

# **Using mathematics to structure laws**

Don Saari

UCI, Institute for Math Behavioral Sciences

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Provide introduction

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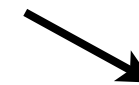
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Cycle! Who gets the  
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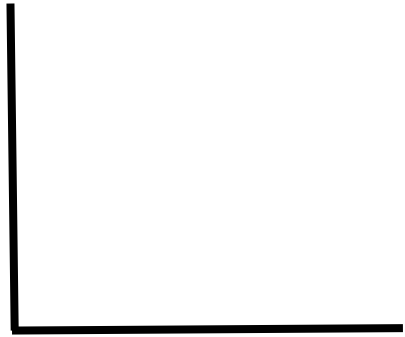
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———— Priority: registering

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Good intensions

Protection against exploitation



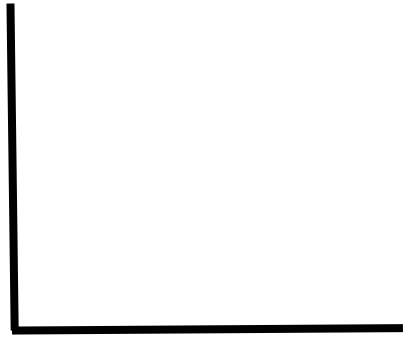
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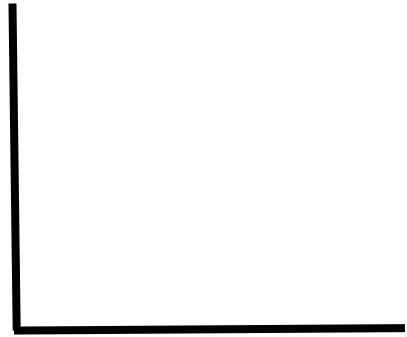


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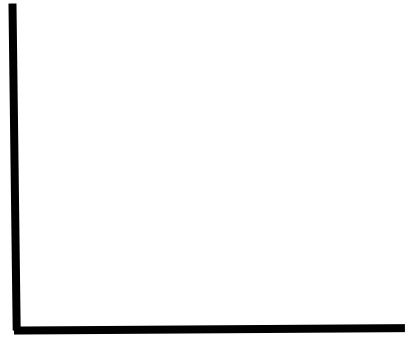
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**Core:**

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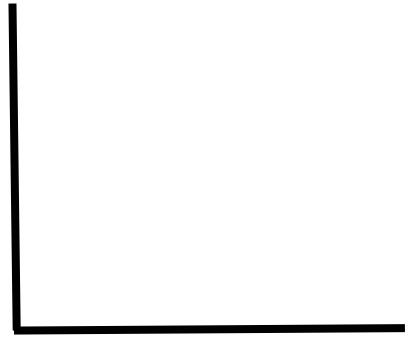
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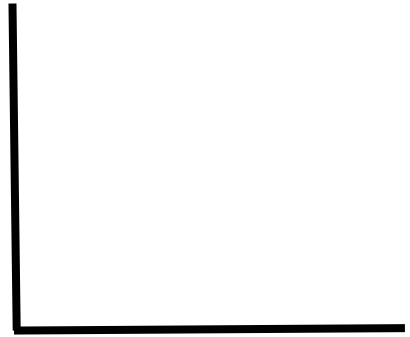
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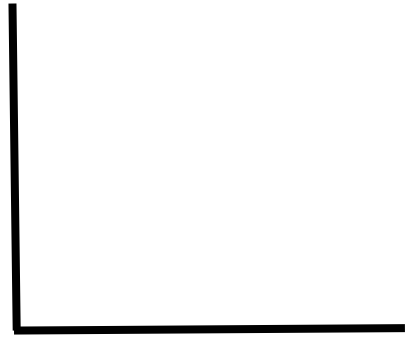
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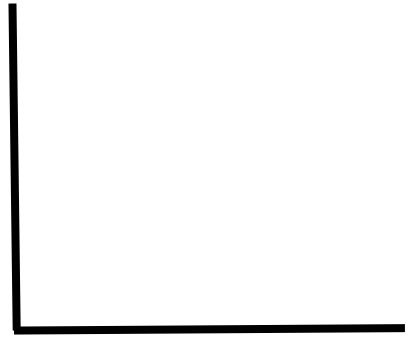
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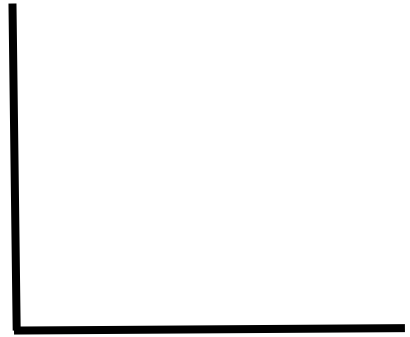
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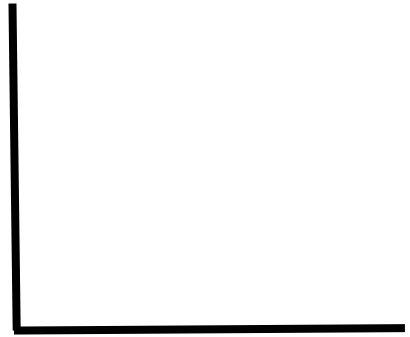
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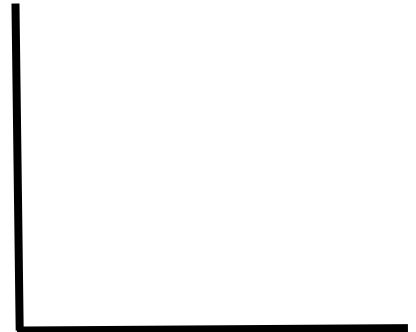
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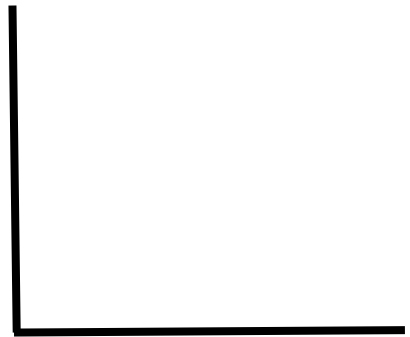
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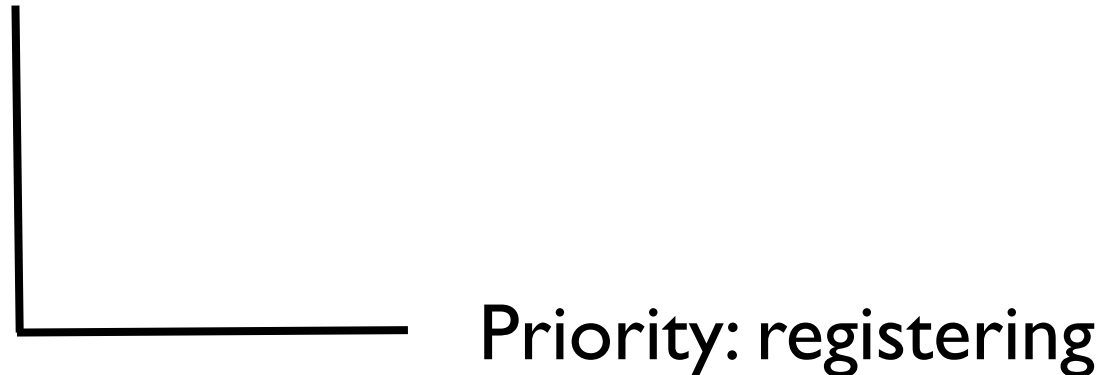
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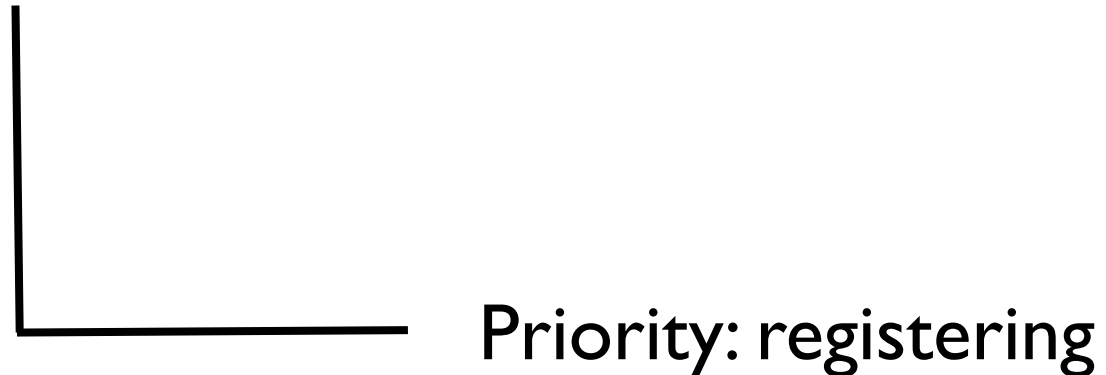
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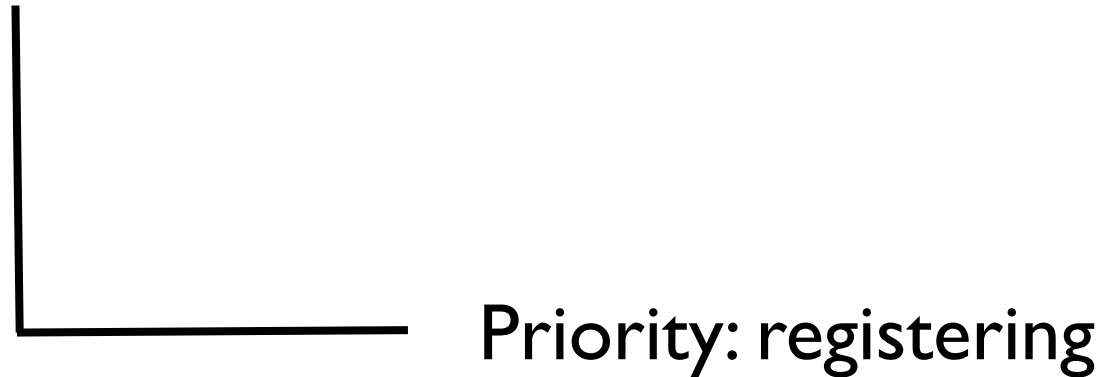
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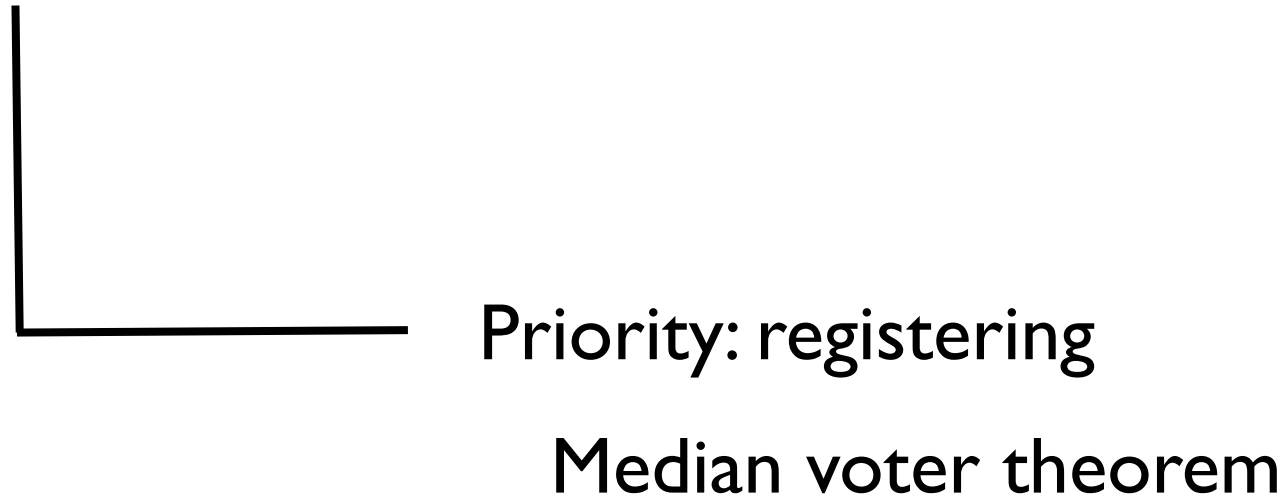
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One issue, core always exists



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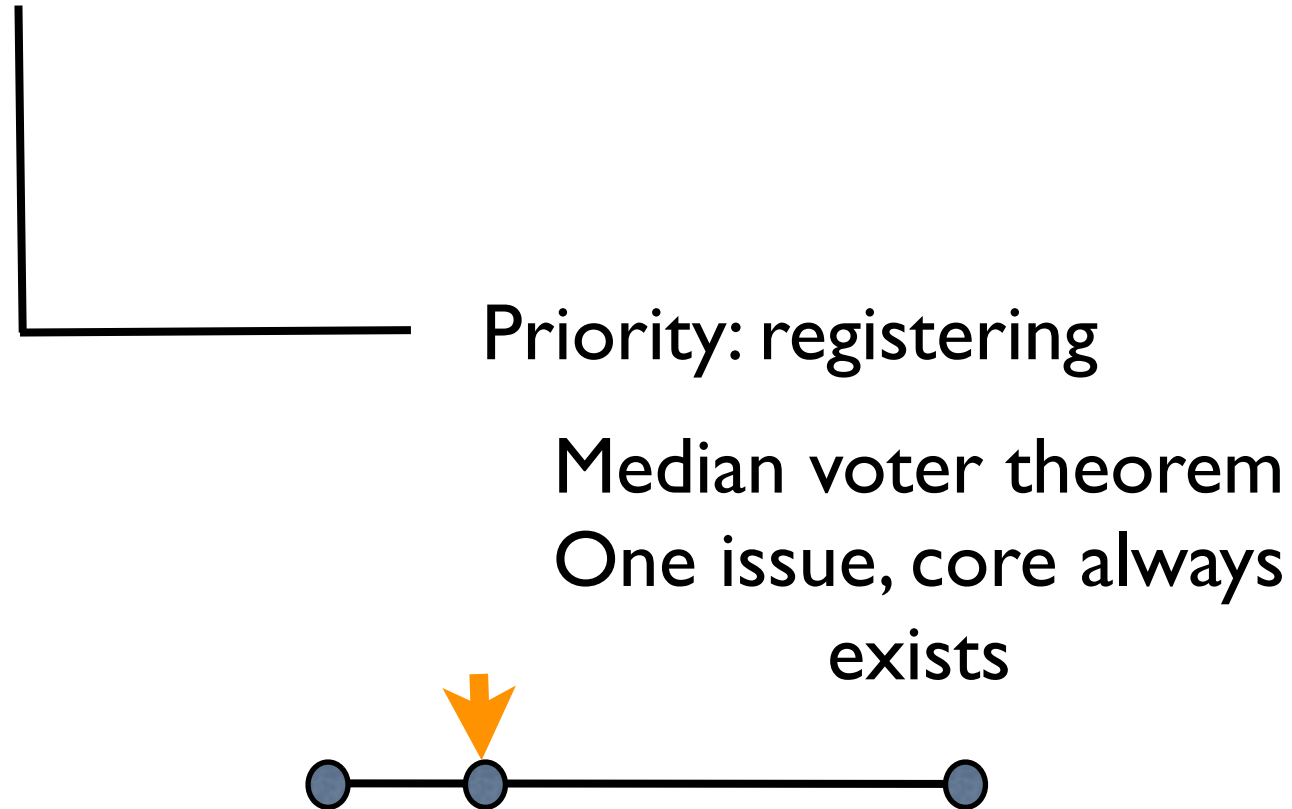
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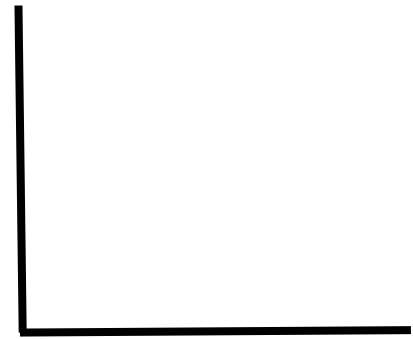
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Majority vote -- a point that can be all others with a majority vote.



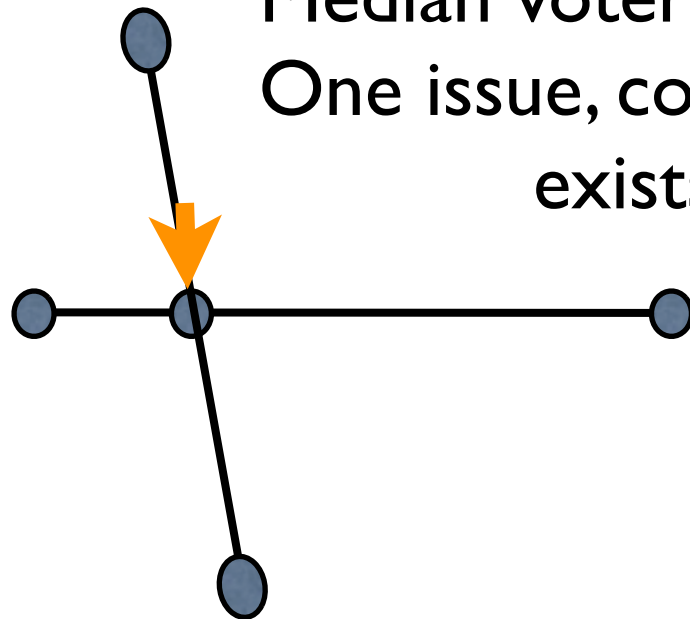
# Dimensionality of issues

Good intentions

Protection against exploitation

Priority: registering

Median voter theorem  
One issue, core always exists



**Avoiding cycles:**

**Core:**

Paired comparisons:

A core point is a point that  
with the specified rule,  
cannot be beaten

so, no cycles

Example:

Computer -- core is empty

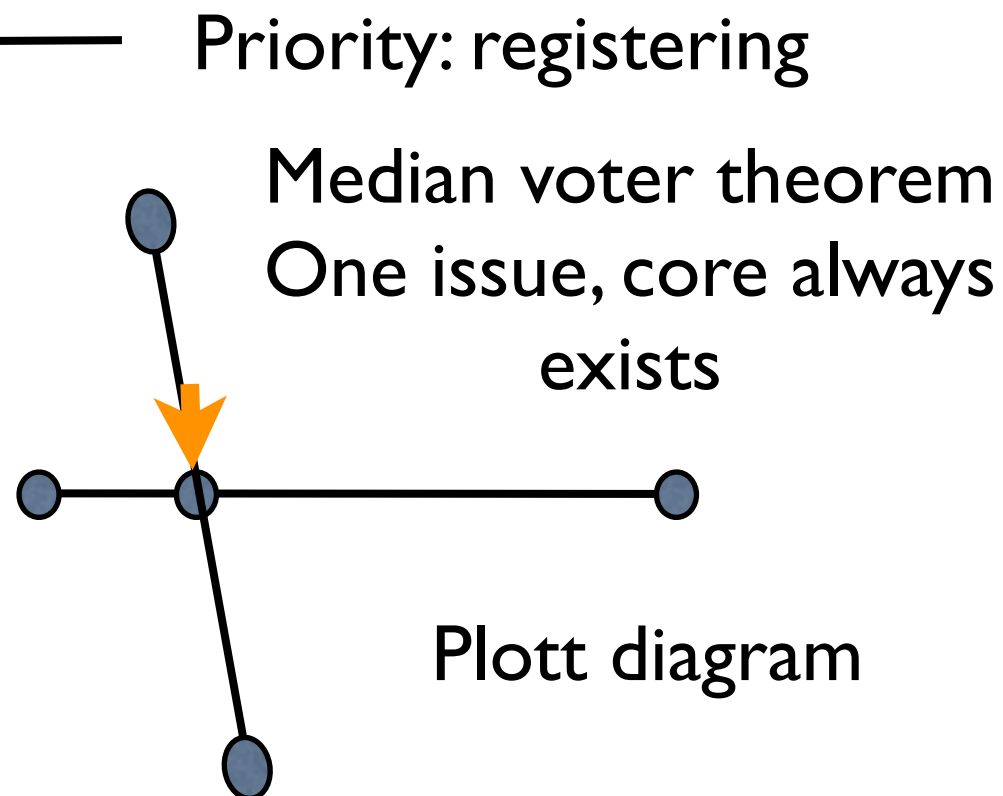
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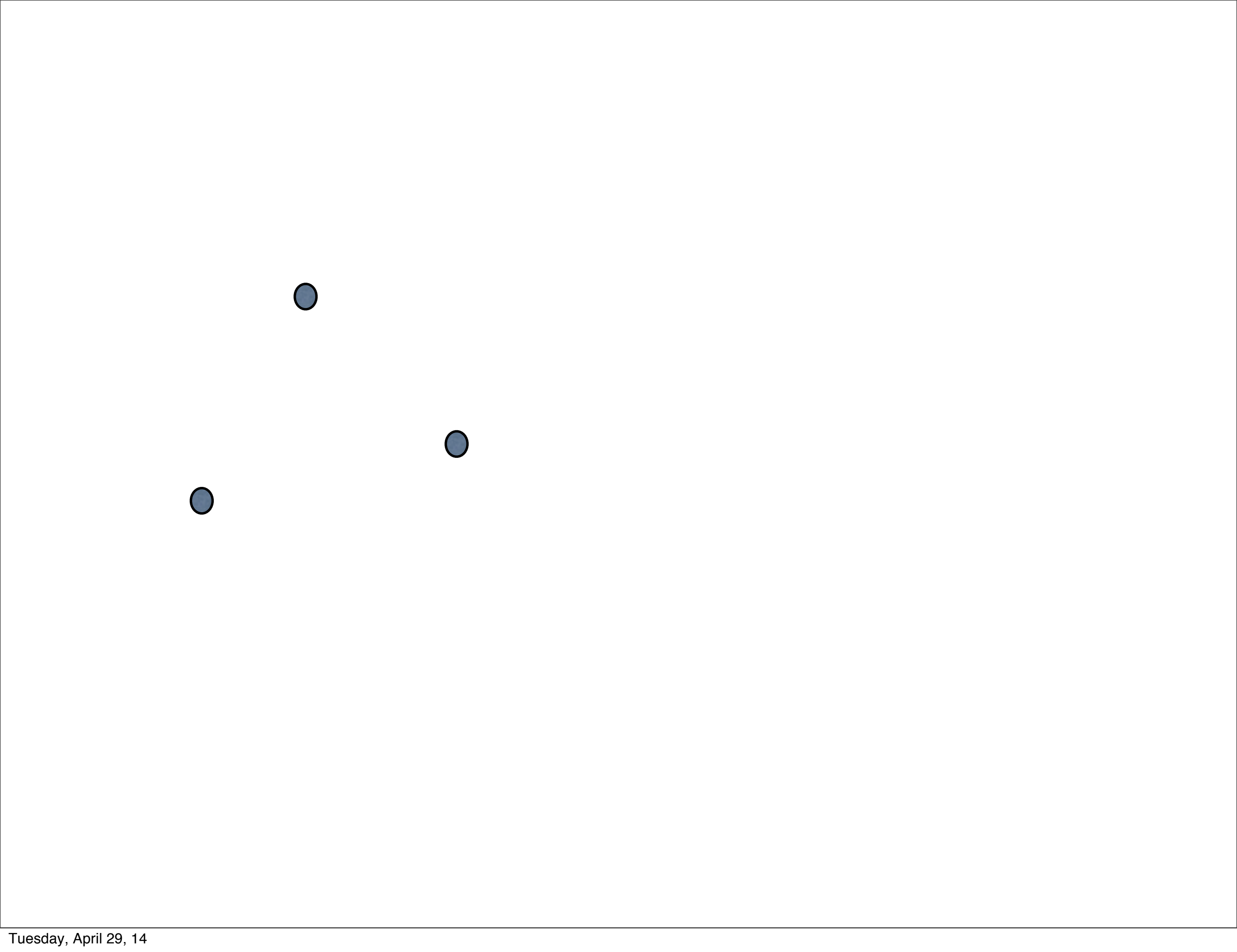
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Example:

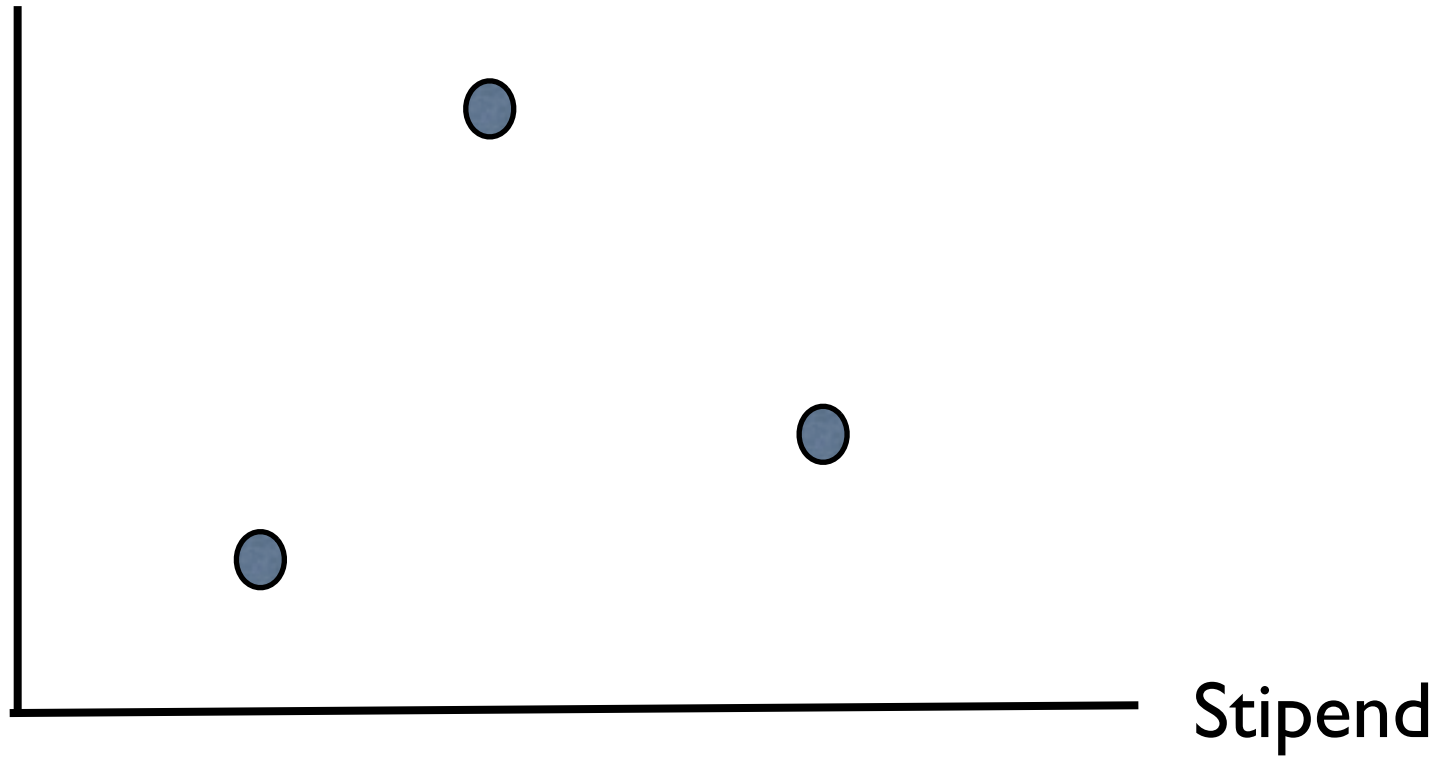
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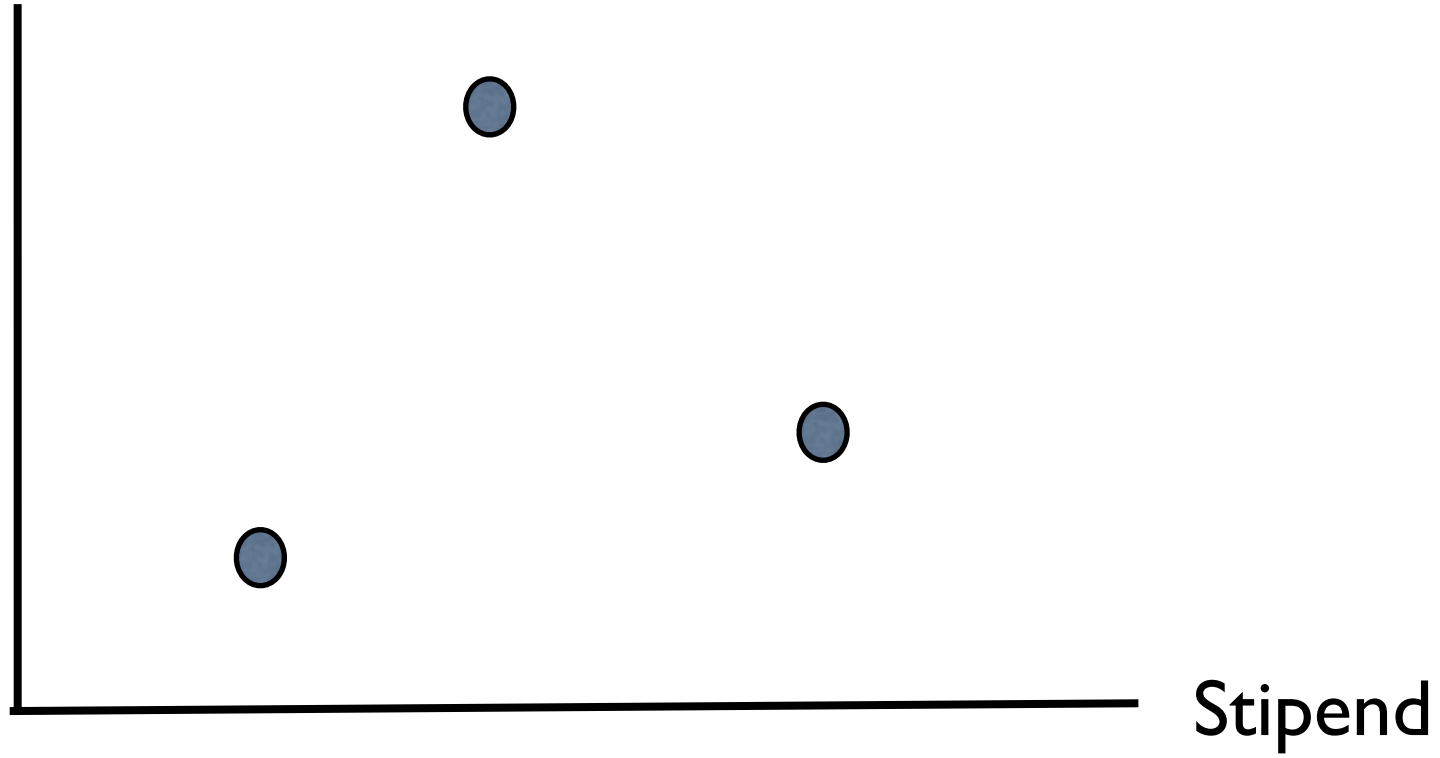
Hours



Stipend

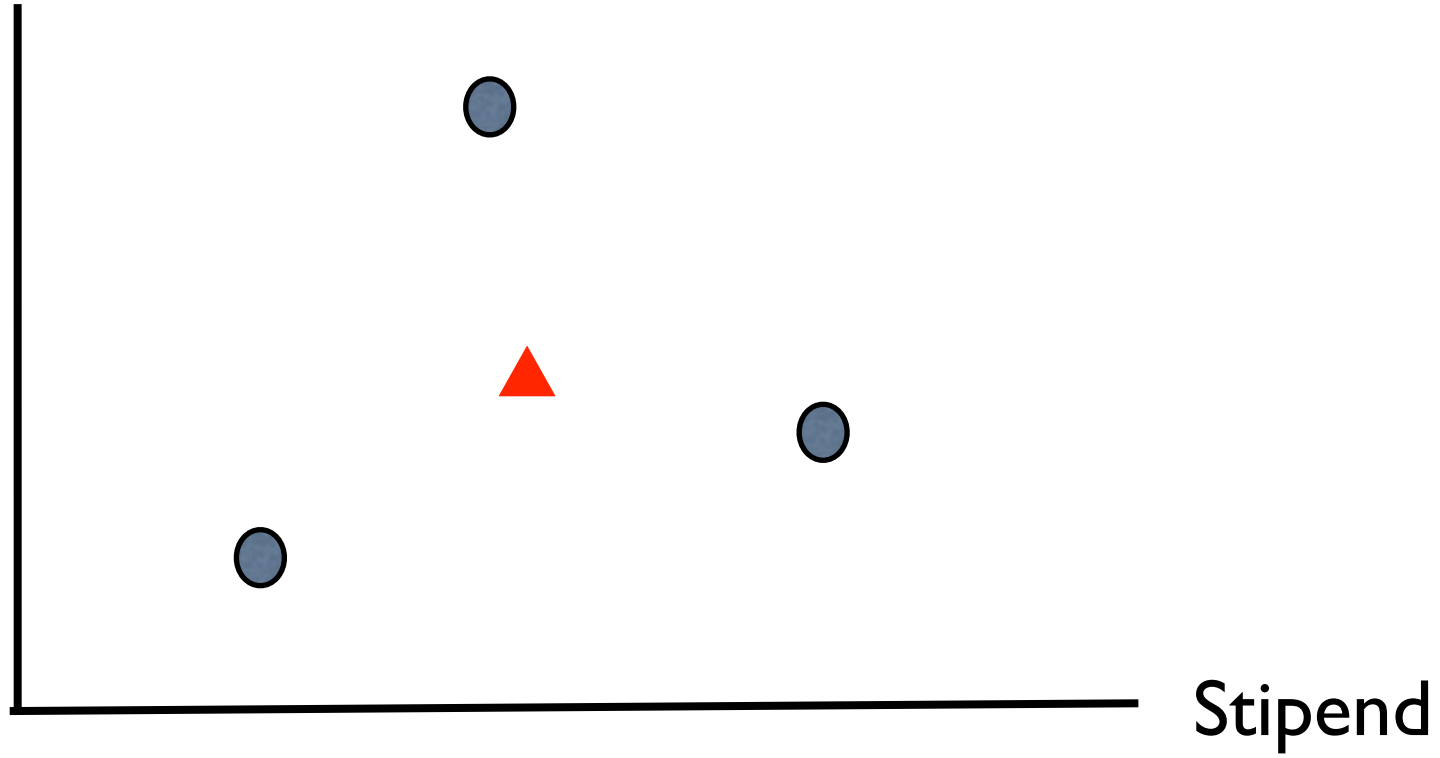
Core?

