

# Deep Generative Models for Graphs

VAEs, GANs, and Reinforcement Learning for *de novo* drug discovery

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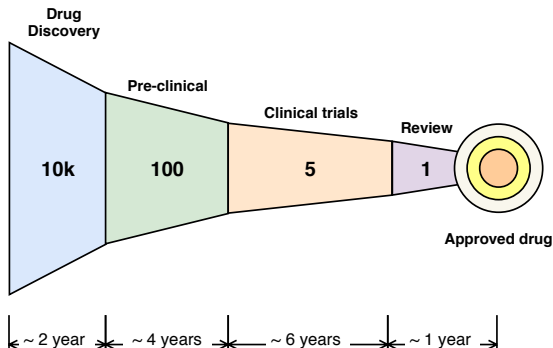
The University of Edinburgh  
and University of Amsterdam

September 24, 2019

# The drug design processes

Which problem do we want to solve?

- **Drug design** is the process of finding new drugs



- The first step is **Drug Discovery**
  - screening large compound libraries
  - designing of **new unknown molecules** (*de novo*)

How others proposed to study the problem?

- Generating **SMILES** representations [Gómez-Bombarelli et al., 2016]
- Generating **labeled graphs** [Simonovsky and Komodakis, 2018]

How do we study the problem?

- Using **labeled graphs**
- **Likelihood-based vs. likelihood-free** methods (VAE vs. GAN)
- Biasing the process using **reinforcement learning**

# Background

# Variational Auto-Encoders

**Likelihood-based** generative process [Kingma and Welling, 2013]

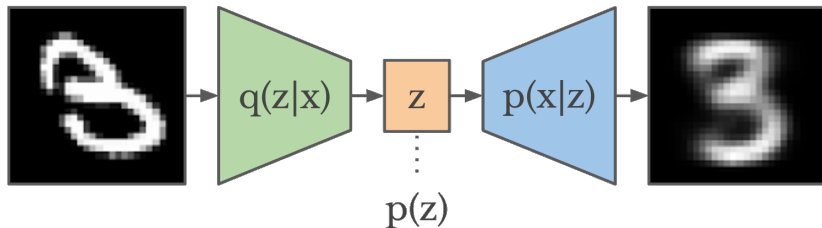


Image credit [Hafner, 2018]

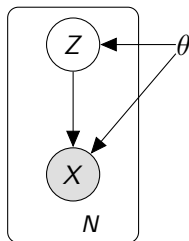


Figure: Graphical model of a simple VAE.

Trained to **maximize the log-evidence**:

$$\log p_{\theta}(\mathbf{x}^{(i)}) = \log \int p_{\theta}(\mathbf{x}^{(i)}, \mathbf{z}) d\mathbf{z}$$

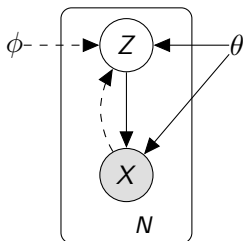


Figure: Graphical model of a simple VAE.

## Evidence Lower Bound (ELBO)

Optimizing a **lower bound** of the loss makes the problem feasible:

$$\log p_{\theta}(\mathbf{x}^{(i)}) \geq \mathbb{E}_{q_{\phi}(\mathbf{z}|\mathbf{x}^{(i)})} \left[ \log p_{\theta}(\mathbf{x}^{(i)}|\mathbf{z}) \right] - D_{KL} \left[ q_{\phi}(\mathbf{z}|\mathbf{x}^{(i)}) \parallel p_{\theta}(\mathbf{z}) \right]$$

# Generative Adversarial Networks

**Likelihood-free** generative process [Goodfellow et al., 2014]

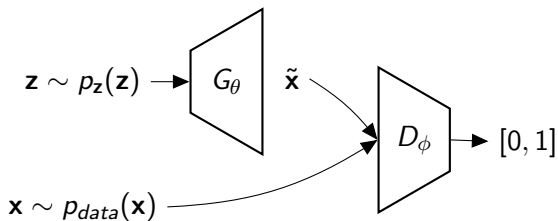


Figure: Schema of **GAN** architecture.



