Interaction of laminar and turbulent effect in the dynamics of the Solar Tachocline

Shear, rotation, stratification, magnetic fields, turbulence, …

Some background and a report on research work done at

University of California, Santa Cruz

by

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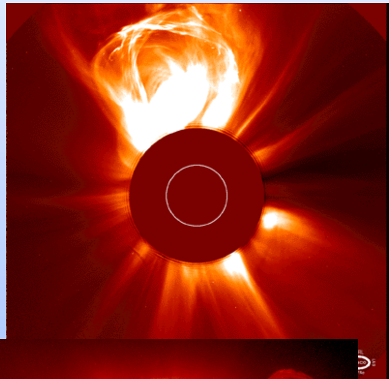
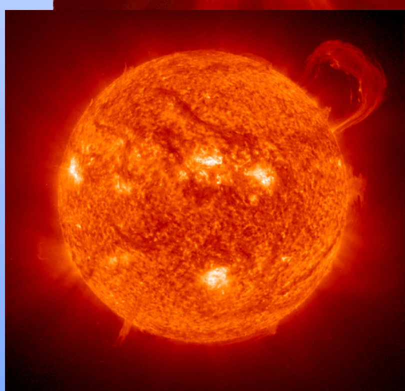
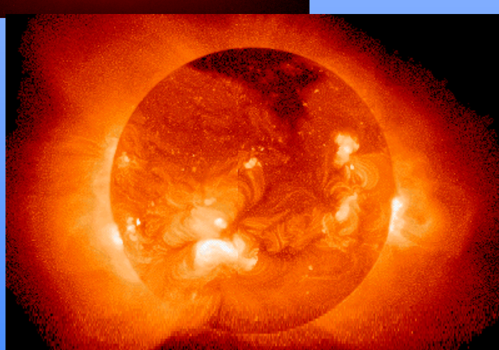
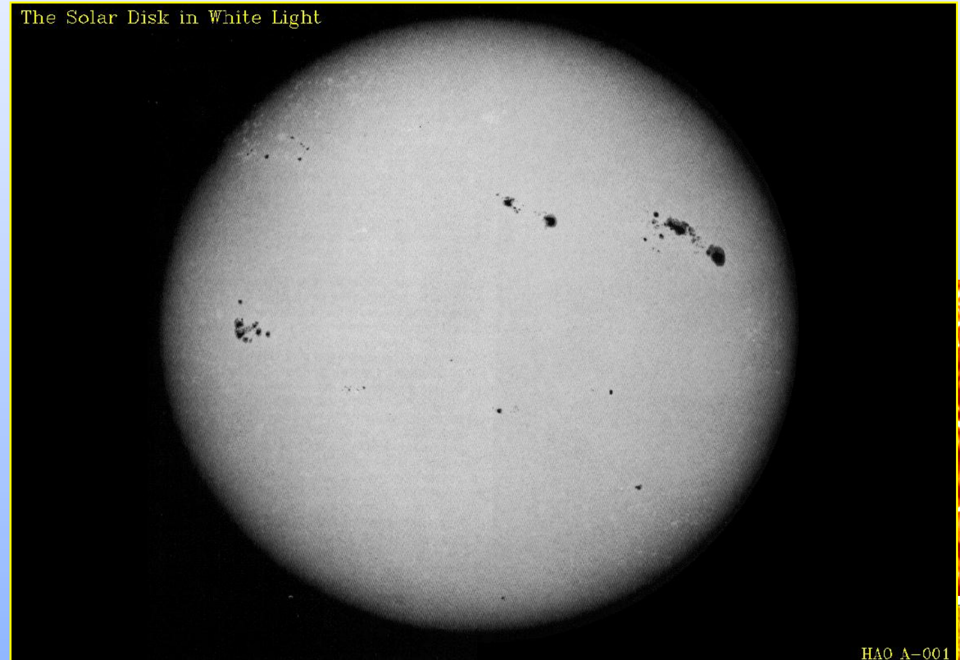
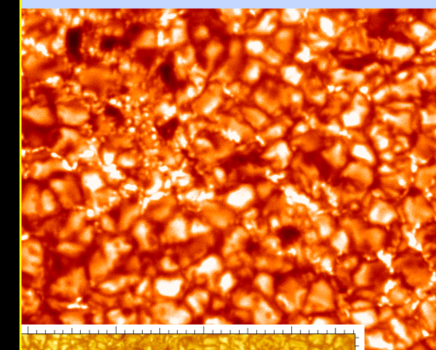
Motivation: Solar Physics

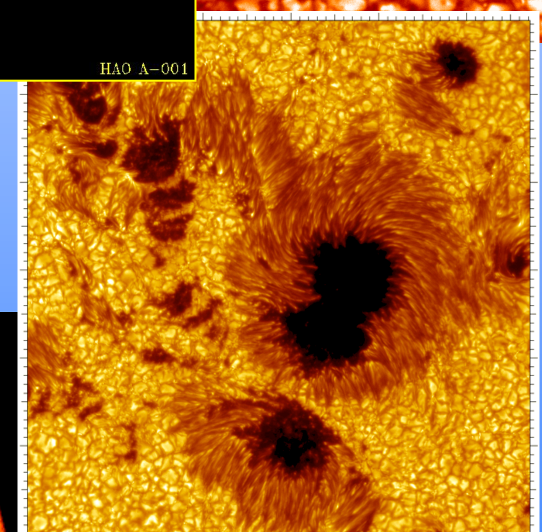


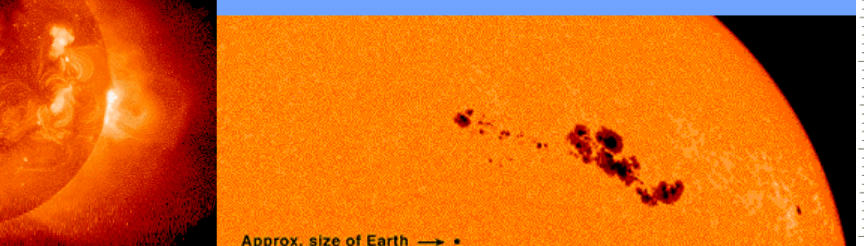






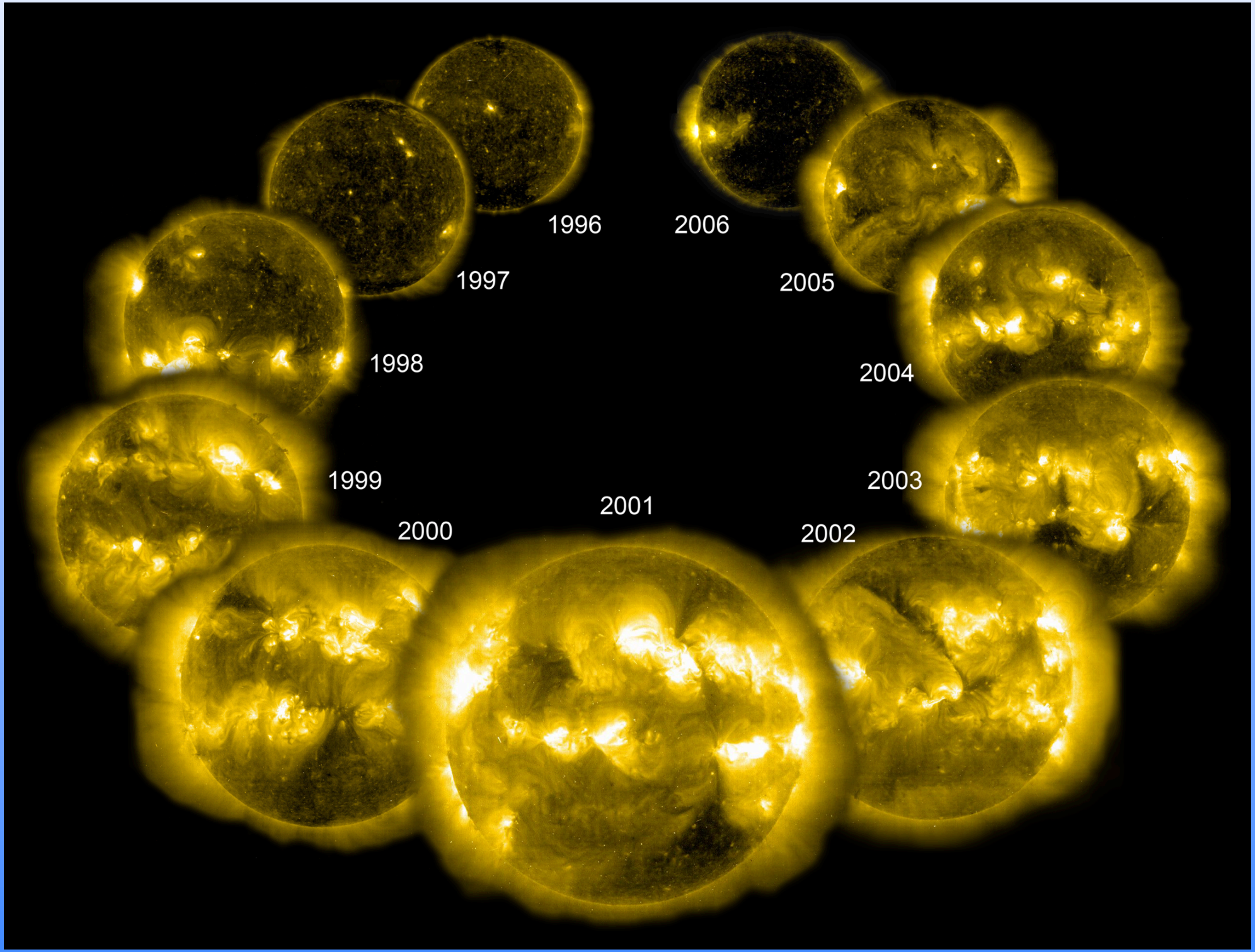




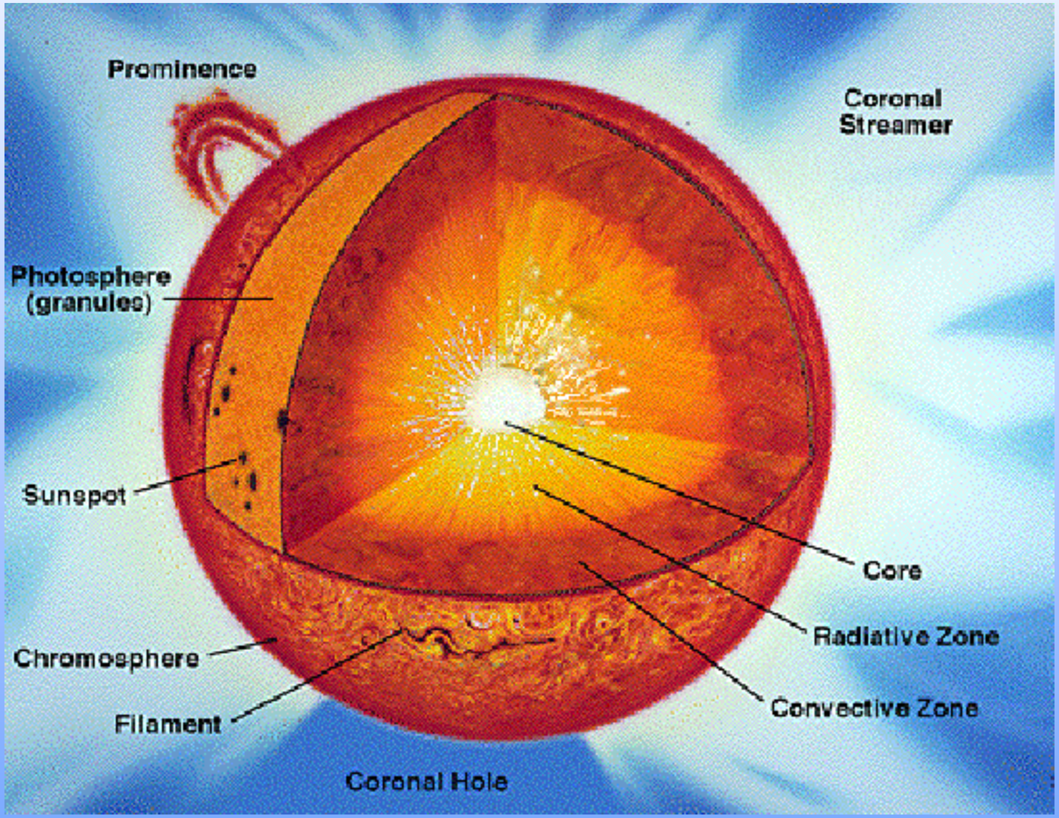


Motivation: Solar Physics

Most of this activity related to solar magnetic activity. Solar magnetic activity is ***VARIABLE***



Basic solar structure



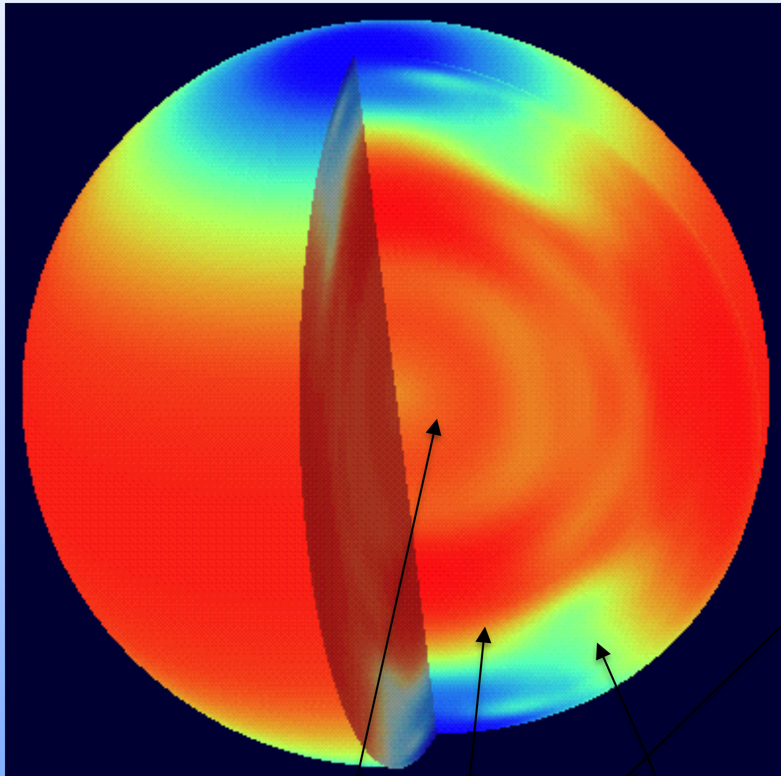
Solar Interior

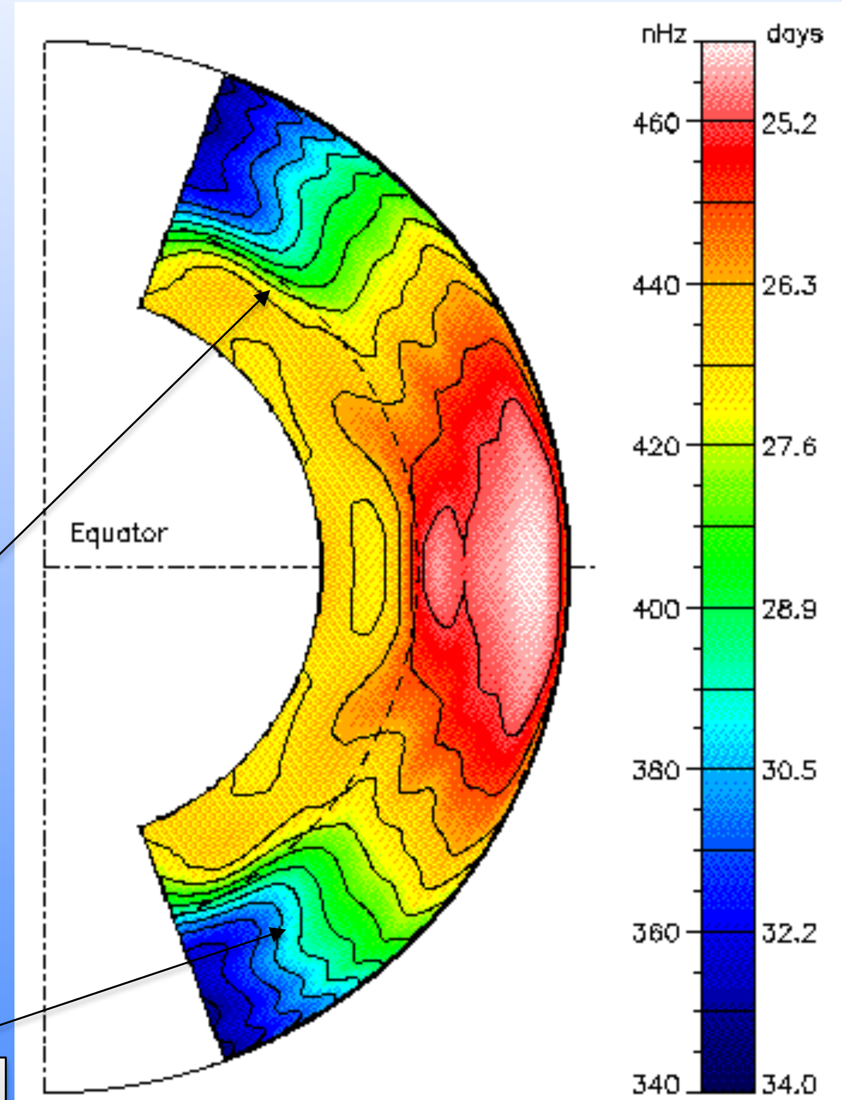
1. Core
2. Radiative Interior
3. (Tachocline)
4. Convection Zone

