Semiclassical Phonon Transport in the Presence of Rough Boundaries

Irena Knezevic University of Wisconsin - Madison

DOE BES, Award No. DE-SC0008712 NSF ECCS, Award No. 1201311



Phonons in Nanostructures

 Phonons scatter with boundaries (mesoscale), imperfections (nanoscale) and isotope/dopant atoms (atomic scale).



Biswas et al., Nature 489, 414 (2012)

- Strong interactions with rough boundaries or interfaces between materials
- Need to understand thermal transport in the presence of both roughness and internal scattering

C. J. Vineis et al., Adv. Mater. 22, 3970 (2010)



Lessons from electronic transport



Μ \mathbf{O} e С \mathbf{O} m р С a е

Phonon transport

Semiclassical (phonons BTE and derivatives))

Heat diffusion equation BTE in the RTA Ensemble Monte Carlo Discrete ordinate method Direct solutions to BTE

Great recent work to extract accurate rates from DFPT to use with Boltzmann transport kernels

Classical (atoms) Molecular dynamics

Μ

0

A

С

С

U

r

a

е

Great recent work on applying ab initio to extract accurate potentials for use in MD

Quantum (phonons)

Nonequilibrium Green's functions

Currently not possible to address phonon-phonon scattering Μ

 \mathbf{O}

С

 \mathbf{O}

m

р

С

a

е

C

Why semiclassical phonon transport?

Pros:

- ✓ Computationally tractable for experimentally relevant sizes
- ✓ A variety of techniques adaptable to geometry
- $\checkmark\,$ All kinds of scattering mechanisms can be incorporated
- ✓ Intuitive (humans like the distribution function)

Cons:

- ↓ Cannot address coherent transport features
- ↓ Does not account for states being affected by interactions
- ↓ Must a priori decide which mechanisms to account for
- \downarrow Only as good as the rates used, rates obtained from elsewhere

Semiclassical transport

From intro-level quantum-mechanics textbooks: The semiclassical (or WKB) approximation holds when the potential vary slowly on the scale of the particle wavelength. The concept of classical trajectories is viable.

When is the semiclassical approximation applicable to transport?

When **relevant length/time scales** on which the single-particle Green's function varies are **much longer than coherence lengths and times**

$$\begin{bmatrix} \omega - (p^2/2m) - U(\mathbf{R}, T) - \operatorname{Re} \Sigma (\mathbf{p}, \omega; \mathbf{R}, T), \ g^{<}(\mathbf{p}, \omega; \mathbf{R}, T) \end{bmatrix}$$

$$+ \begin{bmatrix} \operatorname{Re} g(\mathbf{p}, \omega; \mathbf{R}, T), \ \Sigma^{<}(\mathbf{p}, \omega; \mathbf{R}, T) \end{bmatrix}$$

$$= -\Sigma^{>}(\mathbf{p}, \omega; \mathbf{R}, T) g^{<}(\mathbf{p}, \omega; \mathbf{R}, T)$$

$$+ \Sigma^{<}(\mathbf{p}, \omega; \mathbf{R}, T) g^{>}(\mathbf{p}, \omega; \mathbf{R}, T)$$

$$= g(\mathbf{p}, z; \mathbf{R}, T)$$

$$= \begin{bmatrix} z - (p^2/2m) - U(\mathbf{R}, T) - \Sigma (\mathbf{p}, z; \mathbf{R}, T) \end{bmatrix}^{-1}$$

Kadanoff and Baym, Quantum Statistical Mechanics, W.A. Benjamin, New York, 1962

Example from the electronic world

What scattering does to coherences in a double-barrier tunneling structure



Scattering causes transition from resonant to sequential tunneling

PHYSICAL REVIEW B 78, 165306 (2008)

Wigner Monte Carlo simulation of phonon-induced electron decoherence in semiconductor nanodevices

Damien Querlioz,* Jérôme Saint-Martin, Arnaud Bournel, and Philippe Dollfus Institut d'Electronique Fondamentale, CNRS, University Paris-Sud, 91405 Orsay, France



Scattering kills off spatial coherences (off-diagonal terms)

Semiclassical transport

Semiclassical approximation boils down to transport being diffusive:

