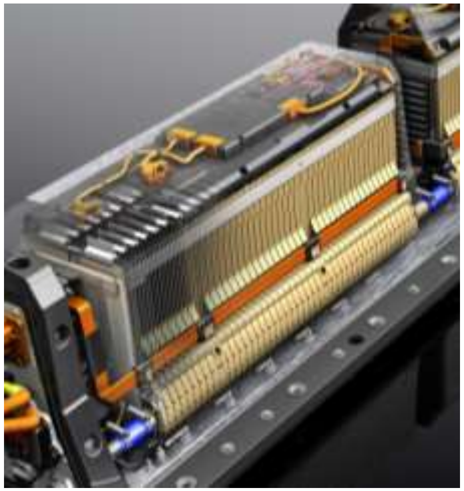
Reduction Modeling of Batteries in Electric Vehicle Cooling System Simulation

A. Hochman, M. Tsuk, S. Asgari,

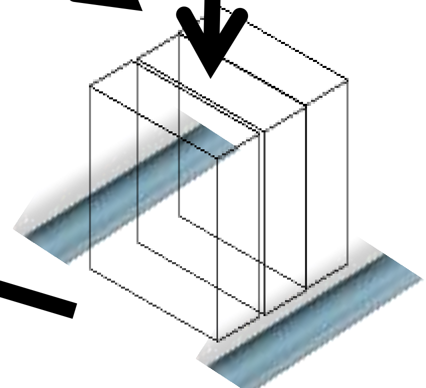
J. White

Cecil H. Green Professor of EECS, MIT

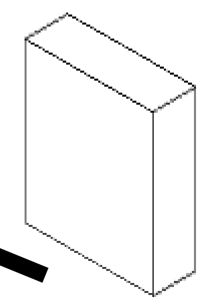
Battery Cooling Systems Not Yet An Application for MOR



Extract
Repeated
Structure



Extract
Battery
Cell



Repeated Structure ROM
Inputs/Outputs
•Pipe Velocities and Pressures
•Surface Temperatures and heat fluxes

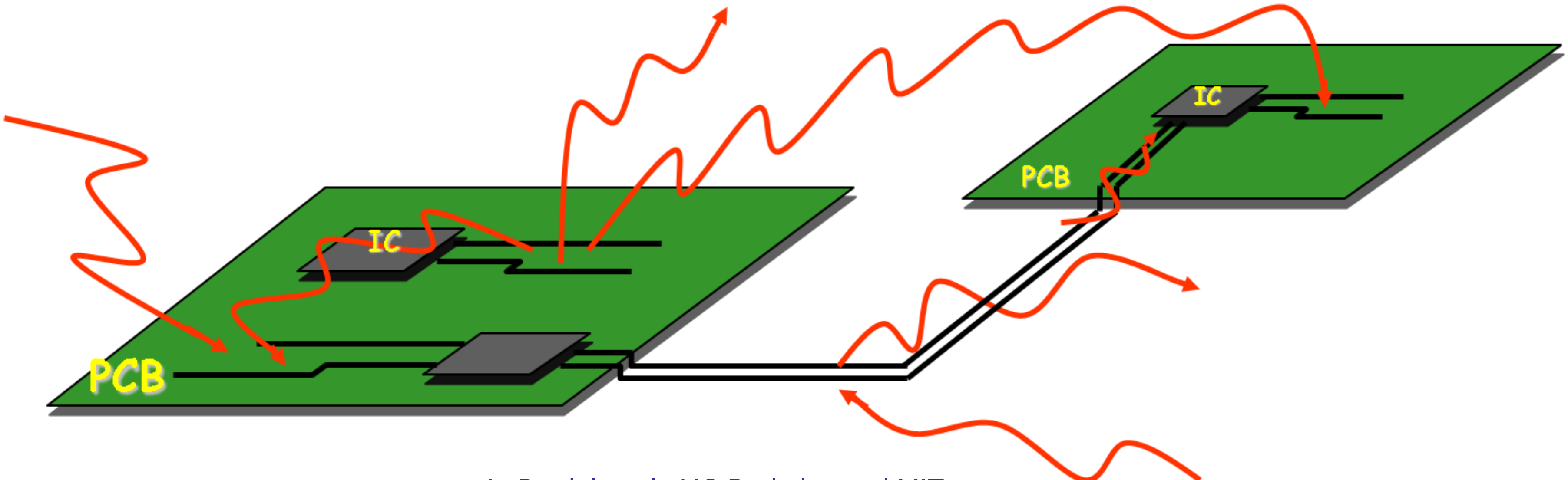
Volume Heat =
f(Terminal Power)

Two Challenges for MOR in Cooling Systems

- Distributed Interfaces
  - Thermal Transport
  - Will NOT Discuss
- MOR for Battery Nonlinearities
  - Thermal-Electrical Coupling
  - Will Discuss “sort of”

“Killer Ap” for MOR – Electronic Systems

Electronic System (ICs on boards with connectors).



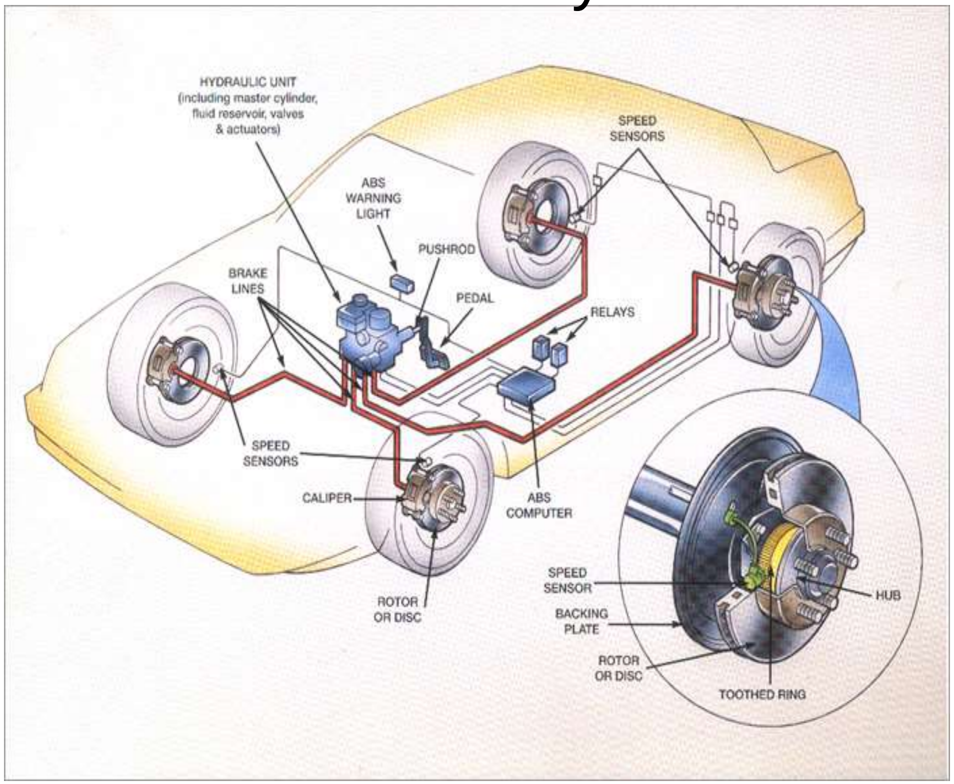
L. Daniel et al., UC Berkeley and MIT.

- Electronic system needs models for:
  - Connectors, PCB interconnect, IC drivers and receivers.

Every IC design, Almost every board design, is simulated using MOR

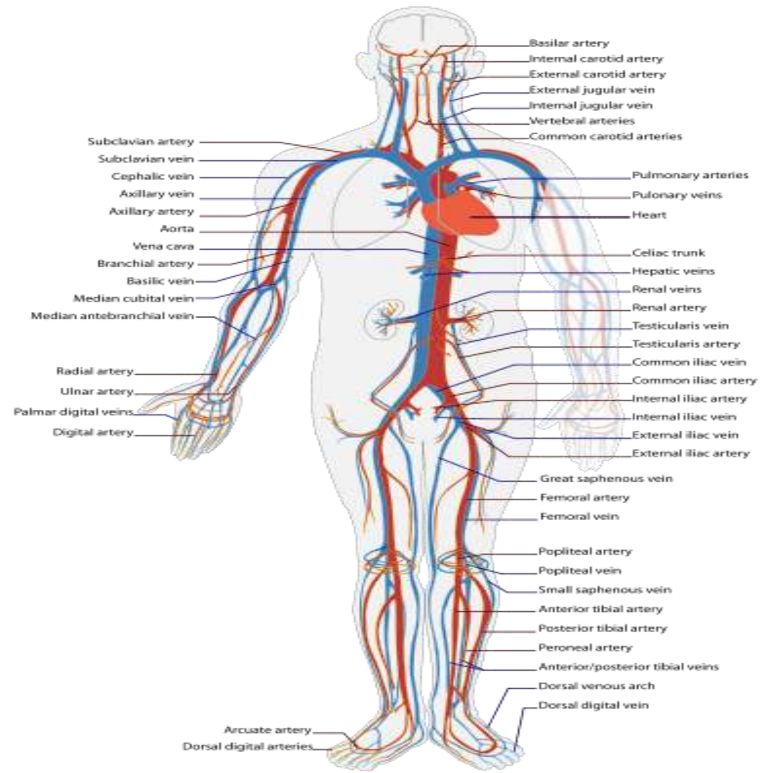
Emerging Applications for Model Reduction

Anti-lock Brake System



http://goodyeartacoma.com/uploads/images/absbrakes.jpg

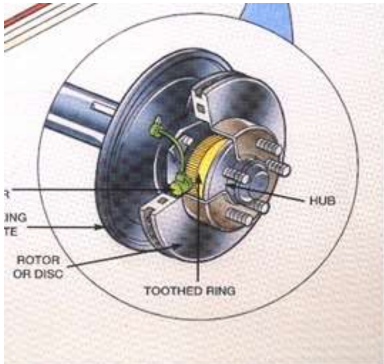
Circulatory System



http://en.wikipedia.org/wiki/File:Circulatory_System_en.svg

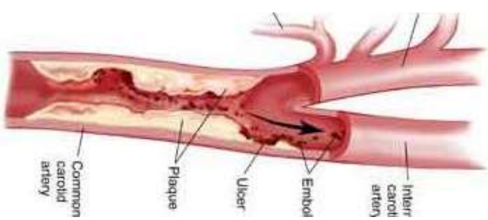
- ABS system needs state-space models for:
  - Master cylinder, disc brake, actuators, control electronics, pipes, etc
- Circulatory system needs models for:
  - Arterial and venous sections, heart

Typical Reduced “Components”



- Disc Brake
  - Input: Hydraulic Pressure, wheel torque
  - Output: disc rotational velocity

- Interconnect Model
  - Input/Outputs
    - Terminal Currents and Voltages



http://www.surgcare.net/Surgery/carotid_artery_disease.jpg

- Artery
  - Input/Outputs
    - Terminal flows and pressures

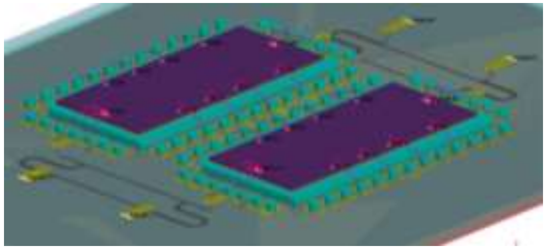
<u>Clear Interface Points!!</u>

Extracting Component Models

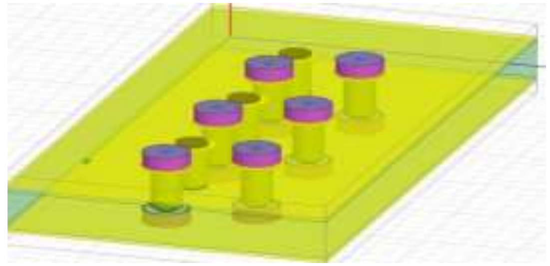
PDE plus boundary conditions → Low Order I/O Differential Equations

- For Electronic Interconnect
  - Maxwell's Equation
  - PDE is Linear
  - Constitutive eqns linear but frequency dependent.
- For Fluid Flow or Mechanical Deformation
  - Continuum Mech (CM) or N-Stokes Eqn (CFD).
  - PDE is nonlinear (geometric nonlinearity or convective term)
  - Constitutive equations often nonlinear
- NOTE: Mech. Dynamics are typically second-order
  - Consistency between velocity and displacement can impact extraction strategies

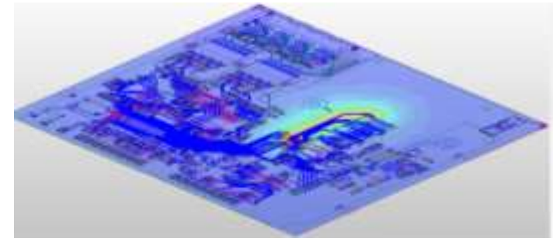
LTI: Rational Fit of Frequency Response



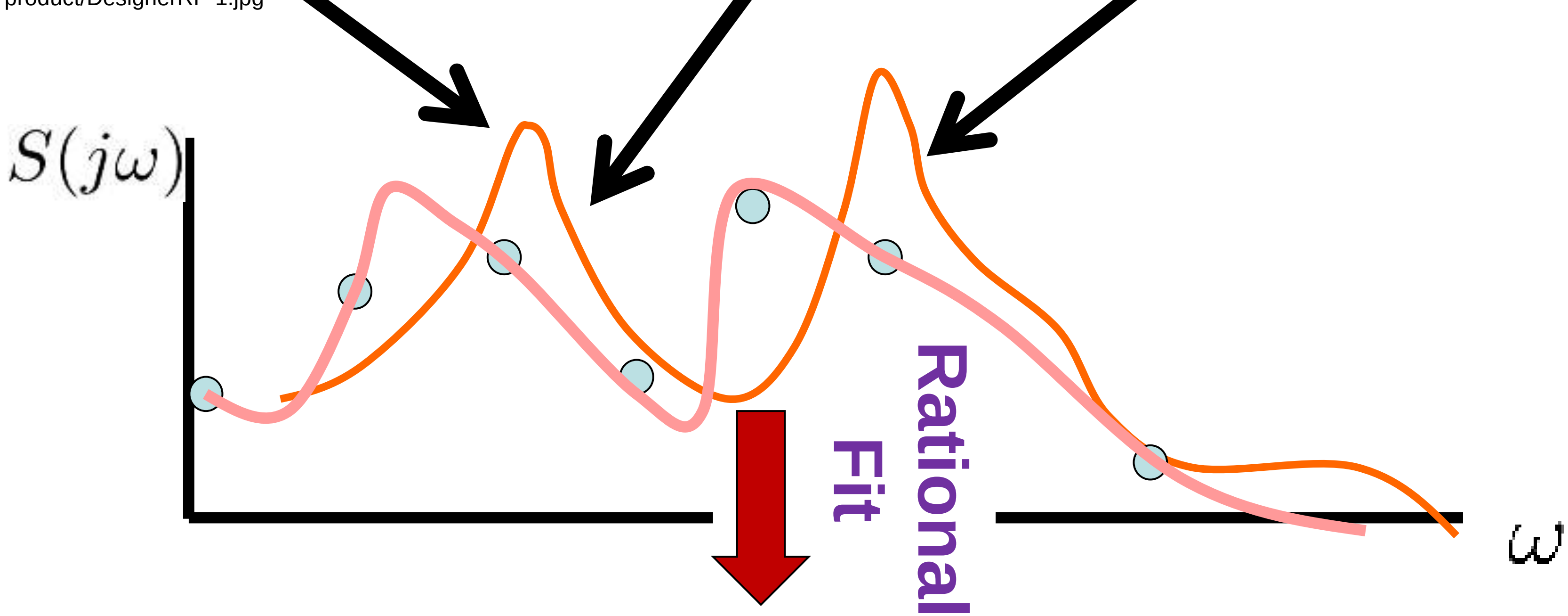
http://www.ansys.com/staticassets/ANSYS/staticassets/product/DesignerRF-1.jpg



Ansys HFSS group



www.edn.com – SI wave



$$S(j\omega) \approx \frac{\sum a_i s^i}{\sum b_i s^i}$$

$$\frac{dx_r(t)}{dt} = \underbrace{A_r}_{qxq} x(t) + \underbrace{b_r}_{qx1} \underbrace{u(t)}_{\text{scalar input}} \qquad \underbrace{y_r(t)}_{\text{scalar output}} = \underbrace{c_r^{\,T}}_{qx1} x_r(t)$$

What we have now

Frequency Domain I/O data

Fitting Algorithm

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

"Medium-sized" State-Space Model where A is "diagonal" by construction.

