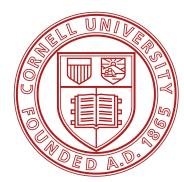
Learning Concepts and Doing Experiments with Language, Code, and Probability

Kevin Ellis

The Learning + Recursion Lab Work with: Wen-Ding Li, Keya Hu, Top Piriyakulkij, Yichao Liang, Cassidy Langenfeld, Hao Tang, Evan Pu, Zenna Tavares



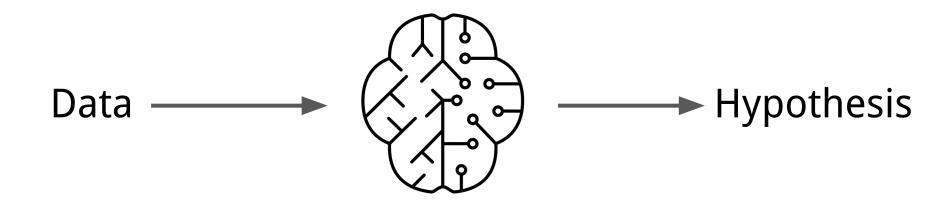
What you'll learn in this talk

1. Computational models of human few-shot learning

2. How to make LLMs better at forming hypotheses and doing experiments

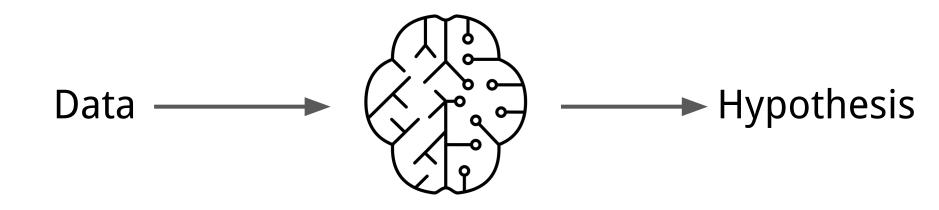
Part 1:

Human Few-Shot Learning

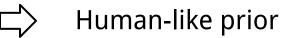


Goal: Generalization to unseen test data , aiming for human-like efficiency and flexibility *

*Few examples; Low-dimensional inputs



Efficient: few examples needed



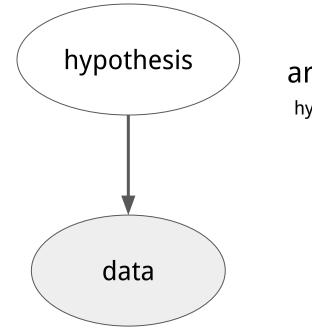
Flexible: infinite, diverse concepts

Compositional hypothesis space

Efficient (v2): little time/energy



Tractability vs Expressivity



argmax p(hypothesis) × p(data | hypothesis)

Tiny Subset of What Humans Learn







Science

Current Issue First release papers Archi

HOME > SCIENCE > VOL. 283, NO. 5398 > RULE LEARNING BY SEVEN-MONTH-OLD INFANTS

REPORTS

Rule Learning by Seven-Month-Old Infants

G. F. MARCUS, S. VIJAYAN, S. BANDI RAO, AND P. M. VISHTON Authors Info & Affiliations

Popular Idea: Compositional Hypothesis Space Recursive + Expressive

"possession of the infinitely many concepts that are expressible in an innate language of thought would be a curse: **the curse of a compositional mind.**"

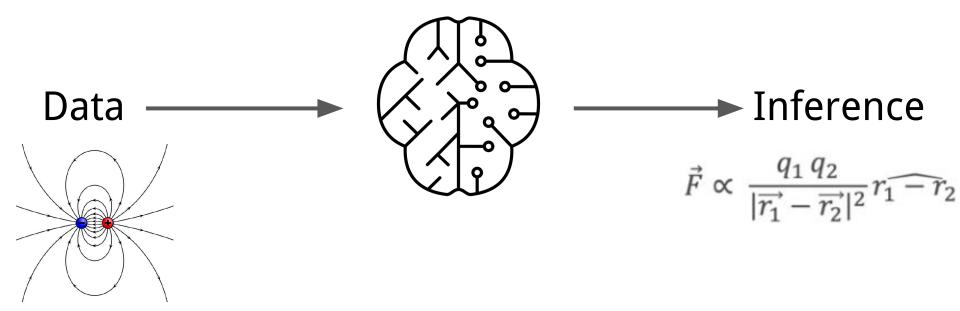
Spelke [2022]

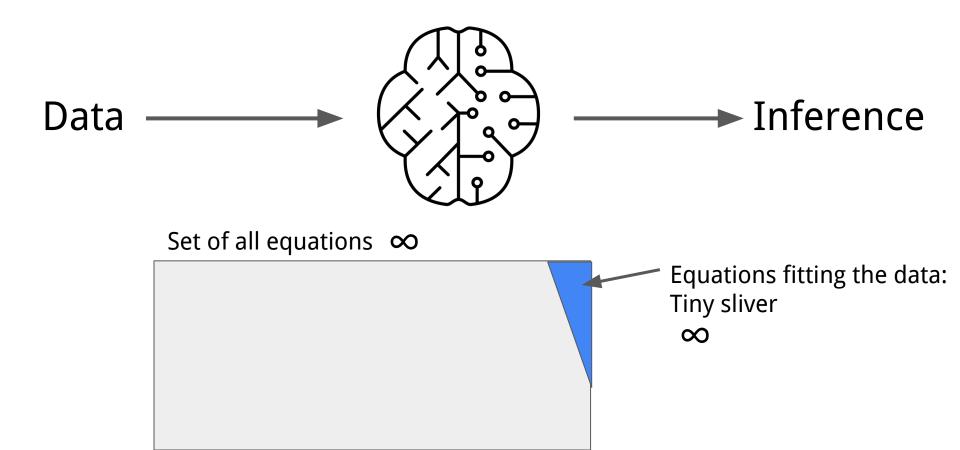
Languages for Composition

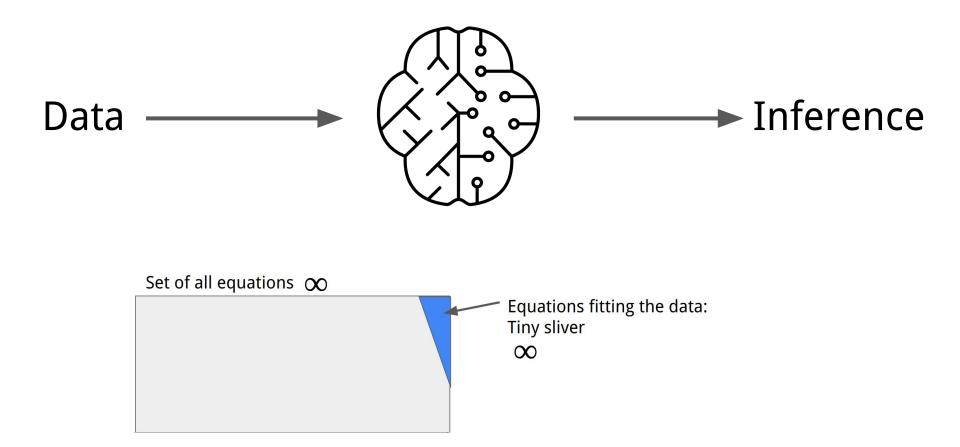
Coulomb's Law

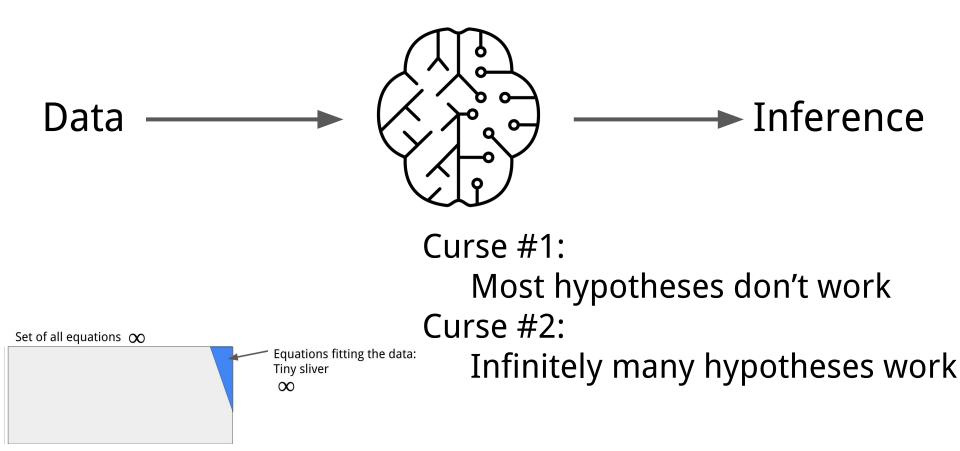
$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2} r_1 \widehat{-r_2}$$

Composed subparts: vector algebra ops



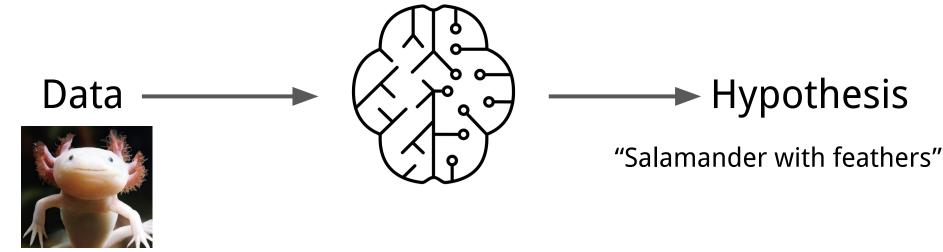






Part 1, Human part of talk:

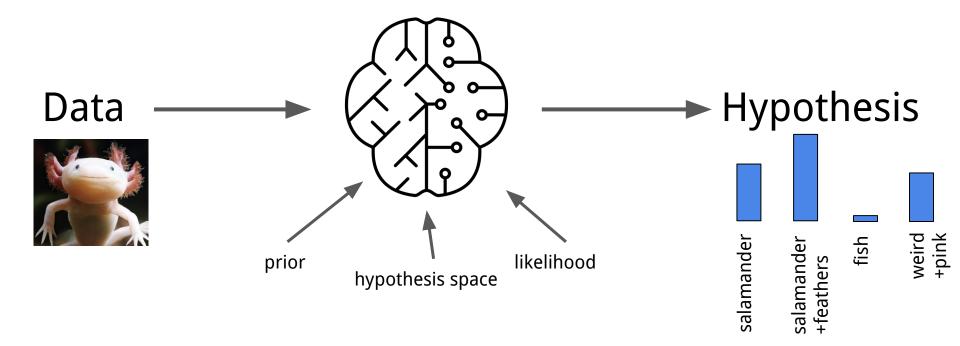
Taming the Curse of Compositionality, using Bayes and Natural Language

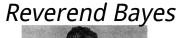


"Axolotl"

