=2000



Homogeneous Shear: Re_λ=100



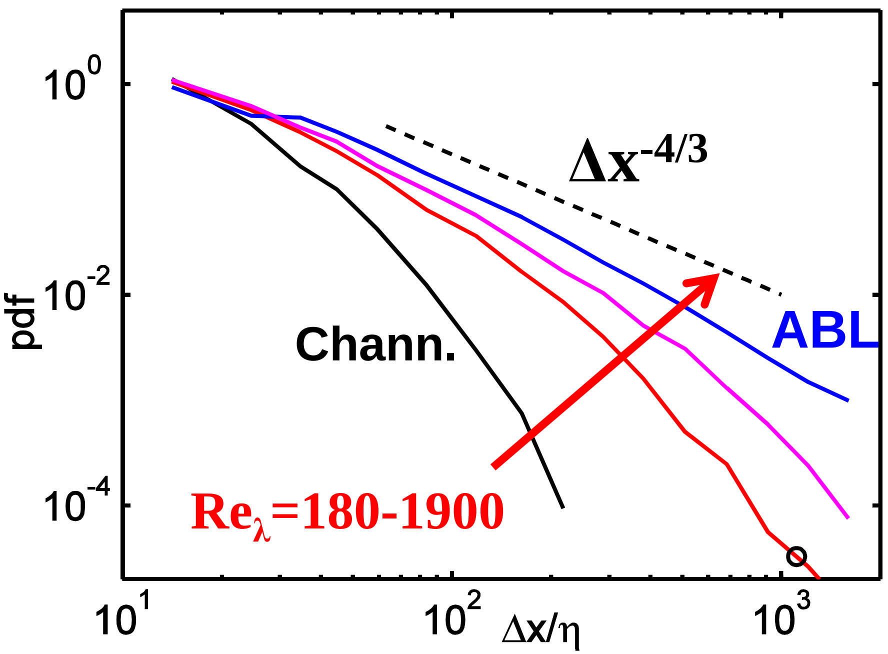
Sweeps + Ejections



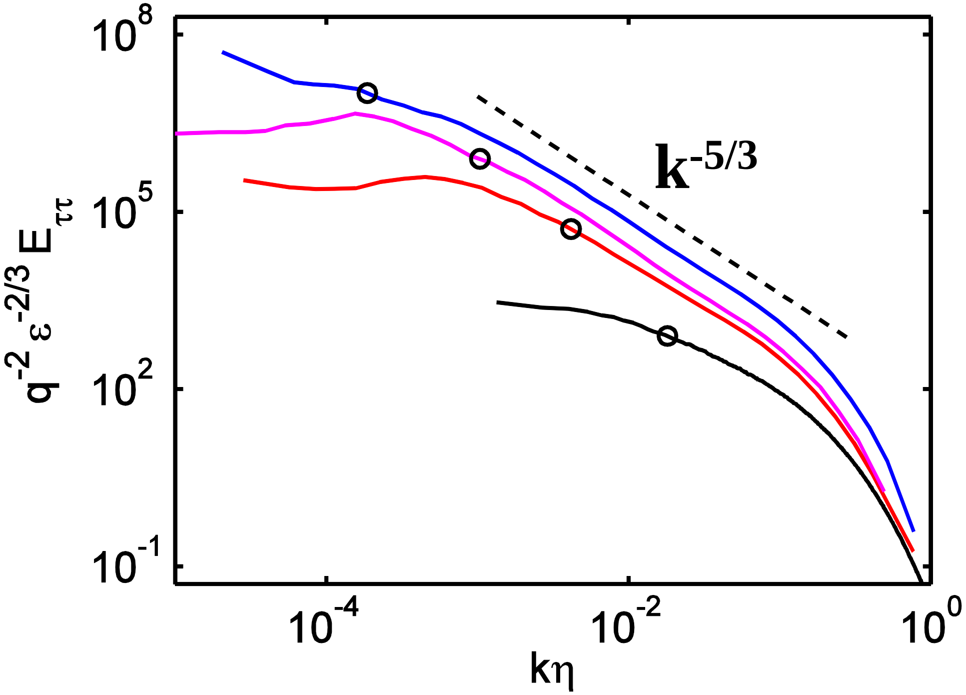
Dong, Sekimoto & J (2013), Lozano-Duran & J (2014)

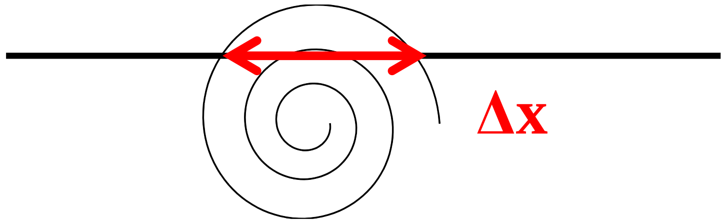
Stress eddies are "Universal"

Length of (uv)



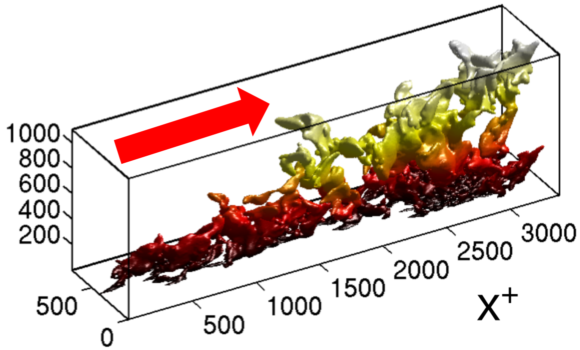
Spectrum of (uv)

