

# Gaming the cancer-immunity cycle by synchronizing the dose schedules

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Key point: Synchronizing and optimizing the chemotherapy and immunotherapy dosing schedules with the underlying periodicity of the cancer-immunity cycle can compensate for lower total dose

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APPLIED MATHEMATICS  
MEDICAL SCIENCES

## Gaming the cancer-immunity cycle by synchronizing the dose schedules

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Affiliations are included on p. 11.

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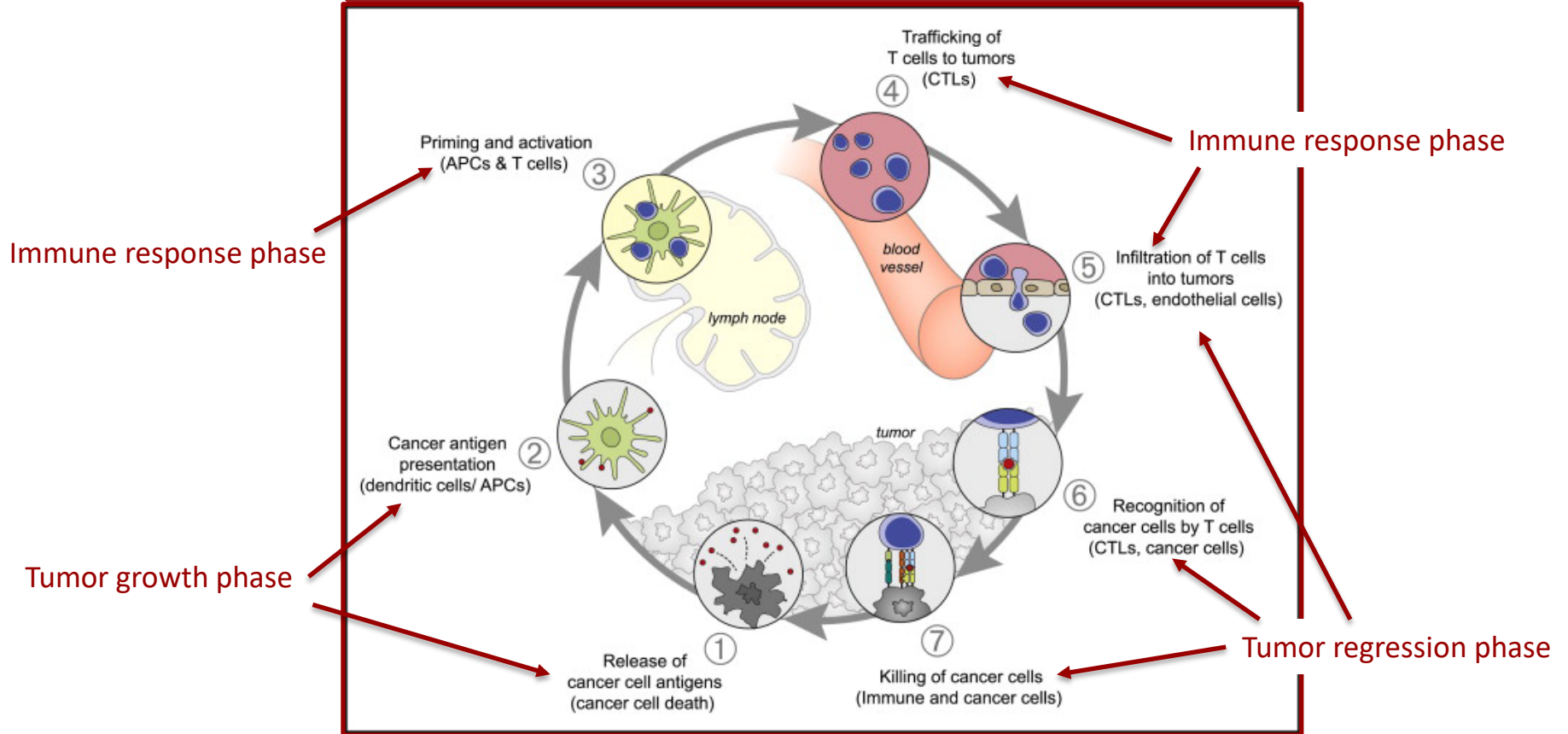
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Ph.D. AME USC 2025

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Mathematics of  
Cancer  
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# The cancer-immunity cycle

