A direct numerical simulation of rotating stably stratified turbulence: Evidence for Bolgiano-Obukhov scaling?

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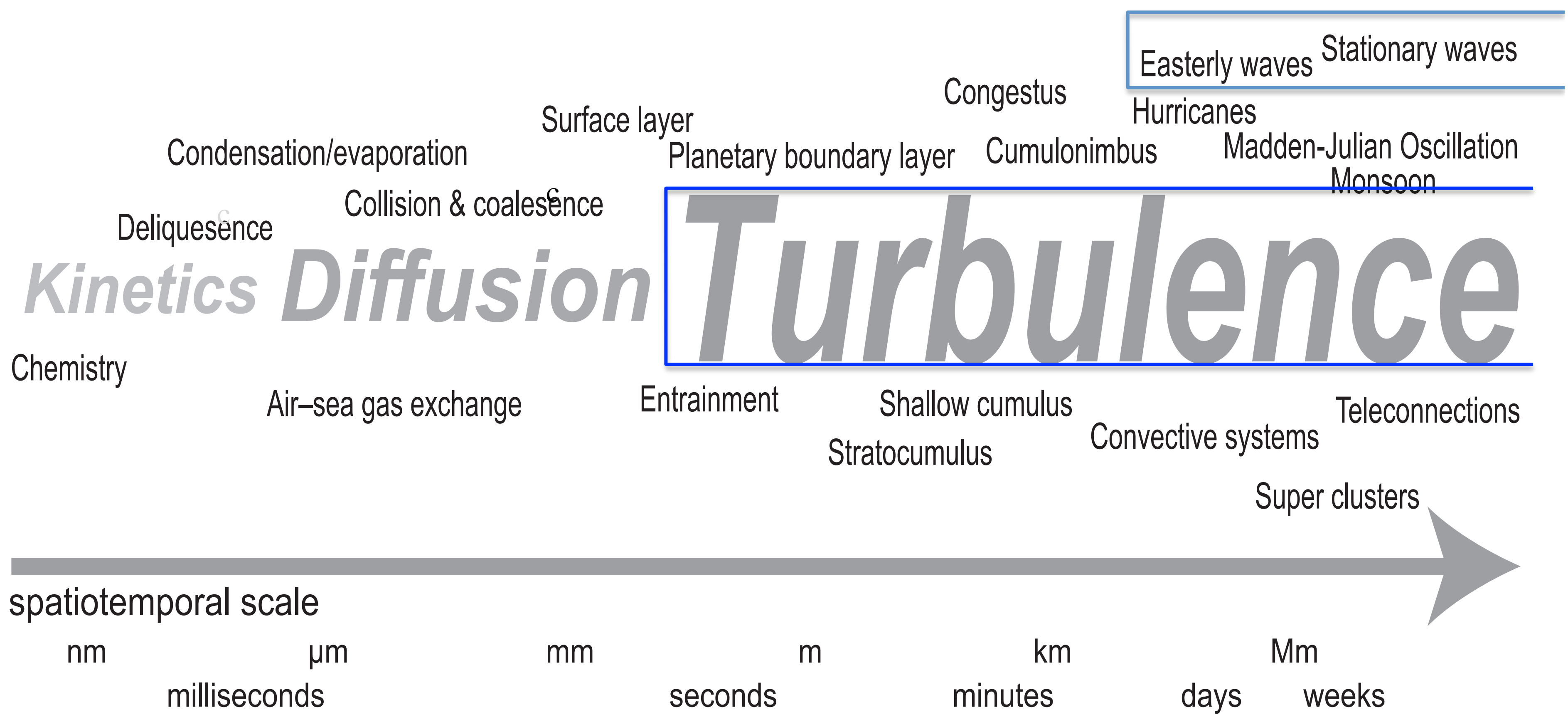
A direct numerical simulation of rotating stably stratified turbulence: Evidence for Bolgiano-Obukhov scaling?

- Turbulence and waves: evidence for multiple regimes
- *Stratification versus* rotation and oceanic values
- A parametric exploration using direct numerical simulations

- The rotating stratified run at Re=55000, N/f=5, Fr=0.024
  - *Temporal dynamics*
  - *Bolgiano-Obukhov scaling: some data*

- Conclusions and questions



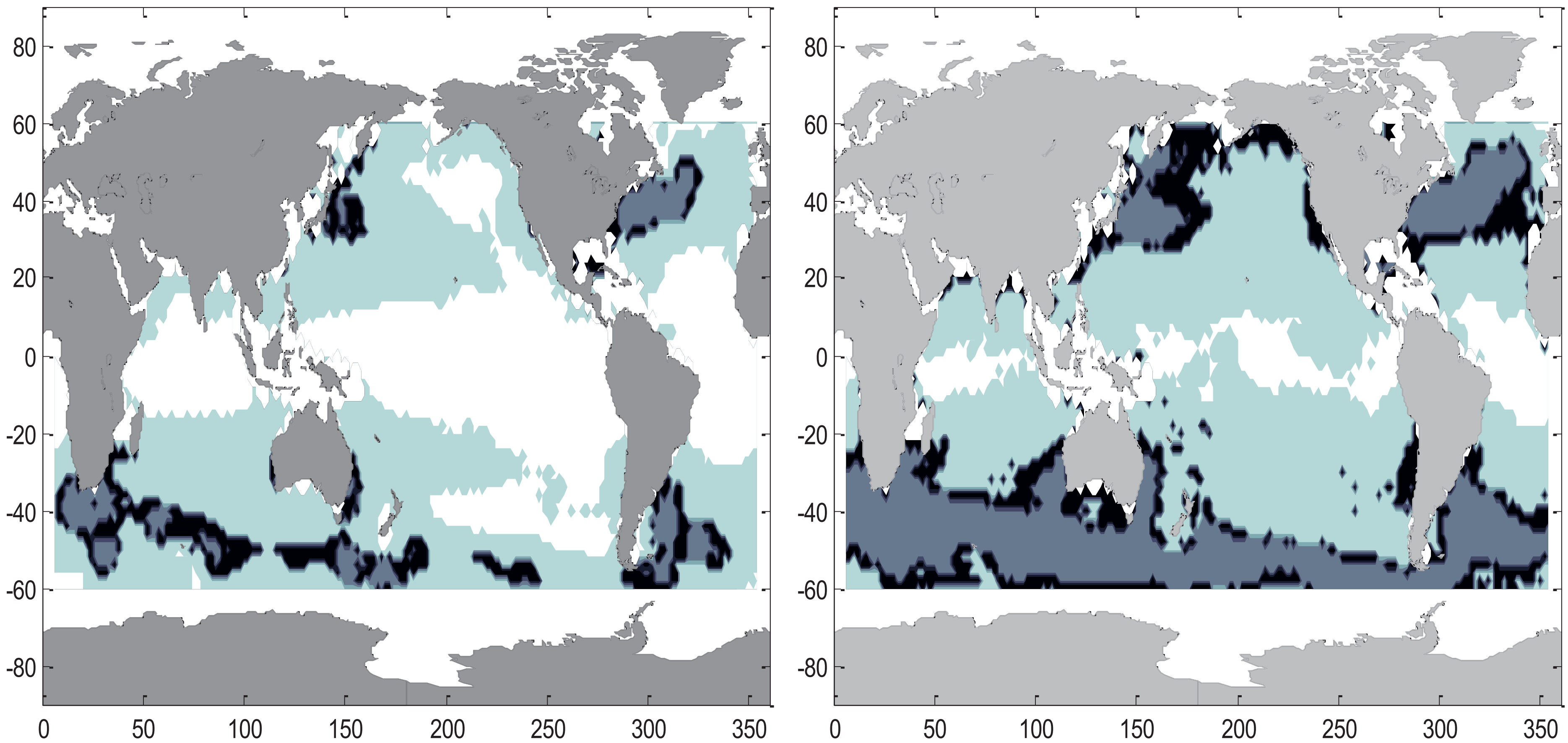
The atmosphere

Bringuier et al., 2009

Kolmogorov:

"*I soon understood that there was little hope of developing a pure, closed theory, and because of absence of such a theory the investigation must be based on hypotheses obtained on processing experimental data.*"

Jason S$_{ea}$S$_{urface}$H: noised & de-noised



$E_V(k) \sim k^x$: Black: $k^{-5/3}$ (``SQG''): edge of major currents w. high eddy energy

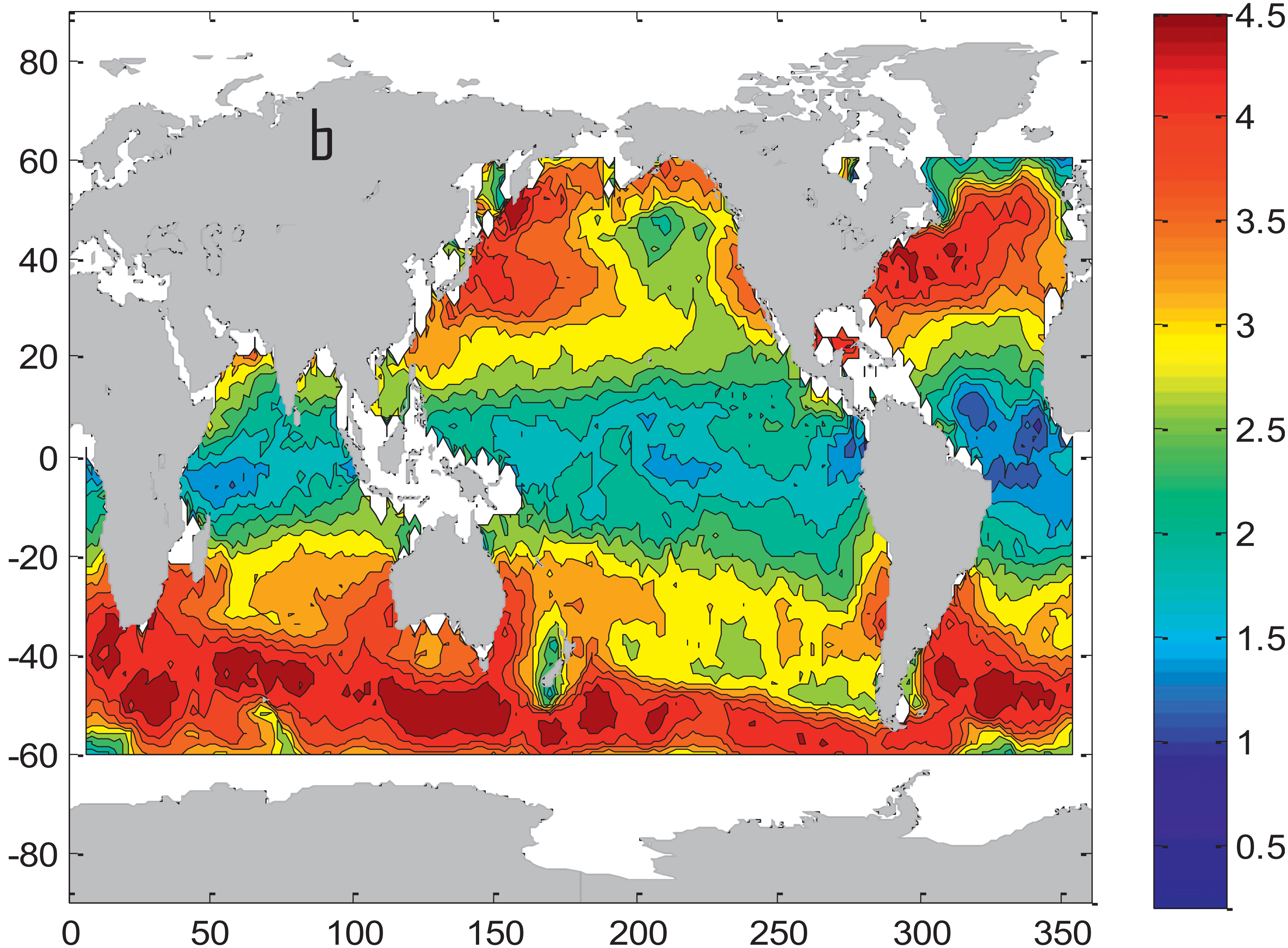
Dark grey: spectrum significantly steeper than -5/3: k^{-2}? Fronts?

Light grey: spectrum flatter than -5/3 but steeper than 0

White: k^0 or more

Xu & Fu JPO 2012

Jason de-noised altimeter SSH



5 k^{-3}

k^{-2}

$k^{-5/3}$

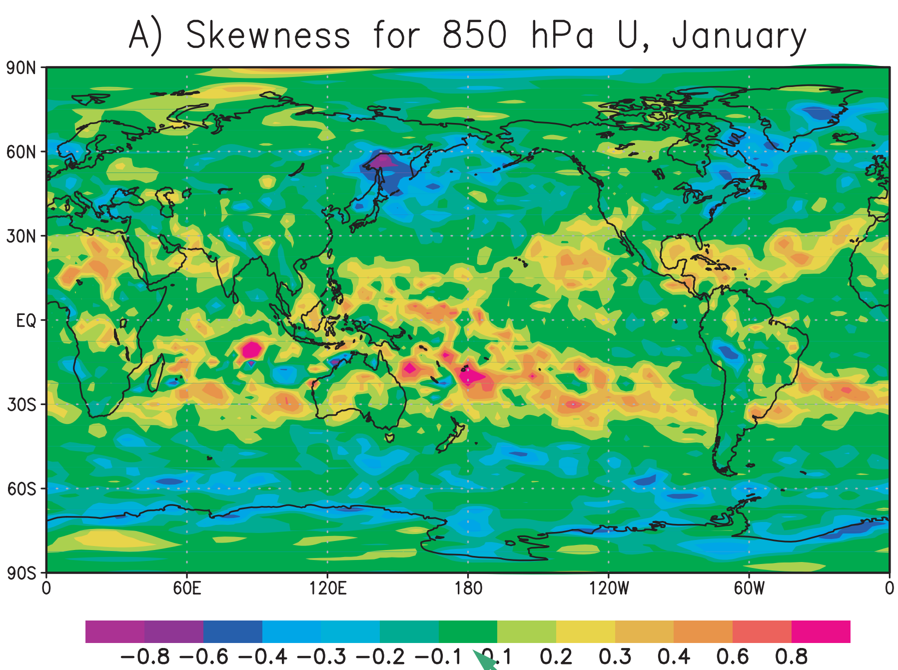
k^{0}

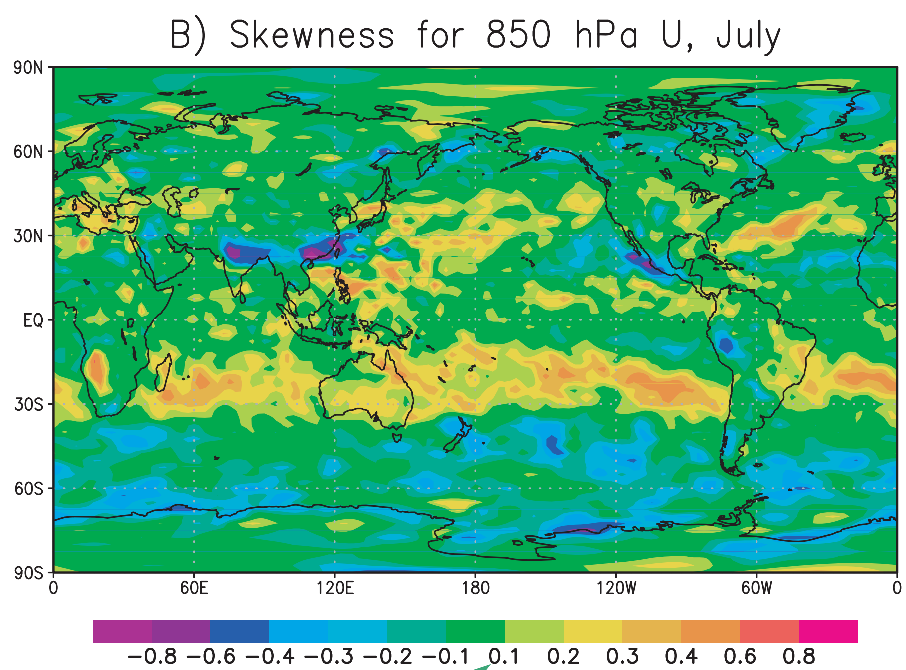
$E_V(k)$

Xu & Fu JPO 2012

Intermittency

Skewness of temperature (ERA40 data)

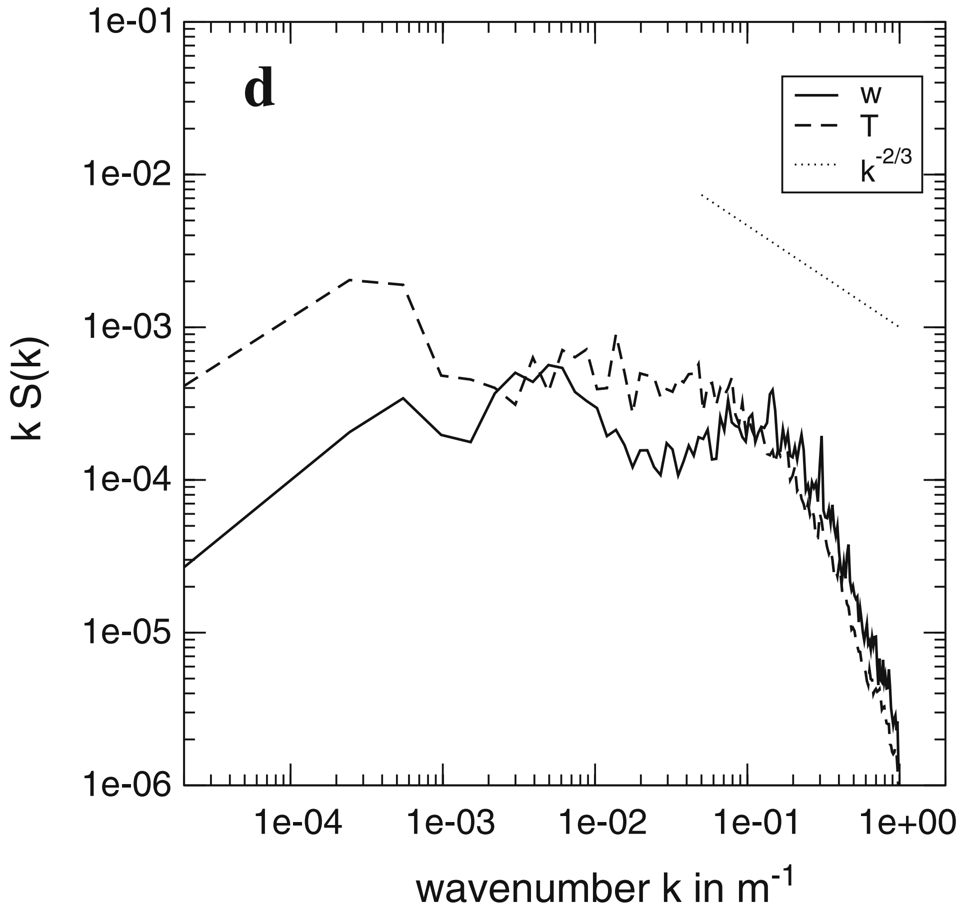




Gaussian

Petoukhov et al., 2008

Boundary layer over Greenland



SBL6, 30 m

Spectra ~ 1/k at large scale

Possibly Kolmogorov-like
at smaller scales,
And then much steeper

Drue Heineman, 2007

Helicity matters: spectrum in the Planetary Boundary Layer

Flat spectrum at night
(when more stable)