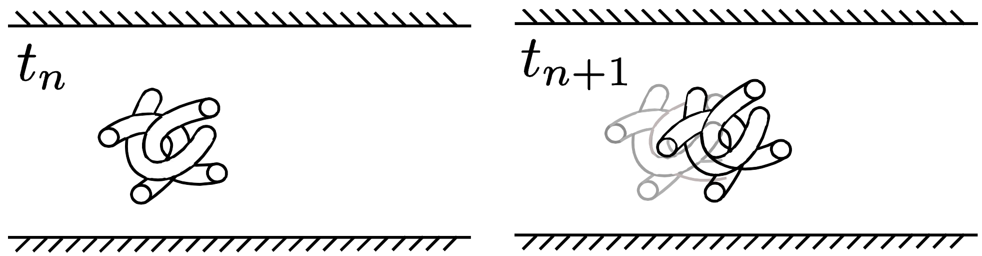


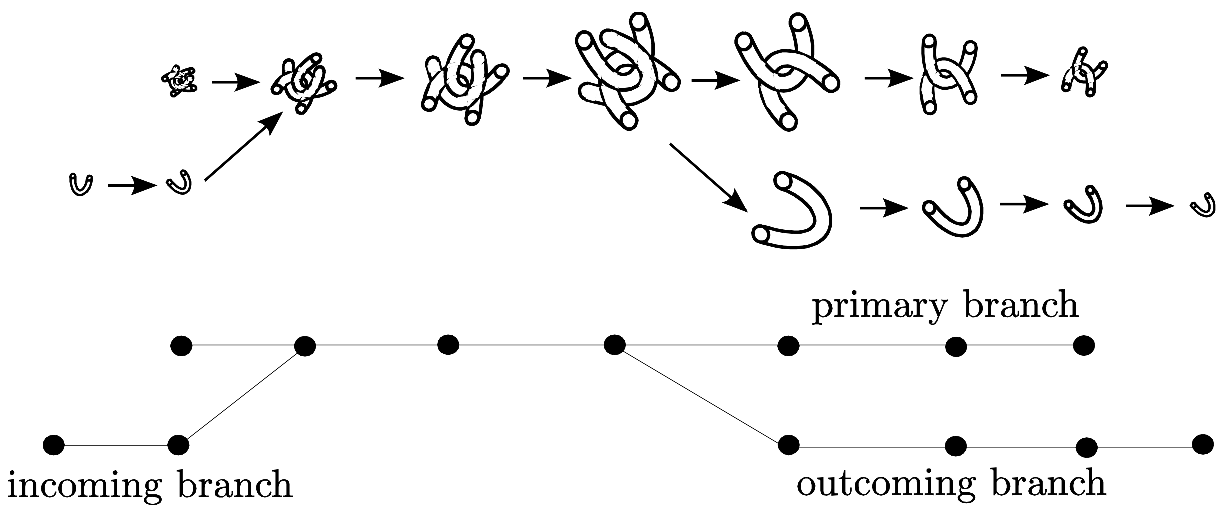


Lozano-Duran, Flores & J (2012)

