Effective wavelet representations in high dimensions

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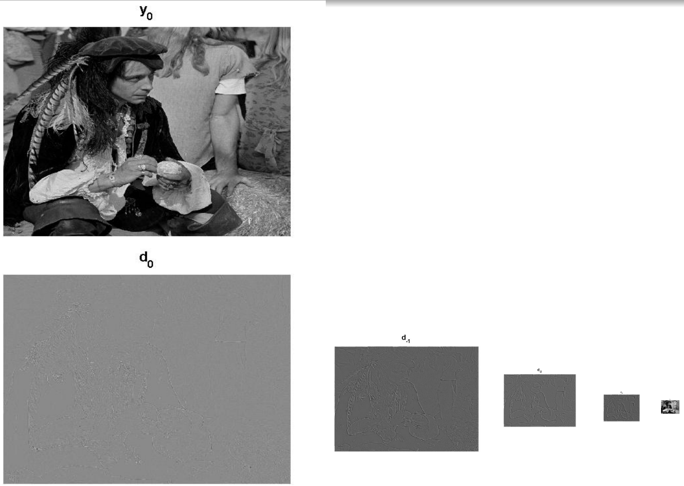
Outline

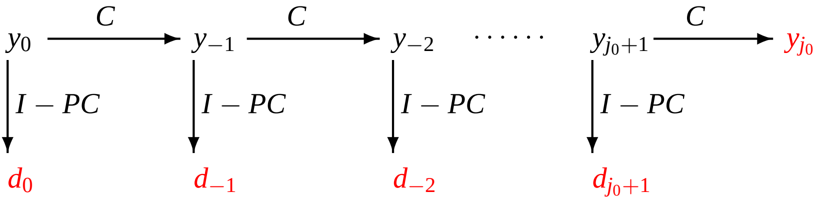
Outline

An example of Laplacian pyramid (CAMP system)



CP: Burt-Adelson

Laplacian Pyramids (Burt and Adelson, 1983)



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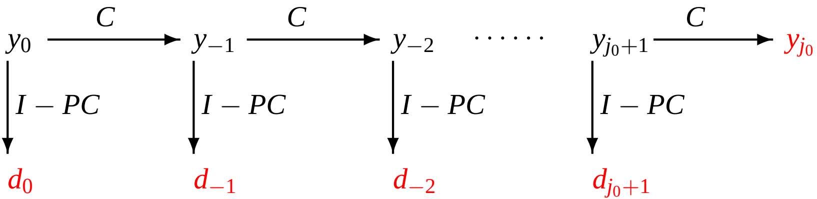

$(y_j)_{j=-\infty}^{\infty} \subset \mathbb{C}^{\mathbb{Z}^n}$ s.t:

$$y_{j-1} = Cy_j := (h_c * y_j)_{\downarrow}, \quad \forall j.$$

C is Compression

y_j is then **predicted** from y_{j-1} by

$$y_j \approx Py_{j-1} := 2^n \left(h_p * (y_{j-1\uparrow})\right).$$

P is Prediction

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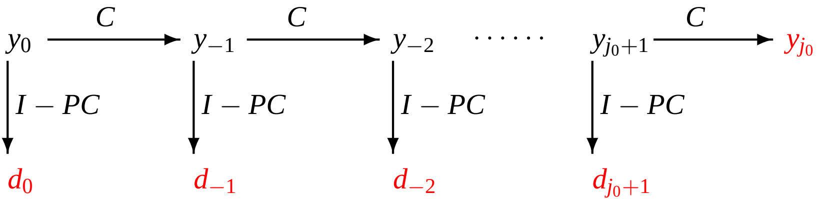


$h_c, h_p : \mathbb{Z}^n \to \mathbb{R}$ are symmetric, normalized, lowpass filters
For each $h := h_c$ and $h := h_p$, $h(k) = h(-k)$, $\sum_{k\in\mathbb{Z}^n} h(k) = 1$.

