Materials for a Sustainable Energy Future

**IPAM** – Tutorials

Sept 10-12, 2013

**Continuum Models of** 

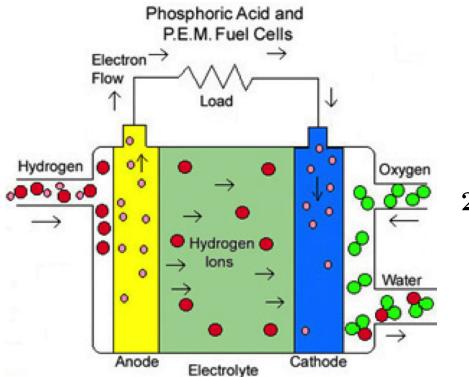
**PEM Fuel Cells** 

Keith Promislow





#### PEM Fuel Cell: Macroview



#### $2H_2+O_2 ightarrow 2H_2O+1.2V$

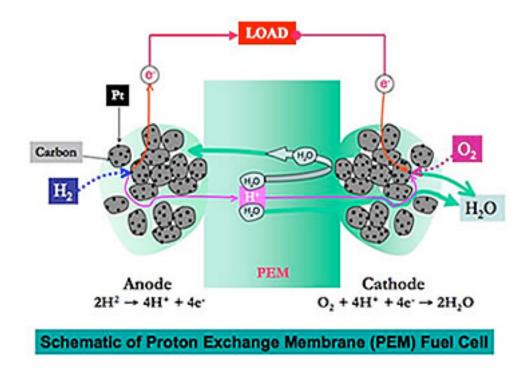
Volts = Energy/electron

This is the WRONG reaction





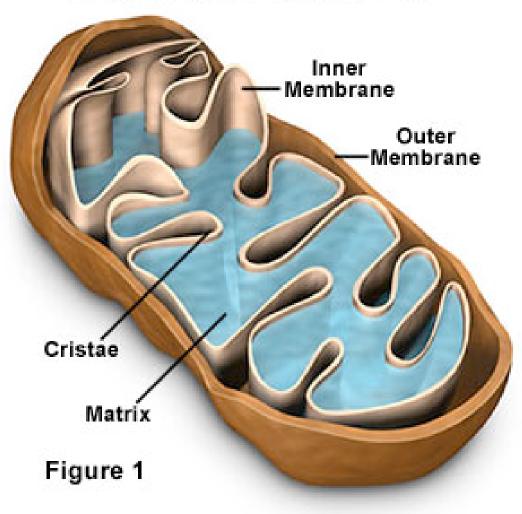
#### Half-Reactions in PEM Fuel Cells



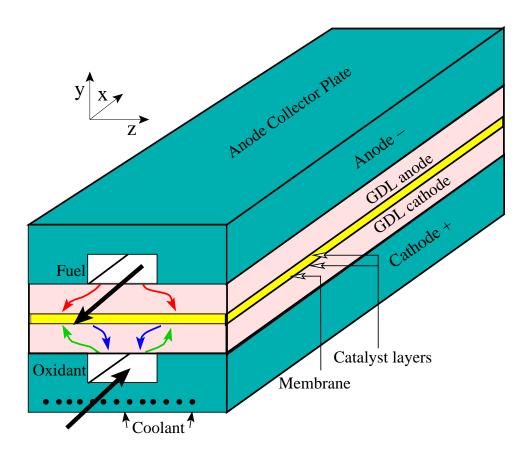
 $H_2 
ightarrow_{
m Pt} 2H^+ + 2e^-$ Hydrogen Oxidation Reaction  $4H^+ + 4e^- + O_2 \rightarrow_{\text{Pt}} 2H_2O + \text{Heat}$ Oxygen Reduction Reaction

## Other Types of Fuel Cells

#### Mitochondria Inner Structure



# 3D View of a PEM Fuel Cell



Consumption of  $O_2$  and fuel along channels Gas diffusion in GDL Production of  $H_2O$  and heat at cathode catalyst layers Build-up of double-layer charge at catalyst interface Ion and water motion in membrane Forms barrier for fuel, oxidant, e<sup>-</sup> Conducts protons as  $H_3O^+$ Condensation/Evaporation of water Heat removal by coolant

The anode reaction

$$\mathrm{H}_2 
ightarrow 2\mathrm{H}^+ + 2\mathrm{e}^2$$

The cathode reaction

$$O_2 + 4H_3O^+ + 4e^- \Rightarrow 6H_2O.$$

## Along-the-Channel Slice

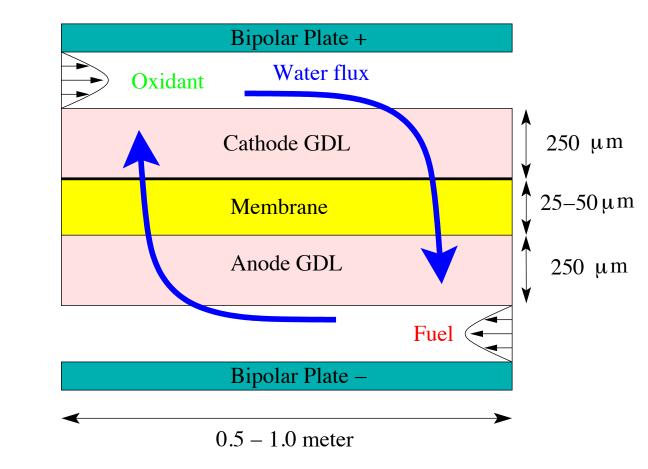
**Dimensionless Quantities** 

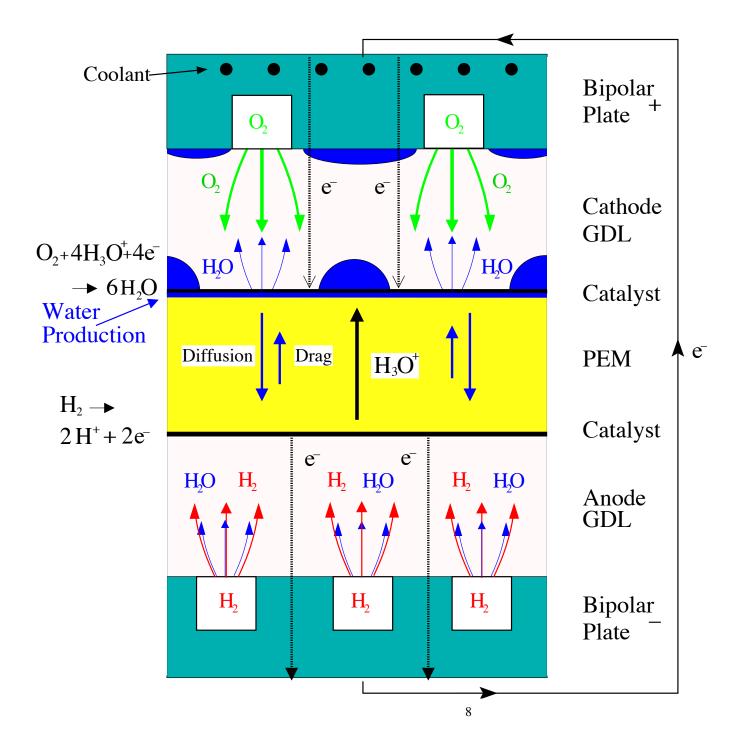
Cathode (Air) Stoich $S_c = rac{Q_{O_2}}{I_T/(4F)}$ 

