Using mathematics to structure laws

Don Saari
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Provide introduction

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UCI, Institute for Math Behavioral Sciences
Using mathematics to structure laws

Provide introduction

Don Saari

UCI, Institute for Math Behavioral Sciences

And politics
Using mathematics to structure laws

Provide introduction

Don Saari

UCI, Institute for Math Behavioral Sciences

Good intensions,
Using mathematics to structure laws

Provide introduction

Don Saari

UCI, Institute for Math Behavioral Sciences

Good intentions,
Unintended consequences:
Using mathematics to structure laws

Provide introduction

Don Saari
UCI, Institute for Math Behavioral Sciences

Good intentions,

Unintended consequences:
Planning on writing the
Great American Novel

And politics
Using mathematics to structure laws

Provide introduction

Don Saari

UCI, Institute for Math Behavioral Sciences

Good intentions,

Unintended consequences:

Planning on writing the

Great American Novel
Using mathematics to structure laws

Provide introduction

Good intensions,

Unintended consequences:

Planning on writing the

Great American Novel

Don Saari
UCI, Institute for Math Behavioral Sciences

Borrow $$

And politics

Tuesday, April 29, 14
Using mathematics to structure laws

Provide introduction
Don Saari
UCI, Institute for Math Behavioral Sciences

Good intentions,
Borrow $$
Ann

Unintended consequences:
Planning on writing the
Great American Novel

And politics
Using mathematics to structure laws

Provide introduction

Don Saari
UCI, Institute for Math Behavioral Sciences

Good intensions,

Unintended consequences:
Planning on writing the
Great American Novel

And politics
Borrow $$
Ann
go broke, Ann gets
computer
Using mathematics to structure laws

Provide introduction
UCI, Institute for Math Behavioral Sciences

Don Saari

Good intensions,

Unintended consequences:
Planning on writing the
Great American Novel
Writing not going well

And politics

Borrow $$
Ann
go broke, Ann gets
computer

Tuesday, April 29, 14
Using mathematics to structure laws

Provide introduction

Don Saari

UCI, Institute for Math Behavioral Sciences

Good intentions,

Unintended consequences:

Planning on writing the

Great American Novel

Writing not going well

Borrow more $$

And politics

Borrow $$

Ann

go broke, Ann gets

computer

Tuesday, April 29, 14
Using mathematics to structure laws

Don Saari
UCI, Institute for Math Behavioral Sciences

Good intentions,

Unintended consequences:
Planning on writing the
Great American Novel
Writing not going well
Borrow more $$
Barb

Borrow $$
Ann
go broke, Ann gets
computer

And politics
Using mathematics to structure laws

Provide introduction

UCI, Institute for Math Behavioral Sciences

Don Saari

Good intentions,

Unintended consequences:

Planning on writing the

Great American Novel

Writing not going well

Borrow more $$

Barb

But Barb registers loan

And politics

Borrow $$

Ann

go broke, Ann gets

computer
Using mathematics to structure laws

Provide introduction

Don Saari
UCI, Institute for Math Behavioral Sciences

Good intentions,

Unintended consequences:

Planning on writing the Great American Novel

Writing not going well

Borrow more $\$

Barb

But Barb registers loan

Rule: go broke, priority of who gets my computer determined by order of registering

Borrow $$

Ann

go broke, Ann gets computer

And politics
Using mathematics to structure laws

Provide introduction

Good intensions,

Unintended consequences:

Planning on writing the

Great American Novel

Writing not going well

Borrow more $$

But Barb registers loan

Rule: go broke, priority

of who gets my computer
determined by order of

registering

BUT

Don Saari

UCI, Institute for Math Behavioral Sciences

Borrow $$

Ann
go broke, Ann gets computer

And politics

IPAM
April 22, 2014

Tuesday, April 29, 14
Using mathematics to structure laws

Provide introduction

UCI, Institute for Math Behavioral Sciences

Don Saari

Good intensions,

Unintended consequences:

Planning on writing the
Great American Novel

Writing not going well

Borrow more $$

Barb

But Barb registers loan

Rule: go broke, priority

of who gets my computer
determined by order of
registering

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did not register.
Using mathematics to structure laws

Good intentions,
Unintended consequences:
Planning on writing the Great American Novel
Writing not going well
Borrow more $$
Barb
But Barb registers loan
Rule: go broke, priority of who gets my computer determined by order of registering
BUT
only if you did not know someone else loaned who did not register.