Hours



Core?

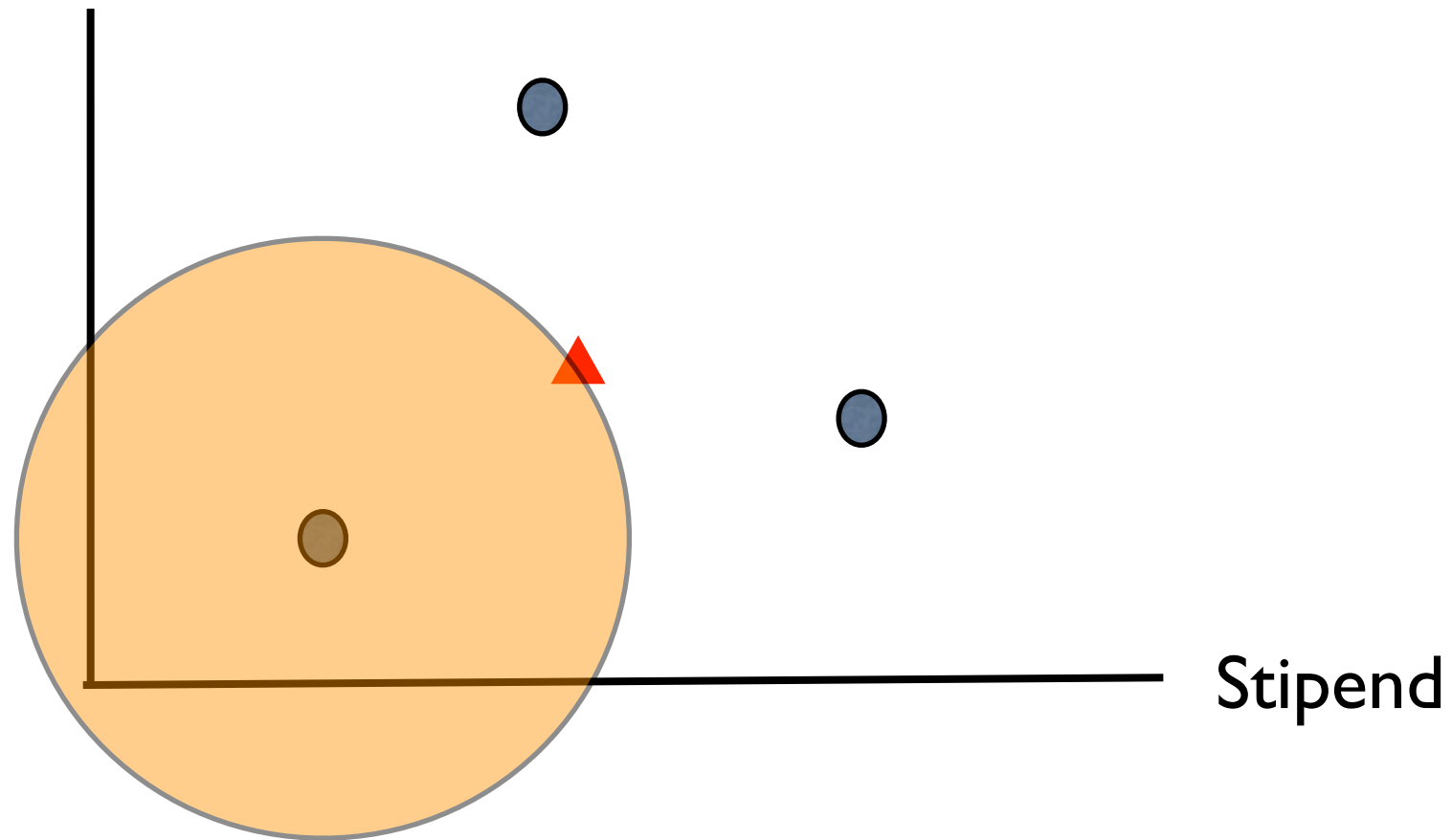
Hours



Stipend

Core?

Hours

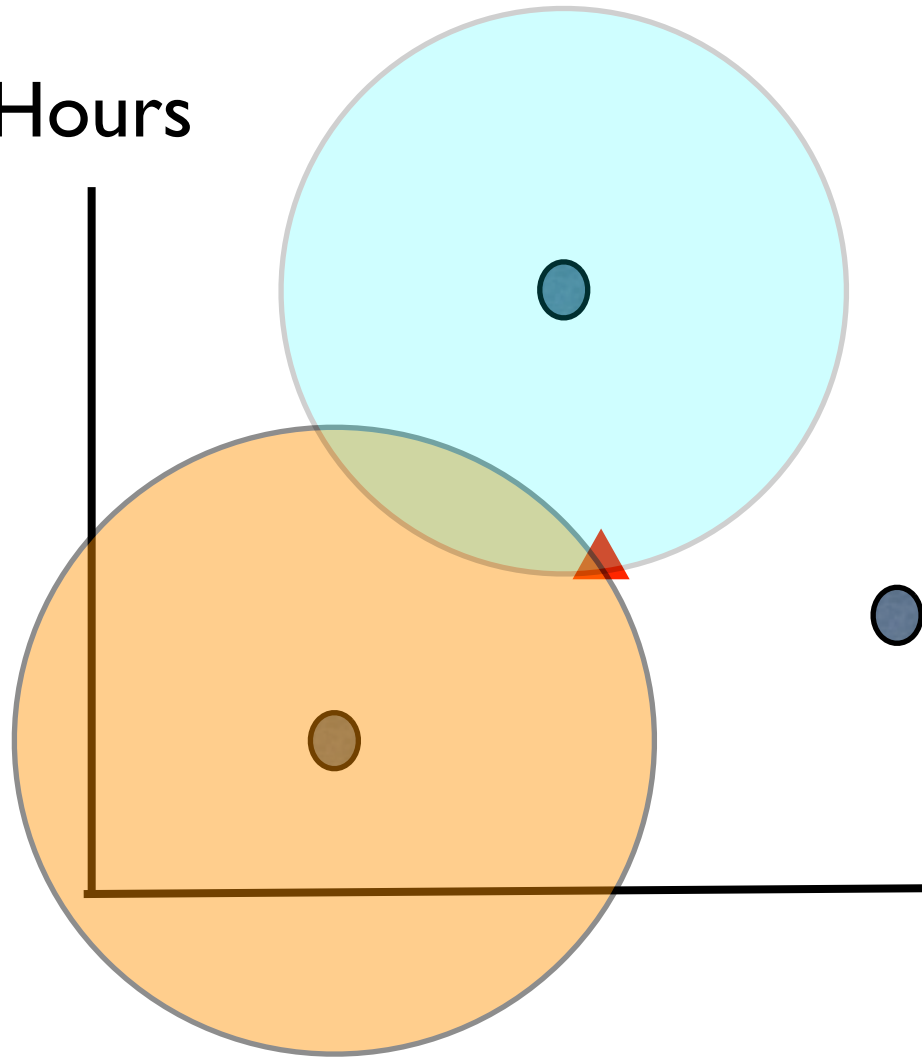


Stipend

Core?

Hours

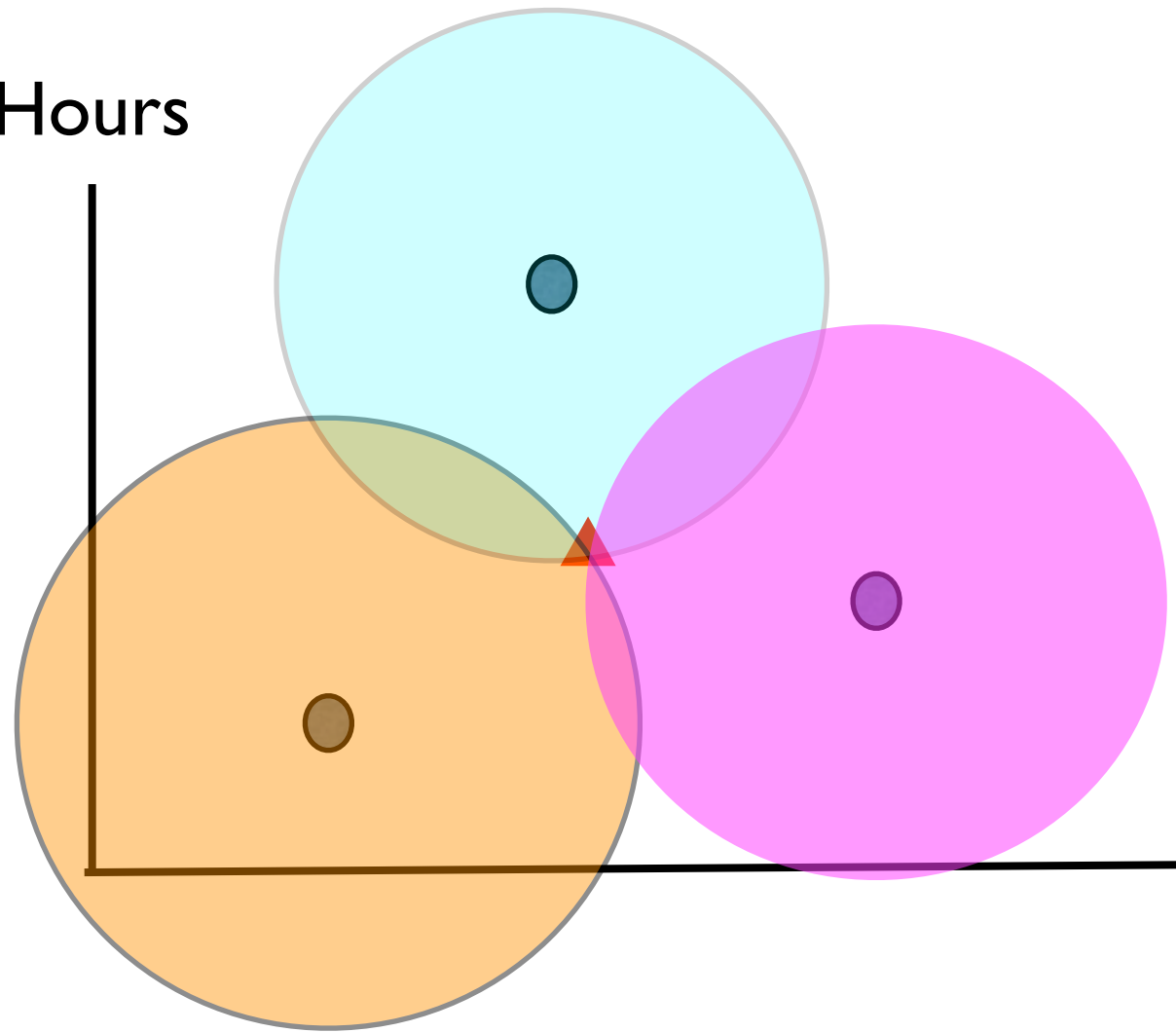
Stipend





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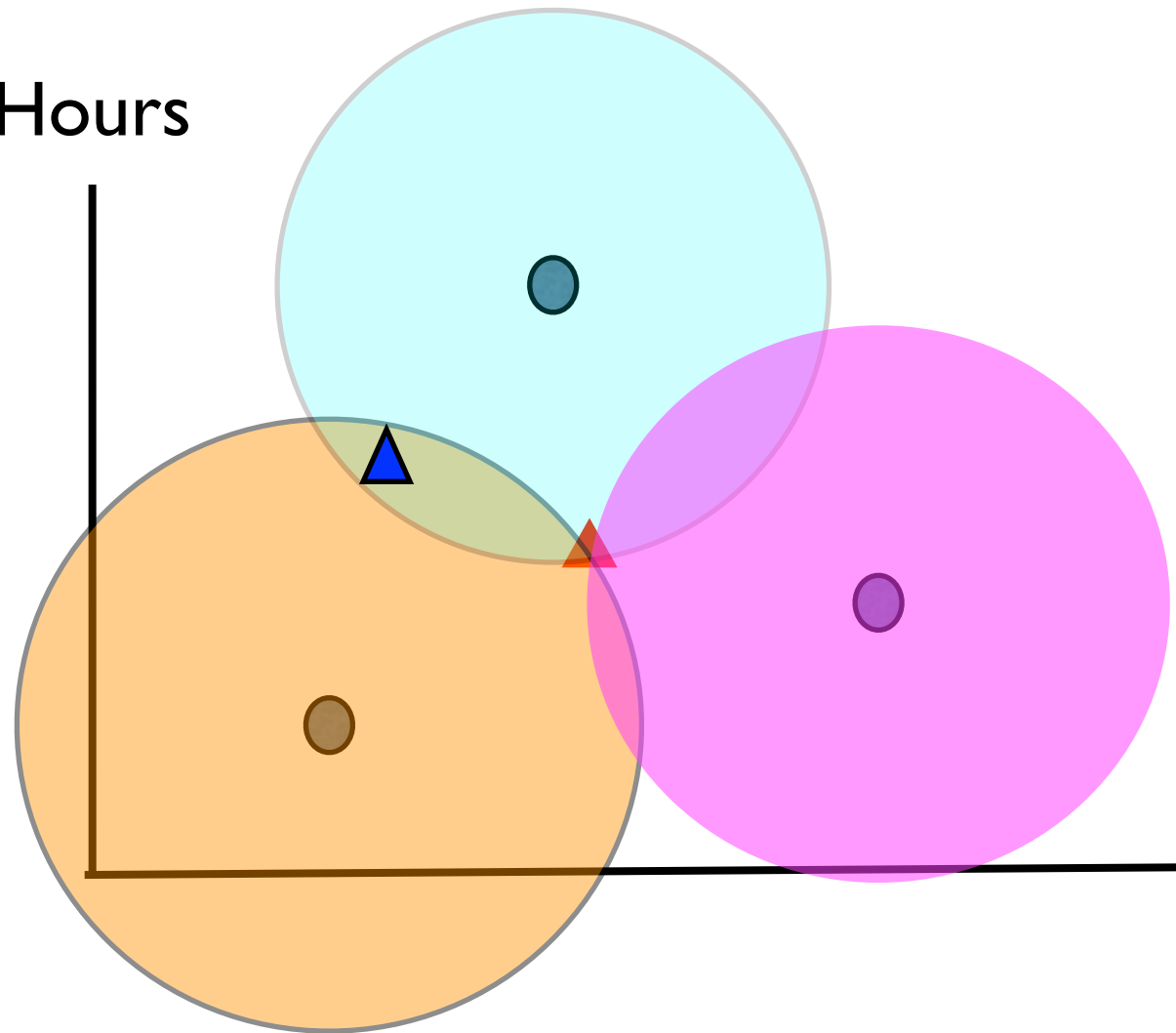
Hours



Stipend

Core?

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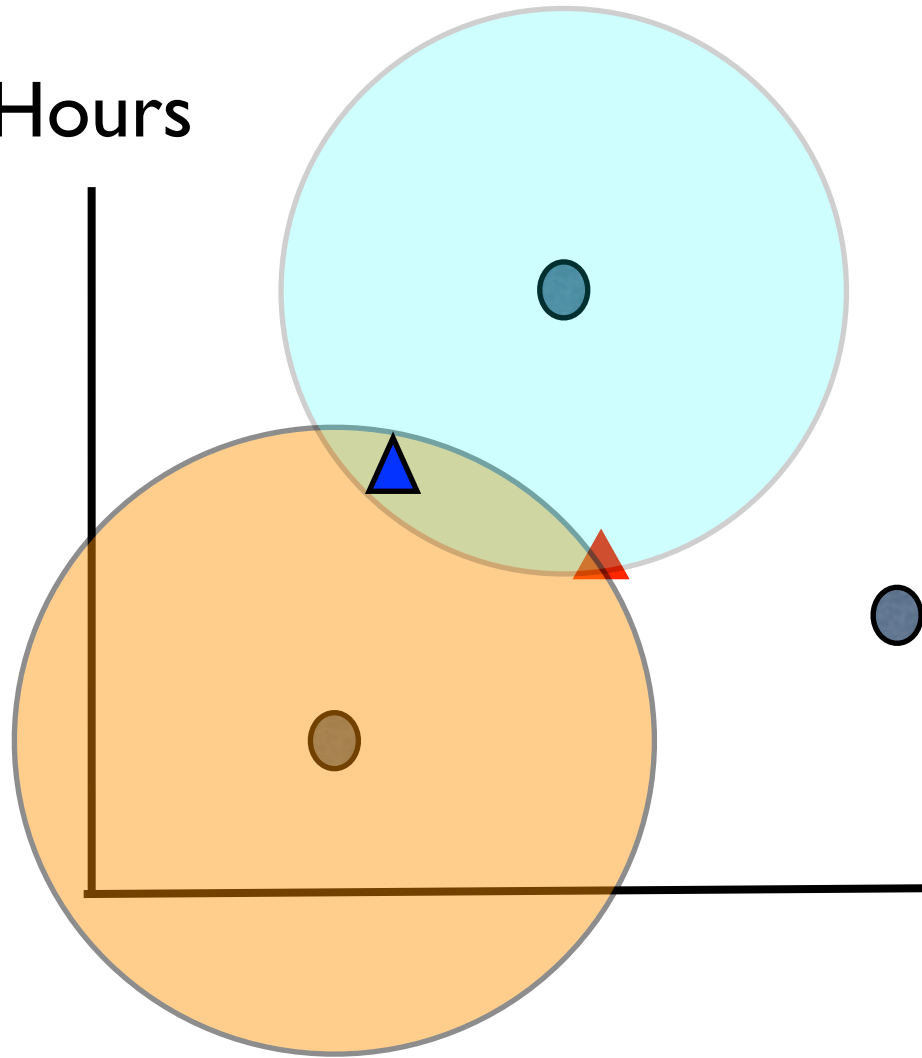


Stipend

Core?

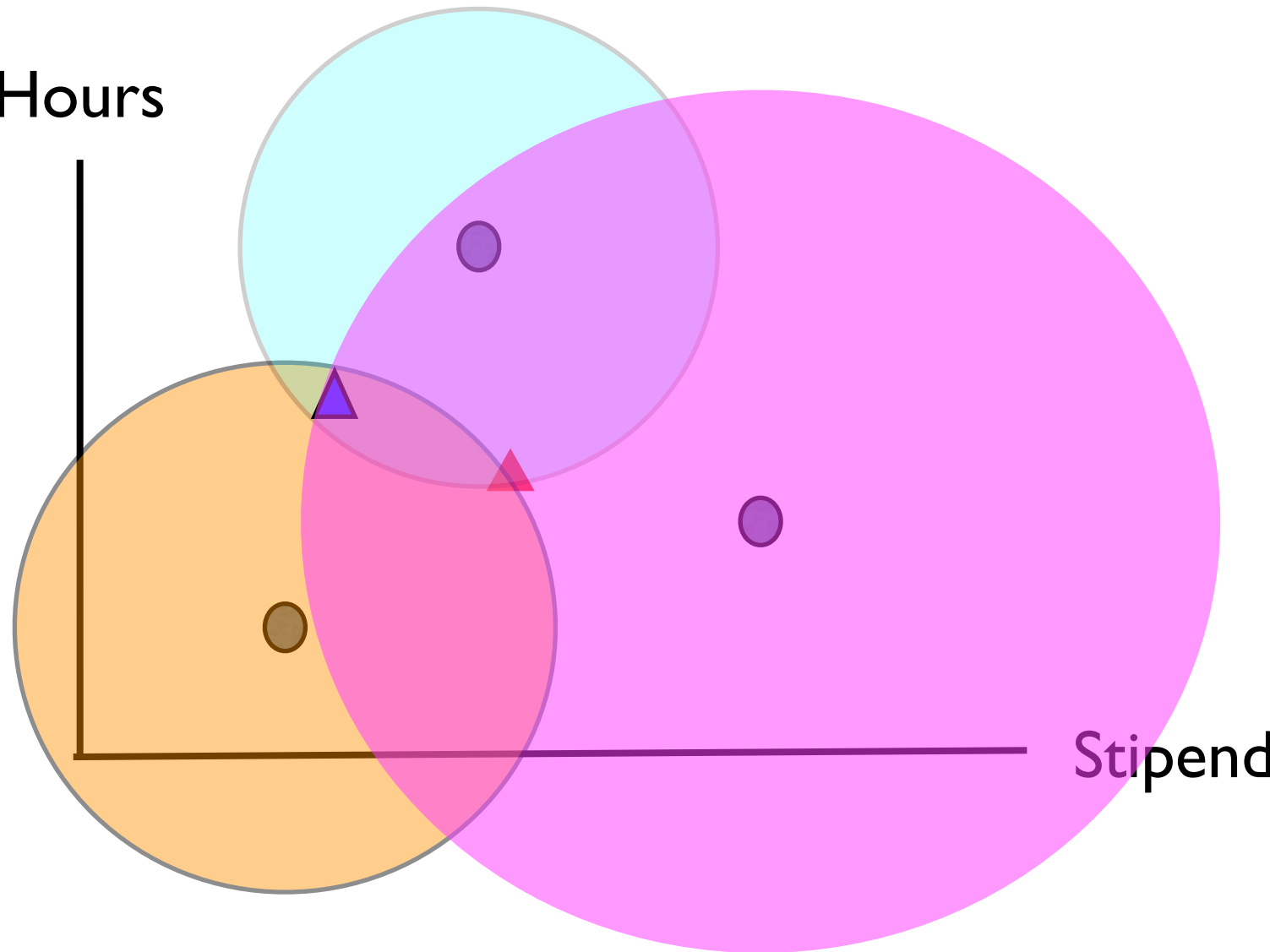
Hours

Stipend



Core?

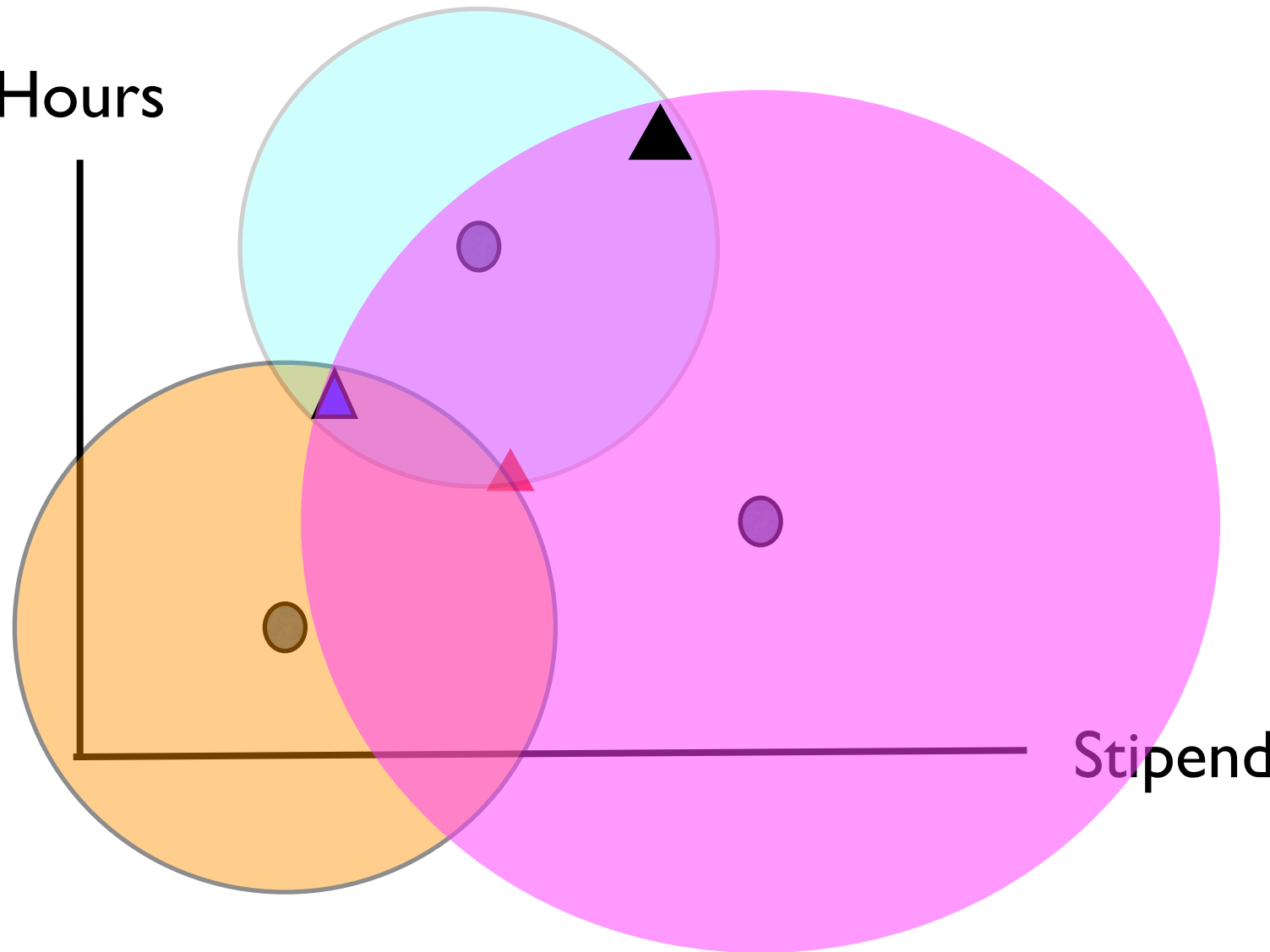
Hours



Stipend

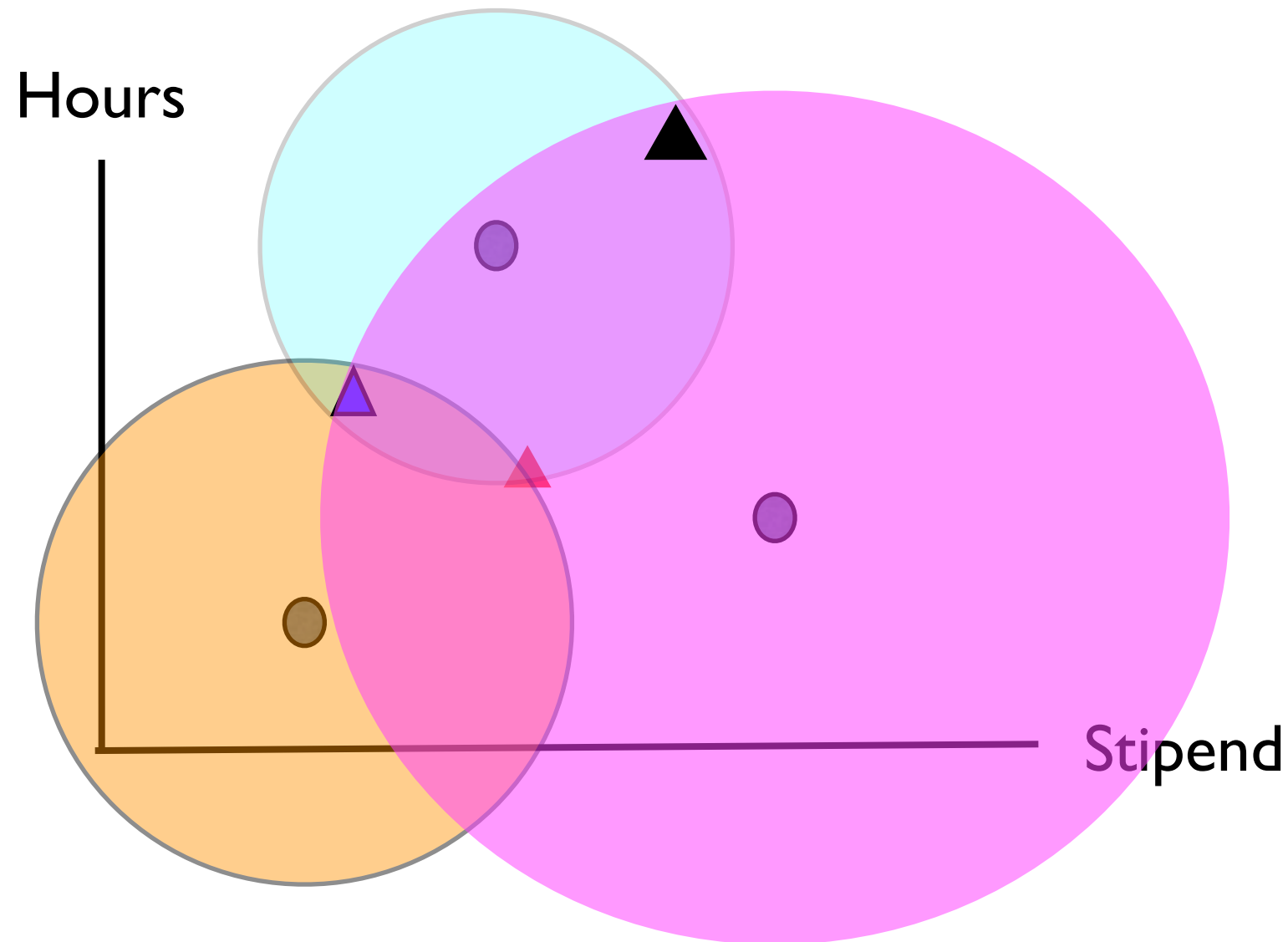
Core?