Originally proposed loss:

$$\min_{\theta} \max_{\phi} \mathbb{E}_{\mathbf{x} \sim p_{data}(\mathbf{x})} [\log D_{\phi}(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})} [\log(1 - D_{\phi}(G_{\theta}(\mathbf{z})))]$$

but there are **better alternatives**:

- f-GAN [[Nowozin et al., 2016](#)];
- WGAN [[Arjovsky et al., 2017](#)];
- WGAN-GP [[Gulrajani et al., 2017](#)] (used in this work).

# Reinforcement Learning

Can we train a **non-differentiable** generative process?

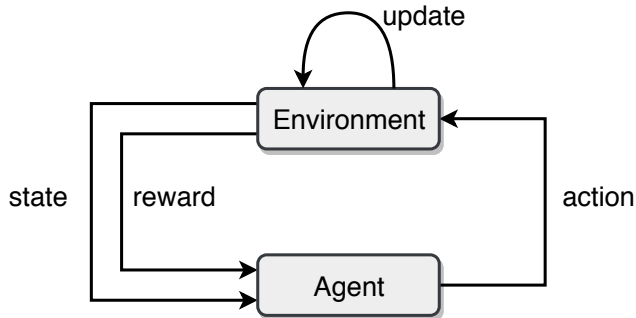
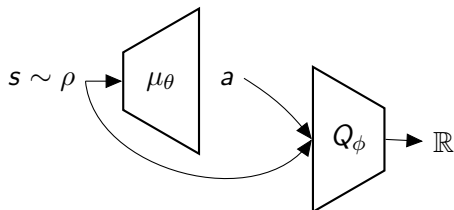


Figure: Reinforcement Learning loop schema.

# Deep Deterministic Policy Gradient (DDPG) I

How to learn a RL policy?

**Deep deterministic policy gradient** [Lillicrap et al., 2016]



## Off-policy deterministic policy gradient

Update  $\theta$  according to [Silver et al., 2014]:

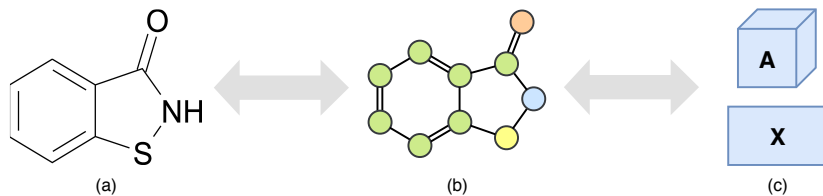
$$\begin{aligned}\mathcal{L}(\mu_\theta) &= \mathbb{E}_{s \sim \rho, a = \mu_\theta(s)} [Q_\phi(s, a)] \\ \nabla_\theta \mathcal{L}(\mu_\theta) &= \mathbb{E}_{s \sim \rho, a = \mu_\theta(s)} [\nabla_a Q_\phi(s, a) \nabla_\theta \mu_\theta(s)]\end{aligned}$$

and  $\phi$  with:

$$\mathcal{L}(Q_\phi) = \mathbb{E}_{s \sim \rho, a = \mu_\theta(s)} [\|Q_\phi(s, a) - r(s, a)\|^2]$$

