Non-linearities in Geodynamo models and their connection to the WEAK-FIELD and STRONG-FIELD branches

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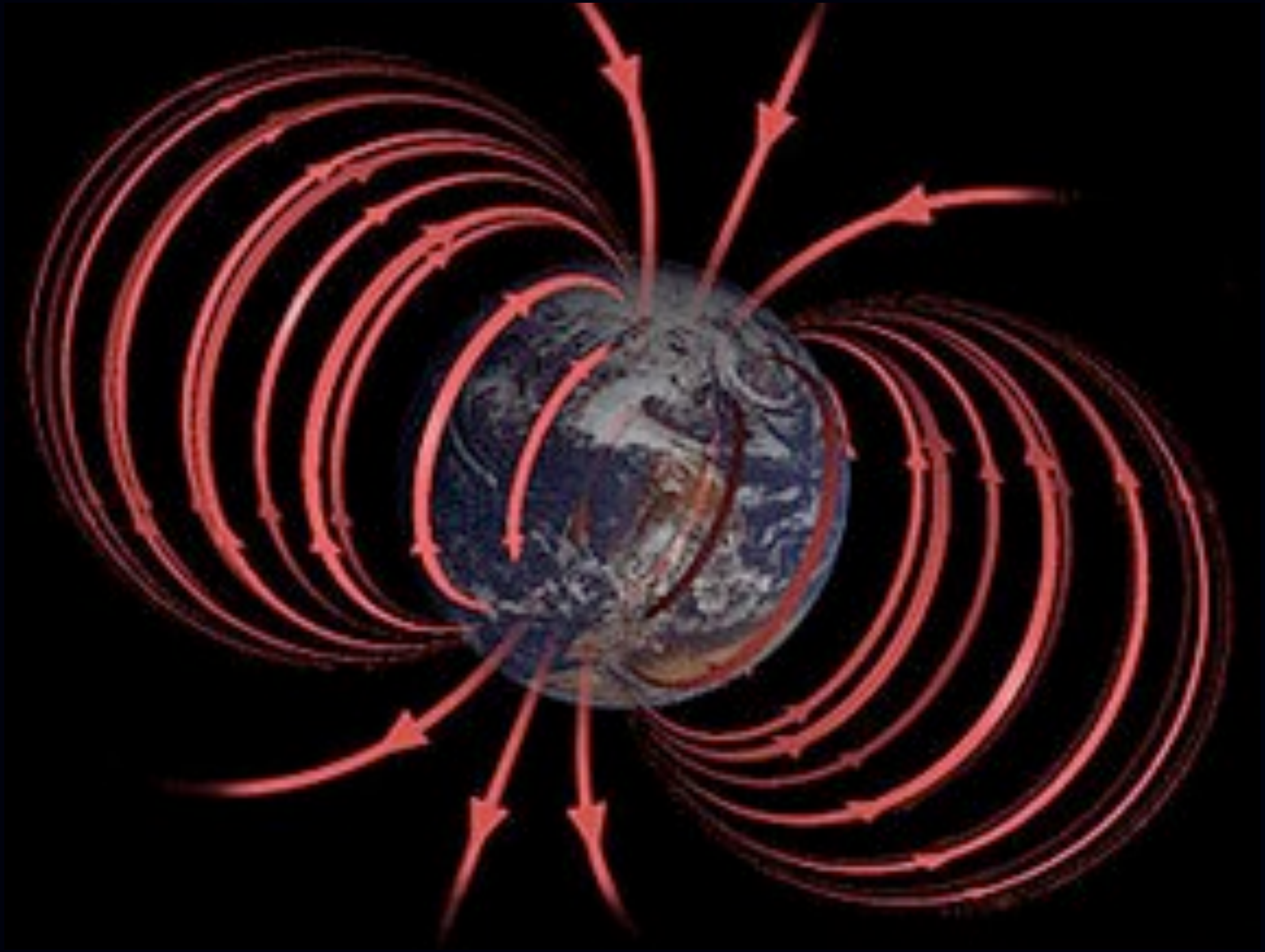
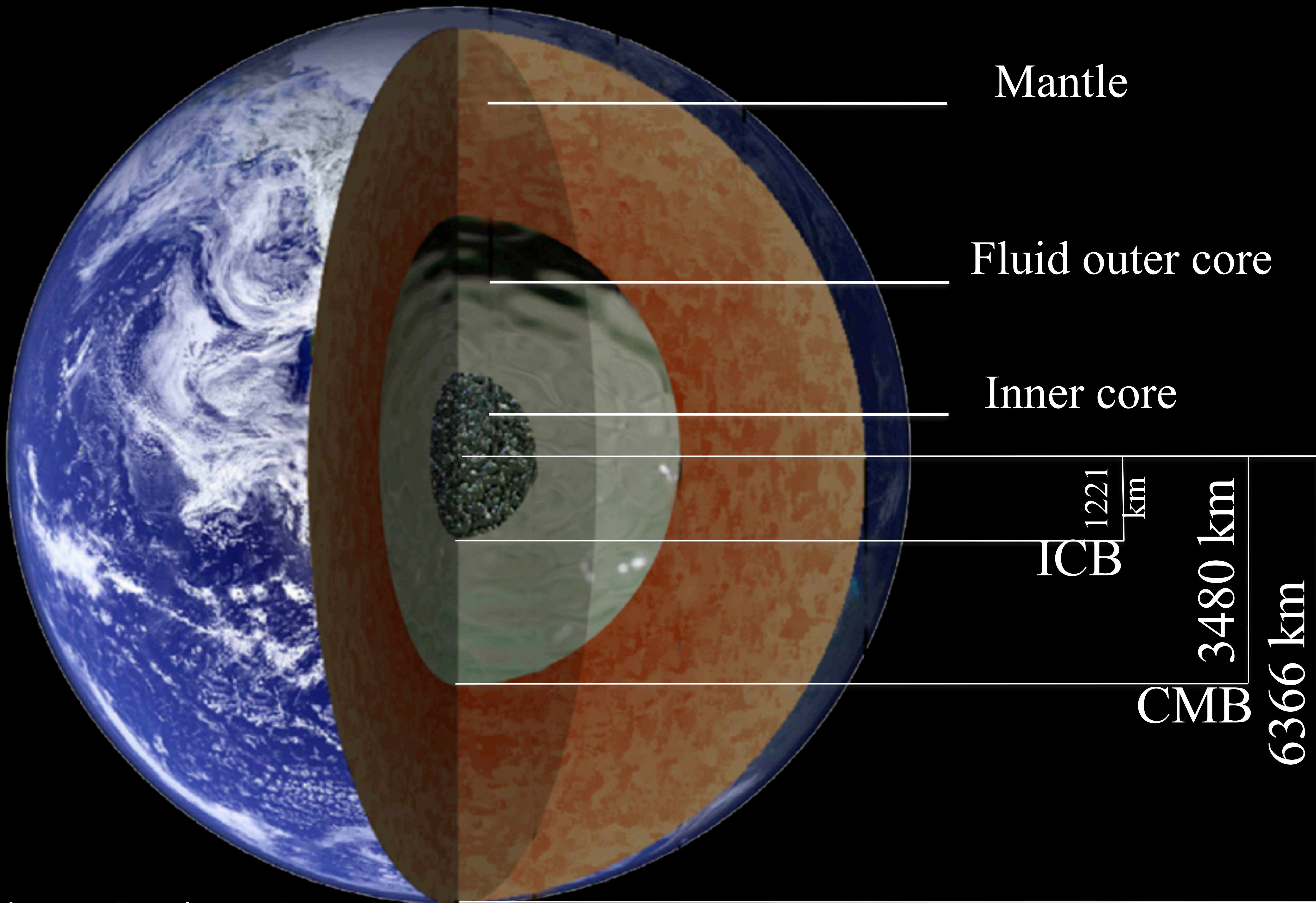


The Earth's internal structure



Roberts & King 2013

Governing Equations

$$\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u}\right) = -\boldsymbol{\nabla}\pi + \nu\,\Delta\mathbf{u} - 2\Omega(\mathbf{e}_z \wedge \mathbf{u})$$

$$-\,\alpha\,T\mathbf{g}$$

$$\boldsymbol{\nabla} \cdot \mathbf{u} = 0\,,$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} T = \kappa\,\Delta T\,,$$

Governing Equations

$$\frac{\partial \mathbf{B}}{\partial t} = -\boldsymbol{\nabla}\times\mathbf{E}\,, \qquad \boldsymbol{\nabla}\cdot\mathbf{B} = 0\,, \qquad \boldsymbol{\nabla}\wedge\mathbf{B} = \mu_0\,\mathbf{j}\,,$$

Governing Equations

$$\frac{\partial \mathbf{B}}{\partial t} = -\boldsymbol{\nabla} \times \mathbf{E}\,, \qquad \boldsymbol{\nabla} \cdot \mathbf{B} = 0\,, \qquad \boldsymbol{\nabla} \wedge \mathbf{B} = \mu_0 \, \mathbf{j}\,,$$

$$\mathbf{j} = \sigma \mathbf{E}' = \sigma \left(\mathbf{E} + \mathbf{u} \wedge \mathbf{B} \right)\,,$$

Governing Equations

$$\frac{\partial \mathbf{B}}{\partial t} = -\boldsymbol{\nabla} \times \mathbf{E}, \qquad \boldsymbol{\nabla} \cdot \mathbf{B} = 0, \qquad \boldsymbol{\nabla} \wedge \mathbf{B} = \mu_0 \, \mathbf{j} \,,$$

$$\mathbf{j} = \sigma \mathbf{E}' = \sigma \left(\mathbf{E} + \mathbf{u} \wedge \mathbf{B} \right) \,,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \wedge (\mathbf{u} \wedge \mathbf{B}) - \boldsymbol{\nabla} \wedge (\eta \boldsymbol{\nabla} \wedge \mathbf{B}) \,.$$

Governing Equations

$$\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u}\right) = -\boldsymbol{\nabla}\pi + \nu\,\Delta\mathbf{u} - 2\Omega(\mathbf{e}_z \wedge \mathbf{u})$$

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$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \wedge (\mathbf{u} \wedge \mathbf{B}) + \eta\,\Delta\mathbf{B}\,,$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0\,.$$

Governing Equations

$$
\begin{cases}
\left(\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u}\right) = -\boldsymbol{\nabla}\pi + \nu\,\Delta\mathbf{u} - 2\Omega(\mathbf{e}_z \wedge \mathbf{u}) \\
\qquad\qquad\qquad - \alpha\, T\mathbf{g} + \dfrac{1}{\mu_0\rho_0}\,(\boldsymbol{\nabla}\wedge\mathbf{B})\wedge\mathbf{B}\,, \\
\boldsymbol{\nabla}\cdot\mathbf{u} = 0\,, \\
\dfrac{\partial T}{\partial t} + \mathbf{u}\cdot\boldsymbol{\nabla}T = \kappa\,\Delta T\,, \\
\dfrac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla}\wedge(\mathbf{u}\wedge\mathbf{B}) + \eta\,\Delta\mathbf{B}\,, \\
\boldsymbol{\nabla}\cdot\mathbf{B} = 0\,.
\end{cases}
$$

$$
\begin{cases}
E_\eta \left(\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} \right) = -\boldsymbol{\nabla}\pi + E\,\Delta\mathbf{u} - \mathbf{e}_z \wedge \mathbf{u} \\
\qquad\qquad - Ra\,T\mathbf{g} + (\boldsymbol{\nabla} \wedge \mathbf{B}) \wedge \mathbf{B}\,, \\
\boldsymbol{\nabla} \cdot \mathbf{u} = 0\,, \\
\dfrac{\partial T}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} T = q\,\Delta T\,, \\
\dfrac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \wedge (\mathbf{u} \wedge \mathbf{B}) + \Delta\mathbf{B}\,, \\
\boldsymbol{\nabla} \cdot \mathbf{B} = 0\,.
\end{cases}
$$

$$
E/Pm = E_\eta = \frac{\eta}{2\Omega\mathcal{L}^2}\,, \quad E = \frac{\nu}{2\Omega\mathcal{L}^2}\,, \quad q = \frac{\kappa}{\eta}\,.
$$

$$\begin{cases} E_\eta \left(\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} \right) = -\boldsymbol{\nabla}\pi + E\,\Delta\mathbf{u} - \mathbf{e}_z \wedge \mathbf{u} \\ \qquad\qquad - Ra\,T\mathbf{g} + (\boldsymbol{\nabla} \wedge \mathbf{B}) \wedge \mathbf{B}\,, \\ \boldsymbol{\nabla} \cdot \mathbf{u} = 0\,, \\ \dfrac{\partial T}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} T = q\,\Delta T\,, \\ \dfrac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \wedge (\mathbf{u} \wedge \mathbf{B}) + \Delta\mathbf{B}\,, \\ \boldsymbol{\nabla} \cdot \mathbf{B} = 0\,. \end{cases}$$

$$E/Pm = E_\eta = \frac{\eta}{2\Omega\mathcal{L}^2}\,, \quad E = \frac{\nu}{2\Omega\mathcal{L}^2}\,, \quad q = \frac{\kappa}{\eta}\,.$$

$$\simeq 10^{-7} \qquad\qquad \simeq 10^{-15} \qquad\qquad \simeq 10^{-7}$$

$$
\begin{cases}
E_\eta \left(\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} \right) = -\boldsymbol{\nabla}\pi + E\,\Delta\mathbf{u} - \mathbf{e}_z \wedge \mathbf{u} \\
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\end{cases}
$$

$\geq 10^{-6}$ $\qquad$ ≥ 1

$$
E/Pm = E_\eta = \frac{\eta}{2\Omega\mathcal{L}^2}\,, \quad E = \frac{\nu}{2\Omega\mathcal{L}^2}\,, \quad q = \frac{\kappa}{\eta}\,.
$$

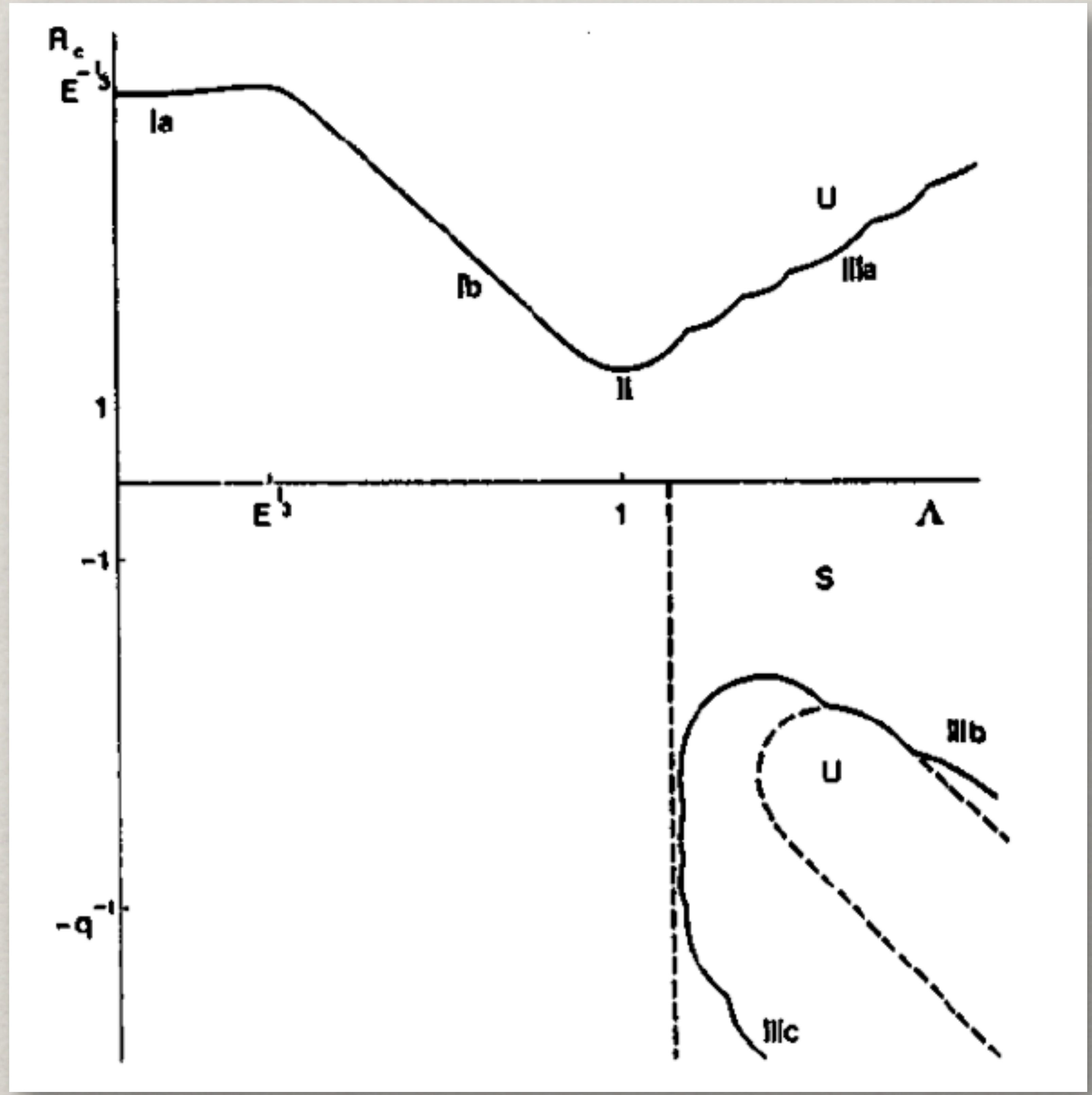
$\simeq 10^{-7}$ $\qquad$ $\simeq 10^{-15}$ $\qquad$ $\simeq 10^{-7}$

Weak and Strong field branches

Chandrasekhar, 1961
Eltayeb & Roberts, 1970
Eltayeb & Kumar, 1977
Roberts & Soward, 1978
Soward, 1979
Fearn, 1979
Fautrelle & Childress, 1982
Proctor & Weiss, 1982

Weak and Strong field branches

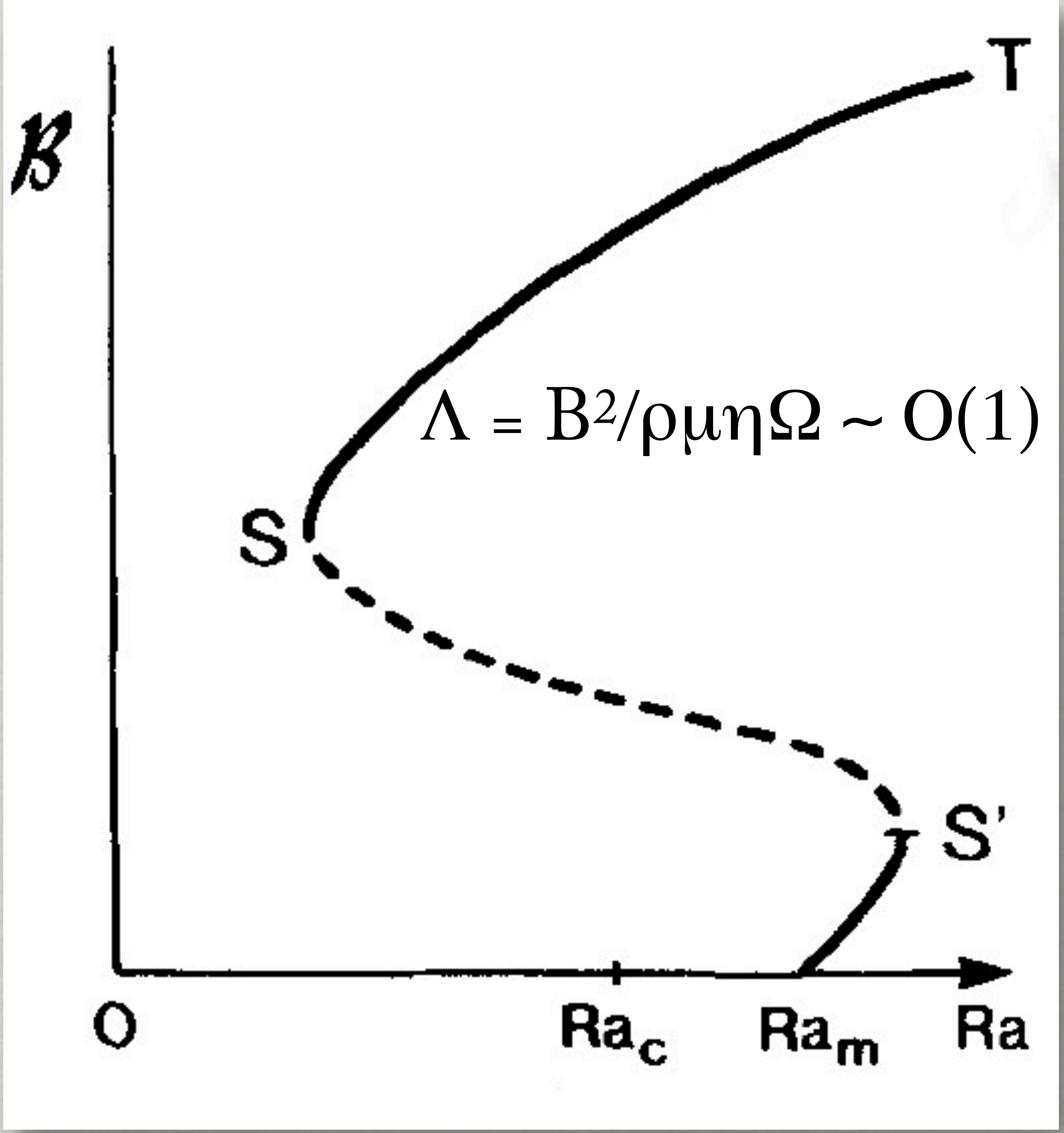
Ra_c



$\Lambda = B^2/\rho\mu\eta\Omega$

Fearn (1979)

Weak and Strong field branches



$\Lambda = B^2/\rho\mu\eta\Omega \sim O(1)$

(Roberts, GAFD, 1988)

$$
\begin{cases}
E_\eta \left(\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} \right) = -\boldsymbol{\nabla}\pi + E\,\Delta\mathbf{u} - \mathbf{e}_z \wedge \mathbf{u} \\
\qquad\qquad\qquad - Ra\,T\mathbf{g} + (\boldsymbol{\nabla} \wedge \mathbf{B}) \wedge \mathbf{B}\,, \\
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\dfrac{\partial T}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} T = q\,\Delta T\,, \\
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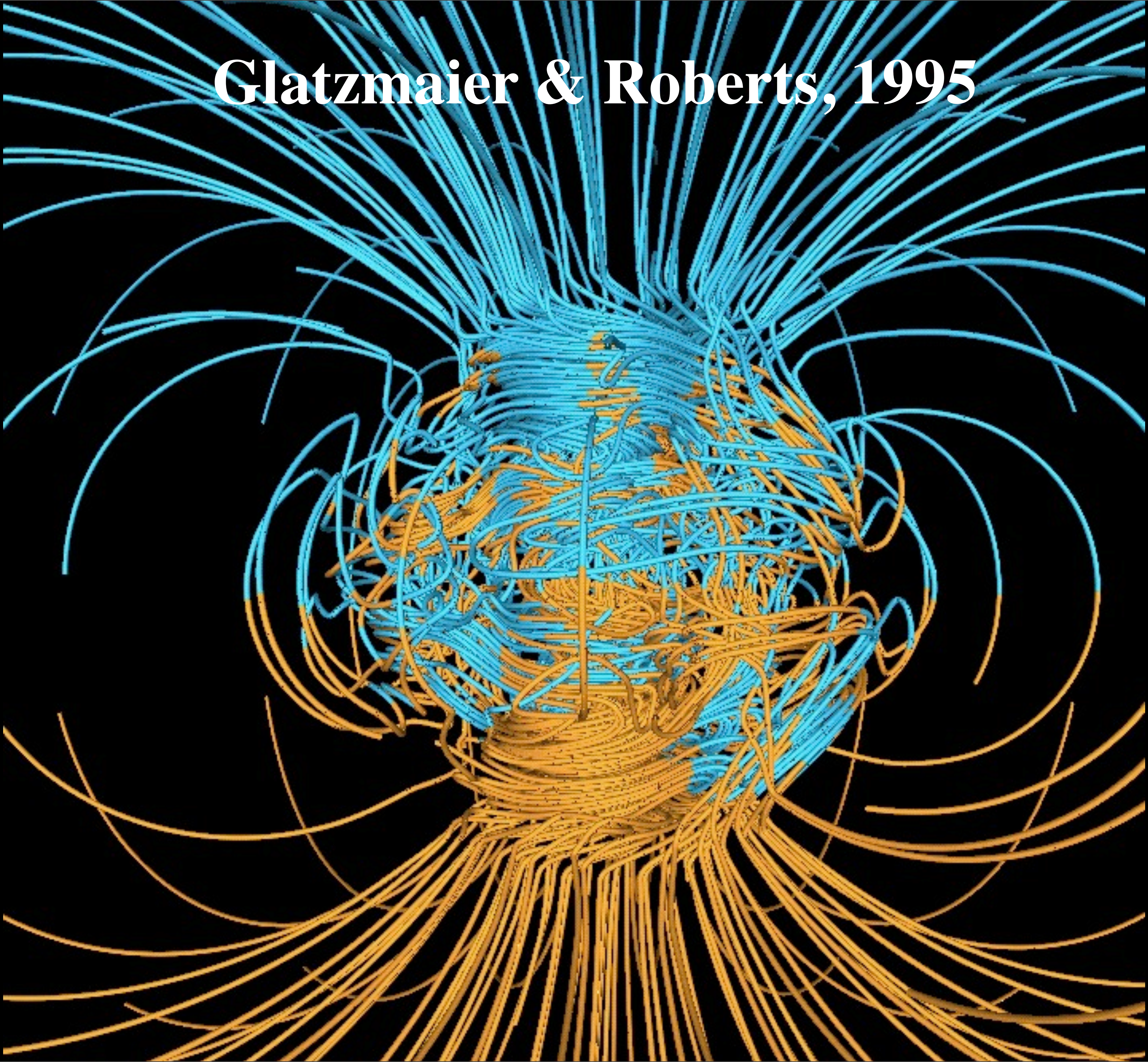
\boldsymbol{\nabla} \cdot \mathbf{B} = 0\,.
\end{cases}
$$