Passivity Enforcer

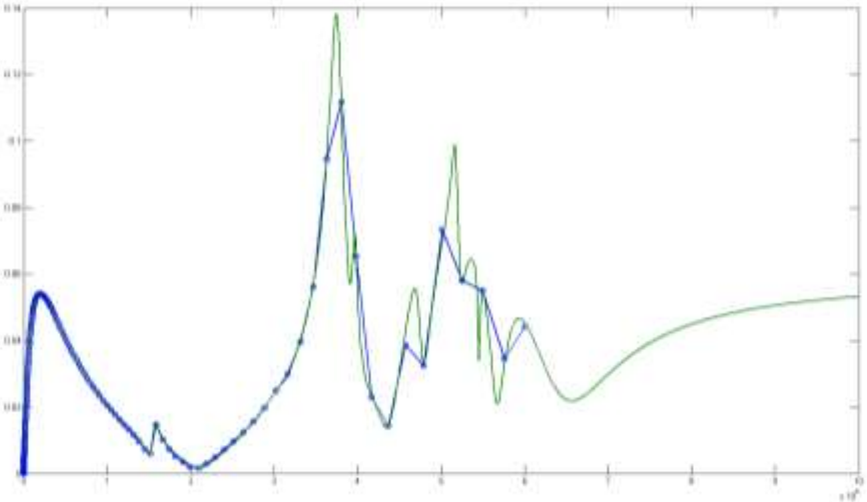
$$\frac{dx_r}{dt} = A_r x_r + B_r u$$
$$y_r = C_r x_r + D_r u$$

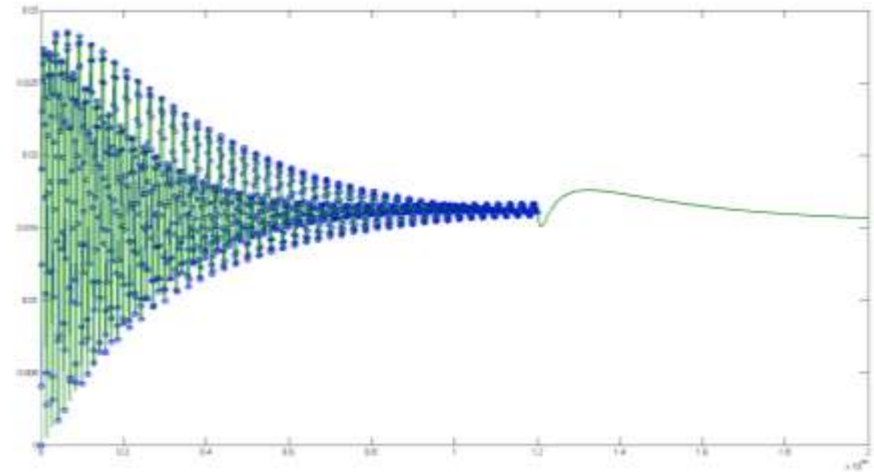
Reduced-order
. passive model.

DAE Integrator (SPICE, DASSL, Matlab)

Typical E-M Examples

Response Magnitude vs. Frequency

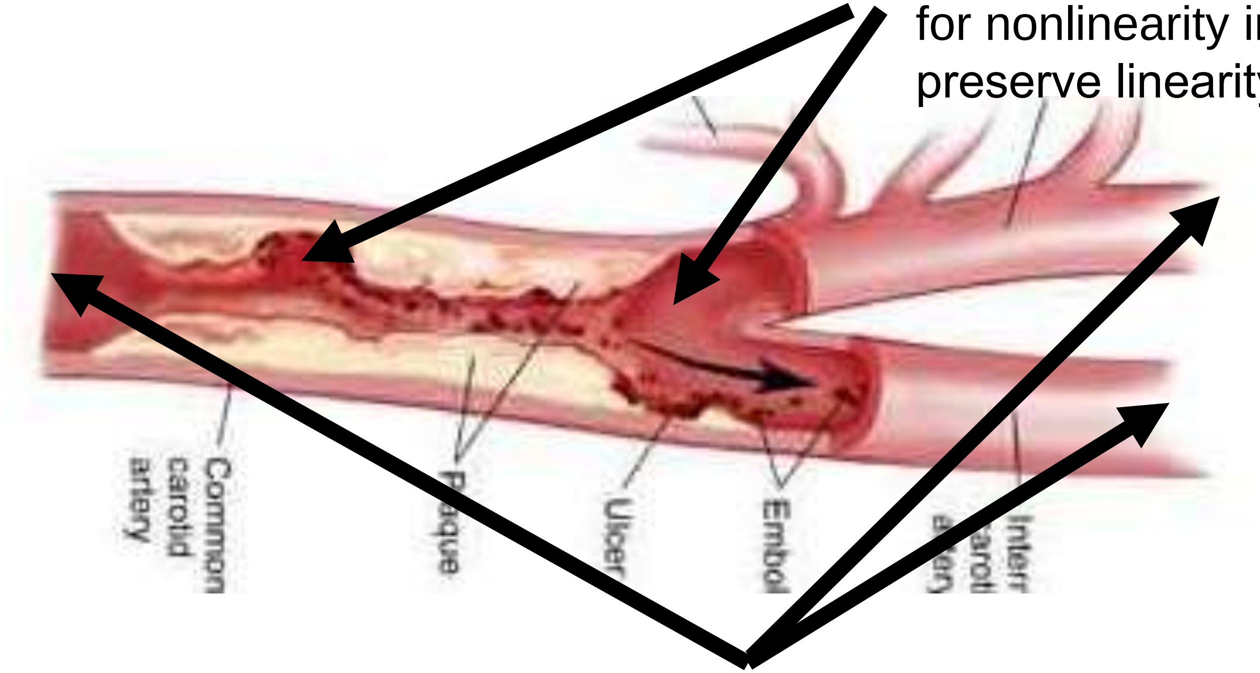




- Left:
  - Mixed power-signal integrity structure: 12 ports, 390 frequencies
- Right:
  - Interconnect structure: 6 ports, 1201 frequencies

Linearized PDE ineffective, but I/O behavior near linear (2nd Easiest case)

Stretching of artery walls may compensate for nonlinearity in interior flow patterns to preserve linearity at “terminals”.



Inputs and Outputs at “Terminals” of the artery

- Terminal Pressure/Flow Relations may be near linear, even if interior dynamics quite nonlinear.
- May not be that common a case.

Linearized PDE Case (Easiest)

- Linearized CFD or CM equations acceptably accurate.
- Two reduction options:

Easier	Compute I/O Frequency response by solving linearized equations with d/dt → iomega	→ Use our fitting strategy as a black box to generate state-space models.
More Efficient	Compute projection vectors by solving linearized system for selected frequencies or timesteps and inputs.	→ Use projection methods to generate state-space models.

Linearized Systems

- Fluids (descriptor)
  - Pressure-Velocity

$$\frac{d}{dt}M\vec{x}(t) = A\vec{x}(t) + B\,\vec{u}(t)$$
$$y(t) = C\vec{x}(t) + D\vec{u}(t)$$

- Mech (2^{nd} Order)
  - Force-Displacement

$$M\frac{d^2}{dt^2}\vec{x}(t) = F\frac{d}{dt}\vec{x}(t) + K\vec{x}(t) + B\,\vec{u}(t)$$
$$y(t) = C\vec{x}(t) + D\vec{u}(t)$$

- Electromag (frequency dependent)
  - Currents-Voltages

$$\omega^2 A(j\omega)\vec{x}(j\omega) = j\omega F(j\omega)\vec{x}(j\omega) + K(j\omega)\vec{x}(j\omega) + B(j\omega)\,\vec{u}(j\omega)$$
$$y(j\omega) = C(j\omega)\vec{x}(j\omega) + D(j\omega)\vec{u}(j\omega)$$

Projection Methods: State-Space Description

- Original Dynamical System - Single Input/Output

$$\frac{dx(t)}{dt} = \underset{NxN}{A}\, x(t) + \underset{Nx1}{b}\, \underset{\substack{scalar\\input}}{u(t)} \qquad \underset{\substack{scalar\\output}}{y(t)} = \underset{Nx1}{c}^{T} x(t)$$

- Reduced Dynamical System q << N, but I/O preserved

$$\frac{dx_r(t)}{dt} = \underset{qxq}{A_r}\, x(t) + \underset{qx1}{b_r}\, \underset{\substack{scalar\\input}}{u(t)} \qquad \underset{\substack{scalar\\output}}{y_r(t)} = \underset{qx1}{c_r}^{T} x_r(t)$$

Projection Framework

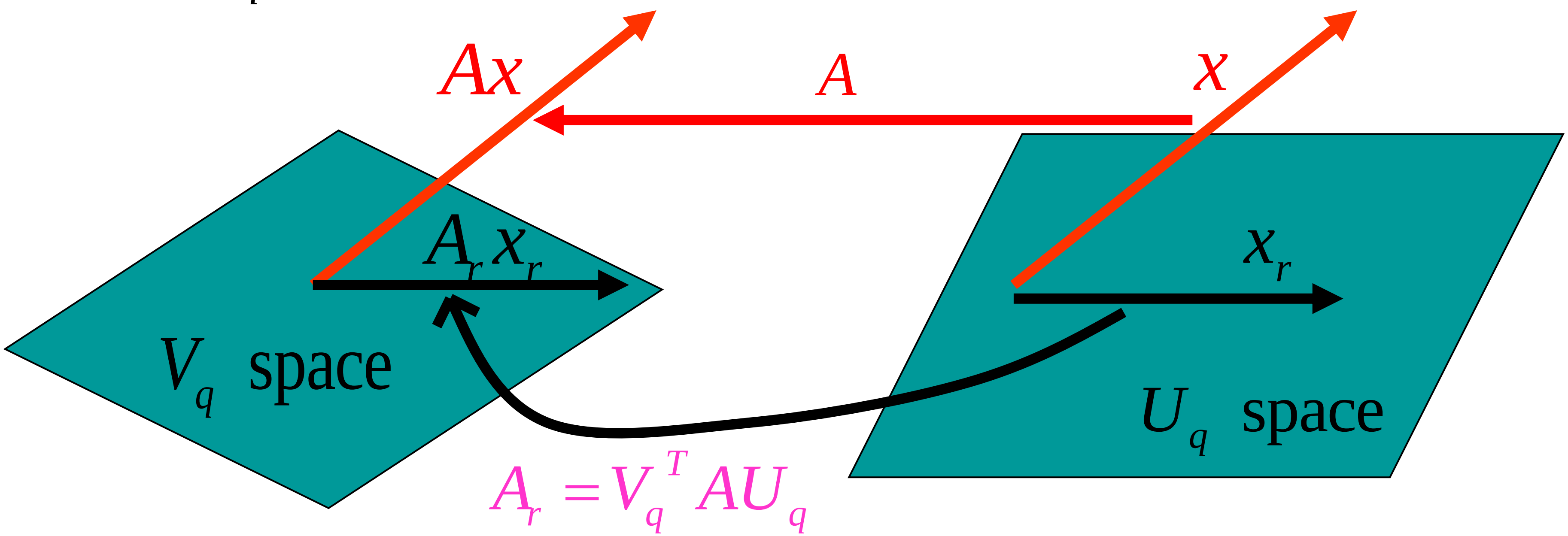
$$\dot{x} = Ax + bu,\ \ y = c^T x \ \Rightarrow \dot{x}_r = A_r x_r + b_r u,\ \ y_r = c_r^{\ T} x$$

<u>Equation Testing</u>

$$V_q^{\ T} A x \approx A_r x_r$$

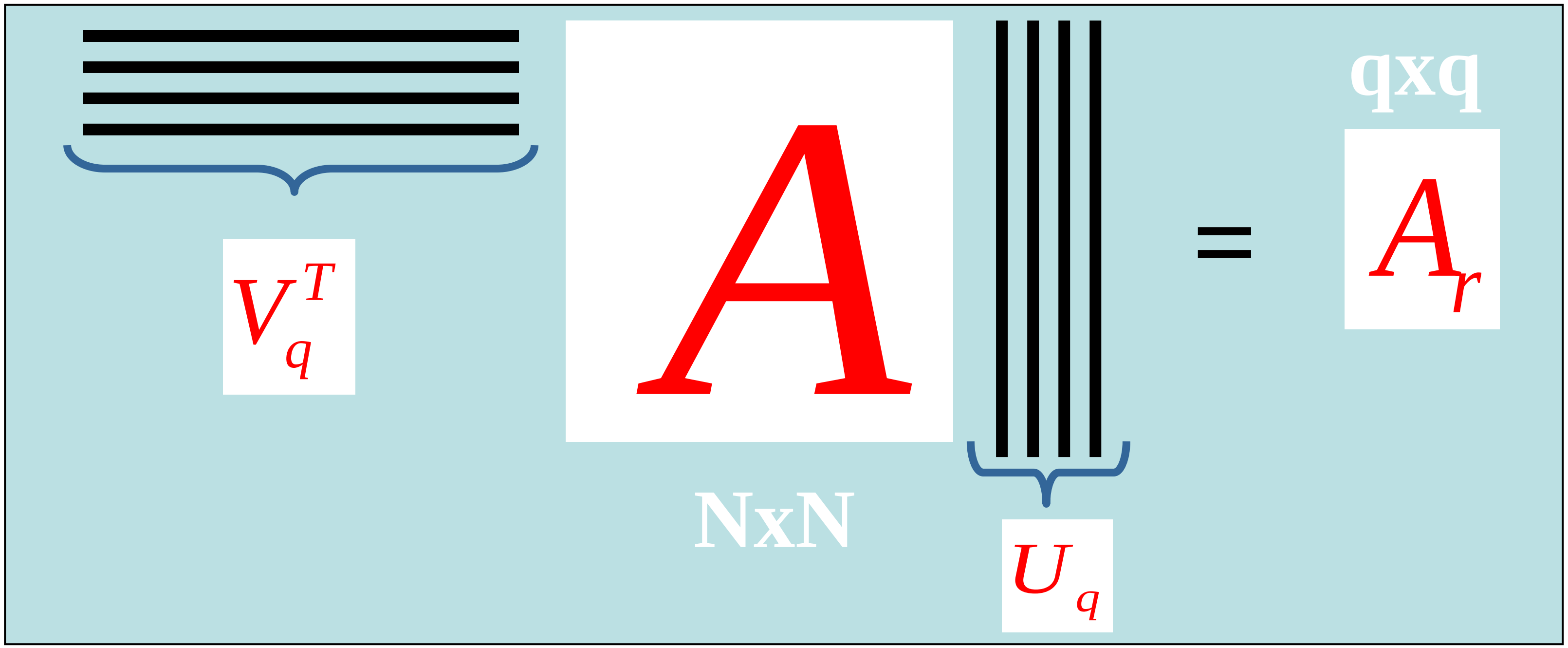
<u>Change of variables</u>

$$x \approx U_q x_r$$



Galerkin $\rightarrow V_q$ space = U_q space

Forming the Reduced Matrix



- No explicit A need, Only Matrix-vector products

For each column of U_q

Multiply by A, then dot result with columns of V_q

Picking U and V

- Use Eigenvectors (Modes)
- Use Time Series Data (Snapshot Method, POD)
  - Use the SVD to pick *q* < *k* important vectors

$$x(t_0), x(t_1), \cdots, x(t_k)$$

- Use Frequency Domain Data (Freq. Domain POD, PMTBR)
  - Use the SVD to pick *q* < *k* important vectors

$$X(s_1), X(s_2), \cdots, X(s_k)$$

- Krylov subspace Vectors
  - Again use SVD to pick q < k important vectors
- Use Singular Vectors of System Grammians (Too Costly)