Length scales longer than the mean free path, timescales longer than relaxation time

Technically, to obtain the Boltzmann equation, two more approximations are needed:

- The Born approximation (scattering event completed before the next one)
- Scattering does little to alter the density of states

$$\omega - (p^{2}/2m) - U(\mathbf{R},T) - \operatorname{Re} \Sigma (\mathbf{p},\omega;\mathbf{R},T), g^{<}(\mathbf{p},\omega;\mathbf{R},T)] + [\operatorname{Re} g(\mathbf{p},\omega;\mathbf{R},T), \Sigma^{<}(\mathbf{p},\omega;\mathbf{R},T)] = - \Sigma^{>}(\mathbf{p},\omega;\mathbf{R},T) g^{<}(\mathbf{p},\omega;\mathbf{R},T) + \Sigma^{<}(\mathbf{p},\omega;\mathbf{R},T) g^{>}(\mathbf{p},\omega;\mathbf{R},T) = [z - (p^{2}/2m) - U(\mathbf{R},T) - \Sigma (\mathbf{p},z;\mathbf{R},T)]^{-1}$$

$$\begin{bmatrix} \frac{\partial}{\partial T} + \frac{\mathbf{p} \cdot \nabla \mathbf{R}}{\mathbf{m}} - \nabla_{\mathbf{R}} U_{\text{eff}}(\mathbf{R}, \mathbf{T}) \cdot \nabla_{\mathbf{p}} \end{bmatrix} \mathbf{f}(\mathbf{p}, \mathbf{R}, \mathbf{T}) \\ = -\int \frac{d\mathbf{p}'}{(2\pi)^3} \frac{d\bar{\mathbf{p}}}{(2\pi)^3} \frac{d\bar{\mathbf{p}}'}{(2\pi)^3} (2\pi)^4 \, \delta(\mathbf{p} + \mathbf{p}' - \bar{\mathbf{p}} - \bar{\mathbf{p}}') \\ \times \, \delta\left(\frac{\mathbf{p}^2}{2\mathbf{m}} + \frac{\mathbf{p}'^2}{2\mathbf{m}} - \frac{\bar{\mathbf{p}}^2}{2\mathbf{m}} - \frac{\bar{\mathbf{p}}'^2}{2\mathbf{m}}\right) (1/2) \left[\mathbf{v}(\mathbf{p} - \bar{\mathbf{p}}) \mp \mathbf{v}(\mathbf{p} - \bar{\mathbf{p}}')\right]^2 \\ \times \left[\mathbf{ff}'(1 \pm \bar{\mathbf{f}})(1 \pm \bar{\mathbf{f}}') - (1 \pm \mathbf{f})(1 \pm \mathbf{f}') \bar{\mathbf{f}} \bar{\mathbf{f}}'\right] \qquad (9-$$

Kadanoff and Baym, Quantum Statistical Mechanics, W.A. Benjamin, New York, 1962

Example of semiclassical phonon transport



PHYSICAL REVIEW B 86, 115328 (2012)

Thermoelectric properties of ultrathin silicon nanowires

E. B. Ramayya,^{*} L. N. Maurer, A. H. Davoody, and I. Knezevic[†] Department of Electrical and Computer Engineering, University of Wisconsin-Madison, Madison, Wisconsin 53706, USA

A long smooth nanowire, room temperature



Length > mean free path

The same nanowire, much lower temperature



Length < mean free path

Phonons and boundary roughness

Depending on wavelength, they are treated as waves or classical objects when interacting with a rough boundary or rms roughness Δ and correlation length L

 $\lambda >> \Delta, L$

Wave limit

Phonons should be treated as waves Interference effects important Specularity parameter OK



"Ray phononics" (particle) limit Semiclassical approximation valid Treat phonons as little balls with classical trajectories

The role of correlation length

JOURNAL OF APPLIED PHYSICS

VOLUME 38, NUMBER 4

15 MARCH 1967

Statistical Model for the Size Effect in Electrical Conduction*

STEPHEN B. SOFFER



FIG. 2. Schematic diagrams of the incident and emerging average flux densities. The arrows represent beams of indicated magnitudes. The polar plots represent the diffuse intensities in the plane of incidence. The plot in (a) is obtained from the Appendix for $h/\lambda = 0.2$, $\theta_0 = 45^\circ$. That in (b) is only schematic. *L*=0: Nearly isotropic diffuse portion of the scattered wave