$\geq 10^{-6}$ ≥ 1

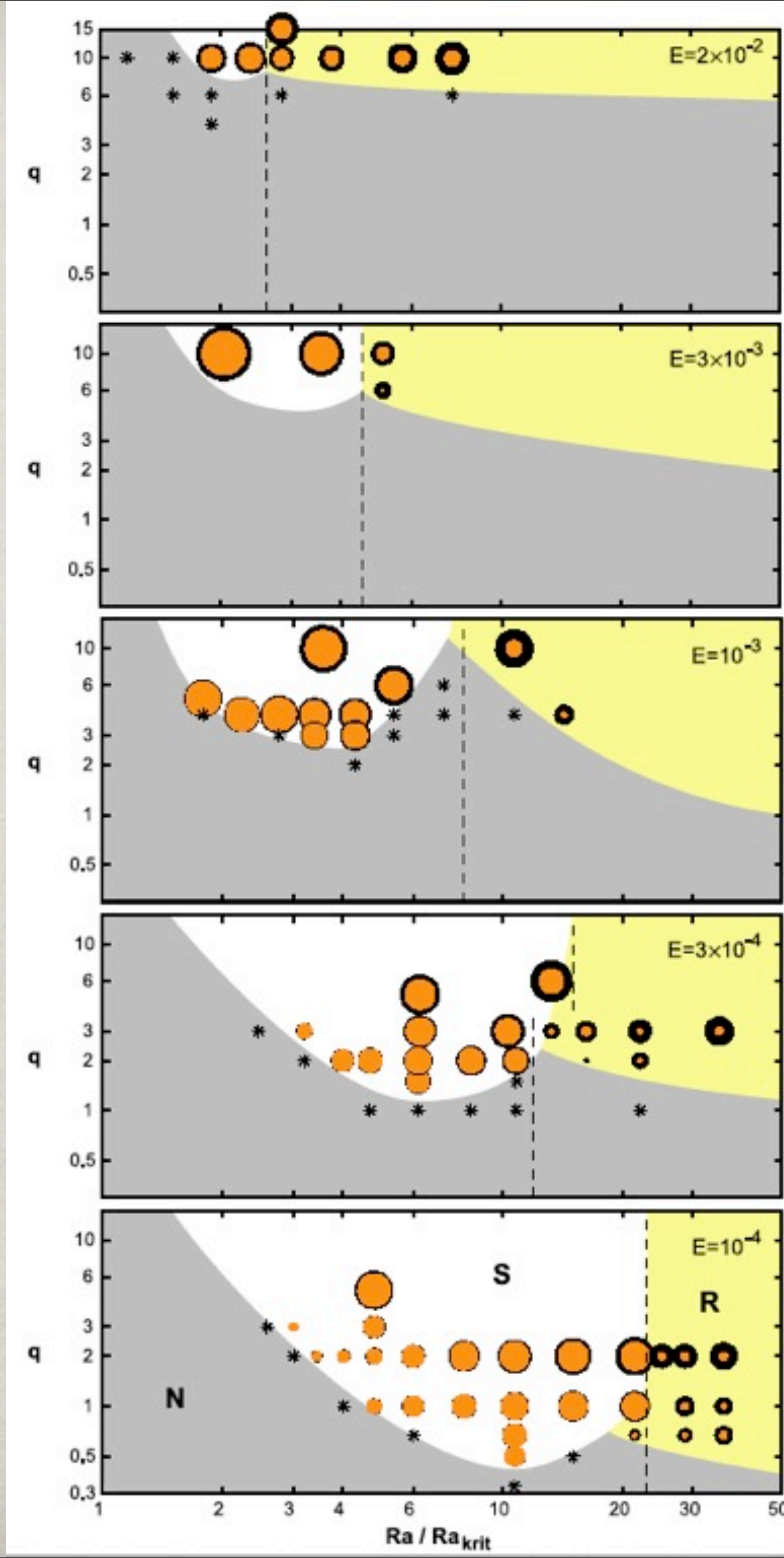
$$
E/Pm = E_\eta = \frac{\eta}{2\Omega\mathcal{L}^2}\,, \quad E = \frac{\nu}{2\Omega\mathcal{L}^2}\,, \quad q = \frac{\kappa}{\eta}\,.
$$

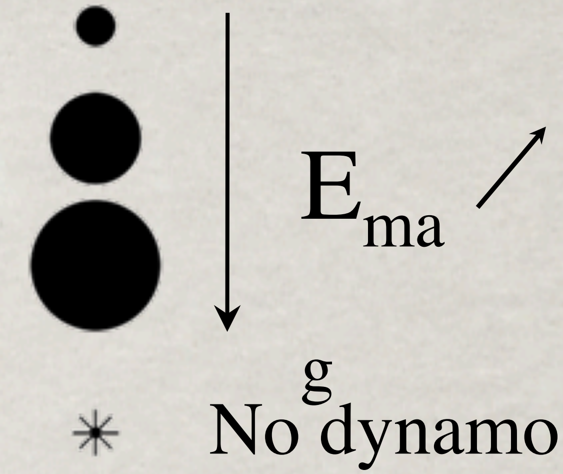
$\simeq 10^{-7}$ $\simeq 10^{-15}$ $\simeq 10^{-7}$

Glatzmaier & Roberts, 1995



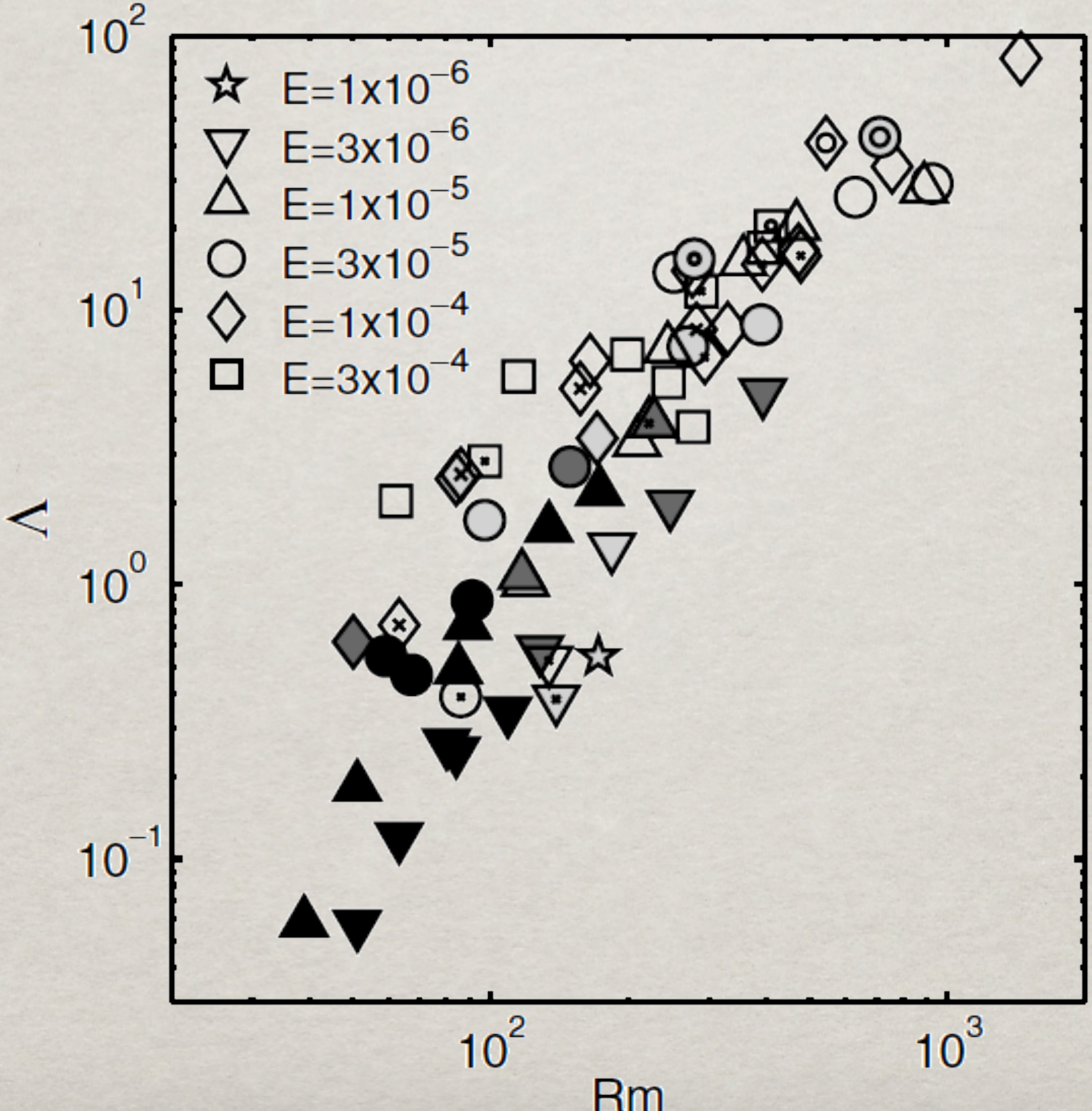
Roberts number
q = Pm / Pr





Carsten Kutzner,
Uli Christensen
2003

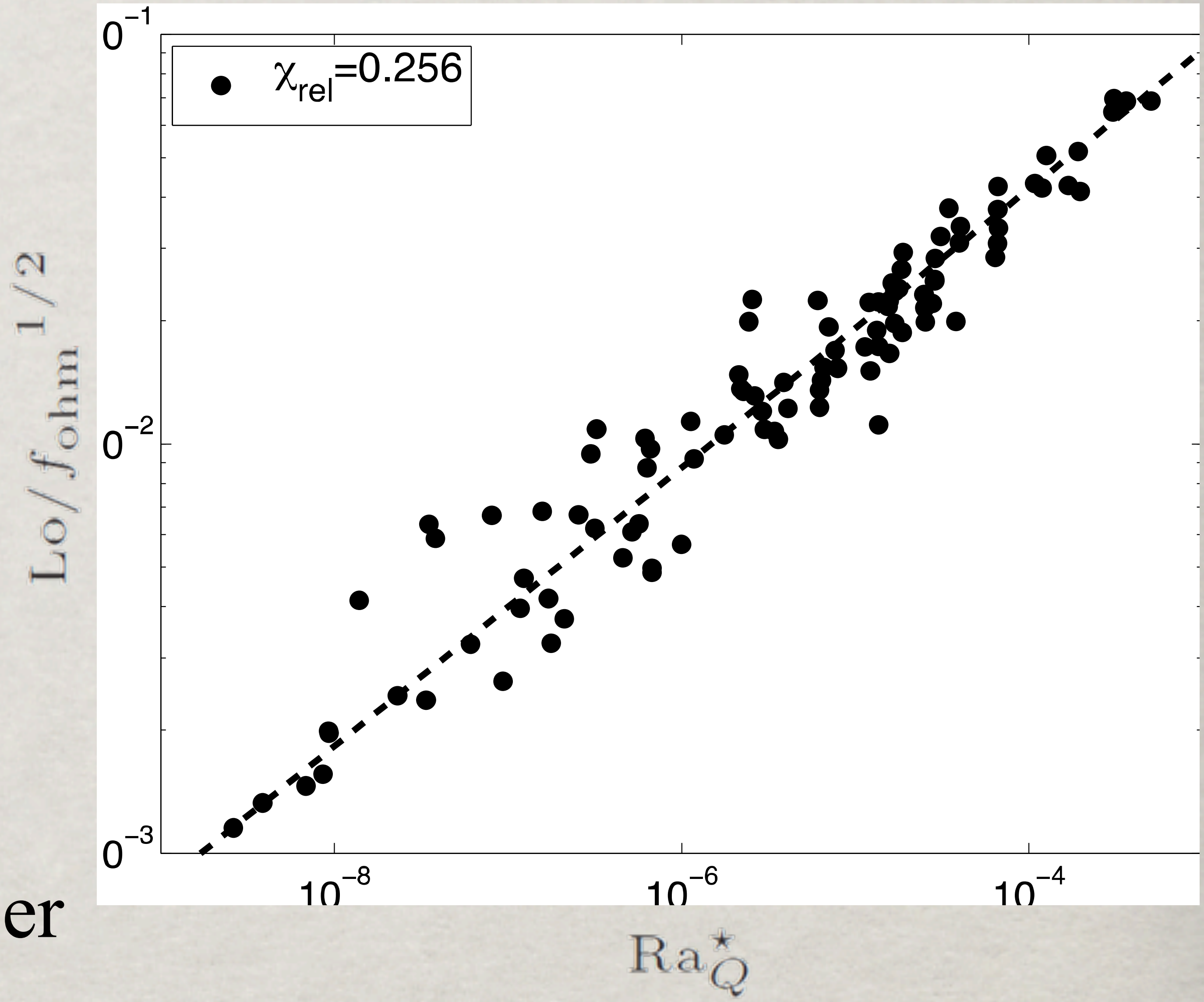
Christensen-Aubert 2006



Power based scaling laws for the magnetic field

$$\mathrm{Lo} \sim {f_{\mathrm{ohm}}}^{1/2}\, {\mathrm{Ra}_Q^\star}^{0.34}$$

(Christensen & Aubert, 2006)



$$\mathrm{Lo} = \frac{B}{\sqrt{\rho\,\mu}\,\Omega L}$$: Lorentz number

$$f_{\mathrm{ohm}} \equiv \frac{D_\eta}{D_\eta + D_\nu}$$: fraction of ohmic dissipation

$\mathrm{Ra}_Q^\star \sim \mathrm{P}^\star$: flux-based Rayleigh number (directly related to the injected power)

Parameter range

$\mathrm{E} \in [5\times10^{-7}, 5\times10^{-4}]$

$\mathrm{Pm} \in [0.04, 33.3]$

$\mathrm{Pr} = 1$

Statistical energy balance between production and dissipation

$$\mathrm{P}^\star = D_\nu^\star + D_\eta^\star \qquad\qquad f_{\mathrm{ohm}}\,\mathrm{P}^\star = D_\eta^\star$$

where $D_\eta^\star = \mathrm{E}_\eta \int_V (\nabla \times \mathbf{B}^\star)^2 \,\mathrm{d}V^\star$

$$f_{\mathrm{ohm}}\,\mathrm{P}^\star = \mathrm{E}_\eta \frac{\mathrm{Lo}^2}{{\ell_B^\star}^2}$$

$\ell_B^\star$: magnetic dissipation length scale ${\ell_B^\star}^2 \equiv \dfrac{\int_V \mathbf{B}^{\star 2}\,\mathrm{d}V^\star}{\int_V (\nabla \times \mathbf{B}^\star)^2\,\mathrm{d}V^\star}$

Oruba & Dormy, GJI, **198**, 828-847 (2014).

Statistical energy balance between production and dissipation

$$f_{\mathrm{ohm}}\, \mathrm{P}^\star = \mathrm{E}_\eta \frac{\mathrm{Lo}^2}{{\ell_B^\star}^2}$$

$$\longrightarrow \quad \mathrm{Lo} = {f_{\mathrm{ohm}}}^{1/2}\, \mathrm{P}^{\star\, 1/2}\, \mathrm{E}_\eta^{-1/2}\, \ell_B^\star$$

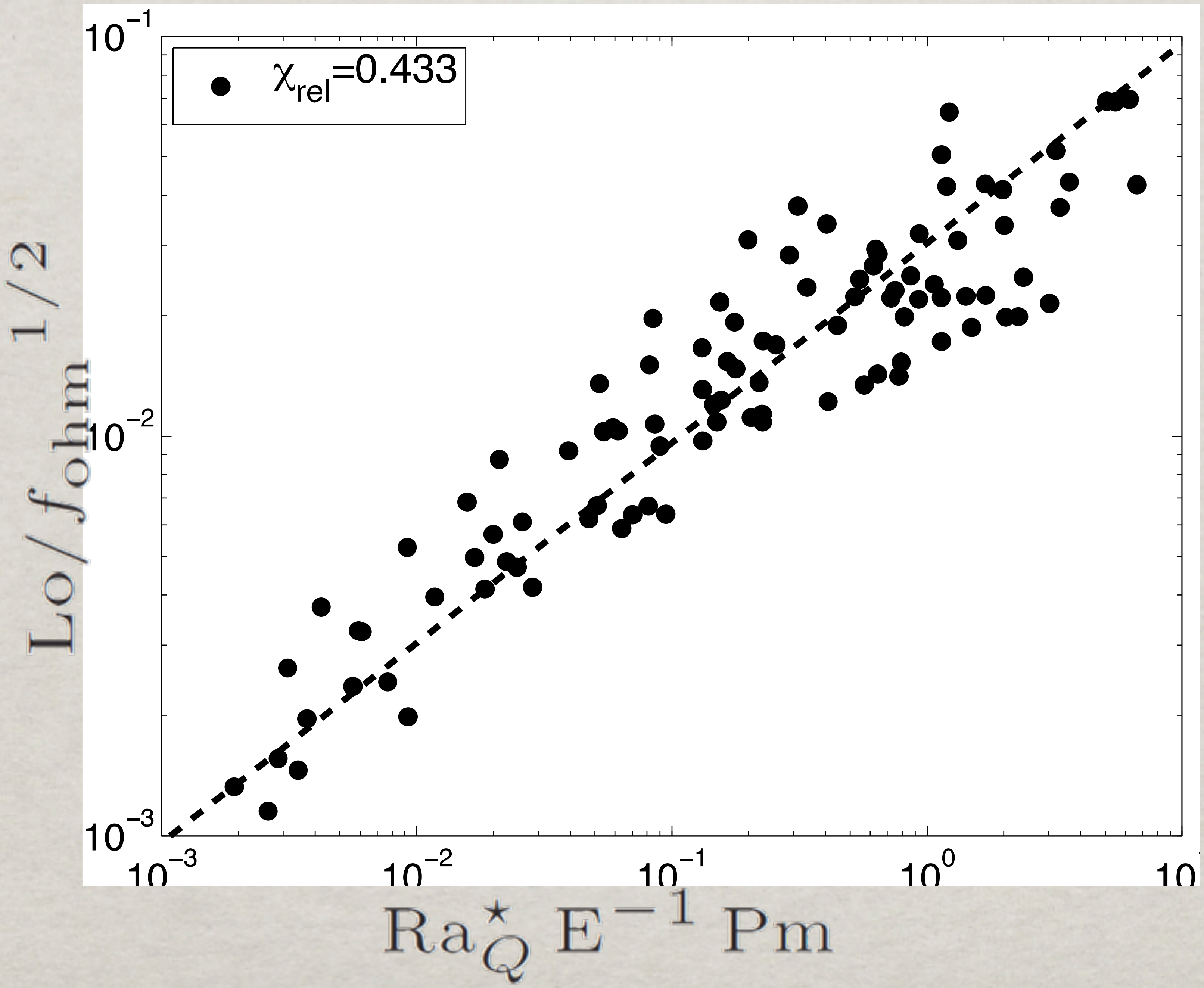
Verified by any statistically steady dynamo

Oruba & Dormy, GJI, **198**, 828-847 (2014).

Statistical energy balance between production and dissipation

- The simplest approximation: $\ell_B^\star$ constant

$$\mathrm{Lo} \sim {f_{\mathrm{ohm}}}^{1/2}\ {\mathrm{Ra}_Q^\star}^{1/2}\ \mathrm{E}_\eta^{-1/2}$$



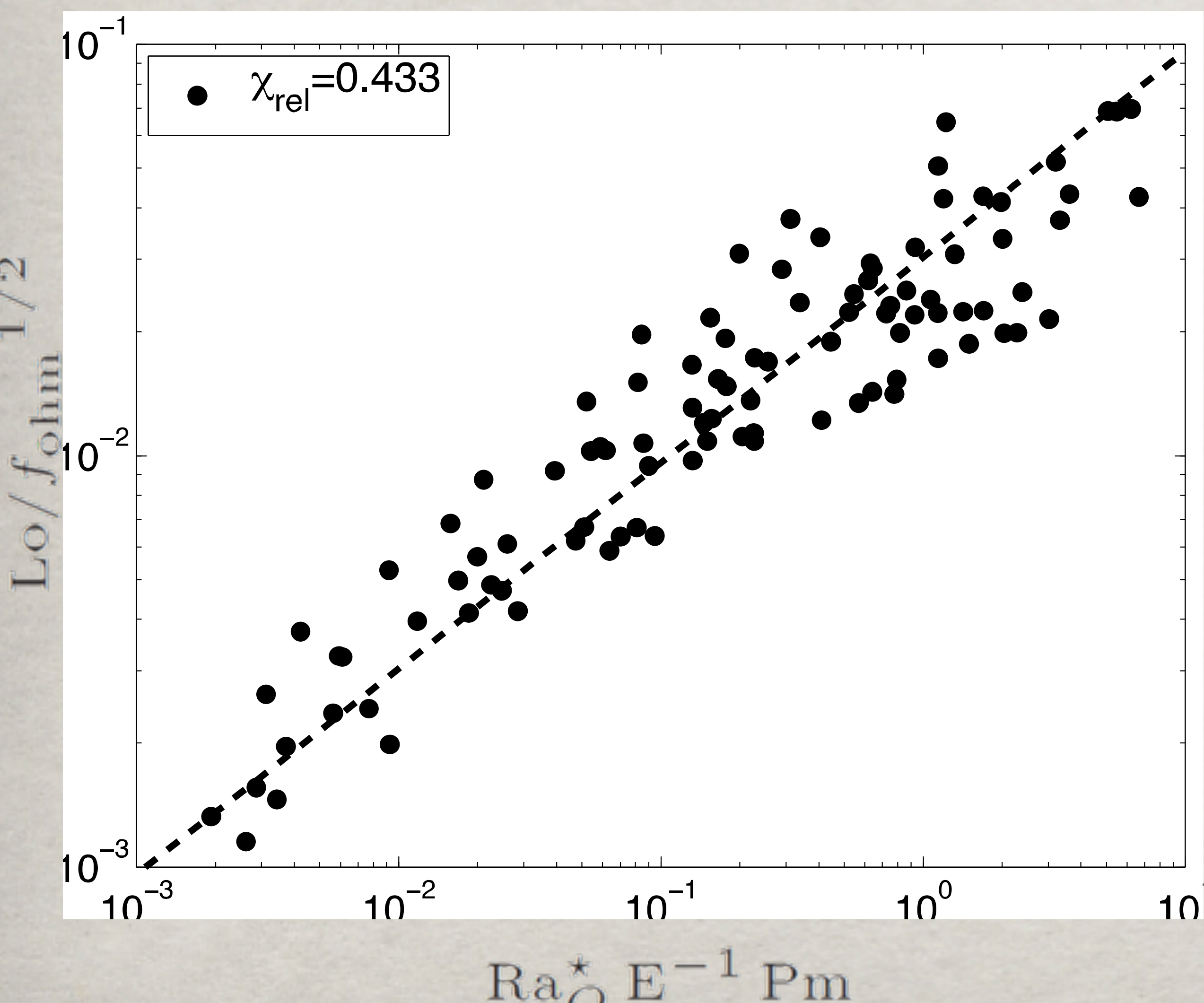
Oruba & Dormy, GJI, **198**, 828-847 (2014).

Power based scaling laws for the magnetic field

- Energy balance

  $+ \ell_B^\star$ constant

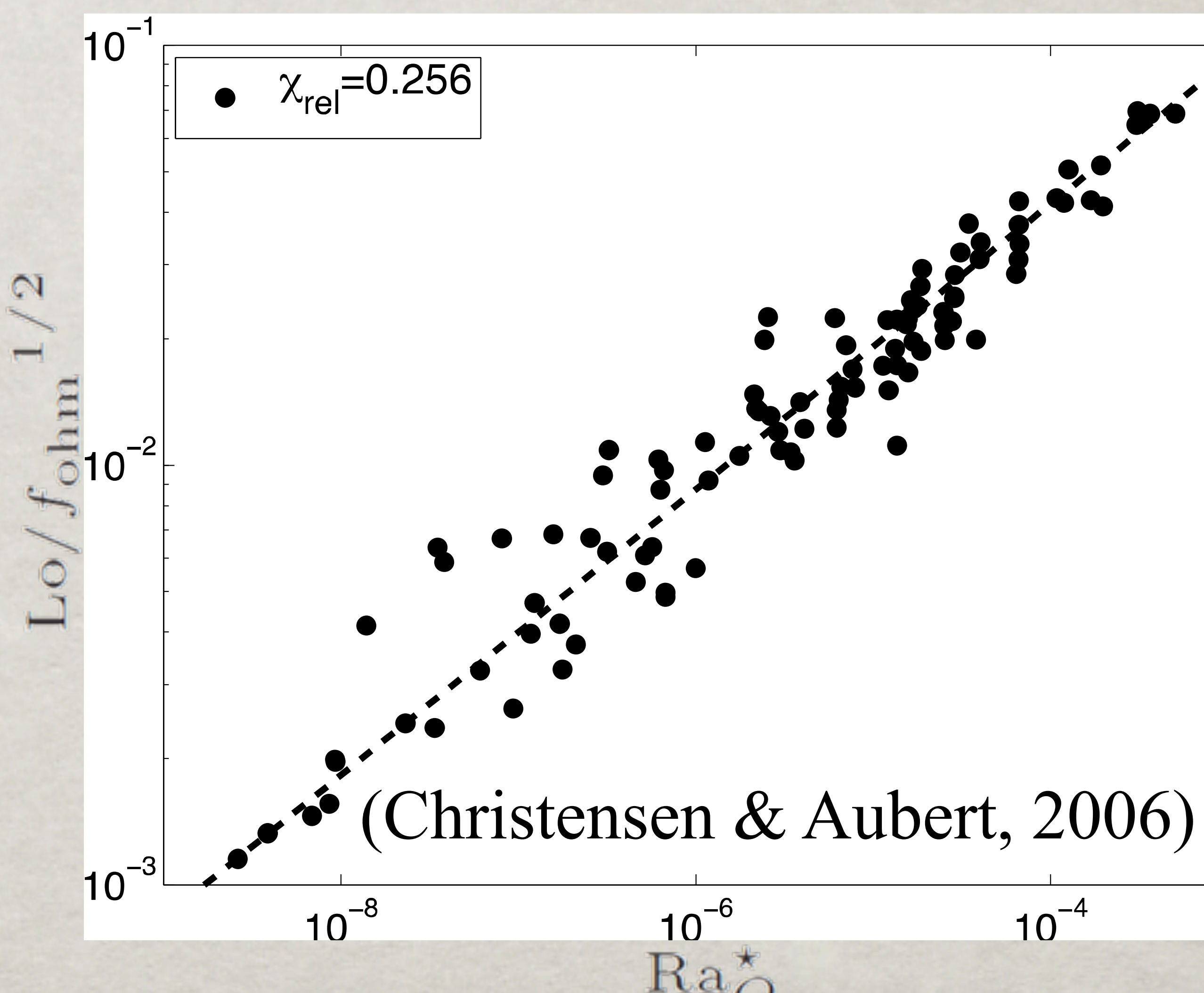
$$\mathrm{Lo} \sim {f_{\mathrm{ohm}}}^{1/2}\ {\mathrm{Ra}_Q^\star}^{1/2}\ \mathrm{E}_\eta^{-1/2}$$



- Energy balance

  + more precise description of $\ell_B^\star$:

$$\mathrm{Lo} \sim {f_{\mathrm{ohm}}}^{1/2}\,{\mathrm{Ra}_Q^\star}^{0.34}$$



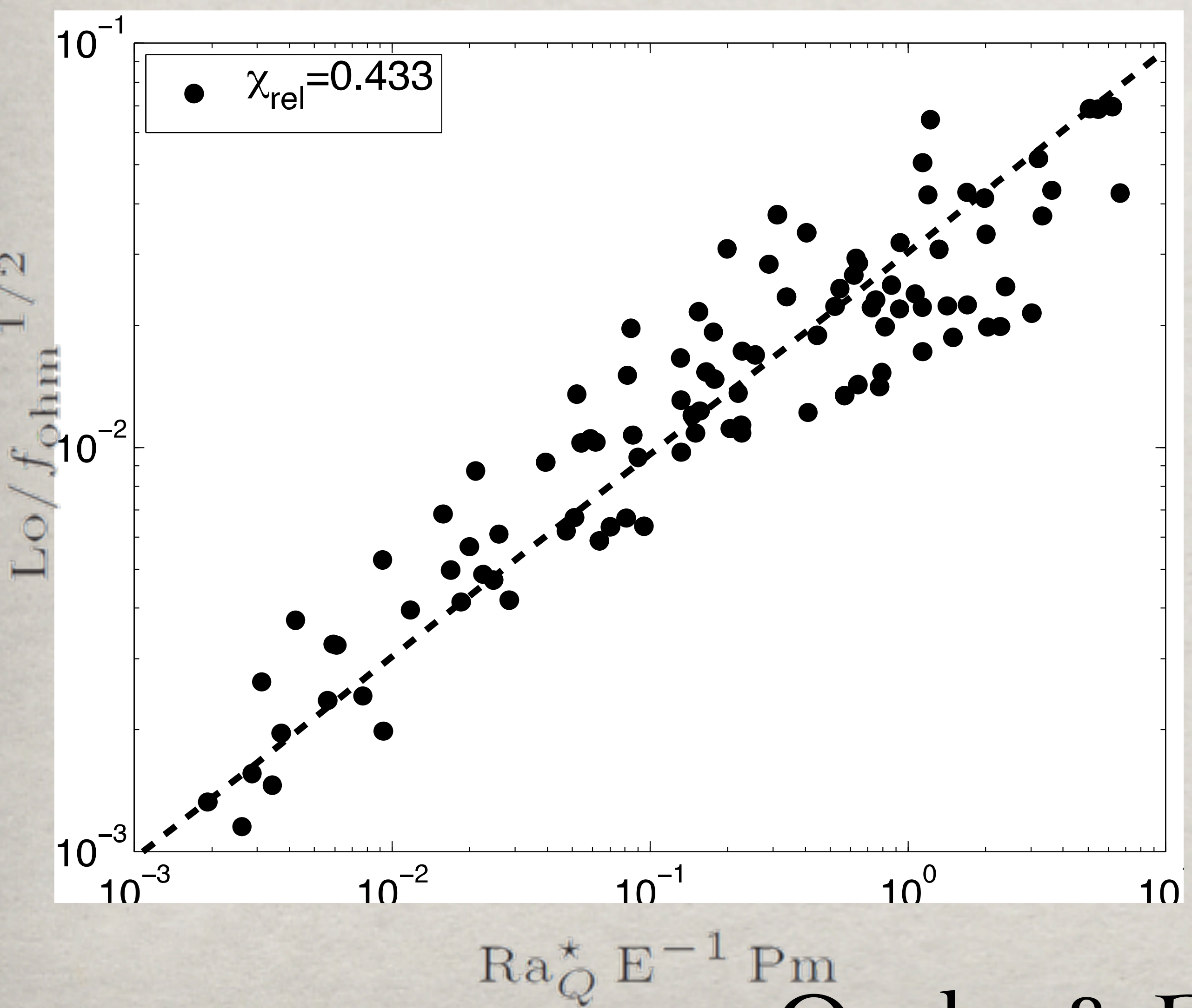
Oruba & Dormy, GJI, **198**, 828-847 (2014).