And politics

Provide introduction
Don Saari
UCI, Institute for Math Behavioral Sciences

Borrow $$
Ann
go broke, Ann gets computer

So, if I go broke,
**Using mathematics to structure laws**

Don Saari

UCI, Institute for Math Behavioral Sciences

Provide introduction

Good intensions,

Unintended consequences:

Planning on writing the

Great American Novel

Writing not going well

Borrow more $$

Barb

But Barb registers loan

Rule: go broke, priority

of who gets my computer
determined by order of

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only if you did not know

someone else loaned who
did not register.

Borrow $$

Ann

go broke, Ann gets

computer

So, if I go broke,

**Ann > Barb**
Using mathematics to structure laws

Provide introduction

Good intensions,
Unintended consequences:
Planning on writing the
Great American Novel
Writing not going well
Borrow more $$
Barb
But Barb registers loan
Rule: go broke, priority
of who gets my computer
determined by order of
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Don Saari
UCI, Institute for Math Behavioral Sciences

Borrow $$
Ann
go broke, Ann gets
computer
Writing not going well

So, if I go broke,
Ann > Barb

And politics
Using mathematics to structure laws

Provide introduction

Don Saari
UCI, Institute for Math Behavioral Sciences

Good intensions,

Unintended consequences:
Planning on writing the Great American Novel
Writing not going well
Borrow more $$
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But Barb registers loan
Rule: go broke, priority of who gets my computer determined by order of registering
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Borrow $$
Ann
go broke, Ann gets computer
Writing not going well
Borrow even more $$

So, if I go broke, Ann > Barb

And politics

Tuesday, April 29, 14
Using mathematics to structure laws

Provide introduction

Don Saari

UCI, Institute for Math Behavioral Sciences

Good intentions,

Unintended consequences:

Planning on writing the

Great American Novel

Writing not going well

Borrow more $$

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go broke, Ann gets

computer

Writing not going well

Borrow even more $$

Carol

So, if I go broke,

Ann > Barb

And politics
Using mathematics to structure laws

Good intentions,

Planning on writing the 
Great American Novel

Writing not going well

Borrow more $$

But Barb registers loan

Rule: go broke, priority of who gets my computer determined by order of registering

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Provide introduction

UCI, Institute for Math Behavioral Sciences

Don Saari

Borrow $$

Ann

go broke, Ann gets computer

Writing not going well

Borrow even more $$

Carol

Carol does not know about any previous loans

And politics

So, if I go broke, Ann > Barb
Using mathematics to structure laws

Good intensions,

Unintended consequences:
Planning on writing the Great American Novel
Writing not going well
Borrow more $$
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Borrow $$
Ann
go broke, Ann gets computer
Writing not going well
Borrow even more $$
Carol
Carol does not know about any previous loans
She registers.

So, if I go broke, Ann > Barb

Provide introduction
Don Saari
UCI, Institute for Math Behavioral Sciences

And politics
IPAM
April 22, 2014

Tuesday, April 29, 14
Using mathematics to structure laws

Provide introduction

Don Saari
UCI, Institute for Math Behavioral Sciences

Good intentions,

Unintended consequences:
Planning on writing the Great American Novel
Writing not going well
Borrow more $$
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Ann
go broke, Ann gets computer

Writing not going well
Borrow even more $$
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So, if I go broke, Ann > Barb
I go broke!

