Minimal Models for Precipitating Turbulent Convection

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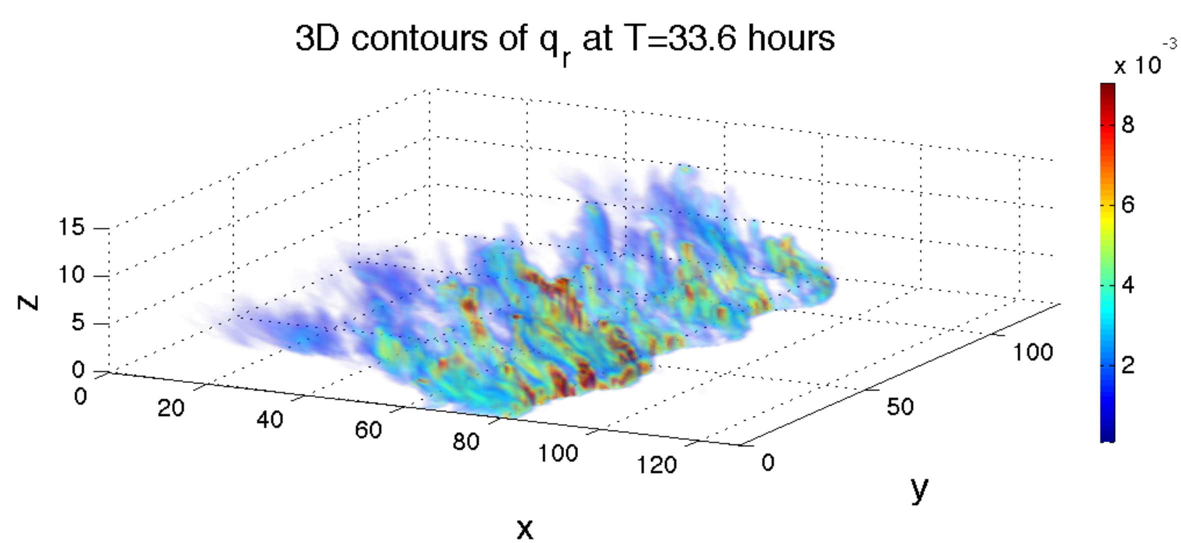


Contours of rain water in a simulated squall line

[a]NSF 1008396

[b]CMG 1025188

Goal

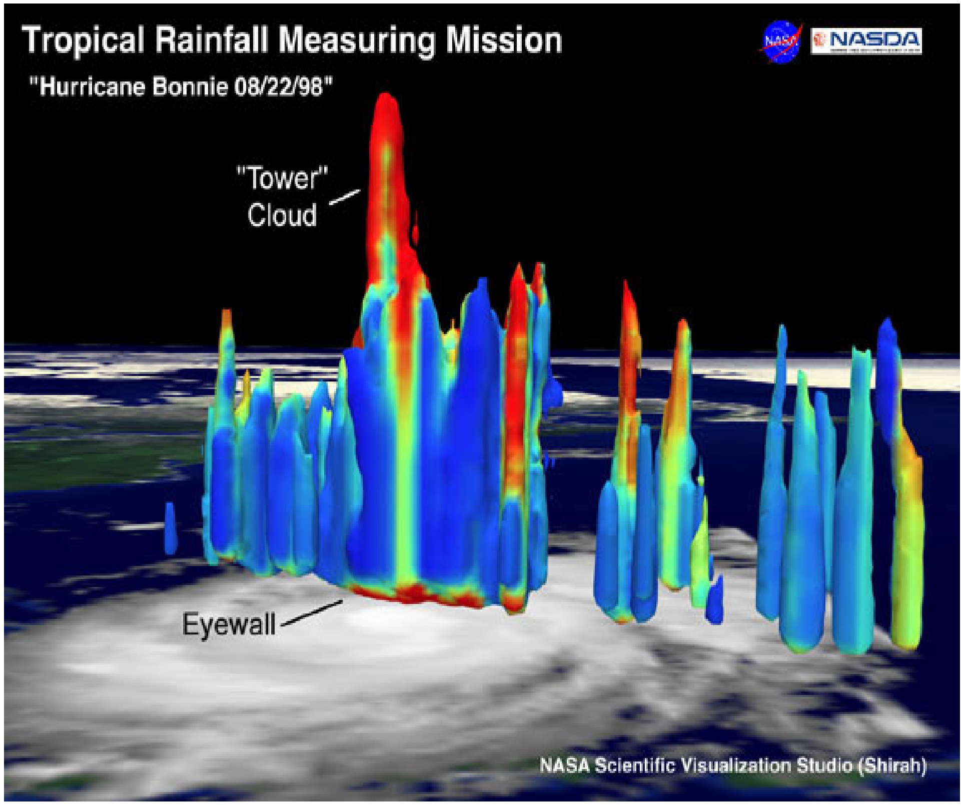
Find the simplest possible model that captures convective organization at many different length scales, for example

vortical hot towers: tall cumulonimbus clouds roughly 10 km (horizontal) by 15-20 km (vertical)

squall lines: horizontal scales $\approx$ 200 km

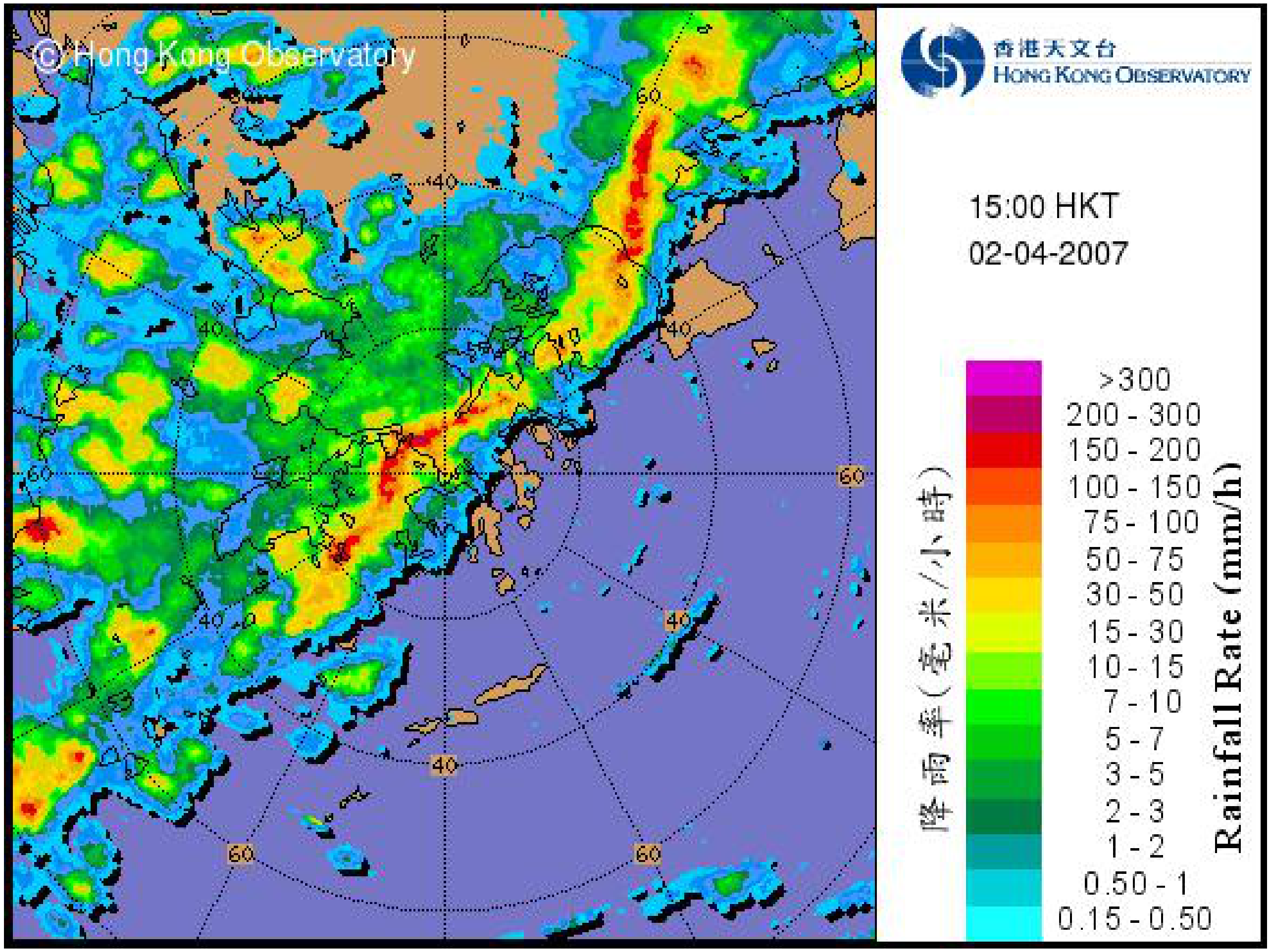
convectively coupled waves: horizontal scales $\approx$ 2000 km

Examples from Observations



Satellite data of “hot towers” during Hurricane Bonnie 1998: colors correspond to surface precipitation; 18 km high; $\approx$ 10 km wide.