Koprov, 2005

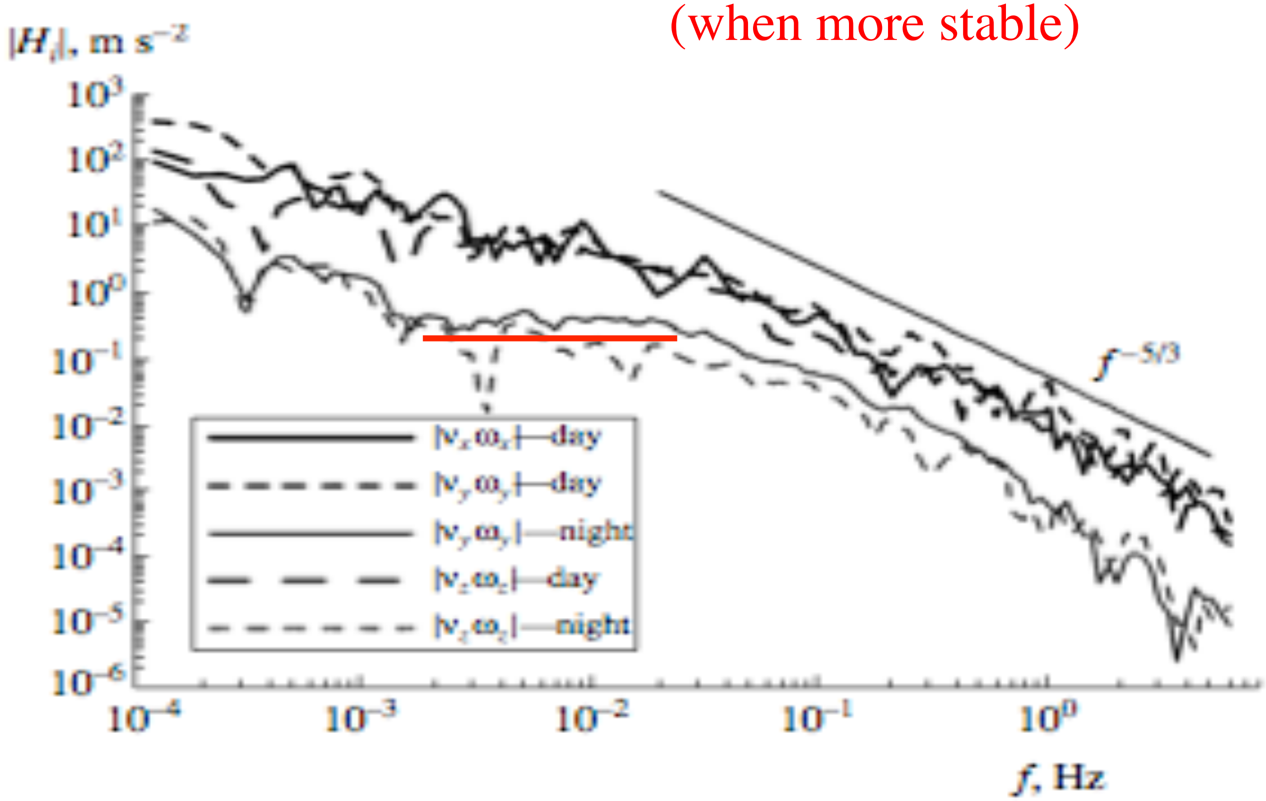
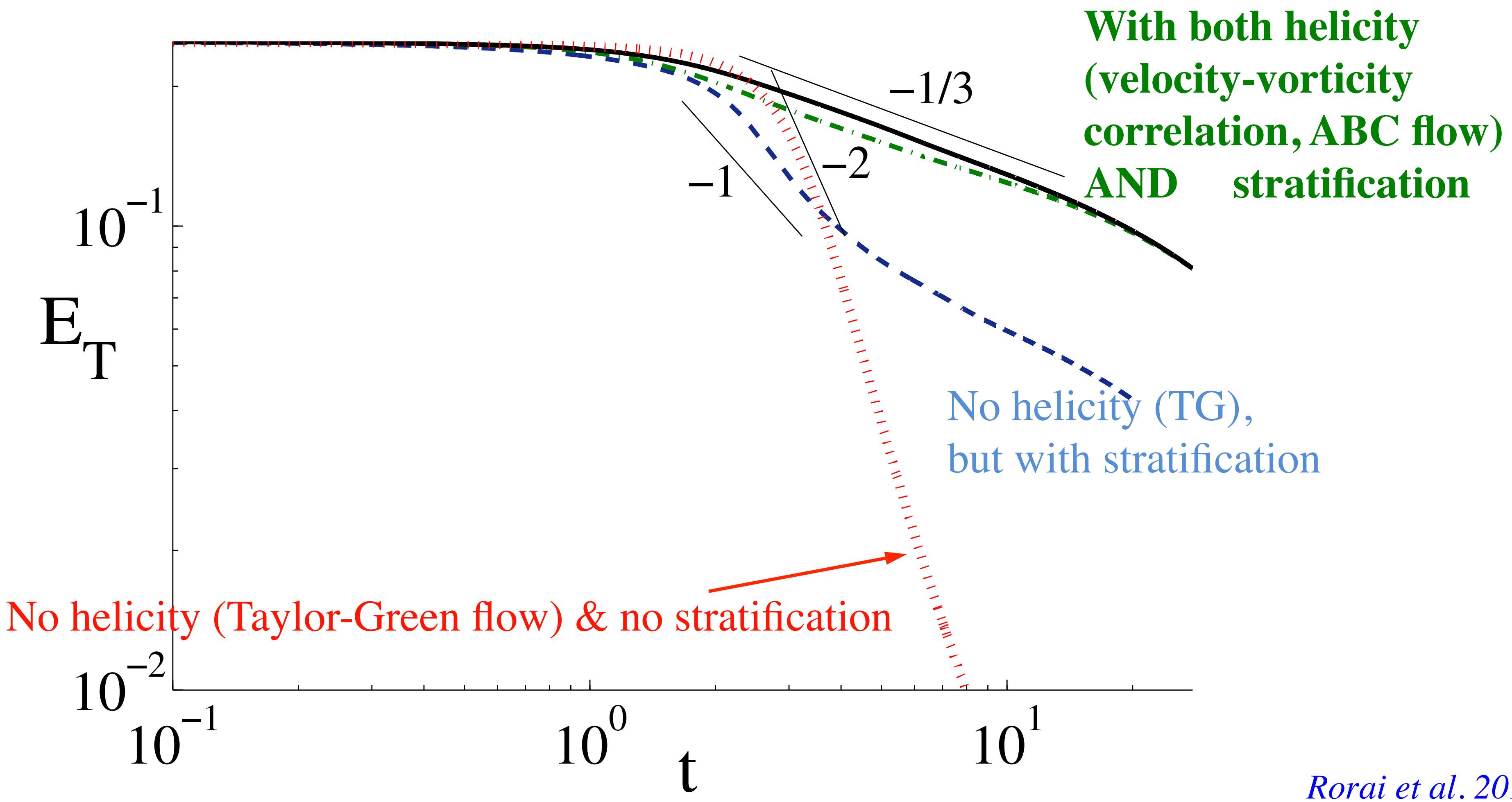


Fig. 4. Spectra of helicity components.

Waves matter: Energy decay (f_u=0), 512^3 resolutions, no rotation, Re~ 3000, Fr~ 0.02, R_B~1



Rorai et al. 2013

Incompressible Boussinesq equations
3D cubic box, periodic boundary conditions

$$\frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\omega} \times \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} = -N\rho \hat{e}_z - \nabla \mathcal{P} + \nu \nabla^2 \mathbf{u} \ ,$$

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = N w + \kappa \nabla^2 \rho \ ,$$

Parameters, with f=2Ω:

$$Re = \frac{U_0 L_0}{\nu}, \ Fr = \frac{U_0}{L_0 N}, \ Ro = \frac{U_0}{L_0 f}, \ Pr = \frac{\nu}{\kappa} \ , \qquad R_B = ReFr^2$$

The 3072^3 and 4096^3 runs:
$k_0 \sim 2.5$, N/f= 4.95, Re= 55000, Fr= 0.024, Ro= 0.12, Pr=1, R_B= 32

Stratification + rotation: geostrophic balance

$$
\begin{aligned}
\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\Delta\mathbf{u} &= \boxed{-\nabla P - Nbe_z - 2\Omega e_z \times \mathbf{u}} + \mathrm{F} \\
\partial_t b + \mathbf{u}\cdot\nabla b - \kappa\Delta b &= Nw\ , \\
\nabla\cdot\mathbf{u} &= 0\ .
\end{aligned}
$$

Take the curl of GB → thermal winds (VSHW)

Then, dot with Coriolis force →

$$\boxed{\langle H_\perp\rangle_\perp \equiv \langle u_\perp \cdot \nabla \times u_\perp\rangle_\perp = \boxed{\frac{N}{f}}\langle b\ \frac{\partial w}{\partial z}\rangle_\perp}$$

$f=2\Omega$

Hide, 1976; recent DNS: Marino et al., 2013

Geophysical High Order Suite for Turbulence *(Gomez & Mininni)*

- Pseudo-spectral DNS, periodic BC cubic *(also 2D), single/double precision*; Runge-Kutta for incompressible Navier-Stokes, SQG & Boussinesq. Includes rotation, passive scalar(s), MHD + Hall term
- *GHOST, from laptop to high-performance, parallelizes linearly up to 100,000 processors, using hybrid MPI/Open-MP (Mininni et al. 2011, Parallel Comp. 37)*
- *3D Visualization: VAPOR (NCAR); and development @ OakRidge (D. Rosenberg)*
- LES: alpha model & variants (Clark, Leray) for fluids & MHD
- Helical spectral (EDQNM) model for eddy viscosity & eddy noise
- *NEW!* Lagrangian particles *(w. A. Pumir)*
- *NEW!* Gross-Pitaevskii & Ginzburg-Landau *(with M. Brachet, ENS)*
- **Data, forced:** 2048^3 Navier-Stokes and 1536^3 & 3072^3 with rotation, both w. or w/o helicity. Rotating stratified turbulence w. 2048^3 grids
- **Data, spin-down MHD**:1536^3 random + 6144^3 ideal & 2048^3 w. T-Green symmetry
- *NEW! Decaying* rotating stratified flow, N/f~5, Re=5.5 10^4, 2048^3 , 3072^3 & 4096^3 grids.

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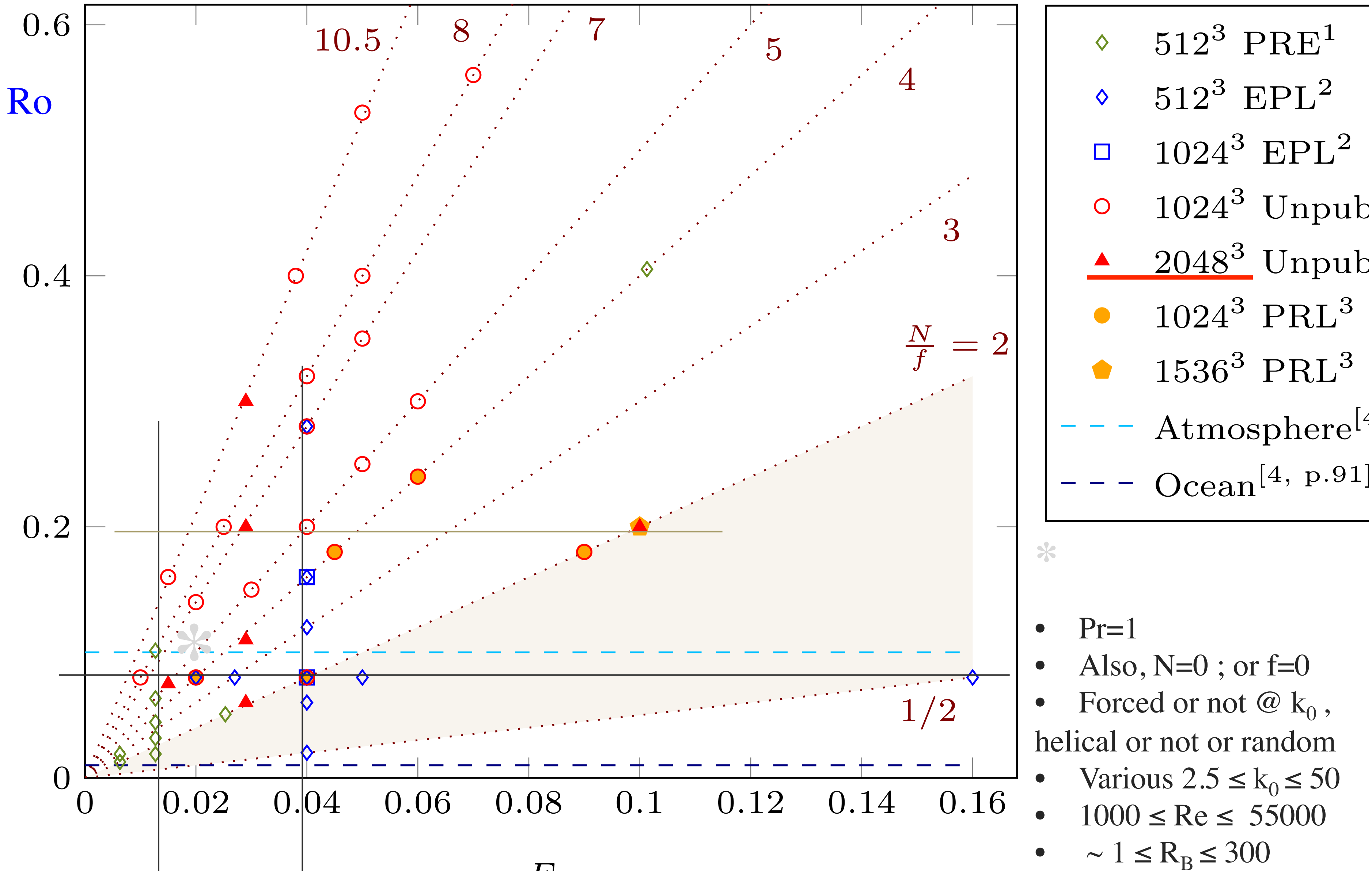
Present rot-strat data *(Marino, NCAR, & Rosenberg, OakRidge, for the ✻ runs)*

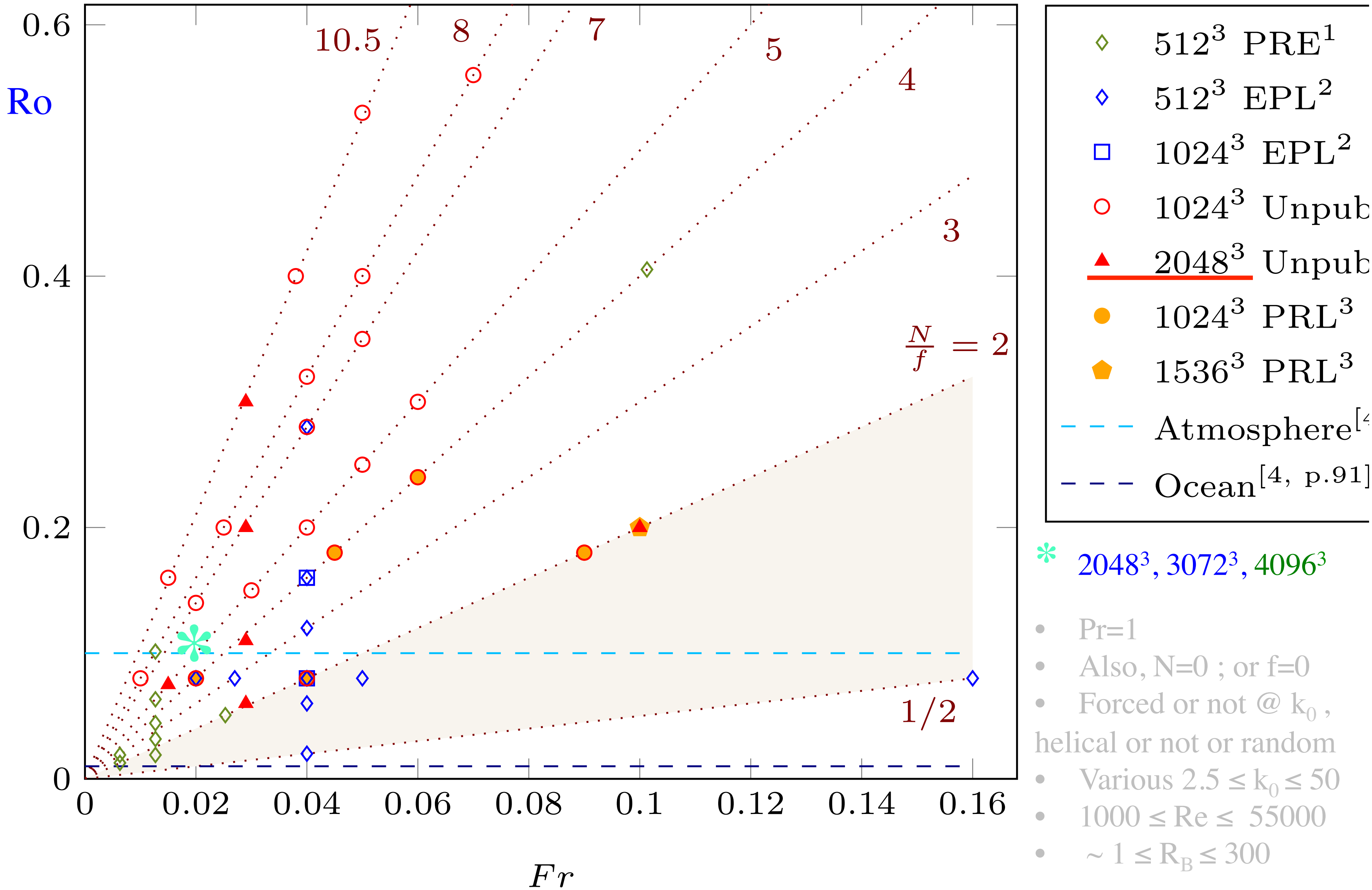


✻

- Pr=1
- Also, N=0 ; or f=0
- Forced or not @ k_0 , helical or not or random
- Various $2.5 \leq k_0 \leq 50$
- $1000 \leq Re \leq 55000$
- $\sim 1 \leq R_B \leq 300$

CPU: NSF (XSEDE & Yellowstone/NCAR); DOE

Present rot-strat data *(Marino, NCAR, & Rosenberg, OakRidge, for the ✻ runs)*



✻ 2048^3, 3072^3, 4096^3

- Pr=1
- Also, N=0 ; or f=0
- Forced or not @ k_0 , helical or not or random
- Various $2.5 \leq k_0 \leq 50$
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CPU: NSF (XSEDE & Yellowstone/NCAR); DOE

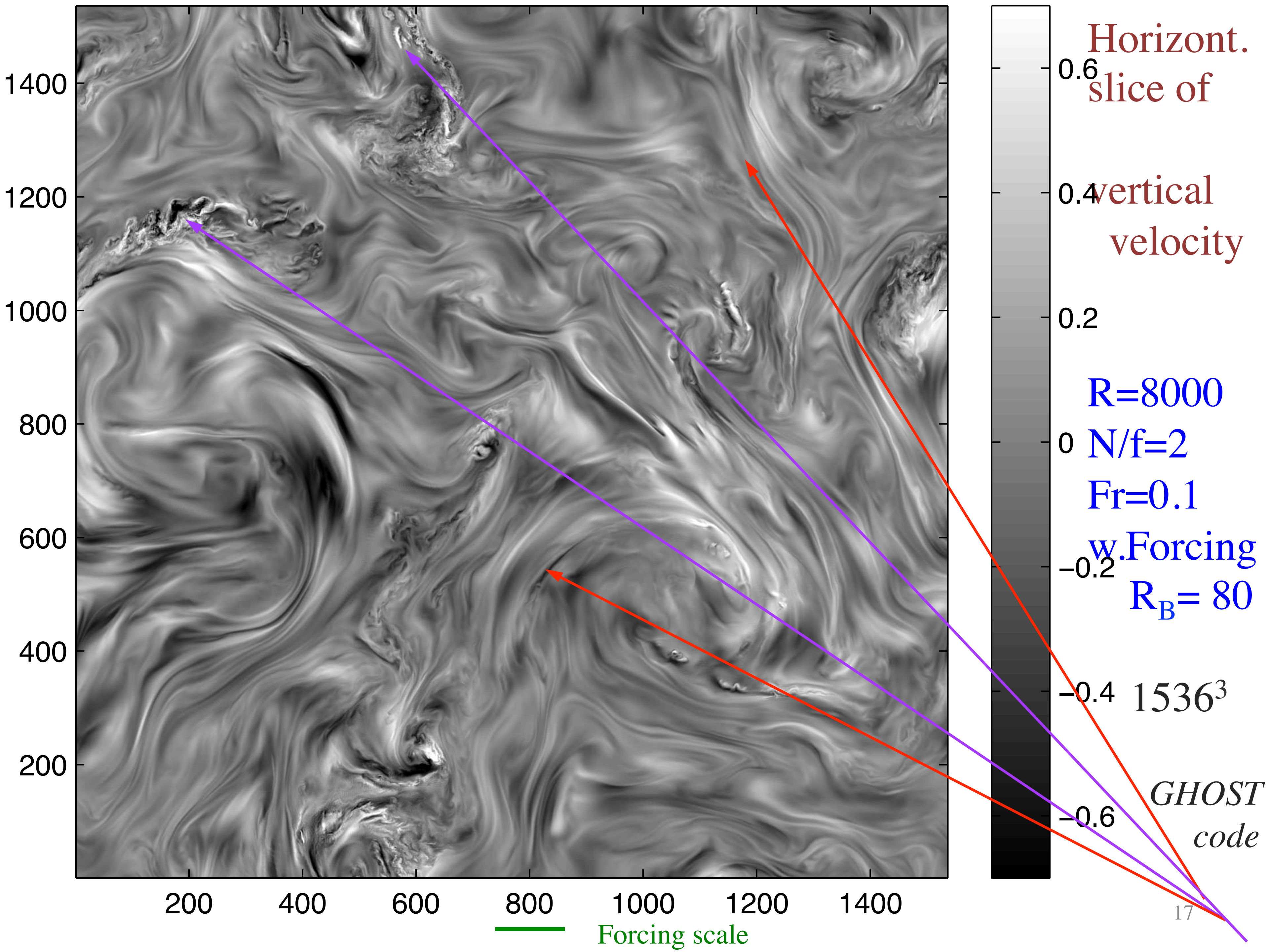
Some of the problems in Rotating Stratified Turbulence *(RST)*

* *First departure from complete isotropy: helicity dynamics, conservation (pure rotation) vs. creation (in RST) [Marino et al. PRE, 2013]*

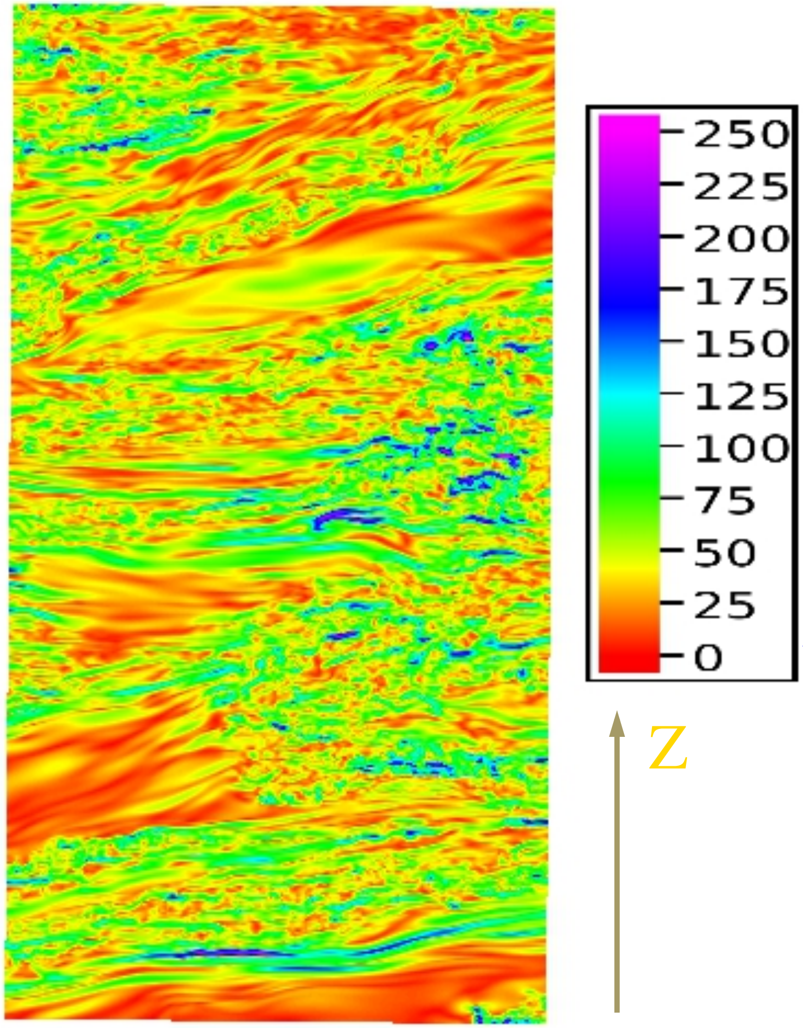
* Identifying scales, directions, relevant diagnostics & dimensionless parameters in rotating and/or stratified turbulence

- *Intermittency of the vertical velocity in stratified turbulence [Rorai et al. PRE 2014]*
- *Inverse energy cascade and anisotropy [Marino et al., EPL 2013, PRE 2014]*
- *Dual constant-flux energy cascades in rotating stratified turbulence [AP & Marino, PRL 2013]*
- Restricted equilibrium and the energy cascade *[Herbert et al., JFM 2014]*

- Bolgiano-Obukhov scaling in RST? *Rosenberg et al., ArXiv 2014*
- …………



200
400
600
800
1000
1200
1400
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
Horizont. slice of vertical velocity
R=8000
N/f=2
Fr=0.1
w.Forcing
R_B= 80
1536^3
GHOST code
Forcing scale



4096^3 run

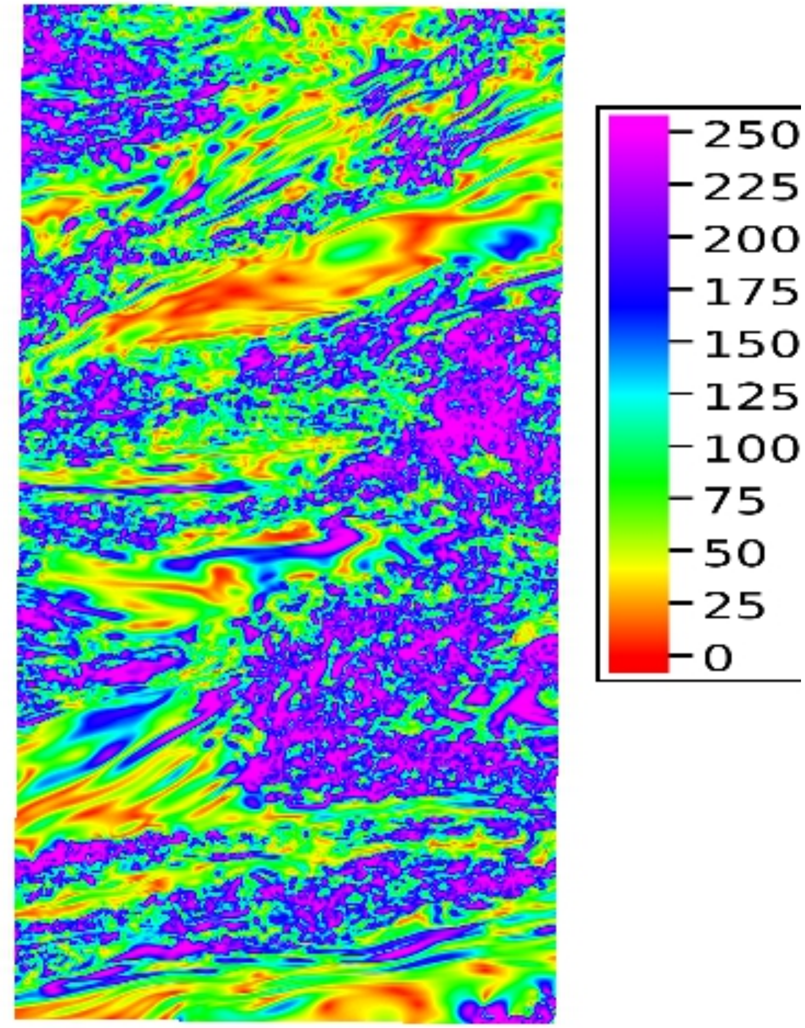
Are these VSHW?