Power based scaling laws for the magnetic field

- Energy balance

  $+\ \ell_B^\star$ constant

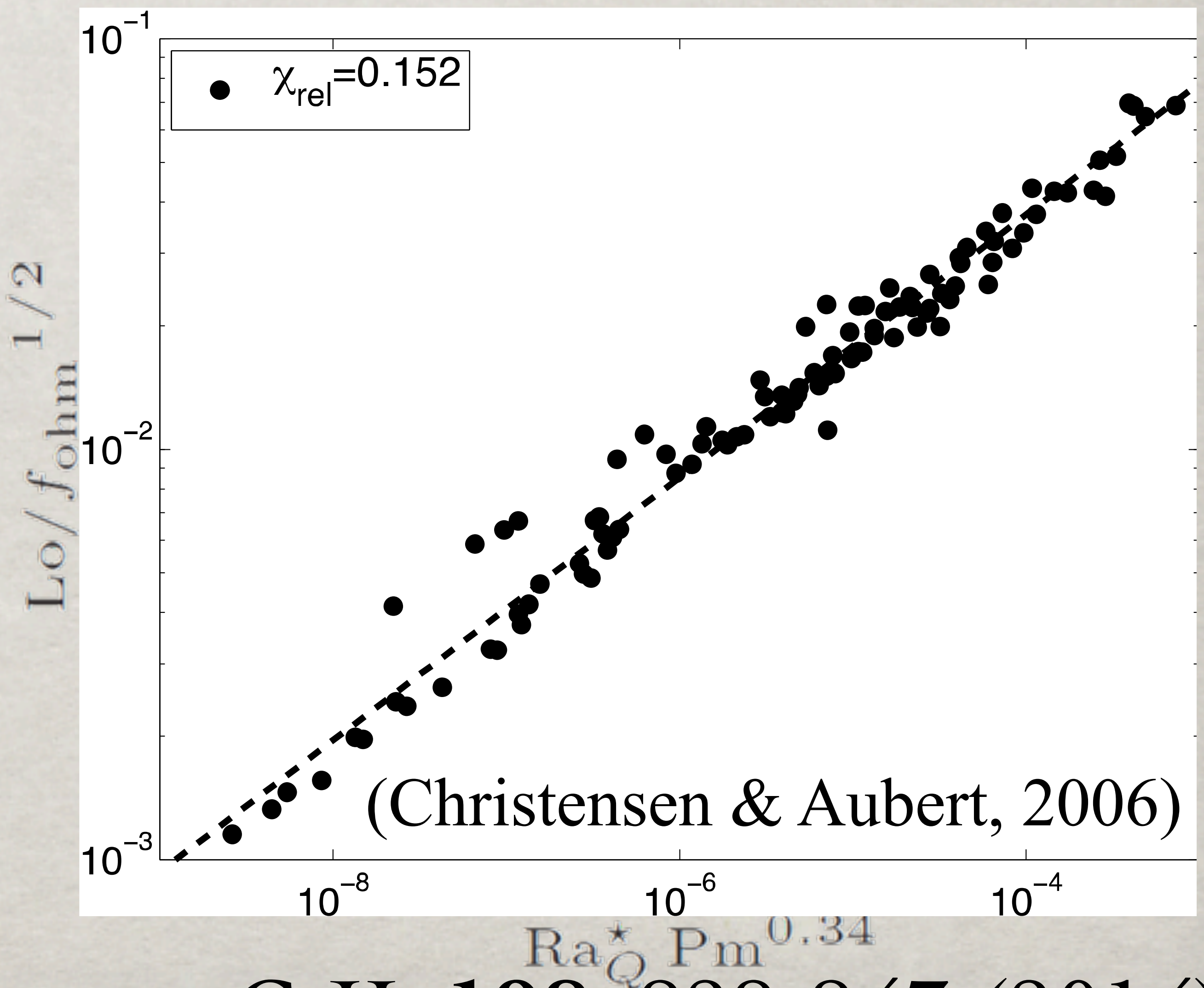
$$\mathrm{Lo} \sim f_{\mathrm{ohm}}{}^{1/2}\ \mathrm{Ra}_Q^{\star\,1/2}\ \mathrm{E}_\eta^{-1/2}$$



- Energy balance

  + more precise description of $\ell_B^\star$:

$$\mathrm{Lo} \sim f_{\mathrm{ohm}}{}^{1/2}\,\mathrm{Ra}_Q^{\star\,0.32}\ \mathrm{Pm}^{0.11}$$



Oruba & Dormy, GJI, **198**, 828-847 (2014).

Power based magnetic field scalings

=

TOO GENERAL:

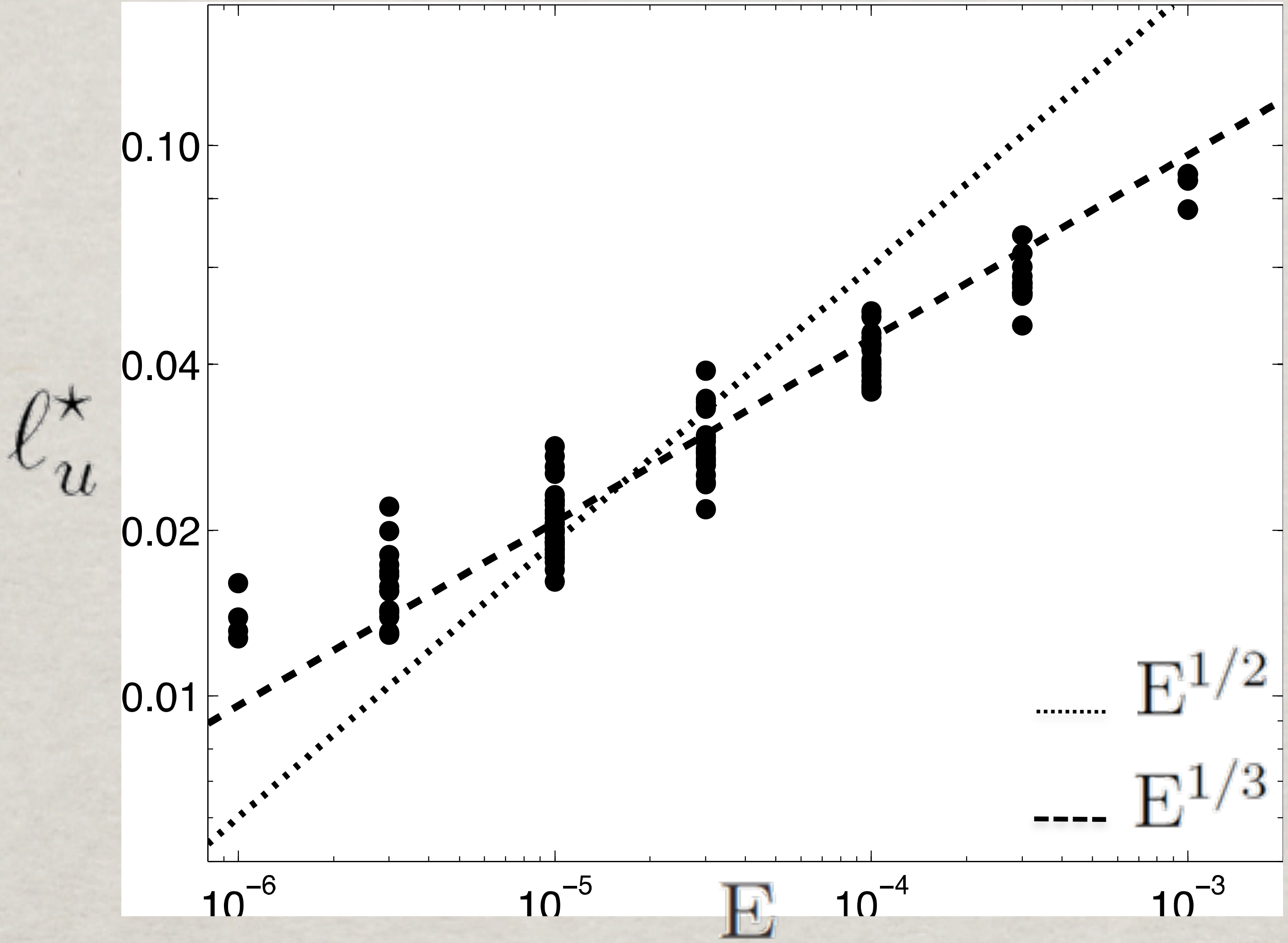
applicable to any model **irrespectively of** the magnetic field generation mechanism

Oruba & Dormy, GJI, **198**, 828-847 (2014).

Viscous length scale of the flow

$$\ell_u^\star \sim \mathrm{E}^{1/3}$$

(see King & Buffett, 2013)



In numerical models:

non negligible viscous effects in the bulk of the flow

Oruba & Dormy, GJI, **198**, 828-847 (2014).

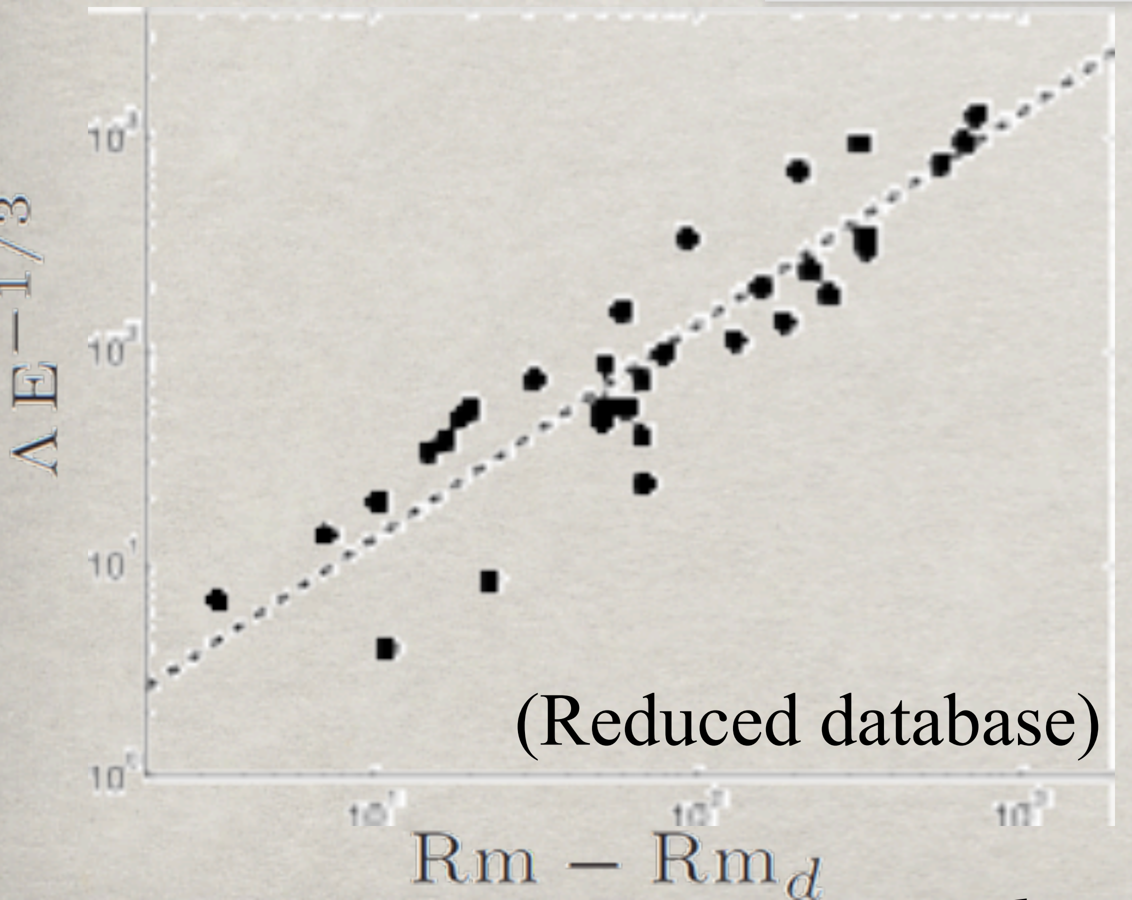
Magnetic field strength

Bifurcation from a laminar flow (Fauve & Pétrélis, 2007):

Lorentz force $\sim$ modified viscous force

with $\ell_u^\star \sim \mathrm{E}^{1/3}$

$$\boxed{\Lambda \sim (\mathrm{Rm} - \mathrm{Rm}_d)\,\mathrm{E}^{1/3}} \qquad \Lambda = \frac{\sigma B^2}{\rho\,\Omega} \text{ : Elsasser}$$



Oruba & Dormy, GJI, **198**, 828-847 (2014).

Magnetic field strength

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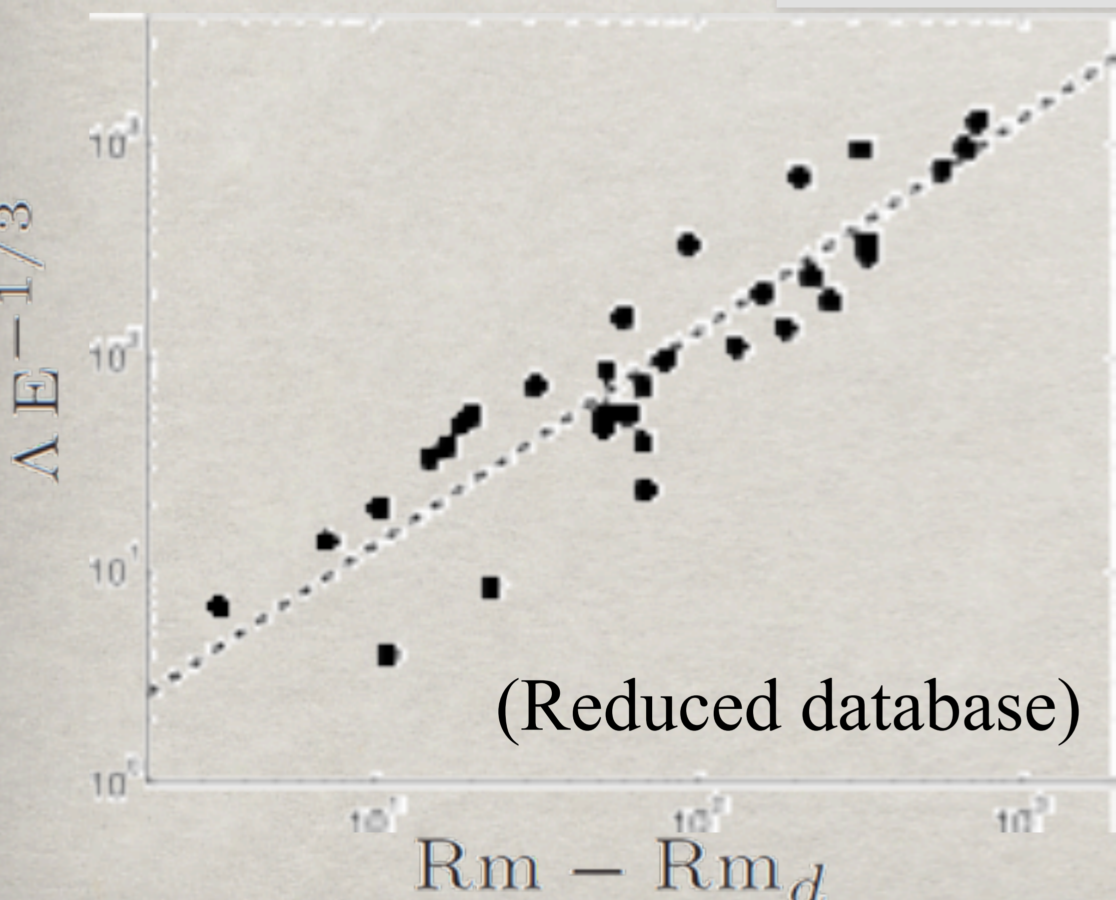


Application to the Earth's core?

$$\mathrm{Rm} - \mathrm{Rm}_d \leq 10^3$$

$$\longrightarrow \Lambda \leq 10^{-2}$$

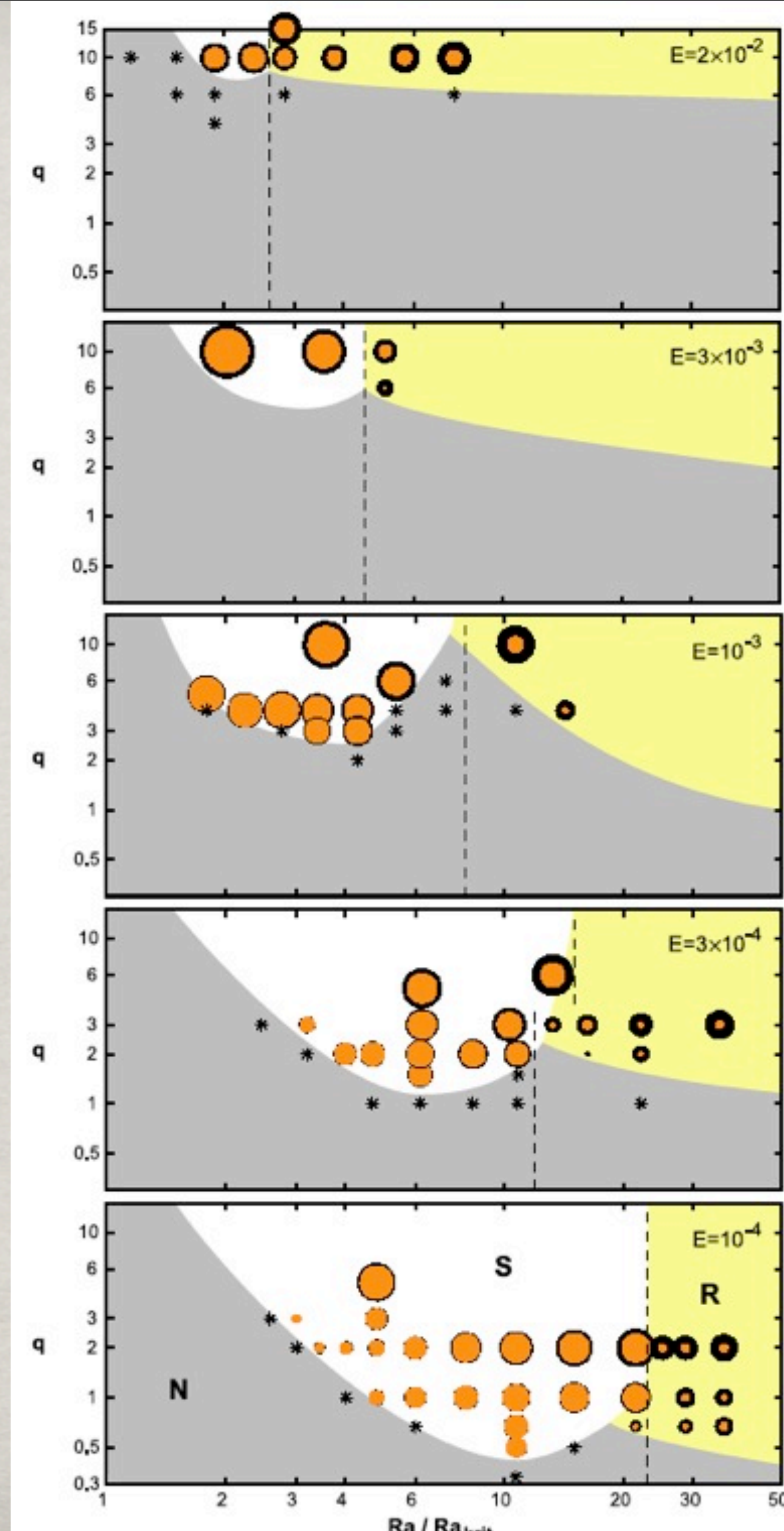
much smaller than $\Lambda_{\mathrm{Earth}} \simeq 1$

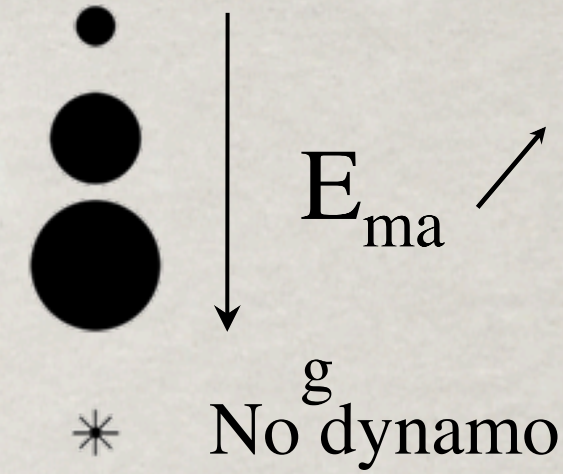
Oruba & Dormy, GJI, **198**, 828-847 (2014).

