

PLANETARY atmosphere: Saturns polar vortex

Proto PLANETARY disk: generation of Vortices



Astrophysical and Geophysical flow 2 x Weather pattern:

Outline:





- Global "Baroclinic" Instability
- Subcritical "Baroclinic" Instability
- Goldreich-Schubert-Fricke Inst.
- Convective Overstability
- Linear theory and non-linear
- Simulations



Gas and dust disks around young stars "Birth places of Planets:"





Synthetic Populations...



Application of recent results on the orbital migration of low mass planets in planetary population synthesis

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...to explore the importance of metallicity, stellar mass, etc...





horizontal Johansen, Henning & Klahr 2006

12/13/2009

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10 cm sized boulders:

t = 0.1

alpha = 0.01WHY DO T TAURI DISKS ACCRETE?

Hartmann et al. 1998, 2006





Turbulent transport of angular momentum

Accretion Energy in rotating systems =>

Turbulence in Disks: Candidates

- Self Gravity until mass too low
- Non-linear Shear Instability at critical Re
- Magneto-Hydro turbulence
- Rossby-Wave Instability (Kelvin-Helmholtz in disks)
- Convective Overstability => Vortices ("subcritical baroclinic instability")
- Goldreich-Schubert-Fricke
- Critical layer ("Zombie Vortices") (Phil Marcus)

MHD Simulations of Stratified Protoplanetary Disk **Turbulence and Accretion in 3D Global**



Concentration of dust in Zonal Flows in MRI Gravoturbulent formation of Planetesimals turbulence

1. Johansen & Klahr 2005.

- 2. Johansen, Klahr &
- Henning 2006, 3. Johansen, Klahr & Mee
- 2006,
- 4. Johansen et al. 2007,
- 5. Flock et al . 2010,
- 6. Dzyurkevich 2011
- 7. Flock et al . 2011a, 2011b,
- 8. Uribe et al . 2011.
- 9. Johansen, Klahr &
- Henning 2011, 10. Flock et al. 2012a, 2012b,
- 11. Dittrich et al . 2013, ...

512 ^2 simulation 64 Mio particles Entire project used 15 Mio. CPU hours.



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revised schematics of protostellar disc

Simulations of thermal convection in disks: 1999 with Peter Bodenheimer

Large Scale 3D - Simulation 90 degree 3.5 - 6.5 AU 102 X 40 X 120 cells => Vortices

3D Global Disk Simulation flux limited Diffusion temperature maintained by artificial (viscous) heating

2





Then a lot of discussion started... Petersen, Stewart and Julien 2007: "Works with the right amount of thermal relaxation!" ...but 4 years later:

-0.001 > Ri > -0.01

Because:

Klahr 2004, Johnson & Gammie 2005, etc. :

somehow...

not a linear instability

Radial Entropy Gradient leads to vortices

Klahr & Bodenheimer 2003

 $Ri^2 = -\frac{2}{3\gamma} \left(\frac{H}{R}\right)^2 \beta_p \beta_s$

Vorticity: Pencil Code: Lyra and Klahr 2011; β = 2; N = 256; τ_c = 1







Lesur and Palaloizou 2010: 3D Unstratified Boussinesq



Lyra and Klahr 2011: 3D Unstratified Compressible + MHD



temperature pert.

density



Including thermal wind / vertical shear: turbulen! How come? 2.5D Local (radial - vertical),



2D axissymetric Pluto Simulation:



Or instantanous cooling: Goldreich & Schubert 1967 - Fricke 1968 Instability Modification of Solberg-Hoiland Criterion, including thermal relaxation: In collaboration with Alexander Hubbard

accretion discs Linear and nonlinear evolution of the vertical shear instability in

Richard P. Nelson¹[★], Oliver Gressel^{1,2,★} and Orkan M. Umurhan^{1,3,★} ¹ Astronomy Unit, Queen Mary University of London, Mile End Road, London El 4NS ² NORDITA, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, 106 91 Stockholm, Sweden ³ School of Natural Sciences, University of California, Merced, 5200 North Lake Rd, Merced, CA 95343, USA