Hours



Stipend

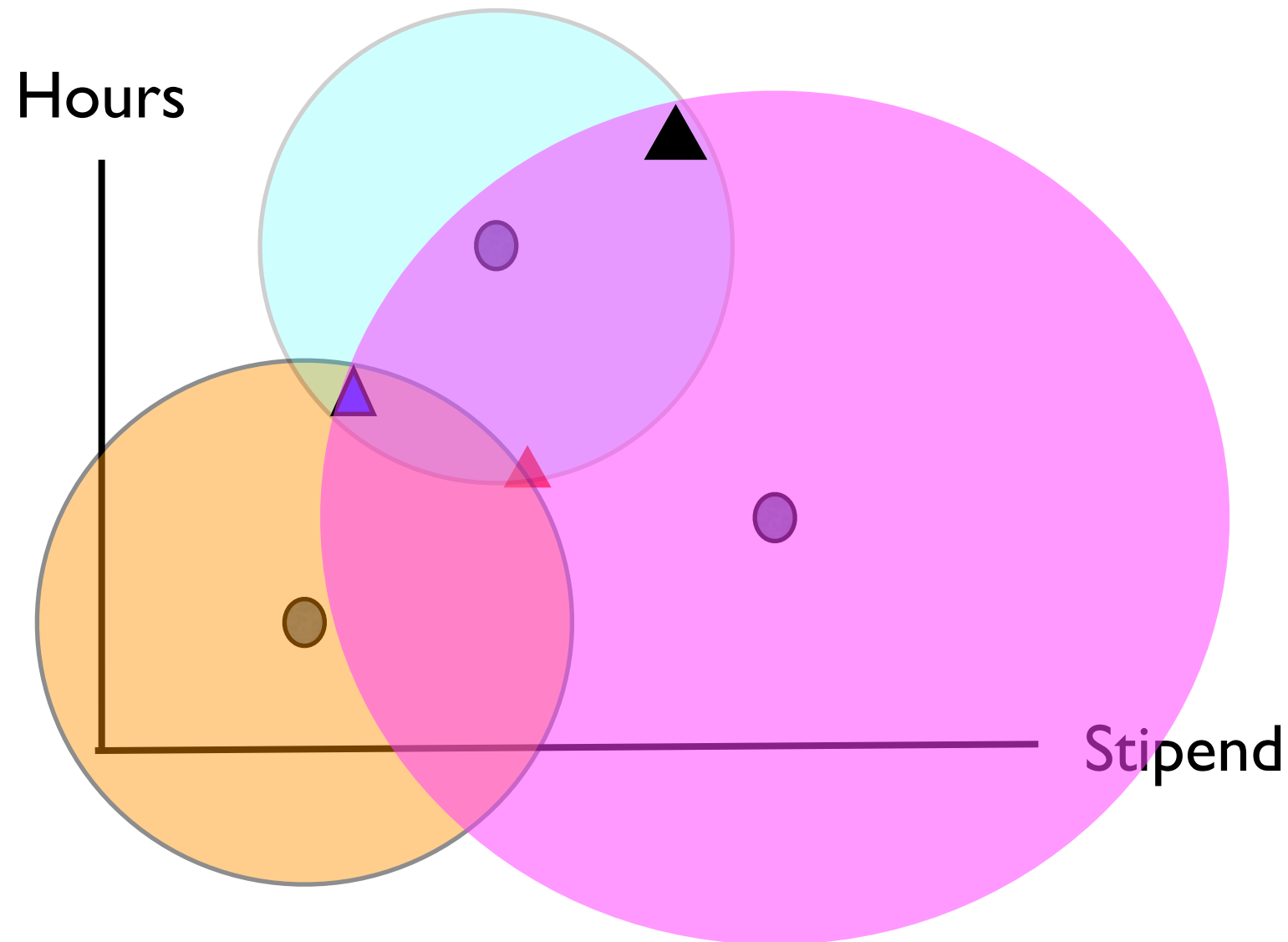
Core?    Here the core is empty



Core?

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McKelvey:

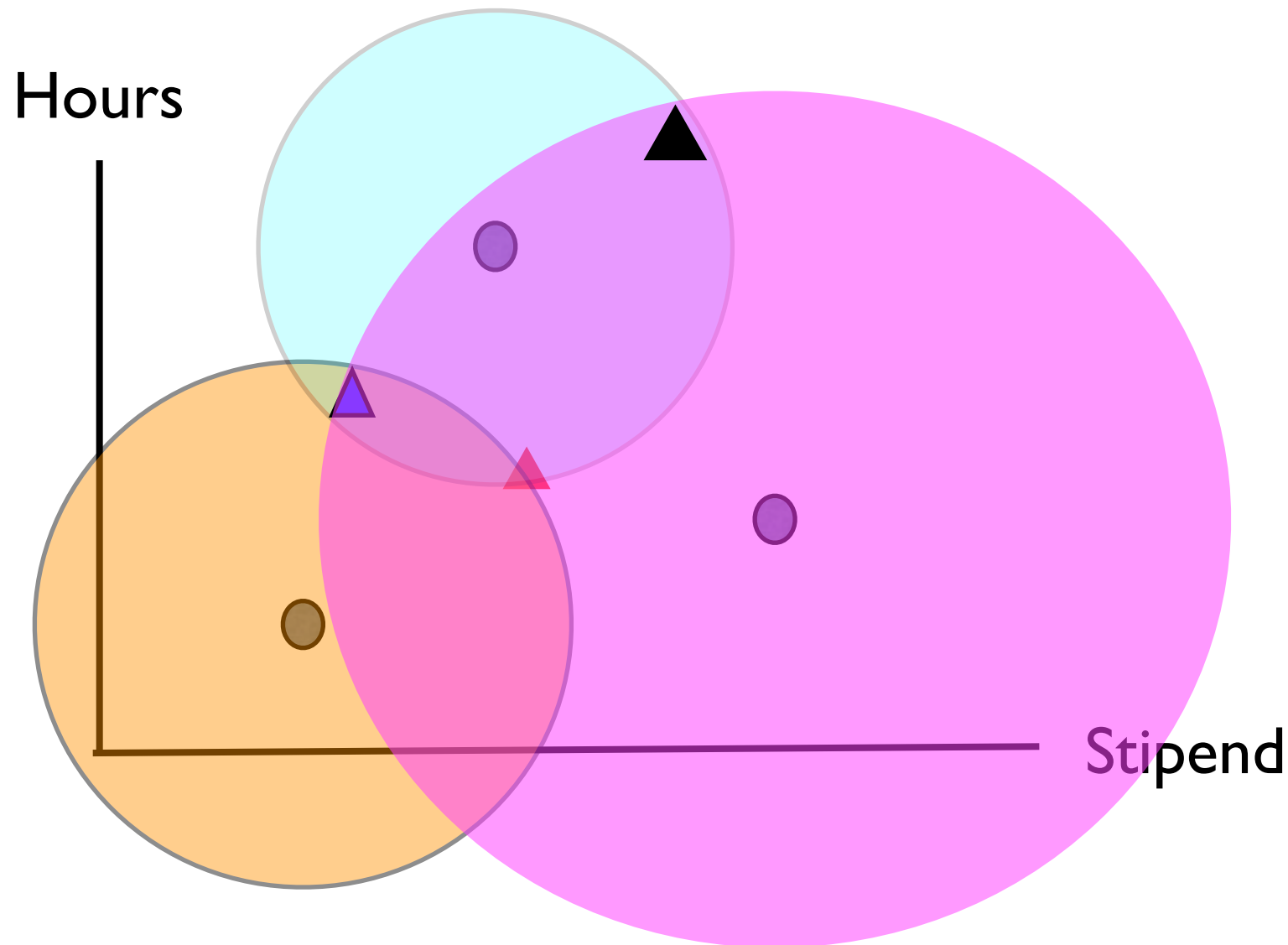


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Here the core is empty

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For majority rule, any number of voters, empty core, select a starting and ending point. There exists an agenda going from one to the other.





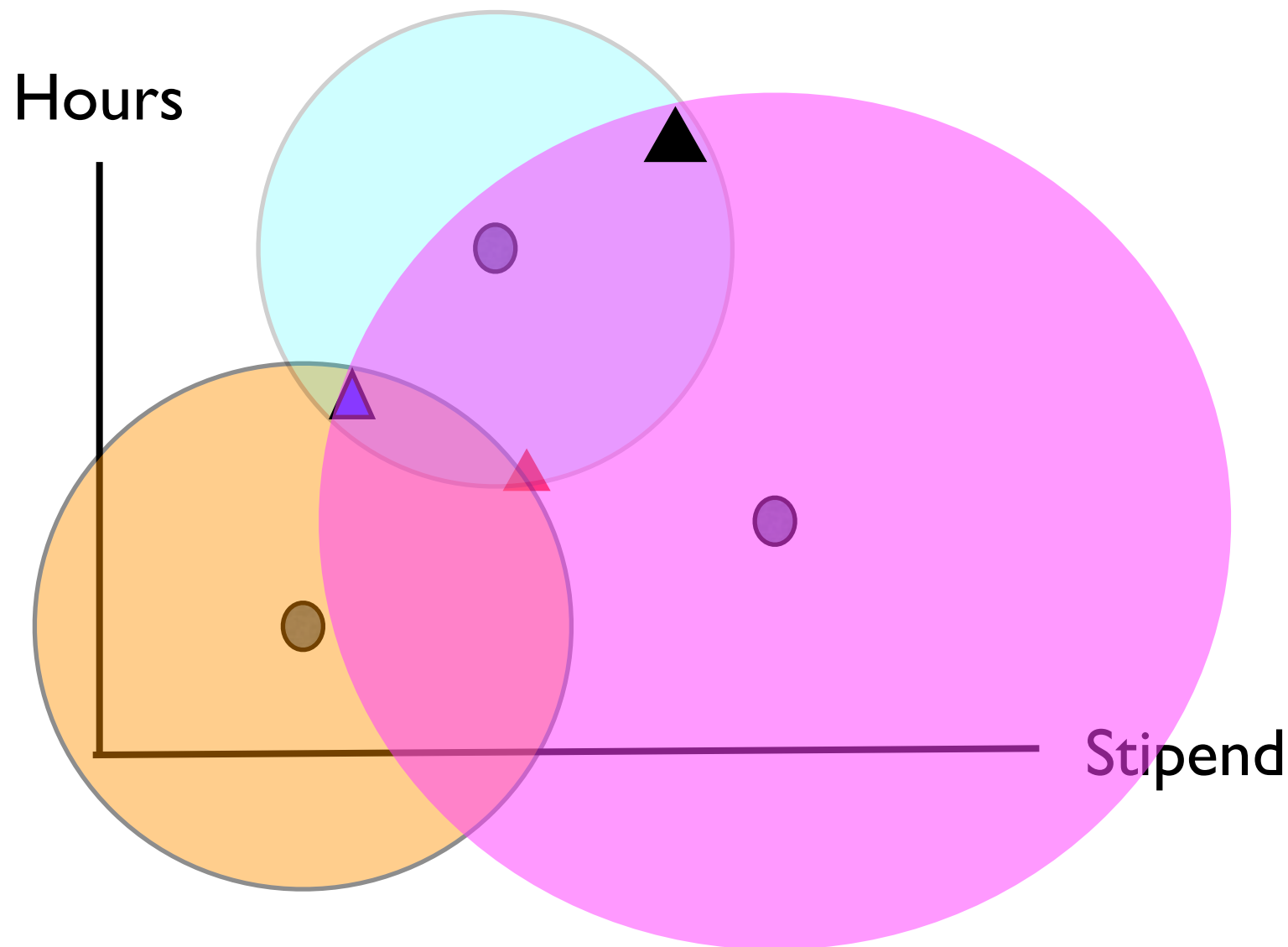
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Idea for proof: Differential topology

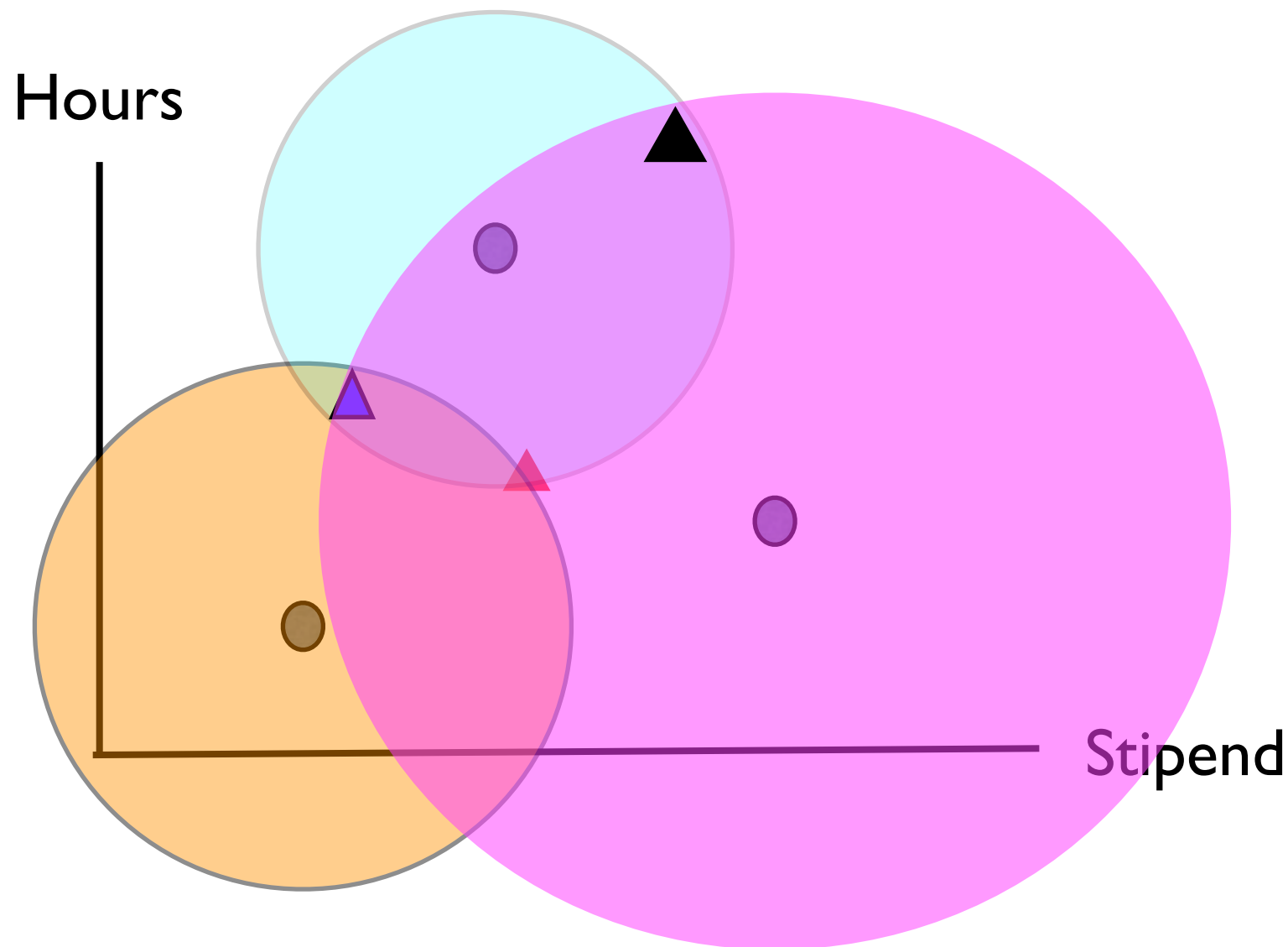


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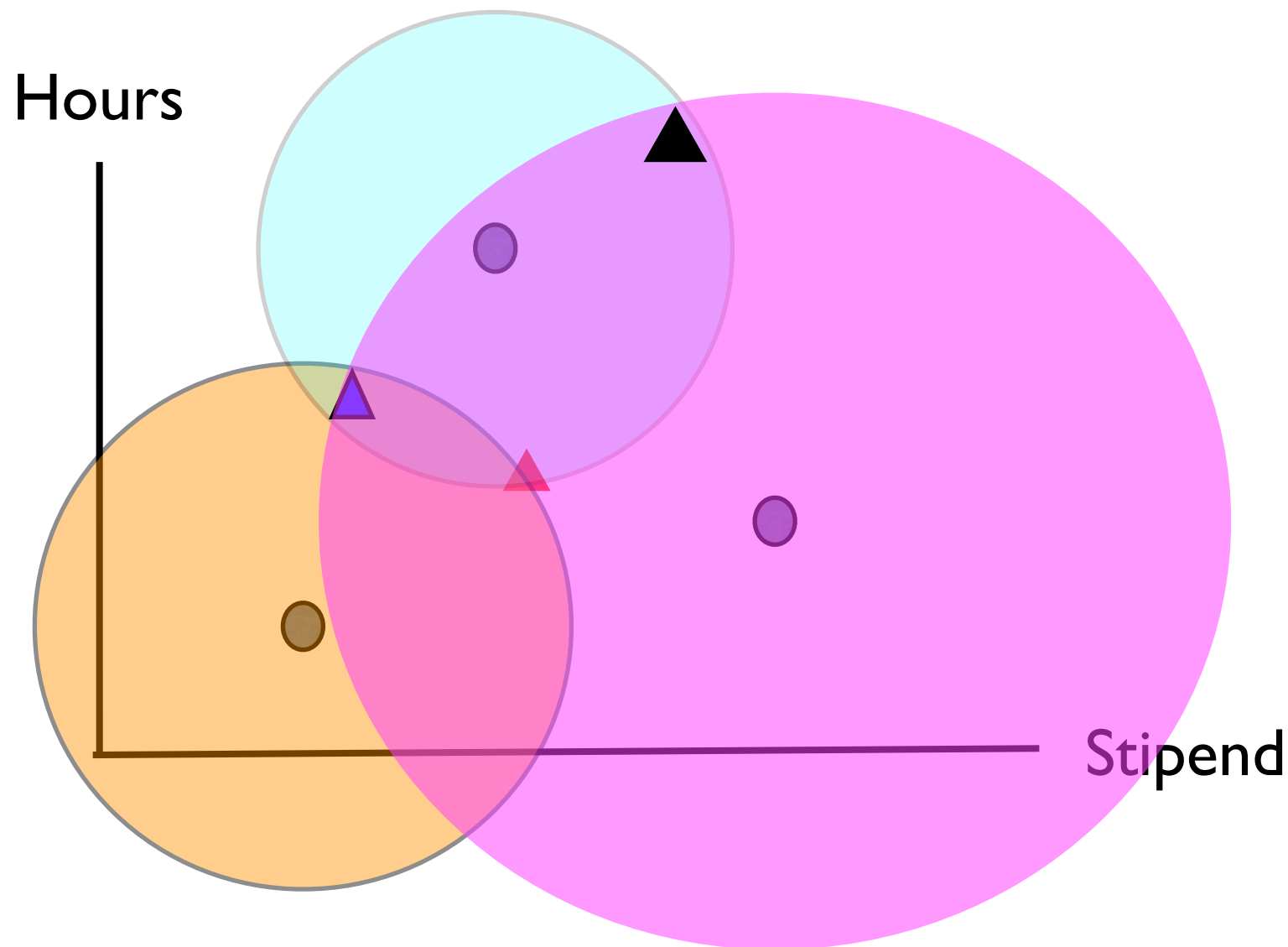


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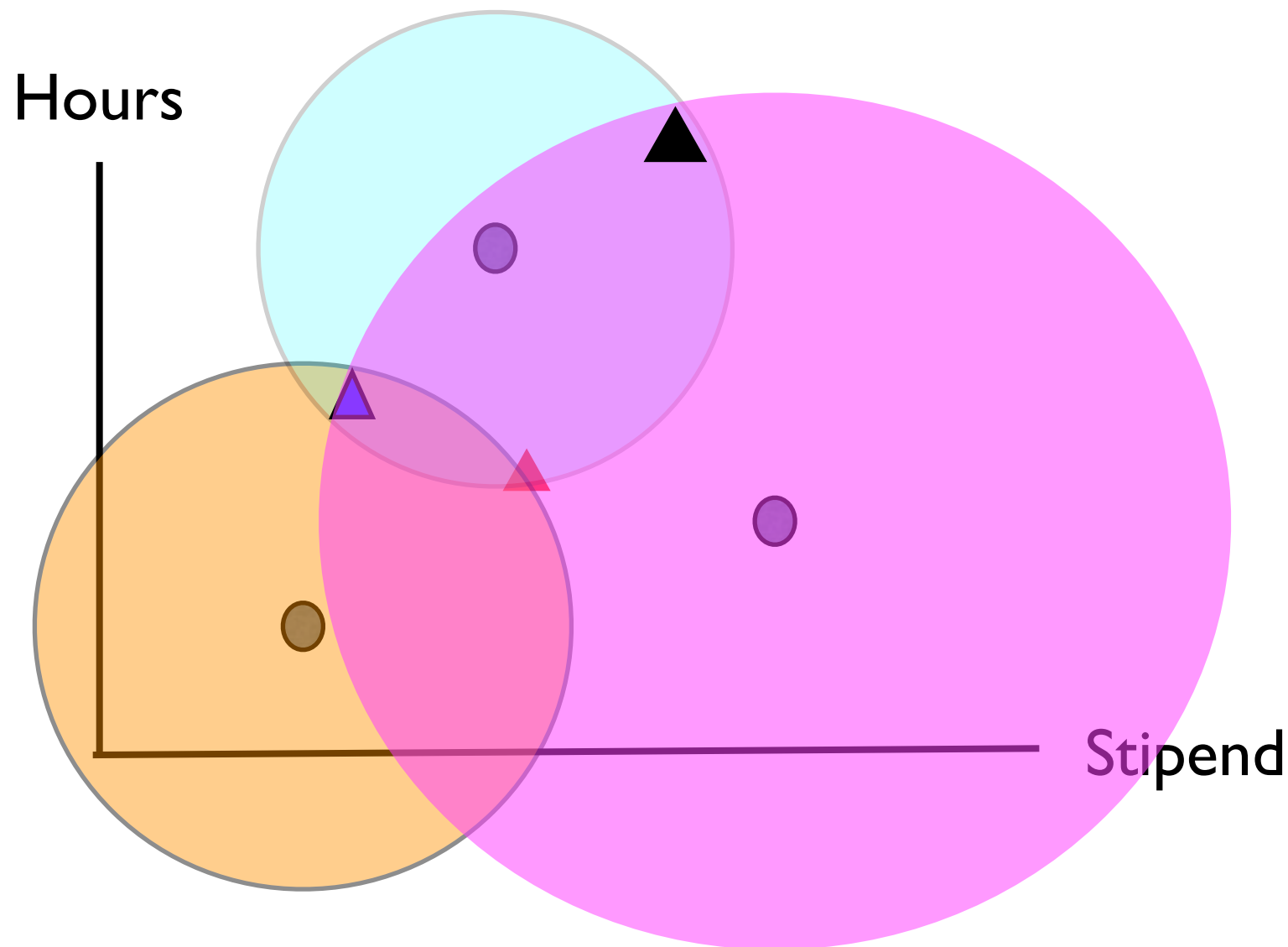
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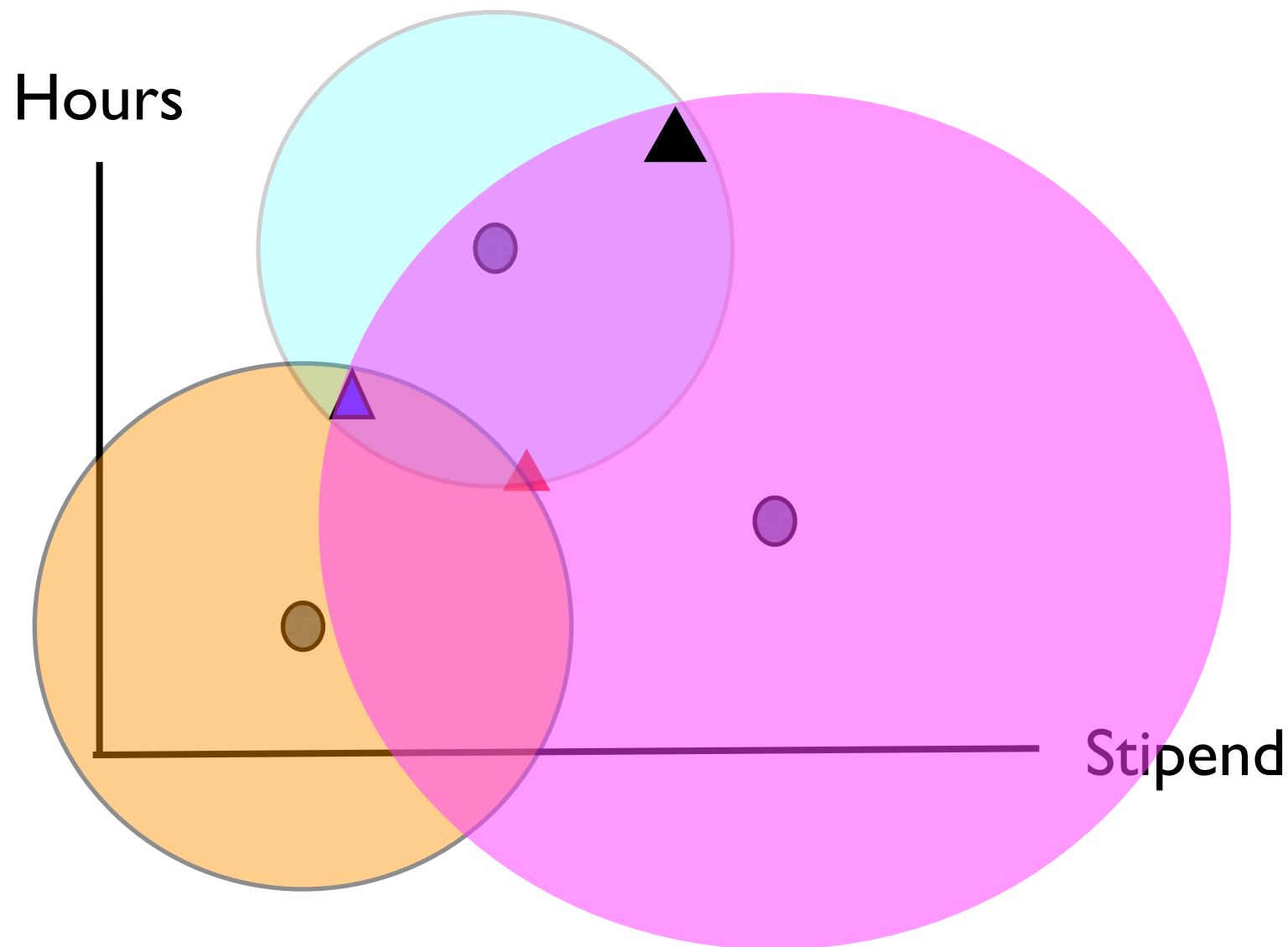
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Shifting coalitions