Examples from Observations



Radar image of a squall line near Hong Kong, roughly 200 km long, propagates normal to the line

Outline

- Derivation of the minimal model
- Test case: squall lines
- Dynamic stability analysis;
  relation to thermodynamic stability

The Anelastic Equations with Warm-Rain Bulk Cloud Physics

$$\frac{D\mathbf{u}}{Dt} + f\sin(\phi)\mathbf{u}_h^{\perp} = -\nabla\left(\frac{p'}{\tilde{\rho}(z)}\right)$$

$$+\mathbf{k}\, g\left(\frac{\theta'}{\tilde{\theta}(z)} + \varepsilon_o q_v - q_c - q_r\right)$$

$$\nabla\cdot(\tilde{\rho}(z)\mathbf{u}) = 0, \qquad \frac{D\theta'}{Dt} + w\frac{d\tilde{\theta}(z)}{dz} = \frac{L\tilde{\theta}(z)}{c_p\tilde{T}(z)}(C_d - E_r)$$

$\rho(\mathbf{x},t) = \tilde{\rho}(z) + \rho'(\mathbf{x},t)$, with $\tilde{\rho}(z)$ prescribed, etc;
valid for $H \approx -\tilde{\rho}(d\tilde{\rho}/dz)^{-1}$ (the density scale height)

Bulk Cloud Physics

$$\frac{Dq_v}{Dt} = -C_d + E_r, \qquad \frac{Dq_c}{Dt} = C_d - A_r - C_r$$

$$\frac{Dq_r}{Dt} - \frac{1}{\tilde{\rho}(z)} \frac{\partial}{\partial z} (\tilde{\rho}(z) V_T q_r) = A_r + C_r - E_r$$

C_d: Condensation $q_v \to q_c$, E_r: Evaporation $q_r \to q_v$

A_r, C_r: Auto-conversion and Collection $q_c \to q_r$

V_T: Rainfall velocity

Now make the following approximations

- Boussinesq $H \ll -\tilde{\rho}(d\tilde{\rho}/dz)^{-1}$ (not true here!), eliminates prescribed backgrounds

- Fast Auto-Conversion of cloud to rain water (in reality 15 minutes), eliminates q_c, A_r, C_r

- Fast Condensation (in reality a few seconds) and Fast Evaporation

The Boussinesq Approximation

un-differentiated background replaced by constants

$$\frac{D\mathbf{u}}{Dt} + f\sin(\phi)\mathbf{u}_h^{\perp} = -\nabla p' + \mathbf{k}\, g\left(\frac{\theta'}{\theta_o} + \epsilon_o q_v - q_r\right)$$

$$\nabla \cdot \mathbf{u} = 0, \qquad \frac{D\theta'}{Dt} + \frac{d\tilde{\theta}(z)}{dz} w = \frac{L}{c_p}(C_d - E_r)$$

$\theta = T(p_o/p)^{R/c_p}$

R, R_v gas constant for air, vapor; $R_v/R = \epsilon_o + 1$

Equations for water vapor and rain

$$\frac{Dq_v}{Dt} = -(C_d - E_r), \qquad \frac{Dq_r}{Dt} - V_T \frac{\partial q_r}{\partial z} = C_d - E_r$$

$$C_d - E_r = 0, \quad q_v \le q_{vs}, \quad q_r = 0 \ \ \text{unsat}$$

$$C_d - E_r = -w\,\frac{dq_{vs}(z)}{dz}, \quad q_v = q_{vs}, \quad q_r > 0 \ \ \text{sat}$$

$q_{vs}(T,p) \approx q_{vs}(z)$ is the saturation profile

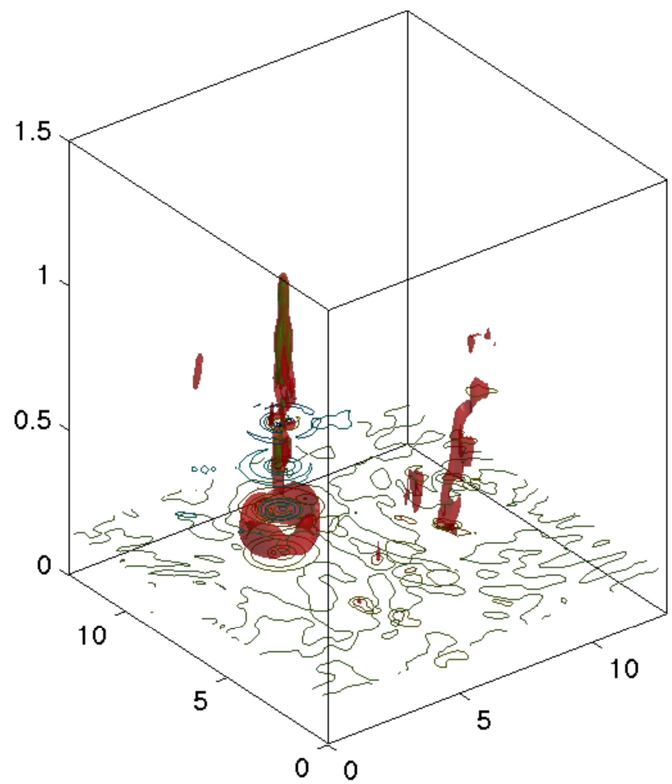
$C_d - E_r > 0$:

$$\frac{D\theta'}{Dt} + Bw = \frac{L}{c_p}(C_d - E_r)$$

$$\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + g\left(\frac{\theta'}{\theta_o} + \varepsilon_o q_{vs} - q_r\right)$$

Standard values: $L^d \approx 2.5 \times 10^6$ J kg^{-1}, $c_p \approx 10^3$ J kg^{-1} K^{-1}, $\theta_o = T_o = 300$ K, $d\tilde{\theta}(z)/dz \equiv B = 3$ K km^{-1}, etc.

Contours of Water Vapor Anomaly ($128 \times 128 \times 15$ km)



Moisture anomaly contour (1.2×10^{-3} kg/kg) in a Vortical Hot Tower; 30 min after moisture bubble injection at low altitude

Constraints

Either Unsaturated:

$$q_v < q_{vs}: \qquad q_r = 0, \qquad q_{\text{tot}} = q_v$$

Or Saturated:

$$q_v = q_{vs}: \qquad q_r = q_{\text{tot}} - q_{vs}$$

$$q_{\text{tot}} = q_v + q_r$$

Using $\theta_e = \theta + \frac{L}{c_p} q_v$, q_{tot}:

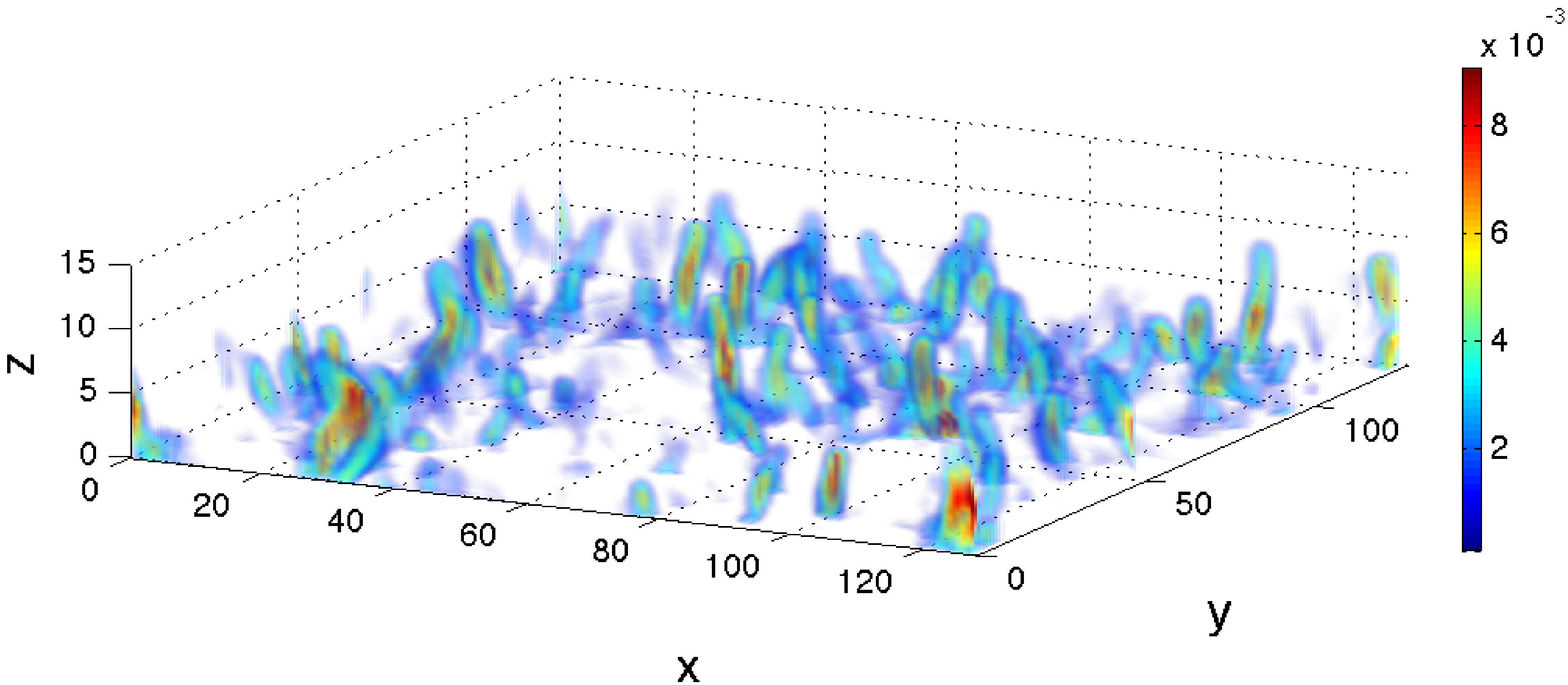
$$\frac{D\theta_e}{Dt} = 0, \qquad \frac{Dq_{\text{tot}}}{Dt} - V_T \frac{\partial q_r}{\partial z} = 0$$

no explicit source terms

Similar models for non-precipitating, shallow convection; q_r replaced by q_c: Grabowski & Clark 1993; Spyksma, Bartello & Yao 2006; Pauluis & Schumacher 2010

