Self-organization of planetary turbulence What can we learn by studying the statistical state dynamics

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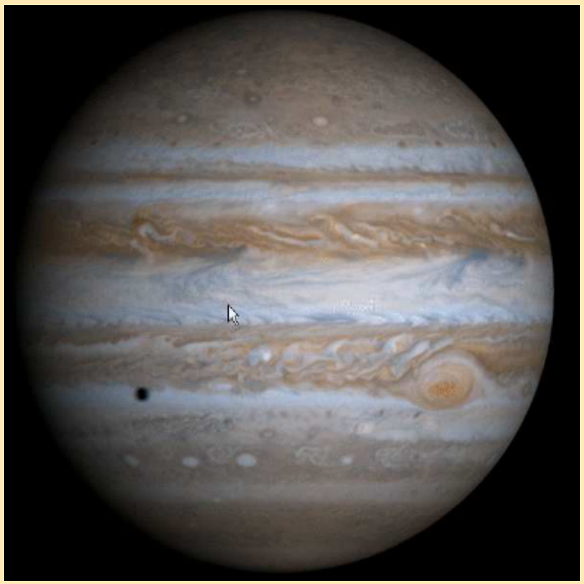
Turbulent flows are organized into vortices and jets

banded jets

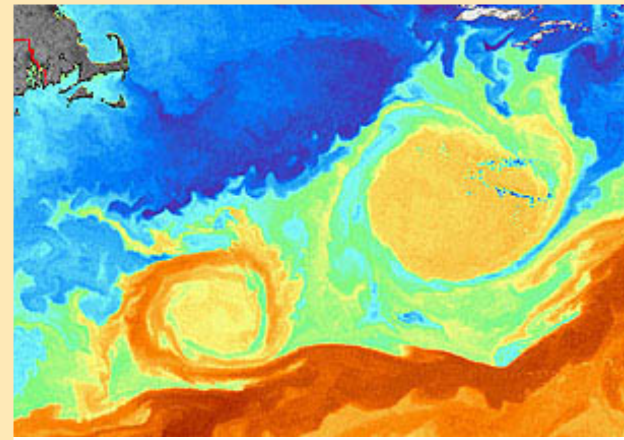


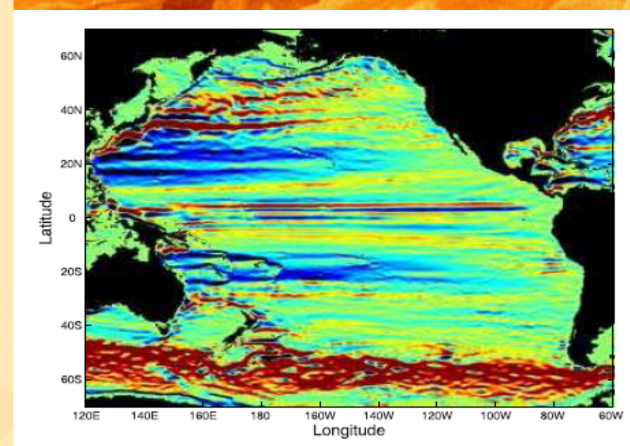
NASA

Great Red Spot

NASA

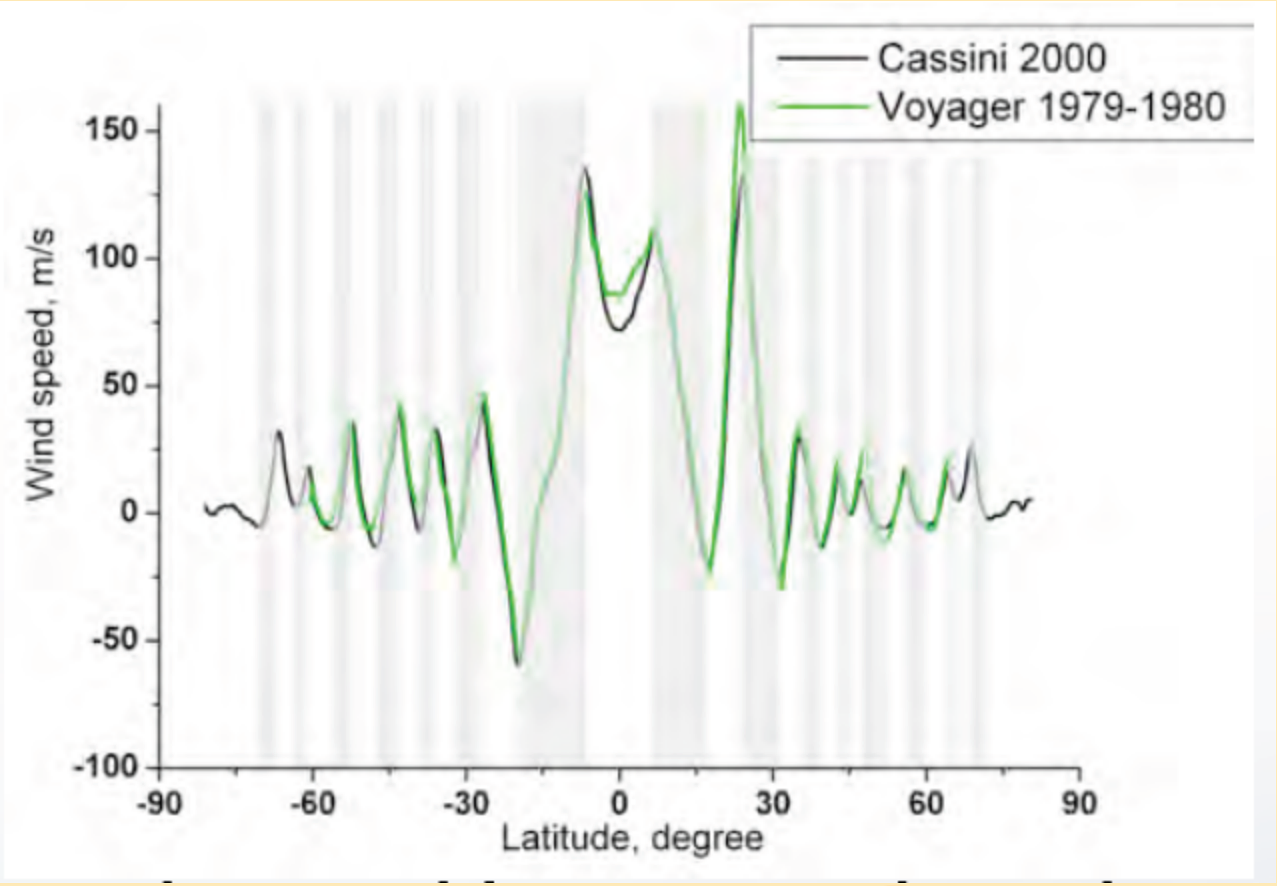
ocean jets & rings





(Richards et al 2006)

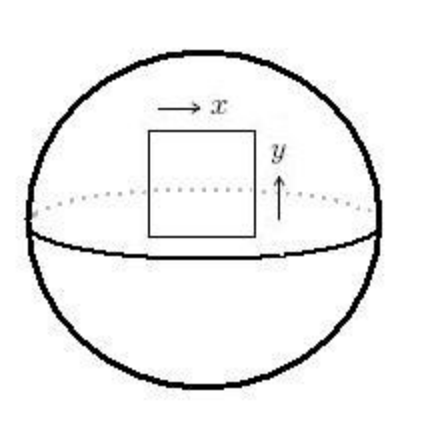
Jets appear "steady" as rock…



Vasavada & Showman 2005

Simplest model: barotropic flow on a beta-plane

- Simplest setting — non-divergent flow in a doubly periodic β-plane channel



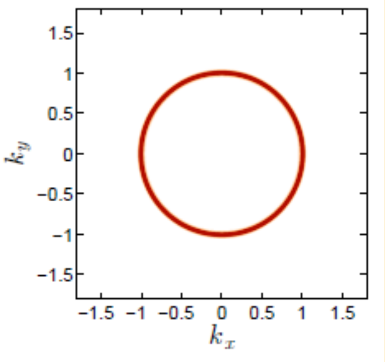
β-plane channel : $f = 2\Omega\sin\theta \approx 2\Omega\sin\theta_0 + 2\Omega\cos\theta_0\left(\theta-\theta_0\right) = f_0 + \beta y$

$$\zeta = \partial_x v - \partial_y u = \Delta\psi \qquad \left(\partial_t + \vec{u}\cdot\nabla\right)\zeta + \beta v = -r\zeta - \nu\nabla^4\zeta + \xi(t)$$

- Spatially homogeneous forcing that is delta-correlated in time

$$\left\langle \xi(x_1,y_1,t_1)\xi(x_2,y_2,t_2)\right\rangle = \delta(t_1-t_2)\Xi(x_1,x_2,y_1,y_2)$$