Projection For Fluids (Descriptor)

$$M\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

$$V_q^T M U_q \frac{dx_r}{dt} = V_q^T A U_q x_r + V_q^T B u$$
$$y = C U_q x_r + Du$$

$$M_r \frac{dx_r}{dt} = A_r x_r + B_r u$$
$$y = C_r x_r + Du$$

where

$$M_r = V_q^T M U_q$$
$$A_r = V_q^T A U_q$$
$$B_r = V_q^T B$$
$$C_r = C U_q$$

Krylov For Fluids and Mech

- Standard Krylov Subspace

$$span\{A^{-1}B, A^{-2}B, A^{-3}B, ..., A^{-k}B\}$$

  - Must back orthogonalize at each step

- Krylov for Descriptor Systems with Singular M

$$span\{A^{-1}MA^{-1}B, \left(A^{-1}M\right)^2 A^{-1}B, \left(A^{-1}M\right)^3 A^{-1}B, ... \left(A^{-1}M\right)^k A^{-1}B\}$$

  - Still must back orthogonalize at each step

- Krylov for Mech

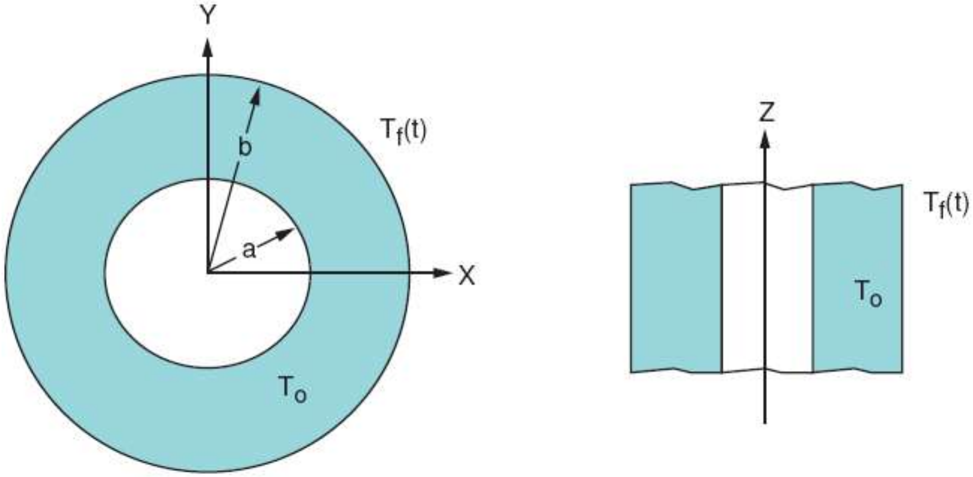
$$M \leftarrow \begin{pmatrix} I & 0 \\ 0 & M \end{pmatrix} \quad A \leftarrow \begin{pmatrix} 0 & I \\ K & F \end{pmatrix}$$

$$span\{A^{-1}MA^{-1}B, \left(A^{-1}M\right)^2 A^{-1}B, \left(A^{-1}M\right)^3 A^{-1}B, ... \left(A^{-1}M\right)^k A^{-1}B\}$$

  - Only Keep Top Half of the vectors

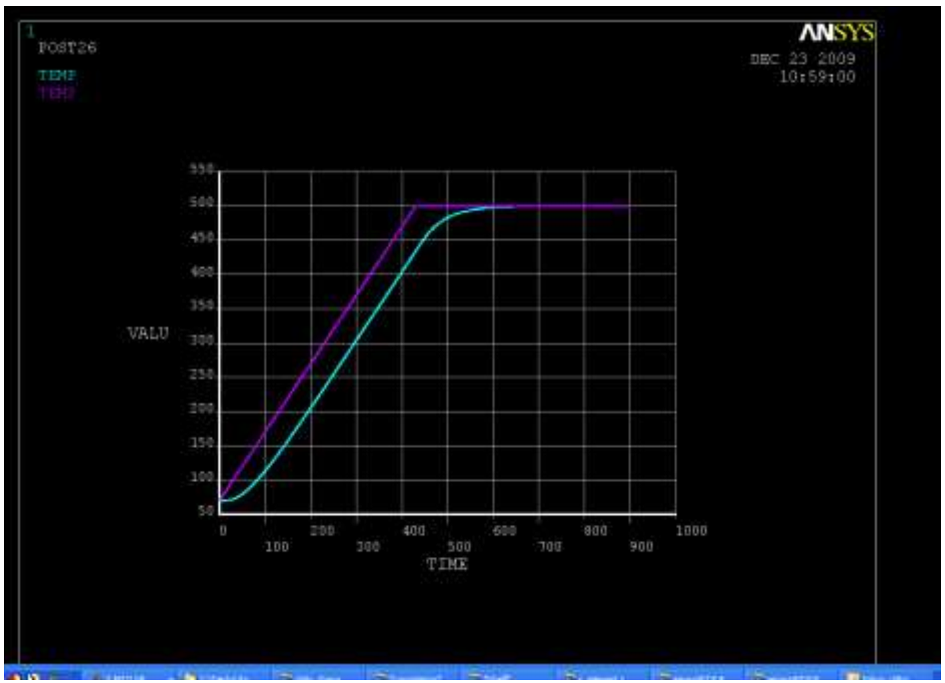
Thermal Expansion of Cylinder

- Example VM33 from ANSYS Verification Manual
- Outer radius of cylinder has temperature ramped up.

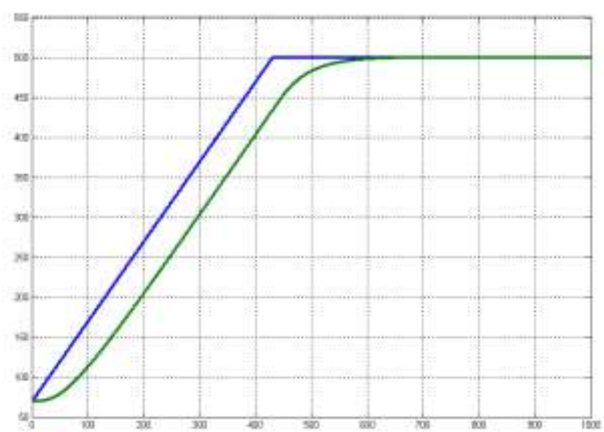


Thermal Expansion: MOR

- Temperature versus time



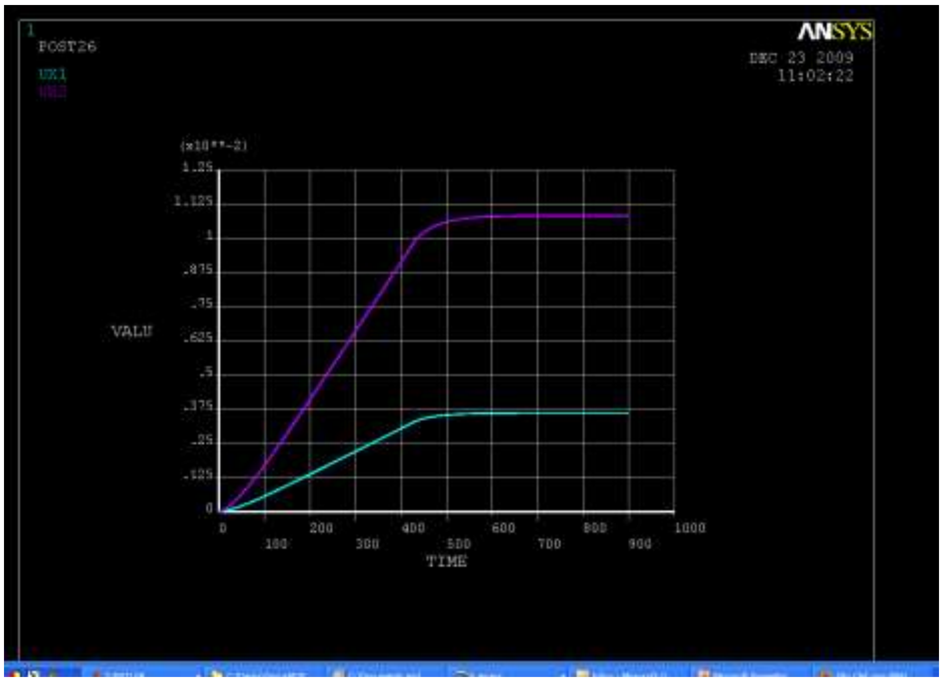
ANSYS



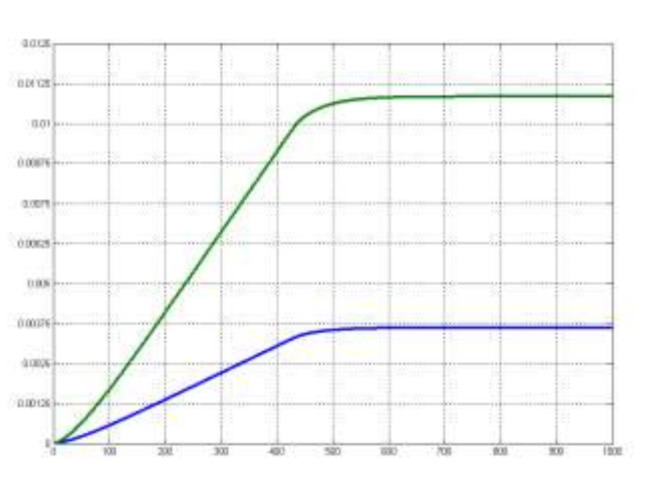
MOR (Matlab)

Thermal Expansion: MOR

- Displacement versus time



ANSYS



MOR (Matlab)

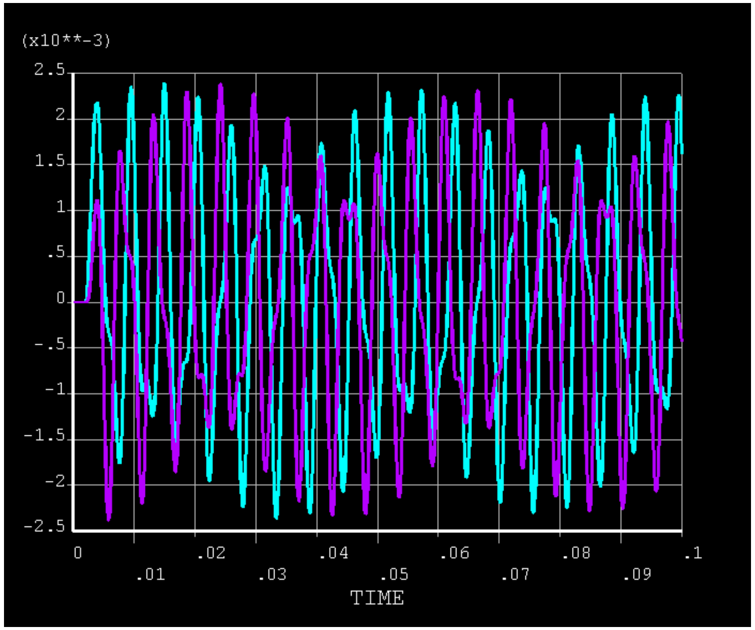
Rotor-bearing system

- Example VM247 from ANSYS Verification Manual
- Problem: only Rayleigh damping can be included in modal analysis
  - Damping proportional to mass and stiffness
  - Non-physical (losses only at bearings)

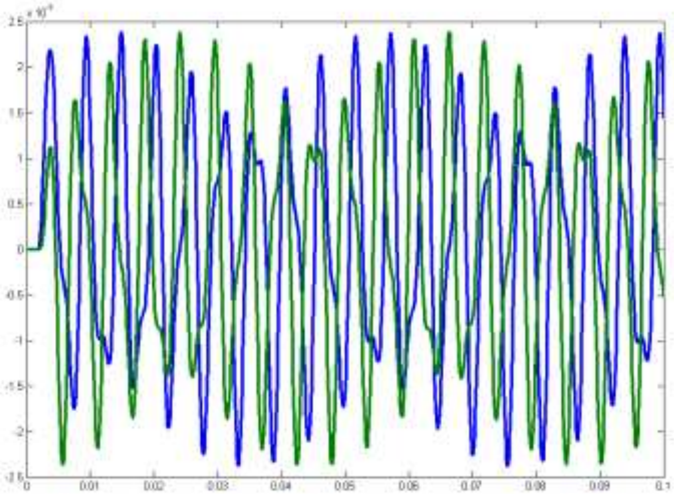
VM247 CAMPBELL DIAGRAMS AND CRITICAL SPEEDS USING SYMMETRIC BEARINGS

Rotor-bearing system: MOR

- Realistic loss included (hard to do with modes)
- Reduction from 184 states to 32
- Perfect agreement!