Visible Sun

1. Photosphere
2. Chromosphere
3. Transition Region
4. Corona
5. (Solar Wind)

Solar structure: Helioseismology



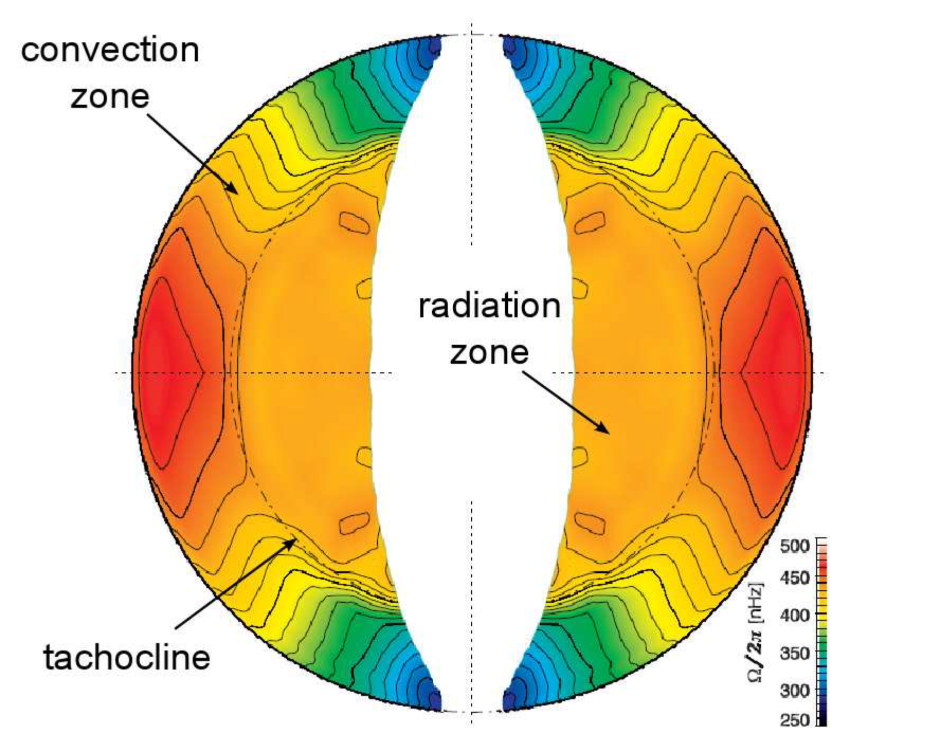


Solid body rotation in the core

Interface layer – the tachocline

Differential rotation in the convection zone

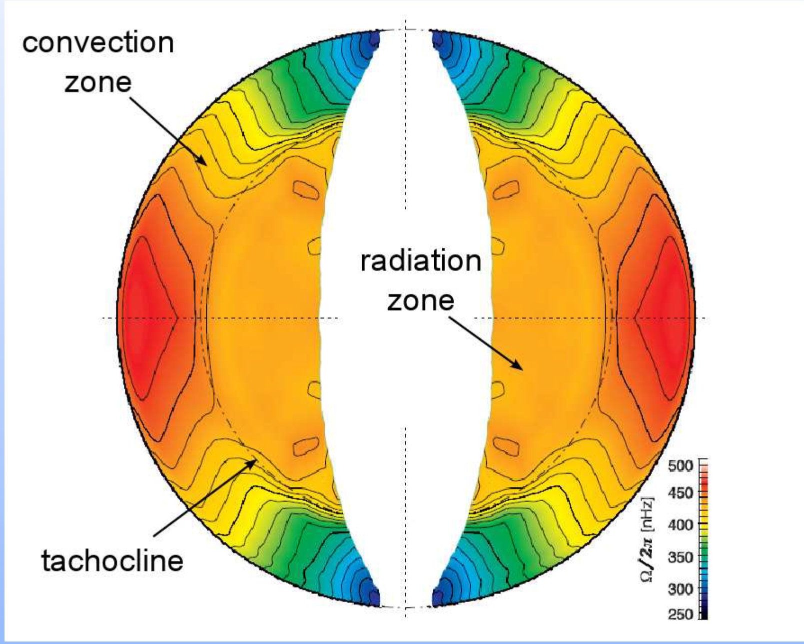
(Some of the) Big questions



Korzennik & Eff-Darwich 2011

How is the magnetic field produced?
Why is the CZ differentially rotating?
Why is the radiative interior in solid body rotation?
Why is the transition region – the tachocline – so thin?

(Some of the) Big questions



Korzennik & Eff-Darwich 2011

Why does the internal rotation profile have this structure?

- Why is the CZ differentially rotating?
- Why is the radiative interior in solid body rotation?
- Why is the transition region – the tachocline – so thin?

(How does this transpire to produce large-scale magnetic field by a dynamo?)

Role of the tachocline

- An essential component of our current dynamo theories
- We therefore need to understand its dynamics
- *Why does the deeper interior exist in the form that it does?*
- Problem: we only know the (time & azimuthally) averaged differential rotation (azimuthal/zonal angular velociy):
  - What other flows exist in the tachocline?
  - How are they generated and sustained?
  - Is there any turbulence?
  - What is the role of magnetic fields?

These are questions of
rotating, sheared, stably-stratified, MHD (turbulence).

The tachocline is surprisingly thin!

4% by radius! Why so thin?

("surprising" in the sense of the following simple, most obvious dynamics)

1. No flow; Purely viscous spreading:

- using microscopic viscosity, in 4.5 Gyrs (lifetime of Sun) flows should have spread 0.05 R.
- Roughly in agreement with helioseimology, BUT this is an absolute LOWER bound.
- Any turbulence => higher effective viscosity => deeper spreading

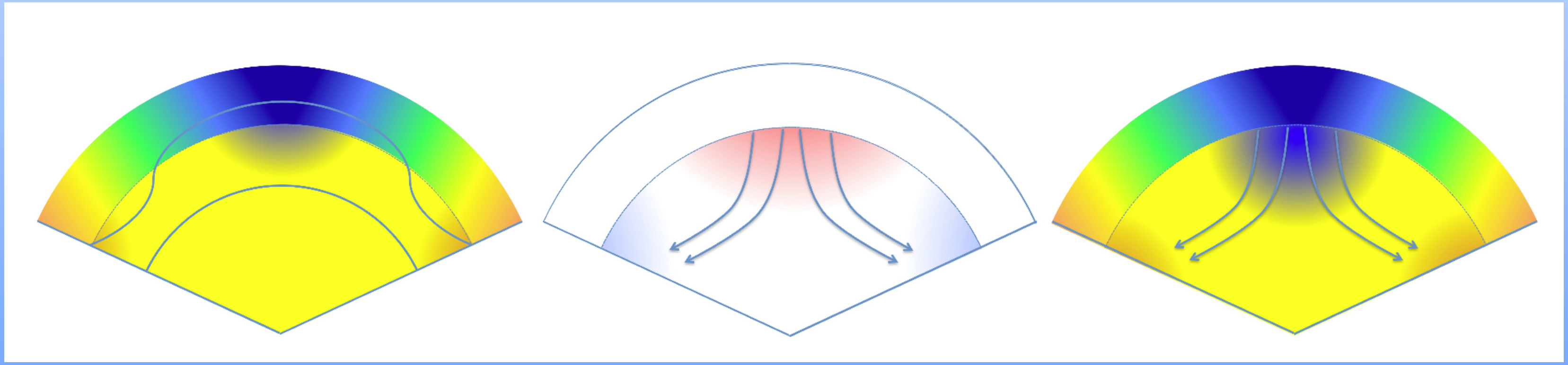
BUT further, we should expect even deeper spreading due to driven flows anyway …

The tachocline is surprisingly thin!

2. Rotational spin-down in a stratified medium: Spiegel & Zahn 1992

(from earlier ideas by Holton 1965, Spiegel 1972, Clark 1975, Haynes et al 1991)

- Rotation dominated (low Ro, low Ek)
- Initial adiabatic adjustment: $t \sim \Omega^{-1}, \frac{h}{R} \sim \frac{\Omega}{N}$
- Followed by radiative spreading:



In meridional force balance, spatial variation of the centrifugal force caused by the differential rotation requires isobars to shift (in tachocline but not below).

Vertical hydrostatic balance implies that the tachocline to warm where the pressure gradient is weakest at high latitudes.

Meridional flows are driven in order to try and restore thermal equilibrium, but which can advect angular momentum too, spreading the differential rotation -- burrowing.

Spiegel & Zahn 1992

Simple model based on earlier ideas from solar spin-down.

Main assumptions:

- Axisymmetric and spherical
- Anelastic
- Rotation dominated:
  - Ro = advection/Coriolis <<1 => linear
  - Geostrophic: Ekman = viscous/Coriolis <<1. Ignore viscous boundary layers. Coriolis balanced by pressure.
- Thin: h << R
- Differential rotation imposed at top of tachocline

Spiegel & Zahn 1992

Equations:

$$r^2 \rho u = \frac{\partial \Psi}{\partial x}, \qquad r \rho \sin\theta \, v = \frac{\partial \Psi}{\partial r}$$

with $x = \cos\theta$.

Define streamfunction (meridional flow)

➔

$$-\frac{1}{\rho}\frac{\partial \widehat{P}}{\partial r} + g\frac{\widehat{T}}{T} = 0$$

hydrostatic

$$-2\Omega r x \widehat{\Omega} = \frac{1}{\rho r}\frac{\partial \widehat{P}}{\partial x},$$

latitudinal velocity eqn = geostrophic

$$\rho r^2 (1-x^2)\frac{\partial \widehat{\Omega}}{\partial t} + 2\Omega x \frac{\partial \Psi}{\partial r} = \frac{(1-x^2)}{r^2}\frac{\partial}{\partial r}\left[\rho \nu_V r^4 \frac{\partial \widehat{\Omega}}{\partial r}\right] + \rho \frac{\partial}{\partial x}\left[\nu_H (1-x^2)^2 \frac{\partial \widehat{\Omega}}{\partial x}\right],$$

inviscid

longitudinal velocity perturbations = differential rotation

$$\frac{\partial \widehat{T}}{\partial t} - \frac{N^2}{g}\frac{T}{\rho r^2}\frac{\partial \Psi}{\partial x} = \frac{1}{\rho C_P r^2}\frac{\partial}{\partial r}\left(\chi r^2 \frac{\partial \widehat{T}}{\partial r}\right).$$

temperature

thermal diffusion

Spiegel & Zahn 1992

Two regimes of adjustment:

1. Initial rapid adiabatic adjustment

- Set thermal diffusivity to zero
- Rapid adjustment of imposed differential rotation to depth

$$\frac{h}{R} \sim \frac{\Omega}{N}$$

- Depth to which rotation overpowers stratification to enforce Taylor-Proudman

Spiegel & Zahn 1992

Two regimes of adjustment:

1. Initial rapid adiabatic adjustment
2. Radiative spreading

- Re-instate radiative (thermal) diffusion
- Radial temperature gradients drive a meridional flow
- Eliminate flow and thermal variables ->

$$\frac{\partial \Omega'}{\partial t} = \upsilon \frac{\partial^2 \Omega'}{\partial r^2} - \left[\frac{R^4}{\lambda^2 t_{ES}}\right]\frac{\partial^4 \Omega'}{\partial r^4}$$

Radial dependence of differential rotation

Viscous spreading

?

Hyperdiffusive spreading

Radiative spreading

Spiegel and Zahn "burrowing"

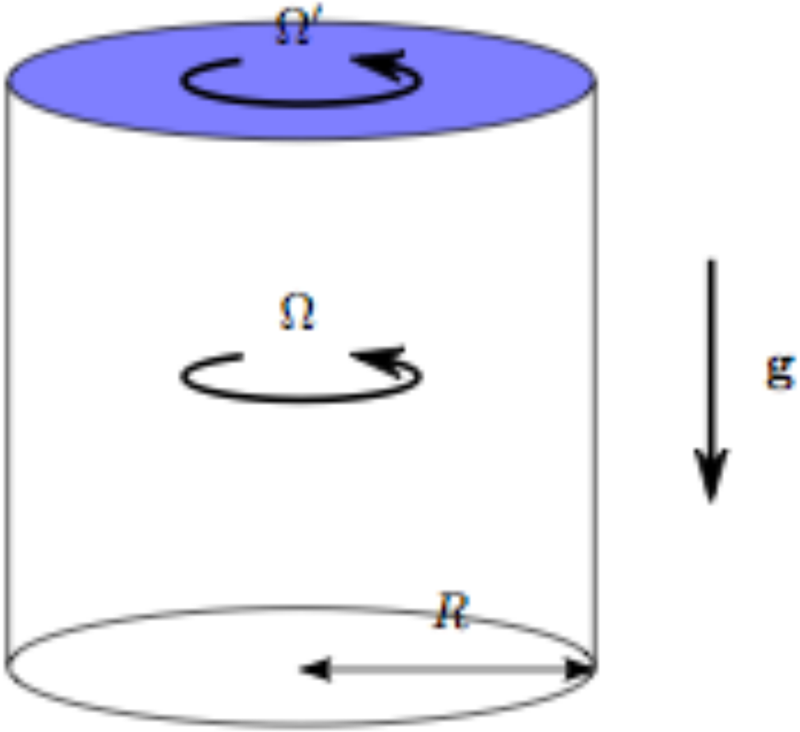
Ignoring viscous term => $\frac{h}{R} \sim \left(\frac{t}{t_{ES}}\right)^{1/4}$ where $t_{ES} = \frac{\mathrm{N}^2}{\Omega^2}\frac{R^2}{\kappa}$

SLOW TIMESCALE

Spiegel & Zahn 1992

Case study: hydrodynamic tachocline burrowing

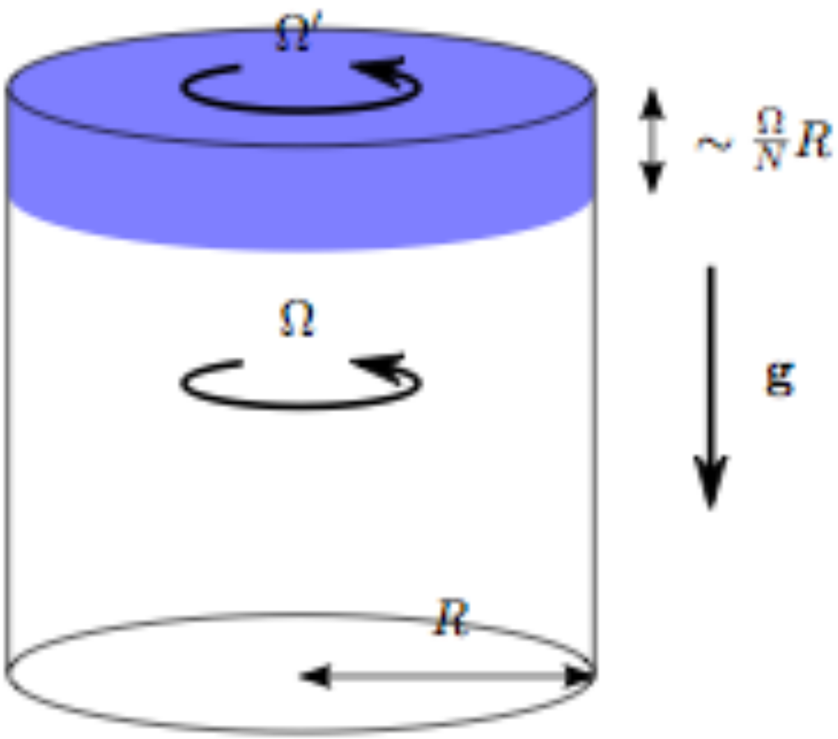
- $t = 0$



Spiegel & Zahn 1992

Case study: hydrodynamic tachocline burrowing

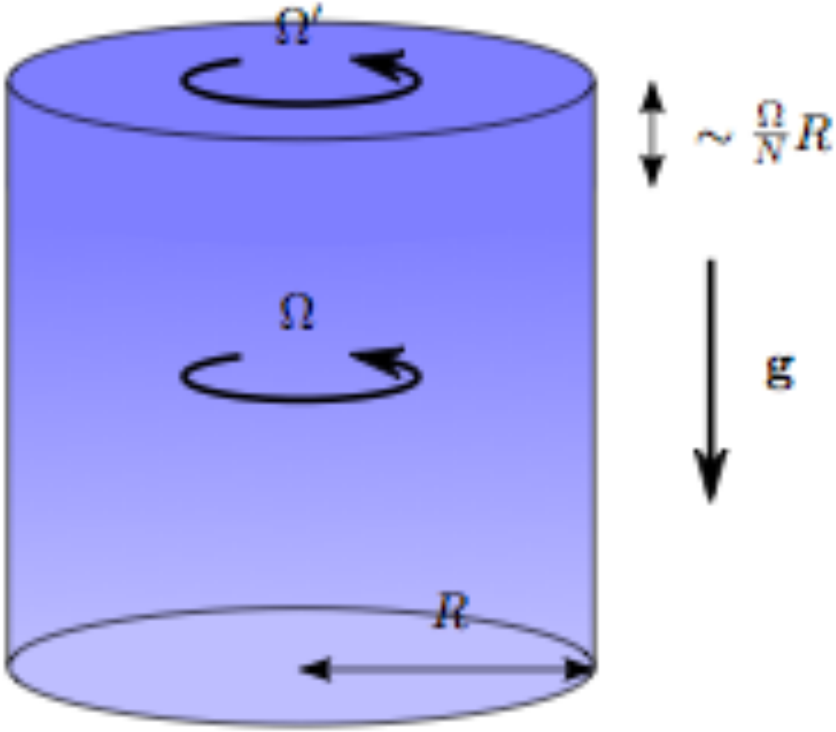
- $t = 0$
- $t \sim \Omega^{-1}$



Spiegel & Zahn 1992

Case study: hydrodynamic tachocline burrowing

- $t = 0$
- $t \sim \Omega^{-1}$
- $t \sim t_{\mathrm{ES}}$
  $= \left(\frac{N}{\Omega}\right)^2 R^2/\kappa$
  (N.B. inviscid)

