Institut de Recherche sur les Phénomènes Hors Équilibre

Understanding exchanges across stratified/convective zones interfaces

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European Commission

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Motivations…

<u>Generic situation:</u> a turbulent convective fluid layer stands above or below a stably stratified one, with a sharp but deformable interface.

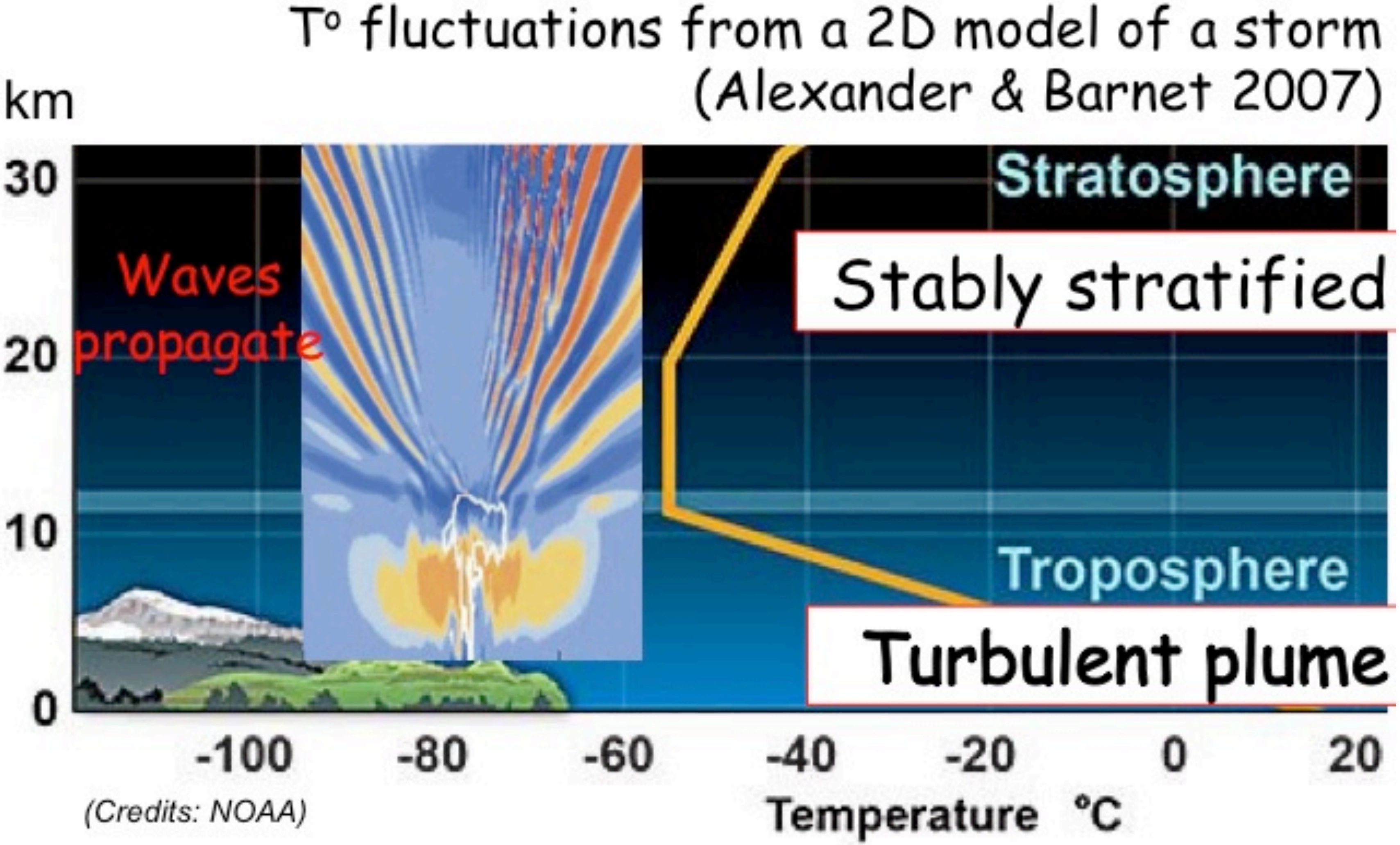
e.g. atmospheres, stars, planetary cores…

Atmospheres...

Turbulent plume

Storm

Atmospheres...



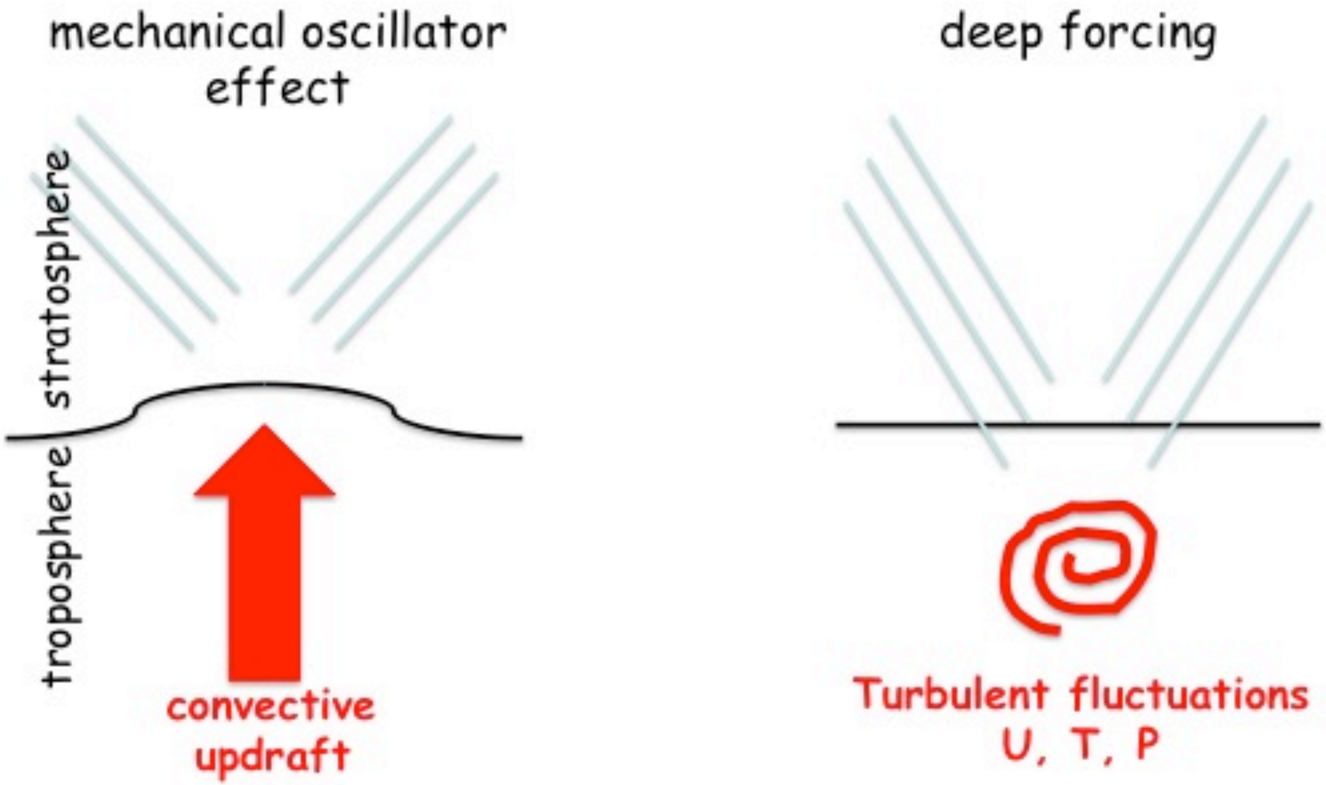
Atmospheres...

How are waves excited?
How and what do they propagate?
What are their consequences?

Atmospheres…

Gravity wave generation by convection

(e.g. Ansong & Sutherland 2010)

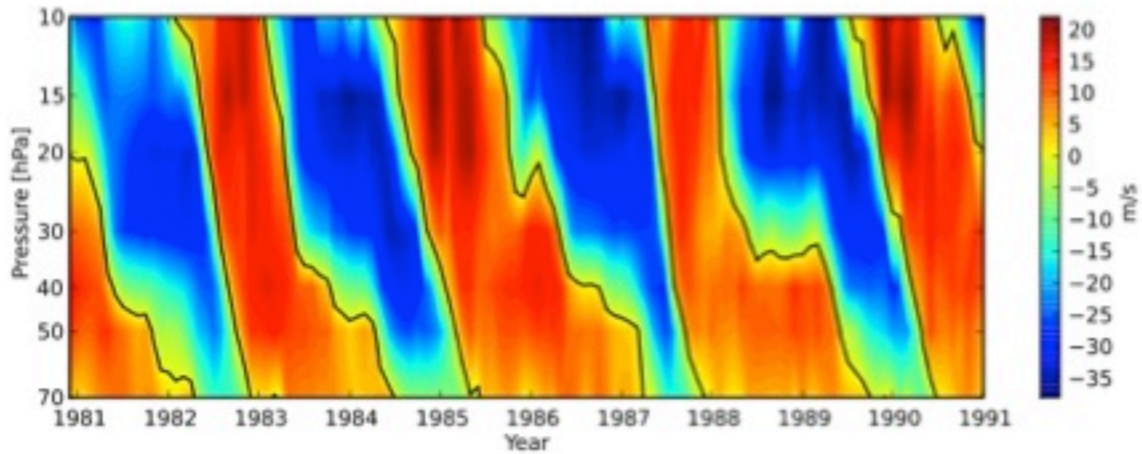


Atmospheres…

- Gravity waves affect the global energy budget
  - ➢ have to be included in general circulation models for accurate predictions of global weather patterns
  - ➢ coarse grids with long time steps => parameterization

- Gravity waves carry momentum:
  - ➢ Breaking and mixing
  - ➢ Non-linear interactions generate zonal flows

    e.g. quasi-biennial oscillation (e.g. Plumb 1977)



monthly-mean zonal-mean equatorial zonal wind in m/s between about 20 and 35 km

Atmospheres…

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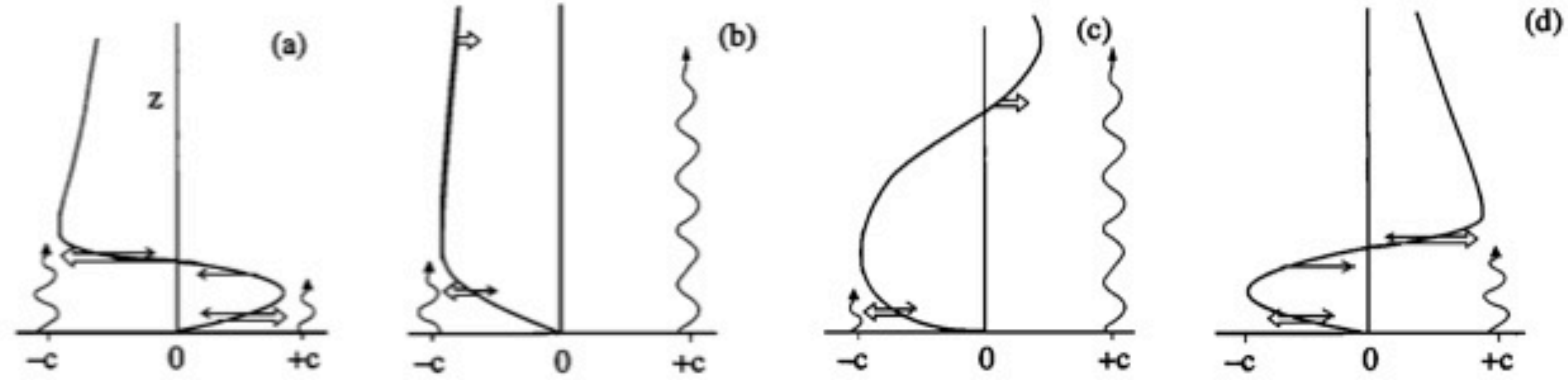
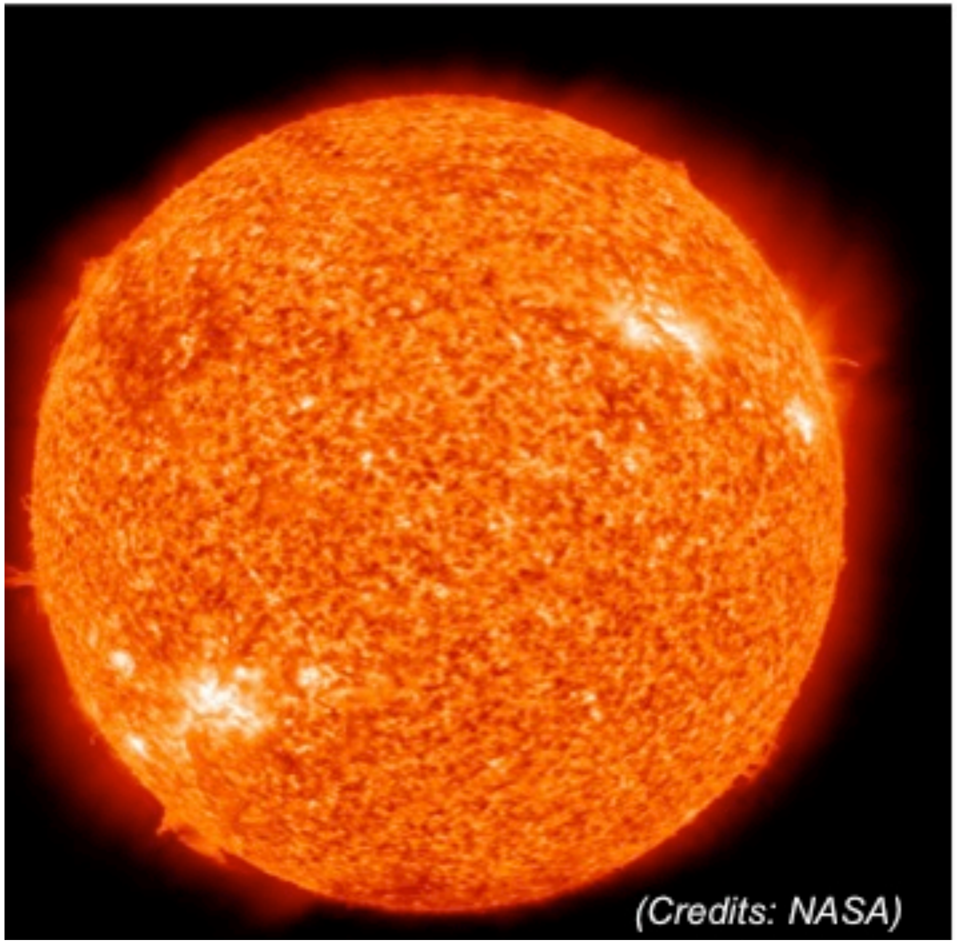
anti-diffusive effect, accentuating angular velocity gradients…



Stars...

Same questions in stellar interiors...