Model

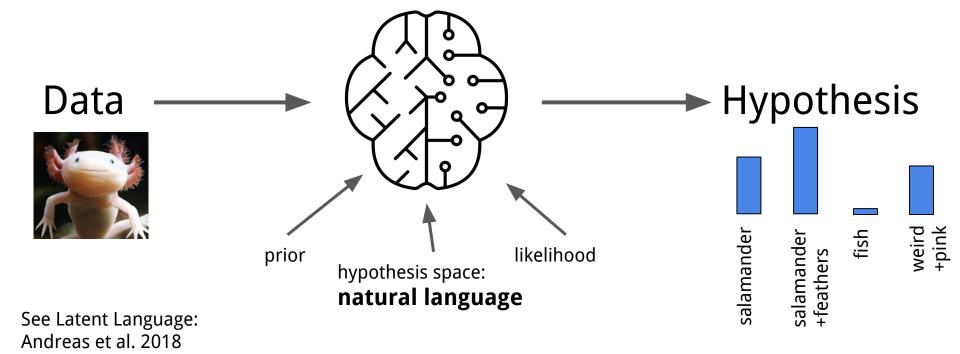
$P(hypothesis | data) \propto P(data | hypothesis) \times P(hypothesis)$





Model

$P(hypothesis | data) \propto P(data | hypothesis) \times P(hypothesis)$

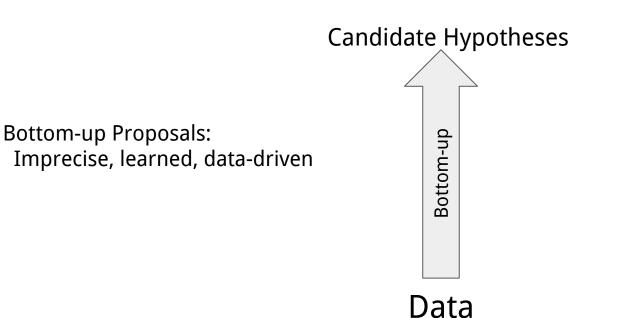


Reverend Bayes

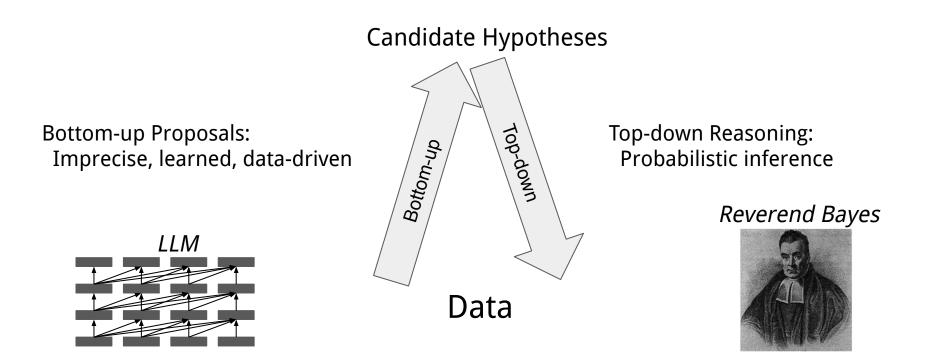
Tractable Approximate Inference:

Top-down + Bottom-up

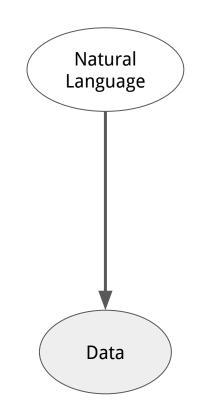
Tractable Approximate Inference



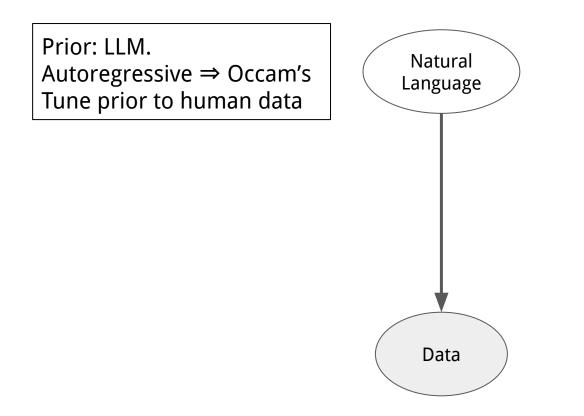
Tractable Approximate Inference



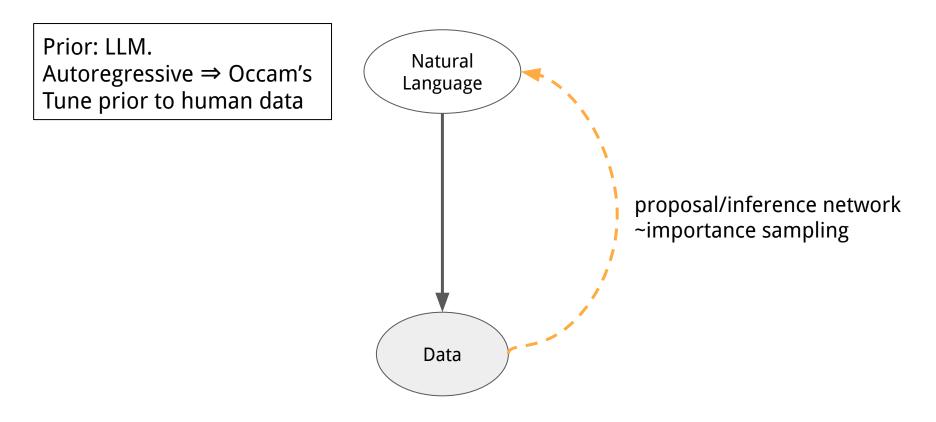
Bayesian Network



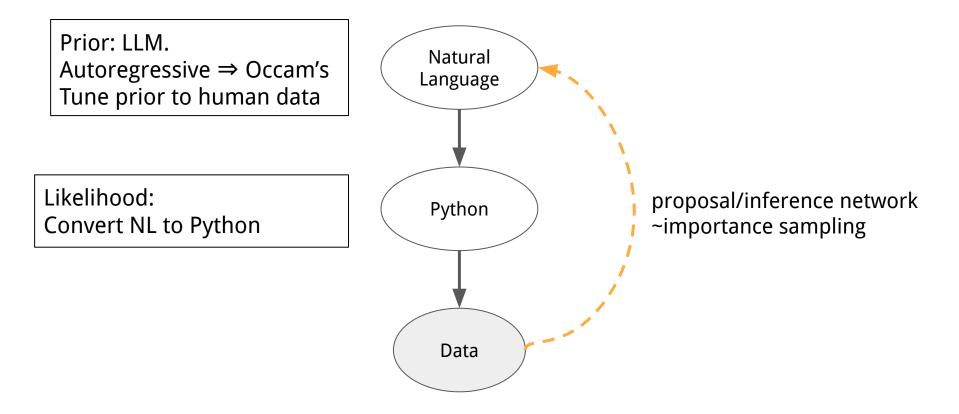
Bayesian Network: Learnable Prior



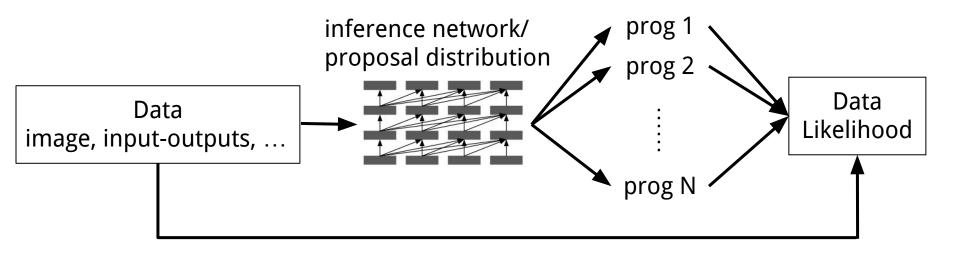
Bayesian Network: Data-Driven Proposal Distribution



Bayesian Network: Likelihood via Python Code Generation



Likelihood filters out bad proposals



Model vs Humans: Logical Concepts

Logical Concepts

"Bachelor"

(Male $\land \neg$ Married)

"Valedictorian"

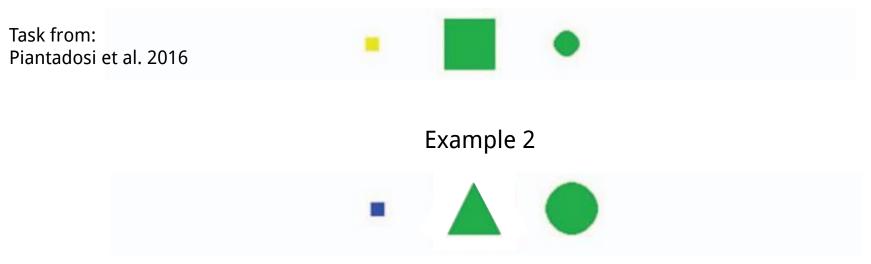
 $Valedictorian(x) \iff (\forall y : School(x) = School(y) \implies GPA(x) \ge GPA(y))$

Task+Data From Piantadosi et al. 2016: 112 concepts, >1k human participants

Example 1: No shapes in concept



Example 1: No shapes in concept



Example 1: No shapes in concept

Task from: Piantadosi et al. 2016



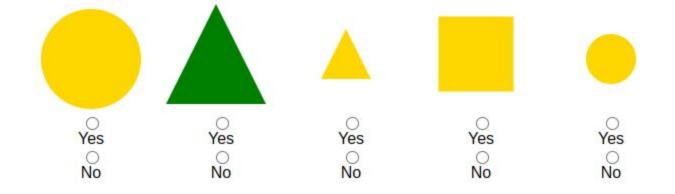
Example 2: Only middle shape in concept

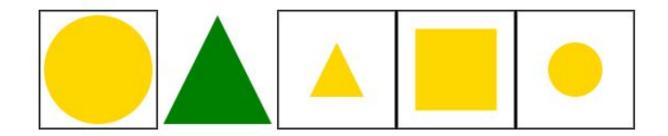


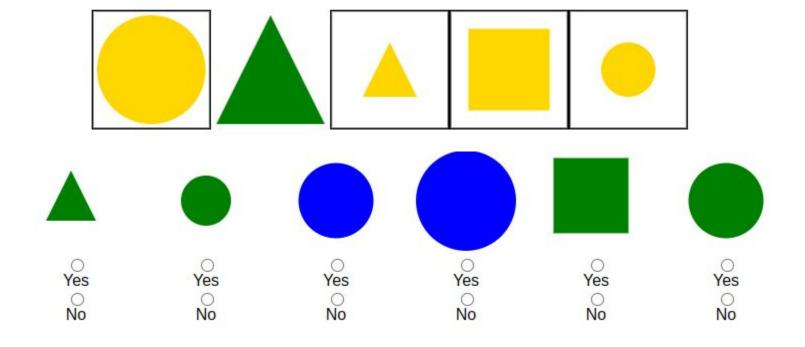
Example 1: No shapes in concept Task from: Piantadosi et al. 2016 Example 2: Only middle shape in concept

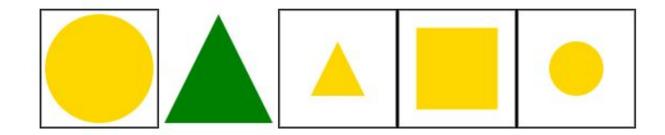
Example 3: Which shapes are in the concept?