0.7 X 0.4

X 0.04

(PVR)

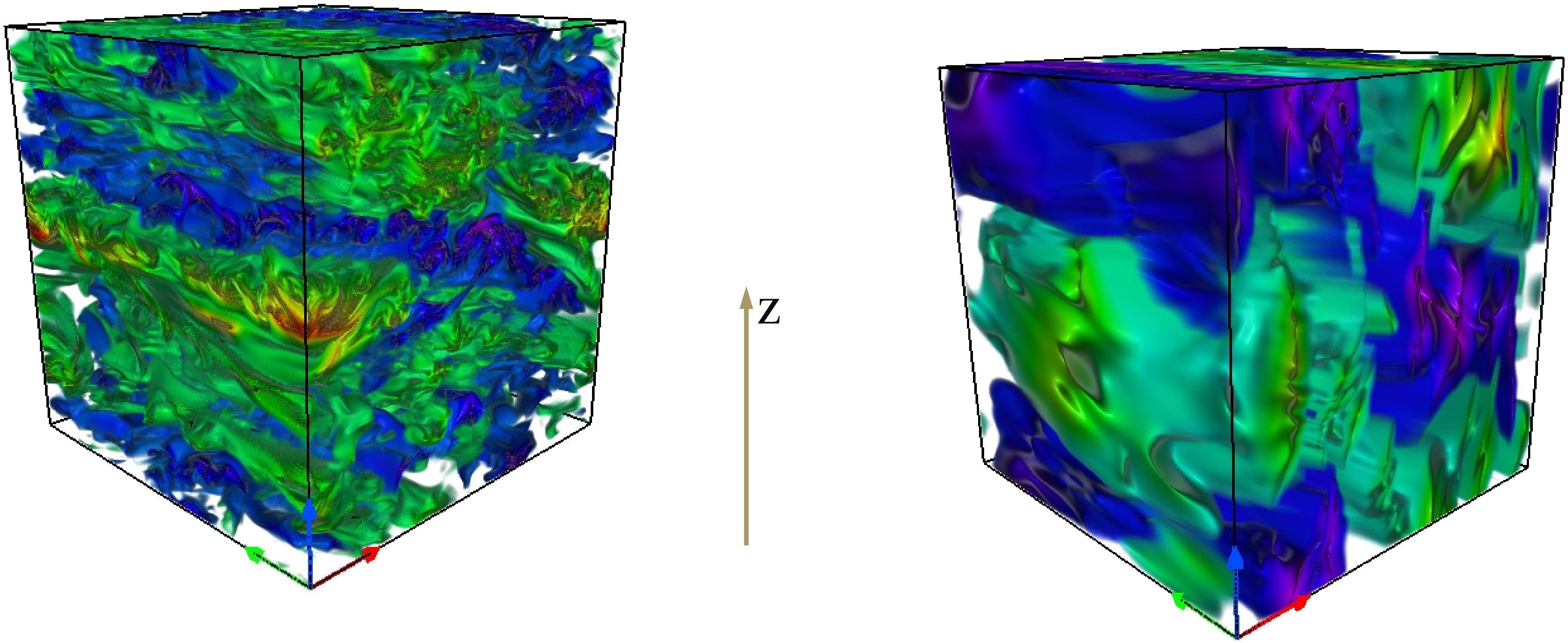
box



? QG

$$\{\partial_z u, \partial_z v\} = [N/f]^* \{-\partial_y \varrho, \partial_x \varrho]$$

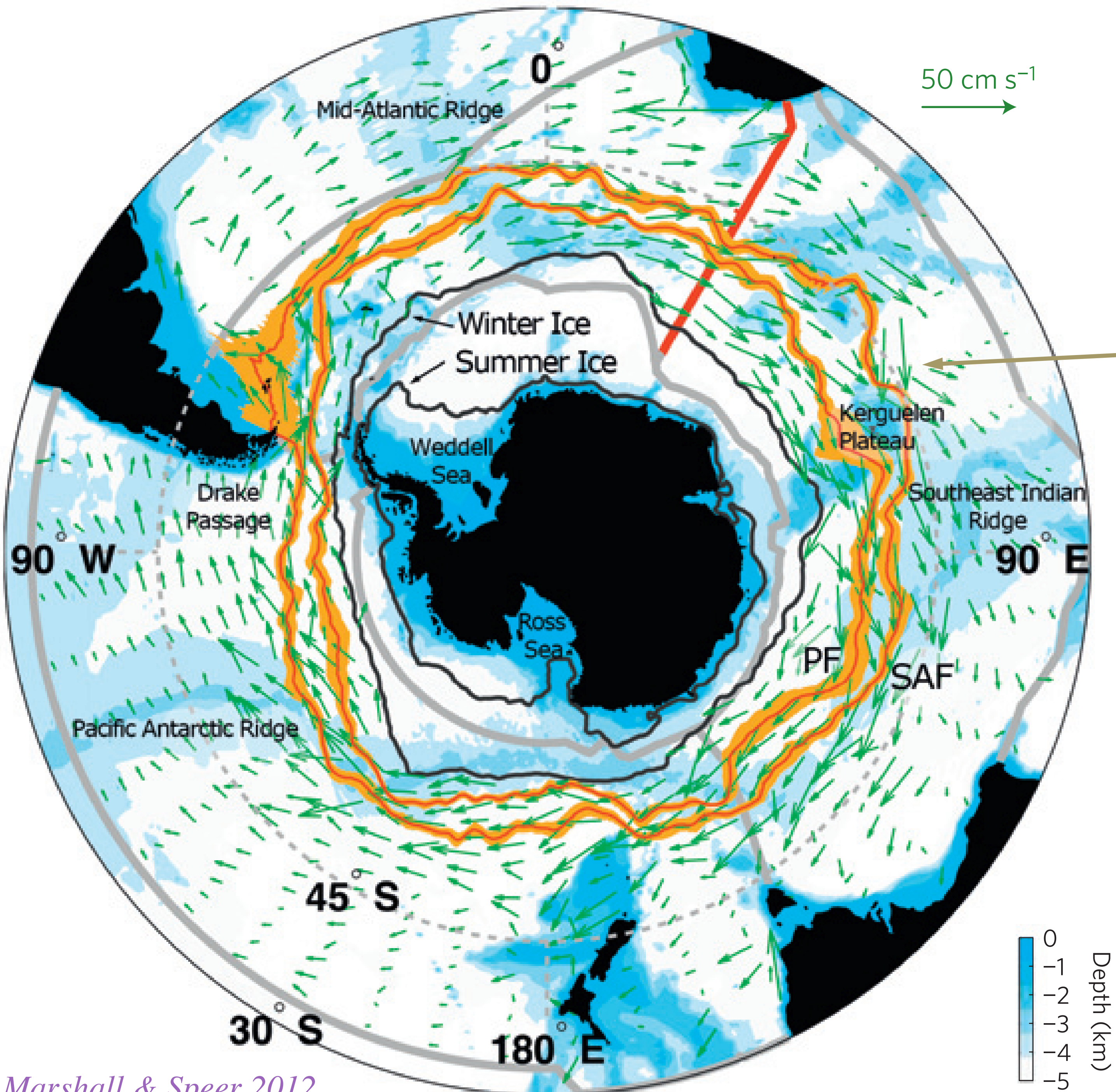
Buoyancy $\quad$ $Re \sim 8000$, 512^3 grids, $\quad$ $R_B = ReFr^2$



$Fr \sim 0.11$, $Ro \sim 0.4$,
$R_B \sim 96$, $N/f \sim 3.6$

$Fr \sim 0.025$, $Ro \sim 0.05$,
$R_B \sim 5$, $N/f = 2$

Marino et al., 2013



Cubic box of turbulence

next to Kerguelen Plateau 45S, 60E

Marshall & Speer 2012

Measurements in in the Southern Artic Ocean

(Drake passage)

← of flow speed

and of buoyancy frequency

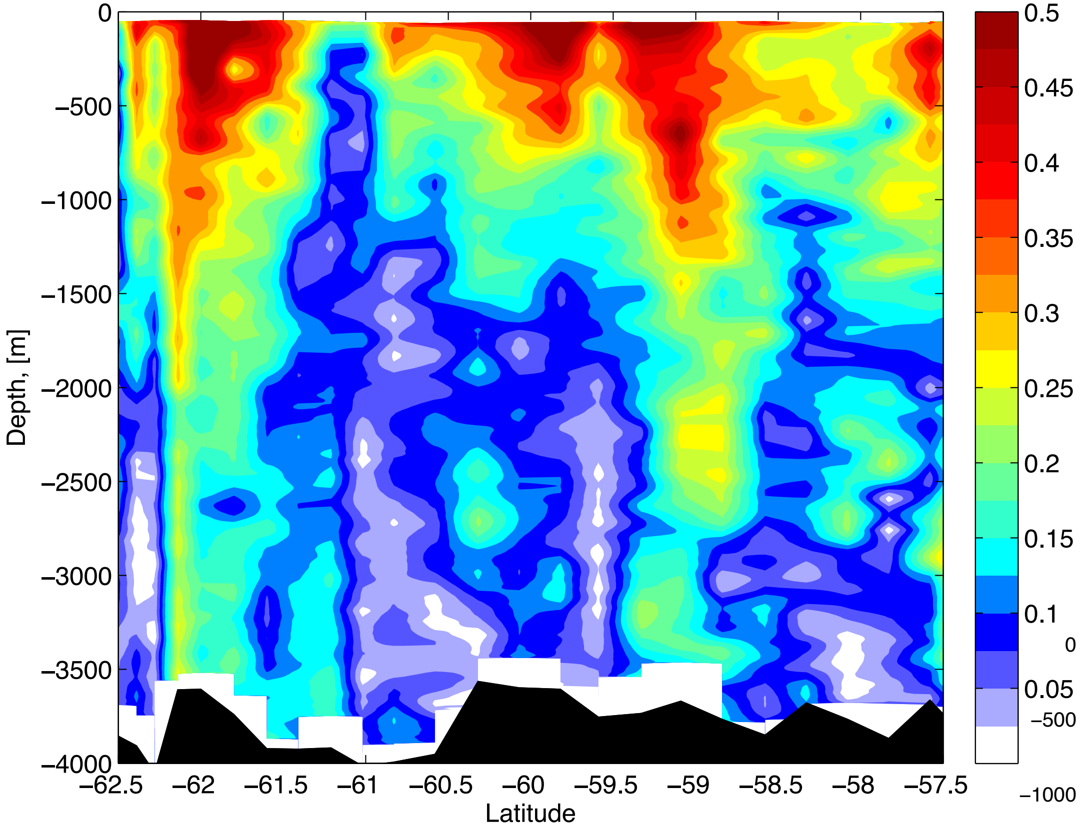


Figure 3-2: Flow speed (m s^{-1}) from the ALBATROSS section, Drake Passage.

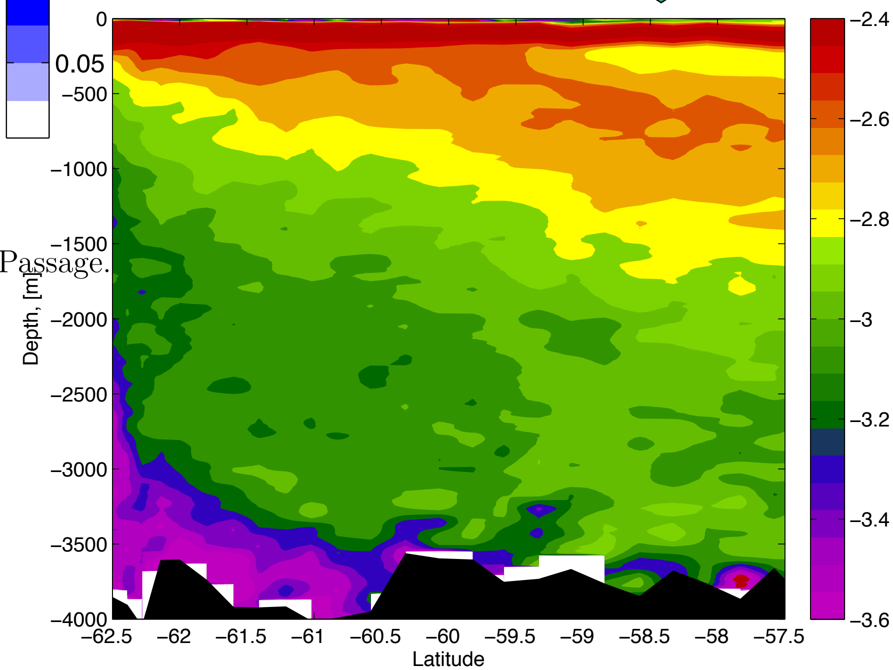
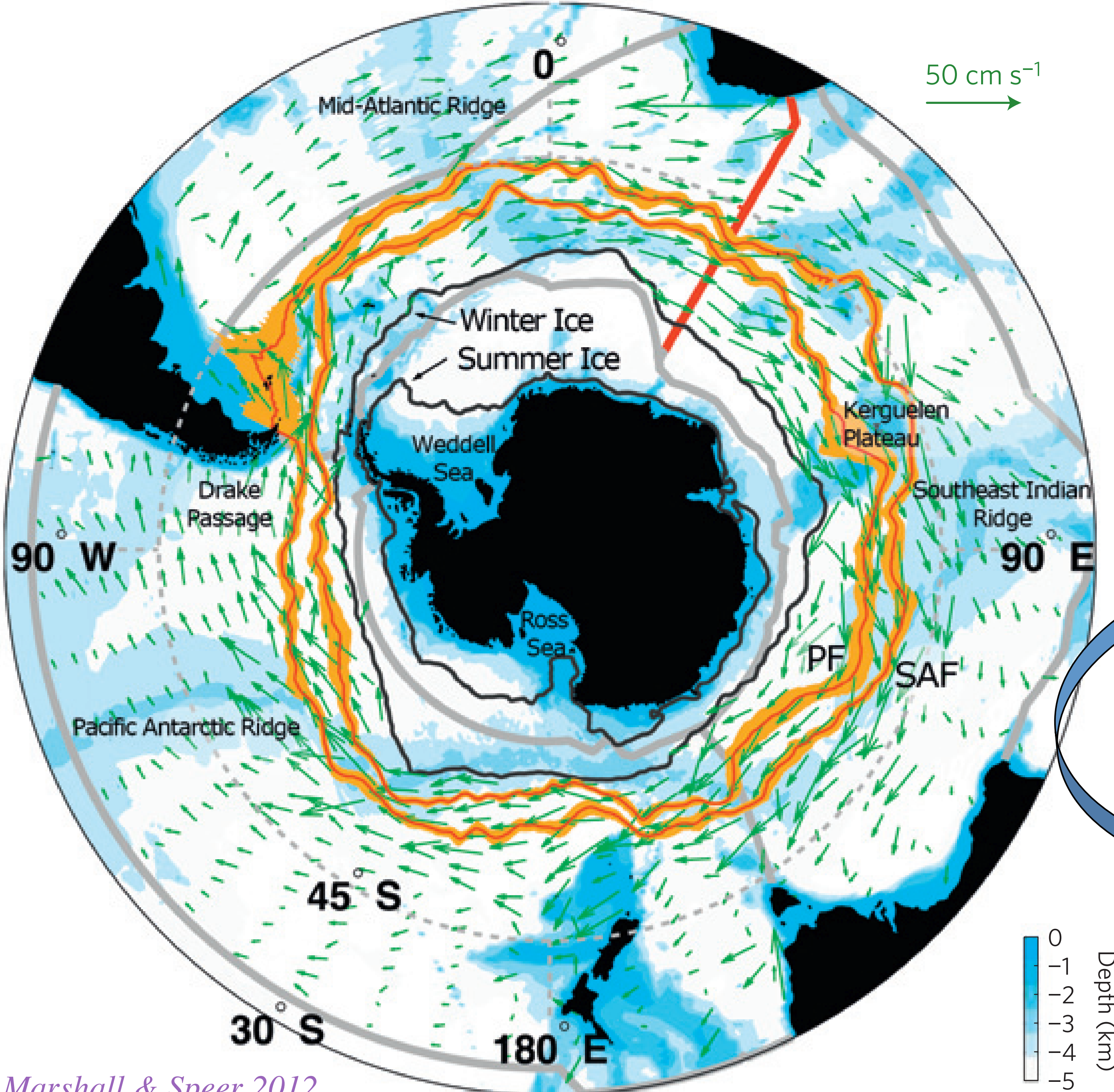


Figure 3-1: Buoyancy frequency (s^{-1}) in logarithmic scale from the ALBATROSS section, Drake Passage.

Nikurashin, 2009



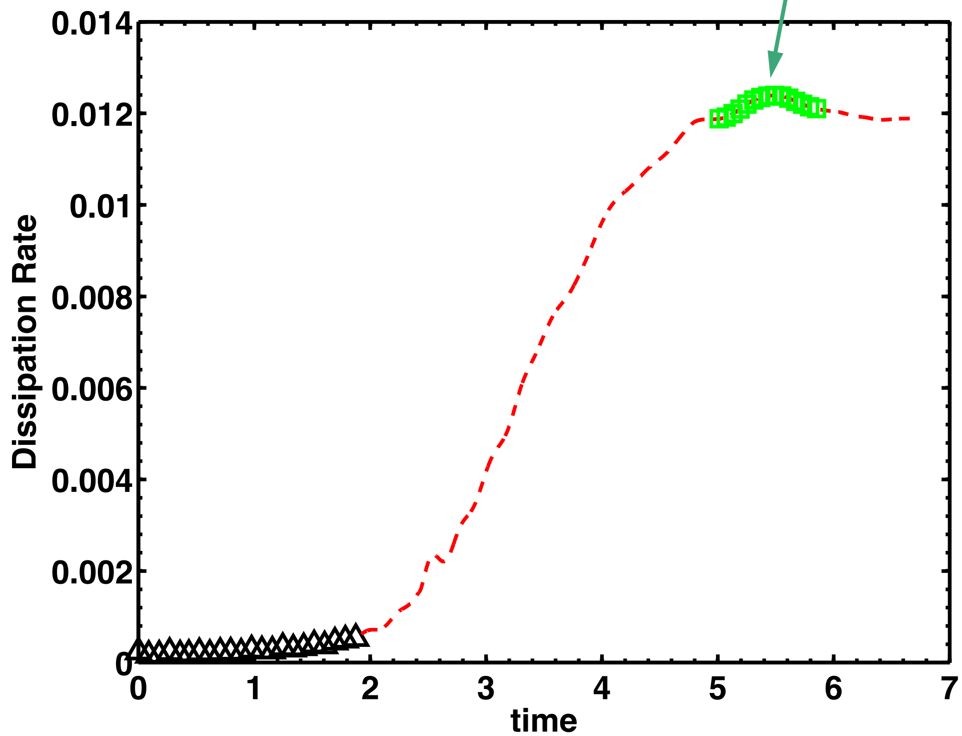
Marshall & Speer 2012

L_0~ 1000m
U~ 0.024m/s
N=0.001 /s
f=N/5
Fr= 0.024
Ro = 0.12
ν = *10^{-6} m²/s*
Re= 2. 10^7
$R_B \sim 10^4$