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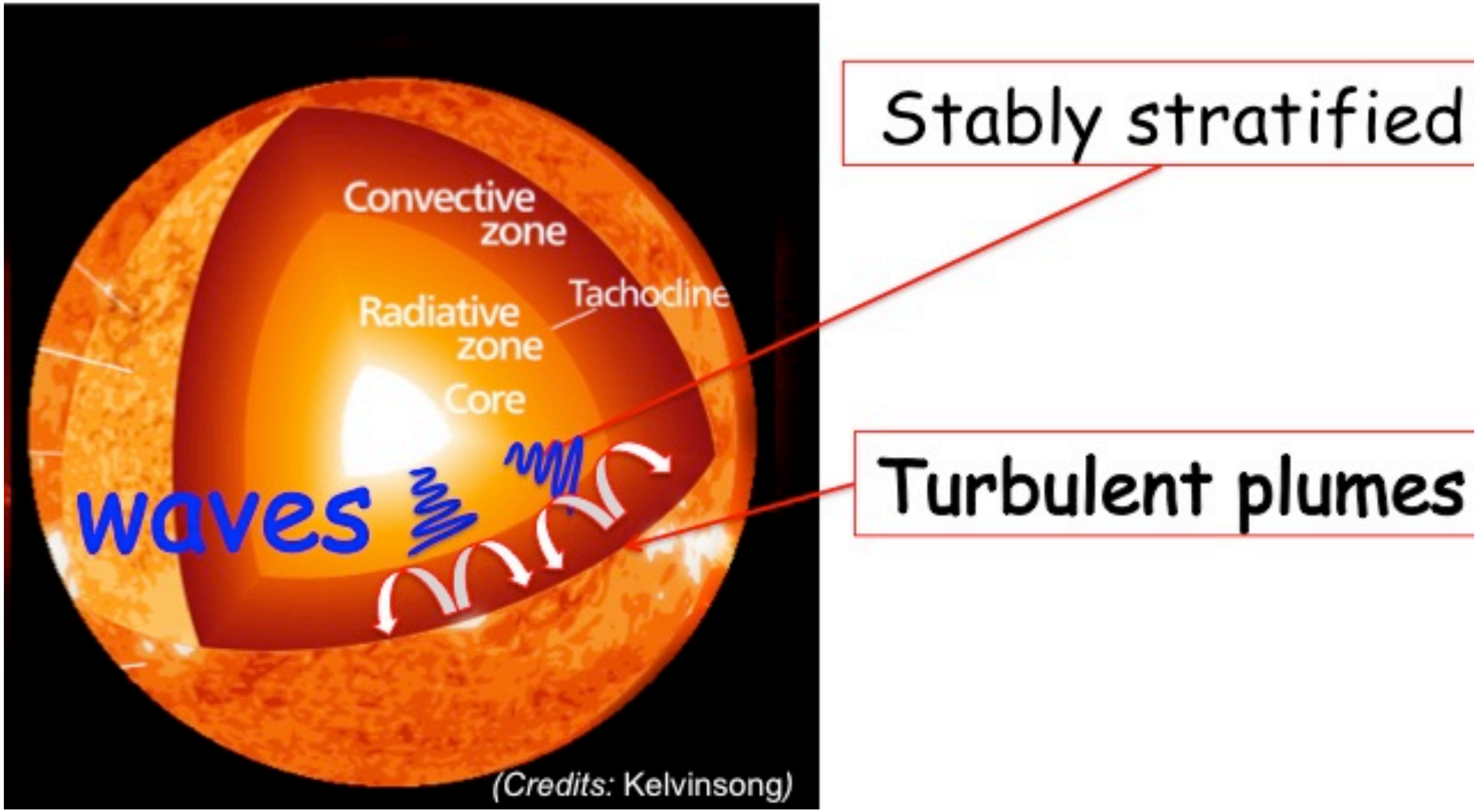
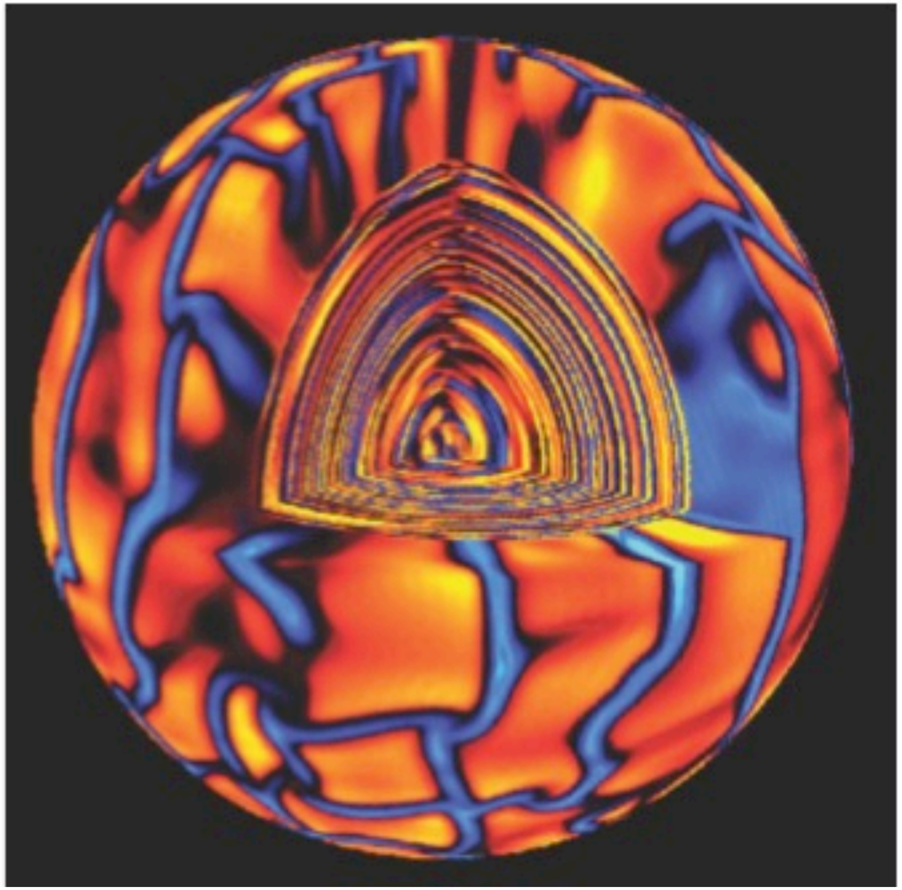


Stars...

Same questions in stellar interiors...

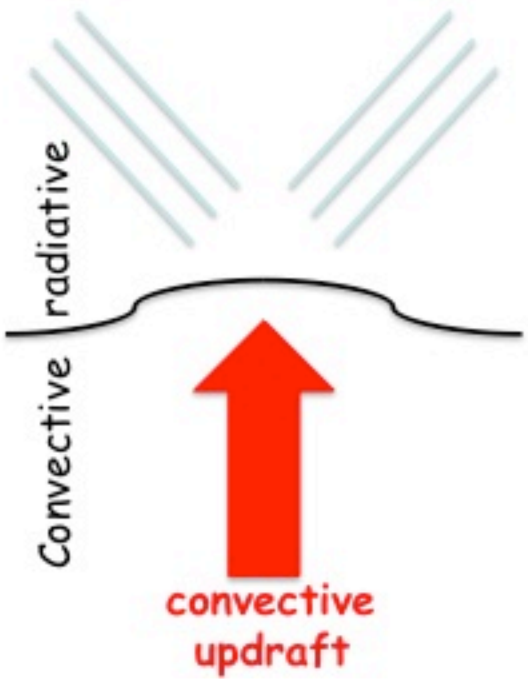


3D numerical simulations of the Sun by Alvan et al. (2012): time snapshot of the radial velocity.

Stars…

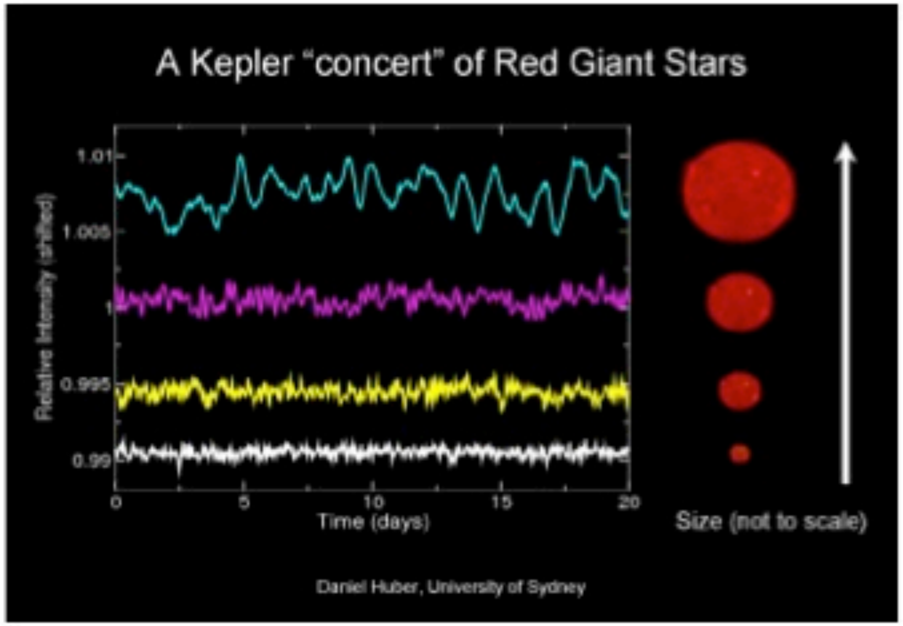
Gravity waves excited

- at the interface by overshooting plumes
- within the convective layer by Reynolds stress and entropy fluctuations



Stars…

Gravity waves = important diagnostic tool of stellar structure in asteroseismology



Oscillations at the surface

⇔

Changes in the propagation

⇔

Internal structure

Stars…

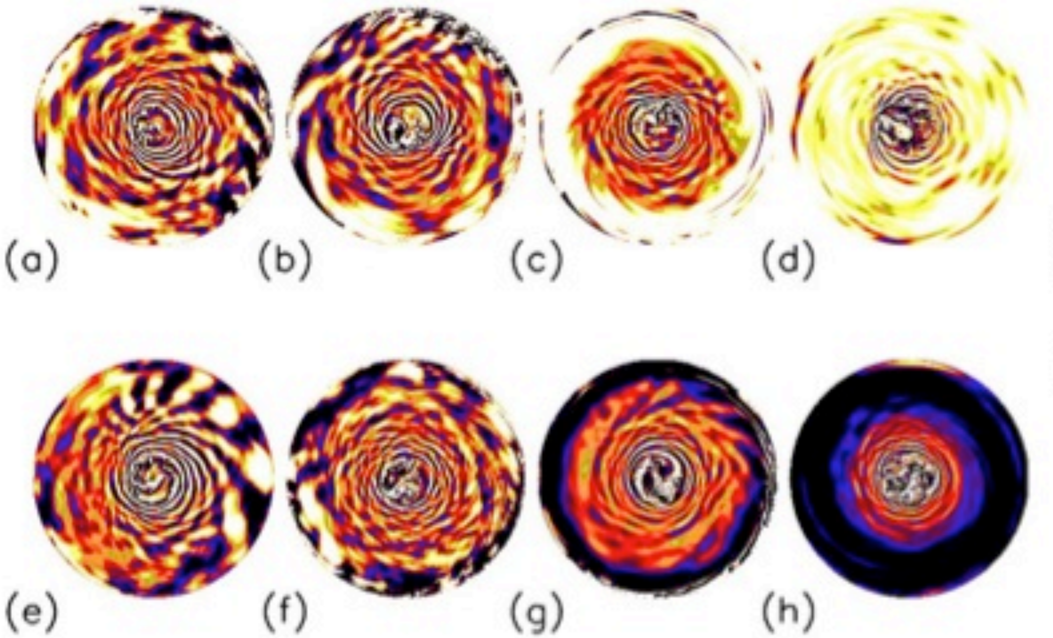
Gravity waves = important diagnostic tool of stellar structure in asteroseismology

Gravity waves transport energy and momentum:

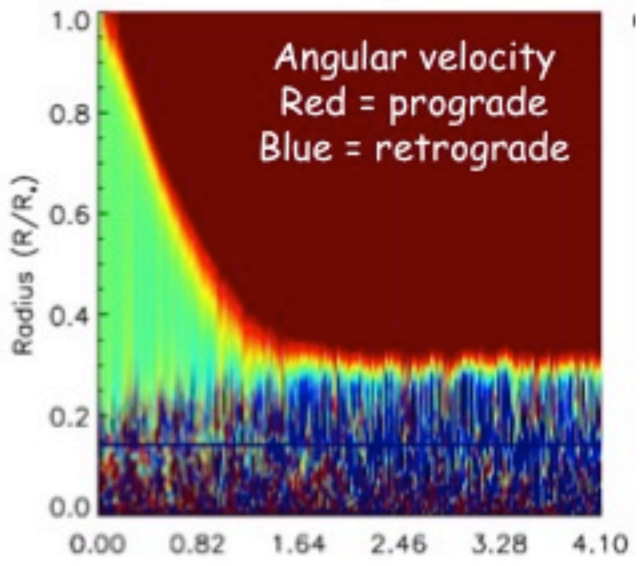
- mixing, enhancing diffusion (e.g. observed lower Li abundance in F-stars, Charbonnel & Talon 2005)
- increased light flux and mass loss at the surface of massive starts (Quataert & Shiode 2012)
- selective damping because of symmetry breaking by rotation
  => angular momentum deposit and modification of rotation profile.

Stars…

Axisymmetric numerical simulations by Rogers et al. (2012)



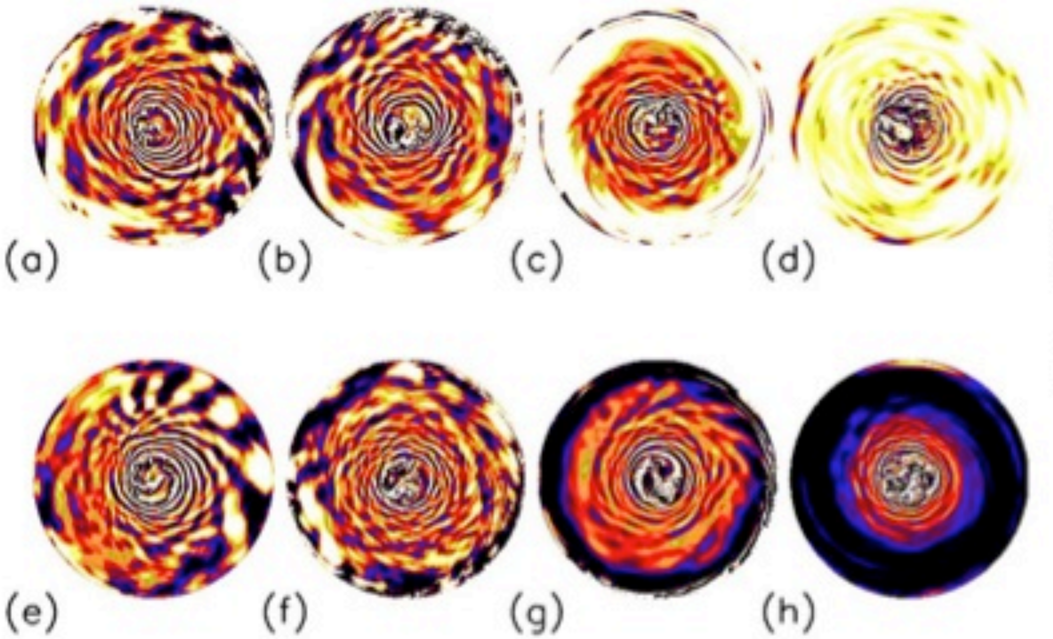
Time snapshots of vorticity at different times
White = positive vorticity, black = negative vorticity



IGWs generally have an anti-diffusive effect, accentuating angular velocity gradients…

Stars…

Axisymmetric numerical simulations by Rogers et al. (2012)



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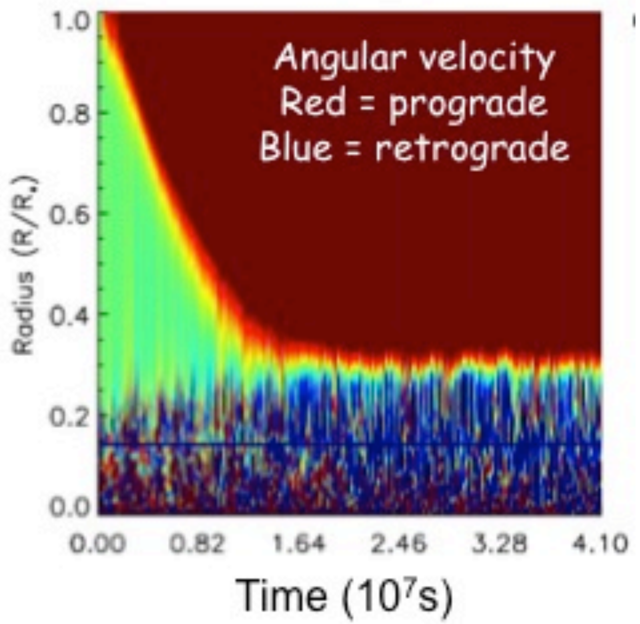
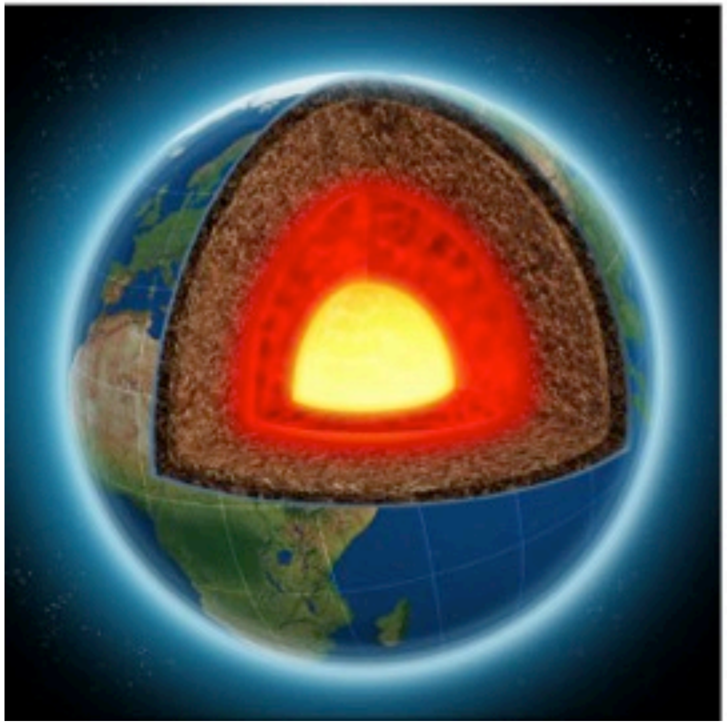


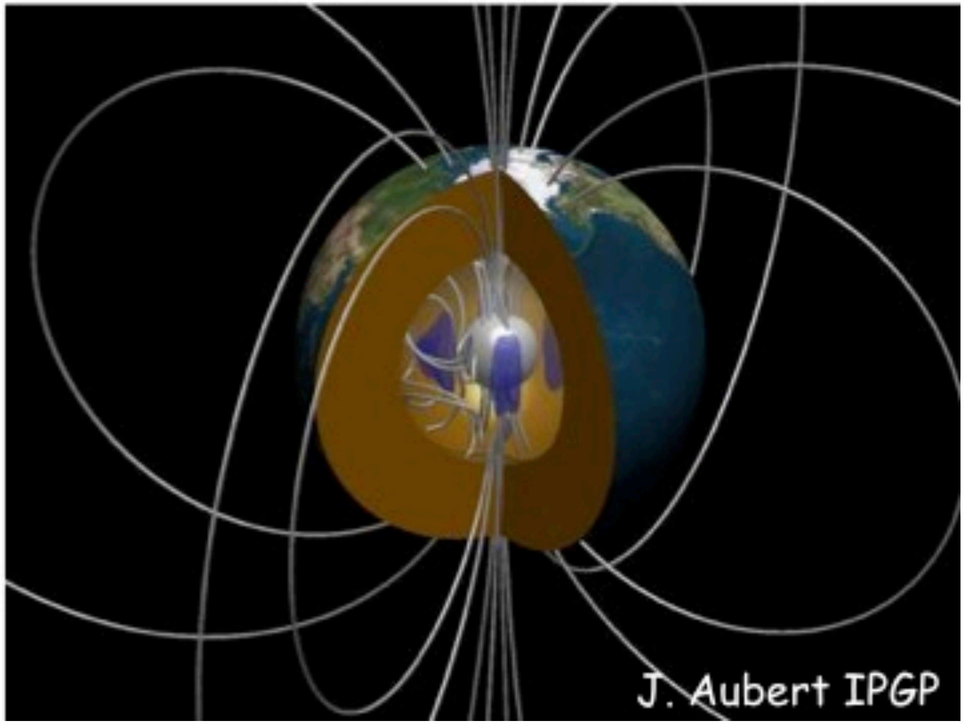
"Internal" mechanism for explaining the observed misalignment between extrasolar planets and their hot host stars

Earth's core…

Standard model :

- convective motions in the outer core
  => mixing => adiabatic T° and well-mixed composition.
- growth of (nearly) pure iron inner core by crystallization
- energy budget = age of the inner core.