Intermediate result

Dipolar Dynamos at moderate forcing are dominated by a balance between viscous and Coriolis forces

Roberts number
$q = Pm / Pr$

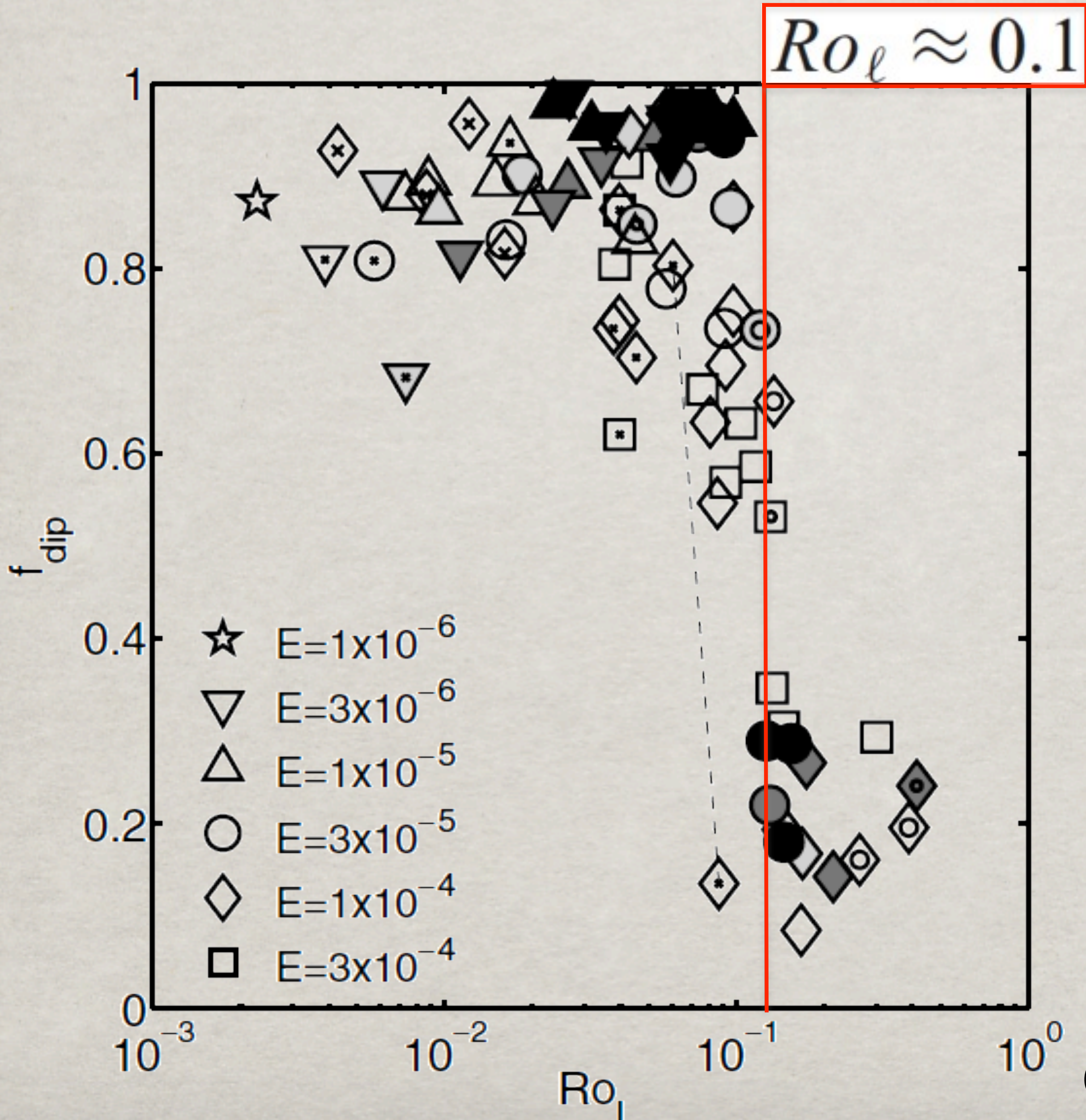




Carsten Kutzner,
Uli Christensen
2003

Transition from dipolar to multipolar regime

Relative dipole field strength vs the local Rossby number:



$$Ro_\ell = Ro \frac{\bar{\ell}_u}{\pi}$$

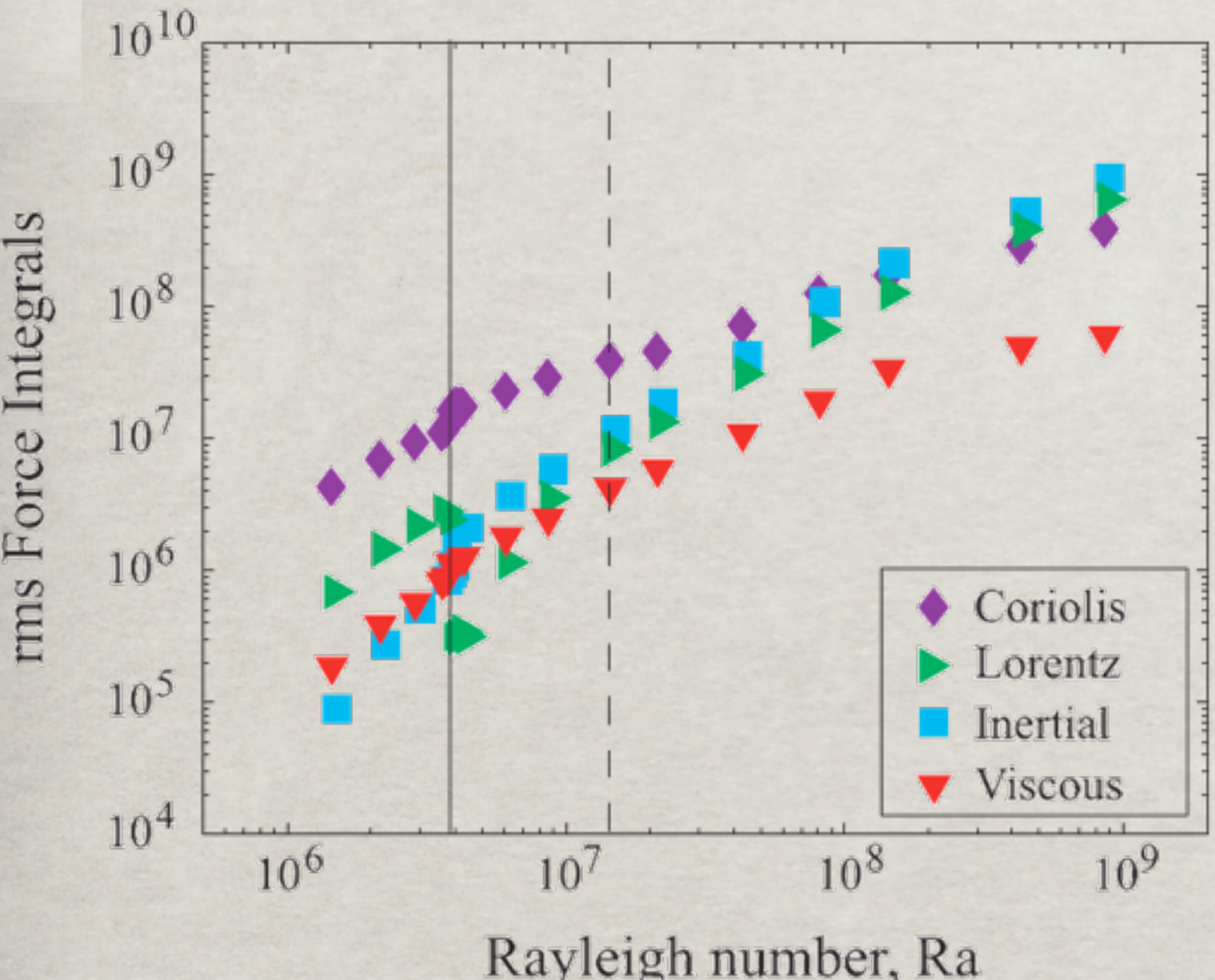
$$Ro = \left(\frac{2E_{\text{kin}}}{V_s} \right)^{1/2}$$

$$\bar{\ell}_u = \frac{\sum \ell \langle \mathbf{u}_\ell \cdot \mathbf{u}_\ell \rangle}{2E_{\text{kin}}}$$

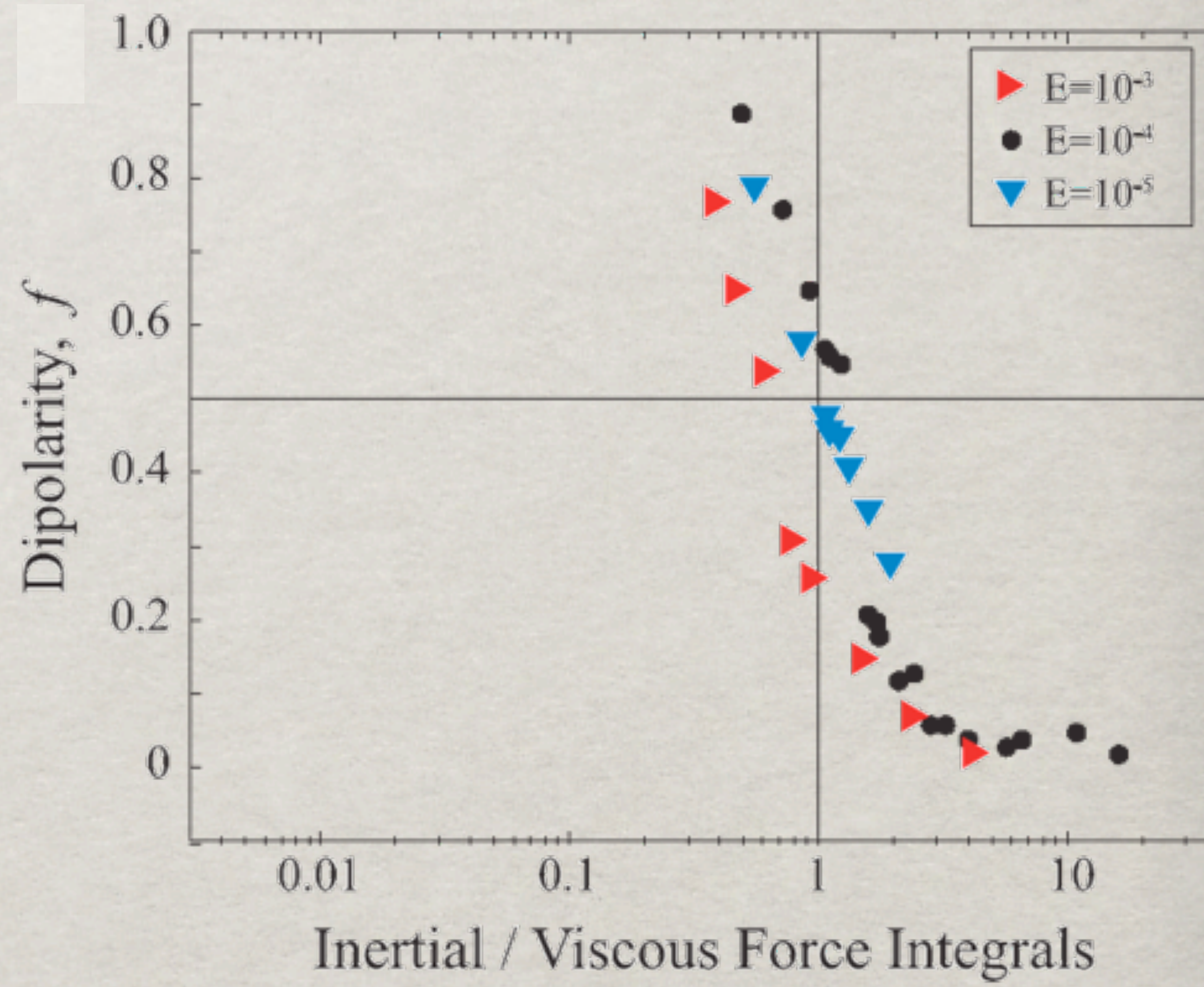
Christensen & Aubert 2006

Transition from dipolar to multipolar regime

Dominance of the Coriolis force in both regimes:



Transition controlled by the relative strength of inertial to viscous forces:



Soderlund *et al* 2012

Inertial forces versus Coriolis

$$\{\nabla \times (\mathbf{u} \times \nabla \times \mathbf{u})\} \sim \{\nabla \times (\mathbf{e}_z \times \mathbf{u})\}$$

$$\frac{\mathrm{u}^2}{\ell\,\ell_u} \sim \frac{\mathrm{u}}{\ell_{//}}$$

Typical length scales

the kinematic dissipation length scale ℓ_u : $\ell_u^2 \equiv \frac{\langle \mathbf{u}^2 \rangle}{\langle (\nabla \times \mathbf{u})^2 \rangle}$

the length scale ℓ (Oruba & Dormy 2014)

the parallel length scale : $\ell_{//} \sim 1$ (quasi-geostrophy)

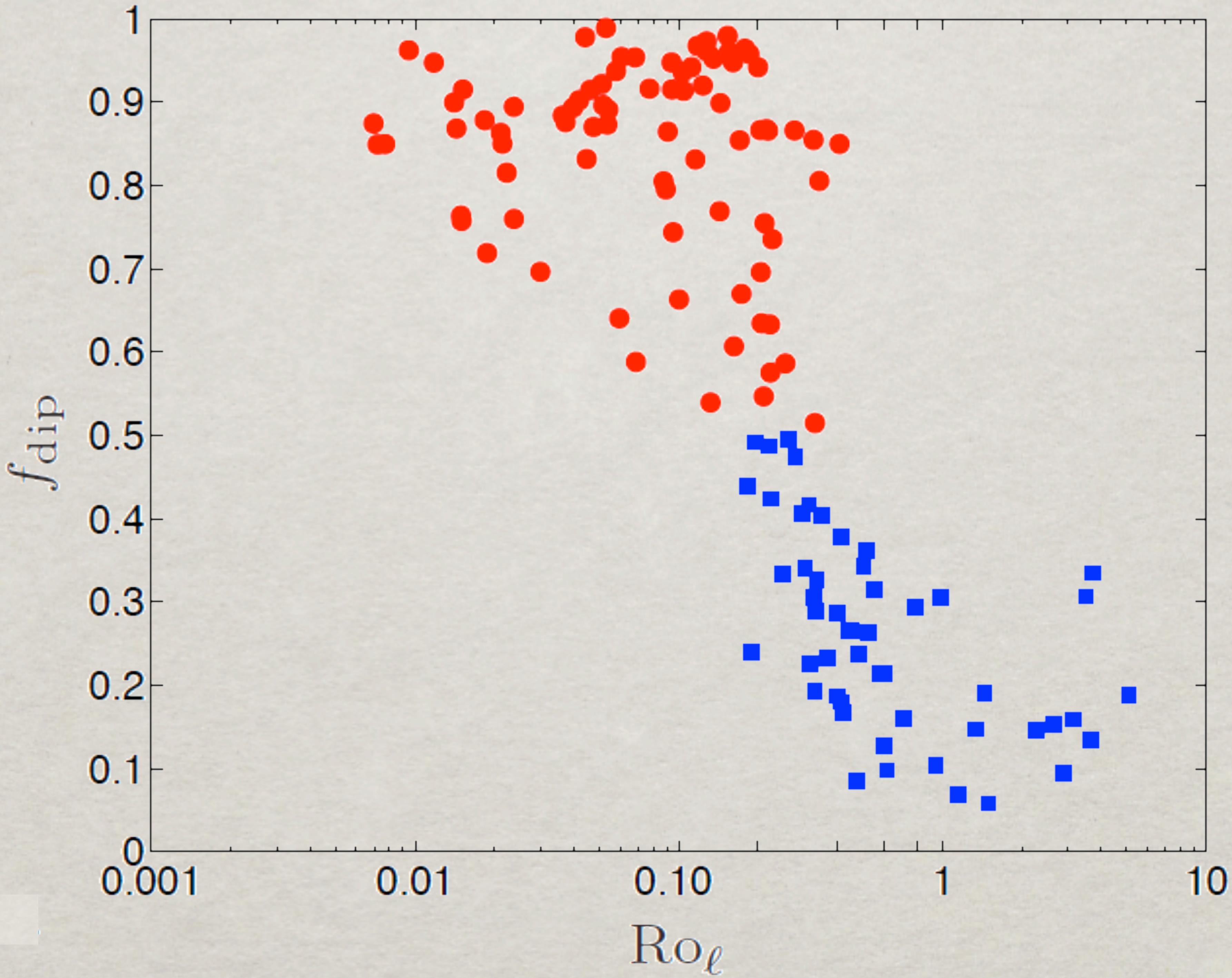
$$\mathrm{Ro}_\ell \sim \ell$$

with $\mathrm{Ro}_\ell = \mathrm{Ro}/\ell_u$

Oruba & Dormy, GRL in press.

Inertial forces versus Coriolis

Test with the numerical database (U. Christensen):



Oruba & Dormy, GRL in press.

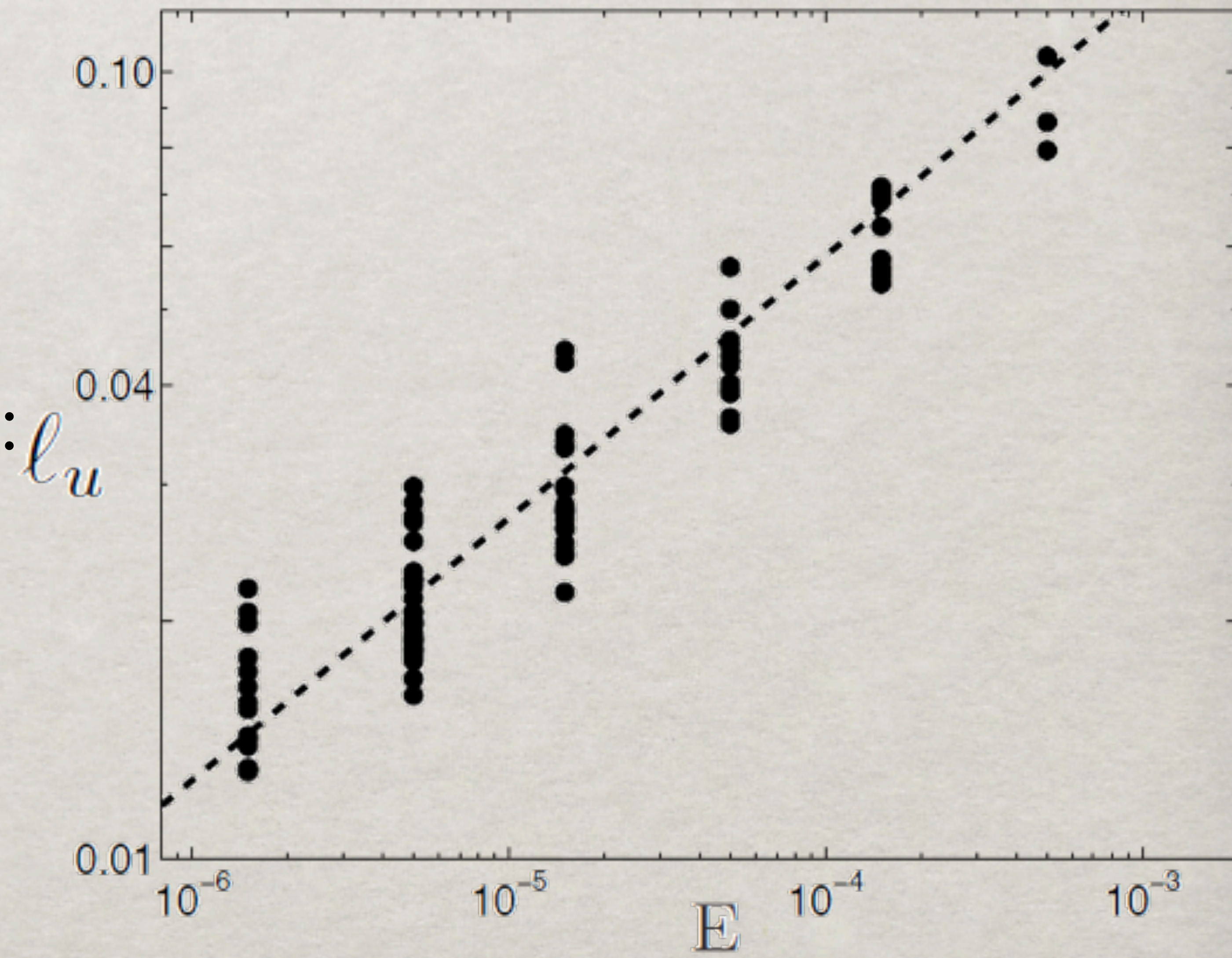
Viscous forces versus Coriolis

$$\{\mathrm{E}\, \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{u}\} \sim \{\boldsymbol{\nabla} \times (\mathbf{e}_z \times \mathbf{u})\}$$

$$\frac{\mathrm{E\,u}}{{\ell_u}^3} \sim \frac{\mathrm{u}}{\ell_{//}}$$

$$\ell_u \sim \mathrm{E}^{1/3}$$

Well verified by dipolar dynamos:



(King & Buffet 2013
and Oruba & Dormy 2014)

Oruba & Dormy, GRL in press.

Inertial versus viscous forces

$$\{\boldsymbol{\nabla} \times (\mathbf{u} \times \boldsymbol{\nabla} \times \mathbf{u})\} \sim \{\mathrm{E}\, \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{u}\}$$

$$\frac{\mathrm{u}^2}{\ell\, \ell_u} \sim \frac{\mathrm{E}\,\mathrm{u}}{{\ell_u}^3}$$

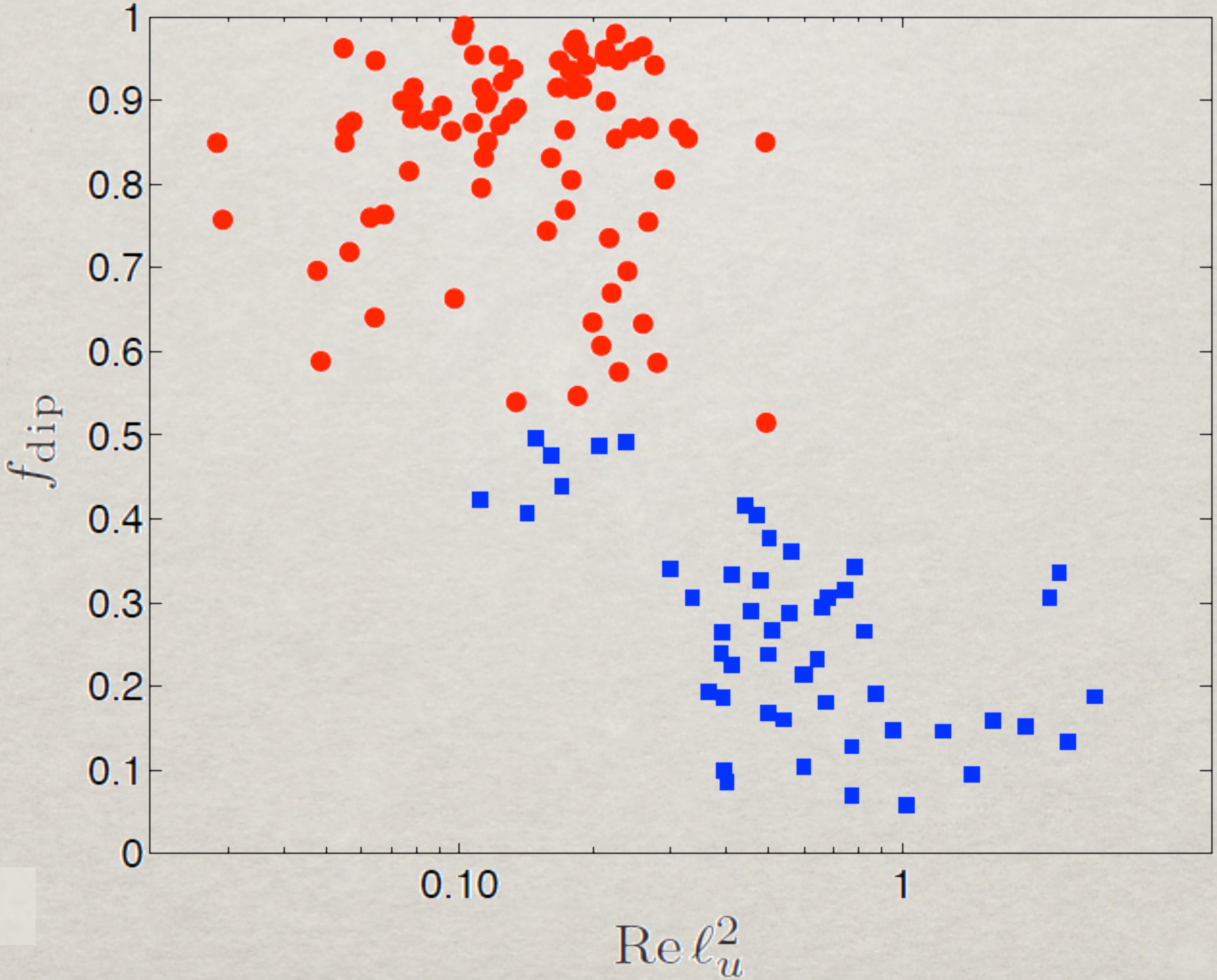
$$\frac{\mathrm{u}}{\mathrm{E}} \ell_u^2 \sim \ell$$

$$\mathrm{Re}\, \ell_u^2 \sim \ell$$

Oruba & Dormy, GRL in press.

Inertial versus viscous forces

Test against the numerical database:



Oruba & Dormy, GRL in press.

A dominant three forces balance at the transition

$$\ell_u \sim \mathrm{E}^{1/3}$$

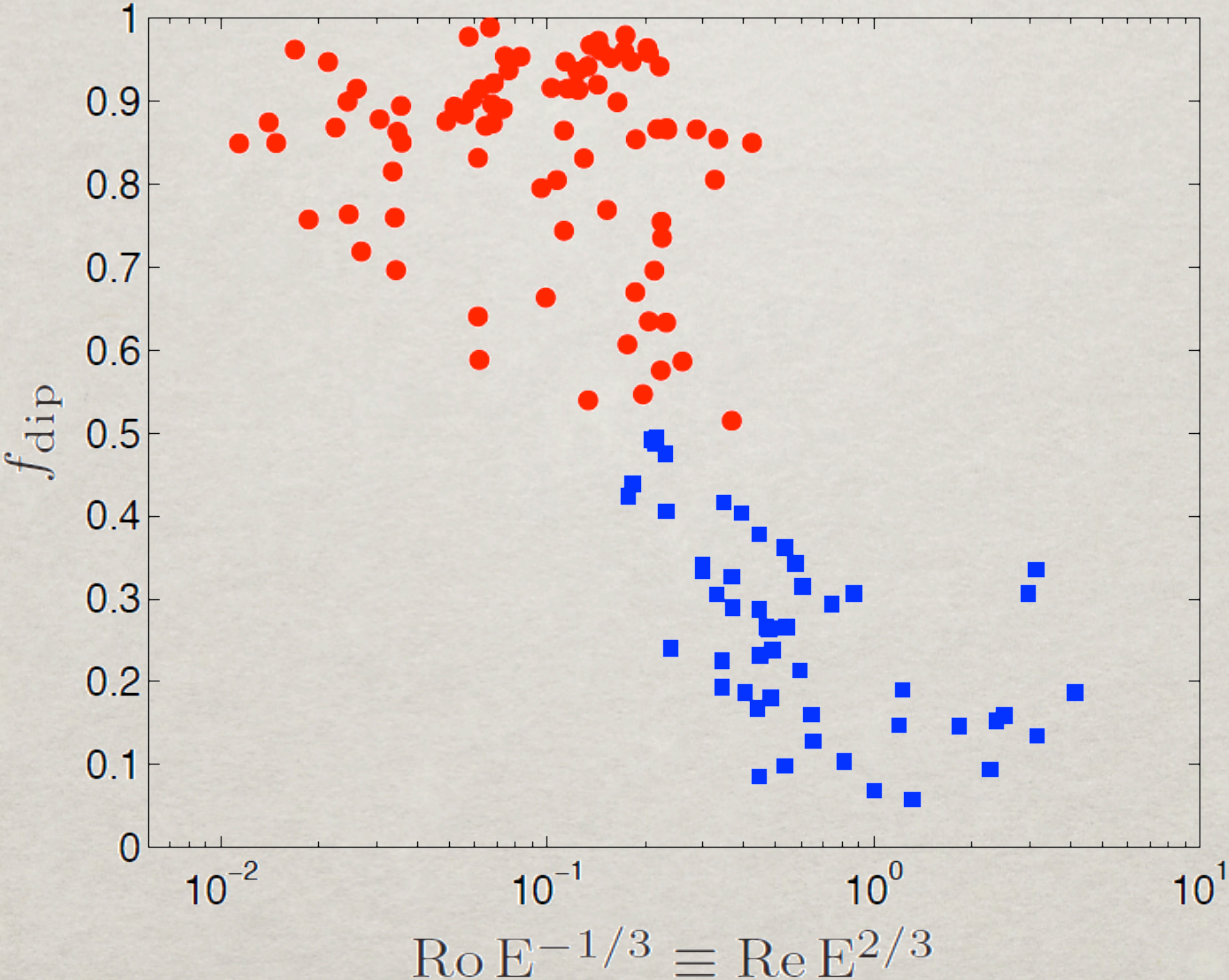
$$\{\boldsymbol{\nabla} \times \mathscr{C}\} \;\sim\; \{\boldsymbol{\nabla} \times \mathscr{I}\} \;\sim\; \{\boldsymbol{\nabla} \times \mathscr{V}\}$$

$$\mathrm{Ro}/\ell_u \sim \ell \qquad\qquad \mathrm{Re}\,\ell_u^2 \sim \ell$$

$$\mathrm{Ro}\,\mathrm{E}^{-1/3} \equiv \mathrm{Re}\,\mathrm{E}^{2/3} \sim \ell$$

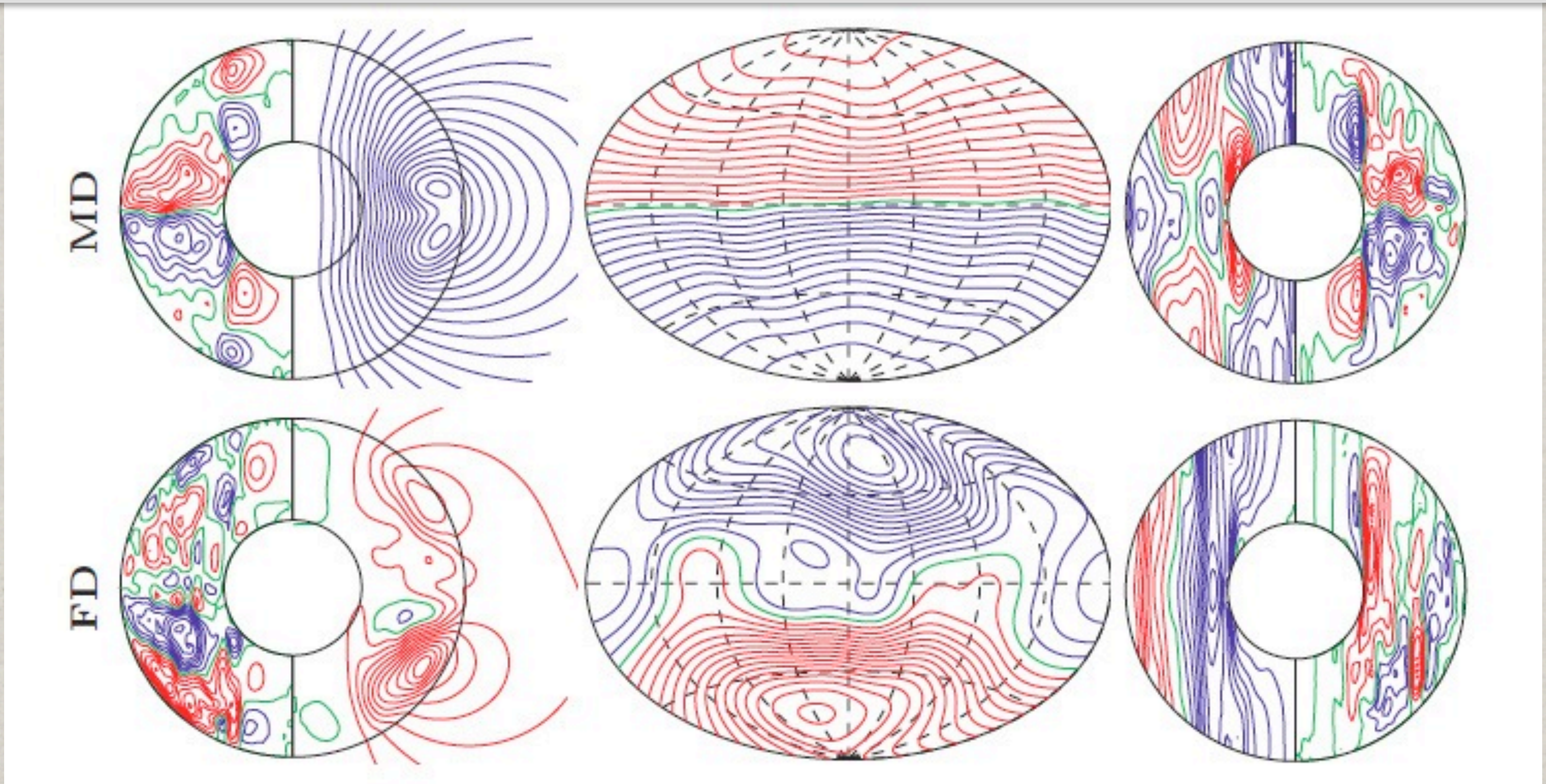
Oruba & Dormy, GRL in press.