DNS run
Boussinesq
4096^3 grid
ν =4.5x10^{-4} m²/s
Re=55000
R_B~ 32

The short 4096^3 run

Peak of dissipation



$N/f=4.95$

$Fr=0.024$, $Ro=0.12$

$Re=55000$, $R_B=32$

Spin-down

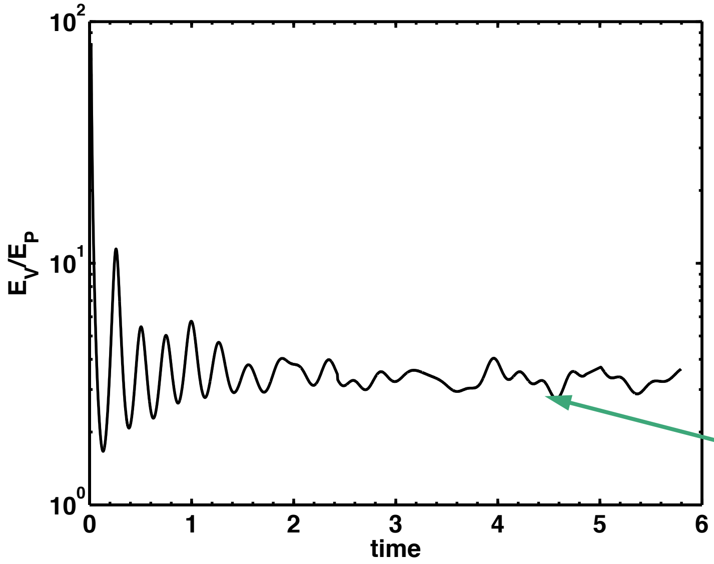
$K_0=2.5$

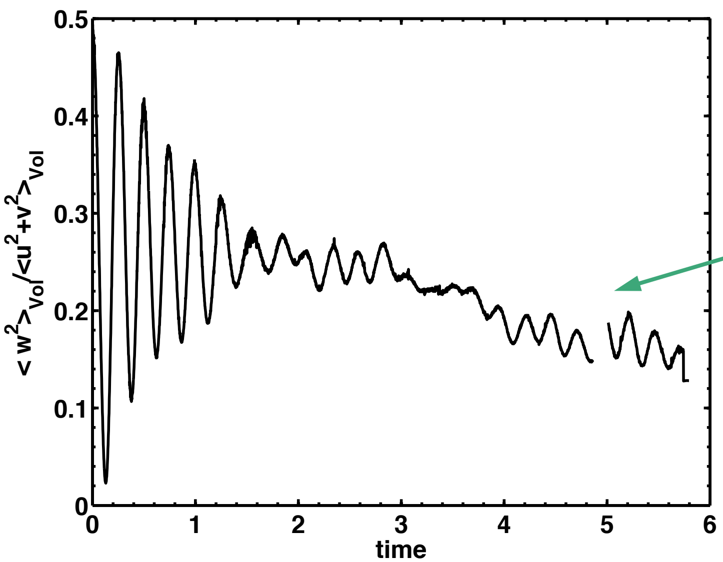
Triangles: 1536^3 grid- NSF

- - - - : 3072^3 grid- NSF

. . . Green: 4096^3 grid- DOE

Energy ratios

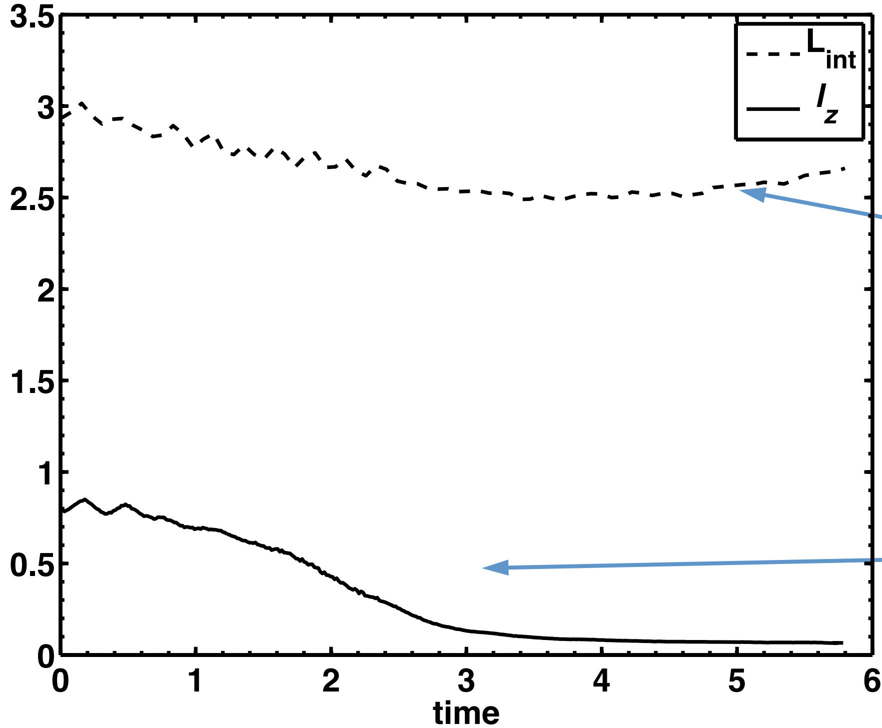




Kinetic to potential: ~ 3

And

Vertical to horizontal: ~ 0.12



N/f=4.95

Fr=0.024, Ro=0.12

Re=55000, R_B=32

K_0=2.5

Spin-down

Integral

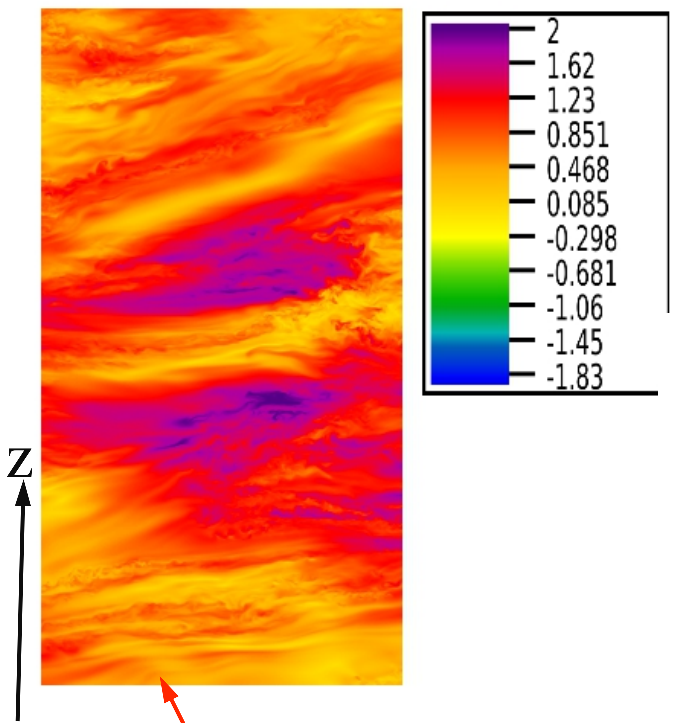
&

Vertical (Taylor)

$$\ell_z = [\langle u_\perp^2 \rangle / \langle (\partial_z u_\perp)^2 \rangle]^{1/2}$$

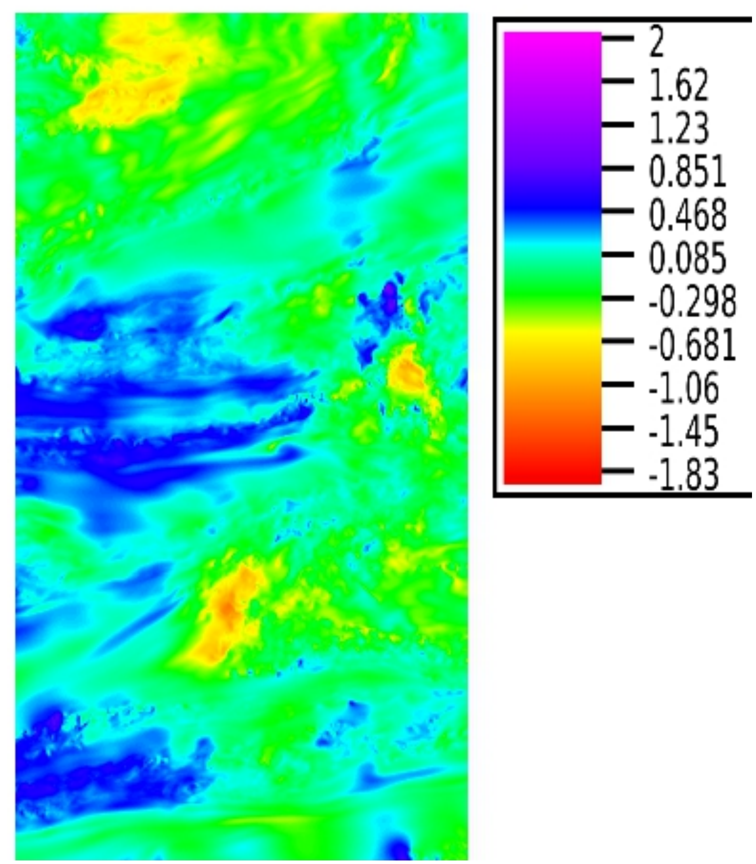
length scales

N/f=4.95, Fr=0.024,
Ro=0.12
Re=55000, R_B=32
K_0=2.5, spin-down

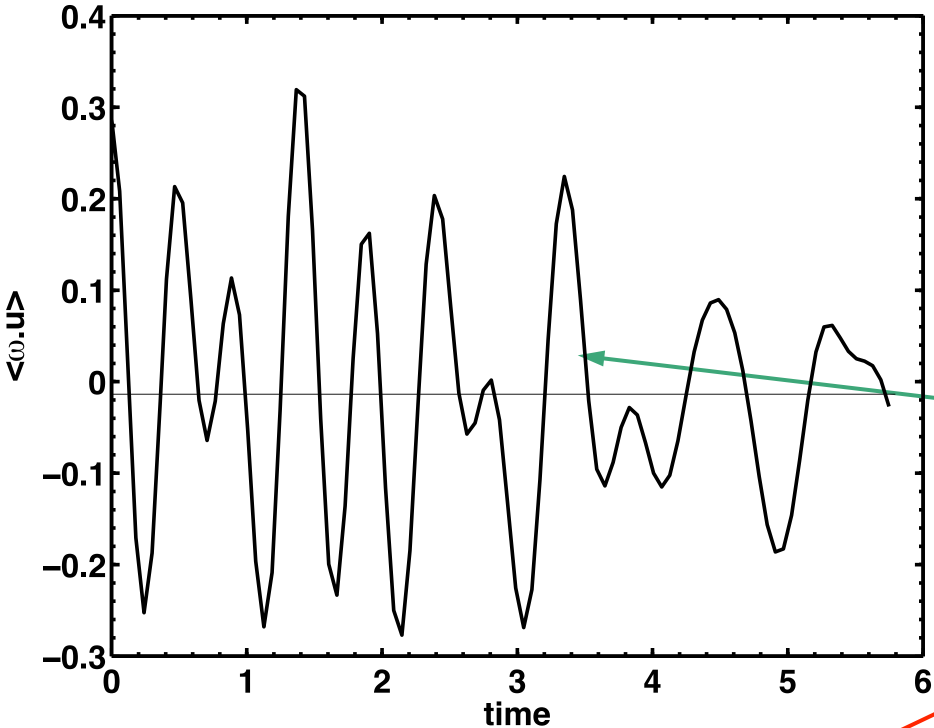


Perpendicular
And
Vertical velocity

Sub-volume: 0.7 X 0.4 X 0.04



4096^3 data



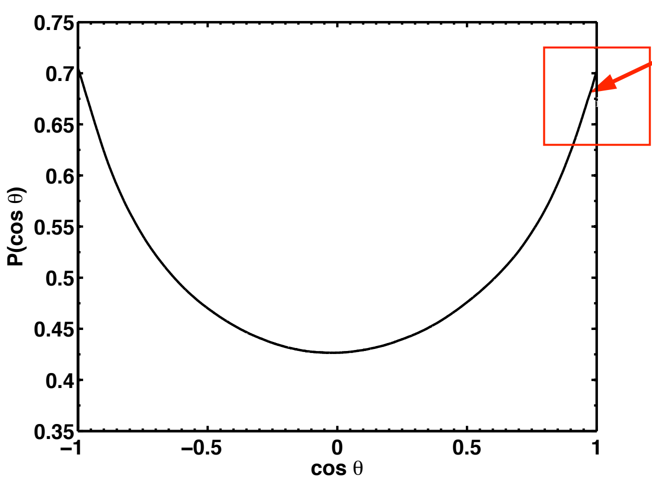
N/f=4.95

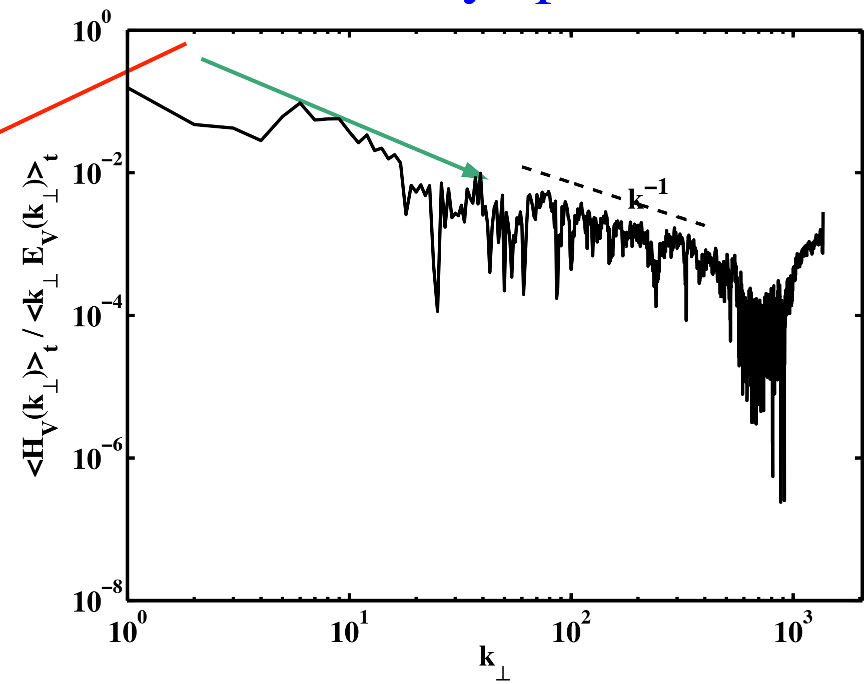
Fr=0.024, Ro=0.12

Re=55000, R_B=32

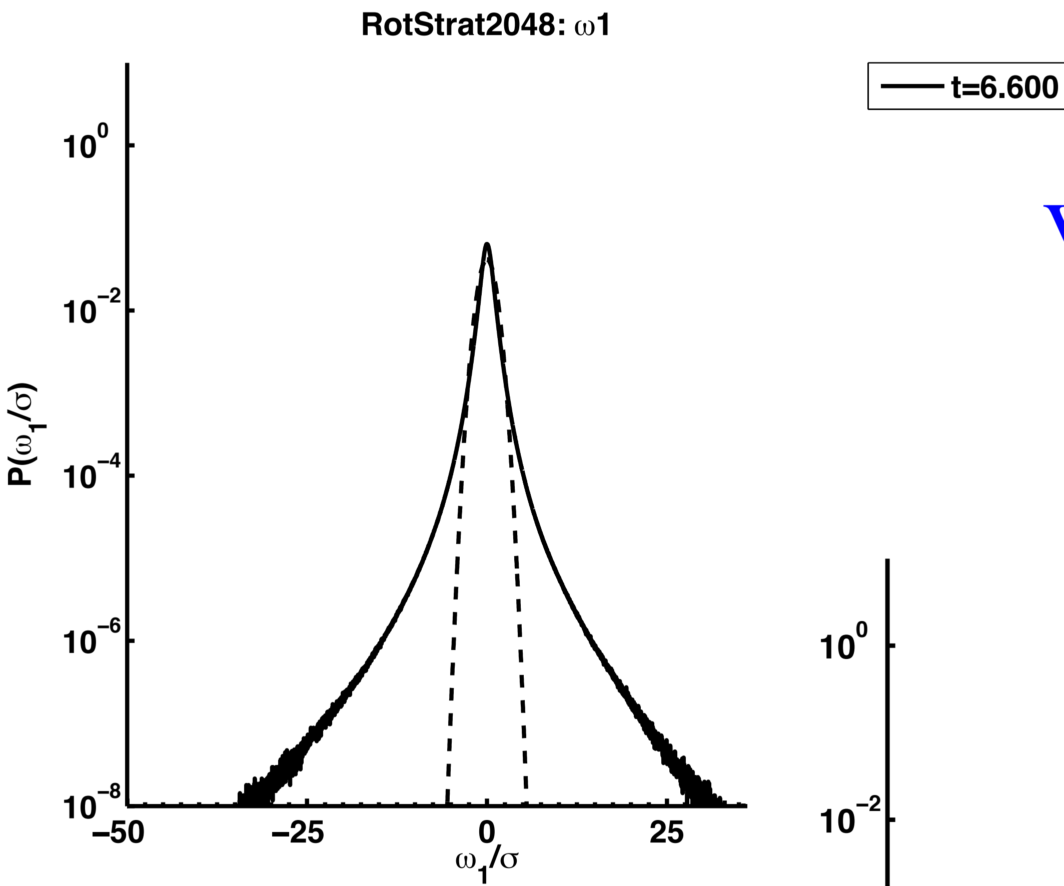
K_0=2.5, spin-down

Total helicity and PDF of relative helicity and helicity spectrum

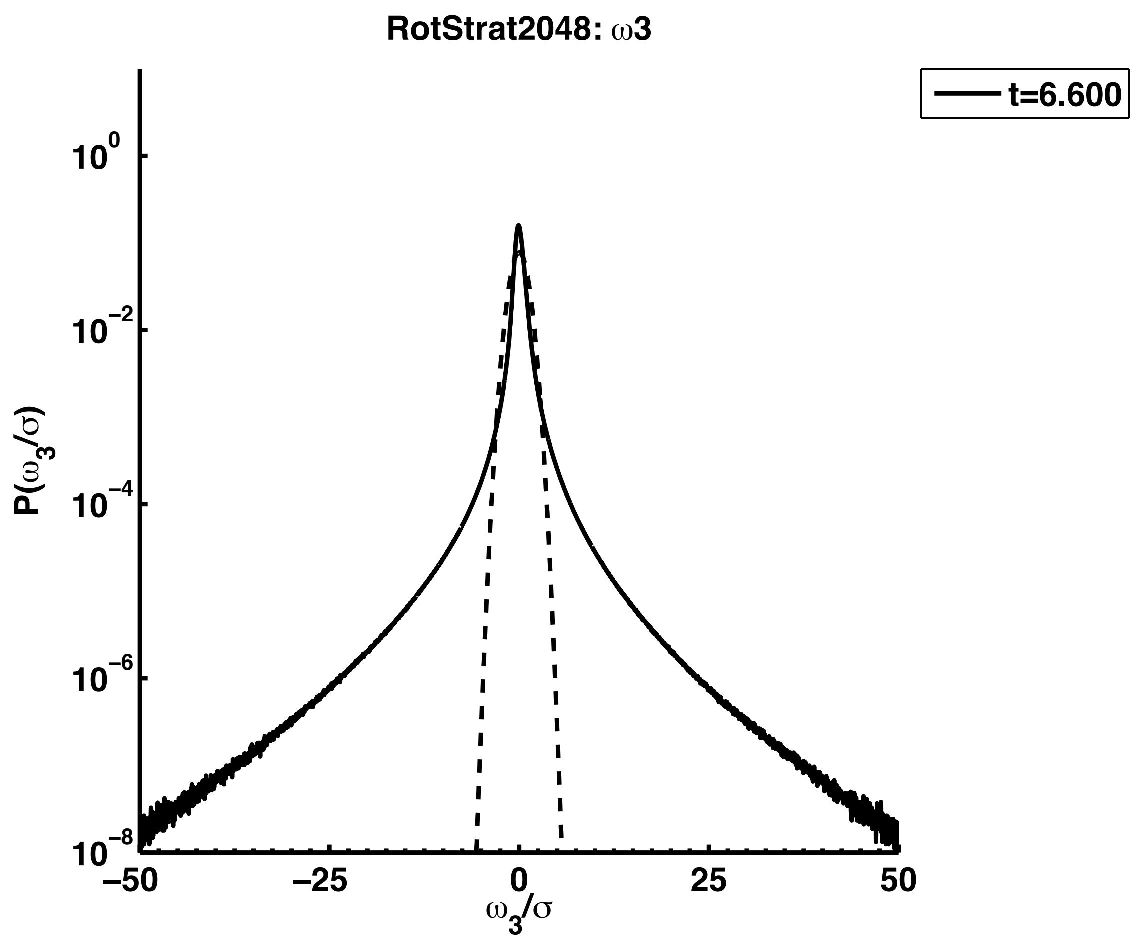


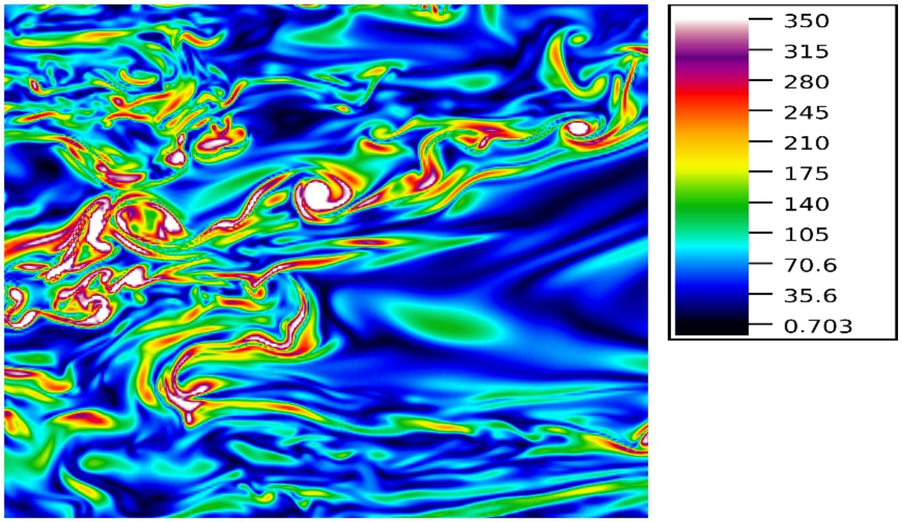


Vorticity components



$\omega_{rms} \sim 17$





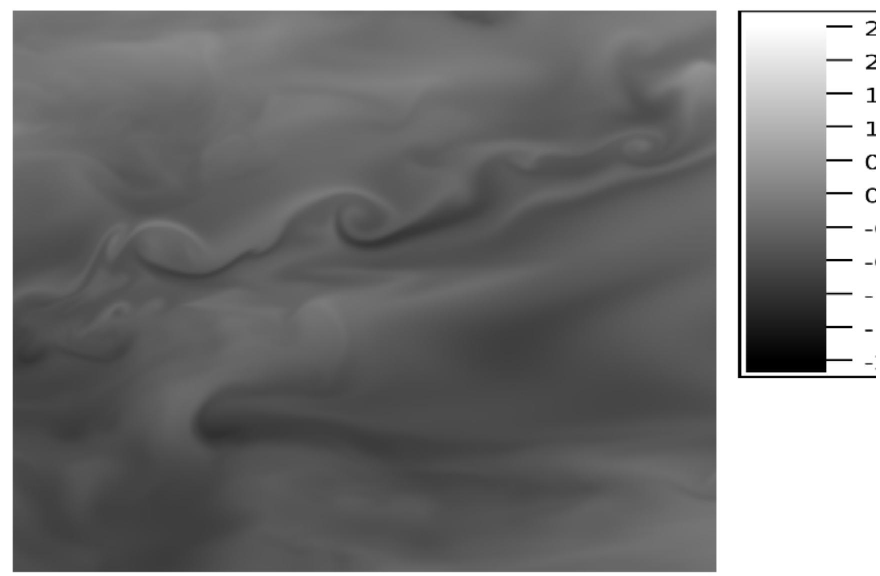
N/f=4.95

Fr=0.024, Ro=0.12

Re=55000, R_B=32

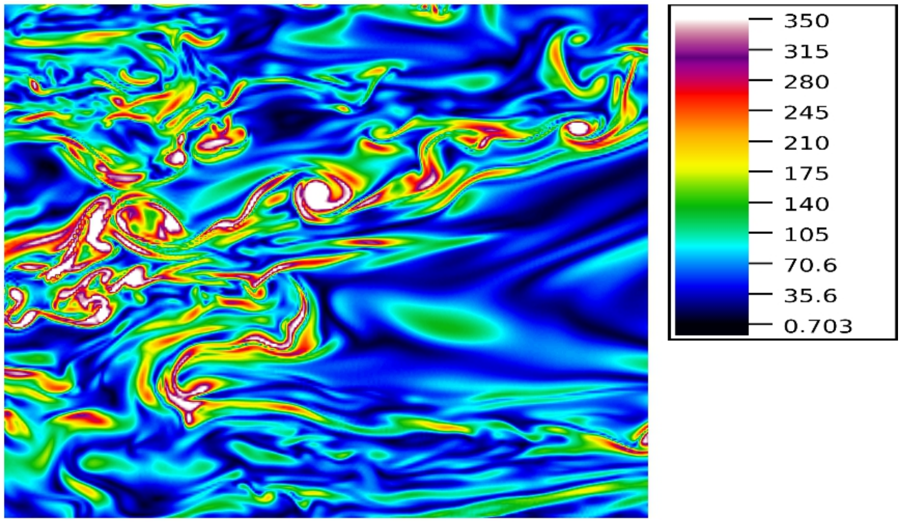
K_0=2.5, spin-down

z



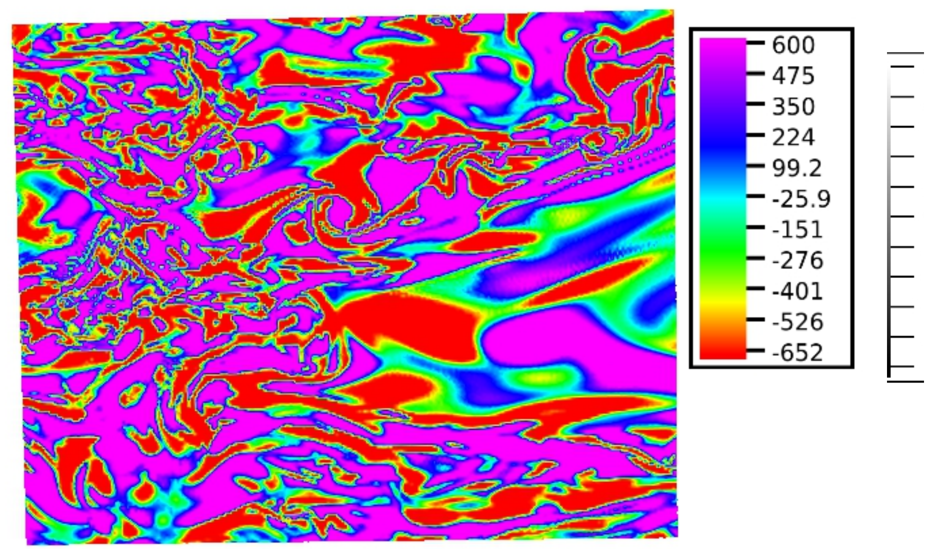
Vertical vorticity & Temperature fluctuations