 $\frac{\partial j^2}{\partial R} - \frac{k_R}{k_Z} \frac{\partial j^2}{\partial Z} < 0.$

Disk with radial temp. profile: $\Gamma \sim R^{-q}$ with q = 1



Disk with radial temp. profile: $\sim R^{-q}$ with q =



Disk with radial temp. profile: $\sim R^{-q}$ with q =



Disk with radial temp. profile: $\sim R^{-q}$ with q =



Movie by Richard Nelson



Is VSI / GSF already at stage 3?

regions (Goldreich & Schubert 1967; Fricke 1968). or effects like meridional circulation or Goldreich-Schubert-Fricke instabilities in radiative the grains are well mixed with the gas by either turbulent motion generated by convection, We assume

Mon. Not. R. astr. Soc. (1980) 191, 37-48

On the structure and evolution of the primordial solar nebula

D. N. C. Lin and J. Papaloizou Institute of Astronomy, Madingley Road, Cambridge CB3 0HA and Board of Studies in Astronomy and Astrophysics, University of California, Santa Cruz, CA 95064, USA

Received 1979 July 30; in original form 1979 April 20

12/13/2009

bert Klahr - Planet Formation - MPLA





Stability of vertically unstratified disk (Klahr and Hubbard 2014)

Stability under the influence of thermal relaxation

$$\Gamma = \frac{1}{2} \frac{-\tau N_R^2}{1 + \tau^2 \left(\kappa_R^2 + N_R^2\right)}$$

$$\Gamma = \frac{1}{2} \frac{-\frac{l^2}{\mu} N_R^2}{1 + \left(\frac{l^2}{\mu}\right)^2 \left(\kappa_R^2 + N_R^2\right)} - \frac{\nu}{l^2}$$

Similar to Lesur and Papaloizou 2010 for finite size vortices

$$\gamma \sim \frac{(-N^2)\sigma^2}{\mu} \phi_{\omega}(S\,\sigma^2/\mu) - \frac{\nu}{\sigma^2}$$

Convective Overstability in radially stratified accretion disks under thermal

relaxation

Hubert Klahr
1 & Alexander Hubbard 2

klahr@mpia.de

ApJ in press



2/13/2009

Hubert Klanr - Planet Formation - .



Analytic and Numerical results: Klahr and Hubbard 2014



12/13/2009

Nati - Franci Formation -Heidelberg

"...the other angles will be similar to the MHD case. "



This guy knew it, but only investigated



CONVECTIVE OVERSTABILITY IN ACCRETION DISKS 3D LINEAR ANALYSIS AND NONLINEAR SATURATION

WLADIMIR LYRA^{1,2,3} ApJ 2014

CONVECTIVE OVERSTABILITY IN ACCRETION DISKS 3D LINEAR ANALYSIS AND NONLINEAR SATURATION

Wladimir Lyra^{1,2,3} ApJ 2014



2/

Stability in vertically and radially stratified accretion disks under thermal relaxation

Hubert Klahr
1 Lingsong ${\rm Ge}^1$ & Alexander Hubbard 2

$$\partial_t \rho + \frac{1}{R} \partial_R R \rho u_R + \frac{1}{R} \partial_\phi \rho u_\phi + \partial_z \rho u_z = 0.$$

$$\partial_t S + u_R \partial_R S + \frac{u_\phi}{R} \partial_\phi S + u_z \partial_z S = -\frac{c_v}{T_0} \frac{T - T_0}{\tau}.$$

$$\begin{split} \partial_t u_R + u_R \partial_R u_R + \frac{u_\phi}{R} \partial_\phi u_R + u_z \partial_z u_R - \frac{u_\phi^2}{R} &= -\frac{1}{\rho} \partial_R p + g_R \\ \partial_t u_z + u_R \partial_R u_z + \frac{u_\phi}{R} \partial_\phi u_z + u_z \partial_z u_z &= -\frac{1}{\rho} \partial_z p + g_z \\ \partial_t u_\phi + u_R \partial_R u_\phi + \frac{u_\phi}{R} \partial_\phi u_\phi + u_z \partial_z u_\phi + \frac{u_\phi u_R}{R} &= -\frac{1}{R\rho} \partial_\phi p, \end{split}$$