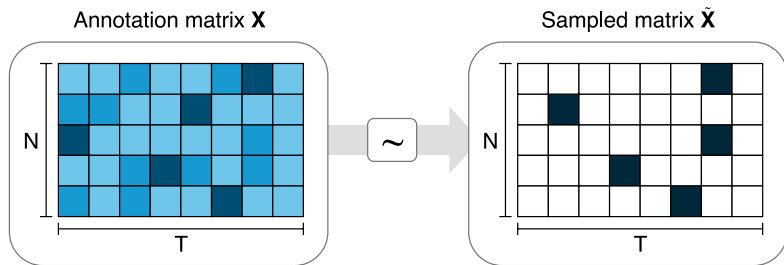
# Models

# Vectorial representation of graphs I

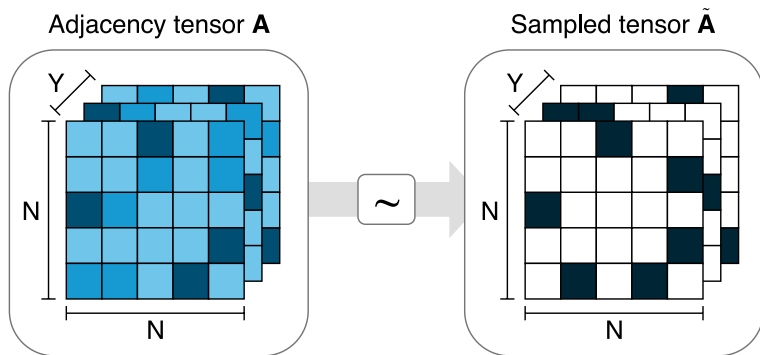


**Figure:** The **molecule** (a) is represented as an **labeled graph** (b) which can be encoded into an **adjacency tensor A** and an **annotation matrix X**.

# Vectorial representation of graphs II



# Vectorial representation of graphs III





Technically:

$$(f * g)(t) = \int_{\mathbb{R}^n} f(\tau)g(t - \tau)d\tau$$

but in practice we do **discrete** convolutions:

$$(f * g)[n] = \sum_{m \in \mathcal{S}} f(m)g(n - m)$$

# Graph Convolutional Networks II

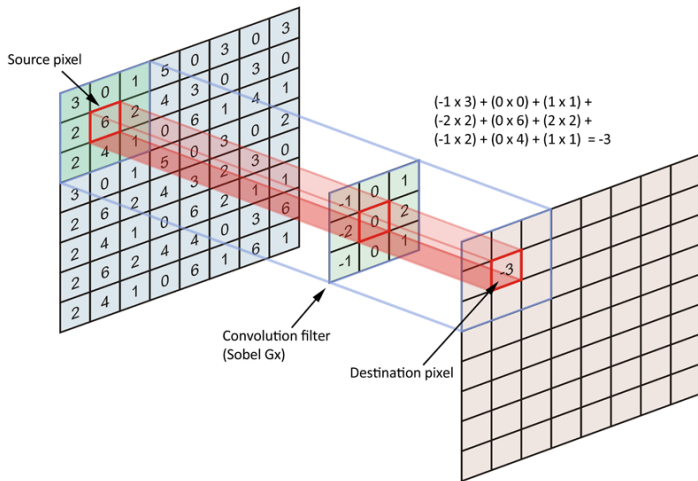
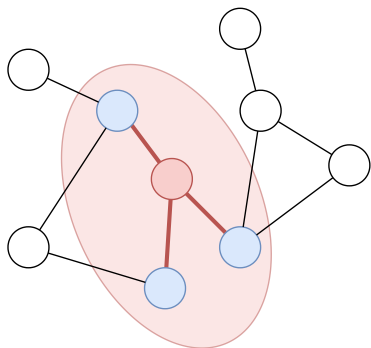
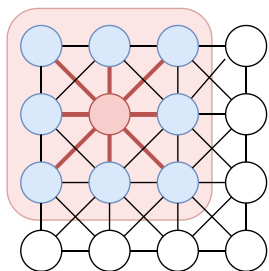


Figure: Graph convolution on an image.