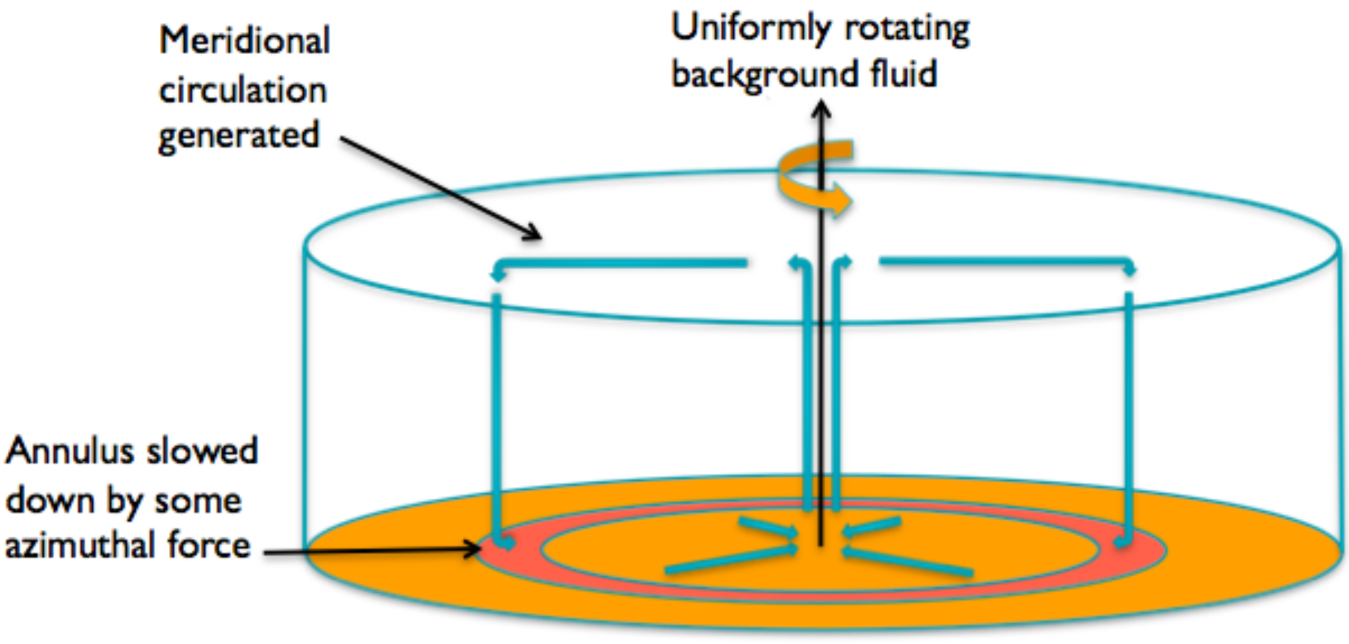


Gyroscopic pumping

Different name, essentially same ideas:

- Slightly different approach to force balance
- Assumes torques create differential rotation

- Gyroscopic pumping, in a rotating system, refers to the **mechanical generation** of **meridional motion** by the application of forces **in the azimuthal direction.**



Gyroscopic pumping: tea leaves

Familiar example: Ekman pumping in a teacup

(movie: Wikipedia)

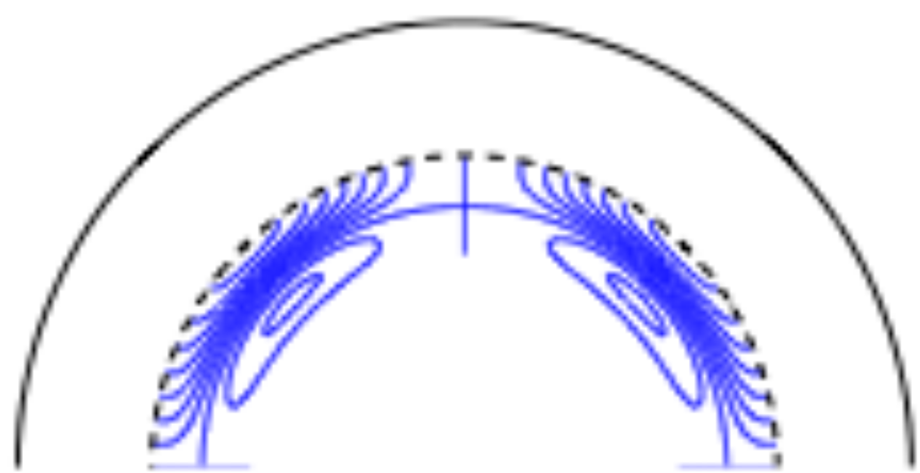
- Viscous stresses at the bottom of the cup exert a retrograde azimuthal force on the fluid, and pump it toward the rotation axis.

Gyroscopic pumping somewhat more general (doesn't need stable stratification)
(=> applies in the convection zone too)
If stable stratification, need thermal relaxation to overcome constraint

Spiegel & Zahn 1992

Ignoring viscous term => $\frac{h}{R} \sim \left(\frac{t}{t_{ES}}\right)^{1/4}$ and $t_{ES} = \frac{N^2}{\Omega^2}\frac{R^2}{\kappa}$ ~ 200 Gyr for Sun

$\Rightarrow$h/R ~ 0.4 after 4.5 Gyr …. DEEP TACHOCLINE!



Spiegel & Zahn 1992

How does tachocline self-organise?

Simple angular momentum transporters should thicken tachocline beyond what is observed

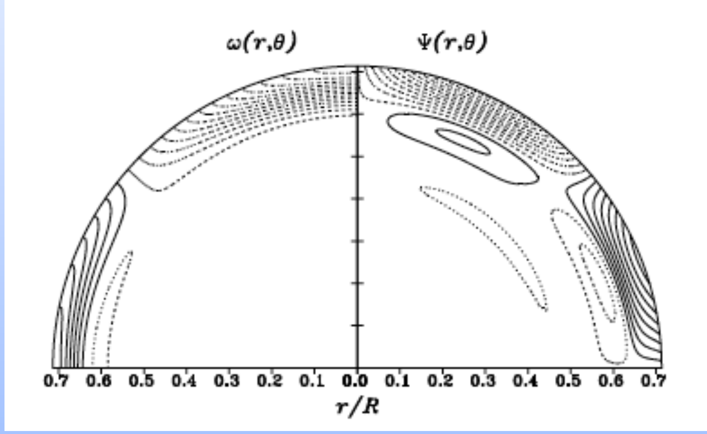
So need something (more complicated) to prevent this.

Two possible approaches have been considered:

1. Purely hydrodynamic - turbulence (and/or waves)

2. Magnetohydrodynamic

Spiegel & Zahn (cont)



$$< u'_r u'_\phi >= -\nu_v r \sin\theta \frac{\partial \Omega}{\partial r}$$

$$< u'_\theta u'_\phi >= -\nu_h r \sin\theta \frac{1}{r}\frac{\partial \Omega}{\partial \theta}$$

PURELY HYDRODYNAMIC MECHANISM

Enhance latitudinal transport of angular momentum and homogenisation by enhanced anisotropic turbulent diffusion:

- Stable stratification → 2 dimensional turbulence (if there is any turbulence)
- 2D turbulence → anisotropic turbulent viscosity (drives system towards constant angular velocity)
- "TURBULENCE ACTS AS A FRICTION" prevents tachocline spreading into the interior.
- TURBULENCE (A FAST PROCESS) IS INVOKED TO STOP THE SPREADING OF THE DIFFERENTIAL ROTATION (A SLOW PROCESS)

Objections to S&Z hydro solution

- Is there an instability leading to turbulence?
- Is this turbulence necessarily (quasi-) 2D?
- Even if this turbulence is quasi-2D does it really act so as to transport angular momentum towards a state of constant differential rotation? -- i.e does the turbulence indeed act as anisotropic viscosity/friction?
  - Gough & McIntyre 1998, McIntyre 2003

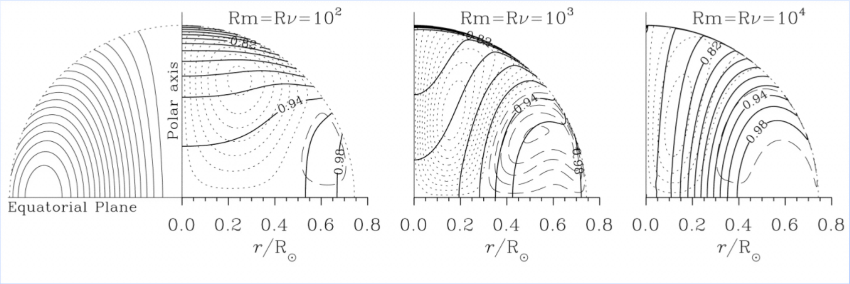
    Analogy with atmospheric models says that (hydrodynamic) 2D turbulence of this type acts so as to mix PV and drive the system away from solid body rotation

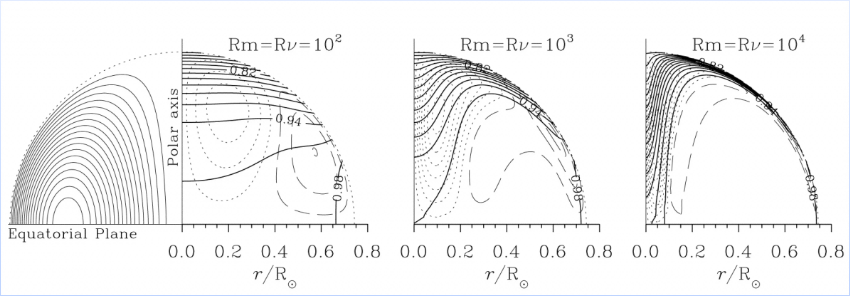
    (ALTHOUGH atmosphere $\nu/\kappa \sim 0.7$, tachocline $\nu/\kappa \sim 10^{-6}$)

? ***BIG QUESTION: Friction (S&Z) or Anti-friction (G&M)?***

…Gough & McIntyre (1998) claimed MUST need magnetic fields to prevent spreading of tachocline

Magnetic models





Some key facts supporting magnetic models:

- If hydrodynamic, differential rotation (tends to) propagate along cylinders (Taylor Proudman)
- If magnetic, then differential rotation (tends to) propagates along field lines (Ferraro)
- A weak relic field in the interior can keep the interior rotating as a solid body (Mestel & Weiss 1987 10^{-3} – 10^{-2} G)
- But if magnetic coupling spins down the radiative interior, why doesn't angular velocity propagate in along field lines?
- MacGregor & Charbonneau (1999) suggested that all the field lines must be contained in the radiative interior (no magnetic coupling)
- BUT how is this achieved? Problem is now one of MAGNETIC CONFINEMENT or self-organisation into regions of magnetic field and no magnetic field.

Magnetic models

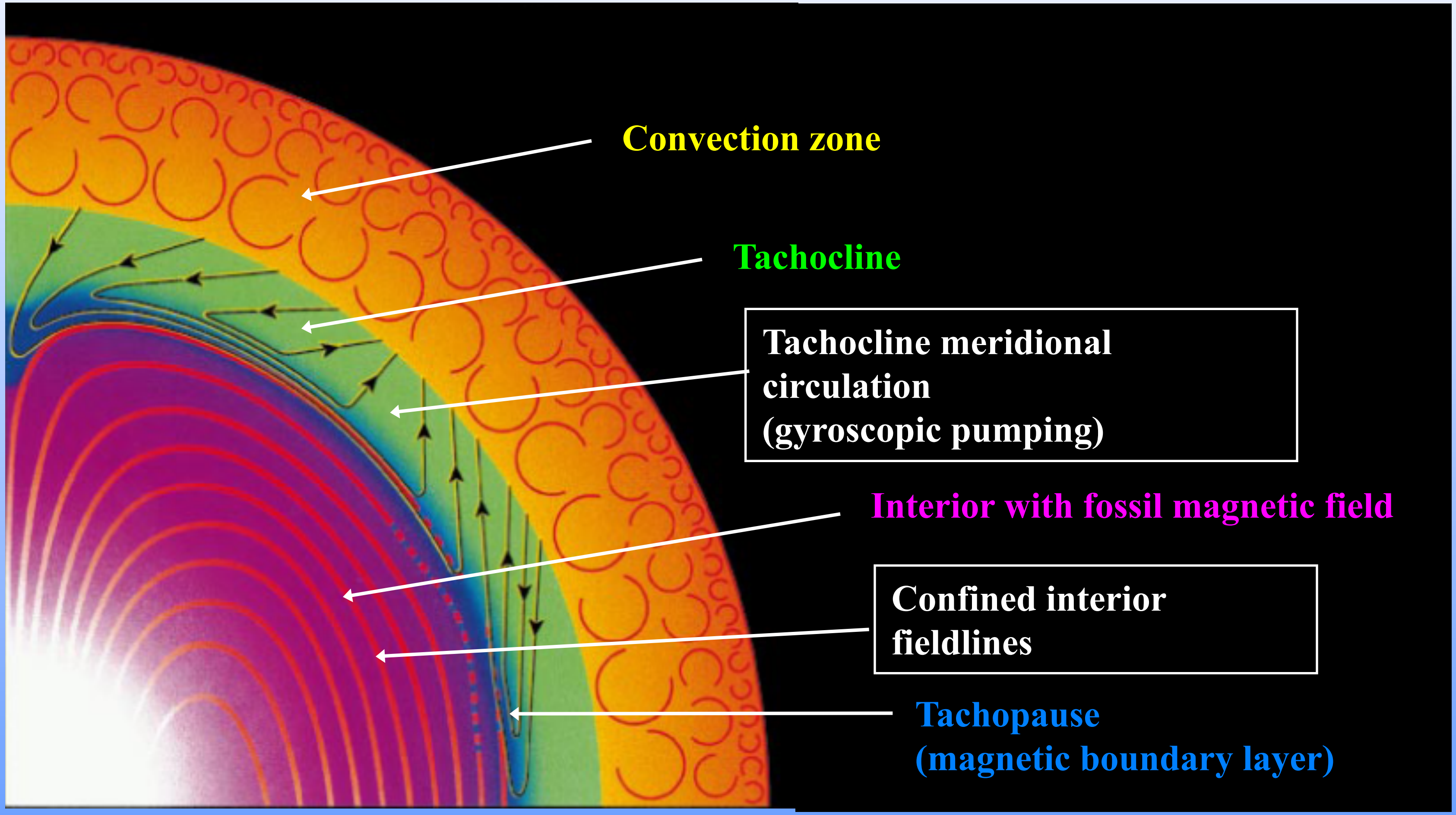
Idea: Can the system self-organise so that there is

- differentially-rotating region (convection zone)
- Radiative region containing magnetic field that homogenises the angular momentum latitudinally and produces a solid body interior
- (Must allow SOME coupling for spin-down, but not too much or propoagation of the differential rotation into the interior)

To do this, need

- some buffer/transition region to keep the two regions apart (tachocline)
- must be relatively (large-scale, horizontal) field-free (otherwise is same as interior)
- so there must exist a mechanism within that region for confining the magnetic field below

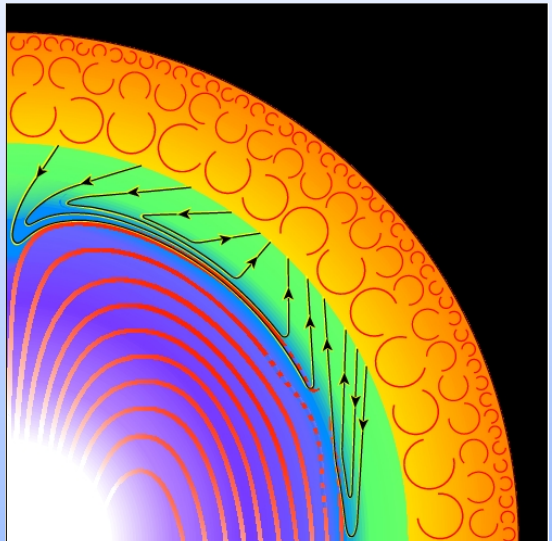
Gough & McIntyre, Nature, 1998



Serendipitous solution:

- Interior forced into isorotation by confined magnetic field
- Magnetic confinement achieved by meridional flows in the tachocline
- Tachocline spread halted by pesence of confined magnetic field

Gough & McIntyre



Gough & McIntyre argued that a meridional circulation due to gyroscopic pumping could contain the field.