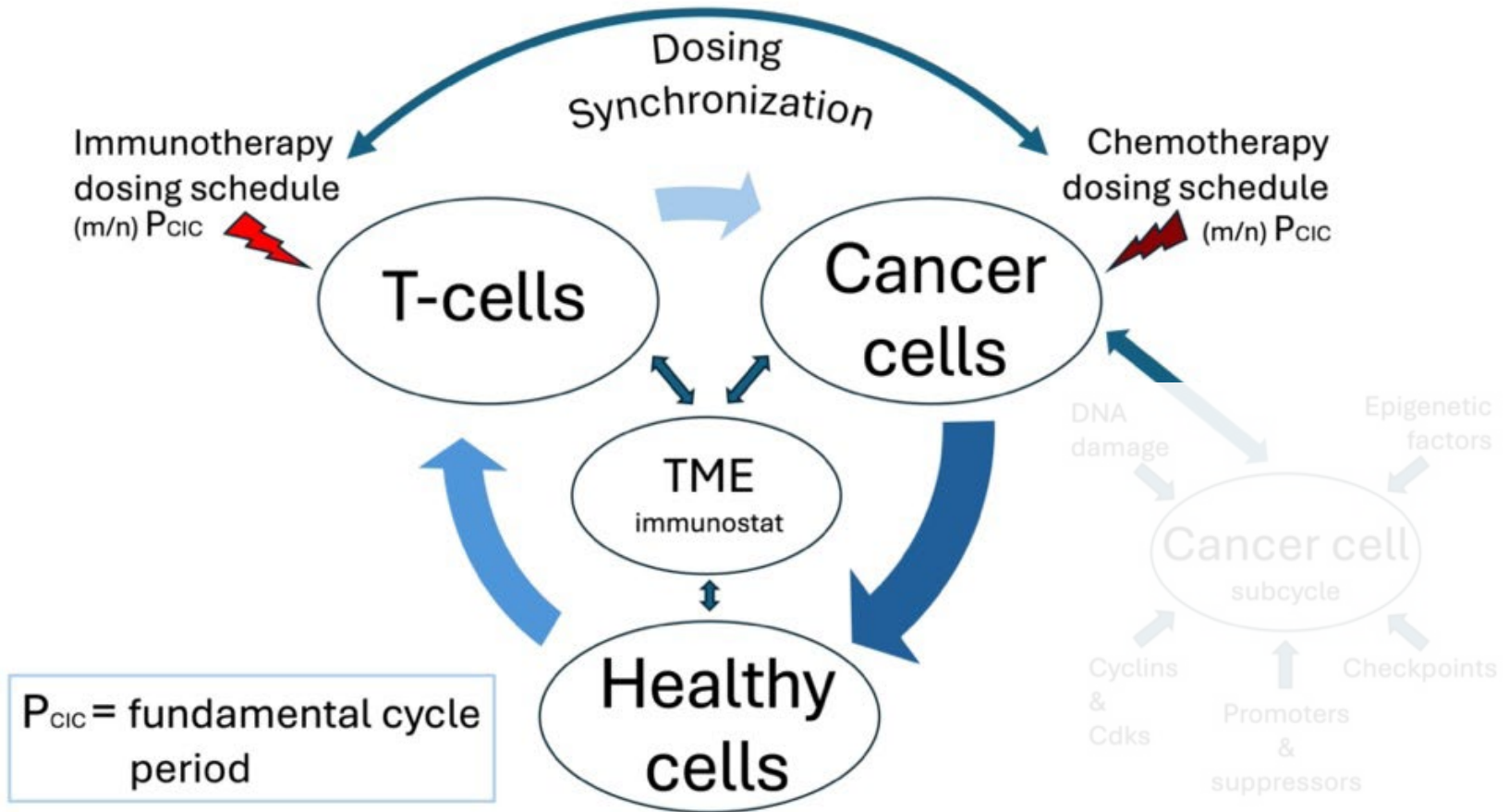


1. Release of cancer cell antigens
2. Cancer antigen presentation
3. Priming and activation of T-cells
4. Trafficking of T-cells to tumor
5. Infiltration of T-cells into tumor
6. Recognition of cancer cells by T-cells
7. Killing of cancer cells

**Oncology meets immunology: The cancer-immunity cycle,**  
D.S. Chen, I. Mellman, *Immunology* (2013)

**The cancer-immunity cycle: Indication, genotype, immunotype**  
I. Mellman, D.S. Chen, T. Powles, S.J. Turley, *Immunology* (2023)

# Key players in the cancer-immunity cycle



# The model: Part 1

## The evolutionary game & biological mechanisms

$$x_1 + x_2 = 1 \text{ (fractions)} \quad 0 \leq V(t) \leq K$$

$$0 \leq n \leq 1$$

Healthy cells

Tumor cells

T-cells

$$\dot{x}_1 = (f_1 - \langle f \rangle)x_1$$

$$\dot{x}_2 = (f_2 - \langle f \rangle)x_2$$

$$\dot{n} = \epsilon n(1 - n)h(x_2; \theta(t)) - S(V(t))$$

T-cell suppression mechanism that increases with tumor burden and adjusts for how immunogenic the tumor is

Replicator dynamics evolutionary game, frequency-dependent fitness, couples to T-cell eqn through fitness functions

$$h(x_2; \theta(t)) = \alpha_G x_2 - x_1$$

immune response timescale

$$x_2 < \frac{1}{1 + \theta} \Rightarrow h < 0$$

$$x_2 > \frac{1}{1 + \theta} \Rightarrow h > 0$$

Activation threshold

$$A_T = \frac{1}{1 + \theta}$$

Immune system growth/decay based on sign of h

$$\dot{V} = (\delta + \alpha_G)V(1 - V/K)$$

Tumor volume growth/decay

Tumor carrying capacity

$$S(V(t)) = \delta V(t) / (1 + \lambda I_t(t))$$

Growth/regression discrepancy parameter

$$I_t(t) = 1 / (\alpha + \beta \exp(-\gamma t))$$

Immunogenicity function to model immune evasion under selection pressure

$\theta(t)$ : Immunotherapy control function



Lowers activation threshold (checkpoint blockade immunotherapy)

$C(t)$ : Chemotherapy control function



Imposes selection pressure on cancer cells

The model: Part 2  
The payoff matrix A

$$\dot{\vec{x}}_i = \underbrace{[(A\vec{x})_i - (\vec{x}^T A\vec{x})]}_{f_i - \langle f \rangle} \vec{x}_i$$

$x_1$  (healthy)  
 $x_2$  (tumor)

$$A(n(t)) = [1 - g(n(t); I_t(t))] A_G + g(n(t); I_t(t)) A_R$$

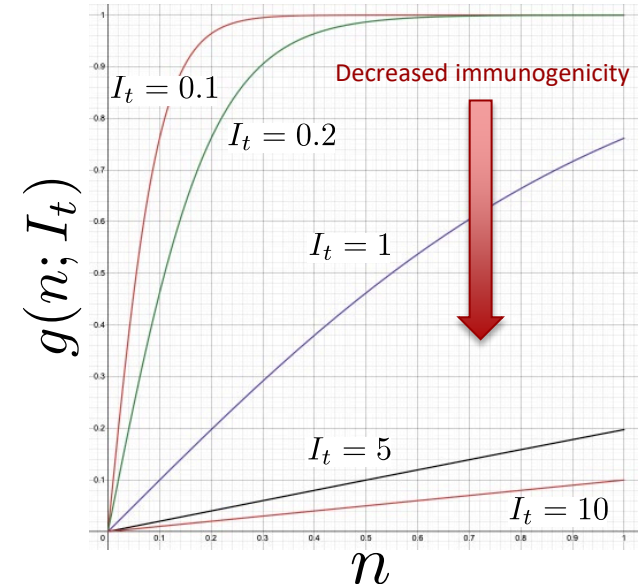
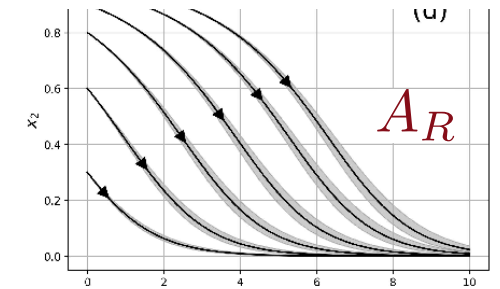
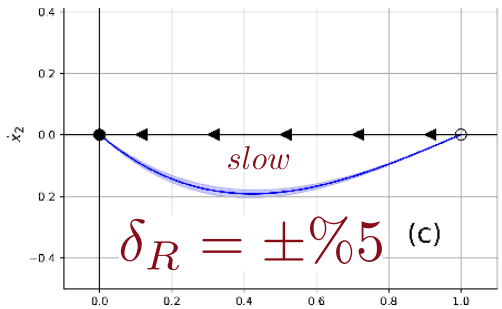
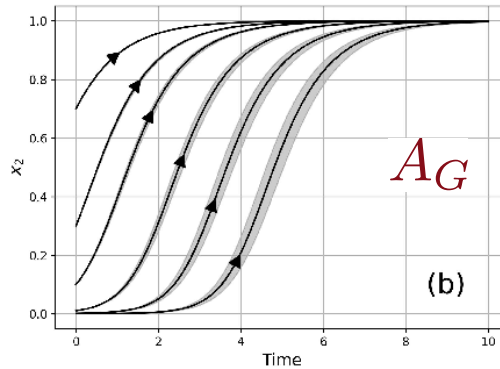
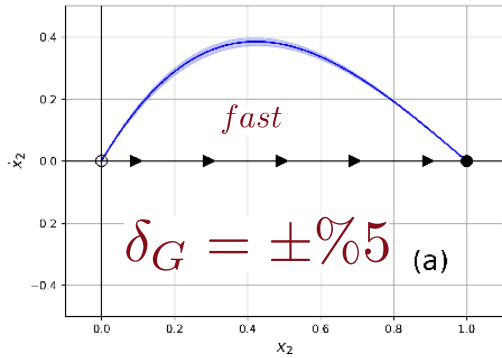
$$g(n; I_t) = \tanh(n(t)/I_t(t))$$