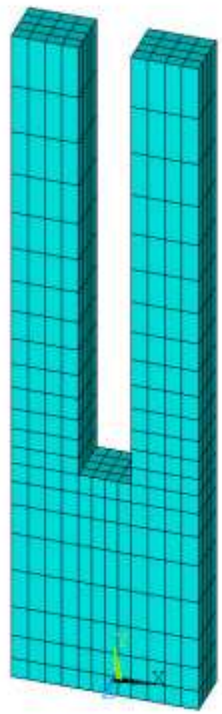


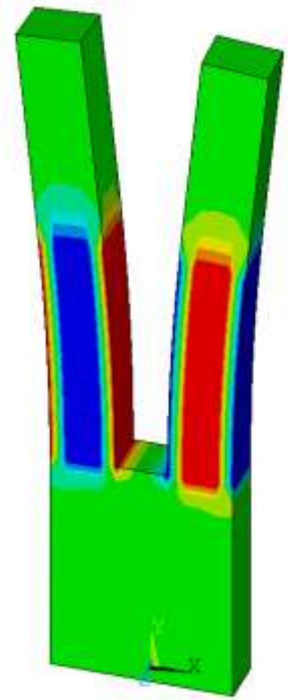
ANSYS



MOR (Matlab)

Tuning Fork



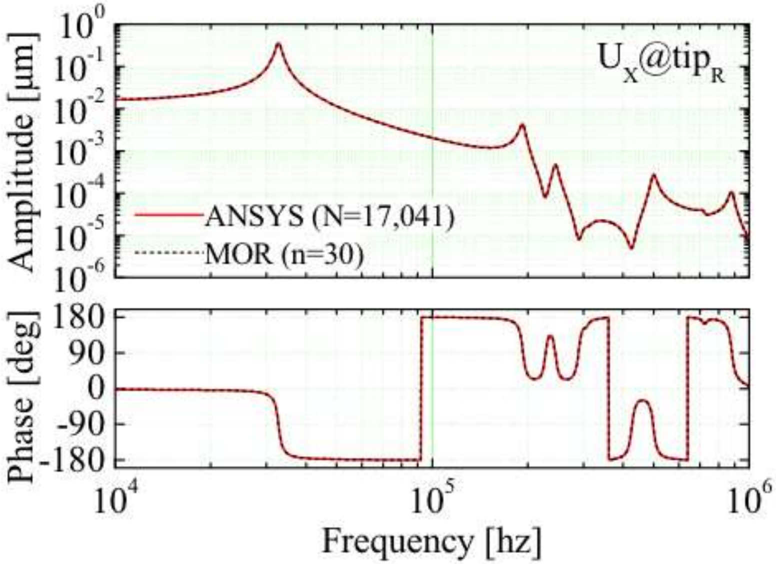


Vibrartion affected by rotation: Coriolis force

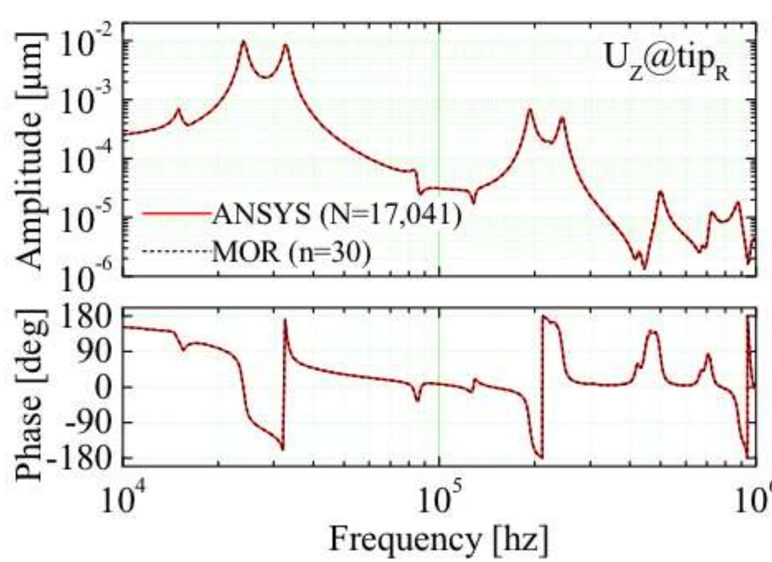
Quartz Tuning Fork

- **Frequency response: out-of-plane vibration ($\Omega_Y=10^3$ rad/s)**

•U_X@tip$_R$

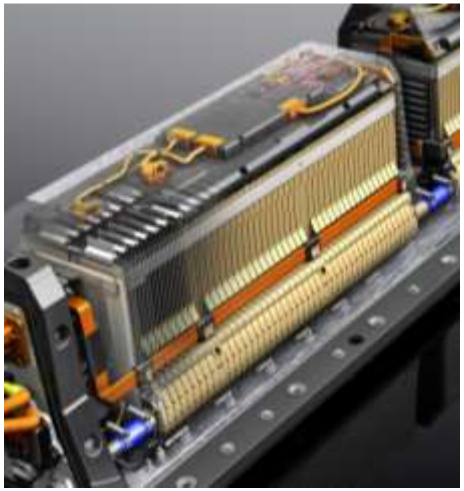


•U_Z@tip$_R$

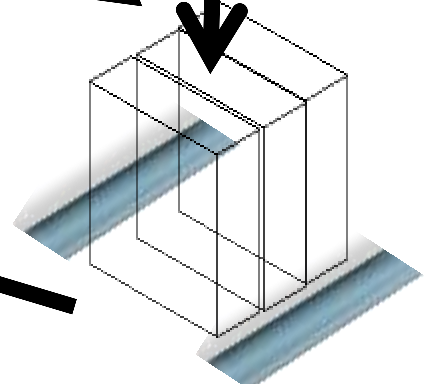


- No. of DOFs: 17,041

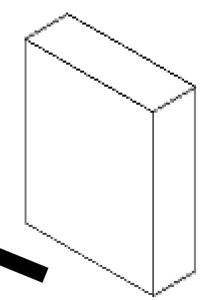
Multiphysics Example – Battery Packs



Extract
Repeated
Structure



Extract
Battery
Cell

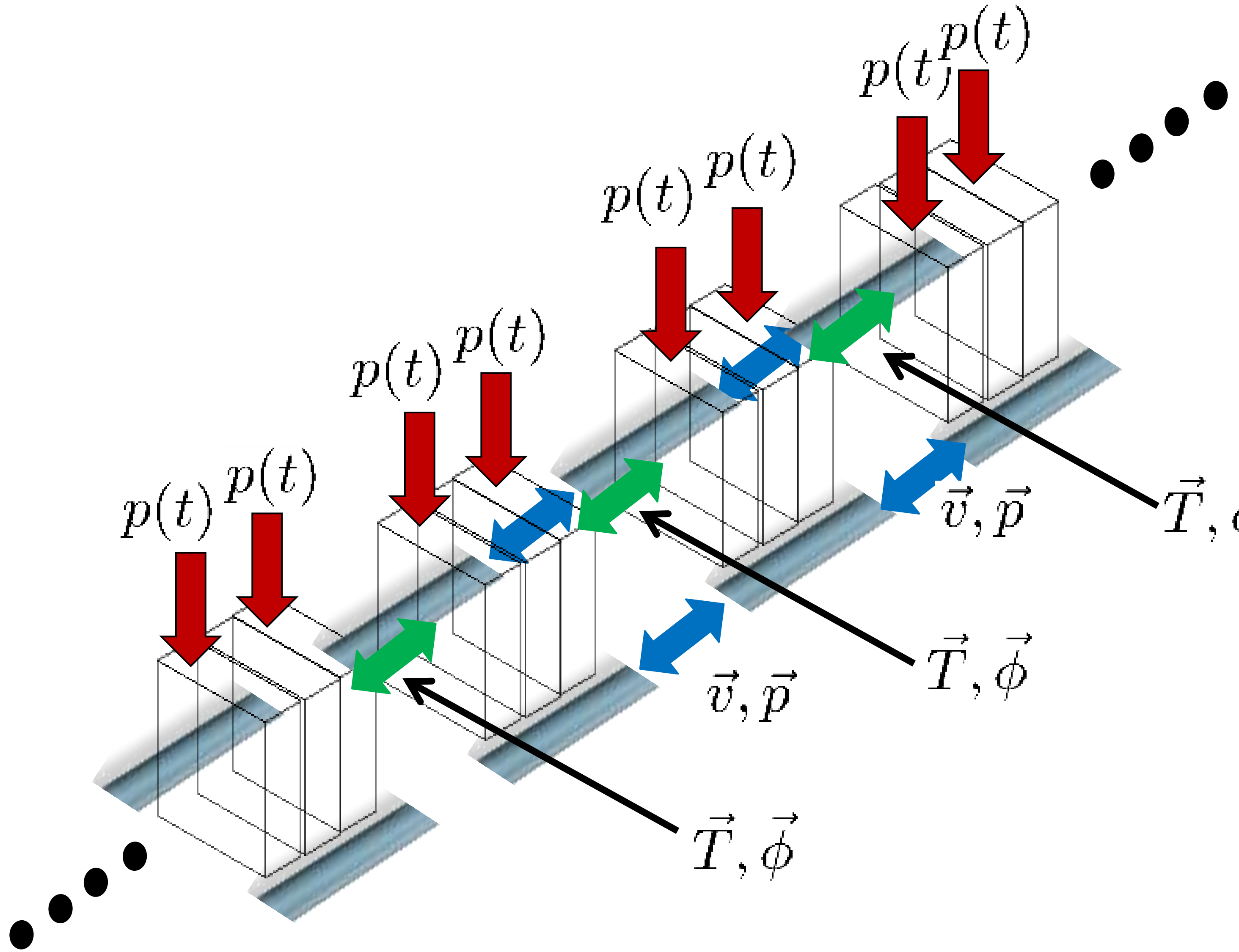


Repeated Structure ROM
Inputs/Outputs

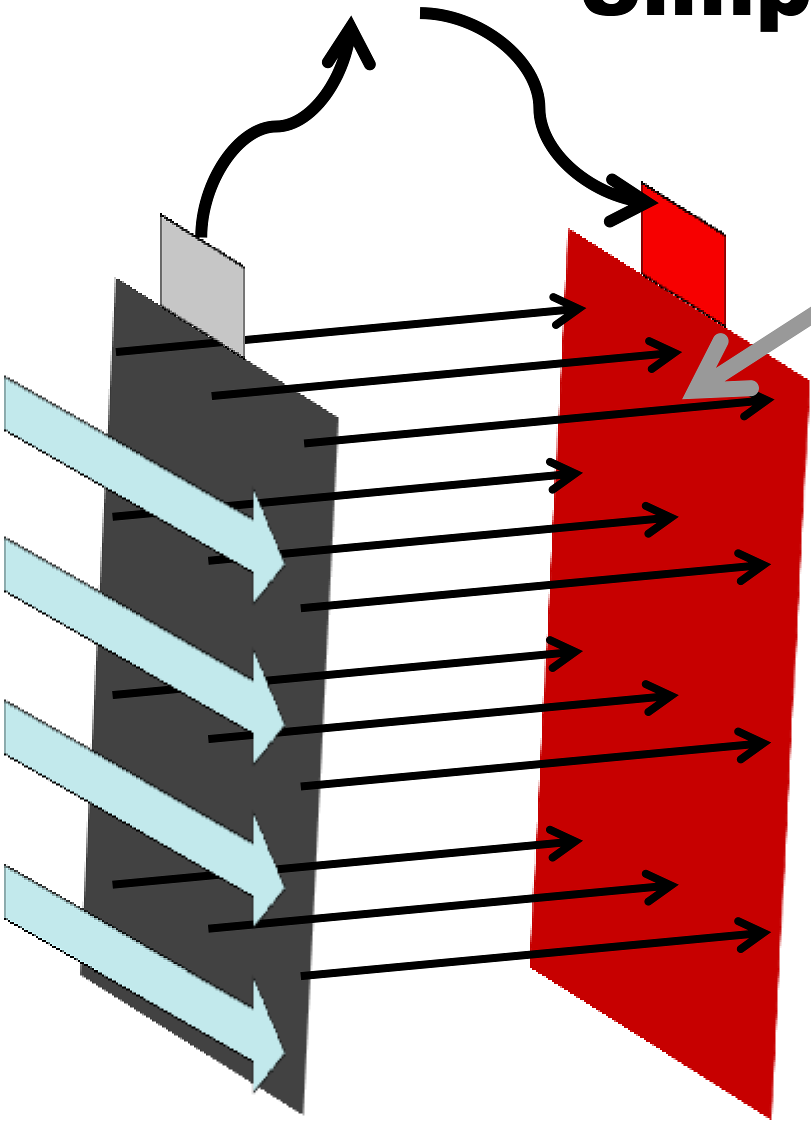
- Pipe Velocities and Pressures
- Surface Temperatures and heat fluxes

Volume Heat =
f(Terminal Power)

A Distributed Interface



Simplified Single Cell



- NTGK Electrochemistry model

$$i = f(\phi_+ - \phi_-, DOD)$$

$$\frac{d}{dt}DOD = f(\phi_+ - \phi_-, DOD)$$

- Electrical Conductivity Model

$$\sigma_e \nabla^2 \phi_+ = i$$

$$\sigma_e \nabla^2 \phi_- = -i$$

- Thermal Conductivity Model

$$\sigma \nabla^2 T = (\phi_+ - \phi_-) * i$$

- Sheet Flow Model

$$\sigma_s \nabla^2 T_s + MFR \cdot \nabla T_s = \alpha \cdot (T_s - T)$$

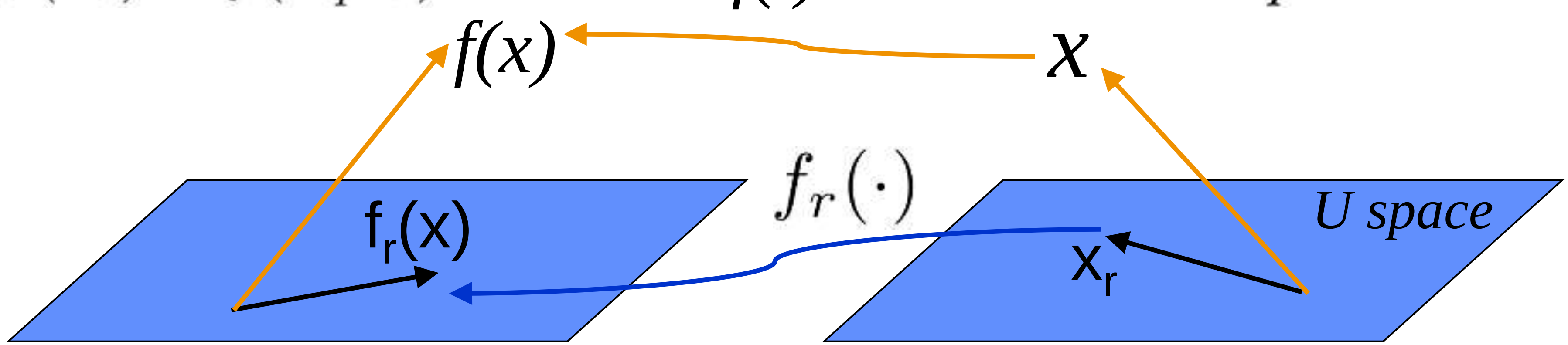
Nonlinear MOR – Representation Problem

- Nonlinear dynamical systems:

$$\frac{dx}{dt} = f(x) + Bu \qquad y = C^T x \qquad x \in R^n$$

- Projection of the nonlinear operator $f(x)$:

$$V_q f_r(x_r) \approx f(U_q x_r) \qquad f(.) \qquad U_q x_r \approx x$$



- How to find $f_r(x_r)$?

Problems with MOR for nonlinear

- Substitute: $x = U_q x_r$ in $\dfrac{dx}{dt} = f(x) + Bu$
- Reduced system: $\dfrac{dx_r}{dt} = \overbrace{V_q^T f(U_q x_r)}^{\hat{f}(x_r)} + V_q^T Bu$
- Problem: $V_q^T f(U_q x_r):\quad \underset{\text{small}}{R^q} \rightarrow \underset{\text{large}}{R^N} \rightarrow \underset{\text{large}}{R^N} \rightarrow \underset{\text{small}}{R^q}$

  $q=10 \qquad N=10^4 \qquad N=10^4 \qquad q=10$
- Using $V_q^T f\left(U_q x_r\right)$ is too expensive!

