Fields and flows in neural dynamics

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W BIOLOGY

uwin

UNIVERSITY *of* WASHINGTON
eScience Institute

2019-11-01
IPAM MLP workshop
UCLA

Steve Brunton
Mech Eng

Nathan Kutz
Applied Math

Rajesh Rao
Computer Sci. & Eng.

Jeff Ojemann
Neurosurgery

Bill Moody
Biology

Washington Research Foundation

Burroughs Wellcome Fund

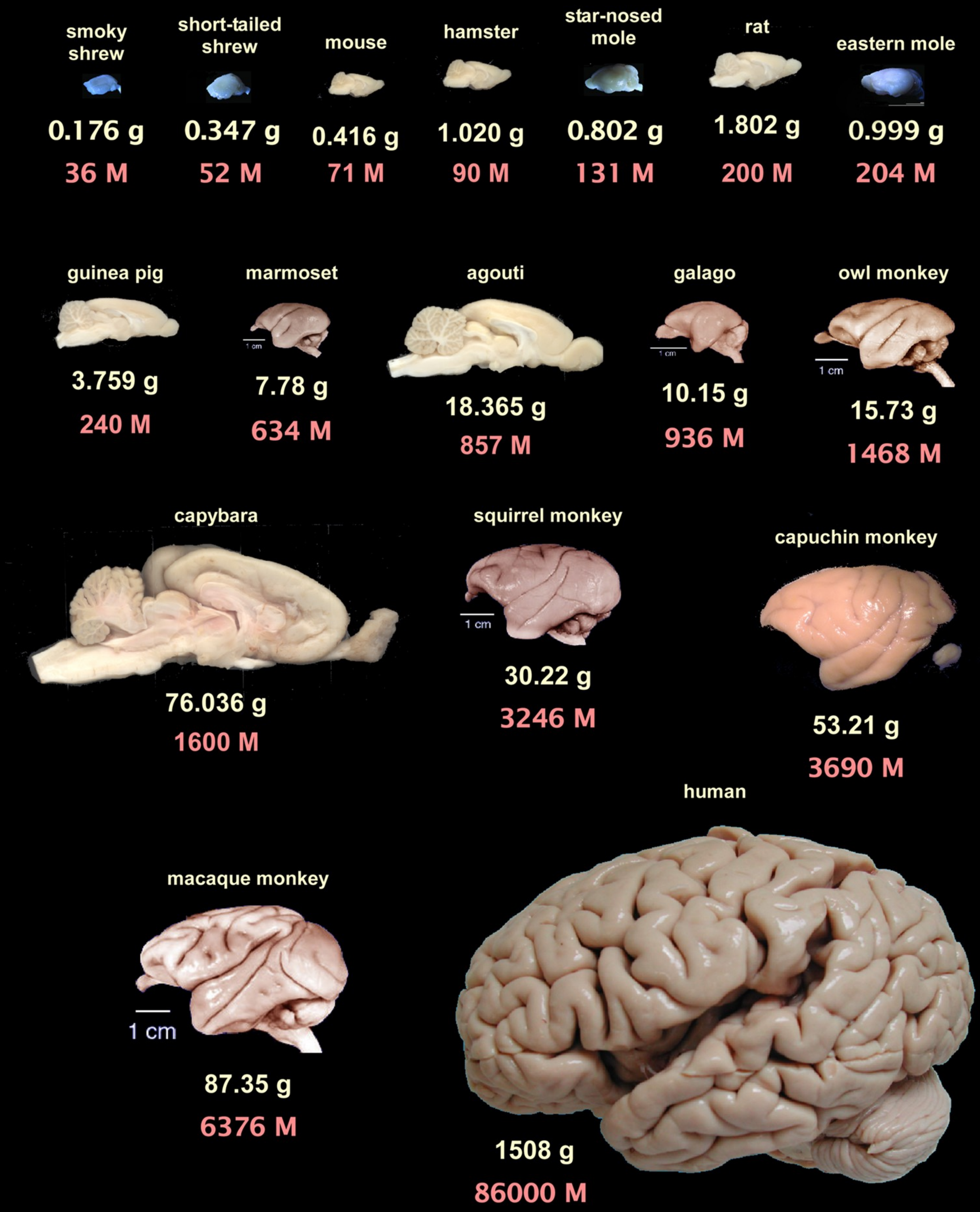
DARPA

Air Force Office of Scientific Research — United States Air Force

NSF

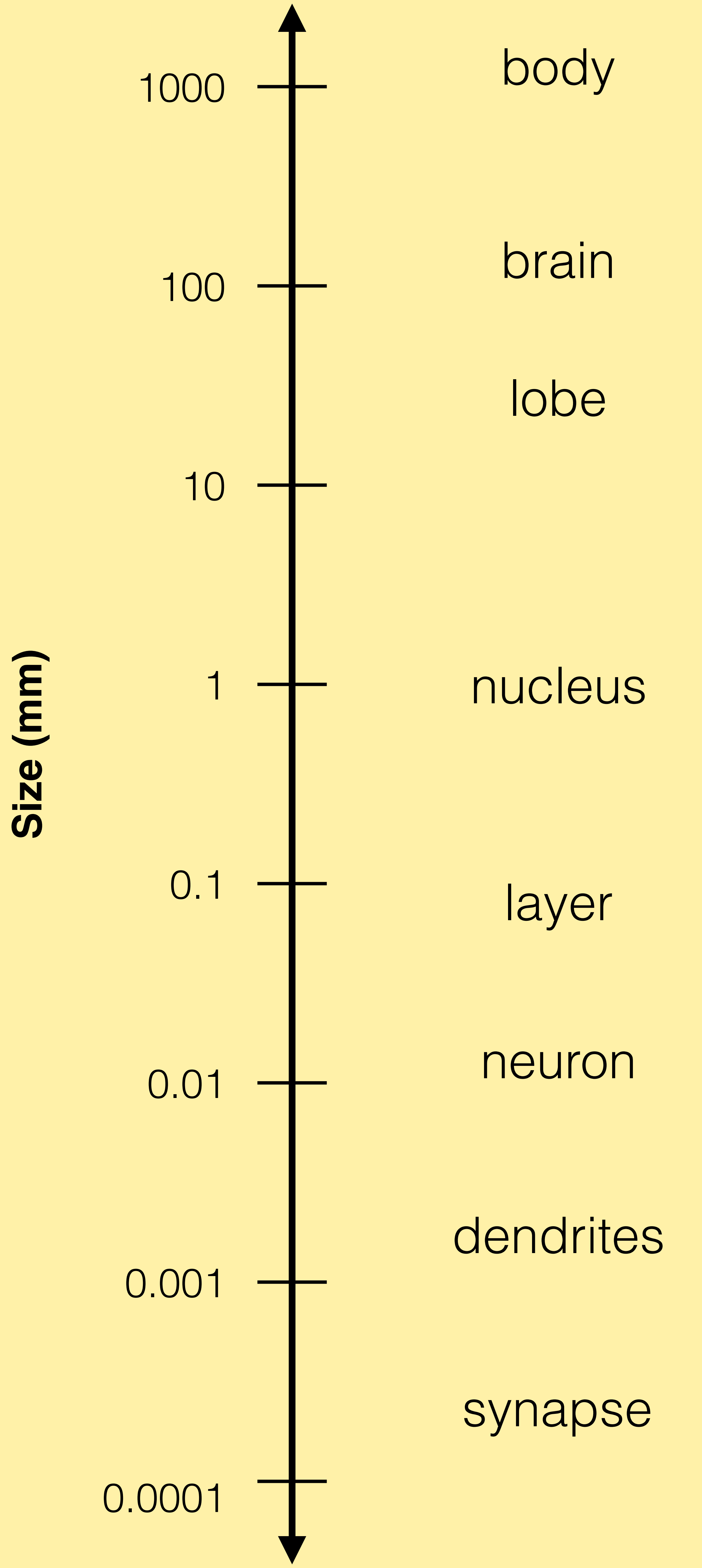
Alfred P. Sloan Foundation

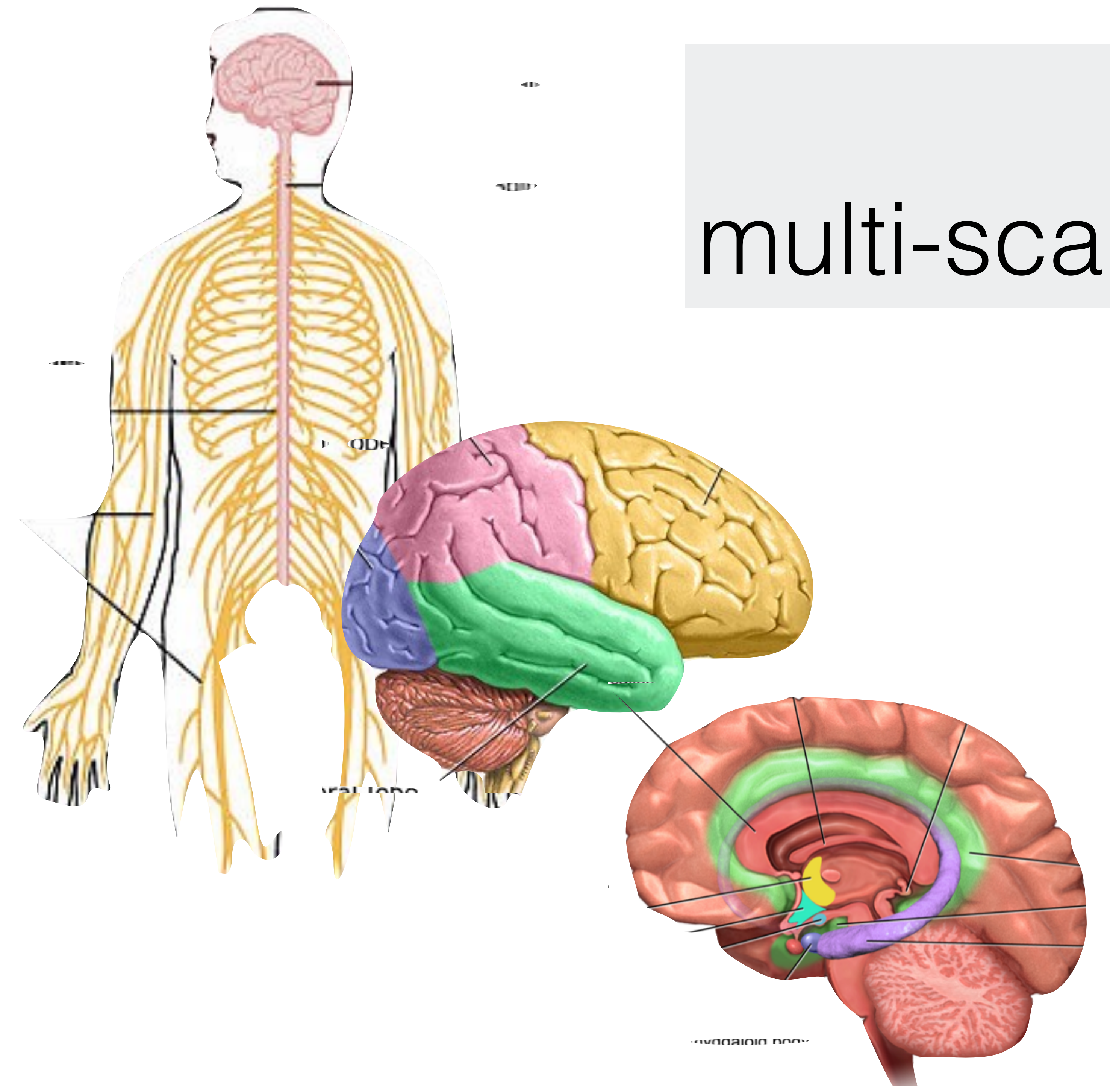
NIH National Institute of Mental Health

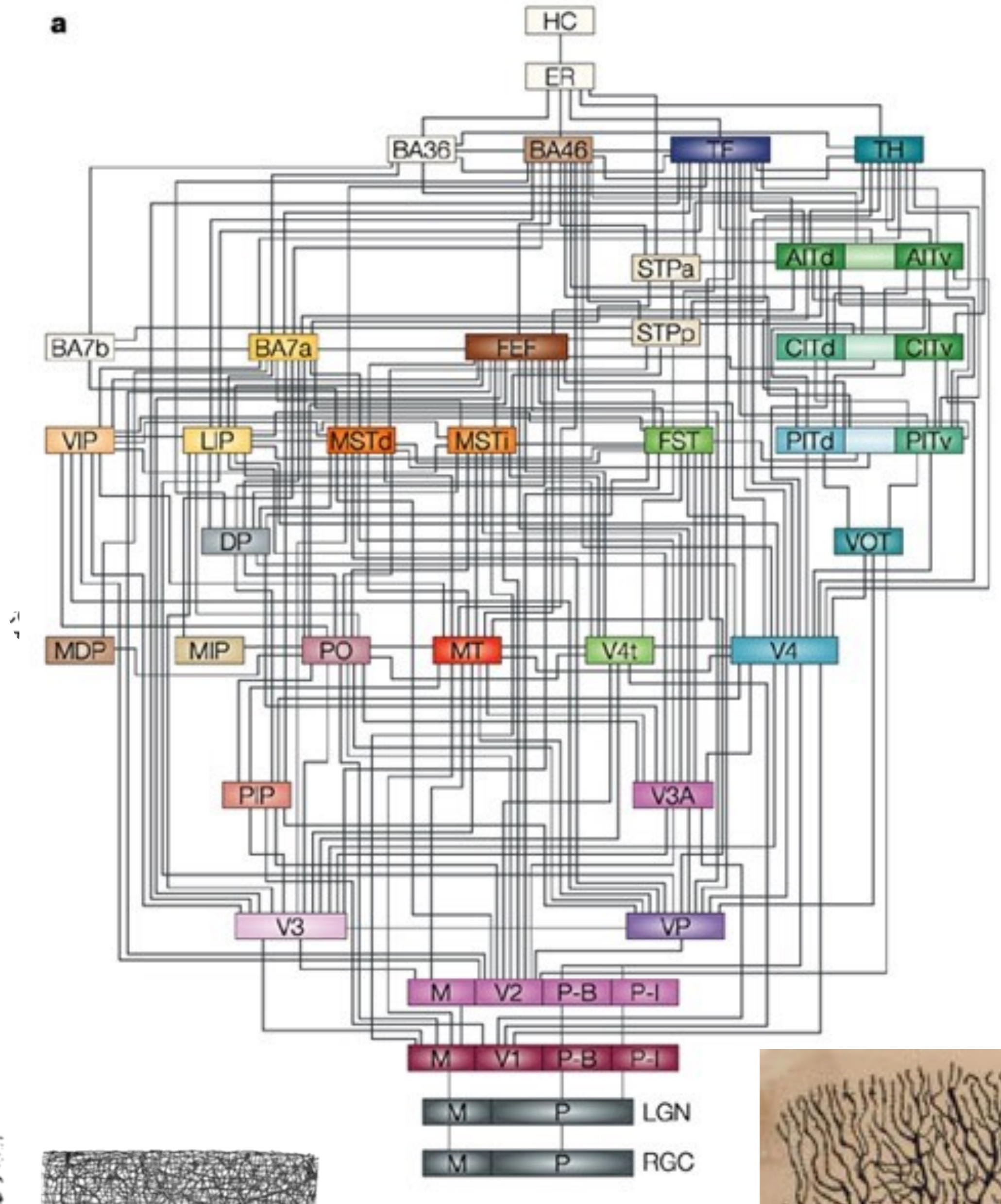


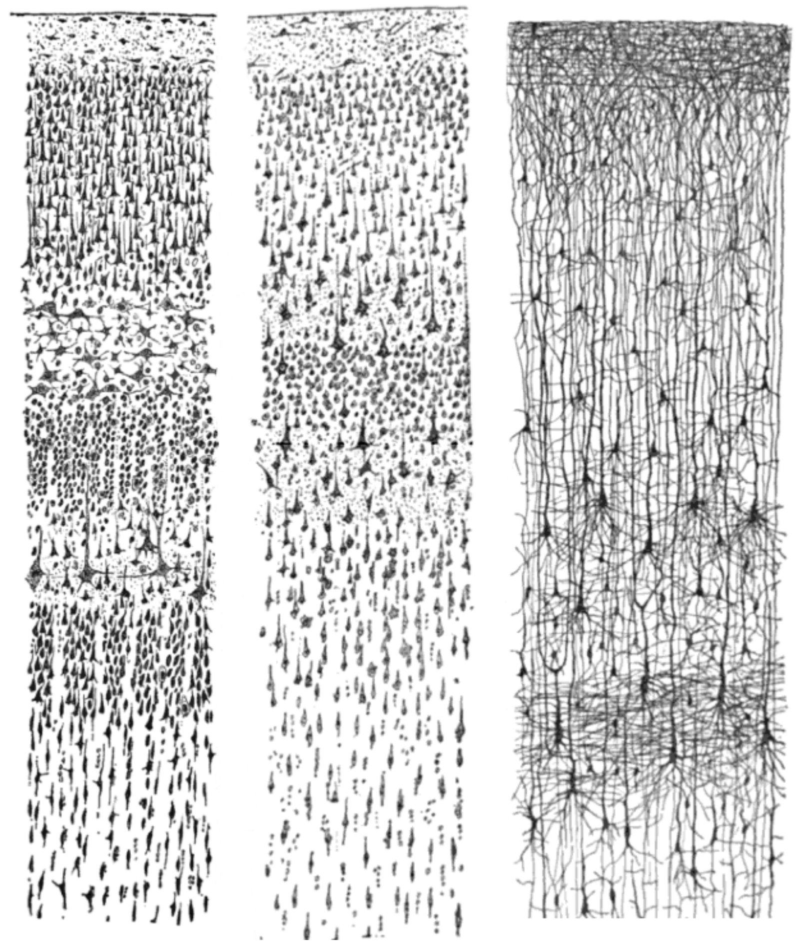
Herculano-Houzel 2009.

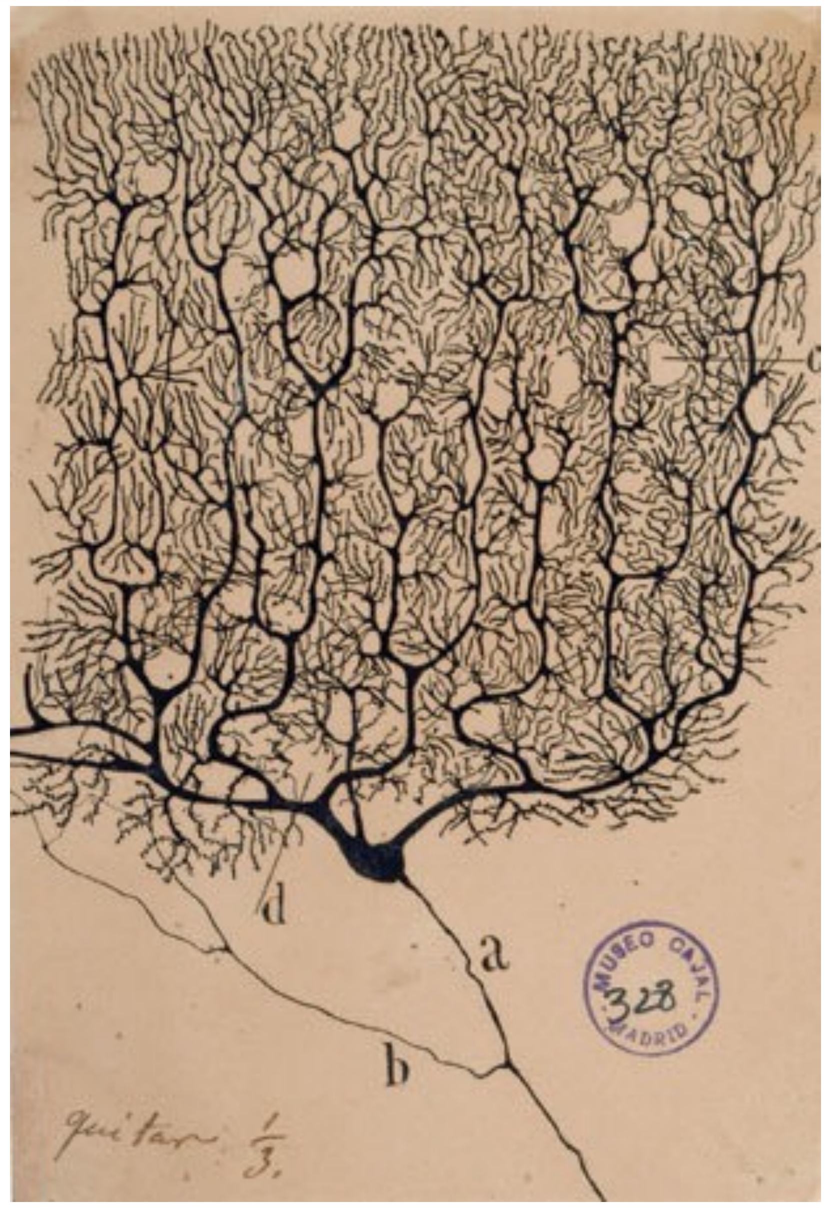
The brain has multi-scale structures in space…

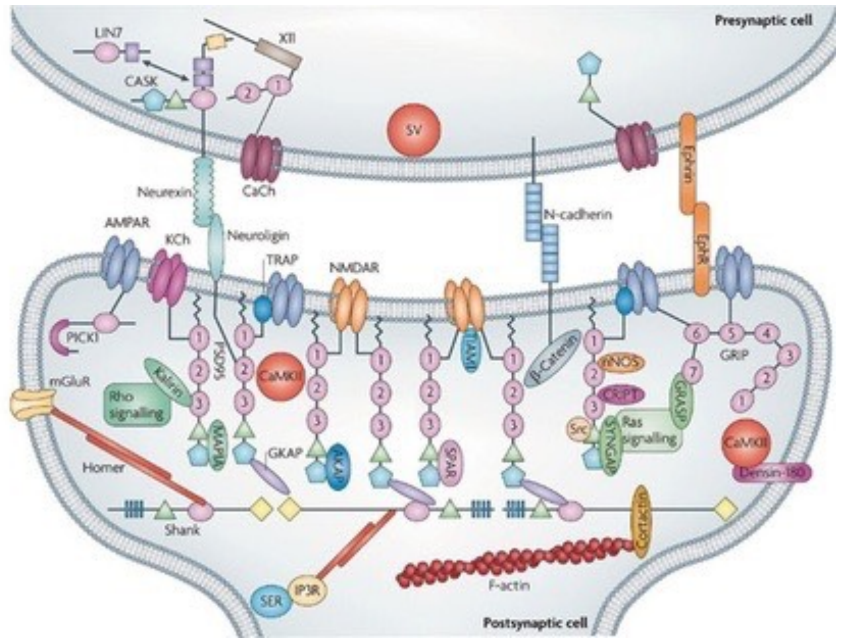






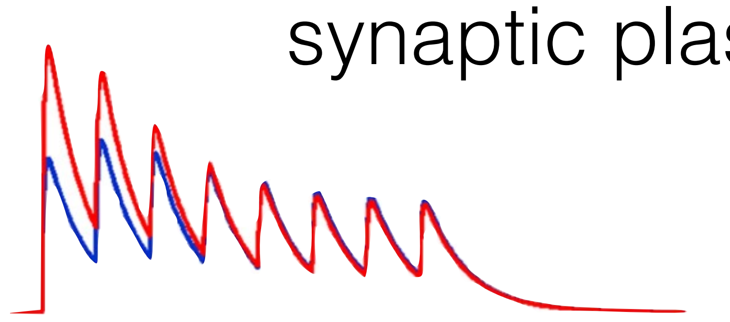






… and multi-scale dynamics in time

synaptic plasticity



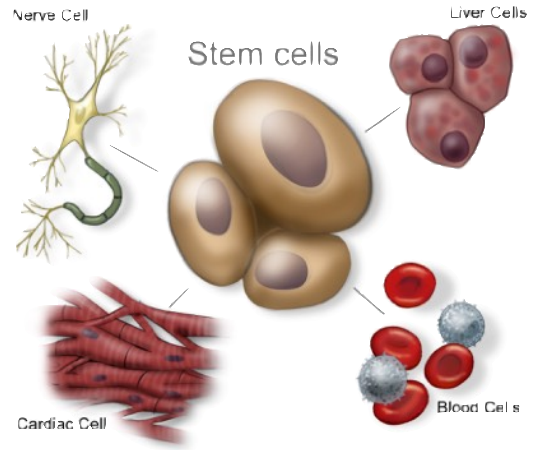
neuromodulation

short-term memory

sensation and movement

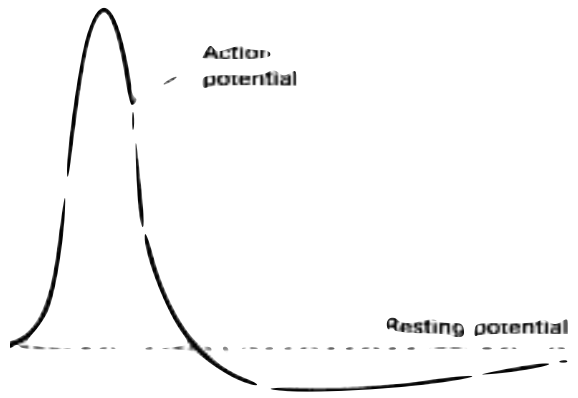
decision-making

neurogenesis

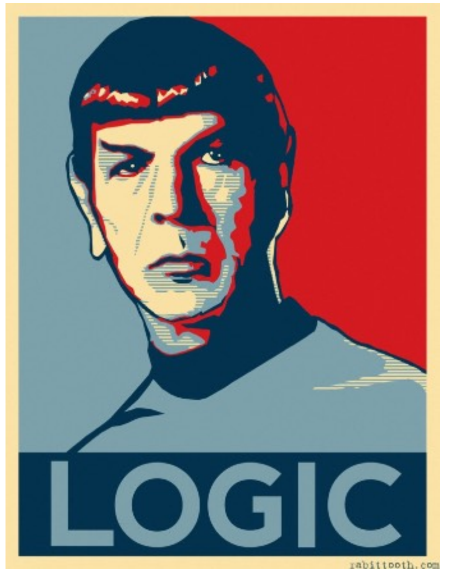


seasonal/hormonal fluctuations

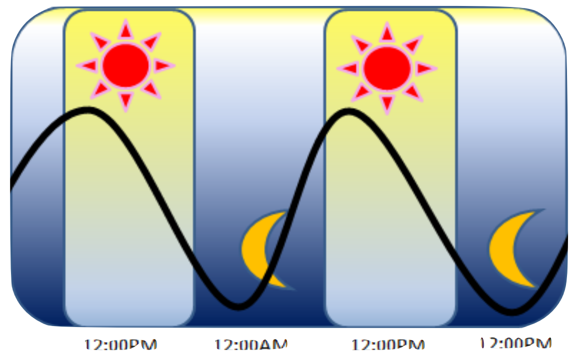
action potentials

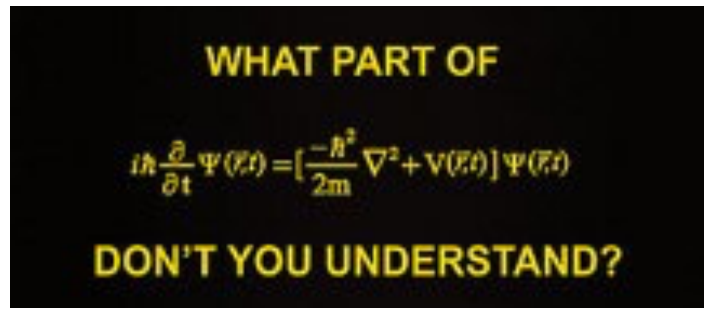


speech

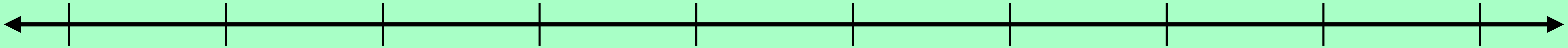


circadian rhythms





long-term learning and memory



milliseconds seconds minutes hours days months years decades

Time

Data-driven dynamic models
to understand neural computations
underlying naturalistic behavior