$$\Xi(x_1,x_2,y_1,y_2) = \sum\hat{\Xi}(k,l)e^{ik(x_1-x_2)+il(y_1-y_2)}$$

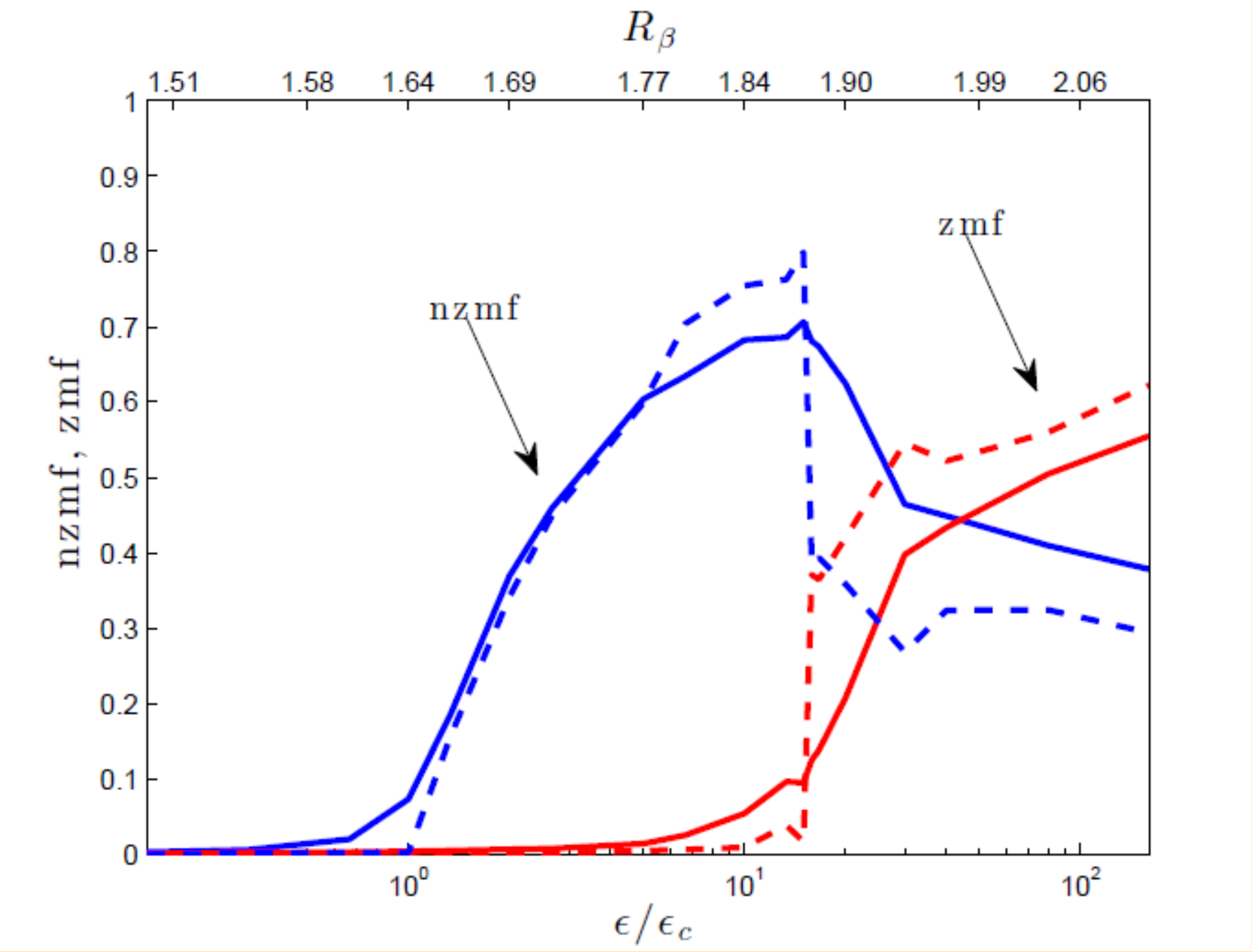


- Isotropic forcing injecting energy at rate *ε* in a narrow ring at K_f

$$K_f = 10\ ,\ \Delta K_f = 1\ ,\ \beta = 10\ ,\ r = 0.01\ ,\ \nu = 10^{-6}$$

Two regime transitions in the flow

$$\mathrm{zmf} = \frac{\sum_{l:l<K_f} \hat{E}(k=0,l)}{\sum \hat{E}(k,l)} \qquad \mathrm{nzmf} = \frac{\sum_{k,l:K<K_f} \hat{E}(k,l)}{\sum \hat{E}(k,l)} - \mathrm{zmf}$$



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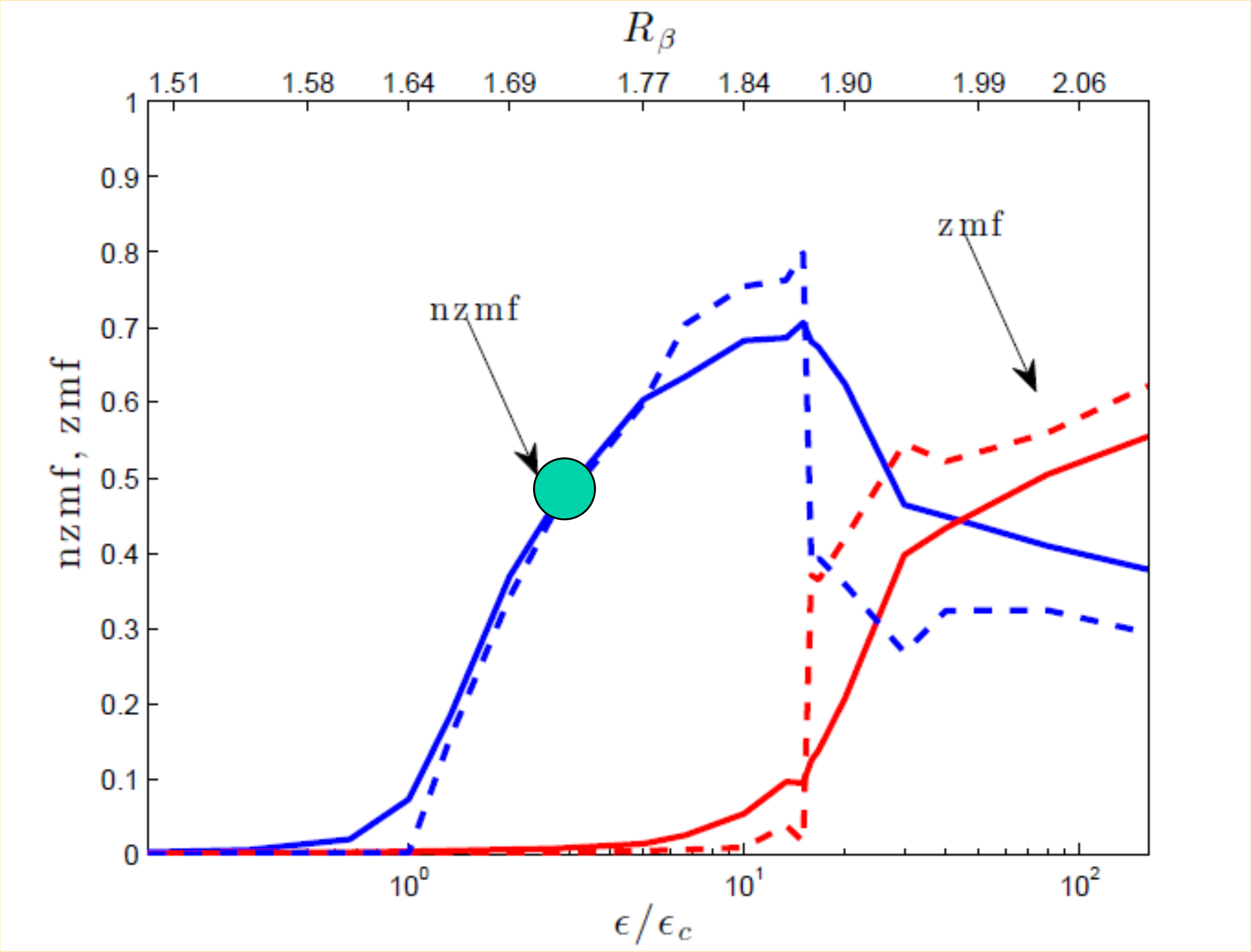