L>0: With increasing correlation length, the diffusely scattered wave collimates along the direction obeying Snell's law

 $L \rightarrow \infty$: Even a rough surface appears completely smooth, as the diffuse and specular reflected waves align

Specularity parameter p

Intuitively plausible: p – probability of specular reflection

1-p – probability of having momentum randomized

Often assumed as a constant, but should be momentum-dependent Technically valid for OK for $L \rightarrow 0$

OK for for phonons whose wavelengths are considerably larger than the correlation length



 $\phi(\vec{q}, \vec{r}) = 2\vec{q} \cdot \hat{s}z(\vec{r}) = 2qz(\vec{r})\cos\Theta_B$ $\langle \phi^2 \rangle = (2q\cos\Theta_B)^2 \langle z^2 \rangle$ $= 4q^2 \Delta^2 \cos^2\Theta_B$

 $p(\vec{q}) = \exp(-4q^2\Delta^2\cos^2\Theta_B)$

 $p(\vec{q}) = \exp(-\langle \phi^2 \rangle)$

Examples: Partially diffuse interfaces

Examples of momentum-dependent specularity parameter $p(\mathbf{q})$ used in conjunction with the single-mode RTA and full dispersions on:



2) Graphene nanoribbons

3) SiGe superlattices

Thin Si Nanomembranes



Normal & umklapp 3-phonon processes, isotope, and boundary scattering



Z. Aksamija* and I. Knezevic[†] PHYSICAL REVIEW B **82**, 045319 (2010)

LA phonons scatter strongly from {111} and {100} TA phonons scatter strongly from {100}

Thin Si Nanomembranes



$$p(\vec{q}) = \exp(-4q^2\Delta^2\cos^2\Theta_B)$$
$$\bar{p}_j = \frac{\int p(\vec{q})N_0[\omega_j(\vec{q})]d^3\vec{q}}{\int N_0[\omega_j(\vec{q})]d^3\vec{q}}$$

Effective specularity parameter is dependent on branch and temperature At high roughnesses, here is still a considerable specular component

Z. Aksamija* and I. Knezevic[†]



Thin Si Nanomembranes





FIG. 3. (Color online) Lattice thermal conductivity of 100 nm (filled circles), 30 nm (open circles), and 20 nm (squares) thick SOI, according to the measurements of Refs. 46 and 47. The curves present our calculations with Δ =0.35 nm for 100 nm SOI, Δ =0.4 nm for 30, and Δ =0.45 nm for 20 nm SOI, in very good agreement with experiment. Thermal conductivity is lower by almost an order of magnitude than the room-temperature bulk value of 148 W/mK, owing primarily to boundary scattering at the rough Si/SiO₂ interfaces.



Z. Aksamija* and I. Knezevic[†]PHYSICAL REVIEW B 82, 045319 (2010)

Casimir limit

When is it OK to treat boundary scattering through an effective scattering rate?

$$\tau_{\mathbf{j},\mathbf{B}}(\vec{q}) = \begin{bmatrix} \frac{1+p(\vec{q})}{1-p(\vec{q})} \end{bmatrix} \begin{bmatrix} L\\ \mathbf{v}_{\mathbf{j},\perp}(\vec{q}) \end{bmatrix}$$

When the presence of boundaries doesn't affect the solution to the BTE, i.e. when boundaries are farther than the mean free path due to internal scattering,

so phonons can thermalize between successive hits.

Casimir rate gives an estimate for how important boundary scattering is

Diffuse (Casimir) limit

$$\lim_{p\to 0} \tau_{\mathbf{j},\mathbf{B}} = \frac{\mathcal{I}}{v_{\mathbf{j},\perp}}$$
Specular limit

 $\lim_{p \to 1} \tau_{\mathbf{j},\mathbf{B}} = \infty$

Examples: Partially diffuse interfaces

Examples of momentum-dependent specularity parameter $p(\mathbf{q})$ used in conjunction with the single-mode RTA and full dispersions on:

1) Thin silicon nanomembranes

2) Graphene nanoribbons

3) SiGe superlattices

Boundary vs. internal scattering

• Model using the steady-state phonon Boltzmann transport equation

$$\vec{v}_{\vec{q}} \cdot \nabla_{\vec{r}} T \frac{\partial N^0_{\vec{q}}(T)}{\partial T} + \upsilon_{\perp}(\vec{q}) \frac{\partial n_{\vec{q}}(y)}{\partial y} = \frac{n_{\vec{q}}(y)}{\tau_{int.}(\vec{q})}$$

- In the absence of boundaries, RTA solution: $R_{\vec{q}} = \tau_{int.}(\vec{q})\vec{v}_{\vec{q}} \cdot \nabla_{\vec{r}}T \frac{\partial N^0_{\vec{q}}(T)}{\partial T}$
- Each boundary contributes one term: $p(\vec{q}) \exp\left[-W/\Lambda_{int.}^{\perp}(\vec{q})\right]$ y κ_{\perp} κ_{\parallel}
- Sum the infinite series:

$$n_{\vec{q}}^{+}(y) = R_{\vec{q}} \left\{ 1 - \frac{[1 - p(\vec{q})] \exp\left[-y/\Lambda_{int.}^{\perp}(\vec{q})\right]}{1 - p(\vec{q}) \exp\left[-W/\Lambda_{int.}^{\perp}(\vec{q})\right]} \right\}$$

Heat flux is weaker near boundaries/interfaces due to roughness scattering

Suspended graphene nanoribbons (GNRs)



Z. Aksamija and I. Knezevic, Appl. Phys. Lett. 98, 141919 (2011)

Thermal transport in supported graphene



Seol et al., Science (2010) and R. Prasher, Science (2010)



Z. Aksamija and I. Knezevic, Phys Rev. B 86, 165426 (2012).



M.-H. Bae, Z. Li, Z. Aksamija, P. N. Martin, F. Xiong, Z.-Y. Ong, I. Knezevic, and E. Pop, "Ballistic to diffusive crossover of heat flow in graphene ribbons," *Nature Communications* **4**, 1734 (2013).

Examples: Partially diffuse interfaces

Examples of momentum-dependent specularity parameter $p(\mathbf{q})$ used in conjunction with the single-mode RTA and full dispersions on:

1) Thin silicon nanomembranes

2) Graphene nanoribbons



SiGe alloy superlattices



Lattice with reasonably thick layers and partially diffuse interfaces

Roughness + mismatch?

Experimentally observed:

- cross-plane/in-plane anisotropy of κ
- increasing *k* with increasing period

Separate roughness from mismatch: Calculate κ for sublattices with rough interfaces; mismatch figures in κ , not the BTE itself



$$\kappa_{\text{in-plane}} = \frac{L_1 \kappa_1^{xx} + L_2 \kappa_2^{xx}}{L_1 + L_2},$$

$$\kappa_{\text{cross-plane}} = \frac{L_1 + L_2}{\frac{L_1}{\kappa_1^{yy}} + \frac{L_2}{\kappa_2^{yy}} + \frac{1}{\sigma_1^{\text{AIM}}} + \frac{1}{\sigma_2^{\text{AIM}}}}$$

Solving the BTE for superlattices



Z. Aksamija and I. Knezevic, Phys. Rev. B 88, 155318 (2013)

Si/Ge and SiGe alloy superlattices

Si/Ge: in-plane vs cross-plane anisotropy



W. Liu, T. Borca-Tasciuc, G. Chen, J. Liu, and K. Wang, J. Nanosci Nanotechnol. 1, 39 (2001).

T. Borca-Tasciuc, W. Liu, J. Liu, T. Zeng, D. W. Song, C. D. Moore, G. Chen, K. L. Wang, M. S. Goorsky, T. Radetic, R. Gronsky, T. Koga, and M. S. Dresselhaus, Superlattices Microstruct. **28**, 199 (2000).

Alloy superlattices \rightarrow

- κ increases w/increasing period
- the role of alloying

6 • 6.5 nm • 5 nm



Si/Ge: κ increases w/increasing period

S.-M. Lee, D. G. Cahill, and R. Venkatasubramanian, Appl. Phys. Lett. **70**, 2957 (1997).