$$A_G = \begin{matrix} & \begin{matrix} x_1 & x_2 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \end{matrix} & \begin{pmatrix} 3 & 0 \\ 5 \pm \delta_G & 1 \end{pmatrix} \end{matrix}$$

fitness advantage

$$A_R = \begin{matrix} & \begin{matrix} x_1 & x_2 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \end{matrix} & \begin{pmatrix} 4 & 1/2 \\ 3 \pm \delta_R & 0 \end{pmatrix} \end{matrix}$$

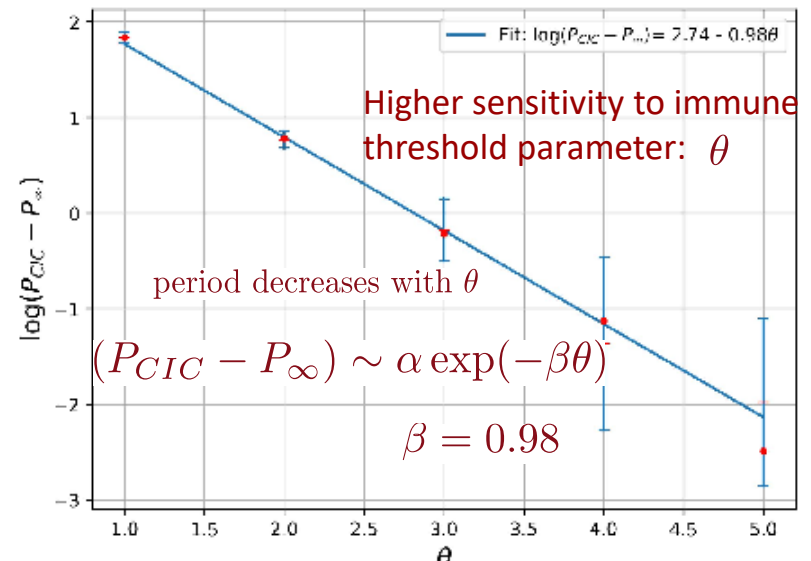
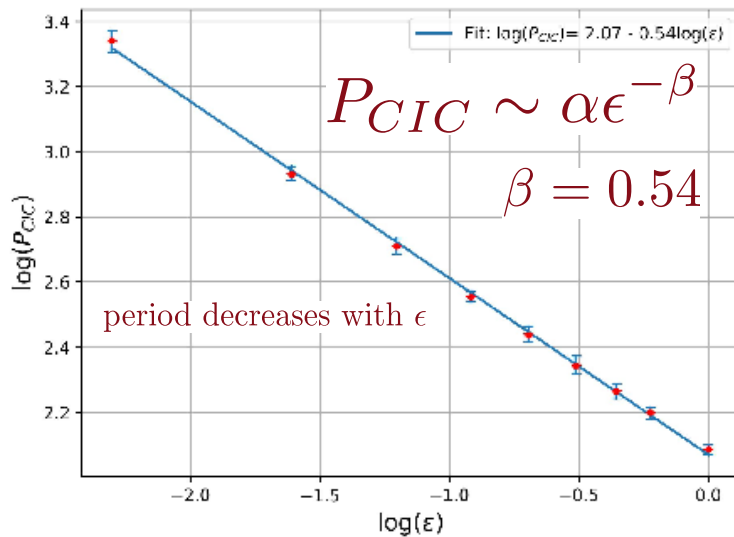
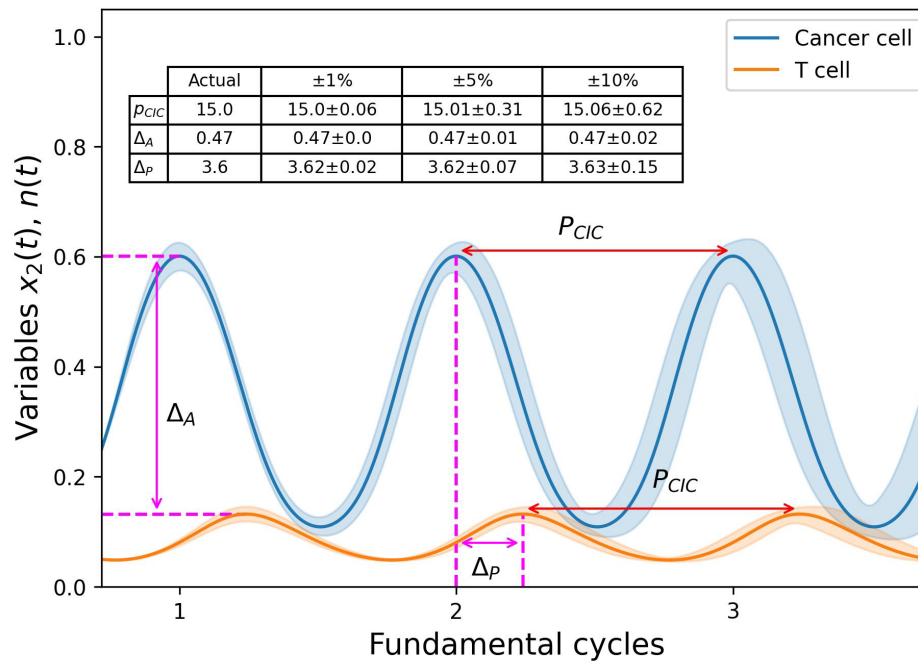
reduced fitness



$$I_t(t) = 1/[\alpha + \beta \exp(-\gamma t)]$$

Tumors can become less immunogenic over time (immune escape)

# The fundamental cycle: results from the interplay of positive and negative feedback mechanisms



$\delta$  growth/regression discrepancy parameter

Volumetric growth

Kuznetsov et. al. 1994

$$V(mP) = \frac{1}{\left[ \left( \frac{1}{V_0} - \frac{1}{K} \right) \exp\left(-\int_0^{mP} (\delta + \alpha_G) dt\right) + \frac{1}{K} \right]}$$

## Questions we address

1. Do chemotherapy and immunotherapy schedules commute?
2. Is it better to combine chemotherapy and immunotherapy dose schedules sequentially or concomitantly?
3. When is the optimal time to start/stop immunotherapy treatment?
4. What is the benefit of using more complicated time-varying dosing schedules compared to pulse-dosing protocols?