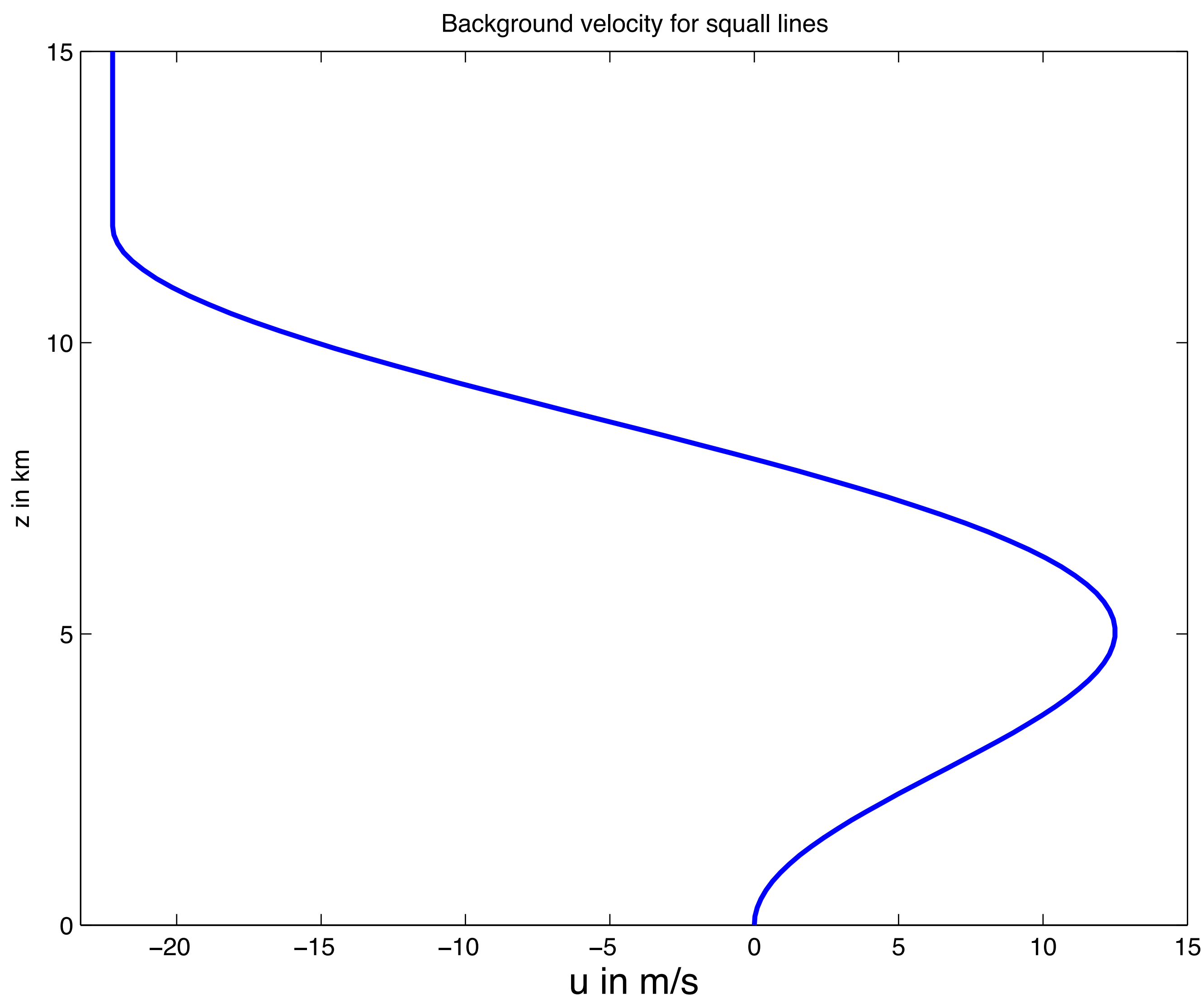
Environmental conditions

- Low-altitude moistening, e.g. over a warm ocean



Scattered convection; 3D contours of rain water

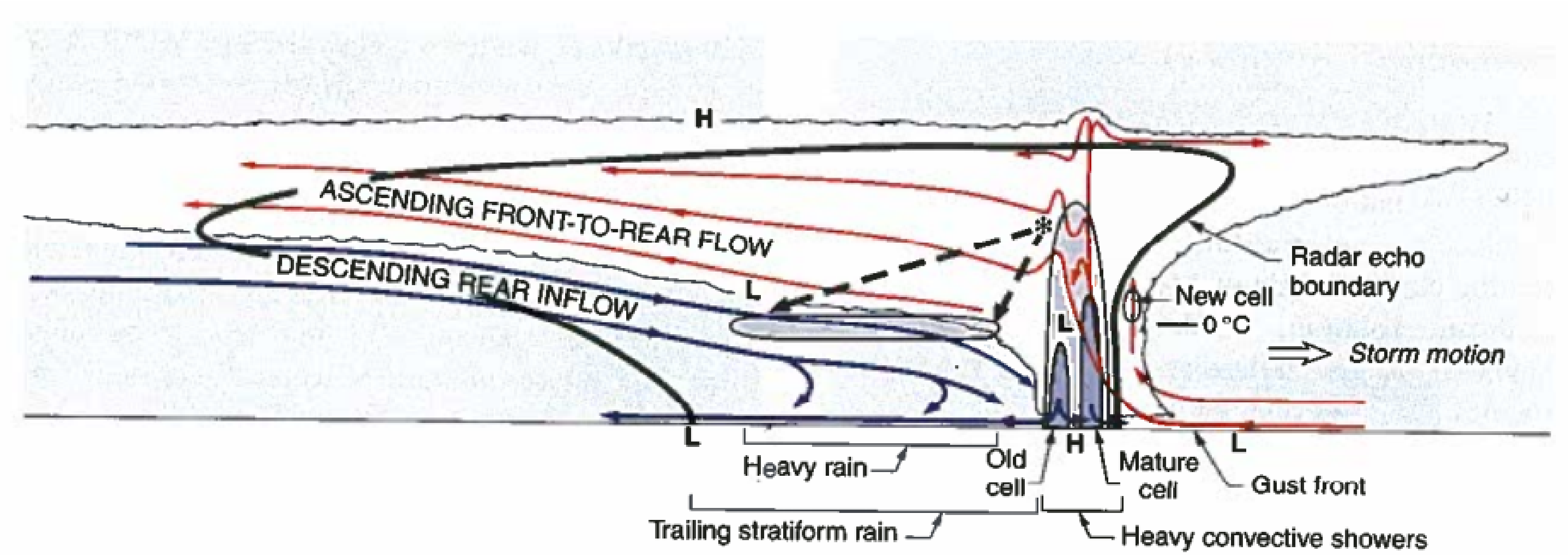
A background wind will organize the convection:



A squall line

Fred Roswald & Judy Jensen, http://wingssail.blogspot.com, 0337-SquallApproachesSumatra.jpg

More Characteristics of Squall Lines (Wallace & Hobbs, 2006)



- Long lasting multi-cell storm
- Tilted profile
- Propagate long distances (Houze 2004)
- Low-altitude cold pool

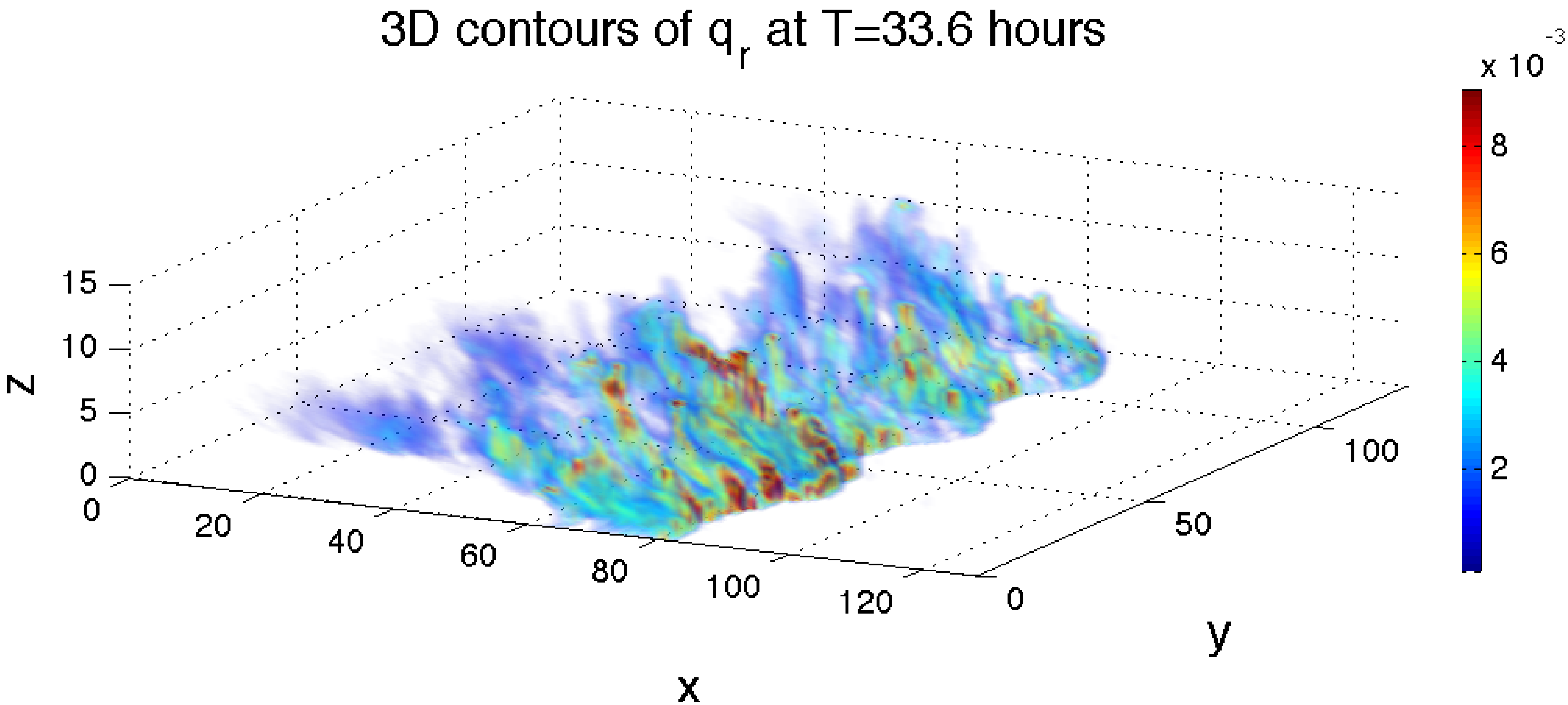
References

Observations, e.g. during GATE: Barnes & Sieckman 1984; during TOGA COARE: Jorgensen, LeMone & Trier 1997; LeMone, Zipser & Trier 1998; Review: Houze 2004.

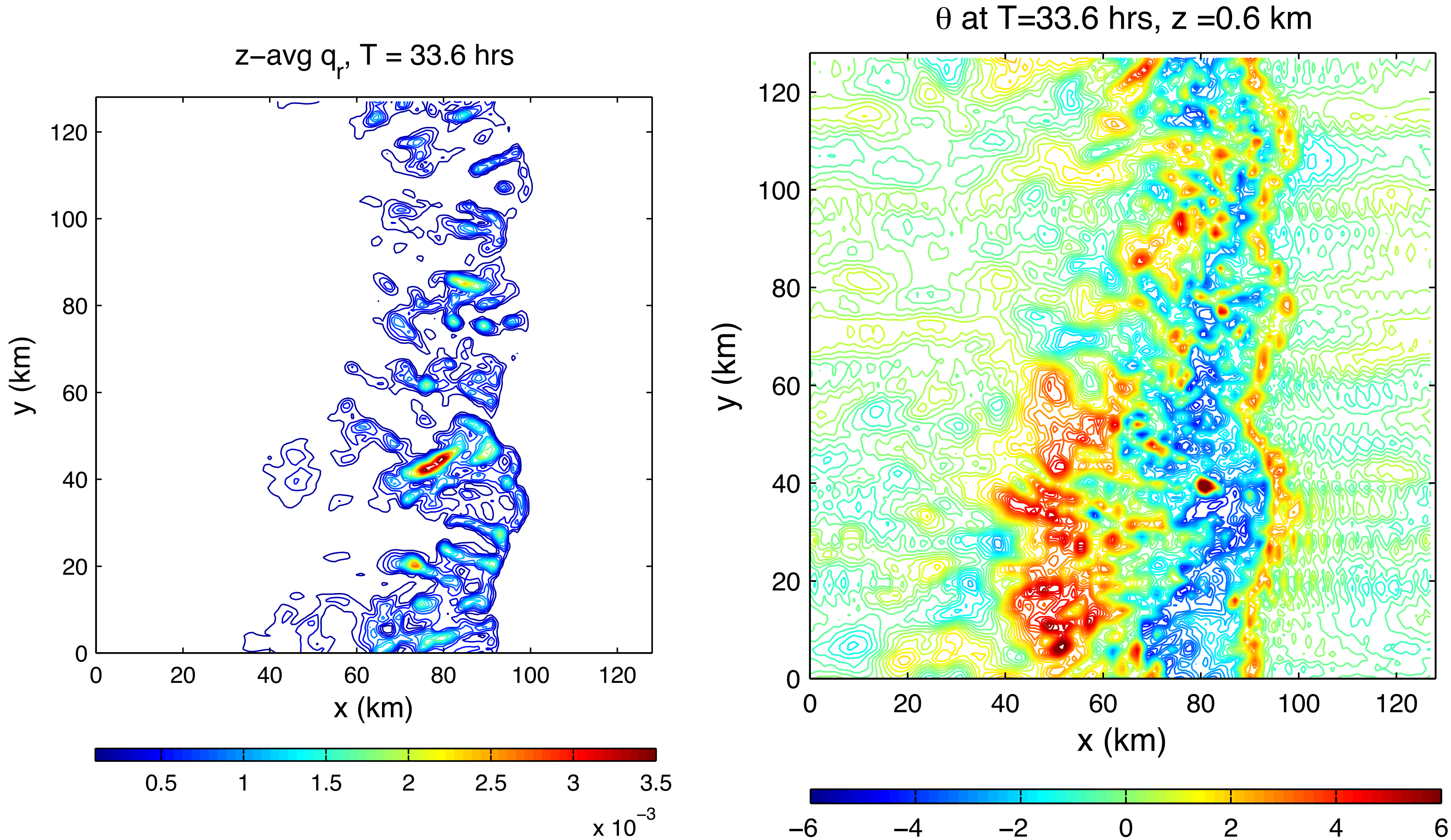
CRMs: Fovell & Ogura 1988; Lafore & Moncrieff 1989; Grabowski, Wu & Moncrieff 1996, 1998; Xu & Randall 1996; Lucas, Zipser & Ferrier 2000; Lu & Moncrieff 2001