Non-zonal westward propagating coherent structures

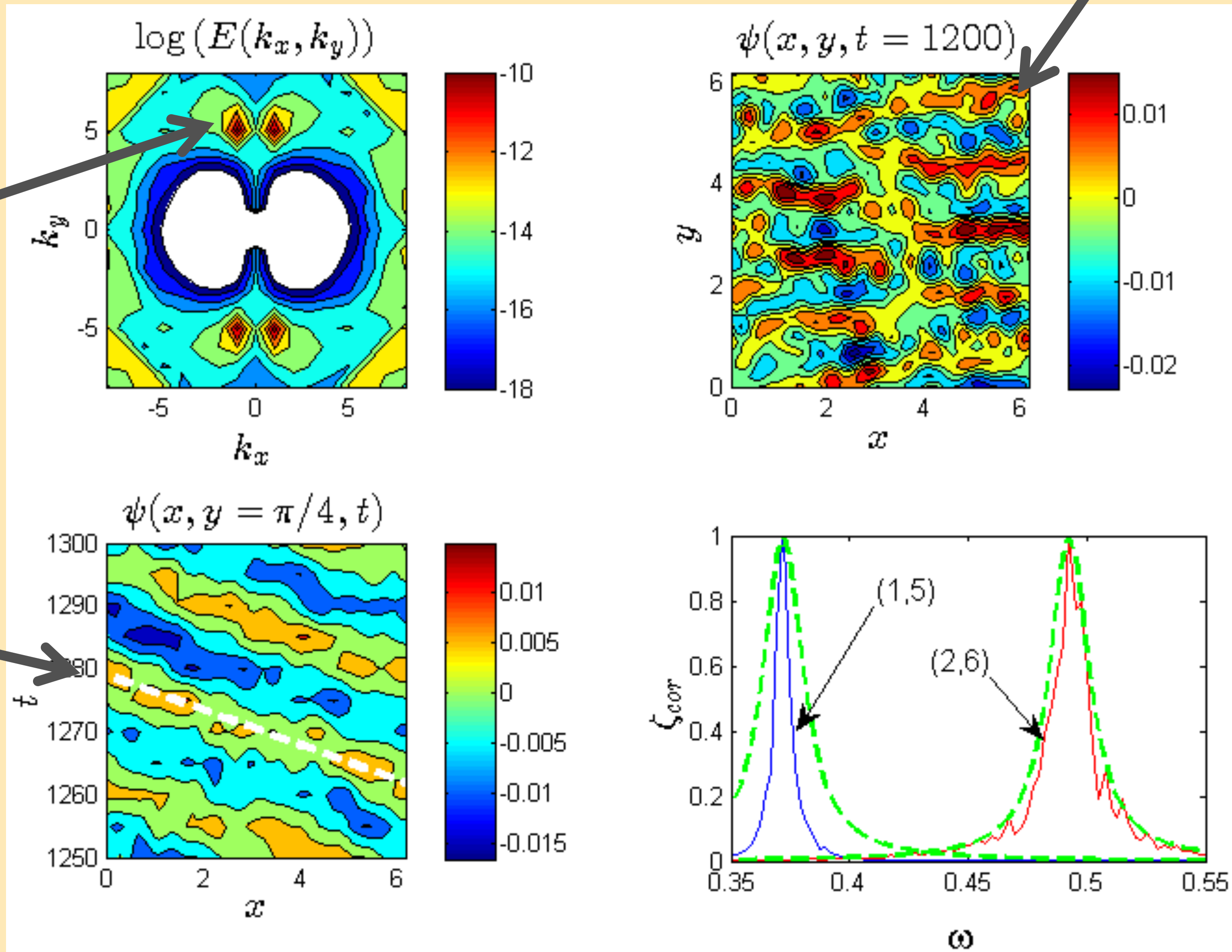
$\varepsilon/\varepsilon_c = 4$

$R_\beta = 1.62$

a $(k_x, k_y) = (1, 5)$ structure

peak at $(k_x, k_y) = (1, 5)$

Rossby wave phase speed



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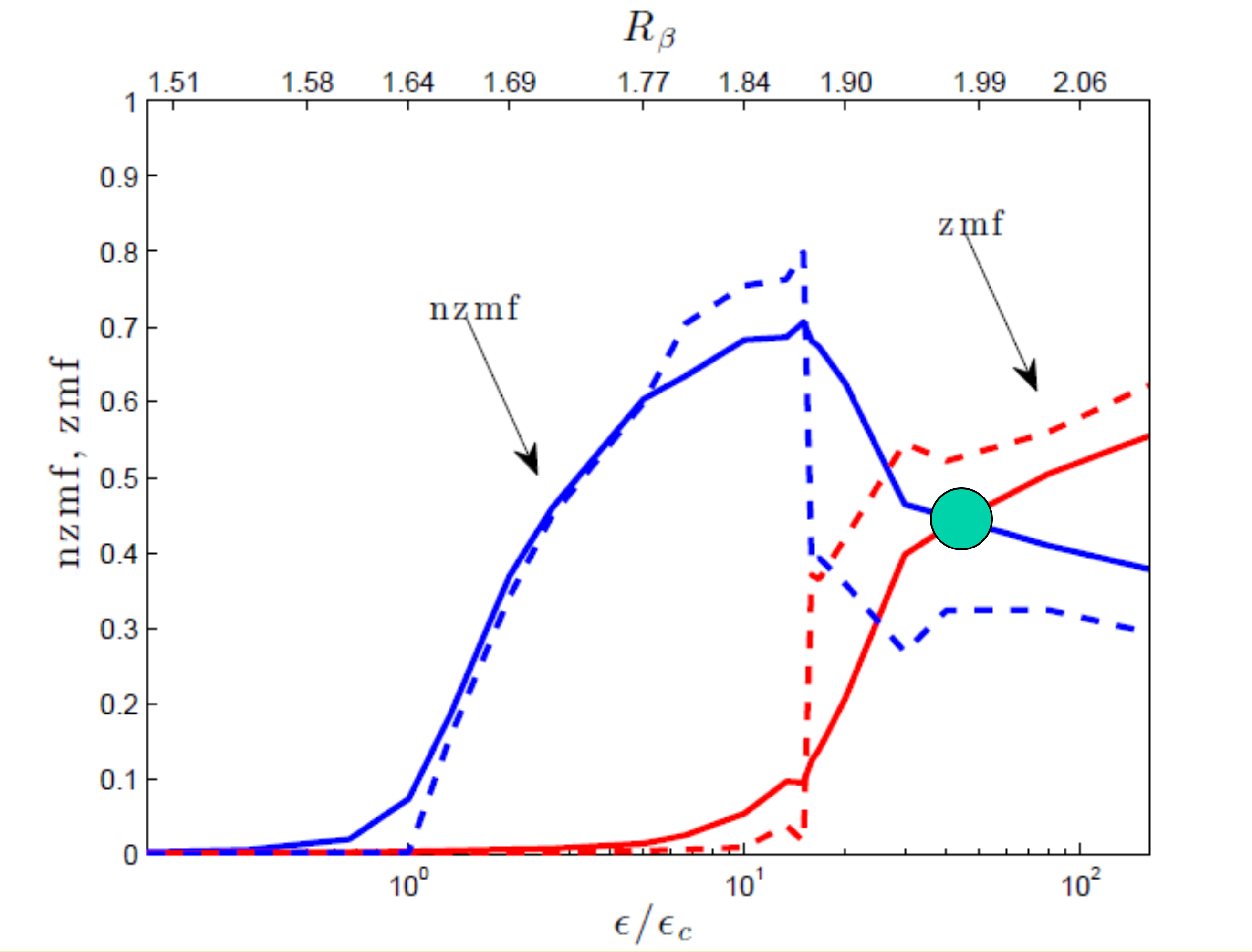
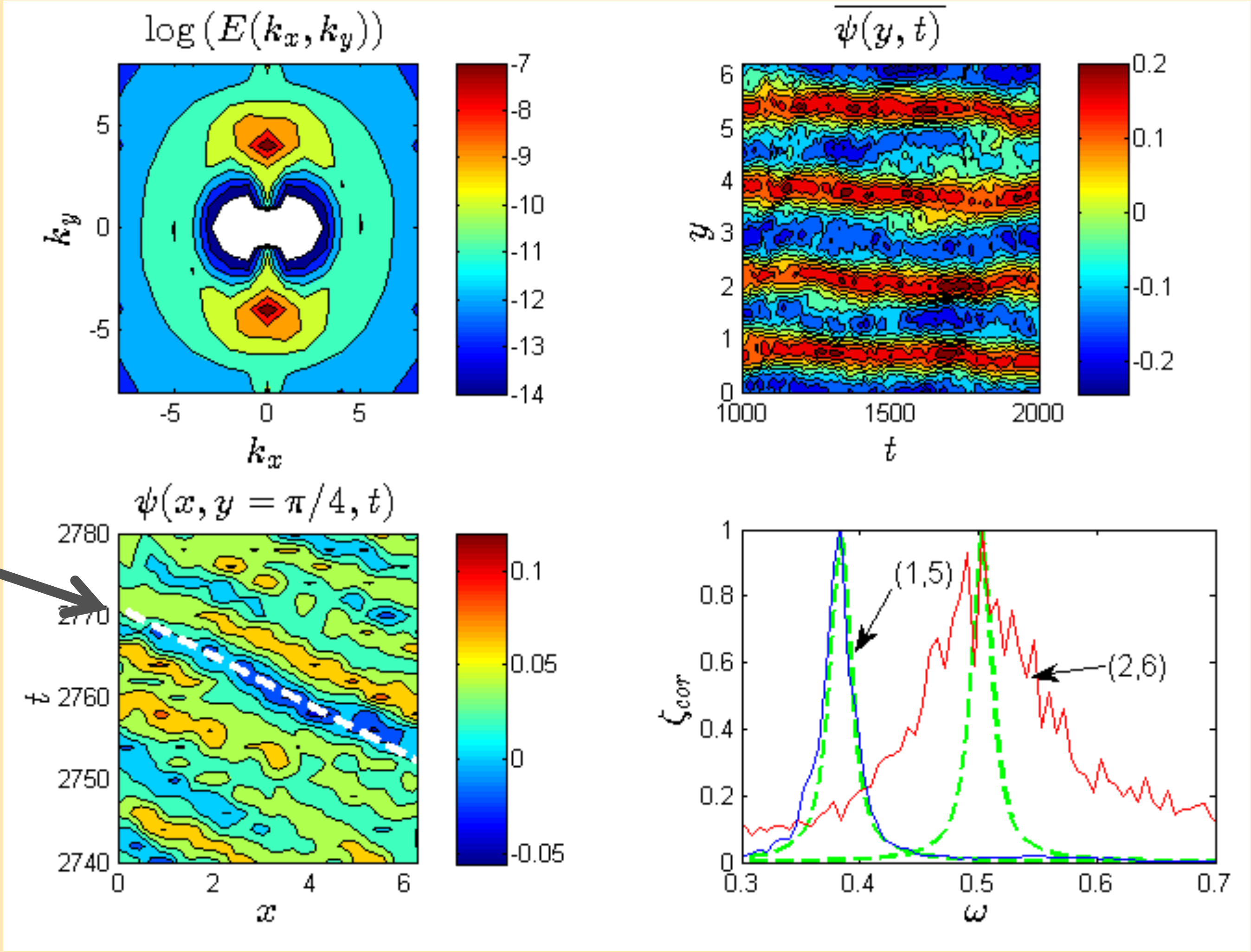