**Cocktails for cancer with a measure of immunotherapy**  
H. Ledford, *Nature* 532 (2016)

Dose combinations

**What, why, where, and when: Bringing timing to immuno-oncology**  
A.M. Rothschilds, K. Dane Wittrup, *Trends in Immunology* (2018)

Dose timing

**The model: Part 3**  
**Control theory**

$$\vec{x}(t) = (x_1(t), x_2(t), n(t)) \text{ (system response variables)}$$

$$\vec{u}(t) = (C(t), \theta(t)) \text{ (controls)}$$

$$0 \leq C \leq 1$$

$$2 \leq \theta \leq 5$$

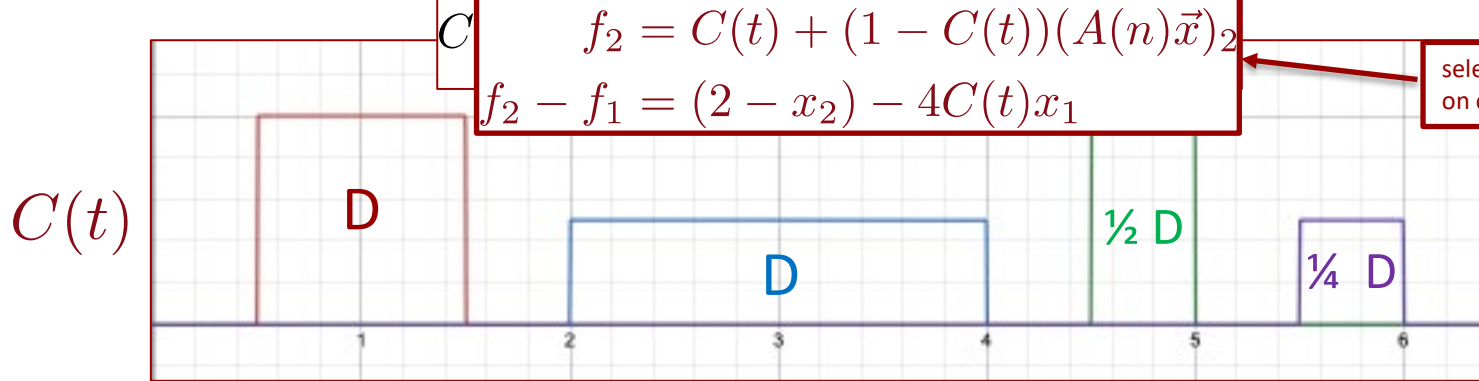
Chemo pulse-dosing

$$f_1 = (A(n))\vec{x}_1$$

$$f_2 = C(t) + (1 - C(t))(A(n)\vec{x})_2$$

$$f_2 - f_1 = (2 - x_2) - 4C(t)x_1$$

selection pressure on cancer cells



$$C(t) = \left(\frac{m}{n}\right) P_{CIC}$$

$$m, n = 1, 2, 3, 4$$

$$b_1 = 0.01$$

$$b_2 = 0.01$$

Begin chemotherapy at peak of cancer cell level when immune response begins to rise

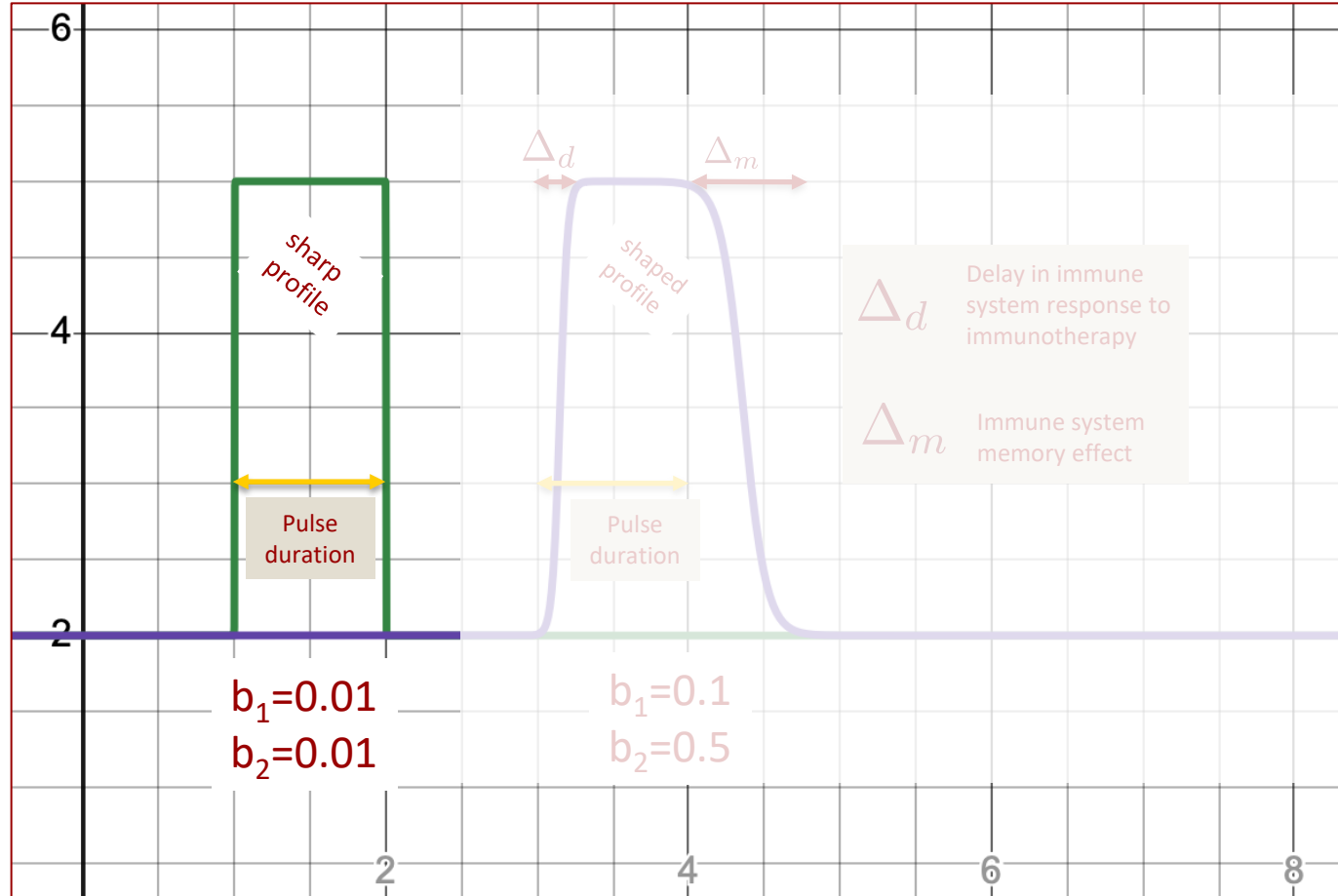
Begin at peak  
 1/4 pulse



Begin at trough



# Immuno-pulse dosing



$$\theta_{eff}(t) = \frac{3}{2} \left[ \tanh\left(\frac{t - a_1}{b_1}\right) - \tanh\left(\frac{t - a_2}{b_2}\right) \right] + 2$$