# Graph Convolutional Networks III



- On images the **topology is regular** and neighbours are **pixels**
- On graphs the **topology is arbitrary** and neighbours are **nodes**

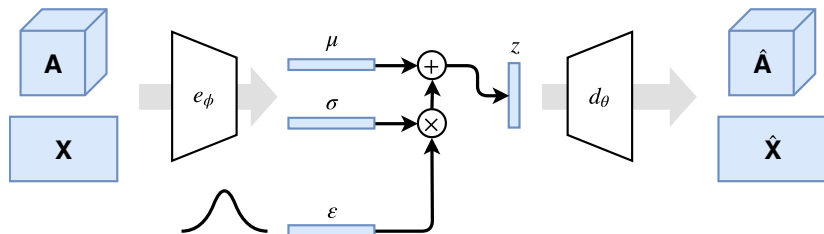
Edge-type-conditioned convolutions based on Relational-GCN [Schlichtkrull et al., 2018]:

$$\mathbf{h}_i^{(\ell+1)} = \tanh \left( f_s^{(\ell)}(\mathbf{h}_i^{(\ell)}, \mathbf{x}_i) + \sum_{j=1}^N \sum_{y=1}^{|\mathcal{X}_E|} \frac{1}{|\mathcal{N}_i|} \mathbf{A}_{jy} f_y^{(\ell)}(\mathbf{h}_j^{(\ell)}, \mathbf{x}_j) \right),$$

and following [Li et al., 2016], we define a graph level representation vector as

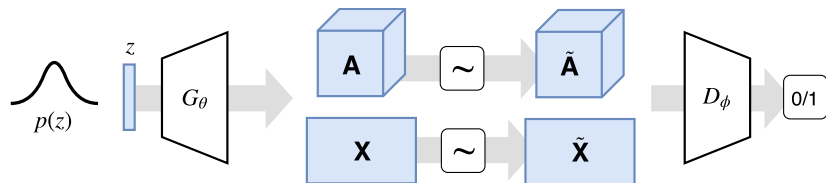
$$\mathbf{h}_G = \tanh \left( \sum_{i=1}^N \sigma \left( f_g \left( \mathbf{h}_i^{(L)}, \mathbf{x}_i \right) \right) \odot \tanh \left( f_r \left( \mathbf{h}_i^{(L)}, \mathbf{x}_i \right) \right) \right).$$

# Molecular graph VAE



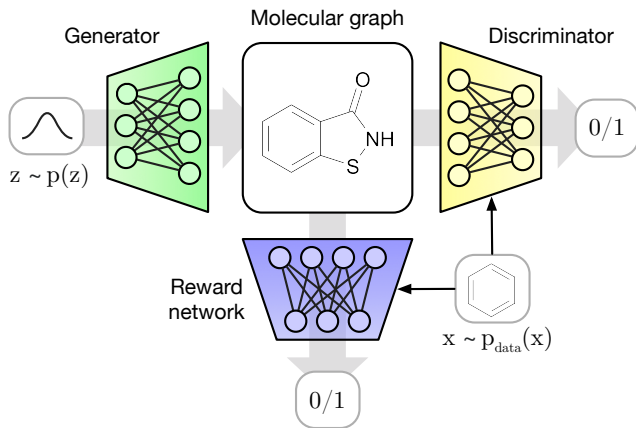
The reconstruction loss is a sum of two **categorical cross entropy** losses.

# Molecular graph GAN



From generator to discriminator with **differentiable sampling**  
[Jang et al., 2017].

# Molecular graph GAN with RL



Architecture from our previous work **MoIGAN** [De Cao and Kipf, 2018]

# Experiments



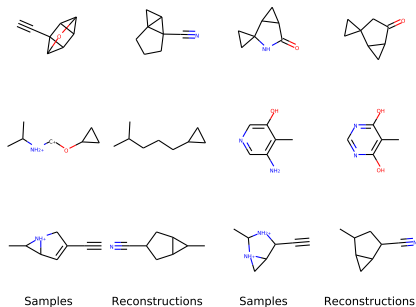
Which questions we would like to answer?

- **likelihood-based vs. likelihood-free** (VAEs vs. GANs)
- the **effect of RL** towards chemical objectives
- is generating a **graph better than a SMILES** representation?

Experiments on QM9 [[Ramakrishnan et al., 2014](#)].