Zonal jets emerge, NZCS persist but slow down

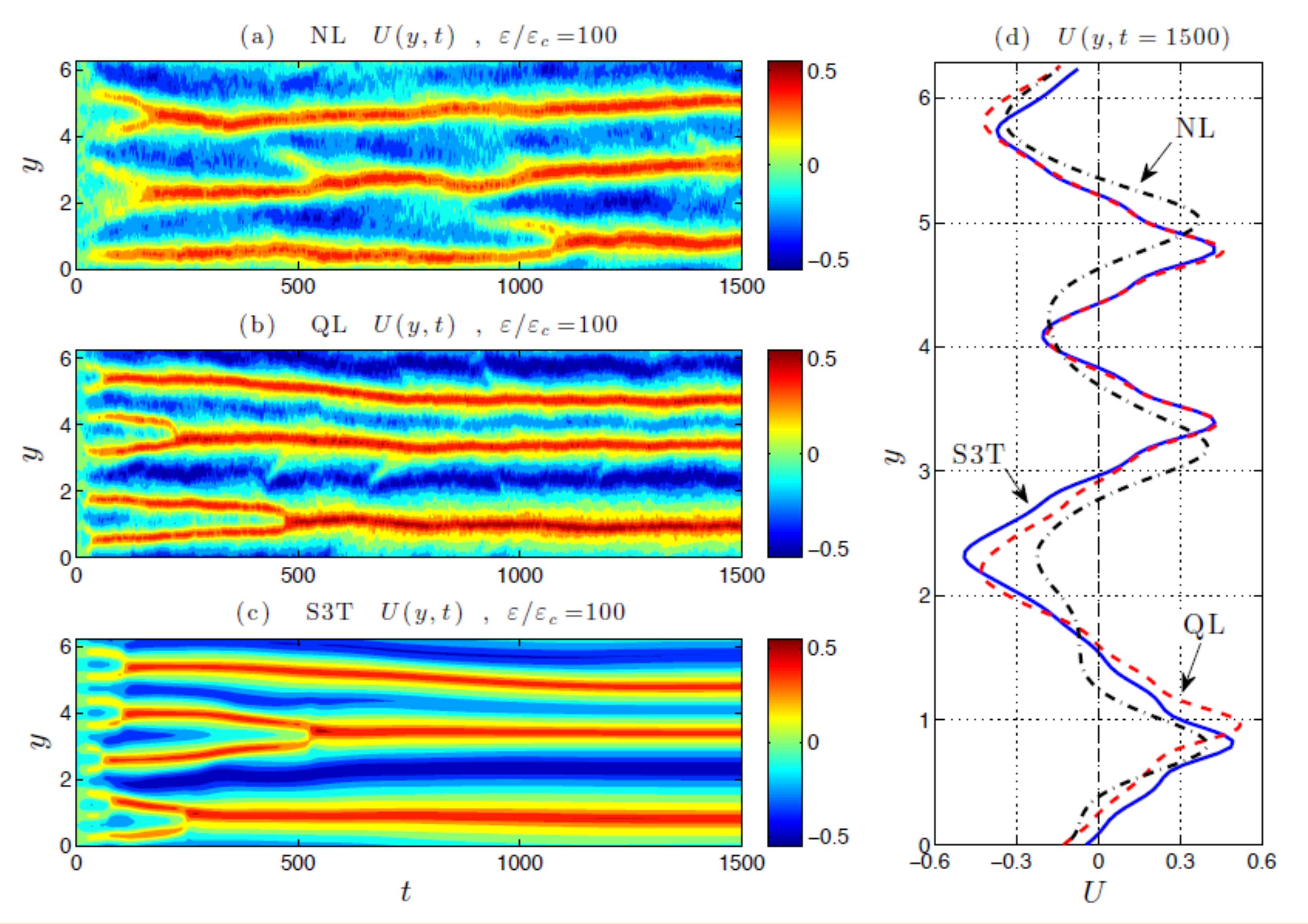
$\varepsilon/\varepsilon_c = 50$

$R_\beta = 1.84$

slower than Rossby wave



Jet merging & asymmetric jet structure



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Our goal

Develop a theory that accurately predicts:

- The regime transitions in the flow (NZCS, jet emergence)
- The characteristics (scale, amplitude, phase speed, form) of the emergent structures
- Captures nuances of the dynamics such as jet merging

Theory for the statistical state dynamics: S3T

- Stochastic Structural Stability Theory (S3T)

  Farrell & Ioannou 2003

  Related: Cumulant Expansion (CE2) Marston et al. 2008

- Variables : ensemble mean + deviation : $\zeta = Z + \zeta'$

$$\left(\partial_t + U\partial_x + V\partial_y\right)\zeta' + \left(\beta + Z_y\right)v' + Z_x u' = \xi - r\zeta' - \nu\nabla^4\zeta' + \left\langle \vec{u}' \cdot \nabla\zeta' \right\rangle - \vec{u}' \cdot \nabla\zeta'$$

$$\left(\partial_t + U\partial_x + V\partial_y\right)Z + \beta V = -\partial_x\left\langle u'\zeta' \right\rangle - \partial_y\left\langle v'\zeta' \right\rangle - rZ - \nu\nabla^4 Z$$

Second order closure of the cumulant expansion

$$\left(\partial_t + U\partial_x + V\partial_y\right)Z + \beta V \;=\; -\partial_x\langle u'\zeta'\rangle - \partial_y\langle v'\zeta'\rangle - rZ - \nu\nabla^4 Z$$

evolution of 1st cumulant $Z = \langle \zeta \rangle$

$$-\partial_x\langle u'\zeta'\rangle - \partial_y\langle v'\zeta'\rangle = F(C)$$

$$C = \langle \zeta_1' \zeta_2' \rangle \;,\; \zeta_i' = \zeta'(x_i, y_i, t)$$

$$\boxed{\frac{dZ}{dt} = -\left(U\partial_x + V\partial_y\right)Z - \beta V - rZ - \nu\nabla^4 Z + F(C)}$$

$$\partial_t \zeta_i' = A_i \zeta_i' + \xi_i + \langle \vec{u}_i' \cdot \nabla \zeta_i' \rangle - \vec{u}_i' \cdot \nabla \zeta_i'$$

$$A_i = -\left(U_i \partial_{x_i} + V_i \partial_{y_i}\right) - \left(\beta + Z_{y_i}\right)\partial_{x_i}\nabla_i^{-2} + Z_{x_i}\partial_{x_i}\nabla_i^{-2} - r - \nu\nabla_i^4$$

evolution of 2nd cumulant $C = \langle \zeta_1' \zeta_2' \rangle$

$$\frac{dC}{dt} = \left(A_1 + A_2\right)C + \Xi + G\left(\langle \zeta_1' \zeta_2' \zeta_3' \rangle\right)$$

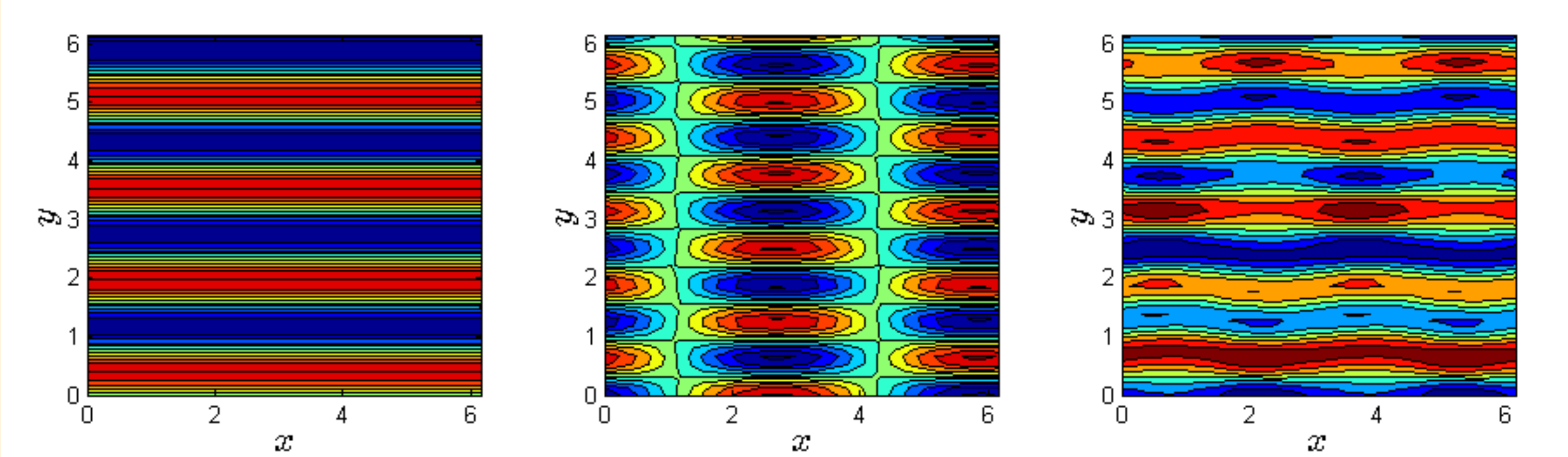
$G = 0$ or $G = \Xi' - rC$

$$\boxed{\frac{dC}{dt} = \left(A_1 + A_2\right)C + \Xi}$$

Examples of S3T equilibria

$$\left.\begin{aligned} \frac{dC}{dt} &= \left(A_1 + A_2\right)C + \Xi = 0 \\ \frac{dZ}{dt} &= -\left(U\partial_x + V\partial_y\right)Z - \beta V - rZ + F(C) = 0 \end{aligned}\right\} \rightarrow Z^E, C^E$$

S3T dynamical system : find fixed points & study their stability



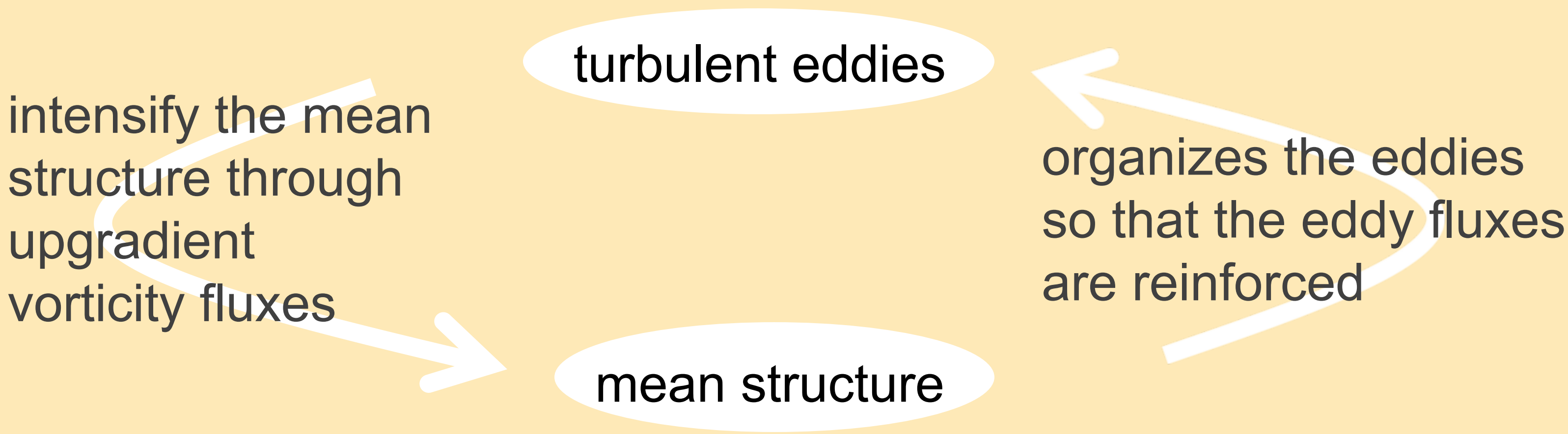
zonal jets | non-zonal structures | mixed-states

Regime transitions in turbulence as an instability

$$\left.\begin{aligned} \frac{dC}{dt} &= \left(A_1 + A_2\right)C + \Xi = 0 \\ \frac{dZ}{dt} &= -\left(U\partial_x + V\partial_y\right)Z - \beta V - rZ + F(C) = 0 \end{aligned}\right\} \rightarrow Z^E, C^E$$