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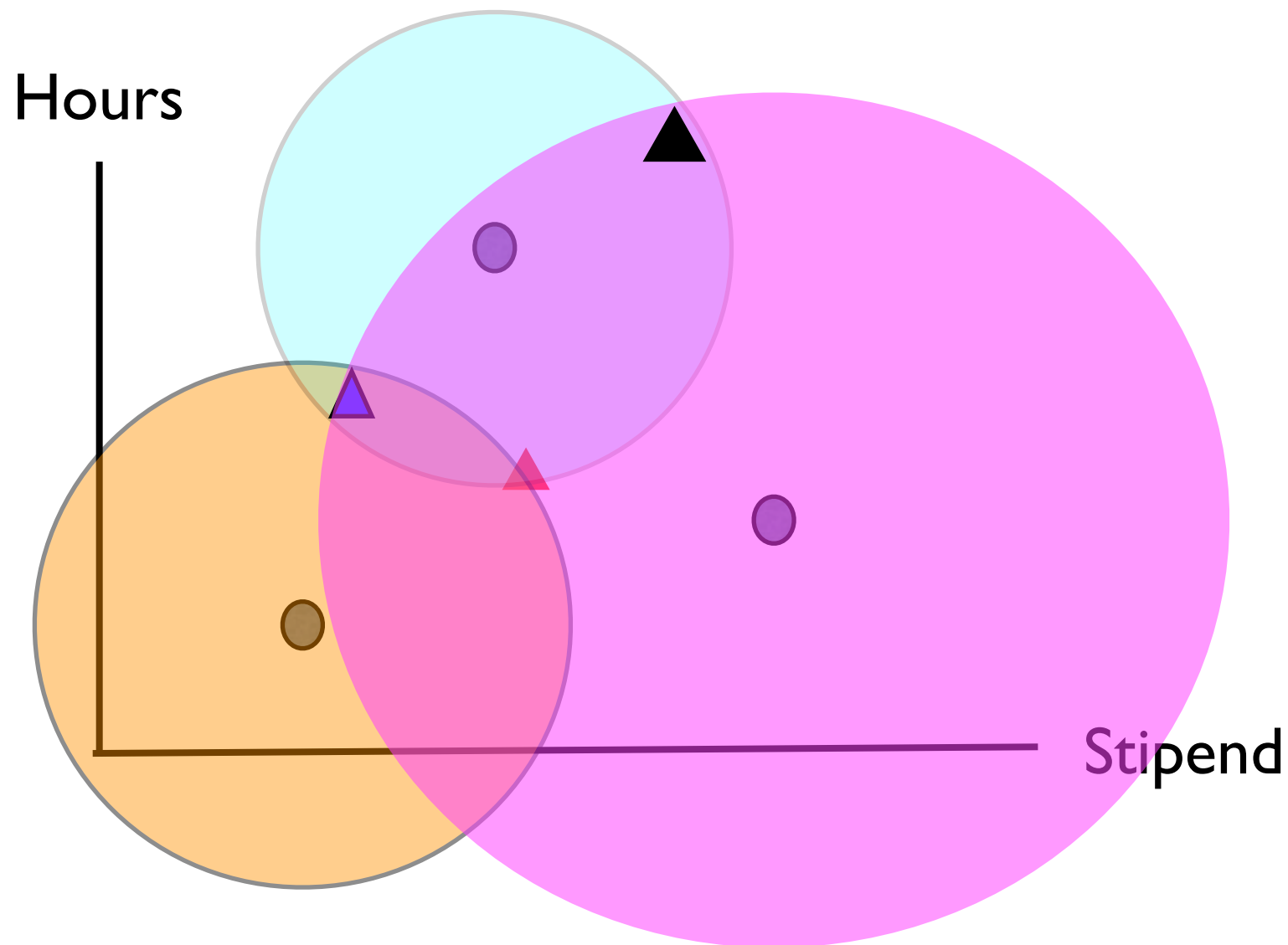
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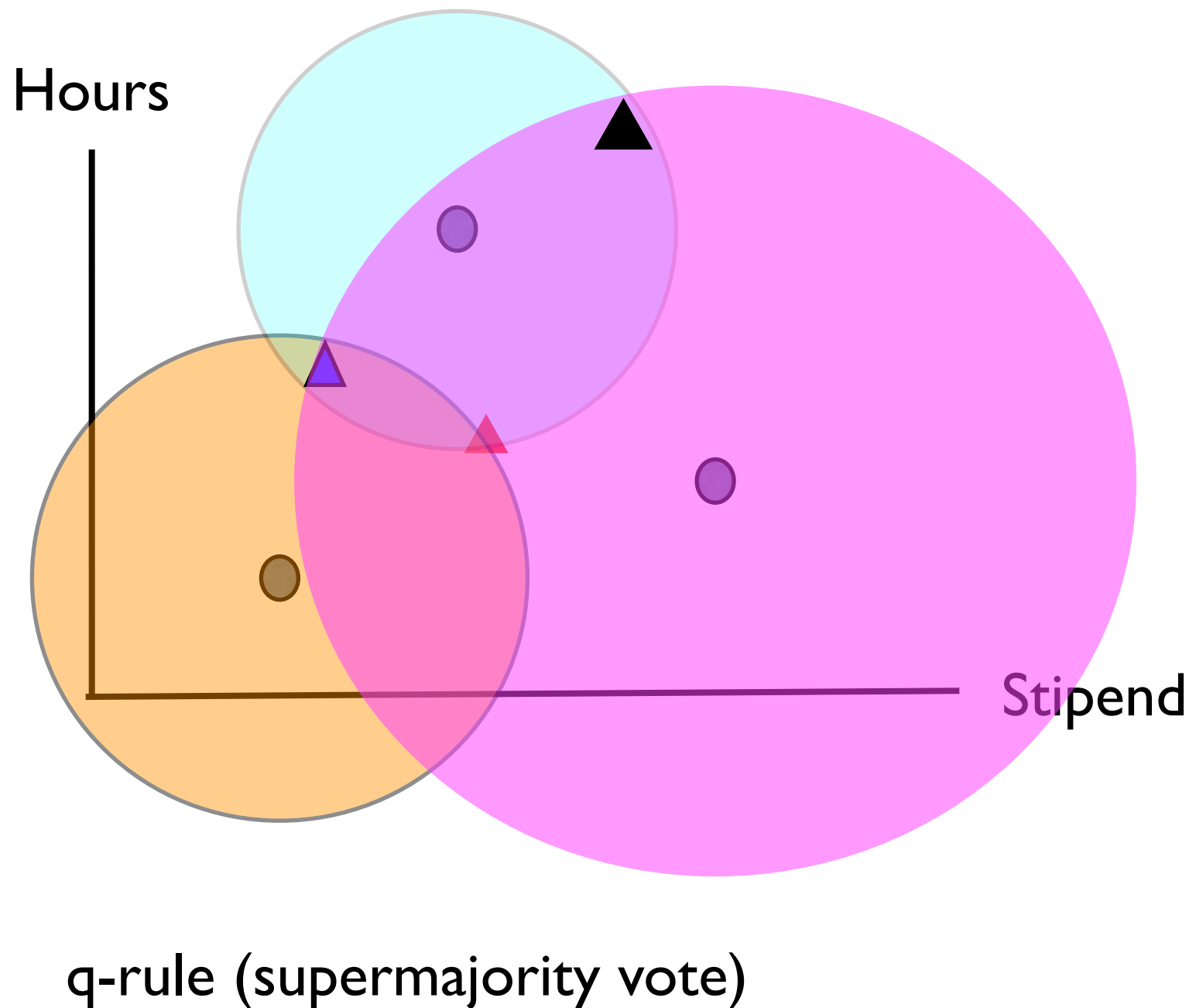
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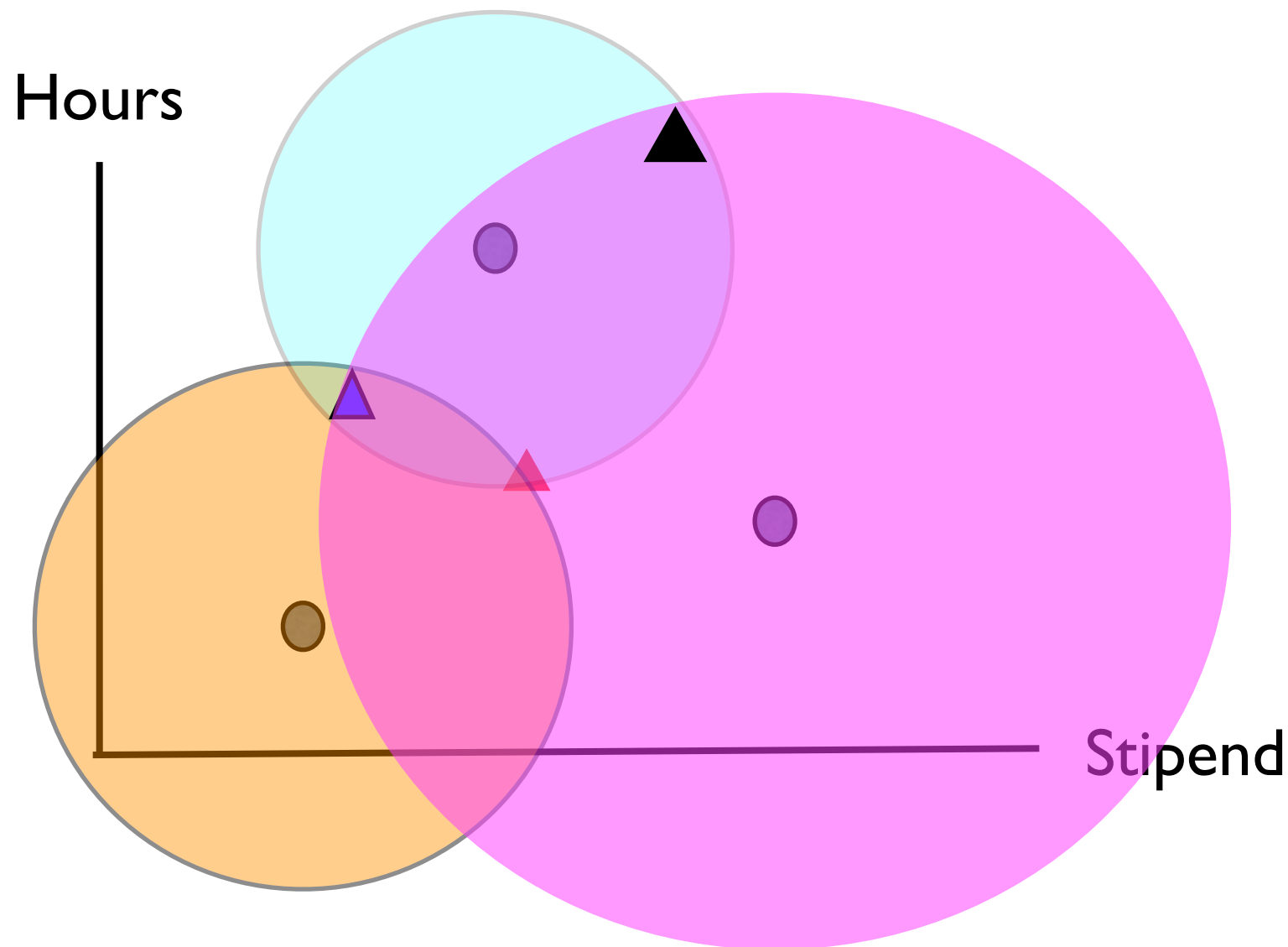
NSF, engineering, with  
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q-rule (supermajority vote)  
to win, need at least  $q$  (quota)  
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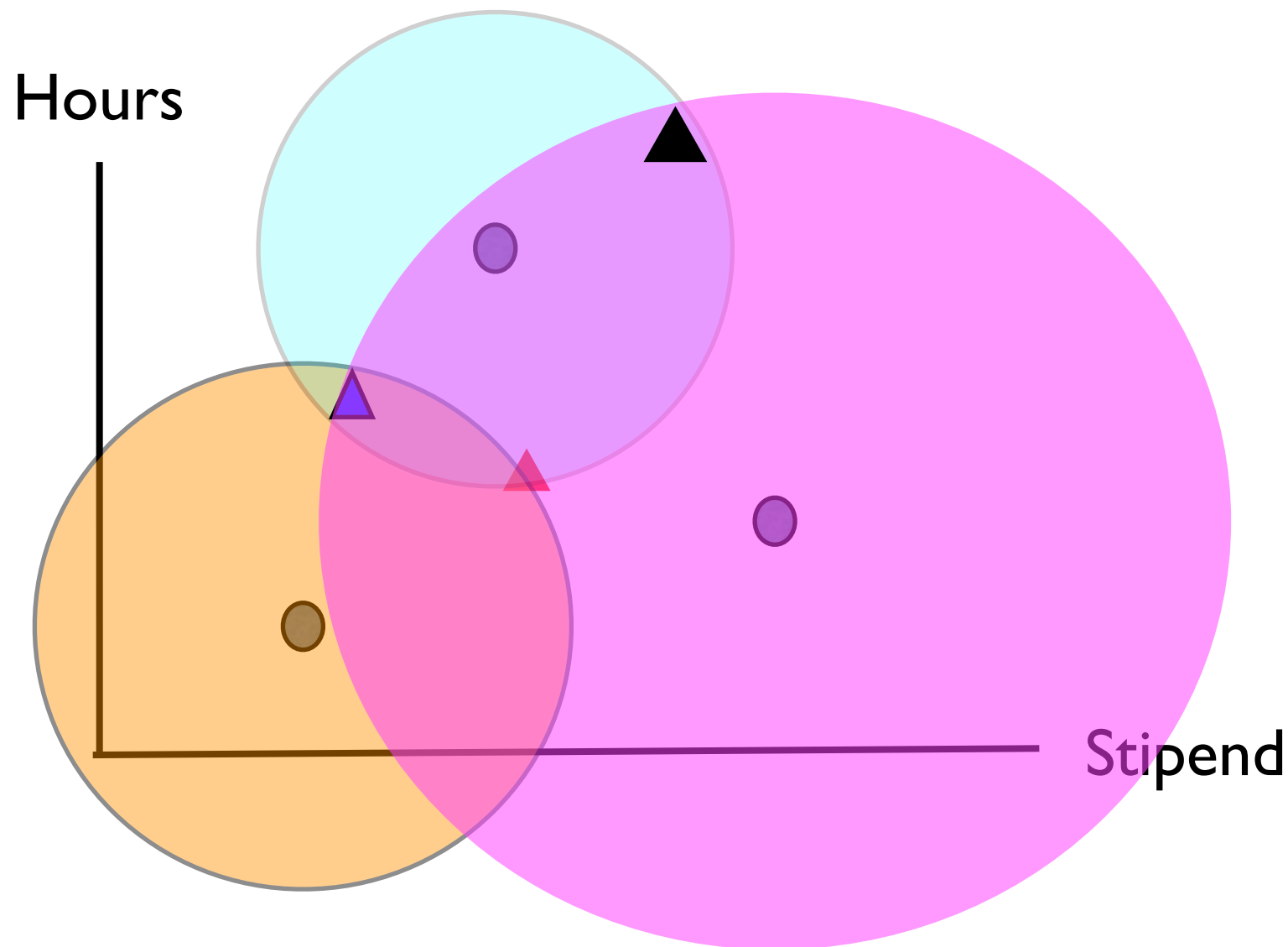
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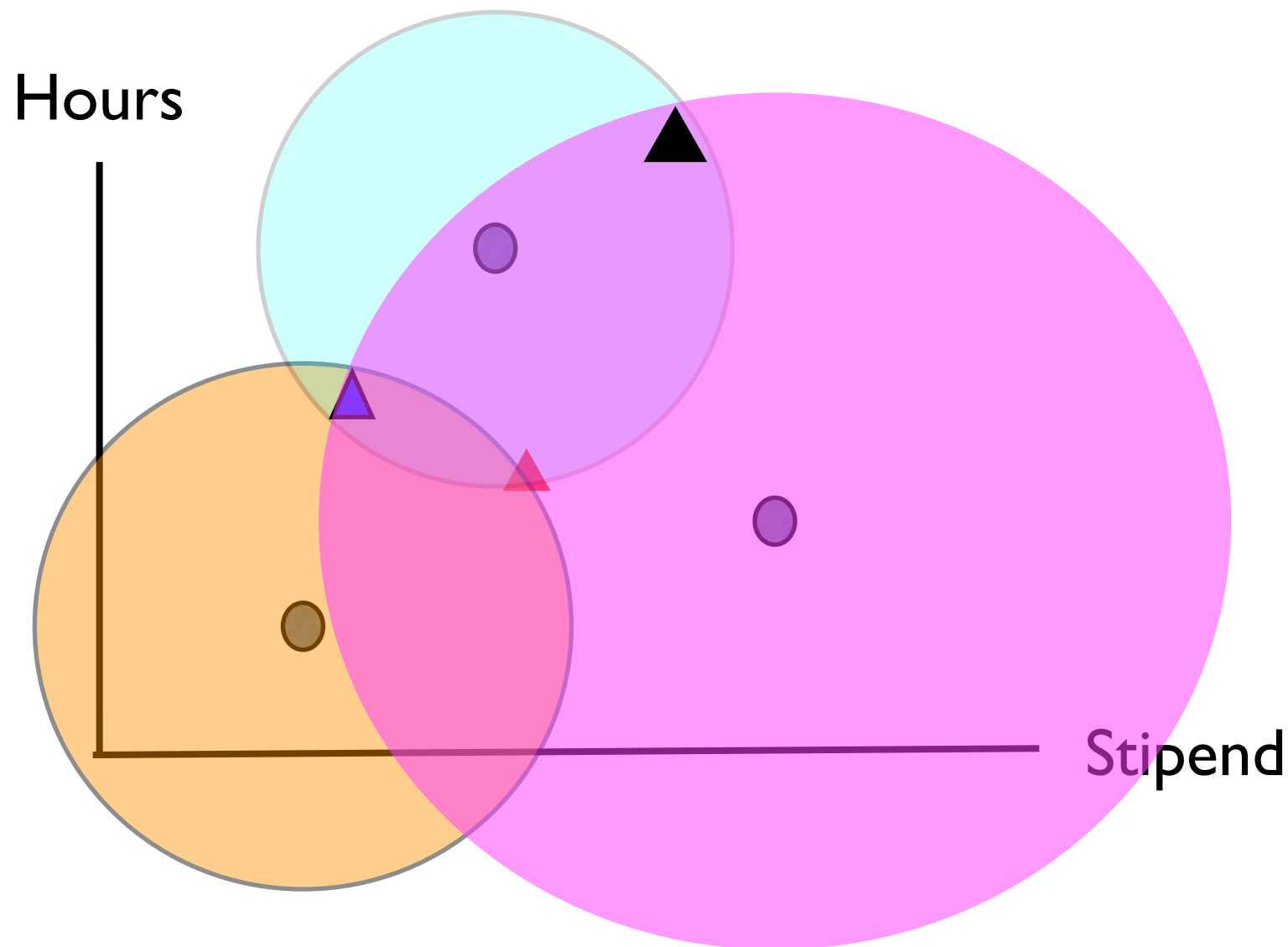
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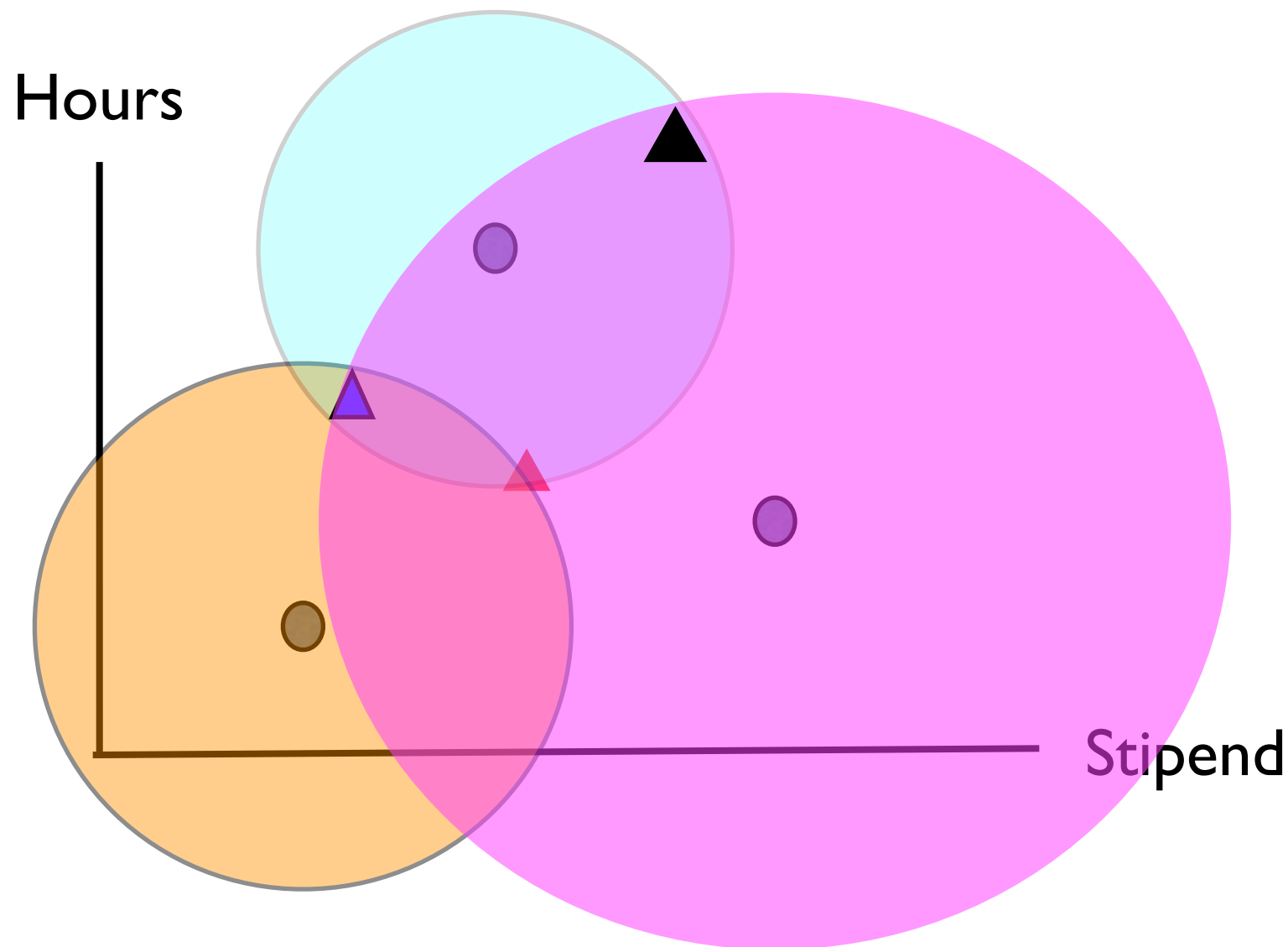
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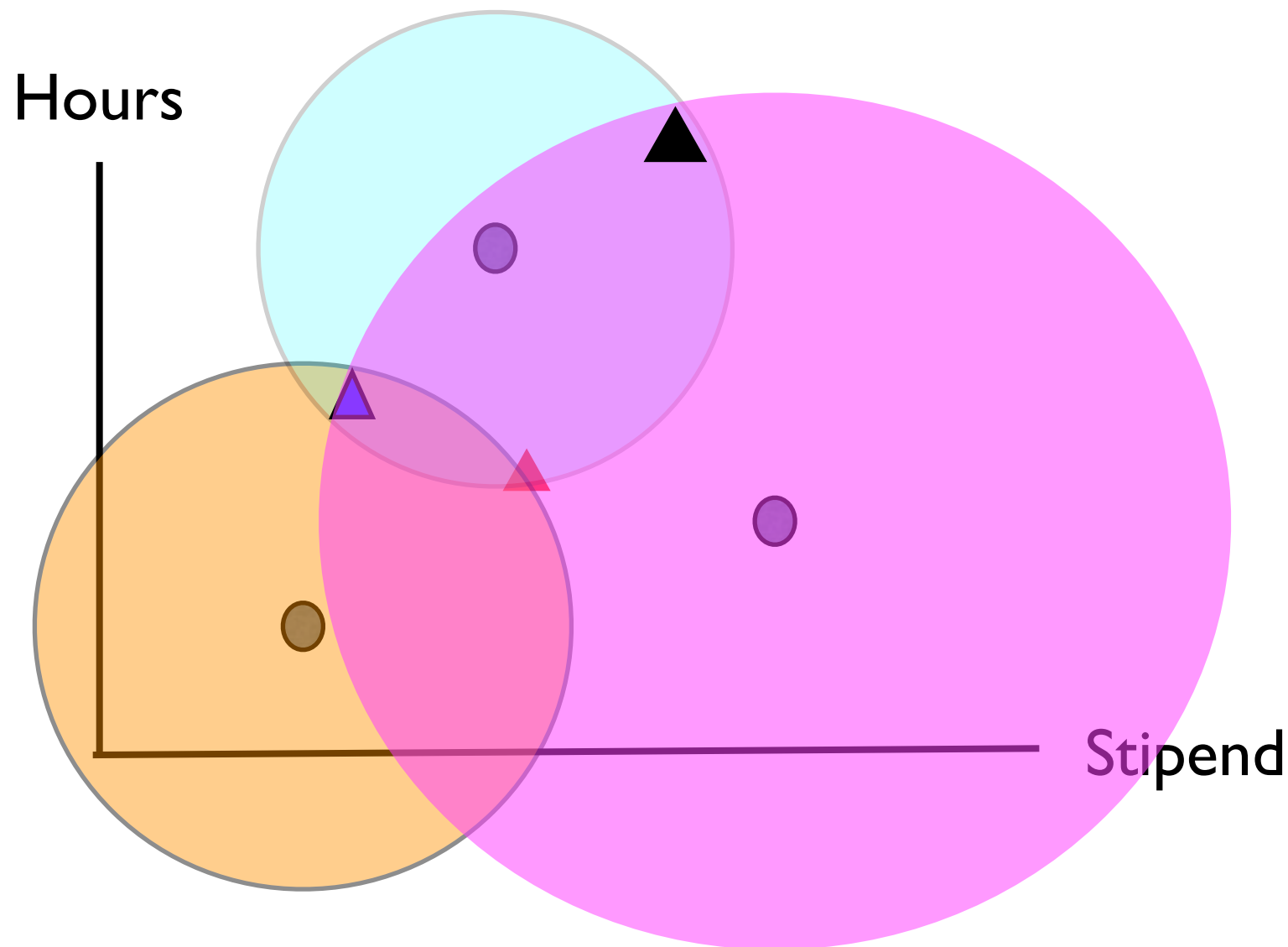
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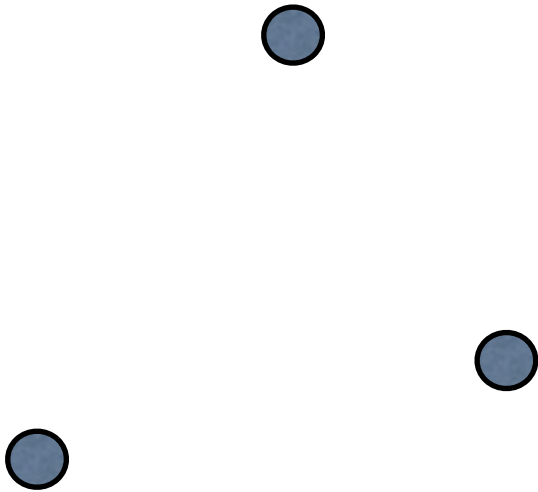
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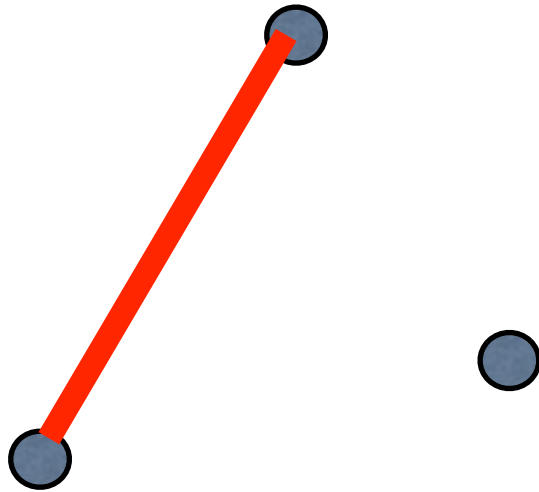
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Idea of proof:  
symmetry group orbits of sets