$\downarrow, \uparrow$ are downsampling & upsampling:

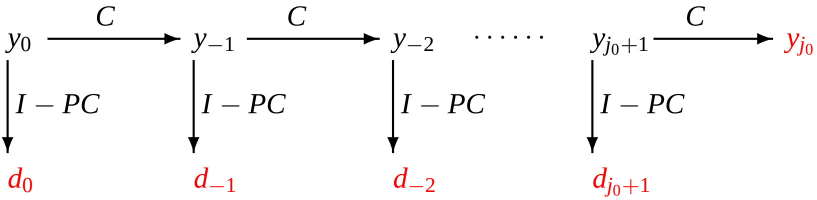
$$y_\downarrow(k) = y(2k), \quad k \in \mathbb{Z}^n.$$

$$y_\uparrow(k) = \begin{cases} y(k/2), & k \in 2\mathbb{Z}^n, \\ 0, & \text{otherwise.} \end{cases}$$

Laplacian Pyramids: summary

The **pyramidal algorithm**:

1. Define the detail coefficients $d_j := (I - PC)\, y_j = y_j - P\, y_{j-1}$.
2. Replace y_j by the pair (y_{j-1}, d_j).
3. Continue iteratively, i.e. do Step 1 with y_{j-1}.



Reconstruction. Recovering y_0 from $y_{j_0}, d_{j_0+1}, \ldots, d_0$ is trivial:
$y_{j_0+1} = d_{j_0+1} + P y_{j_0}$, $y_{j_0+2} = d_{j_0+2} + P y_{j_0+1}$ and so on.

More on CP

- Highpass filter size is the filter size of $h_c * h_p$:
  Note $d_j := (I - PC)\, y_j$.
- Redundancy ratio $\approx \dfrac{2^n}{2^n - 1}$:
  Note that we replace y_j by the pair (y_{j-1}, d_j).
  Let the size of y_0 be $\approx N$.
  After 1 step, the total size is $\left(1 + \frac{1}{2^n}\right) N$
  since the size of d_0 is $\approx N$ and the size of y_{-1} is $\approx \frac{N}{2^n}$.
  At the final step, the total size is

$$\left(1 + \frac{1}{2^n} + \frac{1}{2^{2n}} + \cdots + \frac{1}{2^{-j_0 n}}\right) N \approx \frac{2^n}{2^n - 1} N$$

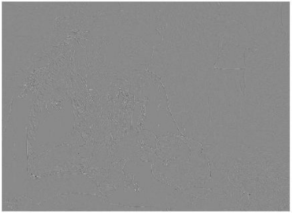
Laplacian Pyramid: example (cont'ed)

y_0

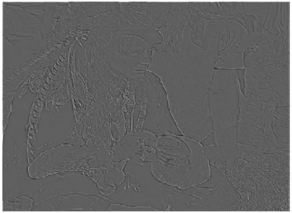
y_{-4}

Laplacian Pyramid: example (cont'ed)

d_0

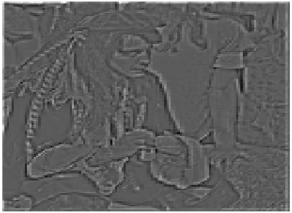


d_{-1}



d_{-2}

d_{-3}



Derivation of wavelets from CP pyramids

The Fast Wavelet/Framelet Transform (Mallat 1987/ DHRS 2003)

Decompose the detail map $I - PC$: $I - PC = \sum_{i=1}^{r} R_i D_i$

$$D_i : y_j \mapsto (h_i * y_j)_{\downarrow} =: w_{i,j-1}, \quad R_i : y \mapsto 2^n \left(h_i * y_{\uparrow}\right)$$

with h_i a real, symmetric, highpass: $\sum_{k \in \mathbb{Z}^n} h_i(k) = 0$.

We can recover y_0 from $y_{j_0}, w_{1,j_0}, \ldots, w_{r,j_0}, \ldots, w_{1,-1}, \ldots, w_{r,-1}$
since $y_{j_0+1} = \sum_{i=1}^{r} R_i w_{i,j_0} + P y_{j_0}$, $y_{j_0+2} = \sum_{i=1}^{r} R_i w_{i,j_0+1} + P y_{j_0+1}$
and so on.

Laplacian pyramid vs. wavelets: Examples in 2D

Burt-Adelson CP: Let $h_c = h_p = \begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix}$.