Sub-volume: 0.12 X 0.1 X 0.01



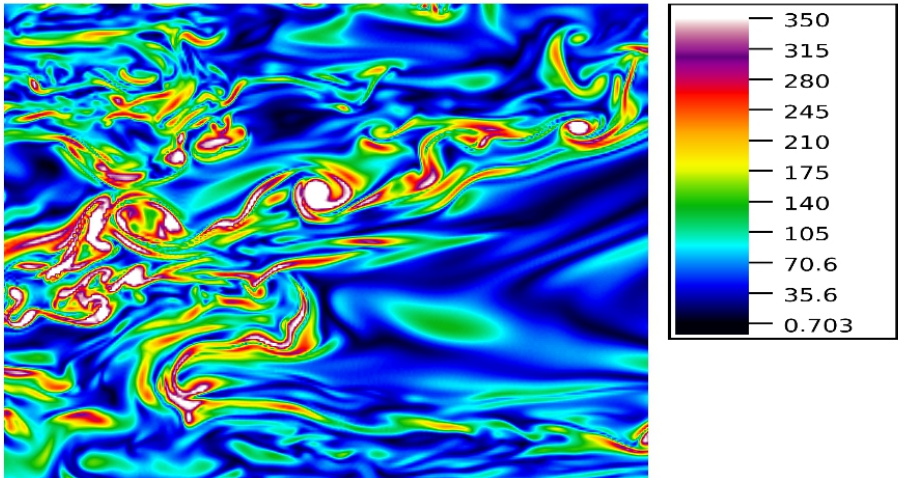
N/f=4.95
Fr=0.024, Ro=0.12
Re=55000, R_B=32
K_0=2.5, spin-down

z

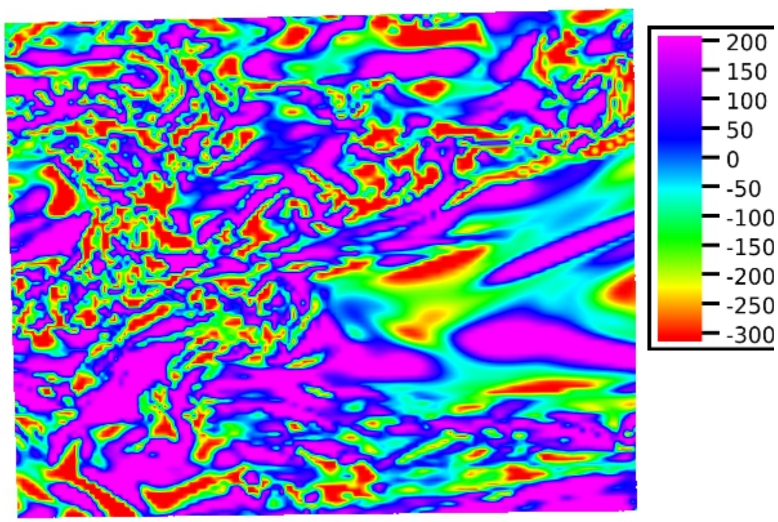


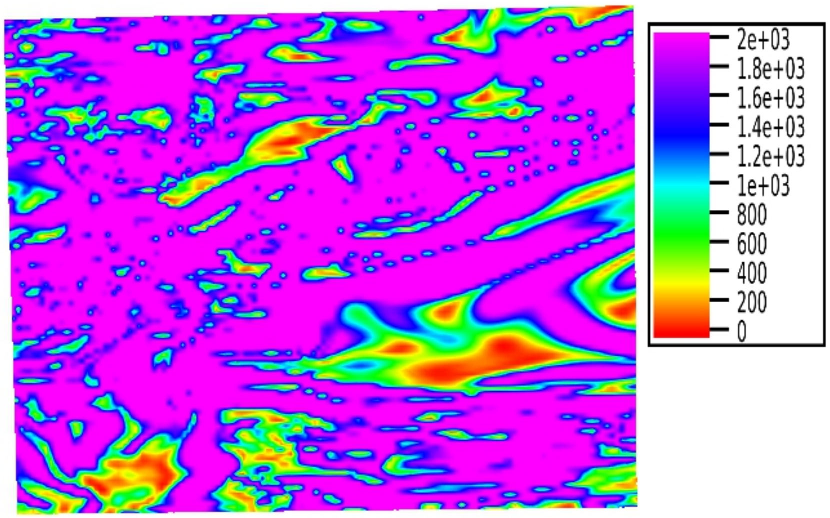
PV

f=2.7, $\omega_{rms} \sim 17$



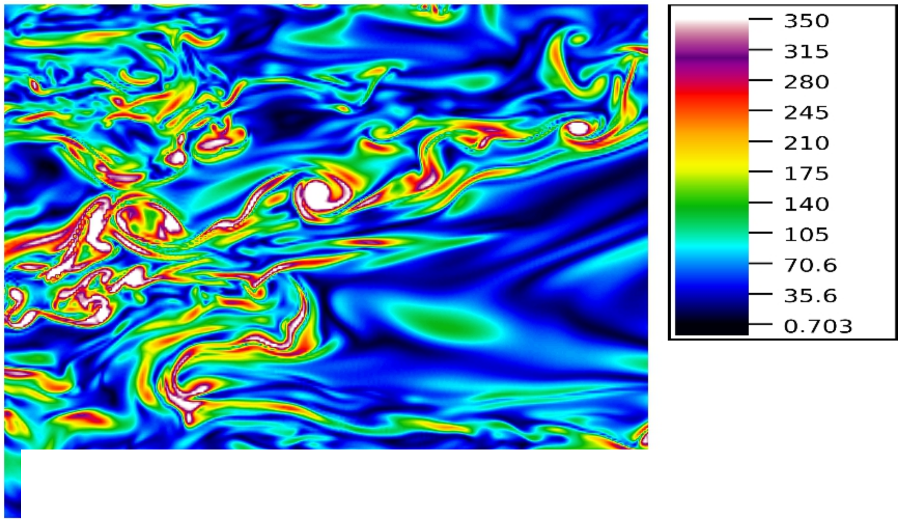
N/f=4.95
Fr=0.024, Ro=0.12
Re=55000, R_B=32
k_0=2.5, spin-down





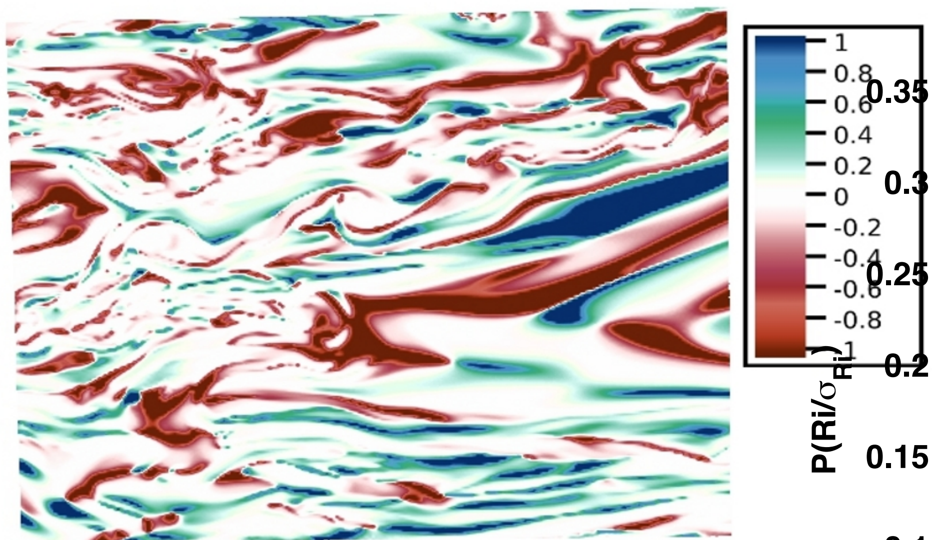
Kinetic and potential energy local dissipation

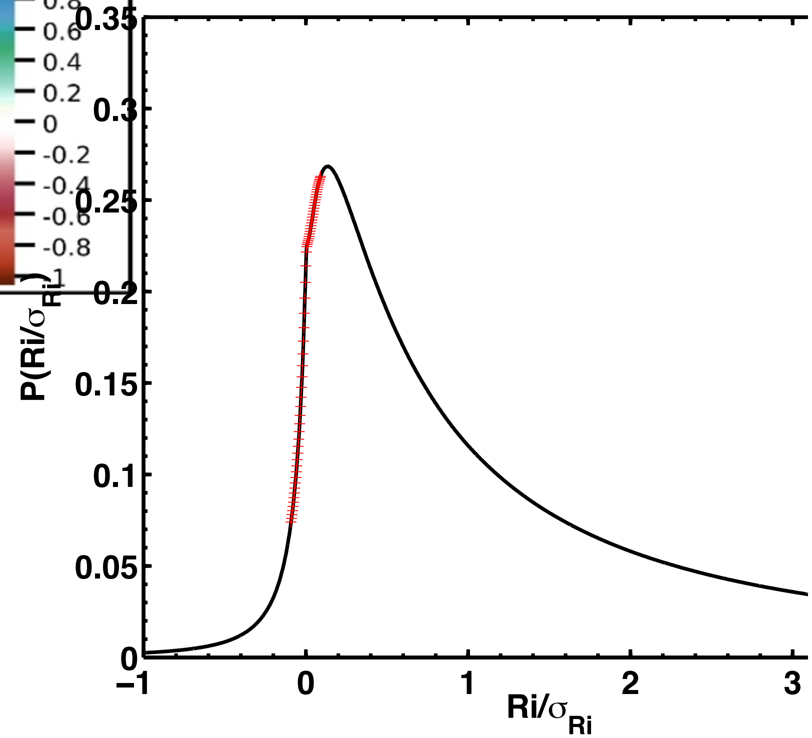
N/f=4.95
Fr=0.024, Ro=0.12
Re=55000, R_B=32
K_0=2.5, Spin-down



Gradient
Richardson
number Ri_g

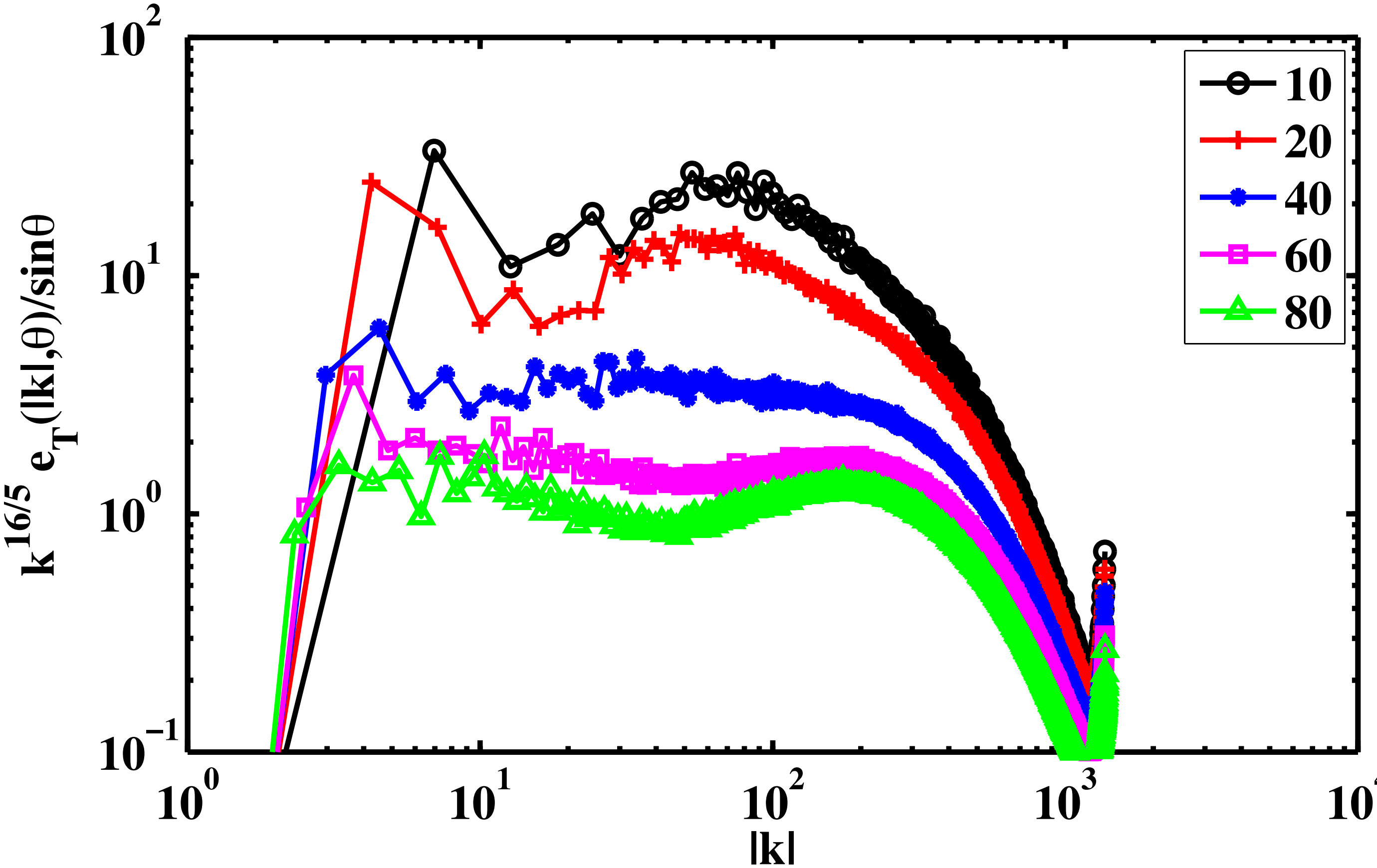
z



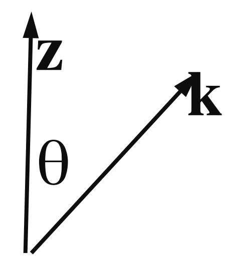


$$Ri_g = N(N - \partial_z \rho)/(\partial_z u_\perp)^2$$

N/f=4.95
Fr=0.024, Ro=0.12
Re=55000, R_B=32
K_0=2.5, spin-down



Compensated
total energy
angular spectra



Rosenberg et al. 2014

Stably stratified turbulence: Bolgiano-Obukhov 1959 scaling

Main hypothesis: Inertial range with a constant buoyancy flux $u\varrho^2/\ell$

$E_{V,P}(k) = f(k, \varepsilon_P)$ with $\varepsilon_P = DE_P/DT$ of dimension m^2s^{-5}

$\rightarrow E_V(k) = \varepsilon_P^{2/5}\ k^{-11/5}$

$\rightarrow E_P(k) = \varepsilon_P^{4/5}\ k^{-7/5}$

$\rightarrow U^2/\ell \sim \varrho$ in the momentum equation

Lohse & Xia, Ann. Rev. Fluid Mech. 2010

R. BOLGIANO, JR.

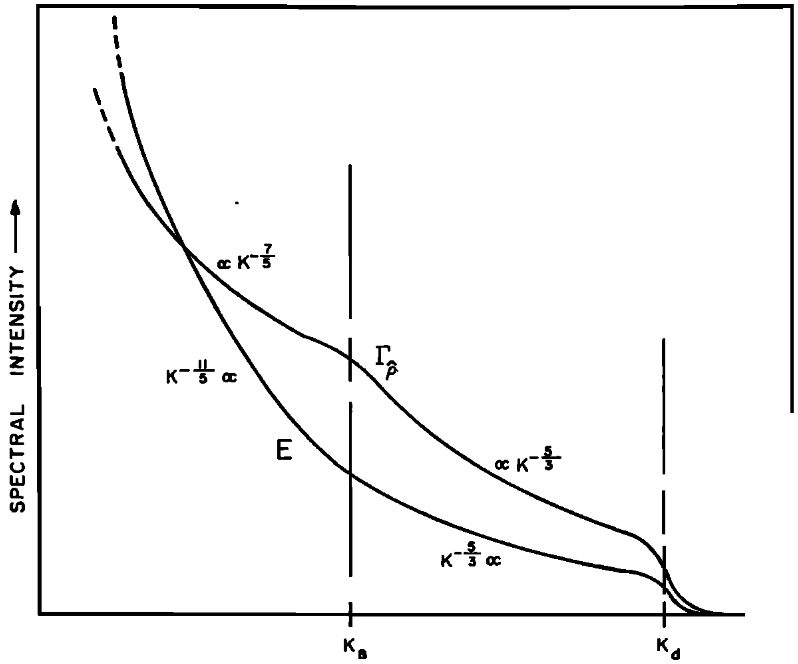


FIG. 1—Spectral forms in a stably stratified atmosphere.