Anode (Fuel) Stoich $S_a = rac{Q_{H_2}}{I_T/(2F)}$ 

Local Water transfer  $lpha = rac{J_w}{I/(2F)}$ 

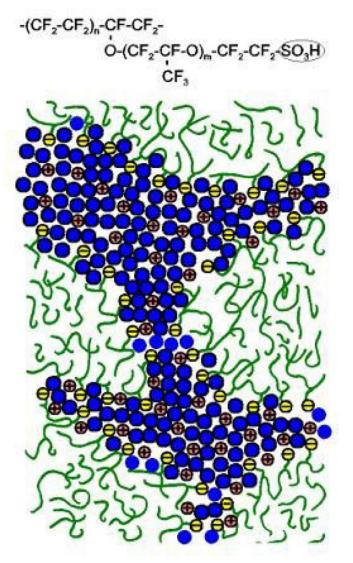
Aspect ratio 2000 : 1

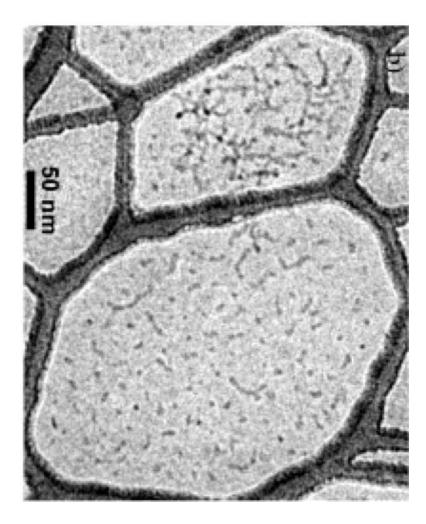




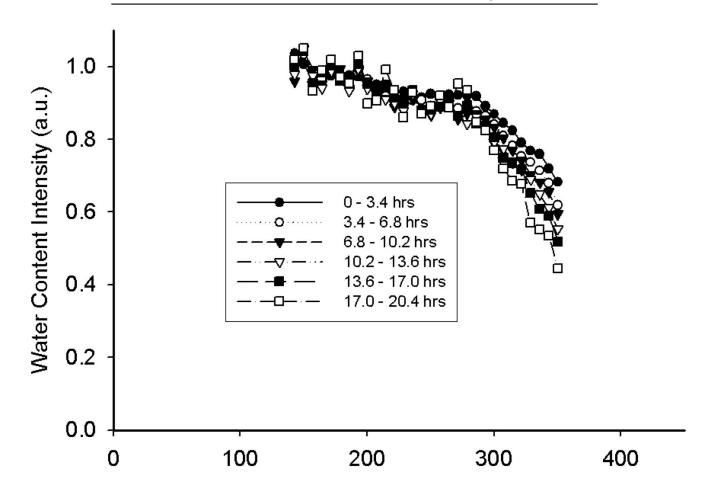
#### **Ionomer Membranes: Network Formation**

NAFION



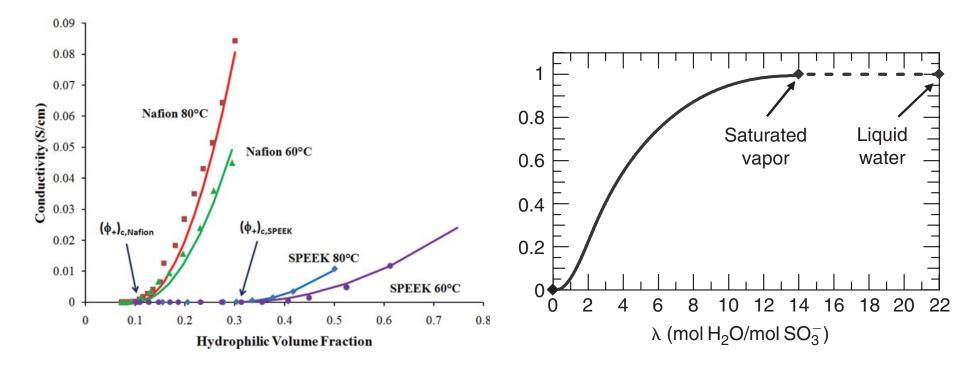


#### Slow Transients in Nafion Hydration



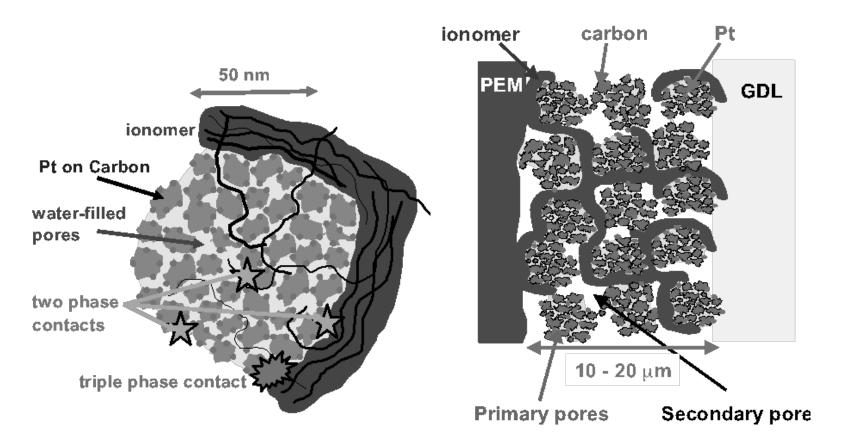
In-situ MRI data of Nafion membrane exposed on one side to liquid water and on the other air at 20% RH.