Tracking Eddies in Time

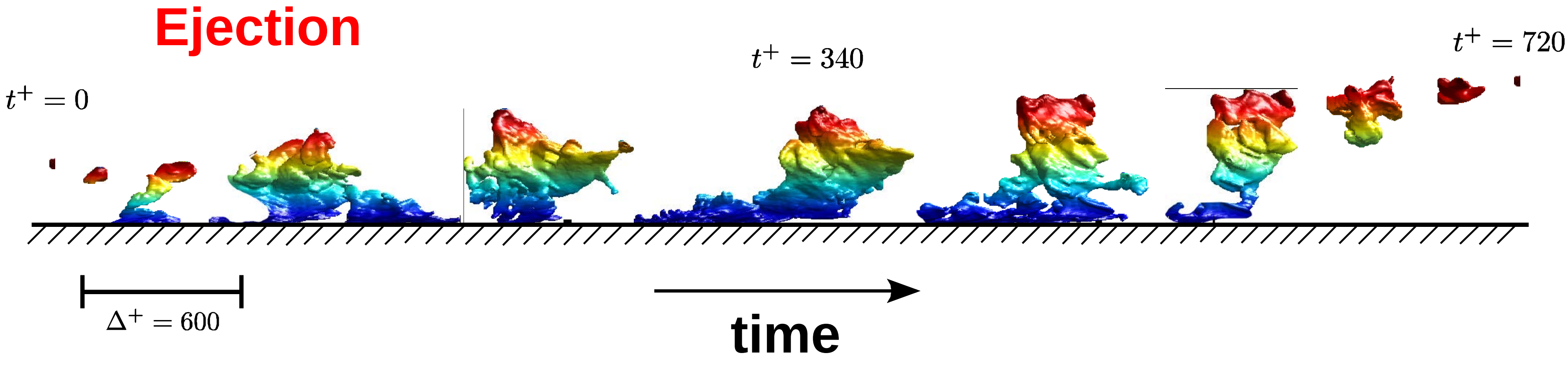


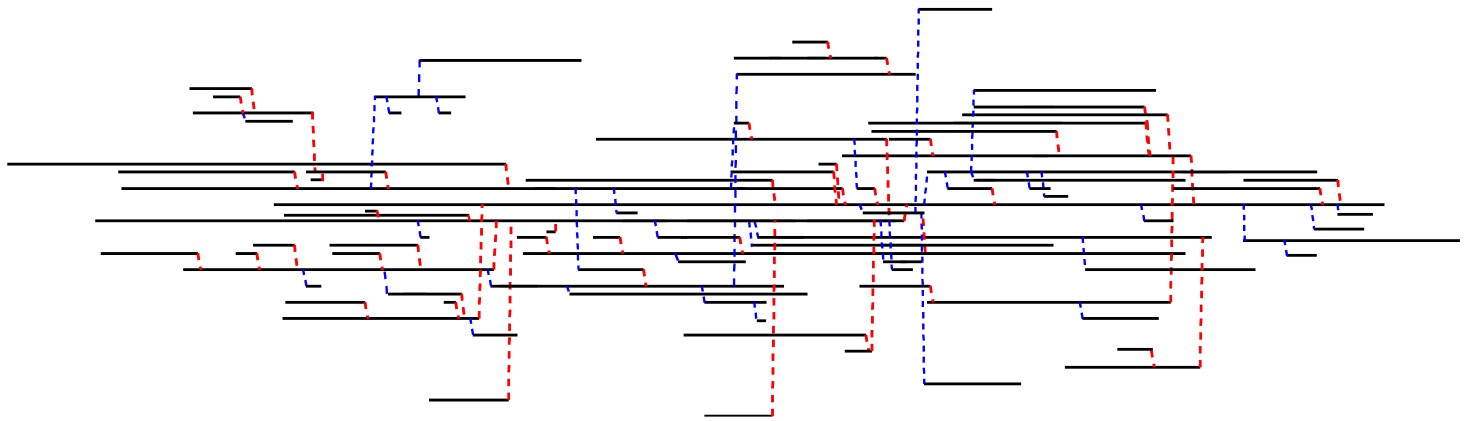




A. Lozano-Durán & J (2010)

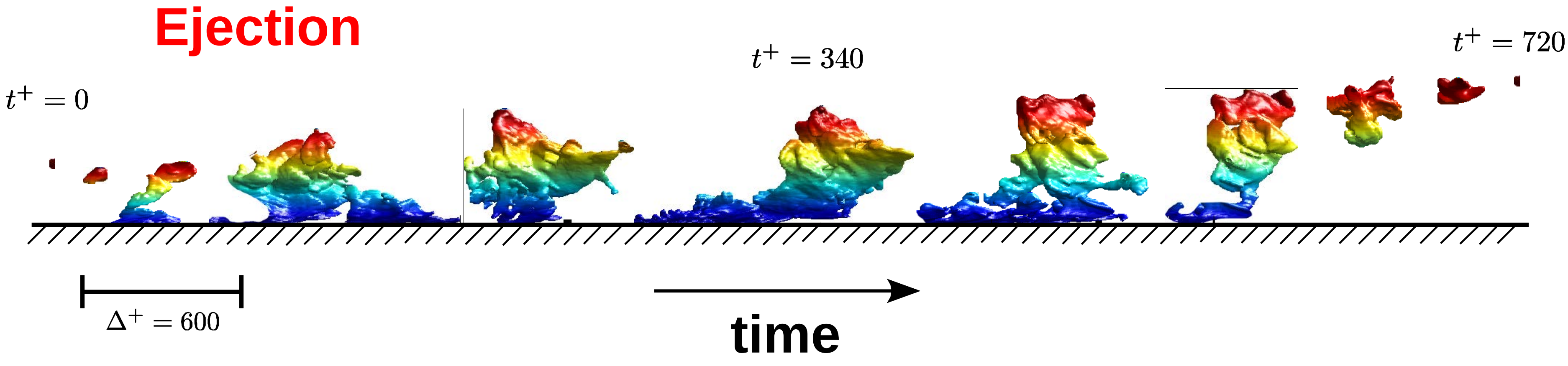
Tracking in Time

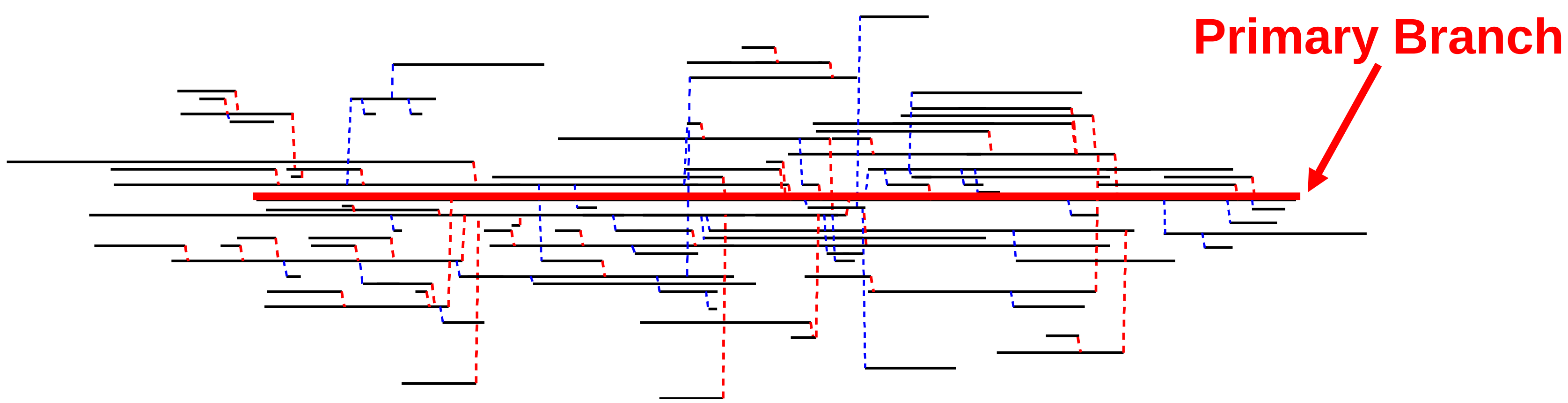




Channel: Re_τ=4200. A. Lozano-Durán

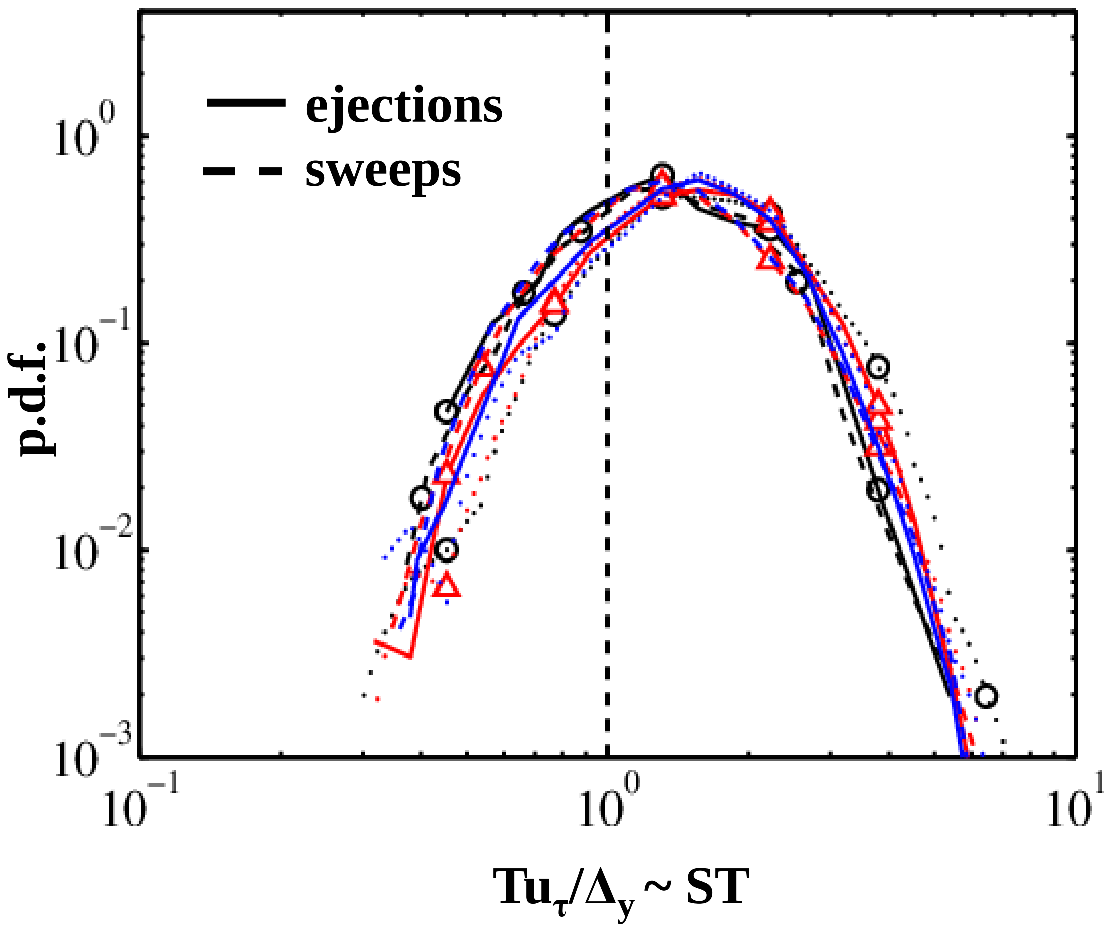
Tracking in Time