Unified description of the transition



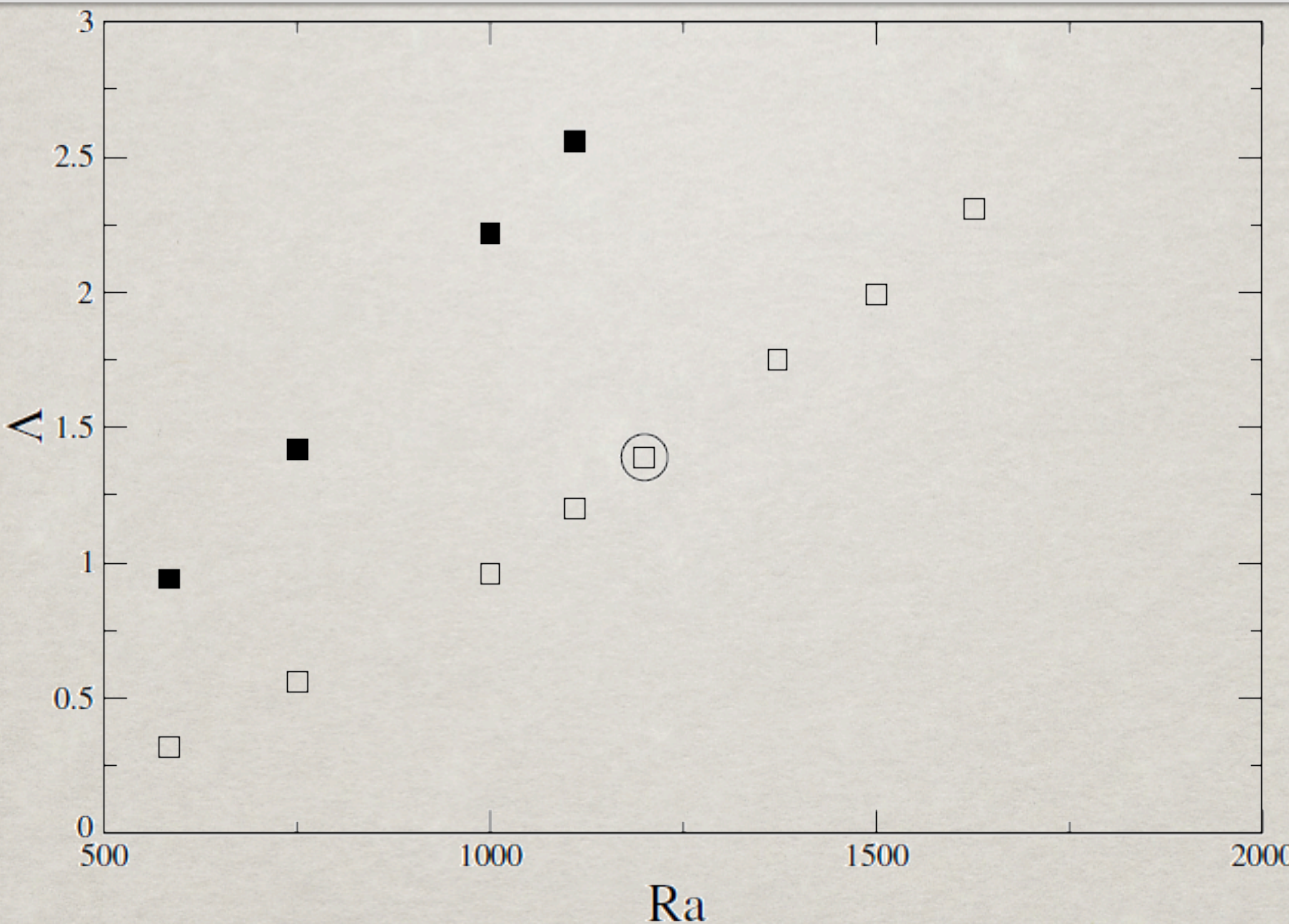
Oruba & Dormy, GRL in press.

Bistability between both branches



Simitev & Busse,
Bistability and hysteresis of dipolar dynamos generated by turbulent convection in rotating spherical shells,
Europhysics Letters (EPL), 85 (2009) 19001

Bistability between both branches



Schrinner, Petitdemange, Dormy, ApJ, 2012

Intermediate results

- Dipolar Dynamos at moderate forcing are dominated by a balance between viscous and Coriolis forces

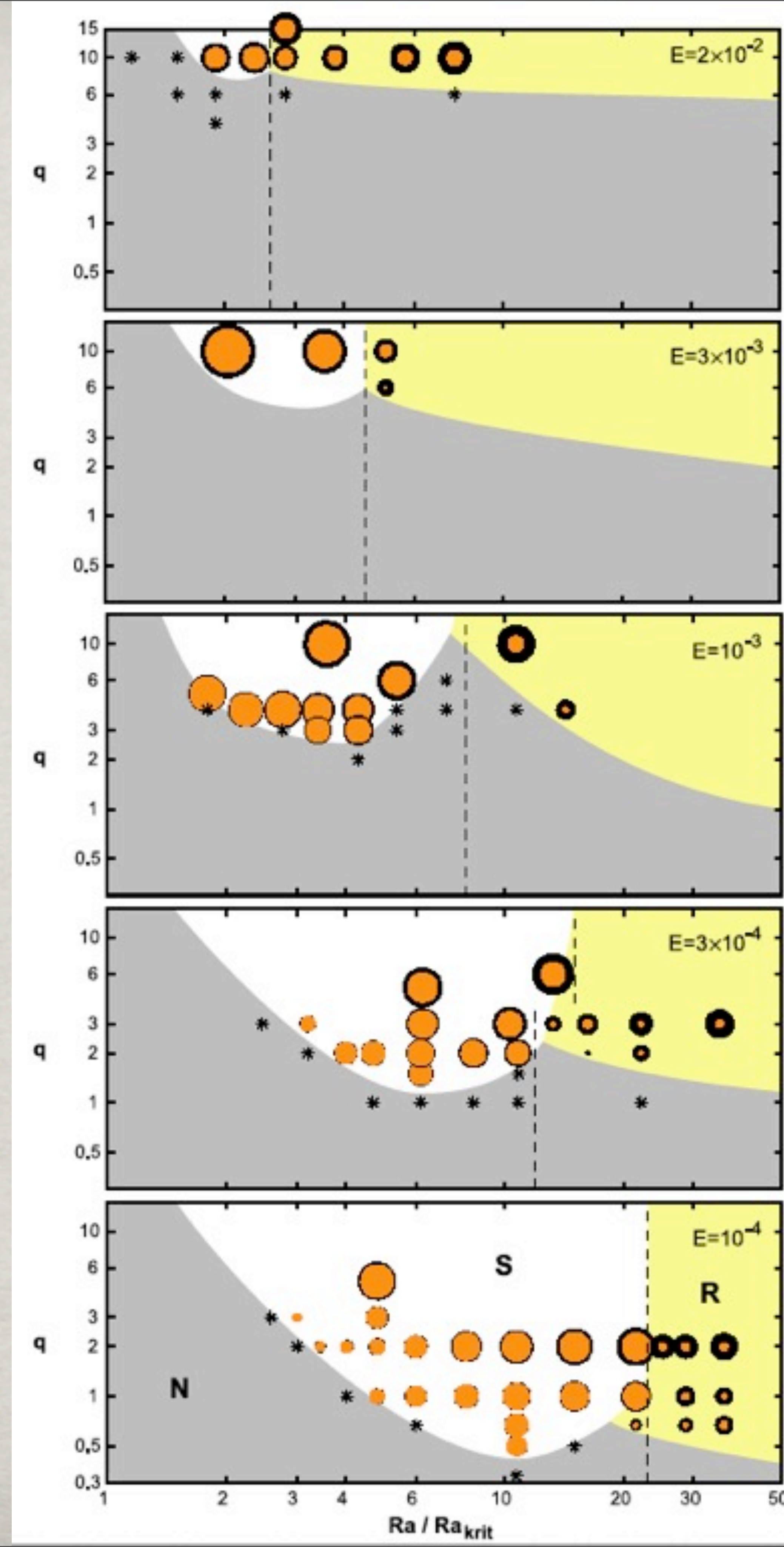
- Loss of dipolarity, and multipolar Dynamos at larger forcing are associated with the increasing strength of inertia

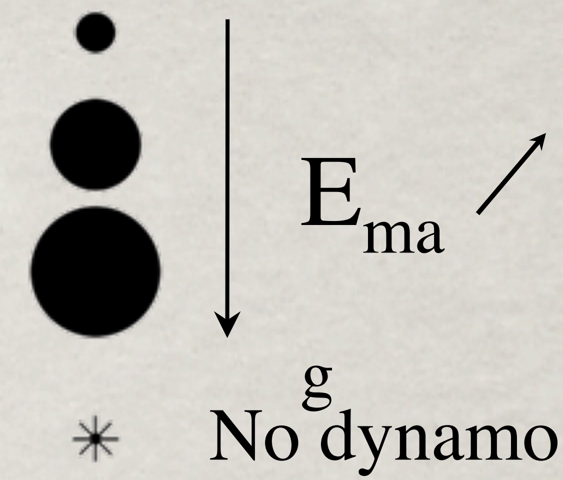
Key question:

Is the Magnetostrophic balance achievable with todays computer?

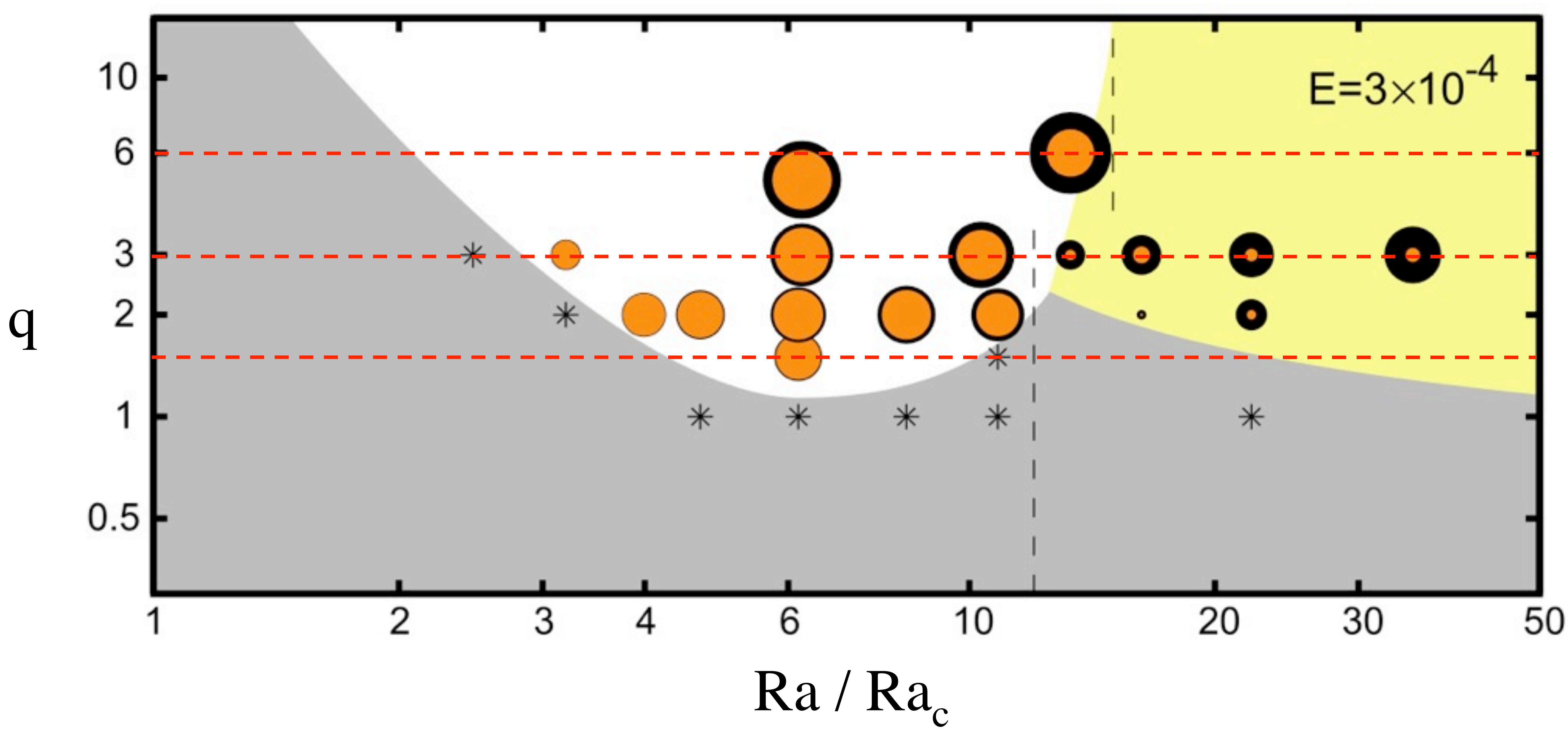
Roberts number

q = Pm / Pr

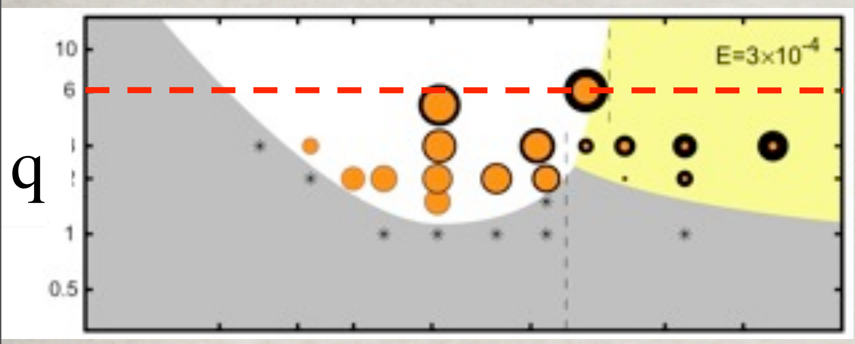


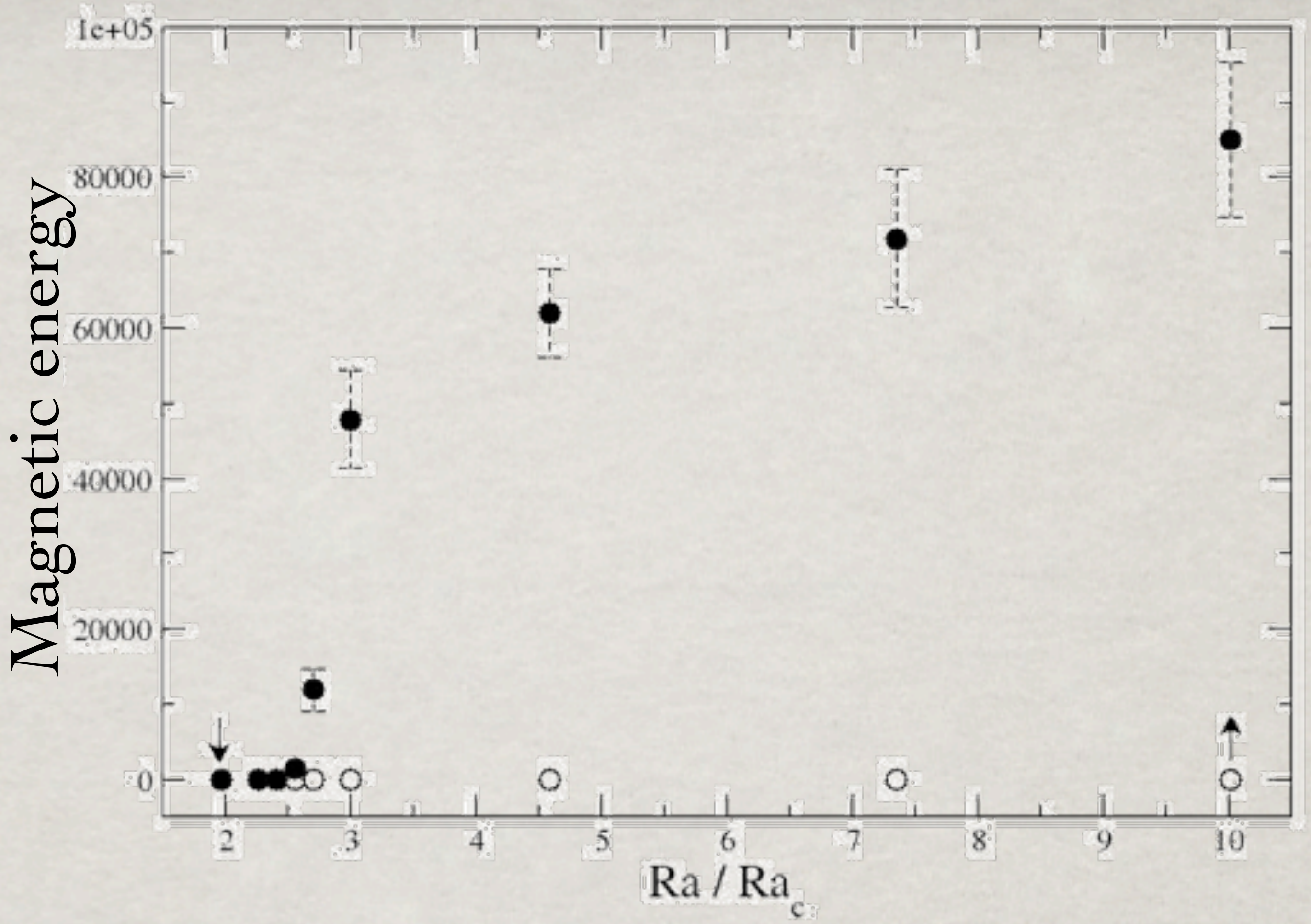


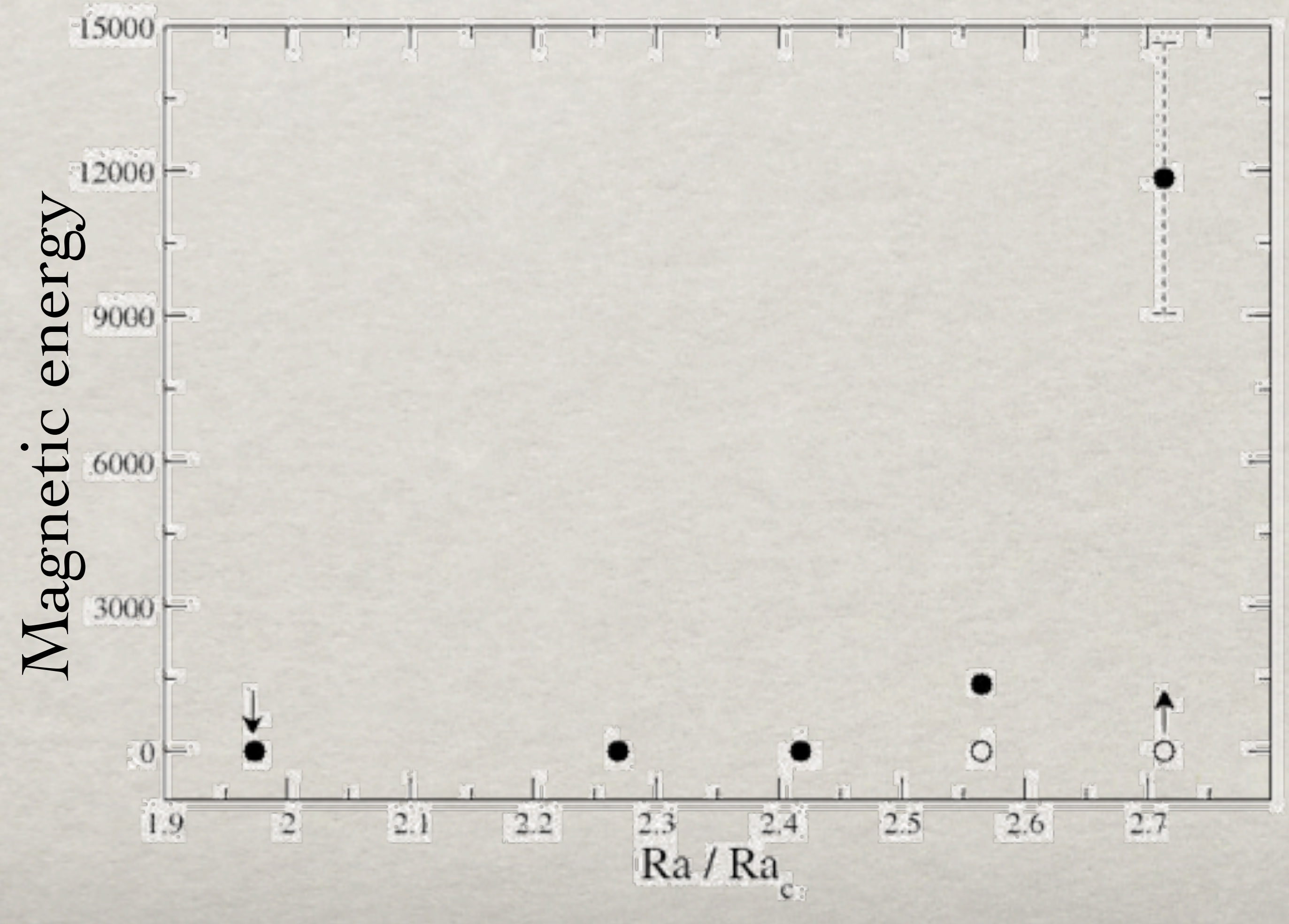
Carsten Kutzner,
Uli Christensen
2003



Morin & Dormy
(2005, 2009)