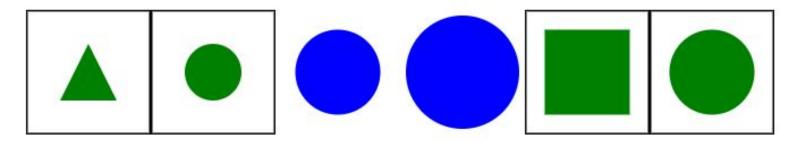


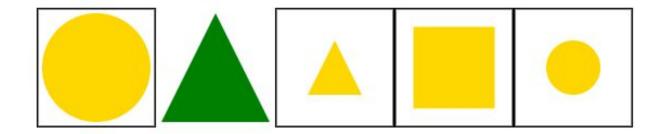


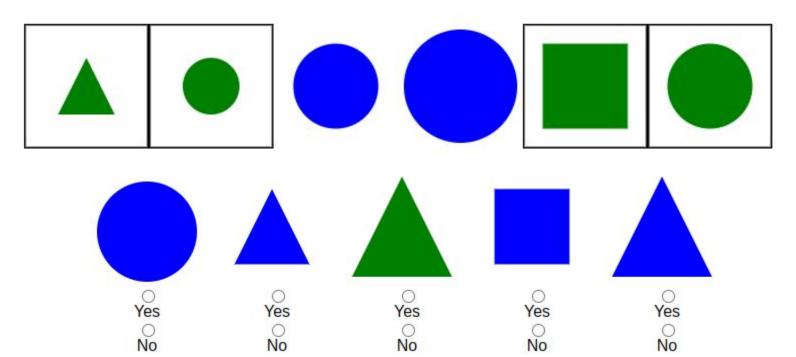


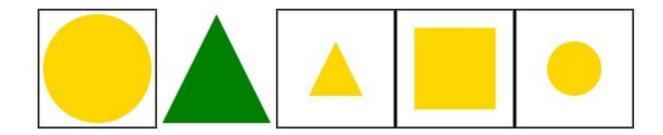


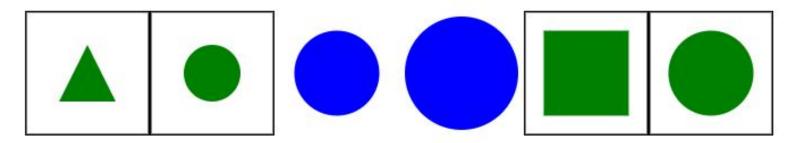


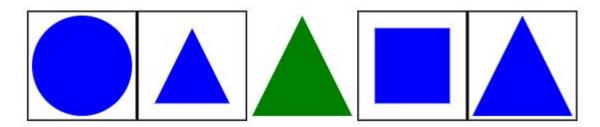






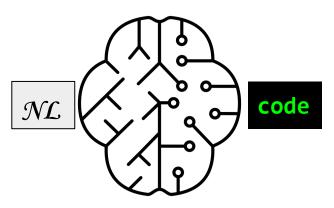






Model

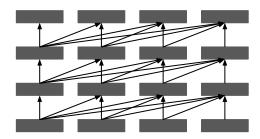
Hypothesis Space



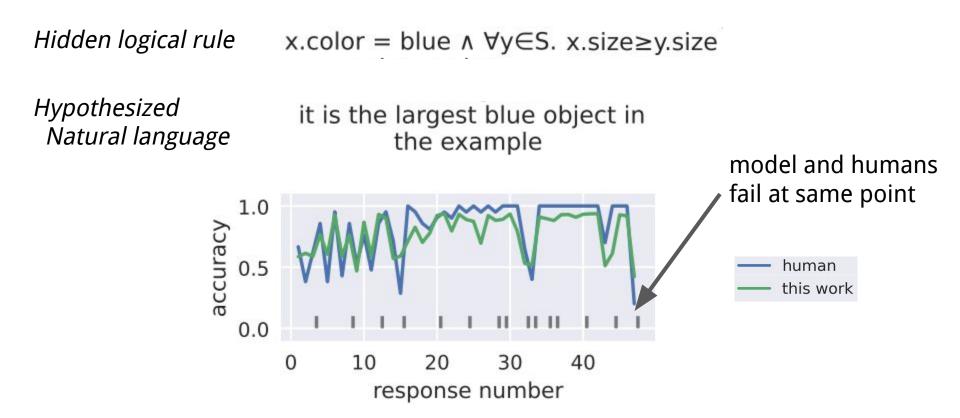
Bayes, learnable neural prior



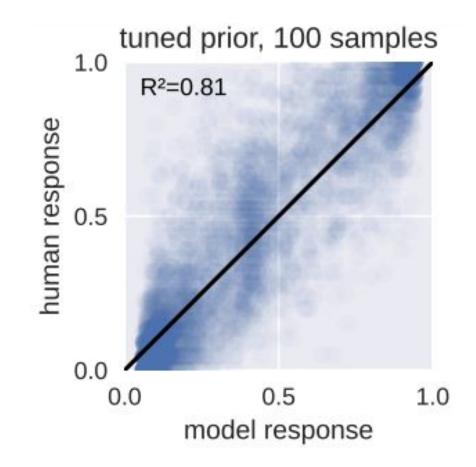
LLM Proposal distribution



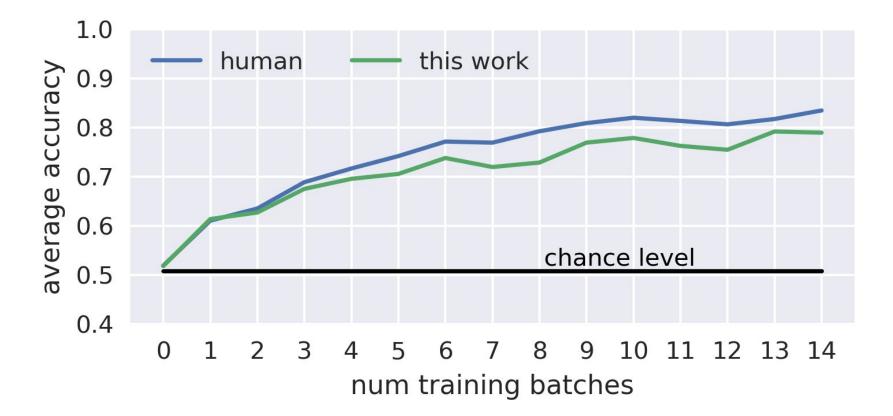
Model Predicts Human Learning Dynamics



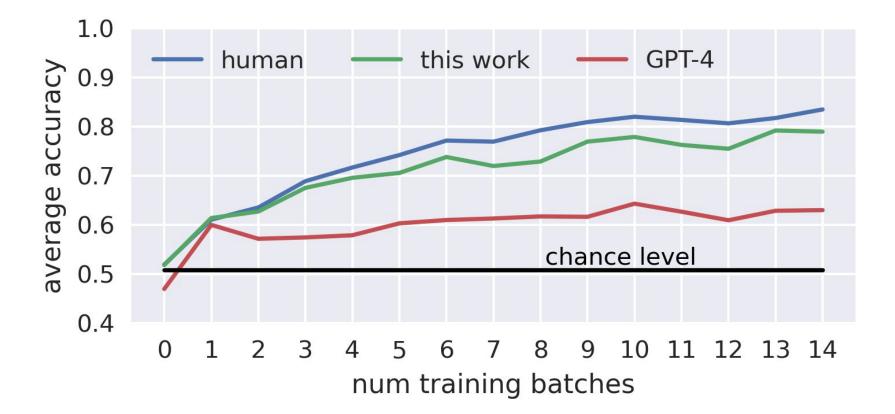
Model is Good but Not Perfect



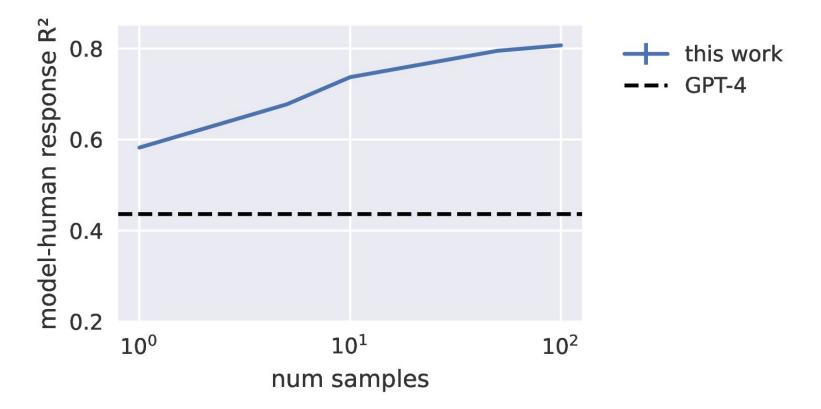
Efficient: Learns from few examples



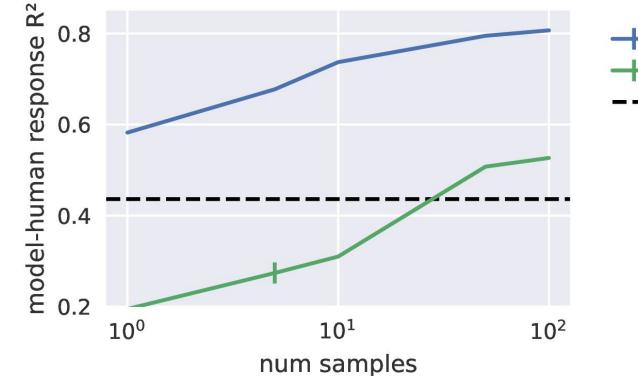
Efficient: Learns from few examples



Efficient: Requires modest compute



Efficient: Requires modest compute



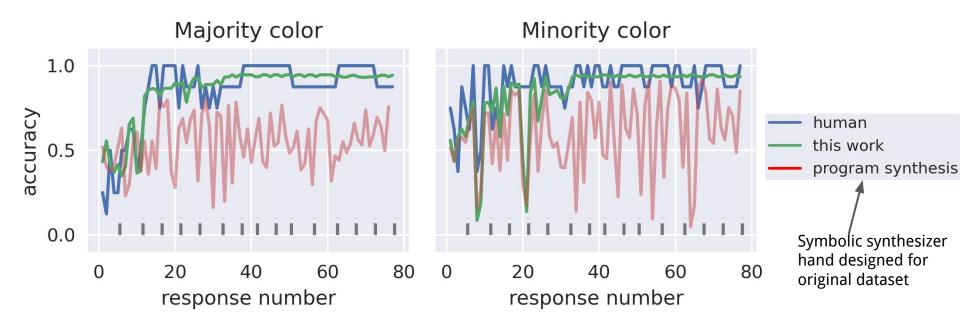
this work
 top-down proposals
 GPT-4

Flexibility test: Replicating to new subjects on out-of-distribution concepts

Majority color

Minority color

Flexibility test: Replicating to new subjects on out-of-distribution concepts



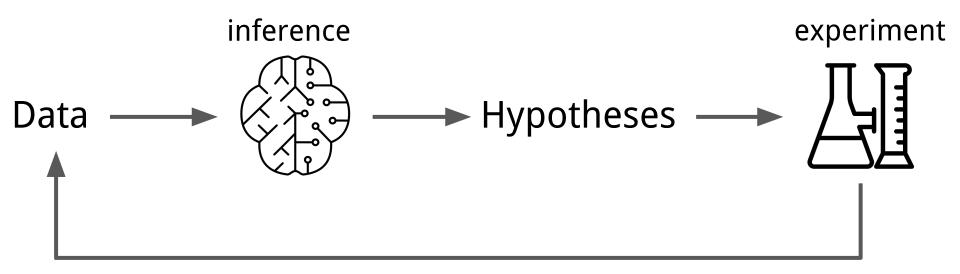
Lessons

LLMs and the Curse of Compositionality:

Tractably index an infinite concept space w/ finite compute

Neural prior gives good inductive bias

Adding Experimentation and Active Learning



Everyday experiments in adulthood:

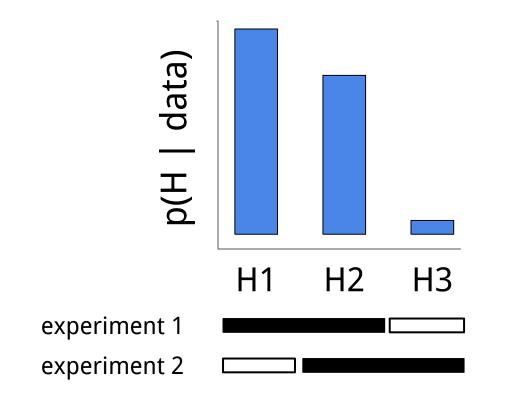
learning to use new devices, webpages, tools, fixing technical problems

Active learning during childhood development: exploratory play; active inference during early visual learning

Probabilistic beliefs are important for active learning

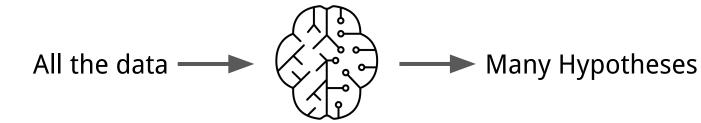
Η

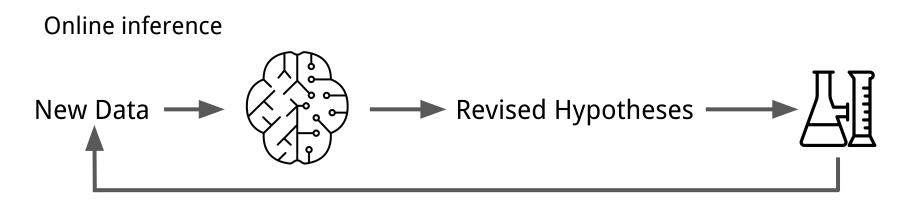
Probabilistic beliefs are important for active learning



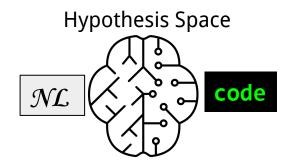
Online inference is important for active learning

Batch inference





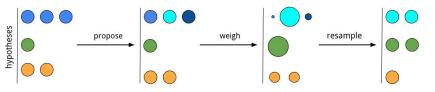
Active Learning Model



Bayes



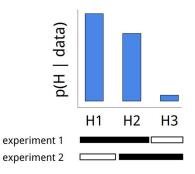
Online LLM-guided SMC



+fuzzy/noisy hypotheses don't immediately "kill" partly correct proposals

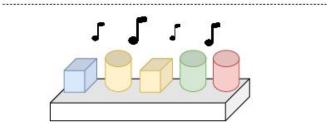
Piriyakulkij et al. NeurIPS '24

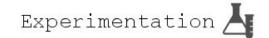
Max InfoGain experiments

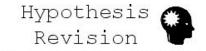


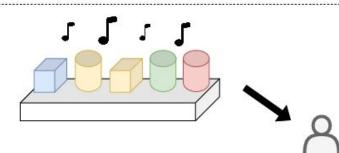
Basic Active Learning Domain: "Blicket Detectors"





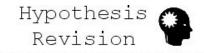


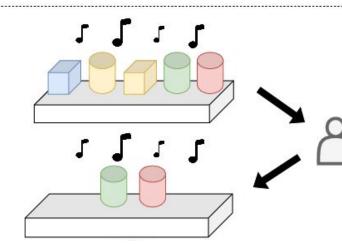




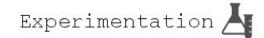
The machine makes sound when **Hypothesis 1**: at least one of them is a yellow object **Hypothesis 2**: more than three objects are present

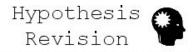


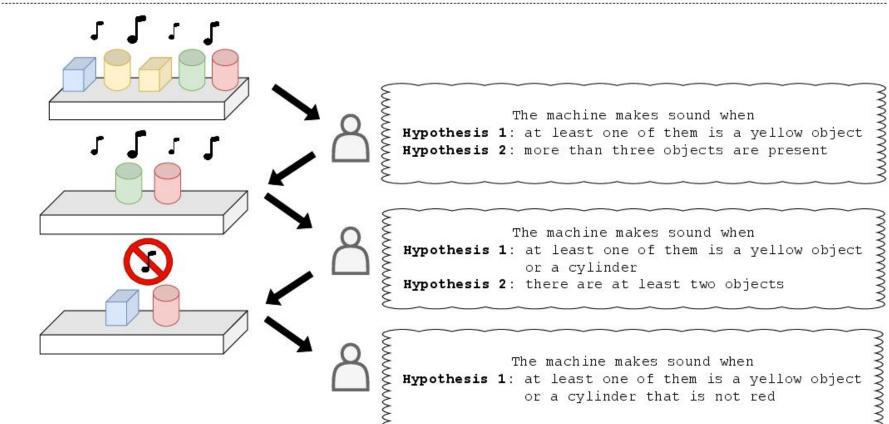




The machine makes sound when **Hypothesis 1**: at least one of them is a yellow object **Hypothesis 2**: more than three objects are present







Time

The Original Blicket Detector

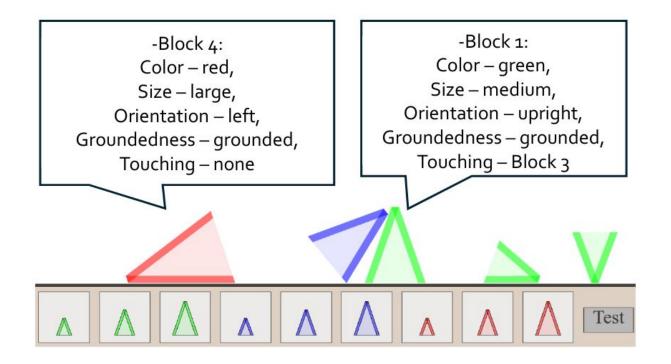


https://www.nytimes.com/video/science/10000002187112/buckets-of-blickets-children-and-logic.html

Zendo: harder Blicket-style task

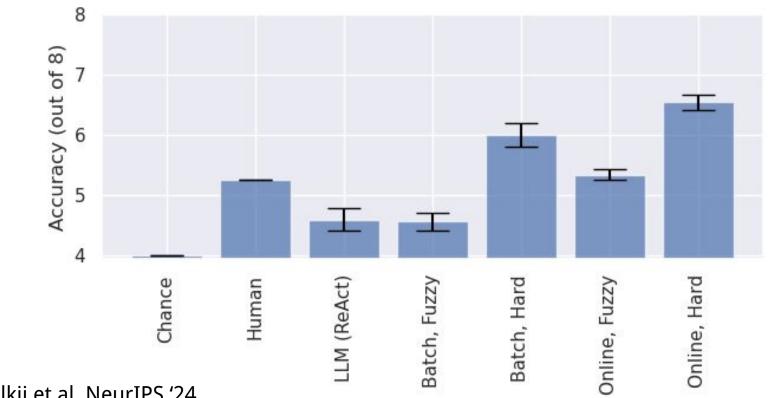


Zendo: harder Blicket-style task

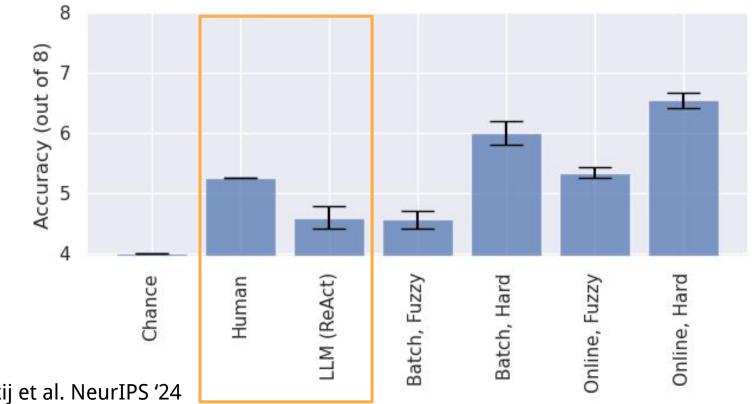


Task & Human Data from Bramley et al. 2018

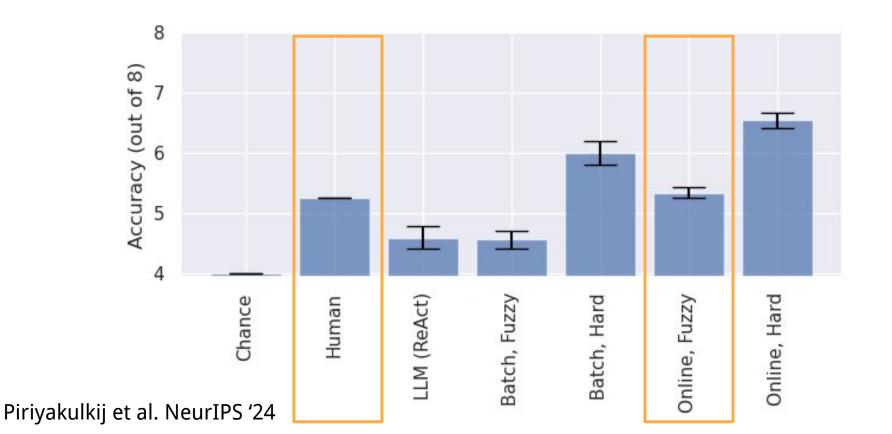
Zendo: performance



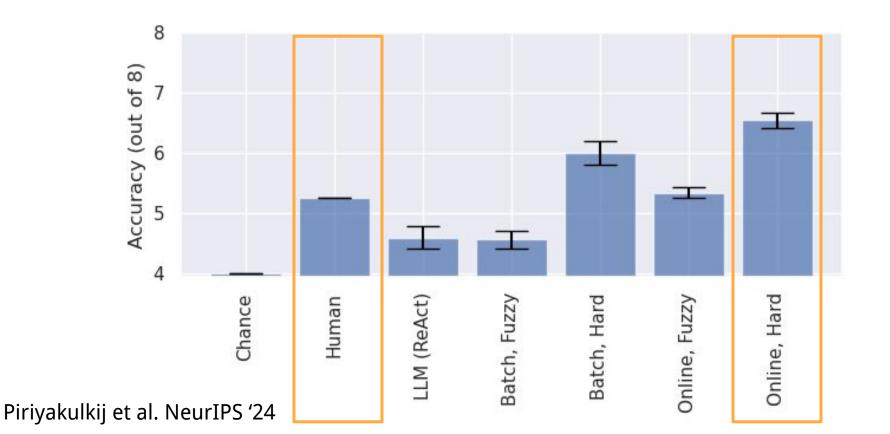
<Human Level with just prompting an LLM



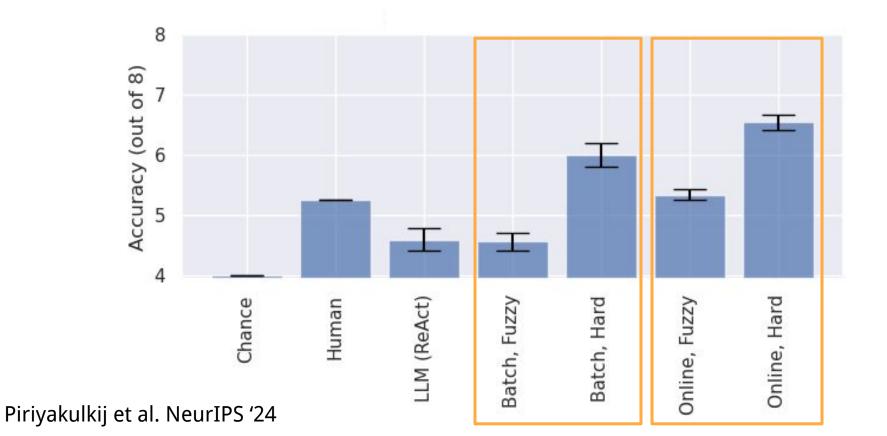
~Human Level with Fuzzy Probabilistic Rules