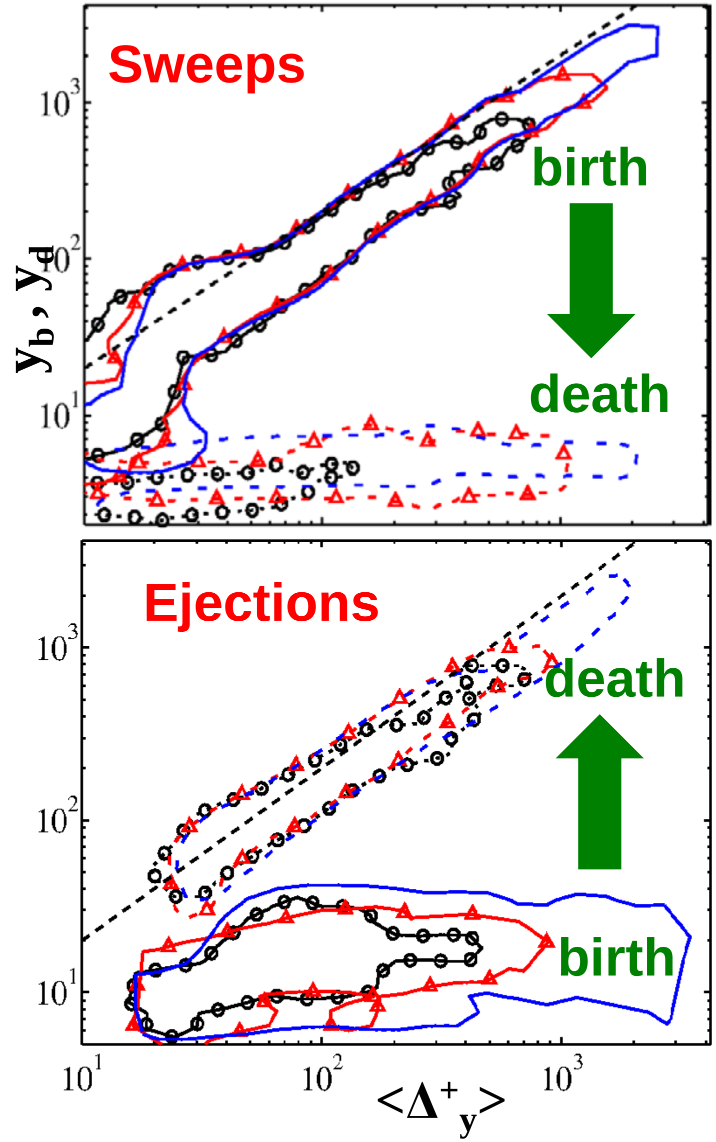




Channel: Re_τ=4200. A. Lozano-Durán

Lifetimes: Attached Sweeps and Ejections

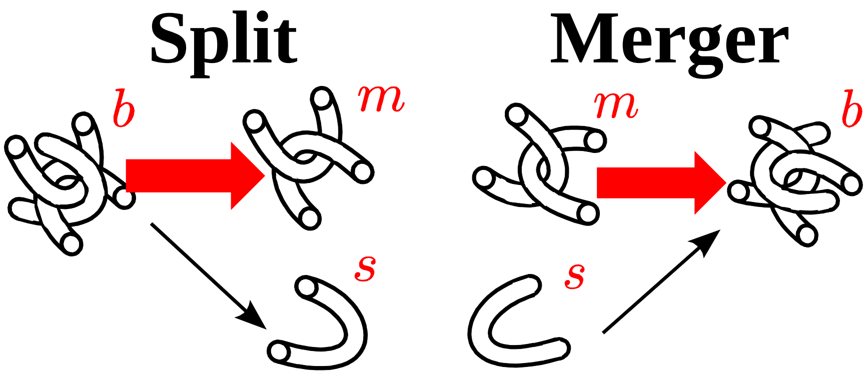




Channels: Re_τ = 950-4200

Lozano-Durán & J (2014)

Splits and Mergers



$V_b \approx V_m + V_s$

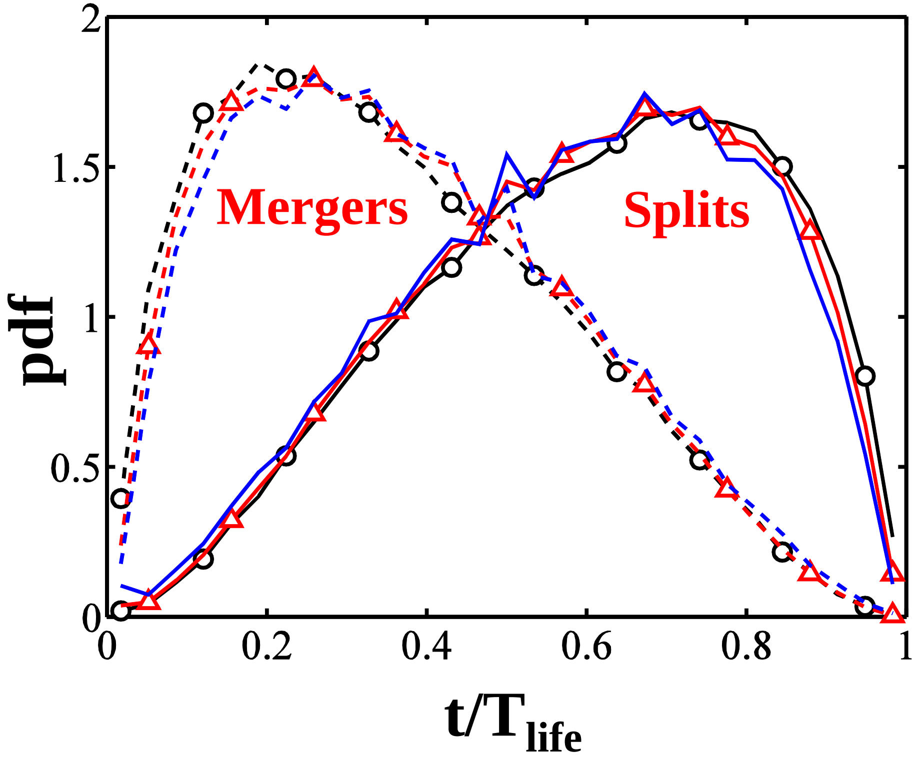
$\Delta V_{BRANCH} = \Sigma\, |\Delta V| \approx \Sigma\, V_s$