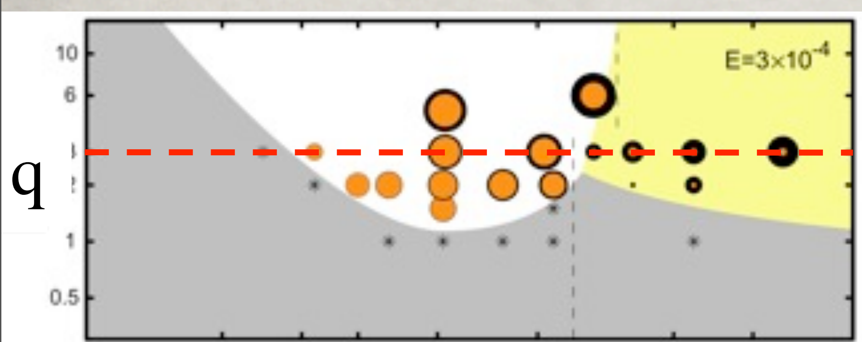


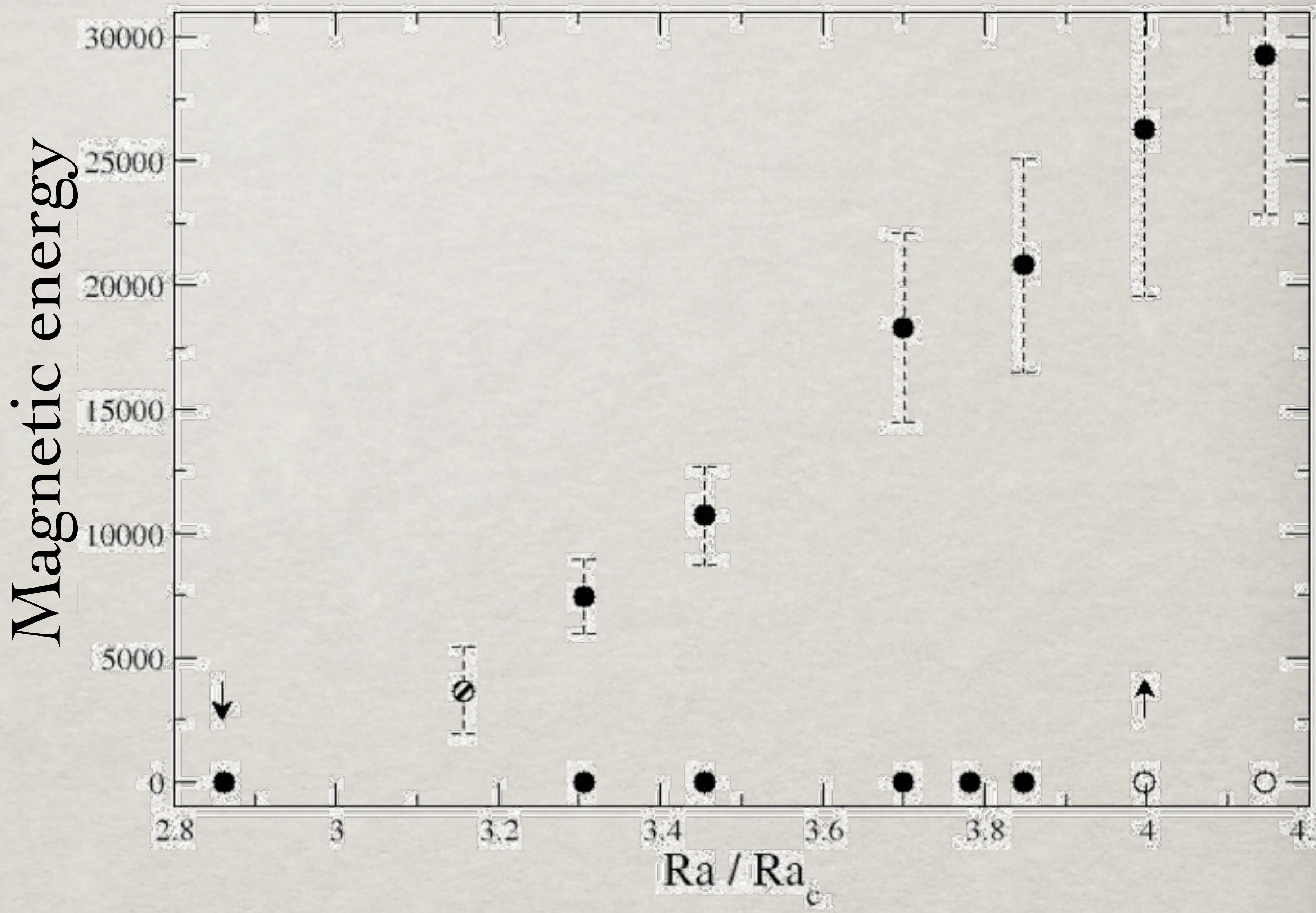




$E=3.10^{-4}$,
$q=6$

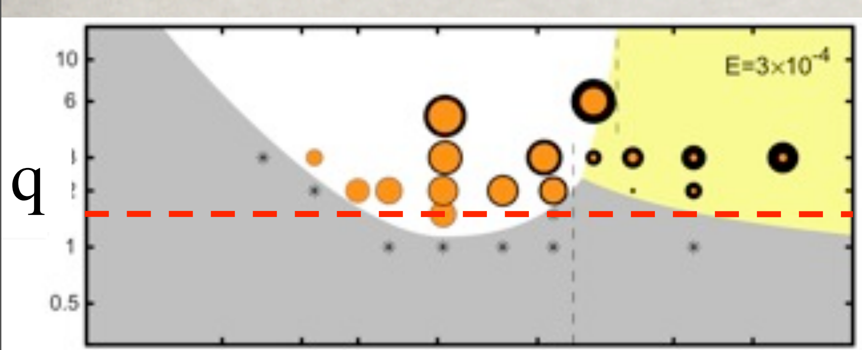
Morin & Dormy
(2005, 2009)



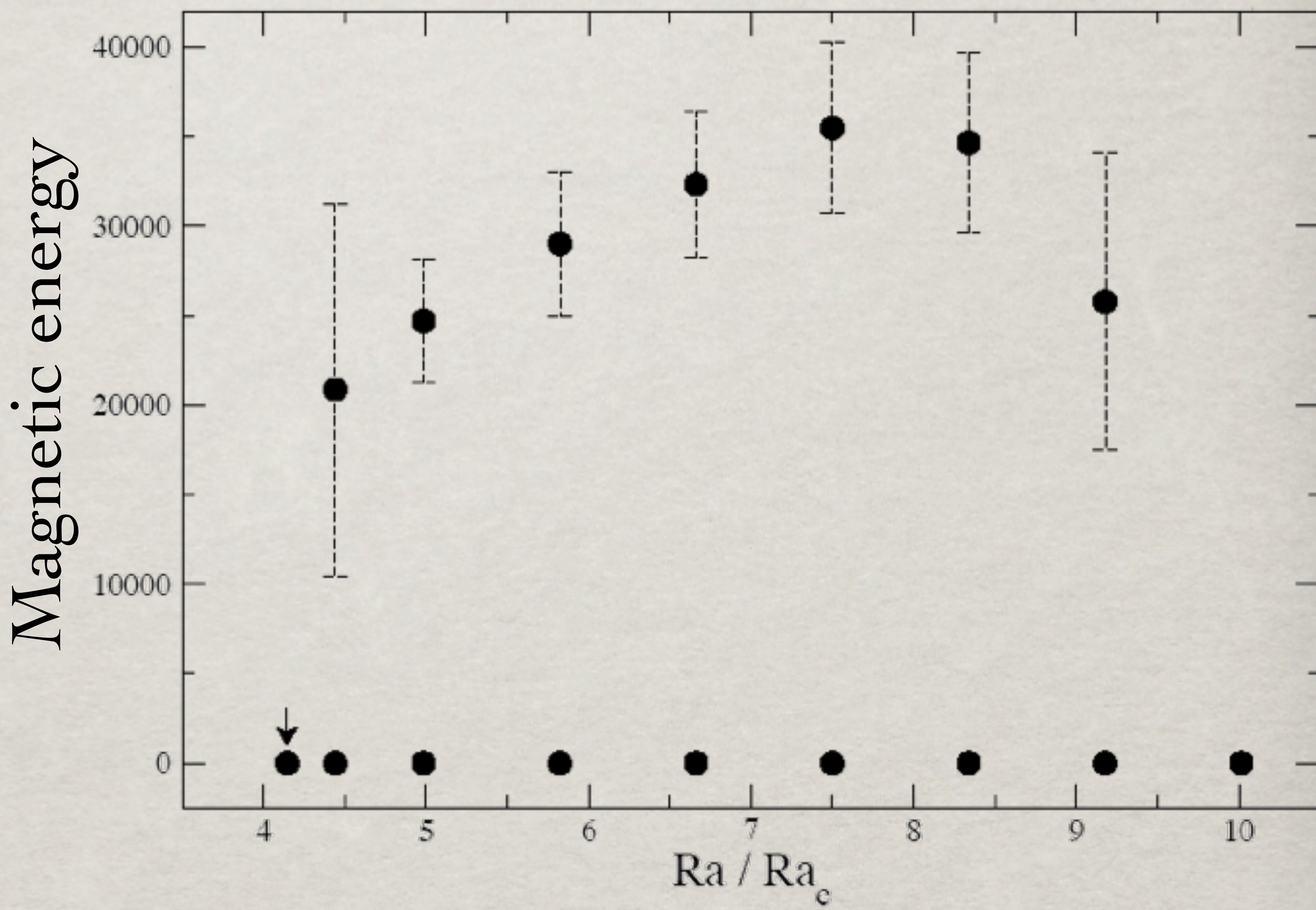


$E=3.10^{-4}$,
q=3

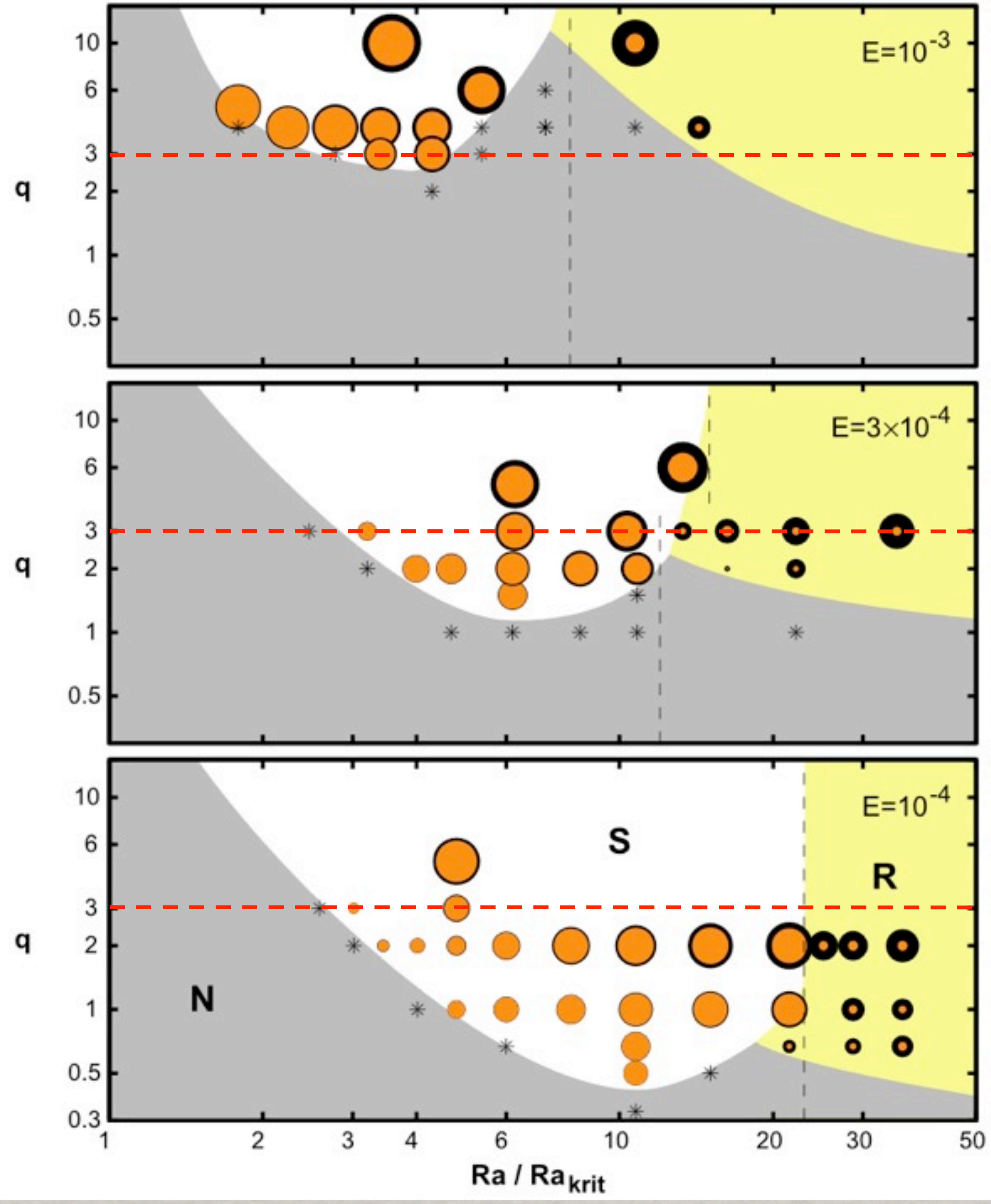
Morin & Dormy
(2005, 2009)



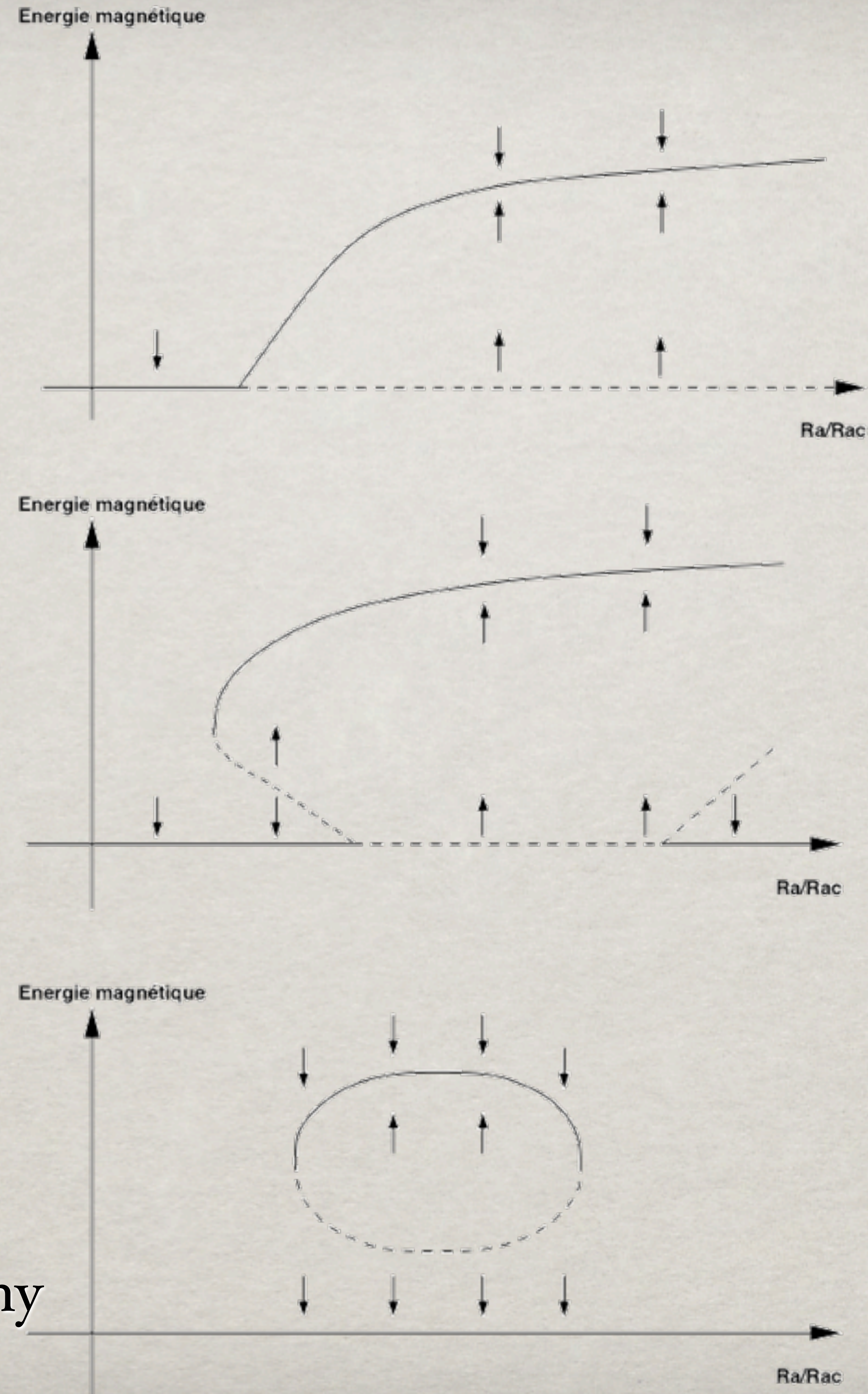
$E = 3.10^{-4}$,
$q = 1.5$



Morin & Dormy
(2005, 2009)



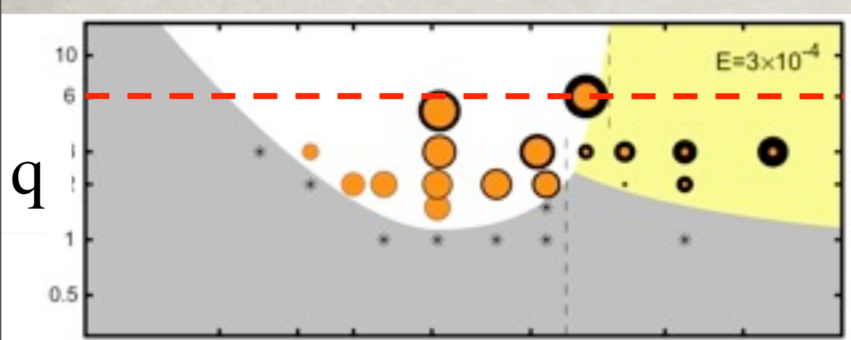
E=10^-3
E=3×10^-4
E=10^-4
S
R
N
q
Ra / Ra_krit

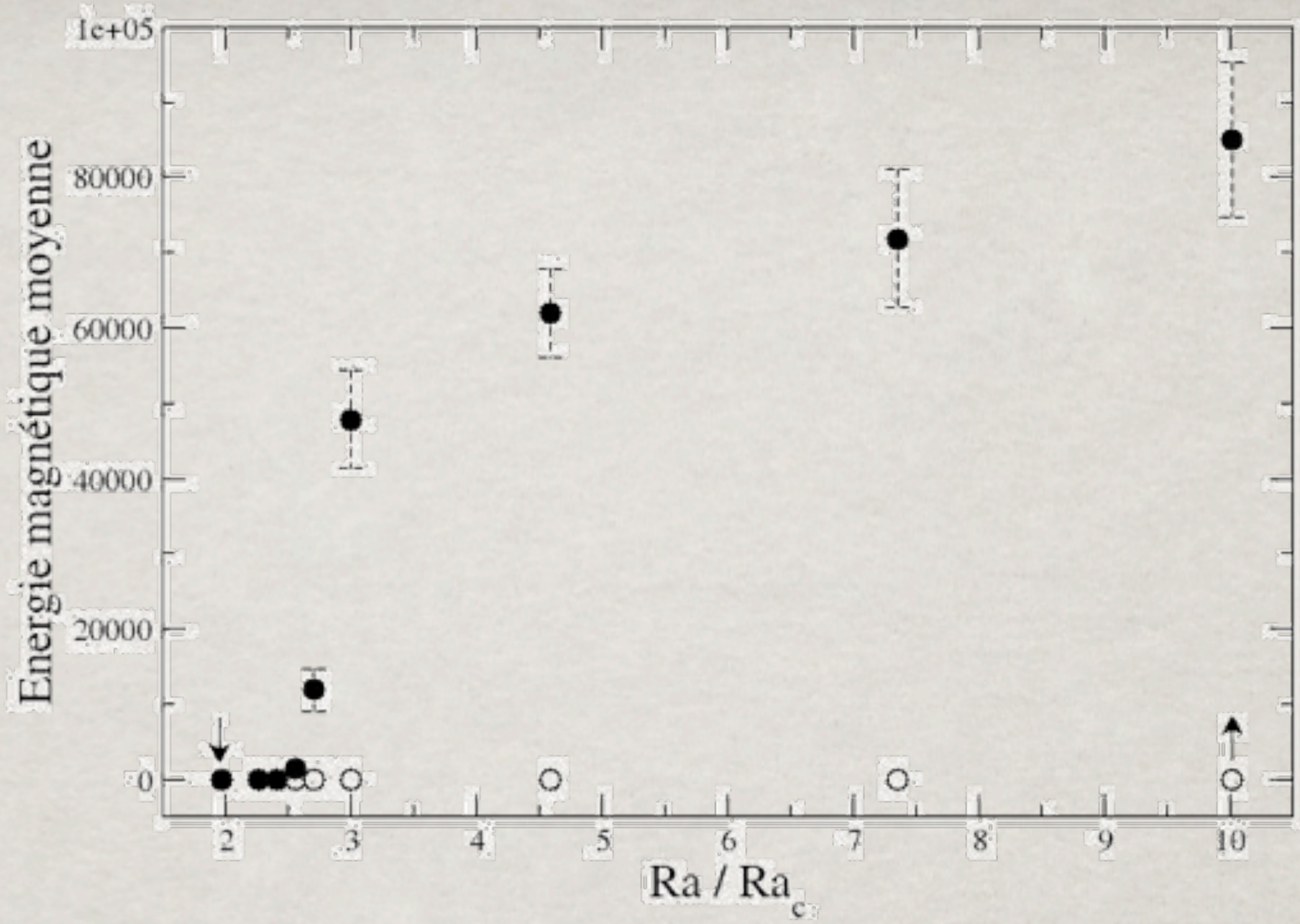


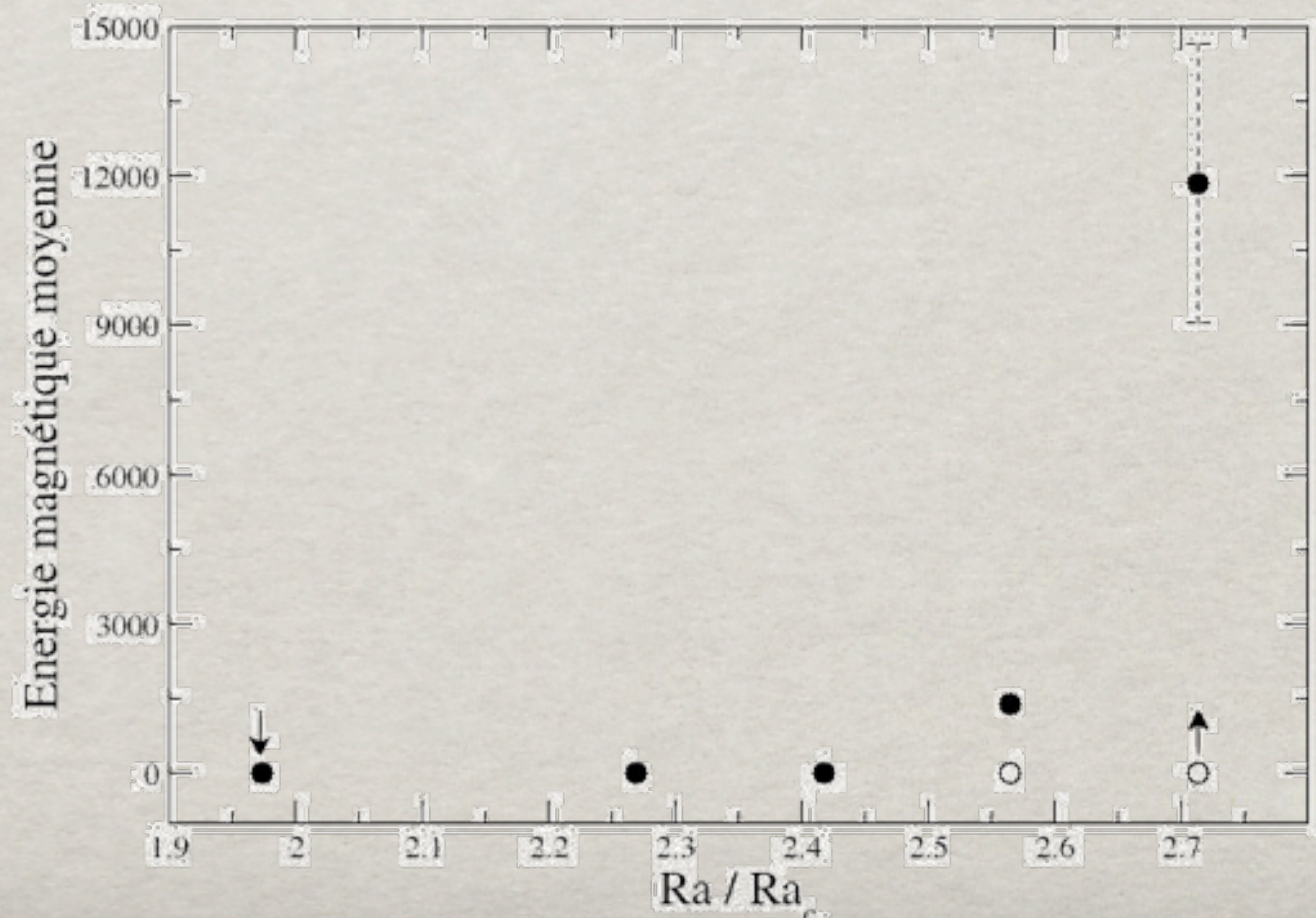
$q \searrow$

$E \searrow$

Morin & Dormy
(2005, 2009)

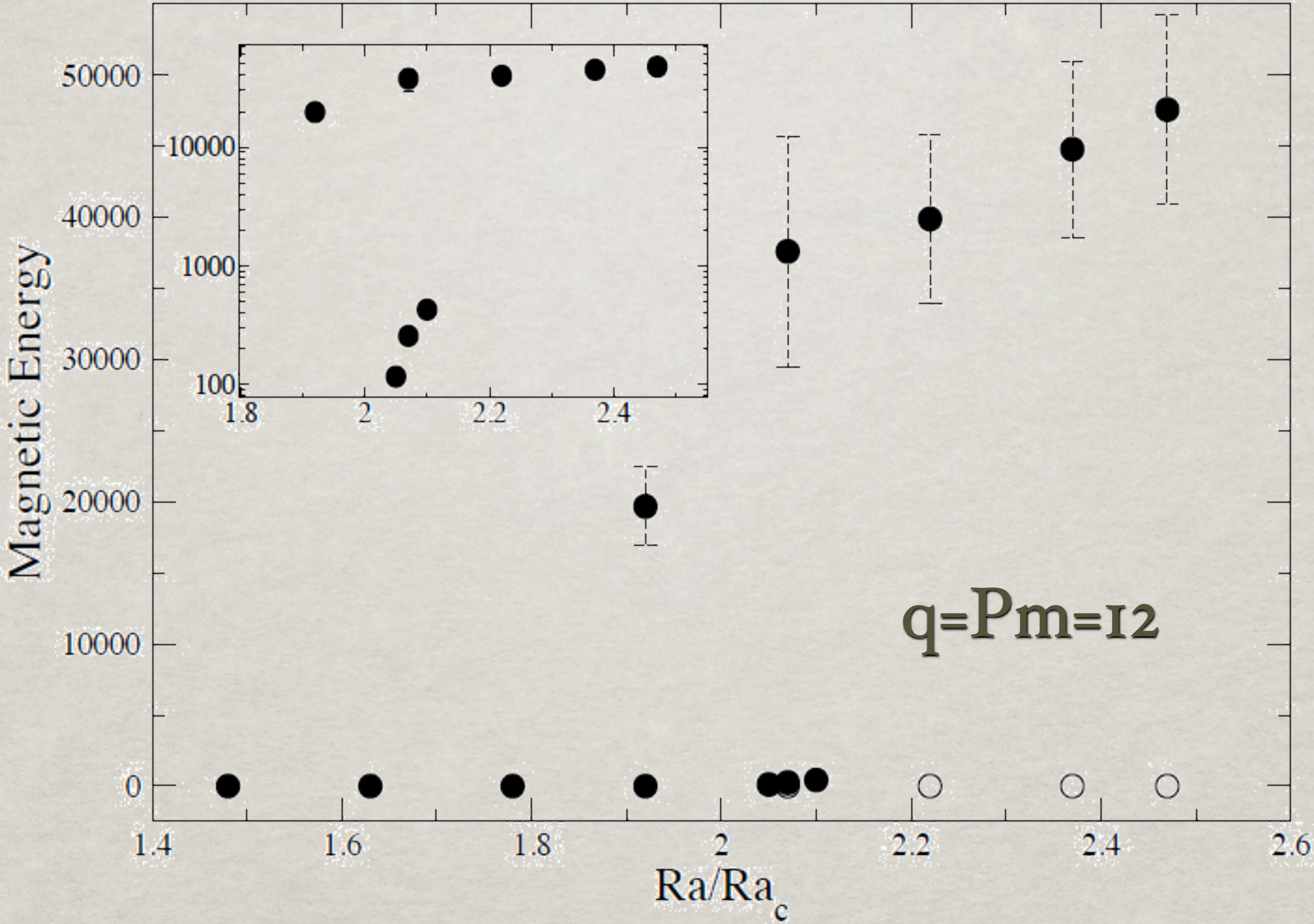

E=3×10-4
q
10
6
1
0.5


Energie magnétique moyenne
1e+05
80000
60000
40000
20000
0
2
3
4
5
6
7
8
9
10
Ra / Rac