>Human Level with Deterministic Rules



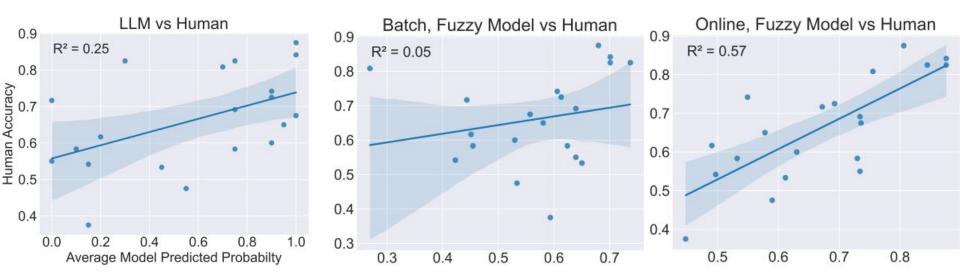
Online Inference beats Batch Inference



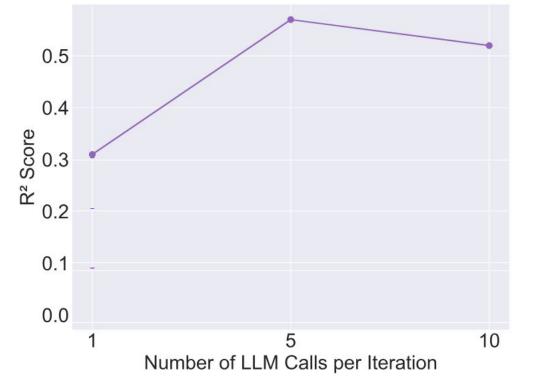
Human-level, Not quite Human-like

Human-level, Not quite Human-like, but Online + fuzzy rules best predicts human responses

Human-level, Not quite Human-like, but Online + fuzzy rules best predicts human responses



Bounded Rationality: Human-Model fit degrades with enough compute budget



*responses binned by problem and ground-truth label

Lessons

Probability important for picking good experiments

Online inference is more effective, and more humanlike

See Top Piriyakulkij & Cassidy Lagenfeld's NeurIPS '24 paper

Why are these models human-like?

Because they approximate rational inference over expressive, flexible representations

NOT because of LLMs: they're just proposal distributions

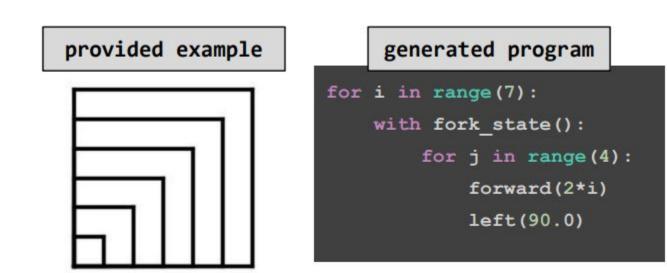
LLMs "just" give you tractable inference in expressive symbolic representations

Part 2:

Engineering Program Learners:

Program Induction in New Domains

What if your pretrained model can't propose good programs?



Finetune for program induction?

Where does the data come from?

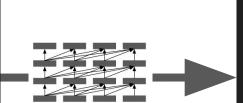
Human-Written Code

a 9-pointed star
for i in range(9):
 forward(16)
 left(180.0 - 40.0)

4 concentric squares
for i in range(5):
 with fork_state():
 for j in range(4):
 forward(2*i)
 left(90.0)

<dozens of examples>

Li & Ellis, NeurIPS '24



LLM-Written Remix

5 rectangle perimeter with a long dash and a small background color rectangle for i in range(5): forward(2) left(90.0) penup() forward(2) left(0.0) pendown() for i in range(2): forward(4) left(90.0) forward(16) left(90.0)

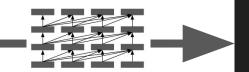
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Li & Ellis, NeurIPS '24

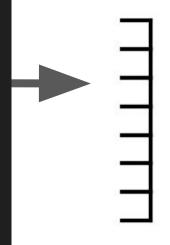


LLM-Written Remix

a spiral staircase
for i in range(7):
 forward(2)
 left(90.0)

forward(2)
left(90.0)

forward(2)
left(180.0)



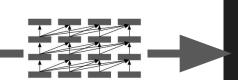
Human-Written Code

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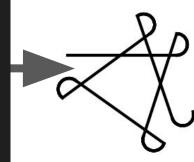
<dozens of examples>

Li & Ellis, NeurIPS '24



LLM-Written Remix

5 sided snowflake with a
medium line and a small
semicircle as arms
for j in range(5):
 forward(10)
 for i in range(HALF_INF):
 forward(EPS_DIST*1)
 left(EPS_ANGLE)
 forward(0)
 left(72.0)



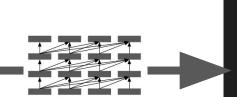
Human-Written Code

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for i in range(9):
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4 concentric squares
for i in range(5):
 with fork_state():
 for j in range(4):
 forward(2*i)
 left(90.0)

<dozens of examples>

Li & Ellis, NeurIPS '24



LLM-Written Remix

series of increasingly
rotated hexagonal shapes
for i in range(1, 8):
 for j in range(6):
 forward(4-i)
 left(60.0)
 penup()
 forward(2)
 pendown()

Human-Written Code

a 9-pointed star
for i in range(9):
 forward(16)
 left(180.0 - 40.0)

4 concentric squares
for i in range(5):
 with fork_state():
 for j in range(4):
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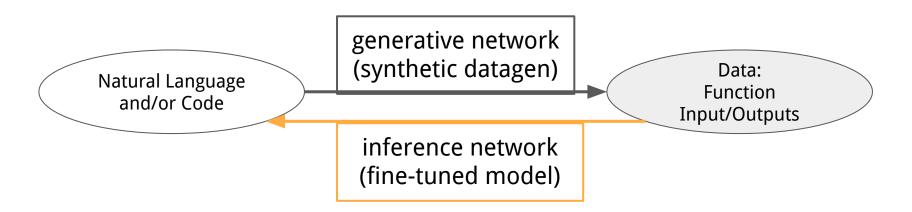
<dozens of examples>

Li & Ellis, NeurIPS '24

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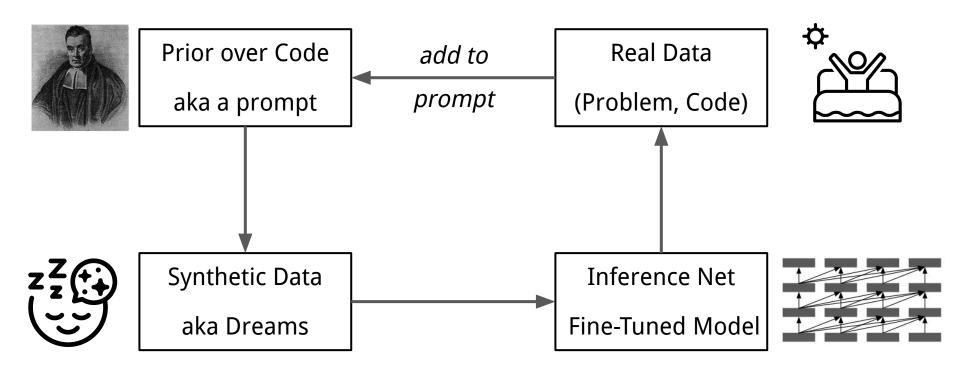
Wake-Sleep



Two models that train each other:

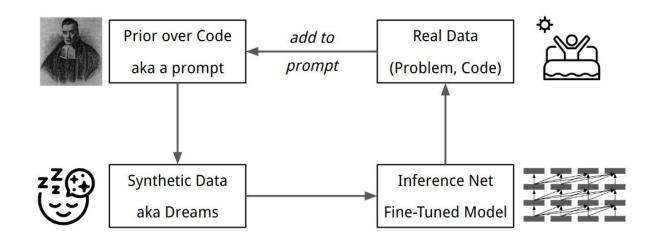
Generative path makes synthetic data

Inference network updates prompt for generative path



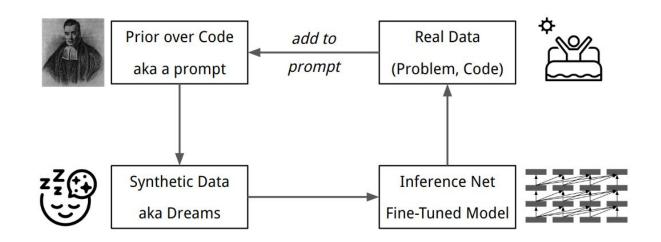
Data Efficient:

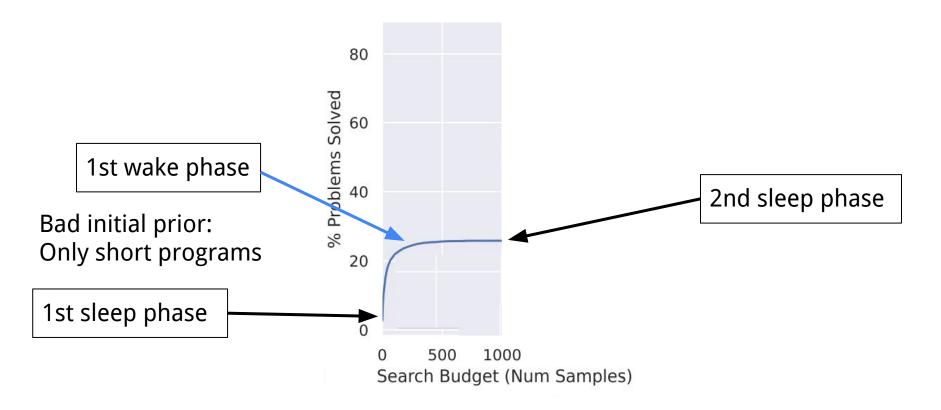
Needs relatively little non-synthetic data