Δ_{y}/η	Inertial $\Delta_s > 100\eta$	Viscous $\Delta_s < 100\eta$	Smooth Growth
0-50	0%	23%	77%
50-200	28%	23%	49%
200-400	54%	8%	38%
>400	94%	5%	1%

$Re_\tau = 4200$; "detached"

Lozano-Durán & J (2014)

Growth and Decay

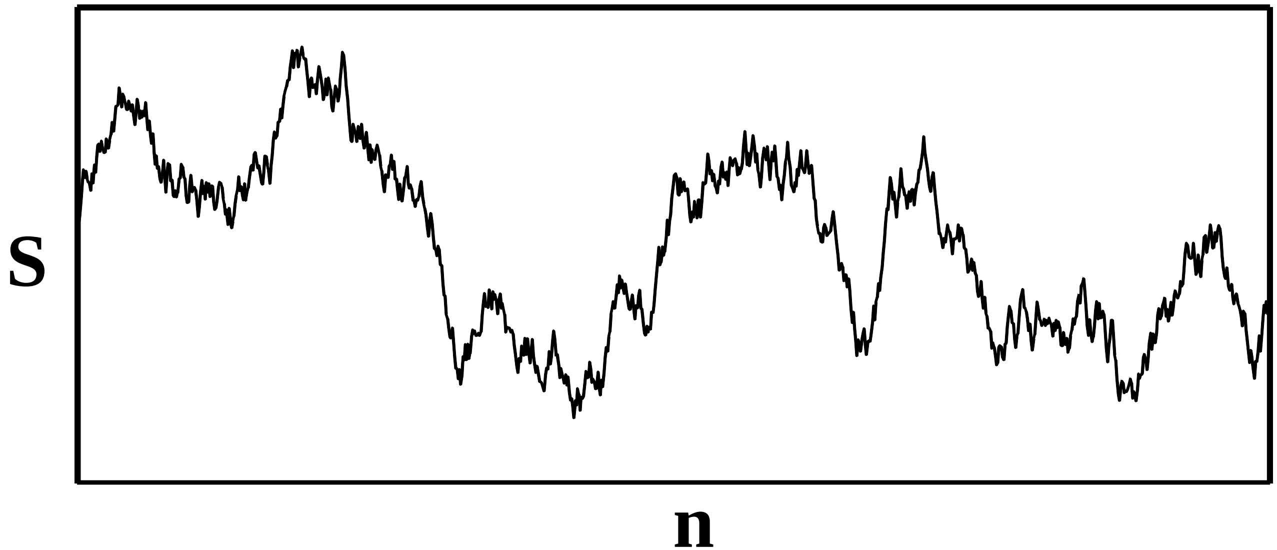


Re_τ= 950-4200; "attached"

Lozano-Durán & J (2014)

A fair coin toss
(martingale)

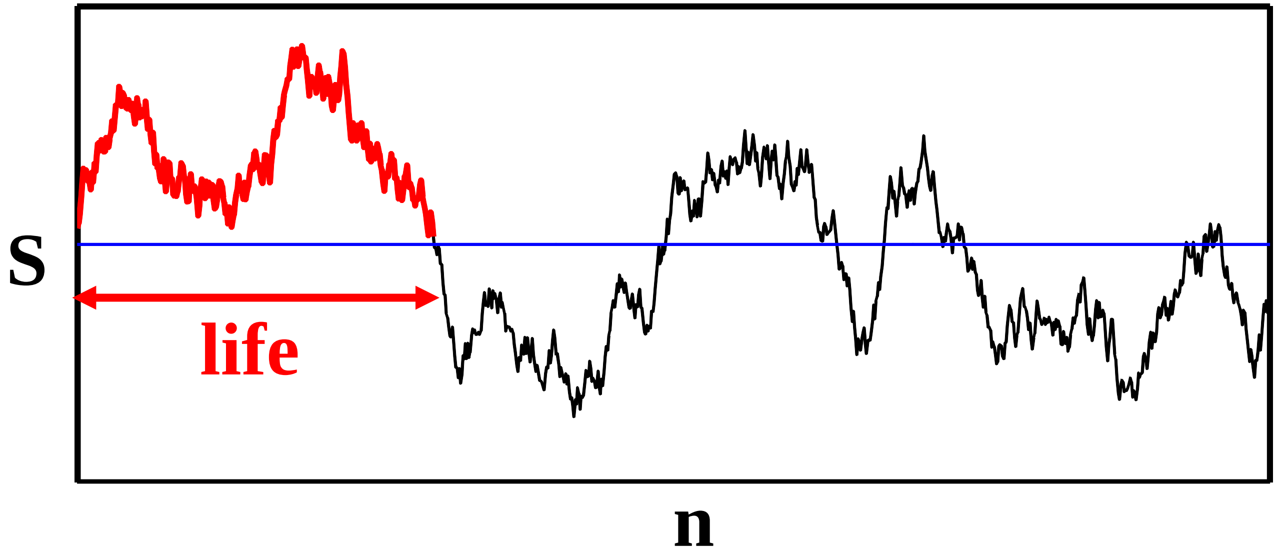
$S_n = S_{n-1} \pm 1\$$



Lozano-Durán & J (2014)