Conceptual & Minimal Models: Moncrieff & Green 1972; Moncrieff & Miller 1976; Moncrieff 1981; Emanuel 1986; Rotunno, Klemp, Weisman 1988; Majda & Xing 2010

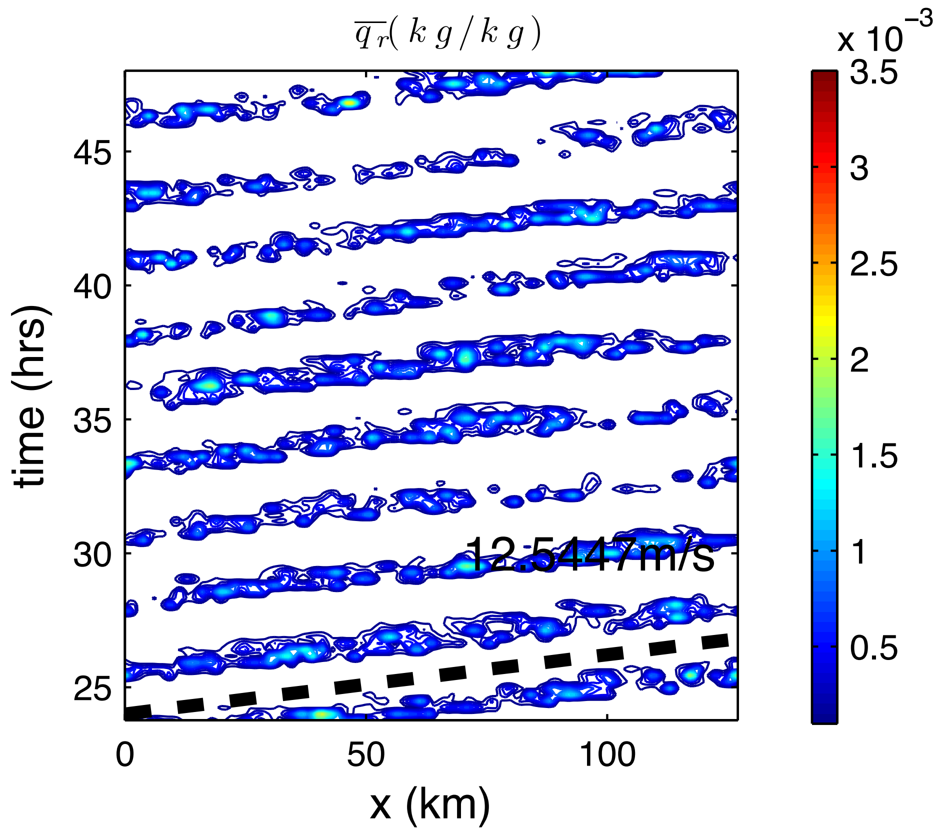
FARE squall line: 3D Contours of Rain Water