## Immuno-pulse dosing

$$\theta_{eff}(t) = \frac{3}{2} \left[ \tanh\left(\frac{t - a_1}{b_1}\right) - \tanh\left(\frac{t - a_2}{b_2}\right) \right] + 2$$

Begin at peak  
 $\frac{1}{4}$  pulse



Begin immunotherapy at  
cancer cell trough when immune  
system level is in decline

Begin at trough  
 $\frac{1}{2}$  pulse



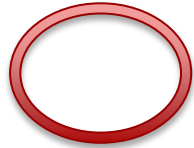
## Multi-pulse sequential therapy

Sequential pulse-therapies  
do not commute

Why?

The system response to the  
first immuno-pulse is different  
from the first chemo-pulse

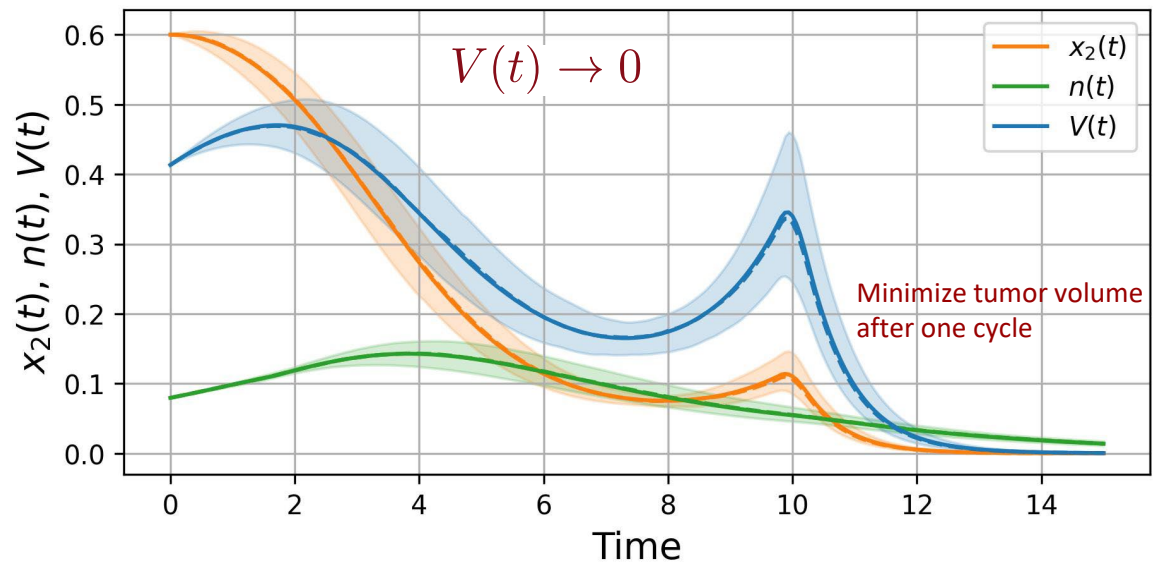
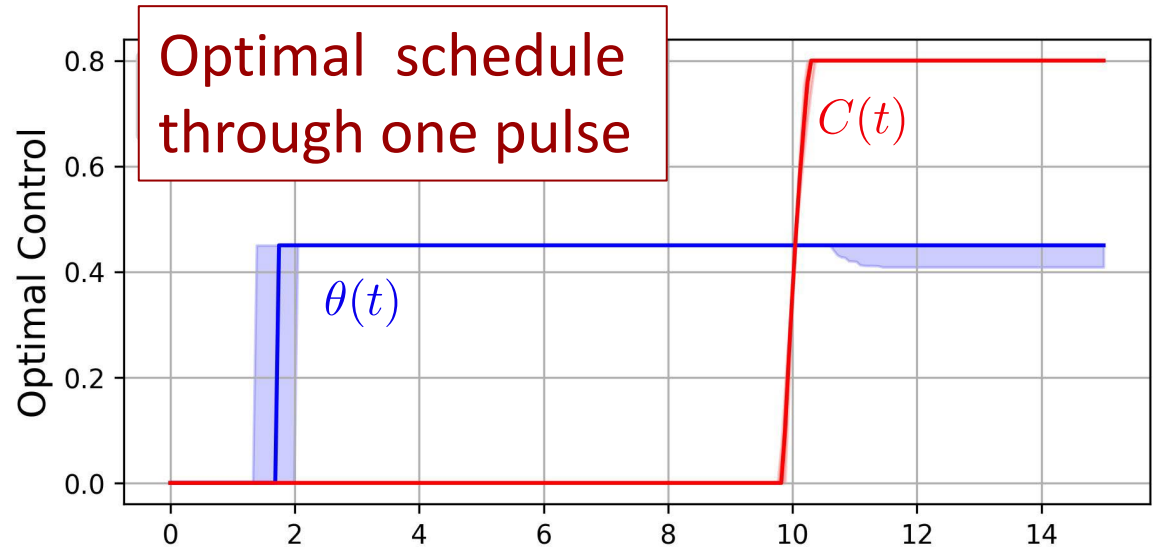
# Concomitant pulse therapy



$\frac{1}{4}$  pulse chemotherapy  
 $\frac{1}{2}$  pulse immunotherapy  
Both start at trough

# Optimized therapy

Immunotherapy should precede chemotherapy and pulse should be (roughly) twice as long



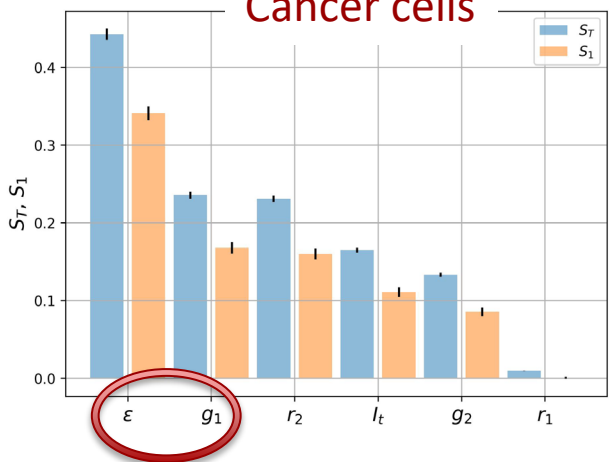
Optimizing

$$J = \underbrace{\int_0^t \mathcal{L}(\vec{x}(t), n(t); \vec{u}(t), V(t), t) dt}_{\text{Running cost}} + \underbrace{\varphi(\vec{x}(t), n(t); \vec{u}(t), V(t))}_{\text{End cost}}$$