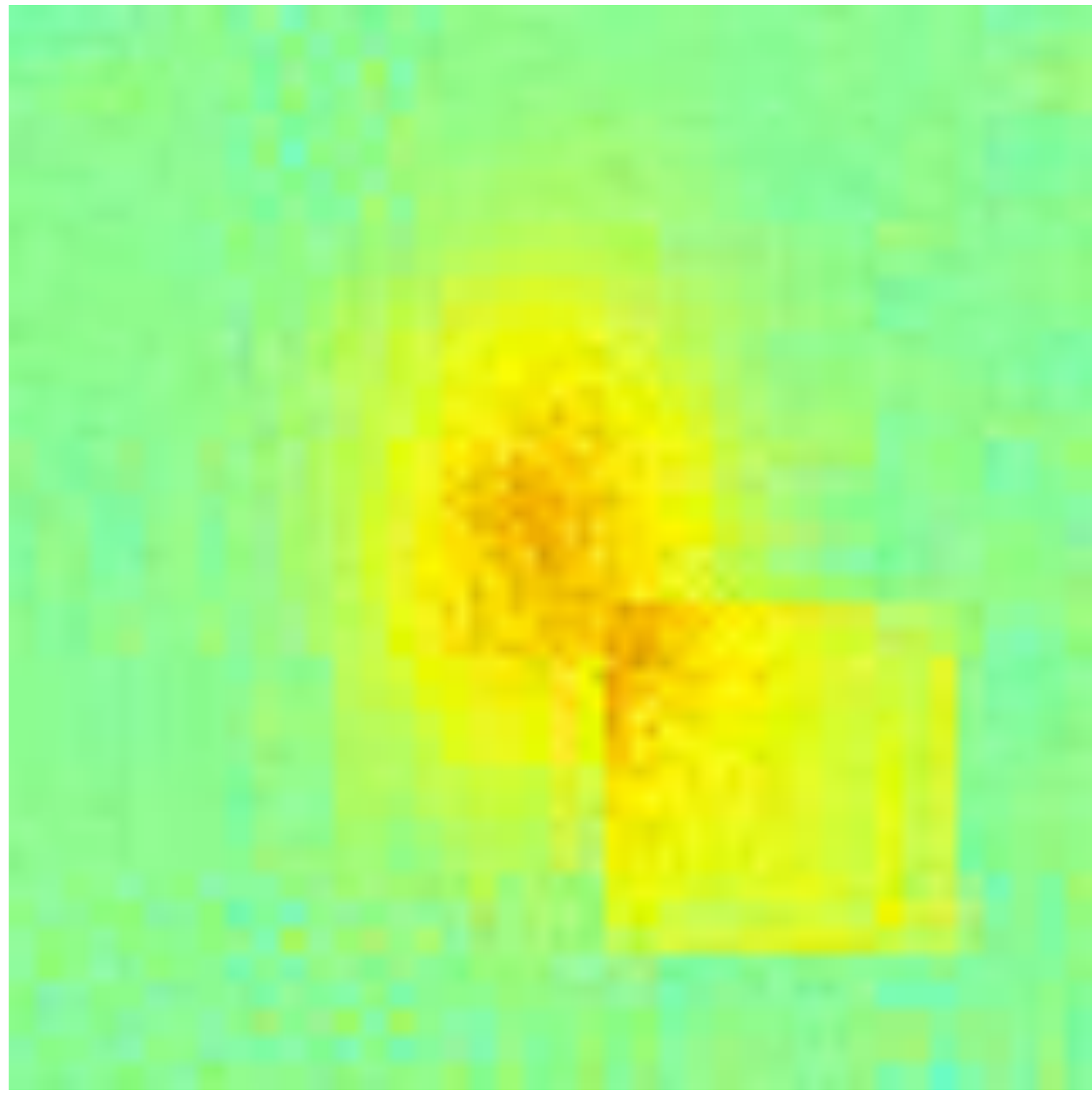


A

Simple Example:
Dimensionality
Reduction

6400-dimensional data,
noisy and varies in time

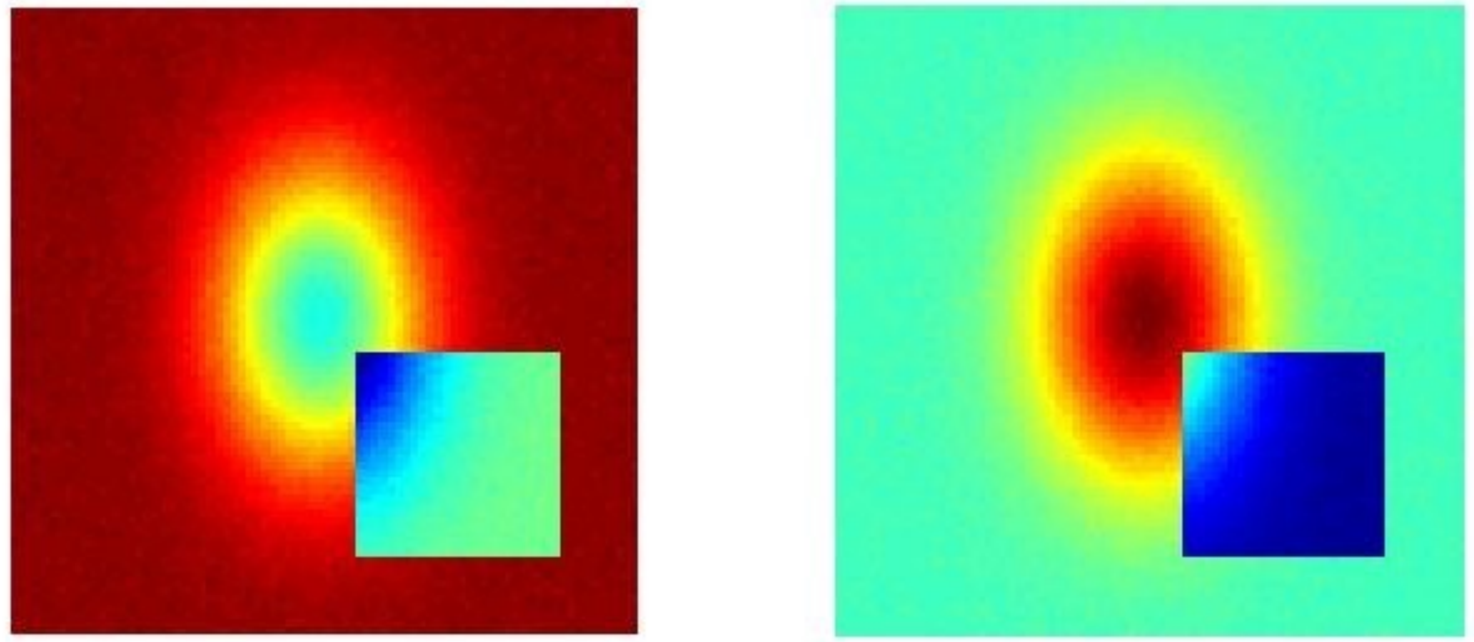
80 pixels



80 pixels

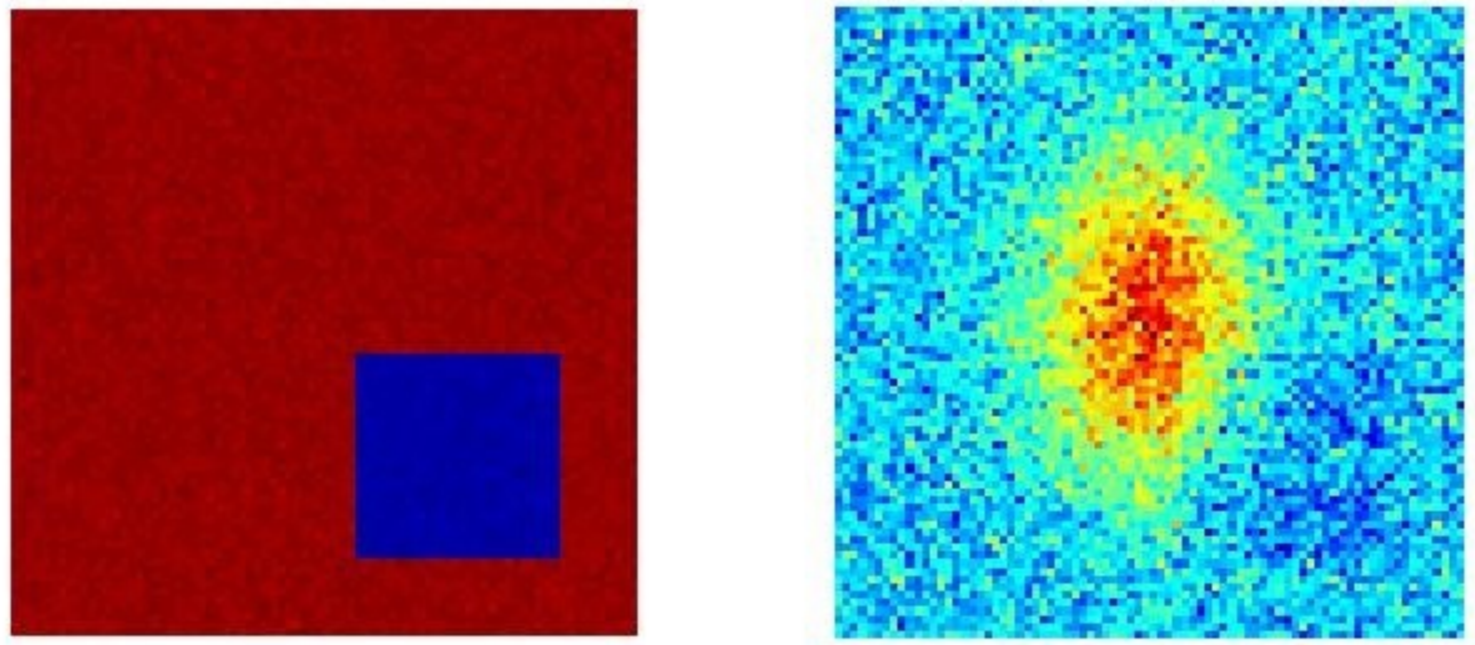
Principal Component Analysis

PCA



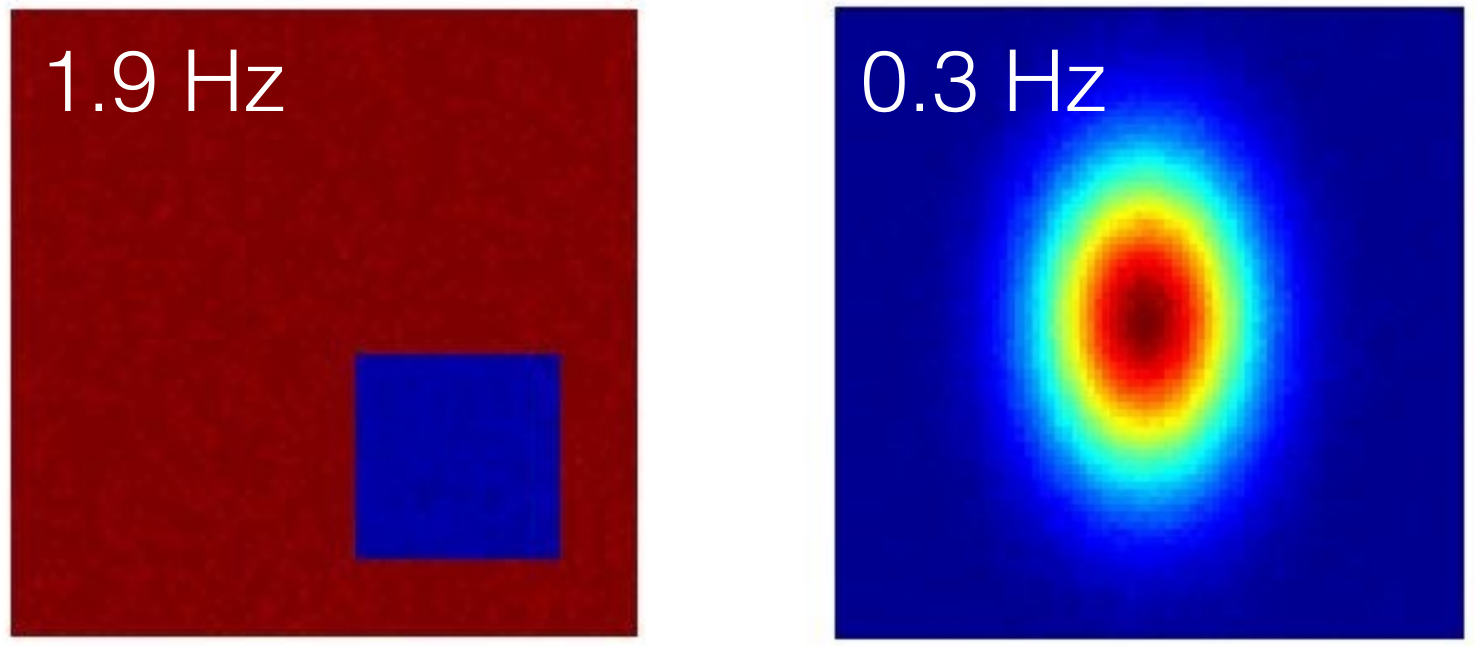
Independent Component Analysis

ICA



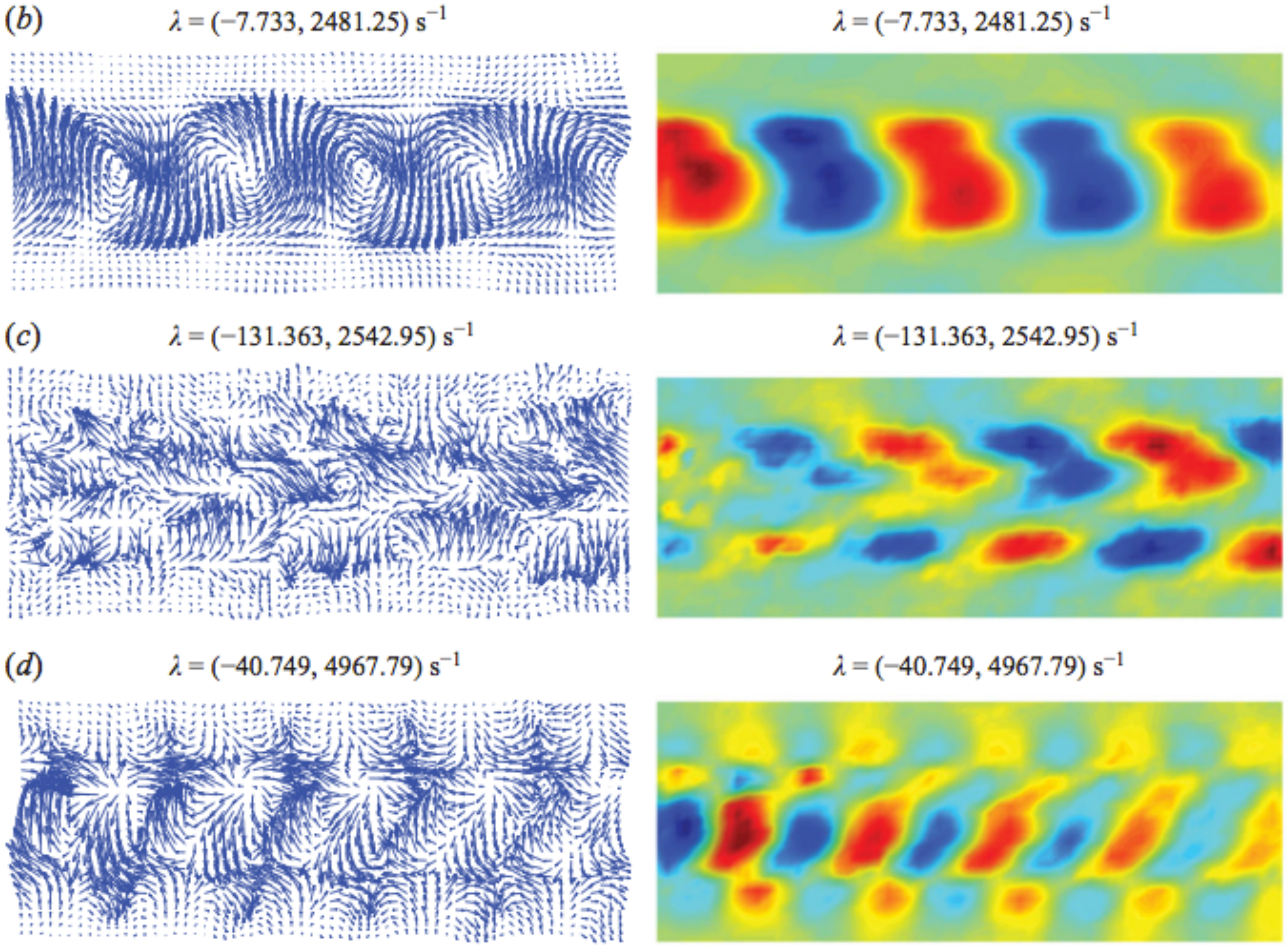
Dynamic Mode Decomposition

DMD



Dynamic Mode Decomposition (DMD)

flow past a
flexible membrane:
velocity field of wake



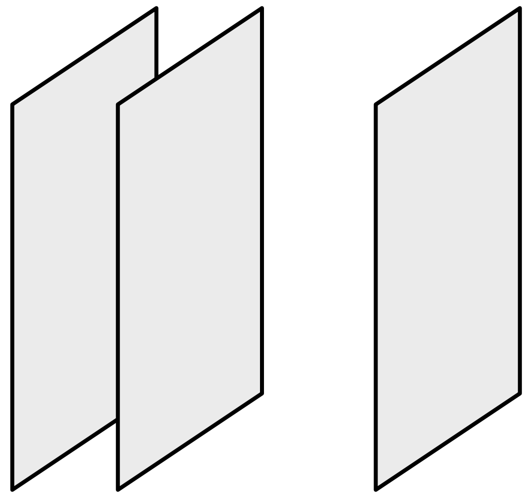
Schmid, *J Fluid Mechanics* 2010.

Rowley *et al.*, *J Fluid Mechanics* 2009.
Schmid, *J Fluid Mechanics* 2010.
Tu *et al.*, *J Comp Dyn* 2014.
Kutz, S. Brunton, BWB & Proctor *SIAM* 2016.

Dynamic Mode Decomposition (DMD):

Dynamical system of coupled spatial-temporal modes

data
snapshots
in time



$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ x_0 & x_1 & \cdots & x_{m-1} \\ | & | & & | \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_m \\ | & | & & | \end{bmatrix}$$

$$\mathbf{Y} = \mathbf{A}\mathbf{X}$$

$$\hat{\mathbf{Y}}(t) = \mathbf{\Phi}\mathbf{\Lambda}^t \mathbf{z}_0$$

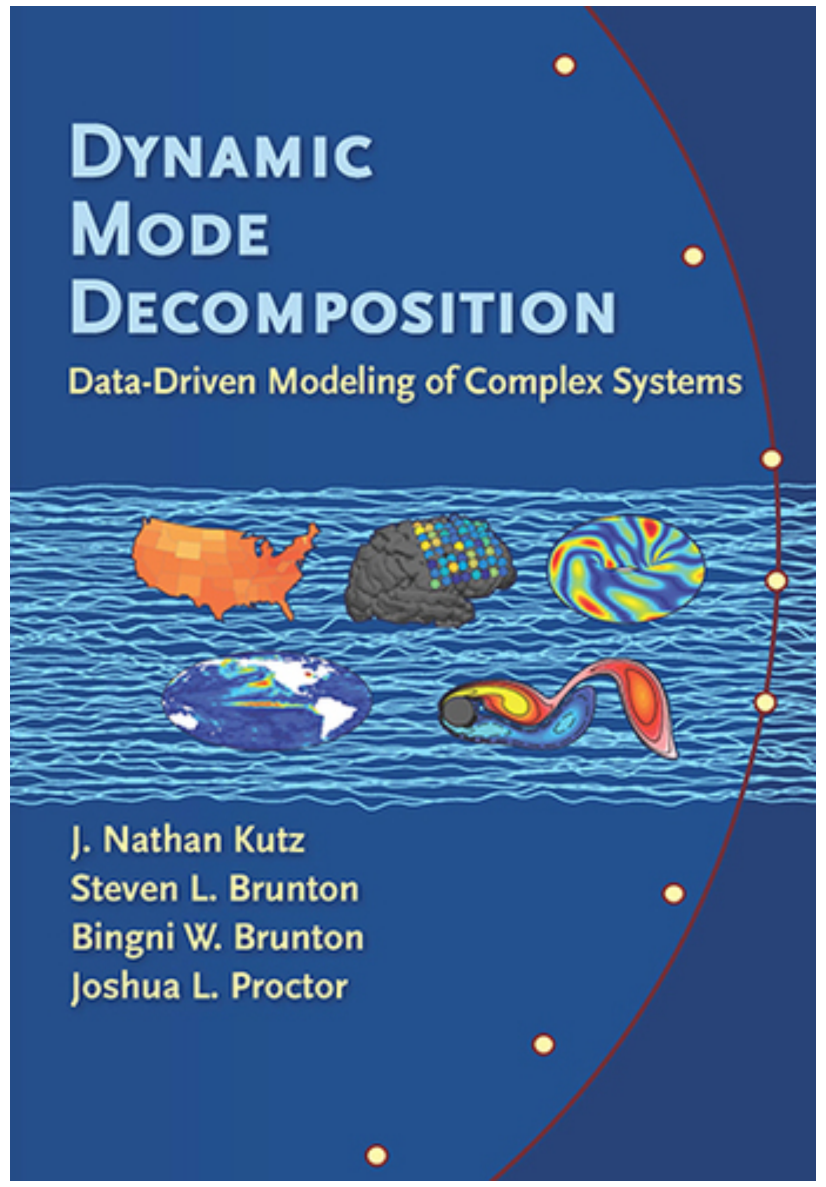
To compute DMD:

svd	1. $\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^* \qquad \mathbf{Y} = \mathbf{A}\mathbf{U}\mathbf{\Sigma}\mathbf{V}^*$
*	2. $\mathbf{U}^*\mathbf{Y}\mathbf{V}\mathbf{\Sigma}^{-1} = \mathbf{U}^*\mathbf{A}\mathbf{U} \equiv \tilde{\mathbf{A}}$
eig	3. $\tilde{\mathbf{A}}\mathbf{W} = \mathbf{W}\mathbf{\Lambda}$
*	4. $\mathbf{\Phi} = \mathbf{Y}\mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{W}$

eigenvalues: growth/decay, oscillations

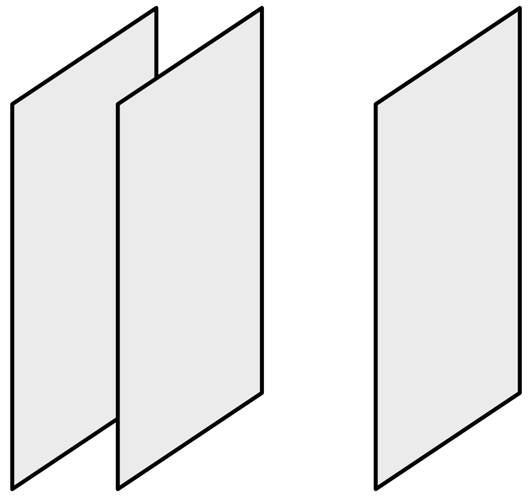
DMD modes: spatial correlations between measurements

Dynamic Mode Decomposition (DMD):
Dynamical system of coupled spatial-temporal modes

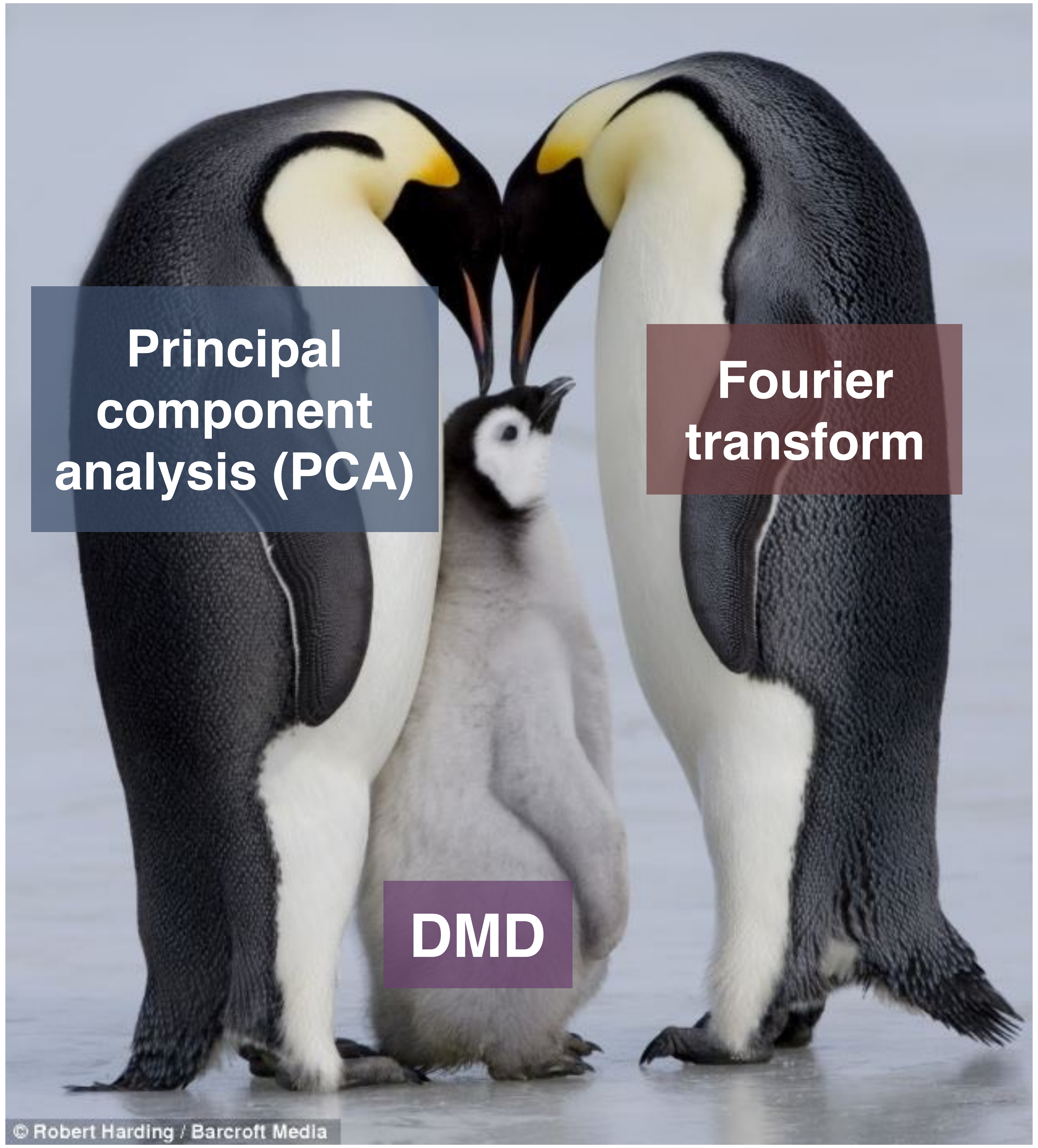


http://dmdbook.com

data snapshots in time



$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ x_0 & x_1 & \cdots & x_{m-1} \\ | & | & & | \end{bmatrix}$$