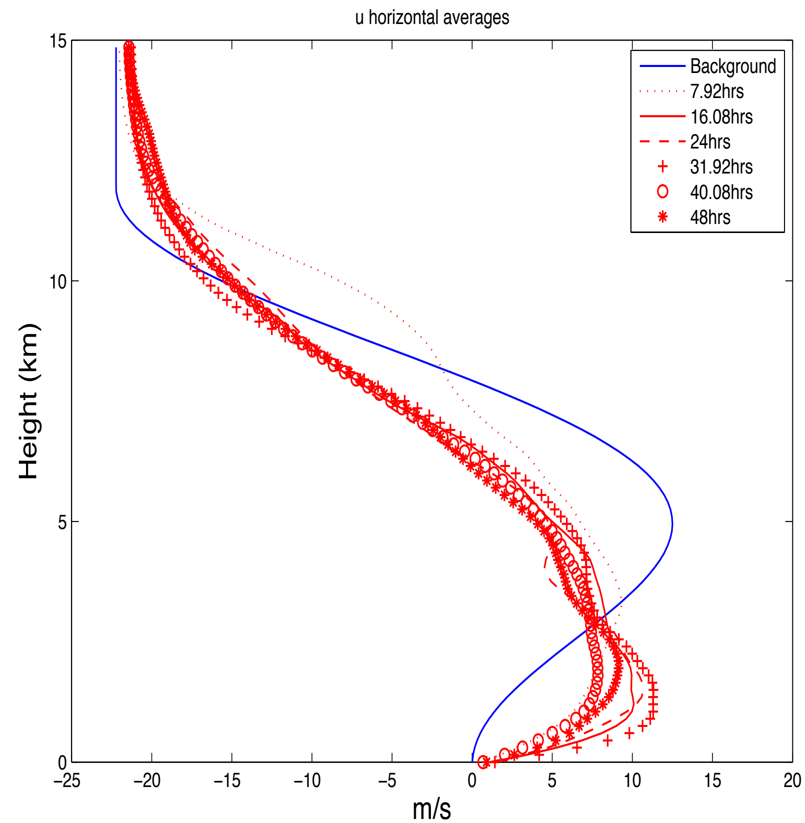


Vertically averaged rain, Low-level cold pool



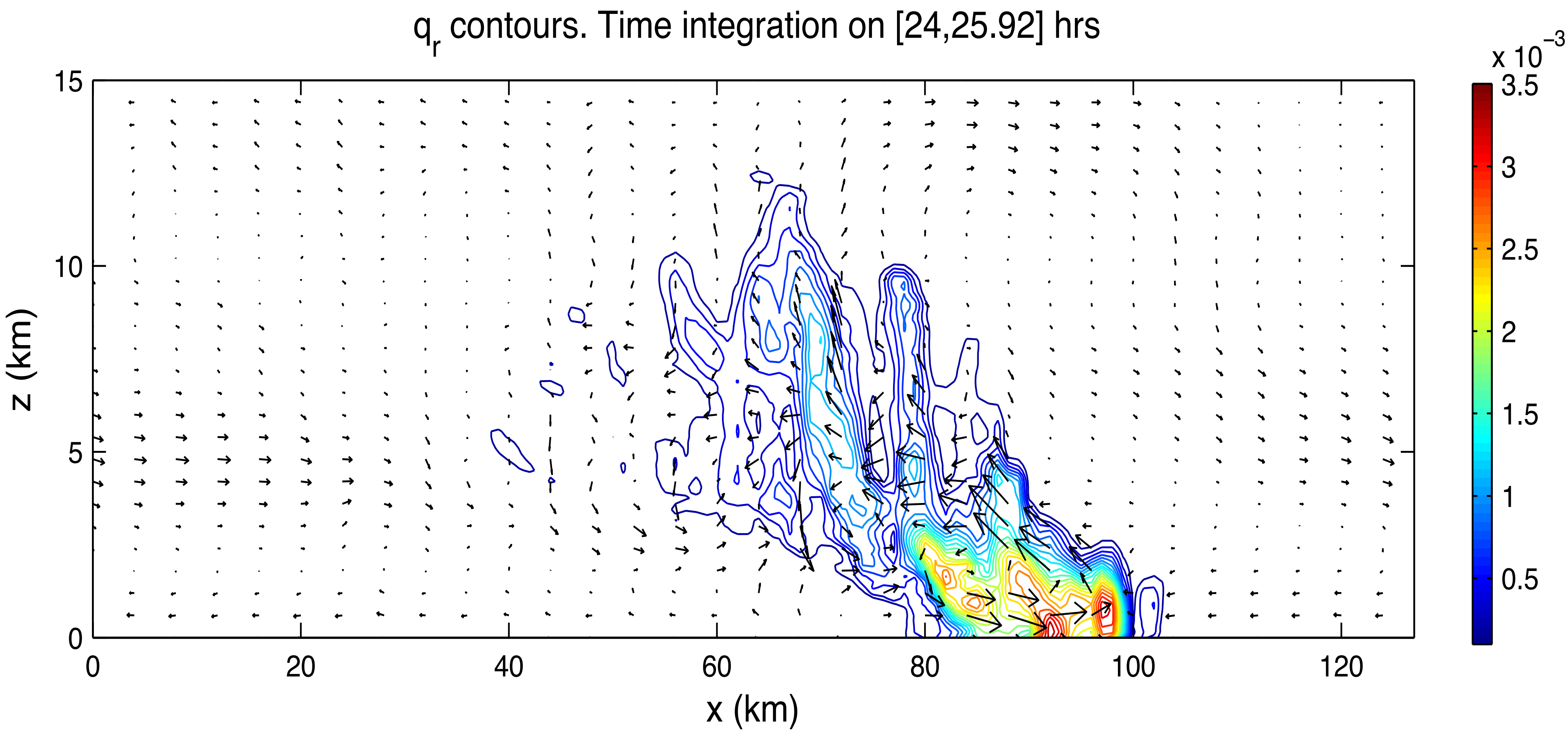
Time Evolution





Propagates at the speed of the jet max (black dash)

Multi-cloud structure

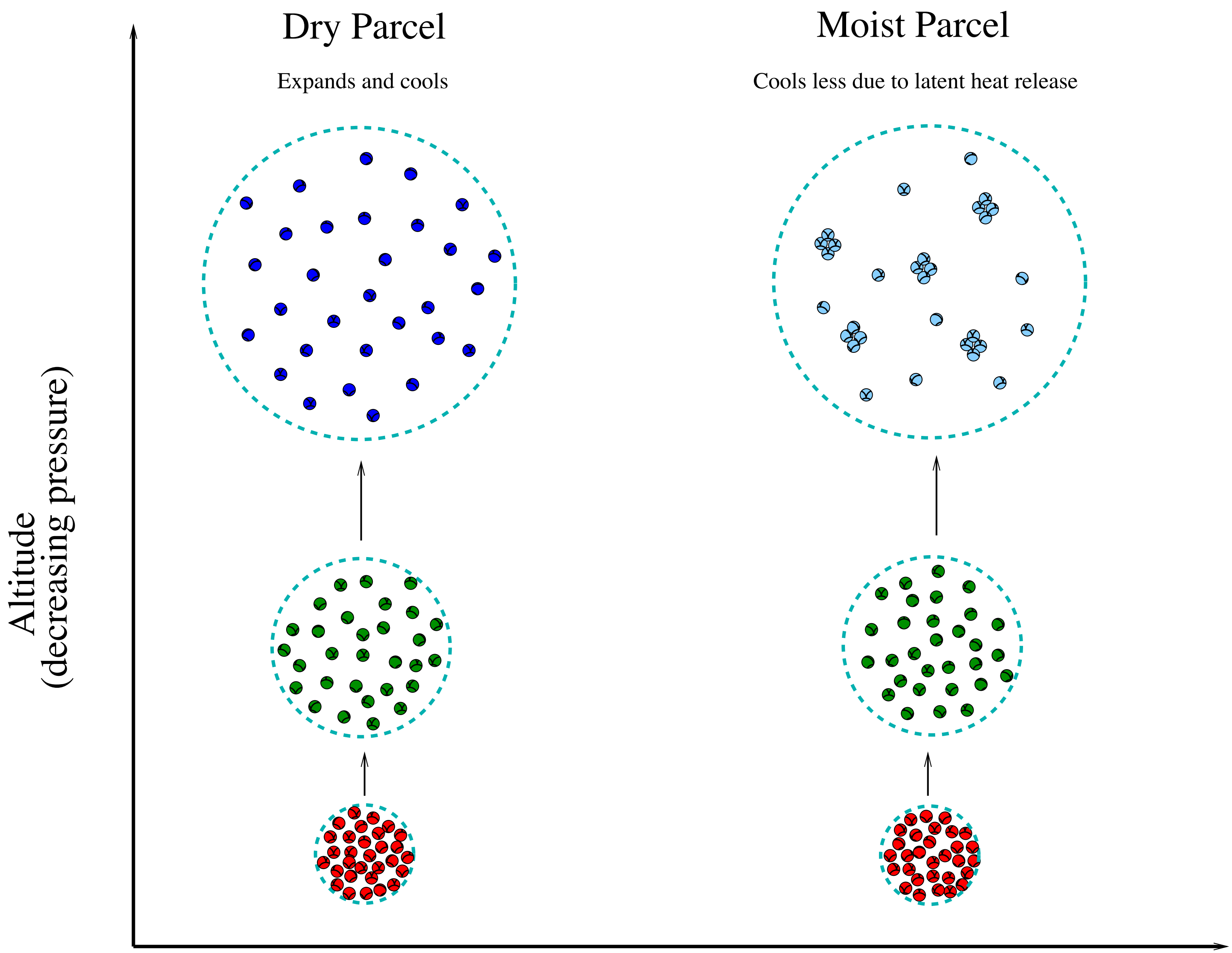


Stability: what is the probability for clouds/precipitation to form?