And politics
Using mathematics to structure laws

Provide introduction

Don Saari
UCI, Institute for Math Behavioral Sciences

Good intensions,

Unintended consequences:

Planning on writing the Great American Novel

Writing not going well

Borrow more $$

Barb

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Rule: go broke, priority of who gets my computer determined by order of registering

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So, if I go broke, Ann > Barb

I go broke!

As Barb registered first
Using mathematics to structure laws

Good intentions,
Unintended consequences:
Planning on writing the Great American Novel
Writing not going well
Borrow more $$
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Don Saari
UCI, Institute for Math Behavioral Sciences

Borrow $$
Ann
go broke, Ann gets computer
Writing not going well
Borrow even more $$
Carol
Carol does not know about any previous loans
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So, if I go broke, Ann > Barb
I go broke!
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Provide introduction

And politics

Tuesday, April 29, 14
Using mathematics to structure laws

Good intentions,

Unintended consequences:
Planning on writing the Great American Novel
Writing not going well
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But, Carol registered without knowing about Ann, so

And politics
Using mathematics to structure laws

Provide introduction

Don Saari
UCI, Institute for Math Behavioral Sciences

Good intensions,

Unintended consequences:
Planning on writing the
Great American Novel
Writing not going well
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And politics

Tuesday, April 29, 14
Using mathematics to structure laws

Provide introduction
UCI, Institute for Math Behavioral Sciences

Good intentions,
Unintended consequences:

Planning on writing the Great American Novel
Writing not going well
Borrow more $$
Barb

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Borrow $$
Ann
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Writing not going well
Borrow even more $$
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Cycle! Who gets the computer???

So, if I go broke,
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Using mathematics to structure laws

Provide introduction

Don Saari
UCI, Institute for Math Behavioral Sciences

Good intensions,

Unintended consequences:
Planning on writing the Great American Novel
Writing not going well
Borrow more $$
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Borrow $$
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go broke, Ann gets computer
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Borrow even more $$
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So, if I go broke,
Ann > Barb
I go broke!
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Recent conference at IMBS,
Using mathematics to structure laws

Provide introduction

Good intensions,

Unintended consequences:

Planning on writing the Great American Novel

Writing not going well

Borrow more $$

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Borrow $$

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Writing not going well

Borrow even more $$

Carol

Carol does not know about any previous loans

She registers.

Cycle! Who gets the computer???

Recent conference at IMBS, discovered this problem is surprisingly common

So, if I go broke, Ann > Barb

I go broke!

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But, Carol registered without knowing about Ann, so Carol > Ann

And politics
Dimensionality of issues
Dimensionality of issues

Priority: registering
Dimensionality of issues

Good intensions
Protection against exploitation

Priority: registering
Dimensionality of issues

- Good intentions
- Protection against exploitation

Priority: registering

Avoiding cycles:
Dimensionality of issues

Good intensions
Protection against exploitation

Priority: registering

Avoiding cycles:
Core:
Dimensionality of issues

Good intensions
Protection against exploitation

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Avoiding cycles:
Core:
Paired comparisons:
Dimensionality of issues

Good intentions
Protection against exploitation

Avoiding cycles:
Core:
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A core point is a point that

Priority: registering
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Good intensions
Protection against exploitation

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Avoiding cycles:
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A core point is a point that with the specified rule,
Dimensionality of issues

- Good intensions
- Protection against exploitation

Priority: registering

Avoiding cycles:

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Dimensionality of issues

Good intentions
Protection against exploitation

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Dimensionality of issues

Good intentions
Protection against exploitation

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Dimensionality of issues

Good intensions
Protection against exploitation

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Computer -- core is empty
Dimensionality of issues

Good intensions
Protection against exploitation

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Would like structure of laws to have core
Dimensionality of issues

Good intensions
Protection against exploitation

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Majority vote -- a point that can be all others with a majority vote.
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Good intentions
Protection against exploitation