Earth's core…

But

- ill-constrained composition of the core
  - ⇒ Density jump and temperature at ICB?
  - ⇒ Radiogenic heating?
- CMB heat flux over time? Present T° (3800-4200K)?
- Extremely tight energy budget to drive convection:
  - density difference $\delta\rho/\rho \sim 10^{-9}$
  - $\delta T_{lateral} \sim 10^{-4}$ at CMB (Hirose et al 2013)
- Physical constants? (e.g. thermal conductivity)

possibility of stably stratified regions

- **at the bottom and/or top of the outer core**
- **at intermediate depth…**

Earth's core…

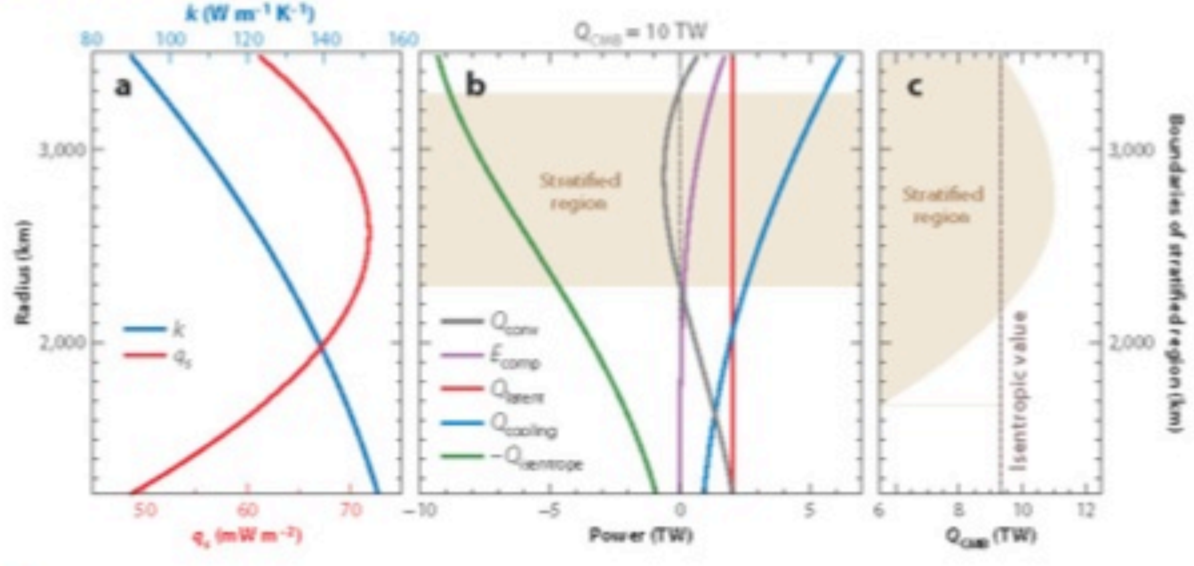


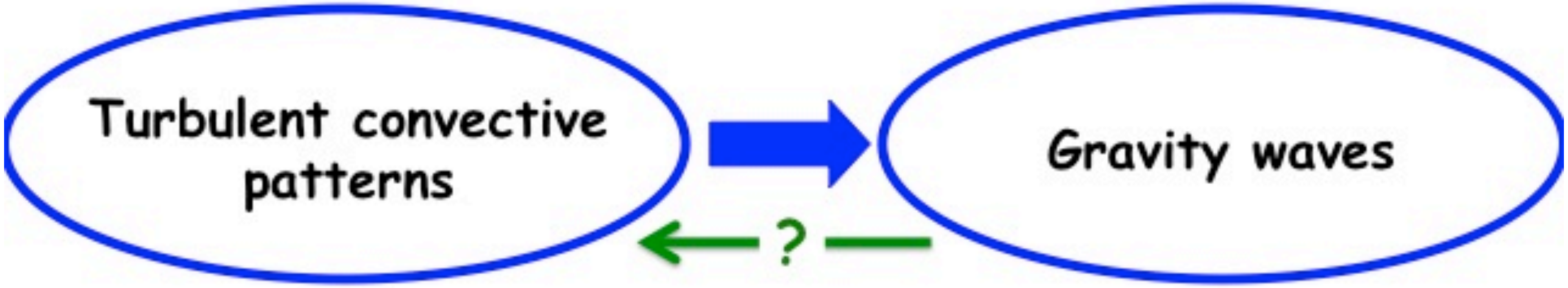
Figure 8

(*a*) Profiles of thermal conductivity (k) and heat flux along the isentrope (q_s) in the core according to Gomi et al. (2011). (*b*) Convective heat flux (Q_{conv}, *gray*) is computed using an energy balance between the inner core boundary and each value of radius, for a core-mantle boundary (CMB) heat flow of $Q_{CMB} = 10$ TW. The balance writes as $Q_{conv} + Q_{isentrope} = E_{comp} + Q_{latent} + Q_{cooling}$, where E_{comp} is the compositional energy due to light element transport in the chemical potential gradient, Q_{latent} is the latent heat of inner core freezing, and $Q_{cooling}$ is the secular cooling of the shell. The region where the convective heat flow is negative (i.e., downward) tends to become stratified. The extent of this region is represented as a function of Q_{CMB} in panel *c*.

Hirose et al. 2013

Consequences on the geomagnetism?

Open questions…

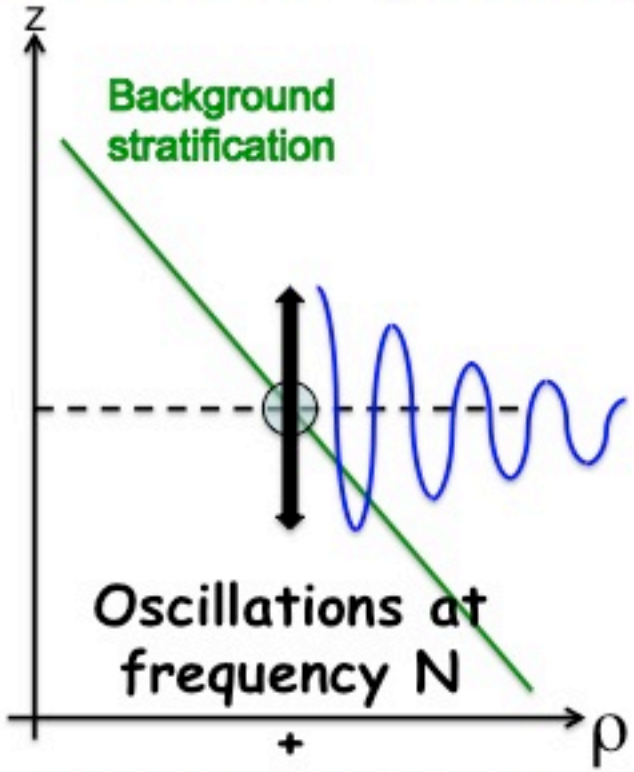


- character of the wave field?
- amount of energy/momentum carried away?
- possible retro-action on the turbulence?

- Global integrated model & non-linear couplings
- Turbulence, stratification and waves
- Length and time scales spanning many orders of magnitude

> ➢ Numerics = very challenging
> ➢ **Experimental study and analytical model**

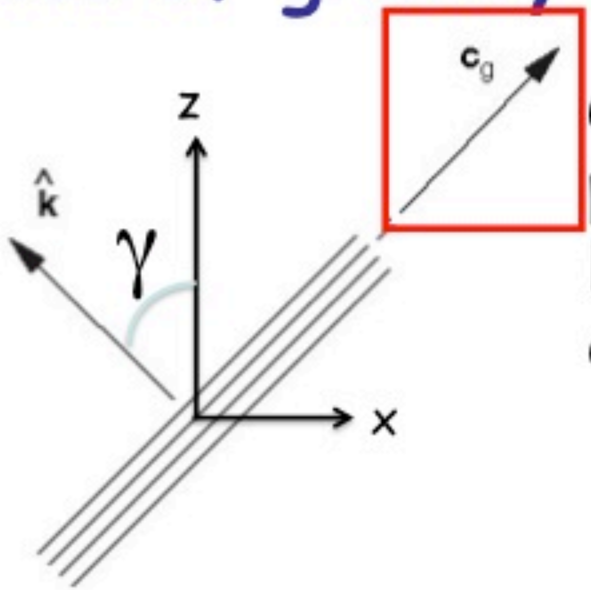
Basics of gravity waves…



Diffusive damping

Fluid linearly stratified with density profile $\rho=\rho_0(1-N^2/g.z)$, N being the buoyancy frequency

Basics of gravity waves…



Generalisation: looking for plane wave solutions of the linearised Navier-Stokes equations

solutions = gravity waves

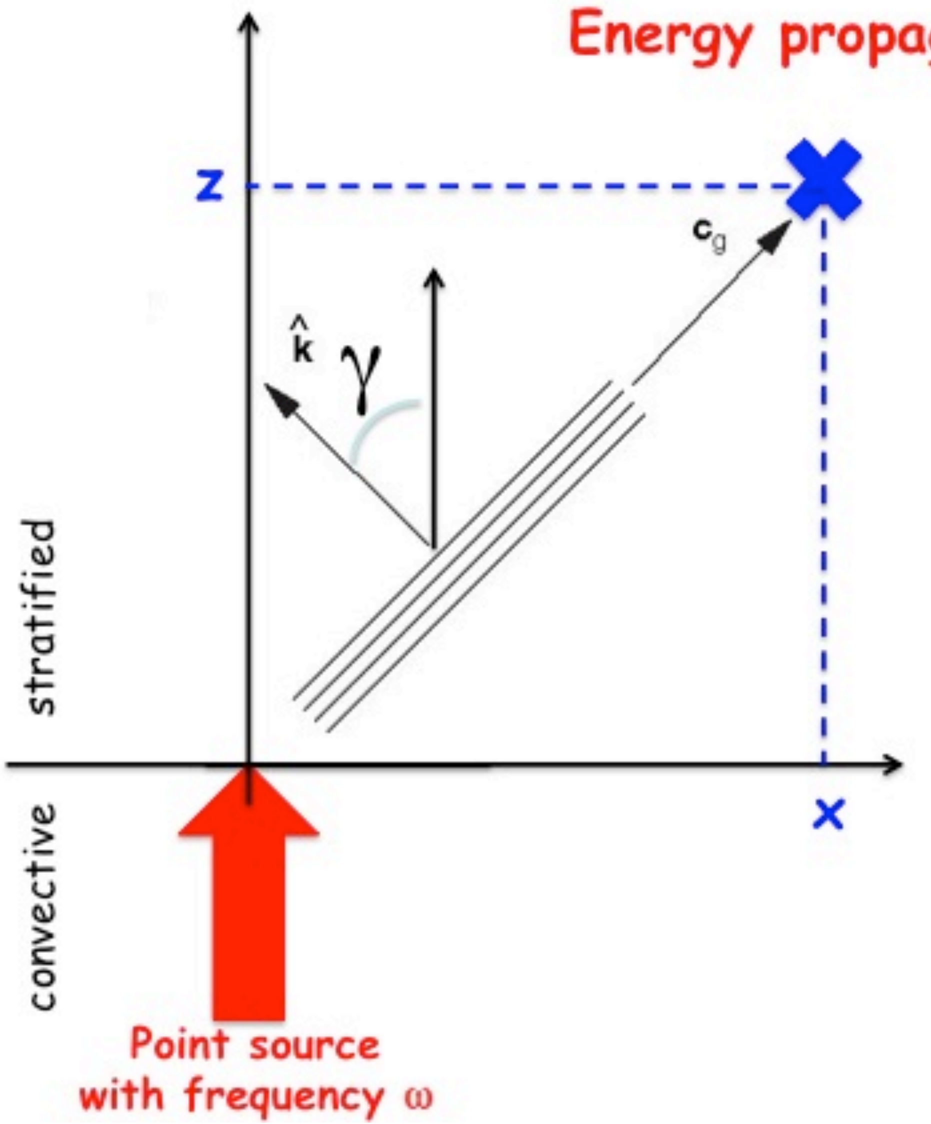
$(\mathbf{u},\rho')=(\mathbf{u_0},\rho_0')\, e^{i(\mathbf{k}.\mathbf{r}-\omega t)}$

dispersion relation $\omega^2 = N^2 \sin^2(\gamma)$

Group velocity
perpendicular to wavevector

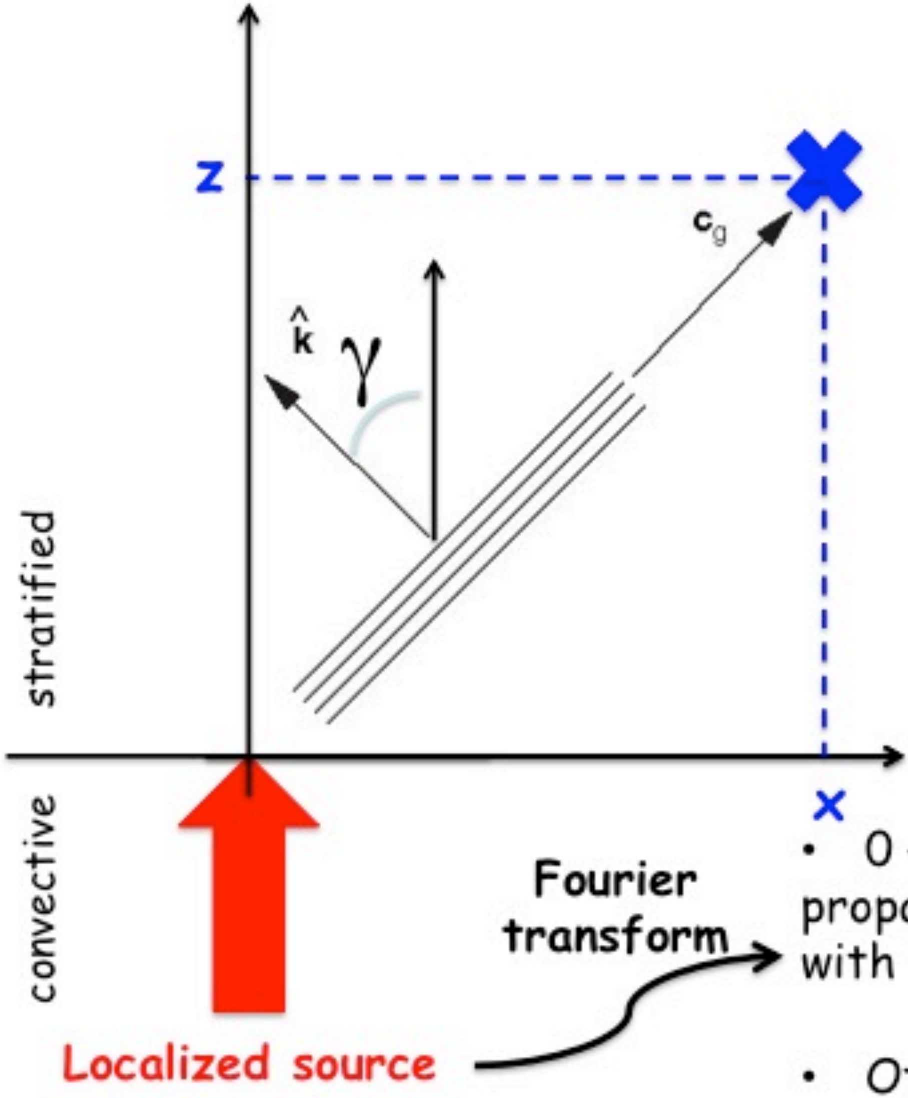
$$\boldsymbol{k} \cdot \frac{\partial \omega}{\partial \boldsymbol{k}} = 0$$