## Membrane Macroscopic Properties



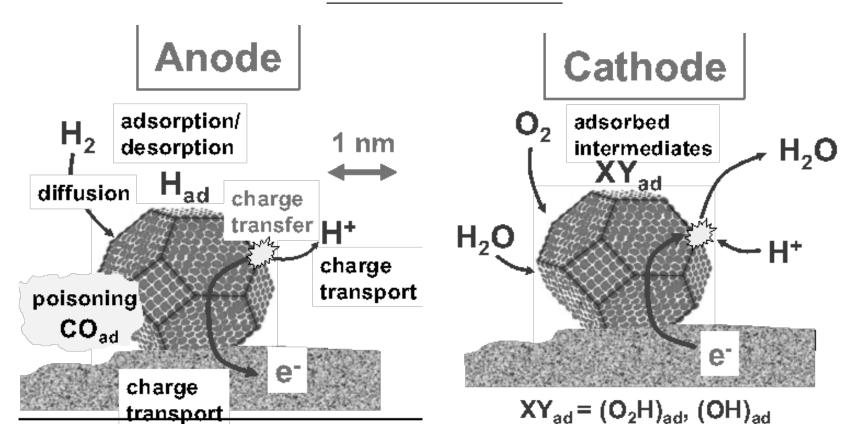
(left) Conductivity of Nafion is sensitive to pre-treatment, water uptake, and temperature, but generally exceeds that of other ionic conducting membranes.(right) Water uptake of Nafion increases with ambient RH, and is discontinuous from saturated vapor to liquid water.

## Catalyst Layer



Carbon support material forms agglomerates (left) which are interpenetrated by ionomer material (right).

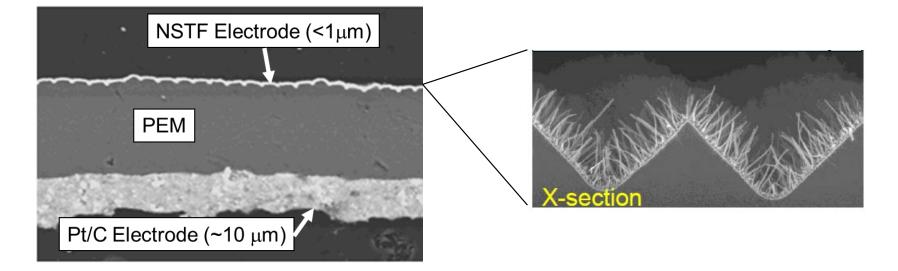
#### Platinum Catalyst



At the anode  $H_2$  is catalyzed into  $H^+$ , which is conducted by the ionomer and  $e^-$  which travels through the Pt and carbon support.

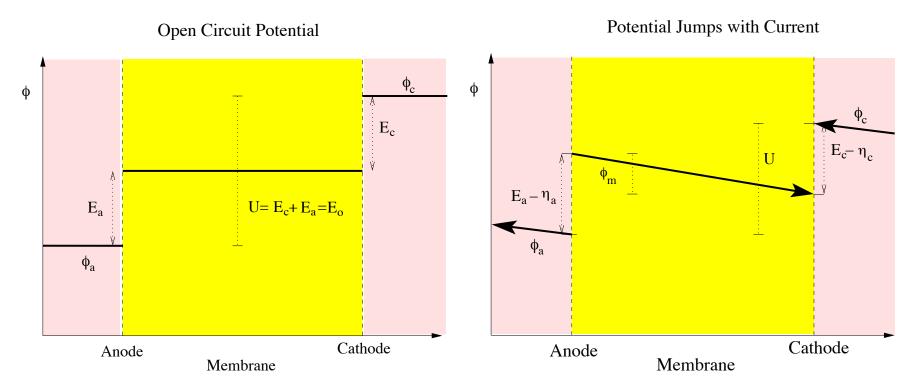
At the cathode  $H^+$  from the ionomer,  $e^-$  from the external circuit, and  $O_2$  from the GDL meet on the Pt surface, completing the reaction.

## MEA and 3M Anode Catalyst



Cross-sectional view of Membrane-Electrode Assembly (MEA) showing the ionomer flanked by the anode and cathode catalyst layers. The cathode catalyst is a traditional carbon-support/Pt agglomerate while the anode catalyst is the 3M Nanostructured thin film (NSTF) from Pt-Ni alloy.

#### **Butler-Volmer Equations**



Over-potentials  $\eta_a$  and  $\eta_c$  describe non-equilbrium loss of voltage

$$egin{aligned} I_c \ &= \ i_{o,c} \left(rac{c_O}{c_{O,\mathrm{ref}}}
ight)^{e_c} \left[\exp\left(rac{lpha_c F}{RT}\eta_c
ight) - \exp\left(-rac{(1-lpha_c)F}{RT}\eta_c
ight)
ight] \ I_a \ &= \ i_{o,a} \left(rac{c_H}{c_{H,\mathrm{ref}}}
ight)^{e_a} \left[\exp\left(rac{lpha_a F}{RT}\eta_a
ight) - \exp\left(-rac{(1-lpha_a)F}{RT}\eta_a
ight)
ight] \end{aligned}$$

## Cell Voltage– Polarization Curve

The membrane resistance is a function of membrane water content

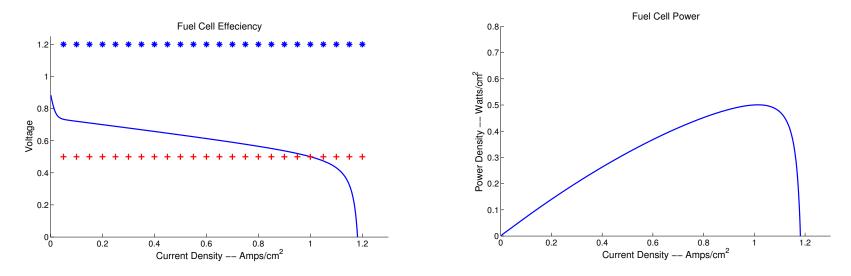
 $R_m=R_m(\lambda),$ 

The catalyst oxygen conc. is a function of current and water content (flooding)

$$C_O(cat) = C_O(ch) - I\delta.$$

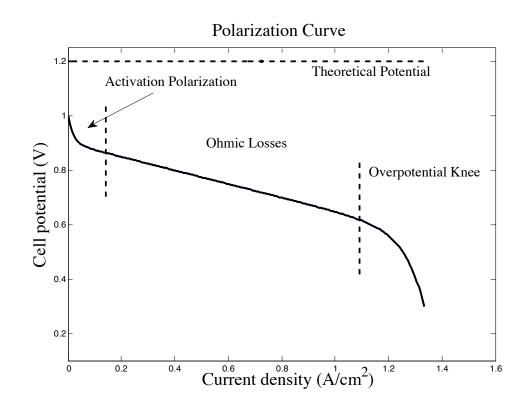
The cell voltage is given by





Below the red line is the Cheetah regime Fuel Cell Power: 0.5 Watts/cm<sup>2</sup>  $\times$  300 cm<sup>2</sup> = 150 Watts/cellt