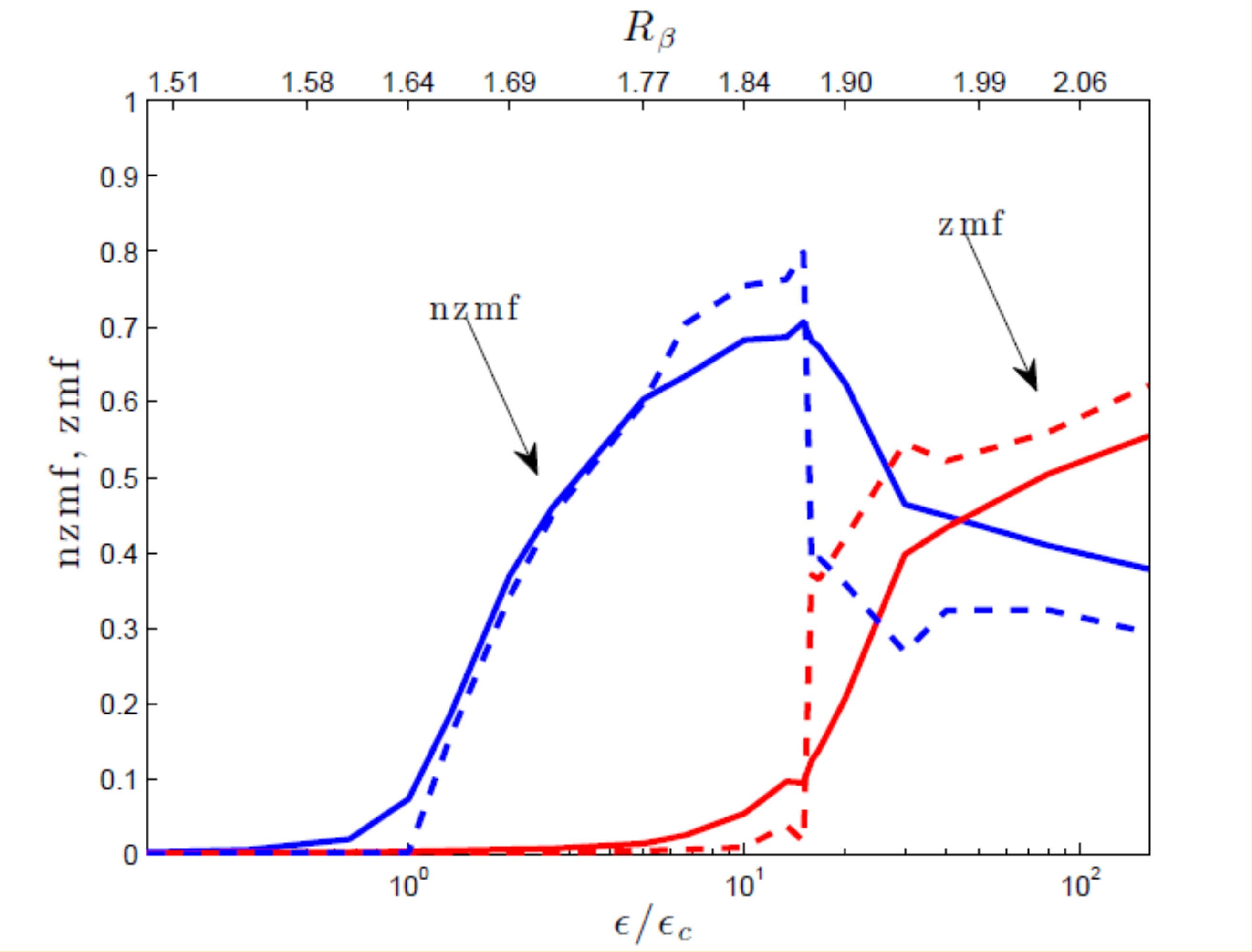
- Linearization: study the evolution of small perturbations in the mean structure δZ and in the eddy statistics δC :

$$\frac{d}{dt}\begin{pmatrix} \delta Z \\ \delta C \end{pmatrix} = L(Z^E, C^E)\begin{pmatrix} \delta Z \\ \delta C \end{pmatrix}$$



Can we address the regime transitions ?

$$\mathrm{zmf} = \frac{\sum_{l:l<K_f} \hat{E}(k=0,l)}{\sum \hat{E}(k,l)} \qquad \mathrm{nzmf} = \frac{\sum_{k,l:K<K_f} \hat{E}(k,l)}{\sum \hat{E}(k,l)} - \mathrm{zmf}$$



Emergence of structures as a structural instability

$$\left.\begin{array}{l} Z^E = 0 \\ C^E = \dfrac{\Xi}{2r} \end{array}\right\} \rightarrow \quad \delta Z = e^{in_x x + in_y y + \sigma t}, \ \delta C = e^{in_x (x_1 + x_2)/2 + in_y (y_1 + y_2)/2 + \sigma t}$$

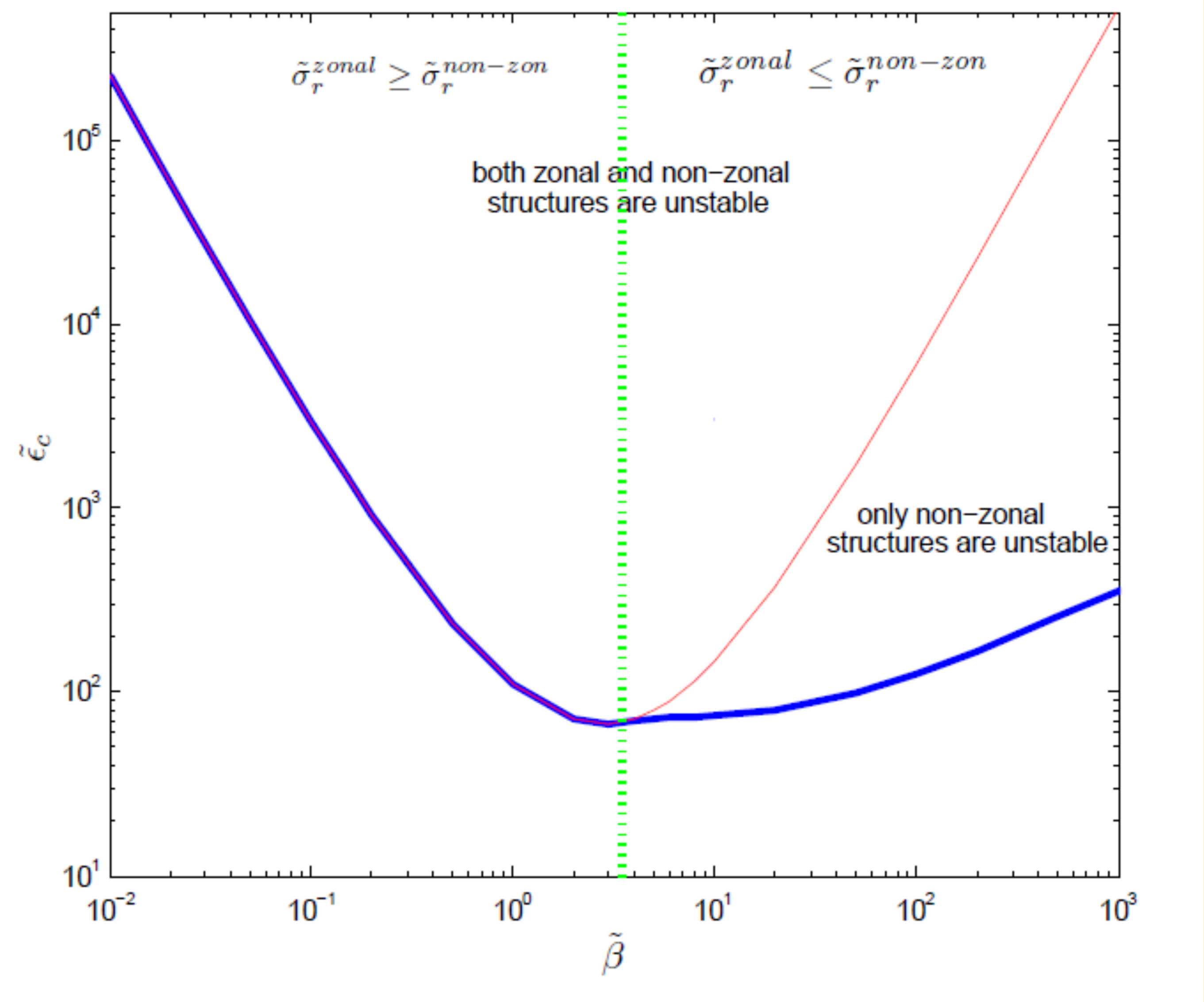
- Linearization: study the evolution of small perturbations in the coherent structure δZ and in the eddy statistics δC :

$$\frac{d}{dt}\begin{pmatrix} \delta Z \\ \delta C \end{pmatrix} = L(Z^E, C^E)\begin{pmatrix} \delta Z \\ \delta C \end{pmatrix} \rightarrow \ \tilde{\sigma}_{\tilde{n},\tilde{m}} = g\left(\tilde{\beta}, \tilde{\varepsilon}\right)$$

$$\tilde{\sigma} = \frac{\sigma}{r} \ , \ \tilde{\beta} = \frac{\beta}{rK_f} \ , \ \tilde{\varepsilon} = \frac{\varepsilon K_f^2}{r^3} \ , \ \tilde{n} = \frac{n}{K_f}$$