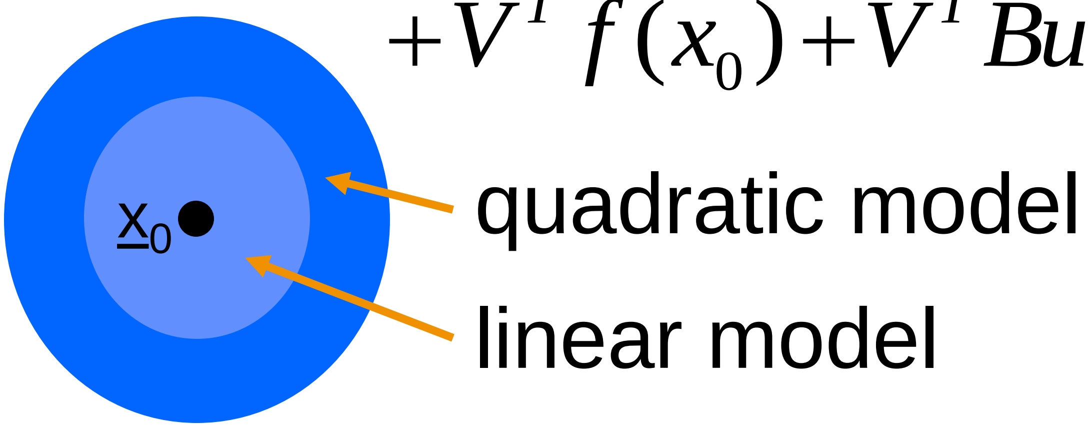
Volterra Approach

- Use Taylor's expansions to approximate *f(x)*:

$$f(x) = f(x_0) + J(x - x_0) + W((x - x_0) \otimes (x - x_0)) + \ldots$$

- Linear, quadratic reduced order models [Chen, Phillips 2000]:

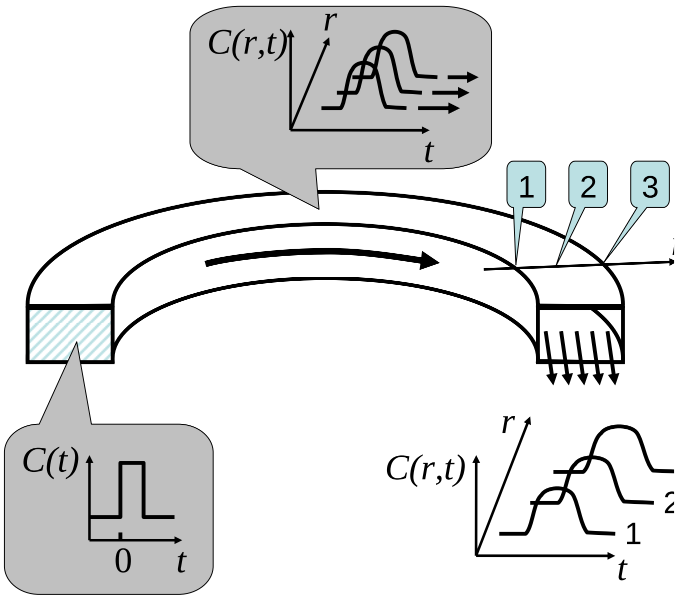
$$\frac{dz}{dt} = \overbrace{V^T J V}^{\hat{J}}(z - z_0) + \overbrace{V^T W V \otimes V}^{\hat{W}}(z - z_0) \otimes (z - z_0) + V^T f(x_0) + V^T Bu$$

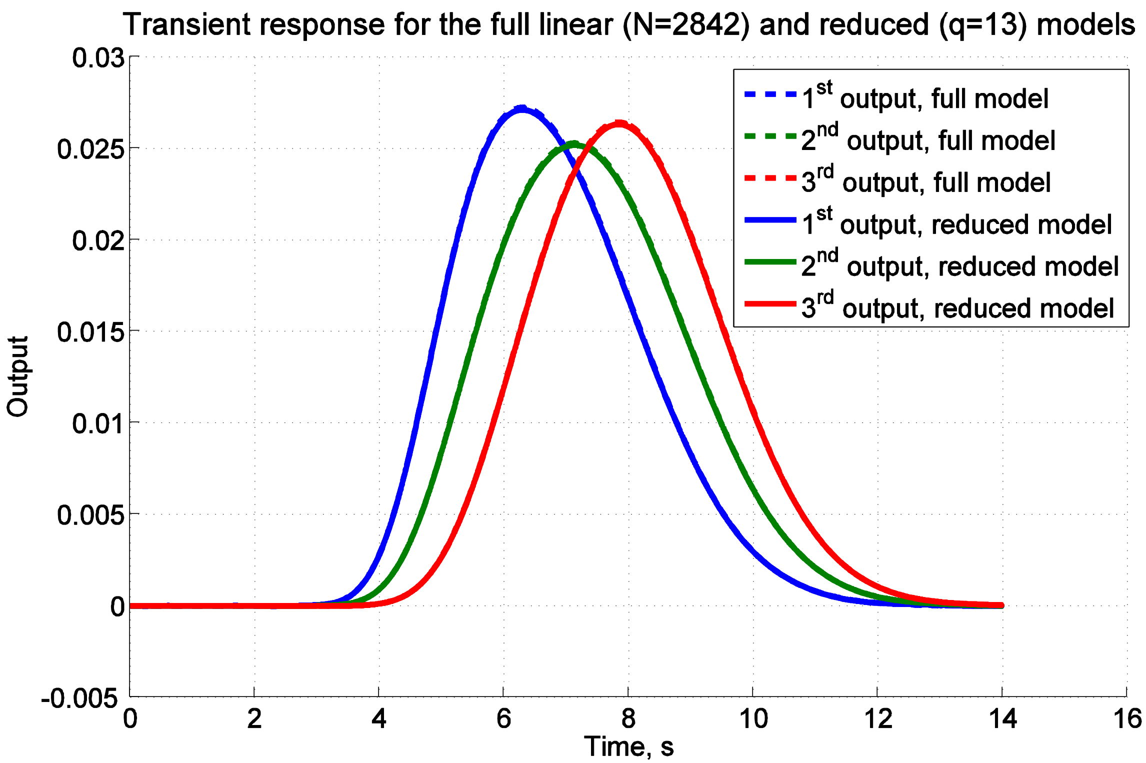


Convection-Diffusion Example

$$\frac{\partial C}{\partial t} = -\mu F \cdot \nabla C + D\nabla^2 C$$

- Easy Problem, nearly 3000 states to 13 states





Nonlinear Convection-Diffusion

$$\frac{\partial C}{\partial t} = -(F \cdot \nabla C)(C\frac{d\mu}{dC} + \mu) + D\nabla^2 C + \frac{dD}{dC}\|\nabla C\|^2$$

Linearization ineffective, because of propagating wave.

Consider a <u>*quadratic problem:*</u>

μ and D *linearly dependent on* C:

$$\mu(C) = \mu_1 C + \mu_0$$

$$D(C) = D_1 C + D_0$$

Kronecker Form

Quadratic Convection-Diffusion:

$$\begin{cases} \frac{dx}{dt} = \underbrace{Ax(t) + Bu(t)}_{\text{linear}} + \\ \qquad +T(x(t) \otimes x(t)) + K(x(t) \otimes u(t)) + L(u(t) \otimes u(t)), \\ y(t) = Cx(t) \end{cases}$$

- Matrices T, K, L, contain coefficients of the quadratic terms $x_i x_j$, $x_i u_j$, $u_i u_j$.
- Reduction can be performed by projection.

Projecting the Quadratic Form

$$\begin{cases} \frac{dx}{dt} = \underbrace{Ax(t) + Bu(t)}_{\text{linear}} + \\ \qquad\qquad +T(x(t) \otimes x(t)) + K(x(t) \otimes u(t)) + L(u(t) \otimes u(t)), \\ y(t) = Cx(t) \end{cases}$$

Substituting $x(t) \approx Ux_r(t)$ and projecting residual using V^T:

$$\begin{cases} \frac{dx_r}{dt} = V^T AUx_r(t) + V^T Bu(t) + \\ \qquad +V^T T(Ux_r(t) \otimes Ux_r(t)) + V^T K(Ux_r(t) \otimes u(t)) + V^T L(u(t) \otimes u(t)), \\ y(t) = CUx_r(t) \end{cases}$$

Therefore, for the reduced system matrices, we have:

$$A^r = V^T AU, \quad B^r = V^T B, \quad C^r = CU,$$

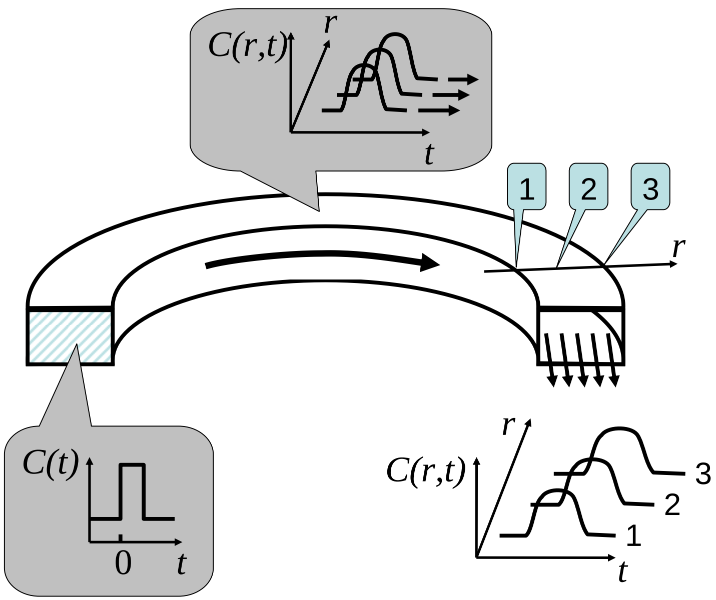
(as for any linear system)

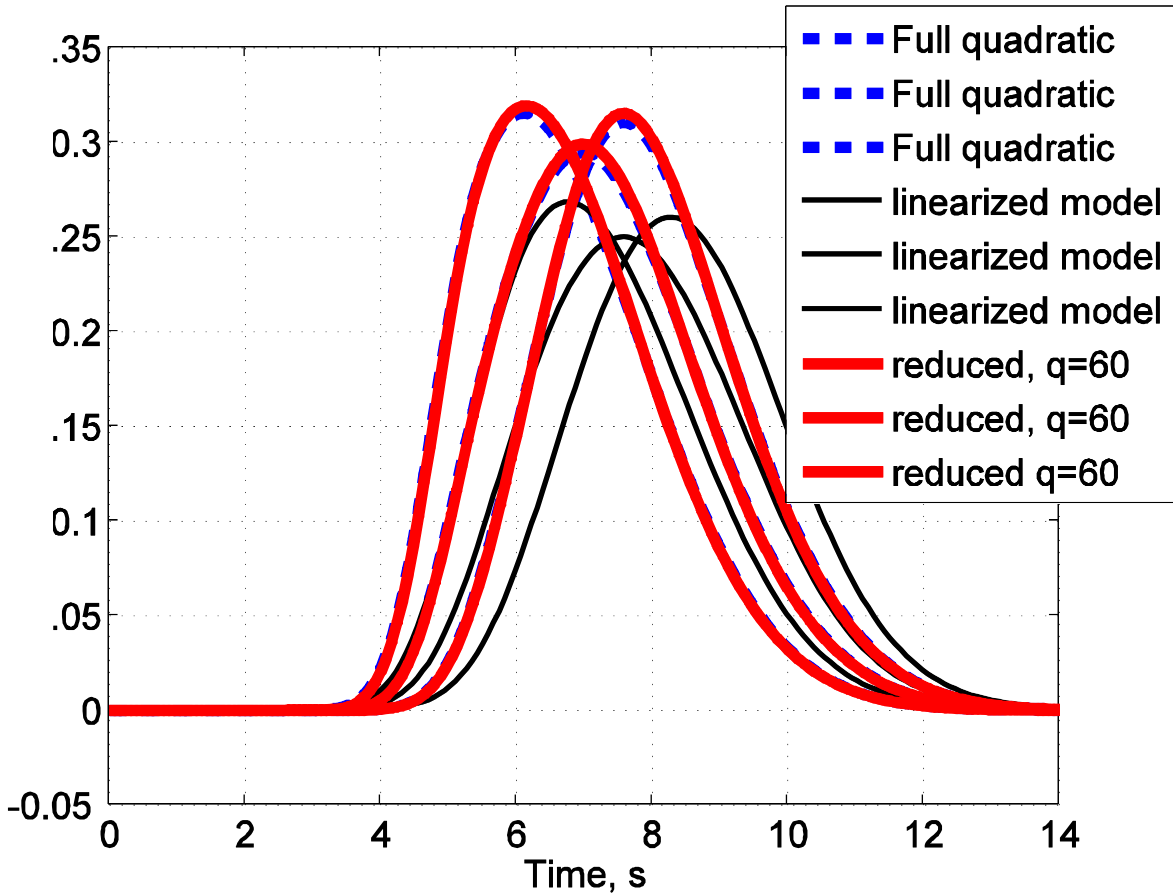
$$T^r = V^T T(U \otimes U), \quad K^r = V^T KU, \quad L^r = V^T L,$$

(projection rules for quadratic terms)

Example Results

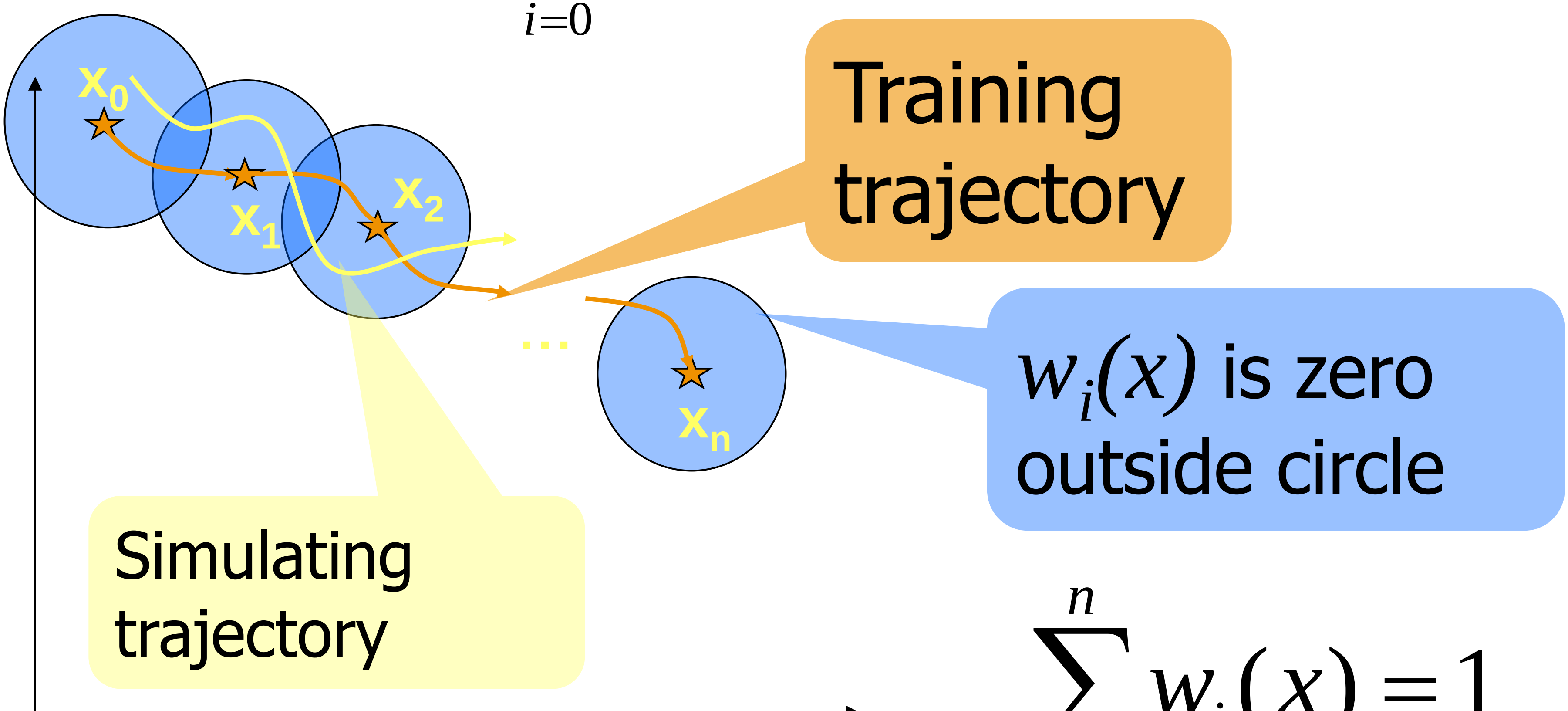
$$\frac{\partial C}{\partial t} = (\nabla\Phi\cdot\nabla C)(C\mu_1+\mu_0) - D_0\nabla^2 C - D_1\|\nabla C\|^2$$





Trajectory Piecewise Linear approximation of *f*.