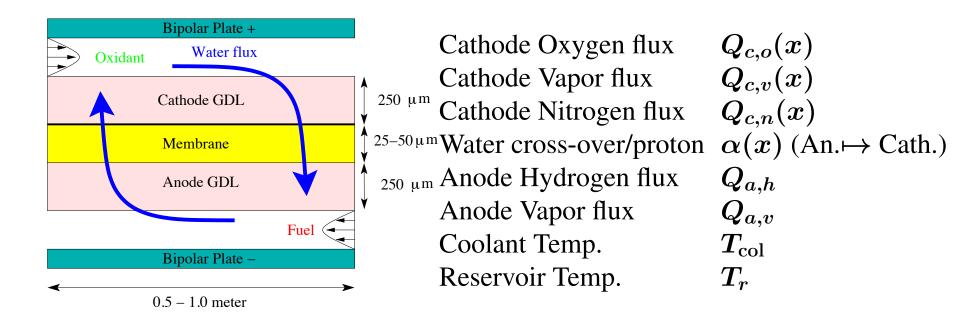
#### Cathode Overpotential



At non-zero currents the cathode over potential takes the form

$$\eta_c pprox rac{\widetilde{RT}}{lpha_c F} ~~ \ln\left(rac{I}{i_{o,c}}\left(rac{c_{O,\mathrm{ref}}}{c_O(\ch) - \delta I}
ight)
ight)$$

# 1+1D Unit Cell Models – Quasi-Steady Along the Channel



Ignore build up of liquid water in channels.

Exploit aspect ratio: couple 1D, steady-state transport for gas phases in channel with time dependent, 1D transport through MEA and accumulation of heat in coolant phase.

#### Channel Equations at Equilibrium

The through-plane current I = I(x) drives the gas composition in channel

$$egin{aligned} rac{dQ_{c,o}}{dx} &= -rac{I(x)L_w}{4F}, & C_{c,o} &= rac{P_c}{\mathcal{R}T_c(x)}rac{Q_{c,o}}{Q_{c,o}+Q_{c,v}+Q_{c,n}}, \ rac{dQ_{c,v}}{dx} &= (1+lpha(x))rac{I(x)L_w}{2F}, \ C_{c,v} &= \mathrm{Min}\left\{rac{P_c}{\mathcal{R}T(x)}rac{Q_{c,v}}{Q_{c,o}+Q_{c,v}+Q_{c,n}}, rac{P_{\mathrm{sat}}(T_c)}{\mathcal{R}T_c(x)}
ight\}, \ rac{dQ_{c,n}}{dx} &= 0, & C_{c,n} &= rac{P_c}{\mathcal{R}T_c(x)}rac{Q_{c,n}}{Q_{c,o}+Q_{c,v}+Q_{c,n}}, \ rac{dQ_{a,h}}{\mathcal{R}T_c(x)}rac{Q_{a,h}+Q_{a,v}}{\mathcal{R}T_c(x)}, \ rac{dQ_{a,h}}{\mathcal{R}T_a(x)}rac{Q_{a,h}+Q_{a,v}}{\mathcal{R}T_a(x)}, \ rac{dQ_{a,h}+Q_{a,v}}{\mathcal{R}T_a(x)}
ight\}, \end{aligned}$$

The coolant temperature is transient and coupled to resevoir

$$egin{aligned} &rac{\partial}{\partial t}\left(
ho cT_{ ext{col}}
ight)+\partial_x\left(
ho cT_{ ext{col}}v_g
ight)=N_TL_w, \ \ T_{ ext{col}}(0,t)=T_r(t), \ &rac{dT_r}{dt}=q(T_{ ext{col}}^{ ext{out}}-T_r)-r(T_r-T_{ ext{amb}}) \end{aligned}$$

## GDL: Transient Multiphase flow

Degenerate transport of liquid water Disparate time scales:

 $10^{-6}s$  for pressure  $10^3 s$  for liquid flow  $C_o$  Oxygen Molar Conc. T Temperature

- $C_v$  Vapor Molar Conc.  $\beta$  Liquid Vol. Frac.
- $C_n$  Nitrogen Molar Conc. C Total Gas Conc.

Conservation of Energy and Mass: ( $\Gamma$  denotes liquid-vapor phase change rate)

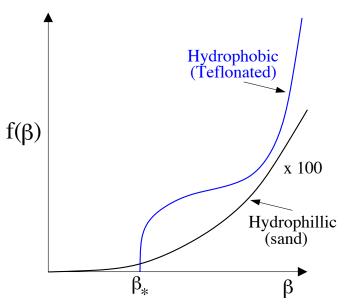
$$egin{aligned} &rac{\partial}{\partial t}((1-eta)C)+\partial_y(\overbrace{CU_g}^{N_g})=-\Gamma,\ &rac{\partial}{\partial t}(\overline{
ho c}T)+\partial_y((\overbrace{
ho c\overline{U}})T-\widetilde{\kappa}\partial_yT)=h_{lg}\Gamma,\ &rac{\partial}{\partial t}((1-eta)C_o)+\partial_y(\overbrace{C_oU_g+J_o}^{N_o})=0,\ &rac{\partial}{\partial t}((1-eta)C_v)+\partial_y(\overbrace{C_vU_g+J_v}^{N_v})=-\Gamma,\ &rac{\partial}{\partial t}(c_leta)+\partial_y(\overbrace{
ho c_l\overline{U}}^{N_l})=\Gamma. \end{aligned}$$

**Constitutive Relations:** 

$$\begin{split} P_g &= CRT, & \text{Ideal Gas Law} \\ U_g &= -\frac{Kk_{rg}(\beta)}{\mu_g} \partial_y P_g, & \text{Darcy's Law-Gas} \\ U_l &= -\frac{Kk_{rl}(\beta)}{\mu_l} \partial_y P_l, & \text{Darcy's Law-Liquid} \\ [J_i] &= M^{-1}[\partial_y C_i] & \text{Maxwell Stefan Flux} \\ P_c &= P_g - P_l = L(\beta), & \text{Leveret-like Capillary Pressure.} \\ \Gamma &= \boldsymbol{H}(\beta)(C_v - C_{\text{sat}}(T)), & \text{Condensation-Saturation} \end{split}$$

Capillary pressure and relative permeability form nonlinear diffusivity

$$egin{aligned} f(eta) &= eta k_{rl}(eta) L'(eta). \ &rac{\partial}{\partial t}(eta) - D\partial_y(eta(eta) \partial_yeta) &= \Gamma/c_l \end{aligned}$$