Critical curve

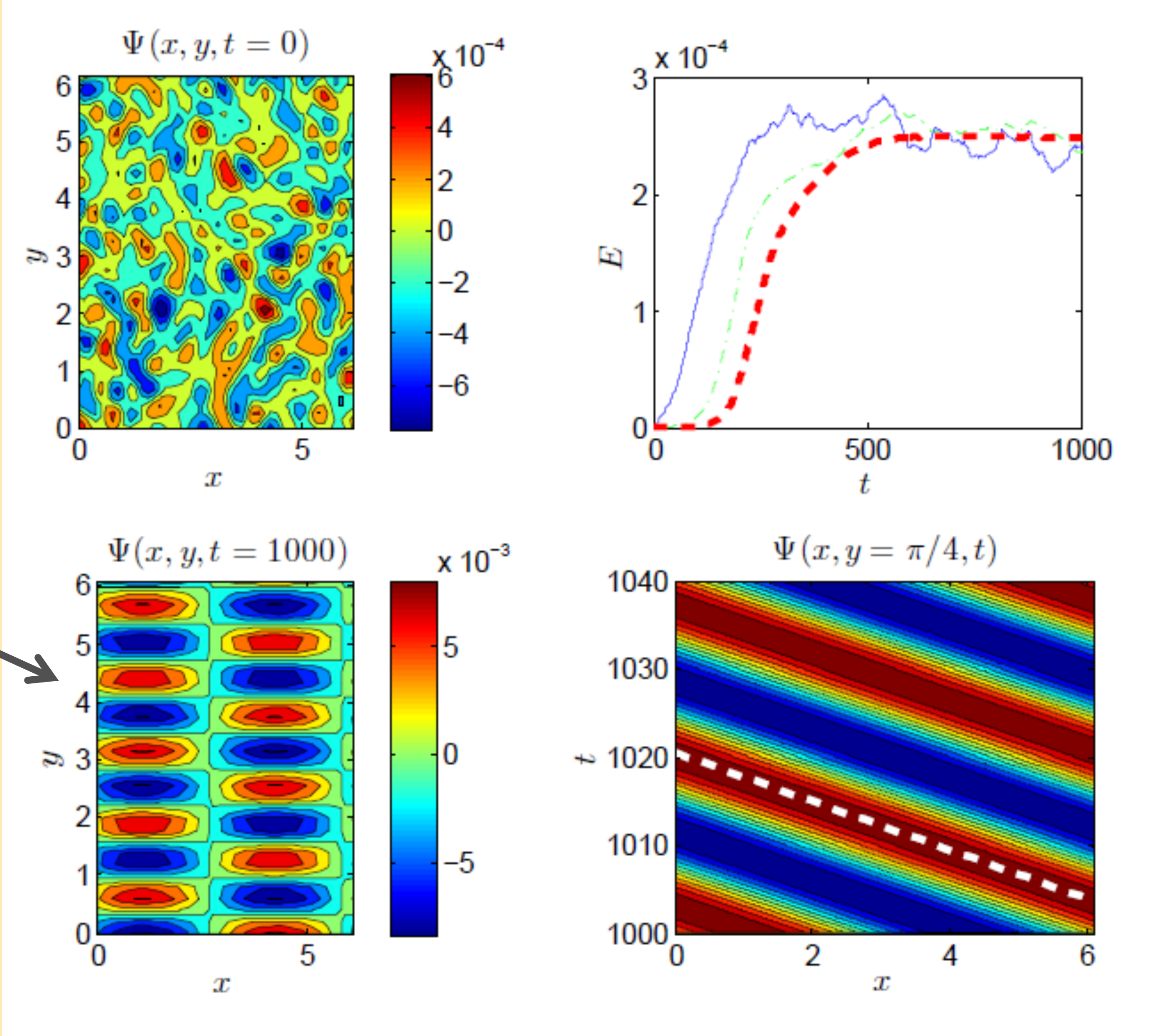
$$\widetilde{\varepsilon}_t : \mathrm{Re}\left(\widetilde{\sigma}_{\widetilde{n},\widetilde{m}}\right) = 0$$
$$\widetilde{\varepsilon}_c = \min_{\widetilde{n},\widetilde{m}} \widetilde{\varepsilon}_t$$



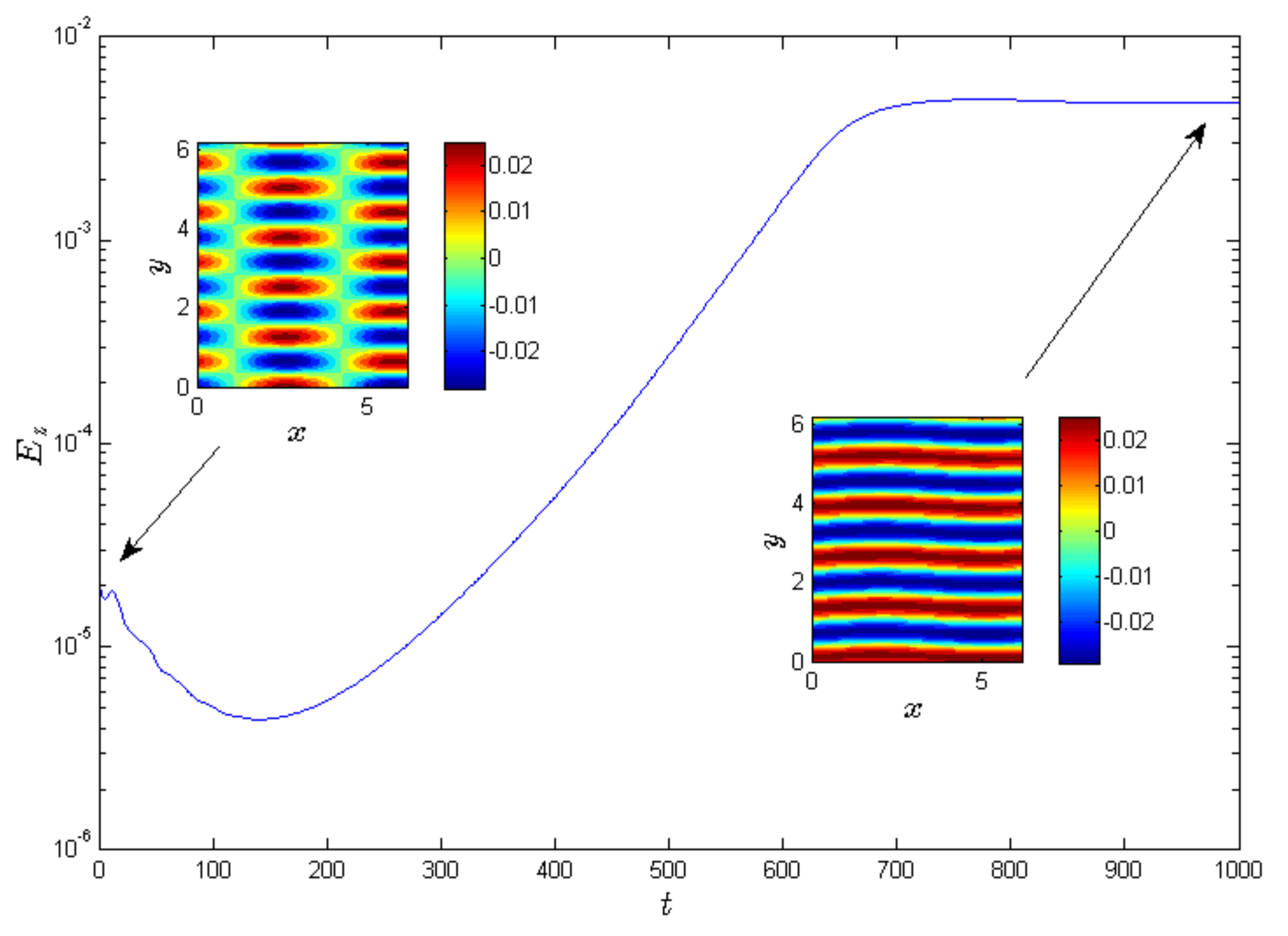
Equilibration of instabilities: emergent structures

$\tilde{\beta} = 100$
$\tilde{\varepsilon}/\tilde{\varepsilon}_c = 4$
$R_\beta = 1.62$

$(n_x, n_y) = (1, 5)$



Are the traveling wave states stable ?



the finite amplitude traveling wave states become unstable to zonal jets

Are the zonal jets (mixed states) stable ?

Some analytical progress close to the critical curve

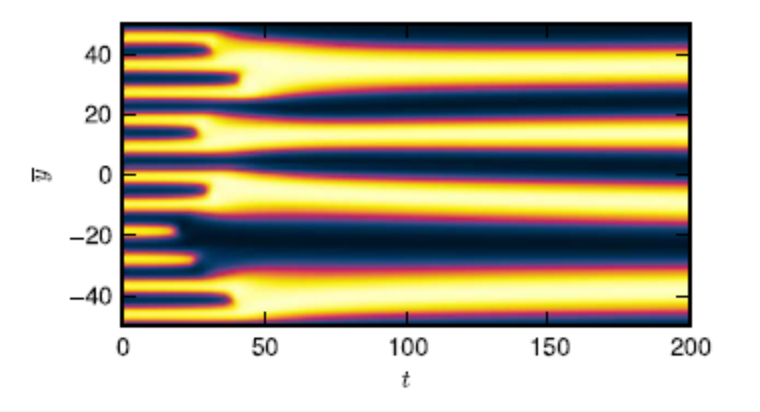
$$U = A(Y,T)e^{imy}$$

$$\lambda_1 \partial_T A = \lambda_2 A + \lambda_3 \partial^2_{YY} A - \lambda_4 |A|^2 A$$

GL equation

$\downarrow$

Eckhaus instability
leading to jet mergers !!!