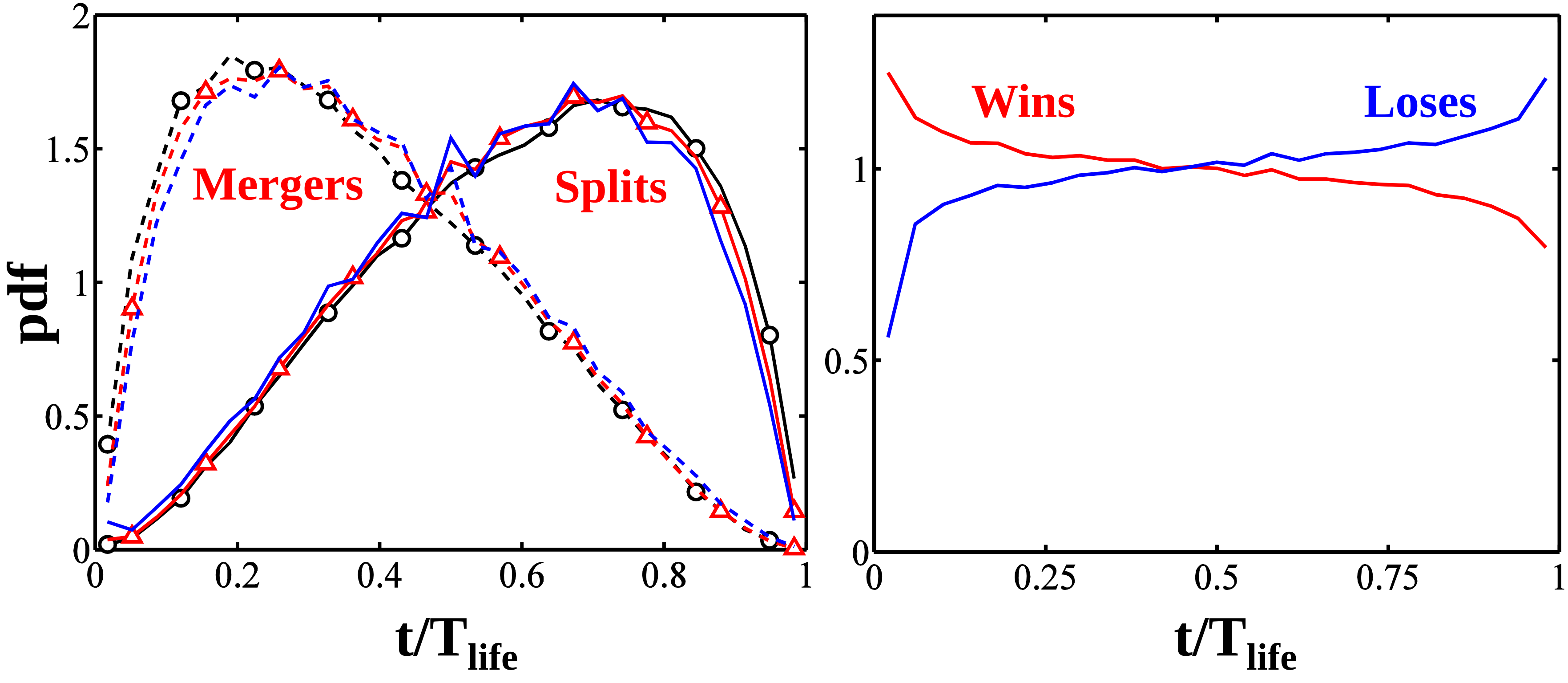
A fair coin toss
(martingale with ruin)

$$S_n = S_{n-1} \pm 1\$$$



Lozano-Durán & J (2014)

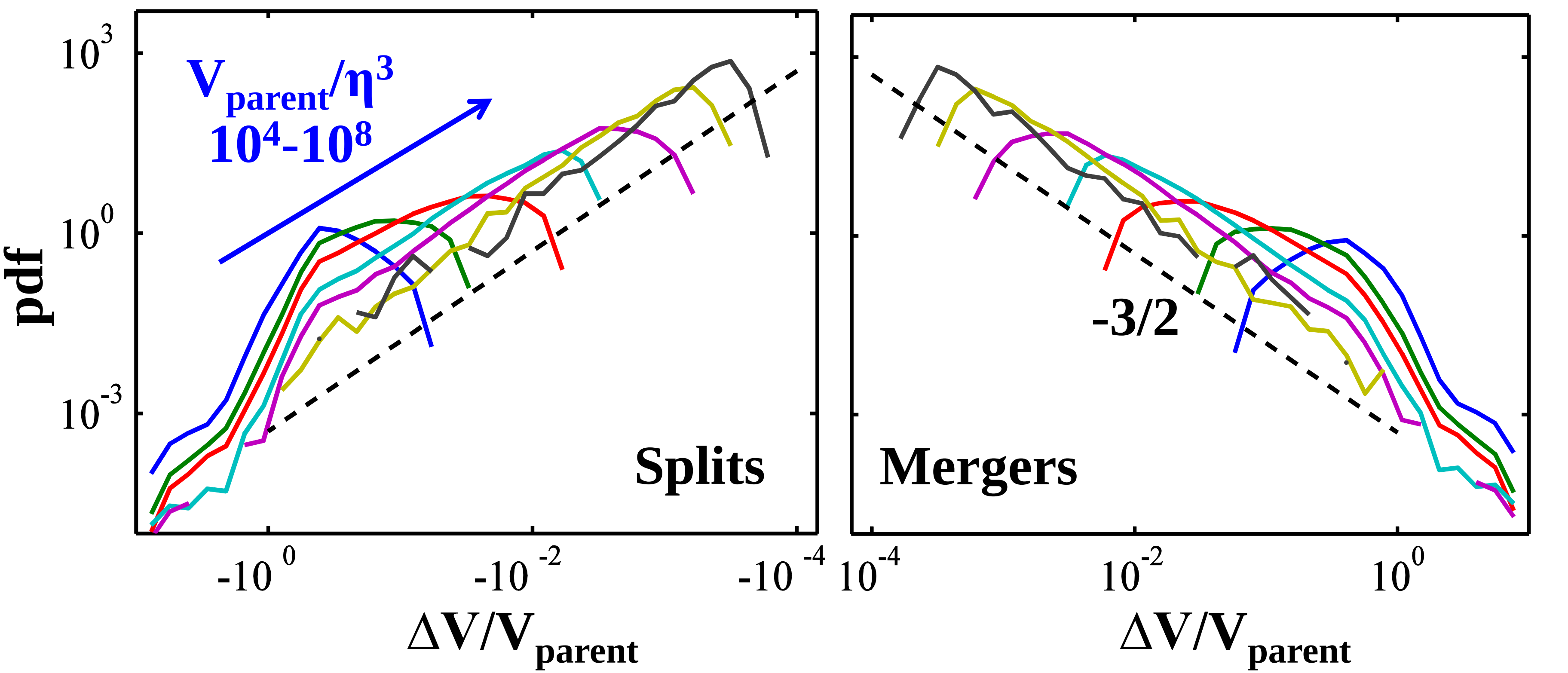
Growth and Decay



Channel **Martingale**

Lozano-Durán & J (2014)