# VAE advantages and disadvantages

VAEs train an **encoder!**



- **VAE objective:** reconstruction loss and divergence
- **RL objective:** sampled molecules should maximize a score

There is a **mismatch between these two!**

# Synthetic accessibility score (SAS) distributions I

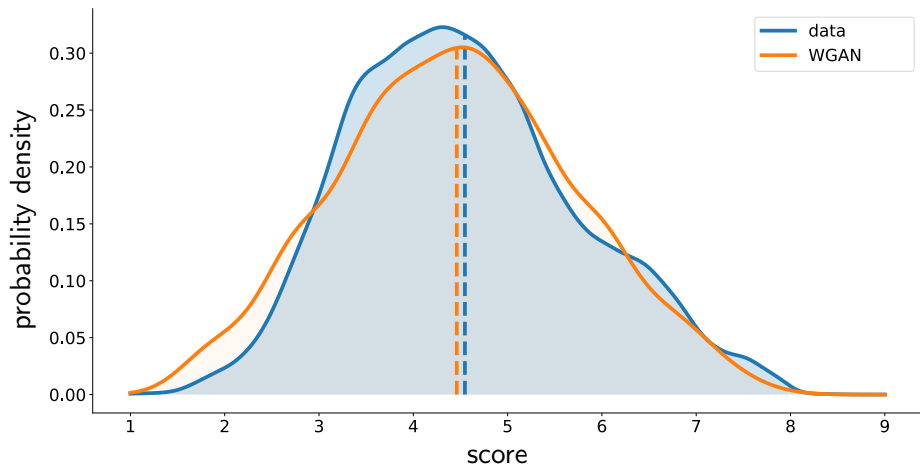
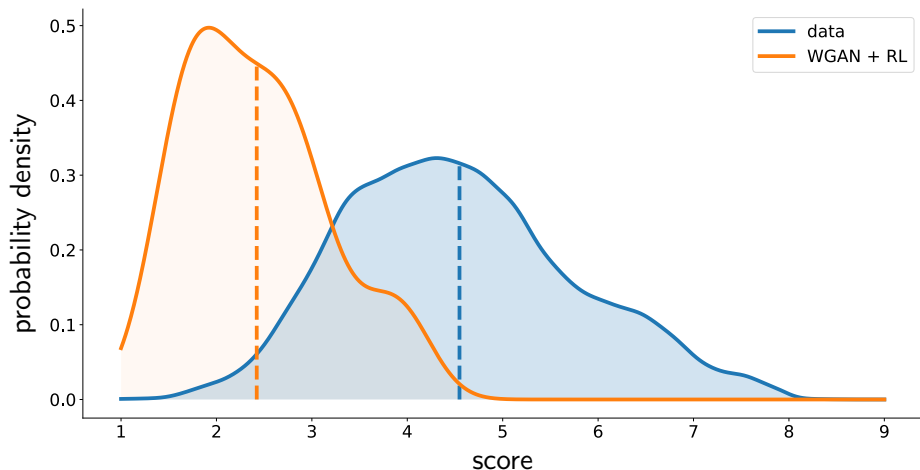


Figure: WGAN matches the data distribution of the synthetic accessibility score [Ertl and Schuffenhauer, 2009].

# Synthetic accessibility score (SAS) distributions II



**Figure:** WGAN in combination with RL push the distribution of the synthetic accessibility score (SAS) to be as low as possible.

# Trade-off between WGAN and RL

Method	validity	uniqueness	QED*
$\lambda = 0.0$ (full RL)	<b>100.00</b>	3.16	<b>0.61</b>
$\lambda = 0.125$	<b>100.00</b>	7.21	<b>0.61</b>
$\lambda = 0.25$	99.80	10.16	<b>0.61</b>
$\lambda = 0.375$	99.90	11.11	0.60
$\lambda = 0.5$	99.40	31.29	0.56
$\lambda = 0.625$	97.20	49.69	0.51
$\lambda = 0.75$	93.70	64.35	0.51
$\lambda = 0.875$	89.40	<b>69.69</b>	0.50
$\lambda = 1.0$ (no RL)	90.10	63.91	0.50

Table: WGAN and RL objectives trade-off. \*QED is the quantitative estimate of drug-likeness [Bickerton et al., 2012].