S. T. Huxtable, A. R. Abramson, C.-L. Tien, A. Majumdar, C. LaBounty, X. Fan, G. Zeng, J. E. Bowers, A. Shakouri, and E. T. Croke, Appl. Phys. Lett. **80**, 1737 (2002).

S. T. Huxtable, Ph.D. thesis, University of California, Berkeley, 2002.

Z. Aksamija and I. Knezevic, Phys. Rev. B 88, 155318 (2013)

When is p not enough?

Wavelength-independent:



- Diffuse scattering angle from $\cos \theta \, d\Omega$ probability distribution
- 100% diffuse scattering called the Casimir Limit
- Thermal conductivities below this limit have been measured in Si nanowires
- H. B. G. Casimir, *Physica* **5**, 459 (1938)
- R. Berman, F. E. Simon, and J. M. Ziman, *Proc. Roy. Soc.* A 220, 171 (1953)

Wavelength-dependent:



• For zero correlation length, and large wavelength:

$$p(\theta_i) = e^{-(2q\sigma\cos\theta_i)^2}$$

- Diffuse scattering angle from $\left[1 e^{-(2q\sigma\cos\theta)^2}\right]\cos\theta \,d\Omega$
- Same limit of 100% diffuse scattering as previous model
- Z. Aksamija and I. Knezevic, *Phys. Rev. B.* 82, 45319 (2010)
- S. Soffer, J. Appl. Phys. 38, 1710 (1967)
- H. Davies, *Proc. IEEE part IV* **101**, 209 (1954)

When things get really really rough....









- Rough Si nanowires show greatly reduced κ vs bulk
- Potential thermoelectric applications
- Lower than the Casimir limit, showing limitation of the specularity model
- Depends on σ and L

J. Lim, K. Hippalgaonkar, S. C. Andrews, A. Majumdar, and P. Yang, Nano Lett. 12, 2475 (2012)

Phonon EMC



E. B. Ramayya, L. N. Maurer, A. H. Davoody, and I. Knezevic, Phys. Rev. B 86, 115328 (2012)

- R. B. Peterson, J. Heat Transfer 116, 815 (1994)
- S. Mazumder, and A. Majumdar, J. Heat Transfer 123, 749 (2001)
- D. Lacroix, K. Joulain, and D. Lemonnier, Phys. Rev. B 72, 064305 (2005)

 $\phi = -\kappa \frac{dT}{dx}$

Real-space rough surfaces



- Generates a random rough surface from an autocorrelation function
- Captures important features, like multiple scattering
- Allows lower thermal conductivities





Autocorrelation functions

 $L = 50, \sigma = 2$



We can make a surface to match any ACF.

 $ACF(\vec{r}) = \sigma^2 e^{-|\vec{r}|/L}$

C. Buran, M. G. Pala, M. Bescond, M. Dubois, and M. Mouis, *IEEE Trans. Electron Devices* **56**, 2186 (2009)

Models at room temperature



Temperature dependence



Dependence on rms roughness

Exponential autocorrelation



in prep (2013).

Gaussian autocorrelation



6 nm correlation length Exponential ACF 14 nm correlation length Exponential ACF 6 nm correlation length Gaussian ACF 14 nm correlation length Gaussian ACF 10 0.5 1.5 2 4.5 5.5 1 2.5 3 3.5 5 4 RMS Roughness (nm)

Dependence on correlation length

Exponential autocorrelation

Gaussian autocorrelation



Phonon trapping



70 nm thick, **3 nm rms roughness**,10 nm correlation length



L.N. Maurer and I. Knezevic, in prep (2013).



70 nm thick, 6 nm rms roughness,10 nm correlation length



Conclusions

- Semiclassical transport simulations of great practical utility
- Specularity parameter limited to moderately rough interfaces with very low correlation
- Very rough or correlated surfaces require a different approach
- High roughness phonon trapping



Thanks to

Thermal transport gang



Leon



Amirhossein



Song



Nishant



Farhad



Dr. Zlatan Aksamija (ECE, Umass)



Dr. Edwin Ramayya (Intel, OR)



Yanbing



Olafur



Gordon

DOE BES, Award No. DE-SC0008712 NSF ECCS, Award No. 1201311