Similar arguments to S&Z but obtain steady state in boundary layer analysis by balancing spreading with magnetic diffusion

$$2\Omega_i s \left(\frac{\partial \bar{\Omega}}{\partial z}\right)_s \approx \frac{g}{rT}\left(\frac{\partial \bar{T}}{\partial \theta}\right)_r$$

Thermal wind

$$\frac{N^2 T u}{g} \approx \frac{1}{\rho c_p r^2}\frac{\partial}{\partial r}\left(r^2 K \frac{\partial \bar{T}}{\partial r}\right)$$

Advection-radiative diffusion

$$2\Omega_i v \cos\theta \approx \frac{B_0}{\mu_0 \rho r \sin\theta}\frac{\partial}{\partial \theta}(B_\phi \sin\theta)$$

Coriolis-Lorentz

$$-B_0 \sin\theta \frac{\partial \bar{\Omega}}{\partial \theta} \approx \eta \frac{\partial^2 B_\phi}{\partial r^2}$$

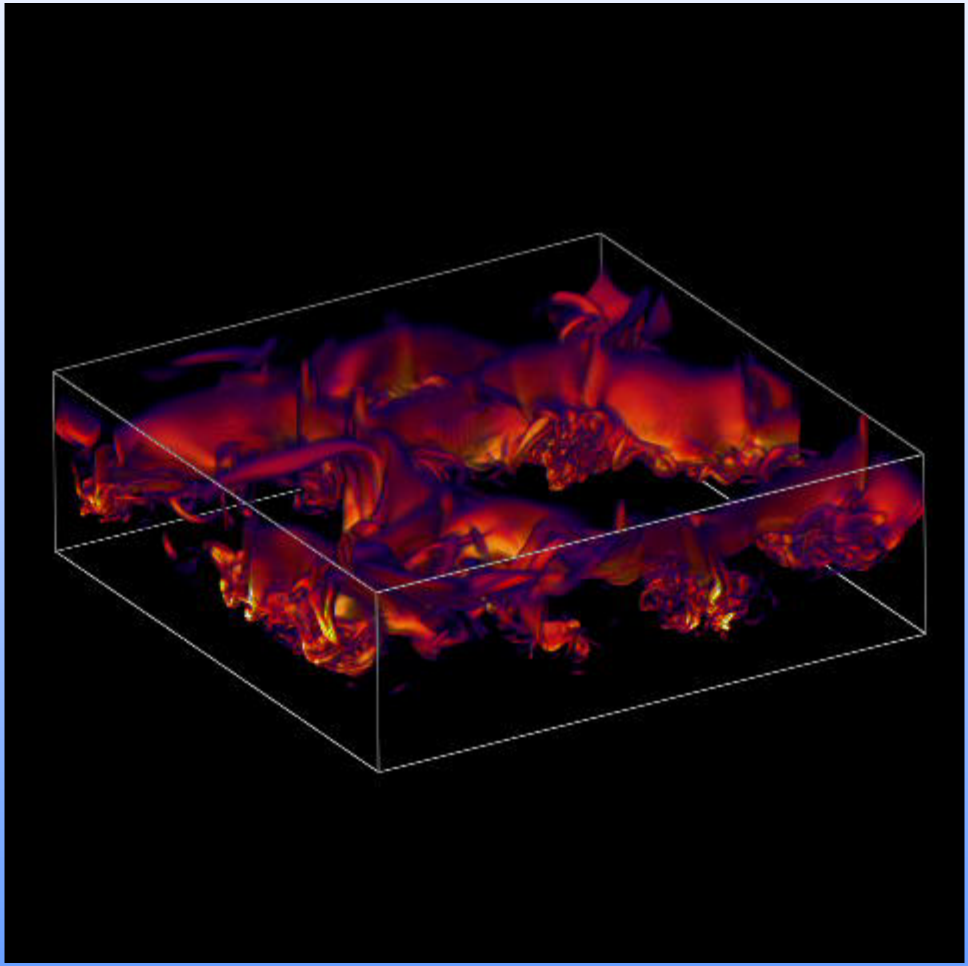
Induction

$$\Rightarrow \frac{\partial^6 \bar{T}}{\partial r^6} - \delta^{-6}\bar{T} \approx 0$$

$$\Rightarrow \delta = \left(\frac{2\mu_0 \rho \eta \kappa \Omega_i^2}{B_0^2 r_c^2 N^2 L^4}\right)^{1/6} r_c$$

Boundary layer thickness

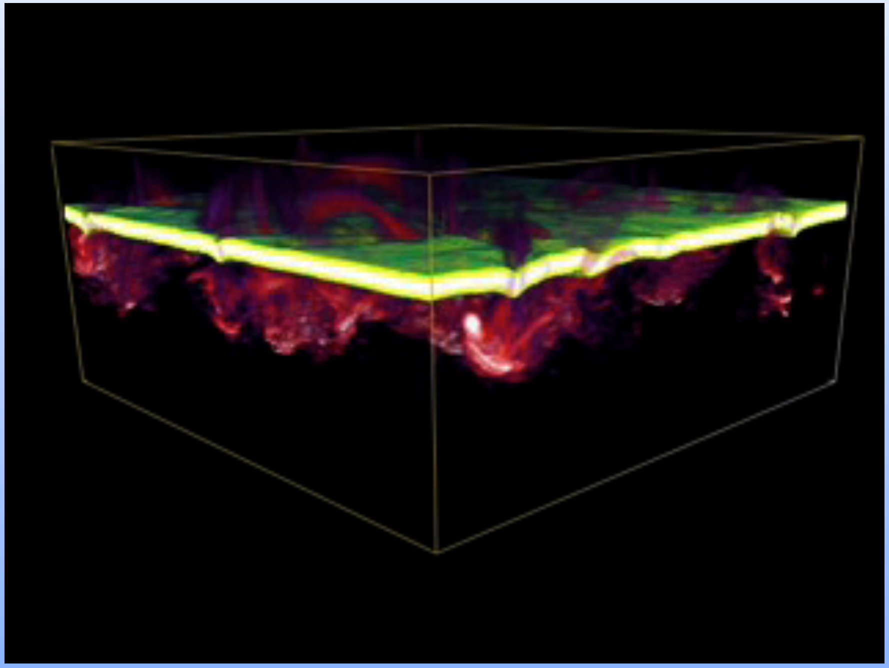
What else confines B?: Turbulent transport



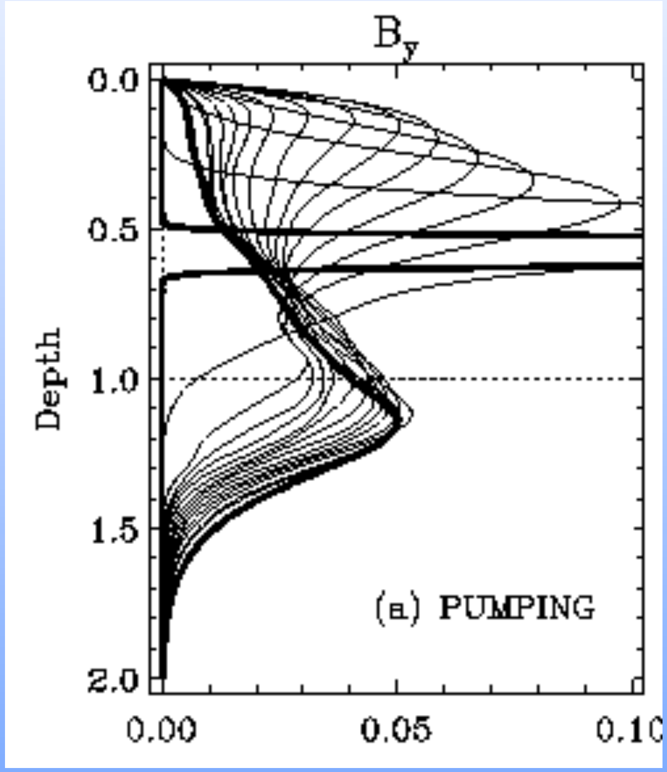
Brummell, Clune & Toomre 2002

- Penetrative convection
  - o Extension of unstable motions into a stable regime = transport of momentum across barriers

What else confines B?: Turbulent transport



Tobias, Brummell, Clune & Toomre 1998

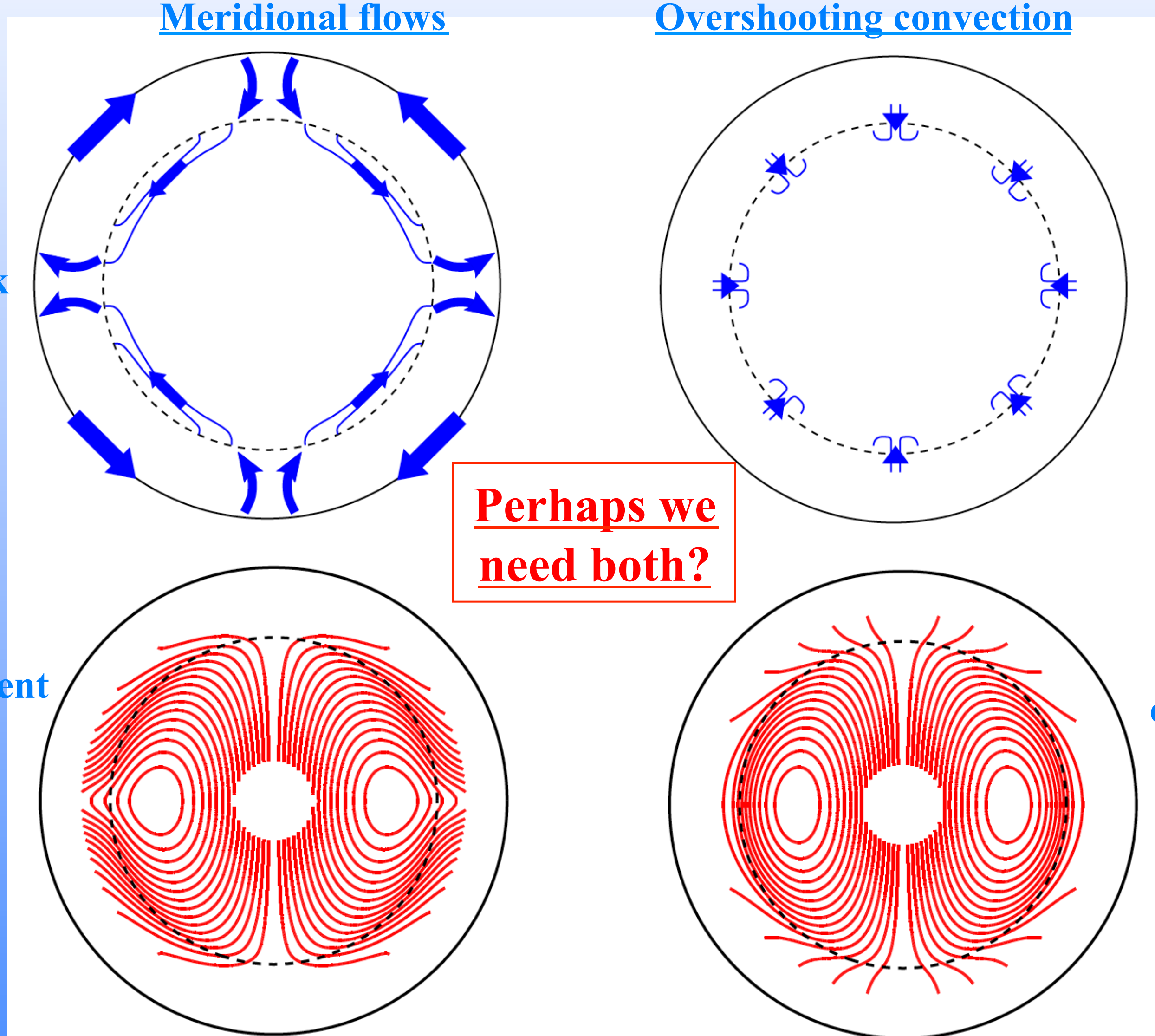


Magnetic pumping

- Turbulent transport of magnetic fields by penetrative convection

What is the effect of convection + waves in the stable region?
FAST time scale issues.

Slow vs. fast?



Slow
Might work at poles

Fast
Works on horizontal field

Polar confinement

Equatorial confinement

Can these magnetic confinement ideas work?

<u>Are gyroscopically-pumped meridional flows generated?</u>

- How deep do they penetrate and how strong are they?

<u>Does the Gough and MacIntyre type balance ever hold?</u>

- Can differential rotation generate a meridional circulation that burrows into the tachocline and confines a magnetic field?

<u>Does convective turbulence/waves help or hinder the situation?</u>

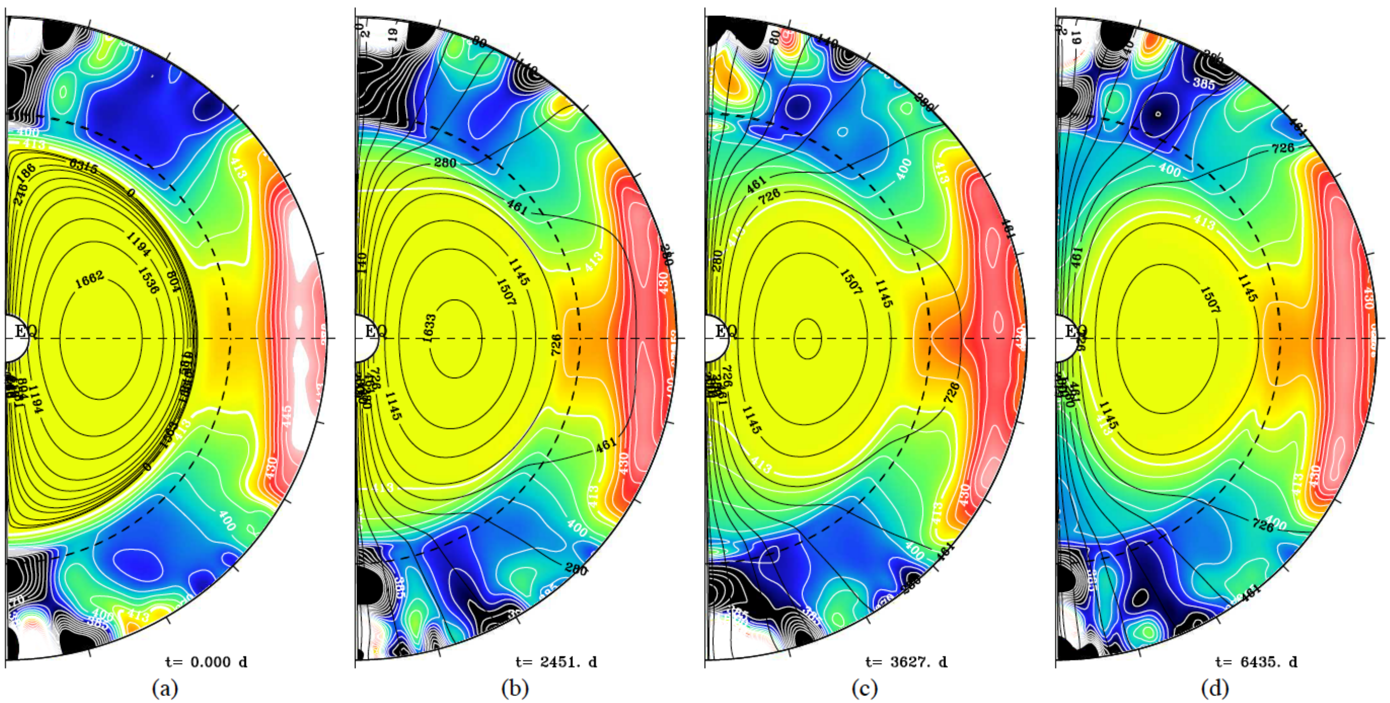
People have tried to answer this question by performing numerical experiments:

- Garaud 2002
- Brun & Zahn 2006
- Garaud & Garaud 2008
- Strugarek, Brun & Zahn 2011

<u>Mamy have tried and failed to reproduce the G&M balanced scenario!</u>

Latest global numerical simulations

e.g. Strugarek, Brun & Zahn 2011 (A&A, 5332,34)
Global numerical MHD simulations of a deep spherical shell using ASH



Strugarek et al. 2011
(the ASH code)

<u>No confinement found</u>

New ideas from simple models

However, through a series of simple models and new simulations, we believe we now have an idea as to why the previous numerical simulations may have failed to find the G&M balance, and what may be necessary to achieve magnetic confinement.

Models:

- Garaud & Brummell 2008
- Garaud & Acevedo-Arreguin 2009
- Wood, McCaslin & Garaud 2011

New simulations:

- Cartesian, compressible : Wood & Brummell (2012, 2014)
- Spherical, Boussinesq : Guervilly, Wood, Garaud, Brummell + students, in progress

Briefest synopsis of new ideas from simple models

Meridional flows are generated by gyroscopic pumping due to forced differential rotation

Hydro, unstratified: (Acevedo-Arreguin & Garaud)
Must be Taylor-Proudman in interior
Existence of vertical flow therefore depends on lower boundary layer
No-slip -> Ekman pumping -> allows completion of penetrating meridional flow
No-stress -> no meridional flow possible
Boundary stresses required to complete/allow meridional penetration

Magnetic, unstratified: (Wood, McCaslin &Garaud)
Existing (pre-confined) magnetic fields can provide stresses and therefore allow penetrating flow.

Stratification:
Strong stratification ($\sigma > 1$) tends to prevent penetration of flow
Moderate stratification ($\sigma < 1$) allows moderate penetration of flow - tachocline plus tachopause

Overall:
Two constraints:
✓ one mechanical (stresses to communicate flow). Magnetic field will do!
✓ one thermal (stratification)

Must have all this right for G&M scenario!

TEST THESE IDEAS OUT WITH NEW NUMERICAL MODELS?