# Tools and a question

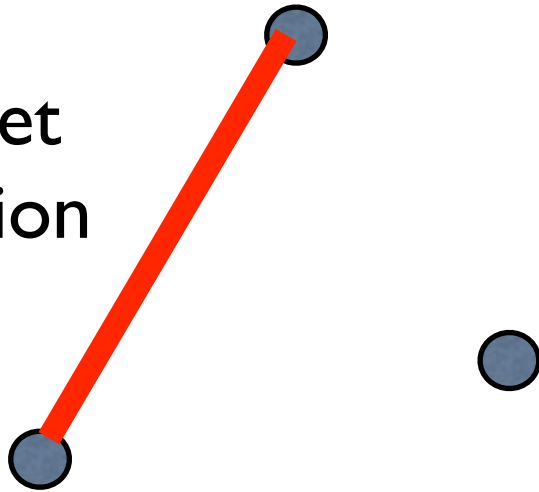


# Tools and a question



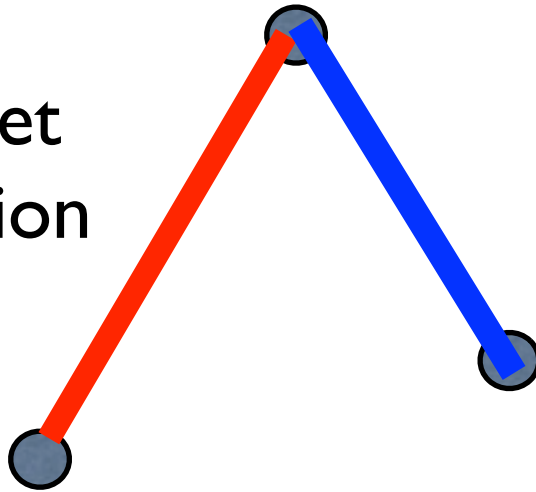
# Tools and a question

Pareto set  
for coalition



# Tools and a question

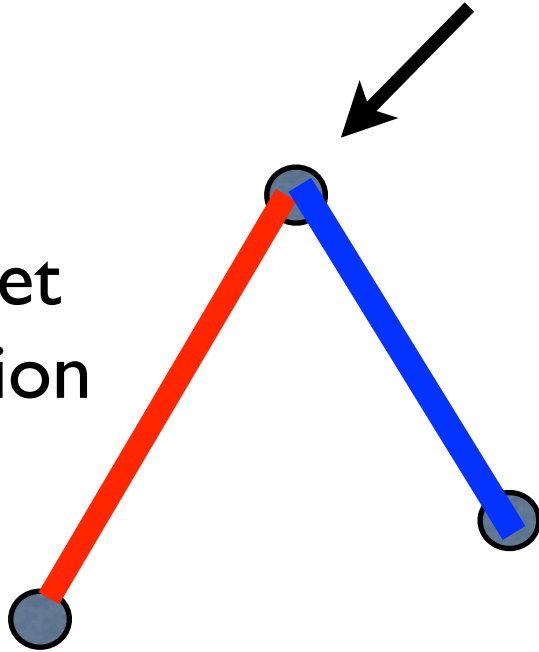
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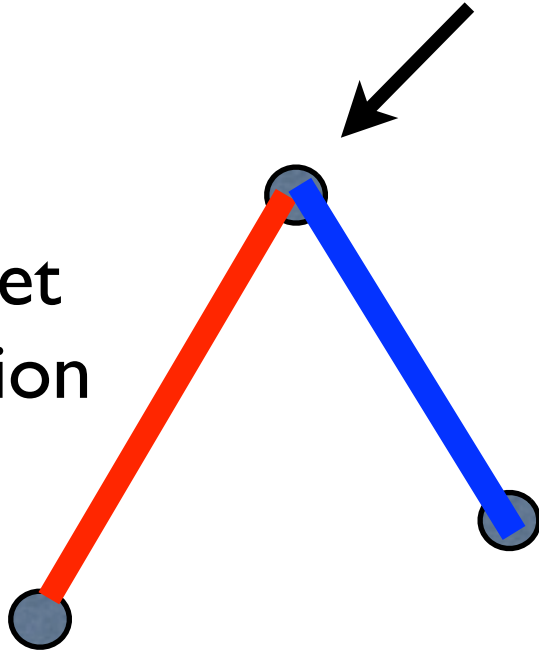
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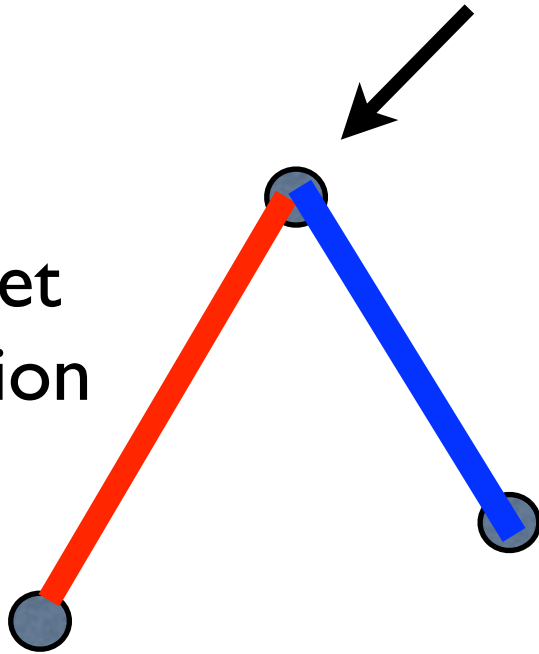
Sen. Jeffords, VT

Pareto set  
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## Tools and a question

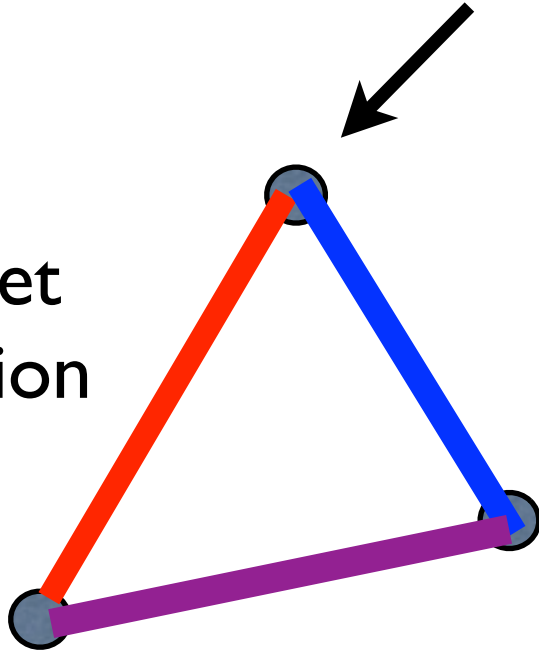
Pareto set  
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Sen. Jeffords, VT  
Core is the intersection of  
Pareto sets

## Tools and a question

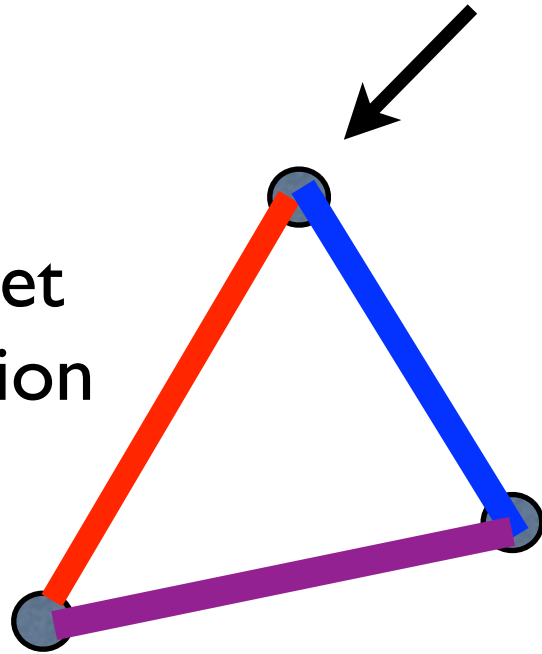
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Pareto set  
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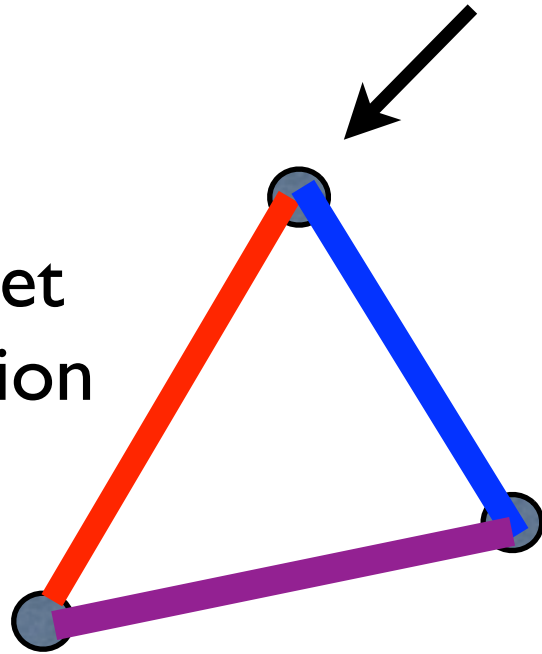


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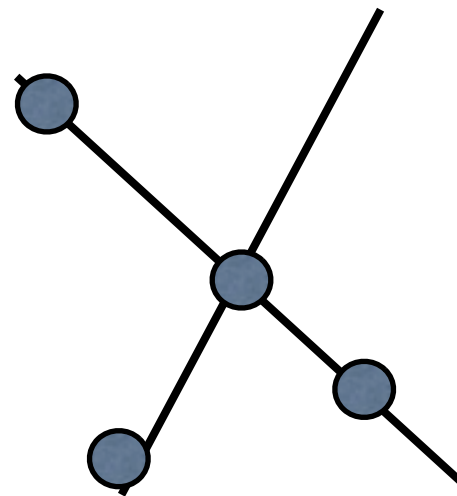
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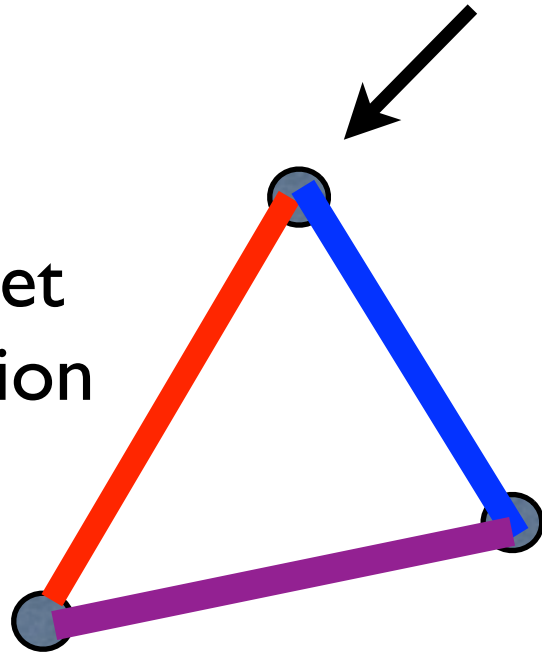
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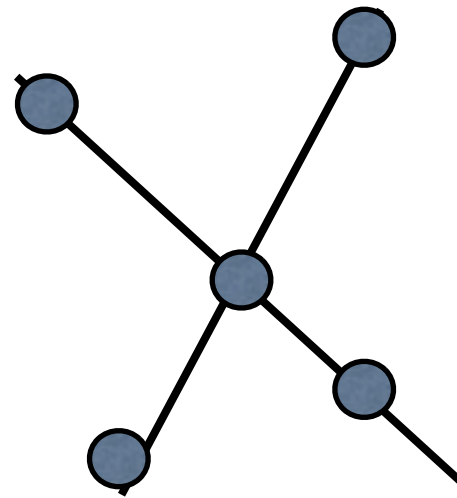
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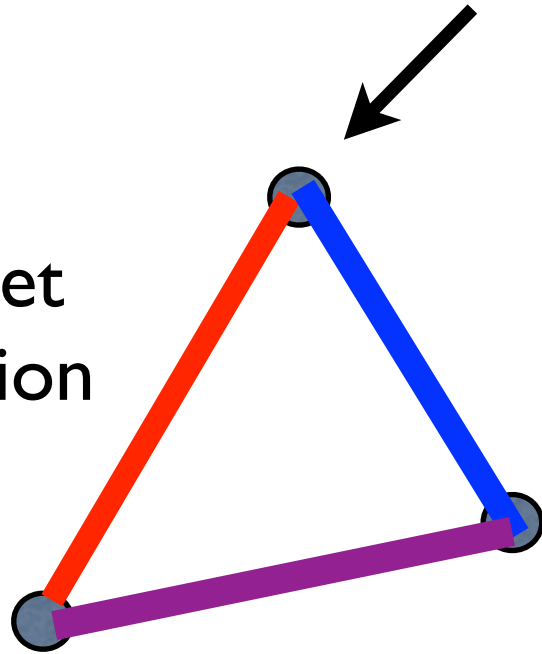
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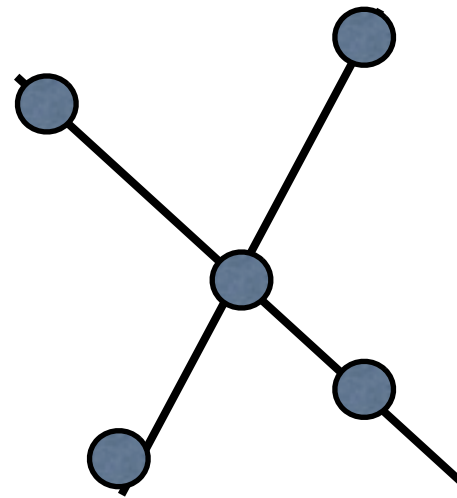
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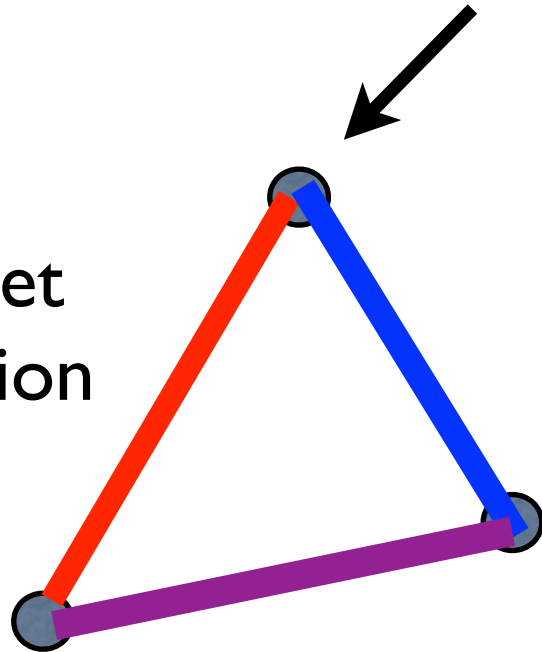


Pareto sets:



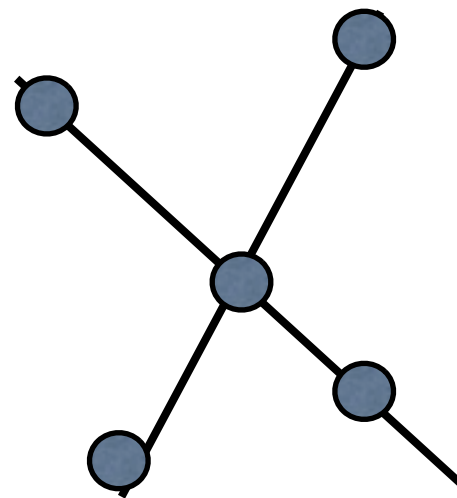
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Pareto set  
for coalition



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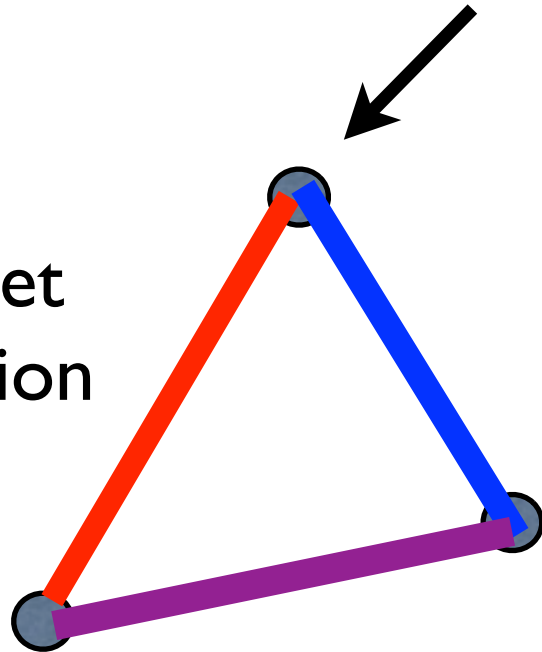
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Pareto sets:  
winning coalition has three  
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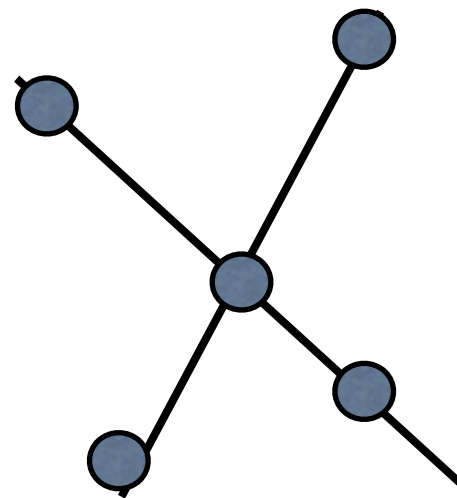
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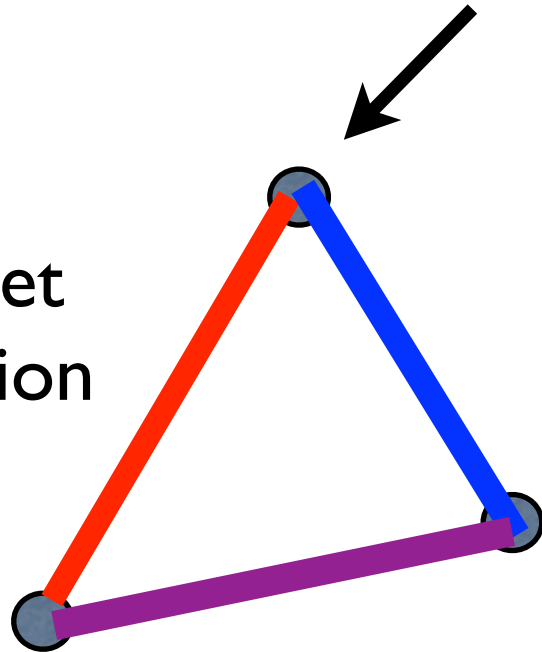
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Pareto sets:  
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triangles

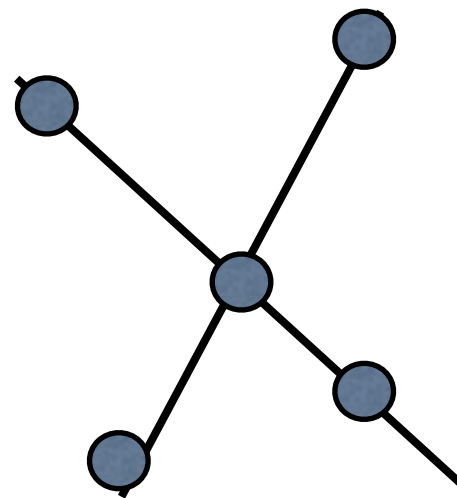
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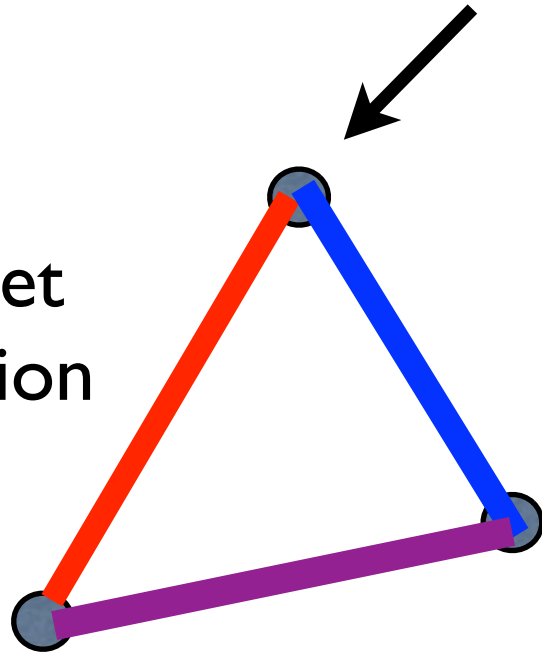
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Pareto sets:  
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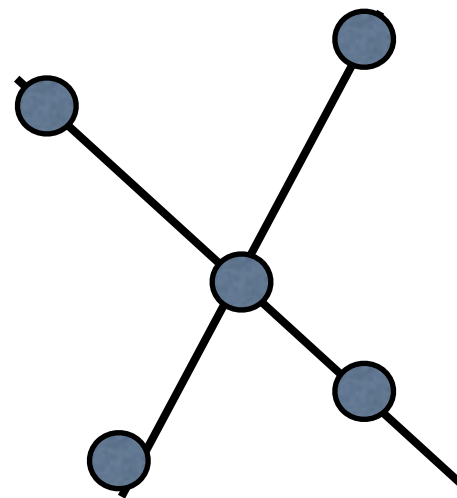
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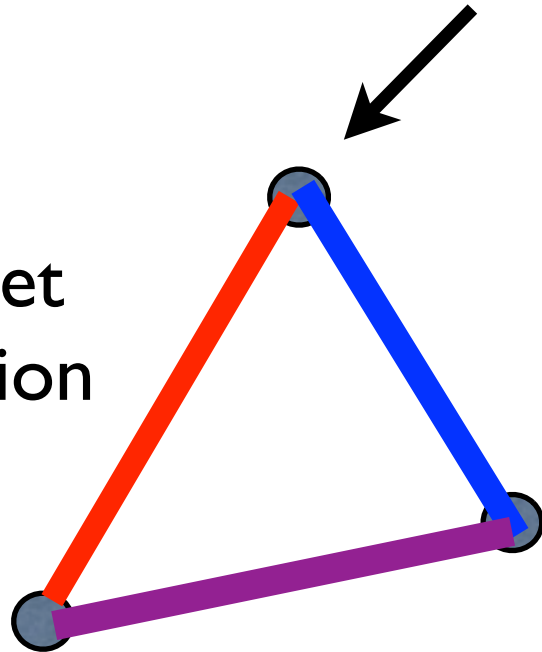
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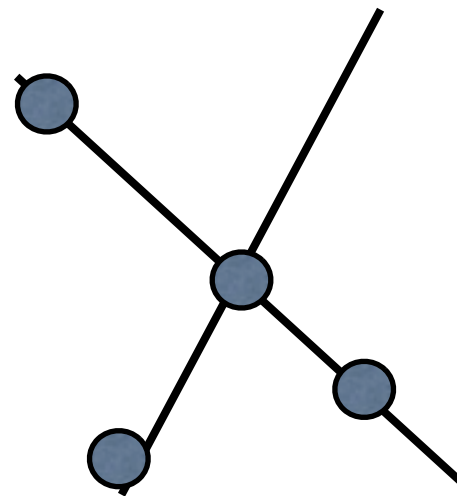
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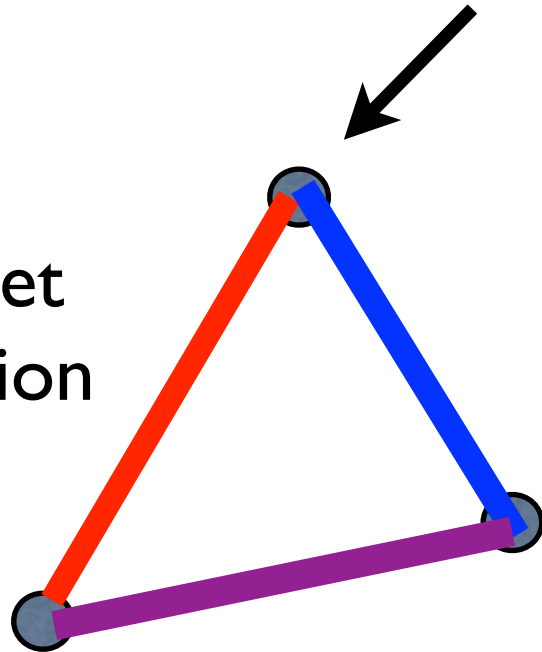
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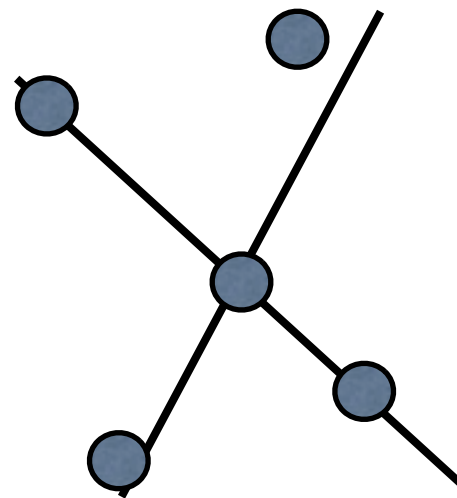
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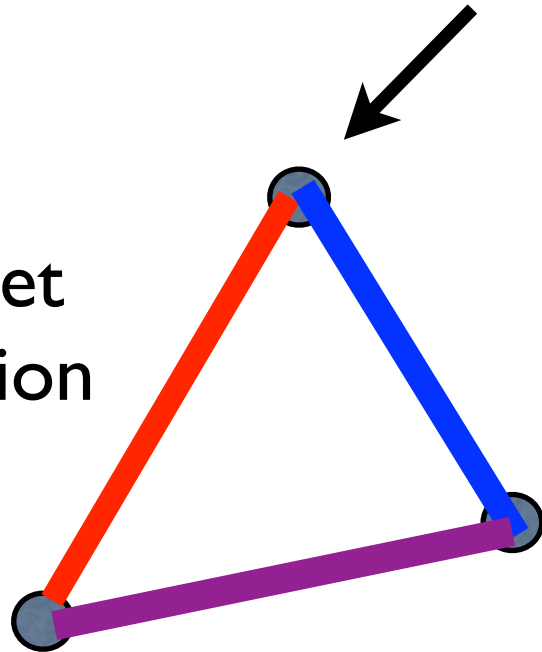
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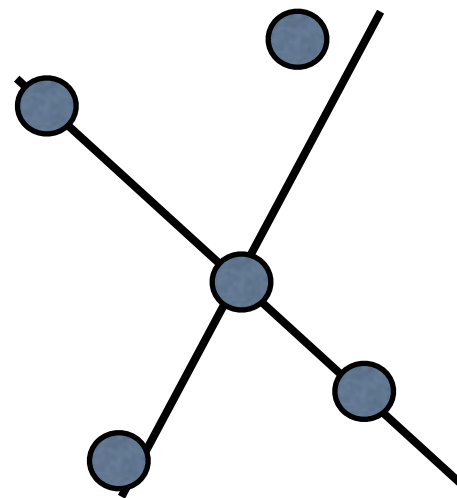
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Core is empty!

Now, triangles do not  
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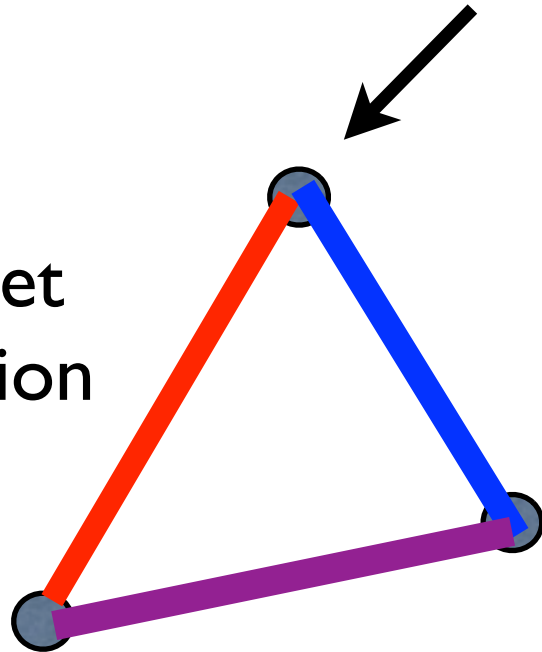
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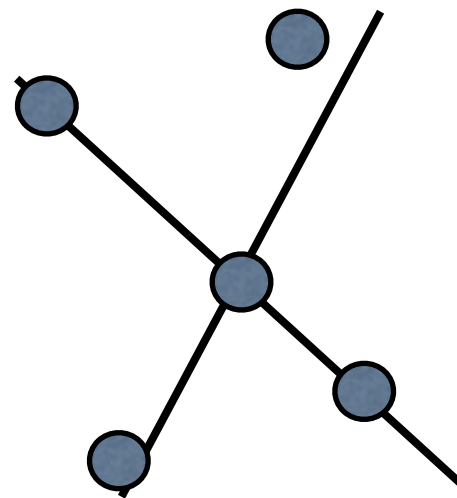
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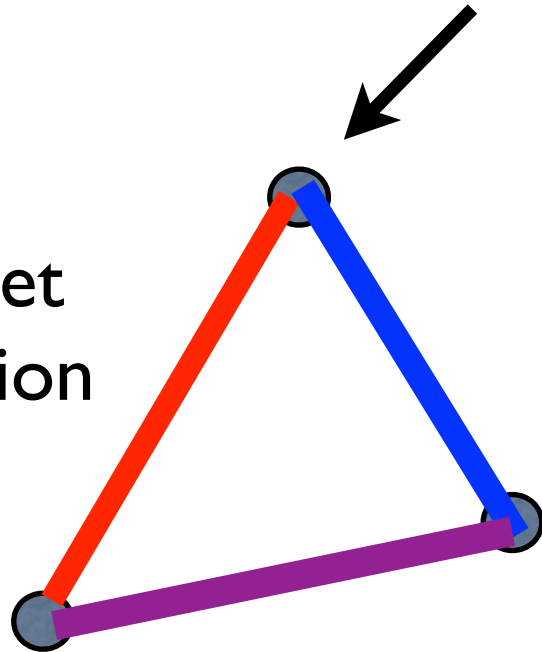


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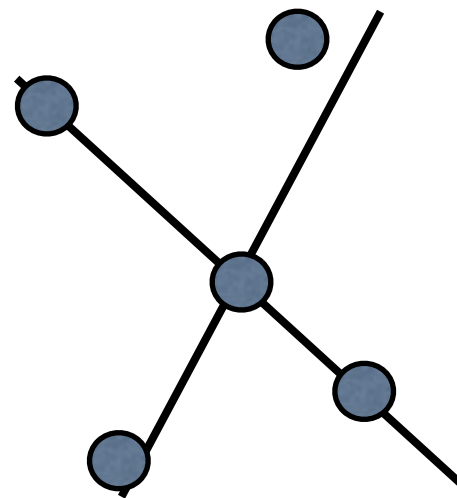
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For core to be meaningful  
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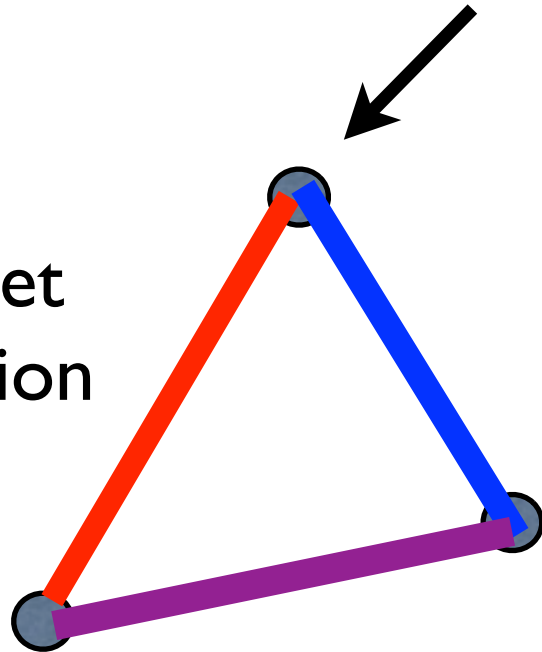
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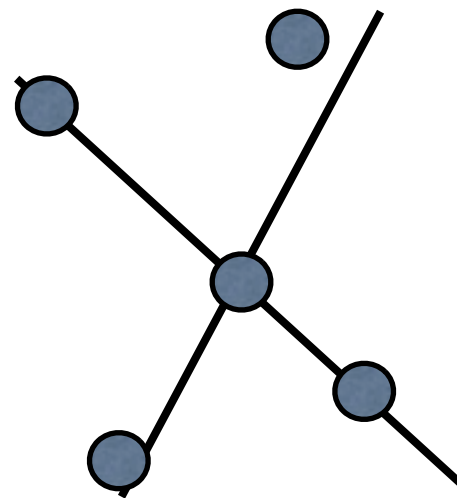
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**structurally stable**

Sen. Jeffords, VT

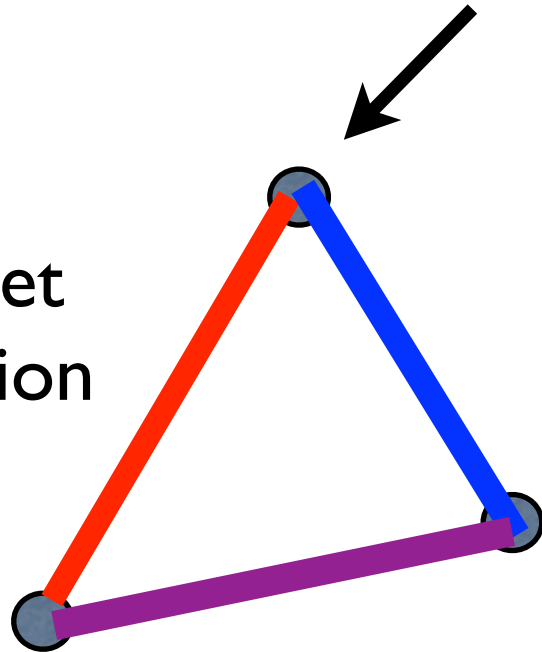
Core is the intersection of  
Pareto sets



Pareto sets:  
winning coalition has three  
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triangles  
All intersect at midpoint  
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## Tools and a question

Pareto set  
for coalition



Core is empty!