Emission from a localized source



$$\omega^2 = N^2 \sin^2(\gamma)$$

Basic results from ray theory



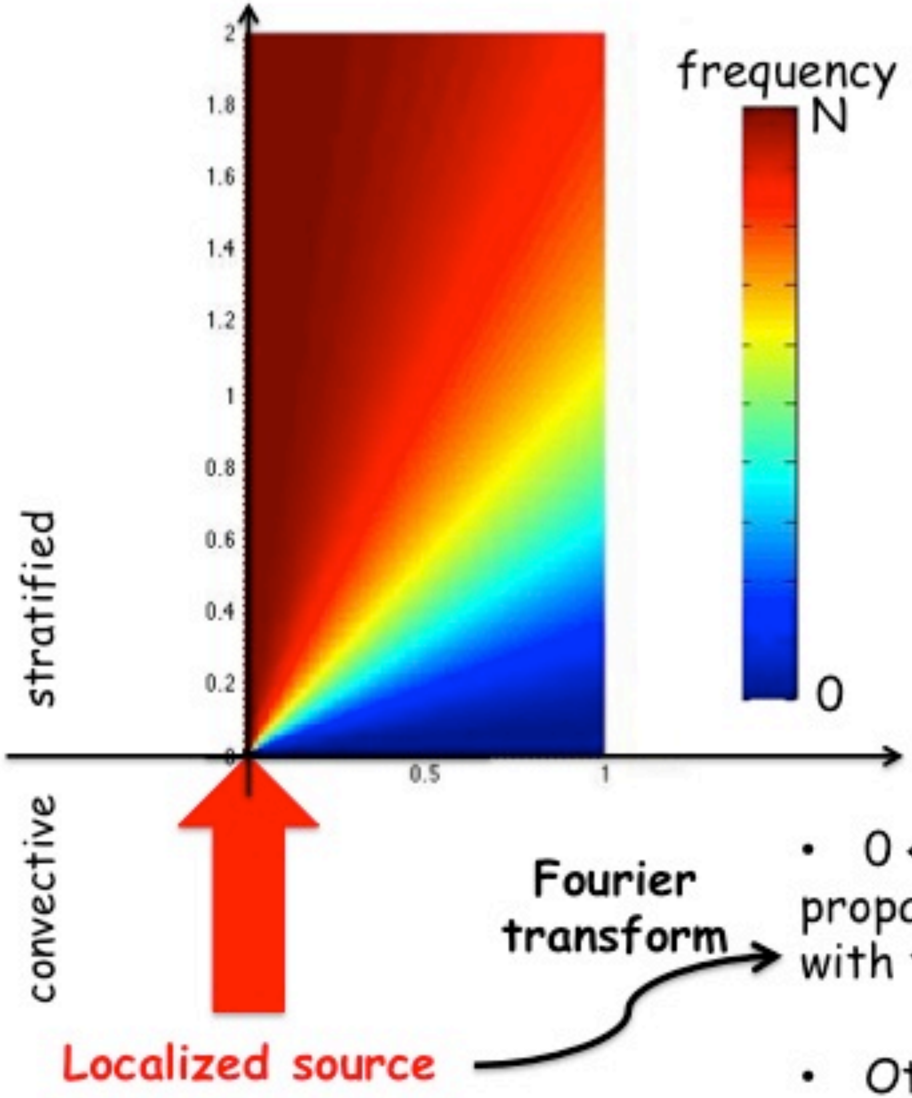
$$\omega^2 = N^2 \sin^2(\gamma)$$

- 0 < ω < N:

propagation in the determined direction with the corresponding amplitude

- Other frequencies = evanescent

Basic results from ray theory



$$\omega^2 = N^2 \sin^2(\gamma)$$

- $0 < \omega < N$:
propagation in the determined direction with the corresponding amplitude

- Other frequencies = evanescent

Basic results from ray theory

Including diffusive effects…

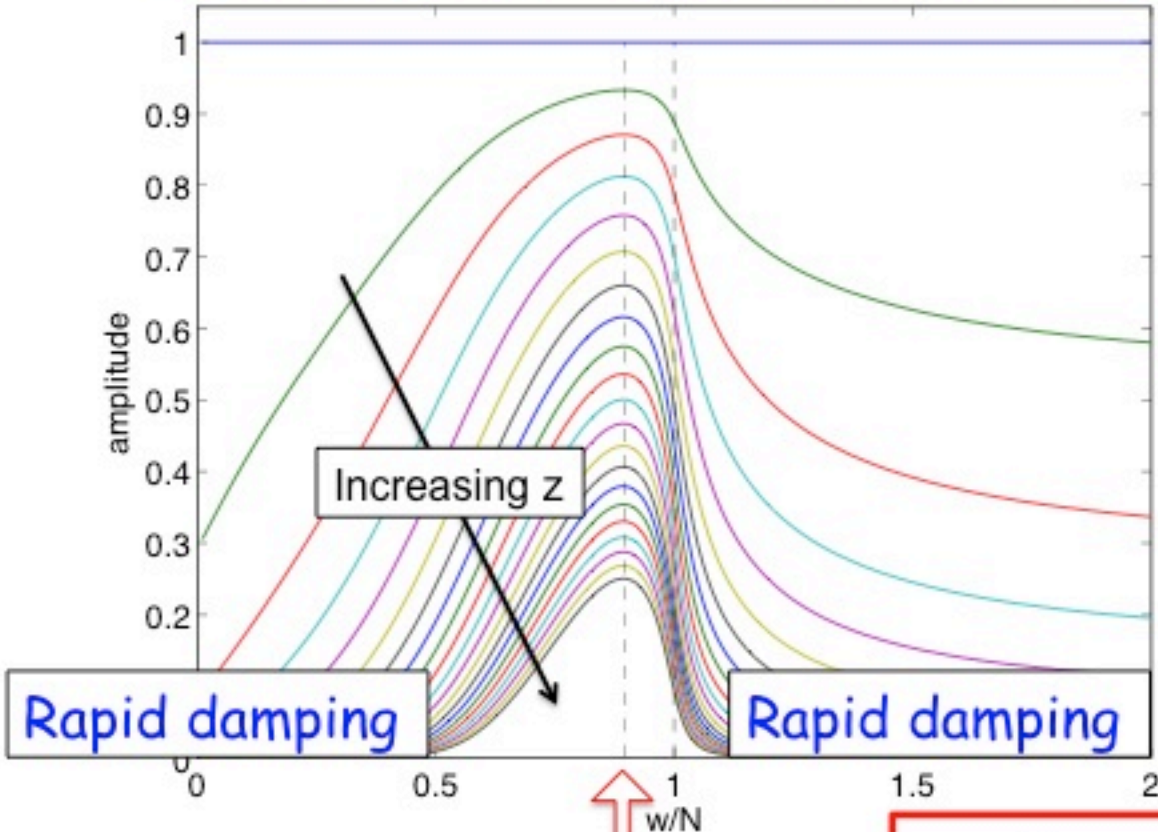
Diffusive dispersion relation: $$\left(i\omega+\nu k^2\right)^2+N^2\frac{\left(i\omega+\nu k^2\right)}{\left(i\omega+\kappa k^2\right)}k_x^2=0$$

Knowing the source signal (i.e. ω and k_x at z=0), we solve for k_z

$$u=u_0\mathrm{e}^{\mathrm{i}(\mathrm{k.r}-\omega \mathrm{t})}\ \mathrm{e}^{-Att(\nu,\kappa,\gamma)z}$$

Basic results from ray theory

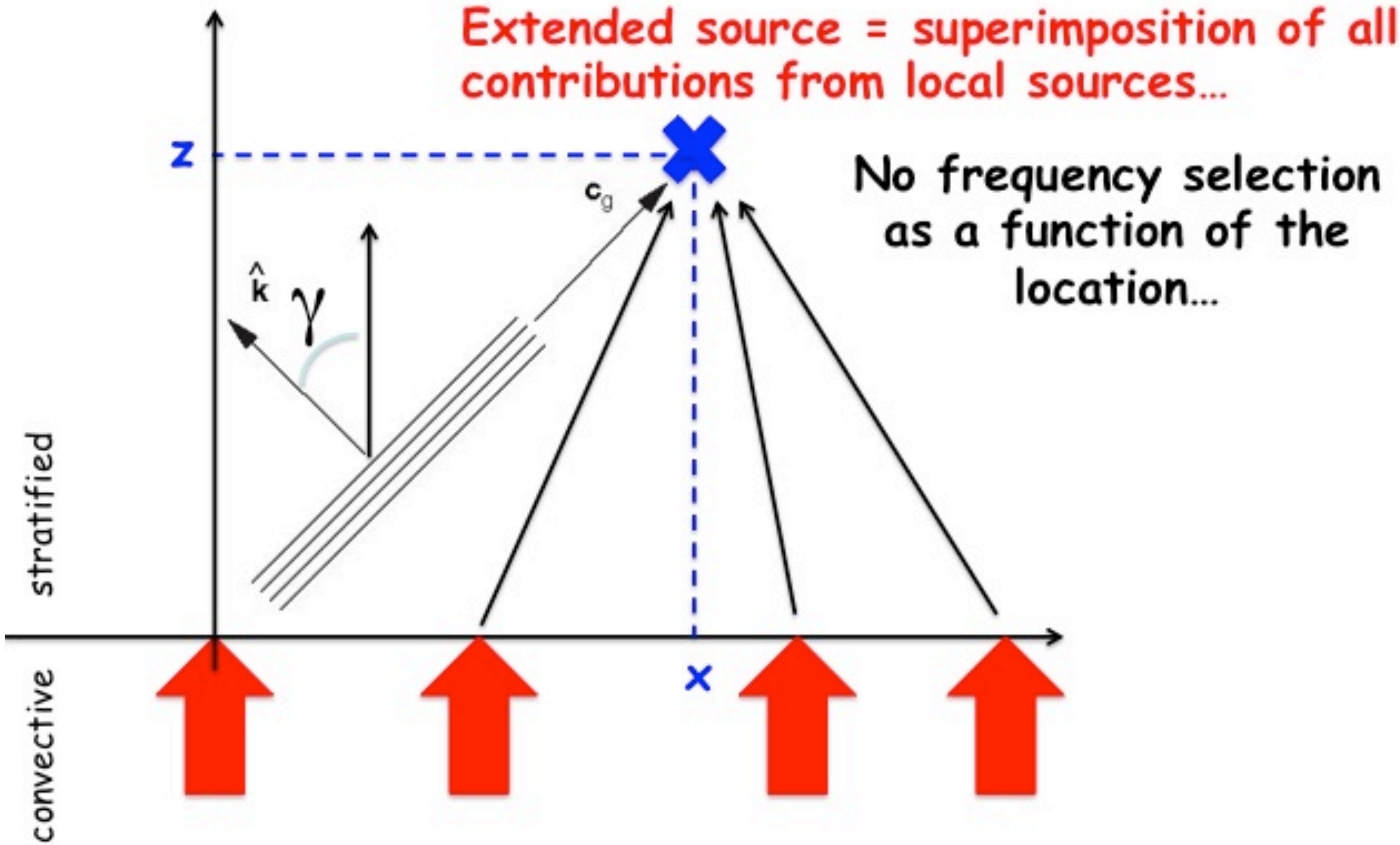
Wave amplitude at a given depth z as a function of ω starting from a uniform excitation



Max = less attenuated

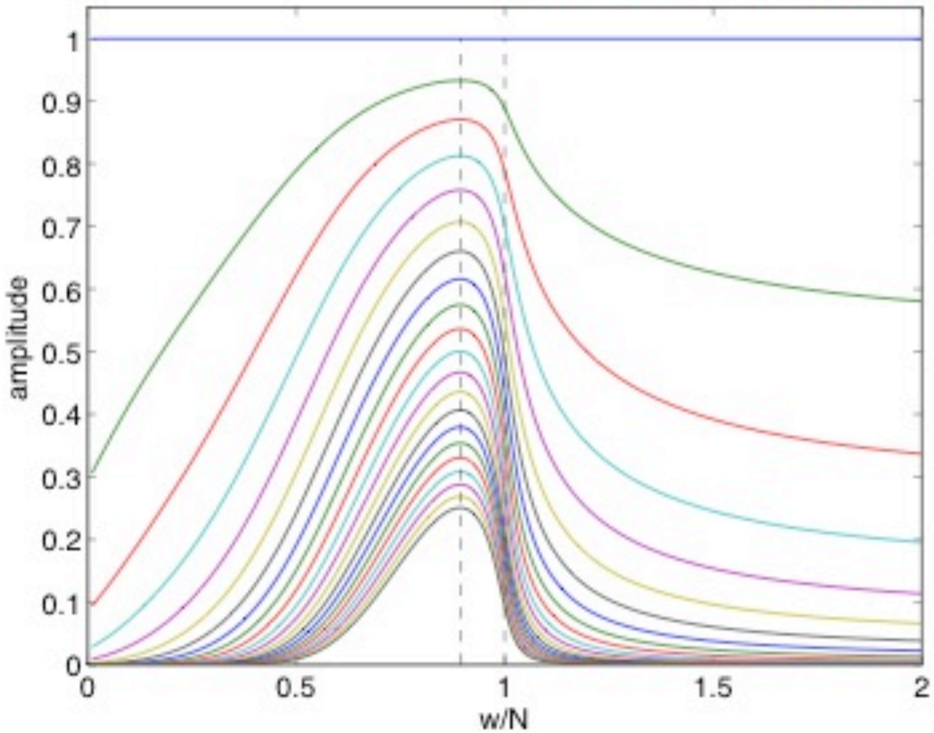
$$\omega = \frac{2}{\sqrt{5}} N \Leftrightarrow \gamma \approx 63^o$$

Basic results from ray theory



Basic results from ray theory

Extended source = superimposition of all contributions from local sources…



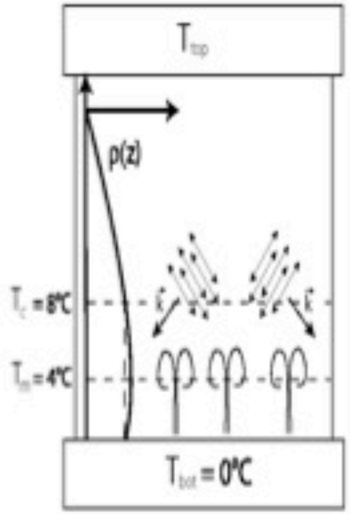
No frequency selection as a function of the location…

But still attenuation with depth and selective damping

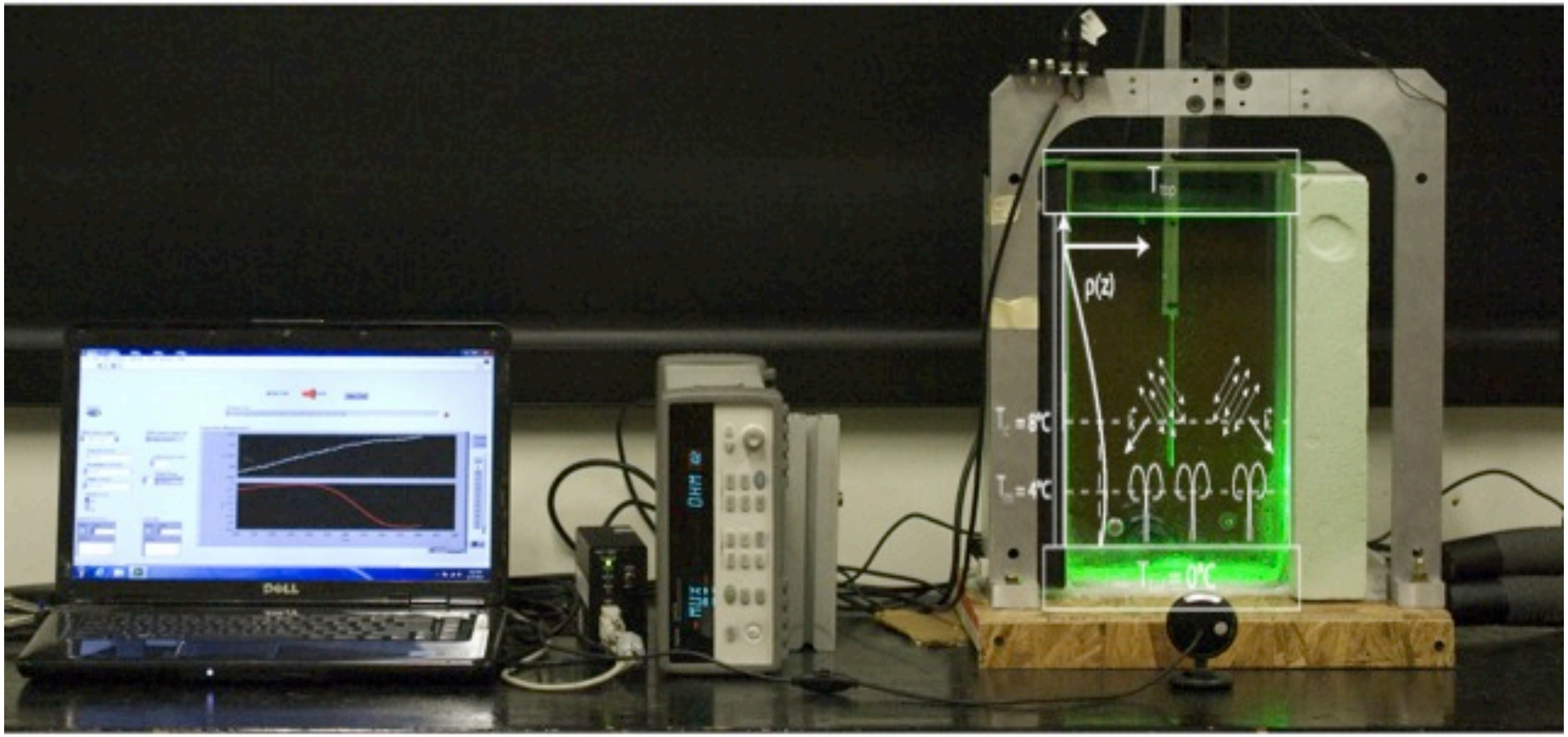
Validation in a simple, "self-organized", experimental system?