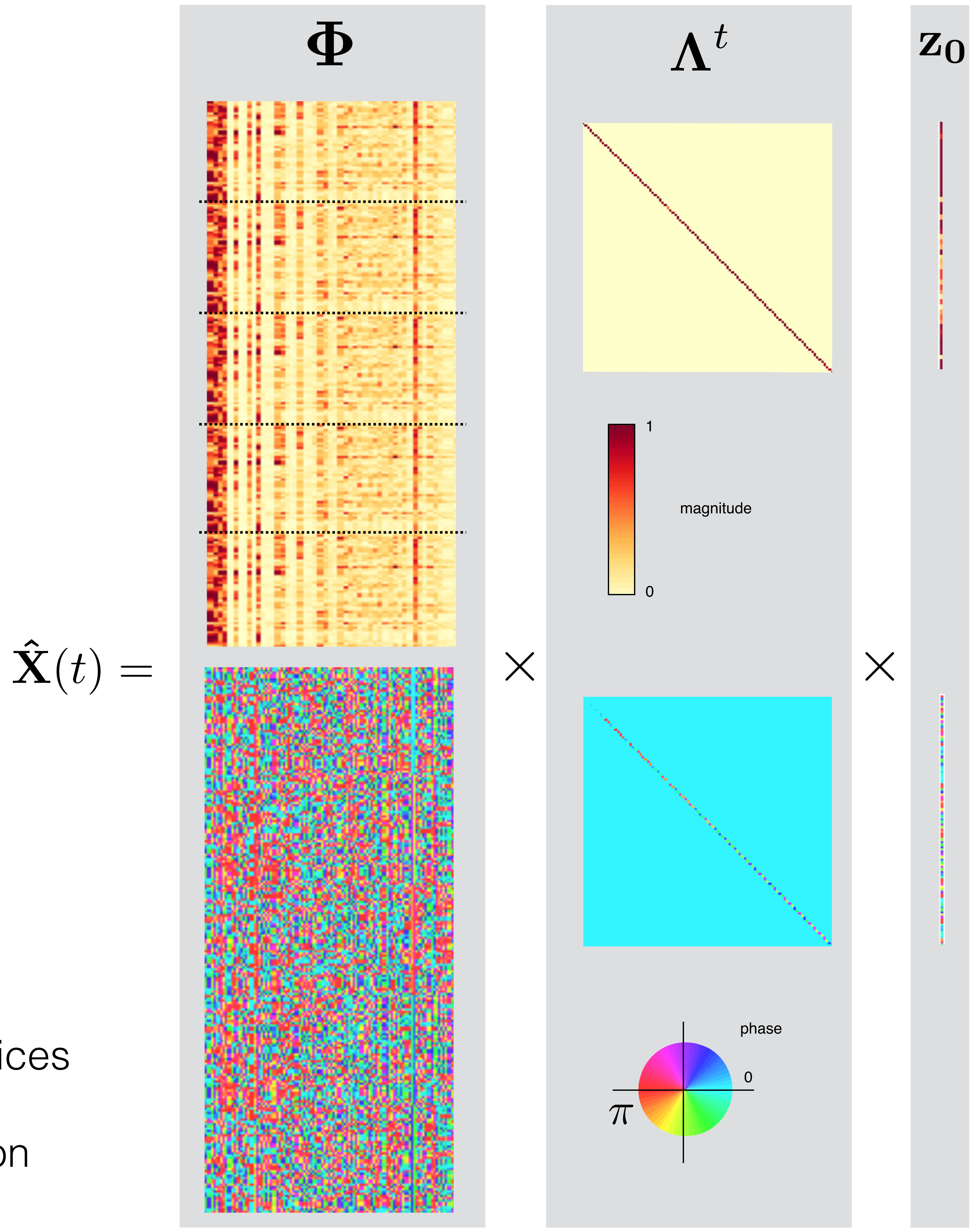
When the rank of X is insufficient to capture dynamics:

$$X_{\text{aug}} = \begin{bmatrix} | & | & & | \\ x_0 & x_1 & \cdots & x_{m-h-1} \\ | & | & & | \\ | & | & & | \\ x_1 & x_2 & \cdots & x_{m-h} \\ | & | & & | \\ & & \vdots & \\ | & | & & | \\ x_{h-1} & x_h & \cdots & x_{m-1} \\ | & | & & | \end{bmatrix}$$

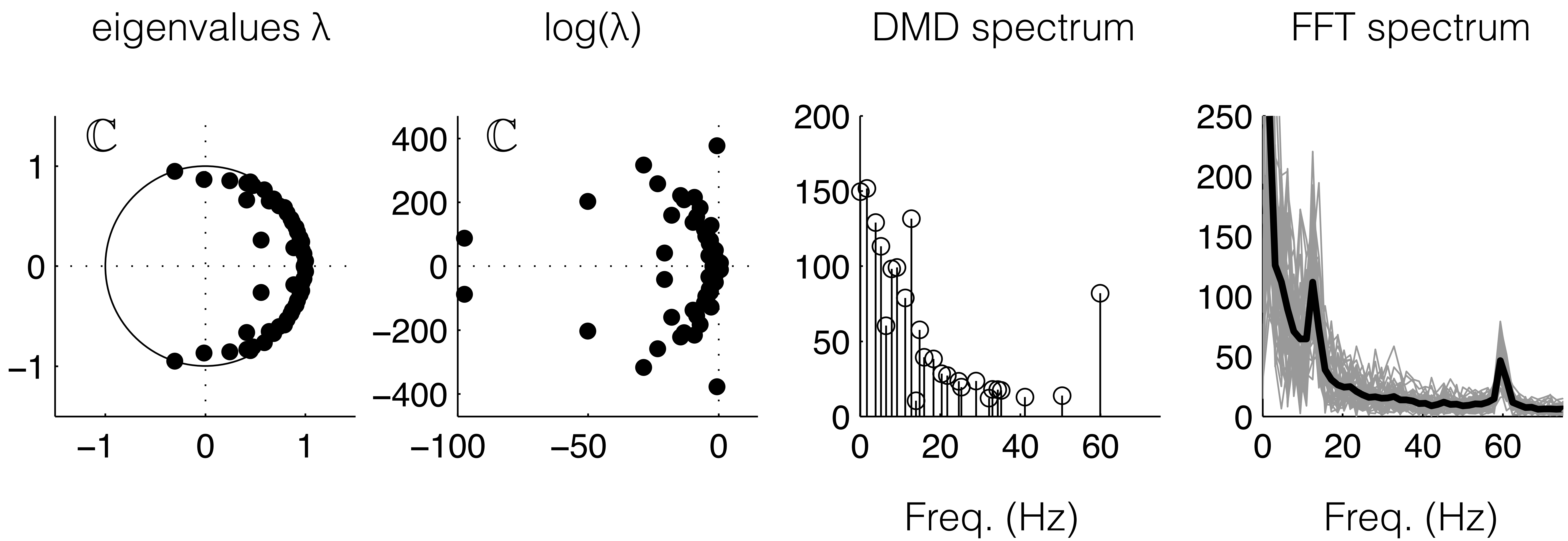
shift and stack
(delay coordinates)

$$Y_{\text{aug}} = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_{m-h} \\ | & | & & | \\ | & | & & | \\ x_2 & x_3 & \cdots & x_{m-h+1} \\ | & | & & | \\ & & \vdots & \\ | & | & & | \\ x_h & x_{h+1} & \cdots & x_m \\ | & | & & | \end{bmatrix}$$

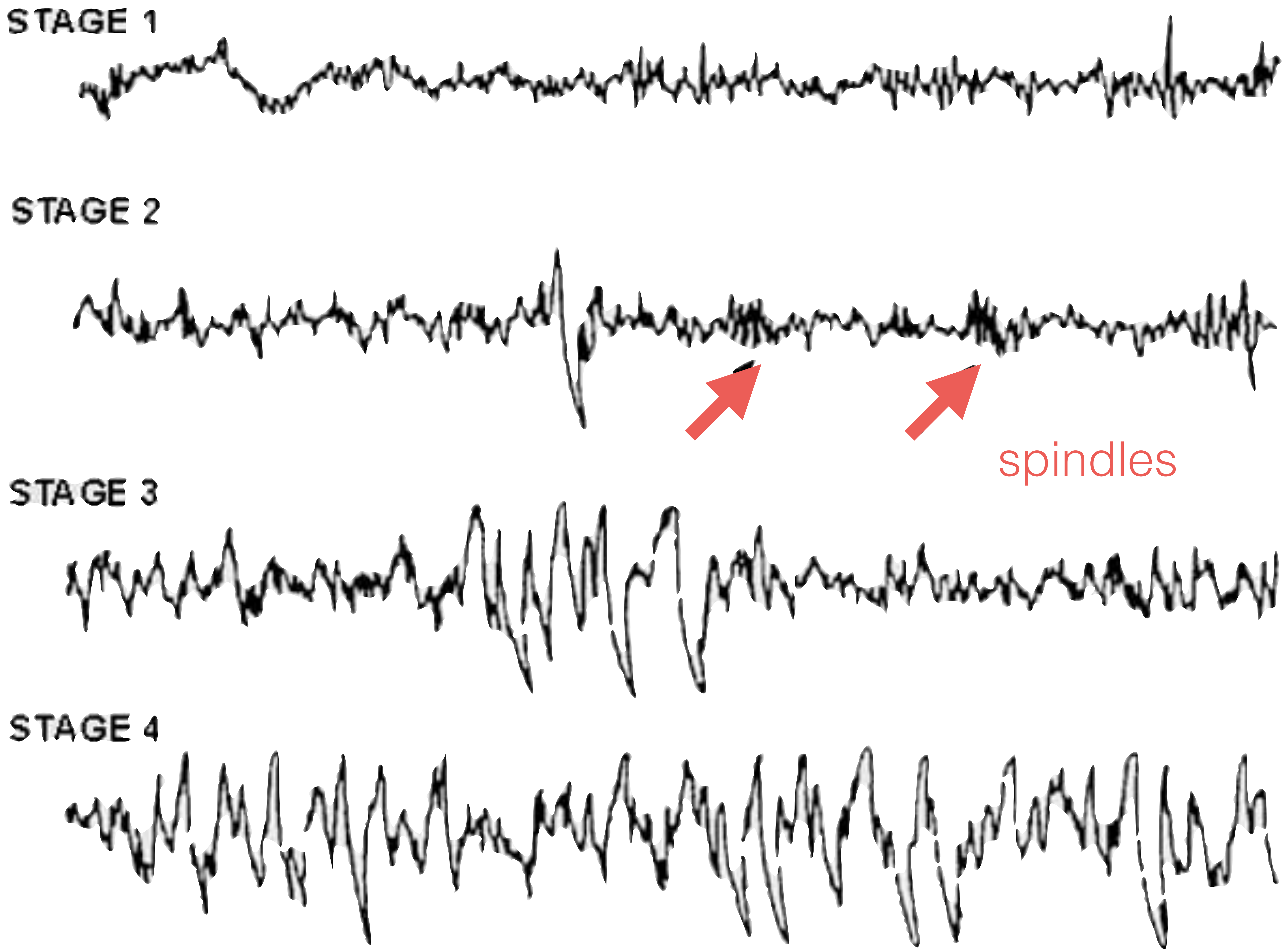
(inspired by Hankel matrices
as constructed in the
Eigensystem Realization
Algorithm ERA)



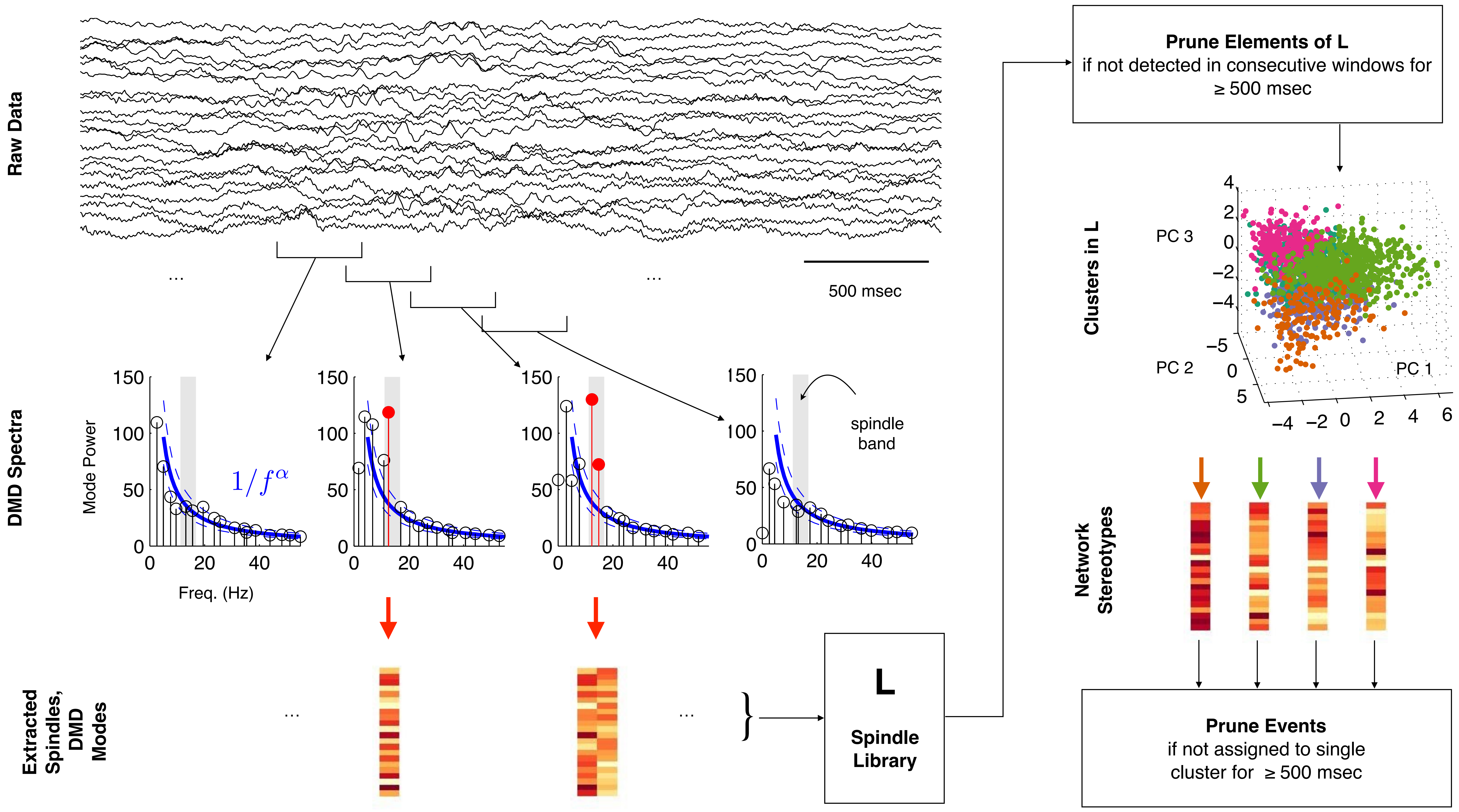
The DMD Spectrum



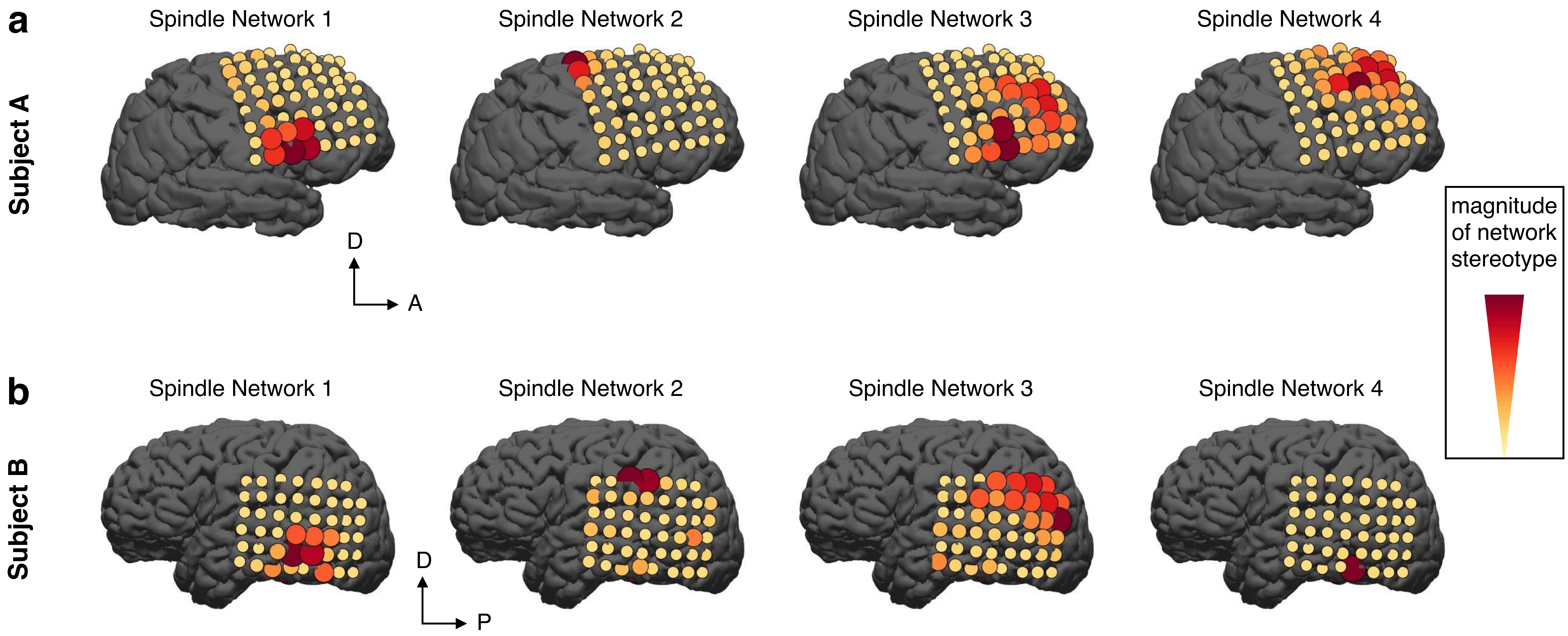
Sleep spindle networks in ECoG



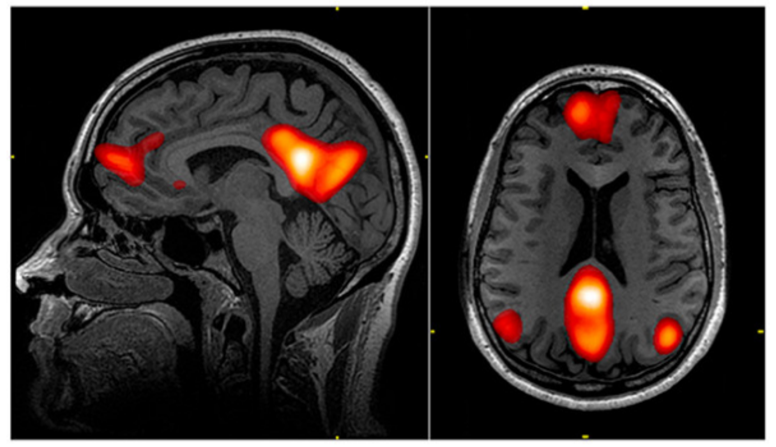
Extracting spindle network stereotypes by clustering

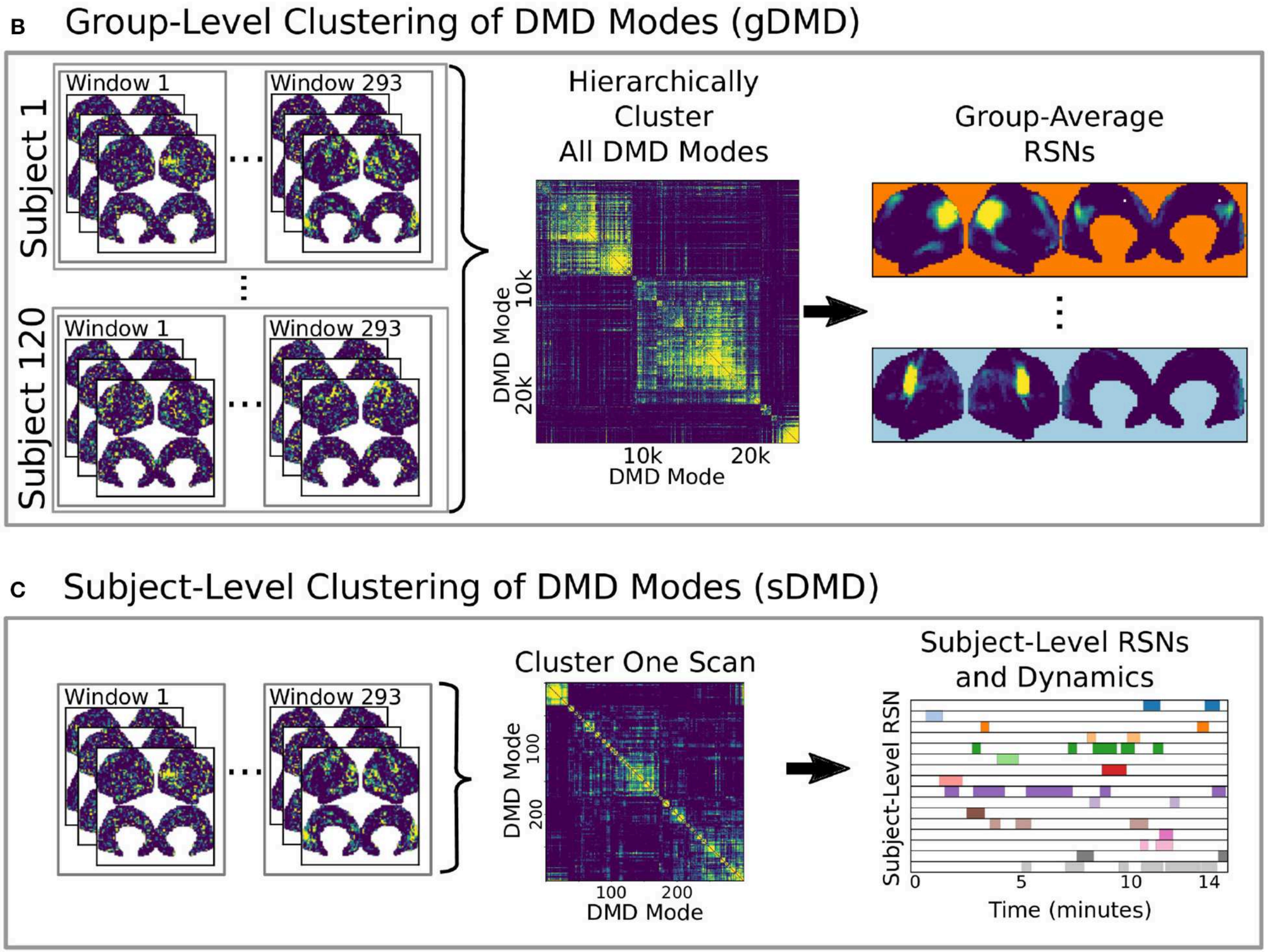


BWB, Johnson, Ojemann & Kutz, *J Neurosci Methods 2016.*



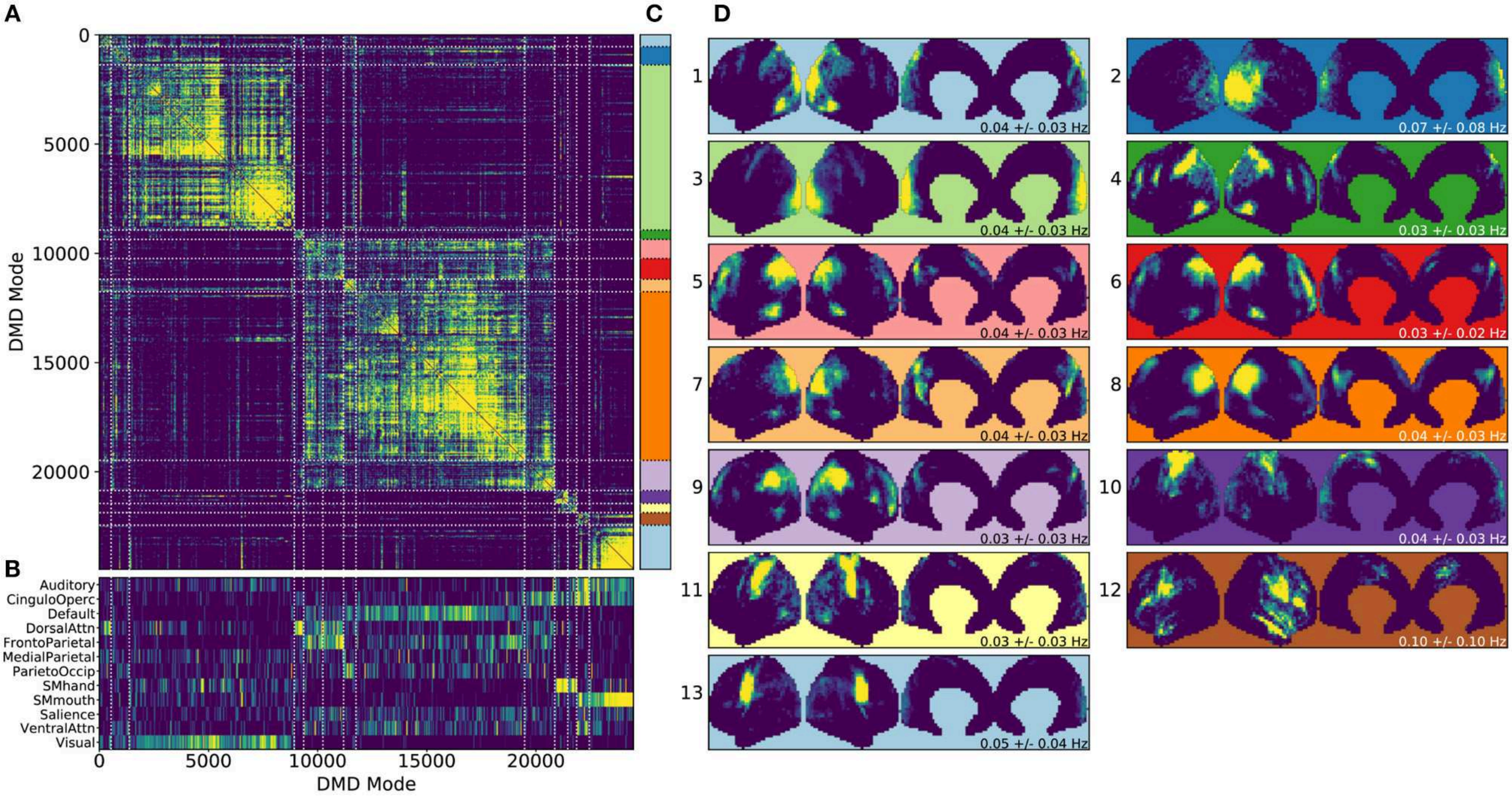
BWB, Johnson, Ojemann & Kutz, *J Neurosci Methods 2016.*





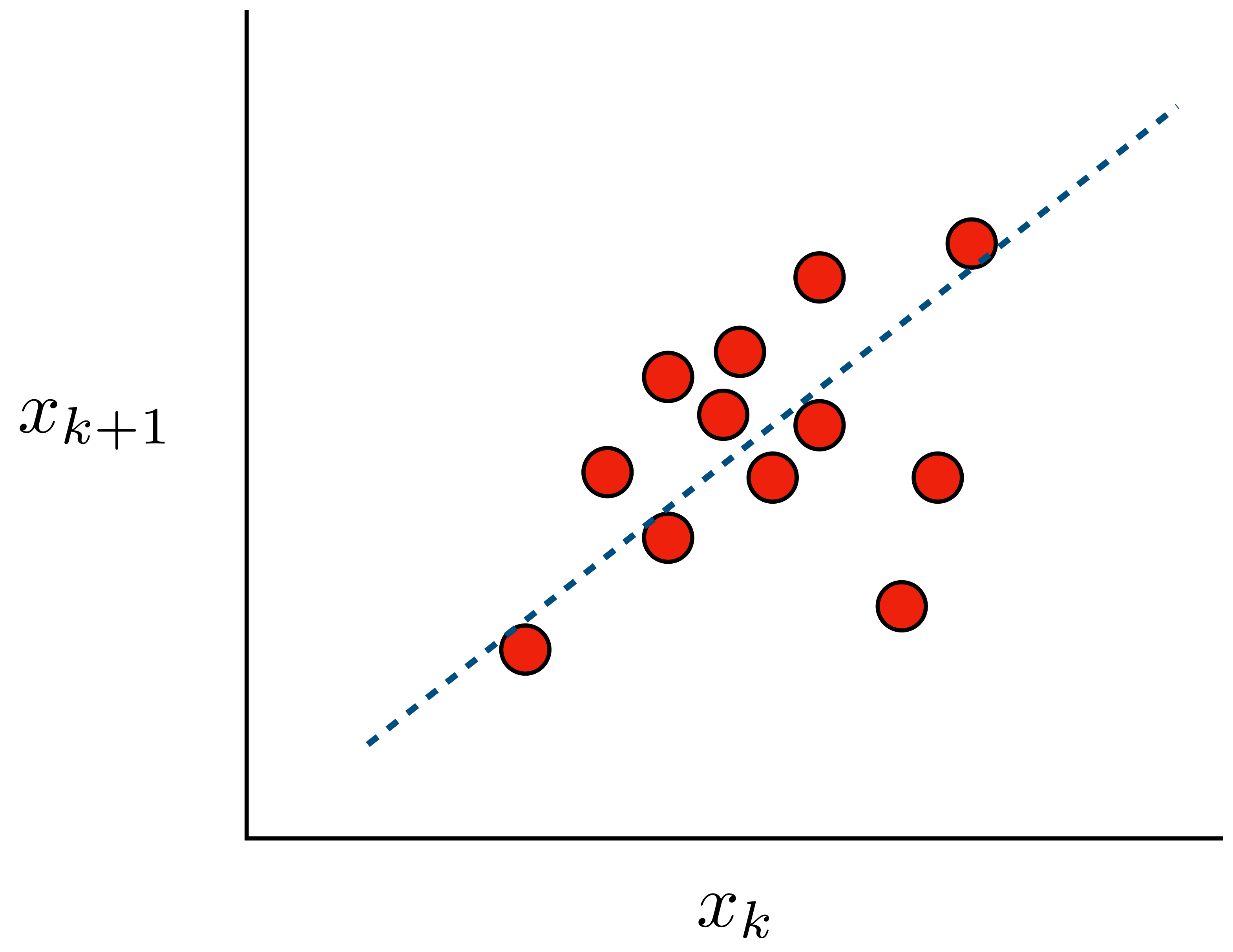
Extracting time-resolved resting-state networks using DMD

Kunert-Graf, … BWB, Frontiers Comp Neurosci 2019.