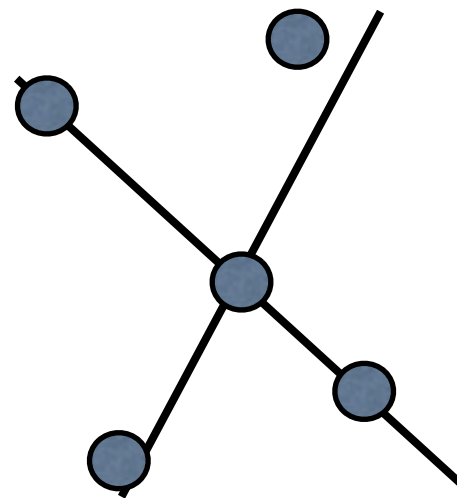
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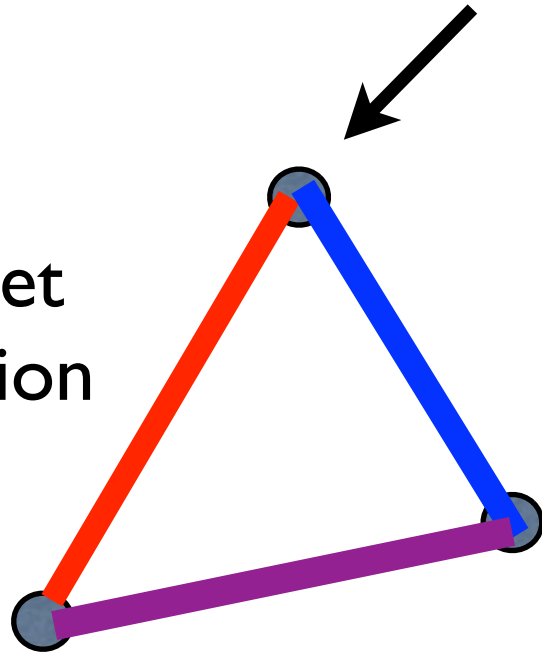


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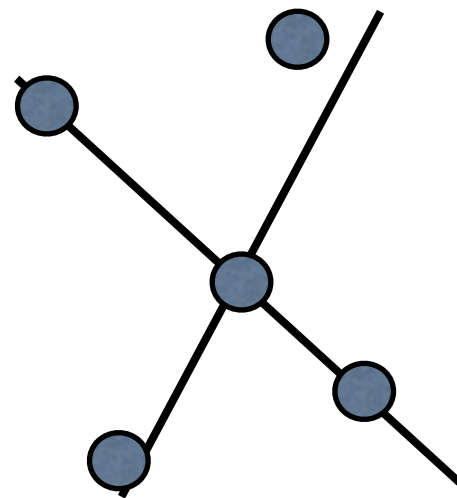
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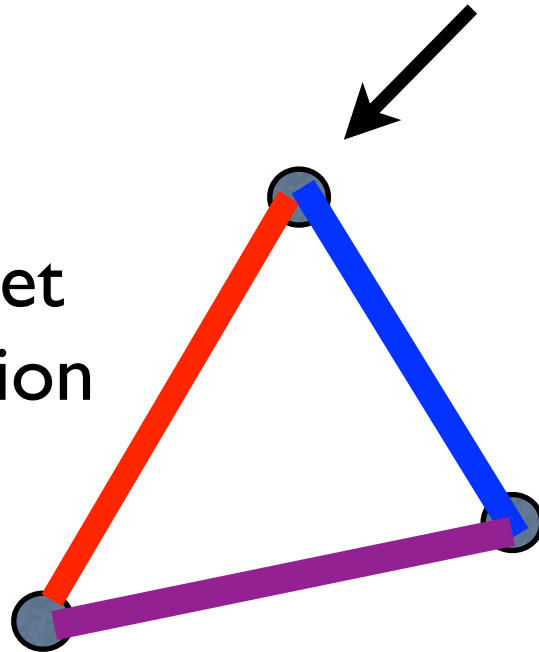
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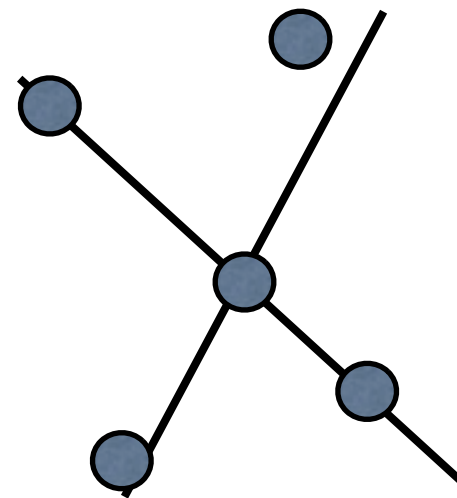
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Early attempts by McKelvey and  
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$\frac{2}{3}$  rule So, if 120 voters, need 80 to win, 40 on losing side.

$k = 2q - n = 2(80) - 120 = 40$ , or the number needed to change outcome.

For a price .....

For a price .....

I will come to your group before your next election.

For a price .....

I will come to your group before your next election.  
You tell me who you want to win.

For a price .....

I will come to your group before your next election.  
You tell me who you want to win.  
After talking to everyone in your group, I will design a  
“fair” election rule, which includes all candidates.



For a price .....

I will come to your group before your next election.  
You tell me who you want to win.  
After talking to everyone in your group, I will design a  
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Your candidate will win!

For a price .....

10  $A > B > C > D > E > F$

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10 A>B>C>D>E>F      **Everyone** prefers C to D to E to F  
10 B>C>D>E>F>A  
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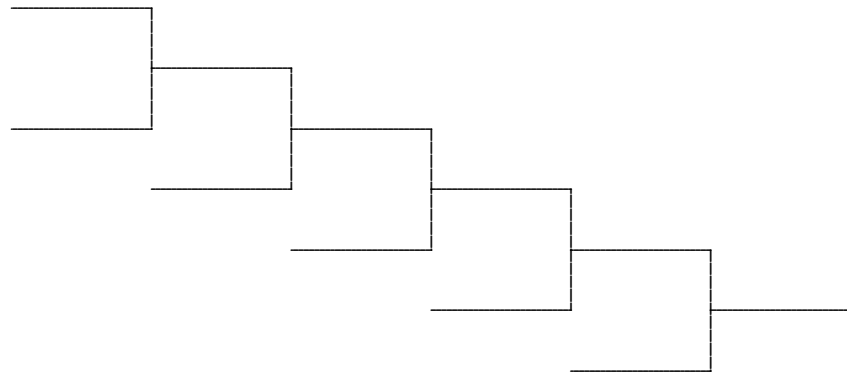
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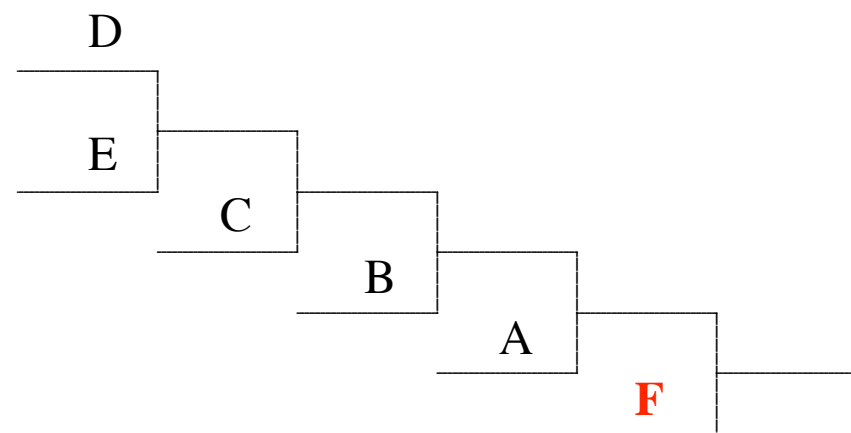
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10 A>B>C>D>E>**F**

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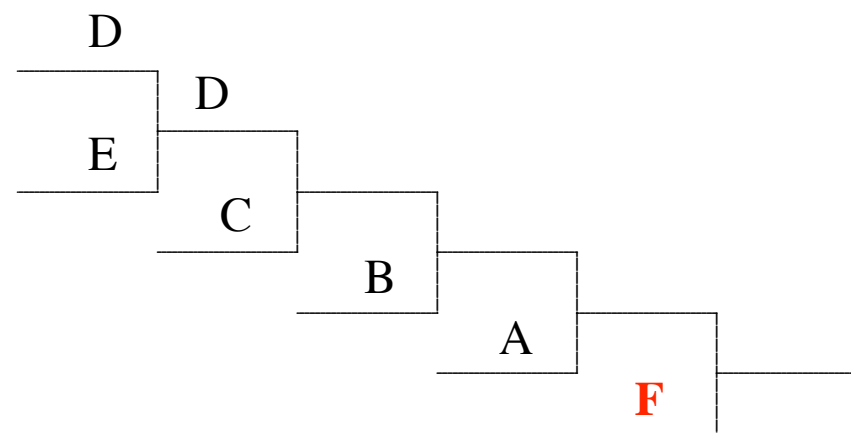
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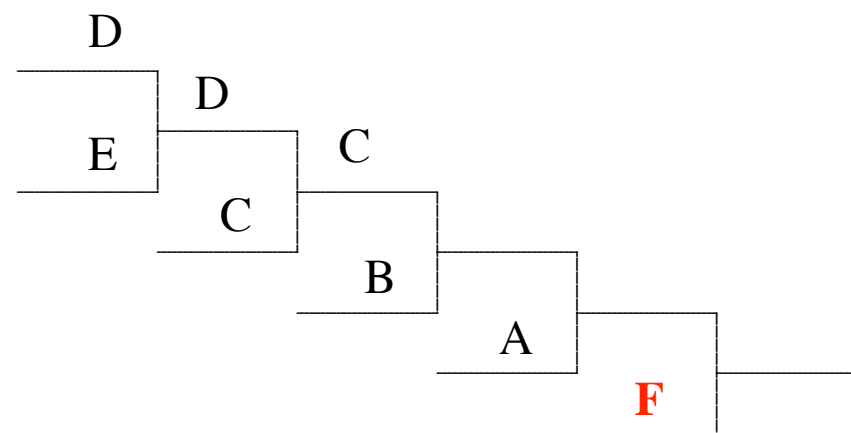
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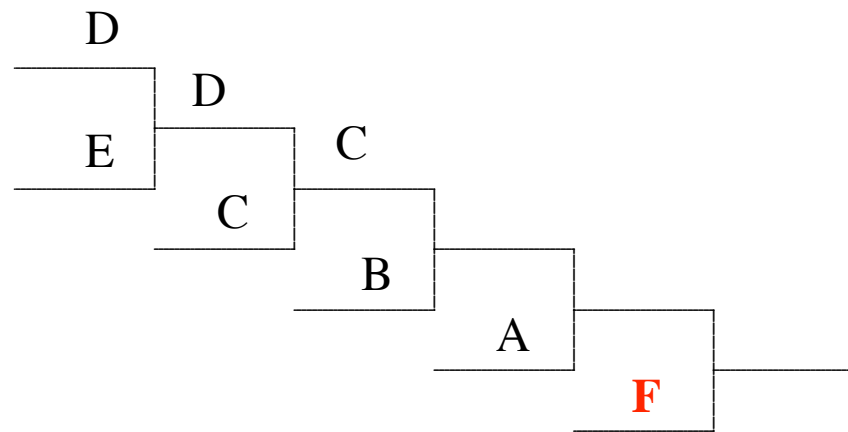
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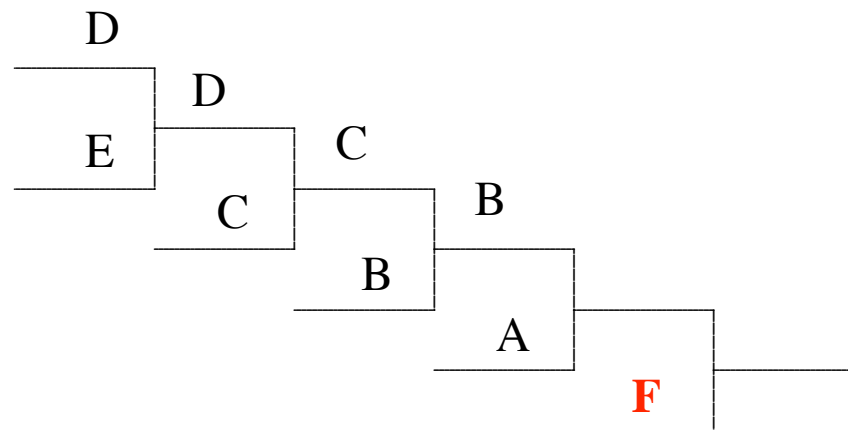
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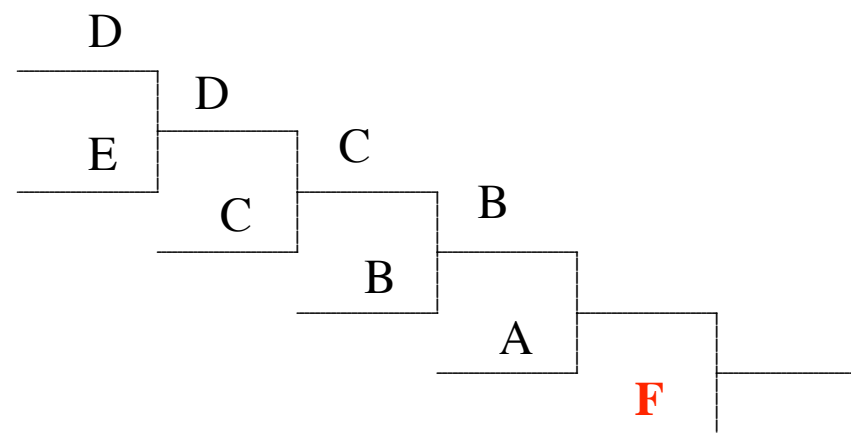
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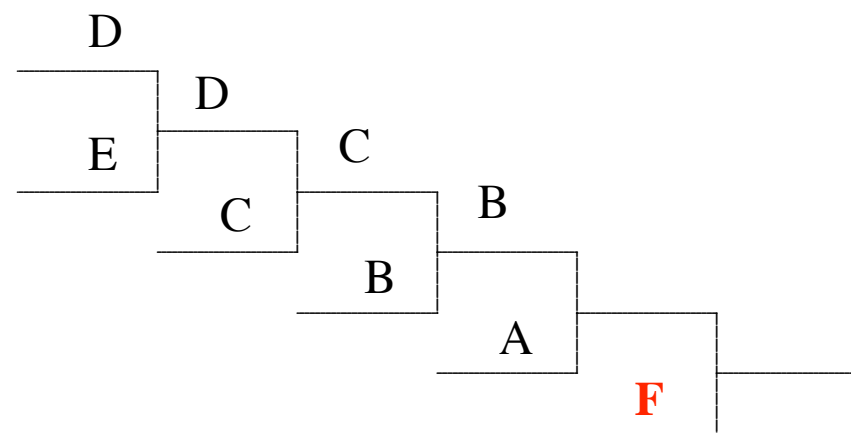
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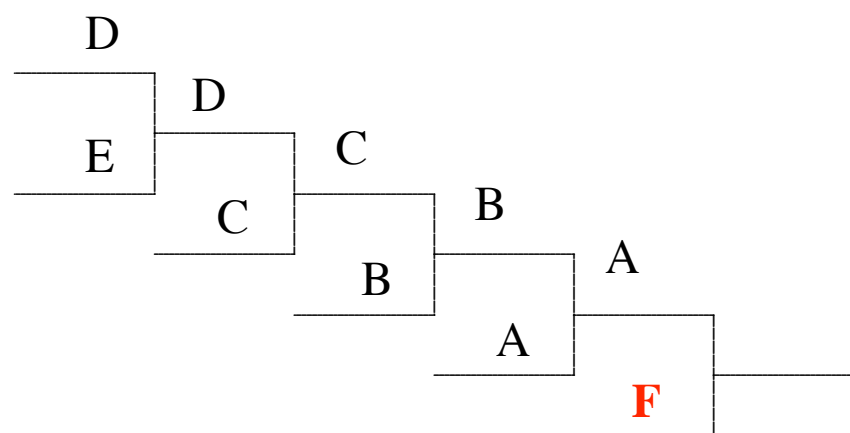
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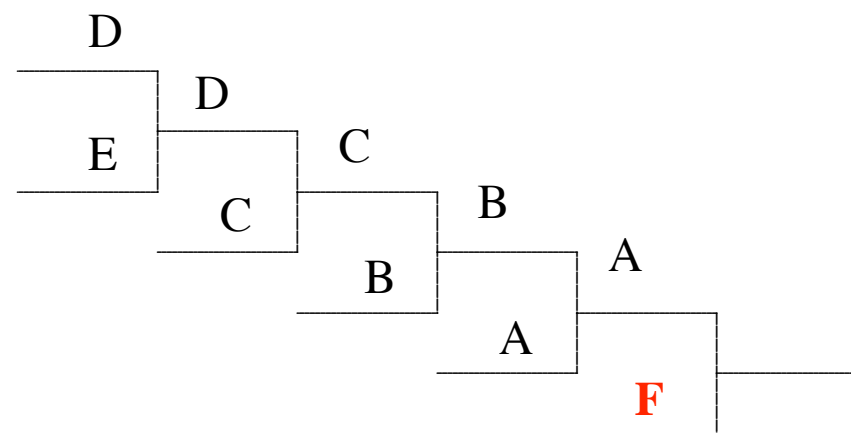
10 A>B>C>D>E>**F**  
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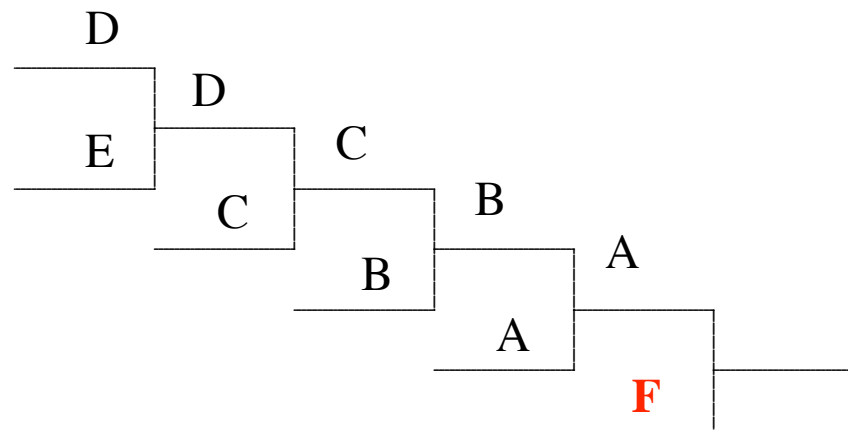
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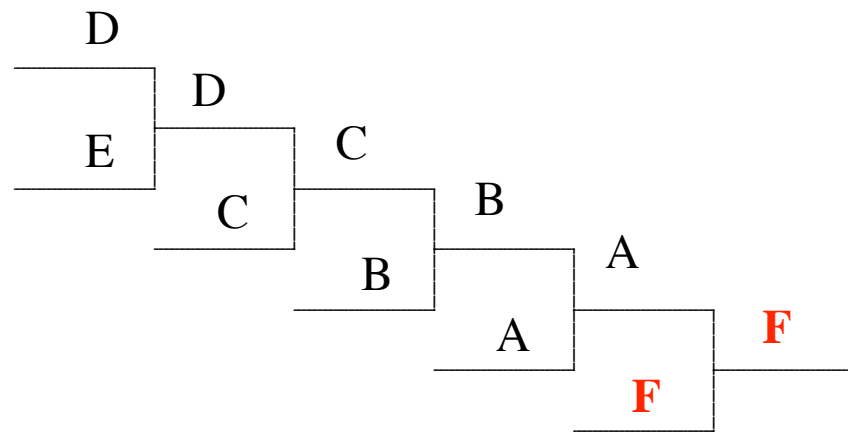
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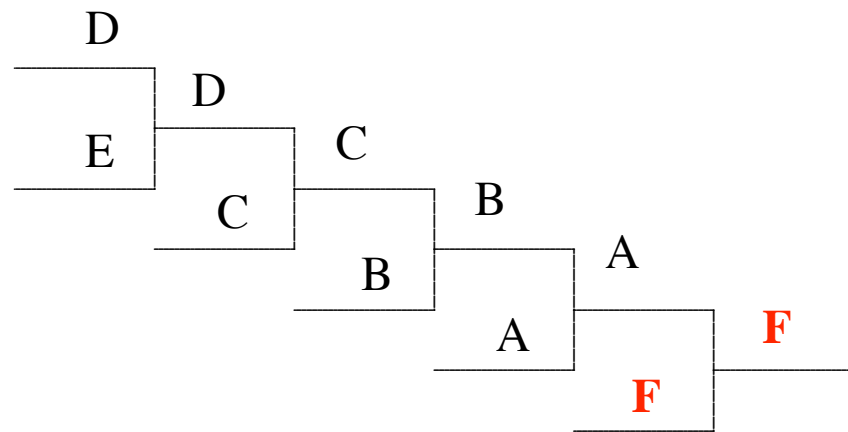
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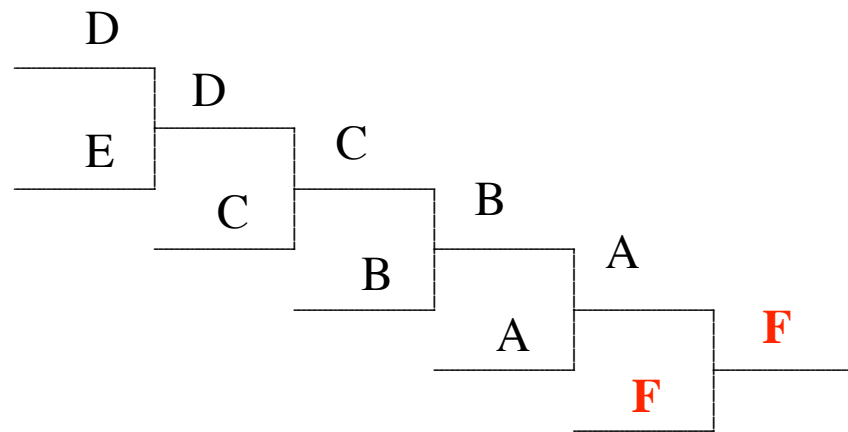
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**Fred wins by a landslide!!**

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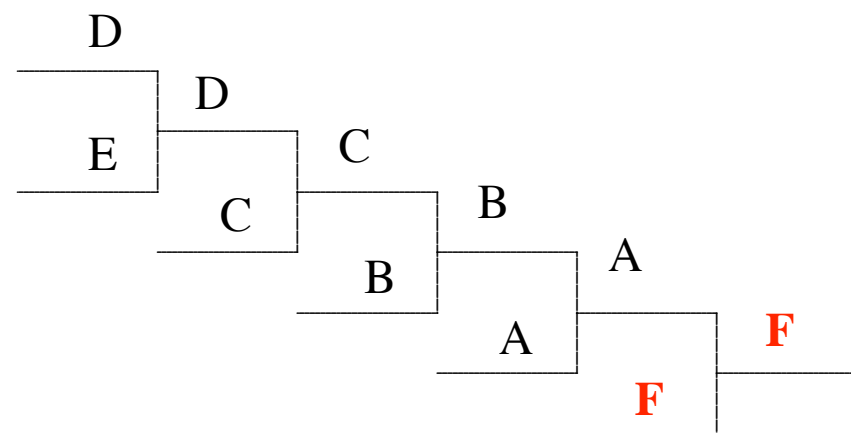
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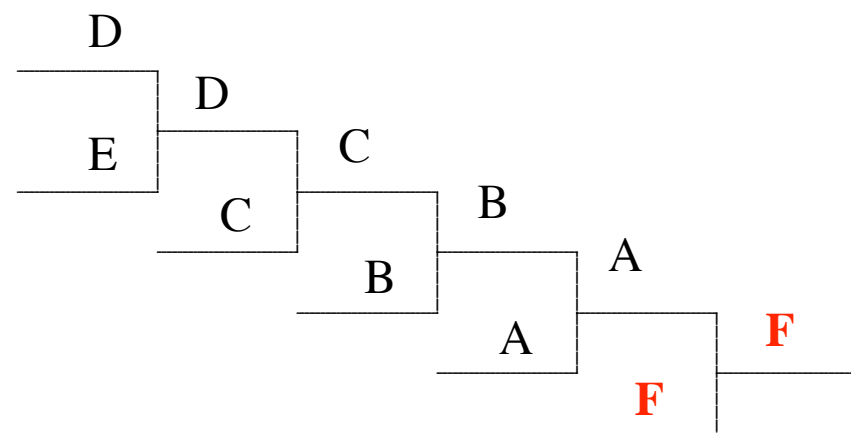
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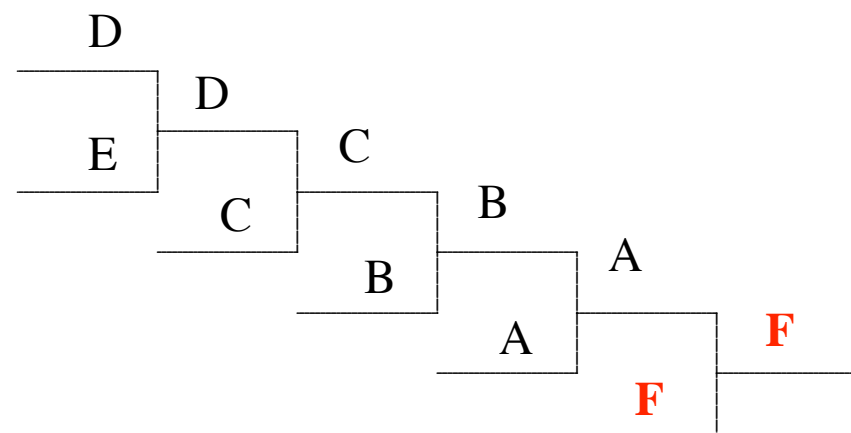
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*Mathematics?*

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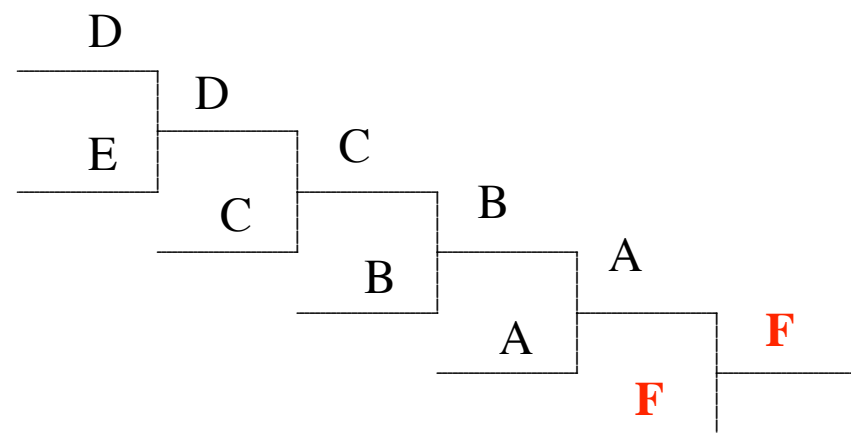
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**Ranking Wheel**

