

TURBULENCE IN ENGINEERING APPLICATIONS II

INSTANTANEOUS FIELDS IN WALL-BOUNDED TURBULENCE

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OUTLINE

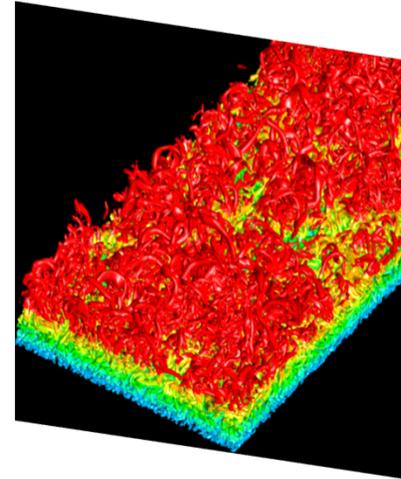
- Introduction
- Review of the state of the art – data and scaling
- Modeling of observations: what do the models tell us about the underlying math?
- Challenges and opportunities

I will not address:

- Mean profile
- Higher order statistics
- Structure functions
- Scalar mixing, compressibility, wall roughness, non-Newtonian fluids, etc...
- Control

INTRODUCTION – BASIC CONCEPTS

- Definitions, terminology
- What is different about wall turbulence?
- Regions and scalings
- Data and experimental limitations



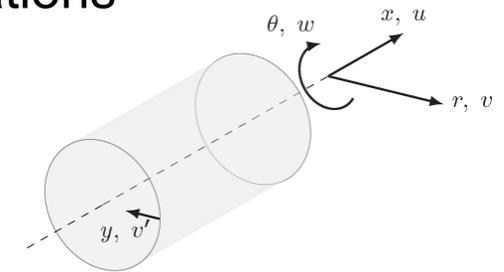
Reference papers

- *Marusic et al Phys. Fluids 22, 2010*
- *Smits, McKeon and Marusic, Annu. Rev. Fluid Mech. 43, 2011*

EQUATIONS OF MOTION

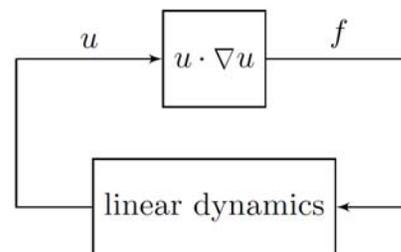
- Define an instantaneous velocity field and “fluctuations” relative to the mean

$$\tilde{\mathbf{u}}(t) = \mathbf{U} + \mathbf{u}(t)$$



- Then the momentum equation reduces to:

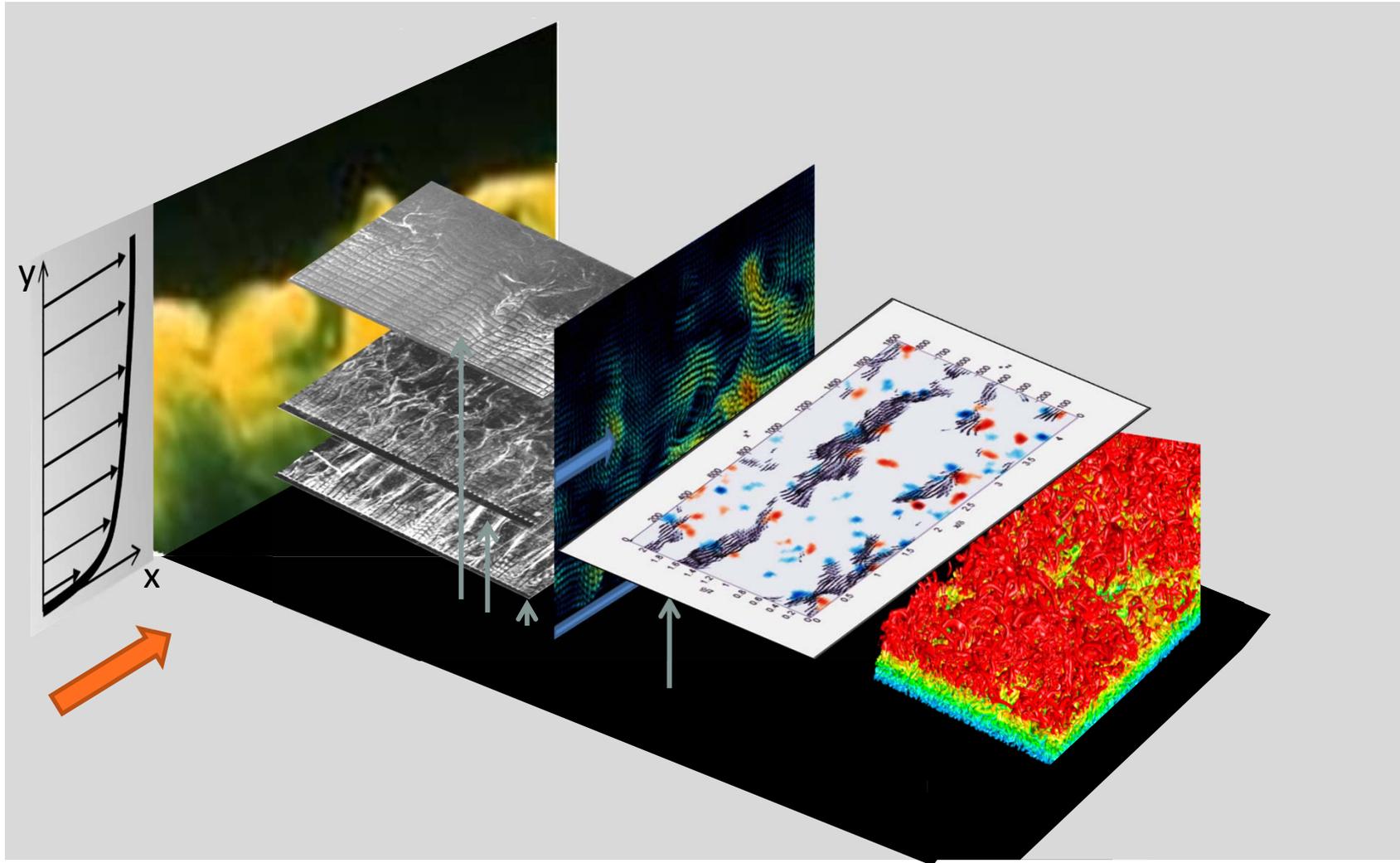
$$\mathbf{u}_t + (\mathbf{u} \cdot \nabla)\mathbf{U} + (\mathbf{U} \cdot \nabla)\mathbf{u} + \nabla p - (1/Re_\tau)\nabla^2\mathbf{u} = \overbrace{-(\mathbf{u} \cdot \nabla)\mathbf{u}}^{\mathbf{f}}$$



- Plus continuity

Introduction

WALL TURBULENCE HAS SPECIAL STRUCTURE



von Karman (1930)
Millikan (1938)
Coles (1952)

Gad-el-Hak
<http://efluids.com>

Kline, Reynolds, Schraub & Runstadler
J. Fluid Mechanics (1967)
Copyright © (1967) Cambridge University Press.
Reprinted with permission.

LeHew, Guala & McKeon
Expts. in Fluids (2011)

Wu & Moin
<http://ctr.stanford.edu>

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REGIONS AND SCALES

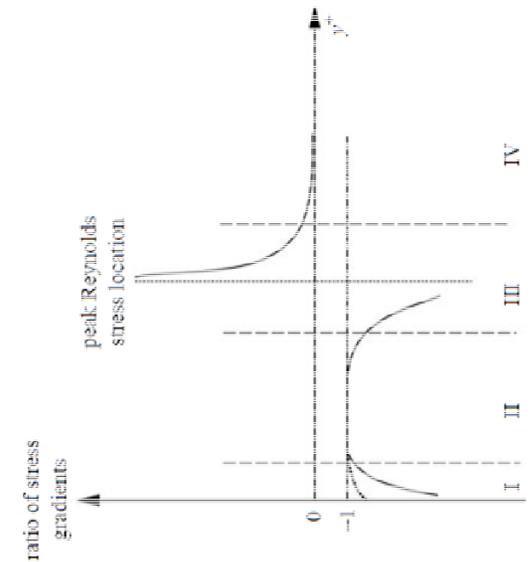
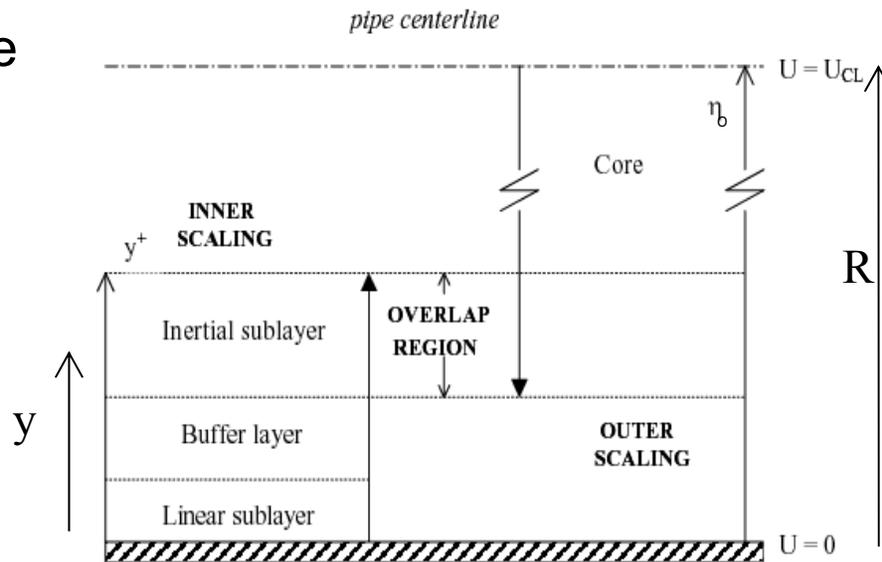
Physical space

$$y^+ = \frac{yu_\tau}{\nu}$$

$$\eta_o = \frac{y}{R}$$

$$u_\tau = \sqrt{\frac{\tau}{\rho}}$$

$$R^+ = \frac{Ru_\tau}{\nu}$$



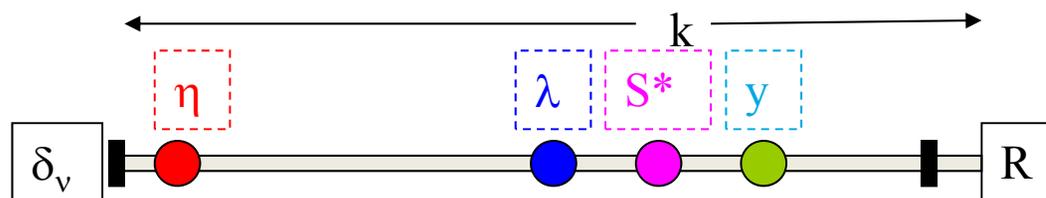
Klewicki, Wei, Fife & McMurtry,
Phil. Trans. R. Soc. A 2007

Spectral space

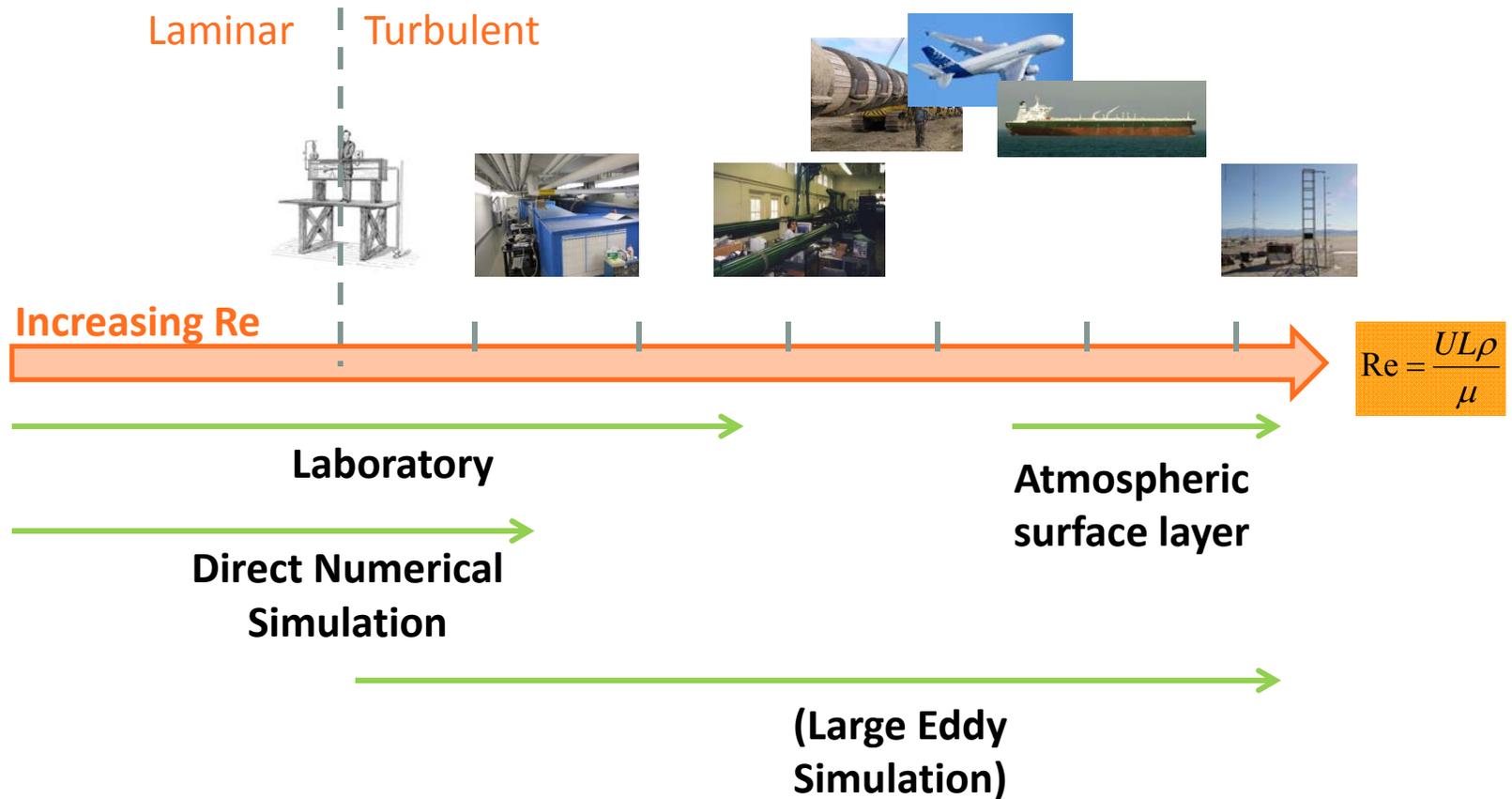
$$\delta_v = \frac{\nu}{u_\tau}$$

$$\lambda = \sqrt{u^2 / (\partial u / \partial x)^2}$$

$$S^* = \sqrt{\frac{\varepsilon}{S^3}} \quad s = \frac{\partial U}{\partial y}$$



WHAT DATASETS ARE AVAILABLE?