R

	ıd D. Shalybkov ^{1,2}	G. Rüdiger ¹ , R. Arlt ¹ , and	
	ion disks under the combined density stratification	Hydrodynamic stability in accretio influence of shear and d	
	Astronomy Astrophysics	A&A 391, 781–787 (2002) DOI: 10.1051/0004-6361:20020853 © ESO 2002	
$+ u_z \frac{1}{c_v} \partial_z S_0 = 0$	$(1) + rac{1}{\gamma au} \left(\gamma rac{ ho_1}{ ho_0} + u_R rac{1}{c_v} \partial_R S_0 \right)$	$i(\omega-m\Omega)+rac{1}{ au} ight)rac{p_1}{p_0}-igg(-i(\omega-m\Omega))$	$\left(-i\right)$
			and
$k_z u_z = 0$	${}_{\mathrm{R}}+\partial_z\log ho_0 u_z+ik_R u_R+i_k$	$-i(\omega-m\Omega)rac{ ho_1}{ ho_0}+\partial_R\log ho_0 u_H$	
	$+ik_z\frac{p_1}{\rho_0} - \frac{\rho_1}{\rho_0^2}\partial_z p_0 = 0$	$-i(\omega-m\Omega)u_z$	
0	$\partial_R (R^2 \Omega) u_R + R \partial_z (\Omega) u_z = 0$	$-i(\omega-m\Omega)u_{\phi}+rac{1}{R}\hat{c}$	
_	$\partial u_{\phi} + ik_R rac{p_1}{ ho_0} - rac{ ho_1}{ ho_0^2} \partial_R p_0 = 0$	$-i(\omega-m\Omega)u_R-2 \Omega$	
tz: m << k _{R,z}	WKB Ansa	aves $\exp[i(k_rR + k_zz + m\phi - \omega t)].$	plain wa

$\begin{split} D = & \kappa_R^2 \left[\frac{c_s^2 k_z^2}{\gamma} + \frac{1}{\rho_0^2} \frac{\partial \rho_0}{\partial z} \frac{\partial p_0}{\partial z} - i \frac{k_z c_s^2}{\rho_0 \gamma} \frac{\partial \rho_0}{\partial z} + i \frac{k_z}{\rho_0} \frac{\partial p_0}{\partial z} \right] \\ & + \kappa_z^2 \left[\frac{c_s^2 k_z k_R}{\gamma} + \frac{1}{\rho_0^2} \frac{\partial \rho_0}{\partial R} \frac{\partial p_0}{\partial z} - i \frac{k_z c_s^2}{\rho_0 \gamma} \frac{\partial \rho_0}{\partial R} + i \frac{k_R}{\rho_0} \frac{\partial p_0}{\partial z} \right] \end{split}$	$C = \left(\frac{k_R}{\rho_0}\frac{\partial p_0}{\partial z} - \frac{k_z}{\rho_0}\frac{\partial p_0}{\partial R}\right) \left[\frac{k_R c_s^2}{c_v \gamma}\frac{\partial S_0}{\partial z} - \frac{k_z c_s^2}{c_v \gamma}\frac{\partial S_0}{\partial R} + \frac{i}{\rho_0^2}\left(\frac{\partial p_0}{\partial z}\frac{\partial \rho_0}{\partial R} - \frac{\partial p_0}{\partial R}\frac{\partial \rho_0}{\partial z}\right)\right] + \kappa_R^2 (k_z^2 c_s^2 + \frac{1}{\rho_0^2}\frac{\partial \rho_0}{\partial z}\frac{\partial p_0}{\partial z}) - \kappa_z^2 \left[k_R k_z c_s^2 + \frac{1}{\rho_0^2}\frac{\partial \rho_0}{\partial z}\frac{\partial p_0}{\partial z} + i\left(\frac{k_R}{\rho_0}\frac{\partial p_0}{\partial z} - \frac{k_z}{\rho_0}\frac{k_z}{\partial z}\frac{\partial \rho_0}{\partial z}\right)\right]$	$B = -i\left(rac{k_z}{ ho_0}rac{\partial p_0}{\partial z} + rac{k_R}{ ho_0}rac{\partial p_0}{\partial R} - rac{k_z c_s^2}{ ho_0 \gamma}rac{\partial ho_0}{\partial z} - rac{k_R c_s^2}{ ho_0 \gamma}rac{\partial ho_0}{\partial R} ight) onumber \ - \left(rac{k^2 c_s^2}{\gamma} + rac{1}{ ho_0^2}rac{\partial p_0}{\partial R}rac{\partial ho_0}{\partial R} + rac{1}{ ho_0^2}rac{\partial p_0}{\partial z}rac{\partial ho_0}{\partial z} + \kappa_R^2 ight)$	$A=k^2c_s^2+rac{1}{ ho_0^2}rac{\partial p_0}{\partial R}rac{\partial ho_0}{\partial R}+rac{1}{ ho_0^2}rac{\partial p_0}{\partial z}rac{\partial ho_0}{\partial z}+\kappa_R^2$	$\omega_m^5 + \frac{i}{\tau} \omega_m^4 - A \omega_m^3 + B \frac{i}{\tau} \omega_m^2 + C \omega_m + \frac{i}{\tau} D = 0$
	$\left[- rac{k_z}{ ho_0} rac{\partial p_0}{\partial R} ight) ight]$			