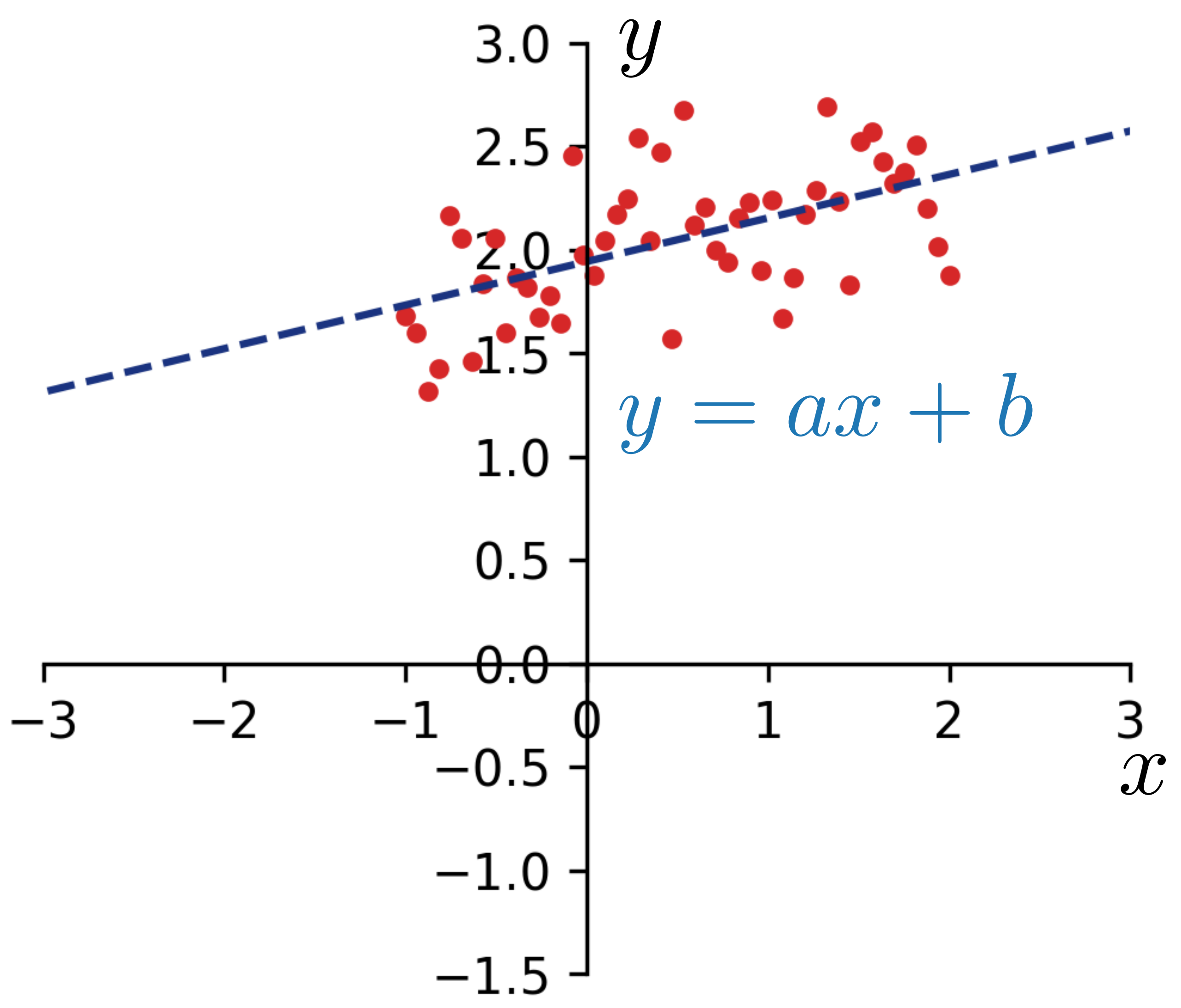
Kunert-Graf, … BWB, Frontiers Comp Neurosci 2019.

Dynamic Mode Decomposition (DMD):
Regression model on the dynamics

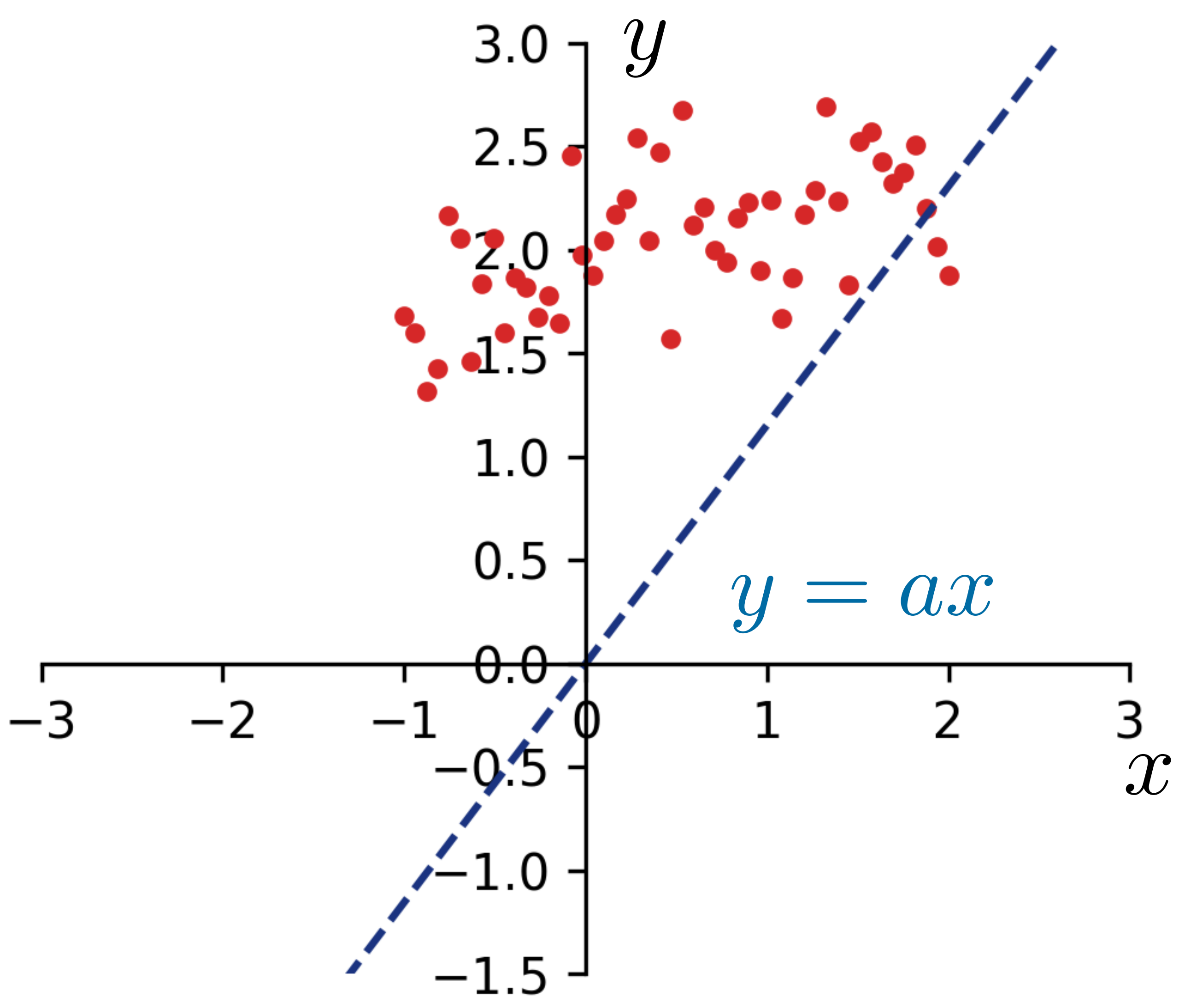


… however, often we are interested in perturbations **around fixed points or equilibria**

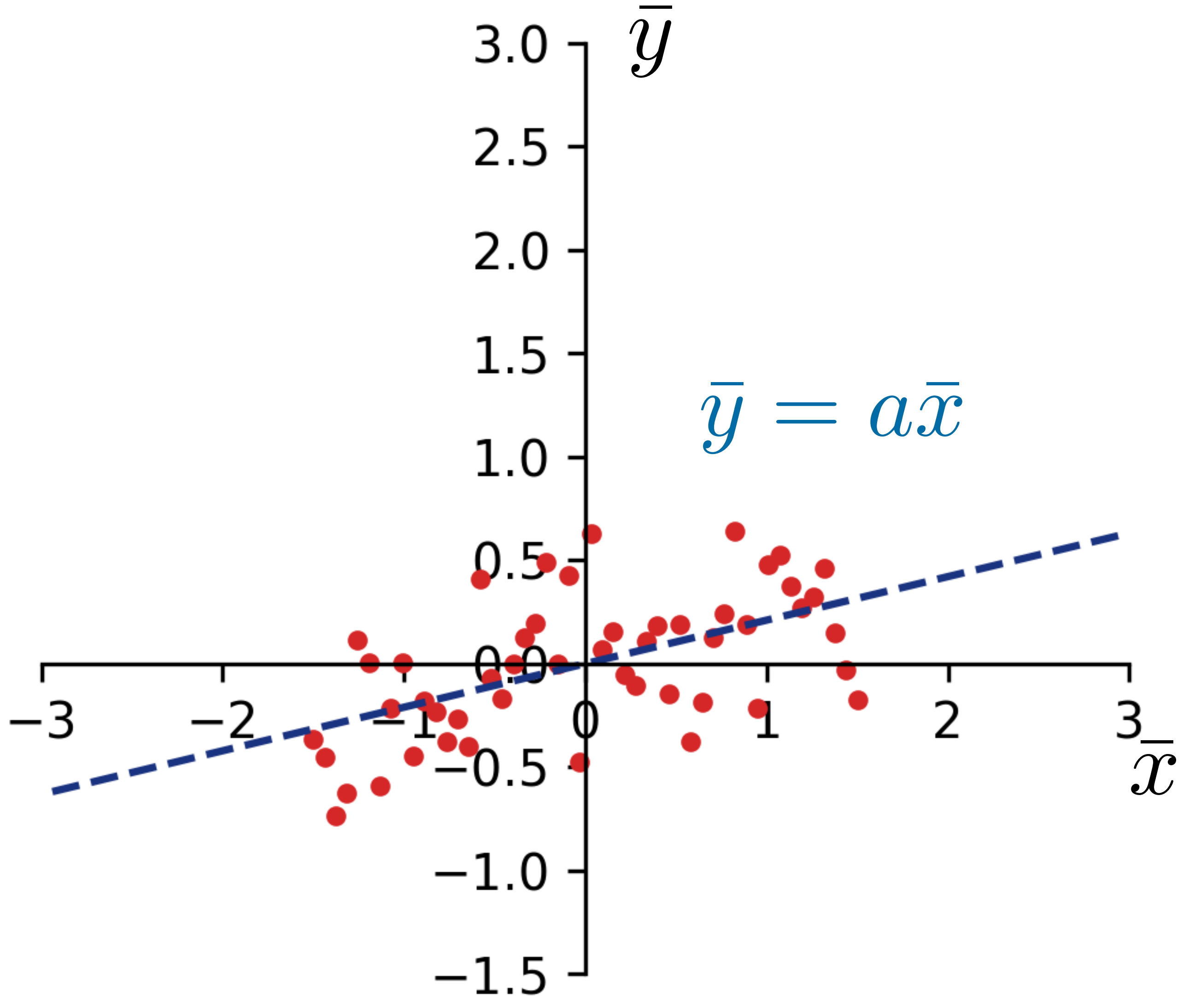
Model with affine term



Model without affine term



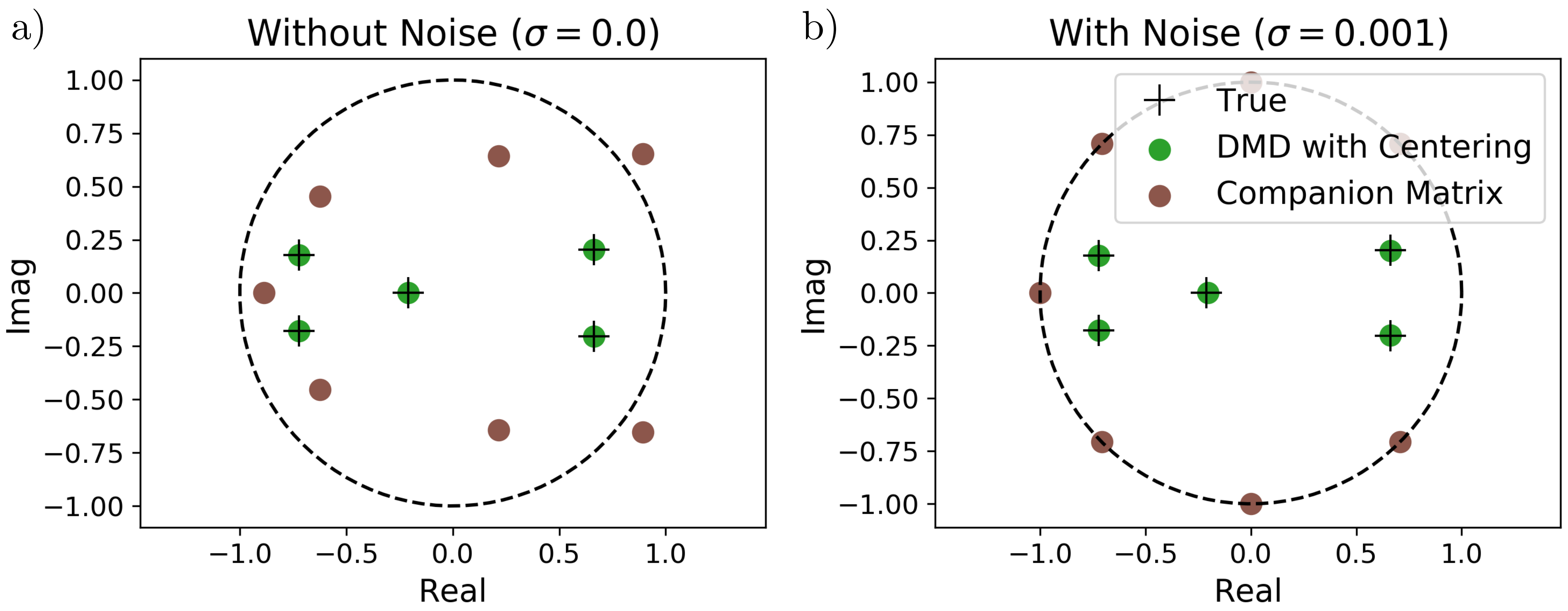
Model on centered data



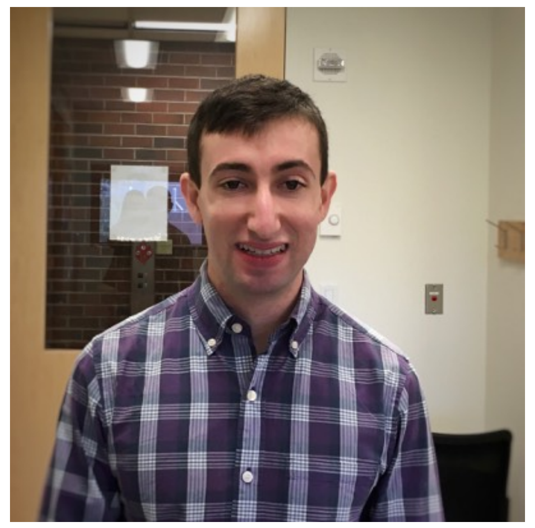
Method		Data Generation: Linear System $\boldsymbol{x}_{j+1} = \boldsymbol{A}\boldsymbol{x}_j$	Data Generation: Affine System $\boldsymbol{x}_{j+1} = \boldsymbol{A}\boldsymbol{x}_j + \boldsymbol{b}$
	DMD w/o centering	✓ (Theorem 4.5)	Sometimes (Theorem 5.3)
	DMD w/ centering	✓ (Theorem 5.2)	✓ (Theorem 4.7)

Centering improves the dynamic mode decomposition

Hirsh, Harris, Kutz & BWB, arXiv:1906.05973.



Chen, Tu & Rowley, J Nonlinear Sci 2012.

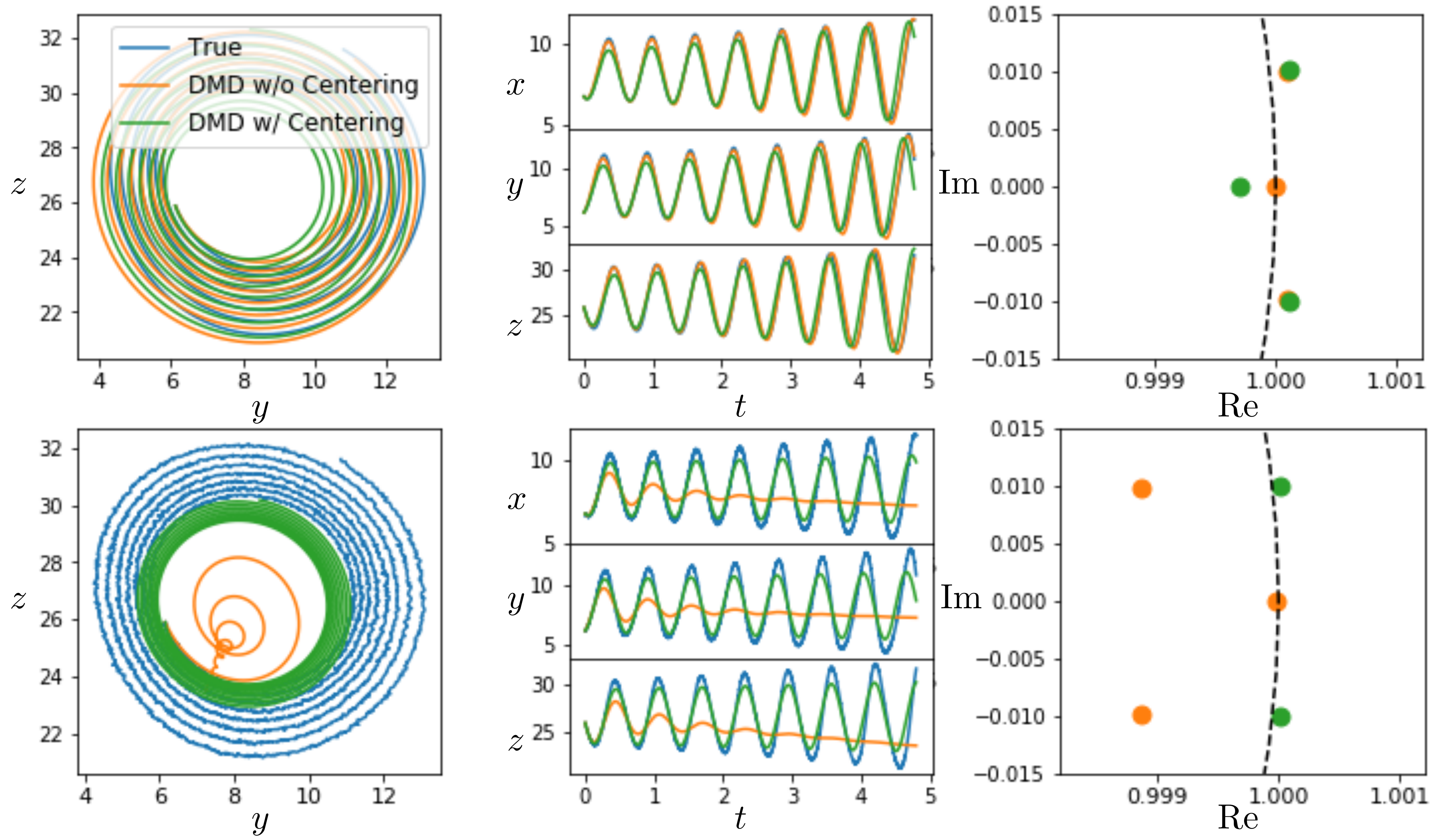


Seth Hirsh

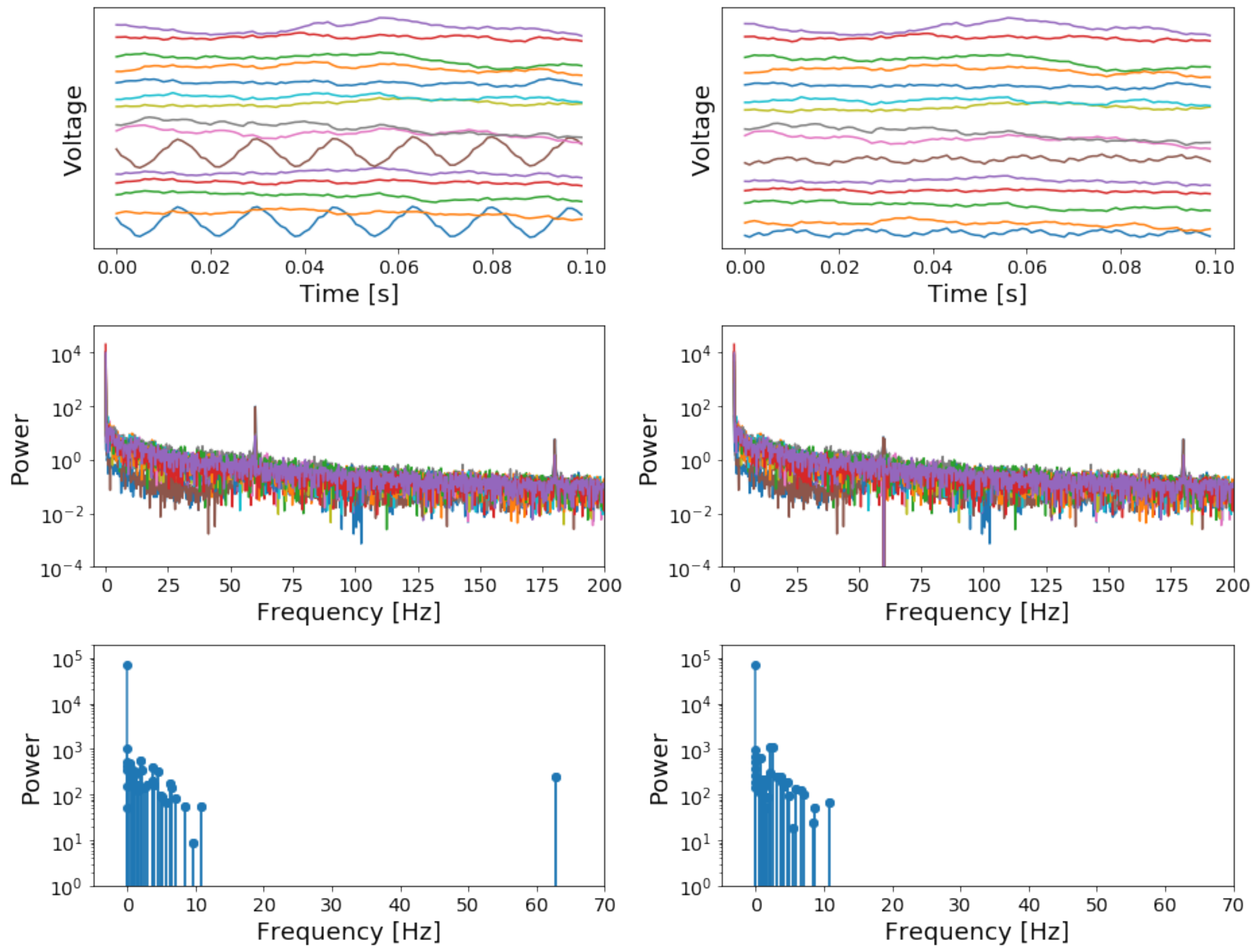
Kameron Decker Harris

Applications to nonlinear dynamical systems

Lorenz: subtract center of data

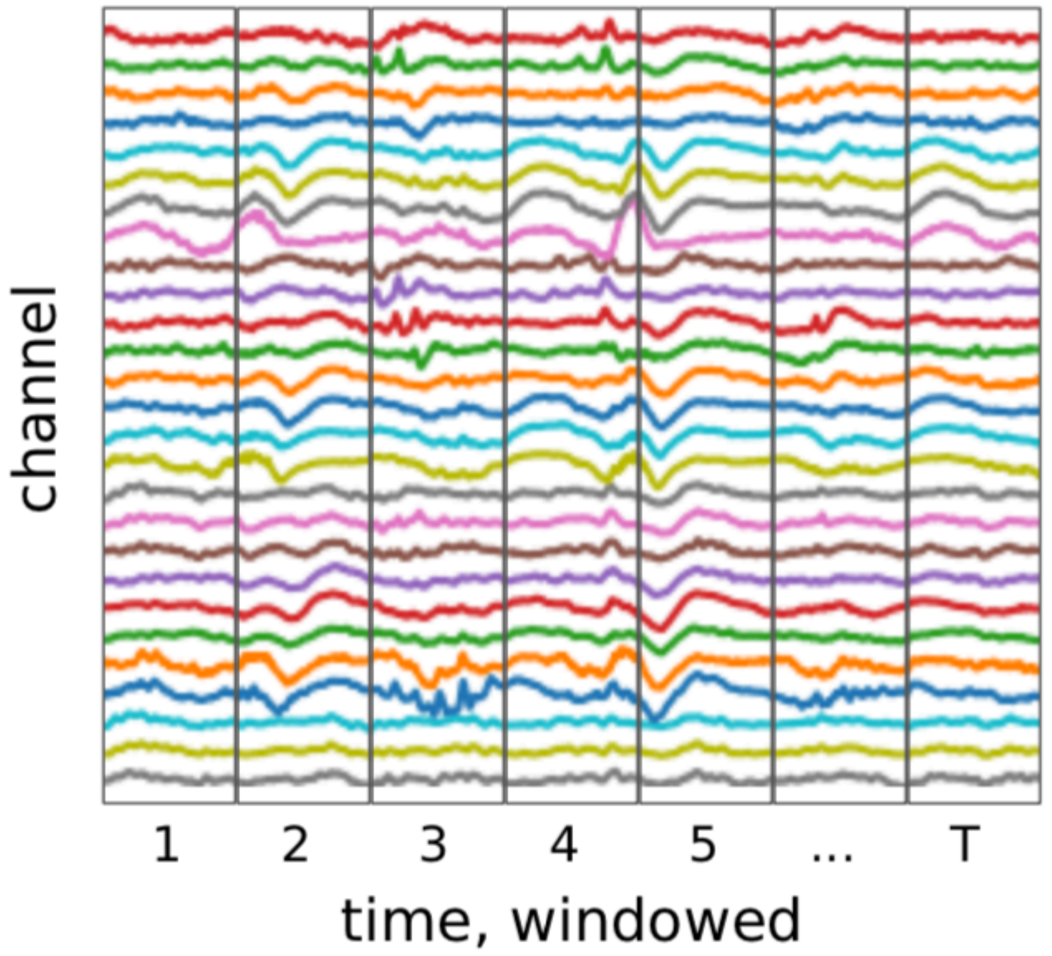


Brain recordings: subtract known 60Hz noise



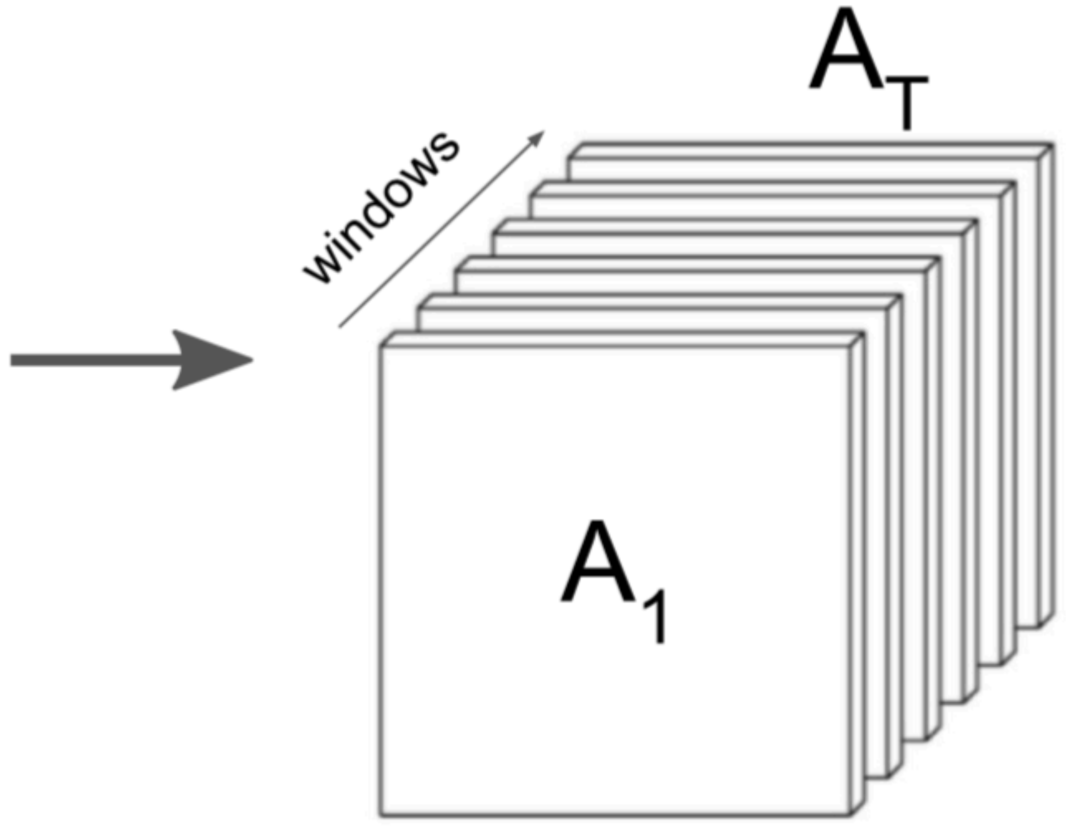
Hirsh, Harris, Kutz & BWB, arXiv:1906.05973.