There are four (hidden) highpass filters:

$$\begin{bmatrix} +3/4 & -1/4 \\ -1/4 & -1/4 \end{bmatrix}, \begin{bmatrix} -1/4 & +3/4 \\ -1/4 & -1/4 \end{bmatrix}, \begin{bmatrix} -1/4 & -1/4 \\ +3/4 & -1/4 \end{bmatrix}, \begin{bmatrix} -1/4 & -1/4 \\ -1/4 & +3/4 \end{bmatrix}$$

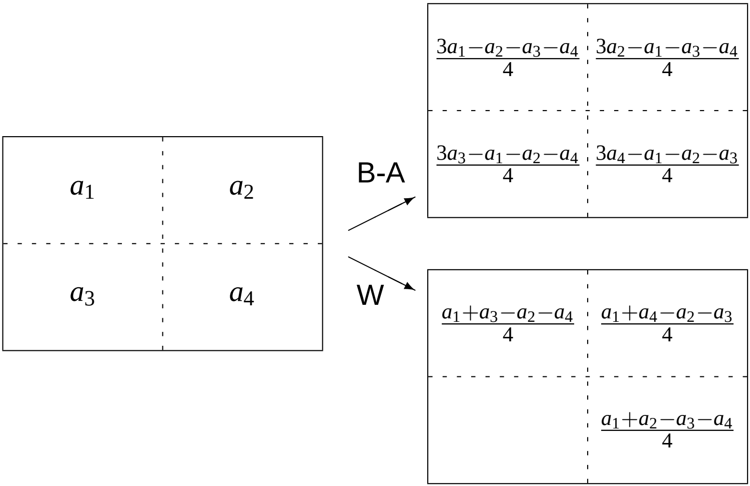
2D Haar wavelets: There are three highpass filters:

$$\begin{bmatrix} +1/4 & -1/4 \\ +1/4 & -1/4 \end{bmatrix}, \begin{bmatrix} +1/4 & -1/4 \\ -1/4 & +1/4 \end{bmatrix}, \begin{bmatrix} +1/4 & +1/4 \\ -1/4 & -1/4 \end{bmatrix}$$

In both cases, average filter size is 4.

back to algorithms

Laplacian pyramid vs. wavelets: example cont'ed



Outline

Wavelet Why: Laplacian pyramids vs. wavelets

Question: why or why not decomposing the detail map $I - PC$:

$I - PC = \sum_{i=1}^{r} R_i D_i$

Pros

1. Reducing the size of the filters
2. Making it possible to be non-redundant: $r = 2^n - 1$
3. Making it possible to be highly redundant: $r >> 2^n - 1$
   for applications in feature detection and denoising
4. Solid mathematical theory in terms of performance

Cons

1. Non-trivial to do.
   Intrinsic factorizations in high-D are essentially impossible.

Neutral

1. Later: not all wavelet constructions are obtained in this way.

How to construct a wavelet representation?

1. Mallat's algorithm, '87 :
   1D only. Imposes very stringent conditions on C and P.
   Daubechies' systems fall here.
2. Cohen-Daubechies-Feauveau' algorithm, '92:
   Still 1D only.
   Imposes milder, still stringent, conditions on C and P.
   1D bi-orthogonal systems (5/3, 7/9) fall here.
3. The Unitary Extension Principle (UEP), R-Shen, '97:
   All spatial dimensions. No restrictions on C and P.
   The wavelet decomposition of $I - PC$ results in a frame.
4. The Oblique Ext. Pr. (OEP), Daubechies-Han-R-Shen '03:
   All spatial dimensions.
   Cannot be derived by factoring Laplacian pyramids.
   The most general construction of wavelet frames.

back to theory

Wavelets: mathematical theory I

Mathematical interpretation of the wavelet representation

$$L_2 := \{f : \mathbb{R}^n \to \mathbb{R} : \int_{\mathbb{R}^n} |f|^2 < \infty\}.$$

1. Step I: Find refinable function $\phi \in L_2$ with mask h_c:

$$\widehat{\phi}(2\cdot) = \widehat{h_c}\widehat{\phi}, \quad (\widehat{\phi}(0) = 1), \qquad (h_c\text{: refinement filter}).$$

2. Step II: Define mother wavelets $\Psi := \{\psi_1, \cdots, \psi_r\}$ by the rule

$$\widehat{\psi}_i(2\cdot) = \widehat{h_i}\widehat{\phi}, \qquad (h_i\text{: wavelet filter}).$$

3. Step III: Assume that $y_0(k) = \langle f, \phi(\cdot - k)\rangle$, $k \in \mathbb{Z}^n$

Then, upon using the above filters in Decomposition, we get

$$w_j^i(k) = 2^{j\frac{n}{2}} \langle f, (\psi_i)_{j,k}\rangle, \quad (\psi_i)_{j,k} := \psi_i(2^j \cdot -k).$$

Wavelets: mathematical theory I, cont'ed

$\Psi \subset L_2$ is finite. The wavelet system $X(\Psi)$ is then

$$\psi_{j,k} := 2^{j\frac{n}{2}}\psi(2^j \cdot -k), \quad \psi \in \Psi, j \in \mathbb{Z}, k \in \mathbb{Z}^n$$