ISSUES
Resolution issues – space and time – in experiments
Computational expense (and scaling) in simulation

REVIEW OF DATA AND SCALING

- Form and scaling of the fluctuation intensities
 - u, v, w
 - uv and implications for TKE production
- Spectral arguments
- Importance of the very long scales
- “Amplitude modulation”
 - or the phase relationship between large and small scales
- Structure
 - Hairpin vortices in the overlap region
 - Near-wall cycle

Spectral vs. physical picture, Tennekes and Lumley’s “typical eddy”

SCALING OF THE SECOND ORDER STATISTICS: u, v, w

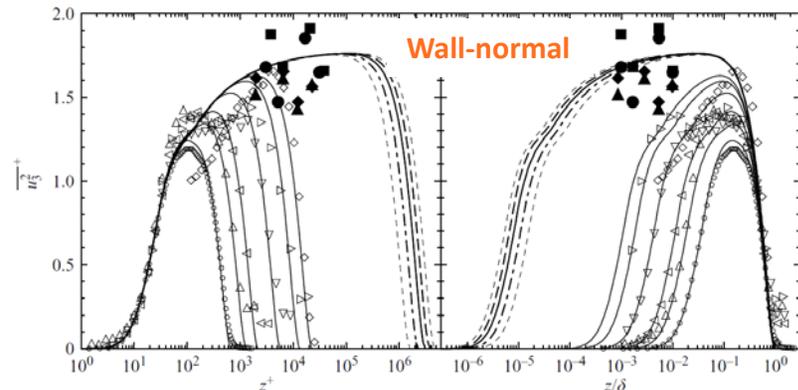
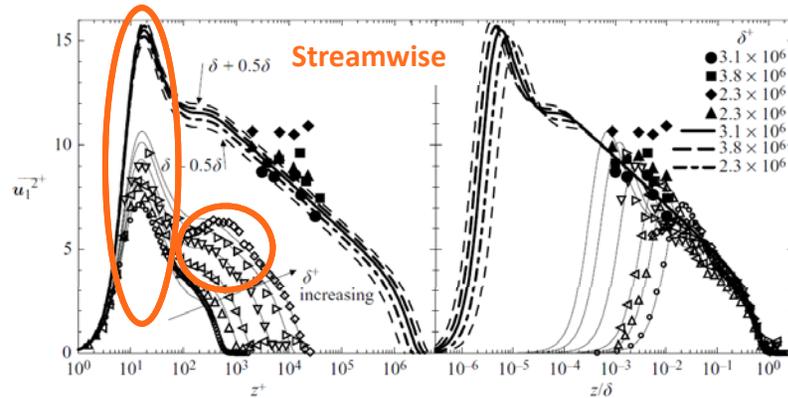
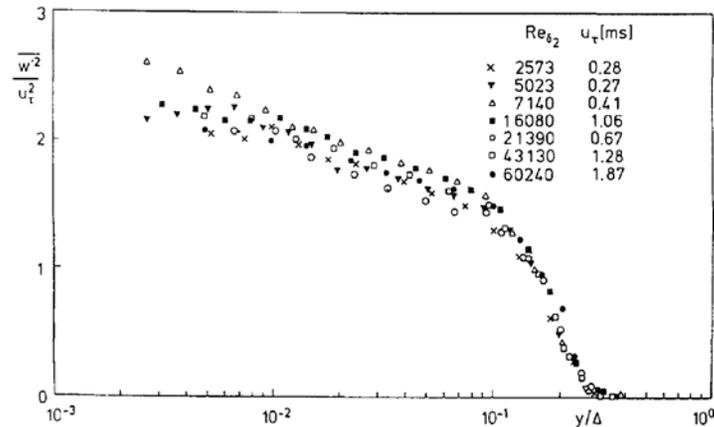
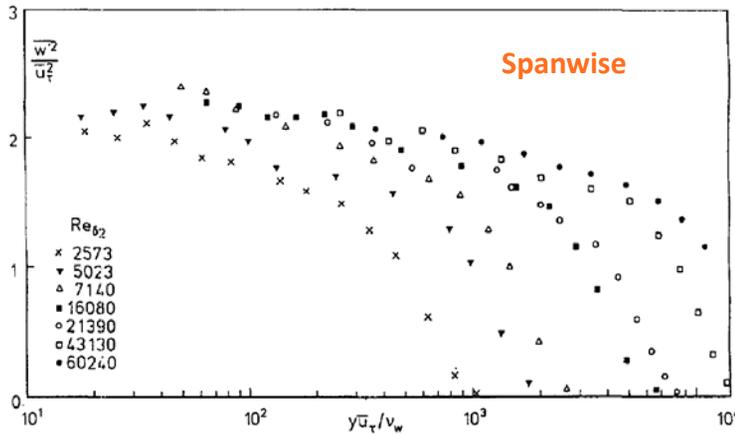
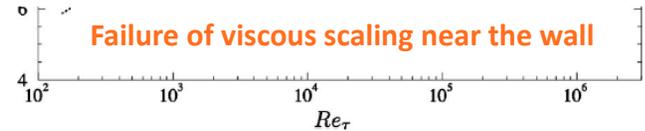


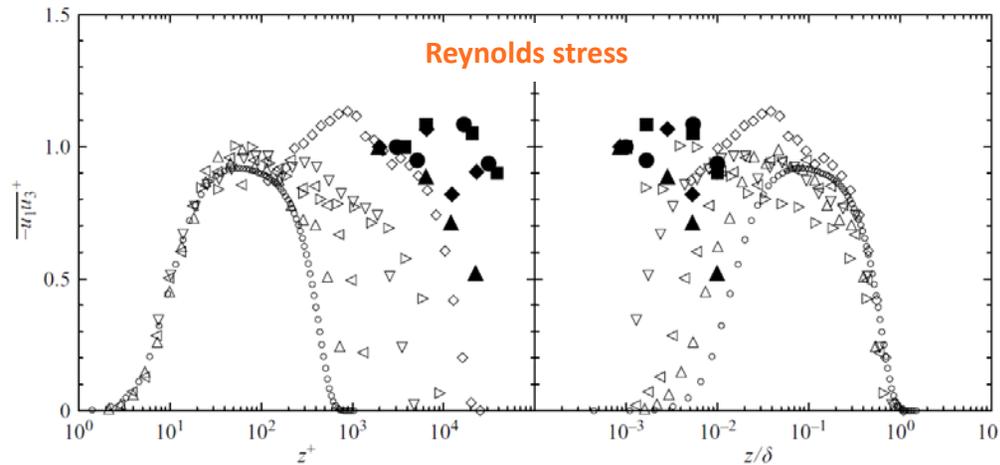
FIGURE 4. Streamwise turbulence intensities. Solid symbols are atmospheric data. Open symbols are laboratory data; $\delta^+ = 689$ (\circ)—Spalart (1988); 1335 (Δ), 2217 (∇), 13 490 (\triangleright)—DeGraaff & Eaton (2000); 23 013 (\diamond)—Fernholz *et al.* (1995). Solid, dashed and dot-dashed lines are smooth-wall similarity formulations (equation (2.1)) that are also valid for rough walls in the outer region. Lighter dashed lines are similarity formulation for $\delta^+ = 3.1 \times 10^6$ if boundary-layer thickness is 50% larger or smaller.



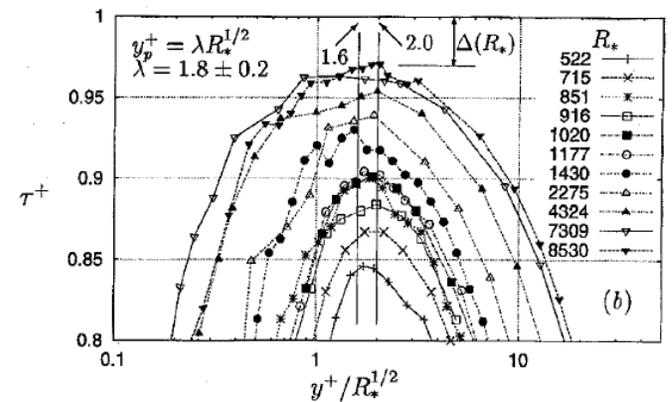
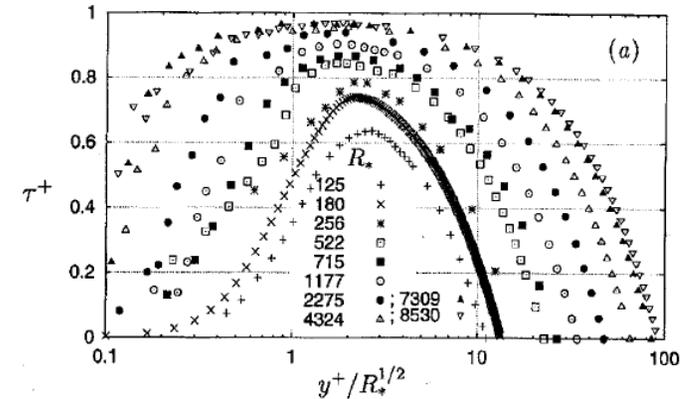
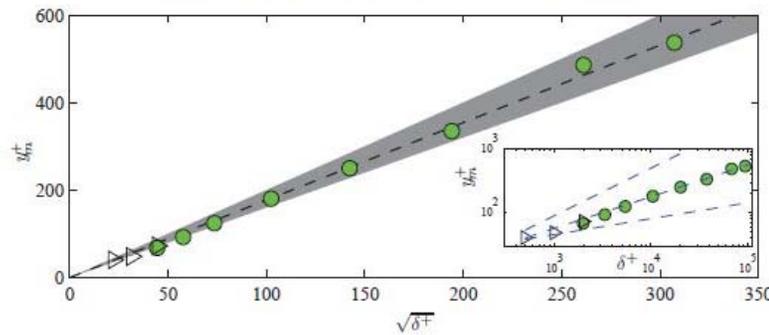
Kunkel & Marusic, *J. Fluid Mech.* 2006
 Marusic *et al*, *Phys. Fluids* 2010
 Fernholz & Finley, *Prog. Aero. Sci.* 1996

Non-simple scaling of the second order moments
 (made obvious with increasing Reynolds number)

SCALING OF THE SECOND ORDER STATISTICS: UV



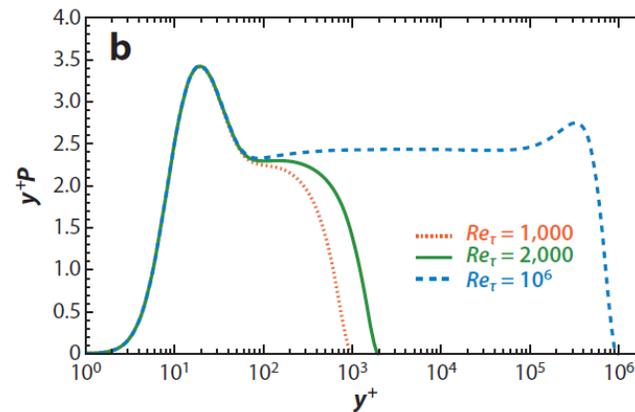
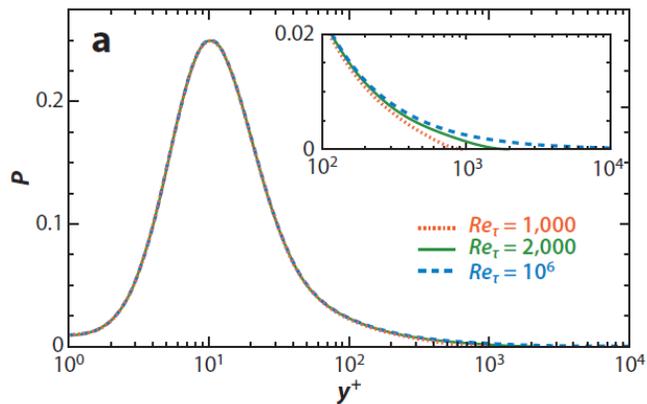
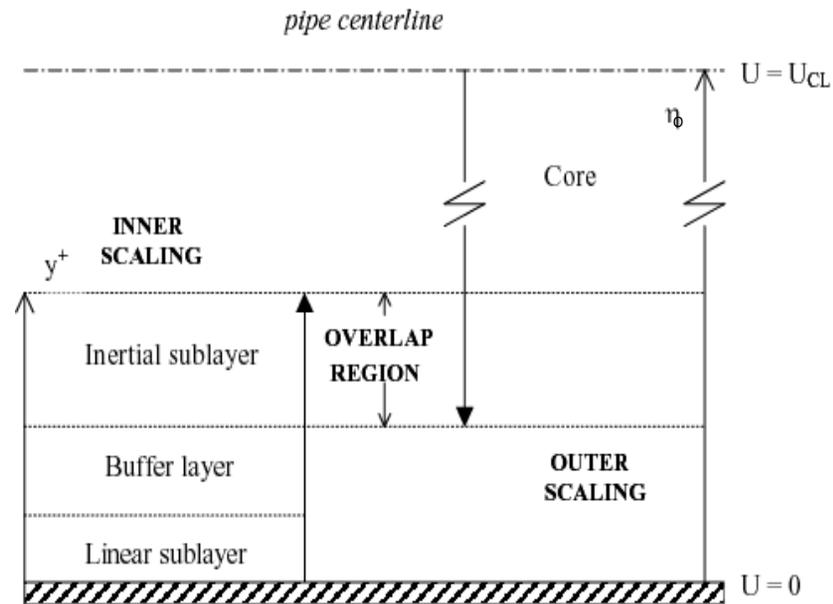
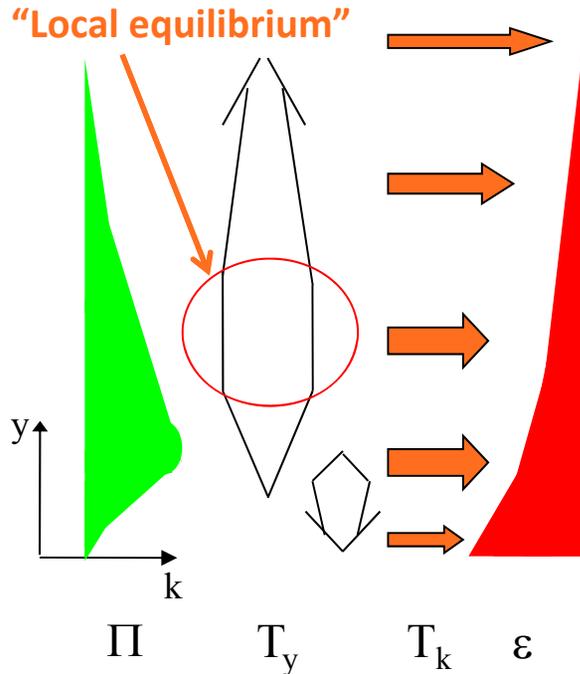
C. Chin, J. Philip, J. C. Klewicki, A. Ooi and I. Marusic



Kunkel & Marusic, *J. Fluid Mech.* 2006
 Sreenivasan & Sahay, 1997
 Chin et al, *J. Fluid Mech.* to appear

Reynolds number variation of the peak Re stress location leads to inconsistencies in the classical scaling

IMPLICATIONS FOR TURBULENCE PRODUCTION



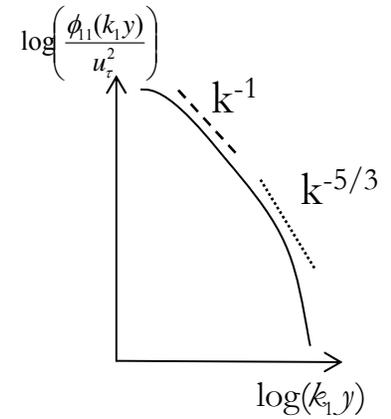
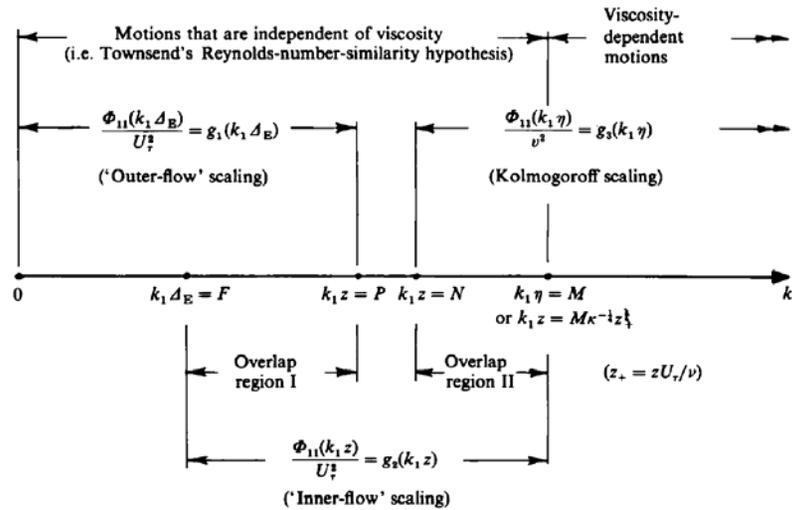
Peak production corresponds with near-wall cycle only for low Reynolds number