Convection in water around 4°C…

Water = maximum density at 4°C
(Townsend 1964)

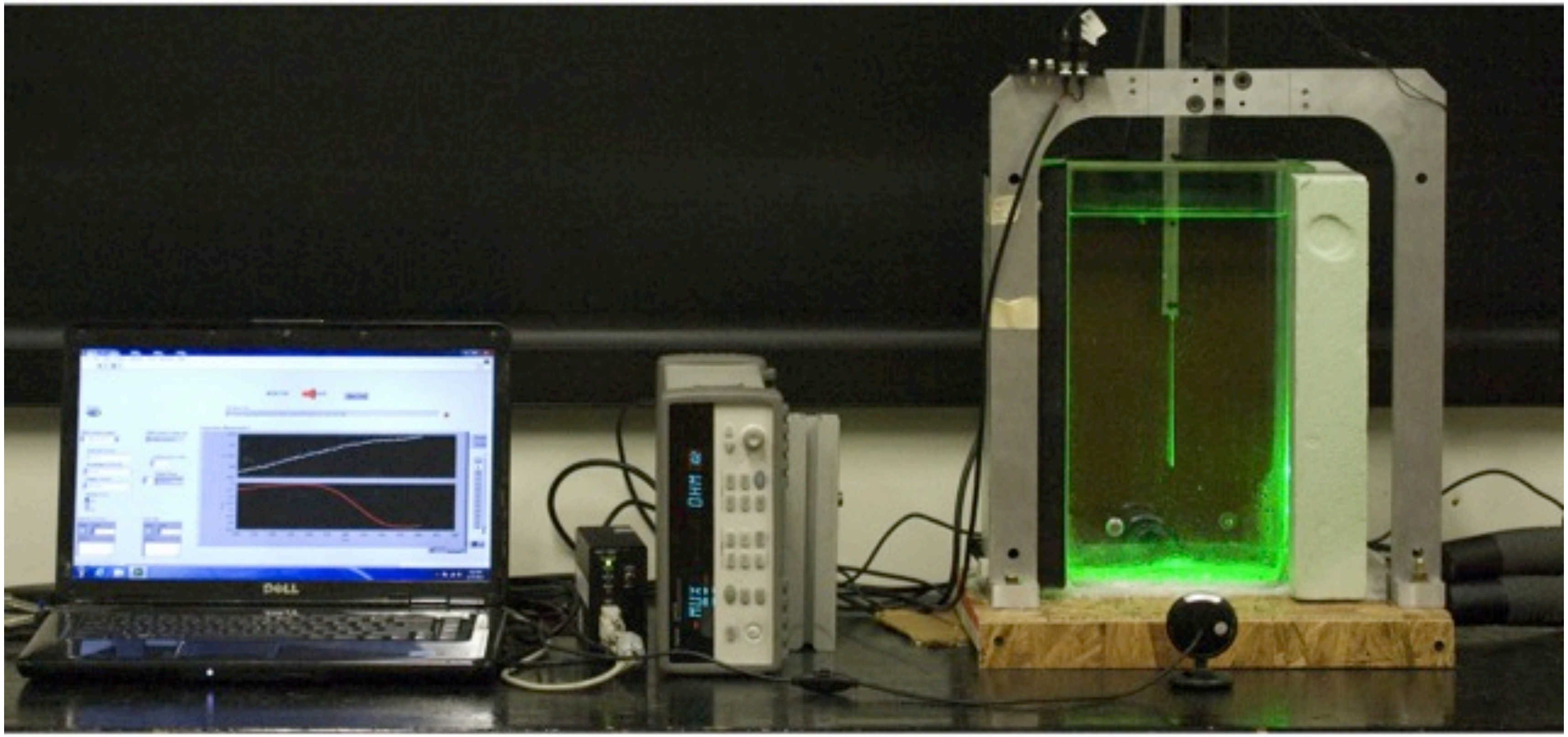


Convection in water around 4°C…



Dimensions: 20 x 4 x 35 cm^3

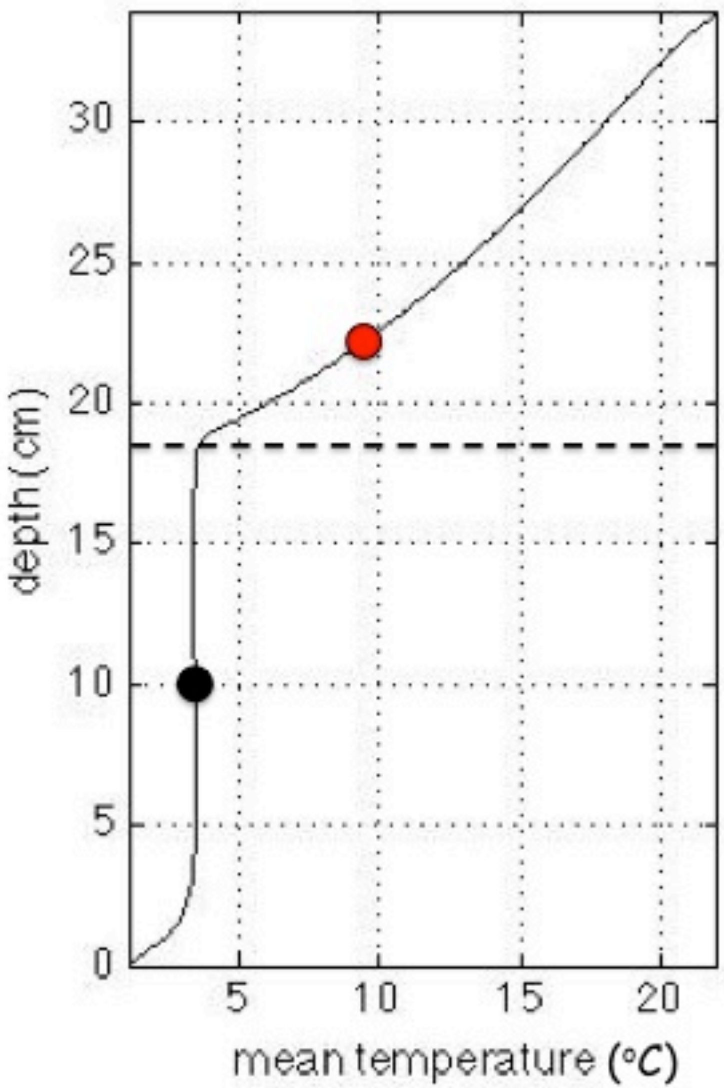
Convection in water around 4°C...



- Local T° measurements = high precision thermistors
- PIV measurements in the convective and stratified zones

$Ra \sim 2\times10^7 - 2\times10^8$

Convection in water around 4°C…

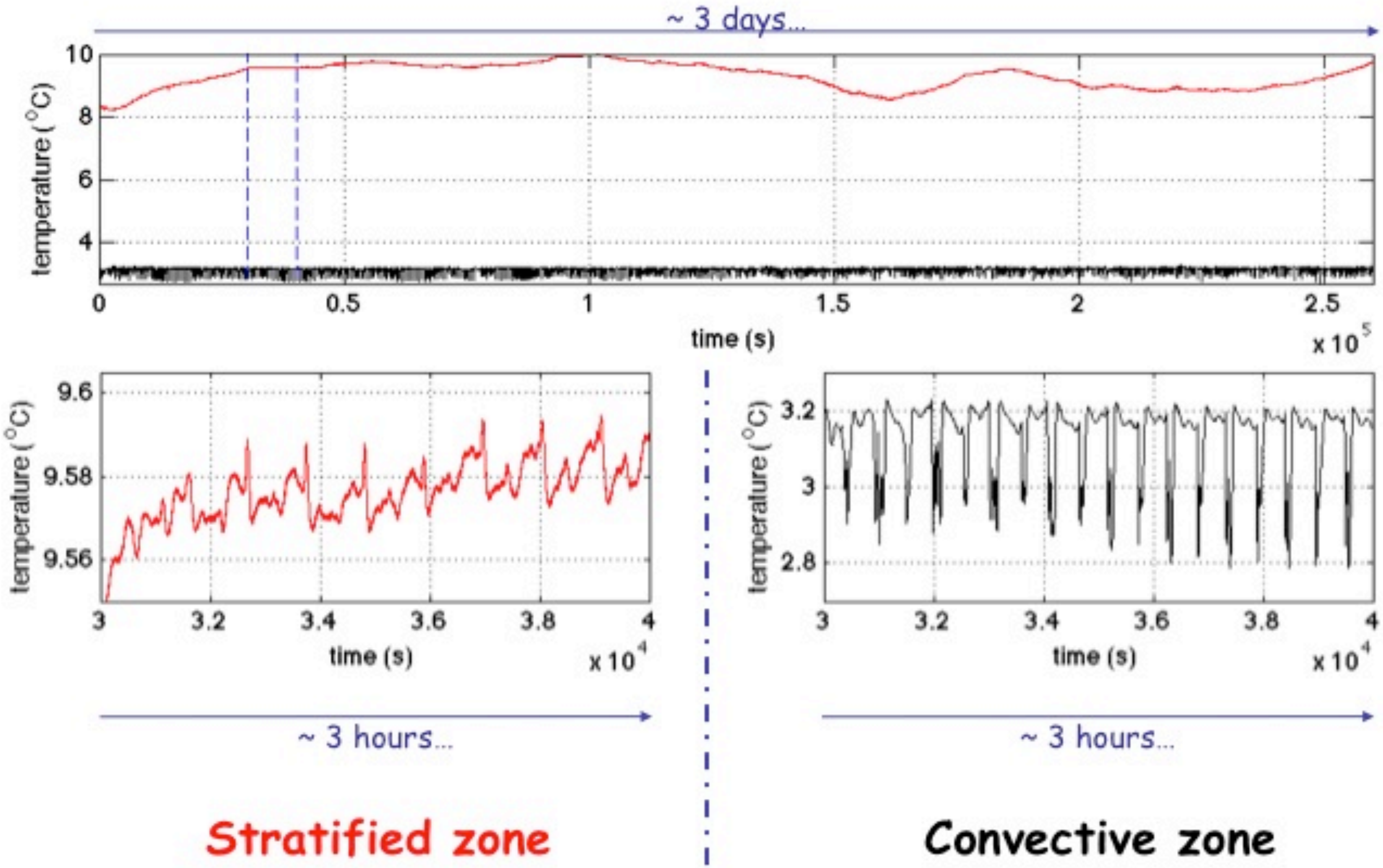


Stratified zone
= heat transfer by conduction

Interface (location function of
imposed T° and heat losses)

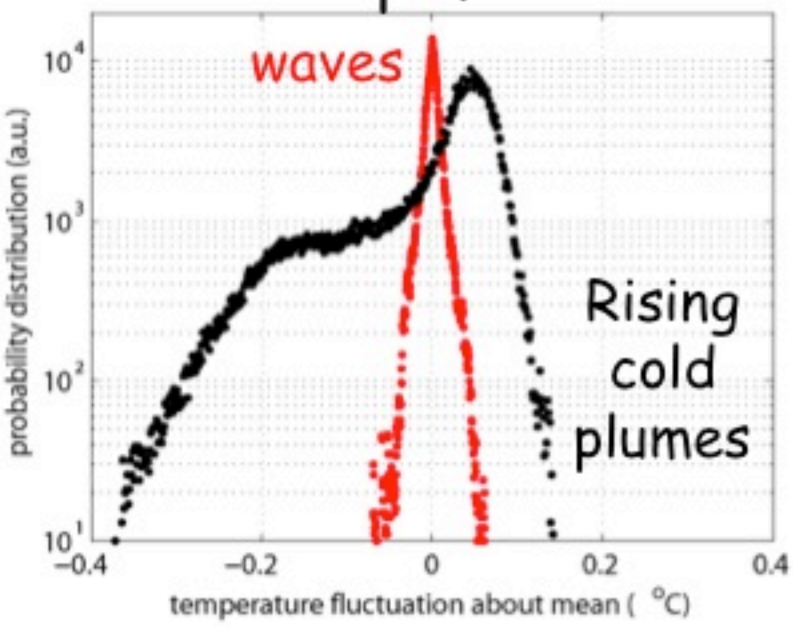
Convective zone
= heat transfer by convection
Mean T° ~3.4°C

Convection in water around 4°C…

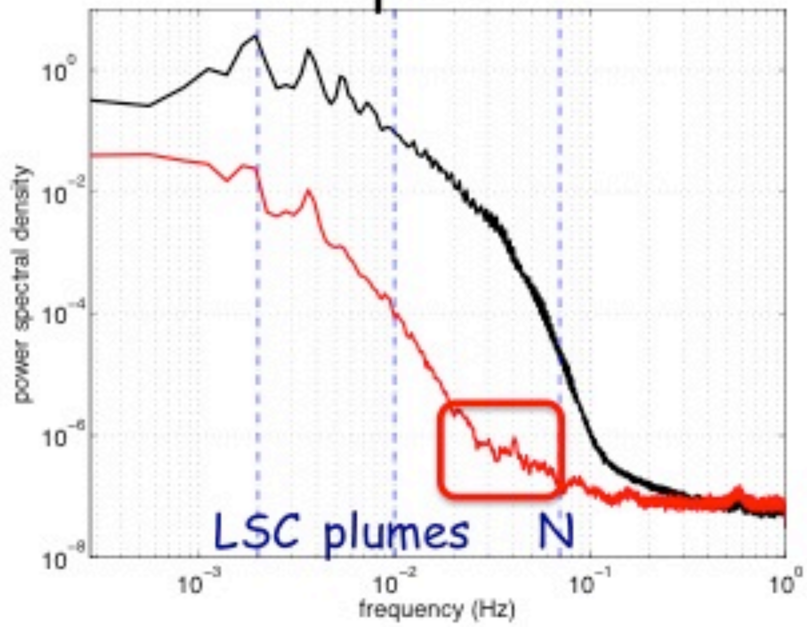


Convection in water around 4°C…

pdf



psd

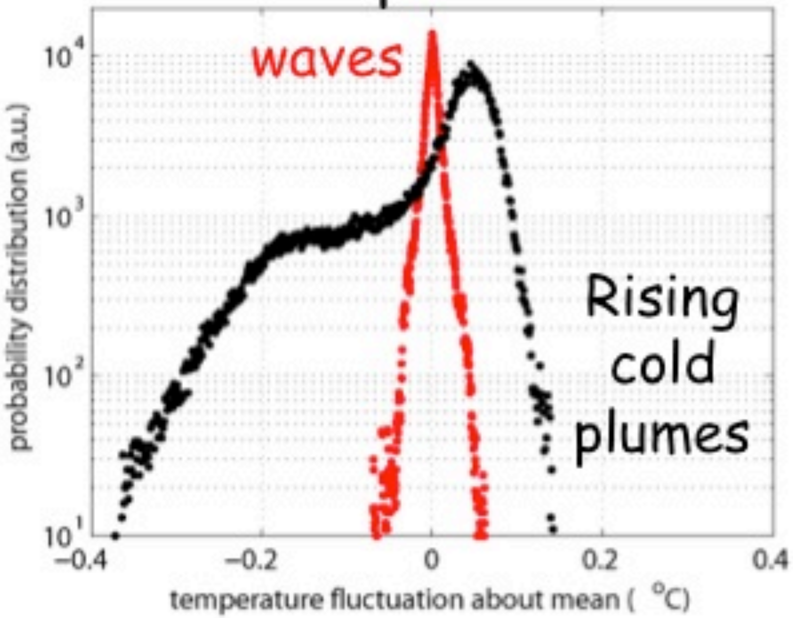


- Local excitation by random cold plumes
  → Gravity waves propagation

- Most energy in low frequency waves ← convective excitation
- No wave for f>N
- Less damping for high frequency waves