SYMPOSIUM ON TURBULENCE IN GEOPHYSICS

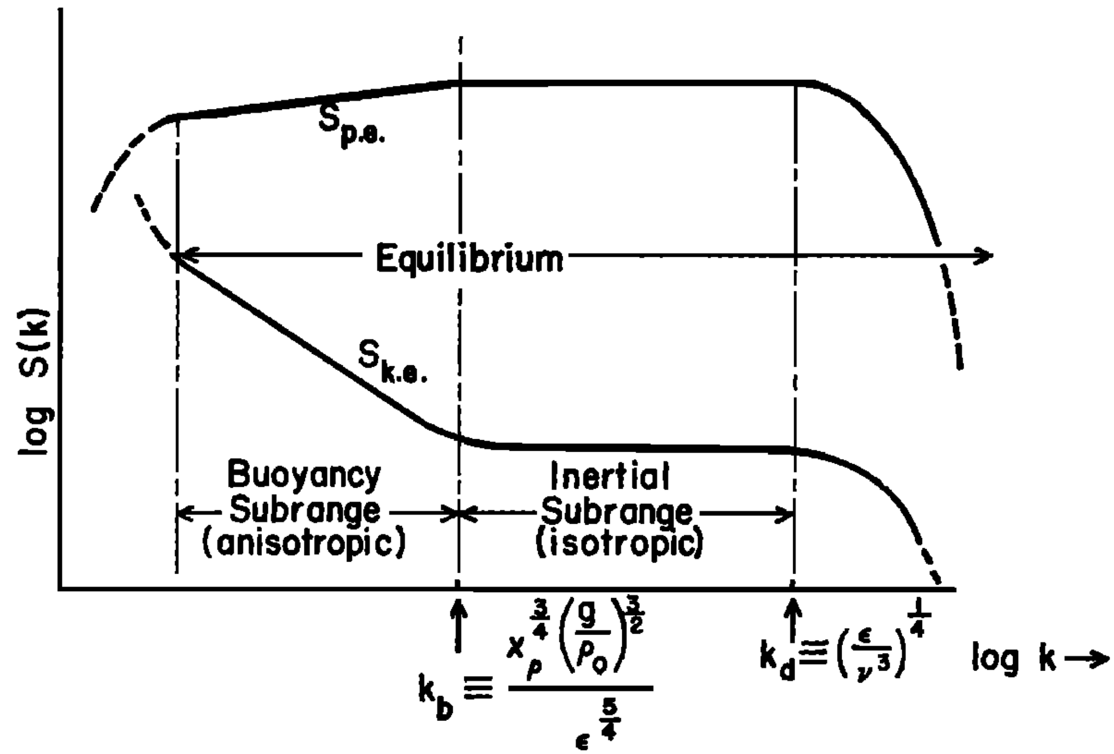
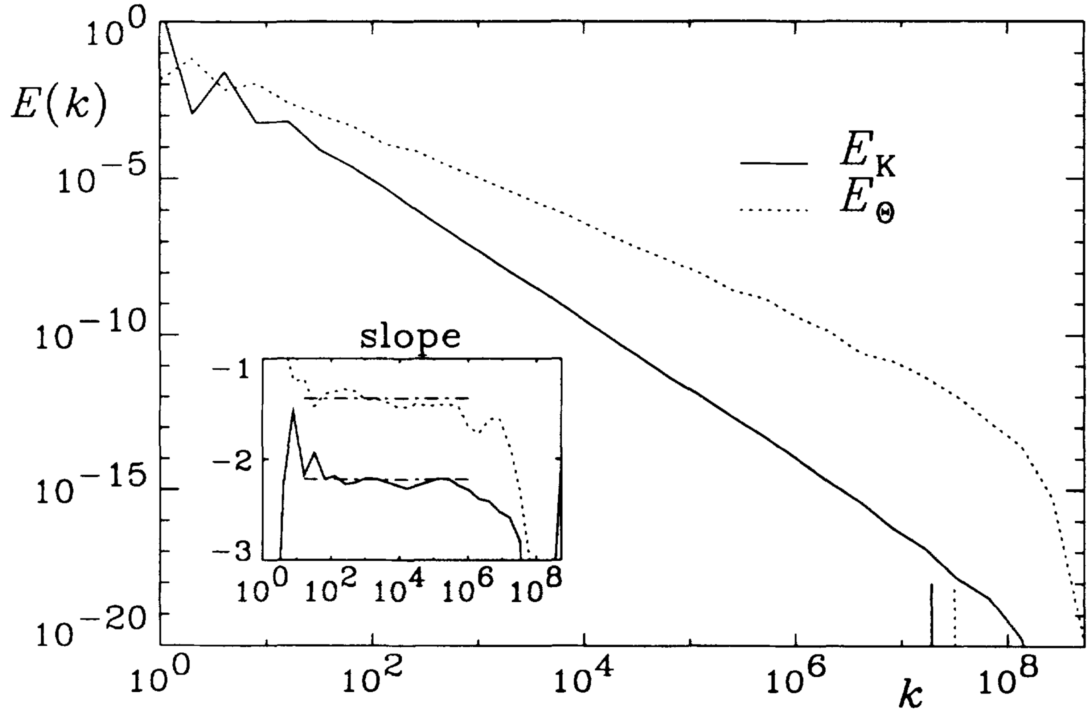


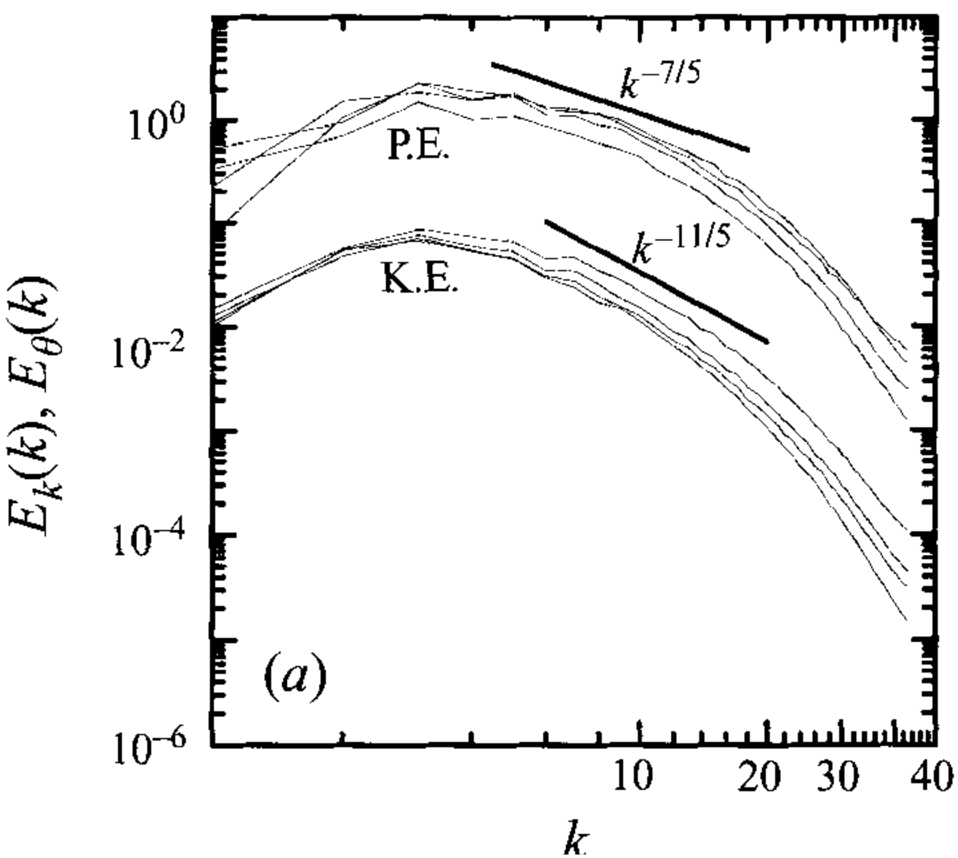
Fig. 2. Representation of energy transfer in stratified turbulence.

Bolgiano 1959, 1962

Brandenburg (1992) scalar model



Kimura & Herring (1996) 128^3 run, no rotation



N=10
Re=150
Spin-down

Kinetic & potential energy spectra

Stably stratified turbulence: Bolgiano-Obukhov 1959 scaling

Main hypotheses: * an inertial range with a constant buoyancy flux

* and isotropy

$E_{V,P}(k) = f(k, \varepsilon_P)$ with $\varepsilon_P = DE_P/DT$ of dimension L^2T^{-5}

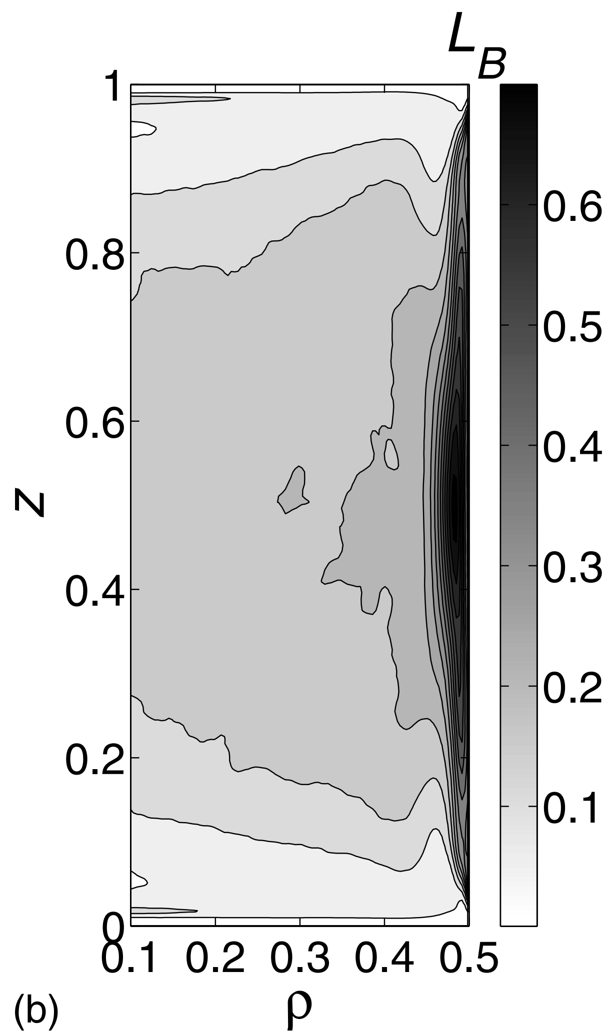
→ $E_V(k) = \varepsilon_P^{2/5} \; k^{-11/5}$
→ $E_P(k) = \varepsilon_P^{4/5} \; k^{-7/5}$

Bolgiano-Obukhov wavenumber (transition to a Kolmogorov spectrum):

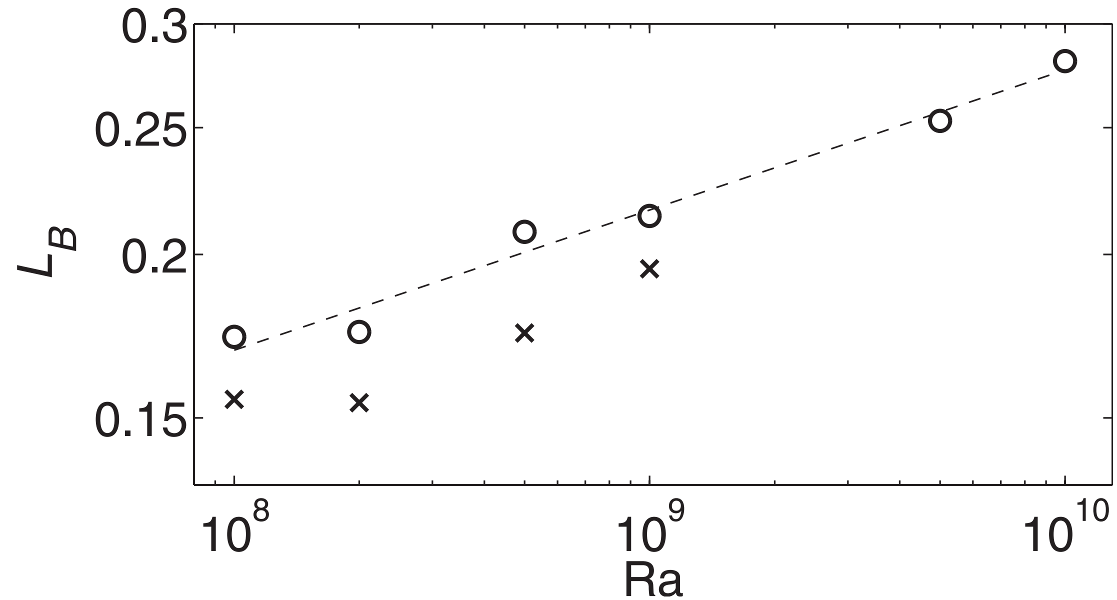
$$K_{BO} \sim \varepsilon_P^{3/4} \varepsilon_V^{-5/4}$$

Bolgiano-Obukhov scale in convective flows

$$K_{BO} \sim \varepsilon_P^{3/4} \varepsilon_V^{-5/4}$$



(b)



Kunnen et al. 2008

Bolgiano-Obukhov scale in convective flows

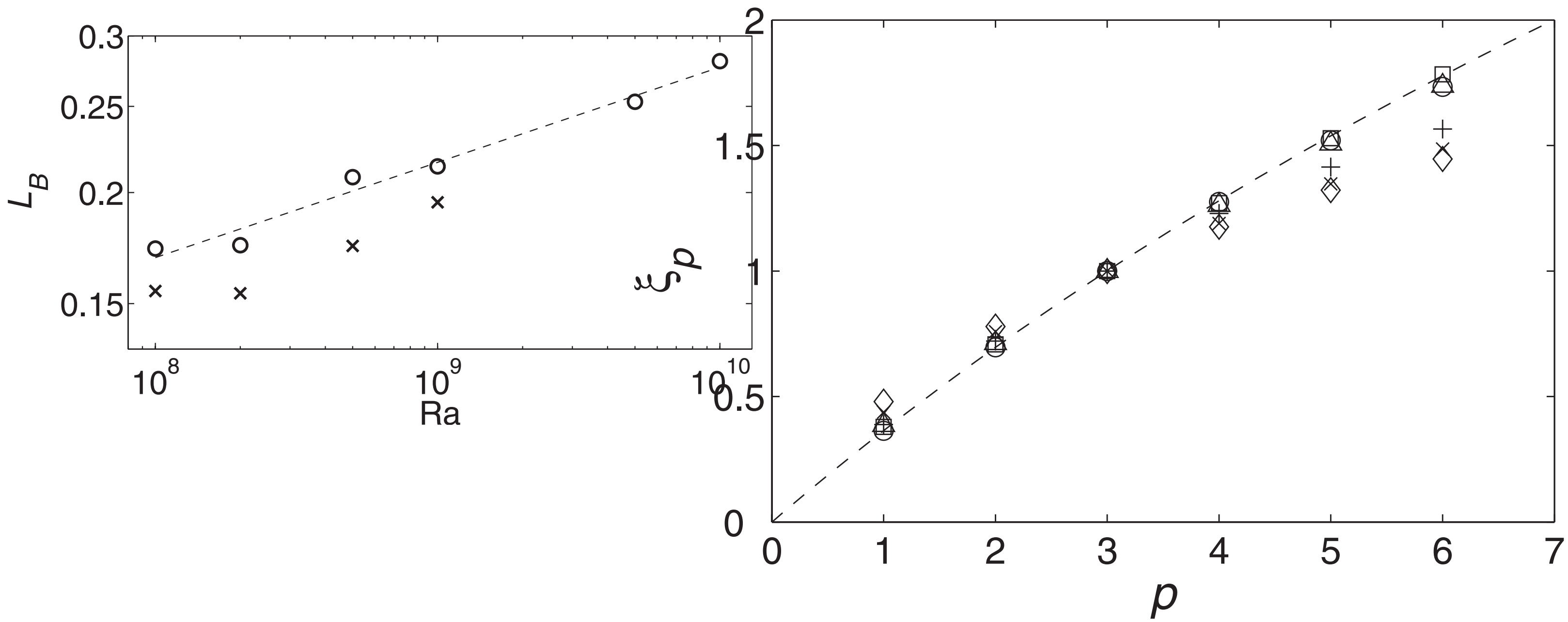
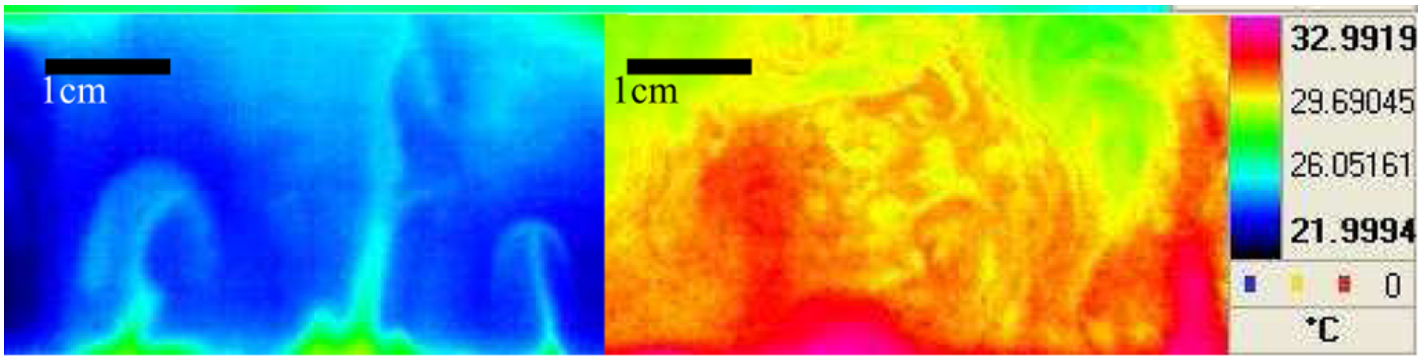
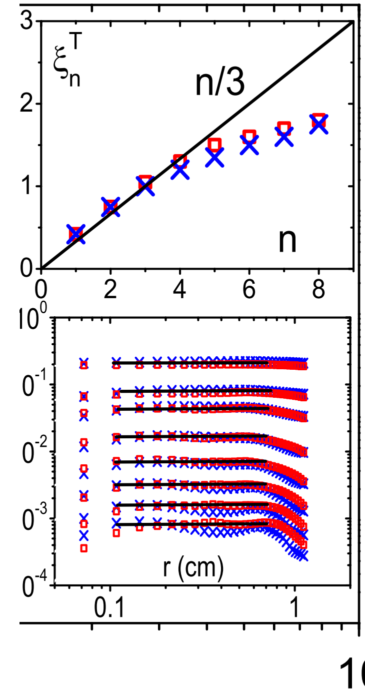


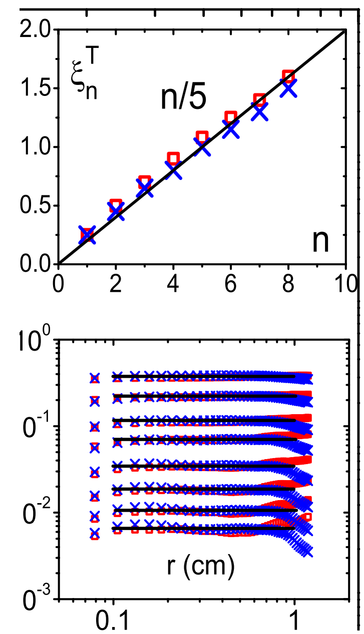
FIG. 17. Scaling exponents ξ_p as a function of the order p from the ESS plots. Non-BO range: triangles are for Ra$=1.11\times10^8$, squares for Ra$=3.34\times10^8$, and circles for Ra$=1.11\times10^9$. BO range: diamonds are for Ra$=1.11\times10^8$, crosses for Ra$=3.34\times10^8$, and pluses for Ra$=1.11\times10^9$. The dashed line indicates the hierarchical-shell model of Ref. [40].

Kunnen et al. 2008

Seychelles et al. (2010) 2D soap bubble experiment







ΔT= 21C or 50C

Boffetta et al. (2012): shell model of quais 2D turbulence

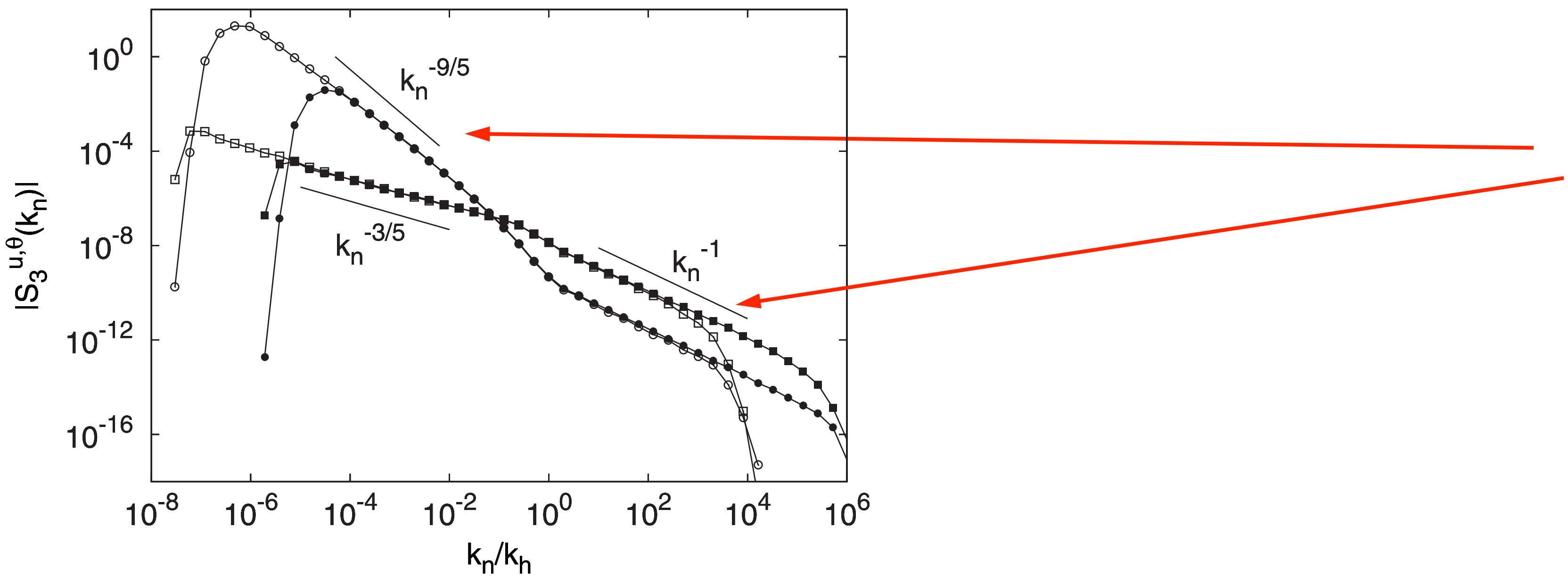
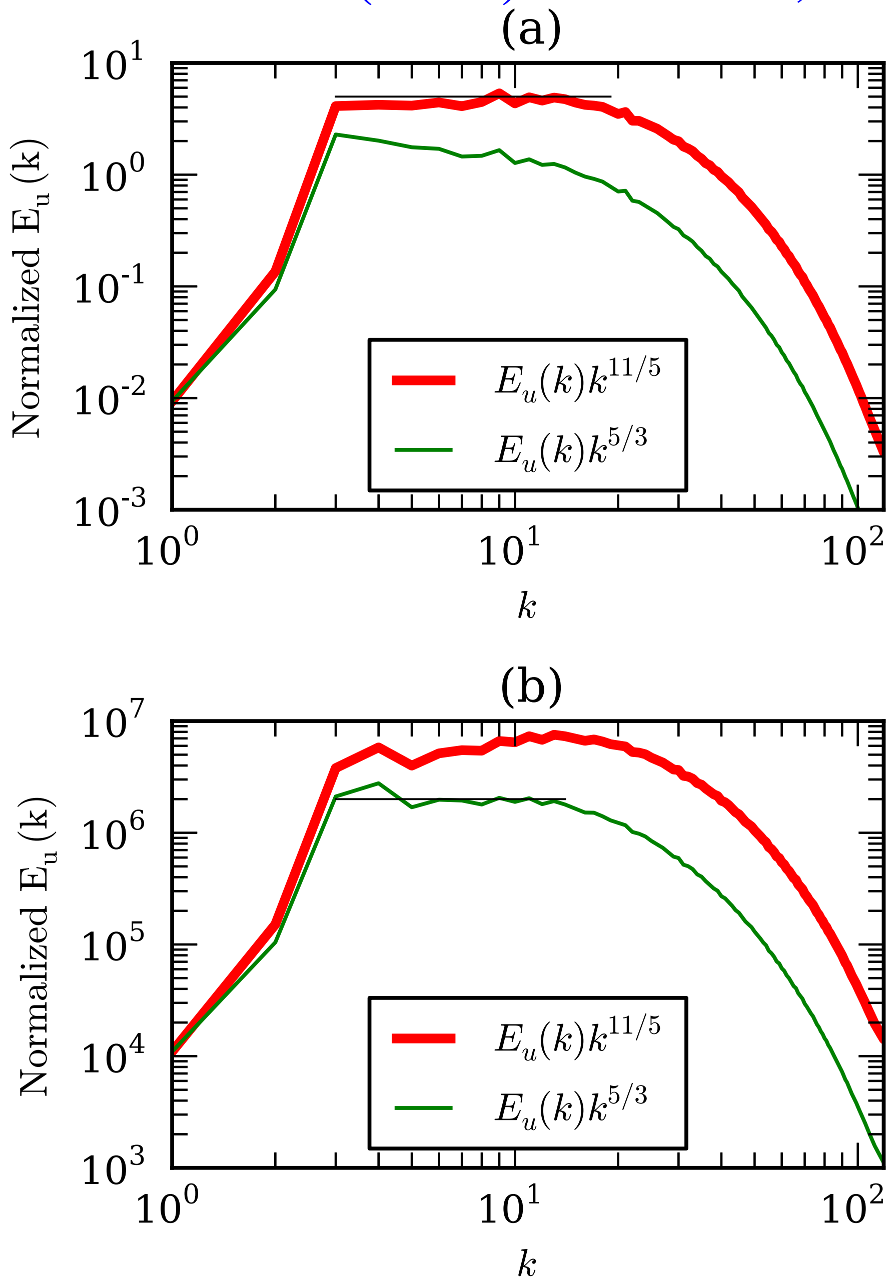


FIG. 3. Absolute value of the third order structure function $|S_3(k_n)|$ for the velocity u (circles), and temperature θ (squares), for $n_h = 20$ (black symbols) and $n_h = 26$ (empty symbols). S_3^{θ} has been multiplied by a factor 10^{-3} for plotting purposes.