Marusic, Mathis & Hutchins,
Int J. Heat Fluid Flow, 2010

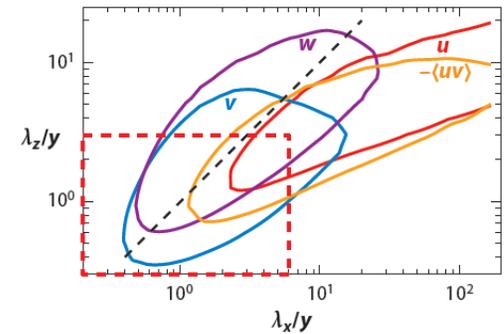
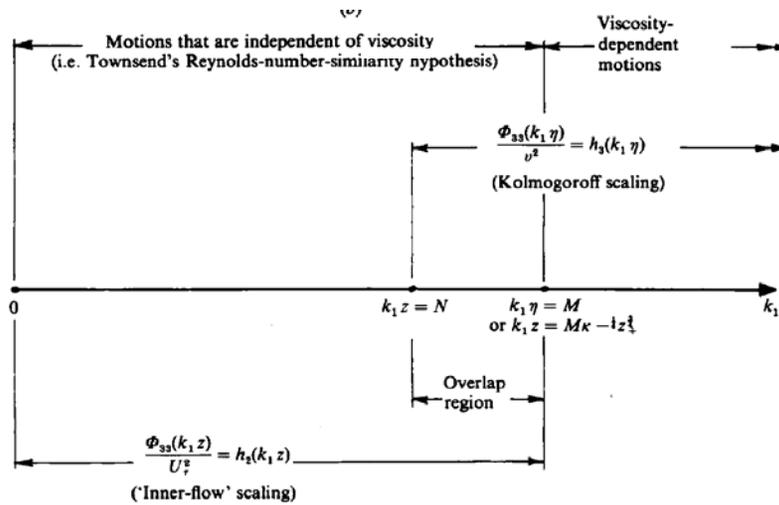
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SPECTRAL ARGUMENTS

Wall-parallel

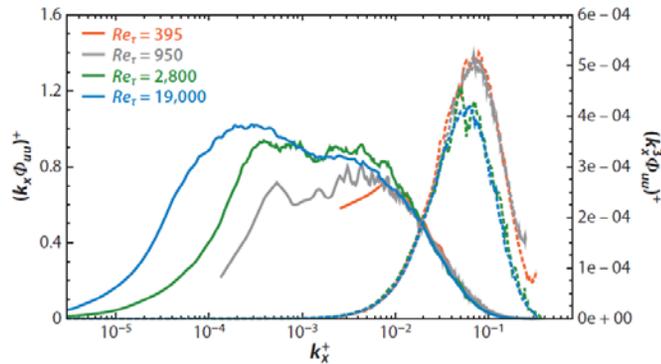


Wall-normal

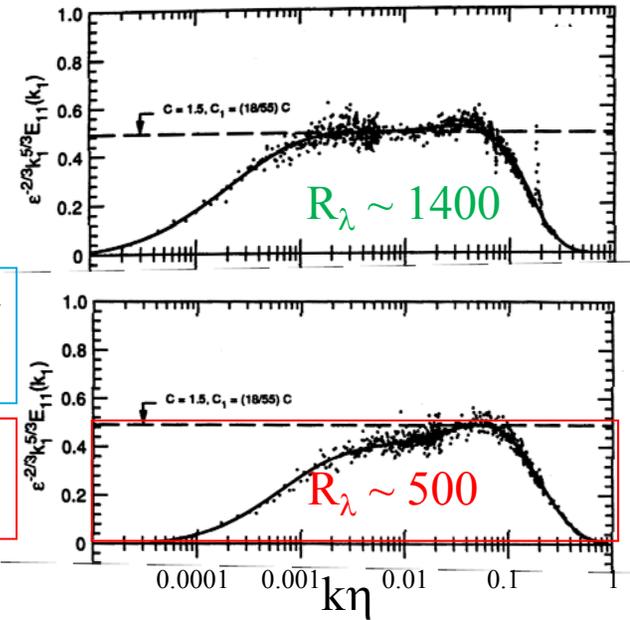
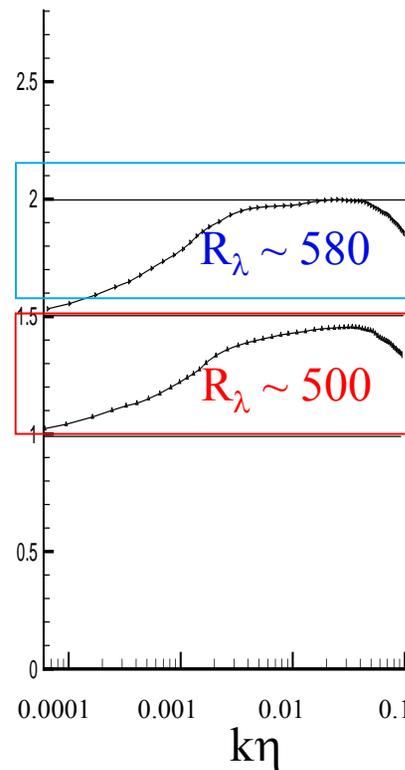


KOLMOGOROV-TYPE SCALING?

$S^*/\eta = 110$ $S^*/\eta = 160$ $S^*/\eta = 140$ $S^*/\eta = 440$



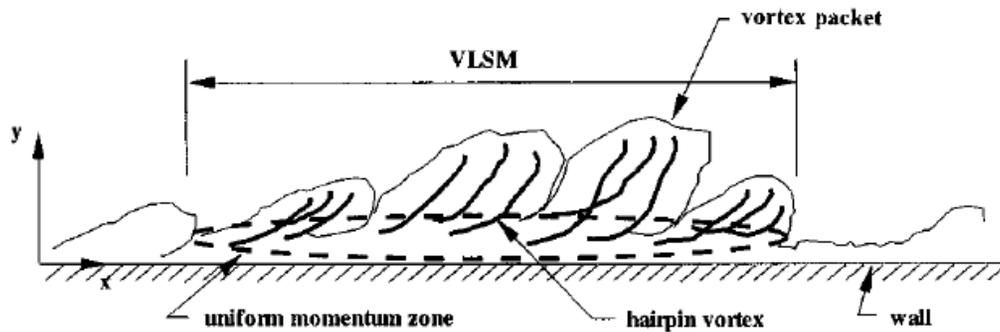
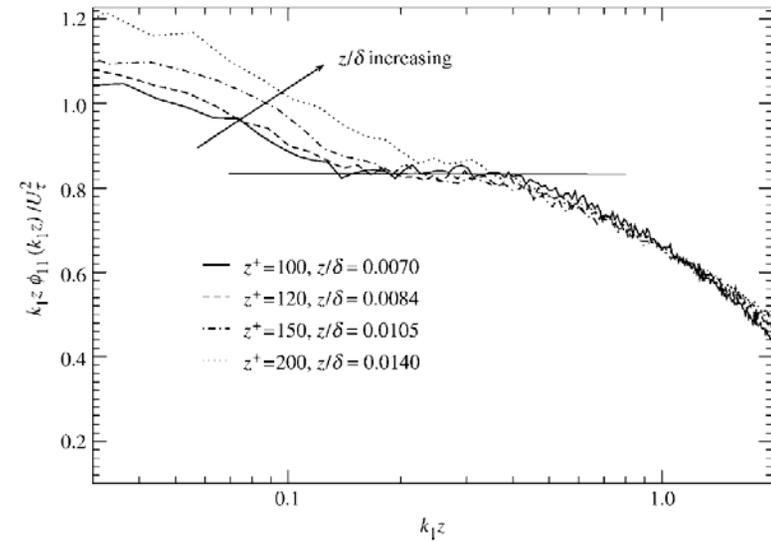
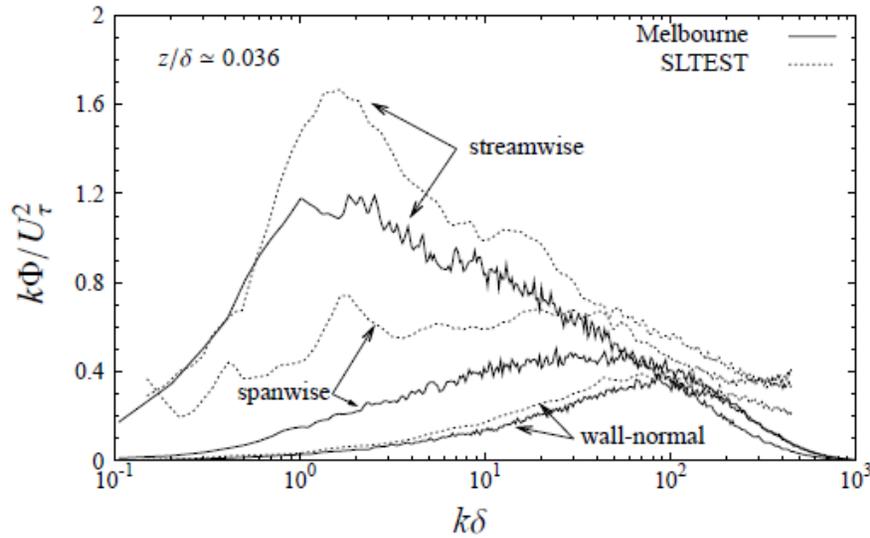
(Some sort of) scale separation is achieved



Superpipe, $Re_D = 1 \times 10^6$
($y/R < 0.1$)

Turbulent boundary layer
($y/R = 0.1$)

k^{-1} SCALING?



Marusic, Mathis & Hutchins, *Int J. Heat Fluid Flow*, 2010
 Nickels, Marusic, Hafez, Hutchins & Chong, *Phil. Trans. R. Soc A* 2007
 Kim & Adrian, *Phys. Fluids*, 2000

LOGARITHMIC SCALING OF u'^2

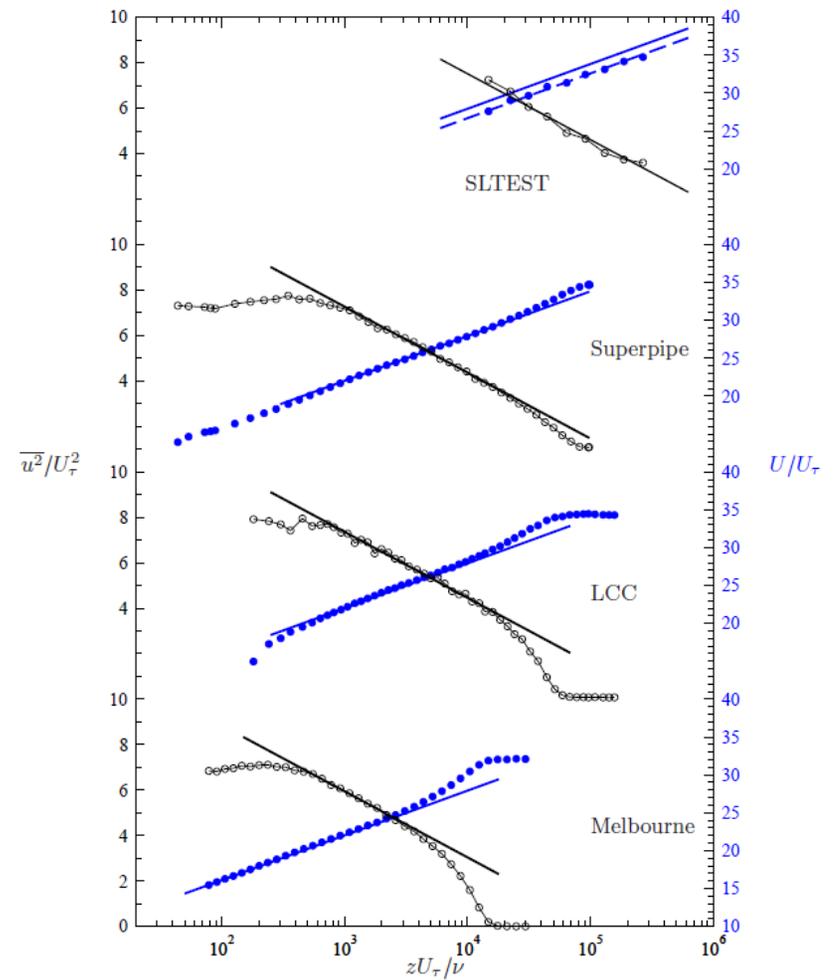
Attached eddy hypothesis (more later) can be used to show that

$$\frac{\overline{u^2}}{U_\tau^2} = C_1 + D_1 \log\left(\frac{\delta}{z}\right)$$

$$\frac{\overline{v^2}}{U_\tau^2} = C_2 + D_2 \log\left(\frac{\delta}{z}\right)$$

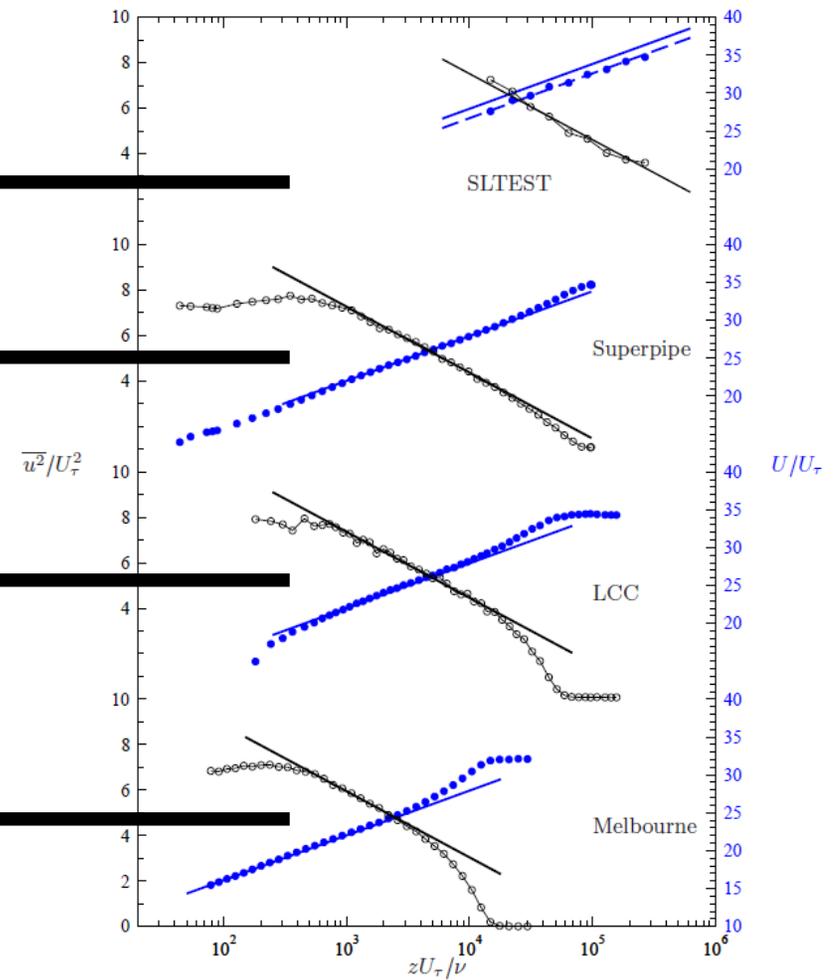
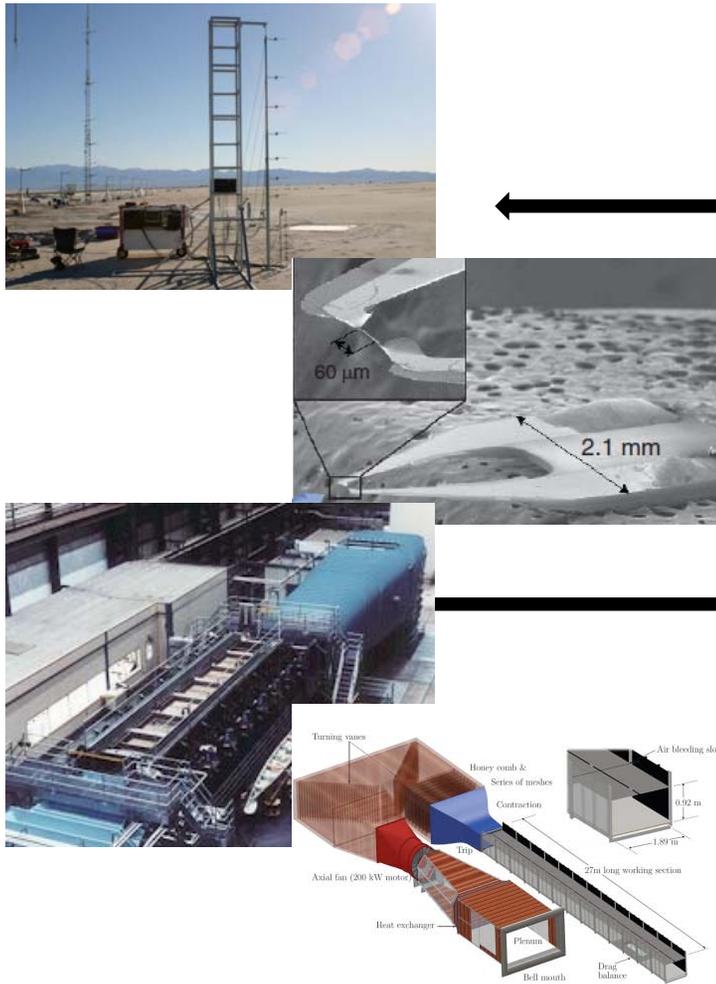
$$\frac{\overline{w^2}}{U_\tau^2} = C_3,$$