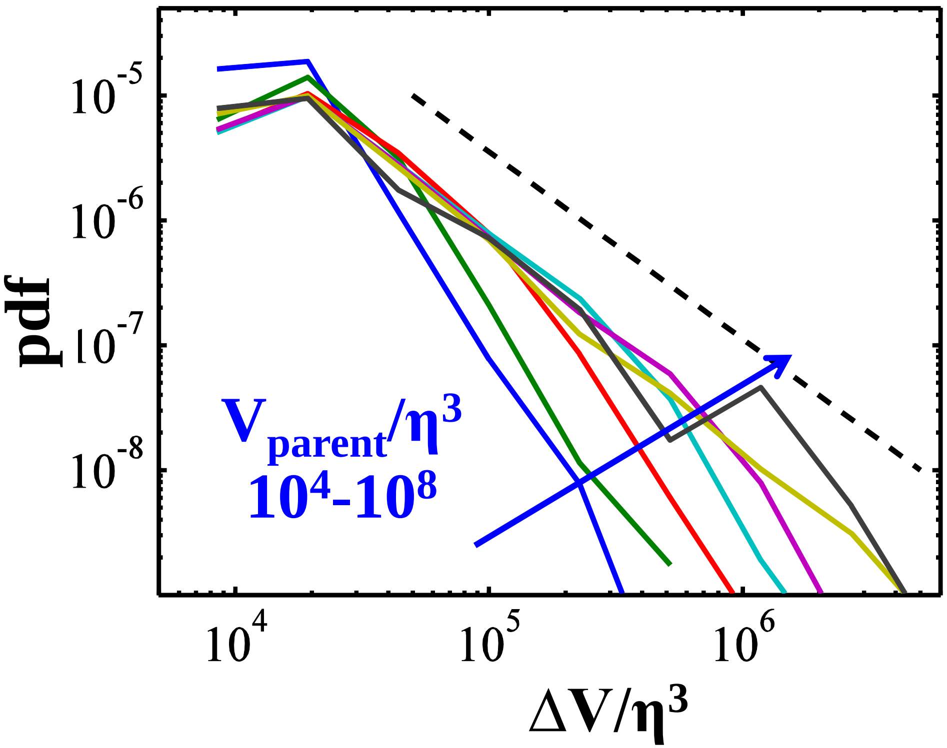
Relative Volume Increments

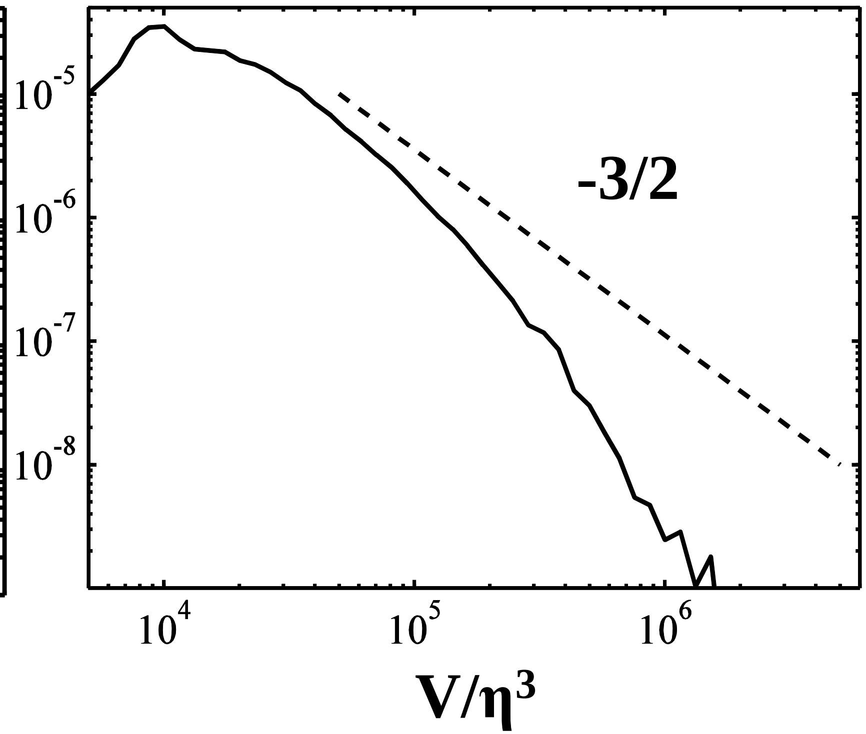


Re$_\tau$= 4200; "detached"

Lozano-Durán & J (2014)

