Economics and dynamics in networking: three case studies

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Outline:
1. Congestion control with multipath routing.
2. Controlling fairness through number of TCP connections.
1. Congestion control with multipath routing

Optimal WELFARE problem:
\[
\max \sum U_k(x^k) \quad \text{subject to } y \leq c.
\]

Source-destination pair ("commodity") $k$, input rate $x^k$ (bps)
Utility $U_k(x^k)$.

Fraction of traffic destined to node $j$, $\alpha_{i,j}^d \geq 0$, $\sum_j \alpha_{i,j}^d = 1$.
A simple network

The "customer":
- elastic traffic source, rate follows "demand curve" \( x = f(q), f = U^{-1} \)
- \( q = \alpha p_1 + (1 - \alpha)p_2 \), mean price.

The "broker": multipath router, adapts routing fraction \( \alpha \) slowly in direction of cheaper prices, \( \dot{\alpha} = \beta(p_2 - p_1) \).

Optimal welfare equilibrium:
- \( x = c_1 + c_2 = f(q) \),
- \( \alpha = \frac{c_1}{c_1 + c_2} \), \( p_1 = p_2 = q \).

Does the system reach this equilibrium?

The resources: link capacities. Prices \( p_1, p_2 \) (e.g. queueing delays) indicate their scarcity.
We implemented this in the packet simulator ns2:
- Source runs TCP-FAST, responds to delay.
- Router split traffic, adapt split to measured delays.

Sources rate reaches desired equilibrium.

but....

Queues and routing splits oscillate!
Can we explain this with flow models? A commonly model for delay is the "latency function" $p_l = \phi(y_l)$, where $y_l$ is the link rate.

This model implies global convergence to equilibrium in $\alpha, p, x$, using a Lyapunov argument.

Something's wrong.... The latency model, from queueing theory in *steady state*, not relevant to dynamic studies.
Another fluid model for queueing delay

\[ p_l = \frac{1}{c_l} [y_l - c_l]_+ \]

We can prove \( x \to c_1 + c_2 \).

Take \( x = c_1 + c_2 \), around equilibrium we have the linear dynamics

\[
\begin{bmatrix}
\delta \dot{\alpha} \\
\delta \dot{p}_1 \\
\delta \dot{p}_2
\end{bmatrix} =
\begin{bmatrix}
0 & -\beta & \beta \\
\gamma_1 x^* & 0 & 0 \\
-\gamma_2 x^* & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta \alpha \\
\delta p_1 \\
\delta p_2
\end{bmatrix}
\Rightarrow \text{UNSTABLE (imaginary modes)}
\]

Mass-spring like system, frequency matches packet simulations.

**Equilibrium:**
\[ x^* = c_1 + c_2, \quad \alpha^* = \frac{c_1}{x^*}, \]
\[ p_1^* = p_2^* = q^* = U'(x^*). \]

Conclusion:

- Route oscillations are not trivial to avoid.
- A naive equilibrium viewpoint (most of econ) misses all this.
- Beware of simplistic models for delay!
Solving the problem

Adapt $\alpha$ based on anticipated (rather than current) price $\pi_i = p_i + \nu \dot{p}_i$

In control terms, add derivative action. Same equilibrium. Simulations:

Theorems [P'-Mallada, to appear in IEEE ToN]
the equilibrium point (optimum $\max \sum U^k(x^k)$)
is locally asymptotically stable in an arbitrary network

Packet implementation: variants of TCP-FAST and RIP.
2. Stability and user-level fairness

[Ferragut-P’, CISS ’08, Stochastic Networks ’08]

User rate $\rho_k$

Utility $U_k(\rho_k)$

Single path routing matrix:

$$R_{lk} = \begin{cases} 
1 & \text{if user } k \text{ uses link } l \\
0 & \text{otherwise}
\end{cases}$$

$$y = R \rho$$

**KELLY’S SYSTEM PROBLEM**

$$\max \rho \sum_k \underbrace{U_k(\rho_k)}_{\text{USER UTILITY FUNCTION}}, \quad \text{subject to } \underbrace{R \rho \leq c}_{\text{LINK CAPACITY CONSTRAINTS}}$$
Contrast with flow-level fairness of TCP

Rate $x_k$ per flow $\rightarrow$
Utility $U_{TCPk}(x_k)$
n_k flows per user.

TCP utility $U_{TCPk}$ (per flow) determined by the protocol, e.g., $U'_{TCPk}(x) = \kappa_k x^{-\alpha}$ for $\alpha = 2$.