Energie magnétique moyenne
15000
12000
9000
6000
3000
0
1.9
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
Ra / Rac

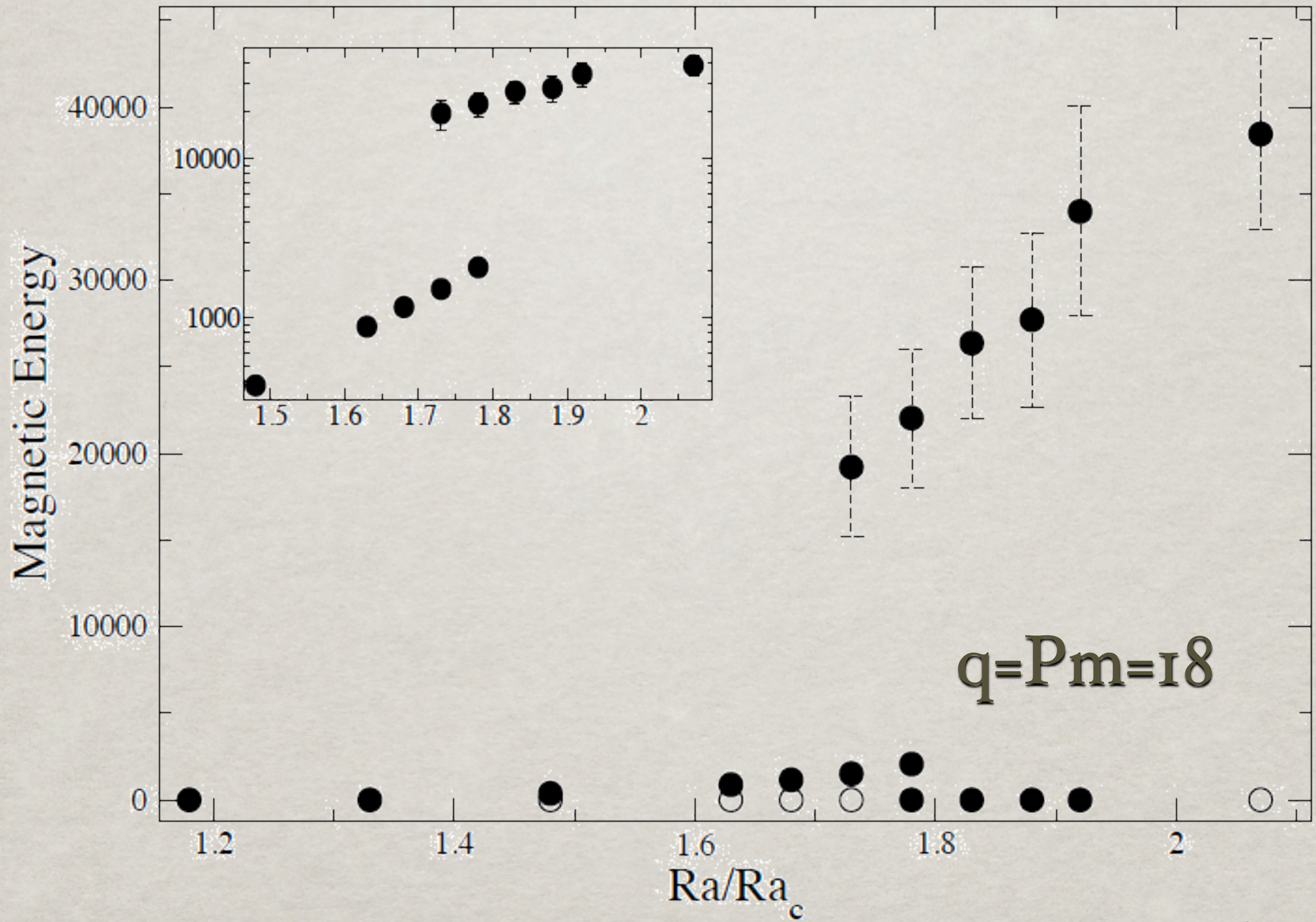
$E=3.10^{-4}$,
$q=6$

Weak and Strong field branches



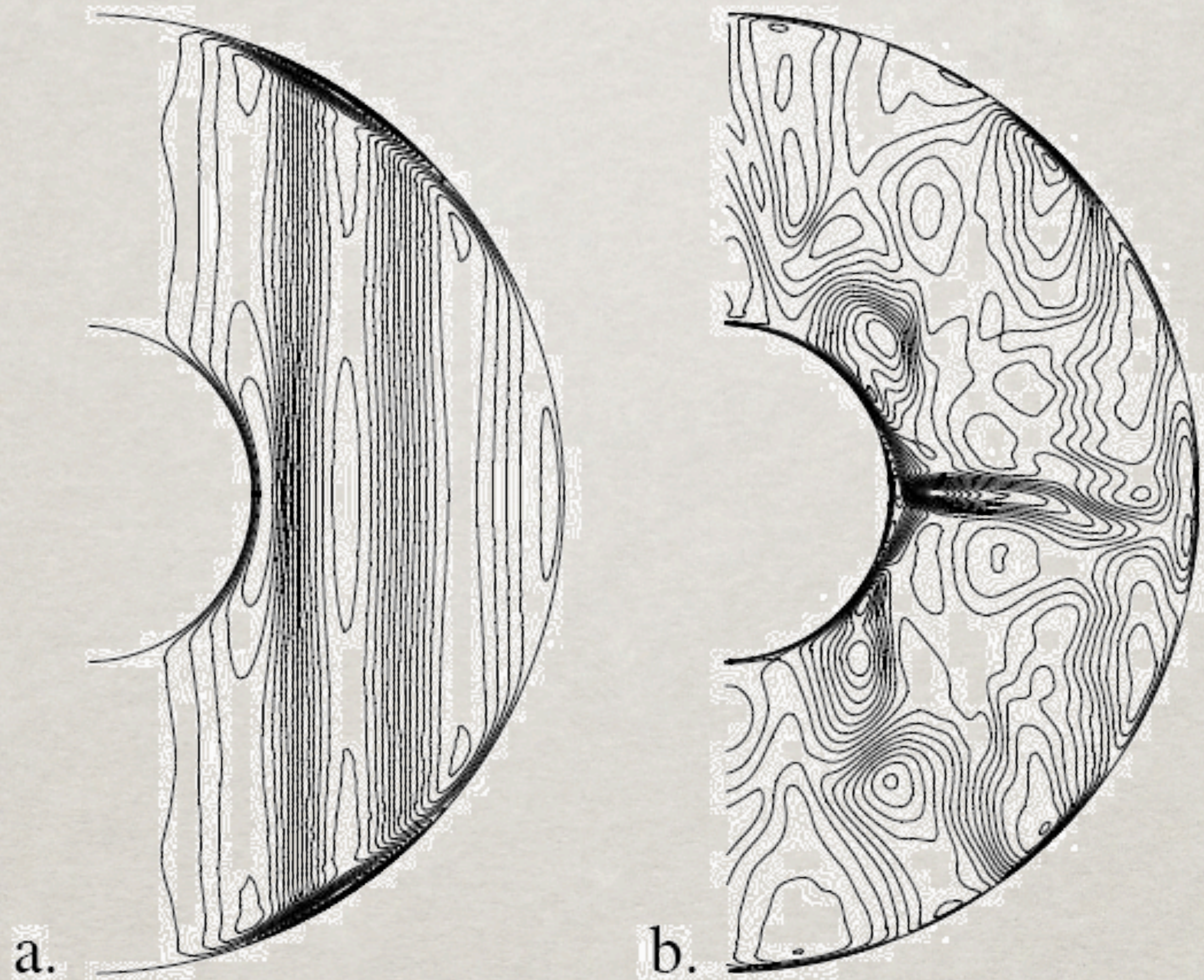
(Dormy, in prep)

Weak and Strong field branches



(Dormy, in prep)

Weak and Strong field branches



V_ϕ, E = 3 10^{-4}, Pm = 18, Ra/Rac = 1.72

(Dormy, in prep)

$$\begin{cases} E_\eta \left(\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} \right) = -\boldsymbol{\nabla}\pi + E\,\Delta\mathbf{u} - \mathbf{e}_z \wedge \mathbf{u} \\ \qquad\qquad - Ra\,T\mathbf{g} + (\boldsymbol{\nabla} \wedge \mathbf{B}) \wedge \mathbf{B}\,, \\ \boldsymbol{\nabla} \cdot \mathbf{u} = 0\,, \\ \dfrac{\partial T}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} T = q\,\Delta T\,, \\ \dfrac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \wedge (\mathbf{u} \wedge \mathbf{B}) + \Delta\mathbf{B}\,, \\ \boldsymbol{\nabla} \cdot \mathbf{B} = 0\,. \end{cases}$$

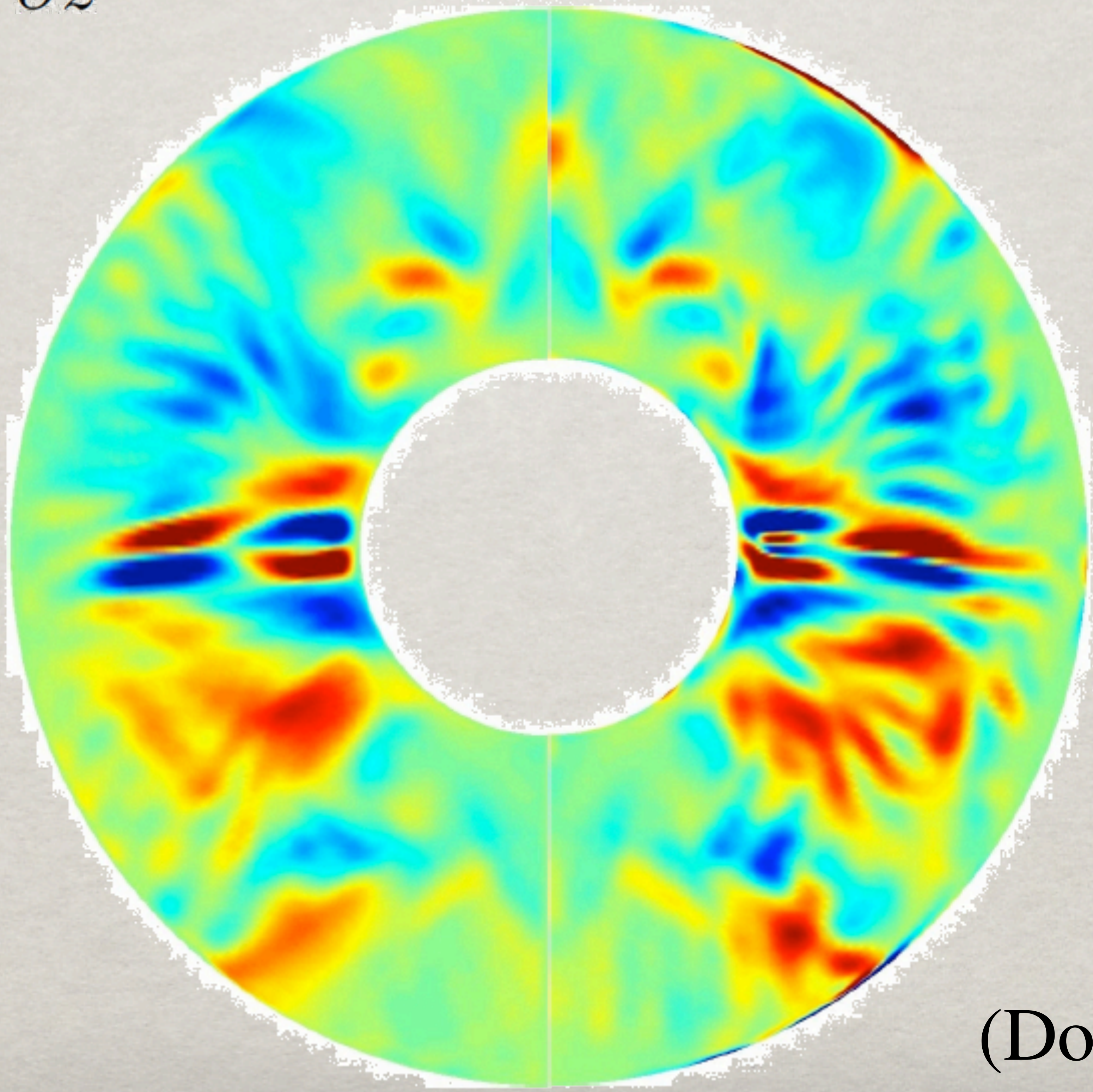
$\geq 10^{-6}$ ≥ 1

$$E/Pm = E_\eta = \frac{\eta}{2\Omega\mathcal{L}^2}\,, \quad E = \frac{\nu}{2\Omega\mathcal{L}^2}\,, \quad q = \frac{\kappa}{\eta}\,.$$

$\simeq 10^{-7}$ $\simeq 10^{-15}$ $\simeq 10^{-7}$

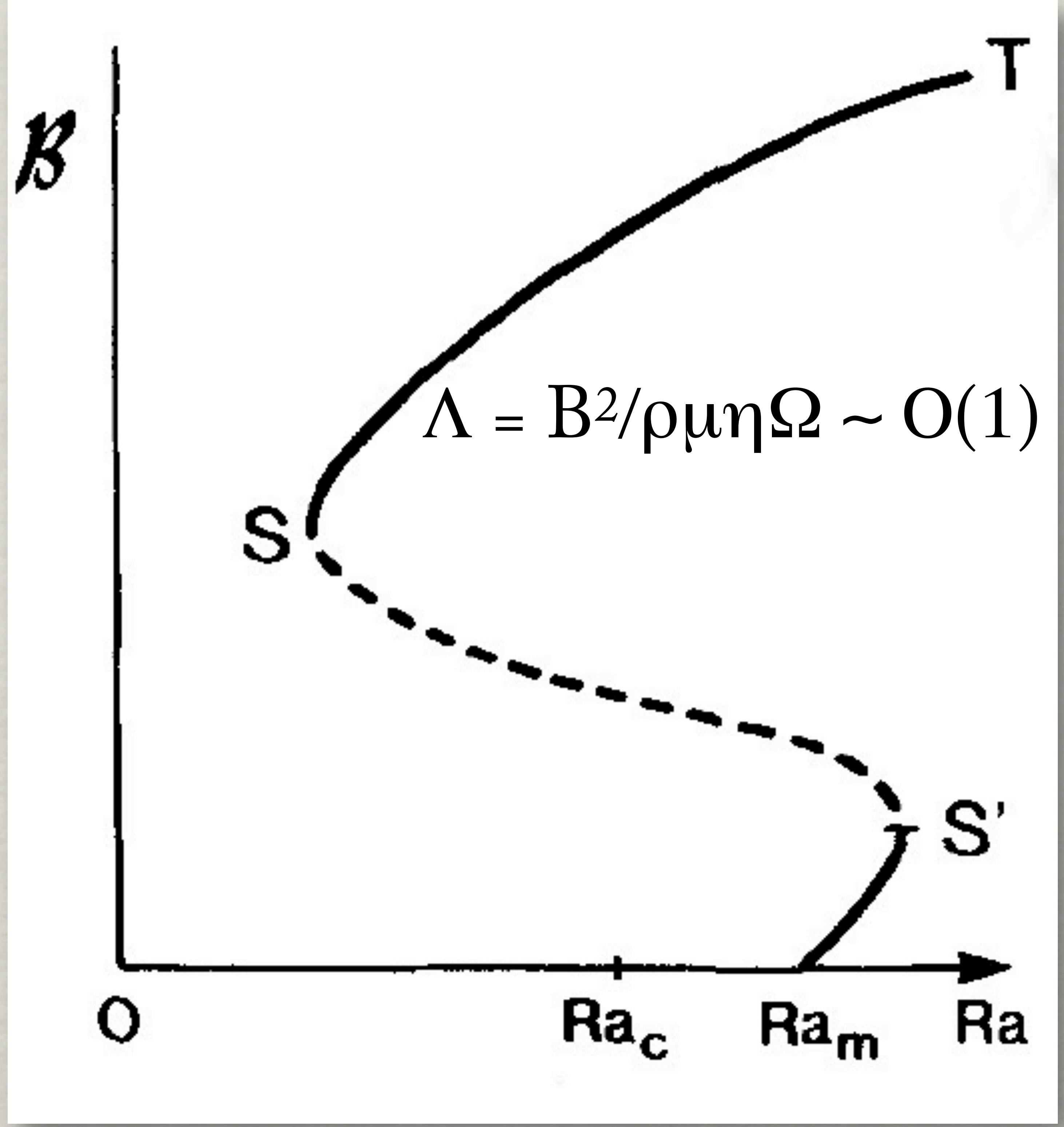
Weak and Strong field branches

$$-2\,\frac{\partial \mathbf{u}}{\partial z}\cdot \mathbf{e}_r \qquad (\boldsymbol{\nabla}\times((\boldsymbol{\nabla}\times\mathbf{B})\times\mathbf{B}))\cdot\mathbf{e}_r$$



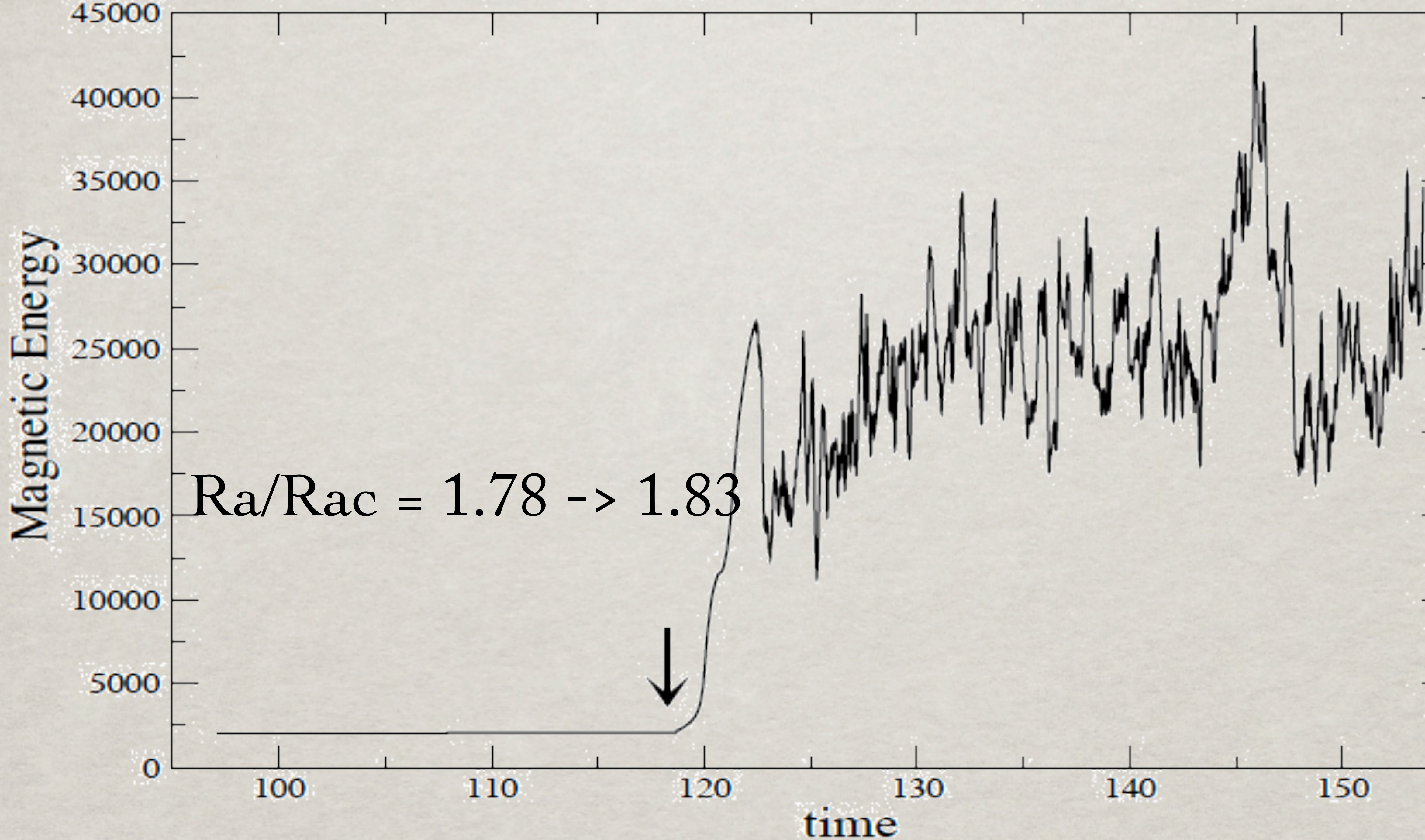
(Dormy, in prep)

Weak and Strong field branches



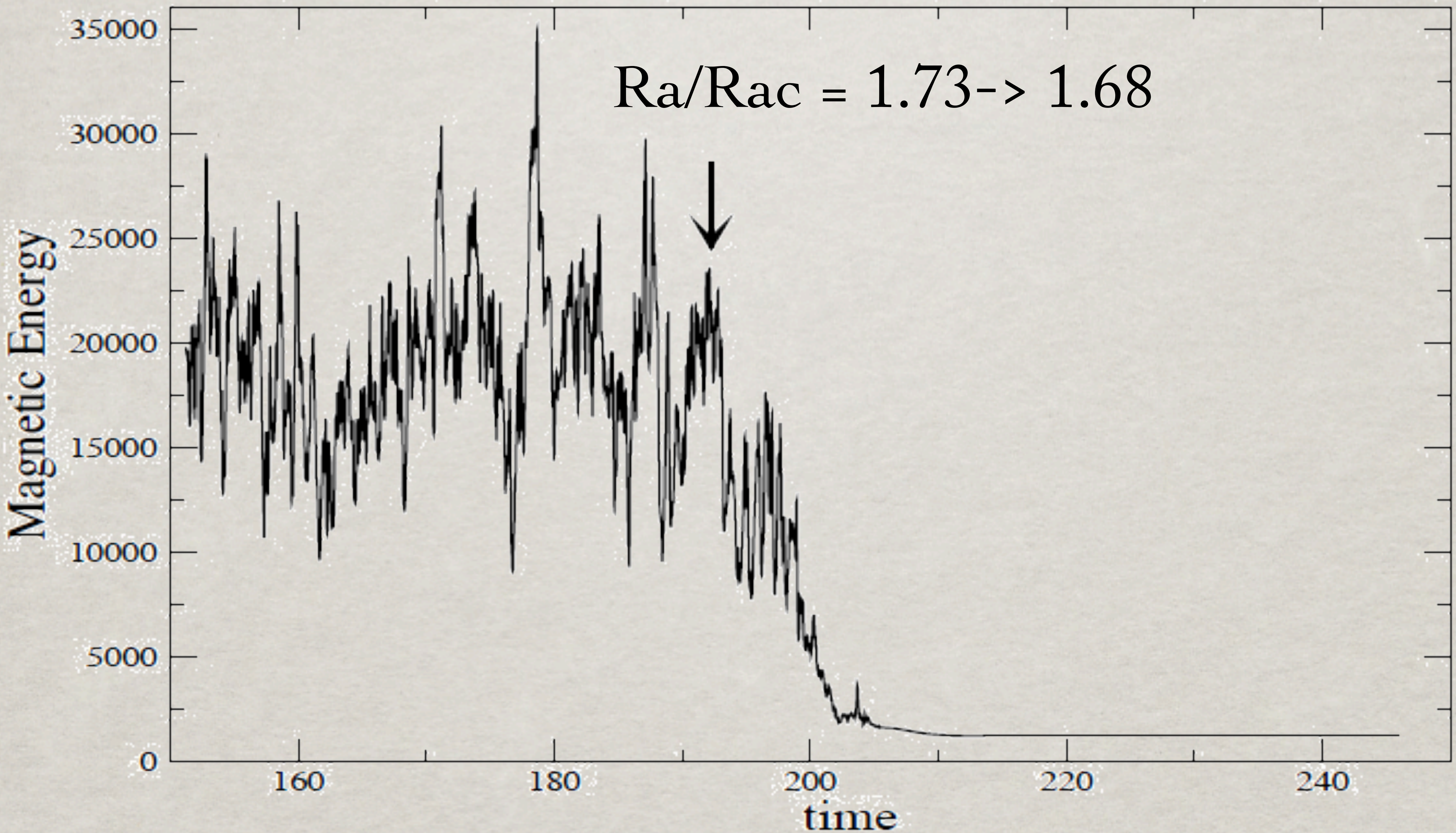
(Roberts, GAFD, 1988)

Weak and Strong field branches



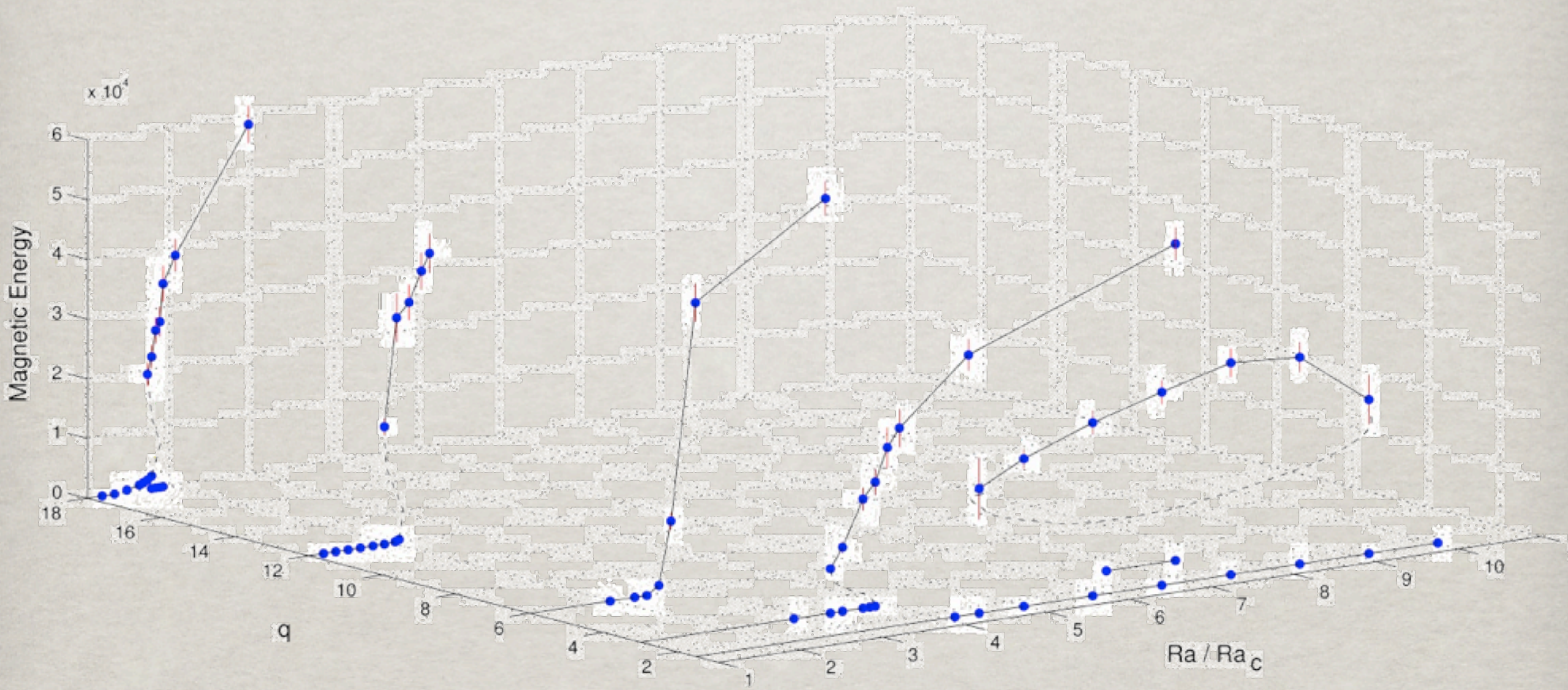
(Dormy, in prep)