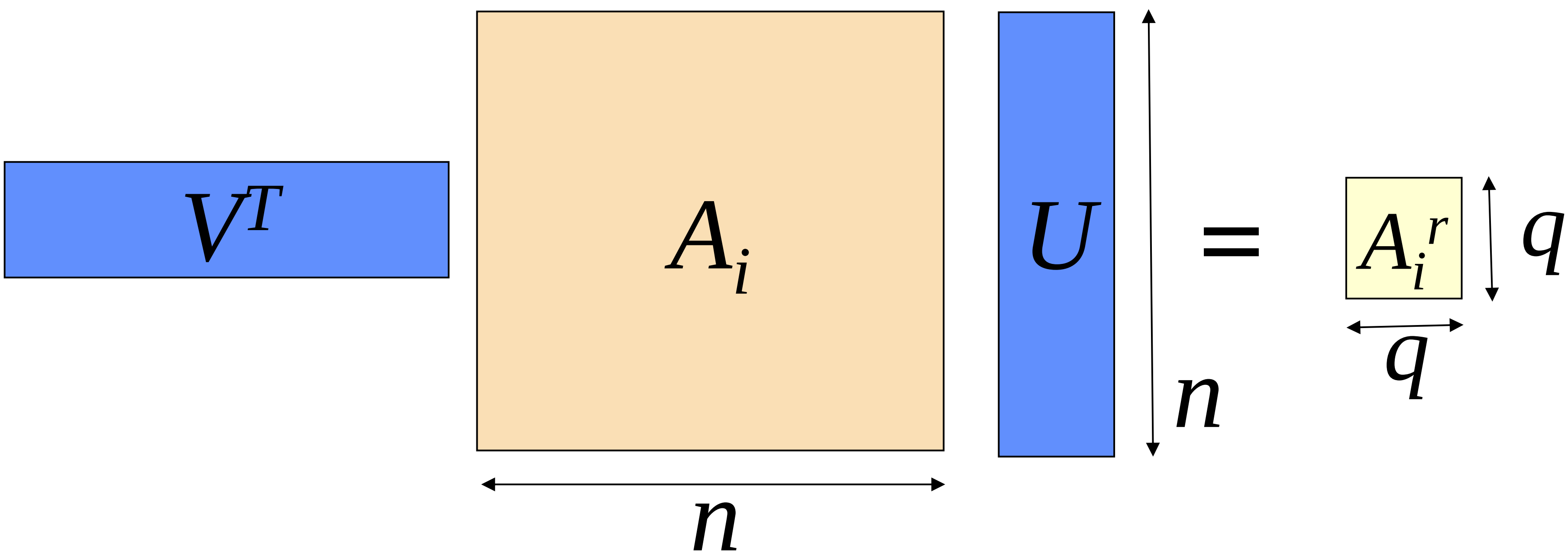
$$f_{TPWL}(x) = \sum_{i=0}^{n} w_i(x)\big(f(x_i) + A_i(x - x_i)\big)$$



$$\sum_{i=0}^{n} w_i(x) = 1$$

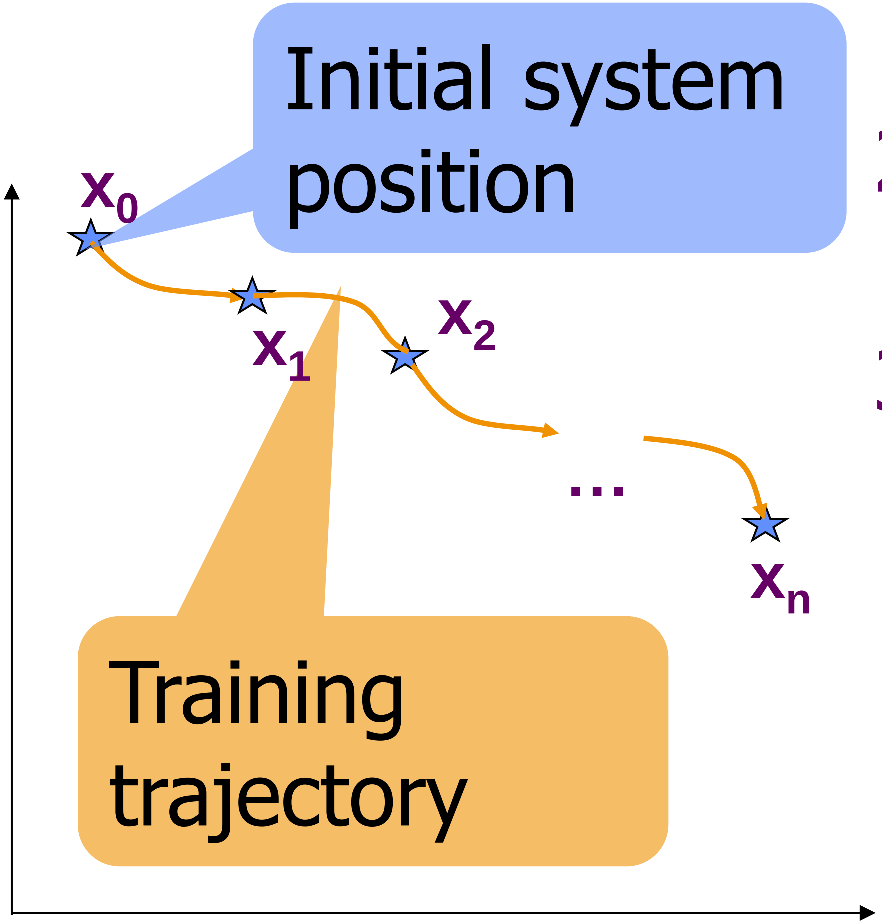
Projection and TPWL approximation yields efficient f^r

$$f_{TPWL}(x_r) = \sum_{i=0}^{n} w_i(x_r)\left(\underbrace{V^T f(x^i)}_{q \times 1} + \underbrace{V^T A_i U}_{A_i^r}(x_r - V^T x^i)\right)$$



Evaluating f^r_{TPWL} only order q^2 operations

TPWL approximation of *f*: Extraction algorithm



Non-reduced state space

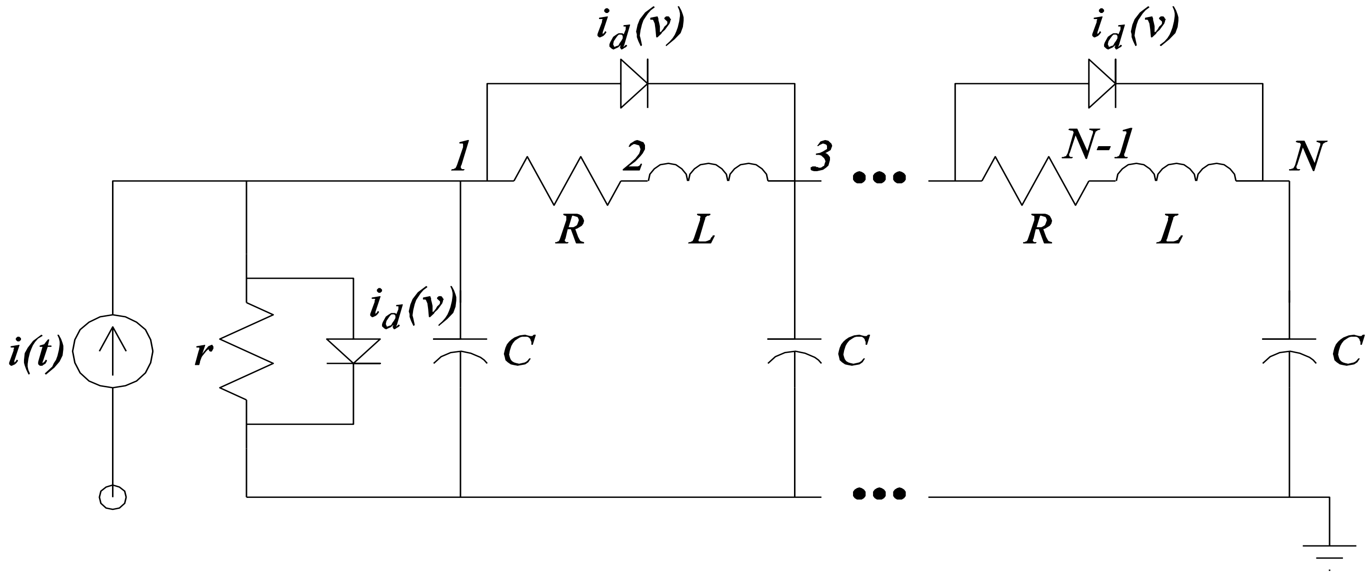
1. Compute A_1
2. Obtain W_1 and V_1 using linear reduction for A_1
3. Simulate training input, collect and reduce linearizations

$$A_i^r = W_1^T A_i V_1$$

$$f^r(x_i) = W_1^T f(x_i)$$

Example problem

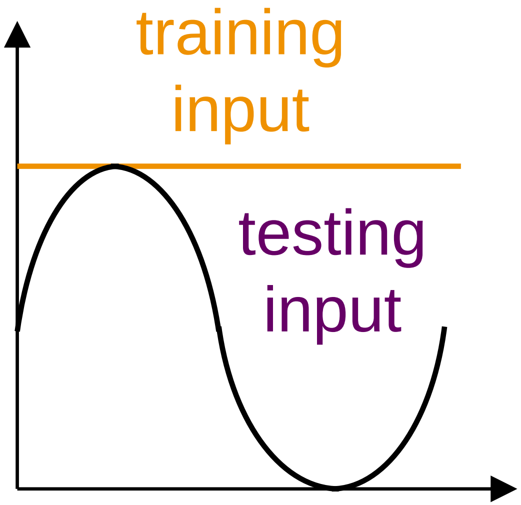
RLC line



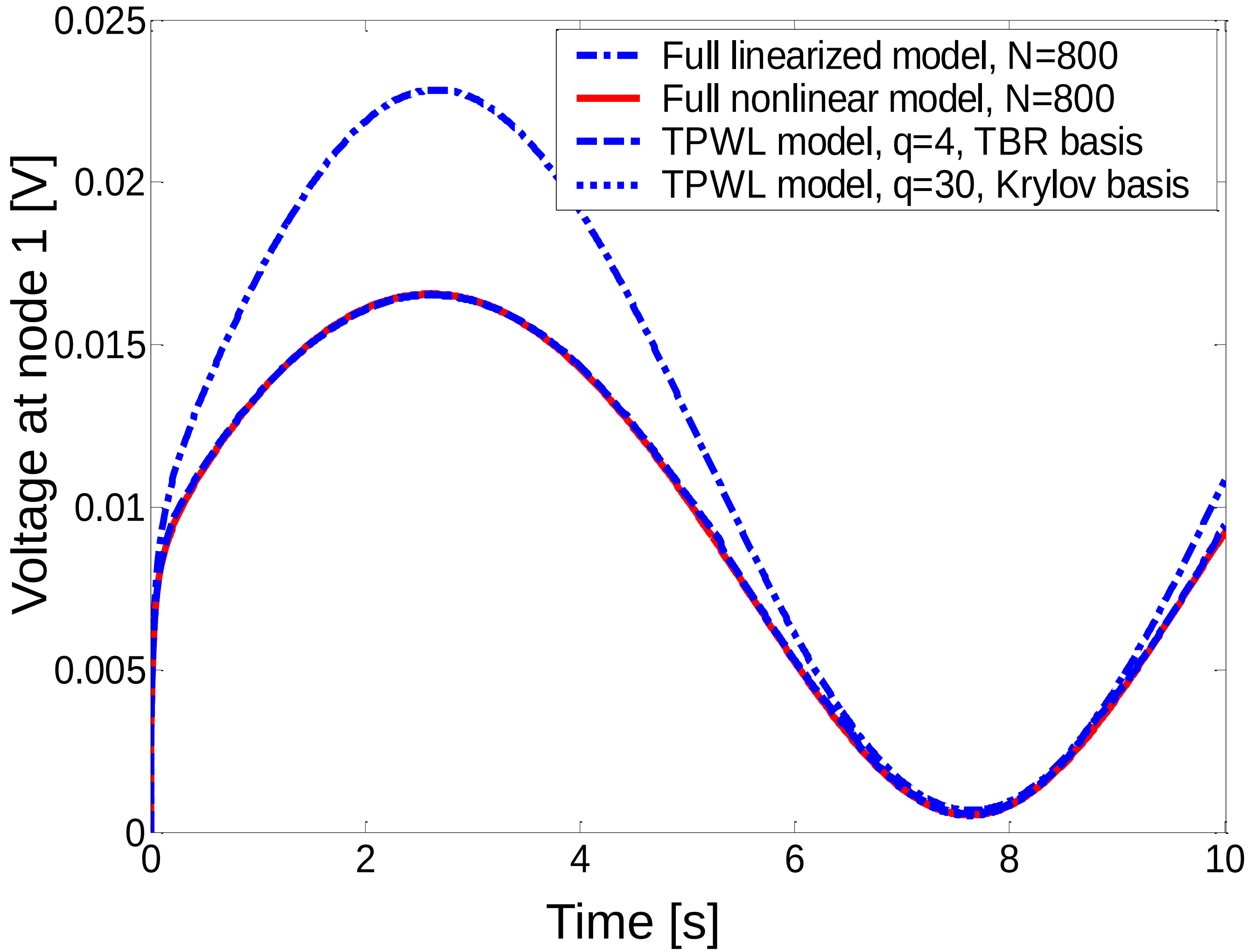
Linearized system has nonsymmetric, indefinite Jacobian

Numerical results
– nonlinear RLC transmission line

- **Input:**



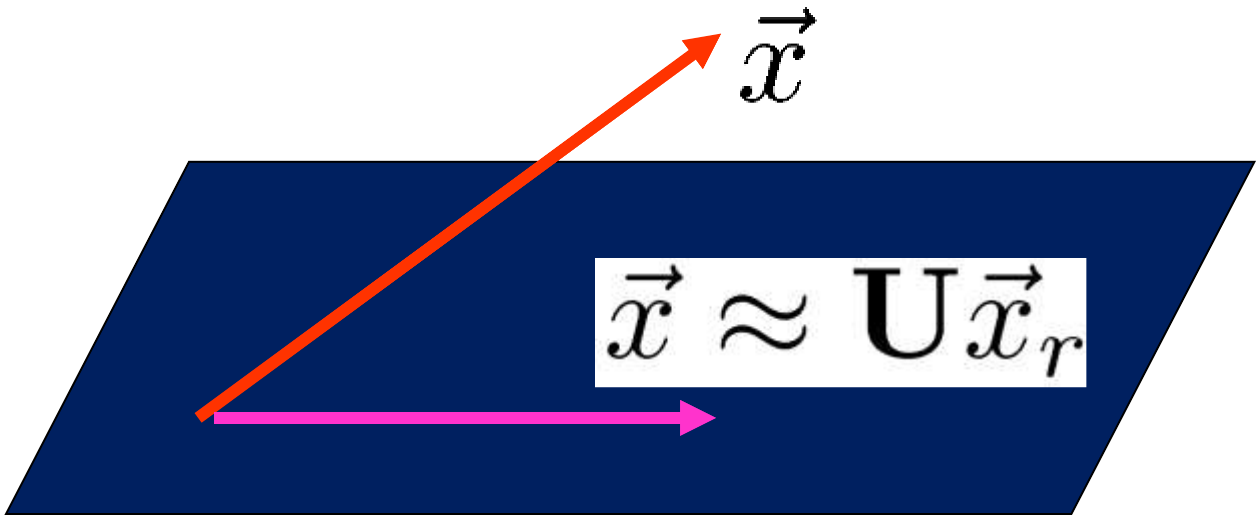
System response for input current $i(t) = (\sin(2\pi/10)+1)/2$



Reminder: Projection Assumption 1

- For all inputs of interest

$$x(t) \approx\in span\{\vec{U}_1\ \vec{U}_2 \ldots \vec{U}_q\}\ \ q << n$$



- U's could be generated from
  - SVD of time series data,
  - Krylov subspaces from linearizations, etc.