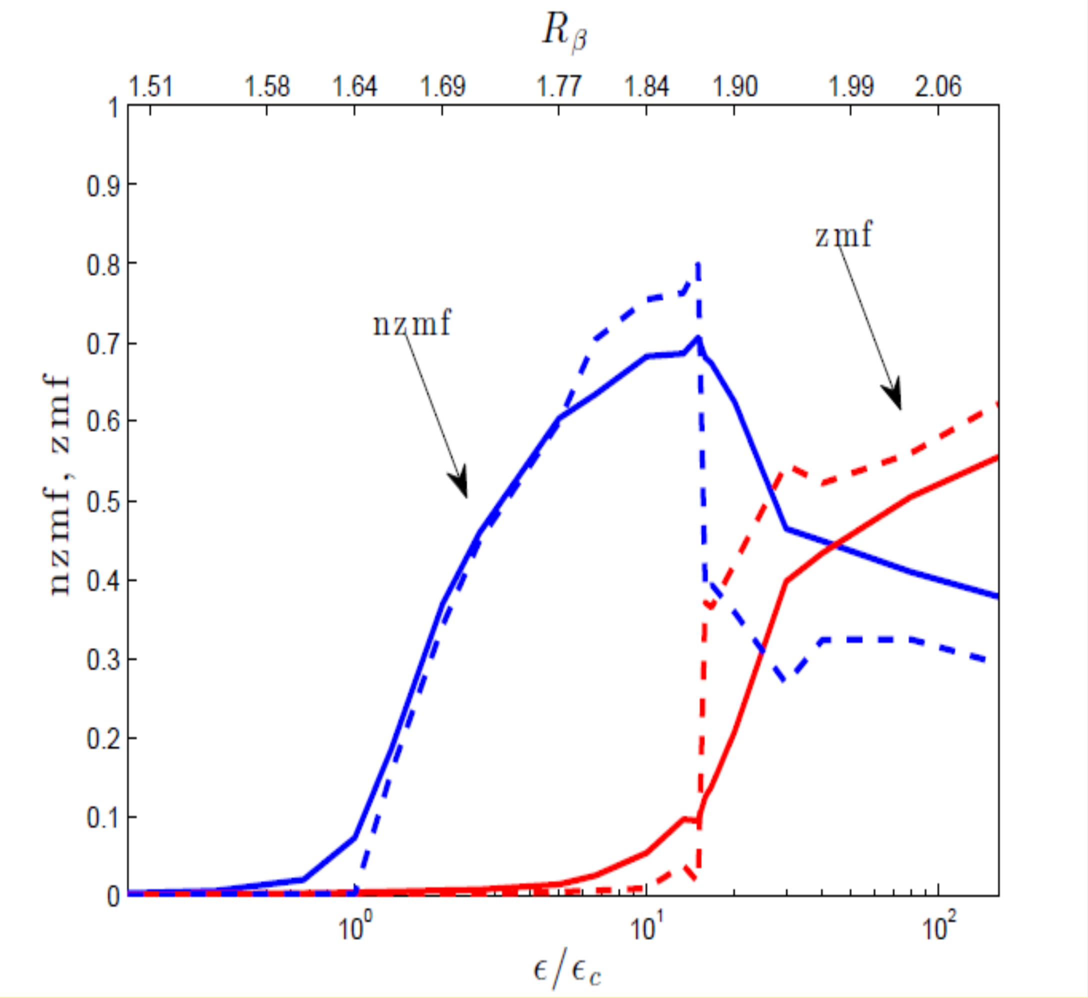
Parker & Krommes 2013

Numerical progress away from it (stay tuned for Navid)

Accurate prediction of regime transition

$$\mathrm{zmf} = \frac{\sum_{l:l<K_f} \hat{E}(k=0,l)}{\sum \hat{E}(k,l)}$$

$$\mathrm{nzmf} = \frac{\sum_{k,l:K<K_f} \hat{E}(k,l)}{\sum \hat{E}(k,l)} - \mathrm{zmf}$$



Accurate prediction of scale, amplitude

$$\mathrm{zmf} = \frac{\sum_{l:l<K_f} \hat{E}(k=0,l)}{\sum \hat{E}(k,l)}$$

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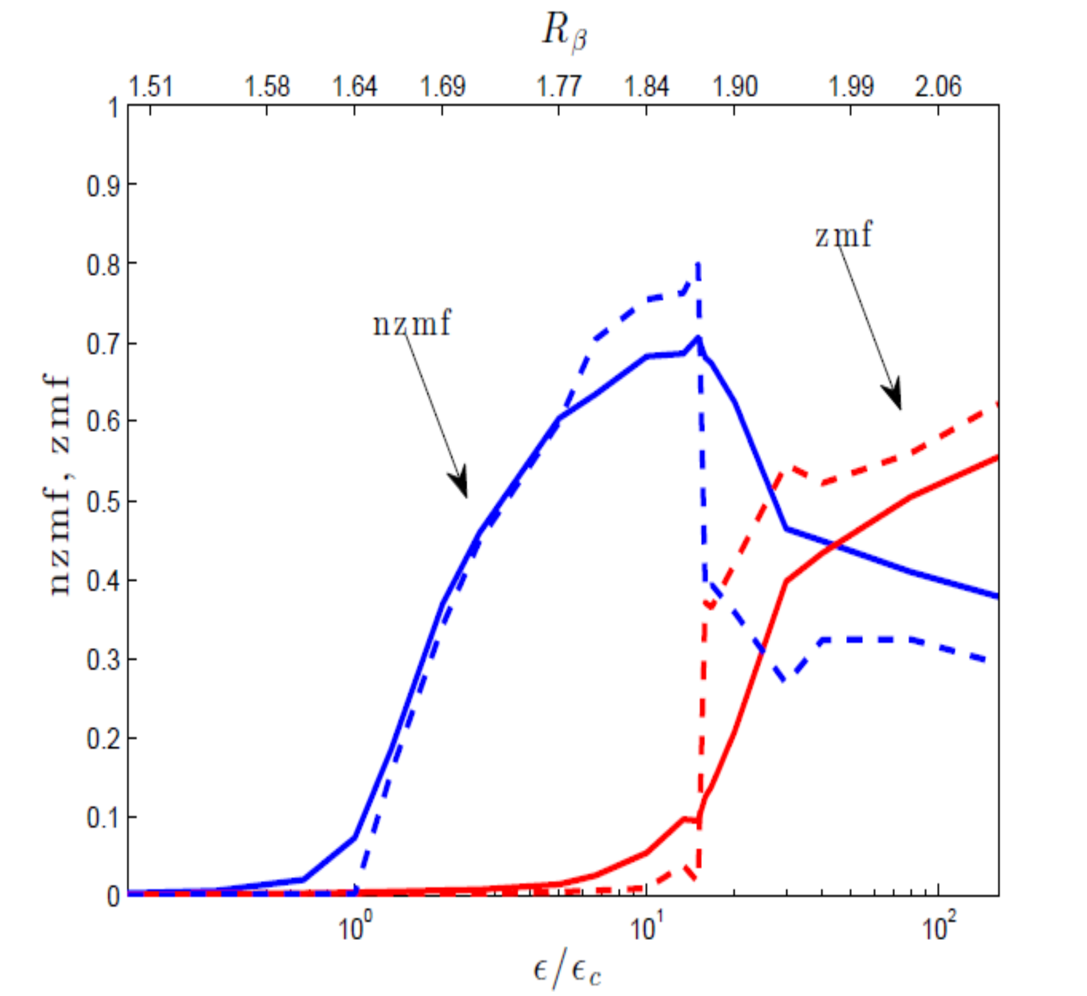

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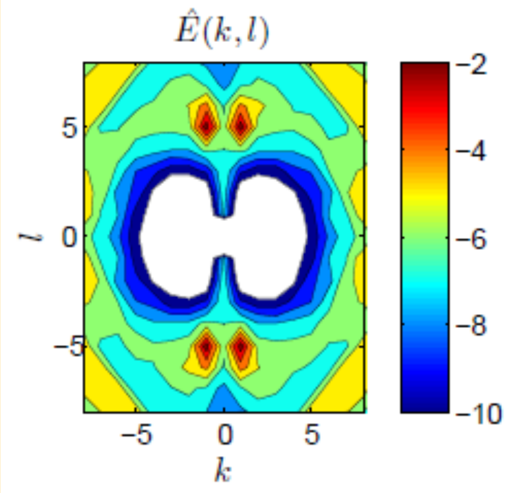
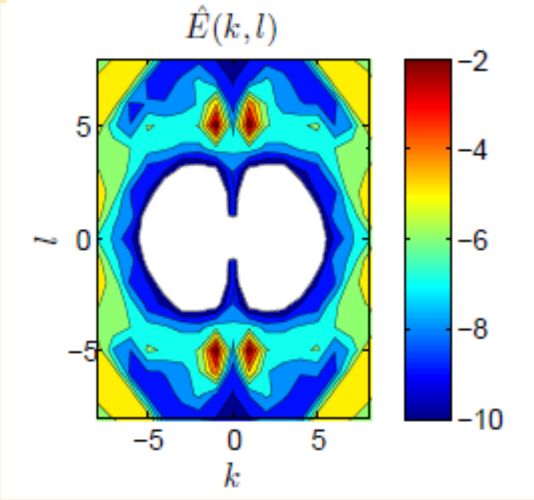
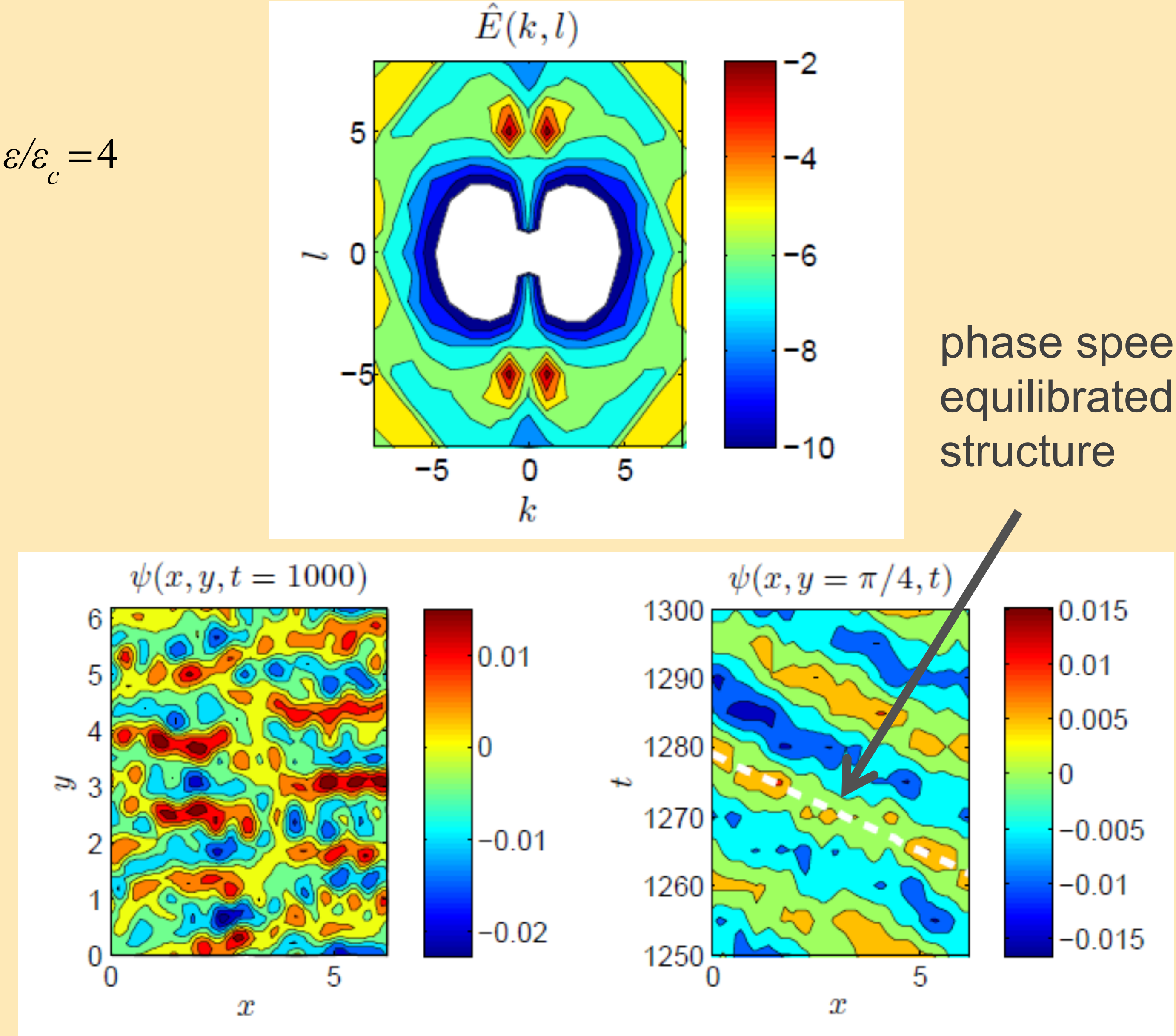