Key issue: Timescales

Propagation of meridional flows governed by many timescales.
For the Sun, the relevant ordering is:

buoyancy time		rotation time		Eddington–Sweet time		viscous time
$\frac{1}{N}$	$\ll$	$\frac{1}{\Omega}$	$\ll$	$\left(\frac{N}{\Omega}\right)^2 \frac{L^2}{\kappa}$	$\ll$	$\frac{L^2}{\nu}$

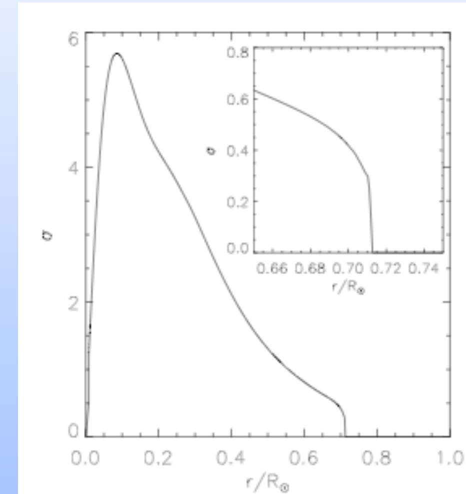
$$\Rightarrow \quad \frac{\nu}{\kappa} \ll \left(\frac{\Omega}{N}\right)^2 \ll 1$$

Determining parameter:

$$\sigma = \frac{N}{\Omega}\sqrt{P_r} << 1$$

Key issue: Timescales

Determining parameter: $\sigma = \frac{N}{\Omega}\sqrt{P_r} << 1$



Real Tachocline

$$N \approx 10^{-3},\ \Omega \approx 10^{-6},\ \mathrm{Pr} \approx 10^{-8} \rightarrow \sigma = \frac{N}{\Omega}\sqrt{\mathrm{Pr}} \approx \mathbf{0.1}$$

(N, Ω): Solar; Pr: Solar

Tachocline simulations

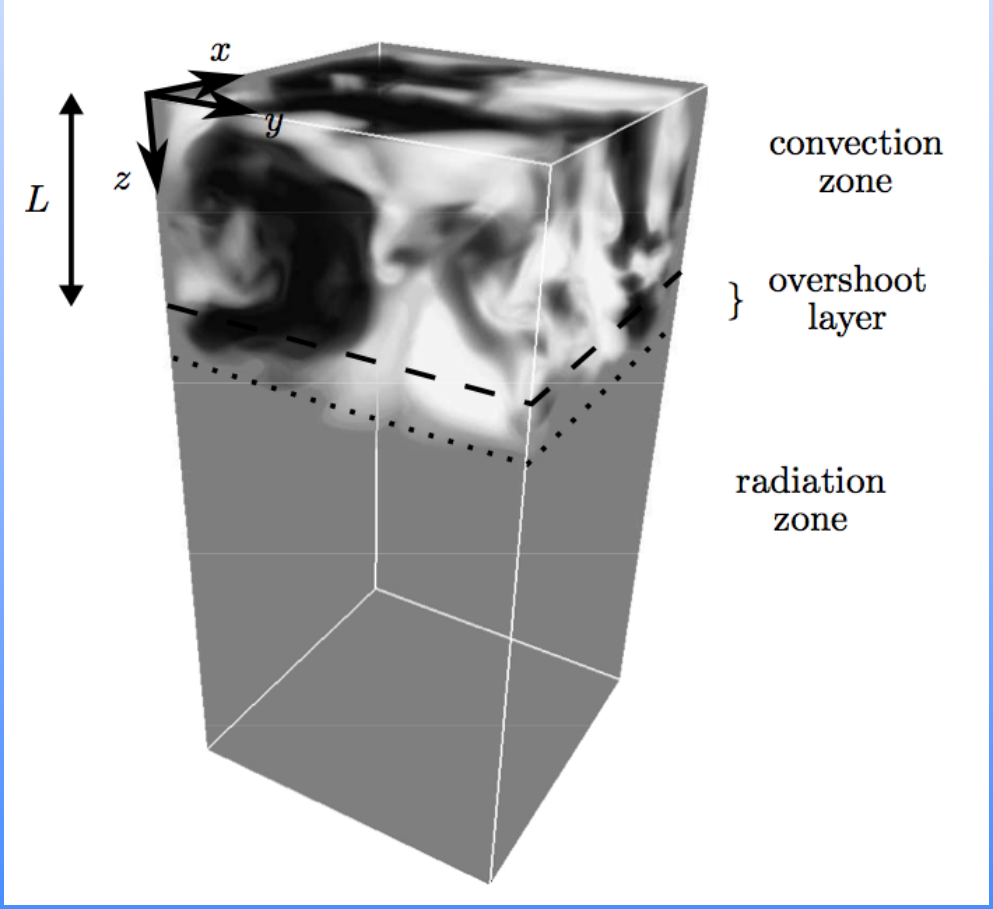
$$N \approx 10^{-3},\ \Omega \approx 10^{-6},\ \mathrm{Pr} \approx 10^{-2} \rightarrow \sigma = \frac{N}{\Omega}\sqrt{\mathrm{Pr}} \approx 100$$

(N, Ω): Solar; Pr: Best you can do with a simulation!

Have to give up on being "as solar as possible"!

Wood & Brummell 2012

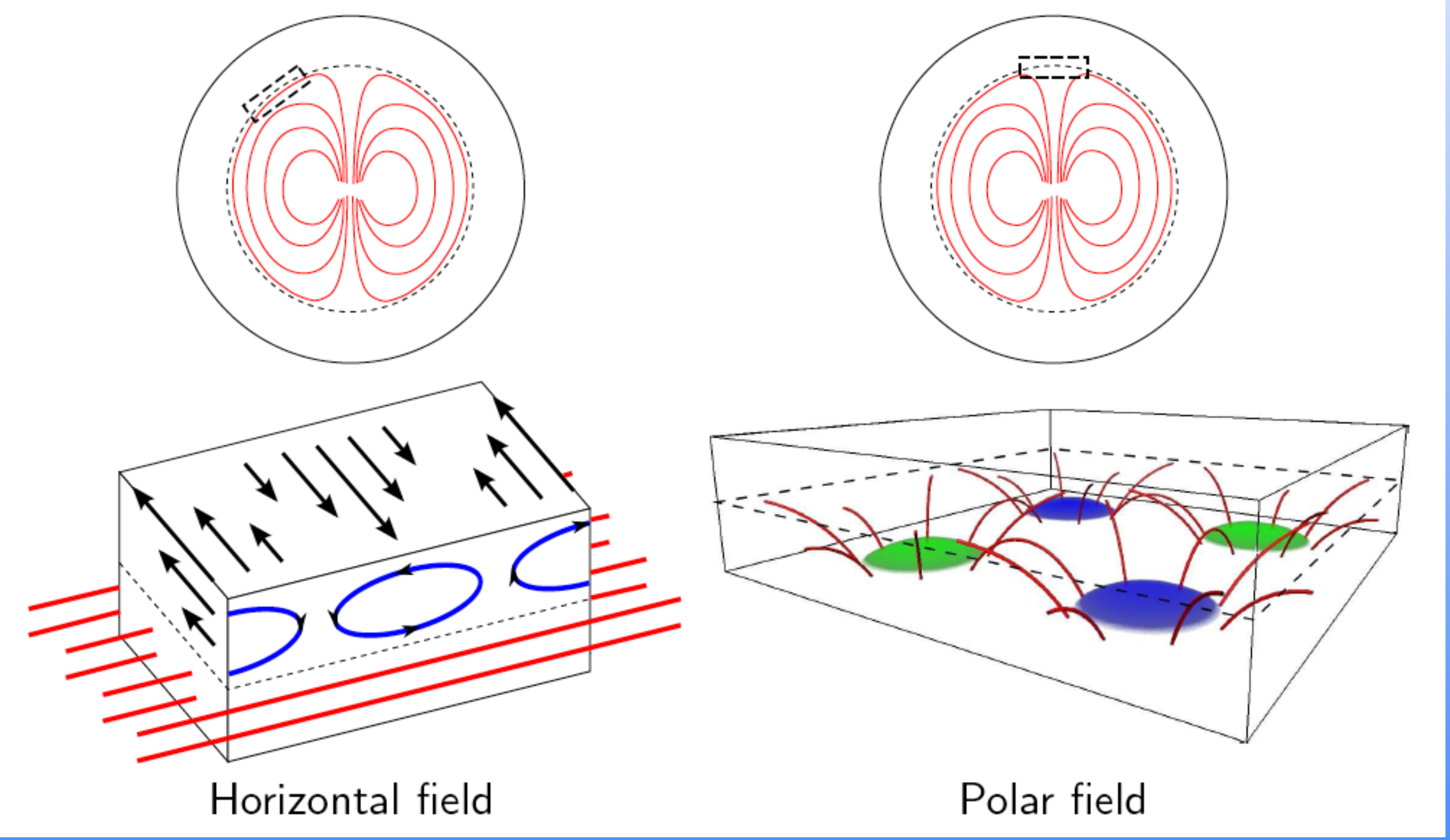
Cartesian simulations: compressible, f-plane, penetrative
Give up on being solar - get σ right
Timescales in right order but wrong separation



Wood & Brummell 2012

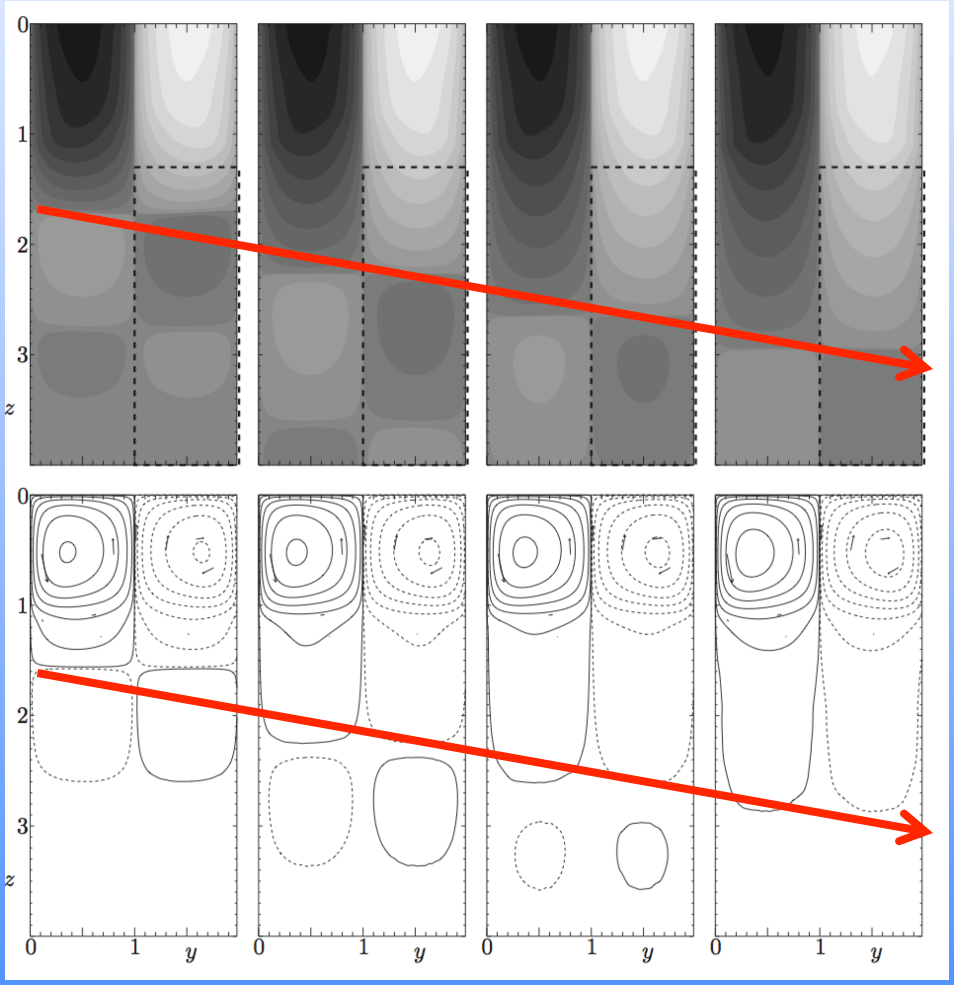
Cartesian simulations: compressible, f-plane

Forced "differential rotation" + initial applied magnetic field: two configurations



Wood & Brummell 2012

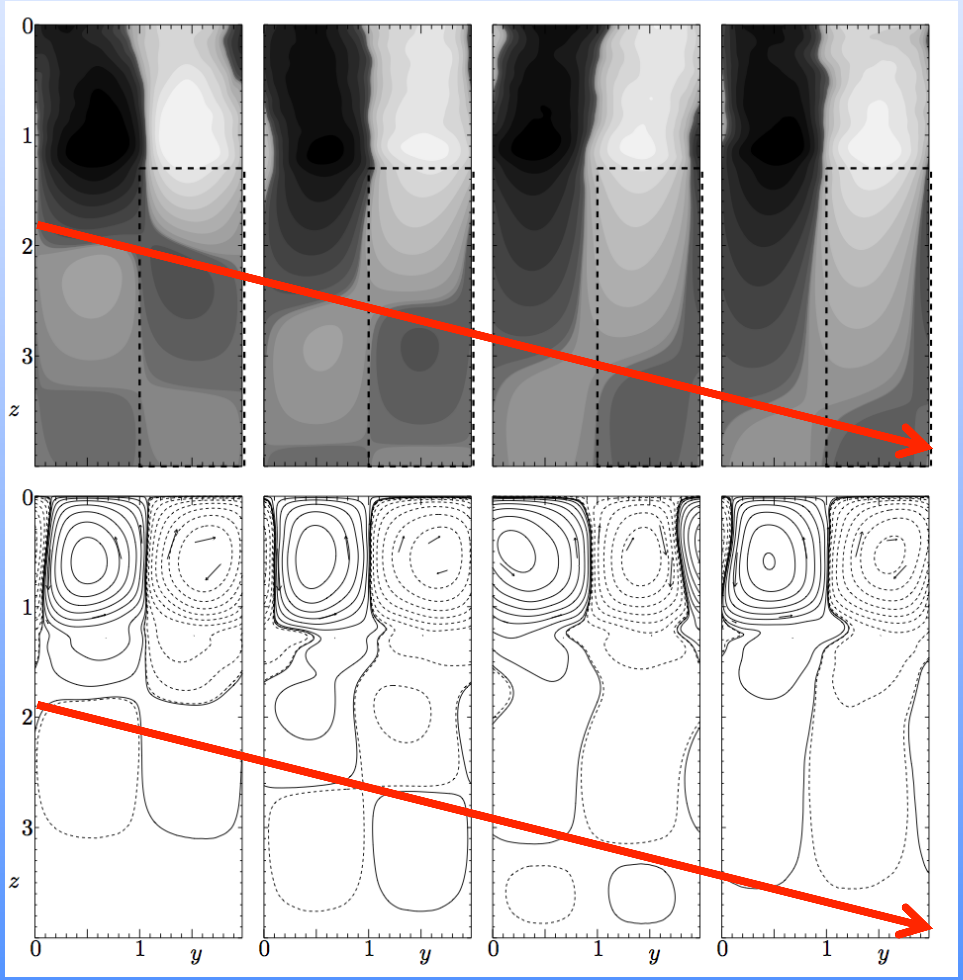
σ<1, no convection, no magnetic field

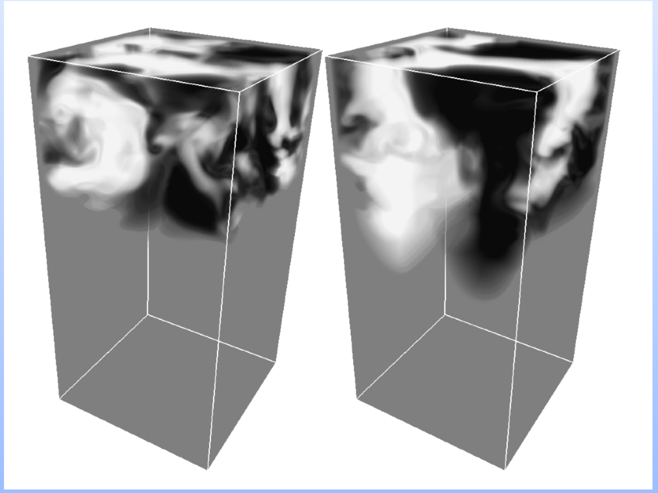


Spiegel & Zahn burrowing!
(as expected)