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Protection against exploitation

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Good intensions
Protection against exploitation

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Good intensions
Protection against exploitation

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Dimensionality of issues

Good intensions
Protection against exploitation

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Median voter theorem

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Good intentions  
Protection against exploitation

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Dimensionality of issues

Good intentions
Protection against exploitation

Priority: registering

Median voter theorem
One issue, core always exists

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Majority vote -- a point that can be all others with a majority vote.
Dimensionality of issues

Good intentions
Protection against exploitation

Priority: registering

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Computer -- core is empty
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Dimensionality of issues

- Good intensions
- Protection against exploitation

Avoiding cycles:
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Computer -- core is empty

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Dimensionality of issues

Good intentions
Protection against exploitation

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Computer -- core is empty
Would like structure of laws to have core
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Priority: registering

Plott diagram
Core?

Hours

Stipend
Core?

Hours

Stipend
Core?

Hours

Stipend
Core?

Hours

Stipend
Core?

Here the core is empty

Hours

Stipend
Core?

Here the core is empty

McKelvey:
McKelvey:
For majority rule, any number of voters, empty core, select a starting and ending point. There exists an agenda going from one to the other.
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Idea for proof: Differential topology
McKelvey:
For majority rule, any number of voters, empty core, select a starting and ending point. There exists an agenda going from one to the other.
Idea for proof: Differential topology
Examples:
McKelvey:
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Idea for proof: Differential topology

Examples:
Iraq
McKelvey:
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Examples:
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Shifting coalitions
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Idea for proof: Differential topology

Examples:
- Iraq
- Shifting coalitions
- Good intensions:
  NSF, engineering, with “continued improvement”
Core?  Here the core is empty

McKelvey:
For majority rule, any number of voters, empty core, select a starting and ending point. There exists an agenda going from one to the other.

Idea for proof: Differential topology

Examples:
Iraq
Shifting coalitions

Good intensions:
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q-rule (supermajority vote)
Core? Here the core is empty

McKelvey:
For majority rule, any number of voters, empty core, select a starting and ending point. There exists an agenda going from one to the other.

Idea for proof: Differential topology
Examples:
Iraq
Shifting coalitions
Good intensions:
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q-rule (supermajority vote)
to win, need at least q (quota) votes.
For majority rule, any number of voters, empty core, select a starting and ending point. There exists an agenda going from one to the other. Idea for proof: Differential topology

Examples:
Iraq
Shifting coalitions
Good intentions:
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q-rule (supermajority vote) to win, need at least q (quota) votes.
Example: US Senate where q=60
Core? Here the core is empty

McKelvey:
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Example: US Senate where q=60

Tataru: For q-rule, empty core, same result as for McKelvey
For majority rule, any number of voters, empty core, select a starting and ending point. There exists an agenda going from one to the other.

Idea for proof: Differential topology

Examples:
- Iraq
- Shifting coalitions
- Good intensions: NSF, engineering, with "continued improvement"

For q-rule, empty core, same result as for McKelvey

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Tataru: For q-rule, empty core, same result as for McKelvey
Idea of proof:
symmetry group orbits of sets

q-rule (supermajority vote) to win, need at least q (quota) votes.
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Tuesday, April 29, 14
Tools and a question
Tools and a question
Tools and a question

Pareto set for coalition
Tools and a question

Pareto set for coalition
Tools and a question

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Tools and a question

Pareto set for coalition

Sen. Jeffords, VT
Tools and a question

Sen. Jeffords, VT
Core is the intersection of Pareto sets
Tools and a question

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Pareto set for coalition

Core is empty!
Tools and a question

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Tuesday, April 29, 14
Tools and a question

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Tools and a question

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Core is empty!