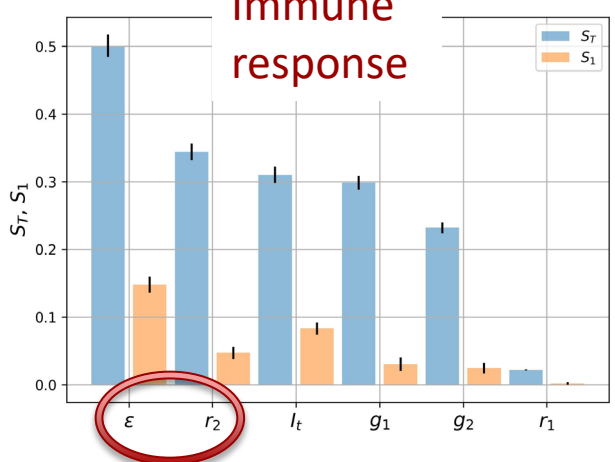
# Sensitivity analysis: Sobol indices

- Variance based UQ method that decomposes output variance into components based on variances of each parameter
- Parameters are sampled stochastically with uniform random distributions

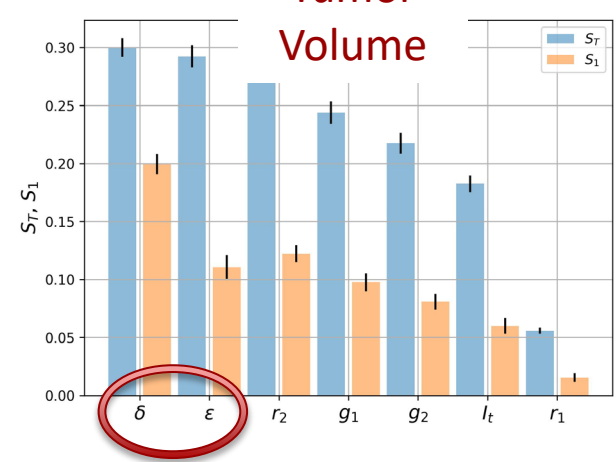
Cancer cells



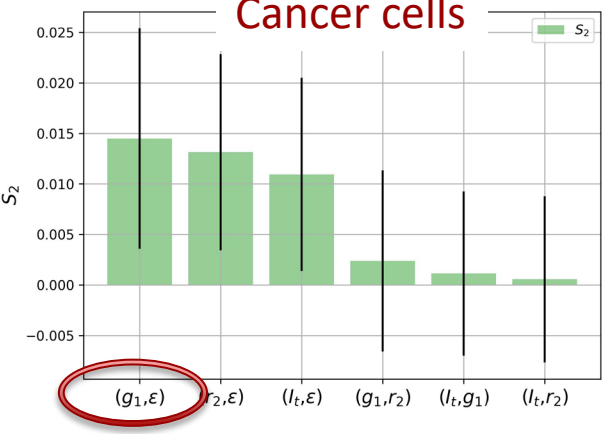
Immune response



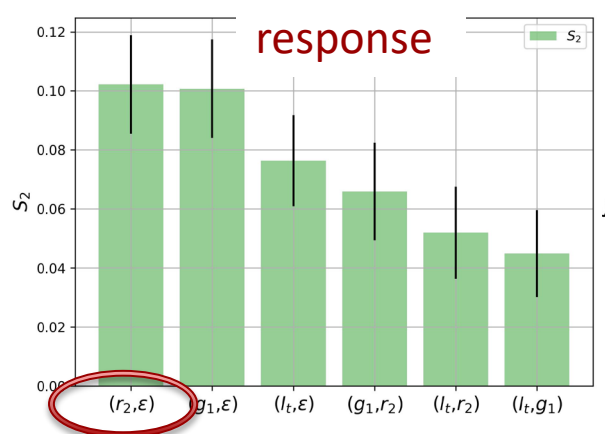
Tumor Volume



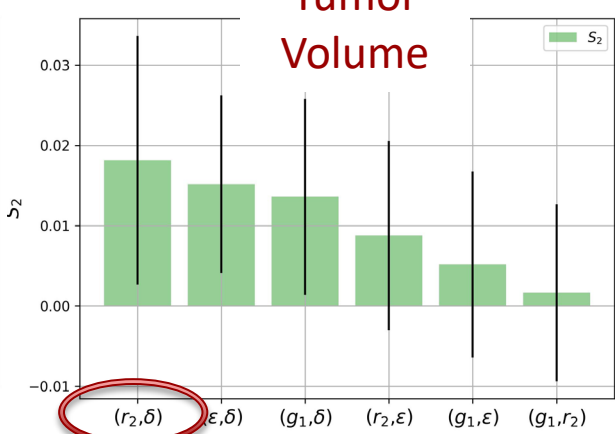
Cancer cells



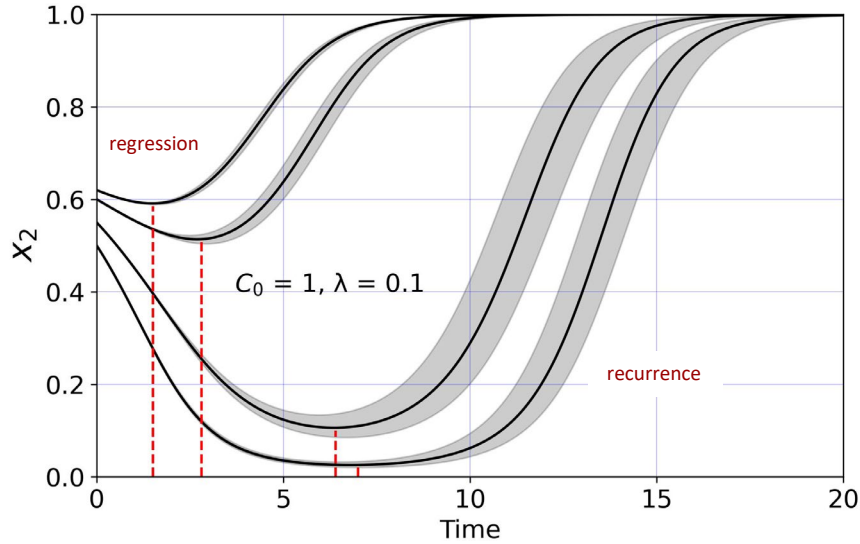
Immune response



Tumor Volume



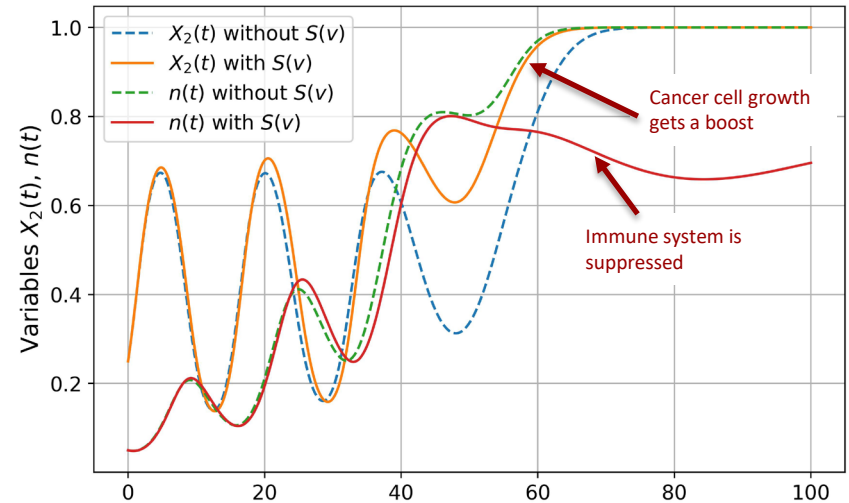
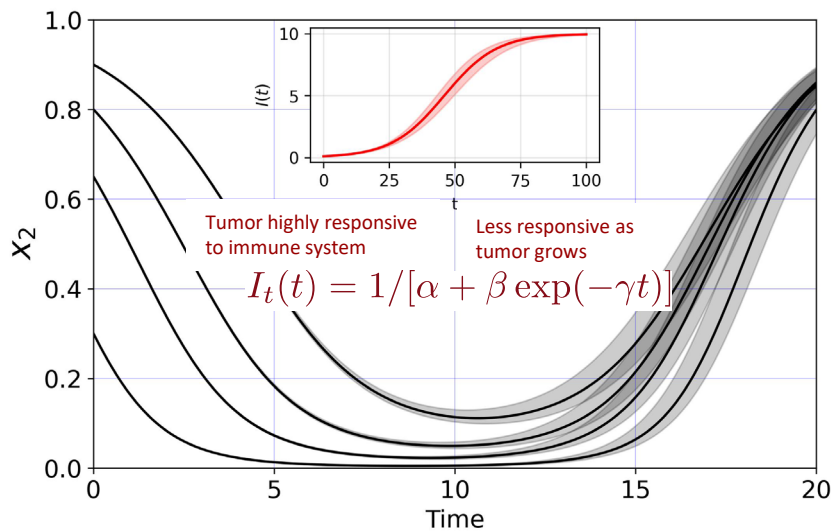