Wood & Brummell 2012

σ<1, convection+waves, no magnetic field

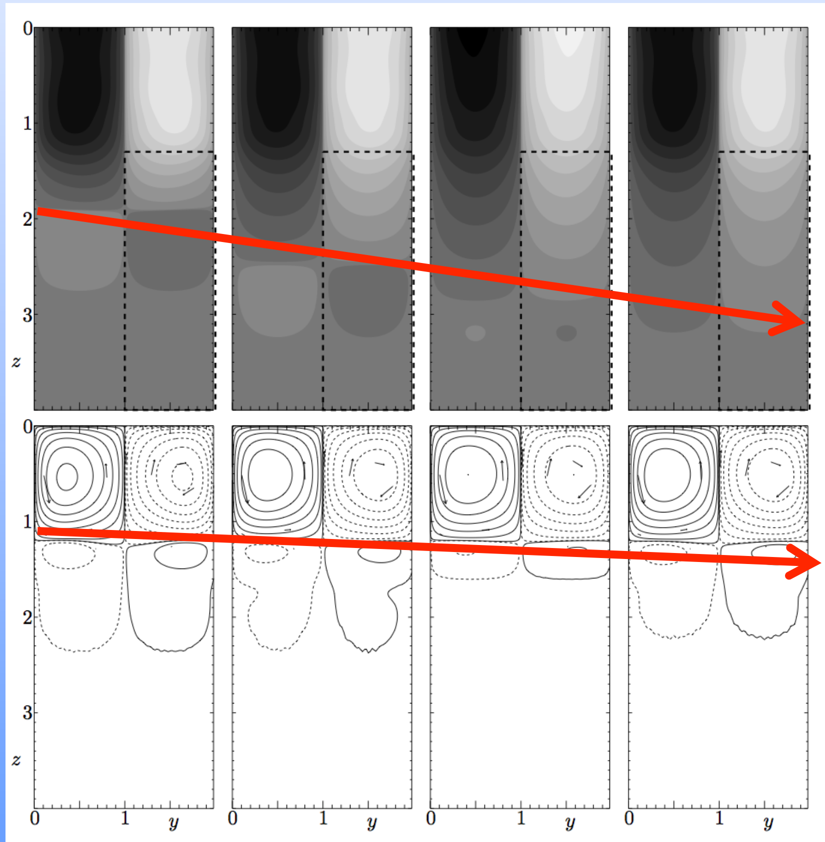


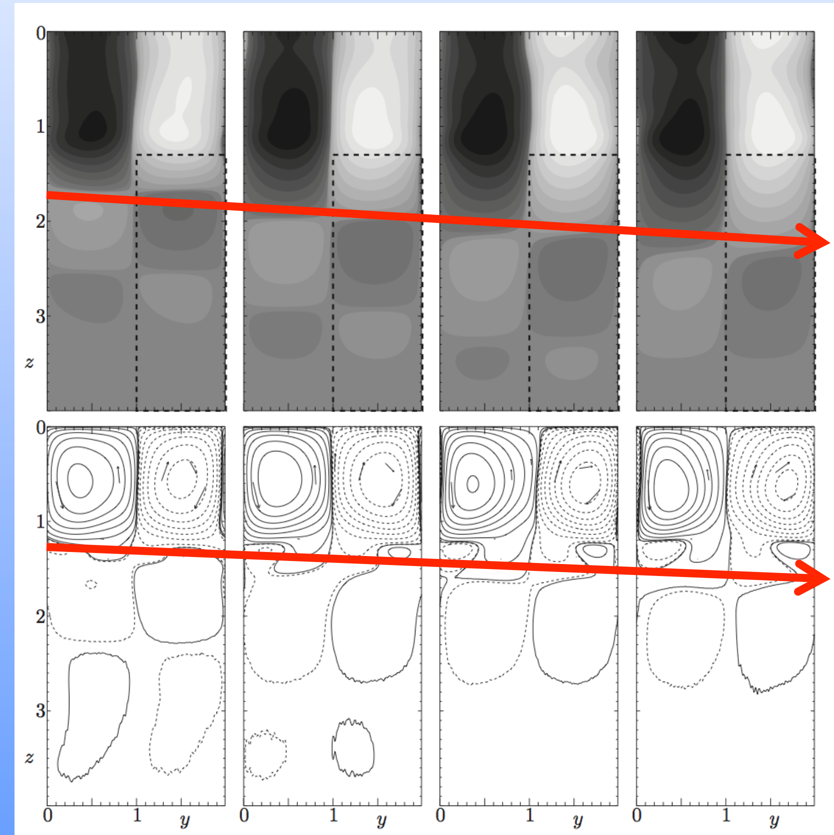


STILL get
Spiegel & Zahn burrowing!
(new!)

Wood & Brummell 2012

σ>1, weak (left) and stronger (right) convection, no magnetic field





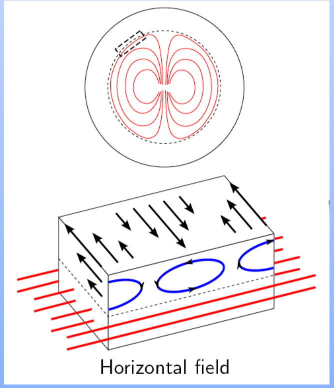
No Spiegel & Zahn burrowing!

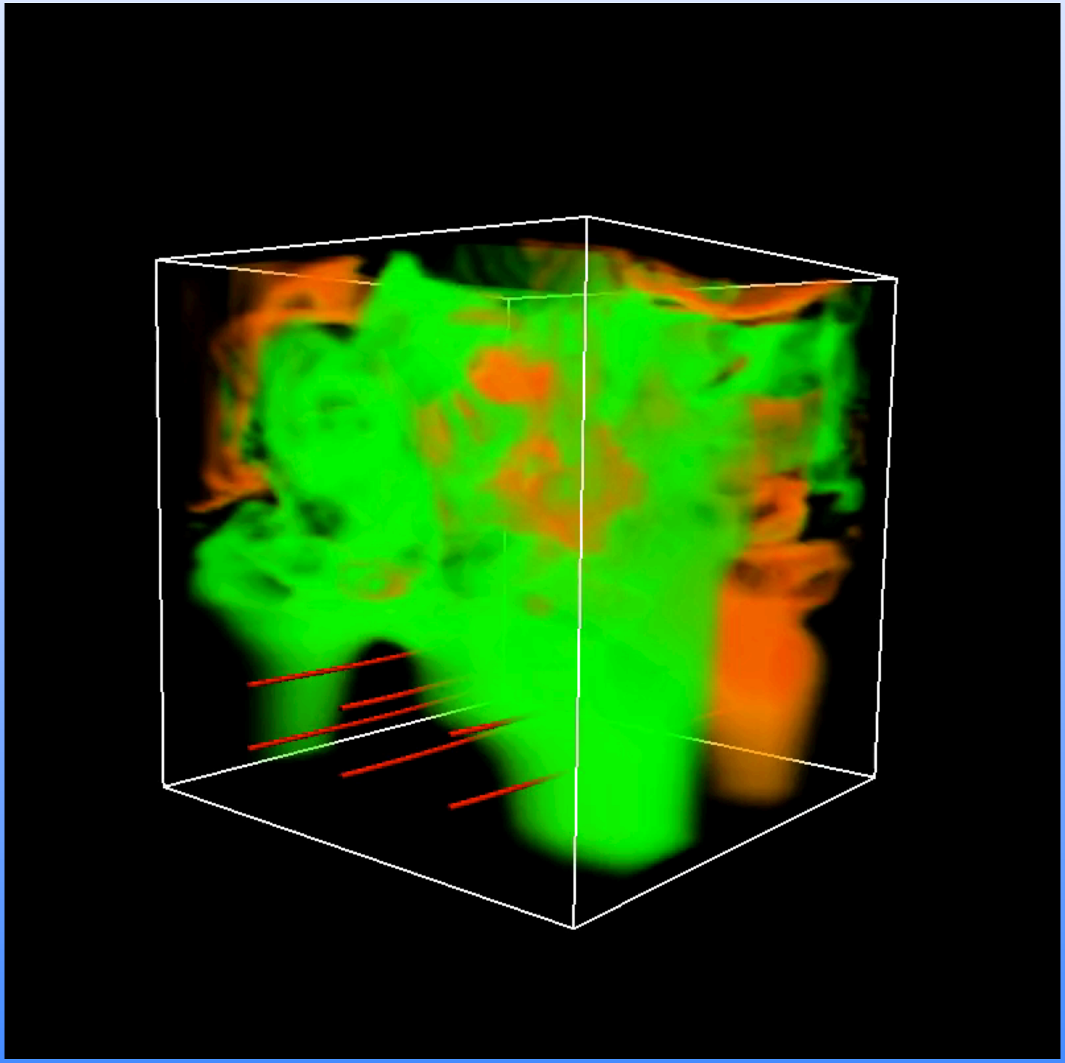
(weak counter rotating meridional cells; weak propagation of shear by viscous diffusion)

Wood & Brummell 2014 (in prep)

σ<1, convection+waves, magnetic field

- Start with burrowed flow
- Insert magnetic field
- Evolve

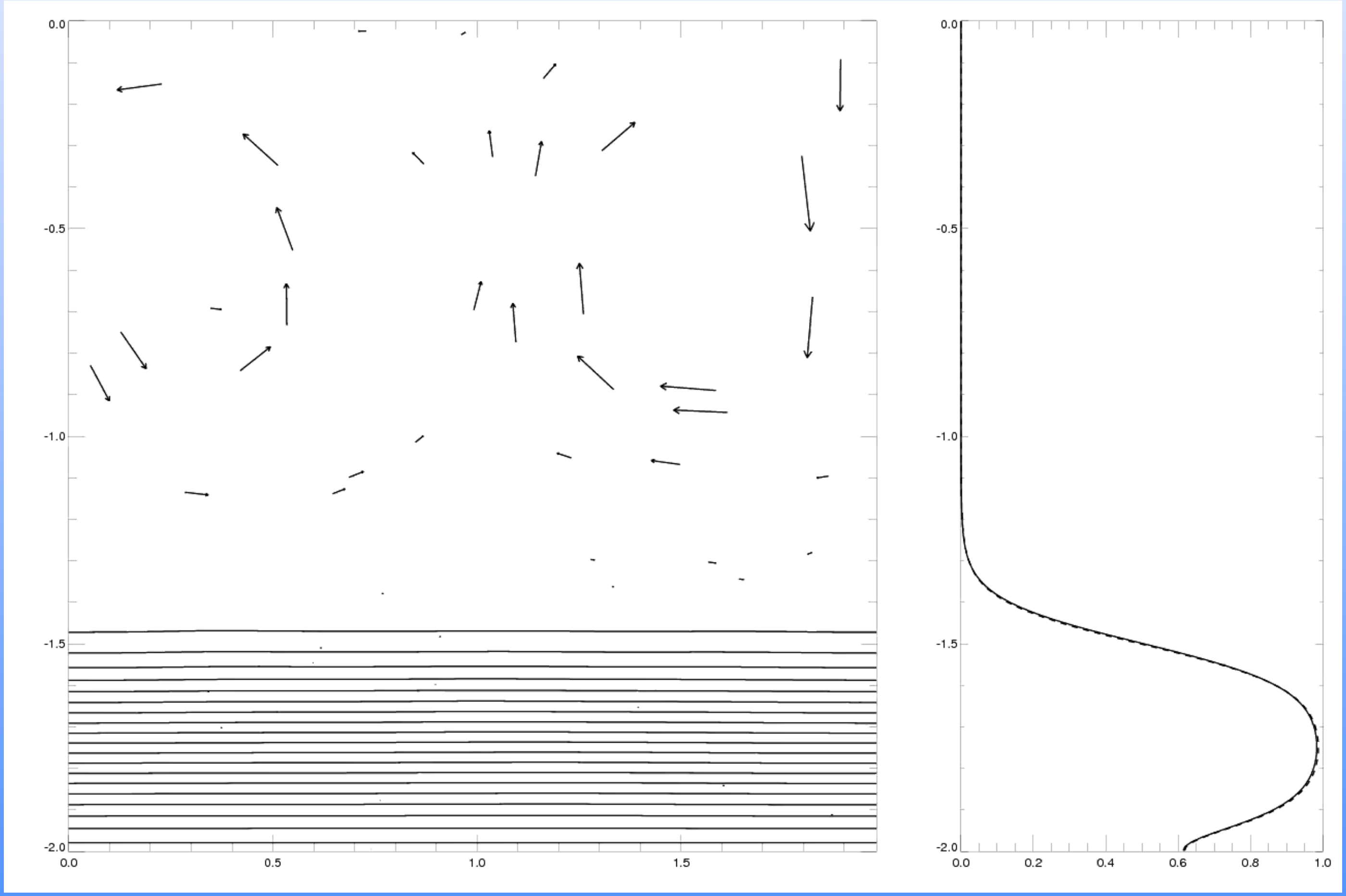




- Horizontal field homogenises the interior (removes differential rotation)

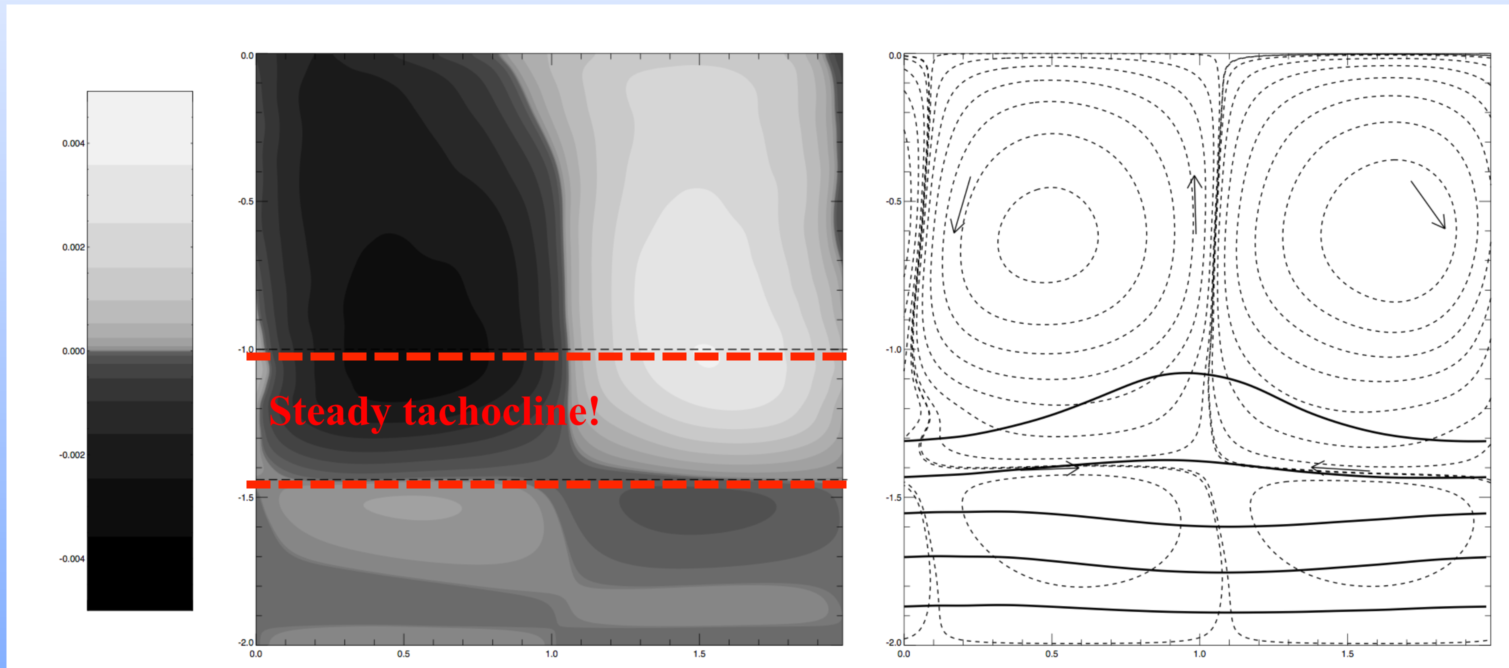
Wood & Brummell 2014 (in prep)

$\sigma<1$, convection+waves, magnetic field

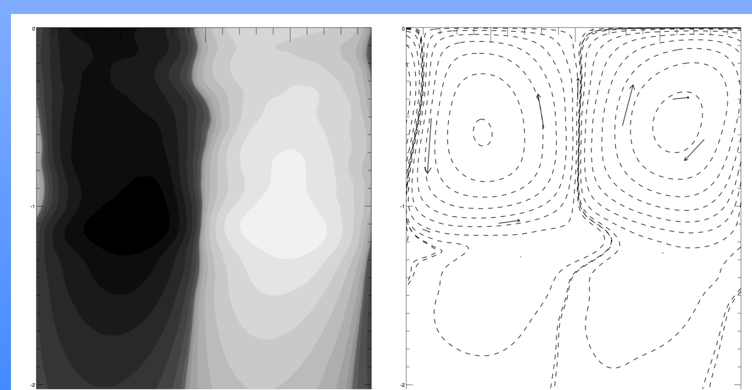


Wood & Brummell 2014 (in prep)