Collect the fluxes and unknowns

 $ec{N} = (N_g, N_T, N_o, N_v, N_l)^t ext{ and } ec{V} = (C, T, C_o, C_v, eta)^t$ 

The full problem can be written as

$$\left(M(ec{V})ec{V}
ight)_ au + \left(\mathcal{D}(ec{V})\partial_yec{V}
ight)_y = ec{S}\,\Gamma,$$

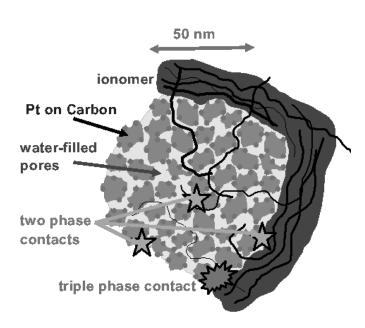
where  $\vec{S} = (-1, 1, 0, -1, 1)^t$  is the scaled stoichiometry vector for phase change and the matrix  $\mathcal{D}$  is given by

$$\mathcal{D} = - egin{pmatrix} R_g k_{rg} T C & R_g k_{rg} C^2 & 0 \ 0 & 0 \ \delta_l \mathcal{C}_{lg} T^2 & \delta_l \mathcal{C}_{lg} C T + R_T \ 0 \ 0 & R_c T f(eta) \ (R_g k_{rg} T - rac{1}{C}) \ C_o & R_g k_{rg} C C_o & 1 \ 0 & 0 \ (R_g k_{rg} T - rac{1}{C}) \ C_v & R_g k_{rg} C C_v & 0 \ 1 & 0 \ R_l eta k_{rl}(eta) T & R_l eta k_{rl}(eta) C & 0 \ 0 \ R_c f(eta)/\delta_l \end{pmatrix}$$

The local current production I is the driving force: fluxes at membrane proportional to I. A large charge flow is a small *molar* flux:

$$I = O(10^{-2})$$

## Membrane-Catalyst Layer Coupling (Boundary conditions)



Membrane equilibrium hydration levels function of GDL RH level r,

$$c^*(r) = 0.043 + 17.81r - 39.85r^2 + 36.0r^3.$$

Flux out of membrane proportional to disequilibrium

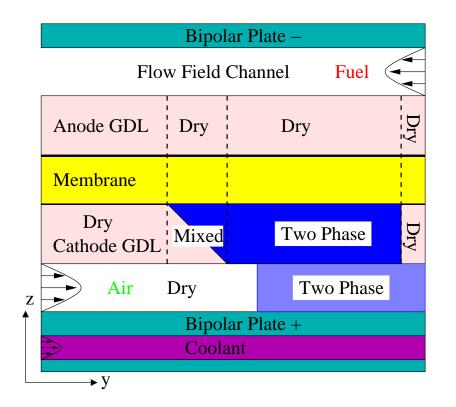
$$\gamma(c_w) \overbrace{(c_{w,a}^*-c_{w,a})}^{ extsf{disequilibrium}} = \overbrace{J_{w,a}^{GDL}}^{ extsf{Flux}} = J_{w,a}^m + rac{I}{F}, 
onumber \ \gamma(c_w)(c_{w,c}-c_{w,c}^*) = J_{w,c}^{GDL} = J_{w,c}^m + rac{3I}{2F}.$$

 $\gamma \ll 1$  controls membrane water loss.

Heat Production in Catalyst

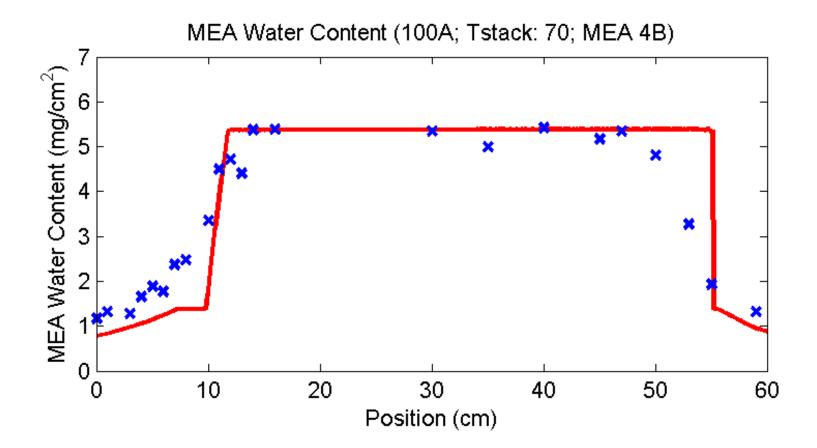
$$Q_{\,\mathrm{heat}}\,=\left(rac{Th_{rc}}{4F}+\eta_c
ight)I_c-h_v\gamma(c_T-c_T^*(r)),$$