# Three additional important biological mechanisms in the model



Chemo-resistance over longer timescales

$$C(t) = C_0(t) \exp(-\lambda t)$$

$$S(V(t); I_t(t)) = \sigma V(t) / (1 + \lambda I_t(t))$$



Immune suppression as a mechanism for immune evasion

Tumor immunogenicity allowing for immune evasion as tumor matures

# Timing trials are beginning...

## Impact of timing on treatment outcomes

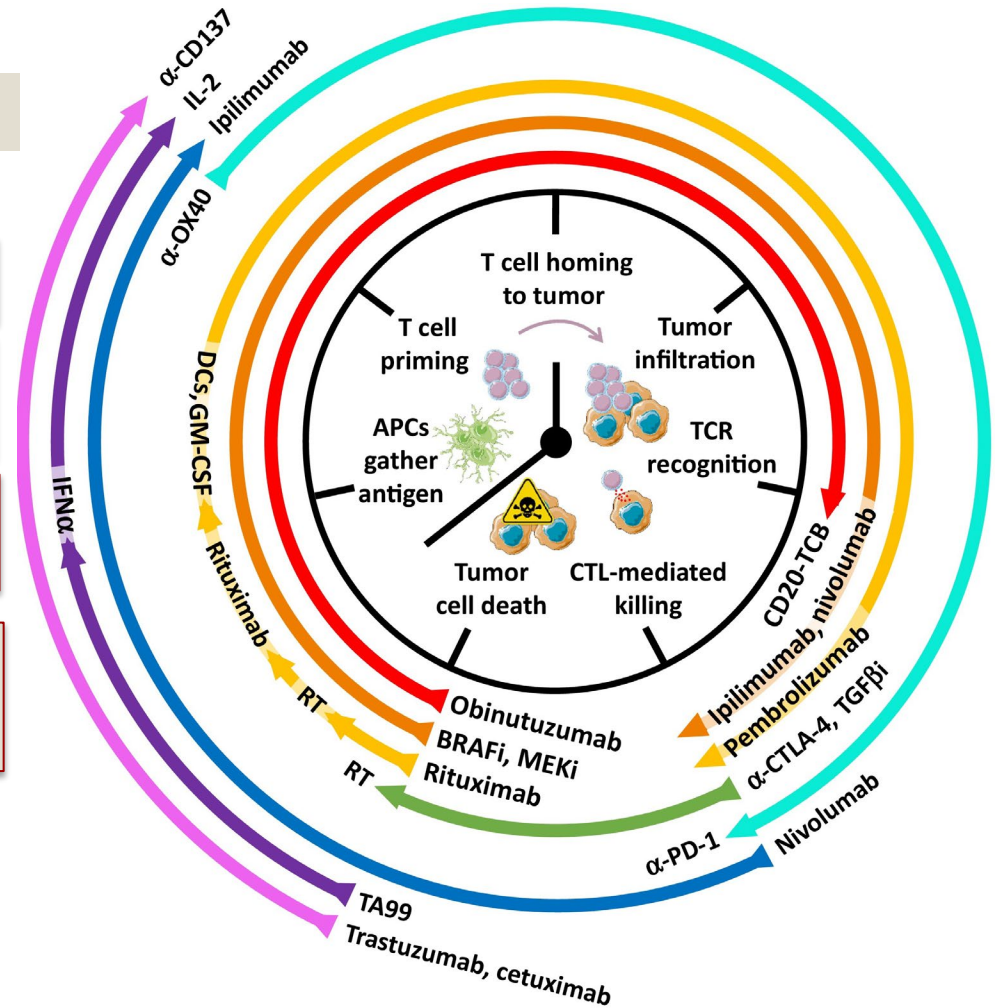
**What, Why, Where, and When: Bringing Timing to Immuno-Oncology**  
A.M. Rothschilds, K. Dane Wittrup, *Trends in Immunology* (2018)

**Temporally programmed CD8+ DC activation enhances combination cancer immunotherapy**  
A. Tzeng, ..., K. Dane Wittrup, *Cell Reports* (2016)