10 A>B>C>D>E>**F**

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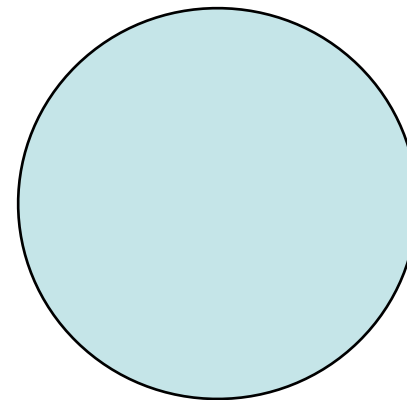
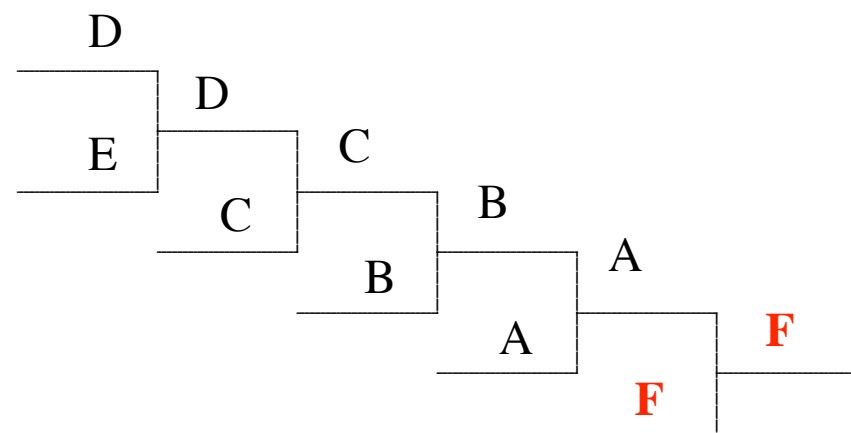
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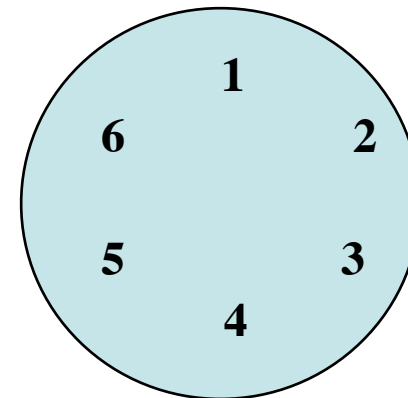
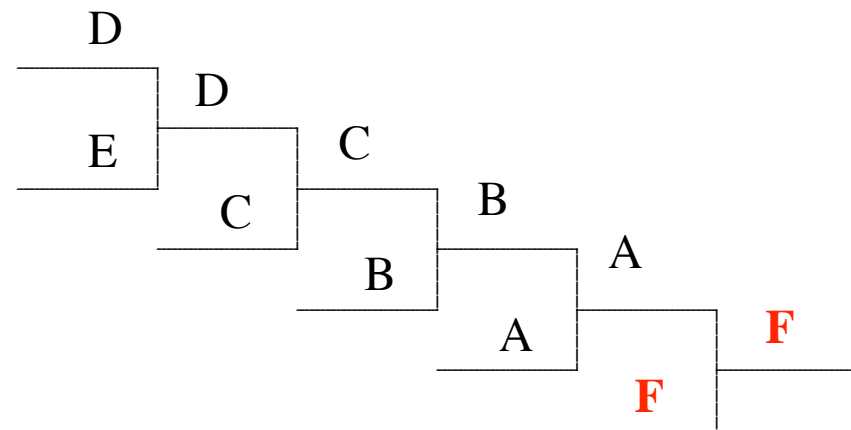
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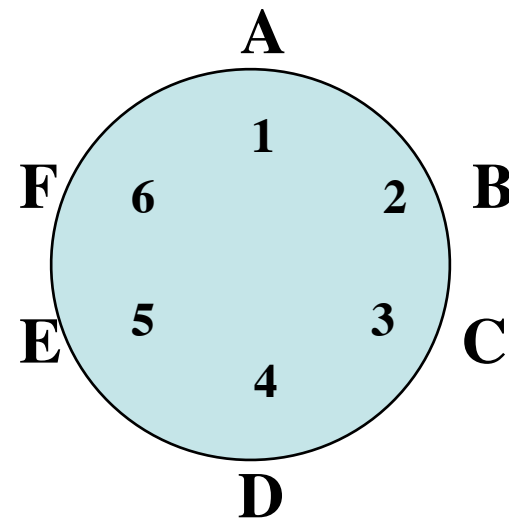
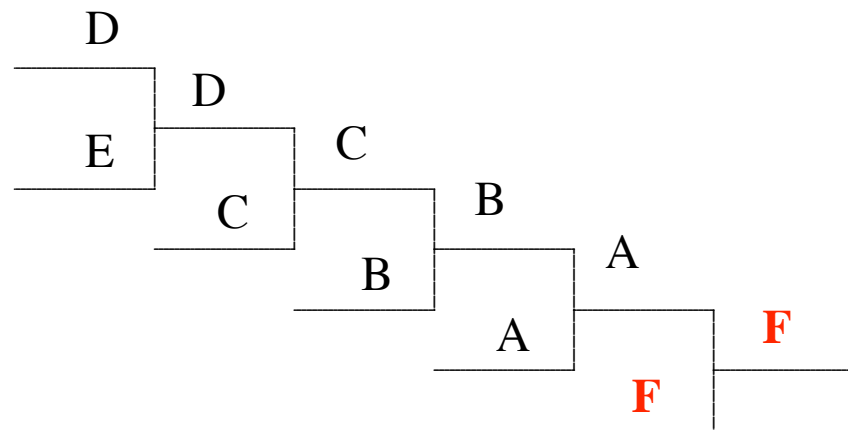
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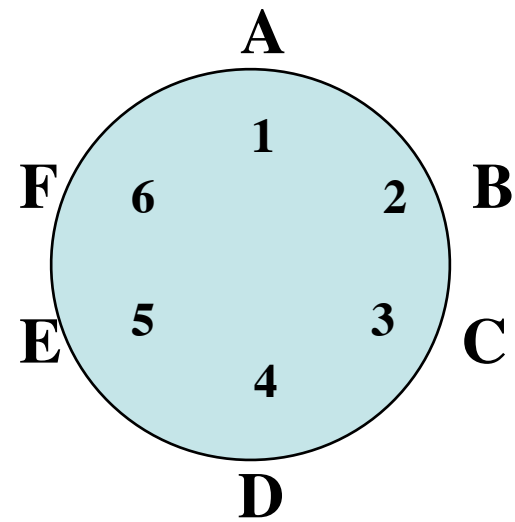
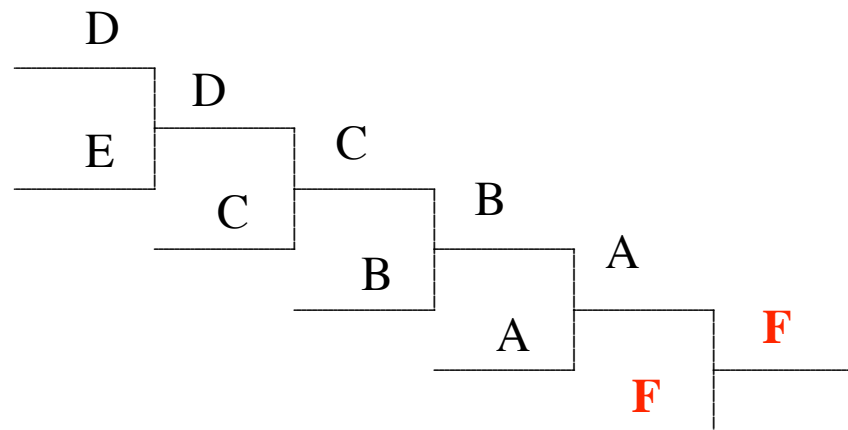
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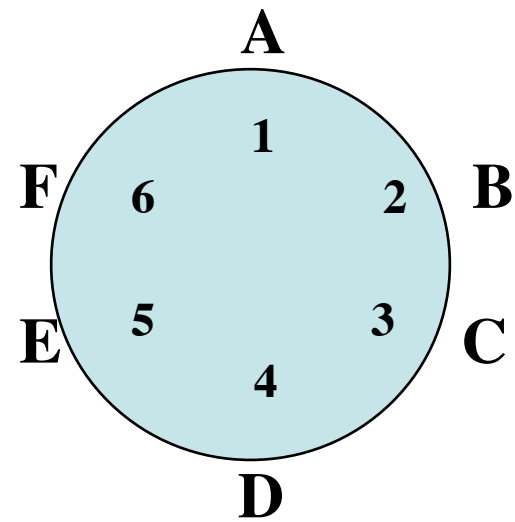
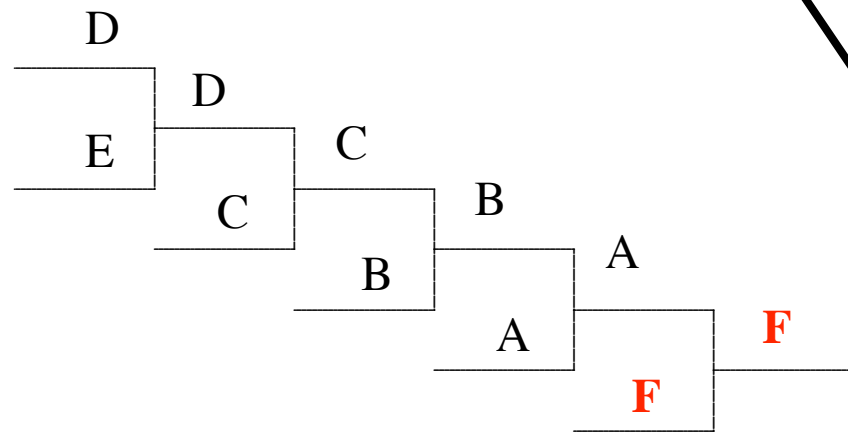
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A>B>C>D>E>F

For a price .....

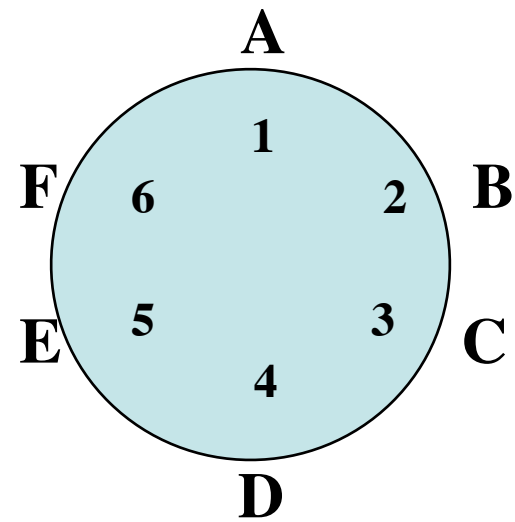
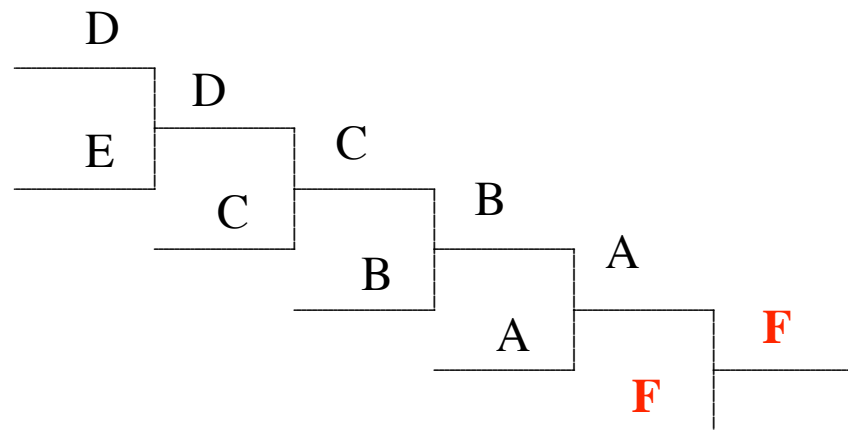
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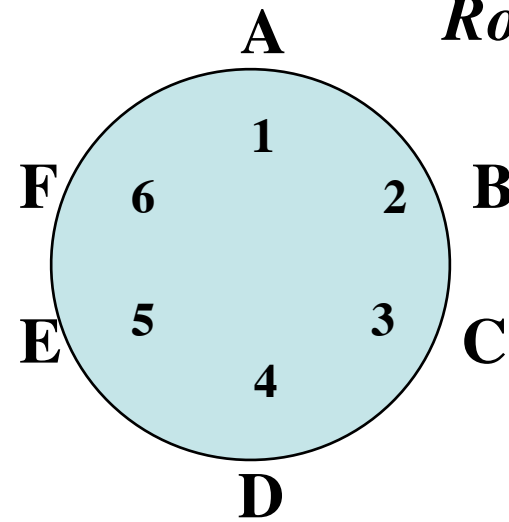
**Ranking Wheel**

10 A>B>C>D>E>**F**

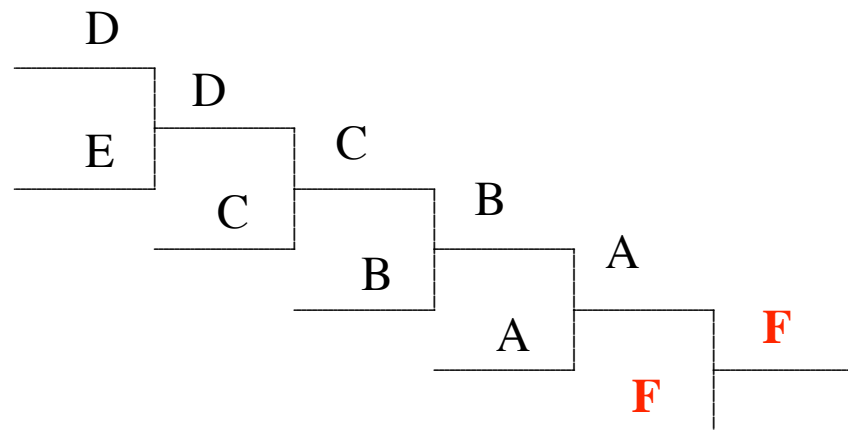
10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B

*Rotate -60 degrees*



A>B>C>D>E>F



For a price .....

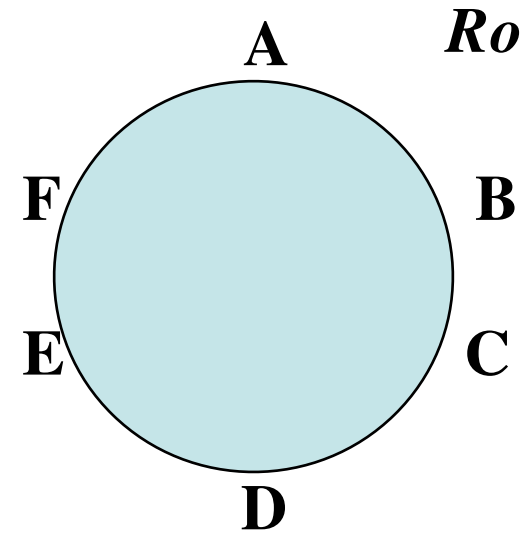
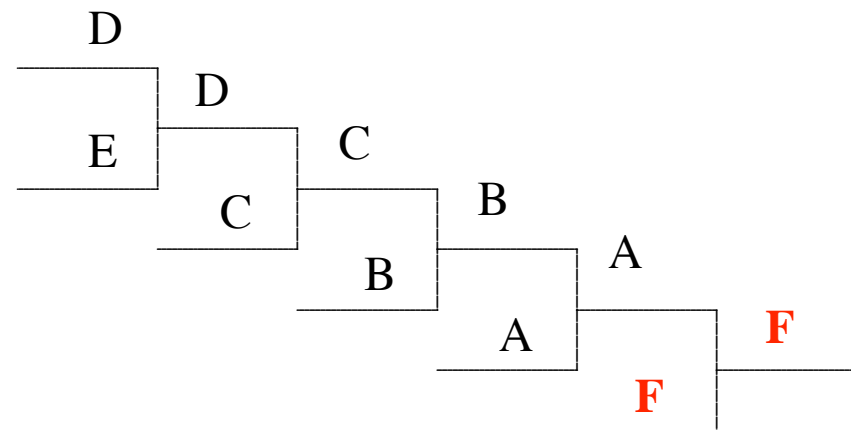
*Mathematics?*

**Ranking Wheel**

10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B



*Rotate -60 degrees*

A>B>C>D>E>F



For a price .....

*Mathematics?*

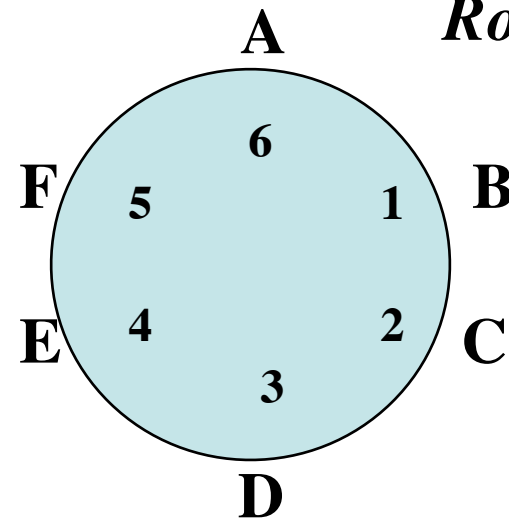
**Ranking Wheel**

10 A>B>C>D>E>**F**

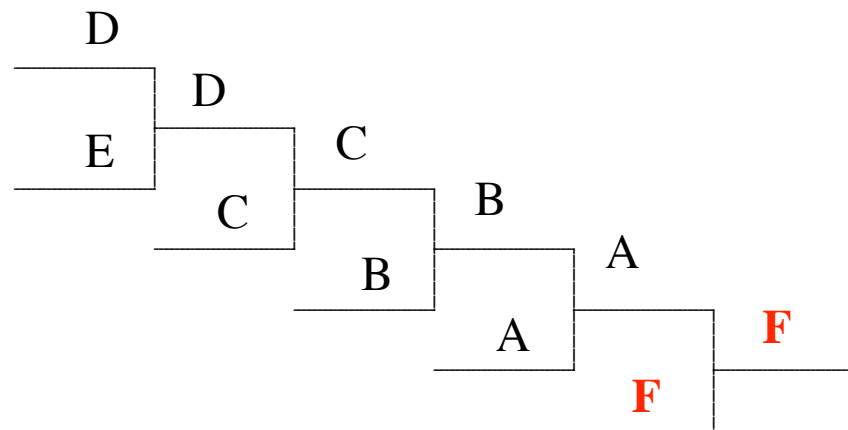
10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B

*Rotate -60 degrees*



A>B>C>D>E>F



For a price .....

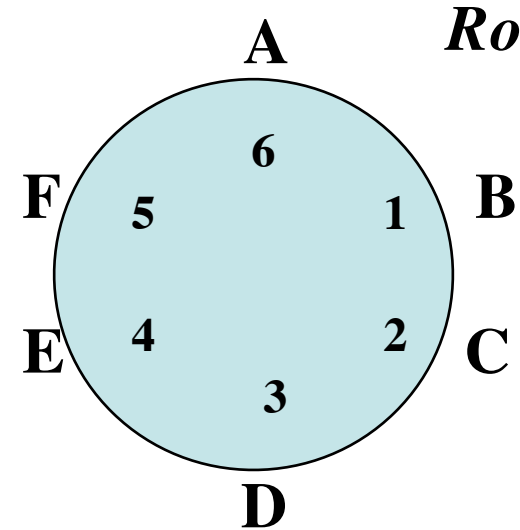
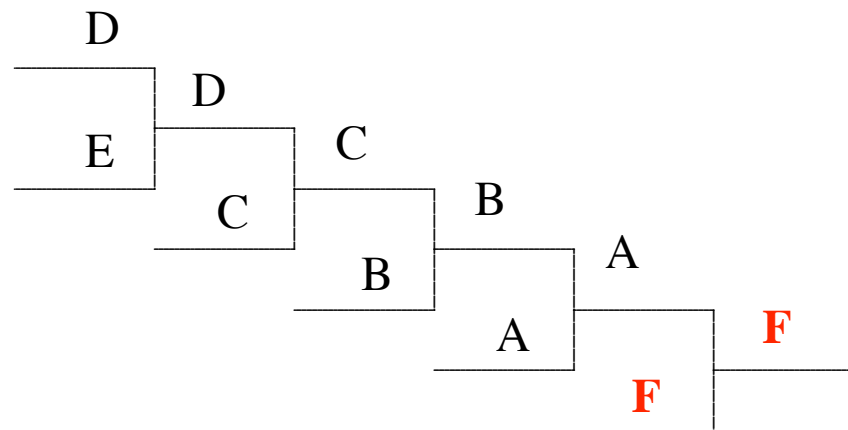
*Mathematics?*

**Ranking Wheel**

10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B



*Rotate -60 degrees*

A>B>C>D>E>F

B>C>D>E>F>A

For a price .....

*Mathematics?*

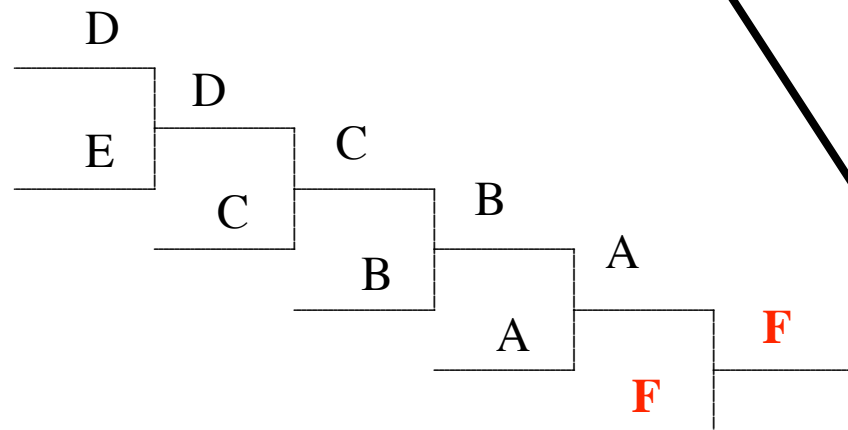
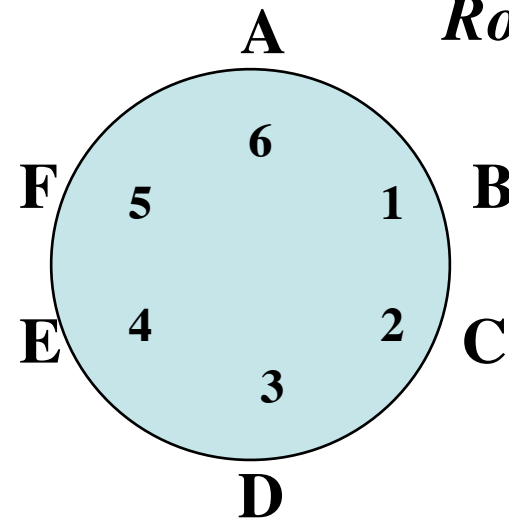
**Ranking Wheel**

10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B

*Rotate -60 degrees*



A>B>C>D>E>F

B>C>D>E>F>A

For a price .....

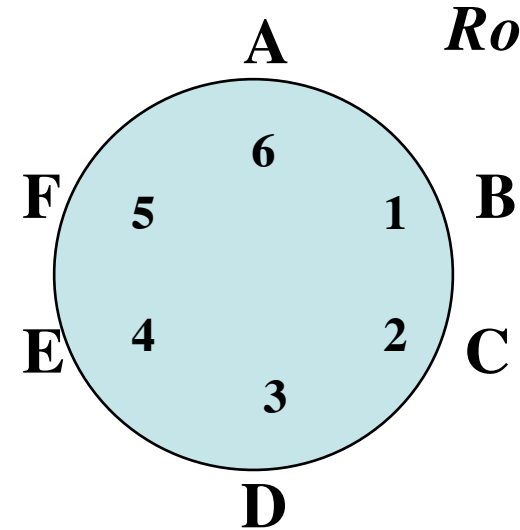
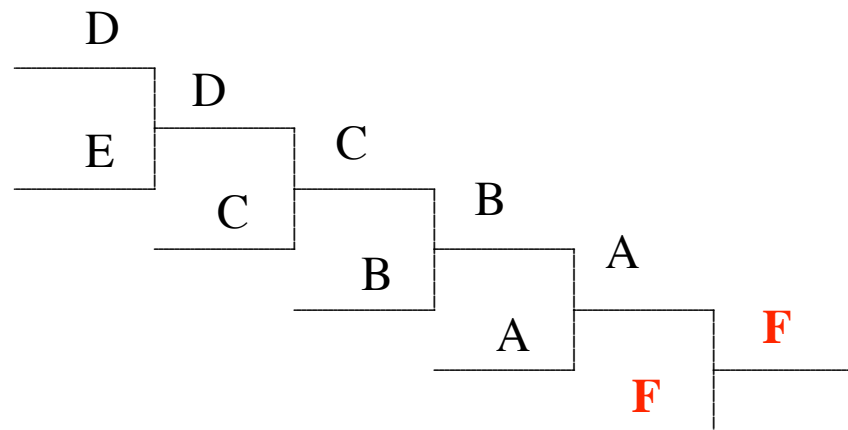
*Mathematics?*

**Ranking Wheel**

10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B



*Rotate -60 degrees*

A>B>C>D>E>F

B>C>D>E>F>A

For a price .....

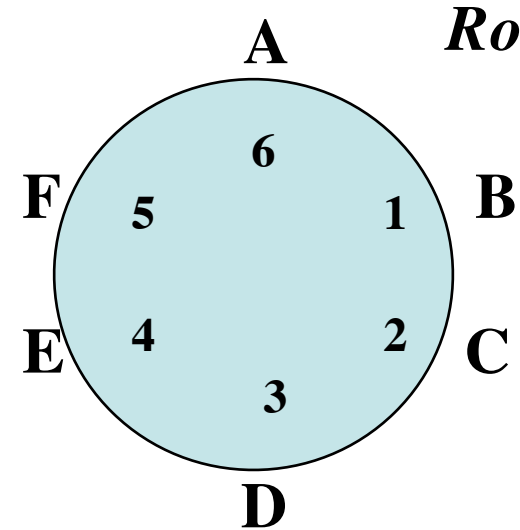
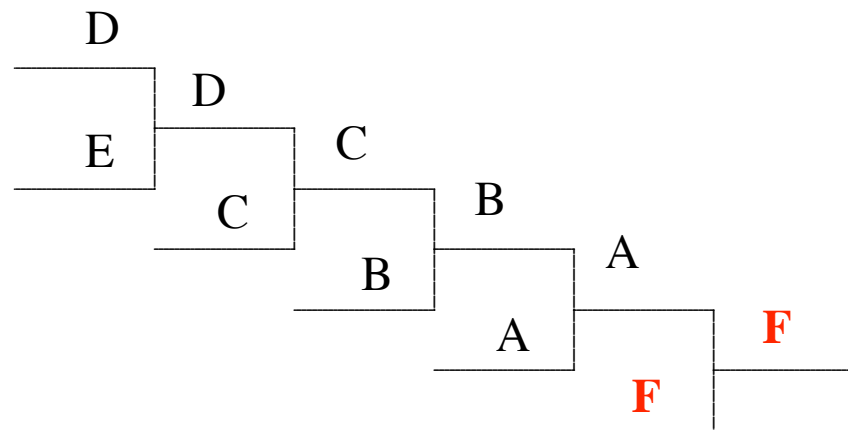
*Mathematics?*

**Ranking Wheel**

10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B



*Rotate -60 degrees*

A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

etc.

For a price .....

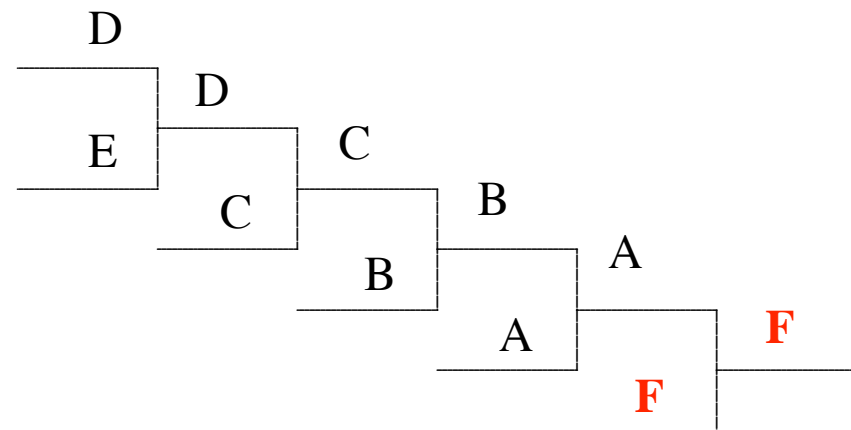
*Mathematics?*

**Ranking Wheel**

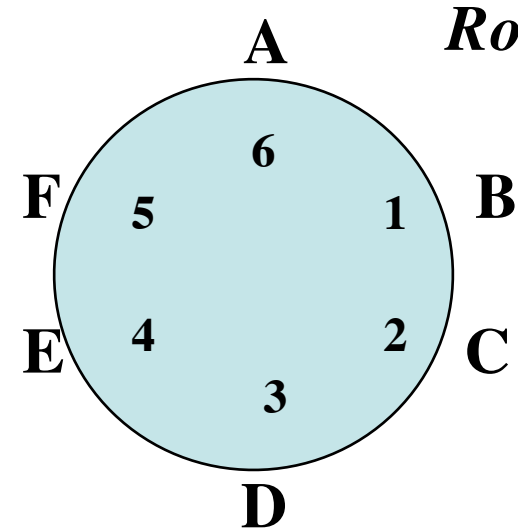
10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B



*No candidate is favored: each is in first, second, ... once.*



A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

etc.

For a price .....