Convection in water around 4°C…

pdf



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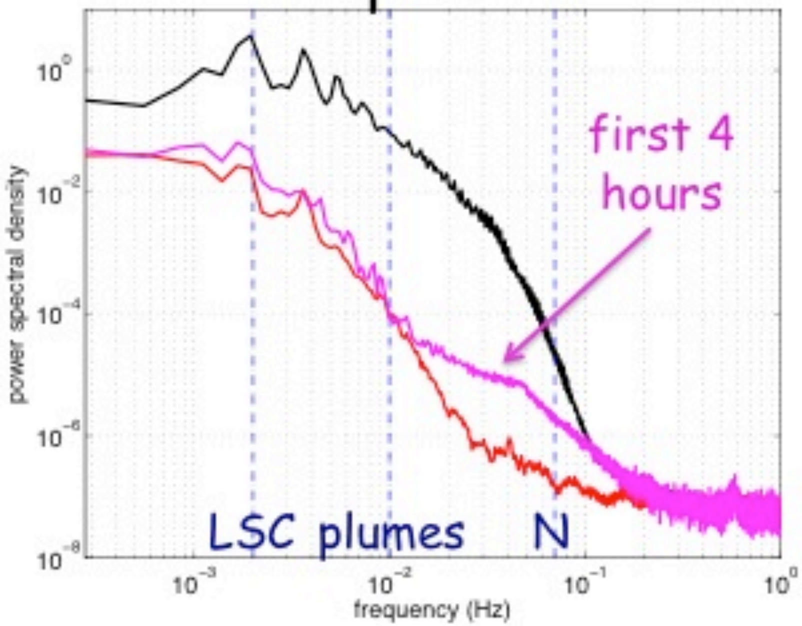
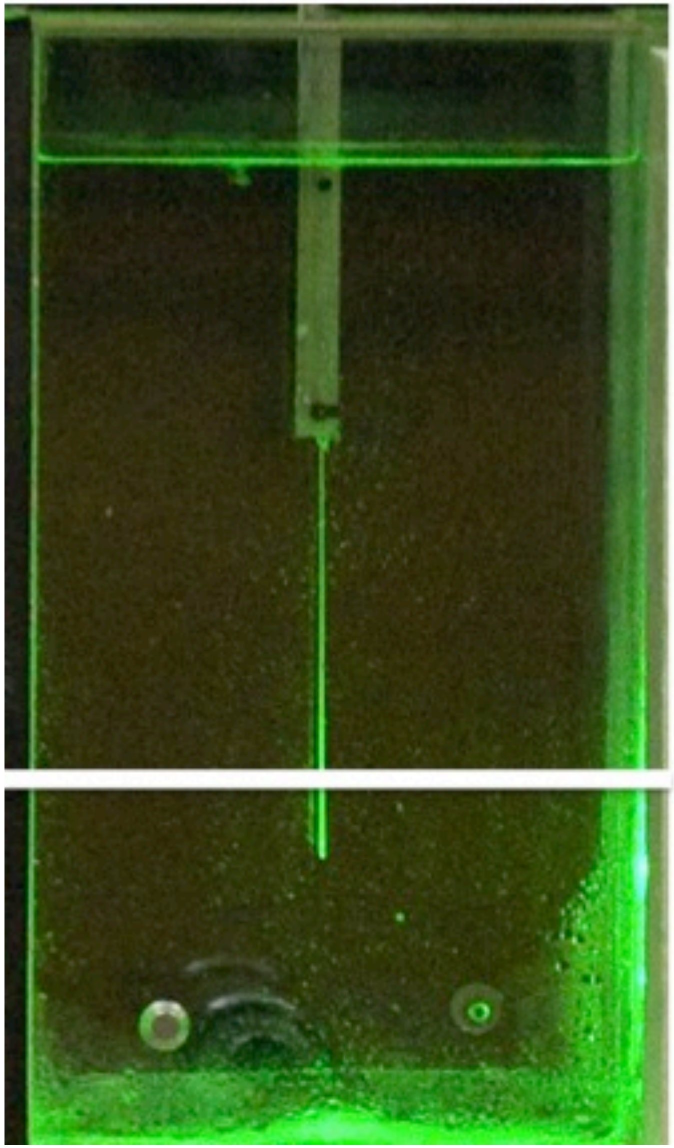


- Local excitation by random cold plumes
  ➡ Gravity waves propagation

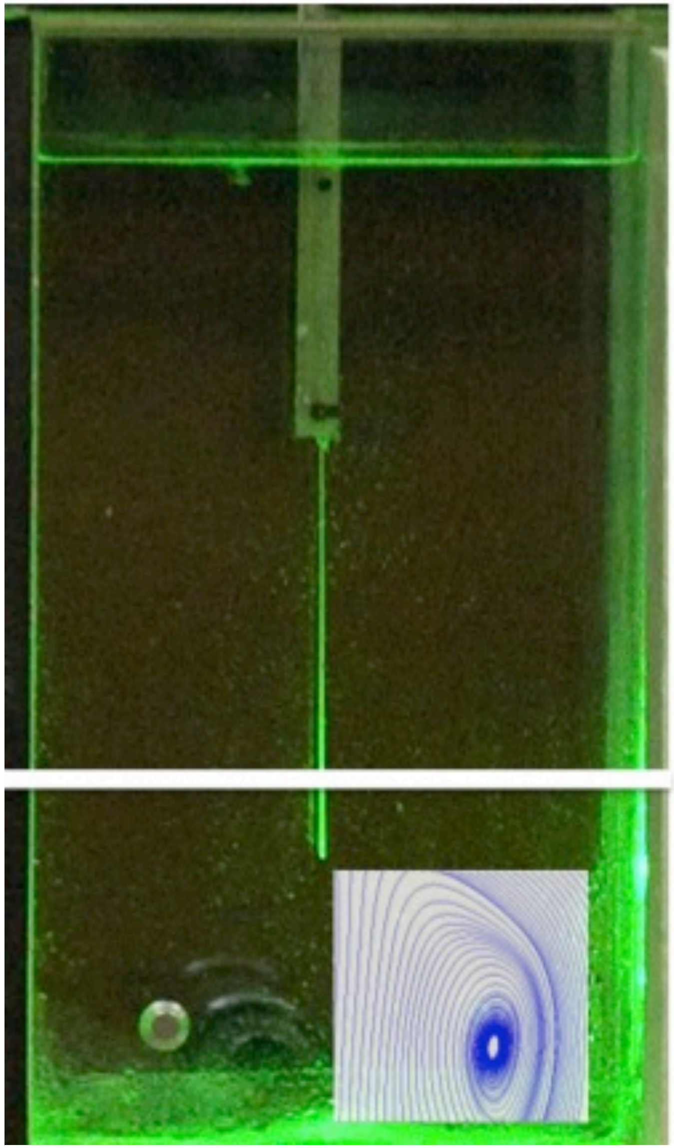
- Most energy in low frequency waves ← convective excitation
- No wave for f>N
- **Less damping for high frequency waves**

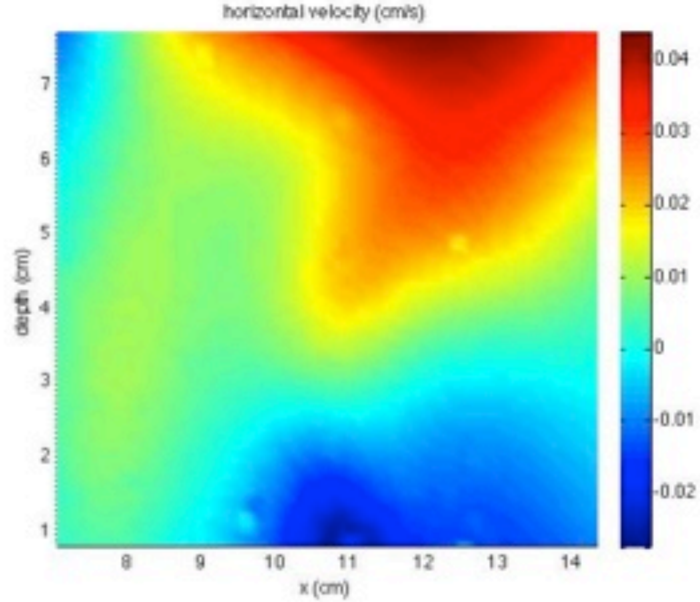
PIV measurements

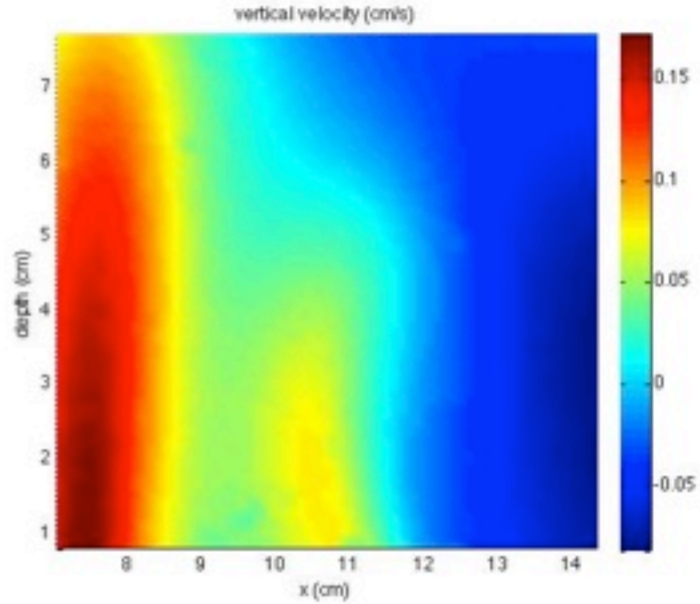


Interface location

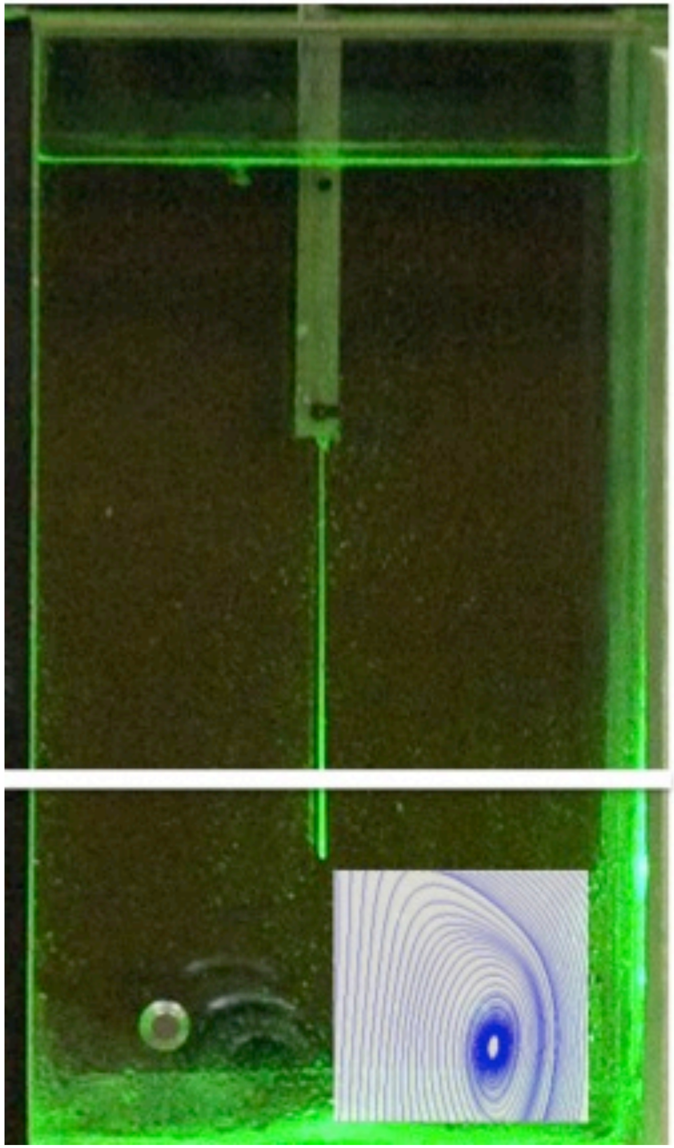
Convective flow



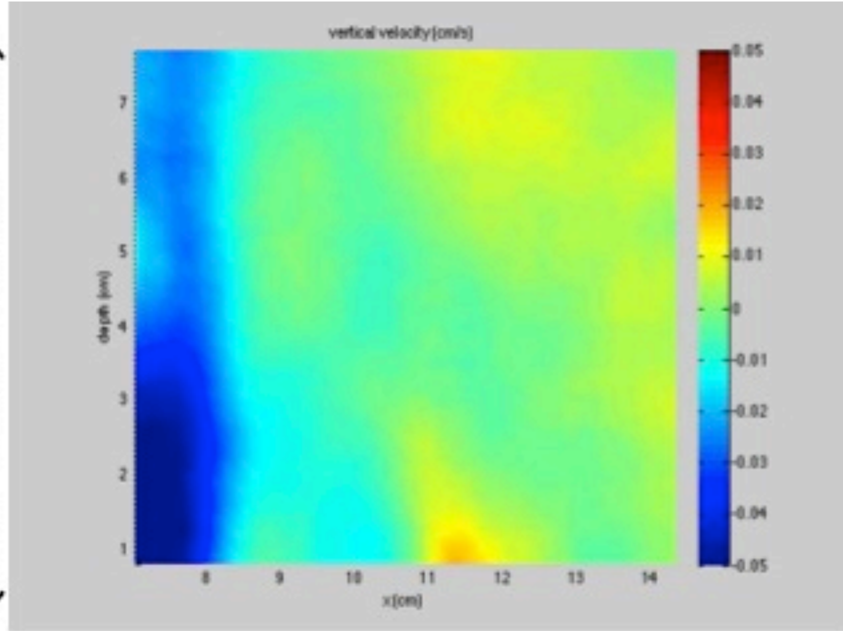


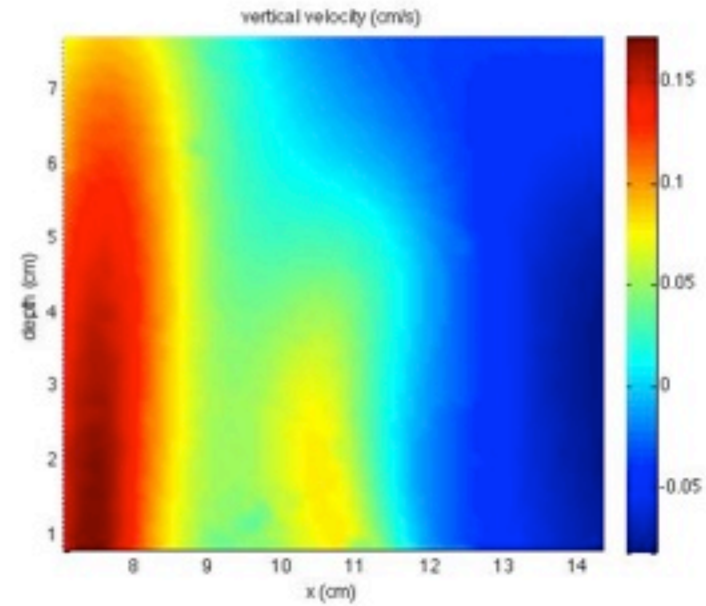


Convective flow

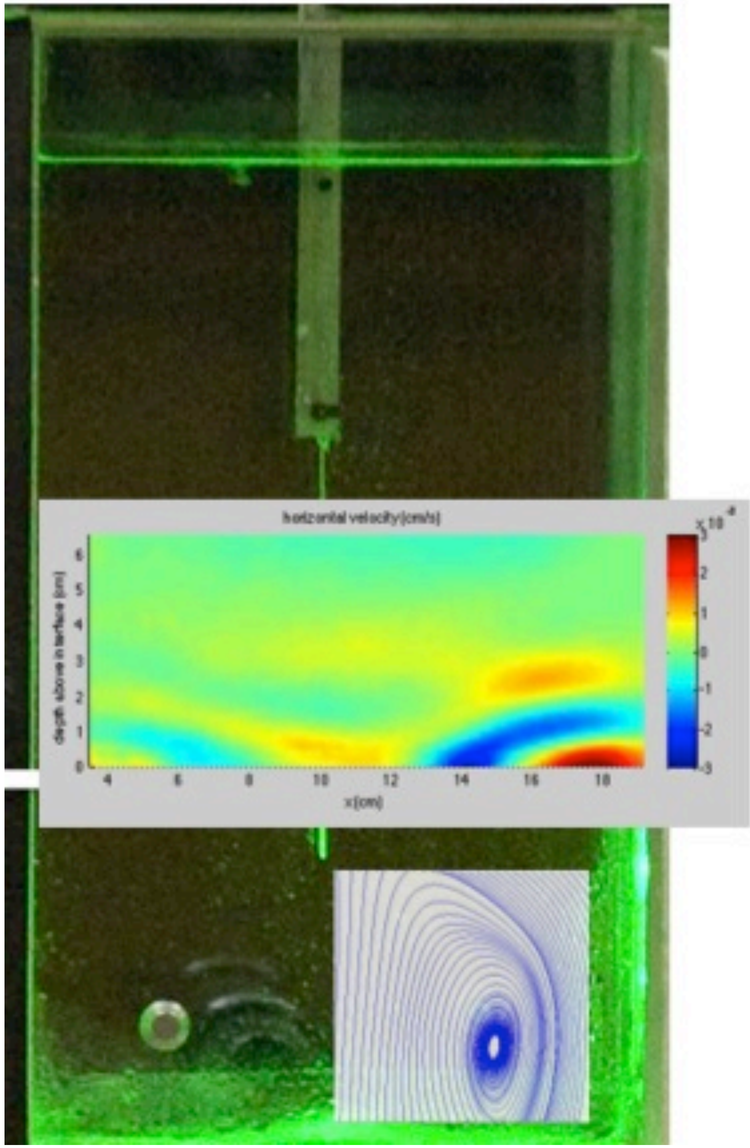


Accelerated x10
(real duration 10 min)