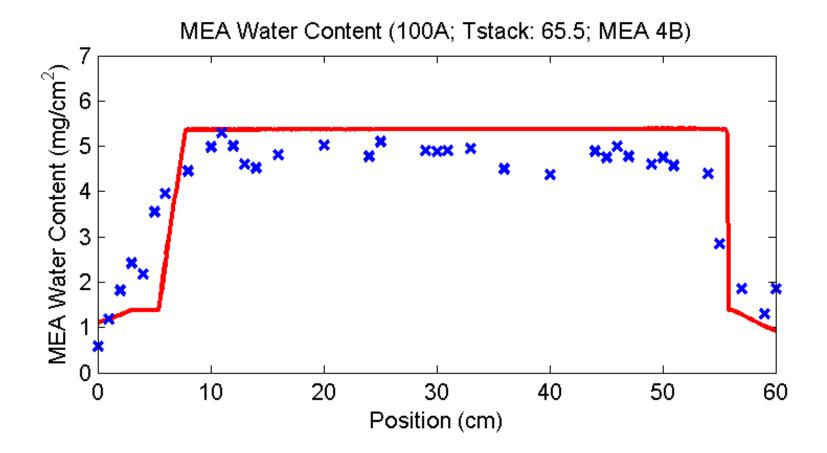
1

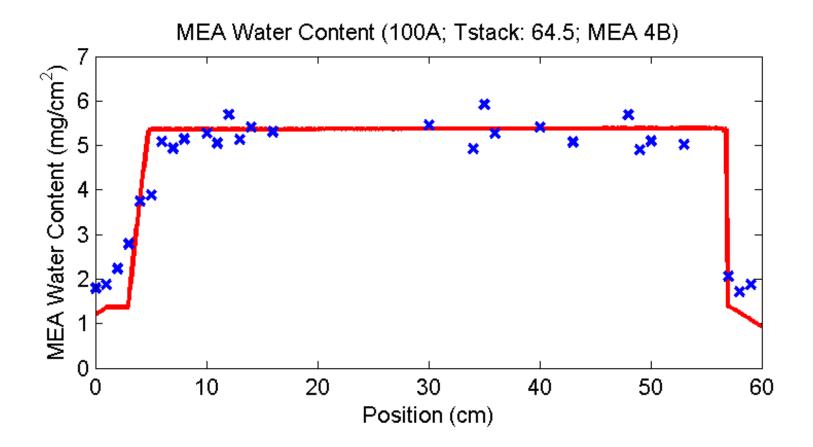


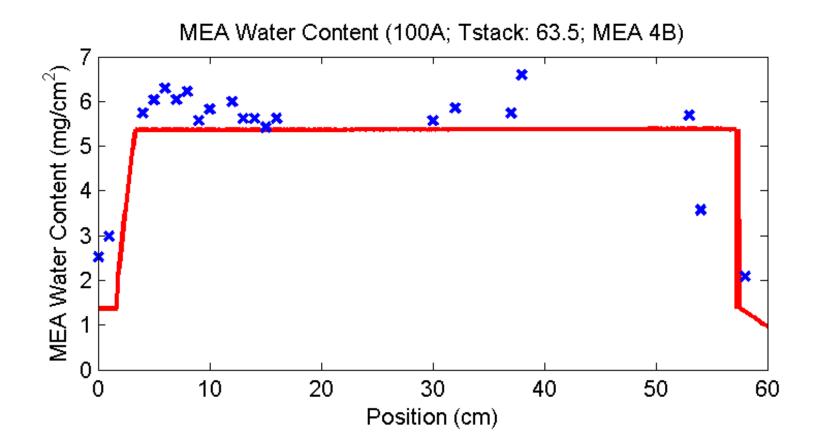
Break PDE into Dry, Two-phase, and a Boundary Layer regime  $O\left(\frac{I}{\sqrt{H}}\right)$ . Exact solutions in Dry and Two-phase yield flux imbalance at wet-dry interface. Leads to explicit ODE for slow front evolution.

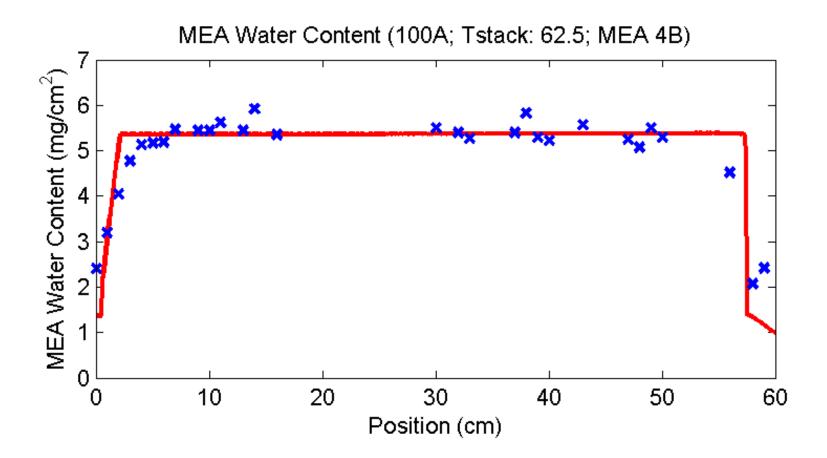
K.P., J. Stockie, B. Wetton, Proc. Roy. Soc. London: Series A 462 (2006).