$\begin{split} D = & \kappa_R^2 \left[\frac{c_s^2 k_z^2}{\gamma} + \frac{1}{\rho_0^2} \frac{\partial \rho_0}{\partial z} \frac{\partial p_0}{\partial z} - i \frac{k_z c_s^2}{\rho_0 \gamma} \frac{\partial \rho_0}{\partial z} + i \frac{k_z}{\rho_0} \frac{\partial p_0}{\partial z} \right] \\ & + \kappa_z^2 \left[\frac{c_s^2 k_z k_R}{\gamma} + \frac{1}{\rho_0^2} \frac{\partial \rho_0}{\partial R} \frac{\partial p_0}{\partial z} - i \frac{k_z c_s^2}{\rho_0 \gamma} \frac{\partial \rho_0}{\partial R} + i \frac{k_R}{\rho_0} \frac{\partial p_0}{\partial z} \right] \end{split}$	$C = \left(\frac{k_R}{\rho_0}\frac{\partial p_0}{\partial z} - \frac{k_z}{\rho_0}\frac{\partial p_0}{\partial R}\right) \left[\frac{k_R c_s^2}{c_v \gamma}\frac{\partial S_0}{\partial z} - \frac{k_z c_s^2}{c_v \gamma}\frac{\partial S_0}{\partial R} + \frac{i}{\rho_0^2}\left(\frac{\partial p_0}{\partial z}\frac{\partial \rho_0}{\partial R} - \frac{\partial p_0}{\partial R}\frac{\partial \rho_0}{\partial z}\right)\right] + \kappa_R^2 (k_z^2 c_s^2 + \frac{1}{\rho_0^2}\frac{\partial \rho_0}{\partial z}\frac{\partial p_0}{\partial z}) - \kappa_z^2 \left[k_R k_z c_s^2 + \frac{1}{\rho_0^2}\frac{\partial \rho_0}{\partial z}\frac{\partial p_0}{\partial z} + i\left(\frac{k_R}{\rho_0}\frac{\partial p_0}{\partial z} - \frac{k_z}{\rho_0}\frac{k_z}{\partial z}\frac{\partial \rho_0}{\partial z}\right)\right]$	$B = -i\left(rac{k_z}{ ho_0}rac{\partial p_0}{\partial z} + rac{k_R}{ ho_0}rac{\partial p_0}{\partial R} - rac{k_z c_s^2}{ ho_0 \gamma}rac{\partial ho_0}{\partial z} - rac{k_R c_s^2}{ ho_0 \gamma}rac{\partial ho_0}{\partial R} ight) onumber \ - \left(rac{k^2 c_s^2}{\gamma} + rac{1}{ ho_0^2}rac{\partial p_0}{\partial R}rac{\partial ho_0}{\partial R} + rac{1}{ ho_0^2}rac{\partial p_0}{\partial z}rac{\partial ho_0}{\partial z} + \kappa_R^2 ight)$	$A=k^2c_s^2+rac{1}{ ho_0^2}rac{\partial p_0}{\partial R}rac{\partial ho_0}{\partial R}+rac{1}{ ho_0^2}rac{\partial p_0}{\partial z}rac{\partial ho_0}{\partial z}+\kappa_R^2$	$\omega_m^5 + \frac{i}{\tau} \omega_m^4 - A \omega_m^3 + B \frac{i}{\tau} \omega_m^2 + C \omega_m + \frac{i}{\tau} D = 0$
	$\left[- rac{k_z}{ ho_0} rac{\partial p_0}{\partial R} ight) ight]$			