Pareto sets: winning coalition has three voters
Tools and a question

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Pareto sets:
winning coalition has three voters triangles
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Pareto sets: winning coalition has three voters triangles
All intersect at midpoint
Tools and a question

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Pareto sets:
winning coalition has three voters triangles
All intersect at midpoint core exists
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Tools and a question

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Core is empty!

Now, triangles do not intersect

Pareto sets: winning coalition has three voters triangles
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For core to be meaningful must be

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For core to be meaningful must be **structurally stable**
Tools and a question

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Pareto set for coalition

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Pareto sets: winning coalition has three voters triangles
All intersect at midpoint core exists

This is a mathematical topic in dynamical systems, and singularity theory
Tools and a question

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Core is empty
For core to be meaningful must be **structurally stable**

Pareto sets: winning coalition has three voters triangles
All intersect at midpoint core exists

This is a mathematical topic in dynamical systems, and singularity theory
Must expect tools from these areas will provide insights
Tools and a question

Sen. Jeffords, VT
Core is the intersection of Pareto sets

Core is empty!

Pareto sets: winning coalition has three voters triangles
All intersect at midpoint core exists

Now, triangles do not intersect
Core is empty
For core to be meaningful must be structurally stable

This is a mathematical topic in dynamical systems, and singularity theory
Must expect tools from these areas will provide insights
Early attempts by McKelvey and Scholfield
Answer:
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Preferences: replace distances with smooth utility functions
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Proof by use of singularity theory, Thom’s results about transverse intersections in Jet space, etc.
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**Theorem:** For a q-rule with n-voters core point to be structurally stable, the number of issues, k, must be less than or equal to
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Majority rule; odd number of voters so \( q = \frac{n+1}{2} \)
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Core point stable ONLY for \( k=(n+1)-n = 1 \) issue
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Even number of voters: $q = \frac{(n+2)}{2}$
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Majority rule; odd number of voters so q = \((n+1)/2\)
Core point stable ONLY for k=(n+1)-n = 1 issue
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Catholic Church and election of pope
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\( \frac{2}{3} \) rule So, if 120 voters, need 80 to win, 40 on losing side.
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\( \frac{3}{4} \) rule So, if 120 voters, need 80 to win, 40 on losing side.
\( k= 2q-n = 2(80) - 120 = 40 \), or the number needed to change outcome.
For a price .....
For a price .....

I will come to your group before your next election.
For a price ..... 

I will come to your group before your next election. You tell me who you want to win.
I will come to your group before your next election. You tell me who you want to win. After talking to everyone in your group, I will design a “fair” election rule, which includes all candidates.
For a price ..... 

I will come to your group before your next election. 
You tell me who you want to win. 
After talking to everyone in your group, I will design a 
“fair” election rule, which includes all candidates. 
Your candidate will win!
For a price ..... 

10 A>B>C>D>E>F

I will come to your group before your next election.
You tell me who you want to win.
After talking to everyone in your group, I will design a “fair” election rule, which includes all candidates.
Your candidate will win!
For a price ..... 

10 A>B>C>D>E>F
10 B>C>D>E>F>A 

I will come to your group before your next election. You tell me who you want to win. After talking to everyone in your group, I will design a “fair” election rule, which includes all candidates. Your candidate will win!
For a price ..... 

10 A>B>C>D>E>F
10 B>C>D>E>F>A
10 C>D>E>F>A>B

I will come to your group before your next election. You tell me who you want to win. After talking to everyone in your group, I will design a “fair” election rule, which includes all candidates. Your candidate will win!
For a price ..... 

Everyone prefers C to D to E to F

I will come to your group before your next election. You tell me who you want to win. After talking to everyone in your group, I will design a “fair” election rule, which includes all candidates. Your candidate will win!
For a price .....

10  A>B>C>D>E>F
10  B>C>D>E>F>A
10  C>D>E>F>A>B

Everyone prefers C to D to E to F
For a price ..... 