Reminder: Projection Assumption 2

- There is a space: $\mathbf{V} = \{\vec{V}_1, \ldots, \vec{V}_q\}$ such that:

- ***If*** the residual is forced orthogonal to $\mathbf{V}$

$$\vec{r}(t) \equiv \frac{d}{dt}\mathbf{U}\vec{x}_r(t) - \left(f\left(\mathbf{U}\vec{x}_r(t)\right) + \vec{b}\,u(t)\right)$$

with $\vec{x}_r(t)$ such that $\mathbf{V}^T\vec{r}(t) = 0$

- ***Then*** the U-restricted DE is almost satisfied

$$\vec{r}(t) \equiv \frac{d}{dt}\mathbf{U}\vec{x}_r(t) - \left(f\left(\mathbf{U}\vec{x}_r(t)\right) + \vec{b}\,u(t)\right) \approx 0$$

U = V a common choice

- In General

$$\mathbf{V}^T\mathbf{U}\frac{d}{dt}\vec{x}_r(t) = \mathbf{V}^T f\left(\mathbf{U}\vec{x}_r(t)\right) + \mathbf{V}^T\vec{b}\,z(t)$$

- If U = V and U^T U = I

$$\frac{d}{dt}\vec{x}_r(t) = \mathbf{U}^T f\left(\mathbf{U}\vec{x}_r(t)\right) + \mathbf{U}^T\vec{b}\,z(t)$$

- Good for systems from self-adoint PDE's:
  - Spatial discretization of nonlinear heat conduction

$$\frac{\partial}{\partial t}\vec{x}(t) = \nabla \cdot f\left(\nabla\vec{x}(t)\right) + \vec{b}\;\; z(t)$$

  - Spatial discretization of the Poisson-Boltzmann

$$\frac{\partial}{\partial t}\vec{x}(t) = \nabla^2 x(\vec{t}) + f\left(\vec{x}(t)\right) + \vec{b}\;\; z(t)$$

Reminder: Nonlinear MOR problem

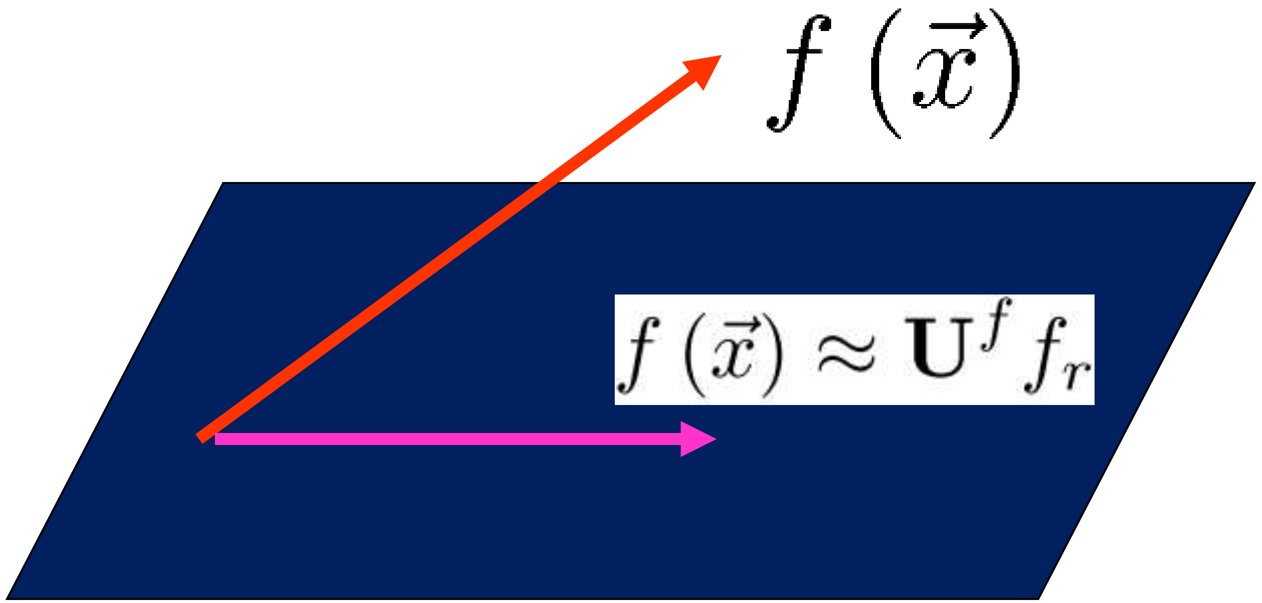
- Substitute: $x = U_q x_r$ in $\dfrac{dx}{dt} = f(x) + Bu$
- Reduced system: $\dfrac{dx_r}{dt} = \overbrace{V_q^T f(U_q x_r)}^{\hat{f}(x_r)} + V_q^T Bu$
- Problem: $V_q^T f(U_q x_r)$: $\underset{\text{small}}{R^q} \rightarrow \underset{\text{large}}{R^N} \rightarrow \underset{\text{large}}{R^N} \rightarrow \underset{\text{small}}{R^q}$

  $q=10$ $\quad N=10^4$ $\quad N=10^4$ $\quad q=10$
- Using $V_q^T f\left(U_q x_r\right)$ is too expensive!

Assumption 3 (For DEIM)

- For x's generated by all inputs of interest

$$f\,(x(t)) \approx\in span\{\vec{U}_1^f\ \vec{U}_2^f \ldots \vec{U}_q^f\}\ \ q << n$$



Assumption 4 (For DEIM)

- We can replace "Galerkin"

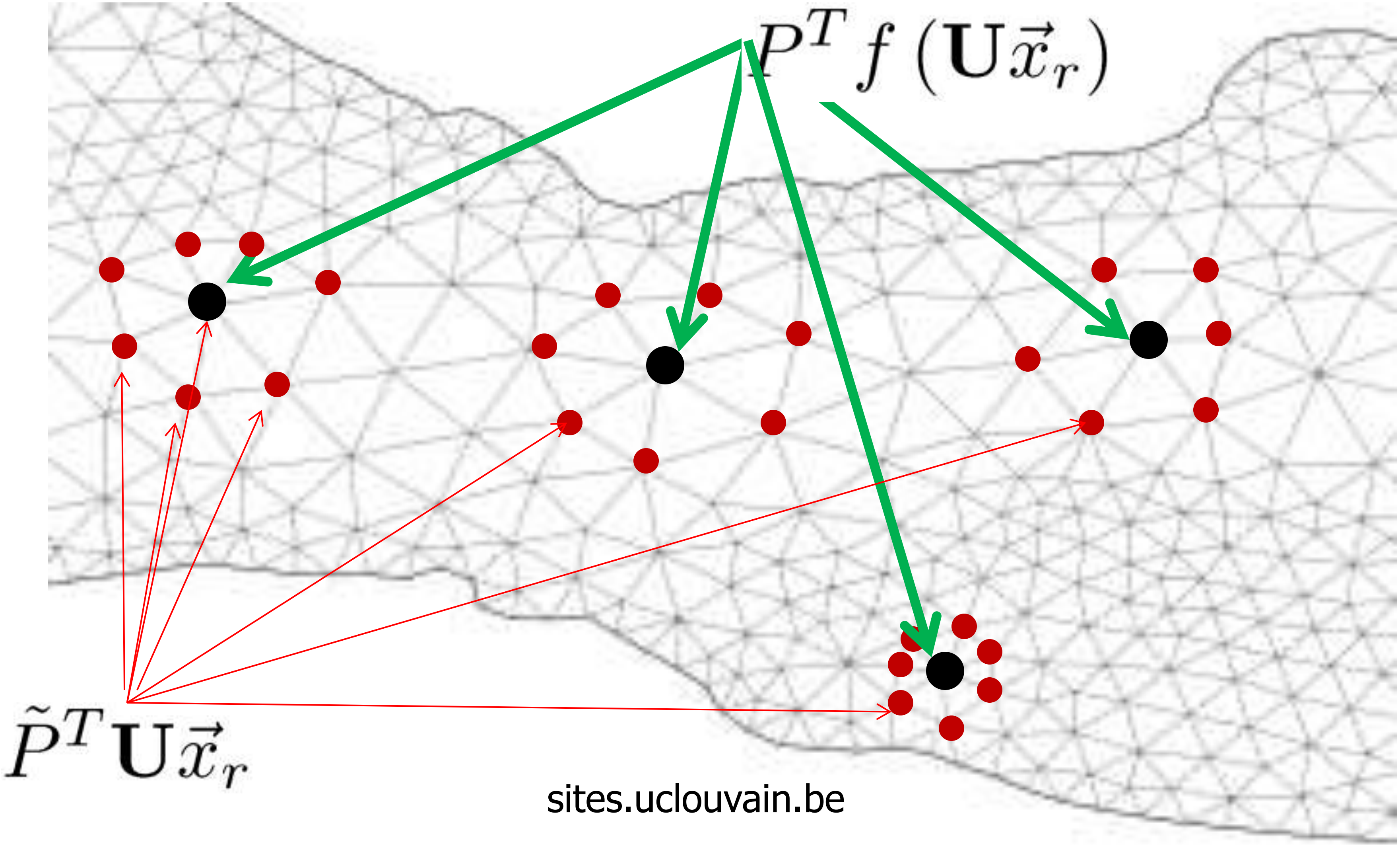
$$f_r = \left(\mathbf{U}^f\right)^T f\left(\mathbf{U}\vec{x}_r\right)$$

- With "Gappy Collocation"

$$P^T \mathbf{U}^f f_r = P^T f\left(\mathbf{U}\vec{x}\right)$$

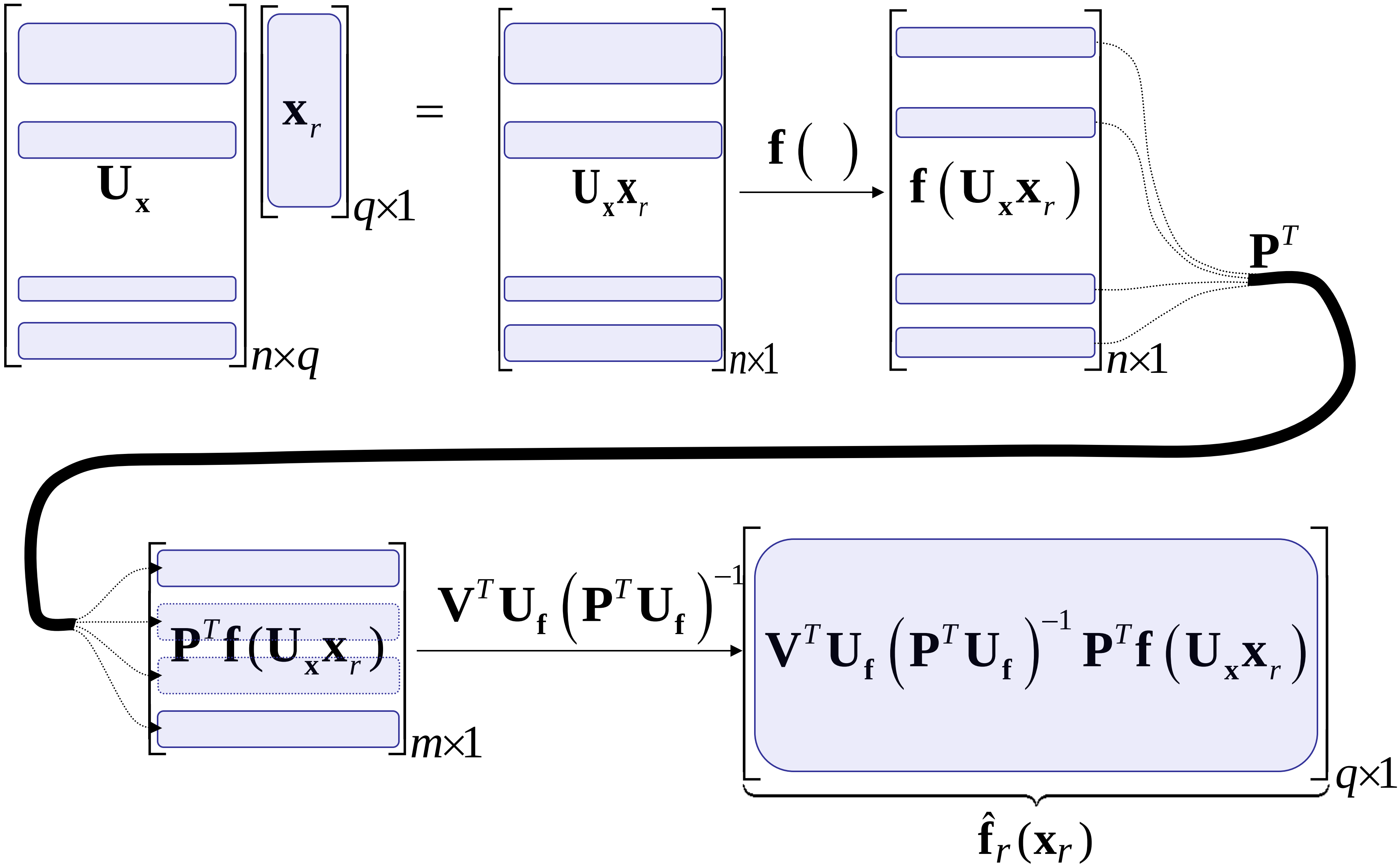
- Where P selects:
  - A few rows of U
  - a few elements of f
    - S. Chaturantabut and D. C. Sorensen, several publications
    - **Empirical interpolation method:** M. Barrault *et al.*, *Comp. Rend. Math.*, 2004.
    - **Missing point estimation:** P. Astrid and A. Verhoeven, Int. Symp. MTNS, 2006.