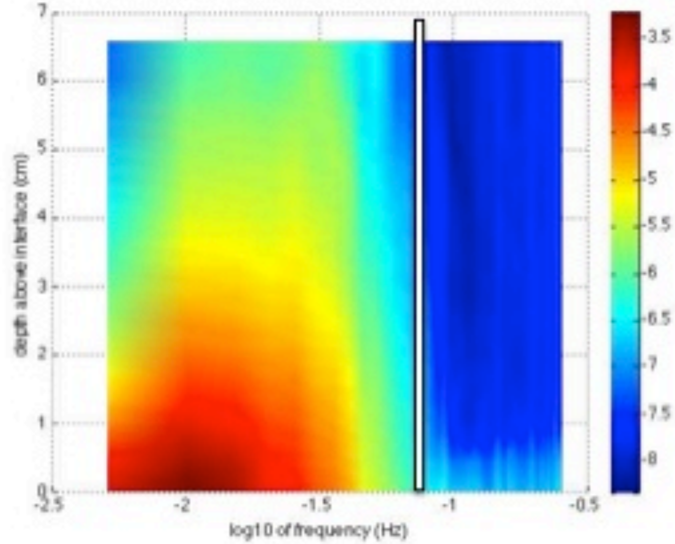




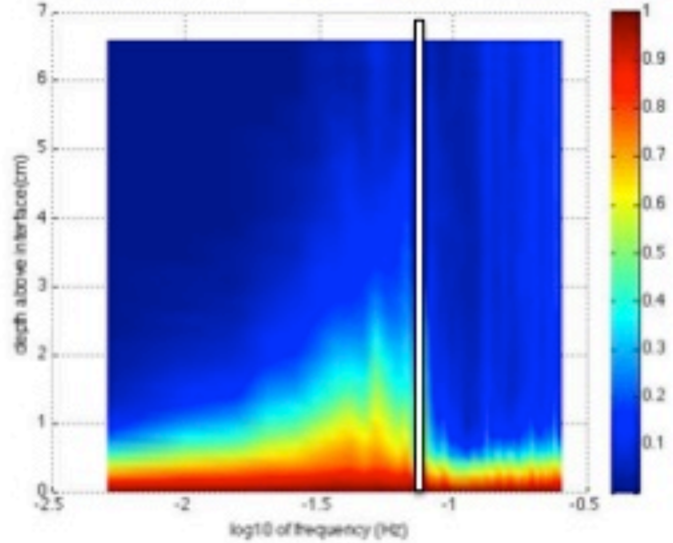
Wave field



Log10 of power spectral density

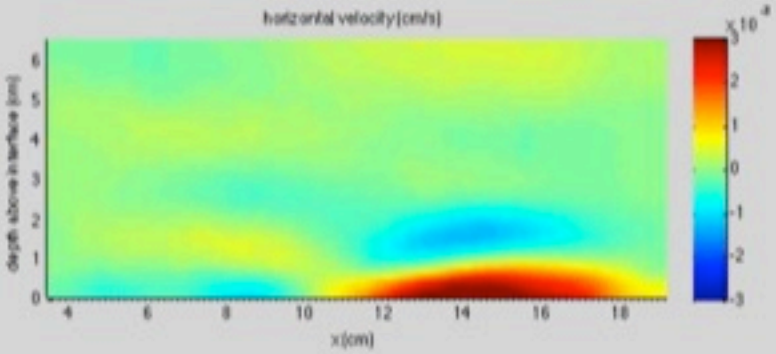


Renormalized power spectral density

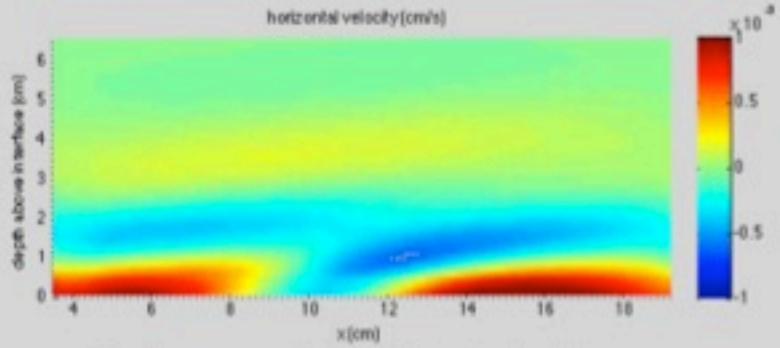


Wave field

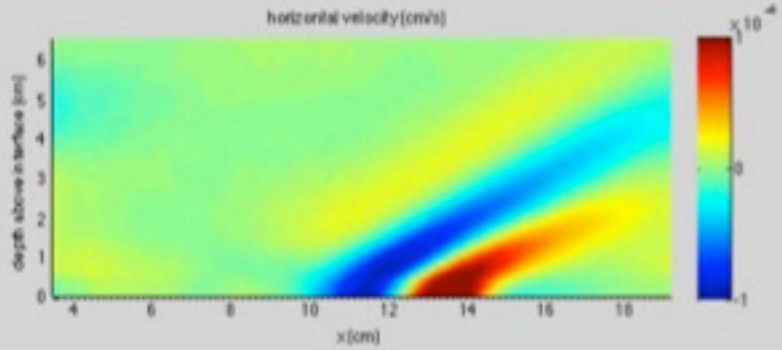
Accelerated x20
(real duration = 13 min)



Complete signal

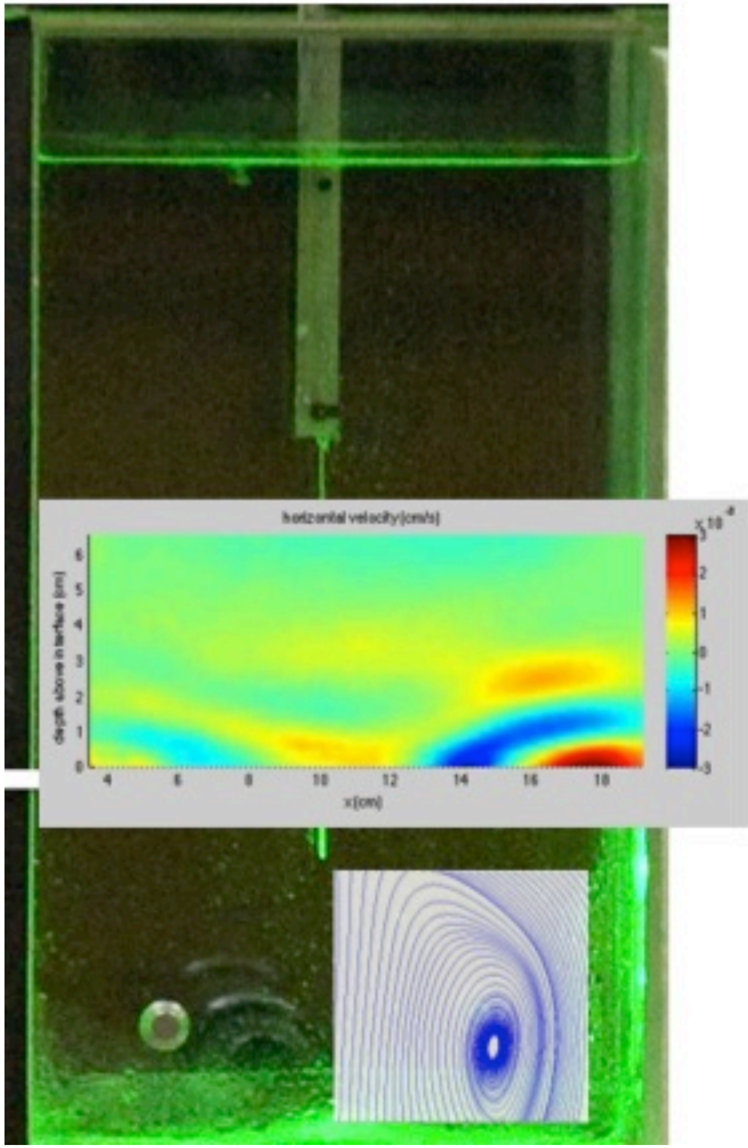


Filtered at f=0.01 Hz
+ right only

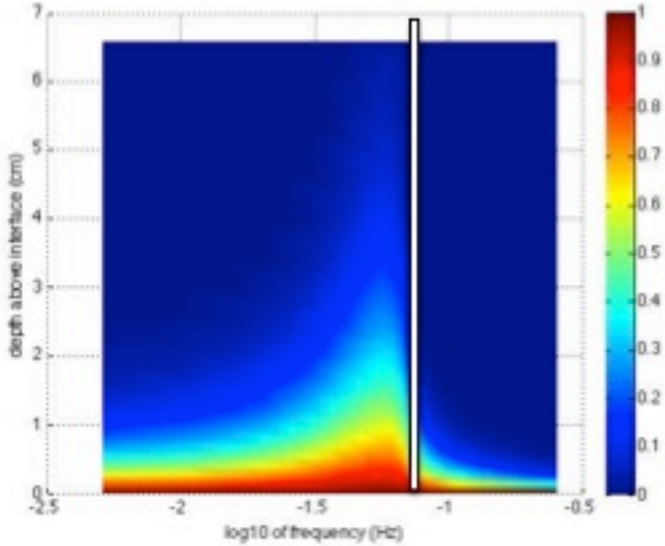


Filtered at f=0.06 Hz
+ right only

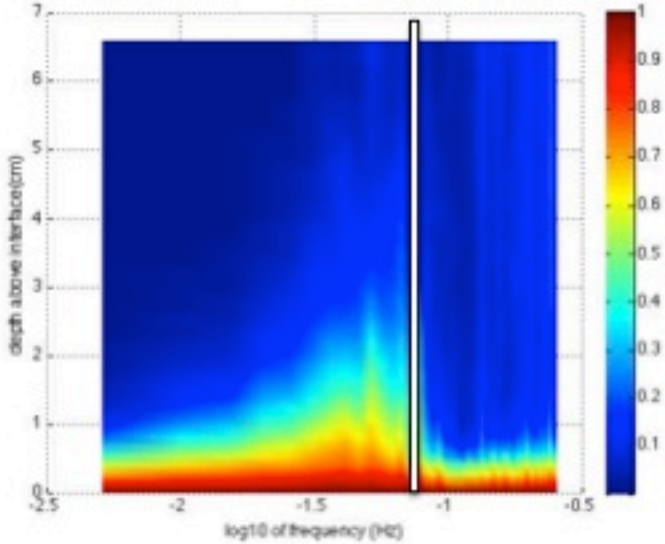
Wave field



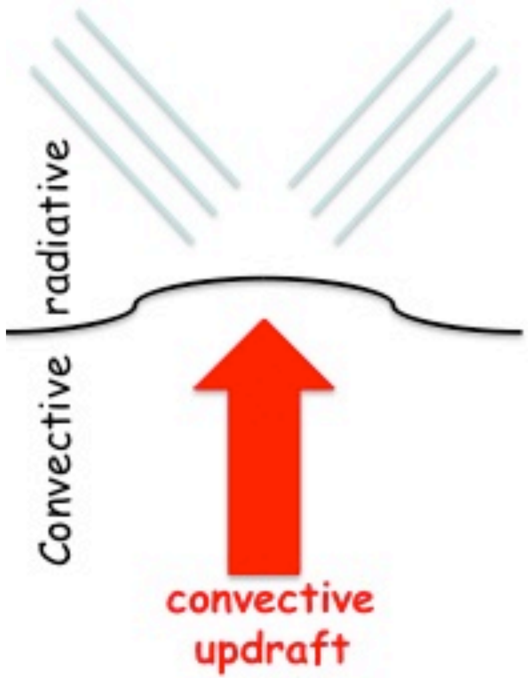
Linear diffusive theory with $k_x \sim f^{0.6}$



Renormalized power spectral density



Excitation mechanism?

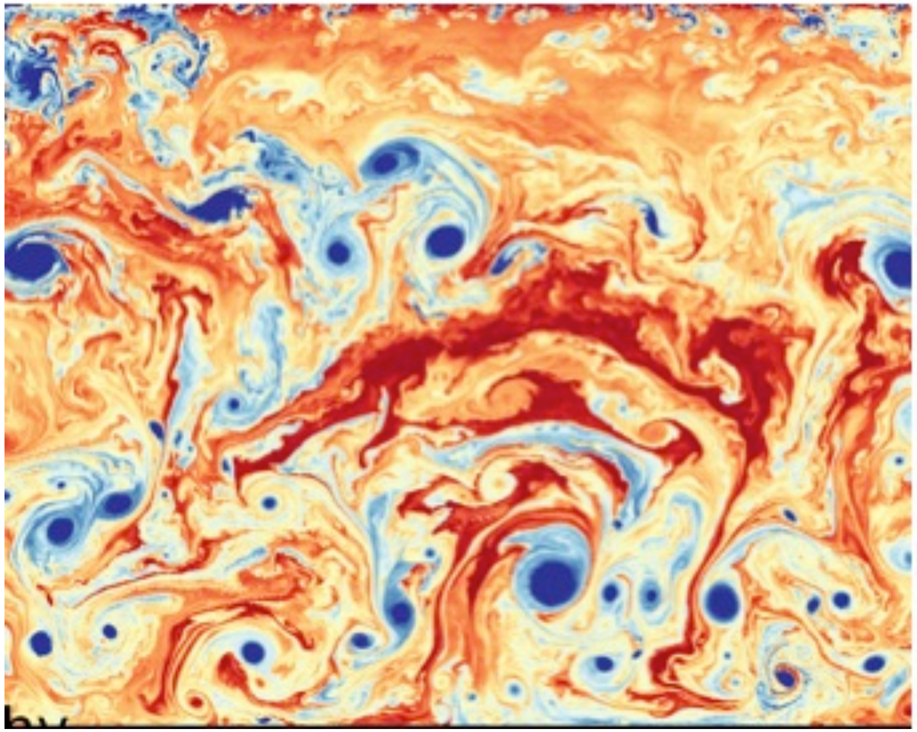


Excitation mechanism?

Numerical simulation...

Dedalus

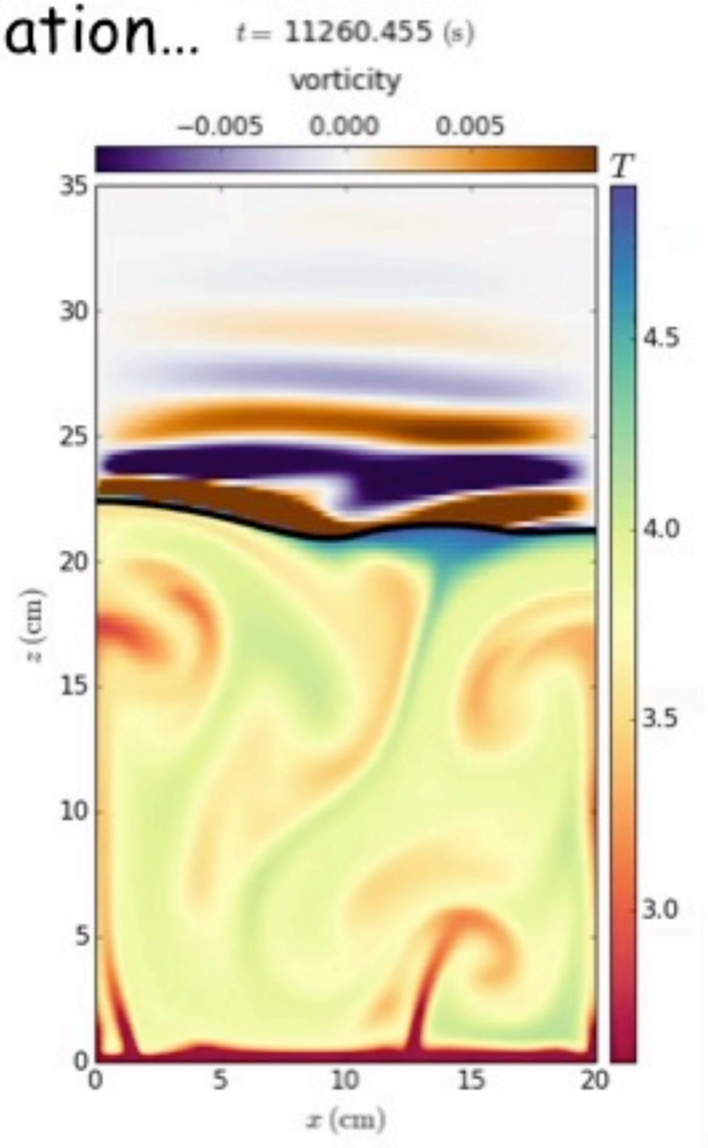
D. Lecoanet (Berkeley), K. Burns (MIT), J. Oishi (AMNH/ SUNY Farmingdale), B. Brown (Colorado), G. Vasil (Sydney)



Pseudo-spectral
Open-source
Python
Very flexible equations

Excitation mechanism?

Numerical simulation…



Excitation mechanism?