Kumar et al. (2014) 1024^3 run, no rotation



Ri=1/Fr^2 = **0.5 & 4x10^{-7}**

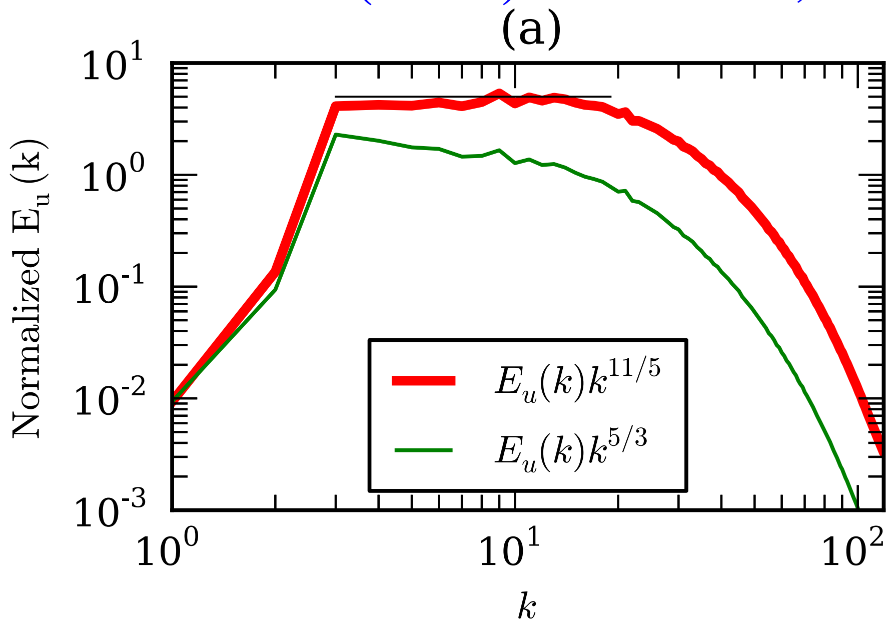
Re~500, forced at large scale

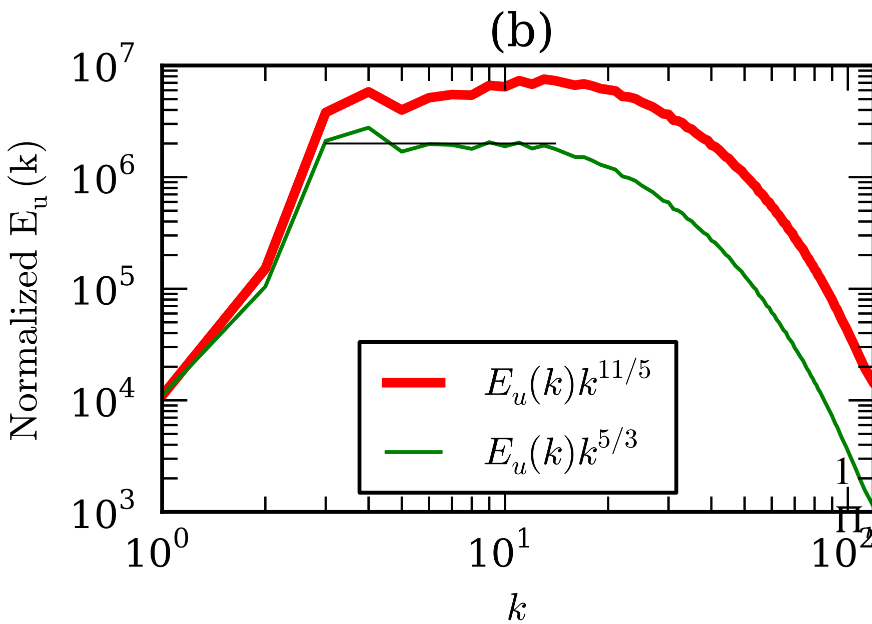
Kinetic energy spectra compensated by K41 (5/3) & BO (11/5)

Kumar et al. (2014) 1024^3 run, no rotation

Ri=1/Fr^2 = **0.5 & 4x10^{-7}**
Re~500, forced at large scale

Kinetic energy spectra compensated by
K41 (5/3) & BO (11/5)





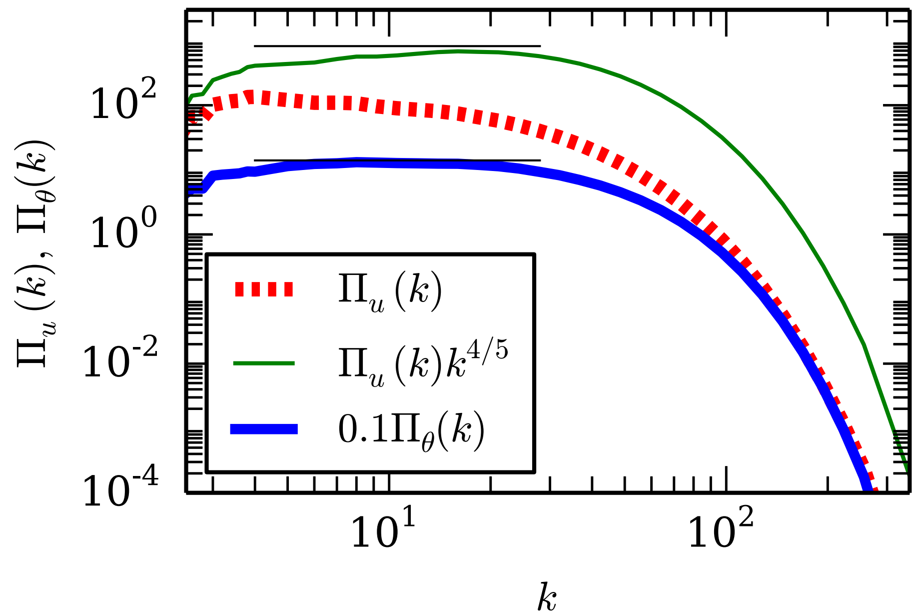
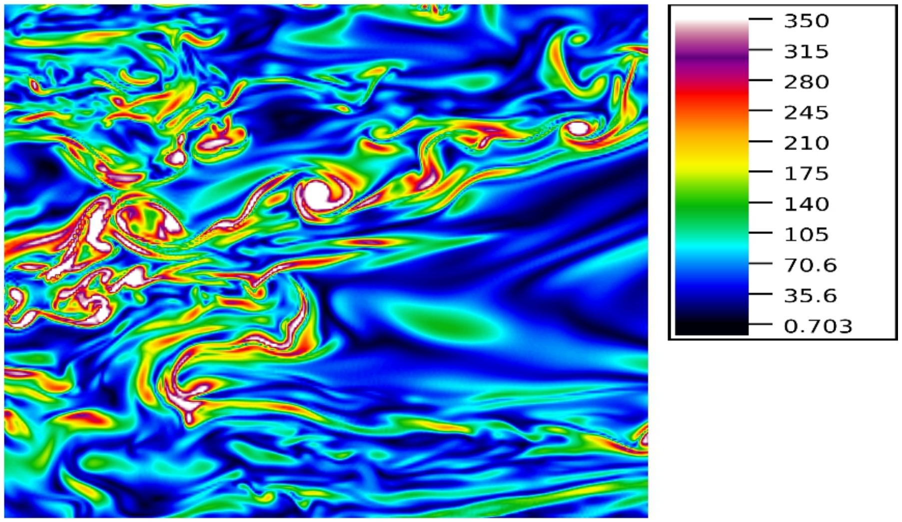


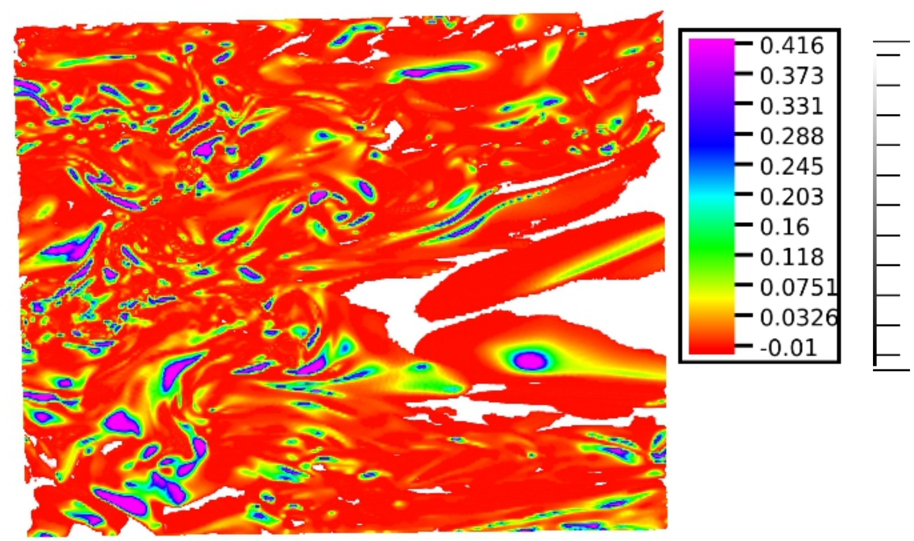
FIG. 3. (Color online) For stably stratified simulation with Pr = 1 and Ri = 0.01, plots of KE flux $\Pi_u(k)$, normalized KE flux $\Pi_u(k)k^{4/5}$, and potential energy flux $\Pi_\theta(k)$.



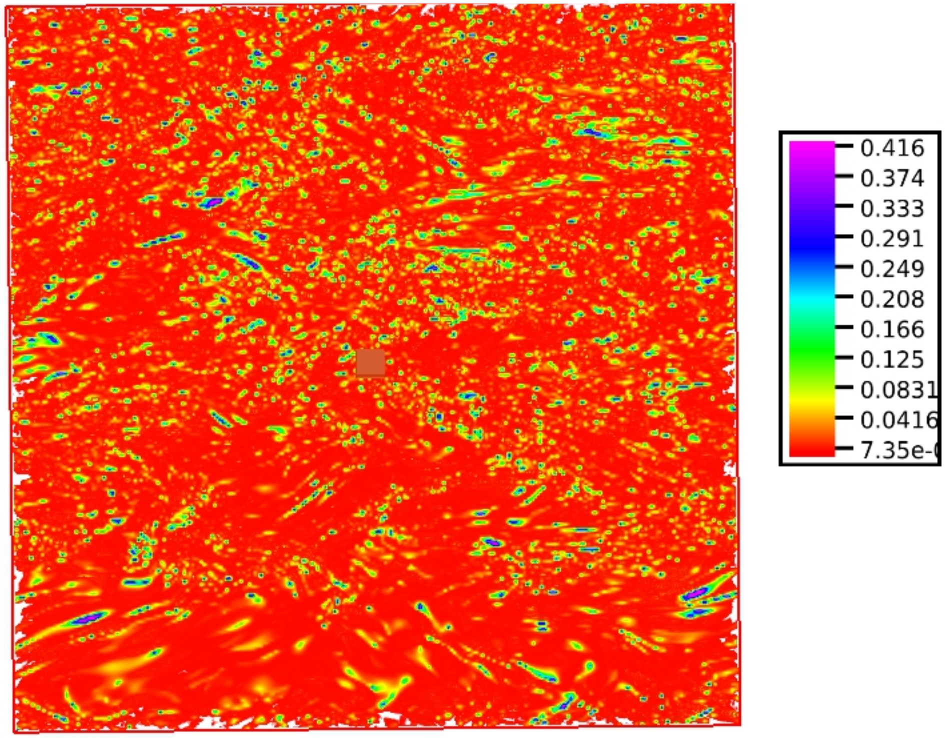
N/f=4.95
Fr=0.024, Ro=0.12
Re=55000, R_B=32
K_0=2.5, Spin-down

z

$$K_{BO} \sim \varepsilon_P^{3/4} \varepsilon_V^{-5/4}$$

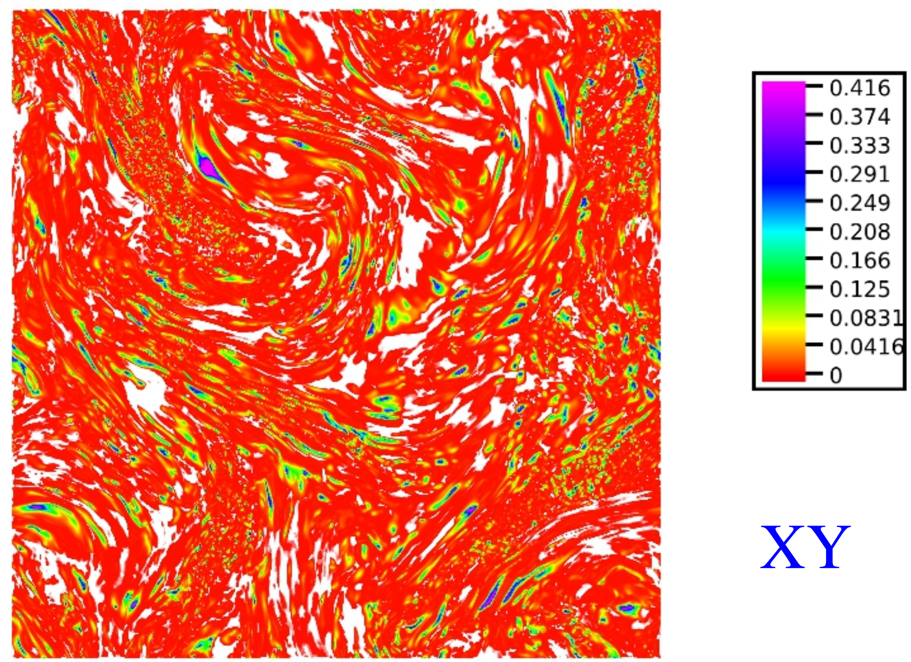


L_{BO}



XZ

Maps of L_{BO}
In our 4k data

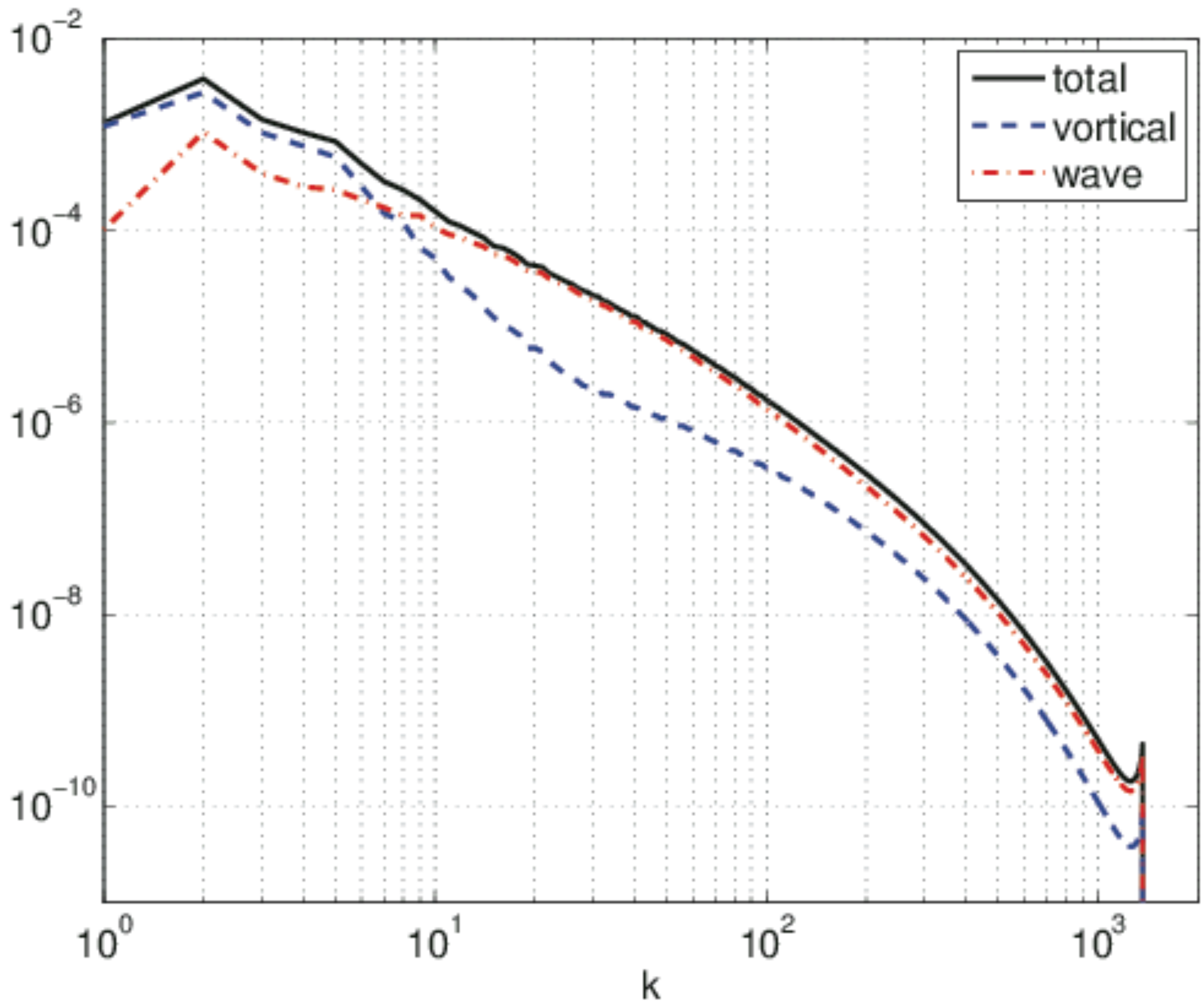


L_{BO}

XY

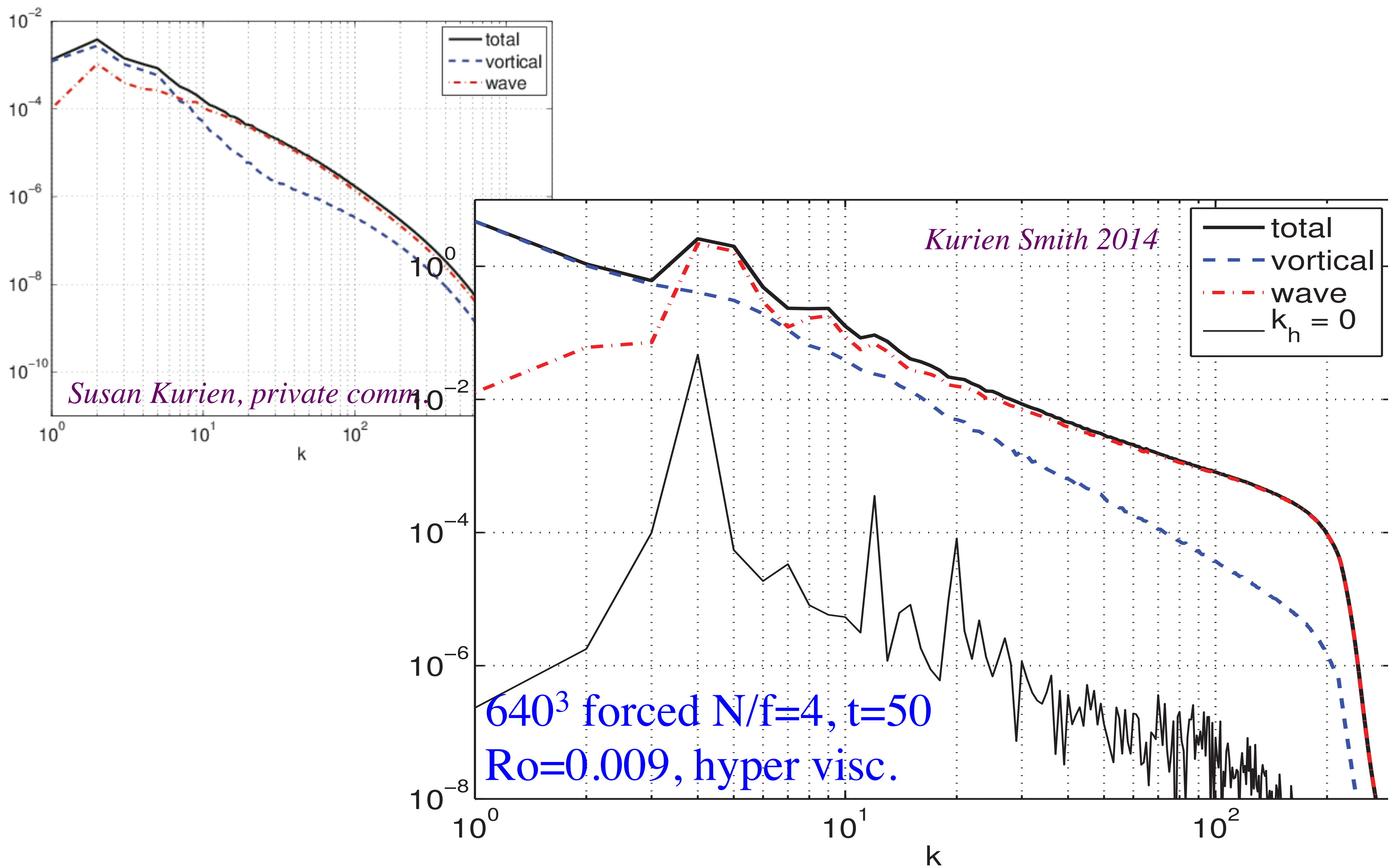
Rosenberg et al. 2014

Normal mode decomposition of the total energy spectrum at peak, R_B=32



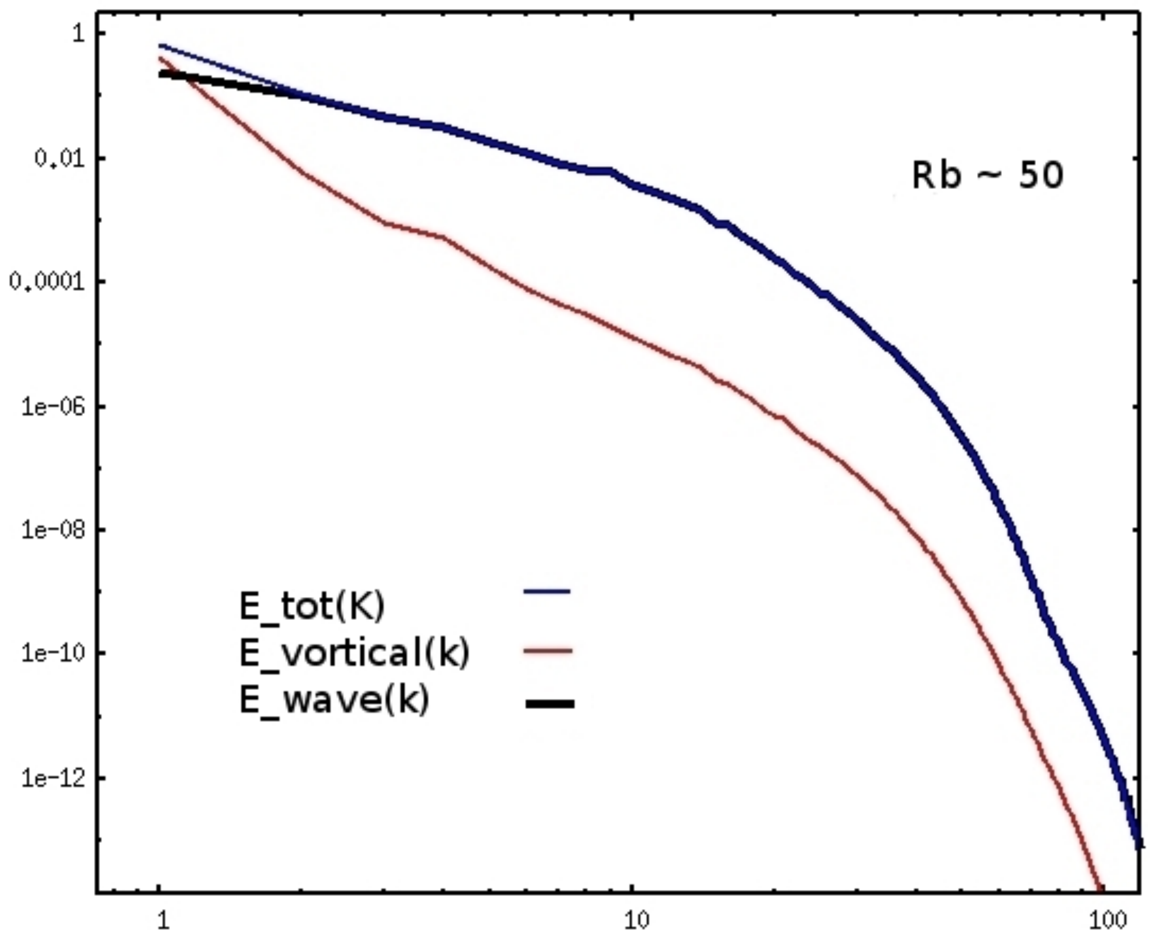
Susan Kurien, private communication, 2014