Marusic, Monty, Hultmark & Smits, *J. Fluid Mech.* 2013



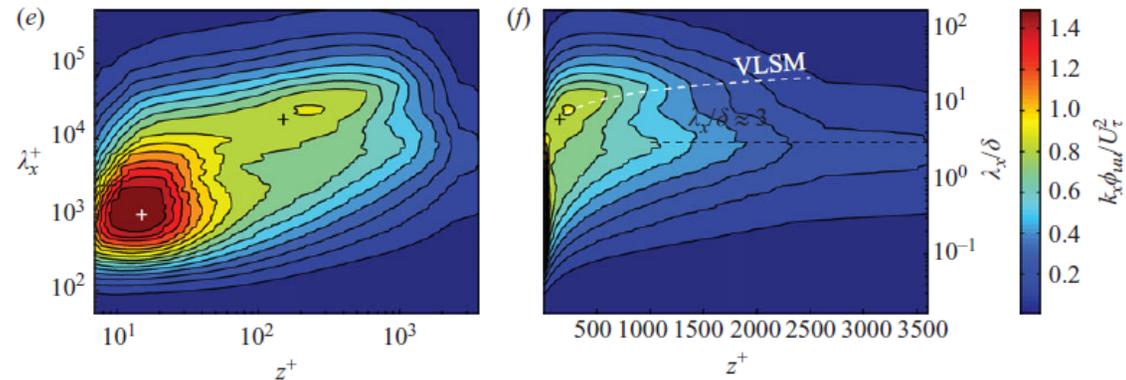
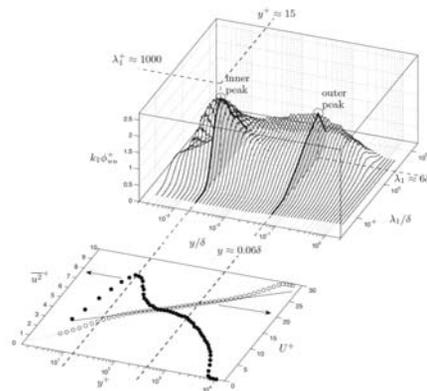
LOGARITHMIC SCALING OF u'^2

Marusic, Monty, Hultmark & Smits, *J. Fluid Mech.* 2013



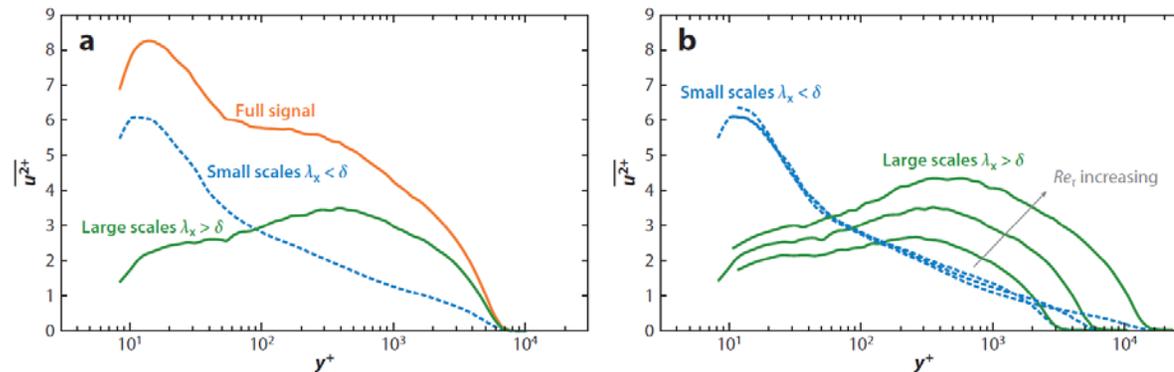
Hultmark et al, *Phys. Rev. Letters* 2012
<http://www.mech.unimelb.edu.au/fluid-mechanics/walter-bassett.html>
https://portal.navfac.navy.mil/portal/page/portal/navfac/navfac_ww_pp/navfac_hq_pp/navfac_bdd_pp/eul/projects/large_cavitation_channel/index.html

FOOTPRINT OF THE SUPERSTRUCTURES/VLSMS



Spectral footprint of the VLSMs is a secondary peak in the streamwise spectrum

- becomes increasingly energetic as the Reynolds number increases
- cohabits with the near-wall peak at low Reynolds number

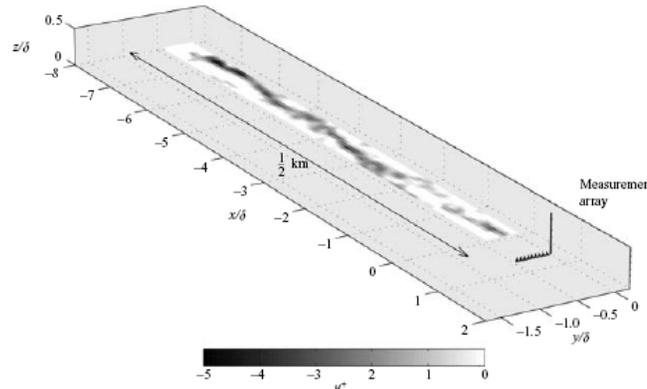
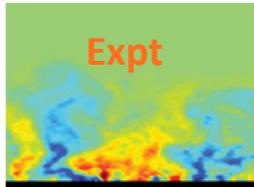


Important implications for scaling & modeling

Things get easier at higher Re!

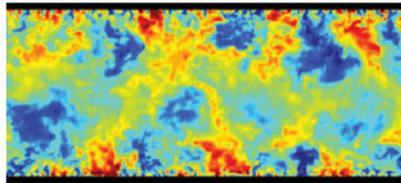
OBSERVATIONS OF SUPERSTRUCTURES/VLSM

Boundary layer



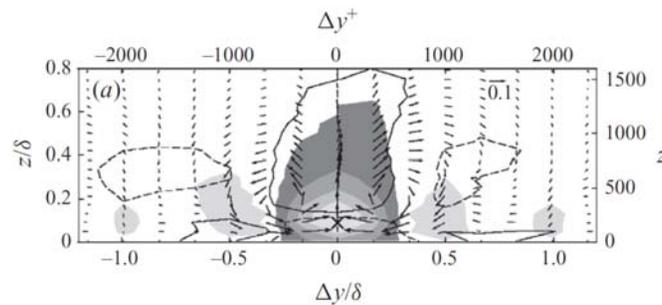
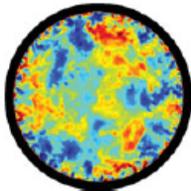
Reconstructed u signal of the superstructures in the ASL

Channel

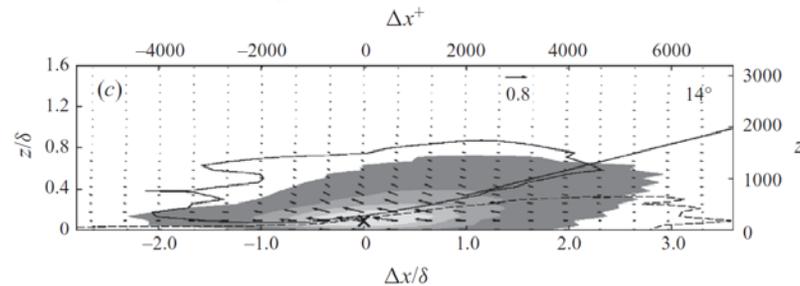


LES

Pipe



Conditionally averaged u signal of the VLSM (filled contours) and envelope of the small scale activity (lines) from LES



MODULATION OF ALL VELOCITY COMPONENTS

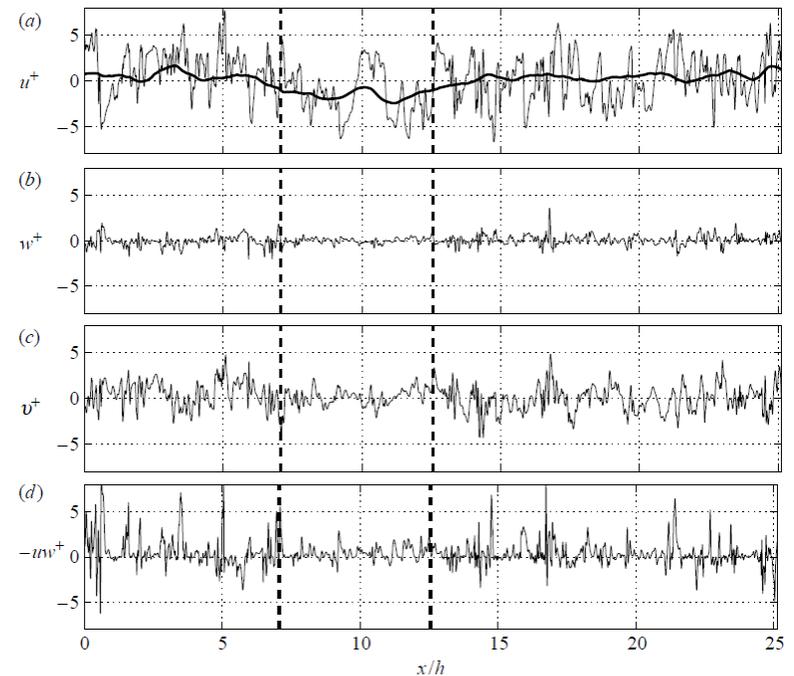
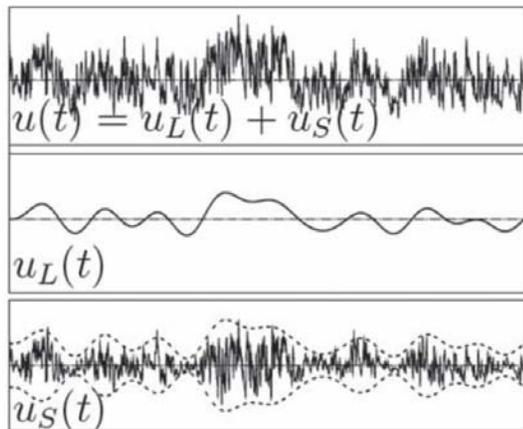
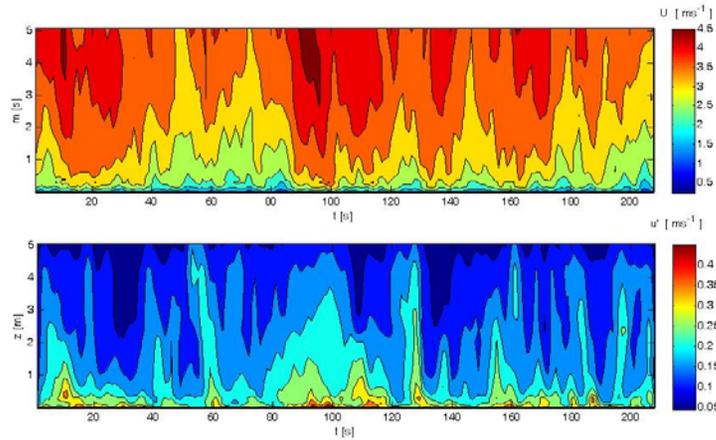
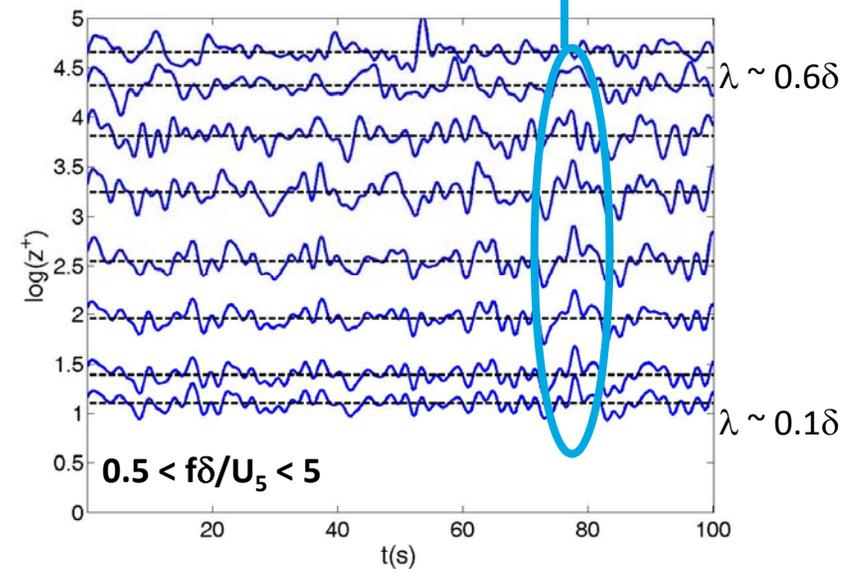
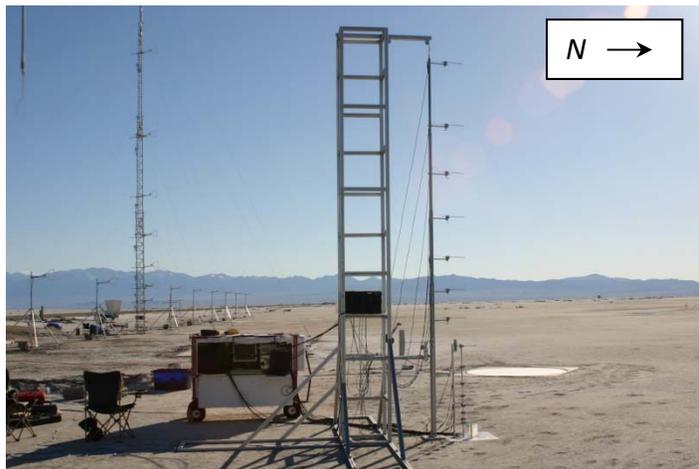
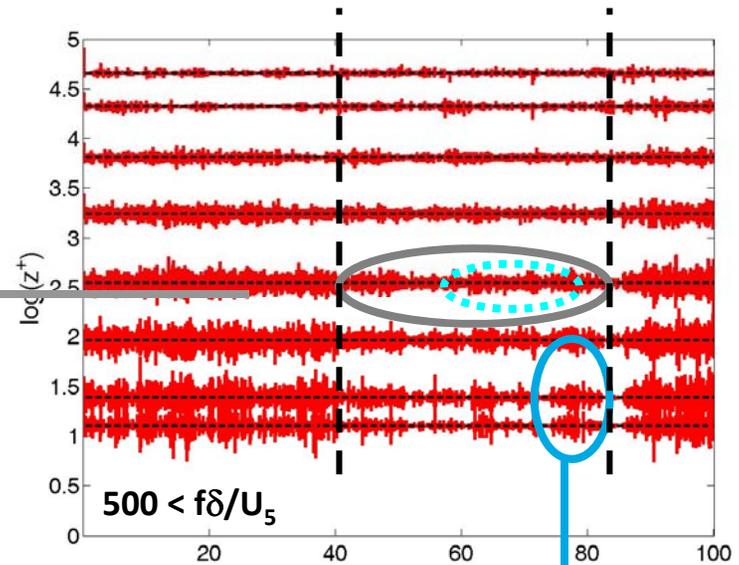
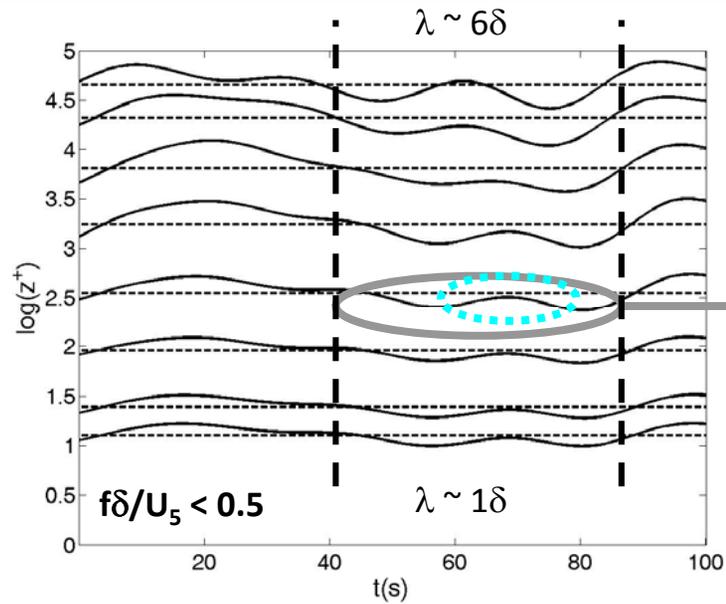