Timeseries data



Linear model
in each window

Time-varying linear model



$$\boldsymbol{x}(t+1) \approx \mathbf{A}_k\, \boldsymbol{x}(t)$$

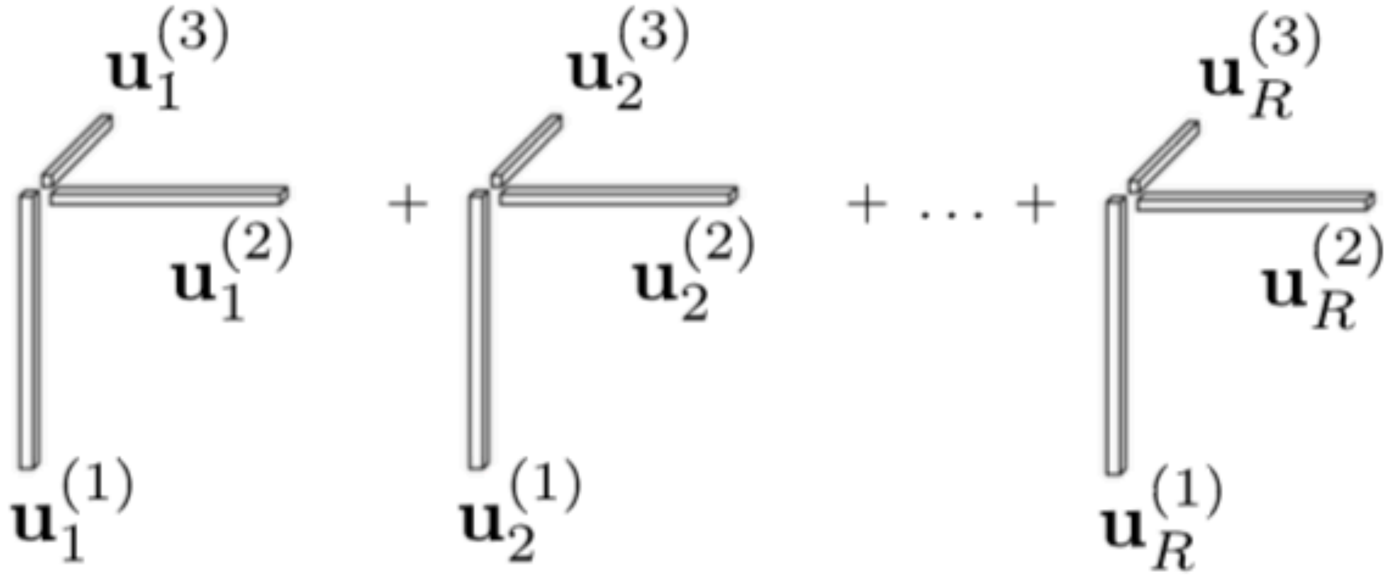
model for data
in window *k*

Tensor of
system matrices

N x N x T
N = # channels
T = # windows

Assumptions on tensor

Low tensor rank



$$\mathbf{Y}_k \approx \mathbf{A}_k \mathbf{X}_k$$

VAR model: reproduce data in
each window (least squares)

$$\mathcal{A} = \sum_{r=1}^{R} \lambda_r\, \mathbf{u}_r^{(1)} \circ \mathbf{u}_r^{(2)} \circ \mathbf{u}_r^{(3)}$$

N x N x T
N = # channels
T = # windows

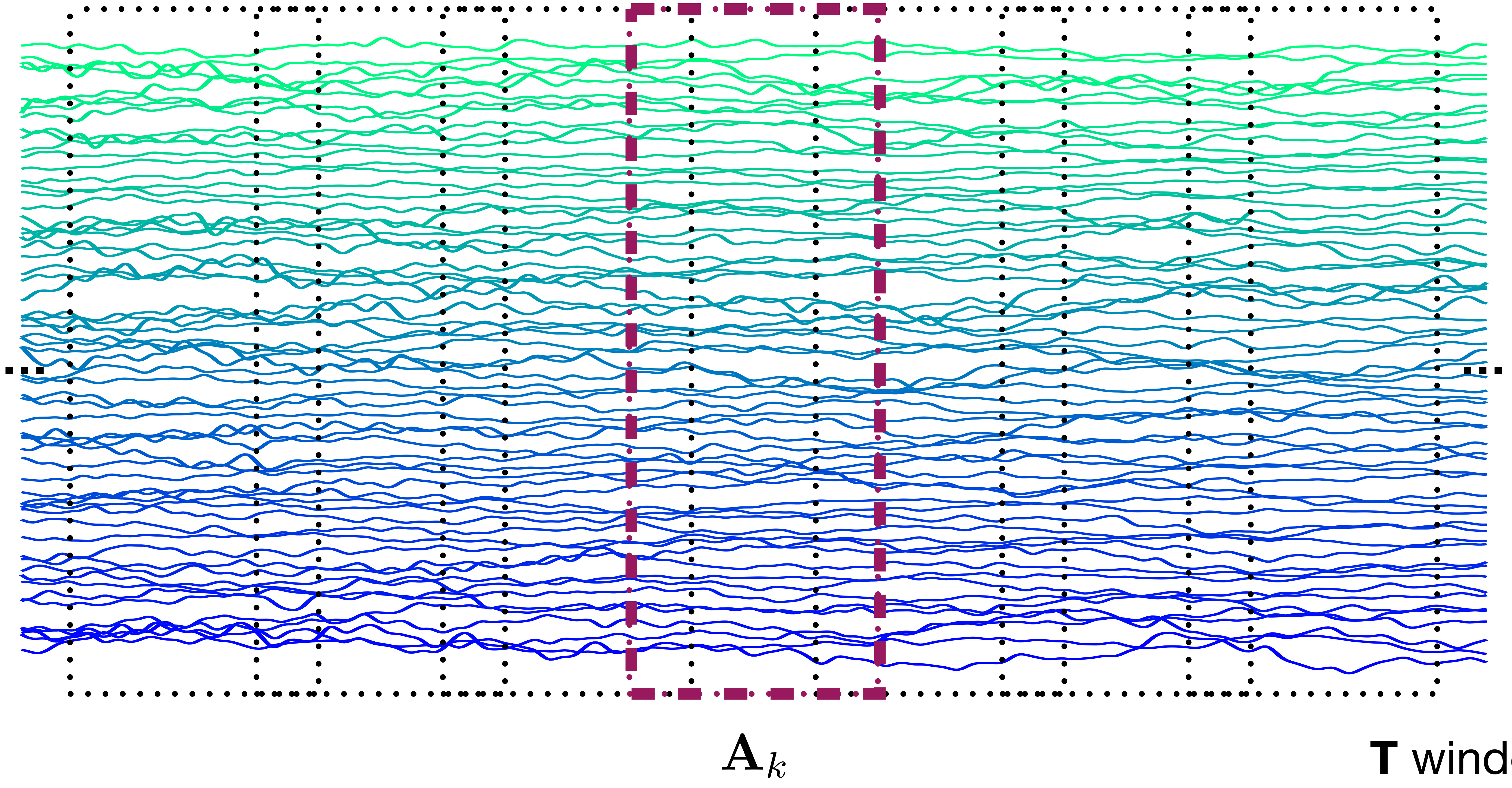
Time-varying autoregression with low-rank tensors (TVART)

Harris, Aravkin, Rao & BWB, arXiv:1905.08389.

Kameron Decker Harris

$$\mathbf{X}_k = \begin{bmatrix} | & | & & | \\ x_0 & x_1 & \cdots & x_{m-1} \\ | & | & & | \end{bmatrix}$$

$$\mathbf{Y}_k = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_m \\ | & | & & | \end{bmatrix}$$



T windows

Least-squares optimization problem:

$$\min_{\mathcal{A}:\ \mathrm{rank}(\mathcal{A})=R} \frac{1}{2} \sum_{k=1}^{T} \|\mathbf{Y}_k - \mathbf{A}_k \mathbf{X}_k\|_F^2 \,.$$

$$\mathcal{A} = \sum_{r=1}^{R} \mathbf{u}_r^{(1)} \circ \mathbf{u}_r^{(2)} \circ \mathbf{u}_r^{(3)}$$

$$\boxed{\begin{gathered} \mathbf{A}_k = \mathbf{U}^{(1)} \mathbf{D}^{(k)} \mathbf{U}^{(2)\,\mathsf{T}}, \\ \text{where} \quad \mathbf{D}^{(k)} = \mathrm{diag}\left(\mathbf{u}_{k:}^{(3)}\right) \end{gathered}}$$

Harris, Aravkin, Rao & BWB, arXiv:1905.08389.

Regularized cost function:

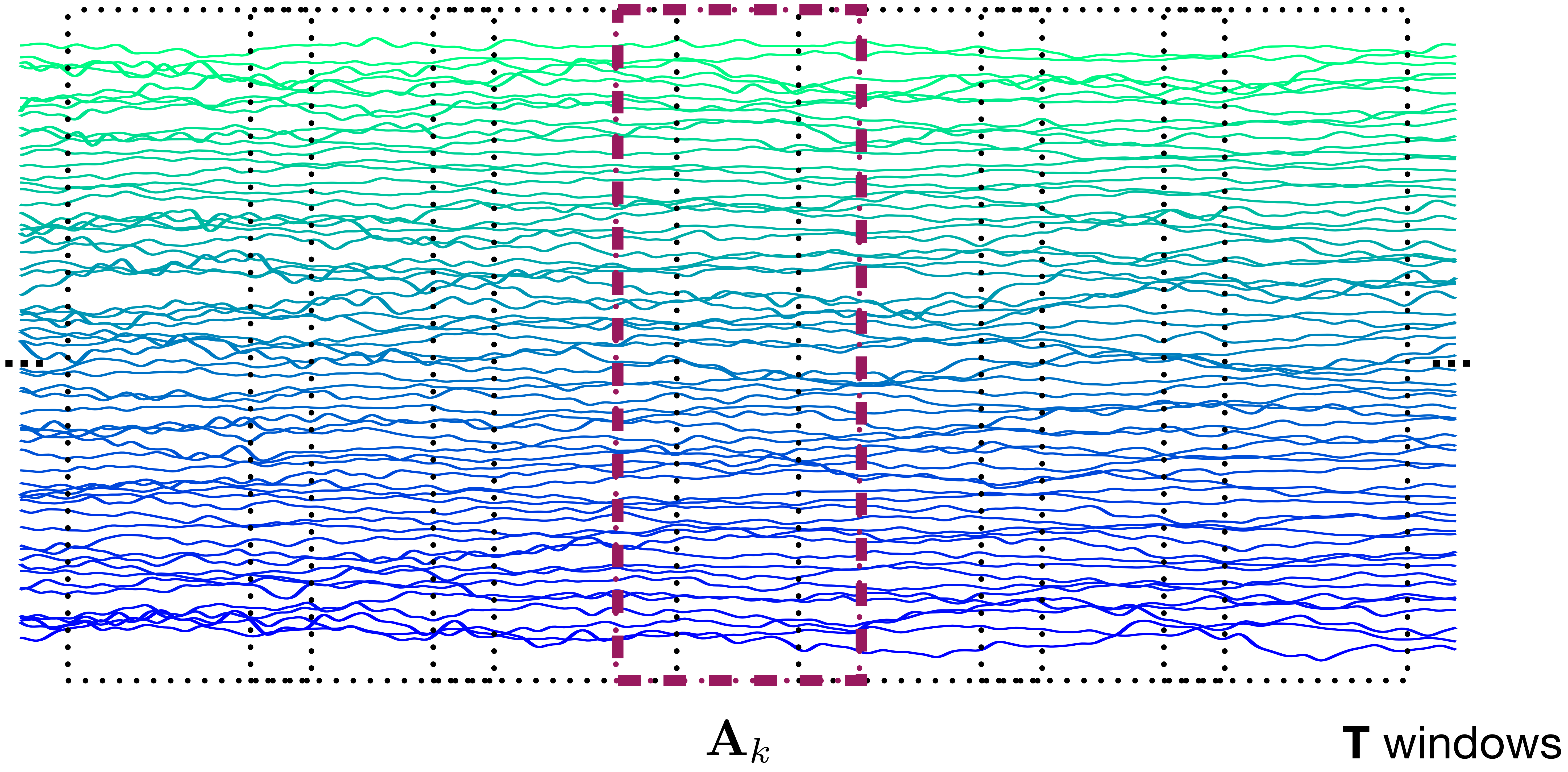
2 Parameters

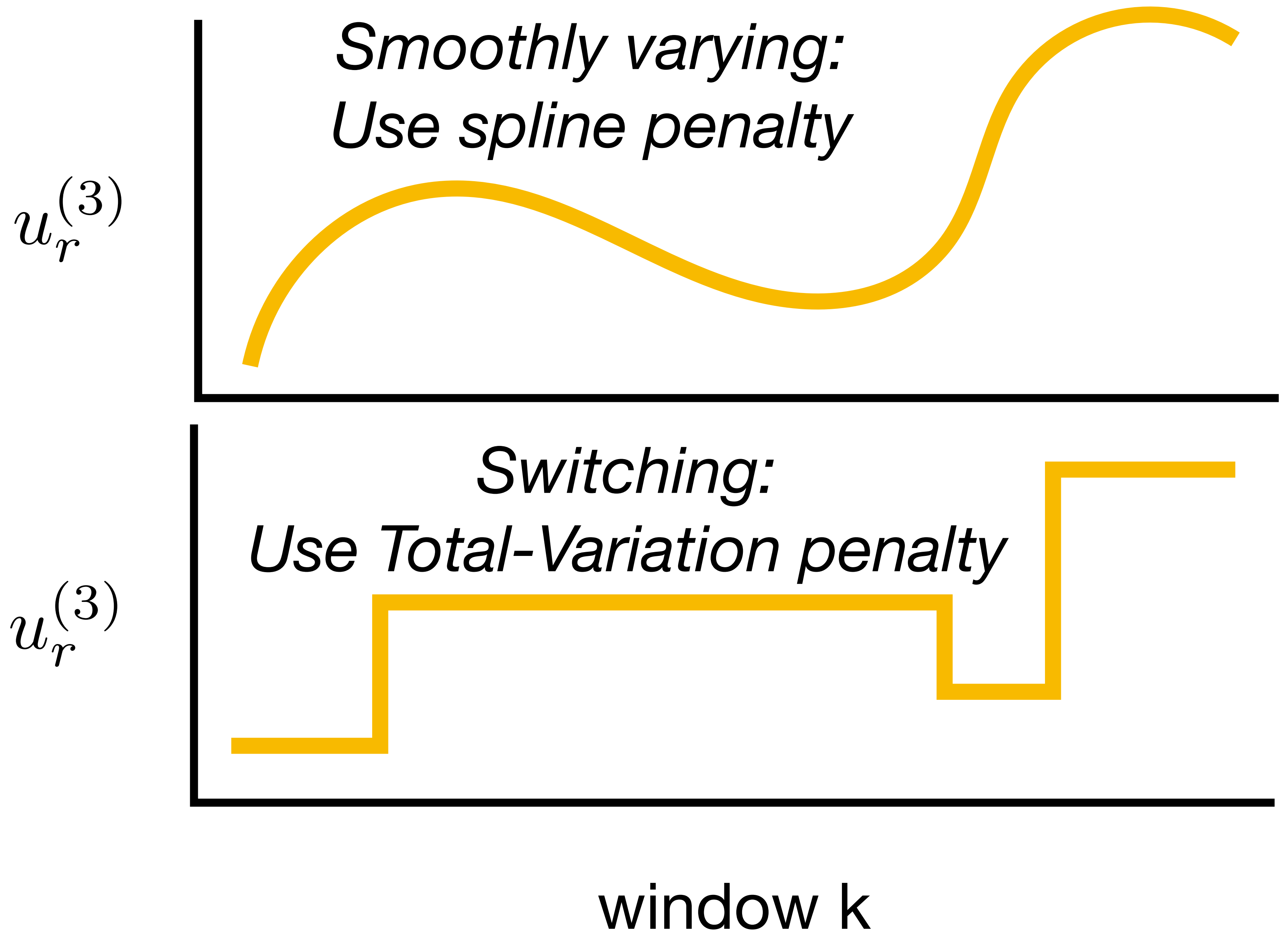
$$C = \frac{1}{2}\sum_{k=1}^{T} \|\mathbf{Y}_k - \mathbf{A}_k\mathbf{X}_k\|_F^2 + \frac{1}{2\eta}\left(\|\mathbf{U}^{(1)}\|_F^2 + \|\mathbf{U}^{(2)}\|_F^2 + \|\mathbf{U}^{(3)}\|_F^2\right) + \beta\,\mathcal{R}(\mathbf{U}^{(3)}),$$