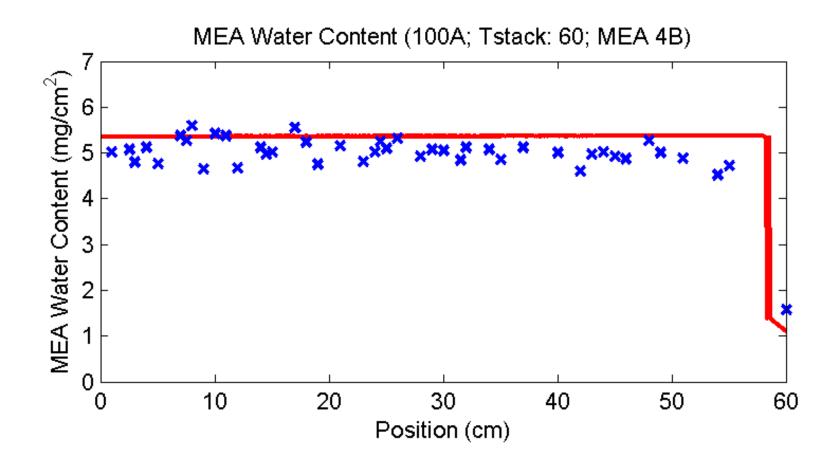


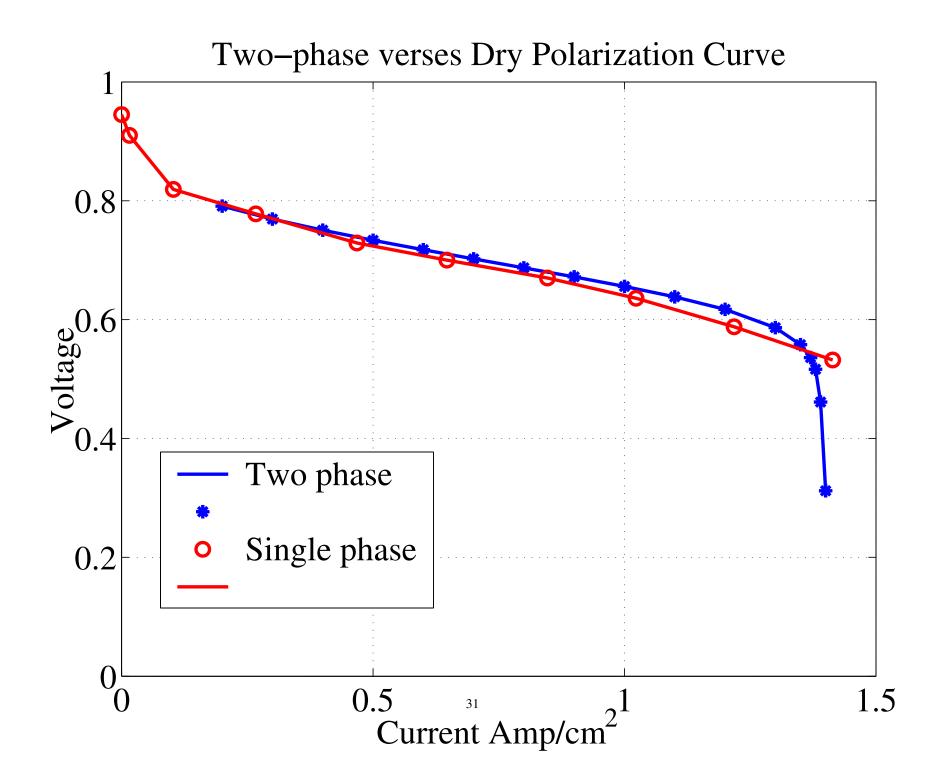






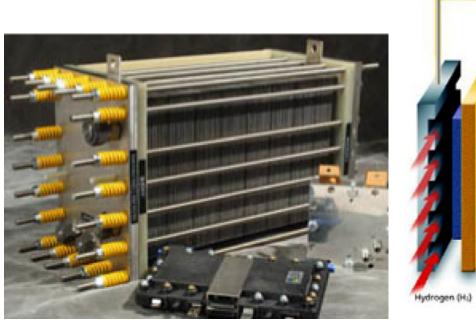


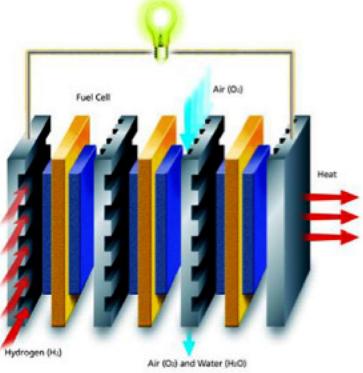




## Fuel Cell Stacks

A single  $60 \text{cm} \times 30 \text{cm}$  cell, operating at 1 Amp/cm<sup>2</sup> and 0.6 Volts produces about a kilowatt in power. Automotive applications, which require almost a MegaWatt of power require Stacks.





#### Carbon Corrosion

Oxygen Reduction/Reverse Oxygen Reduction (o) Hydrogen Oxidation/Hydrogen Evolution (h) Carbon Oxidation (c)

$$O_2 + 4p^+ + 4e^- \rightleftharpoons 2H_2O,$$
  

$$H_2 \rightleftharpoons 2p^+ + 2e^-,$$
  

$$C + 2H_2O \rightharpoonup 4p^+ + 4e^- + CO_2$$

Common Electrode potential, for reaction r=o, h, c, at anode/cathode

$$egin{aligned} E_{a/c} &= E_{r,ref} + N_r(C_o,C_h) + \eta_r(i_r), \ V_{ ext{cell}} &= E_c - E_a - R_\Omega I, \ i_r &= i_{r,ref} \left\{ \exp\left(rac{lpha_r \mathcal{F} \eta_r}{RT}
ight) - \exp\left(-rac{(1-lpha_r) \mathcal{F} \eta_r}{RT}
ight) 
ight\}, \end{aligned}$$

Small current  $i_r \approx \eta_r / R_r$ , Large current  $\eta_r \approx \frac{RT}{\alpha_r \mathcal{F}} \ln \frac{i_r}{i_{r,ref}}$ 

Each reaction competes to provide the local current I(x)

$$I(x) = i_c(x) + i_o(x) + i_h(x)$$

Oxidation reactions (producing electrons) i > 0 and  $\eta > 0$ , Reduction reactions (consuming electrons) i < 0 and  $\eta < 0$ 

## **Kinetic Parameters**

Reaction	Nerst	$E_{ref}$	$i_{ref}$	$lpha_r$
Carbon Ox. (c)	-	0.207V	$1.110^{-5}rac{A}{m^2}$	0.324
Hydrogen Ev. (h)	$\mathrm{C}_{H_2}^{ref} = 100 rac{\mathrm{mol}}{\mathrm{m}^3}$	0	$\eta_h = R_h i_h$	$R_h=0.10 \Omega ext{-cm}^2$
Oxygen Red. (o)	$C_{O_2}^{ref} = 40.9 \frac{\text{mol}}{\text{m}^3}$		$9.310^{-4}rac{A}{m^2}$	1.0
Rev. Ox. Red. $i_o > 0$	-		$\eta_o=R_o \overset{m}{i_o}$	$R_o=0.01\Omega ext{-cm}^2$

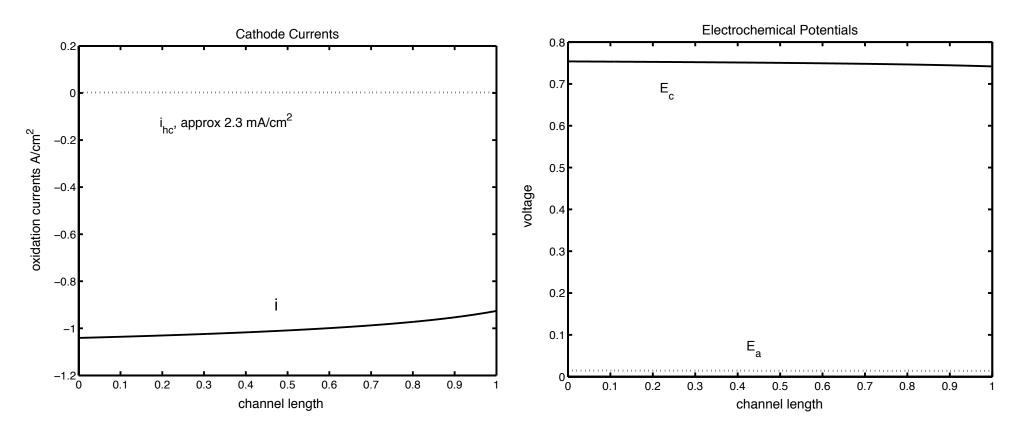
Oxygen/Hydrogen cross over,  $A = 3 \times 10^{-3}$  m/s,