TCP : NETWORK PROBLEM

$\max_x \sum n_k \frac{U_{TCPk}(x_k)}{TCP \text{UTILITY FUNCTION}}$ subject to $\sum_k \frac{R_{lk} n_k x_k}{\rho_k} \leq c_l$

- Without control of number of connections, fairness per flow is moot (Briscoe'07).
- Incentives to employ many TCP flows (e.g., p2p).
Tragedy of the commons?
User: Poisson ($\lambda_k$) arrivals, exp($\mu_k$) workloads.

For each fixed $\{n_k\}$, service rates $x_k$ determined by TCP congestion control $U'_{TCPk}(x) = \kappa_k x^{-\alpha}$ for $\alpha > 0$.

Result: Markov chain $\{n_k\}$ stable if and only if $\sum_k R_{lk} \frac{\lambda_k}{\mu_k} < c_l \quad \forall l$.

Remark: congestion control ensures neither stability nor fairness.

- Both depend solely on users' "open loop" demands $\frac{\lambda_k}{\mu_k}$.
- Fairness choice per flow (e.g., value of $\alpha$) has minimal impact.
  A heavy user will compensate a low TCP rate by increasing $n_k$, until $\rho_k$ serves demand, if feasible. If not $n_k$'s grow without bounds.
Closing the loop on $n_k$ for user-level fairness

Assume that for fixed $n_k$, the flow rate $x_k$ is determined by TCP:

$$x_k = f_{TCPk}(q_k)$$

where $q_k$ is the congestion price seen by the source, and $f_{TCPk} = (U'_{TCPk})^{-1}$, TCP demand curve. The user rate is $\rho_k = n_k x_k$.

Objective: control $n_k$ so that the system converges to an equilibrium where $\rho_k = n_k x_k$ solves $\max_{\rho} \sum_k U_k(\rho_k)$, s.t. $R\rho \leq c$, with utilities defined by users.

Control law for continuous $n_k$:

$$\dot{n}_k = \beta \left( U_{TCPk}^{-1}(q_k) - \rho_k \right).$$
Analysis using dual TCP congestion control,

\[ \dot{n}_k = \beta \left( U_k^{-1}(q_k) - \rho_k \right); \]
\[ \rho_k = n_k x_k; \]
\[ x_k = f_{TCPk}(q_k). \]

Theorem 1 (arbitrary network).

The equilibrium satisfies \( \max_{\rho} \sum_k U_k(\rho_k) \), subject to \( R\rho \leq c \), and is locally asymptotically stable. Proof: passivity argument (as in Wen-Arcak '03).

Theorem 2 (single bottleneck).

Assume time-scale separation: for fixed \( n = \{n_k\} \), let \( \hat{q}_k(n), \hat{x}_k(n) \) be the equilibrium values from dual congestion control, and \( \hat{\rho}_k(n) = n_k \hat{x}_k(n) \).

Then the "slow" dynamics \( \dot{n}_k = \beta \left( U_k^{-1}(\hat{q}_k(n)) - \hat{\rho}_k(n) \right) \) are globally convergent to a point \( n^* \) where the corresponding \( \hat{\rho}_k(n^*) \) are at the optimum welfare point.
From fluid control to admission control.

In practice, $n_k$ is discrete (number of TCP connections). Furthermore:

- Real-time control at sources' (application layer) is impractical, incentives?
- Killing an ongoing TCP connection to reduce $n_k$ is undesirable.

More practical alternative:
- Control increase of $n_k$ (admit new connections), rely on natural termination.
- Admission control carried out by edge router.
- User utility $U_k(\rho_k)$ describes the SLA:
  
  admit new connection $\iff U_k^{-1}(q_k) > \rho_k$

Markov chain model.

Poisson($\lambda_k$) arrivals,  
exp($\mu_k$) workloads.  
Active sessions served with rate $x_k$ obtained from TCP.
Theory and simulations

Fluid limit model,
\[ \dot{n}_k = \lambda_k 1 \left\{ \frac{1}{\mu_k} U_k^{-1}(\hat{\rho}_k(n)) > \hat{\rho}_k(n) \right\} - \mu_k \hat{\rho}_k(n) \]

For $\frac{\lambda_k}{\mu_k} > \rho_k^*$ (optimal fairshare) simulations show convergence to optimal point, we can prove it in simple networks.

If a certain user has $\frac{\lambda_k}{\mu_k} < \rho_k^*$, convergence to solution of $\max \rho \sum_k \overline{U}_k(\rho_k), \ s.t. R \rho \leq c,$

where $\overline{U}_k(\bullet)$ corresponds to a demand curve saturated at rate $\rho_k = \frac{\lambda_k}{\mu_k}$.