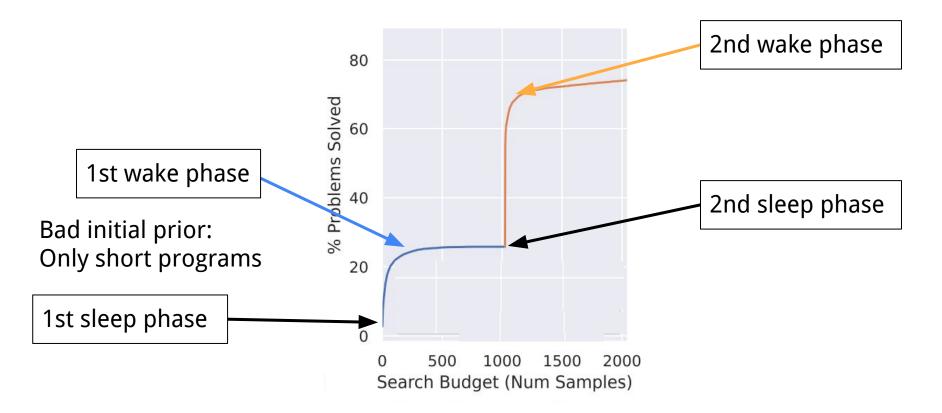


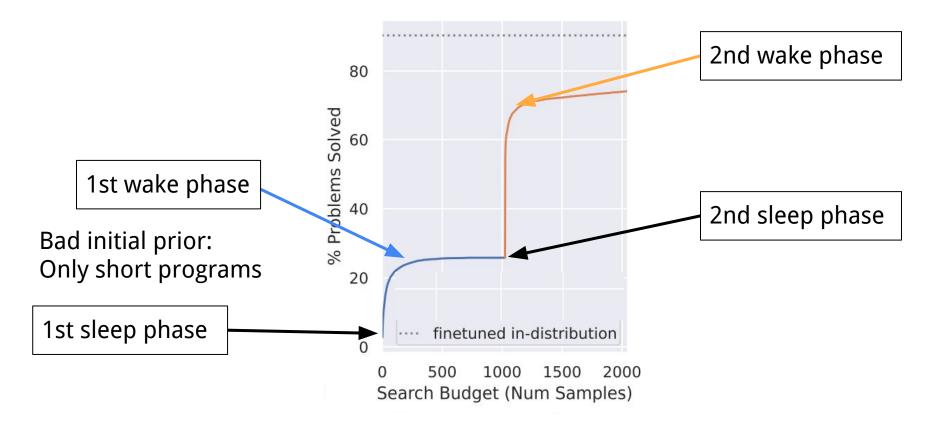
But needs to be "warm started" with a good prompt / prior

Otherwise, might get no learning signal









Why Wake-Sleep

Fine-tuning on synthetic data is conceptually simple [self-instruct]

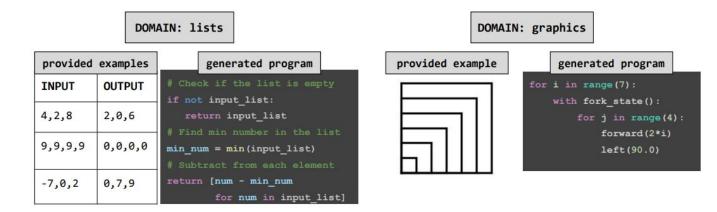
Why make things complicated?

Might not know the distribution of programs we care about

Connections to biological learning

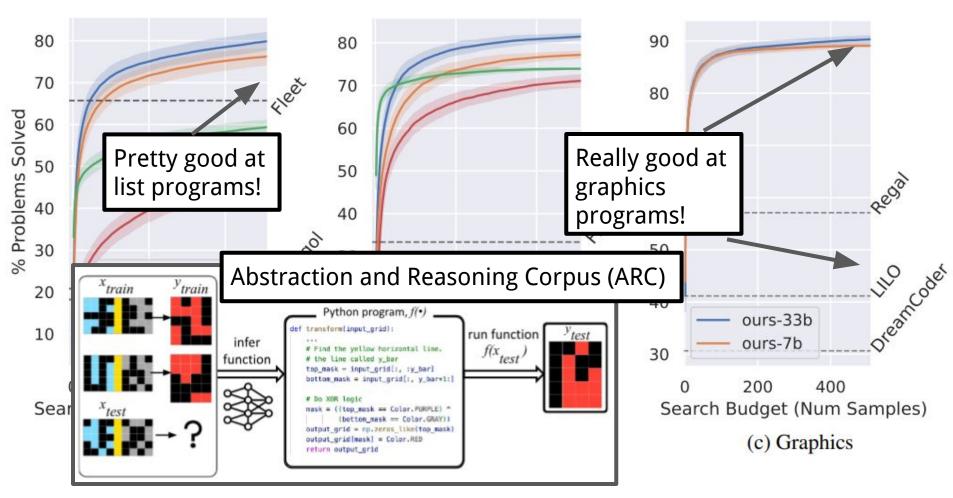
Learning to be a good Bayesian over the timespan of an individual lifetime

Cf. Tom Griffith's talk: learning to be Bayesian via evolution

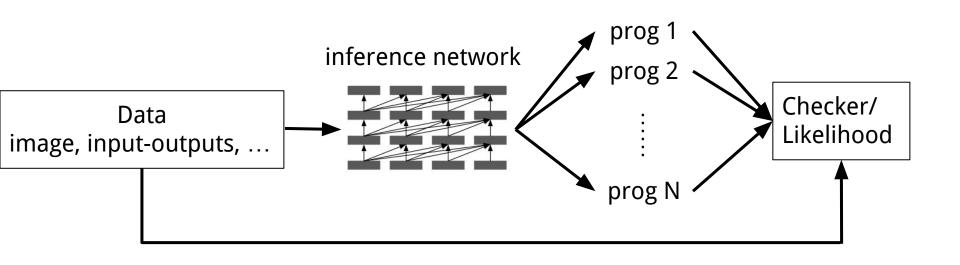


DOMAIN: text editing macros

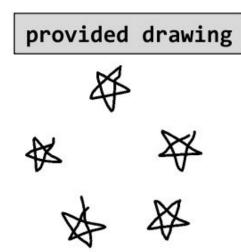
| provided examples | | generated program |
|-------------------|--------------|--|
| INPUT | OUTPUT | <pre>original_time = datetime.strptime(input_str, '%H:%M:%S') hour = original time.hour</pre> |
| 18:25:57 | 6PM to 8PM | <pre>start_hour = hour - (hour % 2) end_hour = start_hour + 2 start_hour_12 = start_hour % 12 or 12 end_hour_12 = end_hour % 12 or 12 start_ampm = "AM" if start_hour < 12 else "PM" end_ampm = "AM" if end_hour < 12 or end_hour == 24 else "PM" return f"{start_hour_12}{start_ampm} to {end_hour_12}{end_ampm}"</pre> |
| 21:44:40 | 8PM to 10PM | |
| 07:00:20 | 6AM to 8AM | |
| 23:34:17 | 10PM to 12AM | |



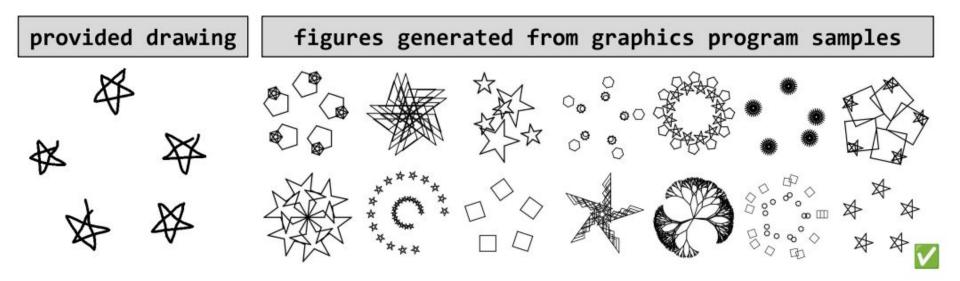
Caveat: Need to be about to check if an answer is correct



Caveat: Need to be about to check if an answer is correct

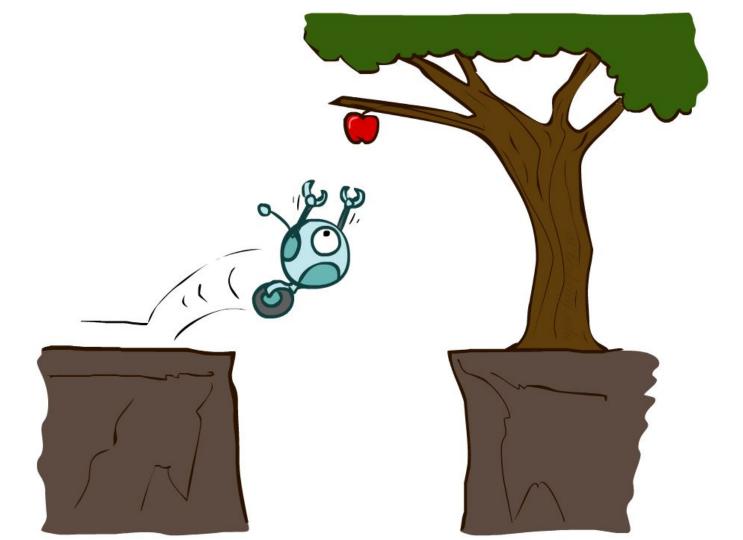


Caveat: Need to be about to check if an answer is correct



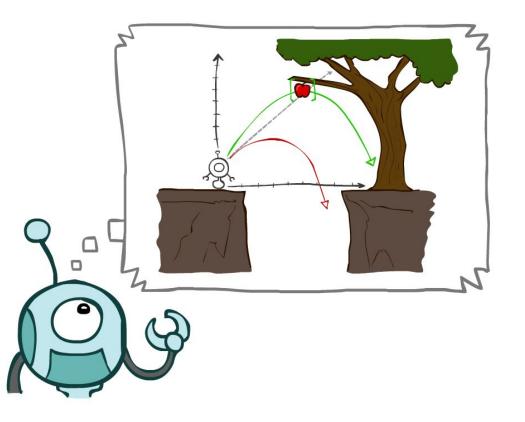
From programs that describe images,

to programs that describe how the world works



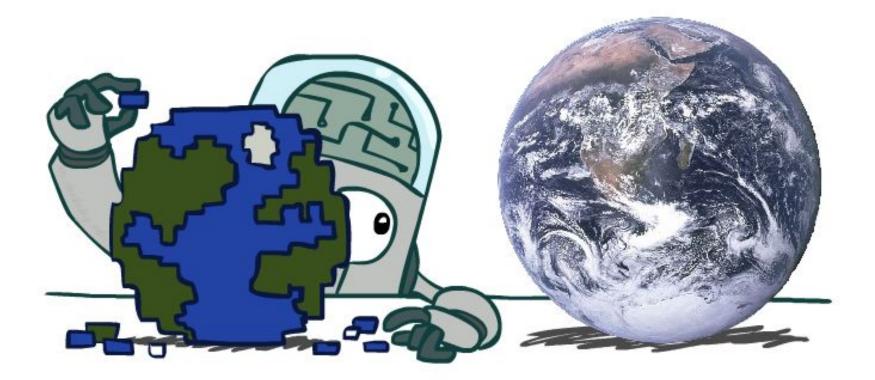
Picture credit: Berkeley CS188

World Models allow imagining the future

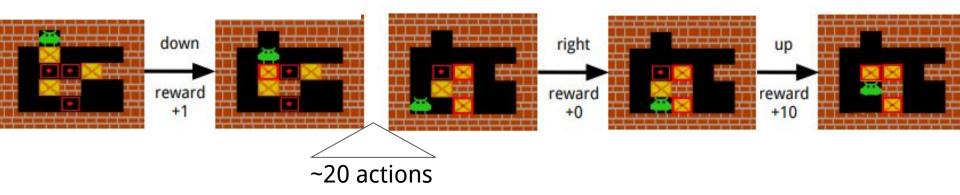


Picture credit: Berkeley CS188

World Models should be learned

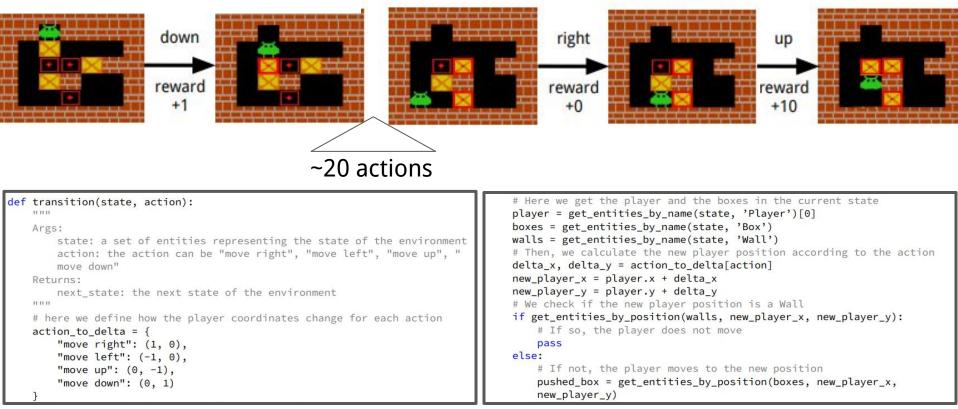


WorldModel : (State, Action) \rightarrow (NewState, Reward)