σ<1, convection+waves, magnetic field



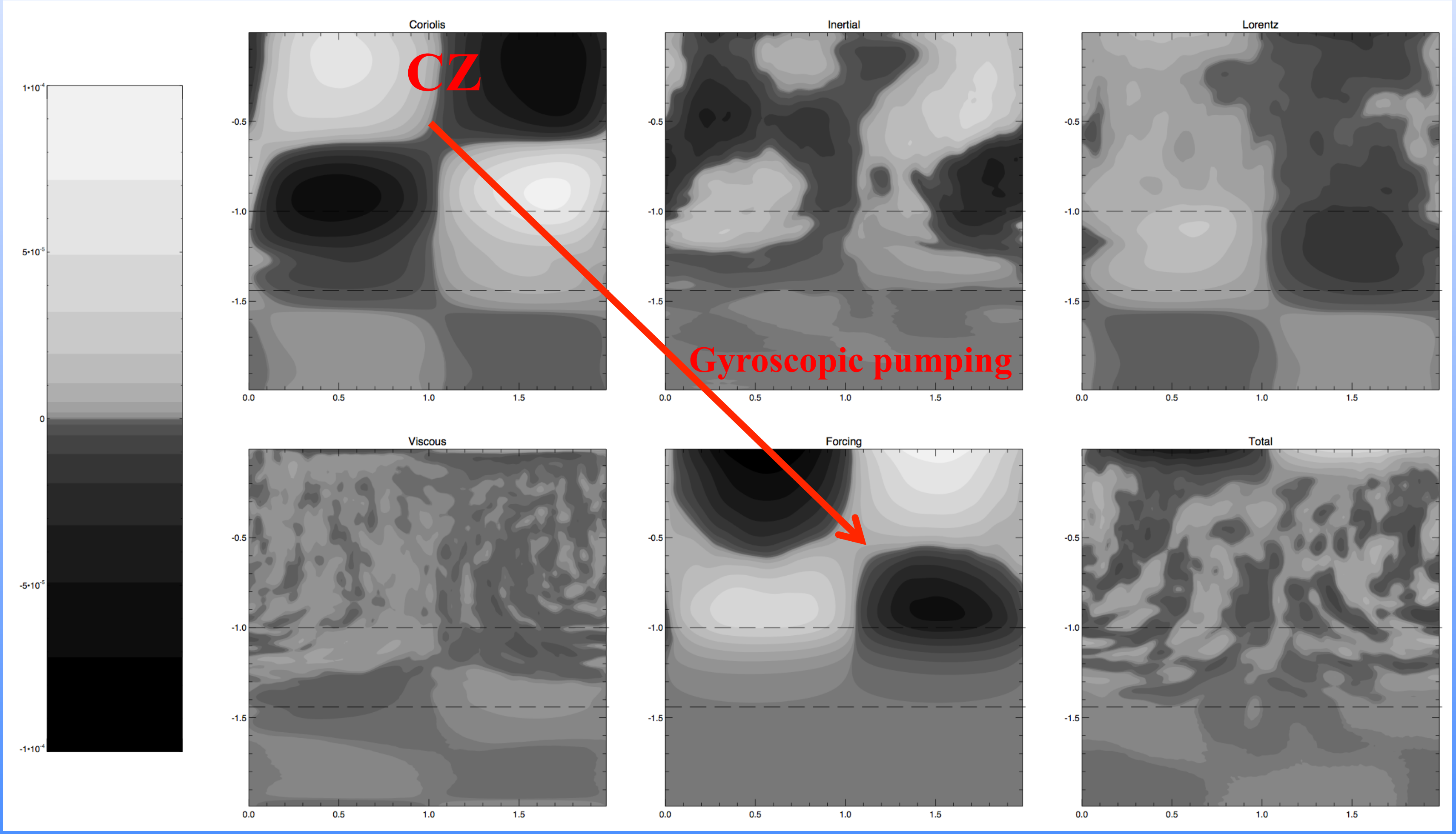
No magnetic field case:



Wood & Brummell 2014 (in prep)

σ<1, convection+waves, magnetic field

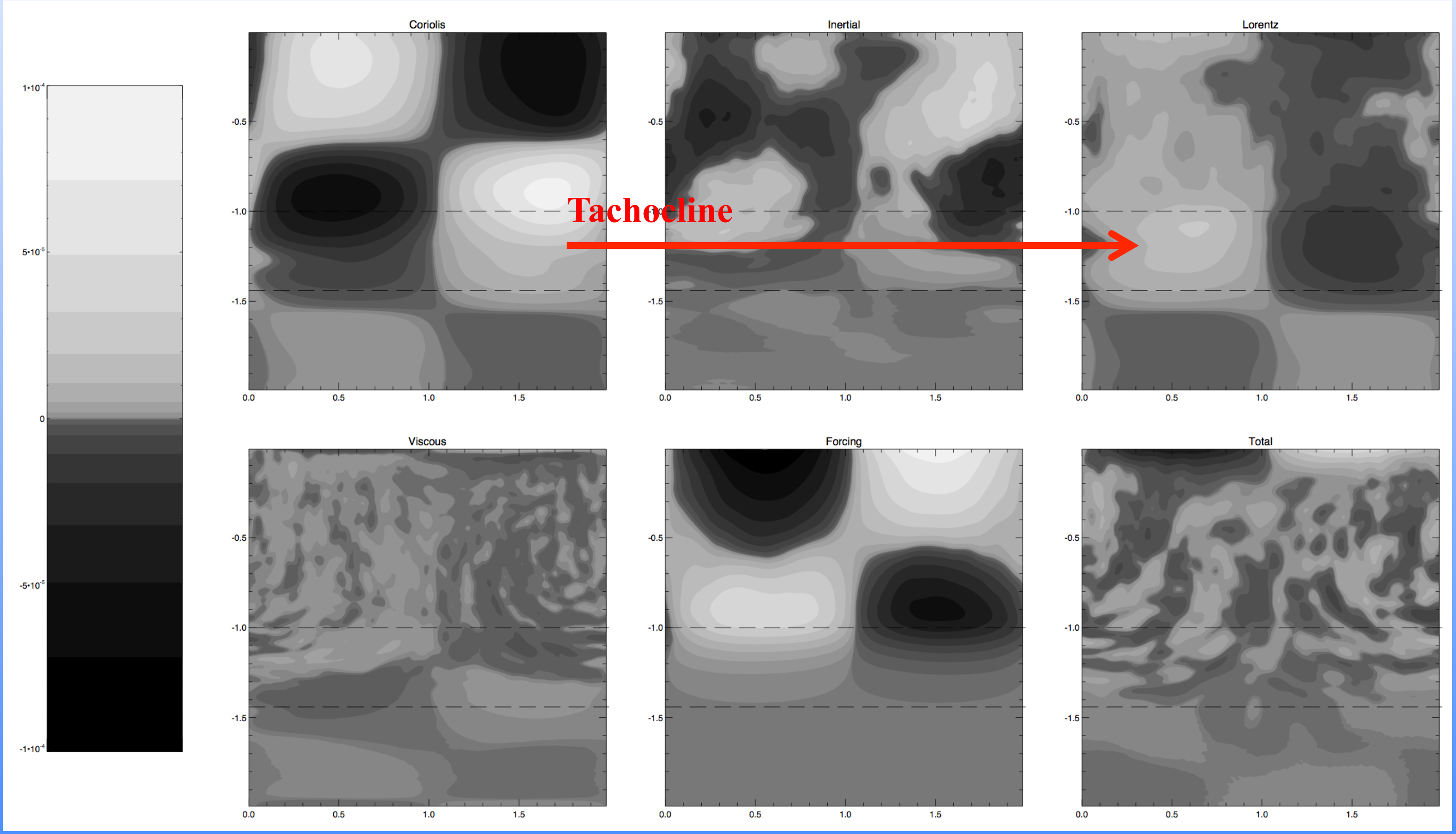
Azimuthal momentum balance



Wood & Brummell 2014 (in prep)

σ<1, convection+waves, magnetic field

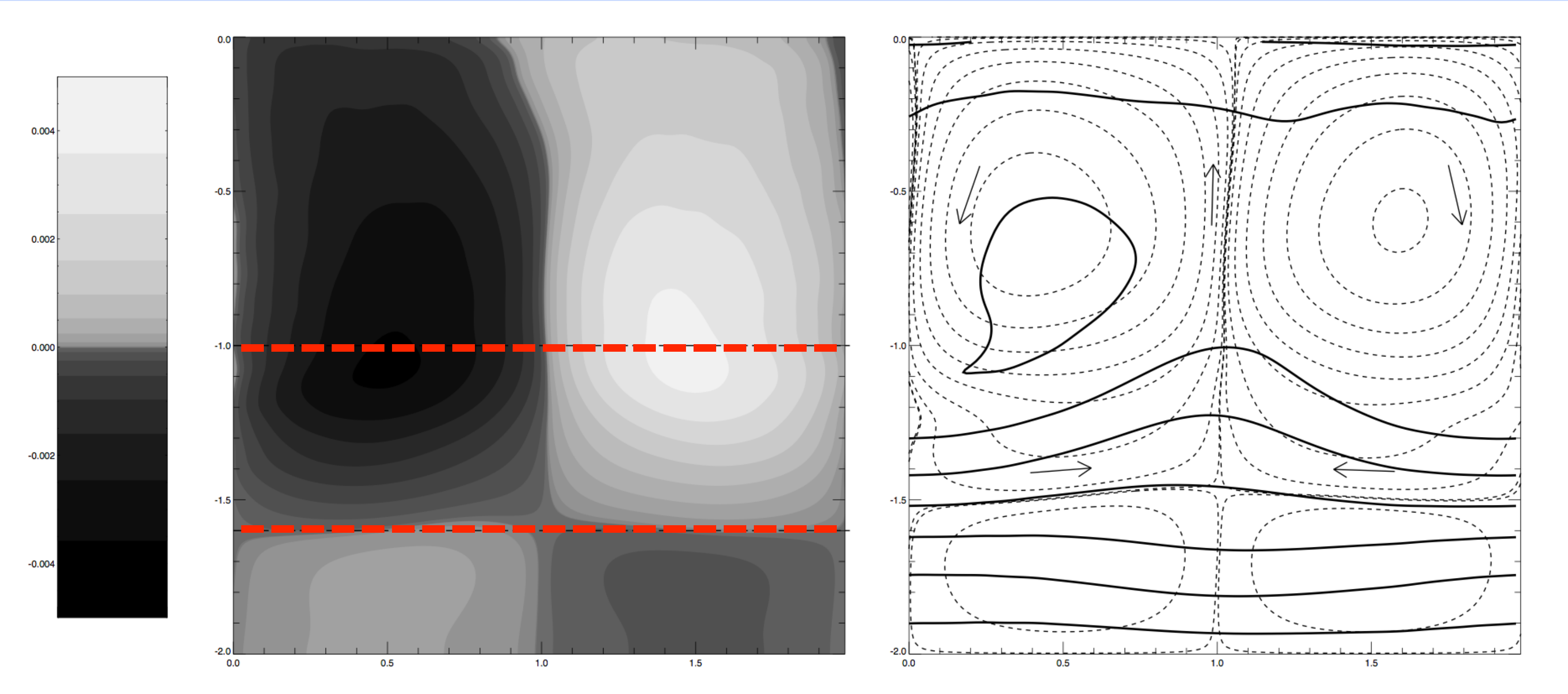
Azimuthal momentum balance



Wood & Brummell 2014 (in prep)

σ<1, convection+waves, magnetic field

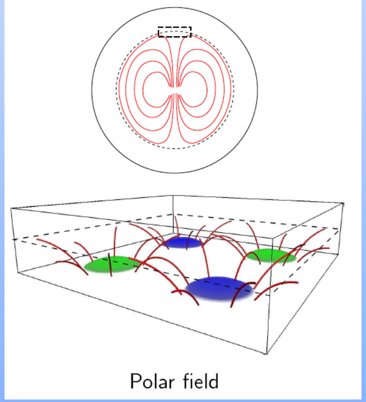
Weaker field, deeper tachocline



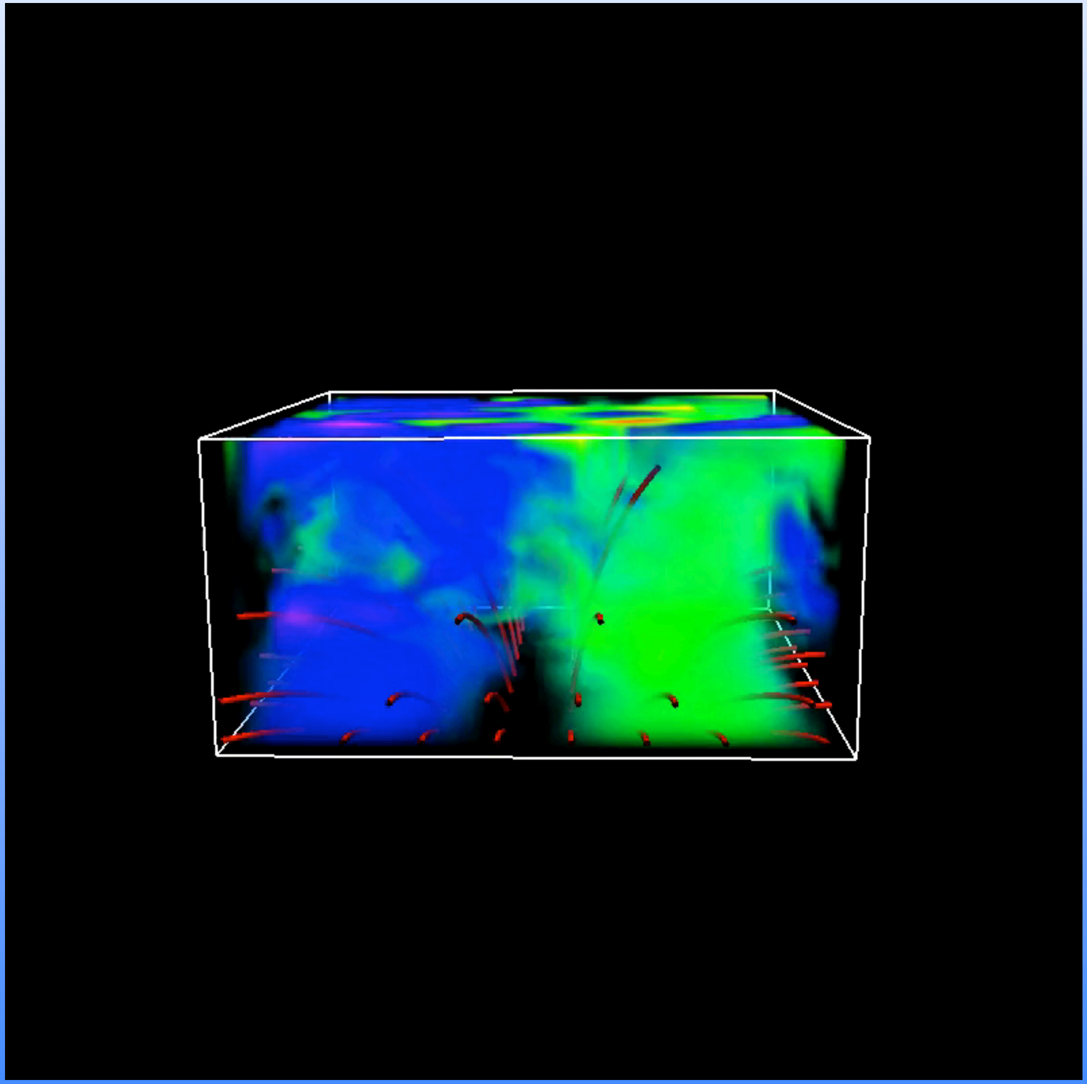
Wood & Brummell 2014 (in prep)

σ<1, convection+waves, magnetic field - polar

- Start with burrowed flow
- Insert magnetic field
- Evolve

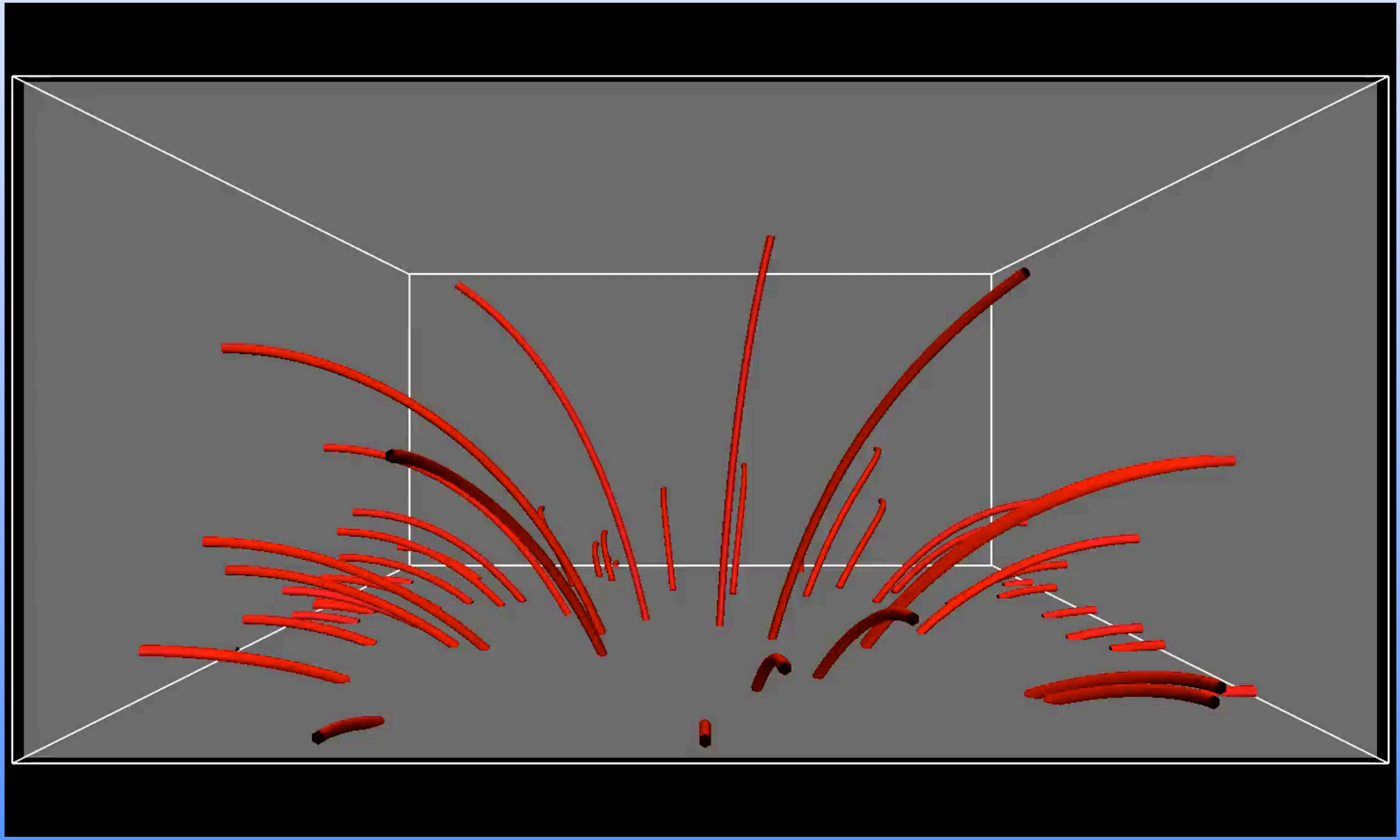


- Polar field homogenises the interior
- Field confined substantially by meridional flows