Picture for a 2-D PDE



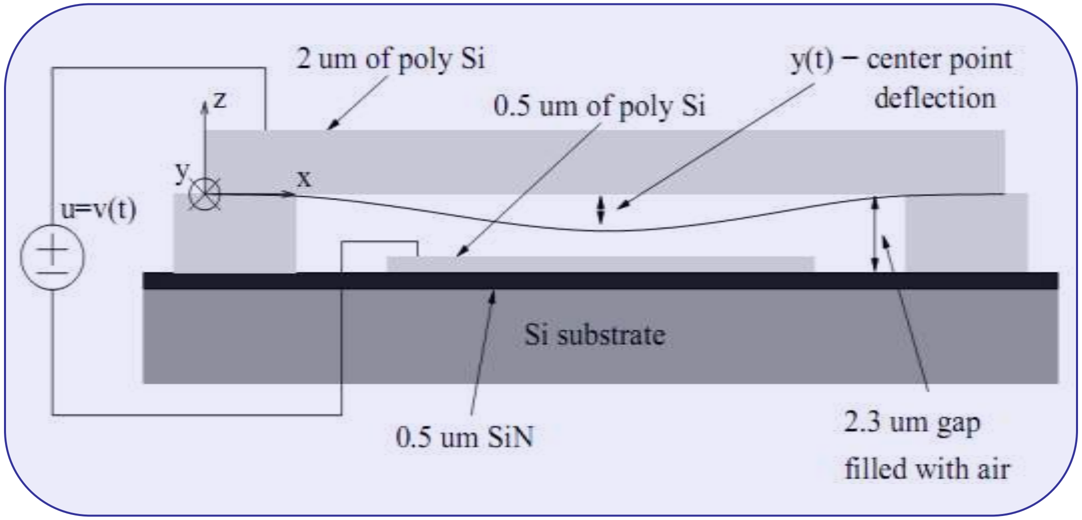
- Evaluate f at approximately q points (black)
- To eval f, need values for x at more points (red)

Discrete Empirical Interpolation Method



Micromachined switch

- Popular benchmark*
- Coupled PDEs are discretized by FD scheme.



Electrostatic force per unit length
$F_{elec} = -w\varepsilon_0 V^2/u^2$

Young modulus
149 GPa

Beam height $u(x,t)$

Euler beam equation:

$$EI\frac{\partial^4 u}{\partial x^4} - S\frac{\partial^2 u}{\partial x^2} = F_{\text{elec}} + \int_0^w (p - p_a)\,dy - \rho\frac{\partial^2 u}{\partial t^2}$$

Moment of inertia
$wh^3/12$

Density (1D)
$2330hw$ Kg/m

Stress coefficient
$-3.7hw$ MPa·m^2

Reynolds squeeze film damping:

$$\nabla \cdot ((1+6K)u^3 p\nabla p) = 12\mu\frac{\partial (pu)}{\partial t}$$

Air viscosity
1.82×10^{5} Kg/(m·s)

Height and width of beam

Knudsen number
$\lambda_{air}/z_0 = 0.028$

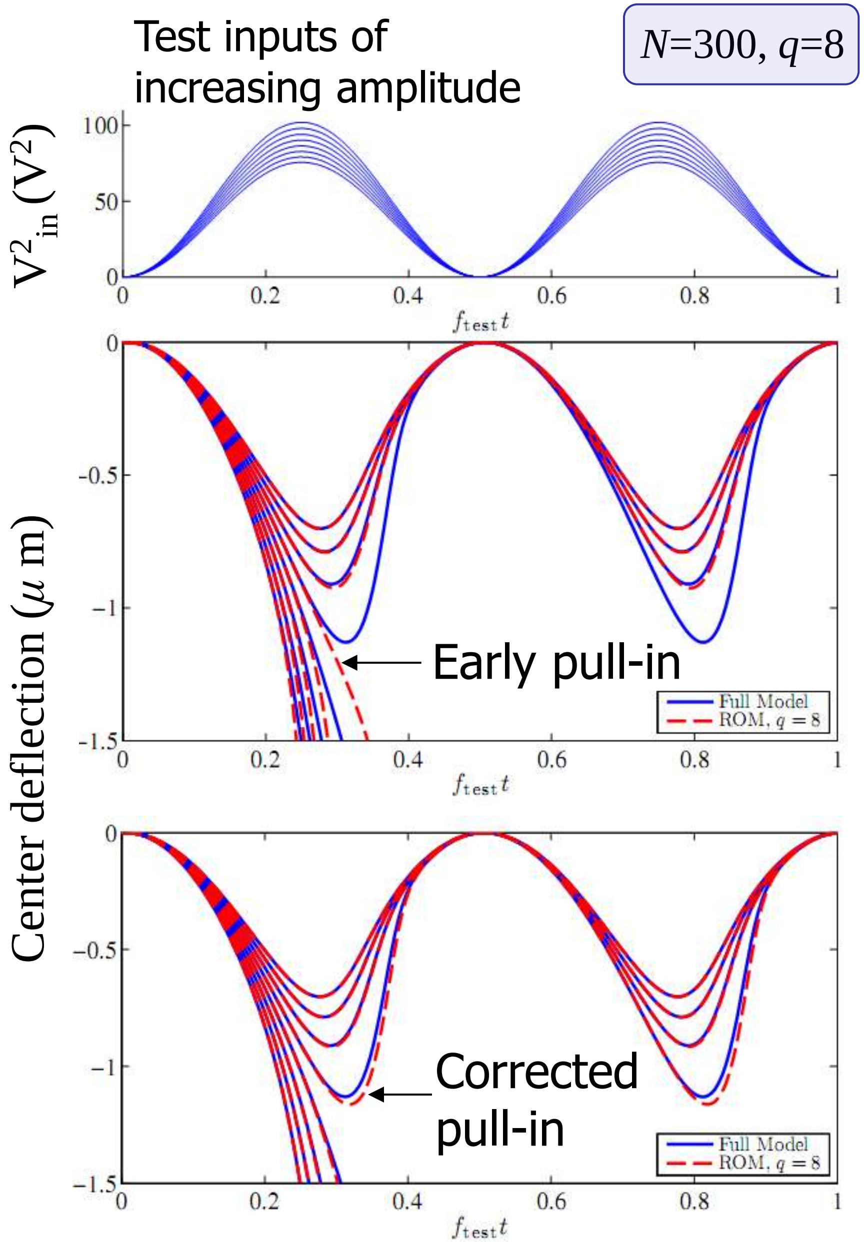
Pressure beneath beam
$p(x,y,t)$

* First analyzed by Hung et al., Int. Conf. on Solid State Sensors and Actuators, 1997

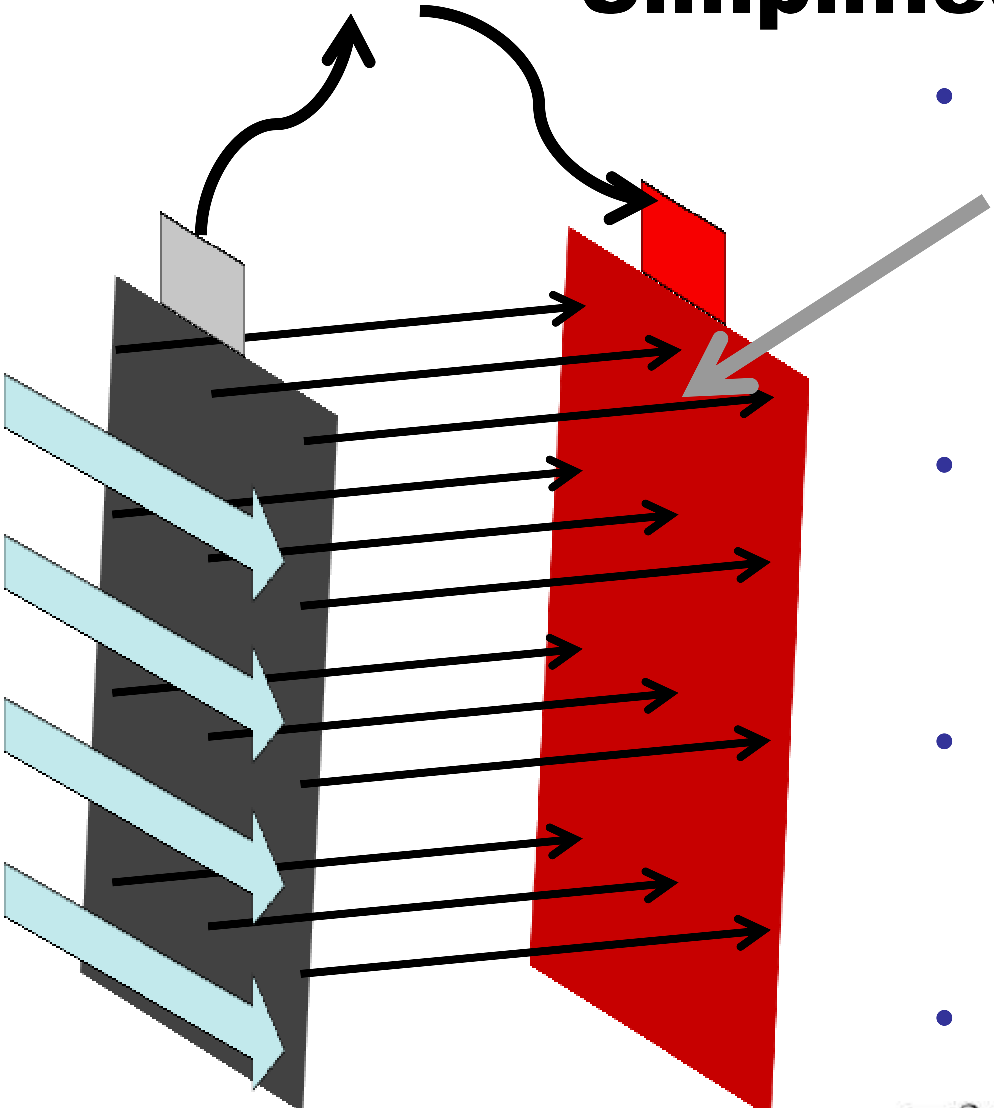
Modeling pull-in

- Parameter of interest: pull-in voltage.
- ROM ***should*** become unstable.
- Make ROM stable at equilibrium and maximum deflection.
- A single (bellow pull-in) training input is used:

$$V_{\text{in}} = 8.5\sin(2\pi f_{\text{train}} t) \;\; (\text{V})$$
$$f_{\text{train}} = f_{\text{test}}/2$$



Simplified Single Cell



- NTGK Electrochemistry model

$$i = f(\phi_+ - \phi_-, DOD)$$
$$\frac{d}{dt} DOD = f(\phi_+ - \phi_-, DOD)$$

- Electrical Conductivity Model

$$\sigma_e \nabla^2 \phi_+ = i$$
$$\sigma_e \nabla^2 \phi_- = -i$$

- Thermal Conductivity Model

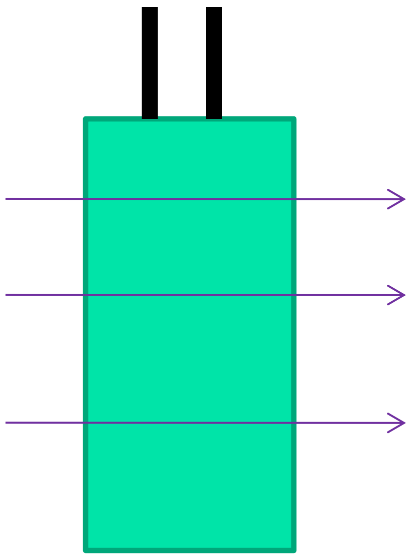
$$\sigma \nabla^2 T = (\phi_+ - \phi_-) * i$$

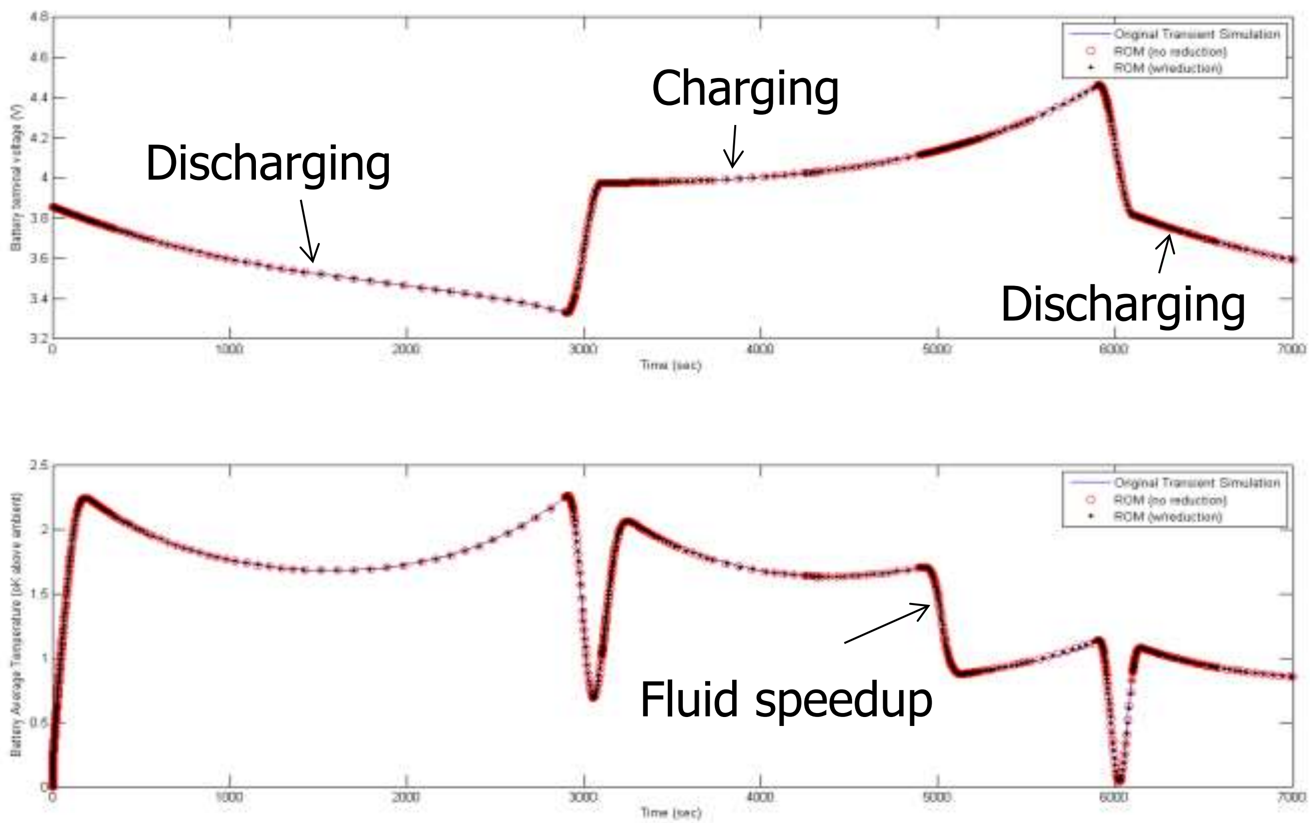
- Sheet Flow Model

$$\sigma_s \nabla^2 T_s + MFR \cdot \nabla T_s = \alpha \cdot (T_s - T)$$

Transient Results for Fluid Cooled Battery

- Inputs are terminal currents and mass flow rate
- Output is average temperature of fluid out





Summary

- Template Examples
  - Need for component models for system analysis
  - Typical input/output descriptions
- Differences between electromagnetics, fluids and continuum mechanics
  - EM is linear, CFD and CM aren’t.
- Linear Problems are easy
  - Fitting and Projection methods for linearizable problems.
- Nonlinear problems are much harder
  - Volterra Series (beyond quadratic, very expensive)
  - Trajectory Methods (Counts on “tube” of paths)
  - DEIM effective ignoring T->Electrochem coupling.