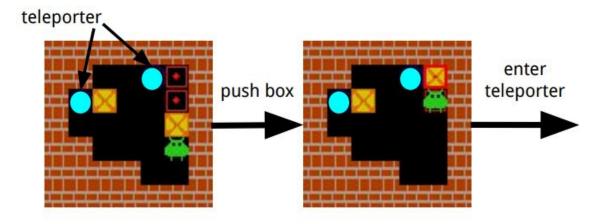
Hao Tang et al. NeurIPS '24

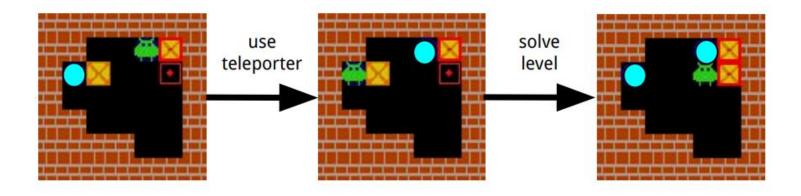
WorldModel : (State, Action) \rightarrow (NewState, Reward)



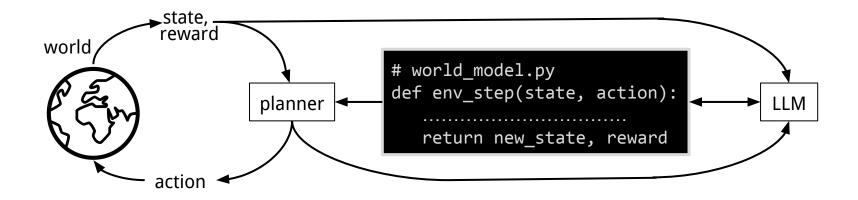
Hao Tang et al. NeurIPS '24

Not in pretraining: Sokoban + Teleporter





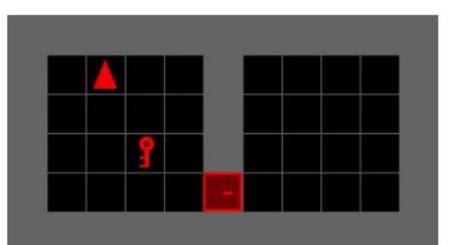
Agent Architecture

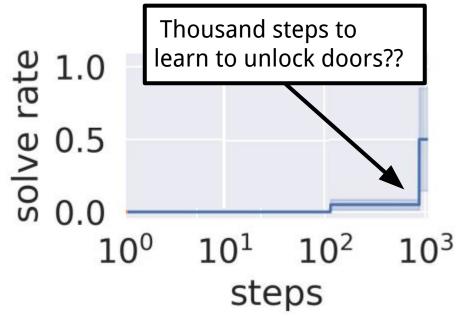


Like EMPA: Tsividis et al. 2021

Hao Tang et al. NeurIPS '24

Learning is Program Synthesis => Sample/Data Efficient ...if you have successful trajectories to learn from

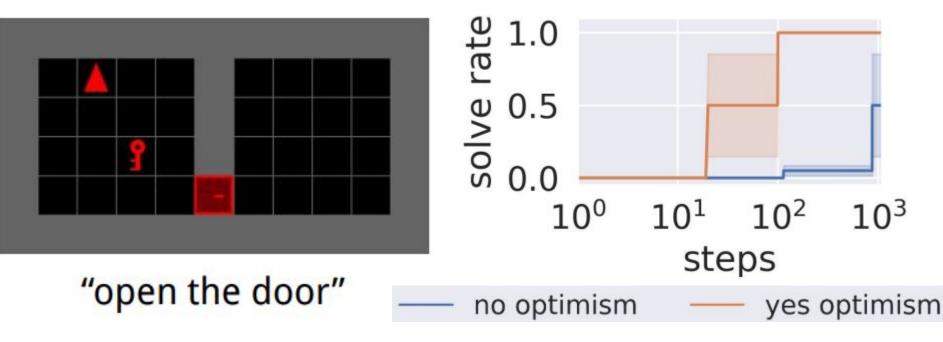




"open the door"

Hao Tang et al. NeurIPS '24

Learning is Program Synthesis => Sample/Data Efficient ...if you have successful trajectories to learn from



+optimism: inductive bias favoring optimistic world models

WorldCoder Lessons

A new prior over programs: **OPTIMISM**

Previously we'd favored simplicity

Online inference BUT no Bayesian framing

...but we're finding those framings useful for harder world models

Hao Tang et al. NeurIPS '24

Learning Abstract World Models

Yichao Liang et al. arXiv '24

Not Abstract World Models

genie: prompt->game



Yichao Liang et al. arXiv '24

+ sora: prompt->video

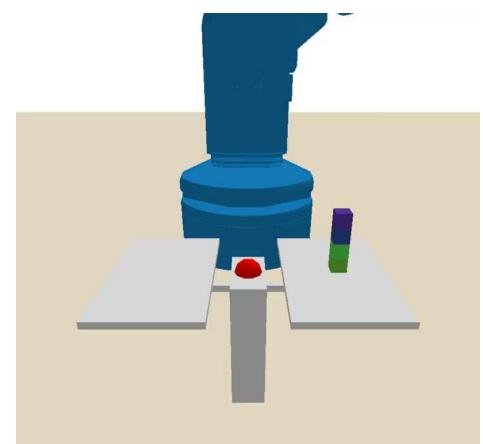
+Model Based RL more generally:

Dreamer, PlaNet, MuZero, ...

Unclear if pixel-learners understand that:

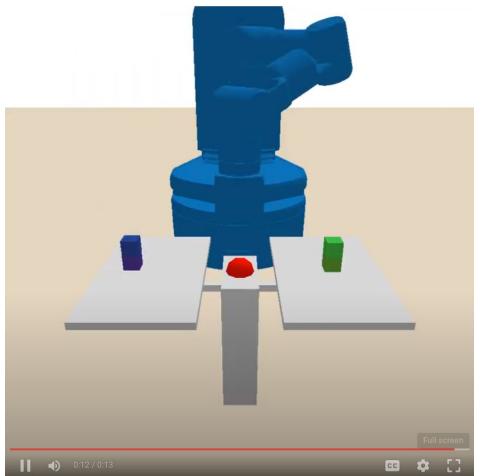
Going to college helps get a good job Balance beams can tell if masses are equal Raising prices lowers demand Water expands when frozen Trees can live a long time

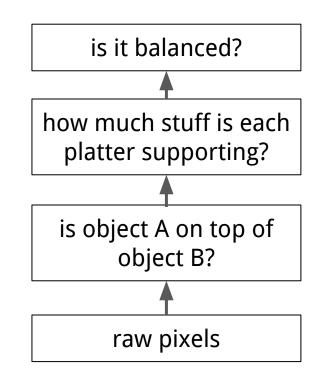
Abstract World Models



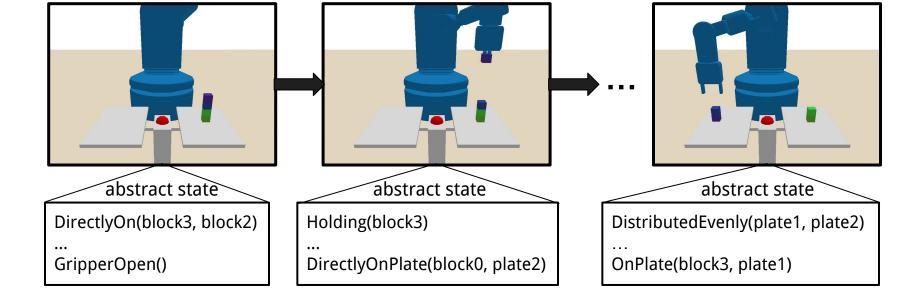
Yichao Liang et al. arXiv '24

Abstract World Models





Yichao Liang et al. arXiv '24



learned state abstraction

```
def DistributedEvenly(state, plate1, plate2):
```

```
if plate1 == plate2: return False
```

```
count1, count2 = 0, 0
```

```
for obj in state.objects:
```

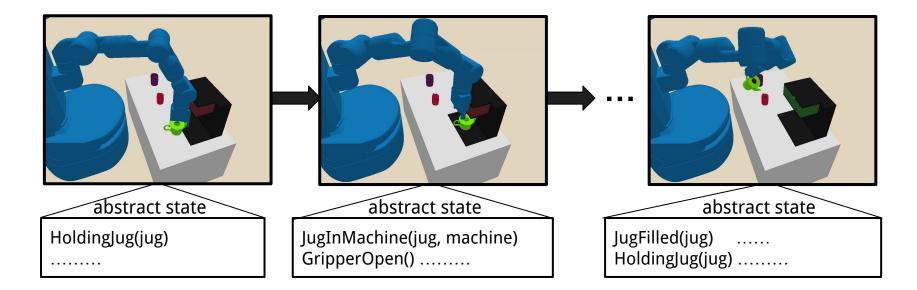
```
if OnPlate(state, obj, plate1): count1 += 1
```

```
if OnPlate(state, obj, plate2): count2 += 1
```

```
return count1 == count2
```

learned abstract state dynamics

```
def press_button(state, plate1, plate2):
    # Precondition
    assert distributed_evenly(state, plate1, plate2)
    # Postcondition
    new_state = state.copy()
    new_state['machine_on'] = True
    return new_state
```



learned state abstraction

def JugInMachine(state, jug, machine):

If the jug is held, it cannot be in the machine.

if Holding(state, state.robot, jug):

return False

Crop to focus on jug and coffee machine

attention_img = state.crop_to_objects([jug,machine])
return attention_img.query_VLM(

f"{jug.name} is placed inside {machine.name}.")