**Order of administration of combination cytokine therapies can decouple toxicity from efficacy in syngeneic mouse tumor models**  
A.M. Rothschilds, ..., K. Dane Wittrup, *Oncoimmunology* (2019)

**SECOMBIT: The best sequential approach with combo immunotherapy and combo target therapy in patients with BRAF mutated metastatic melanoma: A phase II randomized study,**  
P.A. Ascierto, ..., *Annals of Oncology* (2021)

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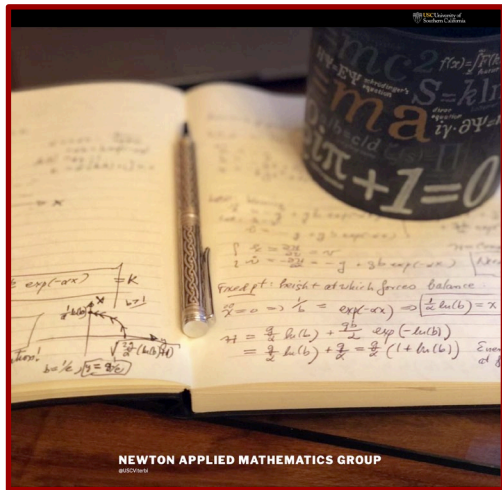
## Important model ingredients

1. Cancer cells – healthy cells: replicator dynamics
2. Cyclic process
3. Stochastic parameter variation
4. Immune evasion mechanisms
5. Immune suppression mechanisms
6. Immune activation threshold
7. Immunotherapy delay response
8. Immunotherapy memory response
9. Chemo-resistance mechanisms
10. Chemotherapy and immunotherapy time-dependent control knobs
11. Single pulse and multi-pulse dosing
12. Optimized dosing

Conclusions:

Lot's more to learn from this model!

*thank you*  
(newton.usc.edu)



John Simon  
Guggenheim  
Memorial Foundation



THE UNIVERSITY OF TEXAS  
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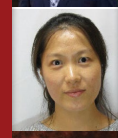
J. West



Z. Hasnain



Y. Ma



J. Park



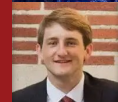
R. Dua



K. Stuckey



S. Mahmoodfar

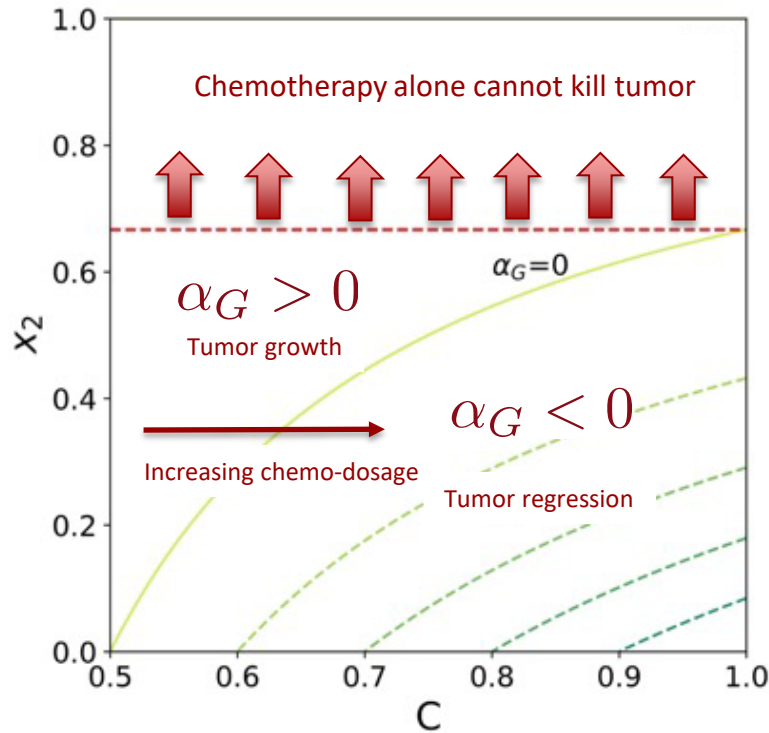


M. Giles

# The model: Part 3

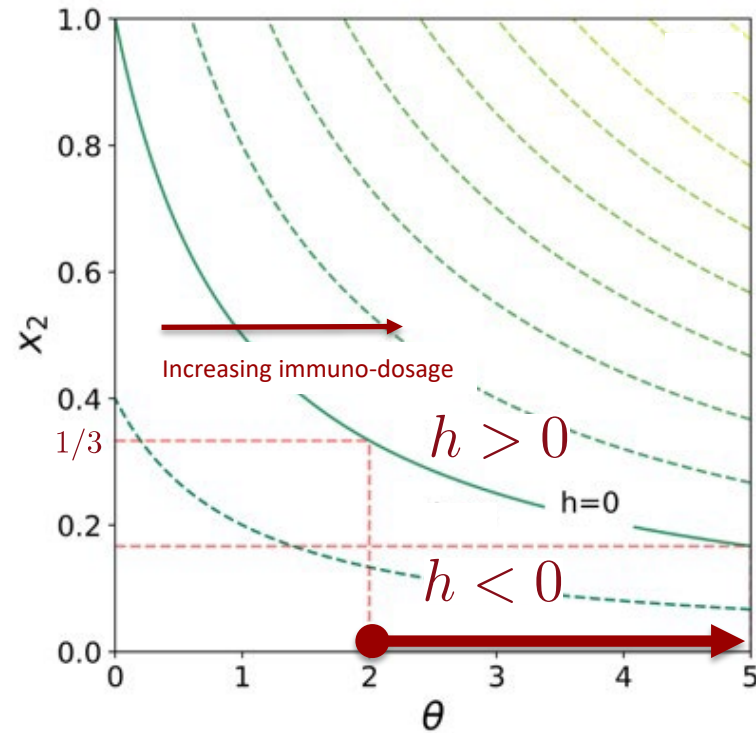
## Control thresholds

$$\alpha_G \equiv f_2 - \langle f \rangle$$



Chemotherapy thresholds

$$x_2 = \frac{1}{1 + \theta}$$



Immunotherapy thresholds

## Fitting and forecasting challenges

### Fitting parameters:

1.  $\epsilon$  Immune response timescale
2.  $\delta$  Volumetric growth/regression discrepancy parameter
3.  $\delta_G$  Tumor growth timescale (payoff matrix)
4.  $\delta_R$  Tumor regression timescale (payoff matrix)
5.  $\sigma$  Immune suppression strength
6.  $A_T$  Immune activation threshold
7.  $K$  Tumor carrying capacity

### Key immune cycle timescale parameters:

1. Tumor growth/immune timescale ratio
2.  $P_{CIC}$  parameters