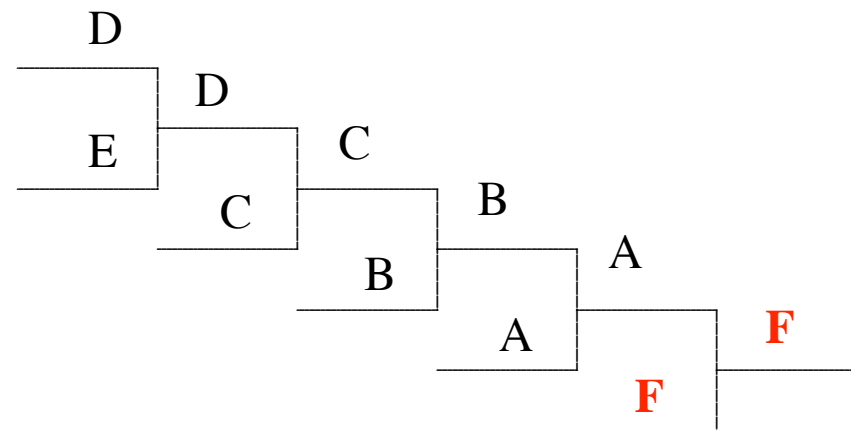
*Mathematics?*

**Ranking Wheel**

10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

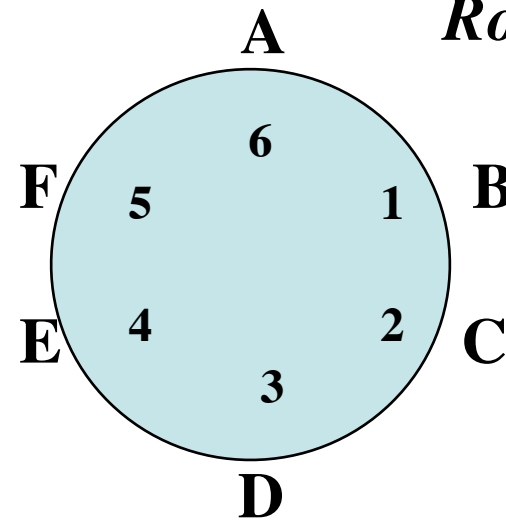
10 C>D>E>**F**>A>B



*No candidate is favored: each is in first, second, ... once.*

*Yet, pairwise elections are cycles!*

*Rotate -60 degrees*



A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

etc.

For a price .....

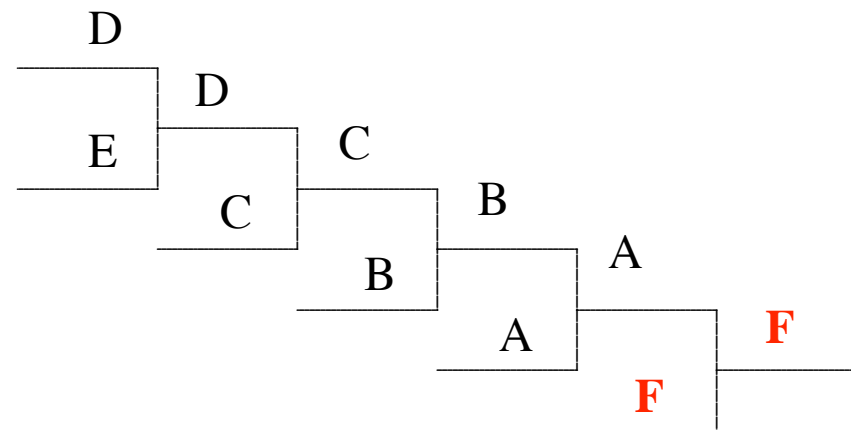
*Mathematics?*

**Ranking Wheel**

10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

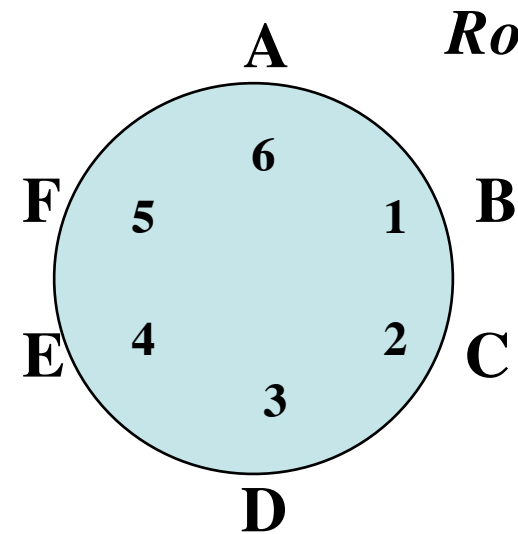
10 C>D>E>**F**>A>B



*No candidate is favored: each is in first, second, ... once.*

*Yet, pairwise elections are cycles!*

**lost information!!**



A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

etc.



For a price .....

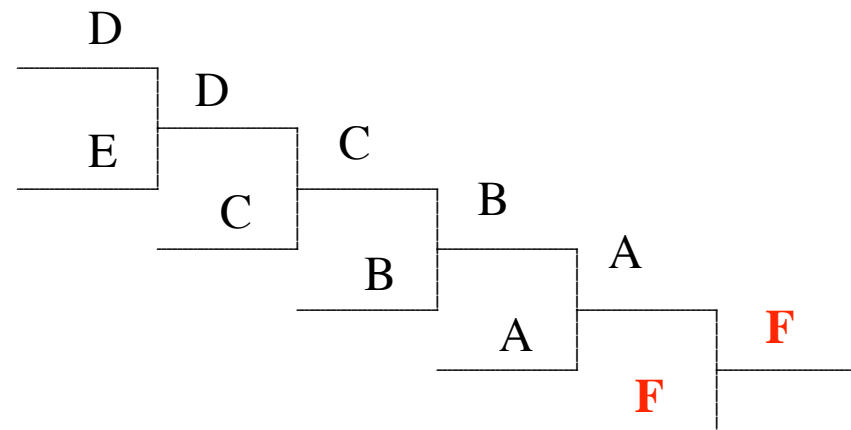
*Mathematics?*

**Ranking Wheel**

10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

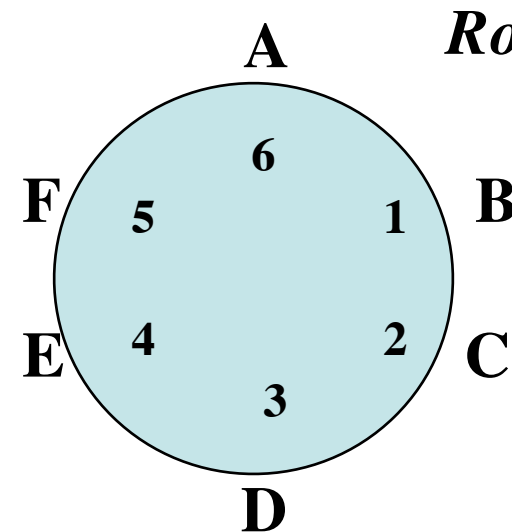
10 C>D>E>**F**>A>B



*No candidate is favored: each is in first, second, ... once.*

*Yet, pairwise elections are cycles!*

**lost information!!**



A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

etc.

For a price .....

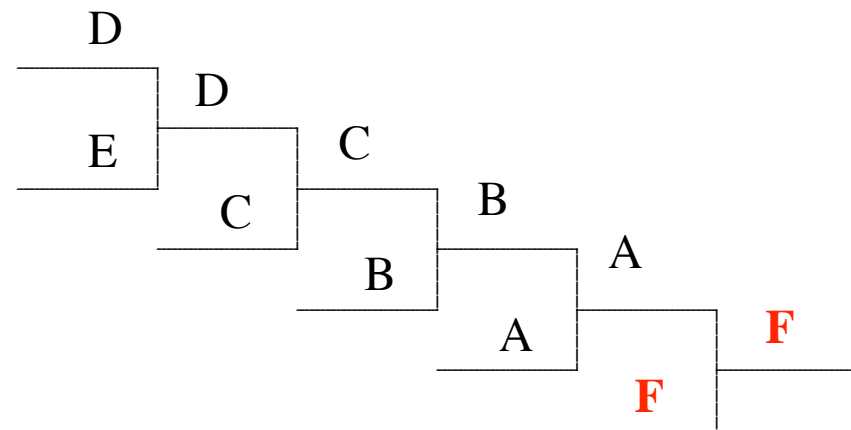
*Mathematics?*

**Ranking Wheel**

10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

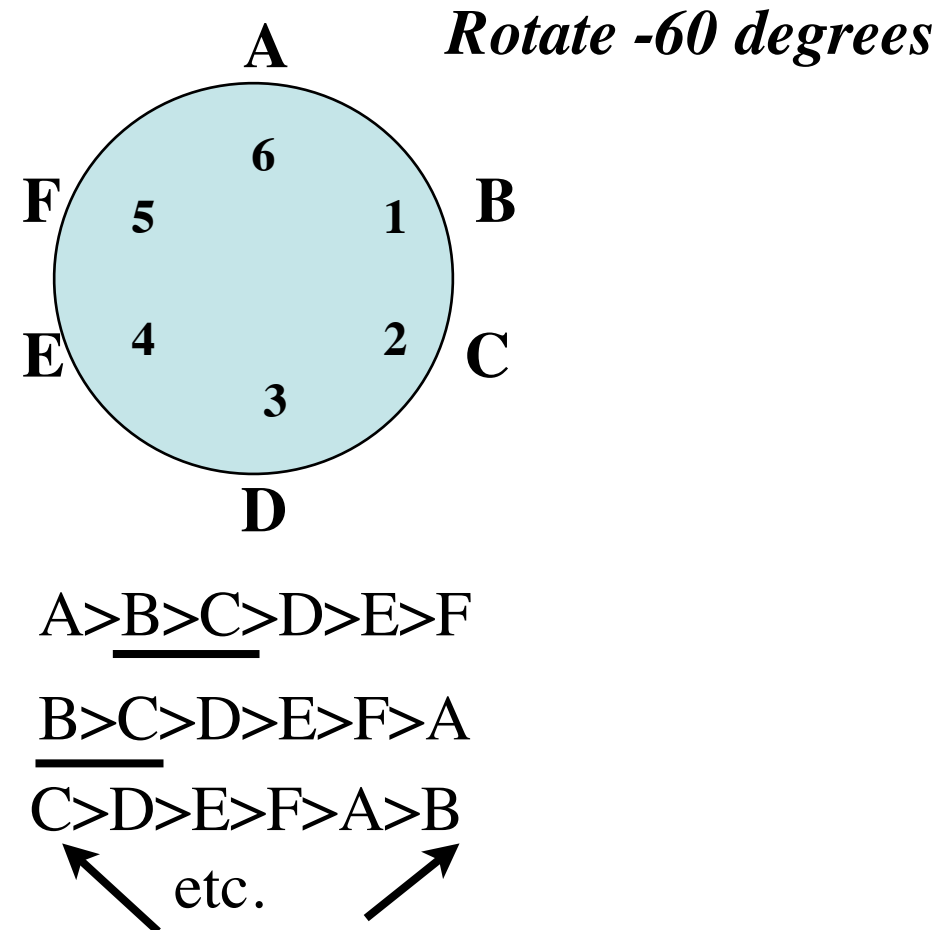
10 C>D>E>**F**>A>B



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**lost information!!**



For a price .....

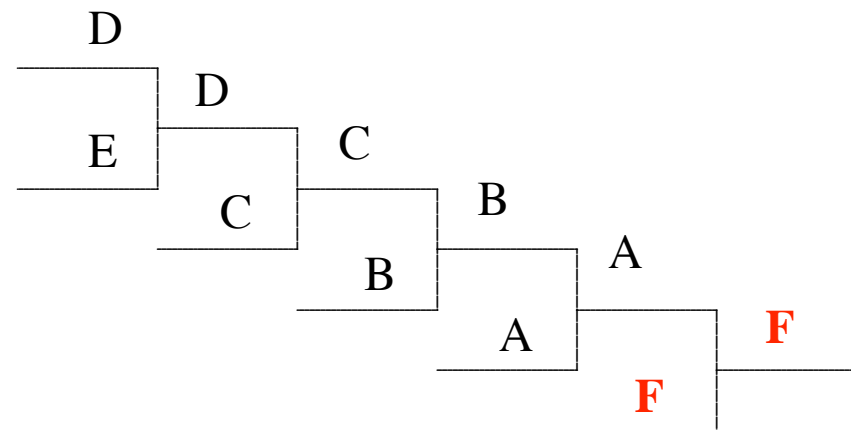
*Mathematics?*

**Ranking Wheel**

10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

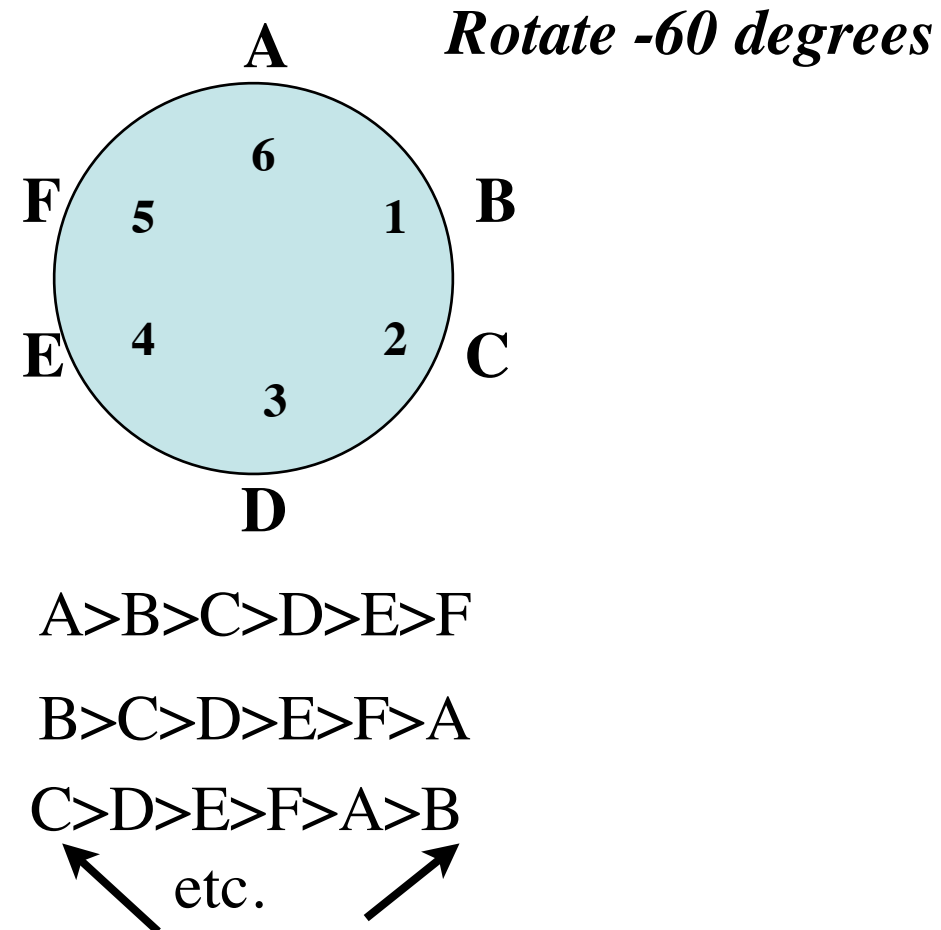
10 C>D>E>**F**>A>B



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**lost information!!**



For a price .....

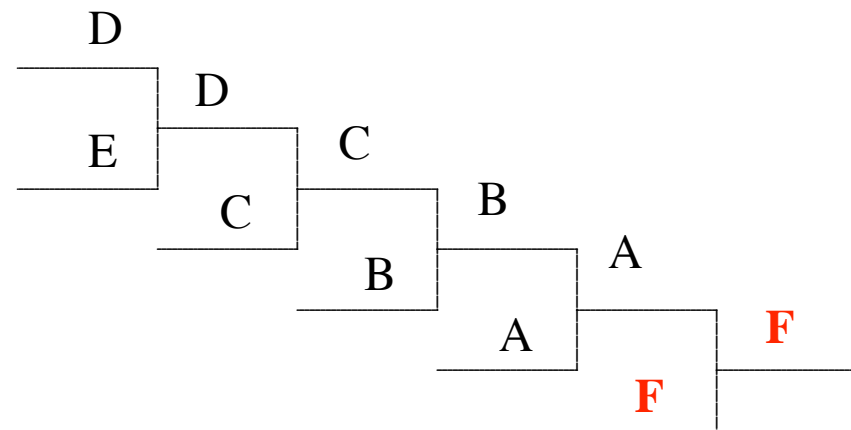
*Mathematics?*

**Ranking Wheel**

10 A>B>C>D>E>**F**

10 B>C>D>E>**F**>A

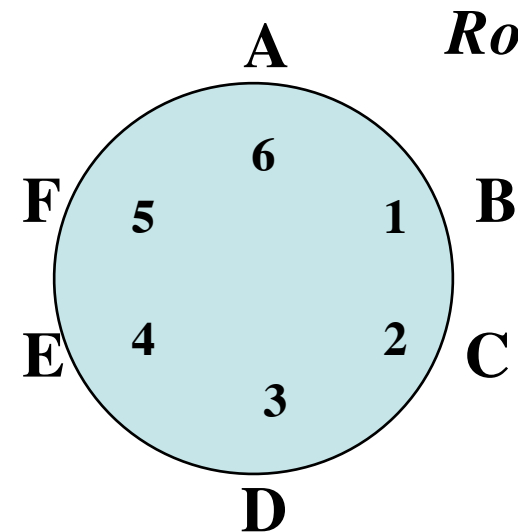
10 C>D>E>**F**>A>B



*No candidate is favored: each is in first, second, ... once.*

*Yet, pairwise elections are cycles!*

**lost information!!**



A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

etc.

For a price .....

10 A>B>C>D>E>**F**

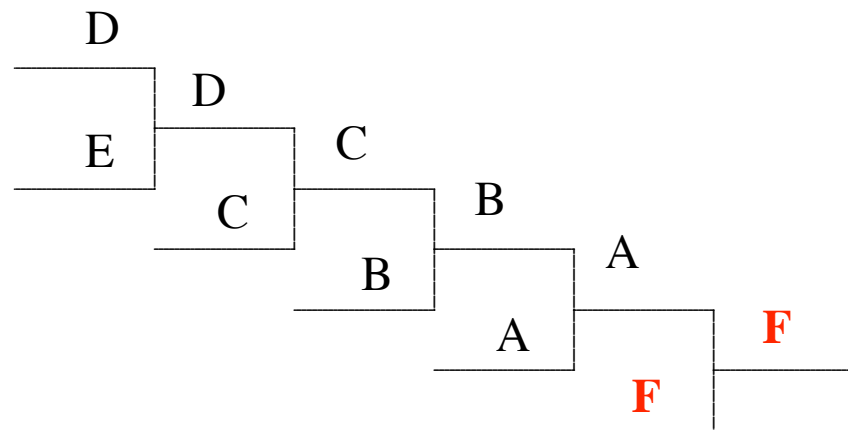
10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B

*Mathematics?*

**Ranking Wheel**

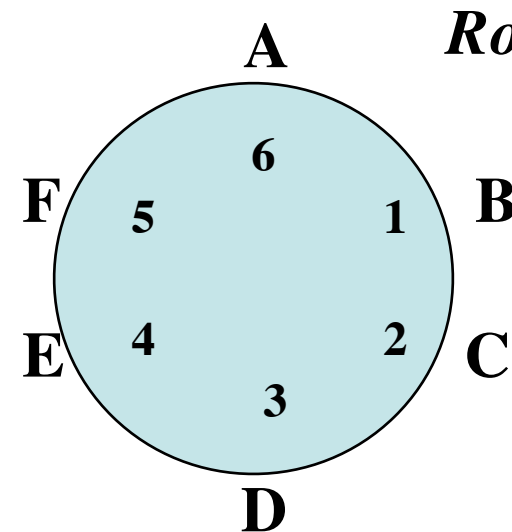
**Symmetry:  $Z_6$  orbit**



*No candidate is favored: each is in first, second, ... once.*

*Yet, pairwise elections are cycles!*

**lost information!!**



A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

etc.

Source of all cycles; voting, statistics, engineering, etc.  
For a price .....

10 A>B>C>D>E>**F**

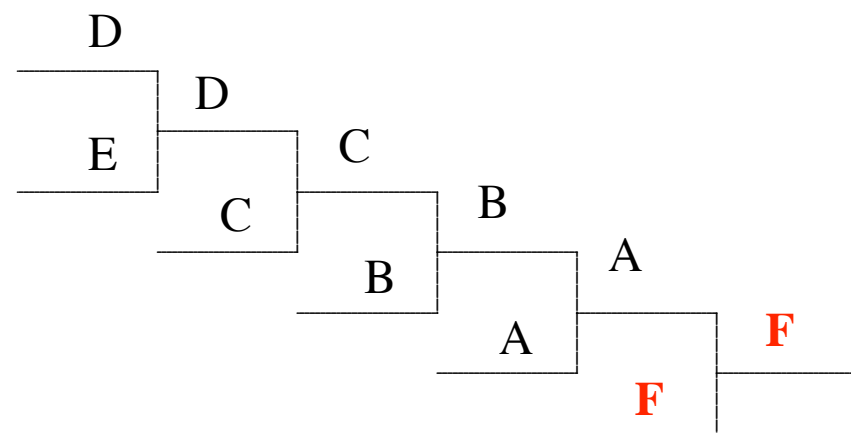
10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B

*Mathematics?*

**Ranking Wheel**

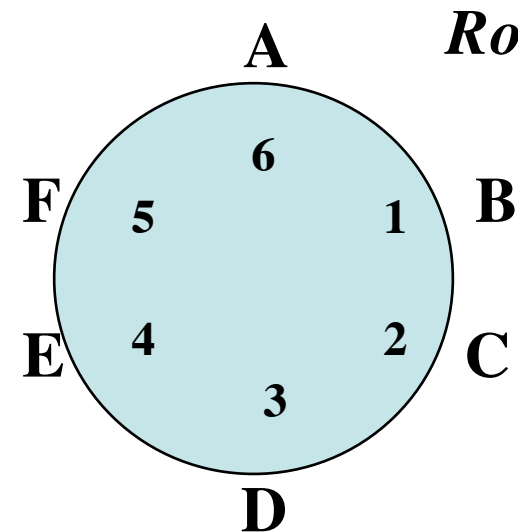
**Symmetry:  $Z_6$  orbit**



*No candidate is favored: each is in first, second, ... once.*

*Yet, pairwise elections are cycles!*

**lost information!!**



*Rotate -60 degrees*

A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

etc.

Source of all cycles; voting, statistics, engineering, etc.  
For a price .....

10 A>B>C>D>E>**F**

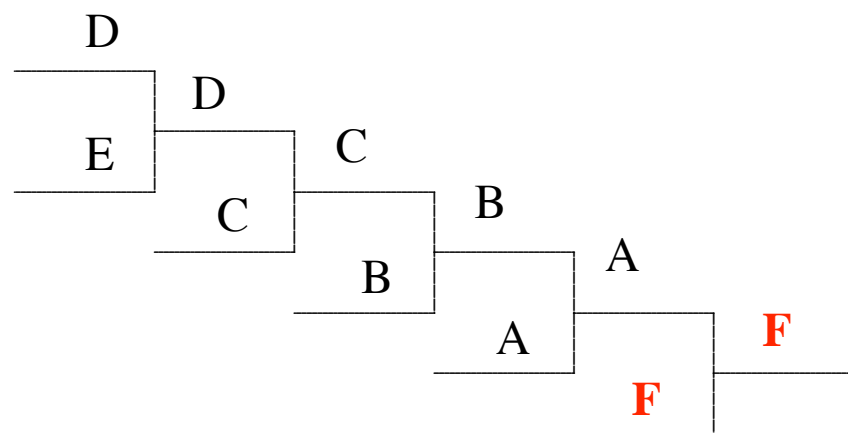
10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B

*Mathematics?*

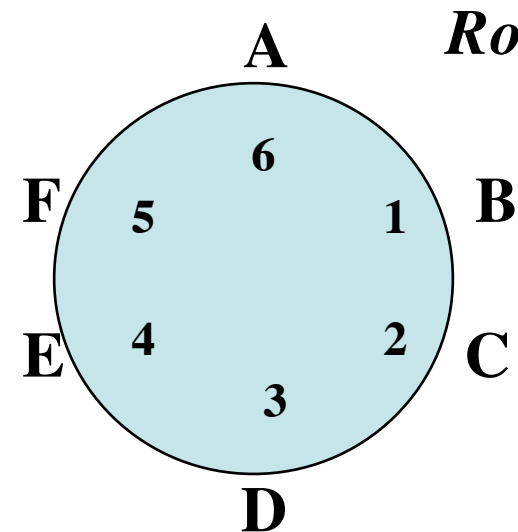
**Ranking Wheel**

**Symmetry:  $Z_6$  orbit**



*No candidate is favored: each is in first, second, ... once.*

*Yet, pairwise elections are cycles!*



*Rotate -60 degrees*

A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

etc.

**lost information!!**  
Everywhere!

Source of all cycles; voting, statistics, engineering, etc.  
For a price .....

10 A>B>C>D>E>**F**

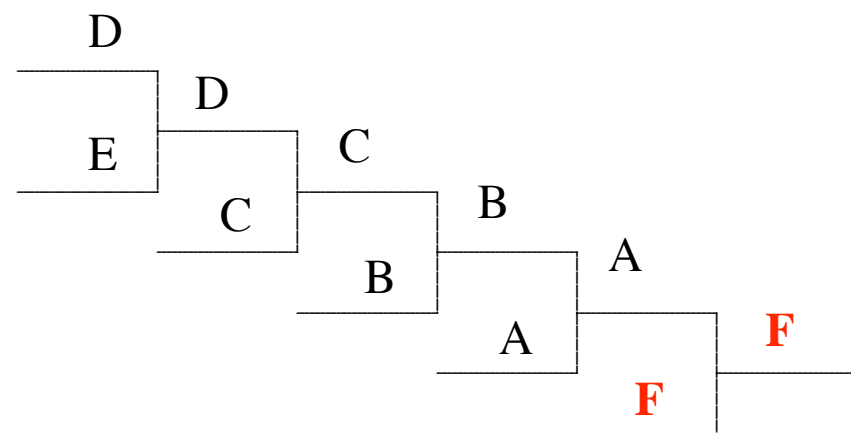
10 B>C>D>E>**F**>A

10 C>D>E>**F**>A>B

*Mathematics?*

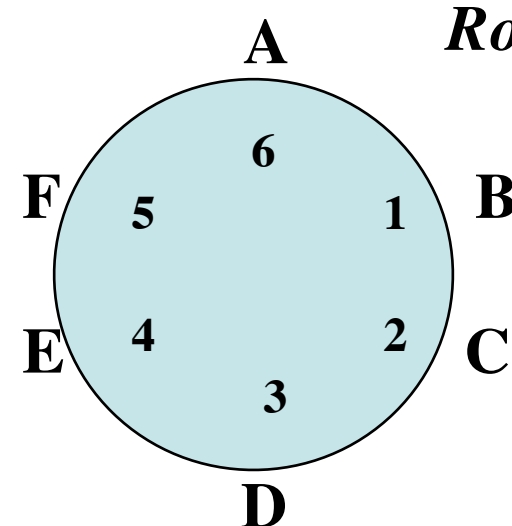
**Ranking Wheel**

**Symmetry:  $Z_6$  orbit**



*No candidate is favored: each is in first, second, ... once.*

*Yet, pairwise elections are cycles!*



*Rotate -60 degrees*

A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

etc.

**lost information!!**

Everywhere!

Manipulation, agenda fixing, all problems



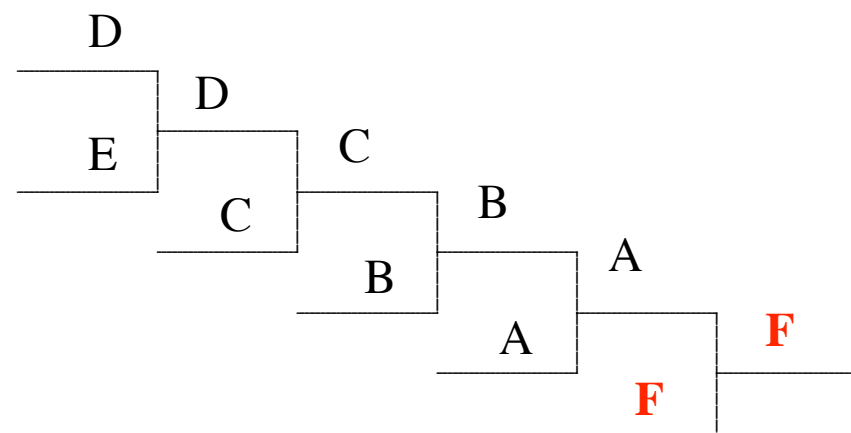
Source of all cycles; voting, statistics, engineering, etc.  
For a price ..... Solutions must address this!

10 A>B>C>D>E>**F**      *Mathematics?*      **Ranking Wheel**

10 B>C>D>E>**F**>A

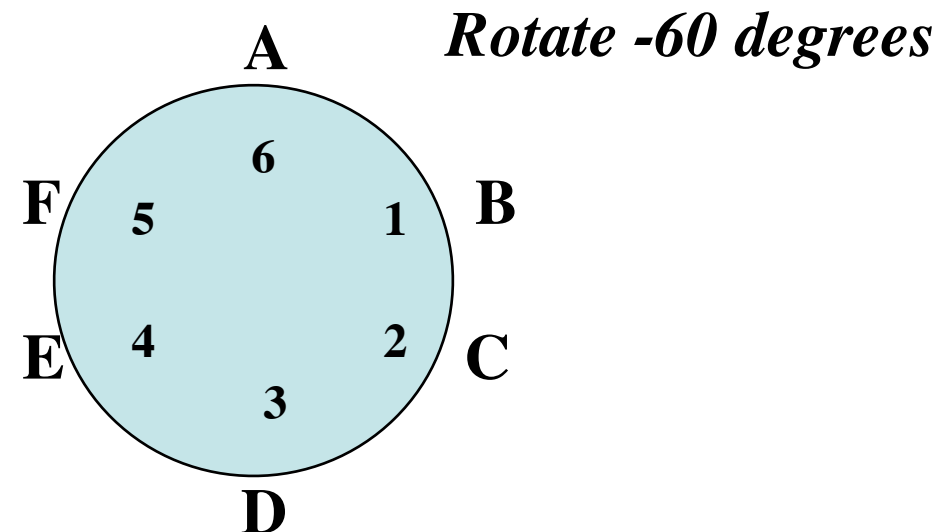
10 C>D>E>**F**>A>B

**Symmetry:  $Z_6$  orbit**



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A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

etc.

**lost information!!**  
Everywhere!

Manipulation, agenda fixing, all problems