10  A>B>C>D>E>F  Everyone prefers C to D to E to F
10  B>C>D>E>F>A
10  C>D>E>F>A>B
For a price ..... 

10  A>B>C>D>E>F  Everyone prefers C to D to E to F
10  B>C>D>E>F>A
10  C>D>E>F>A>B
For a price ..... 

10 A>B>C>D>E>F
10 B>C>D>E>F>A
10 C>D>E>F>A>B

Everyone prefers C to D to E to F
For a price ..... 

10 A>B>C>D>E>F  Everyone prefers C to D to E to F
10 B>C>D>E>F>A
10 C>D>E>F>A>B
For a price.....

10 A>B>C>D>E>F
10 B>C>D>E>F>A
10 C>D>E>F>A>B

Everyone prefers C to D to E to F
For a price ..... 

10 A>B>C>D>E>F
10 B>C>D>E>F>A
10 C>D>E>F>A>B

Everyone prefers C to D to E to F
For a price ..... 

10 A>B>C>D>E>F Everyone prefers C to D to E to F
10 B>C>D>E>F>A
10 C>D>E>F>A>B
For a price ..... 

10  A>B>C>D>E>F
10  B>C>D>E>F>A
10  C>D>E>F>A>B

*Everyone* prefers C to D to E to F
For a price ..... 

Everyone prefers C to D to E to F

10 A>B>C>D>E>F
10 B>C>D>E>F>A
10 C>D>E>F>A>B
For a price ..... 

10  A>B>C>D>E>F  Everyone prefers C to D to E to F 
10  B>C>D>E>F>A 
10  C>D>E>F>A>B 

D
 E
   C
    B
     A
      F
For a price ..... 

10 A>B>C>D>E>F    Everyone prefers C to D to E to F 
10 B>C>D>E>F>A 
10 C>D>E>F>A>B
For a price ....

10 A>B>C>D>E>F  Everyone prefers C to D to E to F
10 B>C>D>E>F>A
10 C>D>E>F>A>B
For a price .....  

$10 \ A>B>C>D>E>F \quad \text{Everyone prefers C to D to E to F}$  

$10 \ B>C>D>E>F>A \quad \text{Fred wins by a landslide!!}$
For a price ..... 

10 A>B>C>D>E>F  Everyone prefers C to D to E to F
10 B>C>D>E>F>A
10 C>D>E>F>A>B

Fred wins by a landslide!!
For a price ..... 

10  A>B>C>D>E>F
10  B>C>D>E>F>A
10  C>D>E>F>A>B

Fred wins by a landslide!!
For a price .....  

10  A>B>C>D>E>F 
10  B>C>D>E>F>A 
10  C>D>E>F>A>B
For a price ..... *Mathematics?*

10  A>B>C>D>E>F
10  B>C>D>E>F>A
10  C>D>E>F>A>B
For a price .....  

Mathematics?  

Ranking Wheel

10  A>B>C>D>E>F  
10  B>C>D>E>F>A  
10  C>D>E>F>A>B
For a price .....  

10 A>B>C>D>E>F
10 B>C>D>E>F>A
10 C>D>E>F>A>B

Mathematics? Ranking Wheel
For a price .....  

10  A>B>C>D>E>F  
10  B>C>D>E>F>A  
10  C>D>E>F>A>B  

Mathematics?  

Ranking Wheel
For a price ..... *Mathematics?*

10 A>B>C>D>E>F
10 B>C>D>E>F>A
10 C>D>E>F>A>B
For a price .....
For a price.....

A > B > C > D > E > F
B > C > D > E > F > A
C > D > E > F > A > B

Mathematics? Ranking Wheel

A > B > C > D > E > F
For a price ..... mathematics? Ranking Wheel

10  A>B>C>D>E>F
10  B>C>D>E>F>A
10  C>D>E>F>A>B

A>B>C>D>E>F
For a price ..... *Mathematics?*   

10 A>B>C>D>E>F  
10 B>C>D>E>F>A 
10 C>D>E>F>A>B 

*Ranking Wheel*

A>B>C>D>E>F

*Rotate -60 degrees*
For a price ..... Mathematics? Ranking Wheel

A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

A>B>C>D>E>F

Rotate -60 degrees
For a price ..... Mathematics? Ranking Wheel

A>B>C>D>E>F
B>C>D>E>F>A
C>D>E>F>A>B

A
B
C
D
E
F

A>B>C>D>E>F

Rotate -60 degrees
For a price .....