 $\Gamma = \frac{1}{2} \frac{1}{1 + \tau^2 \left(\kappa_R^2 + N_R^2\right)}$

Convective Overstability

$$\omega_m^3 + \frac{i}{\gamma \tau} \omega_m^2 - (N_R^2 + \kappa_R^2) \omega_m - \frac{i \kappa_R^2}{\gamma \tau} = 0$$



G.S.F. or V.S. instability

 $-rac{k_R}{k_z}\kappa_z^2)$

 $\omega_m^2 = rac{k_z^2}{k_R^2} (\kappa_R^2 - rac{\omega}{\omega})$

If $k_R \gg k_z$

Incompressible limit: cs -> inf.

$$A'\omega_m^3 - B'\frac{1}{\tau}\omega_m^2 - C'\omega_m - \frac{1}{\tau}D' = 0$$



Evolution of largest velocity in simulation domain:



6000



Plus irradiation: Tstar = 4300; Rstar = 2 Rsolalpha = 0.001; Mdot = 1E-7 Msol/yr; Radial Stratification of acc. disks: Modeling observational data from Sean Andrews



Richardson number & thermal diffusion time

$$\begin{split} N^2 &= -\frac{1}{\gamma} \left(\frac{H}{R}\right)^2 \beta_s \beta_p \Omega^2 \\ Ri &= -\frac{2}{3\gamma} \left(\frac{H}{R}\right)^2 \beta_p \beta_s \end{split}$$

$$D = \frac{\lambda c 4 a_{\rm R} T^3}{\rho(\kappa + \sigma)},$$
$$\mathcal{T}_{therm} = H^2 / \frac{D}{\rho c_v}$$







Self-Replication of Critical Layers in Linearly Stable, Shearing, Philip S. Marcus, Suyang Pei, Chung-Hsiang Jiang, and Pedram Hassanzadeh and Daniel Lecoanet Pedram Hassanzadeh, Space-Filling Lattices of 3D Vortices Created by the $4\tau^2 N_z^2 >$ University of California, Berkeley, California, 94720, USA Department of Mechanical Engineering, Stratified, Rotating Flows (Dated: September 29, 2012) 12 8 0 0 -3 **-** a -3 _0 ა i, Ŀ, <u>_</u> ŗ, L دن 0 N 0 N دت చ ċ 0





Raettig 2012 Vortices: 1-10 AU Huge Concentration in traps: 100-10000 **Basic Picture 2: Starving Mode...** Zonal Flows: > 10 AU Dittrich et al. 2013





A Major Asymmetric Dust Trap in a Transition Disk

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LARGE-SCALE VORTICES IN PROTOPLANETARY DISKS: ON THE OBSERVABILITY OF POSSIBLE EARLY STAGES OF PLANET FORMATION

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Conclusions

GSF: Nelson, Umurhan & Gressel et al . 2013 CO: Klahr & Hubbard 2014, Lyra 2014

- 2 new / rediscovered instabilities that should occur in sufficiently MHD dead zones.
- 3D Properties and fate of vortices?
- 3D full radiation hydro runs...

revisiting: Klahr & Bodenheimer 2003