Non-greedy user is protected.

Multipath extensions are available.
3. Distributed Auctions for Resource Allocation in Overlay Networks

[Belzarena-Ferragut-P’, Net-Coop ’08]

- Scenario: an operator sells premium services (e.g. video-on-demand) over a network.
- QoS is guaranteed by reserving bandwidth in fixed amounts, and controlling access.
- One related proposal Service Overlay Newtork (Zuan et al.): here the service is offered by leasing bandwidth from several ISPs, installing distributed content servers.
- Objective: selling the service to maximize revenue.
End users submit bids for certain end-to-end service to a broker within their domain.

Periodically, an auction is held to decide which users are assigned bandwidth.

Broker decisions are “spatially” coupled.

Also coupled over time: once a circuit is assigned, reserved for entire connection (e.g. movie).
Auctions for a single link

Assume:

• At $k$ – th auction period: $N$ bids $b_{k}^{(1)} \geq b_{k}^{(2)} \geq b_{k}^{(3)} \geq \cdots \geq b_{k}^{(N)}$ for a unit of bandwidth. Link capacity $C$.

• Bids have known probability distribution.

• Revenue from accepting $a_k$ bids (first-price auction) $U_b(a_k) = \sum_{i=1}^{a_k} b_{k}^{(i)}$.

Interpolate to piecewise linear, concave function.

• Expected revenue from $a_k$ bids: $\overline{U}(a_k) = E[U_b(a_k)] = \sum_{i=1}^{a_k} E[b_{k}^{(i)}]$
Successive auctions

\[ p \]: probability that a connection remains active at the end of the period of time \( T \)
\[ x_k \]: number of active connections at \( kT \)
\[ a_k \]: number of accepted connections in auction \( k \)
\[ x_k + a_k \]: active connections in \( kT + \)

**Myopic policy:** \( a_k = C - x_k \), sells all available capacity. May miss better bids in the future.
Optimal revenue problem

Maximize \( \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{a_k} E[U_b(a_k)] \)

- Expectation is w.r.t the bids, and the departure process.
- This is a Markov Decision Process (MDP).
- Solution is a policy \( a_k = a(s_k) \), where the state \( s_k = (x_k, b_k) \)
- \( a(s) \) can be found numerically, large computational cost.

Receding horizon approximation:

\( a_0 = \arg \max_{a \leq C - x_k} [U_b(a) + \mathbb{E}_{x_1} \bar{U}(C - x_1)] \)

- Optimize current revenue+ expected revenue of next auction, assuming all remaining capacity will be sold off at that time.
- Take auction \( a_0 \), and repeat recursively.
Receding horizon policy:  \[ a_0 = \arg \max_{a \leq C-x_k} \left[ U_b(a) + E_{x_1} \bar{U}(C-x_1) \right] \]

- For the second term, \( x_1 \sim \text{Bin}(x_0 + a, p) \).
- Some calculations reduce it to \( -W(x_0 + a) \), where \( W() \) piecewise linear, increasing and convex (cost of missed future opportunities).
- Maximum: intersection of decreasing marginal utilities (bids) with increasing marginal costs (acceptance thresholds, \( w_k \)).
Comparisons (C=1)

A fluid approximation: replace $E_x U(C - x_1)$ by $\bar{U}(C - E[x_1])$

The problem reduces to the convex program

Maximize $\overline{U}(z)$

subject to $x_0 + a \leq C$, $p(x_0 + a) + z \leq C$. 
The network case

Assign a broker to each $r$, route/service in the network.

$a^r$: current allocation, broker $r$

$z^r$: expected next-step allocation.

$R$: routing matrix, as before.

Network allocation problem: Maximize $\sum_r U_{b^r}(a^r) + \bar{U}(z^r)$

subject to $R(x_0 + a) \leq C$, $RP(x_0 + a) + z \leq C$

- Similar to network utility maximization in congestion control
- Additional (one-step ahead) variables and constraints.
  Requires additional price variables.
- Distributed solution via message passing in the control plane (modification to RSVP).
Conclusions

We studied three problems in cross layer control and optimization:
1. Congestion control with multipath routing.
2. Controlling fairness through number of TCP connections.

Common features:
- Economic (utility based) models.
- Dynamics play a non-trivial role.
- Distributed solutions.

Progression from “virtual” to real economics (utilities as protocol representations, versus real monetary utilities).

Grand challenge for the future: an integrated view of network control and economics.