Mathematics?  Ranking Wheel

A>B>C>D>E>F
B>C>D>E>F>A

Rotate -60 degrees
For a price ..... Mathematics? Ranking Wheel

10 A>B>C>D>E>F
10 B>C>D>E>F>A
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Rotate -60 degrees
For a price ..... Mathematics? Ranking Wheel

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10 C>D>E>F>A>B

Rotate -60 degrees

A> B> C> D> E> F
B> C> D> E> F> A
For a price ..... Mathematics? Ranking Wheel

10 A>B>C>D>E>F
10 B>C>D>E>F>A
10 C>D>E>F>A>B

A>B>C>D>E>F
B>C>D>E>F>A
C>D>E>F>A>B

etc.

Rotate -60 degrees
For a price ..... Mathematics? Ranking Wheel

10 A>B>C>D>E>F
10 B>C>D>E>F>A
10 C>D>E>F>A>B

No candidate is favored: each is in first, second, ... once.

Rotate -60 degrees

A>B>C>D>E>F
B>C>D>E>F>A
C>D>E>F>A>B

etc.
For a price .....  

Mathematics? Ranking Wheel

Ranking Wheel

Rotate -60 degrees

No candidate is favored: each is in first, second, ... once.

Yet, pairwise elections are cycles!
For a price ..... 

10  A>B>C>D>E>F
10  B>C>D>E>F>A
10  C>D>E>F>A>B

Mathematics? Ranking Wheel

Rotate -60 degrees

No candidate is favored: each is in first, second, ... once.

Yet, pairwise elections are cycles!

lost information!!
For a price ..... 

Mathematics?

A>B>C>D>E>F

B>C>D>E>F>A

C>D>E>F>A>B

No candidate is favored: each is in first, second, ... once.

Yet, pairwise elections are cycles!

lost information!!
For a price ..... Mathematics? Ranking Wheel

10 A>B>C>D>E>F
10 B>C>D>E>F>A
10 C>D>E>F>A>B

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Yet, pairwise elections are cycles!

lost information!!
For a price .....  

Mathematics?  

Ranking Wheel  

Rotate -60 degrees  

No candidate is favored: each is in first, second, ... once.  

Yet, pairwise elections are cycles!  

lost information!!
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*lost information!!*
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Yet, pairwise elections are cycles!

lost information!!
Source of all cycles; voting, statistics, engineering, etc.
For a price ..... 

10 A>B>C>D>E>F 
10 B>C>D>E>F>A 
10 C>D>E>F>A>B 

Symmetry: Z6 orbit

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For a price .....  

10  A>B>C>D>E>F  
10  B>C>D>E>F>A  
10  C>D>E>F>A>B  

Mathematics?  
Ranking Wheel  

Symmetry: Z6 orbit  

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lost information!!  
Everywhere!
Source of all cycles; voting, statistics, engineering, etc. For a price ..... 

Mathematics? Ranking Wheel 

Symmetry: Z_6 orbit 

No candidate is favored: each is in first, second, ... once. 

Yet, pairwise elections are cycles! 

lost information!! Everywhere! Manipulation, agenda fixing, all problems
Source of all cycles; voting, statistics, engineering, etc.
For a price ..... Solutions must address this!

Mathematics? Ranking Wheel

Symmetry: Z6 orbit

<table>
<thead>
<tr>
<th>10</th>
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