Normal mode decomposition of the total energy spectrum at peak, R_B=32



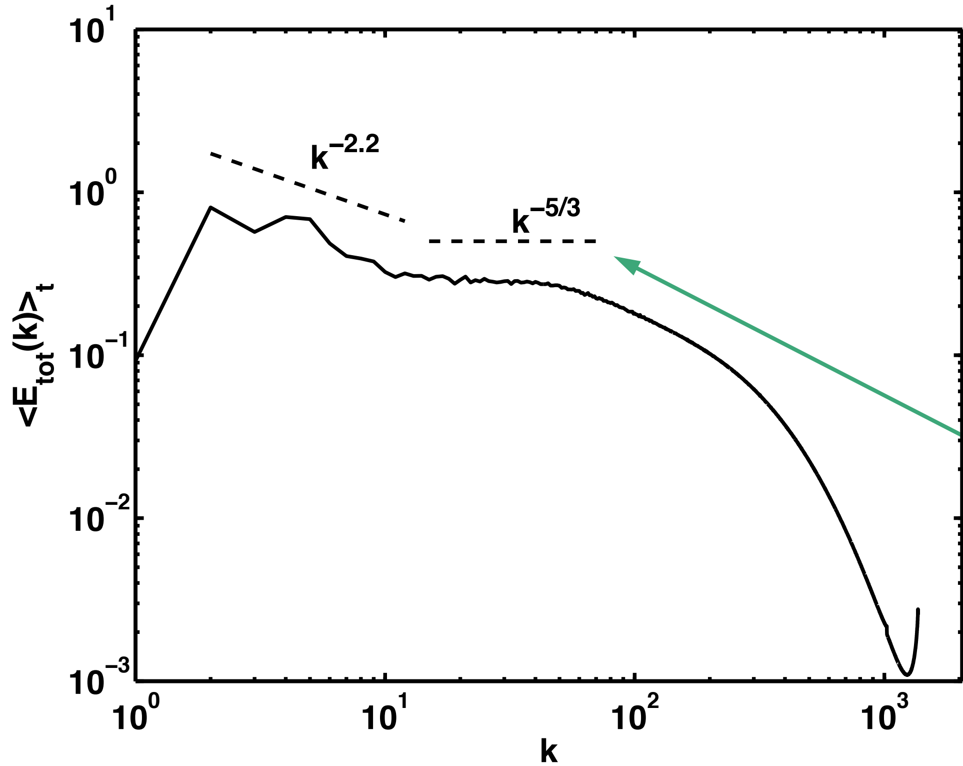
Normal mode decomposition
512^3 runs

Re~ 10^4
Ro=0.038, N/f= ½
Fr-0.076, R_B~50

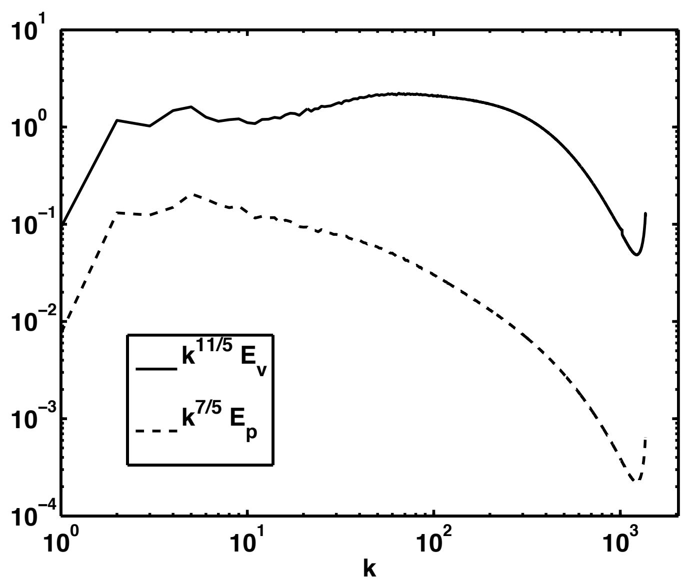


Marino et al., in preparation

N/f=4.95
Fr=0.024, Ro=0.12
Re=55000, R_B=32
K_0=2.5, spin-down



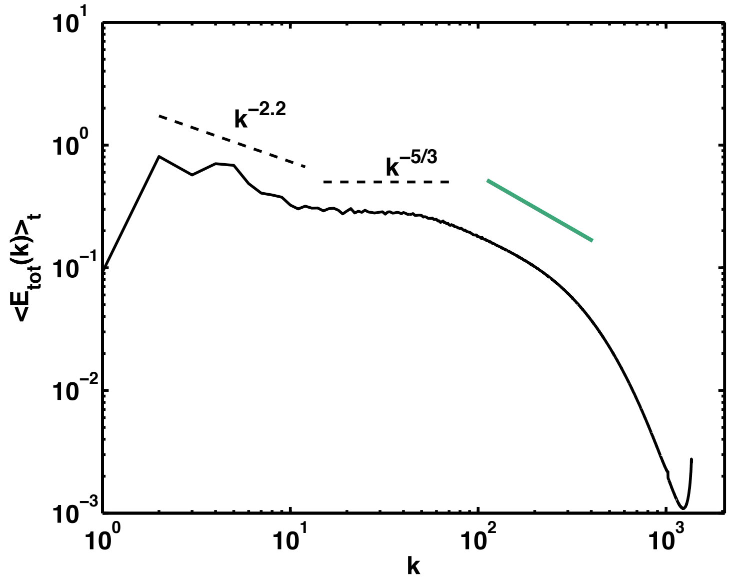
5/3-compensated
total energy
isotropic spectrum

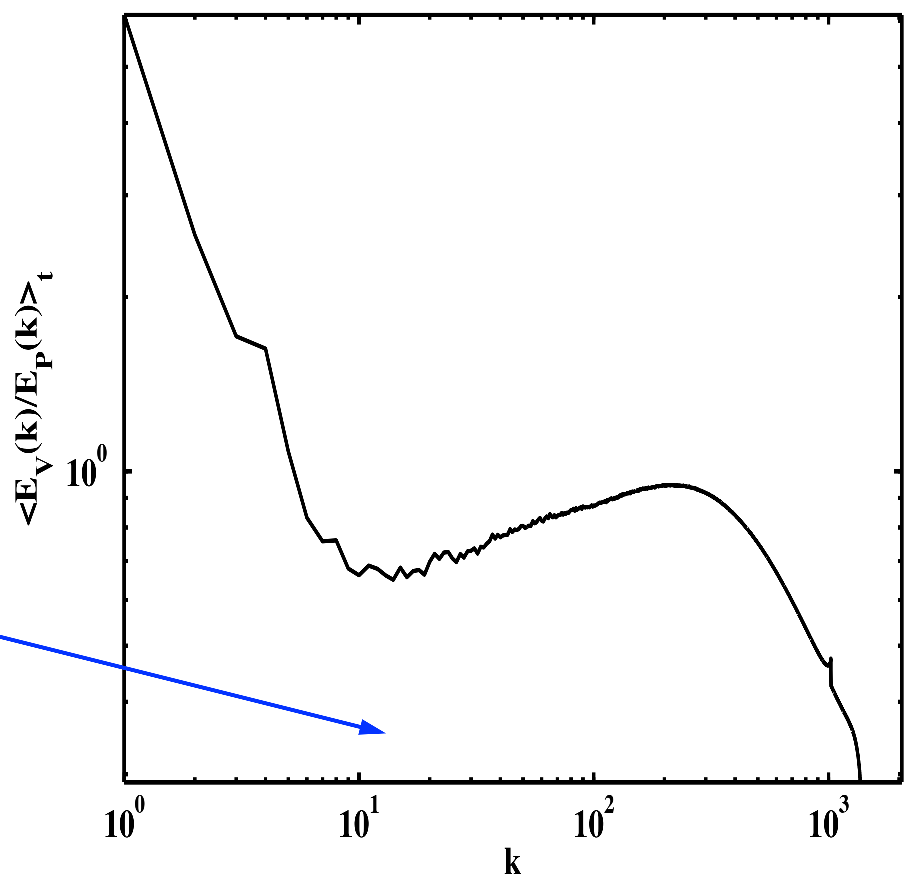


Energy spectra:
kinetic (____) or
potential (- - -)
compensated
by 11/5 or 7/5:
Bolgiano-Obukhov scaling

Energy ratio

$E_V/E_P \sim k^{-4/5}$





$$K_{BO} \sim \varepsilon_P^{3/4} \varepsilon_V^{-5/4}$$

~ 11

Vertical buoyancy flux:

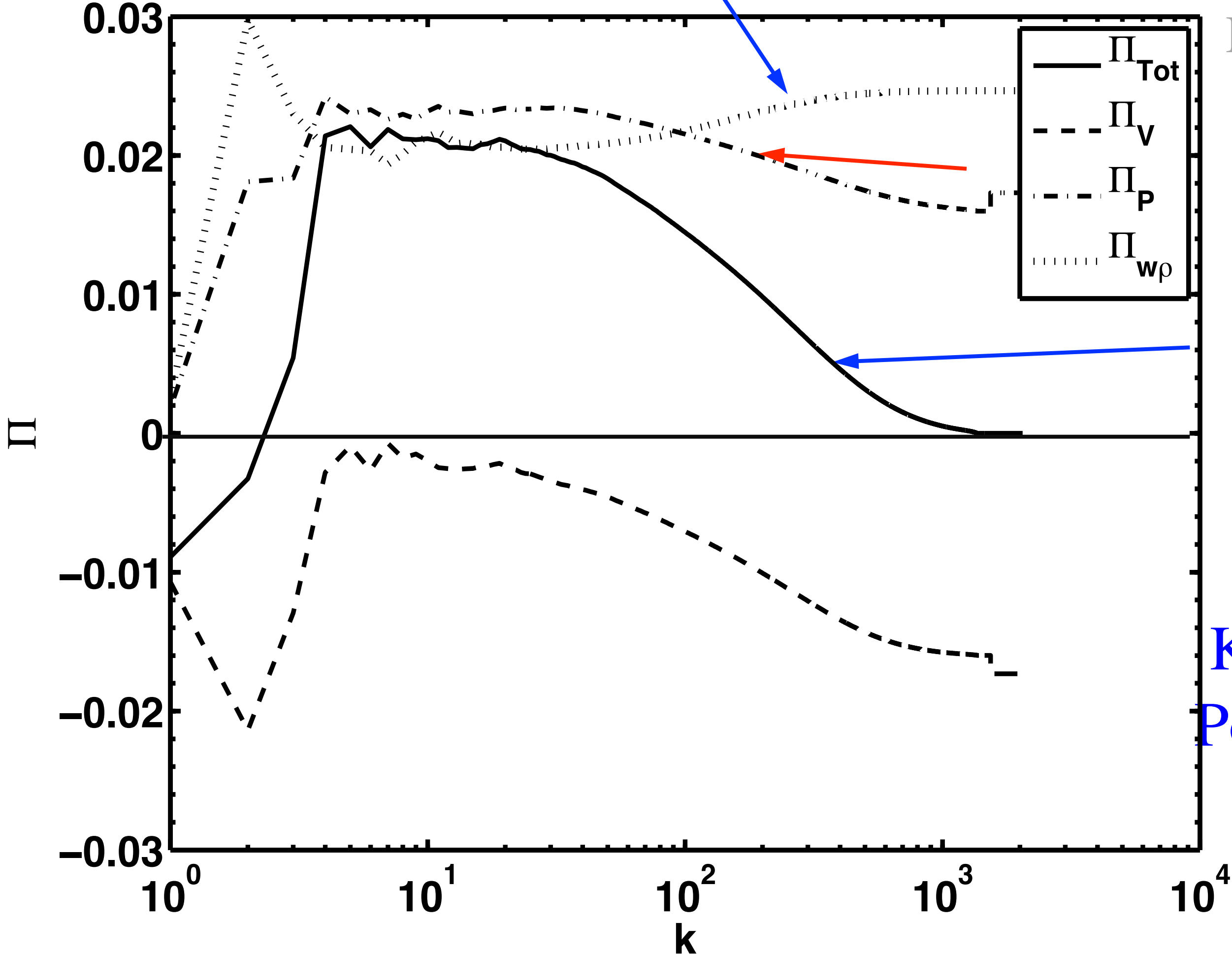
$$\Pi_{w\rho}(k) = \sum_{k'=0}^{k'=k} \sum_{k'<|k''|<k'+1} Re(\hat{w}(\mathbf{k}'')\hat{\rho}(\mathbf{k}'')^*) \ ,$$

4096^3 data, N/f=4.95

Fr=0.024, Ro=0.12

Re=55000, R_B=32

K_0=2.5, spin-down



Isotropic energy flux and its components

Total (____): $\Pi_T \sim \Pi_V + \Pi_P$

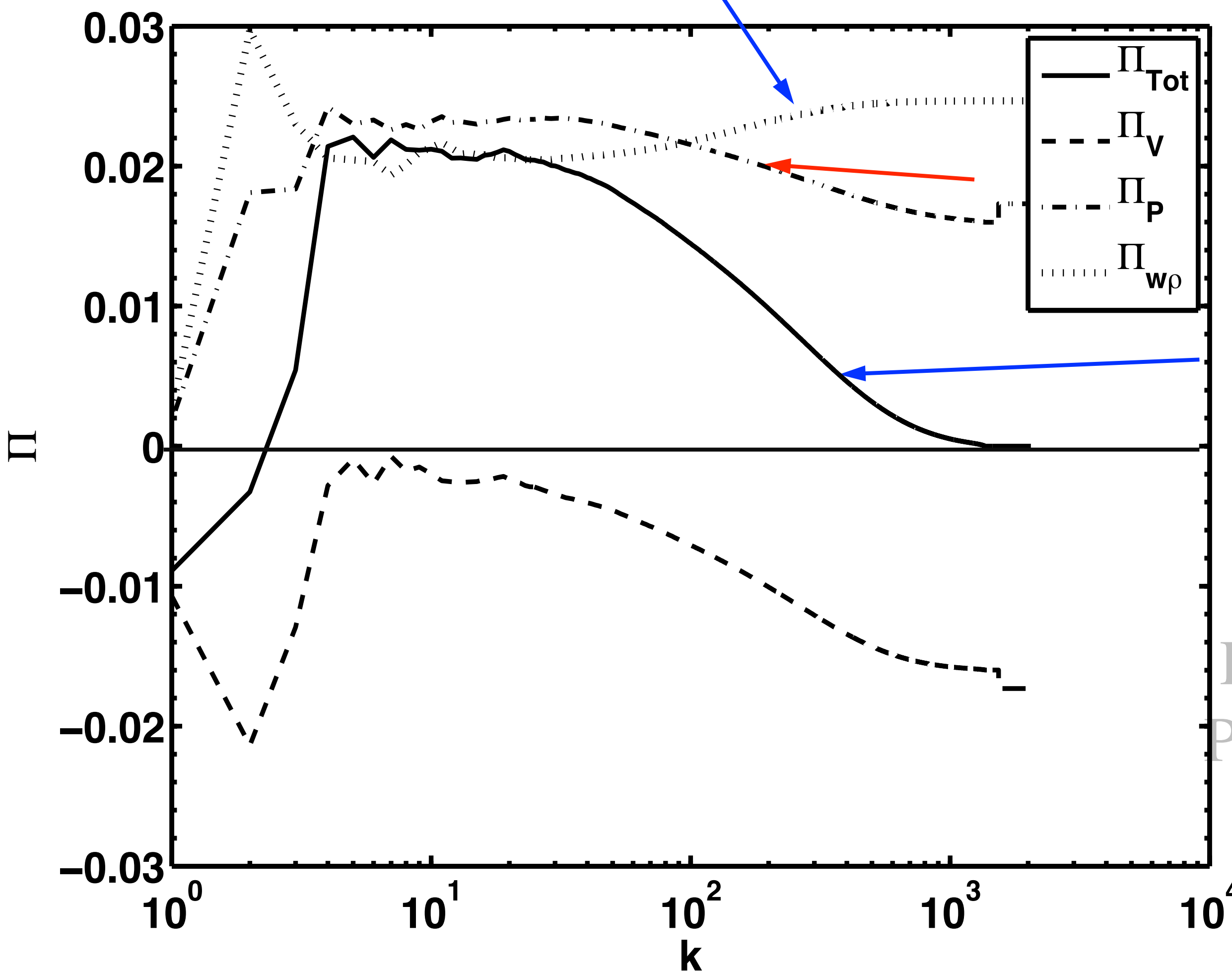
Kinetic (---): $\Pi_V \sim \mathbf{u}.(\mathbf{u}.\mathrm{grad}\ \mathbf{u})$

Potential (-. -.): $\Pi_P \sim \varrho(\mathbf{u}.\mathrm{grad}\ \varrho)$

Buoyancy flux: ……

Vertical buoyancy flux:

$$\Pi_{w\rho}(k) = \sum_{k'=0}^{k'=k} \sum_{k'<|k''|<k'+1} Re(\hat{w}(\mathbf{k}'')\hat{\rho}(\mathbf{k}'')^*) \ ,$$



4096^3 data, N/f=4.95

Fr=0.024, Ro=0.12

Re=55000, R_B=32

K_0=2.5, spin-down

Isotropic energy flux and its components

Total (____): $\Pi_T \sim \Pi_V + \Pi_P$

Kinetic (---): $\Pi_V \sim \mathbf{u}.(\mathbf{u}.\text{grad } \mathbf{u})$

Potential (-. -.): $\Pi_P \sim \varrho(\mathbf{u}.\text{grad } \varrho)$

Buoyancy flux: ……

Π_P constant: OK

Conclusions and questions

- Large resolutions allow for scale separation …
  and thus to distinguish between multiple regimes within a flow
- Evidence for Bolgiano-Obukhov scaling at large scale
  and complex interplay between velocity and buoyancy modes & fluxes
- Local instabilities and strong local variations (dissipation, PV, …)

- Waves and eddies partition
- Local small-scale dynamics
- Role of rotation *in the Bolgiano-Obukhov scaling?*
- Role of inverse cascade *in the Bolgiano-Obukhov scaling?*
- Role of walls and B.C. *in the Bolgiano-Obukhov scaling?*

- Role of forcing (3D *vs*. 2D, vortices *vs*. waves, …)
- Role of non-local interactions
- Role of large-scale friction in the presence of forcing

Bolgiano, Marseille meeting, 1962

``Important progress appears likely in the next few years.”

Thank you for your attention!

Rosenberg et al., ArXiv:1409.4254

Some recent references of ours

- D. Rosenberg, A. Pouquet, R. Marino & P. Mininni, Evidence for Bolgiano-Obukhov scaling in rotating stratified turbulence using high-resolution direct numerical simulations. See ArXiv:1409.4254.
- C. Herbert, A. Pouquet & R. Marino, Restricted Equilibrium and the Energy Cascade in Rotating and Stratified Flows. J. Fluid Mech. 758, 374 (2014)
- R. Marino, P. Mininni, D.L. Rosenberg & A. Pouquet, Large-scale anisotropy in stably stratified rotating flows, Phys. Rev. E 90, 023018 (2014)
- C. Rorai, P. Mininni & A. Pouquet, Turbulence comes in bursts in stably stratified flows, Phys. Rev. E 89, 043002 (2014)
- A. Pouquet & R. Marino, Geophysical turbulence and the duality of the energy flow across scales, Phys. Rev. Lett. 111, 234501 (2013)
- R. Marino, P. Mininni, D. Rosenberg & A. Pouquet, Inverse cascades in rotating stratified turbulence: fast growth of large scales, EuroPhys. Lett.102, 44006 (2013)
- P. Mininni & A. Pouquet, Inverse cascade behavior in freely decaying two-dimensional fluid turbulence, Phys. Rev. E 87, 033002 (2013)
- P. Mininni, D. Rosenberg & A. Pouquet, Isotropization at small scales of rotating helically driven turbulence, J. Fluid Mech. 699, 263 (2012)
- P. Mininni, D. Rosenberg, R. Reddy & A. Pouquet, An hybrid MPI-OpenMP scheme for scalable parallel pseudo-spectral computations for fluid turbulence, Parallel Computing 37, 316 (2011)
- P. Mininni & A. Pouquet, Rotating helical turbulence. Part I. Global evolution and spectral behavior. Phys. Fluids 22, 035105; and Part II, 035106 (2010)

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$T_{transfer} \sim T_{NL} * [T_{NL}/T_{W}]$

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“In this unfolding conundrum of life and history there is such a thing as being too late … We may cry out desperately for time to pause in her passage, but time is adamant to every plea and rushes on. Over the bleached bones and jumbled residue of numerous civilizations are written the pathetic words: "Too late". ‘’
Martin Luther King Jr, 1967, After Clive Hamilton, Utopias in the Anthropocene, American Sociological Association 2012

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Thank you for your attention