Least squares error

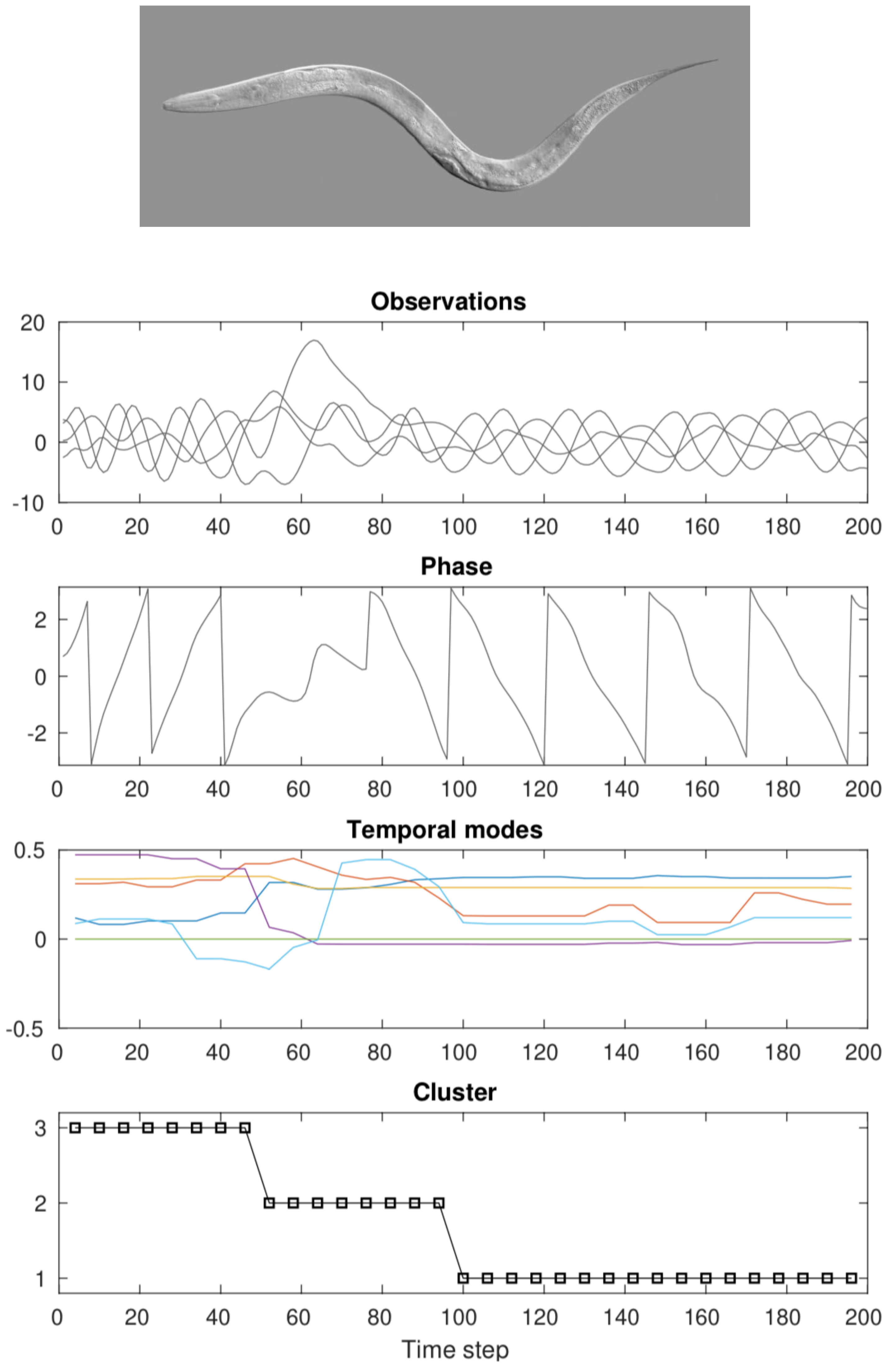
Tikhonov regularizer

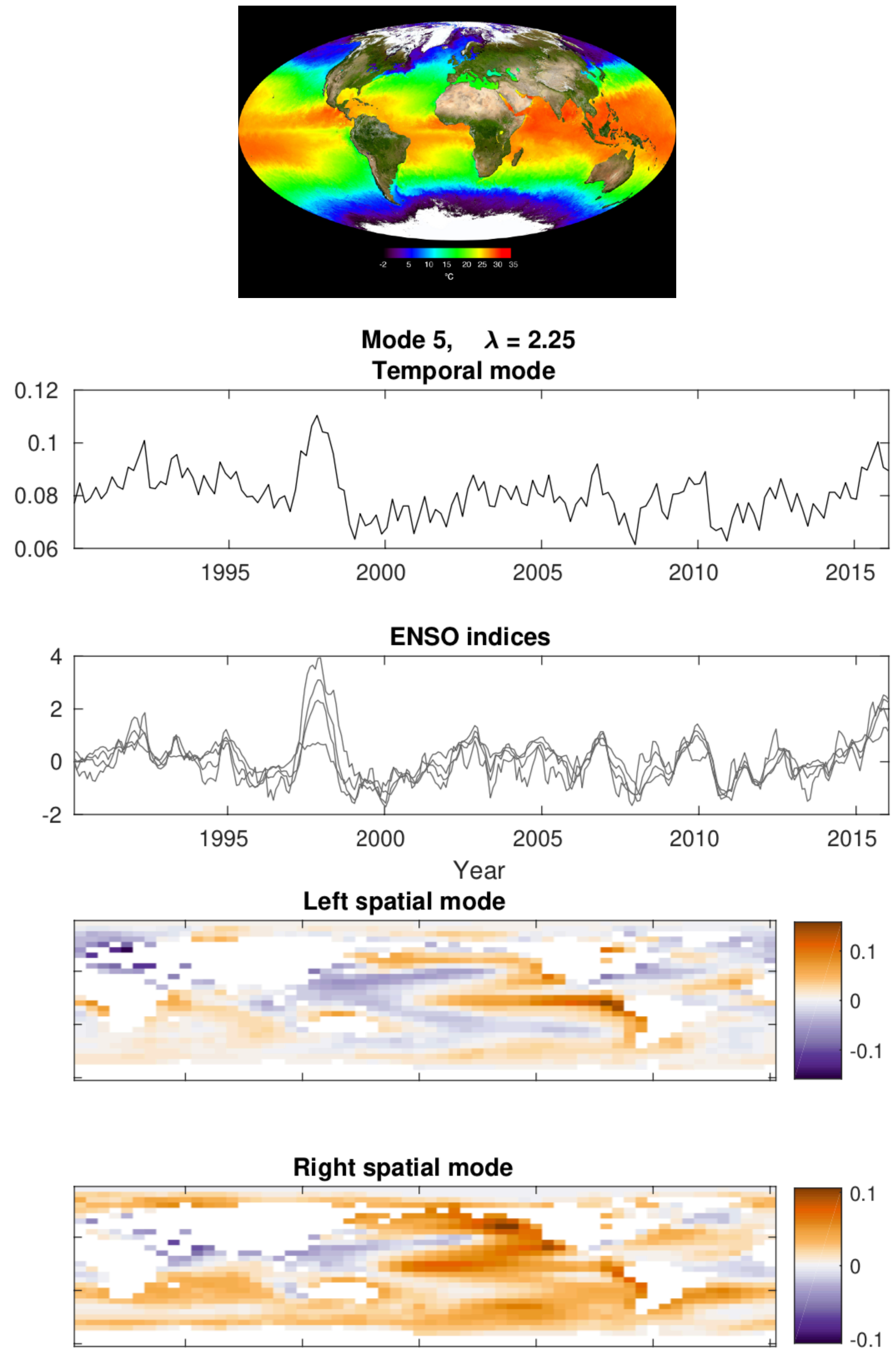
Temporal smoothing regularizer

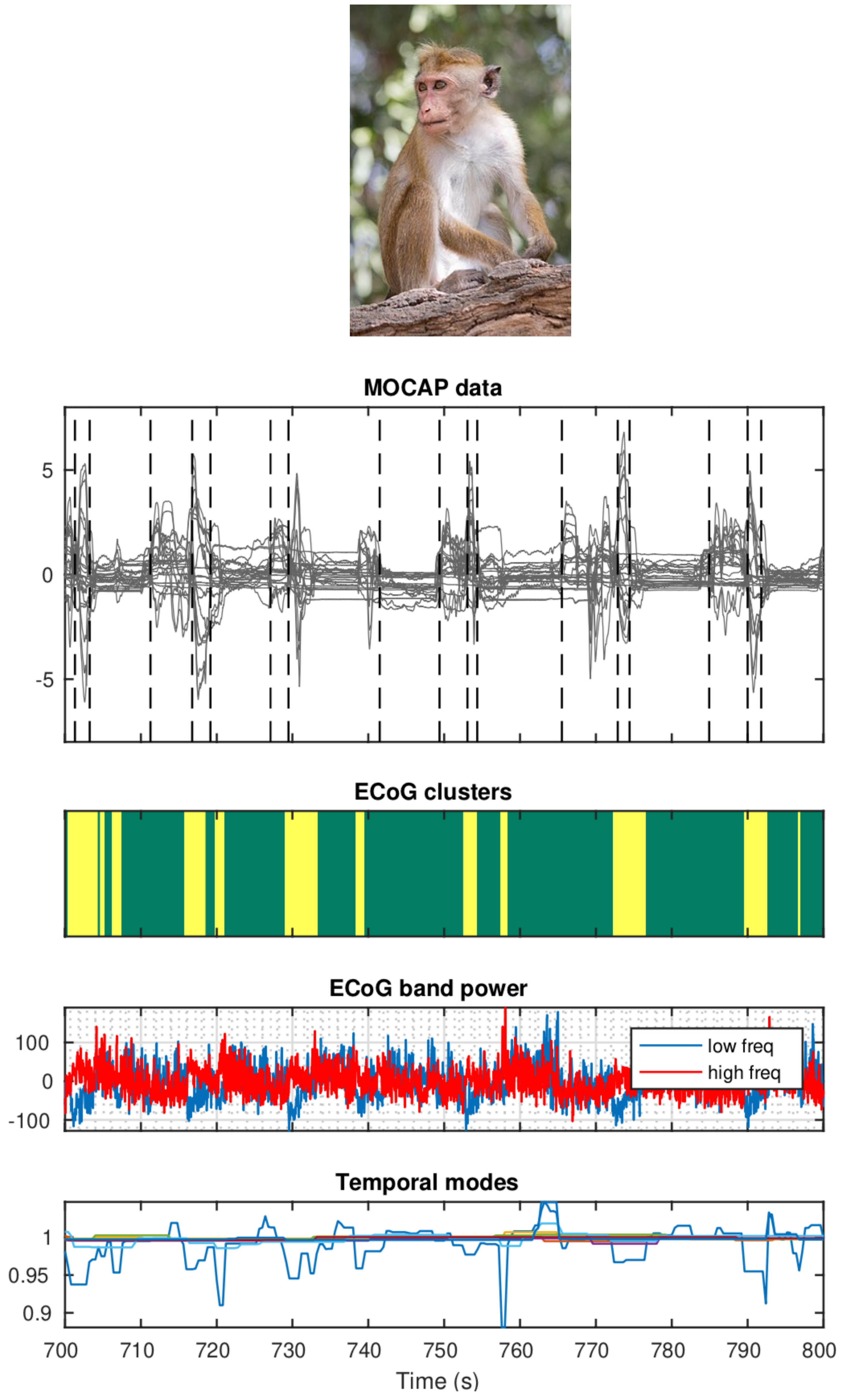




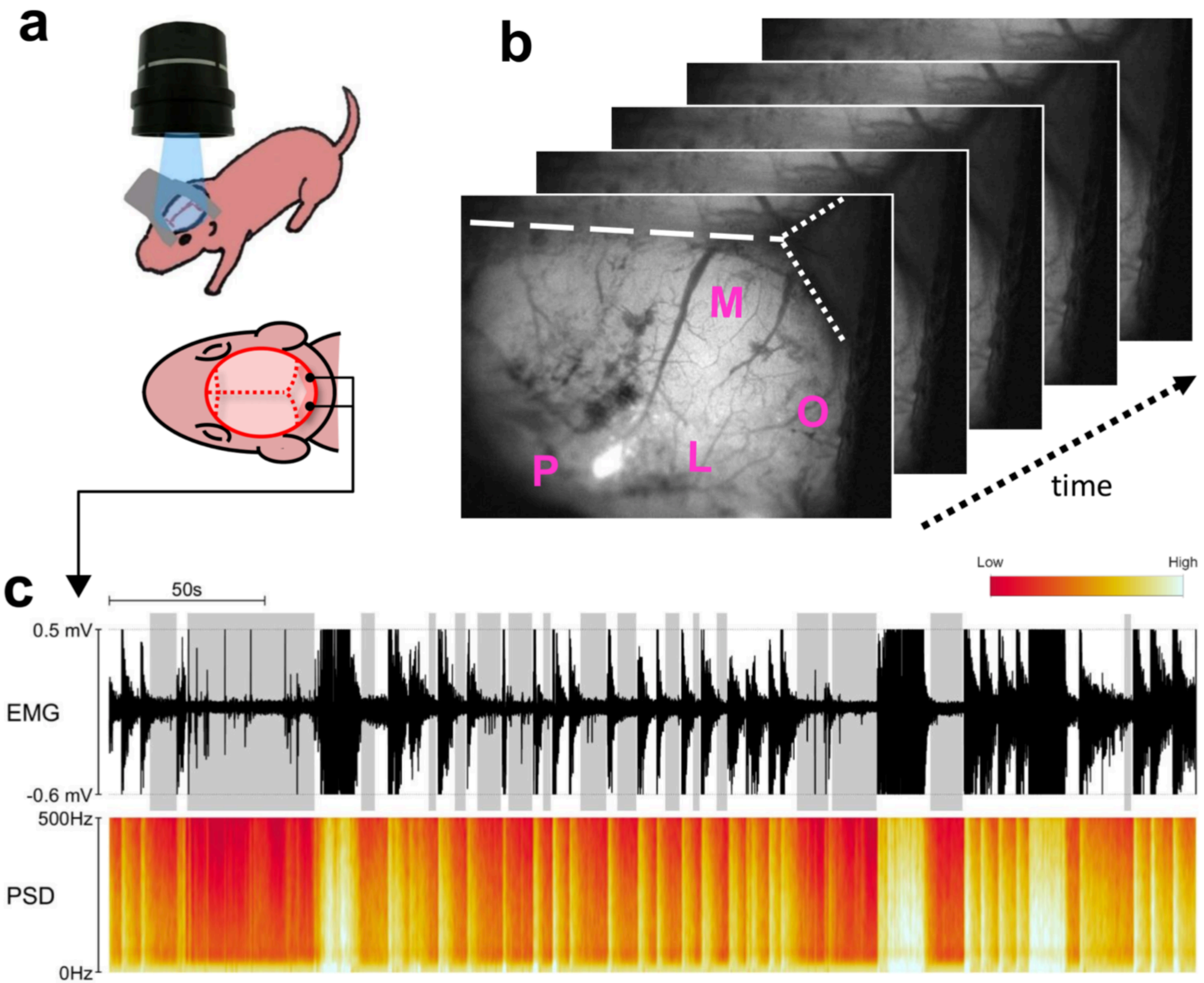
Harris, Aravkin, Rao & BWB, arXiv:1905.08389.





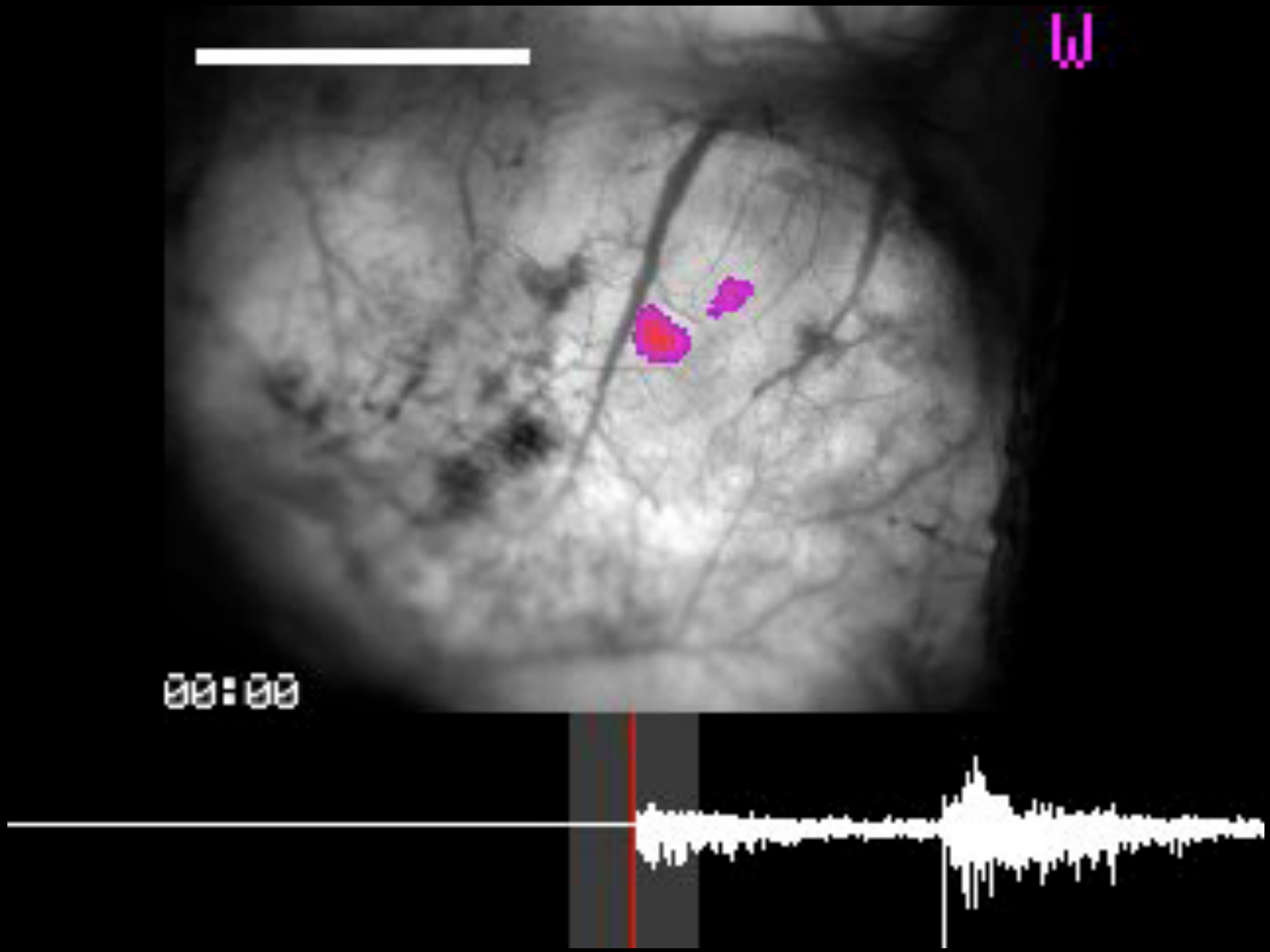


Harris, Aravkin, Rao & BWB, arXiv:1905.08389.

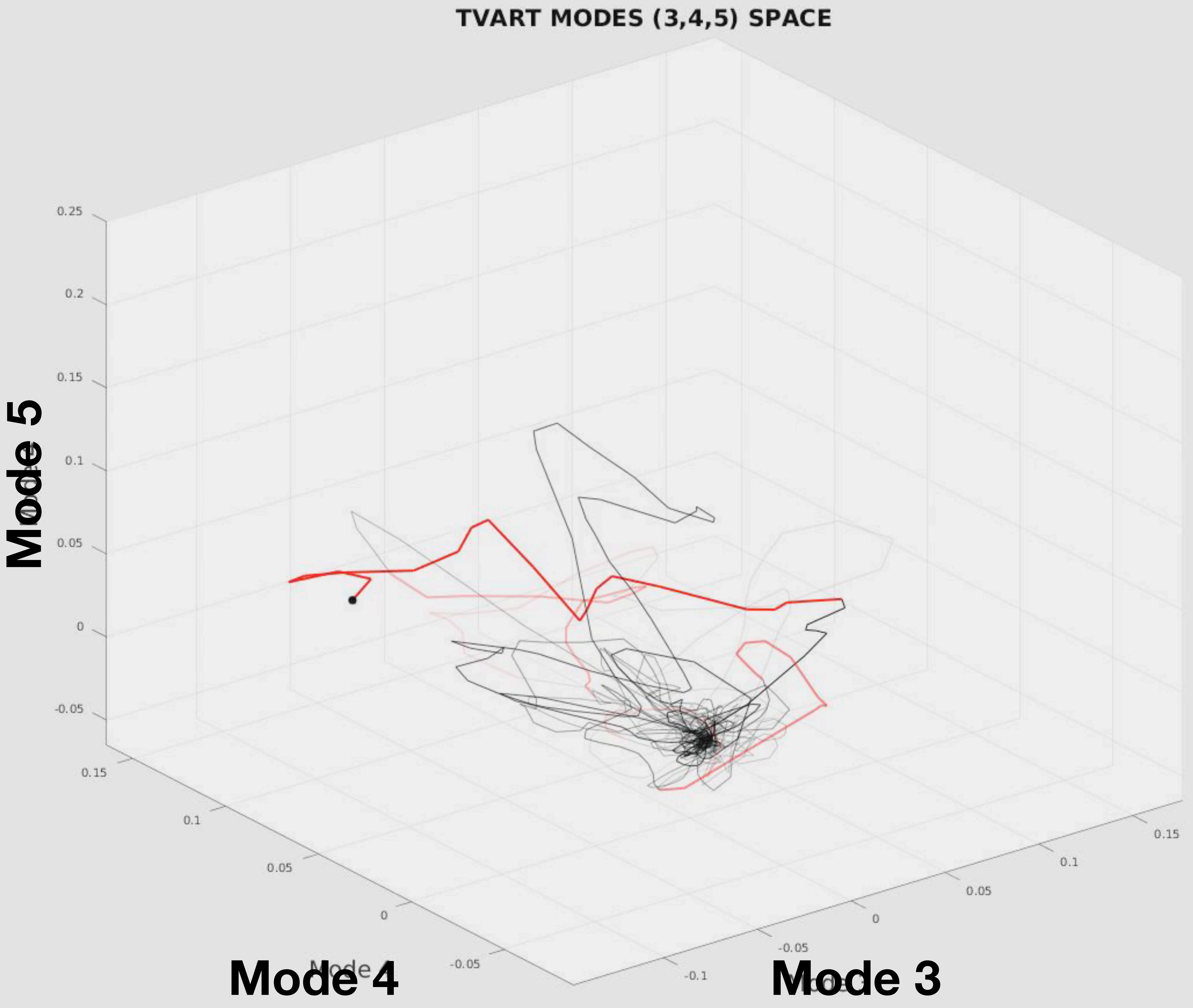


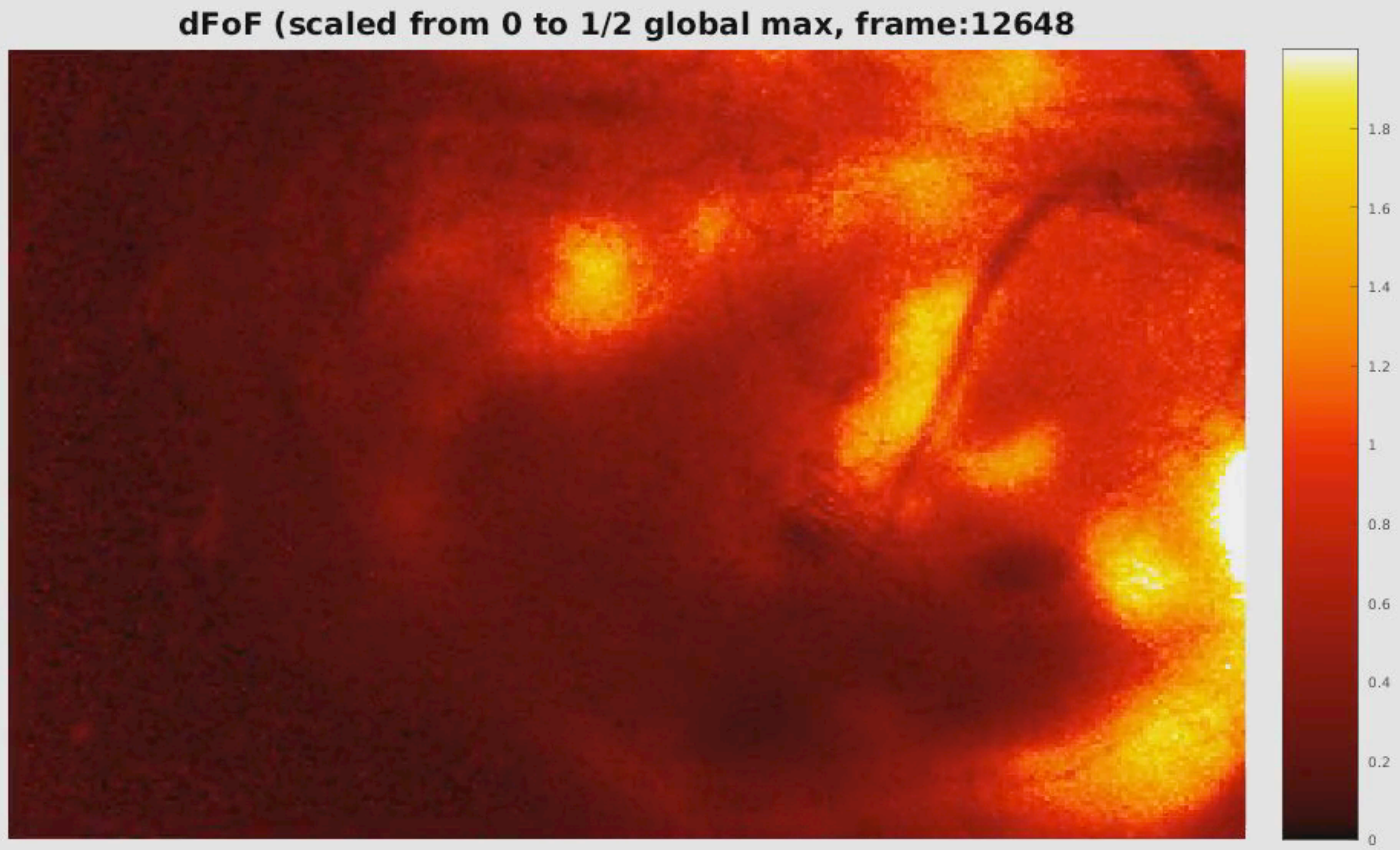
Dennis Tabuena ***Bill Moody***

Tabuena, Huynh, Metcalf, Richner, Stroh, BWB, Moody & Easton, *In review.*



Movie from supplemental material of
Tabuena, Huynh, Metcalf, Richner, Stroh, BWB, Moody & Easton, *In review.*


TVART MODES (3,4,5) SPACE
Mode 5
Mode 4
Mode 3


dFoF (scaled from 0 to 1/2 global max, frame:12648


Mode 3
Mode 4
Mode 5

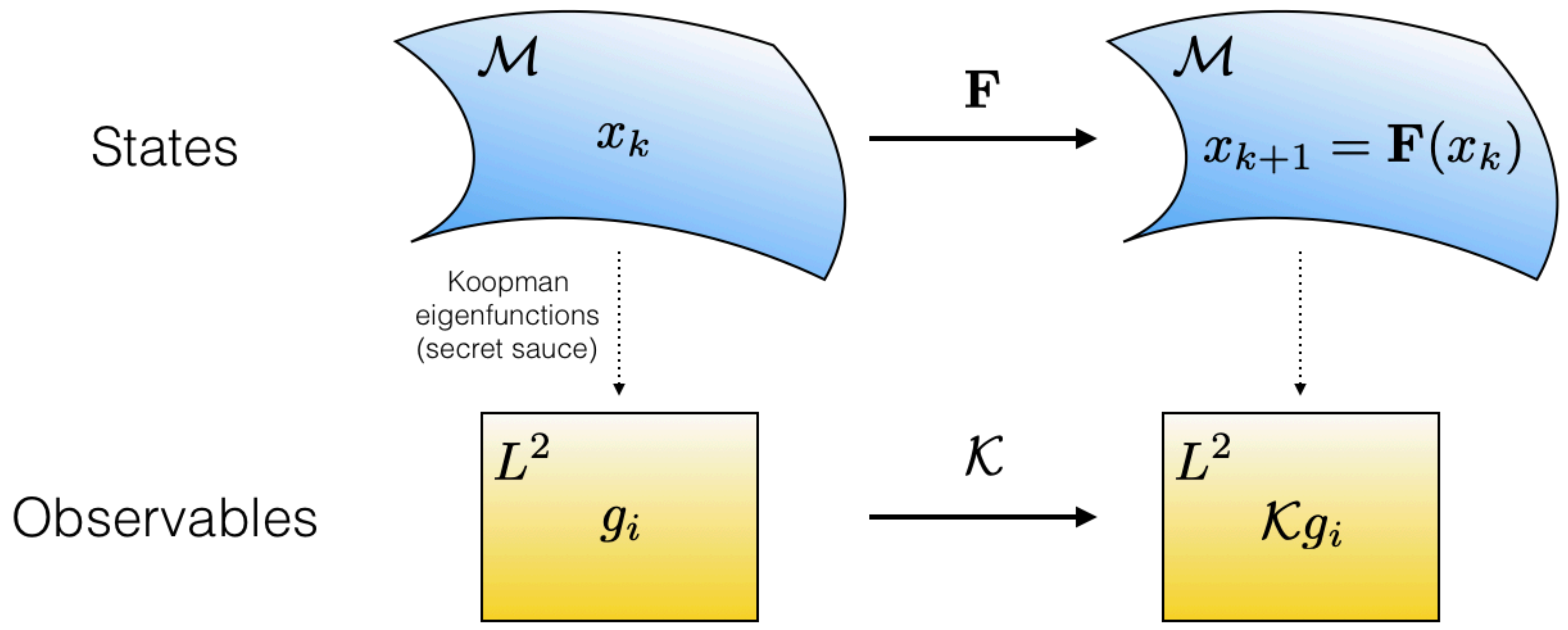
Nate Linden

Networks of neurons and networks of brain areas **expand and contract the effective dimensionality** of their inputs, potentially making their representations **more linear**.

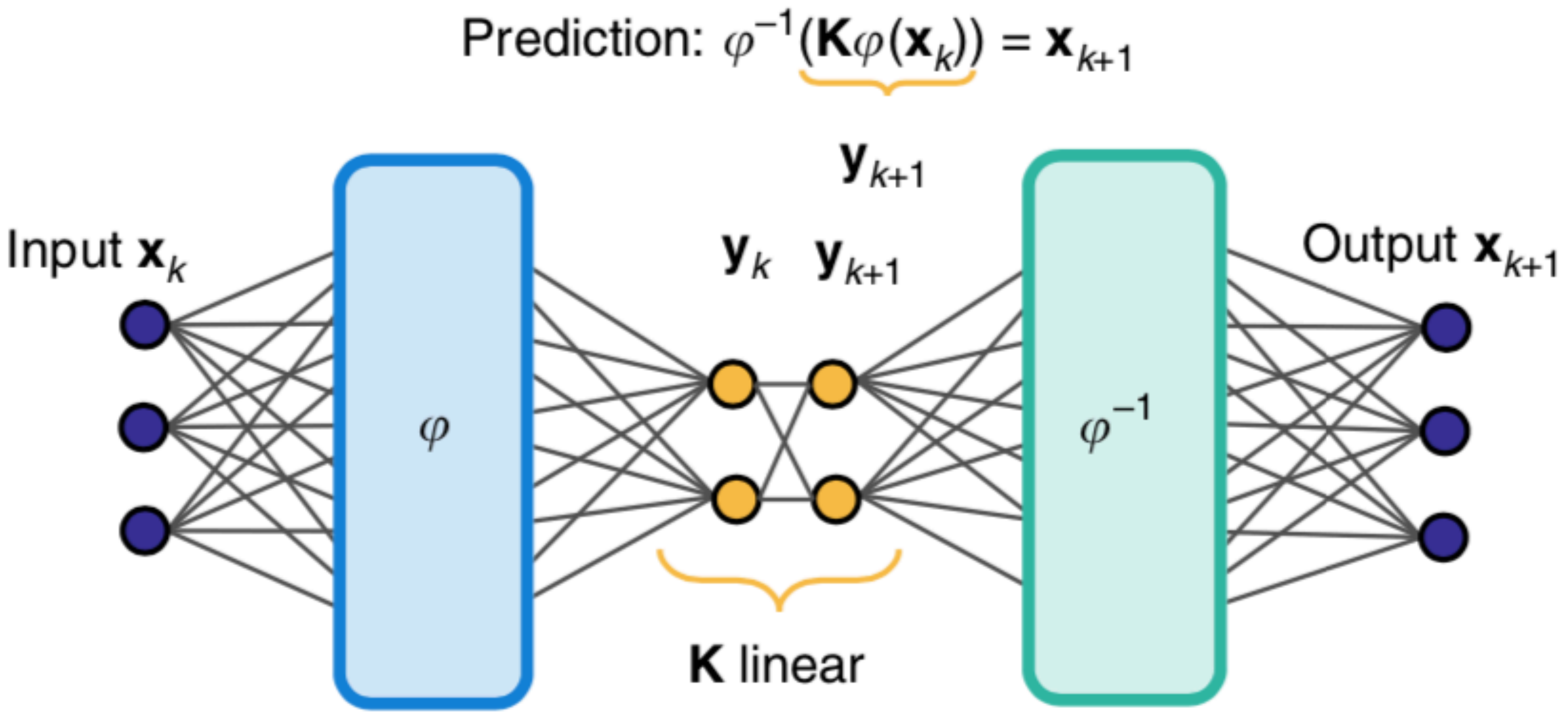
Linearity in representation makes many tasks easier, including **classification**, **modeling of dynamics**, and **control**.

Kernel methods

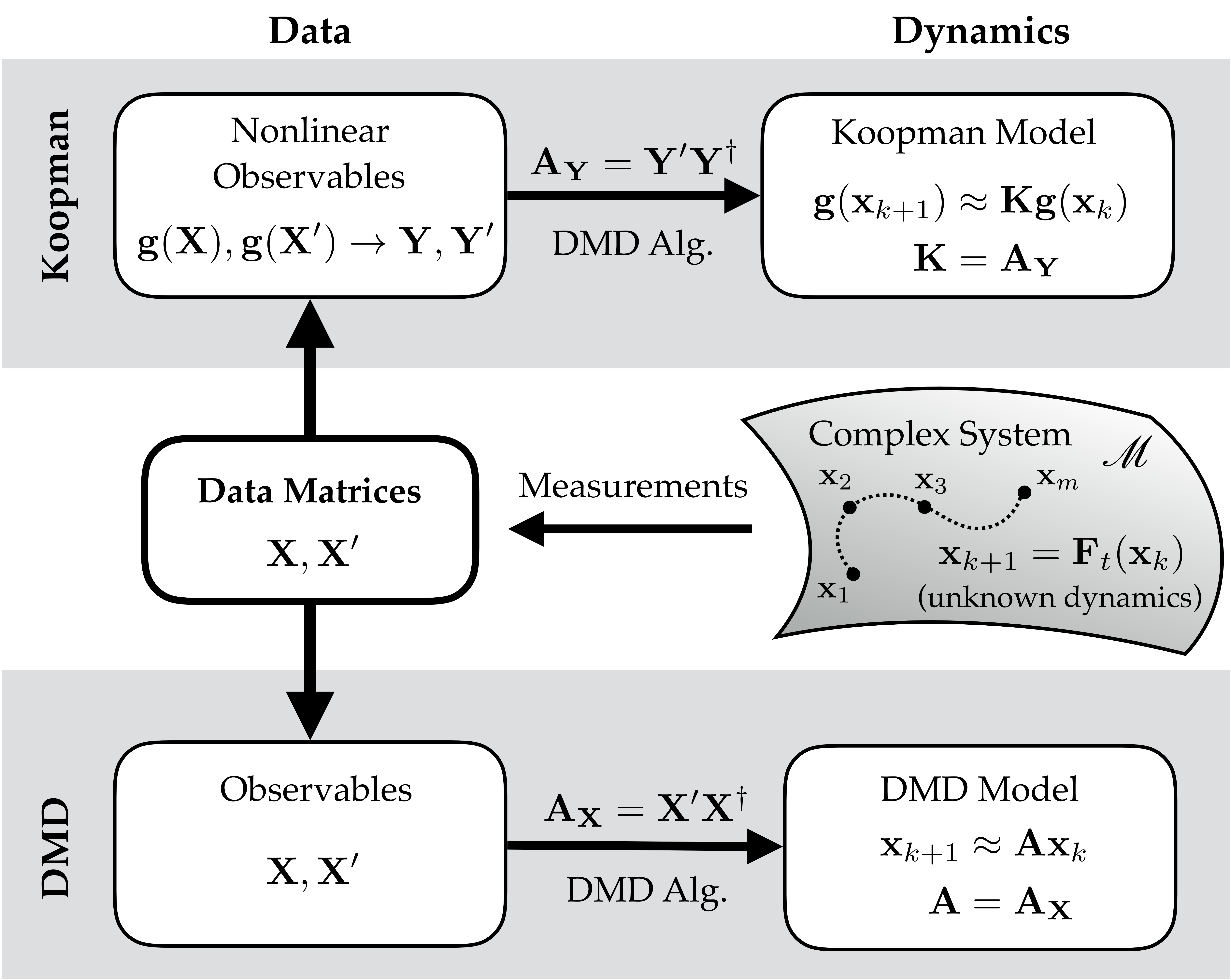
Koopman operator



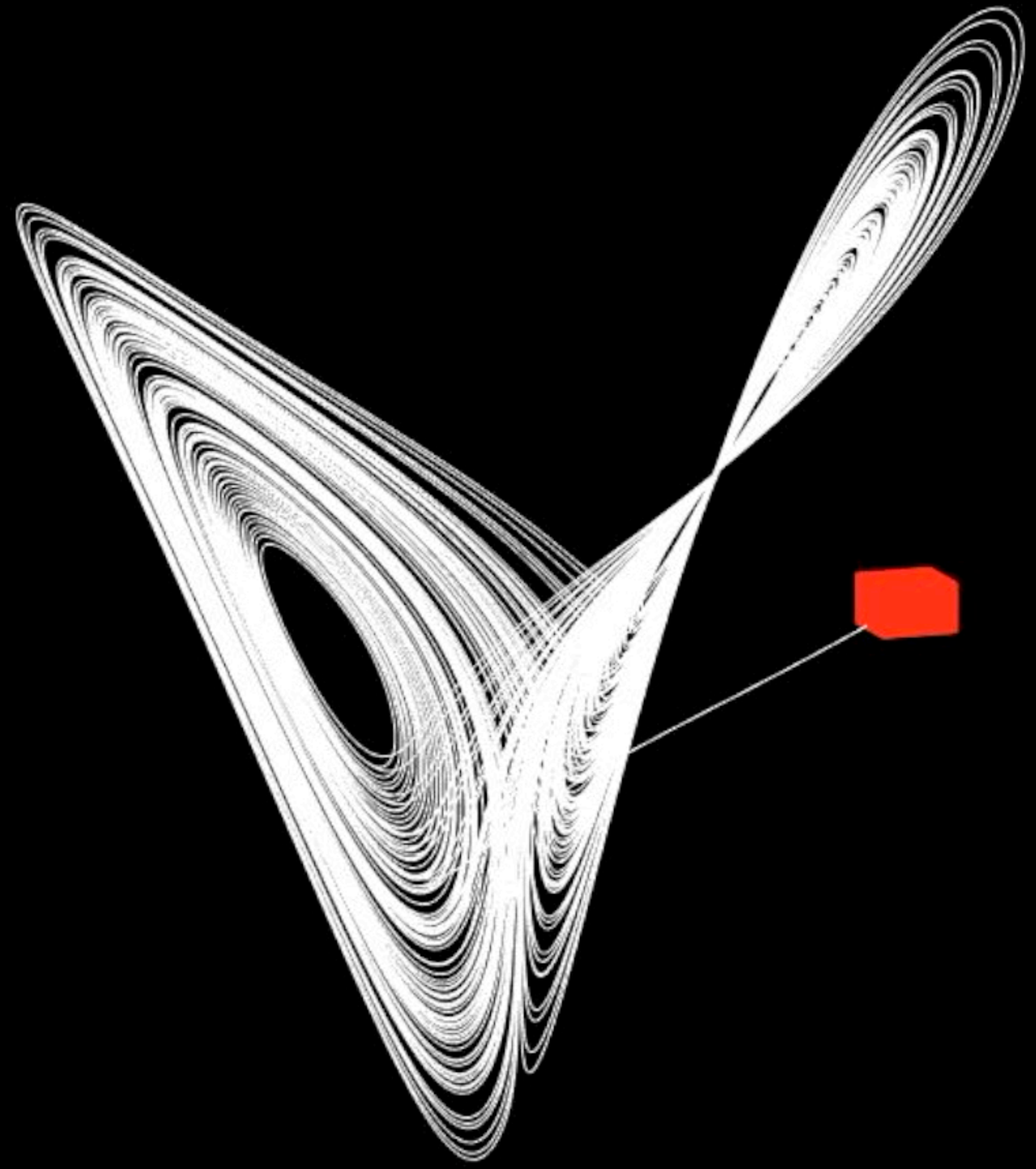
Koopman 1931.