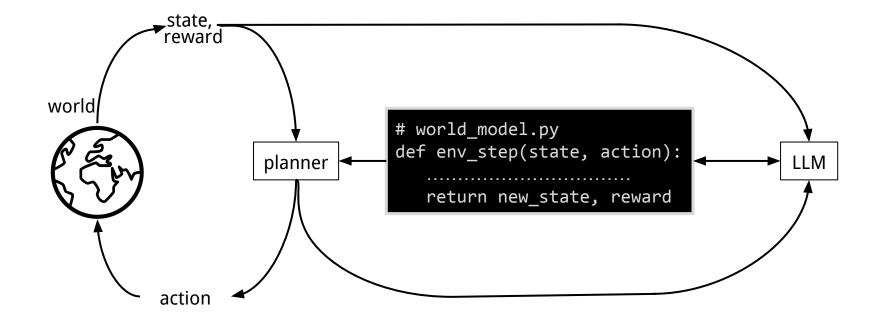
learned abstract state dynamics

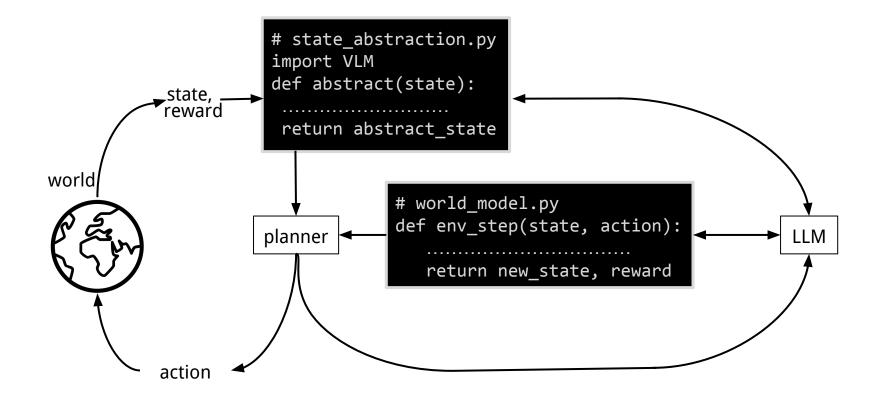
- def turn_on(state, coffee_machine, jug, robot):
 - # Precondition

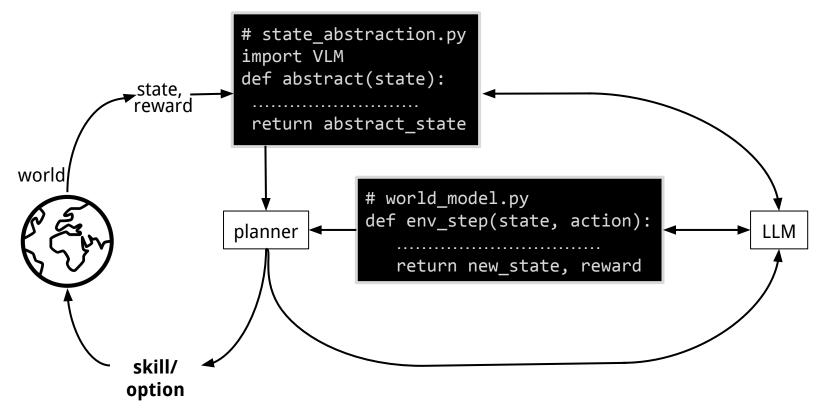
assert JugInMachine(state, jug, coffee_machine)

Postcondition

new_state = state.copy()
new_state['jug_filled'][jug] = True
return new_state







Assume pretrained temporally-extended high-level actions: "Skills"/Option

GenAI World Model vs Abstract World Model

GenAI world models:

Precise model of the world

Requires big training data

Cannot adapt on-the-fly to new dynamics

Abstract world model:

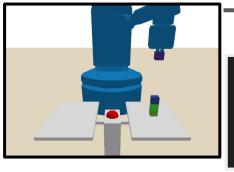
Incomplete model of the world

Requires big **pre**training data

Quickly learn new dynamics

Hierarchical Abstraction

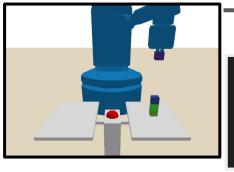
Limitations of this work in particular: full observability, determinism, fixed skills



def DirectlyOn(state, x, y): img = state.crop_to_objects([x,y]) return img.query_VLM(f"{x.name} is on top of {y.name}.")

Hierarchy: State abstractions recursively build on each other

```
def DistributedEvenly(state,plate1,plate2):
  if plate1 == plate2: return False
  cnt1, cnt2 = 0, 0
  for obj in state.objects:
    if OnPlate(state, obj, plate1): cnt1 += 1
    if OnPlate(state, obj, plate2): cnt2 += 1
  return count1 == count2
```



def DirectlyOn(state, x, y): img = state.crop_to_objects([x,y]) return img.query_VLM(f"{x.name} is on top of {y.name}.")

Note:

[+] OnPlate calls itself!

[-] Don't actually learn DirectlyOn

Hierarchy: State abstractions recursively build on each other

Yichao Liang et al. arXiv '24

def DistributedEvenly(state,plate1,plate2):
 if plate1 == plate2: return False
 cnt1, cnt2 = 0, 0
 for obj in state.objects:
 if OnPlate(state, obj, plate1): cnt1 += 1
 if OnPlate(state, obj, plate2): cnt2 += 1
 return count1 == count2

Lessons

Can't actually model the whole world in code!

Build symbolic abstractions of the world, and model those instead

Beyond 2-level hierarchy:

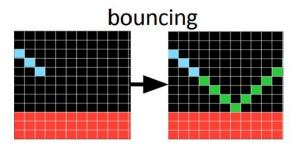
Abstractions can recursively build on top of abstractions

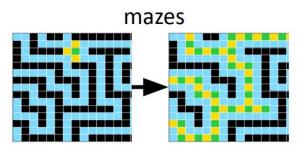
Last part:

How much of the world can/should we model in symbolic code?

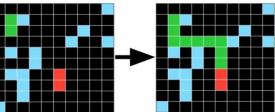
Abstraction and Reasoning Corpus [Chollet 2019]

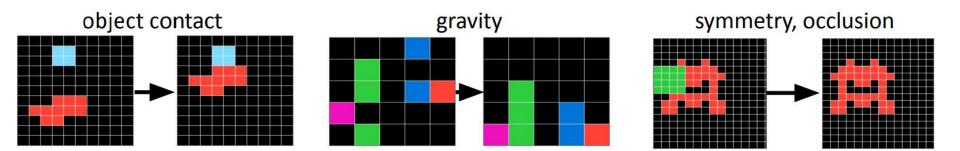
Abstraction and Reasoning Corpus [Chollet 2019]



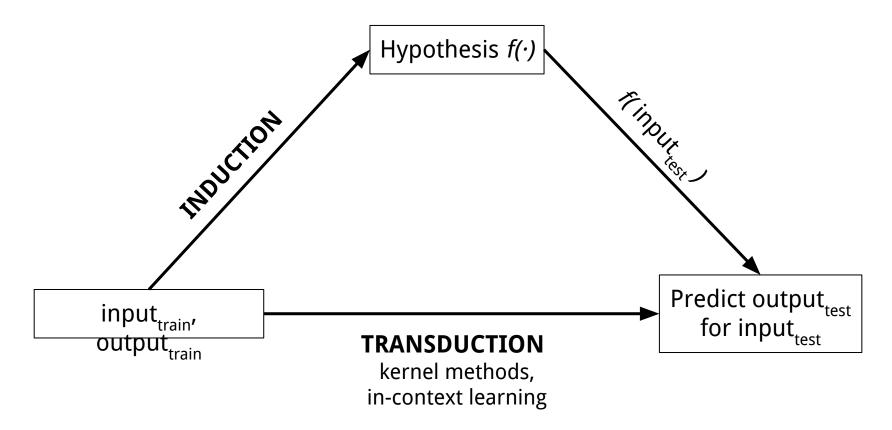




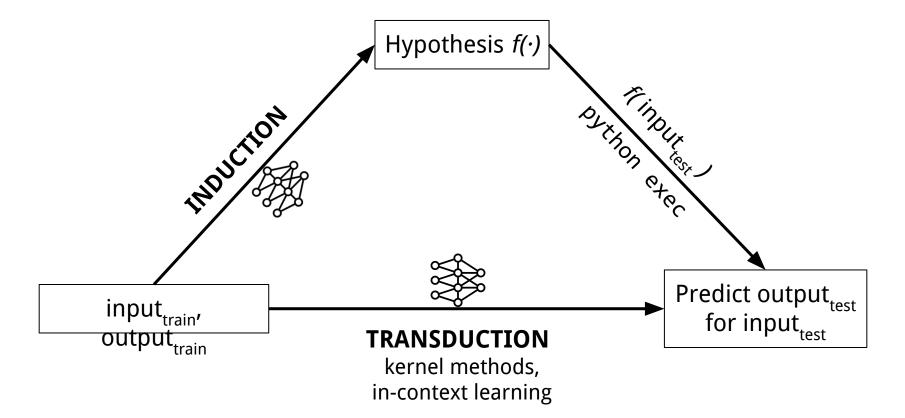




Frameworks for function learning



Neural Networks for Induction and Transduction

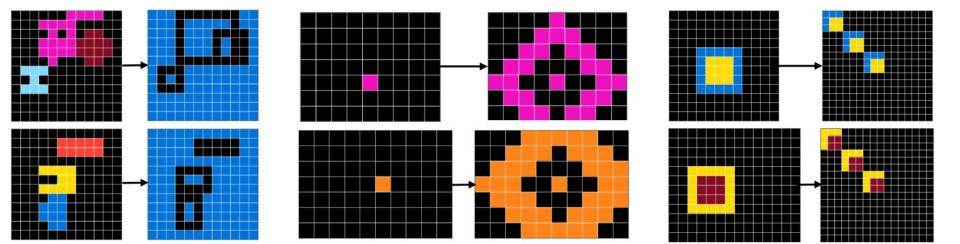


Meta-Learning Induction and Transduction

Metalearning dataset: Tuples of

<input_{train}, *f*(·), input_{test}>

400k synthetic problems, 100% explainable by Python code



Induction (Program Synthesis) VS Transduction (In-Context Learning)

It's a tie!

But solve different problems

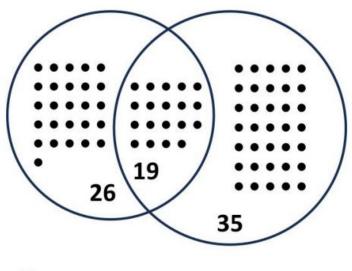
Weird!

- same training problems,

all solvable with programs

- same neural architecture

induction transduction



= 1 problem solved

Combining Induction and Transduction

Generate ONE dataset of few-shot learning problems

Fine-tune TWO models (induction&transduction)

Ensemble them

=> New SOTA on ARC, 54.4% on validation (human=60.2%)

What did we learn?

Generating symbolic hypotheses [Induction]

Imitating symbolic hypotheses [Transduction]

Complementary—*even controlling for neural architecture and training problems!*

Speculation: System 1/2 divide is normative-ish

Not an incidental consequence of architectural decisions

Wen-Ding Li: on industry job market



Keya Hu: applying to PhD's



Zenna Tavares: we're hiring for

next steps



Induction can count, and pinpoint the center of an object Transduction knows qualitative object relations/properties