Figure 6. Example fluctuating velocity signals from the $Re_\tau \approx 950$ channel of del Álamo *et al.* (2004) at $z^+ = 15$. (a) Fluctuating u component, thicker line shows large-scale component, (b) wall-normal w fluctuation, (c) spanwise v fluctuation, (d) Reynolds shear-stress fluctuation. Dashed vertical lines show region of negative large-scale fluctuation.

A HIERARCHY OF SCALE MODULATION

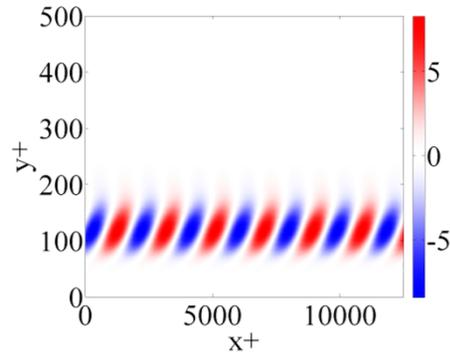


AMPLITUDE MODULATION AS A LINEAR PHENOMENON

(k, n, c, A)

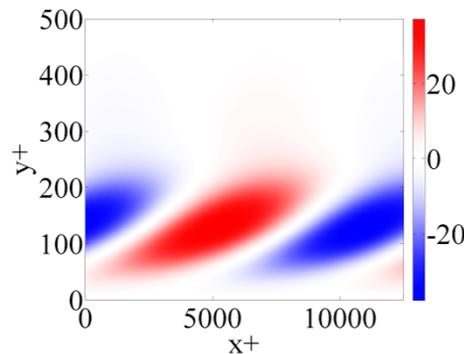
k_1

$(\pm 6, \pm 6, 2/3, -1.00i)$



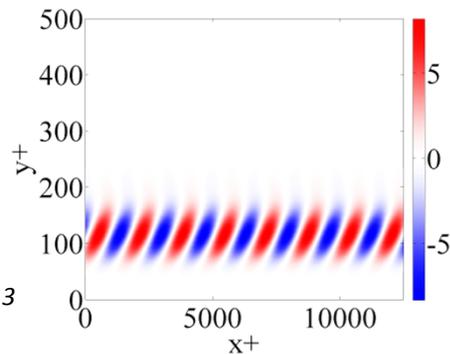
k_2

$(\pm 1, \pm 6, 2/3, 4.50)$



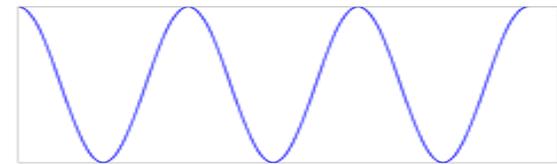
k_3

$(\pm 7, \pm 12, 2/3, 0.83i)$

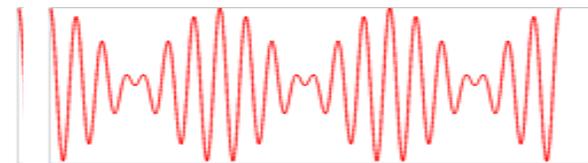


Sharma & McKeon, *J. Fluid Mech.*, 2013

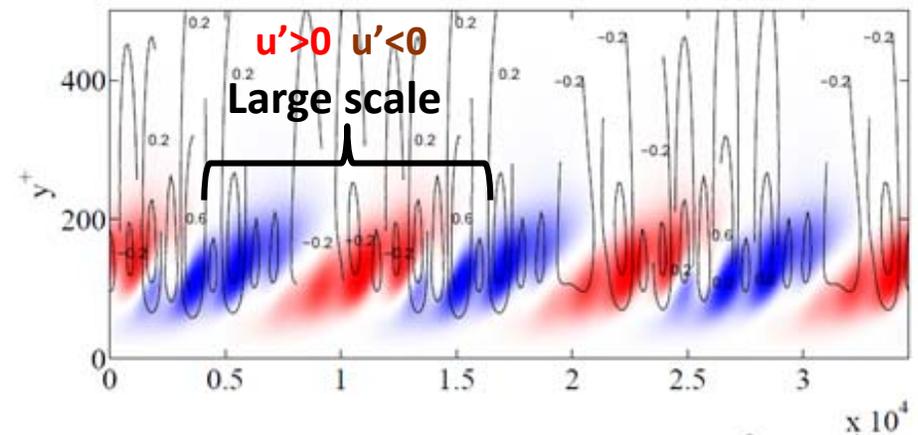
k_2



$k_1 + k_3$

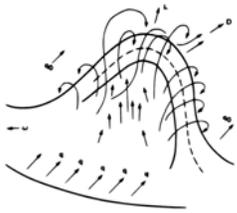


$k_1 + k_2 + k_3$

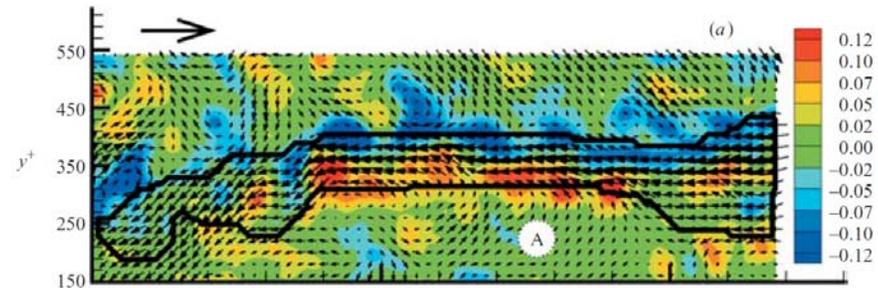
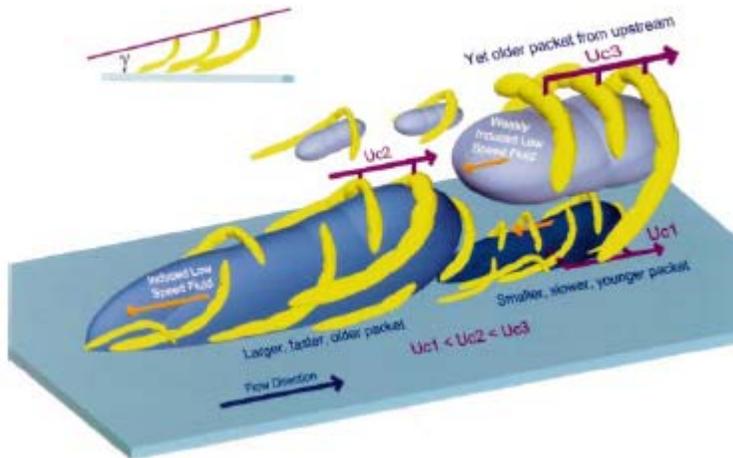
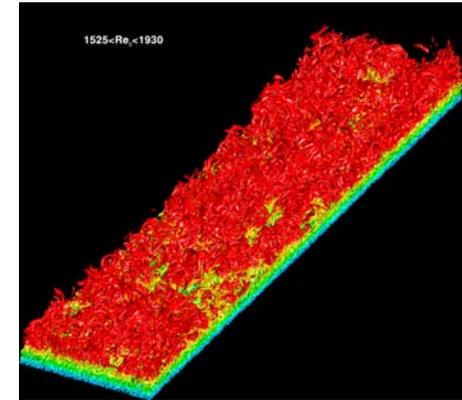
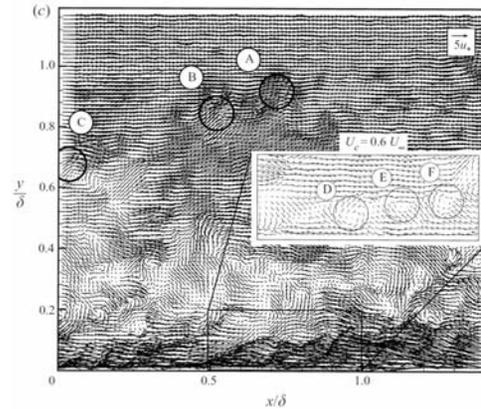


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HAIRPIN VORTEX PARADIGM



Theodorsen's hairpin vortices identified* in experiments and DNS



“Packets of packets”?

Clear Re stress organization around packets

Theodorsen, Proc. Midwest Mechanics Symp. 1952
Adrian, Meinhart & Tomkins, J. Fluid Mech. 2000
Wu & Moin, Phys. Fluids 2010
Kim & Adrian, Phys. Fluids 1999
Ganapathisubramani, Longmire & Marusic, J. Fluid Mech. 2003

BUT is this a low Re phenomenon?

NEAR-WALL ACTIVITY

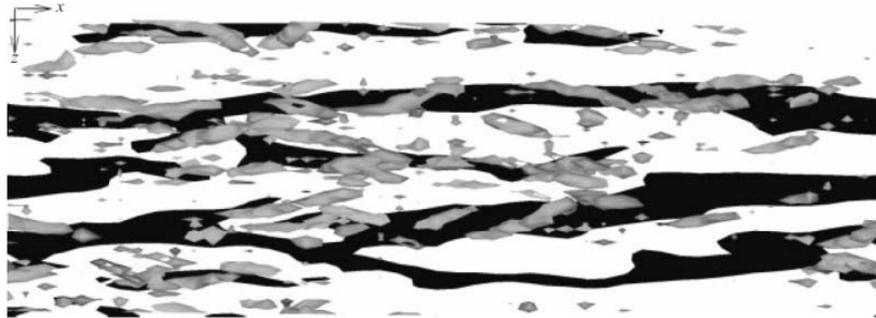
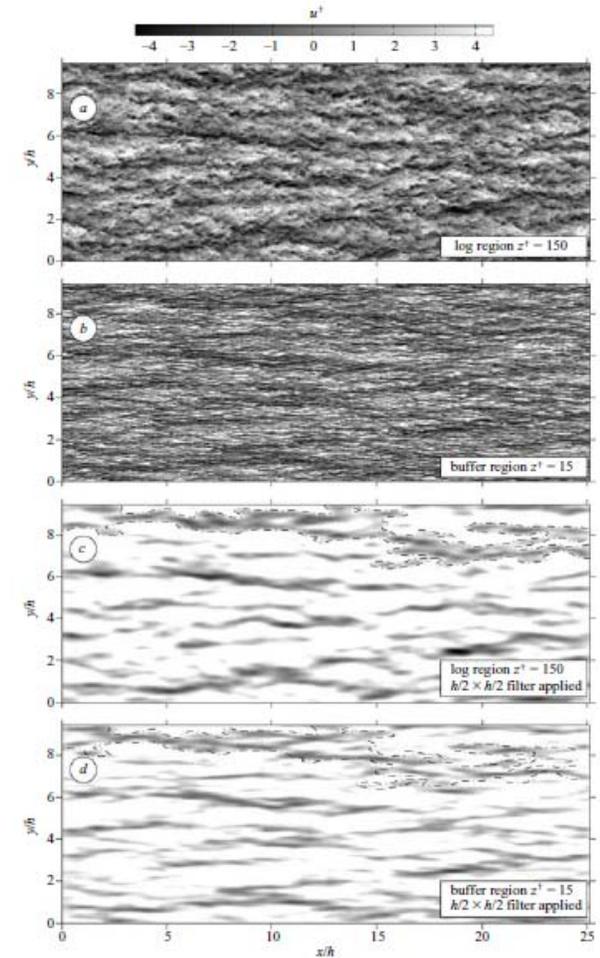
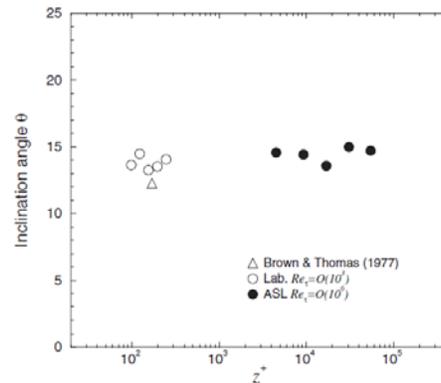
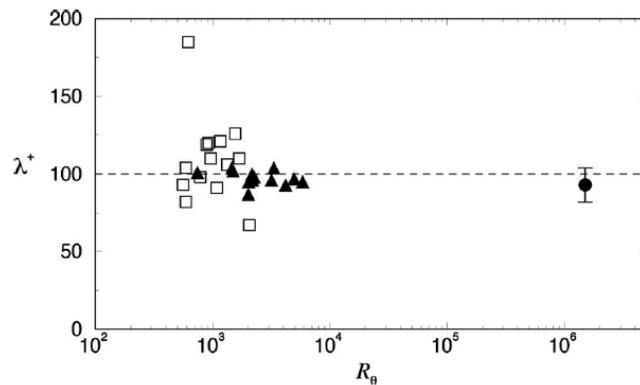


FIGURE 1. Top view of the near-wall region covering $(x^+, z^+) = (1400, 450)$ in the streamwise and spanwise directions. Lifted low-speed streaks (black) denote $u' < 0$ at $y^+ = 20$ and streamwise vortices (grey-shaded) are indicated by the λ_2 vortex definition (Jeong & Hussain 1995) in the region $0 < y^+ < 60$.



Reynolds number insensitivity of near-wall structure BUT also footprint of log layer structures (VLSM)

Schoppa & Hussain, *J. Fluid Mech.* 2002
 Marusic et al, *Phys. Fluids* 2010
 Heuer & Marusic, *Phys. Rev. Letters* 2007
 Hutchins & Marusic, *J. Fluid Mech.* 2007

MODELING CONCEPTS

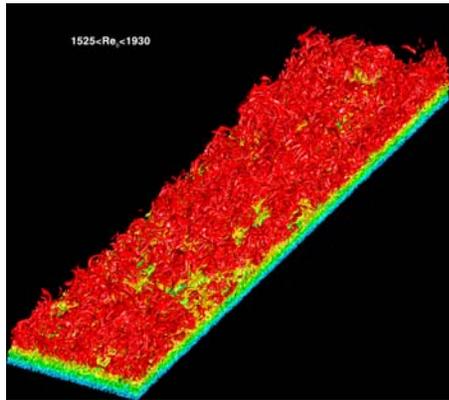
Review of (some) models for regions of the flow

What do the models tell us about the math?