Lusch et al., 2018.

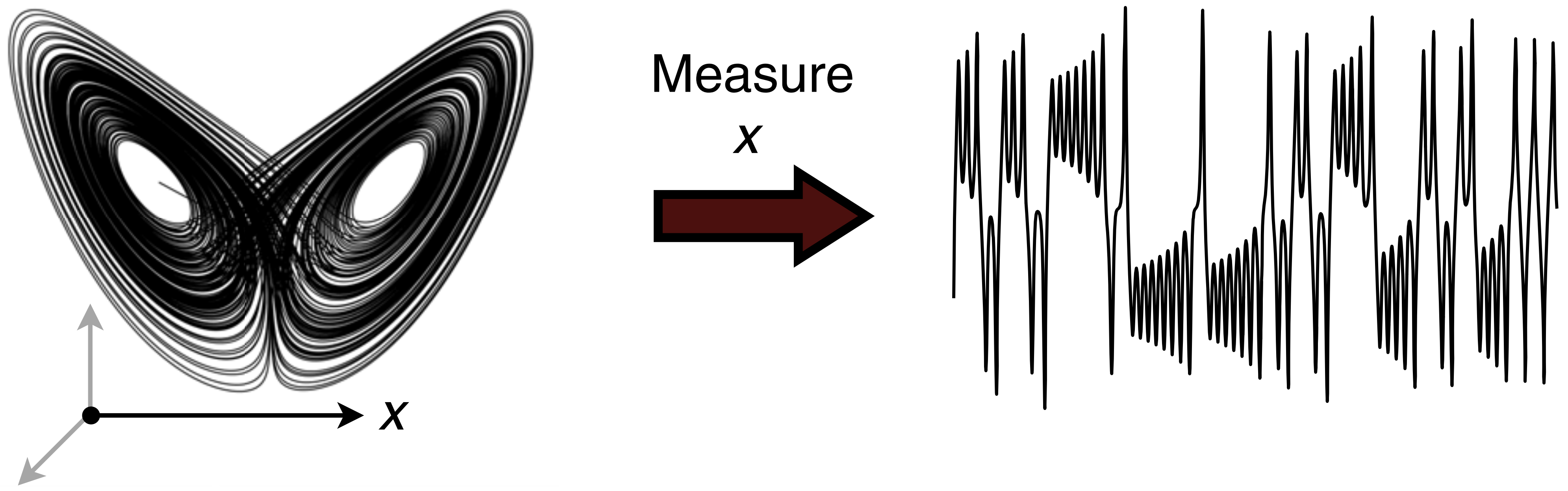


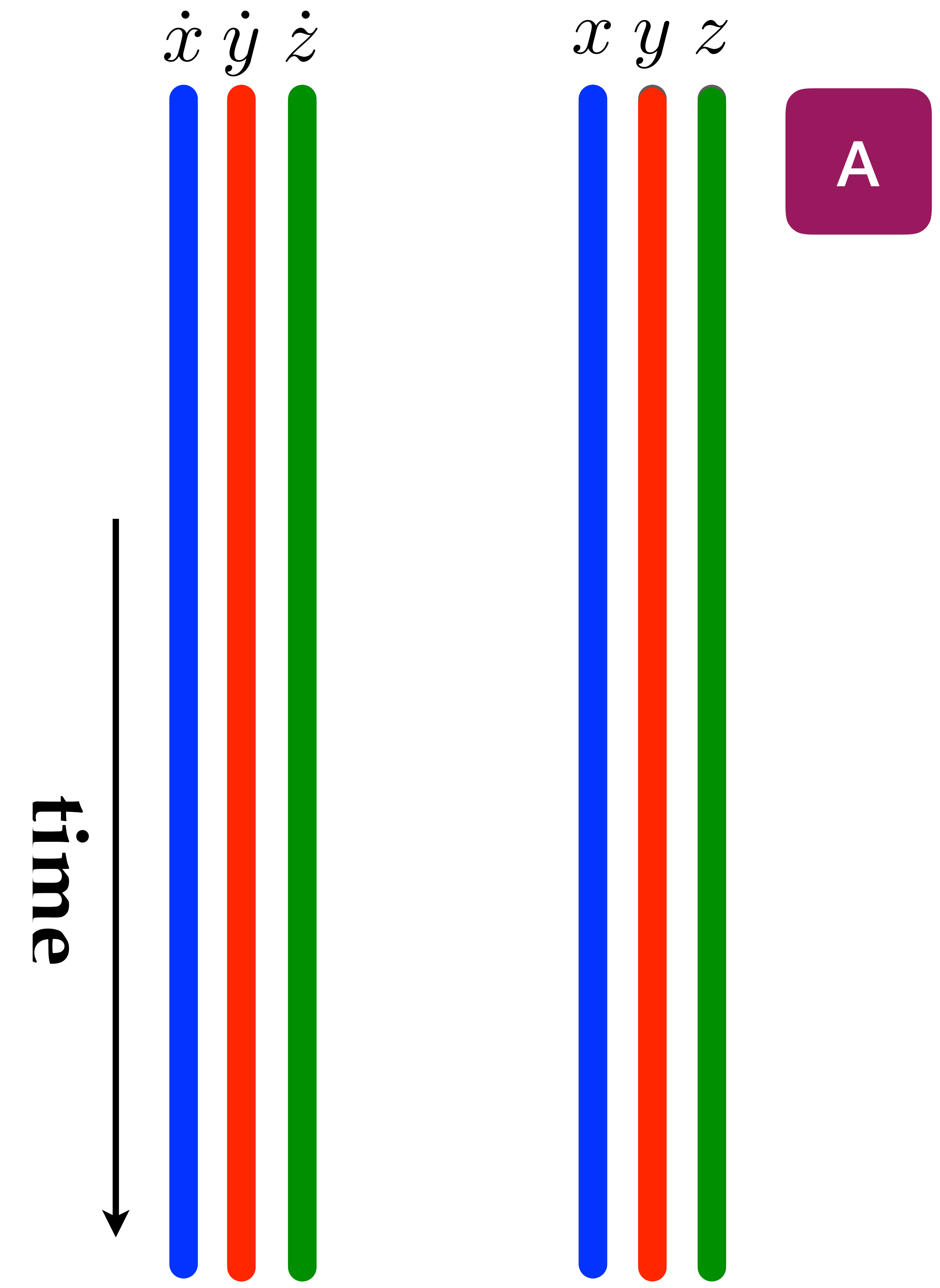
Kutz, Brunton, BWB, & Proctor, SIAM 2016.



Movie by S. Brunton

Hankel alternative view of Koopman (HAVOK) *Representations and Time-Delays*





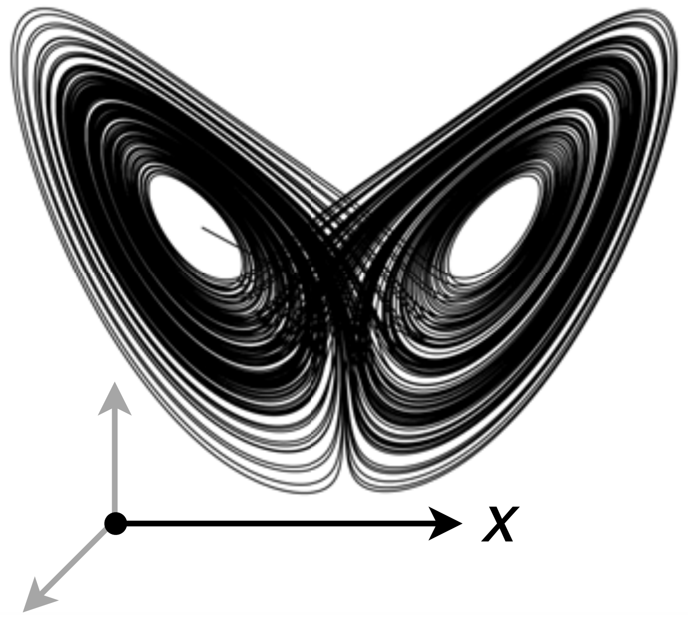
S. Brunton, BWB, Proctor, Kaiser & Kutz, Nat Comm 2017.

Related to:
Takens 1981.
Crutchfield & McNamara, 1987.
Abarbanel et al. 1993.
Sugihara et al. 2012
Ye et al., 2015

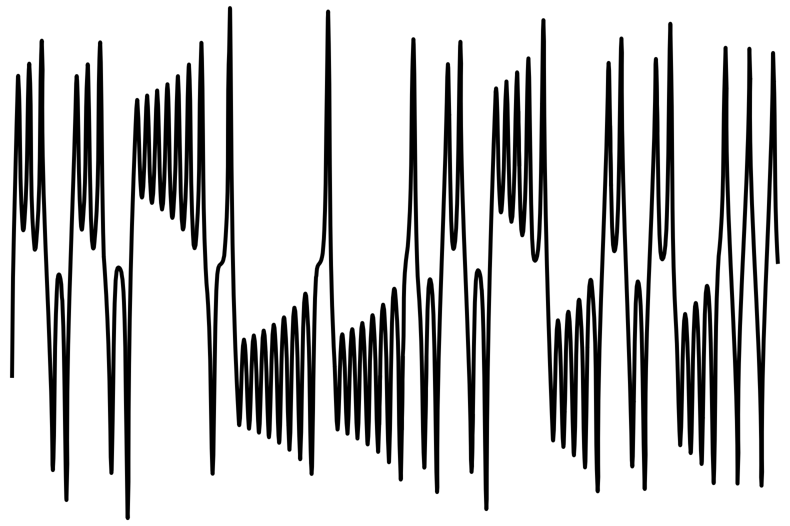
Steve Brunton

Hankel alternative view of Koopman (HAVOK)

Representations and Time-Delays



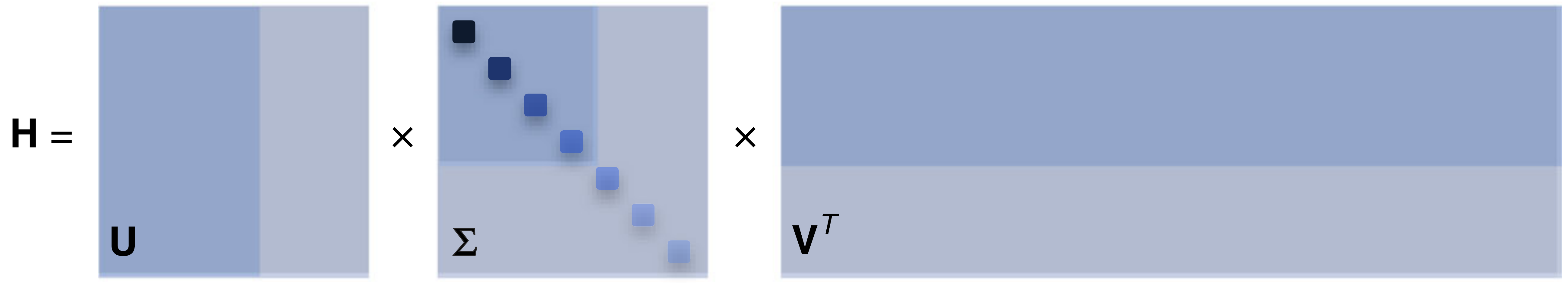
Measure
x



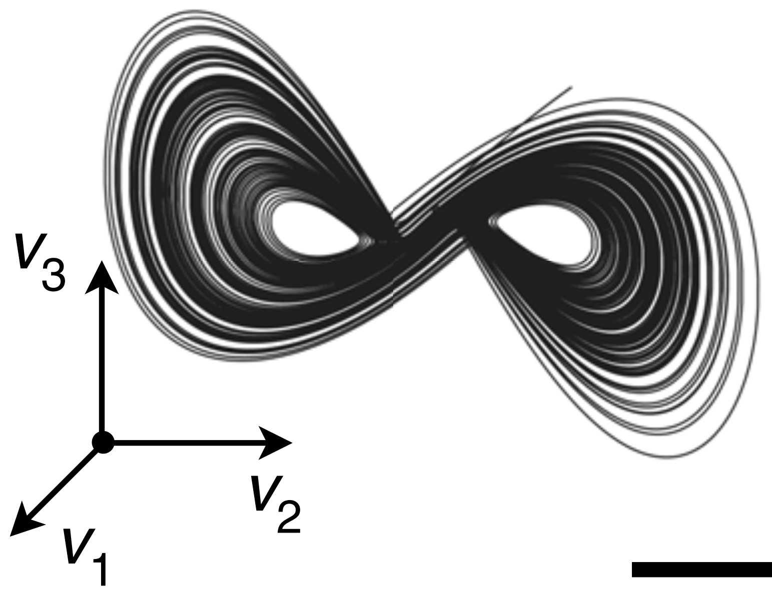
Delay
coordinates

$$\mathbf{H} = \begin{bmatrix} x(t_1) & x(t_2) & x(t_3) & \cdots & x(t_p) \\ x(t_2) & x(t_3) & x(t_4) & \cdots & x(t_{p+1}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x(t_q) & x(t_{q+1}) & x(t_{q+2}) & \cdots & x(t_m) \end{bmatrix}$$

Singular value
decomposition



Delay
embedding



S. Brunton, BWB, Proctor, Kaiser & Kutz, Nat Comm 2017.

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Crutchfield & McNamara, 1987.
Abarbanel et al. 1993.
Sugihara et al. 2012
Ye et al., 2015

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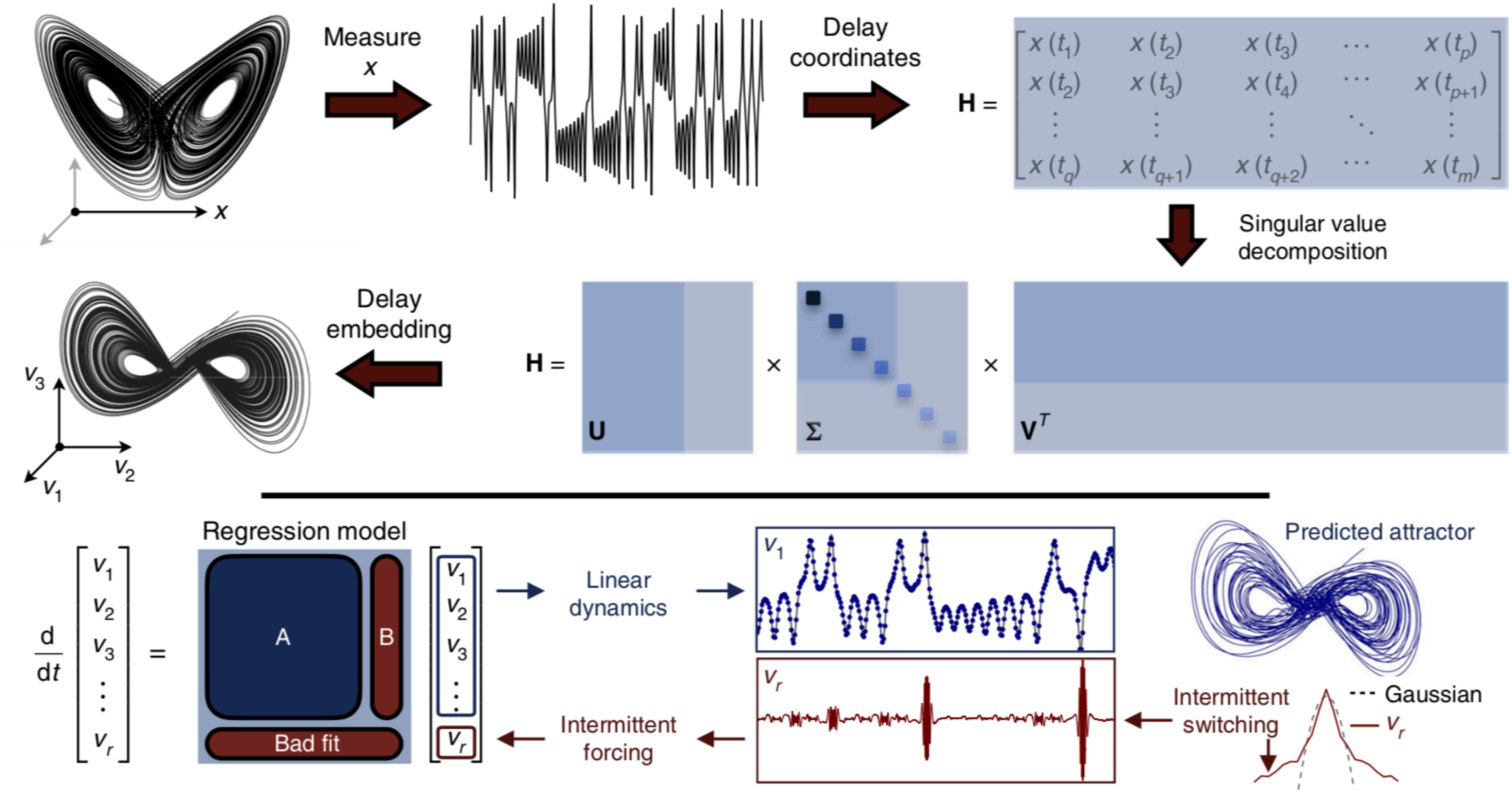
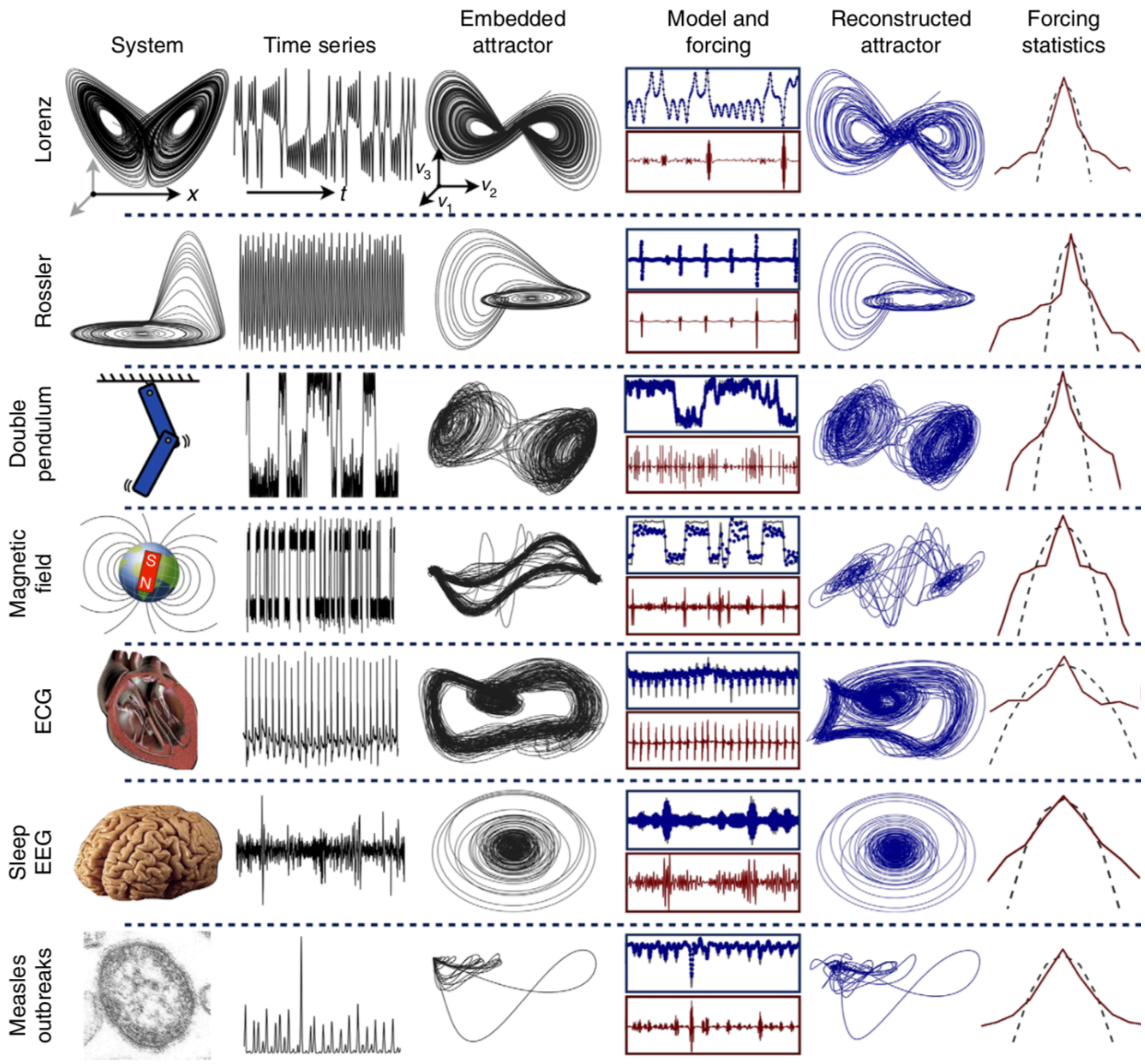
Hankel alternative view of Koopman (HAVOK)

Time-delay coordinates,
Hankel matrices,
and Koopman operators

Chaos as an intermittently forced linear system



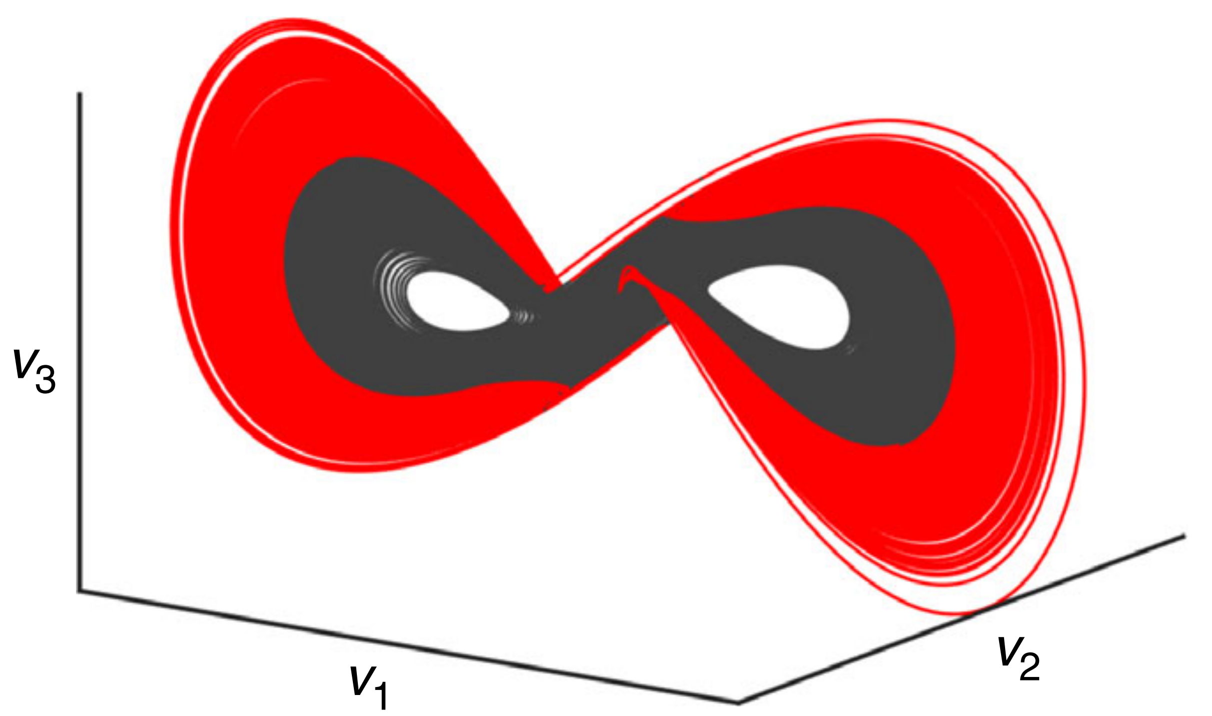
S. Brunton, BWB, Proctor, Kaiser & Kutz, Nat Comm 2017.