Weak and Strong field branches



(Dormy, in prep)

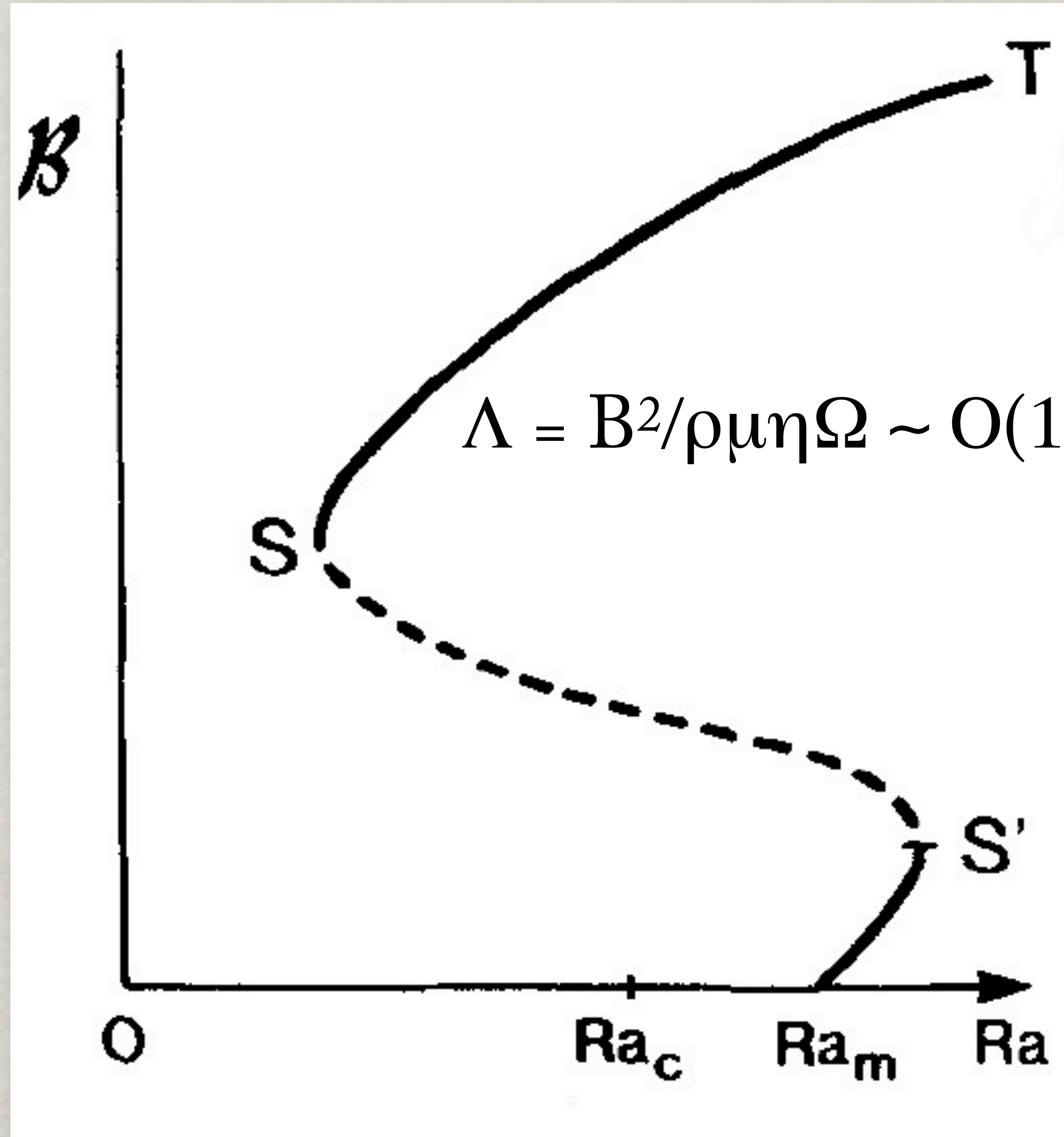
3D bifurcation diagram



E=3 10^{-4}

(Dormy, in prep)

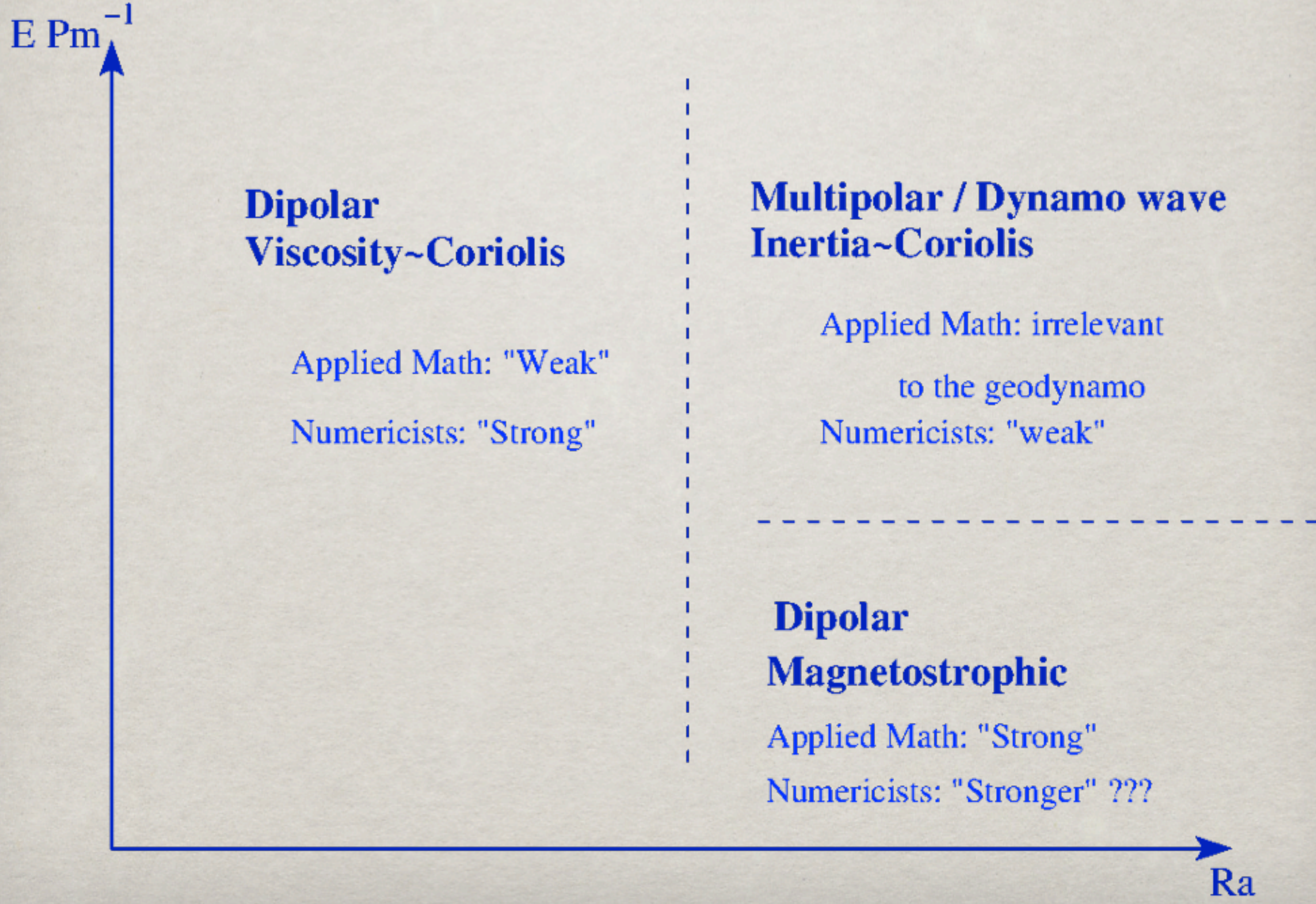
Weak and Strong field branches



$\Lambda = B^2/\rho\mu\eta\Omega \sim O(1)$

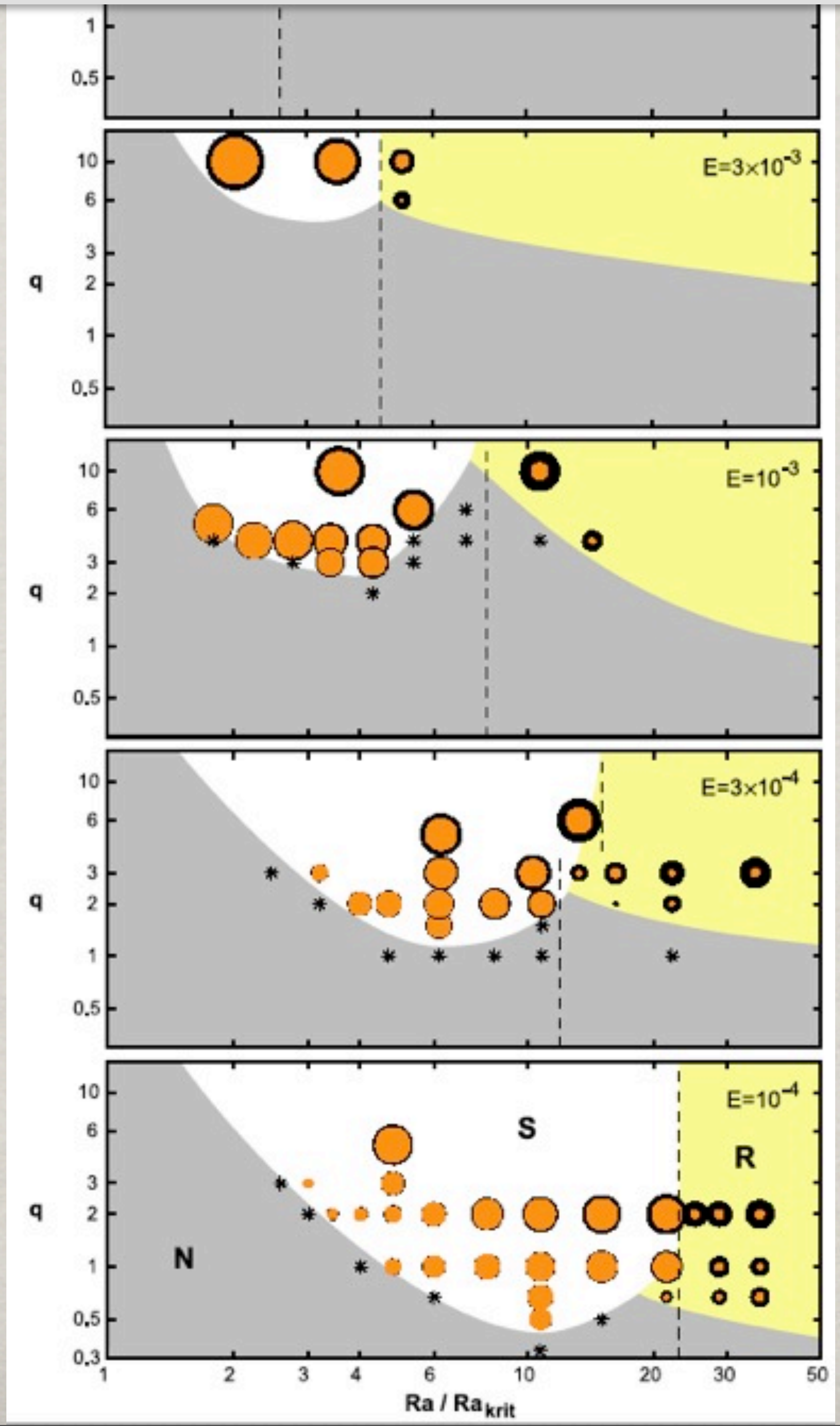
(Roberts, GAFD, 1988)

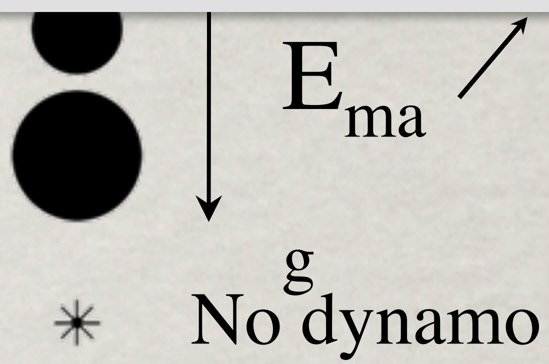
Relevant parameter space



Toward a distinguished limit

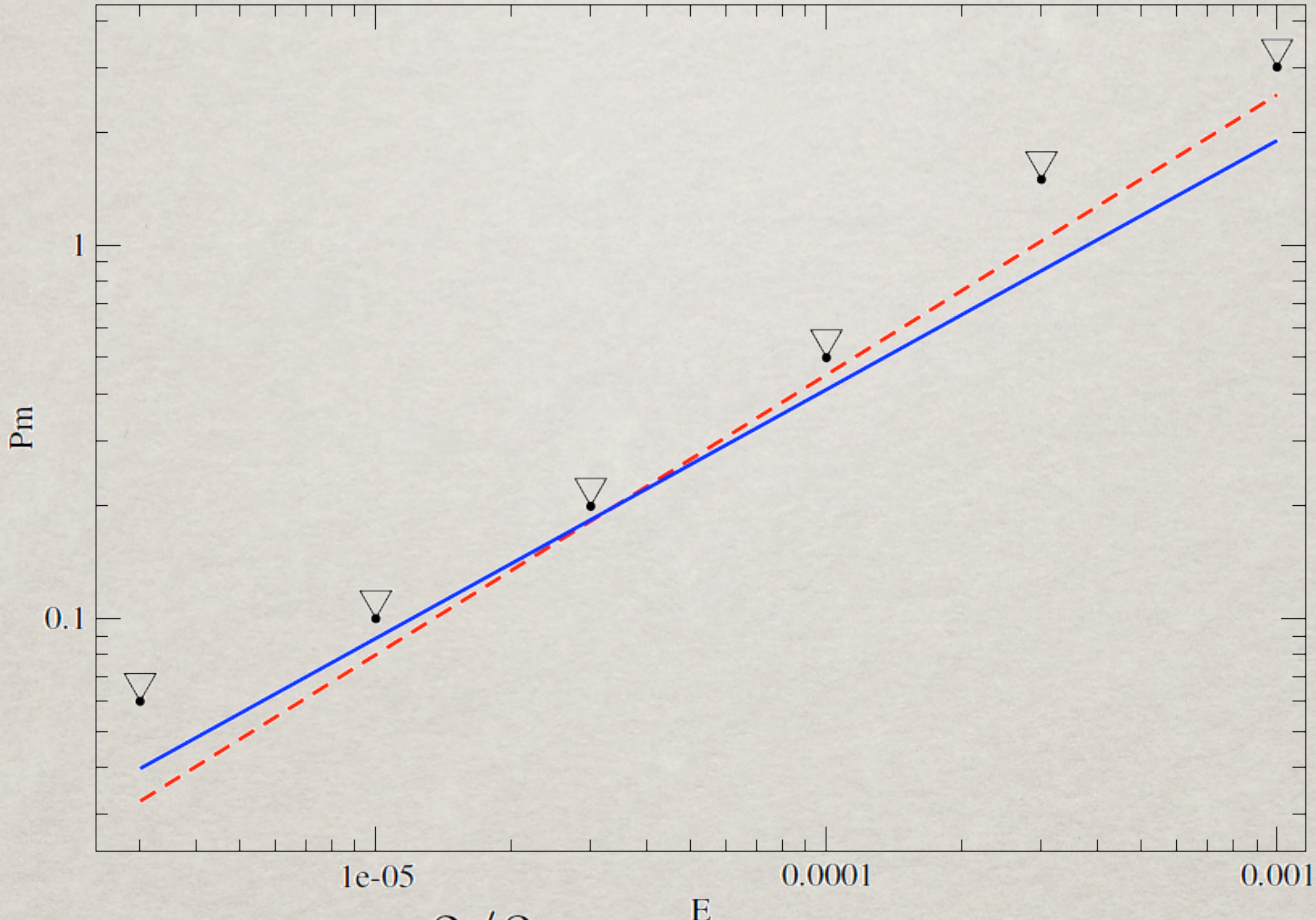
Roberts number
q = Pm / Pr





Carsten Kutzner,
2003

Toward a distinguished limit



$$\mathrm{Pm}_c \sim E^{2/3}$$

Dormy & Le Mouël (2008)

Toward a distinguished limit

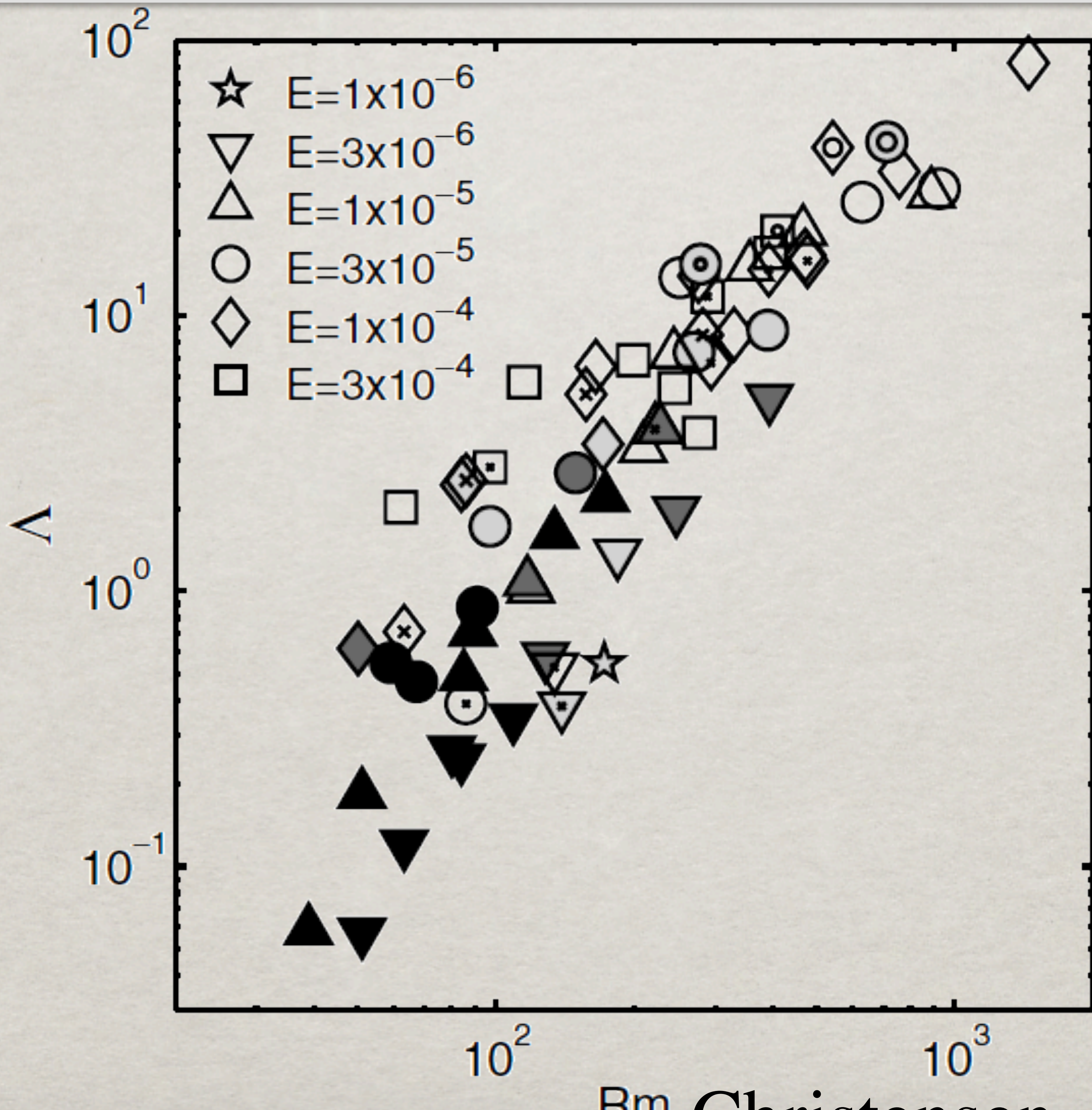
$$\mathrm{Pm}_c \sim E^{2/3}$$

$$\mathrm{Pm}^3 \sim E^2$$

$$\mathrm{Pm} \sim \varepsilon^2 \qquad E \sim \varepsilon^3$$

(Dormy, in prep)

The Elsasser number



Christensen-Aubert 2006

The Elsasser number

$$\Lambda = \frac{\{f_B\}}{\{f_\Omega\}} = \frac{\{\frac{1}{\mu_0 \rho_0}(\vec{\nabla} \wedge \vec{B}) \wedge \vec{B}\}}{\{2\Omega(\vec{e}_z \wedge \vec{u})\}} \simeq \frac{1}{\mu_0 \rho_0} \frac{B^2 \ell^{-1}}{2\Omega u}$$

$$\Lambda = \frac{B^2}{2\Omega \rho_0 \mu_0 \eta}$$

The Elsasser number

Ra	$\mathrm{Ra}/\mathrm{Ra}_c$	Pm	Rm	Λ	$\Lambda/(Rm\,\ell_B)$
105	1.7	18	195	1.14	0.06
105	1.7	18	145	13.2	1.11
125	2.0	18	207	24.0	1.54
150	2.5	12	165	23.7	1.65

$$\Lambda = \frac{2\,E_m}{V_s}\,Pm\,E$$

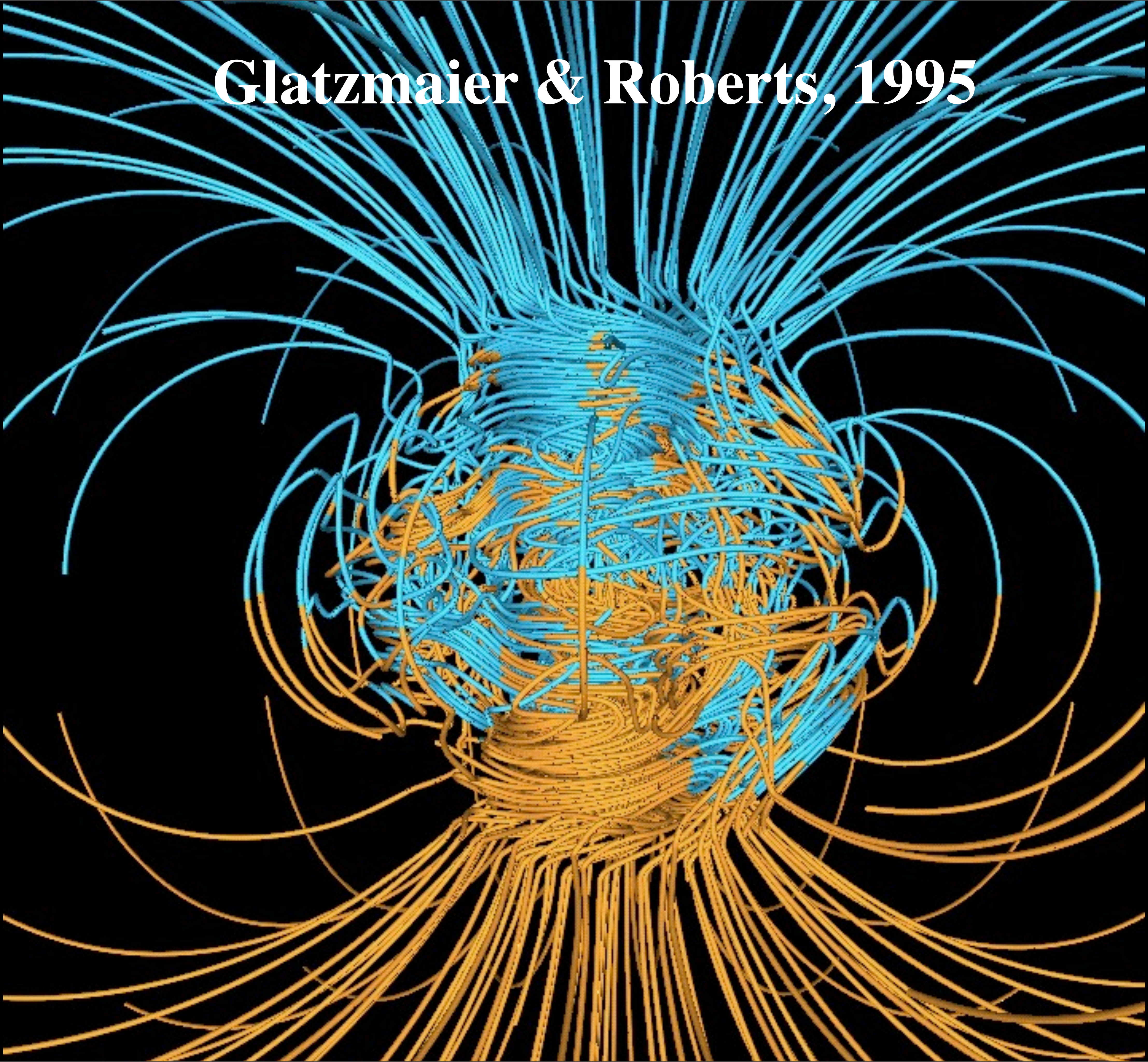
(Dormy, in prep)

CLAIMS

- None of the published spherical dynamo models correspond to a force balance relevant to the geodynamo!
- This force balance can be approached in numerical models.

OPENED ISSUES

- All terms are of similar amplitude, Yet well identified balances seem to emerge. Why?
- How do these models evolve at lower E, and lower Pm?
- Is it possible to sustain dynamo action for $Ra<Ra_c$?


Glatzmaier & Roberts, 1995