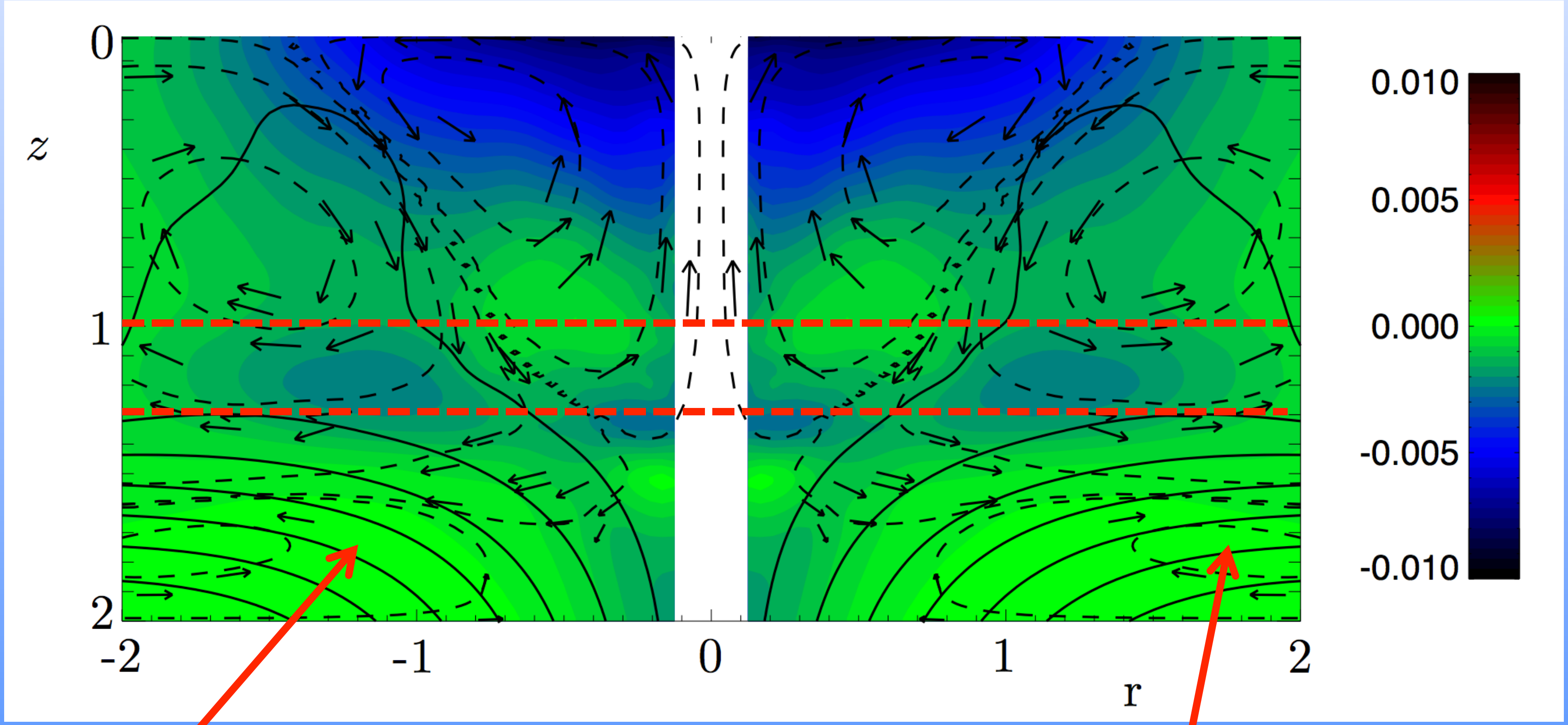
Wood & Brummell 2013 (in prep)

$\sigma<1$, convection+waves, magnetic field - polar



Wood & Brummell 2013 (in prep)

σ<1, convection+waves, magnetic field - polar



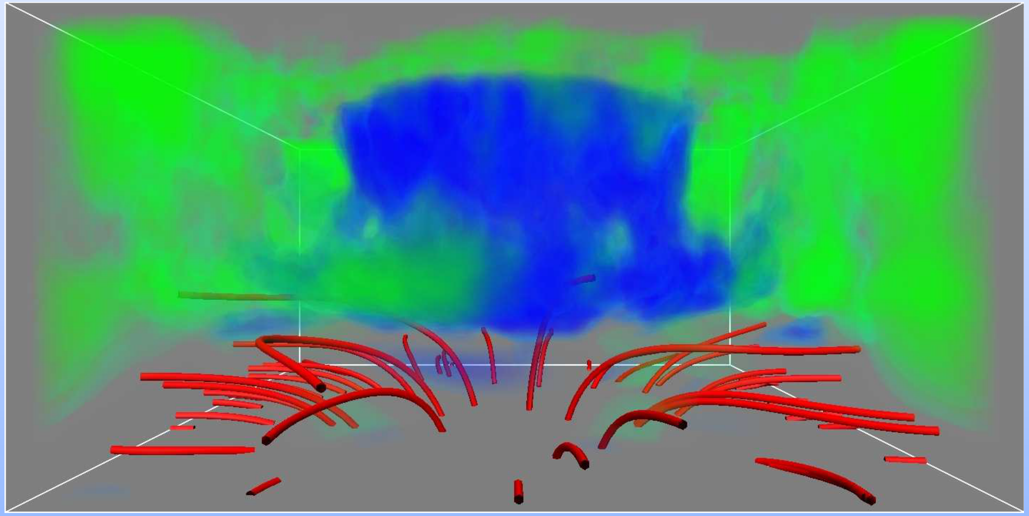
Uniform rotation of radiative zone

Confined field

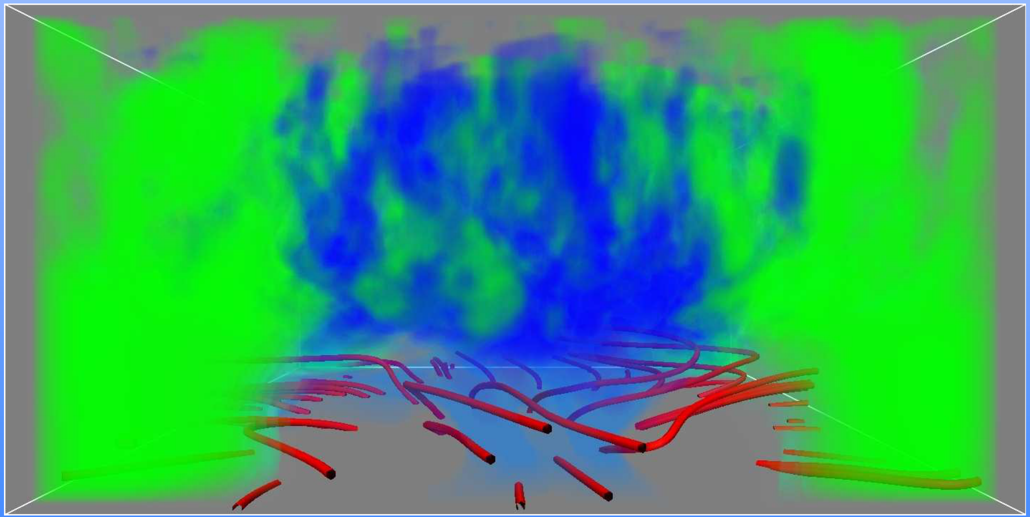
Wood & Brummell 2014 (in prep)

σ<1, convection+waves, magnetic field - polar

Strong field



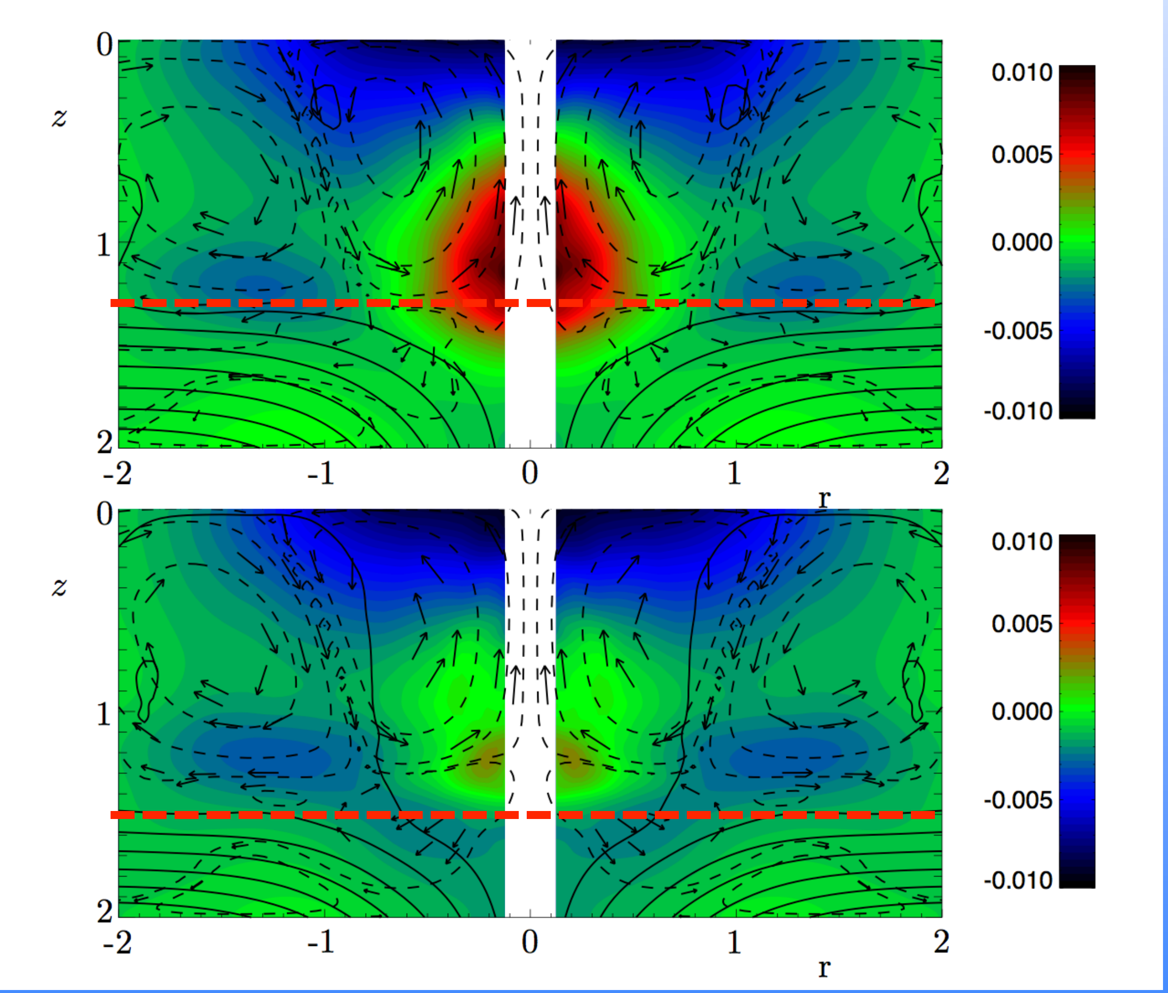
Weak field



Wood & Brummell 2014 (in prep)

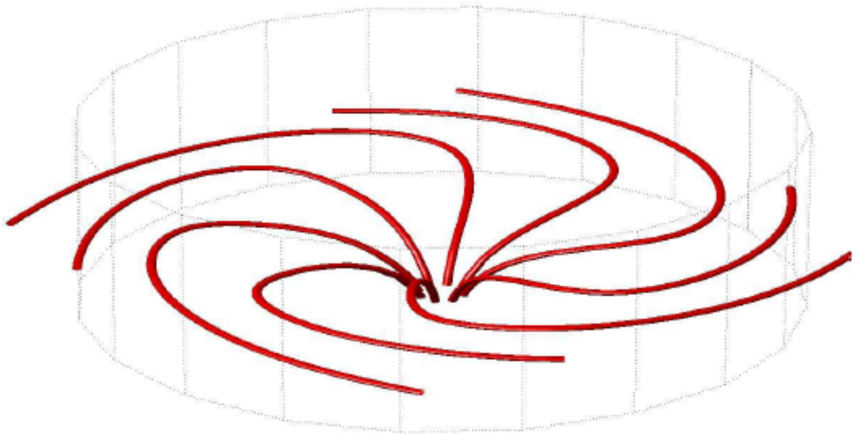
σ<1, convection+waves, magnetic field - polar

Weaker field

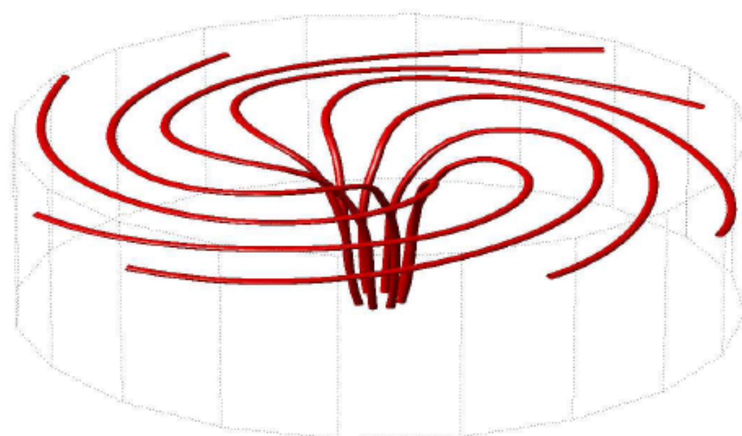


Wood & Brummell 2014 (in prep)

Do we need the slow, laminar, meridional circulation component?



With downwelling



With upwelling

It certainly helps the polar confinement.

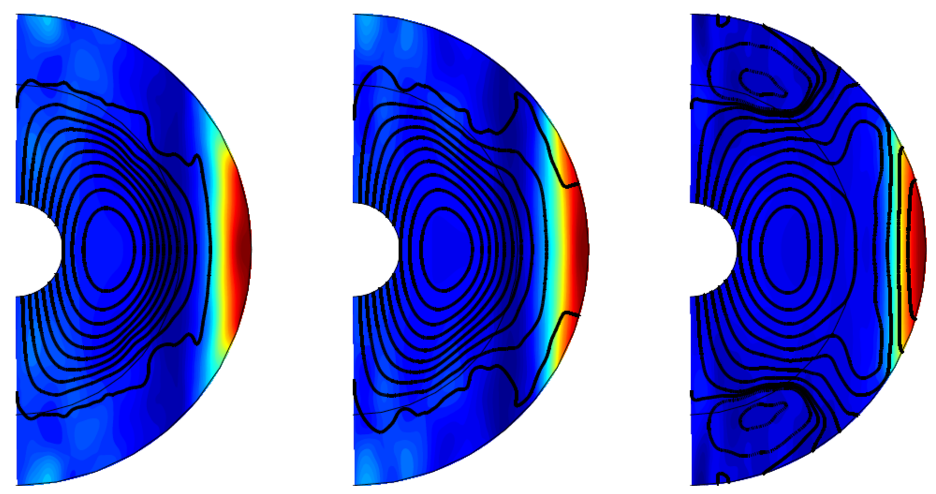
Conclusions

- Gough & McIntyre scenario can work.
- Magnetic field can enforce uniform rotation in the deep interior.
- Magnetic field can be confined (large-scale, non-dynamo field)
- Confinement may be a combination of turbulent and laminar effects
- For laminar effects to work
  - Timescales have to be in the correct order.
  - Burrowing of meridional flow must not be obscured by viscous effects
  - Have to give up being truly solar!

Test ideas further:
Remove artificial geometrical constraints of Cartesian periodic box.

Global Boussinesq simulations (PARODY code) in progress:
Guervilly, Wood, Brummell, Garaud

Clarify role of laminar and turbulent


Guervilly et al. (in prep.)

Current thinking… layers of the tachocline

CONVECTION ZONE

NOMINAL BASE CZ

DEPTH

OVERSHOOTING (AND MAYBE PENETRATIVE) CONVECTION
TURBULENCE IS 3D
MAGNETIC
STRONG MEAN DYNAMO FIELD
BUOYANCY INSTABILITIES

OVERSHOOT ZONE

END OF OVER-SHOOTING

MHD TURBULENCE DRIVEN FROM ABOVE
VERY STABLY STRATIFIED – 2D?
WEAK MERIDIONAL FLOW
=> LATITUDINAL ANGULAR MOMENTUM TRANSPORT
MAINLY FIELD-FREE? WEAK FIELD?

FIELD-FREE ZONE

BASE OF TACHOCLINE

ADJUSTMENT REGION BETWEEN FIELD-FREE TACHOCLINE AND FOSSIL FIELD INTERIOR

TACHOPAUSE

INTERIOR

FOSSIL MAGNETIC FIELD DOMINATED
HOMOGENISES INTERIOR

RADIATIVE INTERIOR

The End