The wavelet representation of $f \in L_2$ is then the discrete set of inner products

$$T^*_{X(\Psi)} f := (\langle f, x\rangle)_{x \in X(\Psi)}, \quad \langle f, g\rangle := \int_{\mathbb{R}^n} f(t)\overline{g(t)}\, dt.$$

The wavelet system $X(\Psi)$ is a frame of L_2 if

$$\sum_{x \in X(\Psi)} |\langle f, x\rangle|^2 \approx \|f\|^2_{L_2}, \quad \forall f \in L_2.$$

Wavelets why II: Desired properties

or, why should anyone still care about new constructions?

1. Localness in space

Quantifying "local": the number of wavelets within a single resolution whose support contains a given generic point $t \in \mathbb{R}^n$.

Note: this is the same as the total volume of the mother wavelets set Ψ:

$$\mathrm{vol}(\Psi) := \sum_{\psi \in \Psi} \mathrm{vol}(\mathrm{supp}\,\psi).$$

2. Localness in frequency: high performance.
3. Speed: Small constants in the linear complexity of the algorithms.

Wavelets why III: Challenges in high-D constructions

or, prevailing approaches go kaput in high-D

The Laplacian pyramid is challenged since:

1. It becomes immensely non-local.
2. There was no rigorous performance analysis, hence lack of mathematical guidance (not even frame analysis).
3. “Feels not right”: after all, the most general wavelet constructions cannot be associated with such pyramid.

Intrinsic wavelet constructions are challenged since:

1. They are in between very difficult and impossible: In n-D, one needs to define $\geq 2^n - 1$ different highpass rules (=mother wavelets).

Simple lifting of univariate wavelets constructions (known as tensor products) are still challenged since:

1. They lead, again, to highly non-local constructs.

Wavelets why IV: Challenges in high-D cont'ed

or, tearful moments for wavelet lovers

Benchmark: Tensor product of biorthogonal 9/7

The tensor biorthogonal 9/7 can analyse $C^{1.70}$-function in $\mathbb{R}^{10}$.
There are 1023 mother wavelets,
each supported in a box of volume.... 562,000,000,
and the total volume is $>$ 575,000,000,000.

	5/3	L-CAMP	L-CAMP	9/7	L-CAMP
s_J	2	2	2	4	4
s_B	1	1.41	2	1.70	2.02
$n = 3$	279	TBA	TBA	2863	TBA
$n = 4$	2145	TBA	TBA	46529	TBA
$n = 5$	15783	TBA	TBA	726607	TBA

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- Step IV: deriving the more local CAMP and L-CAMP. Identifying special classes of CAP pyramids that can be made more local in space, without losing performance.
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- Step V: bi-orthogonal constructions. Finding a way to remove the redundancy from the CAMP and L-CAMP representation

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- Step V: bi-orthogonal constructions. Finding a way to remove the redundancy from the CAMP and L-CAMP representation
- Step VI: numerous bi-products. For example, we had to develop new ways for estimating smoothness of refinable functions in high-D.

L-CAMP: Hallmarks

1. Extreme localness.
2. Works in any spatial dimension.
3. Trivial to construct and implement.
4. Super fast algorithms:
   linear complexity with tiny constants,
   and the constants decay with the dimension!
5. Solid performance theory
   (that shows that, at least in theory, they perform as good as much more complicated wavelets).

L-CAMP: Extreme localness

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A competing L-CAMP system

We construct an L-CAMP system such that it analyses C^2-function in $\mathbb{R}^{10}$.
There are 1024 mother wavelets,
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L-CAMP: The algorithms

Decomposition

Step I: choose three lowpass filters

back to haar

$$h_c := 2^{-n} \sum_{\nu \in \{0,1\}^n} \delta_\nu \quad =: \text{compression filter}$$

h_e := n-dimensional enhancement filter

h := 1-D, supported on the odd integers main filter

Step II: build the MRA

Step III: extract detail coefficients:

L-CAMP: The algorithms
Decomposition

Step I: choose three lowpass filters

Step II: build the MRA

$\downarrow$ is downsampling:

$$y_{\downarrow}(k) \quad = \quad y(2k), \quad k \in \mathbb{Z}^n$$

$(y_j)_{j=-\infty}^{\infty} \subset \mathbb{C}^{\mathbb{Z}^n}$ s.t:

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L-CAMP: The algorithms

Decomposition

Step I: choose three lowpass filters

Step II: build the MRA

Step III: extract detail coefficients:

(1) For $k \in 2\mathbb{Z}^n$, $d_j(k) := y_j(k) - (h_e * y_{j-1})(k/2)$.
(2) For $\nu \in \{0,1\}^n \backslash 0$, and $k \in \nu + 2\mathbb{Z}^n$,

$$d_j(k) = y_j(k) - (h_{J(\nu)} * y_j)(k).$$

$h_{J(\nu)} =?$

back to Theorem

L-CAMP: The algorithms

Decomposition

Step I: choose three lowpass filters

Examples of h:

$$h = [\mathbf{0}, 1], \quad h = [\frac{1}{2}, \mathbf{0}, \frac{1}{2}], \quad h = \frac{1}{16} \times [-1, 0, 9, \mathbf{0}, 9, 0, -1].$$

back to performance

Step II: build the MRA

Step III: extract detail coefficients:

L-CAMP: The algorithms

Reconstruction

Step I: for $k \in 2\mathbb{Z}^n$,

$$y_j(k) := d_j(k) + (h_e * y_{j-1})(k/2).$$

Step II: iteratively, by suitably ordering $\{0,1\}^n \backslash 0$:

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L-CAMP: The algorithms
Complexity

Denote: h_e is A-tap, h is B-tap

Decomposition requires for 2^n details coefficients: $2^n + A + 1 + (B+1) \times (2^n - 1)$.

Reconstruction requires: $A + 1 + (B+1) \times (2^n - 1)$.

Average # of operations per one details coefficient [a]

[a] per one complete cycle of decom-recon

$2B + 3 + 2^{1-n}(A+1)$.

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Wavelets: mathematical theory II: performance

Jackson-type performance: definition

$$W_2^\alpha := \{f \in L_2,\ |f|_{W_2^\alpha} := \|(|\cdot|^\alpha \widehat{f}\,)^\vee\|_{L_2} < \infty\}, \quad \alpha > 0.$$

$$\|c\|^2_{\ell_2(\alpha)} := \sum_{j\in\mathbb{Z},k\in\mathbb{Z}^n} 2^{2j\alpha}|c(j,k)|^2.$$

Jackson-type performance of a frame $X(\Psi)$:

$$s_J := \sup\{\alpha > 0 :\ X(\Psi) \text{ satisfies (1) for the given } \alpha\},$$

$$\sum_{\psi\in\Psi} \|T^*_{X(\psi)} f\|_{\ell_2(\alpha)} \le A_\alpha |f|_{W_2^\alpha}, \quad \forall f \in L_2. \tag{1}$$

s_J is essentially determined by the vanishing moments of $X(\Psi)$.

Wavelets: mathematical theory II: performance

Bernstein-type performance: definition

Bernstein-type performance of a frame $X(\Psi)$:

$$s_B := \sup\{\alpha > 0 : \ X(\Psi) \text{ satisfies (1) and (2) for the given } \alpha\},$$

$$\sum_{\psi\in\Psi} \|T^*_{X(\psi)} f\|_{\ell_2(\alpha)} \geq B_\alpha |f|_{W_2^\alpha}, \quad \forall f \in L_2. \qquad (2)$$

- $s_B \leq s_J$; usually strict inequality holds.
- s_B is not connected directly to any property of the system $X(\Psi)$.
- s_B is essentially determined by s_J, and by the smoothness + Strang-Fix order of the dual system.

L-CAMP: Performance analysis

The key components in the L-CAMP performance analysis

- **The accuracy of the main filter h:**

$$h * P = P, \quad \forall \text{ univariate polynomial } P \text{ of degree } < s_1$$

- **The accuracy of the pair (h_c, h_e):**

$$(h_{e\uparrow} * h_c) * P = P, \quad \forall \text{ multivariate polynomial } P \text{ of degree } < s_2$$

- **The smoothness s_3 of the refinable function ϕ^{dual}** whose mask is

$$\widehat{h_e}\widehat{h}_{tensor},$$

with h_{tensor} the n-dimensional tensor-product of $\frac{\delta + h}{2}$.