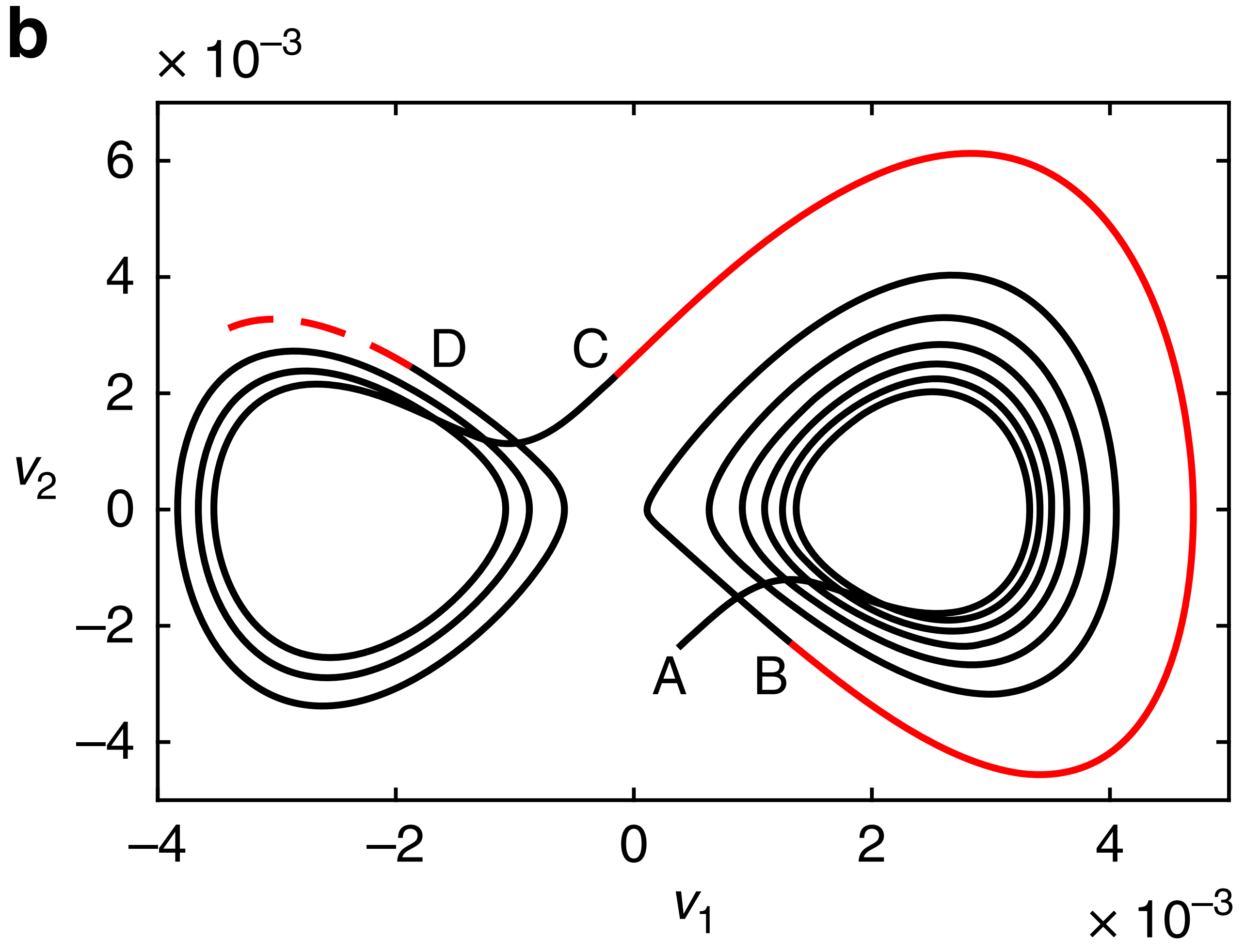
S. Brunton, BWB, Proctor, Kaiser & Kutz, Nat Comm 2017.

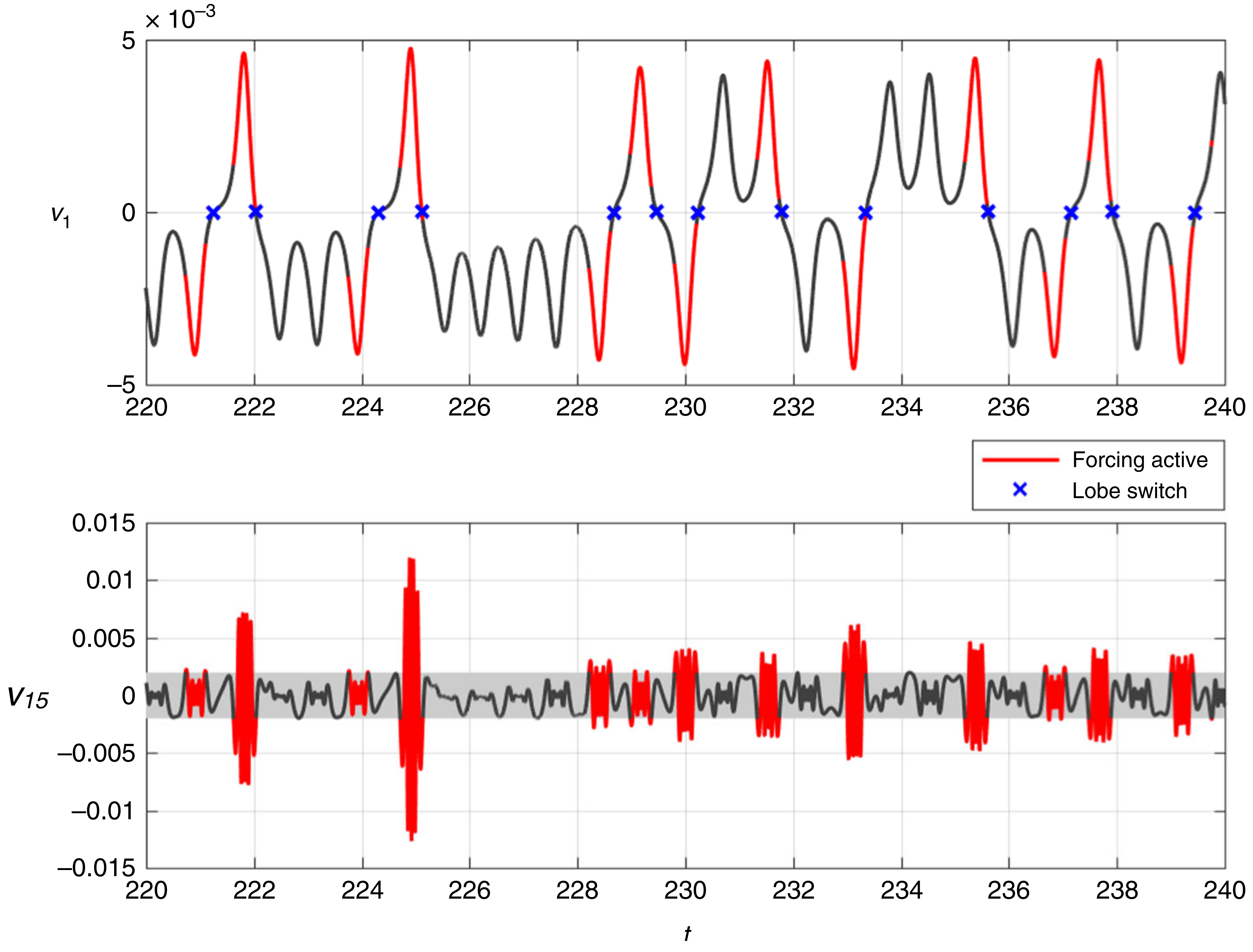
Hankel alternative view of Koopman (HAVOK)
Model predicts lobe switching

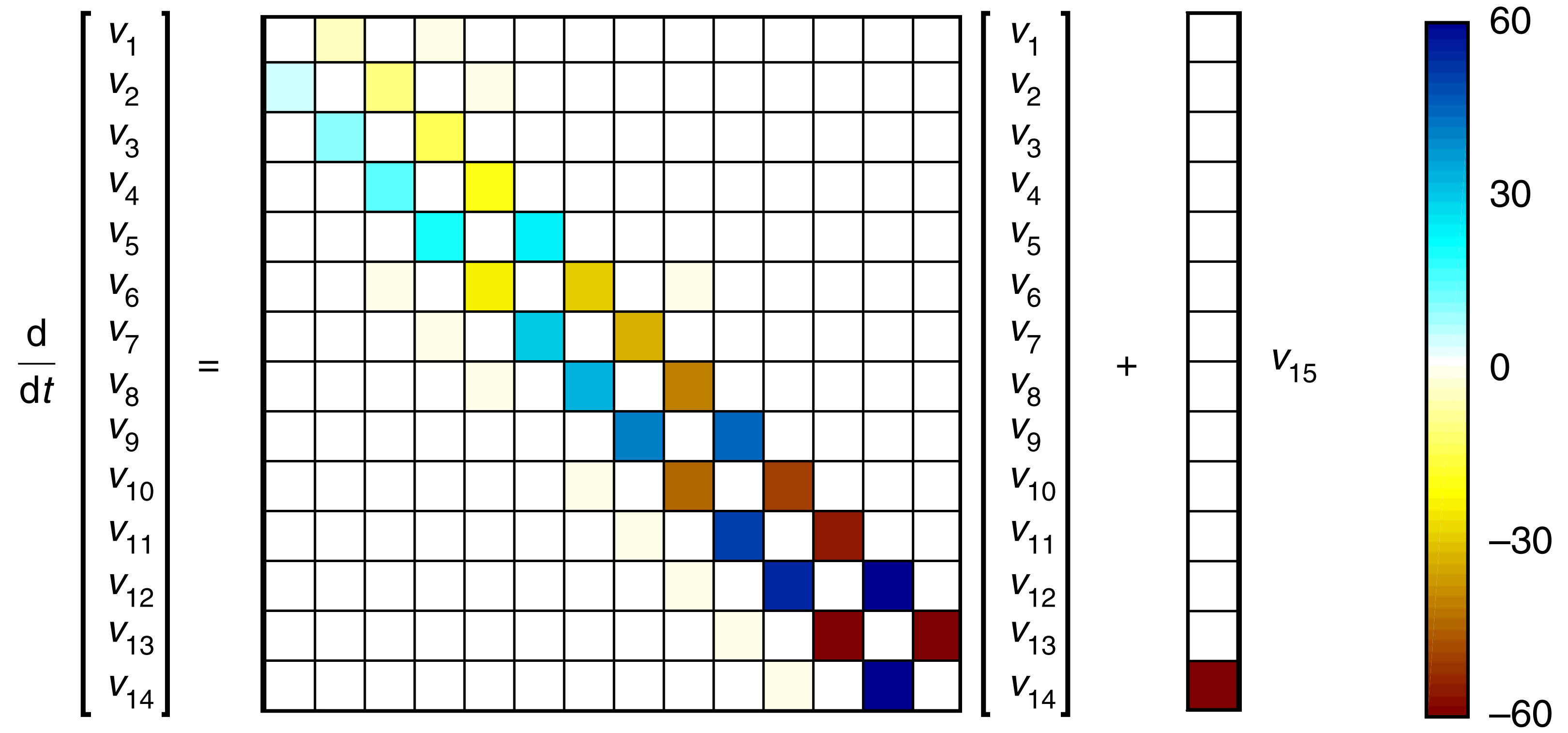
a

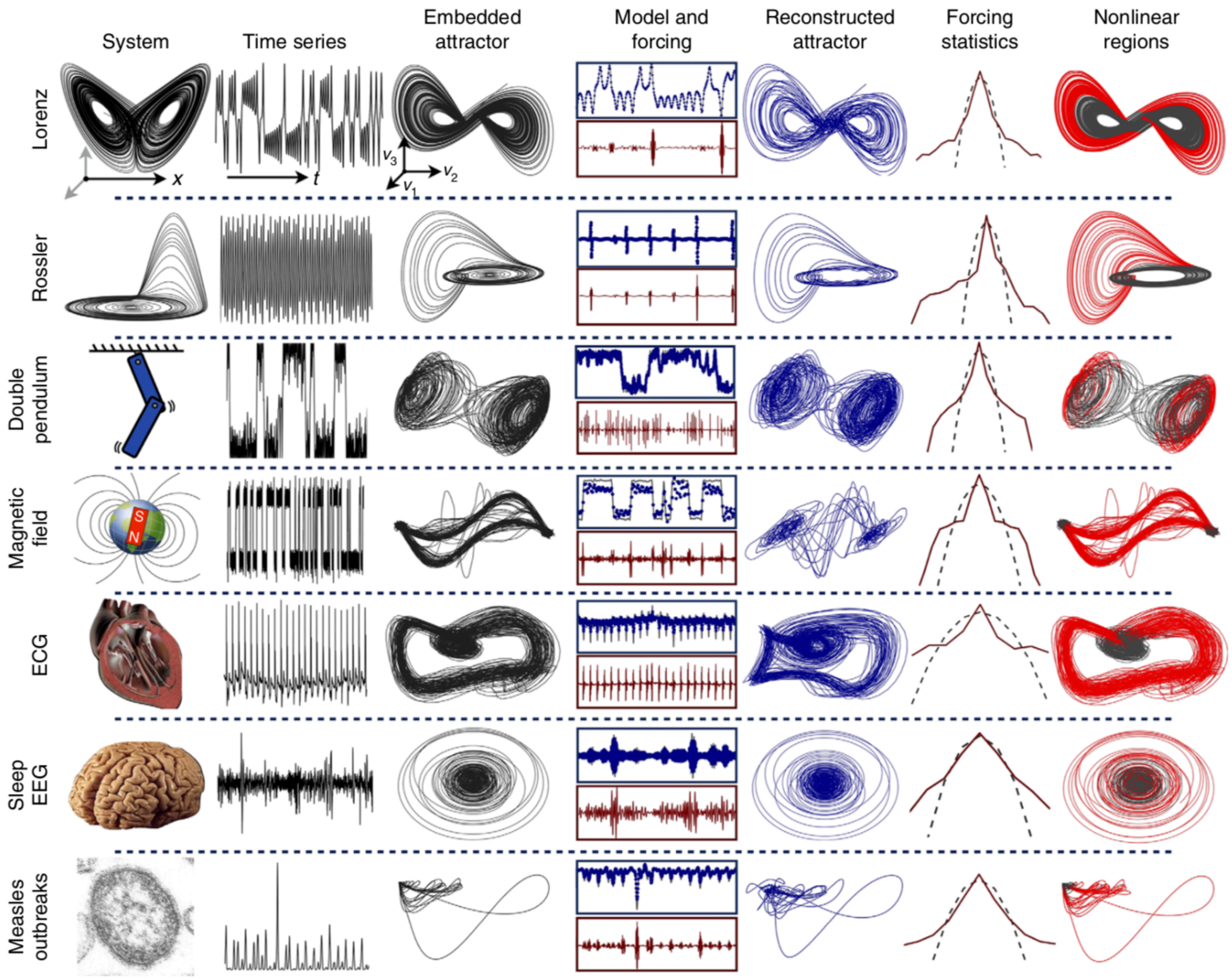


b

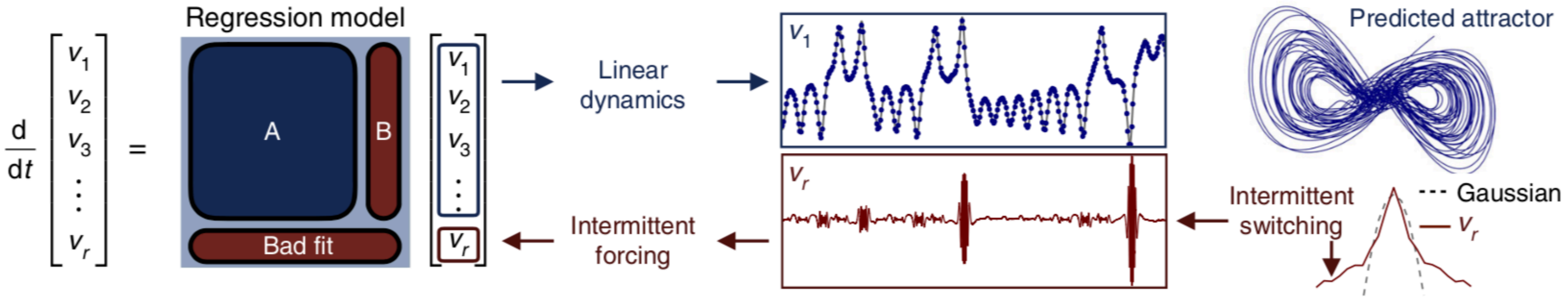






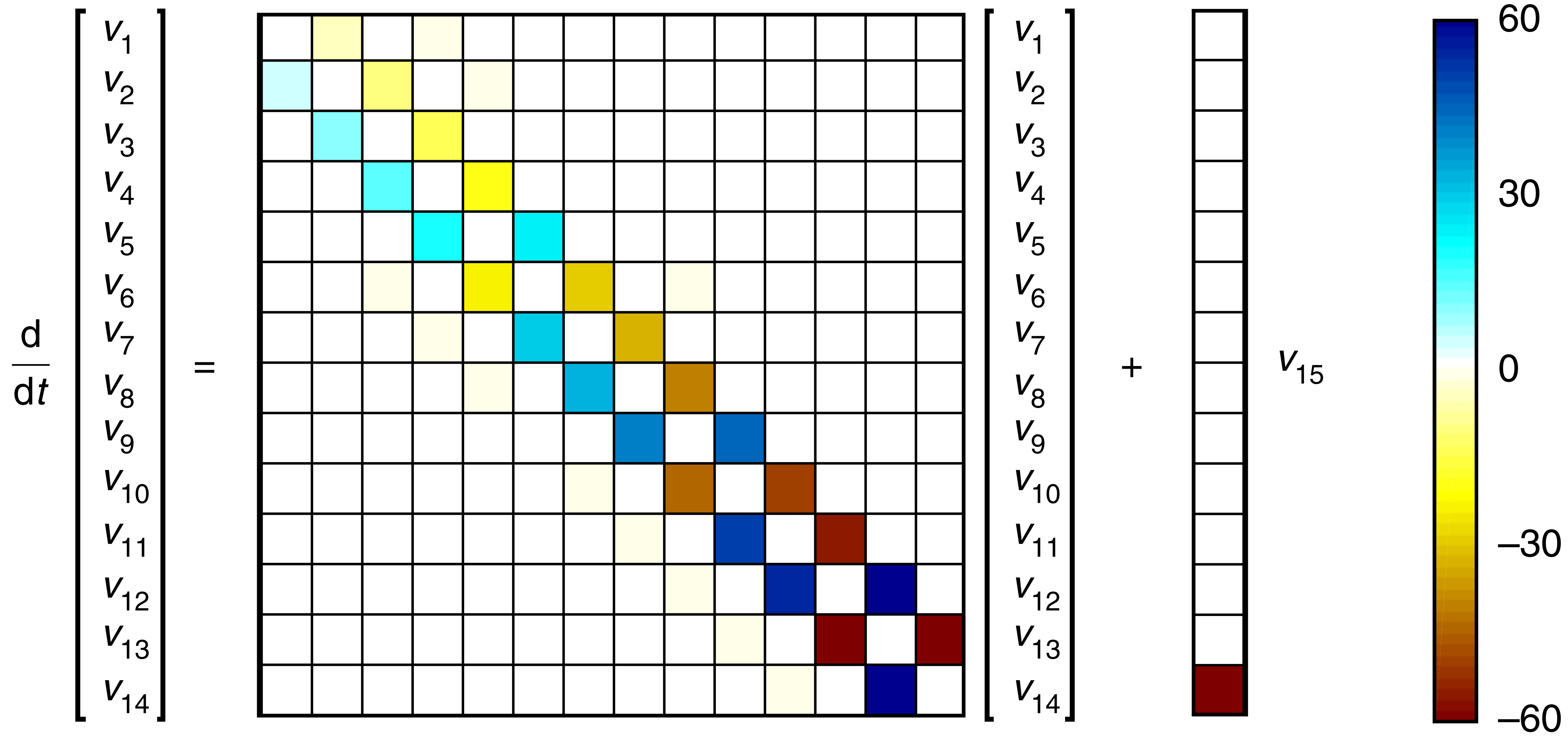


S. Brunton, BWB, Proctor, Kaiser & Kutz, Nat Comm 2017.



Why is this A matrix skew symmetric?

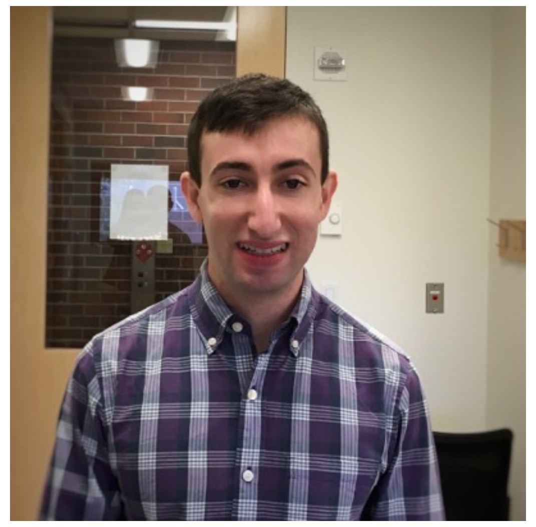
¯_(ツ)_/¯



Connecting **time-delay coordinates** and the **SVD** to **geometric mechanics** and curvature in Frenet-Serret coordinates:

$$\boldsymbol{K}(t) = \begin{bmatrix} 0 & \kappa_1(t) & & 0 \\ -\kappa_1(t) & \ddots & \ddots & \\ & \ddots & 0 & \kappa_{r-1}(t) \\ 0 & & -\kappa_{r-1}(t) & 0 \end{bmatrix}$$

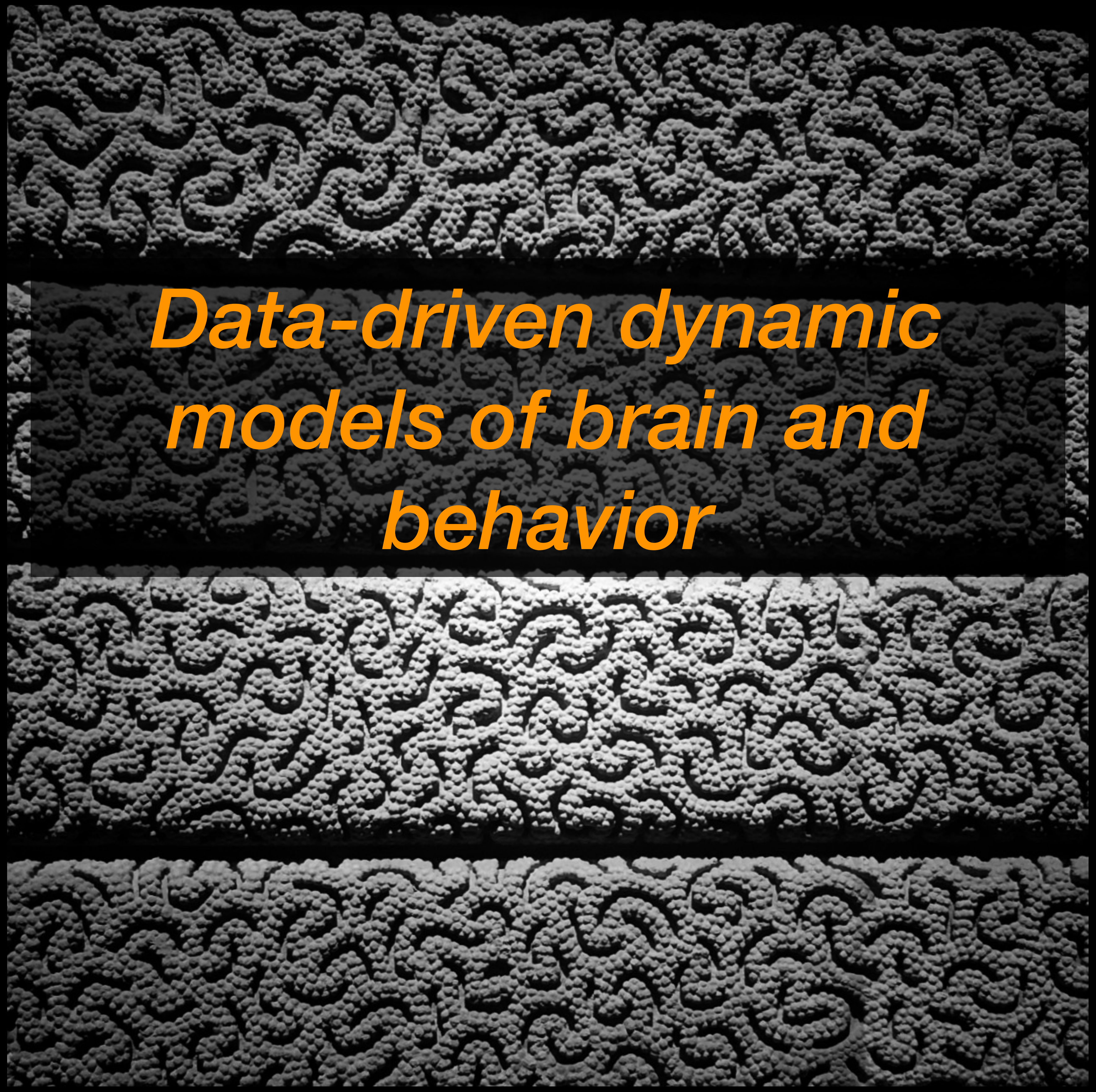
Hirsch et al., preprint soon!

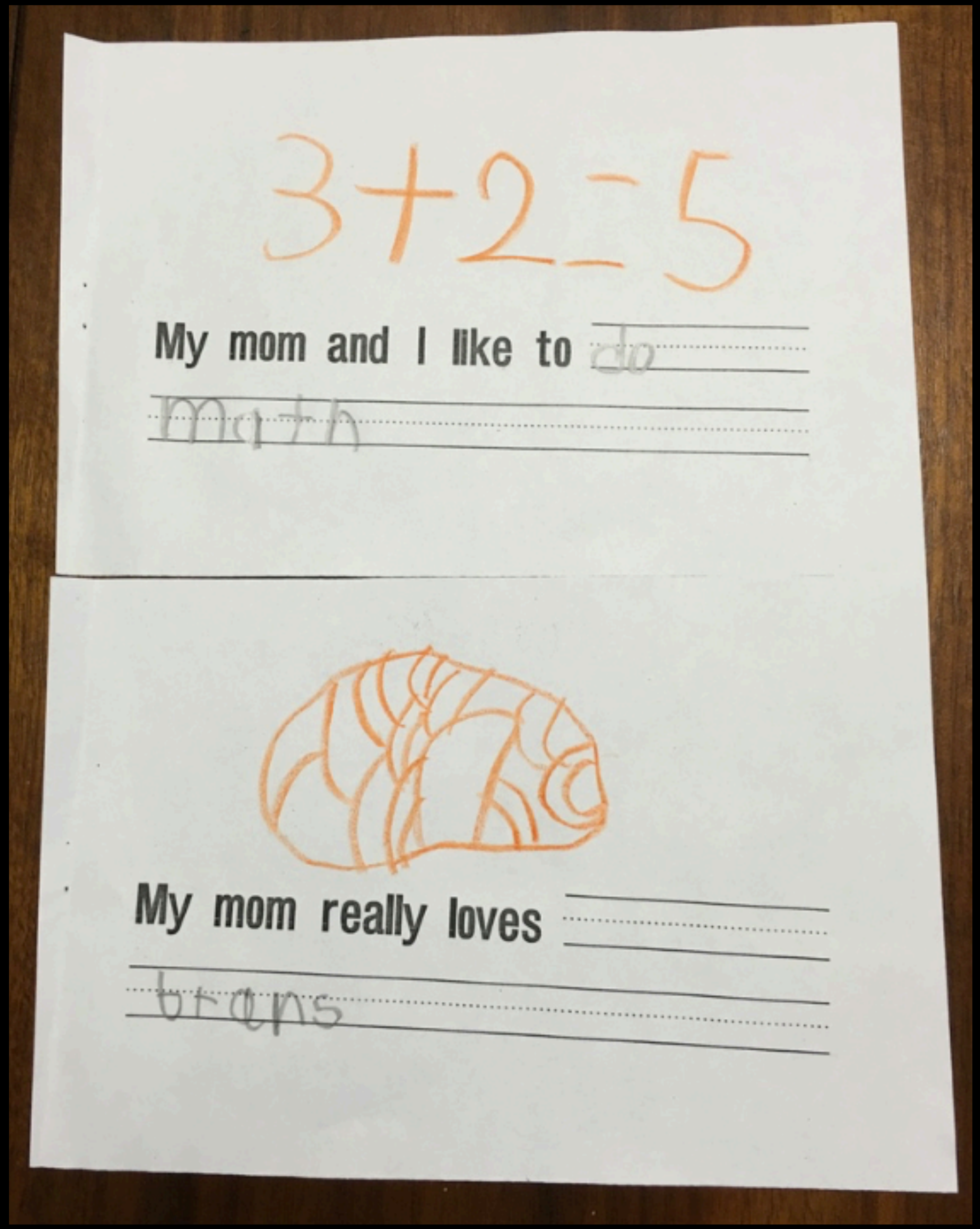


Seth Hirsh

What I think I do:

What my kids think I do:





JAB, 2016

[postdoc] [grad (degree)]

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