S3T



Accurate prediction of phase speed

$\varepsilon/\varepsilon_c = 4$

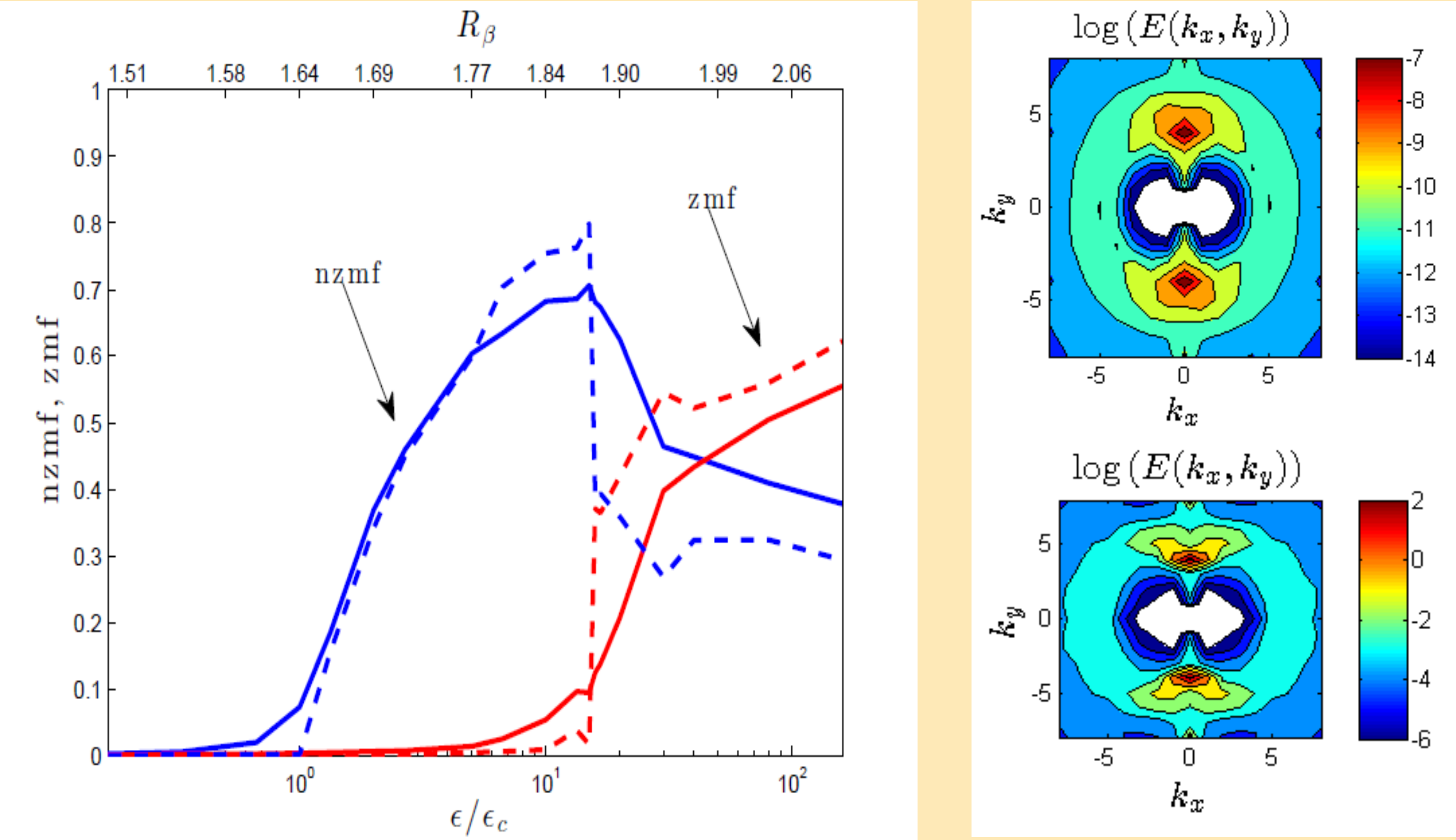


phase speed of equilibrated S3T structure

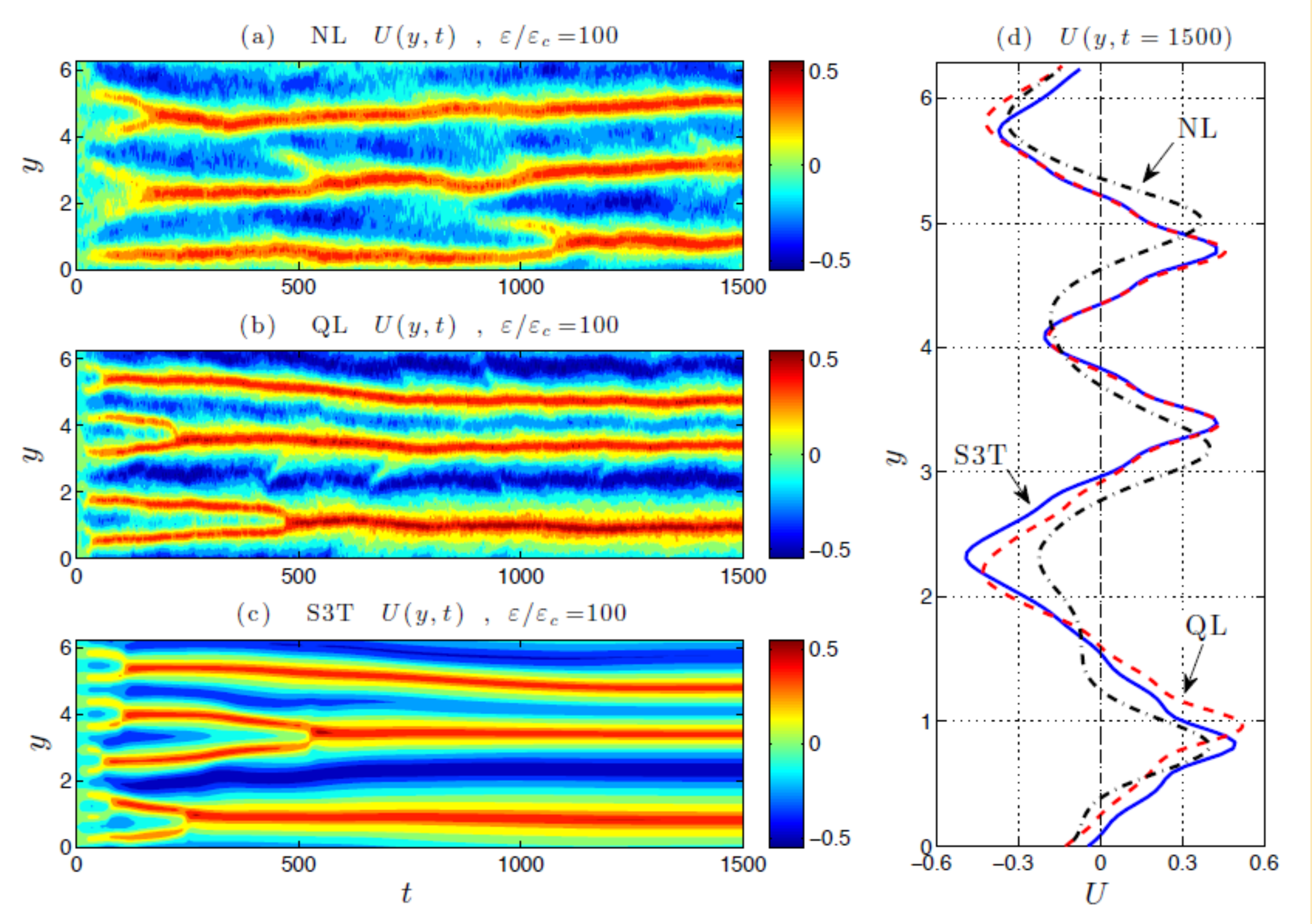
Accurate prediction of the second transition

$$\text{zmf} = \frac{\sum_{l:l<K_f} \hat{E}(k=0,l)}{\sum \hat{E}(k,l)}$$

$$\text{nzmf} = \frac{\sum_{k,l:K<K_f} \hat{E}(k,l)}{\sum \hat{E}(k,l)} - \text{zmf}$$



Accurate prediction of jet merging & jet structure



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Take home messages…

Using S3T, we are able to accurately predict:

- The regime transitions in the flow with the emergence of non-zonal westward propagating coherent structures and the emergence of zonal jets

- The scale, amplitude and phase speed of the emergent coherent structures in the turbulent flow

- Even nuances of the dynamics such as jet mergers

Navid will talk about how within the S3T framework we can shed light into the eddy-mean flow dynamics that lead to the emergence of these structures and support them

Thank you !

Bakas & Ioannou, 2013 : Emergence of large scale structure in planetary turbulence. *PRL,* **110***,* 224501

Bakas & Ioannou, 2014 : A theory for the emergence of coherent structures in beta-plane turbulence. *JFM,* **740***,* 312-341

Bakas & Ioannou, 2015 : Emergence of non-zonal coherent structures. In Galperin, P. and Read P., eds. *Zonal jets*, Cambridge Univ. Press (subm.)