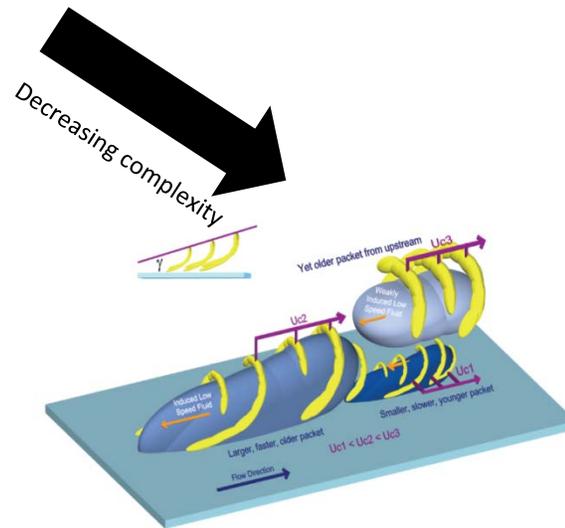
Structure-based models vs. energy norms

- Attached eddy hypothesis (Townsend, Perry, Marusic, et al)
- Near-wall structure, minimal flow unit
- Structure implied by the mean momentum balance
- Properties of the linear(ized) N-S operator
 - Non-normality, transient growth and disturbance amplification
 - Resolvent analysis
- Importance (or not) of exact solutions

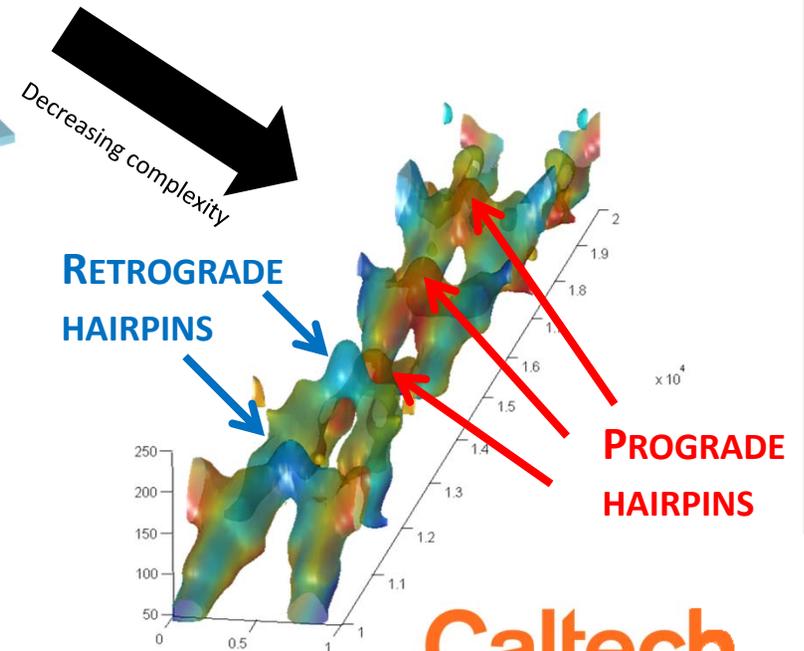
DECONSTRUCTING WALL TURBULENCE



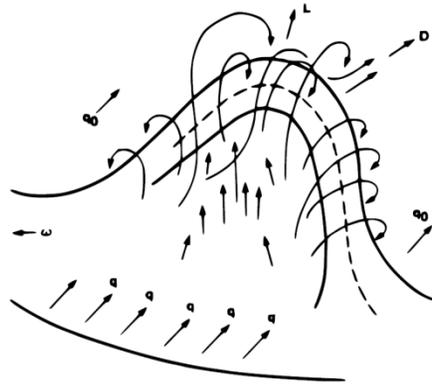
Wu & Moin, <http://ctr.stanford.edu>,
J. Fluid Mech. 2008



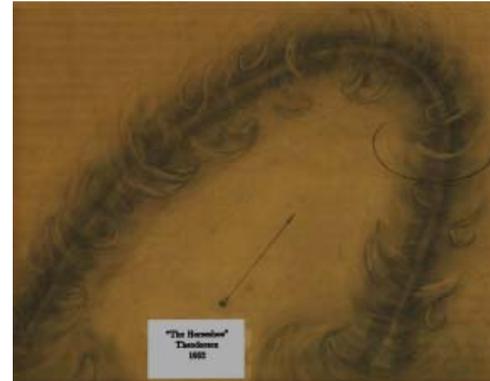
Adrian, Meinhart & Tomkins,
J. Fluid Mech. 2000



ATTACHED EDDY HYPOTHESIS



Theodorsen vortex (1952)

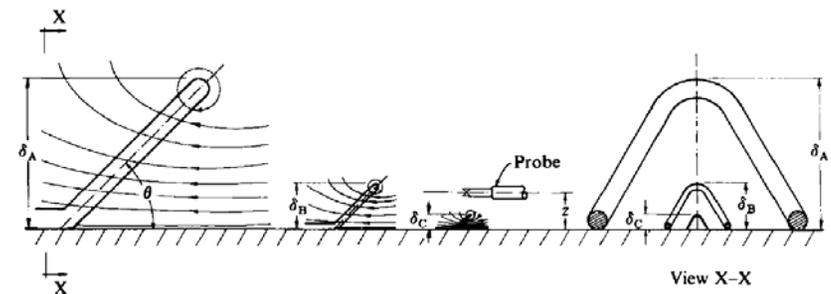


Sketch (Weske) of Theodorsen vortex (1952)

“...the main energy-containing motion of a turbulent wall-bounded flow may be described by a random superposition of such (attached) eddies of different sizes, but with similar velocity distributions.” (AAT, 1956, 1976)

eddies of size y per unit wall area = A/y

i.e. pdf inversely proportional to distance from the wall, or more accurately, eddy size



Townsend, 1956, 1976

Perry, Henbest & Chong, J. Fluid Mech. 1986

THE NEAR-WALL CYCLE

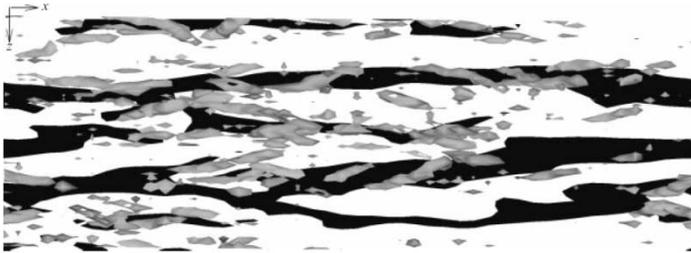
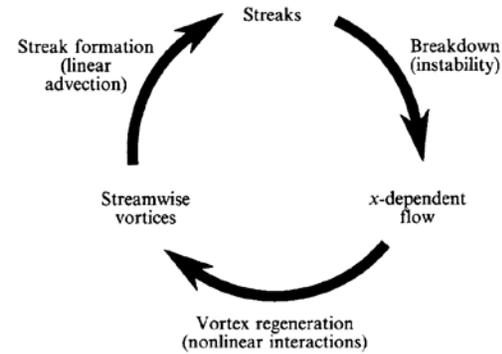


FIGURE 1. Top view of the near-wall region covering $(x^+, z^+) = (1400, 450)$ in the streamwise and spanwise directions. Lifted low-speed streaks (black) denote $u' < 0$ at $y^+ = 20$ and streamwise vortices (grey-shaded) are indicated by the λ_2 vortex definition (Jeong & Hussain 1995) in the region $0 < y^+ < 60$.



Well-developed exact solutions for near-wall activity, captured in minimal flow unit"

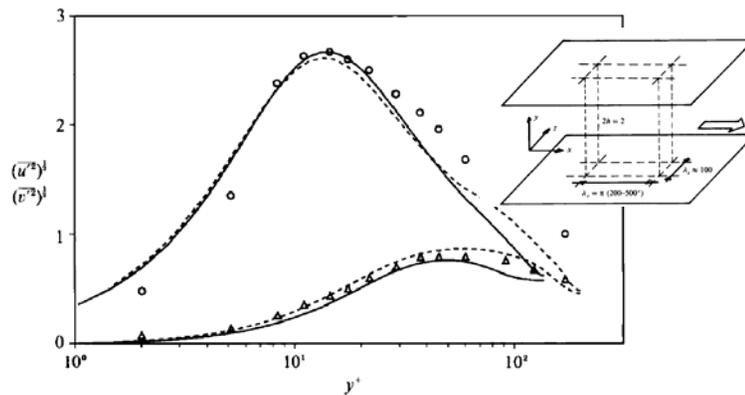


FIGURE 10. Near-wall turbulent intensities in wall coordinates. Solid lines: $Re = 3000$, $\lambda_x \times \lambda_z = \pi \times 0.3\pi$; dashed: $Re = 5000$, $0.6\pi \times 0.18\pi$. Symbols as in figure 9.

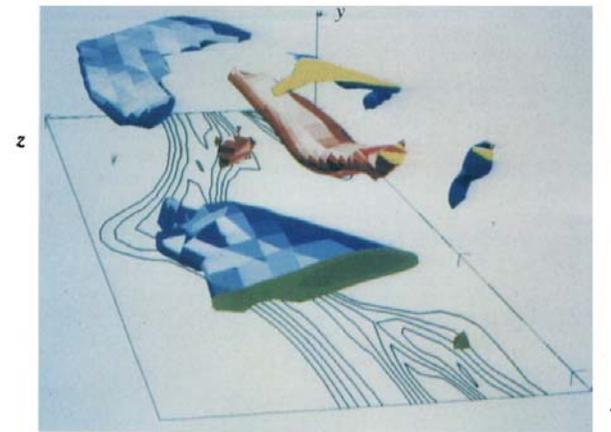
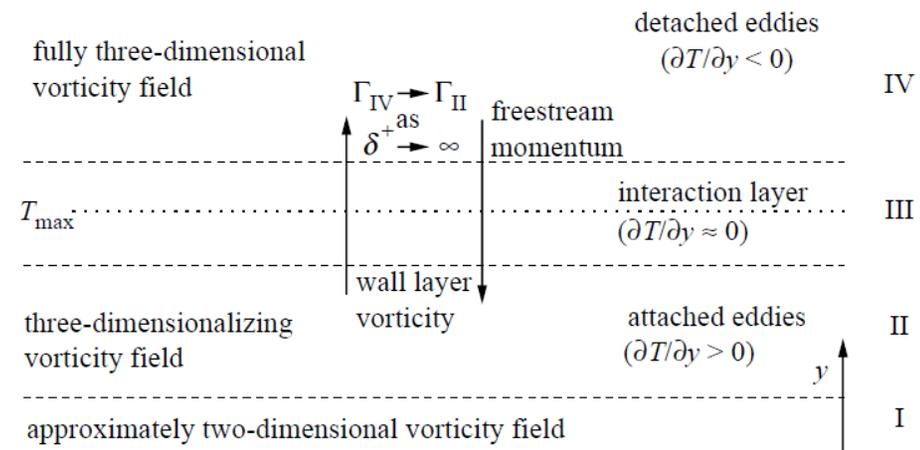
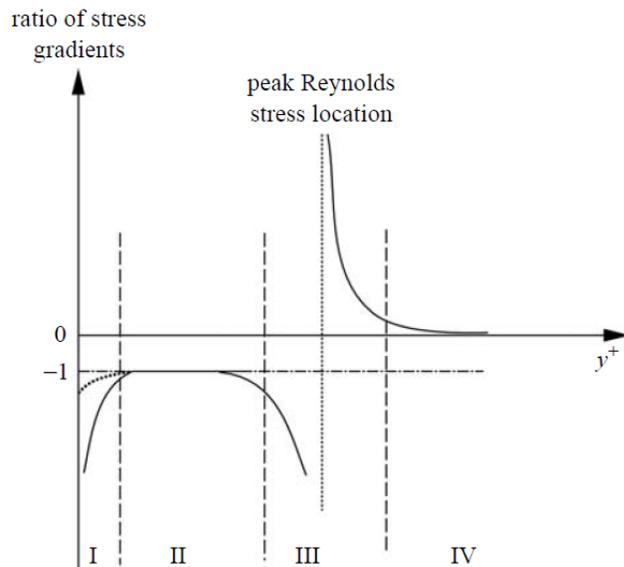


FIGURE 21. Three-dimensional view of the flow field in figure 20. Flow coming out of the page. Horizontal plane is $y^+ = 10$, with isolines for $\omega_x \geq -2.5$ (by 0.25). Surfaces are $\omega_x = -0.75$ (blue), and $\omega_x = 1.1$ (red), between $y^+ = 20$ and 50. Figure displays full horizontal extent of computational box.

Schoppa & Hussain, *J. Fluid Mech.*, 2002
 Hamilton, Kim & Waleffe, *J. Fluid Mech.*, 1995
 Jimenez & Moin, *J. Fluid Mech.*, 1991

STRUCTURE ADMITTED BY MEAN MOMENTUM BALANCE

$$0 = \frac{1}{\delta^+} + \frac{d^2 U^+}{dy^{+2}} - \frac{d\langle uv \rangle^+}{dy^+}$$



TRANSIENT GROWTH AND NON-NORMALITY

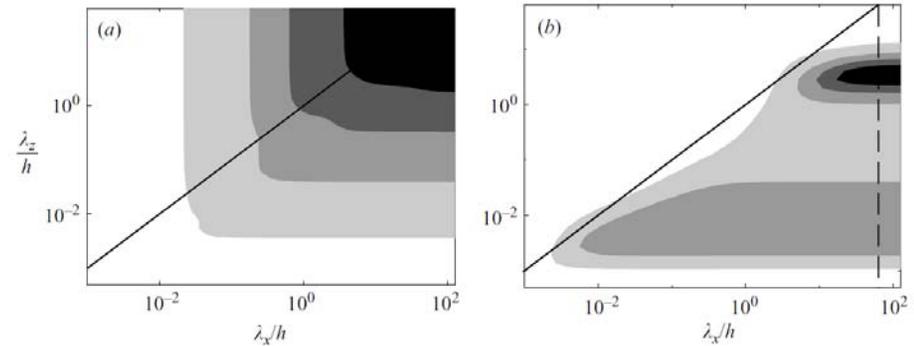
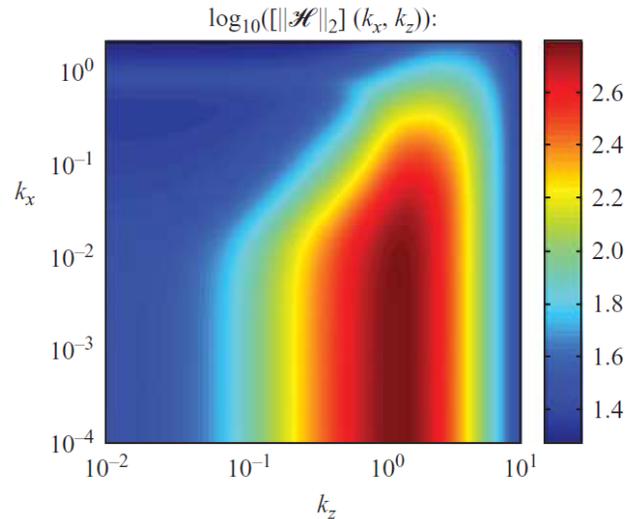


FIGURE 2. (a) Imaginary part $\sigma_{l,1}h/u_\tau$ of the least damped eigenvalue of (2.3)–(2.4), represented as a function of λ_x/h and λ_z/h . The levels represented are, from dark to light, $-1(\times 10)-1000$. (b) Maximum transient growth G as a function of λ_x/h and of λ_z/h . The levels represented are, from light to dark, $1.5(+1)4.5$. The solid straight line is $\lambda_x = \lambda_z$. The dashed vertical line is $\lambda_x = 60h$. In both figures $Re_\tau = 2 \times 10^4$.