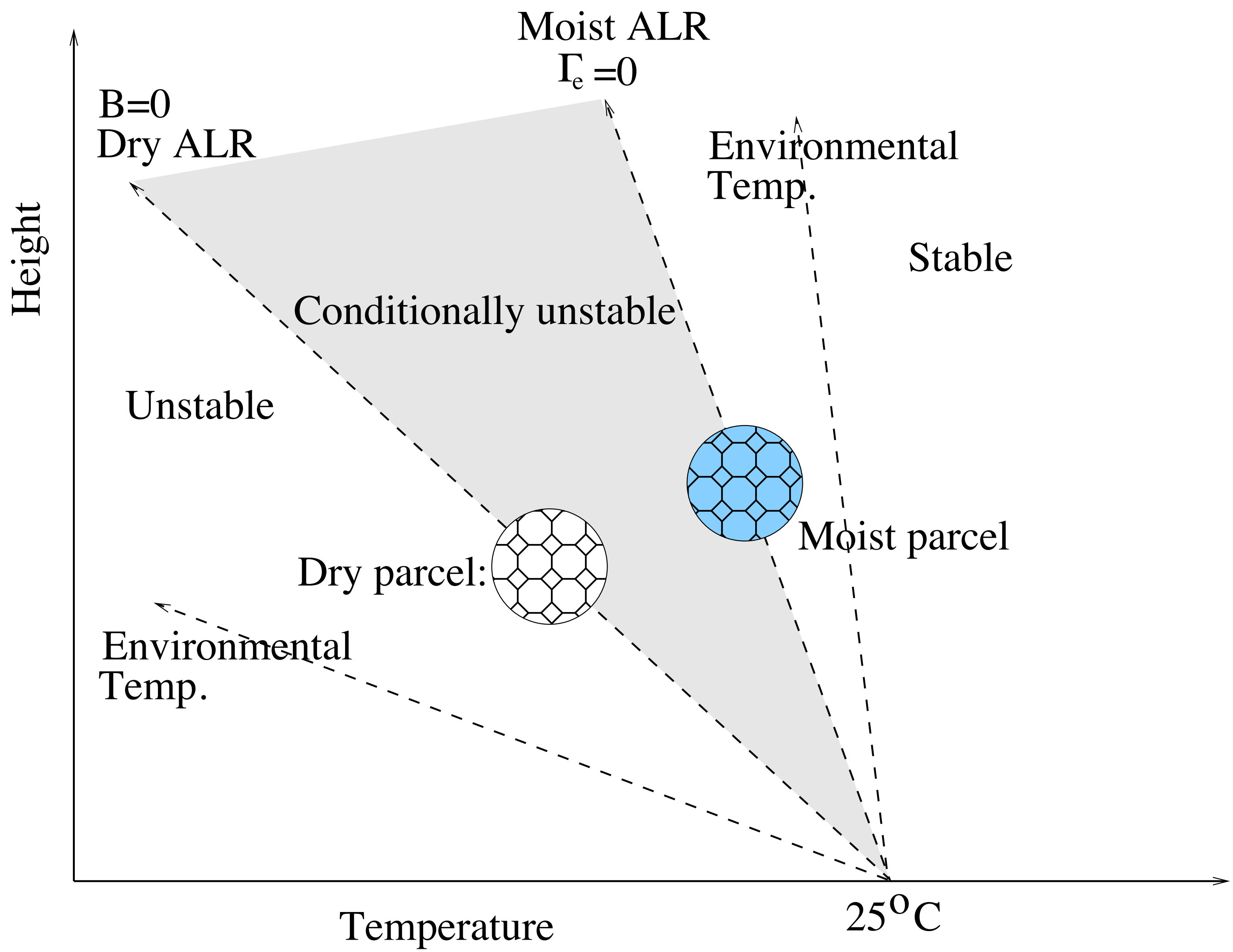
Thermodynamic stability: parcels rising adiabatically, equilibrium thermodynamics

Dynamic stability: linear/nonlinear stability analysis of the dynamical equations

Rising Dry vs Moist Parcels



Thermodynamic Concept of Conditional Stability



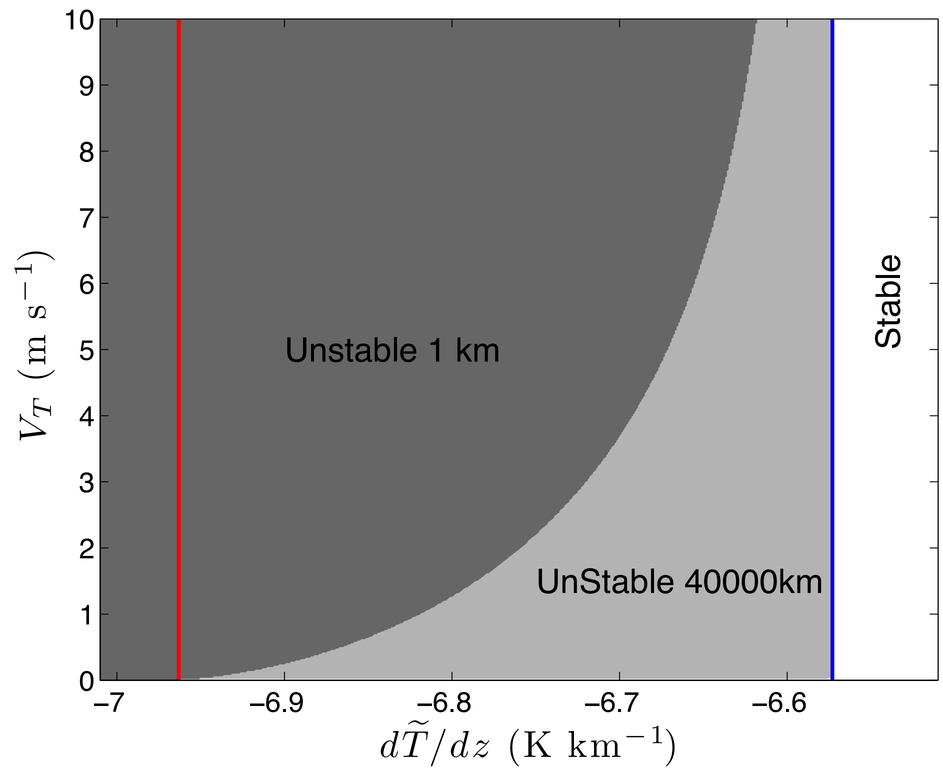
FARE with a saturated background gives the same right stability boundary:

$$\Gamma_e = (g/\theta_o)\, d\tilde{\theta}_e/dz > 0, \qquad \theta_e = \theta + (L/c_p)q_{vs}$$

sufficient for stability;

as well as a left boundary sufficient for instability in the saturated case.

Stability Diagram; fixed $q_{vs}(z)$ profile



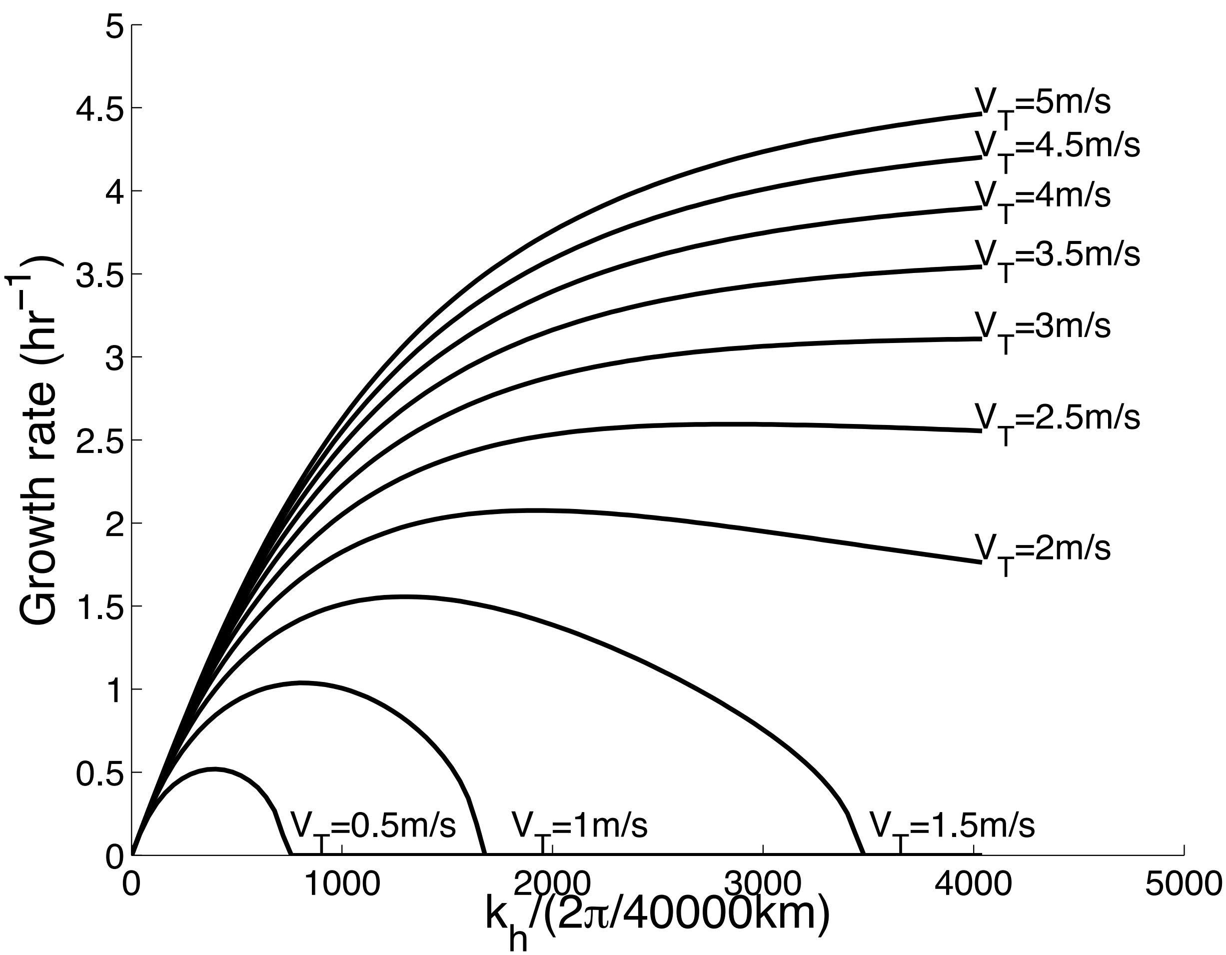
Grey denotes unstable scales;

left of red line guaranteed unstable:

$\Gamma = (g/\theta_o)d(\tilde{\theta}_e - \theta_o\tilde{q}_t)/dz < 0$

right of blue line guartd. stable: $\Gamma_e = (g/\theta_o)d\tilde{\theta}_e/dz > 0$, [E86]

Growth rate of the unstable eigenmode



Fixed background temp/saturation gradients;

$k_z = 2\pi/15\ \mathrm{km}^{-1}$

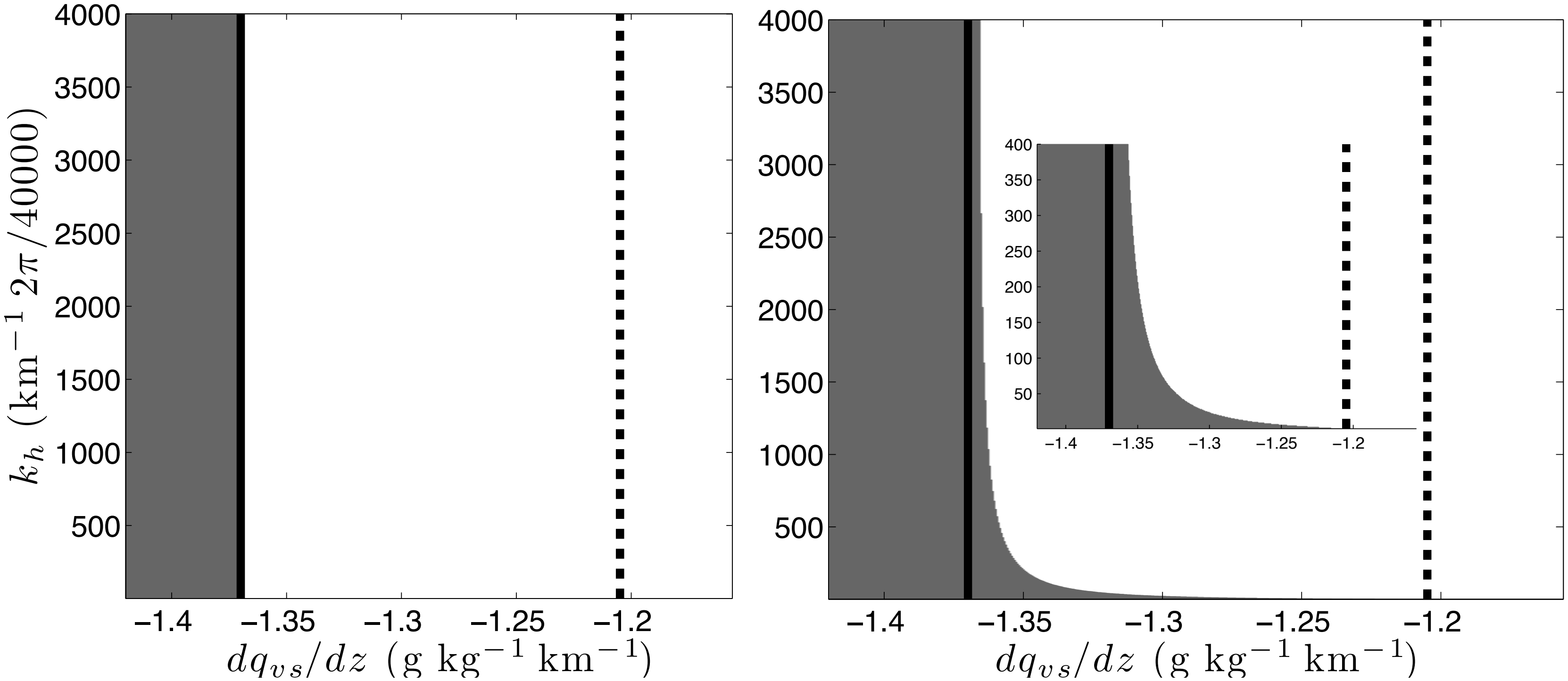
(In)stability boundaries

- Numerically find $\Gamma < 0$, $\Gamma_e > 0$ as sufficient conditions

- Analytically $\Gamma = 0$ is the single stability boundary for $V_T = 0$ (a singular limit)

- Analytically $\Gamma_e = 0$ is the single stability boundary for $V_T \to \infty$ (nonlinear asymptotics)

Singular limit $V_T = 0$



Left: $V_T = 0$ m s^{-1}; Right: $V_T = 0.01$ m s^{-1}; Grey = unstable; $k_z = 2\pi/15$ km^{-1}, $B = 3$ K km^{-1}.

Equation Structure: Saturated with $V_T = 0$ or $V_T \to \infty$

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \hat{\mathbf{k}} b', \qquad \frac{1}{w}\frac{Db'}{Dt} = -\Gamma \quad (-\Gamma_e)$$

with positive definite energy for $\Gamma > 0$:

$$E = \frac{1}{2}\left(||\mathbf{u}||^2 + \frac{(b')^2}{\Gamma} \right)$$

and eigenvalues $\sigma^{\pm} = \pm (k_h/k)\Gamma^{1/2}$

Stability Parameter

- Dry: $b' = g\theta'/\theta_o$, $\Gamma_{\mathrm{dry}} = N^2 = (g/\theta_o)d\tilde{\theta}/dz$
- Saturated, $V_T = 0$: $b' = g(\theta_e'/\theta_o - q_t')$
- Saturated, $V_T \to \infty$: $b' = g\theta_e'/\theta_o$

Energy Conservation in Saturated Regimes, $V_T \neq 0$

$$\partial_t E + \nabla \cdot \left(\mathbf{u}(E+p) \right) = V_T \partial_z \left(\frac{(g q_r')^2}{2(\Gamma - \Gamma_e)} \right)$$

$$E = \frac{1}{2} \left(||\mathbf{u}||^2 + \frac{(g\theta_e'/\theta_o)^2}{\Gamma_e} + \frac{(g q_r'/\theta_o)^2}{\Gamma - \Gamma_e} \right)$$

positive definite for

$$\Gamma_e = \frac{g}{\theta_o} \frac{d\tilde{\theta}_e}{dz} > 0$$

$\Gamma - \Gamma_e$ strictly greater than zero

- The FARE model ignores detailed cloud physics, but retains conservations laws for

$$q_{\rm tot}, \qquad \theta_e = \theta + \frac{L}{c_p} q_v$$

(why it "works")

- Provides a framework for linear and nonlinear analysis

Conclusions

FARE adds V_T to a concept of "conditional instability" in a saturated domain

Next: Derivation of a Moist Balanced Dynamics (the vortical mode $+$ the moisture mode)?

Nonlinear energy analysis

Nonlinear Energy Consistency

$$\frac{\partial}{\partial t}\left(\frac{\mathbf{u}\cdot\mathbf{u}}{2}+\Pi\right)+\nabla\cdot\left[\mathbf{u}\left(\frac{\mathbf{u}\cdot\mathbf{u}}{2}+\Pi+p\right)\right]-\frac{\partial}{\partial z}\left[V_T g(z-a)q_r\right]=-V_T g q_r$$

$$\Pi(\theta_e, q_t, z) = -\int_a^z \frac{g}{\theta_o}\,\theta_v(\theta_e, q_t, \eta)\,d\eta$$

$$\theta_v = \theta_v(\theta_e, q_t, z) = \theta_e - \theta_o q_t + \theta_o\left(\epsilon_o - \frac{L}{c_p\theta_o} + 1\right)\min(q_t, q_{vs}(z)).$$

θ_v linear virtual potential temperature; integration assumes θ_e, q_t fixed; a arbitrary reference height with $q_{vs}(a) = 0$.