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L-CAMP: Performance analysis

L-CAMP based performance results

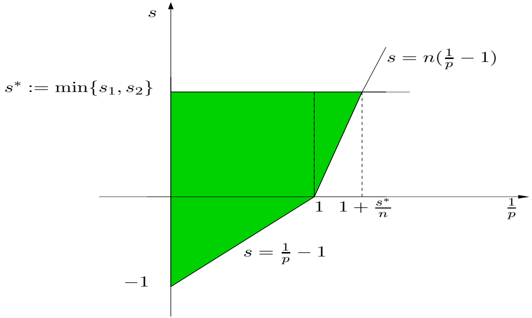
Theorem (Hur-R, 2008)

Assume that we have an L-CAMP system.
Let Ψ be the mother wavelet set associated with the highpass filters in L-CAMP Decomposition.
Let $\min\{s_1, s_2\} \geq 2$.
Let $s_3 > 0$.
Then $X(\Psi)$ has $s_J \geq \min\{s_1, s_2\}$ and $s_B \geq \min\{s_1, s_2, s_3\}$.

L-CAMP: Performance analysis

The Jackson-type performance chart of L-CAMP

performance chart



L-CAMP: Performance analysis

Example 1: extremely local MR representations for C^1 characterization

$$\begin{aligned} h &:= [\frac{1}{2}, \mathbf{0}, \frac{1}{2}], \quad \text{2-tap}, \\ \widehat{h}_e(\omega) &:= \frac{3}{4} + \frac{1}{4} e^{-i\mathbf{1}\cdot\omega}, \quad \text{2-tap}. \end{aligned}$$

- **The accuracy of the univariate filter h:** $s_1 = 2$.
- **The accuracy of the pair (h_c, h_e):** $s_2 = 2$.
- **The smoothness class of the refinable function ϕ^{dual} whose mask is $\widehat{h_e}\widehat{h}_{tensor}$:** $s_3 > 1$ ($s_3 = 1.4$).

Average # of operations: $7 + 3 \cdot 2^{1-n}$.

Total volume of the wavelets' support: < 5.

L-CAMP: Performance analysis

Example 2: extremely local MR representations for C^2 characterization

$$h := [\frac{1}{2}, \mathbf{0}, \frac{1}{2}], \quad \text{2-tap},$$

$$\widehat{h}_e(\omega) := \frac{1}{8}e^{i\mathbf{1}\cdot\omega} + \frac{1}{2} + \frac{3}{8}e^{-i\mathbf{1}\cdot\omega}, \quad \text{3-tap.}$$

- **The accuracy of the univariate filter h:** $s_1 = 2$.
- **The accuracy of the pair (h_c, h_e):** $s_2 = 2$.
- **The smoothness class of the refinable function ϕ^{dual} whose mask is $\widehat{h}_e \widehat{h}_{tensor}$:** $s_3 > 2$ ($s_3 = 2.4$).

Average # of operations: $7 + 4 \cdot 2^{1-n}$.

Total volume of the wavelets' support: < 6.

L-CAMP: Performance analysis

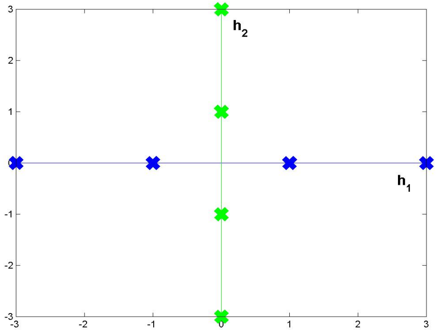
L-CAMP vs. Biorthogonal systems for $n = 3, 4, 5$

We compare the L-CAMP systems with biorthogonal tensor product systems for the spatial dimension $n = 3, 4, 5$.
In the last column, properties of yet another L-CAMP is shown.
In the last 3 rows, the total volume of the mother wavelets is listed for each $n = 3, 4, 5$.

	5/3	**L-CAMP 1**	**L-CAMP 2**	9/7	**L-CAMP 3**
s_J	2	**2**	**2**	4	**4**
s_B	1	**1.41**	**2**	1.70	**2.02**
$n = 3$	279	**4.6**	**5.6**	2863	**14.4**
$n = 4$	2145	**4.8**	**5.8**	46529	**16.7**
$n = 5$	15783	**4.9**	**5.9**	726607	**18.8**

Outline

Orienting the univariate filter



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Meaning of unconditional

unconditional means that only the absolute values of the coefficients count.

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Definition of vanishing moments

Wavelet system $X(\Psi)$ has m vanishing moments if

$$\int t^\beta \psi(t)dt = 0, \quad \forall 0 \le |\beta| \le m-1, \forall \psi \in \Psi.$$

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The 3-tap wavelet in the 5/3 system

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