Simulation of the simulation: use full simulation data as inputs for simplified model simulations

Deep Forcing (Lighthill)

Split full solution into linear convective and linear wave modes:

$$\boldsymbol{u} = \boldsymbol{u}_c + \partial_t \boldsymbol{\xi}$$

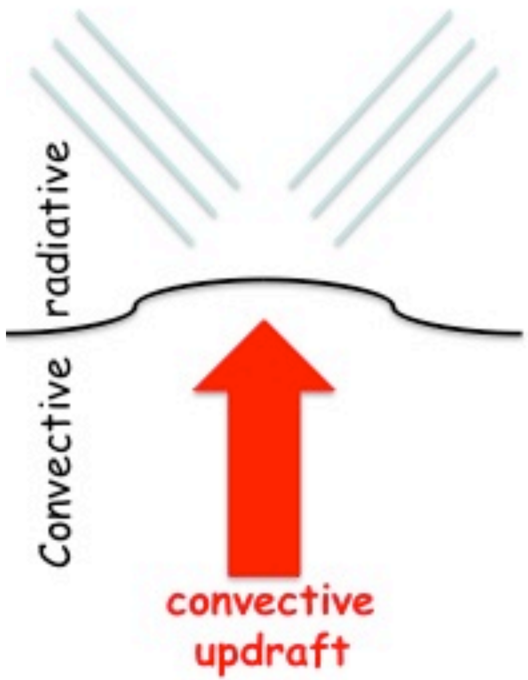
Project onto linear wave mode:

$$\nabla^2 \partial_t^2 \xi_z - N^2(z) \nabla_\perp^2 \xi_z = S$$

from water simulation

Excitation mechanism?

Simulation of the simulation: use full simulation data as inputs for simplified model simulations



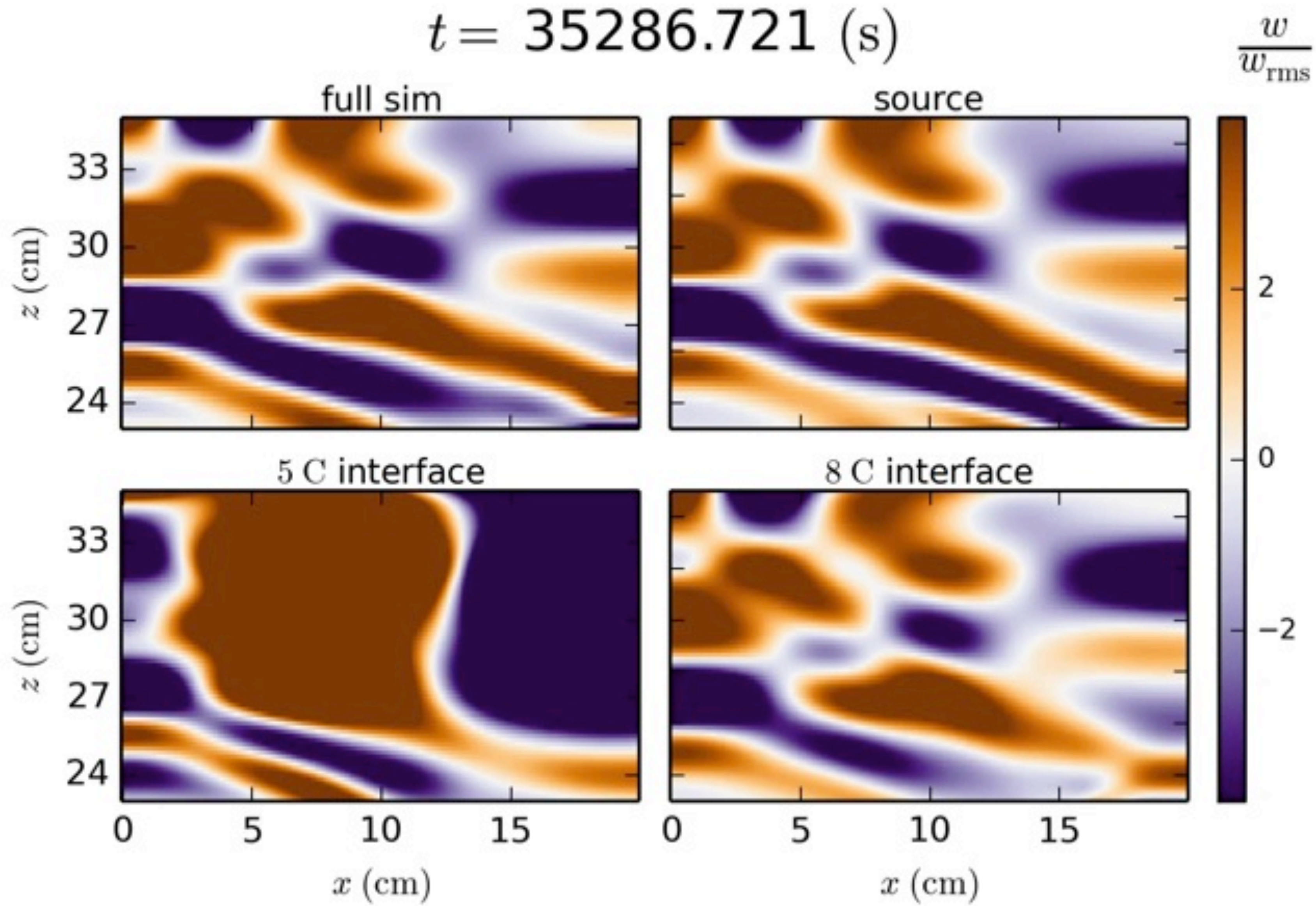
Interface fluctuations

No source term, but force boundaries

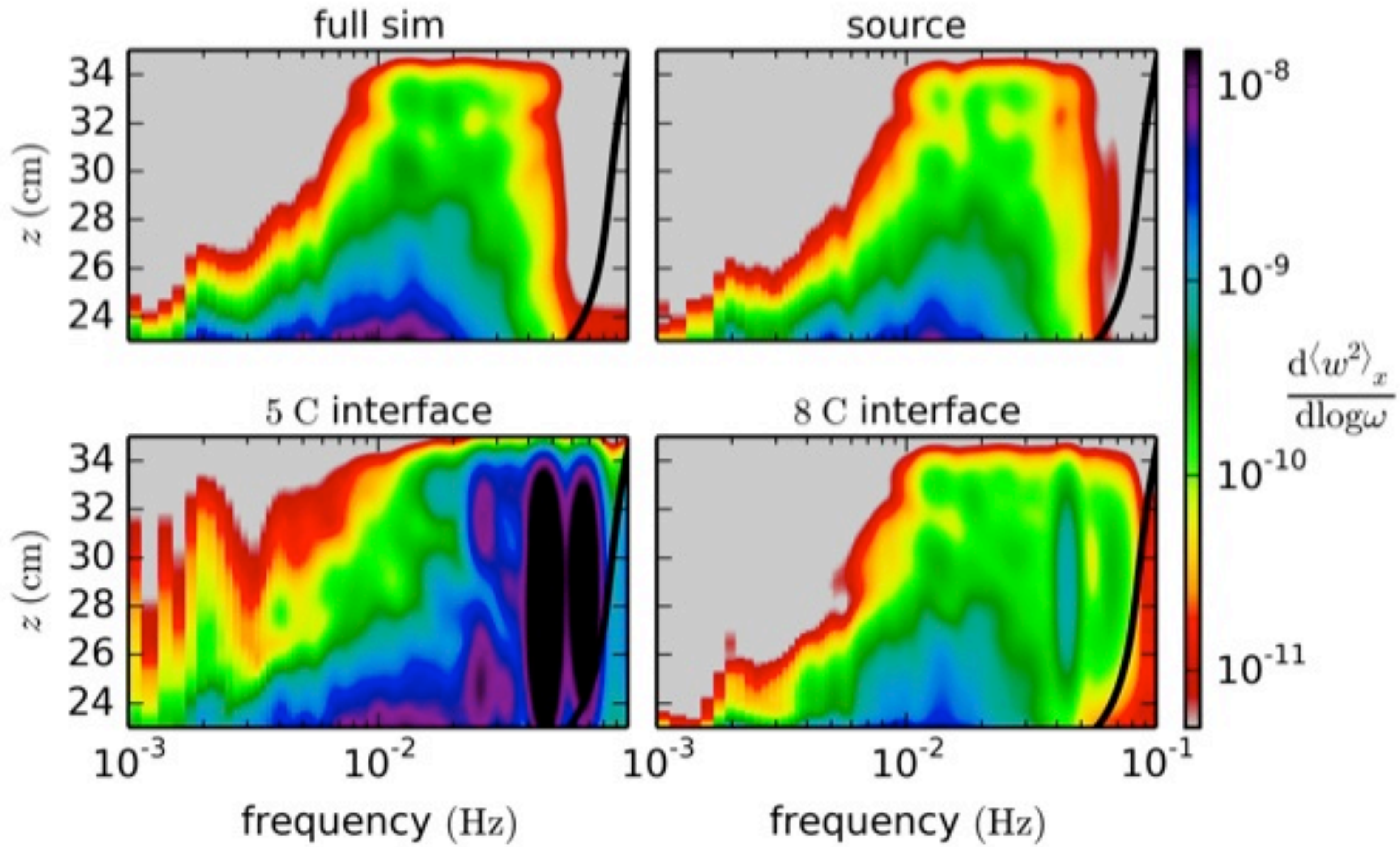
$$\nabla^2 \partial_t^2 \xi_z - N^2(z) \nabla_\perp^2 \xi_z = 0$$

Boundary condition:

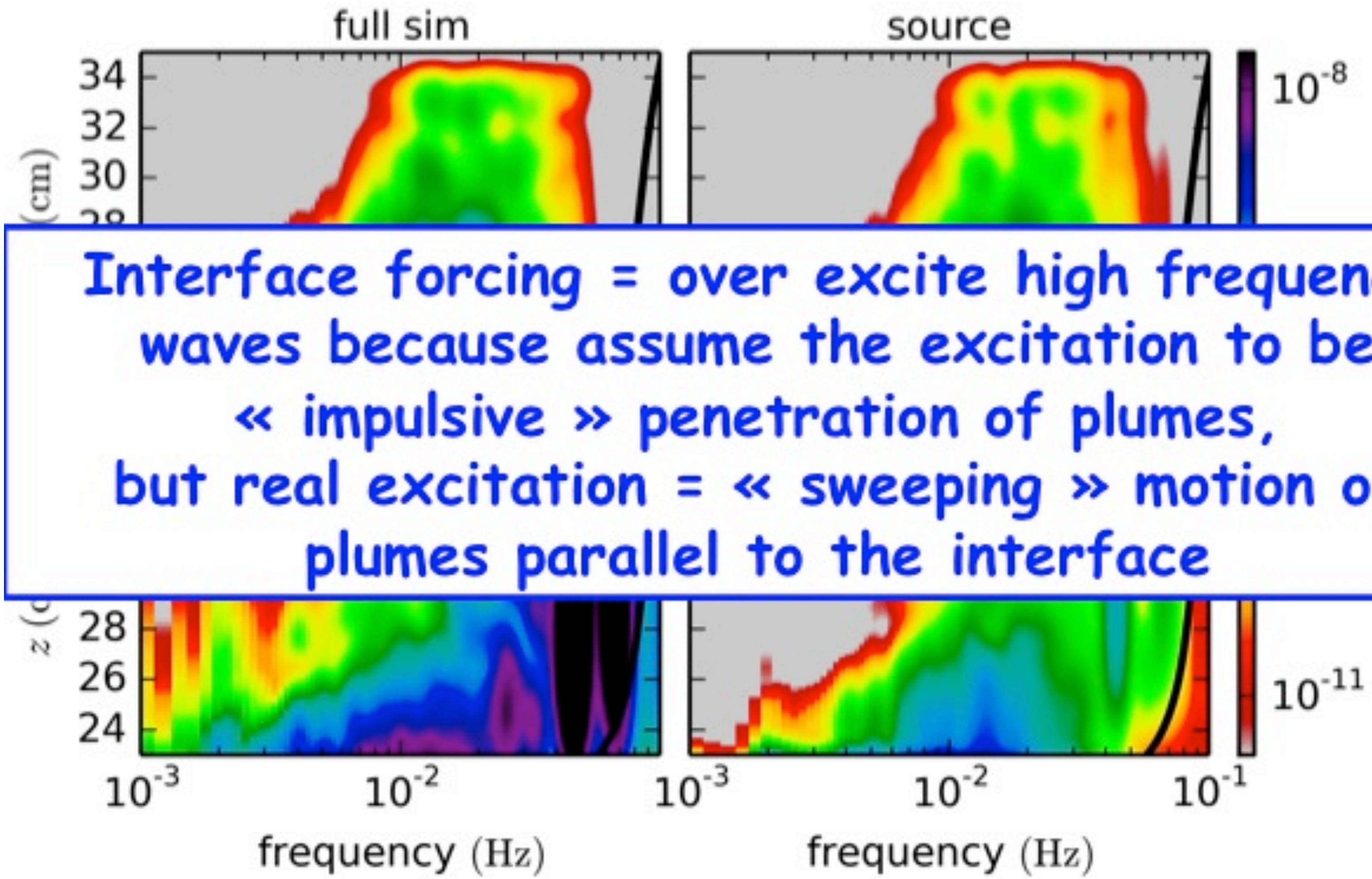
$$\xi_z(x, z_{\rm int}) = z_{\rm int}(x) - \bar{z}_{\rm int}$$



$t = 35286.721$ (s)
full sim
source
5 C interface
8 C interface
$\frac{w}{w_{\mathrm{rms}}}$
z (cm)
x (cm)
33
30
27
24
0
5
10
15
2
0
−2



full sim
source
5 C interface
8 C interface
z (cm)
34
32
30
28
26
24
frequency (Hz)
10^{-3}
10^{-2}
10^{-1}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
$\frac{d\langle w^2\rangle_x}{d\log\omega}$



Interface forcing = over excite high frequency waves because assume the excitation to be « impulsive » penetration of plumes, but real excitation = « sweeping » motion of plumes parallel to the interface

Conclusions

- A stratified region above/below a turbulent one is not motionless, but carries part of the energy

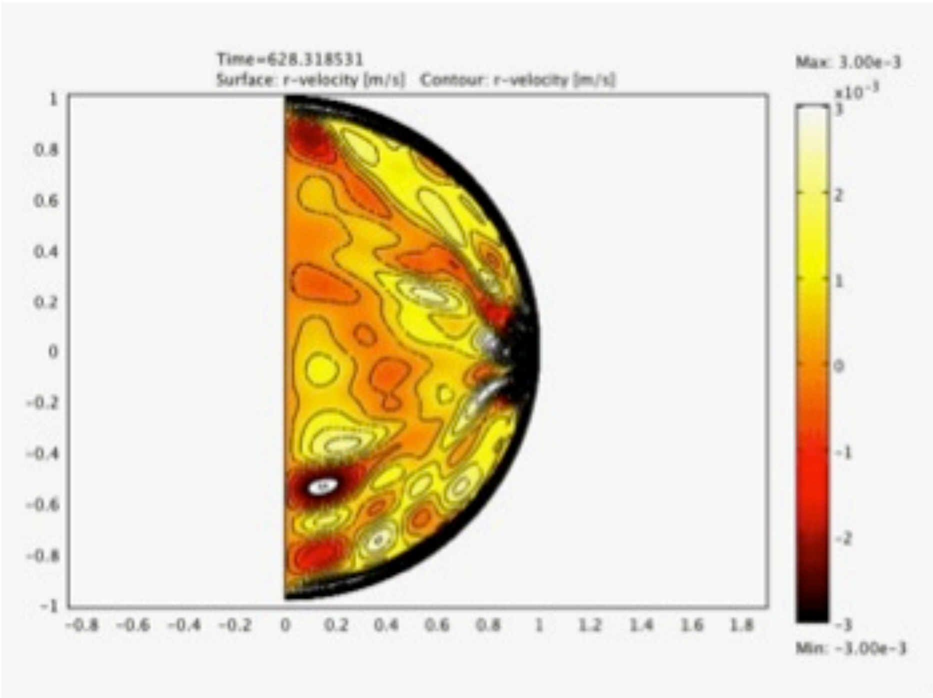
- Required = statistics of the convective source
- then wave amplitude and frequency selection depend on diffusive processes

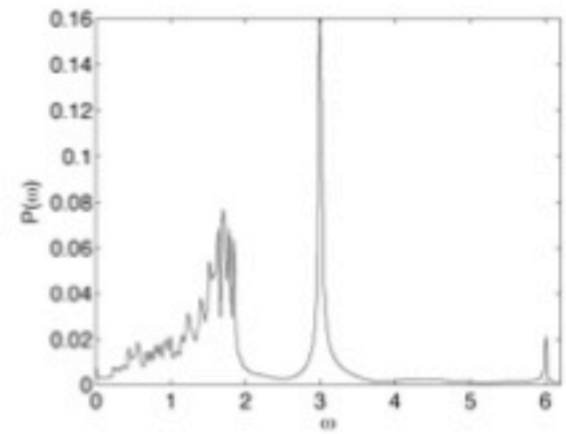
- Main excitation mechanism = Reynolds stress… Can use this to analytically estimate result for more turbulent cases (e.g., stars, see Lecoanet & Quataert 2013)

- Coming studies:
  - generation of a mean flow? Generalization of the QBO…
  - effect of global rotation…

Conclusions

- Generalization to other turbulence sources…
  e.g. boundary turbulence in librating planets





⇒ frequency selection in inertial waves (Sauret et al. 2013)