Absolute Volume Increments





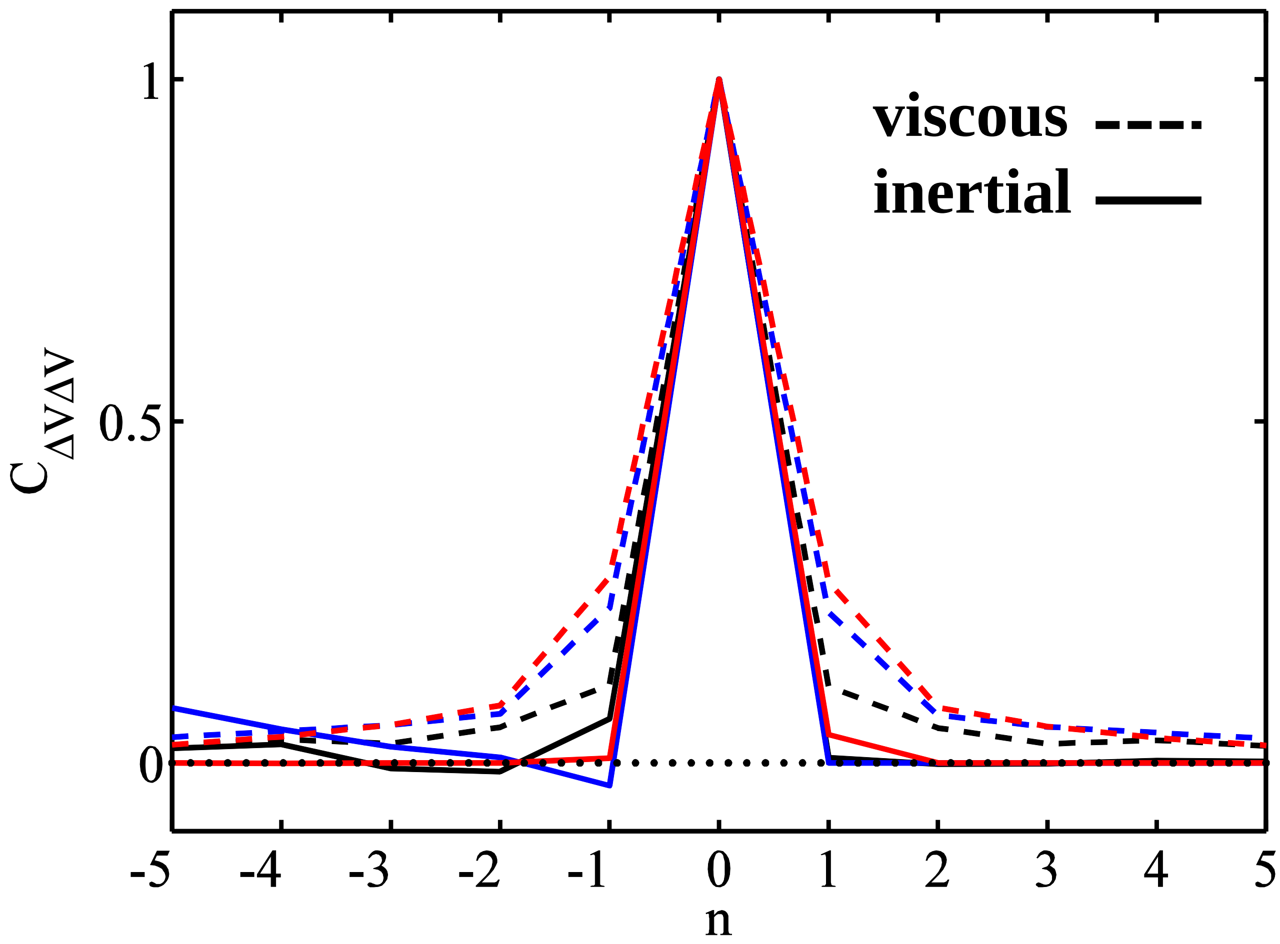
Increments in Mergers

All structures

Re_τ= 4200; "detached"

Lozano-Durán & J (2014)

Mergers and Splits are Markovian



Re_τ= 950-4200; "detached"

Lozano-Durán & J (2014)

Martingales with ruin

Additive: $S_{n+1} = S_n + r_n$

$$S_{max} \sim N_{max}^{1/2}$$

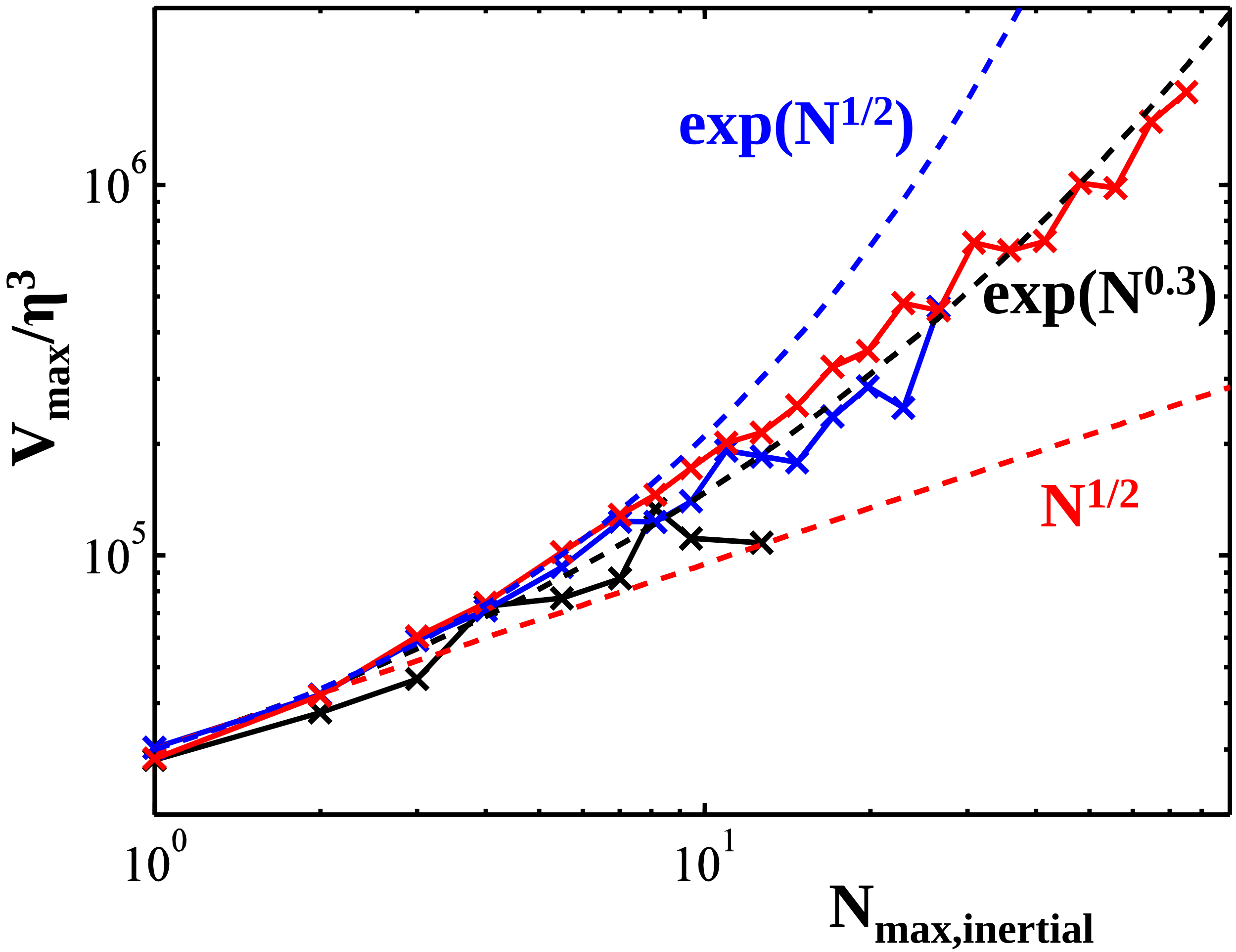
Multiplicative: $S_{n+1} = S_n * r_n$

$$\log(S_{n+1}) = \log(S_n) + \log(r_n)$$

$$S_{max} \sim \exp(N_{max}^{1/2})$$

Lozano-Durán & J (2014)

Volume versus Life



Re_τ= 950-4200; "inertial"

Lozano-Durán & J (2014)

Summary

- There is a forward energy cascade that takes time
- It crosses the inertial range incrementally
- Its velocity is given by the local eddy turnover
- The momentum flux in channels also cascades
- It can be followed in individual eddies
- It "resembles" a martingale with ruin

Thank you

Some talks at the APS

Siwei Dong: Homogeneous shear flow. **A26.8**

Cecilia Huertas-Cerdeira: Another way of analyzing cascades. **H27.3**

Jose Cardesa: The energy flux. **H27.4**

Alberto Vela-Martin: A reversed cascade. **H27.5**