$$J_r = A(C_{r,a} - C_{r,c})$$

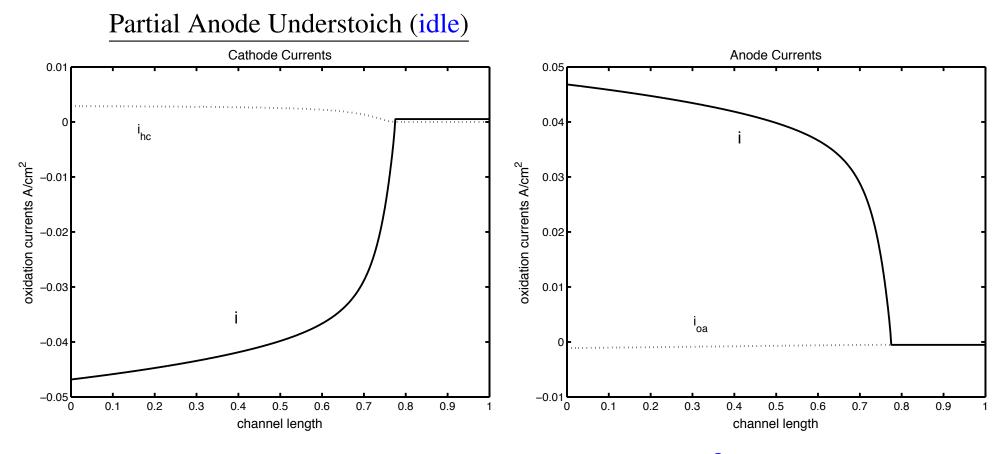
#### Unknowns

Anode and Cathode catalyst layer conc.	$C_{o,a}, C_{h,a}$	$C_{o,c}, C_{h,c}$
Partial currents	$i_{o,a}, i_{h,a}, i_{c,a}$	$i_{o,c}, i_{h,c}, i_{c,c}$
Local current	I(x)	I(x)
Anode and Cathode potentials	$E_{an}$	$E_{cat}$

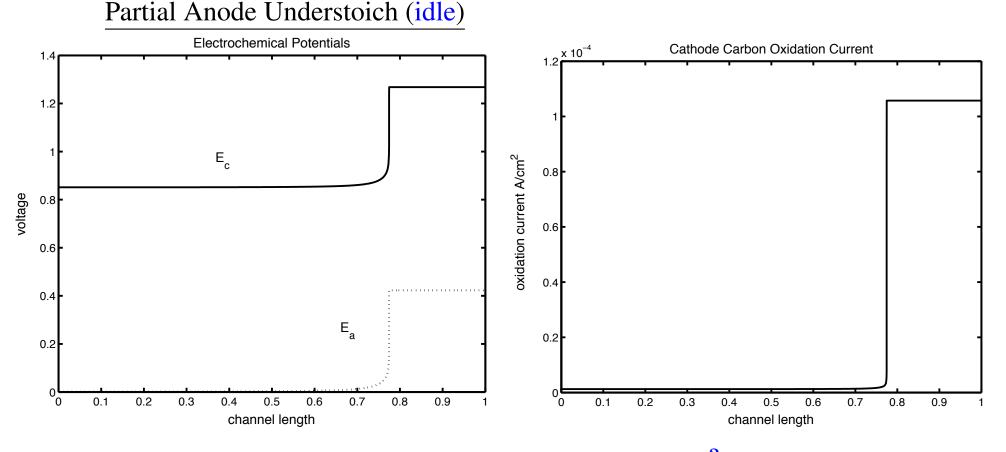
#### Base case– Coflow



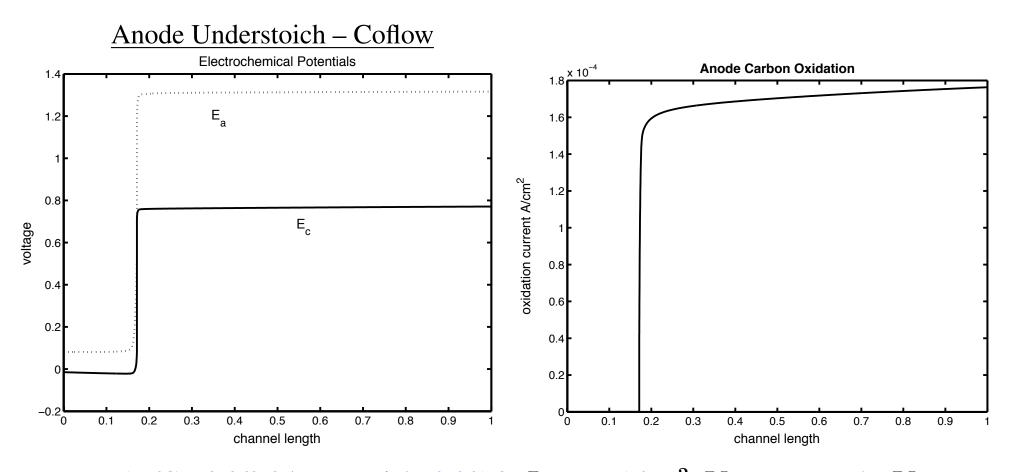
An/Cat 2.2/2.0 barg, stoich=1.2/1.8,  $I_T = 1$  A/cm<sup>2</sup>,  $V_{cell} = 0.635V$ . Small hydrogen oxidation current (2.3 mA/cm<sup>2</sup>) on cathode due to hydrogen crossover. This current is present at open circuit and causes the drop in open circuit voltage from 1.28 to 0.95V.



An/Cat 2.2/2.0 barg, stoich=1.1/1.8,  $I_T = 30 \text{ mA/cm}^2$ ,  $V_{\text{cell}} = 0.845V$ . Hydrogen cross-over drives cell to anode understoich. Oxygen reduction at anode (from crossover) and Reverse Oxygen reduction at cathode. Elevated cathode pot leads to sig. carbon corrosion. (1 mg of carbon/hour =  $6 \times 10^{-4}$ A).



An/Cat 2.2/2.0 barg, stoich=1.1/1.8,  $I_T = 30 \text{ mA/cm}^2$ ,  $V_{cell} = 0.845V$ . Hydrogen cross-over drives cell to anode understoich. Oxygen reduction at anode (from crossover) and Reverse Oxygen reduction at cathode. Elevated cathode pot leads to sig. carbon corrosion. (1 mg of carbon/hour =  $6 \times 10^{-4}$ A).



An/Cat 2.2/2.0 barg, stoich=0.8/1.8,  $I_T = 1$  A/cm<sup>2</sup>,  $V_{cell} = -0.575V$ . Current near inlet is limited only by hydrogen mass transfer on anode. On cathode oxygen reduction (at mass transfer limit) plus hydrogen evolution. After depletion of hydrogen, anode and cathode pot. rise, leading to reverse oxygen reduction on anode, hydrogen oxidation and oxygen reduction on cathode.

#### Fuel Cell Issues

Would like higher temperature operation to avoid CO poisoning, reduce the Cheetah effect, and improve the performance of non-precious metal (cheaper) catalysts.

Requires the development of new (cheaper, higher temperature) polymer electrolyte membranes

Lower Pt loading (mg/cm<sup>2</sup>). Catalyst layers can degrade when run at high voltage, mostly due to understoich (Carbon Corrosion). Pt dissolution is also a long-term degradation worry.

Liquid water builds up in catalyst layers and clogs oxygen transport (Flooding). Also freeze-thaw issues.