Input-output transfer function H

$k_x=0$ mode is always most amplified

Two spanwise wavelengths can be identified

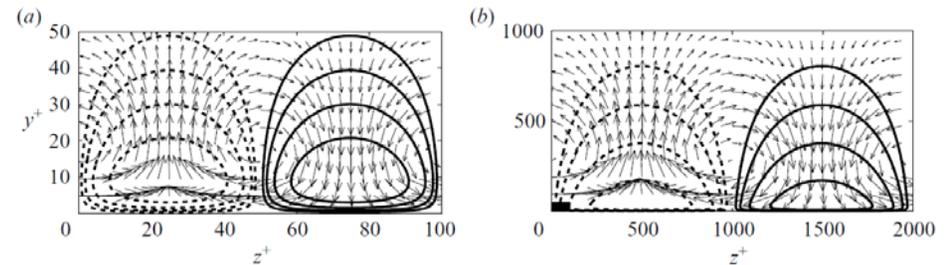


FIGURE 5. Transverse view of the optimal solution for $\lambda_x = 60h$, $Re_\tau = 2 \times 10^4$. The arrows indicate the initial (v, w) field and the longest one is unity in the same arbitrary units used for u . The contours represent the most amplified streamwise velocity and the levels represented are $0.125(\times 2)$. The solid contours are $u > 0$ and the dashed ones are $u < 0$. (a) $\lambda_z^+ = 100$. (b) $\lambda_z^+ = 2000$; the small solid black box in the bottom left corner is the size of the axes in (a).

IMPORTANCE OF LINEAR COUPLING

$$\frac{d}{dt} \begin{bmatrix} \hat{v} \\ \hat{\omega}_y \end{bmatrix} = [A] \begin{bmatrix} \hat{v} \\ \hat{\omega}_y \end{bmatrix} + \begin{bmatrix} \mathcal{N}_v \\ \mathcal{N}_{\omega_y} \end{bmatrix} \quad [A] = \begin{bmatrix} L_{os} & 0 \\ L_c & L_{sq} \end{bmatrix}$$

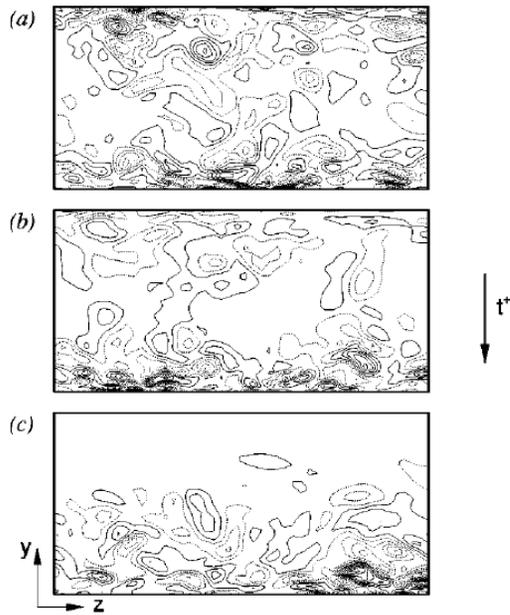


FIG. 2. Contours of streamwise vorticity in $y-z$ plane: (a) $t^+ = 0$; (b) $t^+ = 20$; (c) $t^+ = 200$. $-80 < \omega_x < 80$ with 18 contour levels. Note that $L_c = 0$ only in the upper-half of the channel.

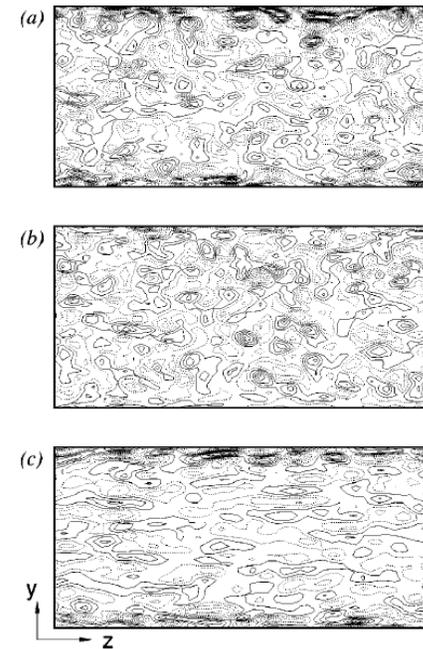
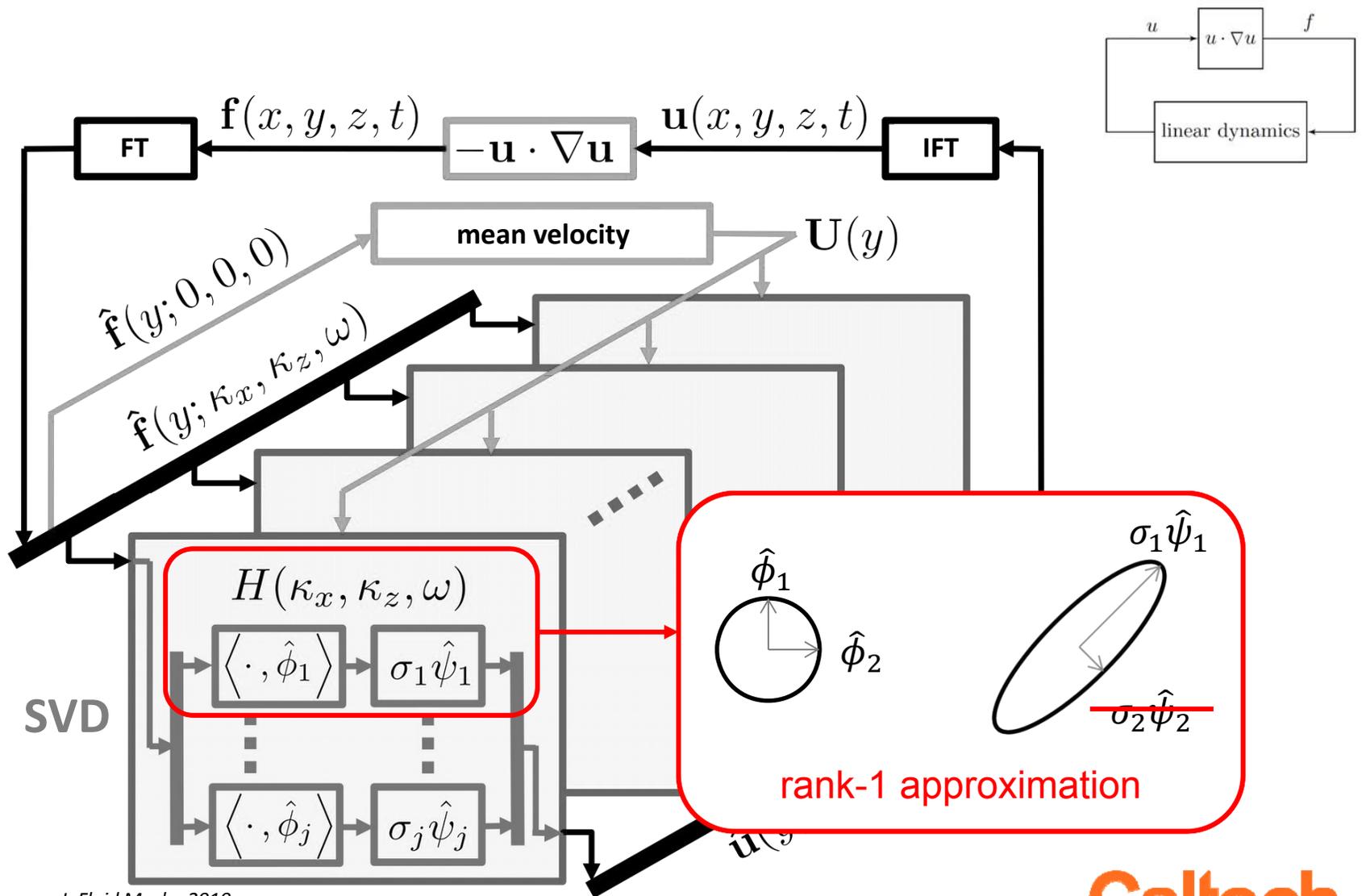


FIG. 4. Contours of streamwise vorticity in $y-z$ plane at $t^+ = 20$, started from an initial random field: (a) Case 1, regular turbulent flow; (b) Case 2, without the linear coupling term, L_c ; (c) Case 3, without the nonlinear terms. Contour levels are the same as Fig. 2.

Linear coupling is responsible for streamwise streaks, nonlinearity for the correct spacing

GAIN-BASED DECOMPOSITION



McKeon & Sharma, *J. Fluid Mech.*, 2010

Moarref, Sharma, Tropp & McKeon, *J. Fluid Mech.*, 2013

EXACT TRAVELING WAVE SOLUTIONS

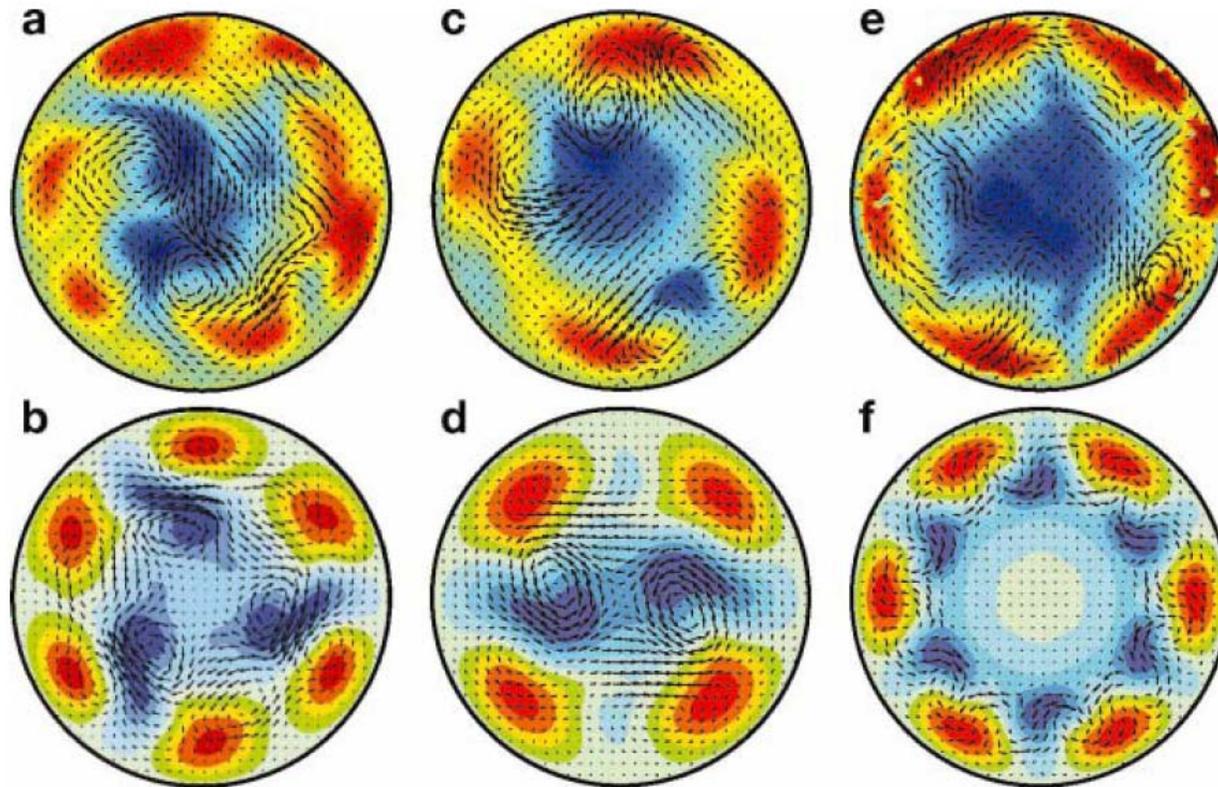


Figure 5

Pairing (*a & b, c & d, e & f*) between flow structures detected in experimental cross sections (*top row*) and numerically determined traveling waves (*bottom row*). The representation of the velocity fields is the same as in **Figure 4**. From Hof, van Doorne et al. (2004).

PROPER ORTHOGONAL (K-L) DECOMPOSITION

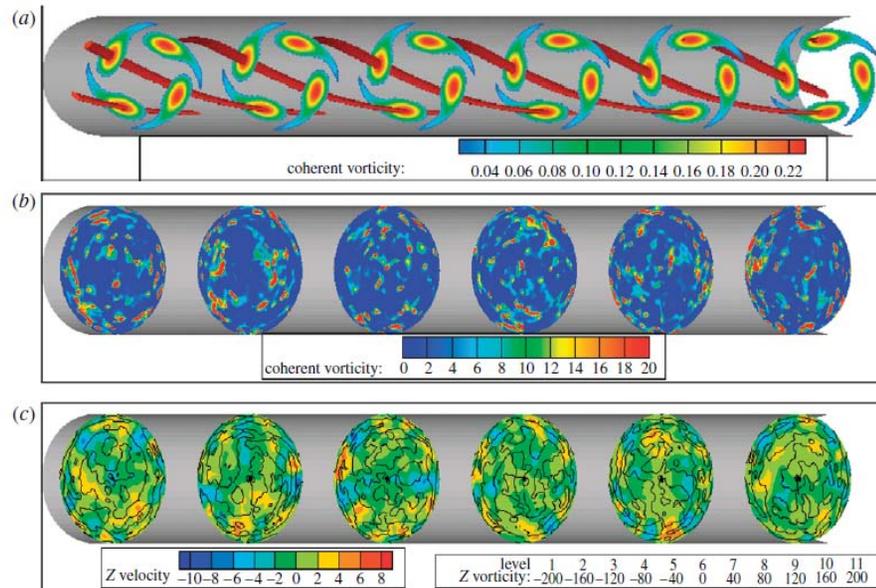


Figure 1. Three-dimensional view of an example (a) Fourier POD mode and (b,c) a method of snapshots POD mode. The Fourier POD mode is the (3, 2, 1) mode, and plotted are six cross sections of coherent vorticity (degree of swirl) and isosurfaces of 95% maximum coherent vorticity. With a streamwise wavenumber of 3, there are three twists of the structure in the streamwise direction. In a similar fashion, cross sections of (b) coherent vorticity and (c) streamwise velocity with contours of streamwise vorticity are shown for the most energetic snapshots mode, showing much more spatial variation in its structure in the streamwise and azimuthal directions compared with the Fourier mode.

Decomposition using an energy norm leads to large d.o.f. representation

	channel $Re_\tau=80$		FV $Re_\tau=80$		channel $Re_\tau=125$		pipe $Re_\tau=150$	
	(m, n, q)	energy (%)	(m, n, q)	energy (%)	(m, n, q)	energy (%)	(m, n, q)	energy (%)
1	(0, 1, 1)	13.00	(0, 3, 1)	6.65	(0, 3, 1)	4.28	(1, 5, 1)	1.62
2	(0, 2, 1)	8.74	(0, 0, 1)	6.08	(0, 1, 1)	3.99	(1, 6, 1)	1.58
3	(0, 3, 1)	7.32	(0, 1, 1)	5.65	(0, 4, 1)	3.27	(1, 3, 1)	1.56
4	(0, 2, 2)	6.93	(0, 2, 1)	5.13	(0, 5, 1)	2.87	(1, 4, 1)	1.42
5	(0, 1, 2)	4.23	(0, 3, 2)	3.67	(0, 4, 2)	2.29	(0, 6, 1)	1.39
6	(0, 3, 2)	4.08	(0, 2, 2)	3.52	(0, 1, 2)	2.20	(1, 7, 1)	1.32
7	(1, 3, 1)	3.27	(0, 4, 1)	3.23	(0, 3, 2)	2.06	(0, 5, 1)	1.28
8	(0, 0, 1)	3.05	(0, 4, 2)	3.13	(0, 2, 1)	1.97	(0, 3, 1)	1.25
9	(1, 3, 2)	2.66	(0, 1, 2)	2.28	(0, 2, 2)	1.88	(1, 2, 1)	1.18
10	(0, 1, 3)	1.88	(1, 3, 1)	1.70	(0, 6, 1)	1.38	(0, 4, 1)	1.11
11	(1, 2, 1)	1.56	(1, 3, 2)	1.50	(0, 5, 2)	1.31	(0, 2, 1)	1.09
12	(0, 0, 2)	1.47	(0, 5, 1)	1.43	(1, 3, 1)	1.25	(2, 4, 1)	1.07
13	(1, 2, 2)	1.37	(1, 4, 1)	1.31	(1, 2, 1)	0.95	(2, 5, 1)	1.00
14	(0, 1, 4)	1.21	(1, 4, 2)	1.28	(1, 4, 1)	0.84	(1, 1, 1)	0.98
15	(0, 2, 3)	1.20	(1, 1, 1)	1.28	(1, 5, 1)	0.83	(2, 7, 1)	0.90

DIRECT LYAPUNOV EXPONENT ANALYSIS

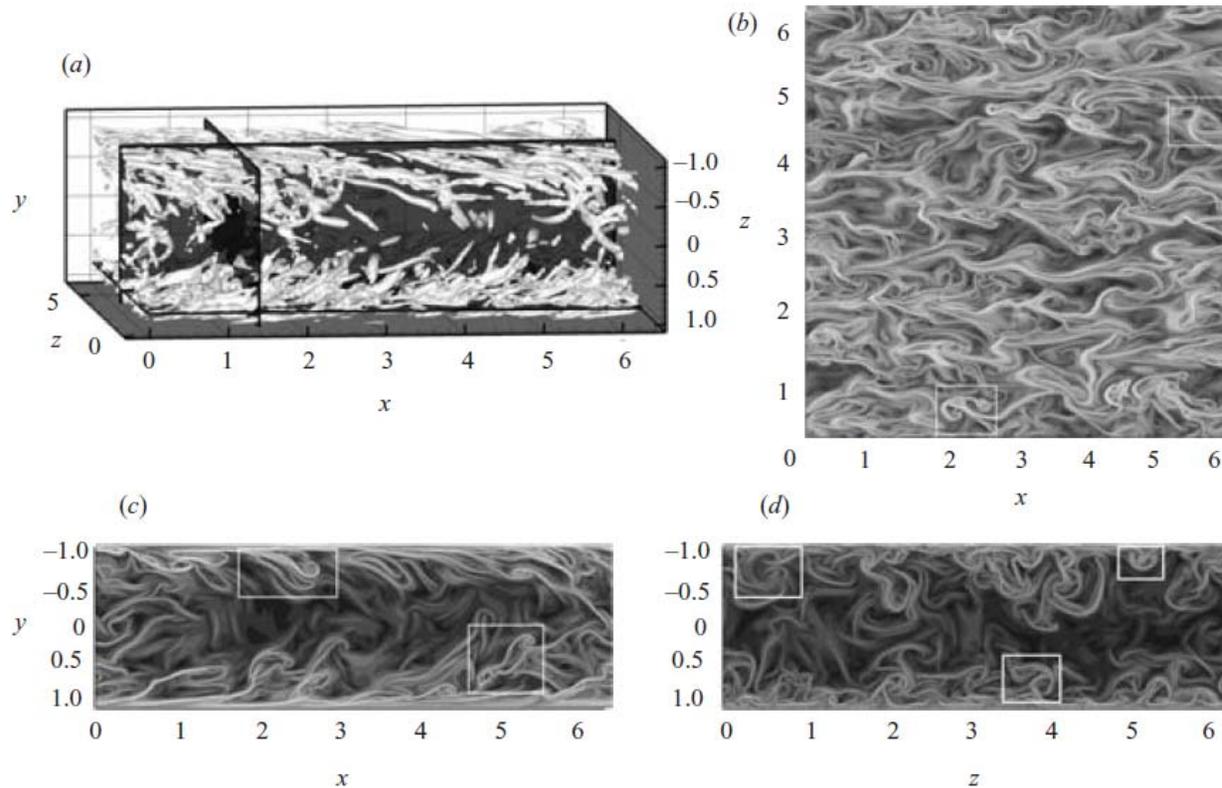


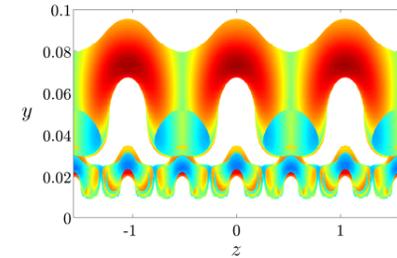
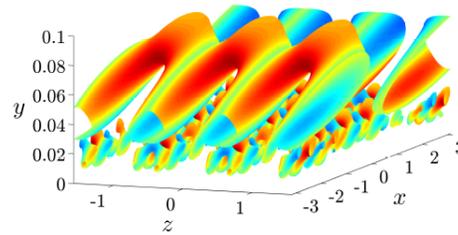
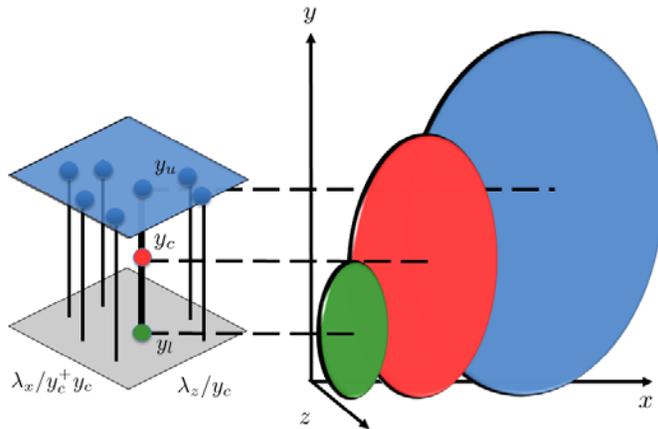
FIGURE 7. Two-dimensional DLE plots of the fully turbulent channel. (a) 1% maximum Q superimposed with the location of the three planes, (b) constant wall-normal cut, (c) constant-spanwise cut ($y^+ = 33$), and (d) constant-streamwise cut.

CHALLENGES & OPPORTUNITIES

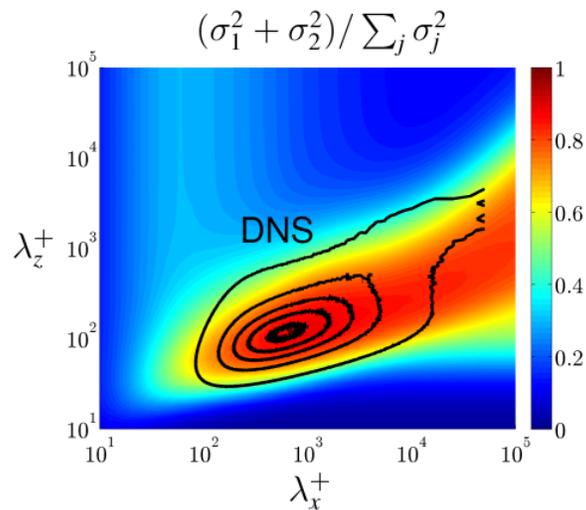
1. Self-sustaining processes. How can these be identified, characterized and compared? What are the connections between such processes at low (transitional) and high Reynolds numbers?
2. Connections between turbulent statistics, coherent structure and the Navier-Stokes equations. How can experimental and numerical observations be reconciled with, or derived from, the equations of motion?
3. Systems approaches to turbulence and control of engineering flows. What mathematical tools are brought into play by such approaches? What system structure is revealed? Can such approaches be used for modeling, prediction and/or control?
4. Low Reynolds number vs. asymptotically high Reynolds number. Which dynamics and phenomena retain their dynamical significance, and which are local phenomena? What can be learned from the opposite extreme in Reynolds number?
5. Modeling, prediction and control. How do we leverage fundamental understanding towards practical advances in engineering applications?

Challenges/opportunities

“SELF-SIMILAR”, “SELF-SUSTAINING”, “ROBUST” ...



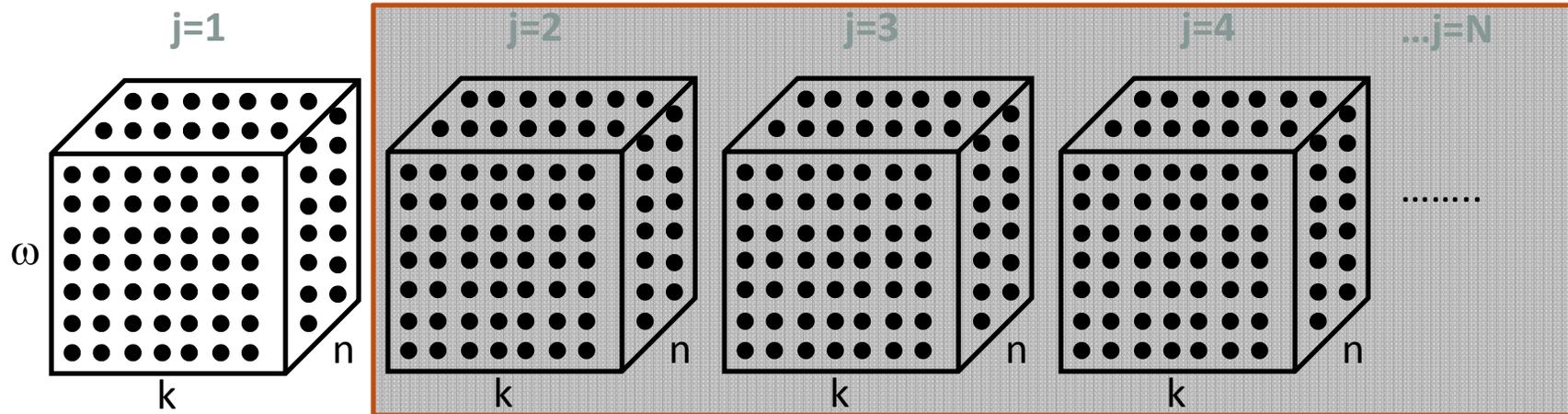
Self-similar structural hierarchies lead to order reduction



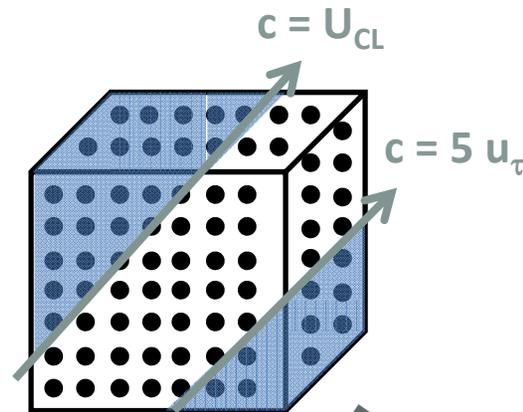
Regions of high amplification (singular value in resolvent SVD) overlay energetic activity

This is a *robust* system

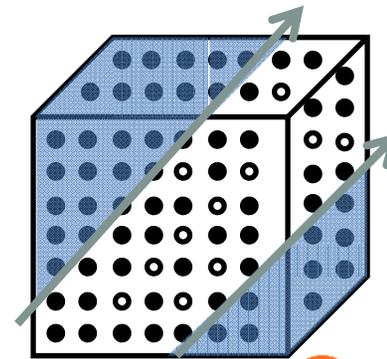
SPARSE



*Simplify
(Limits on c)*

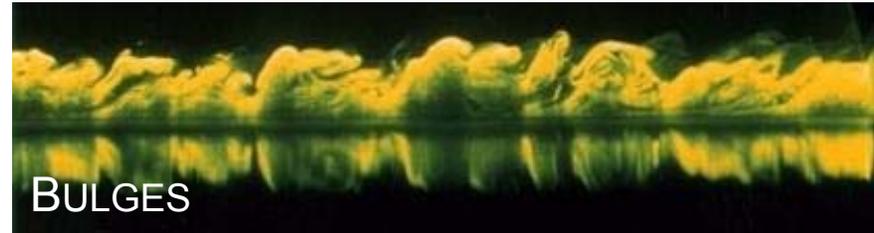
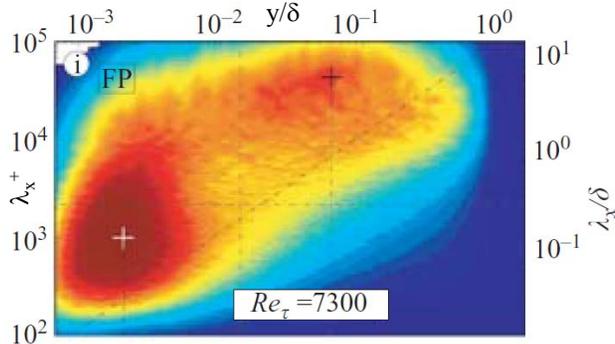


*Simplify
(Sparsity in ω)*

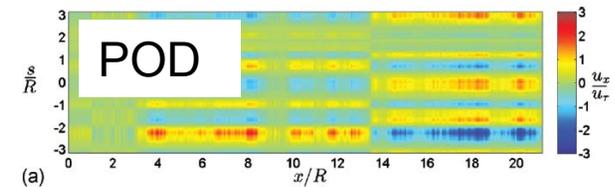


THE BUILDING BLOCKS OF TURBULENCE

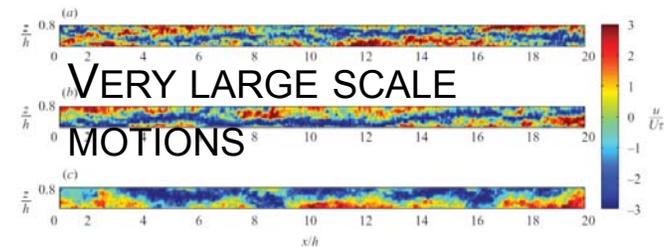
STATISTICS & SPECTRA



BULGES

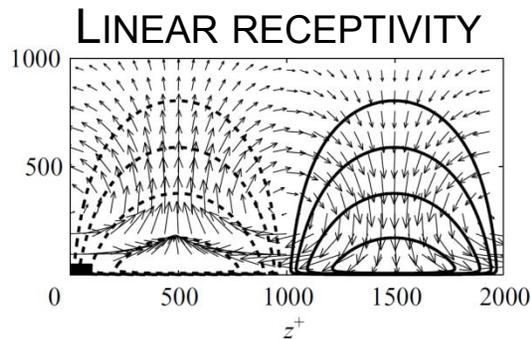


(a)

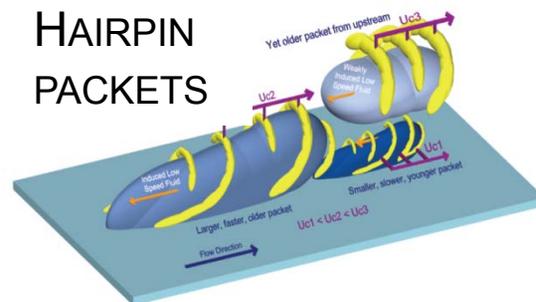


VERY LARGE SCALE MOTIONS

“So, actually the elephant has all the features you mentioned...”



HAIRPIN PACKETS



Credits, CW from top left

Hutchins & Marusic

Gad-el-Hak

Hellstroem, Sinha & Smits

Monty, Stewart, Williams & Chong

Adrian, Meinhart & Tomkins

Del Alamo & Jimenez

THEMES FOR THE WORKSHOP/PROGRAM

1. **Self-sustaining processes.** How can these be identified, characterized and compared? What are the connections between such processes at low (transitional) and high Reynolds numbers?
2. **Connections** between turbulent statistics, coherent structure and the Navier-Stokes equations. How can experimental and numerical observations be reconciled with, or derived from, the equations of motion?
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4. **Low Reynolds number vs. asymptotically high Reynolds number.** Which dynamics and phenomena retain their dynamical significance, and which are local phenomena? What can be learned from the opposite extreme in Reynolds number?
5. **Modeling, prediction and control.** How do we leverage fundamental understanding towards practical advances in engineering applications?