# Evolution of the QED during training

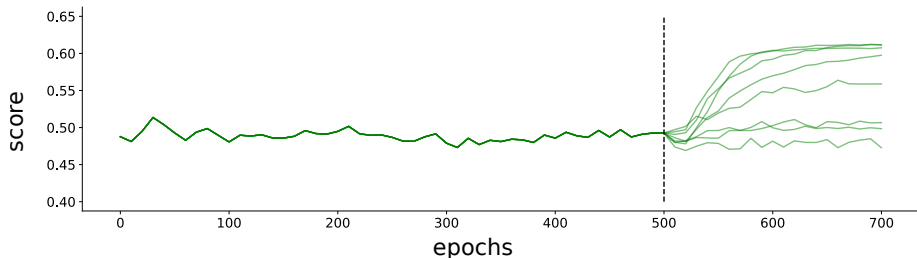


Figure: Evolution of the QED during training with different  $\lambda$  values.

# Comparison with VAE based methods

Method	validity	uniqueness	novelty
CharacterVAE	10.3	67.5	90.0
GrammarVAE	60.2	9.3	80.9
GraphVAE	55.7	76.0	61.6
GraphVAE/imp	56.2	42.0	75.8
GraphVAE NoGM	81.0	24.1	61.0
Our VAE	61.5	<b>97.6</b>	69.1
Our VAE with RL	<b>89.1</b>	11.1	<b>92.3</b>
Our WGAN	<b>89.2</b>	26.5	55.7
Our WGAN with RL	<b>99.6</b>	14.5	<b>97.7</b>

Table: Baseline results from [[Gómez-Bombarelli et al., 2016](#), [Kusner et al., 2017](#), [Simonovsky and Komodakis, 2018](#)]

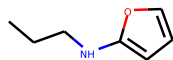
# Comparison with a GAN based method

Method	validity	SAS	time (h)
ORGAN	96.5	0.83	8.7
OR(W)GAN	97.6	0.75	9.6
Naive RL	97.7	0.83	10.6
Our VAE with RL	89.6	0.71	<b>0.09</b>
Our VAE with RL (full QM9)	94.0	<b>0.86</b>	<b>2.2</b>
Our WGAN with RL	<b>100.0</b>	0.70	<b>0.15</b>
Our WGAN with RL (full QM9)	<b>99.8</b>	<b>0.92</b>	<b>3.3</b>

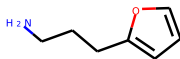
Table: Baseline results from ORGAN [Guimaraes et al., 2017].



# Best QED samples



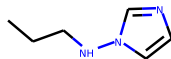
QED=0.67



QED=0.66

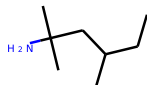


QED=0.65

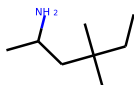


QED=0.65

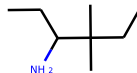
(a) Best VAE with RL molecules.



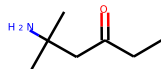
QED=0.62



QED=0.62



QED=0.62



QED=0.62

(b) Best WGAN with RL molecules.

Figure: Top four molecules with QED scores.

# Conclusion and future work

Considering experimental, we identify these further **contributions**:




- recurrent SMILES generation is more **computational expensive**
- likelihood-based models are **difficult to be optimized** with RL

... but keeping in mind and those **limitations**:

- we experimented using compounds of **at most 9 atoms**
- models are susceptible to **mode collapse**

We identify four principal directions for future work:

- address **mode collapse** [[Srivastava et al., 2017](#)]
- combine **variational** approaches with **adversarial** learning to benefit from both approaches [[Mescheder et al., 2017](#), [Rosca et al., 2017](#)]
- train our models on **ChEMBL** [[Gaulton et al., 2011](#)]
- more **realistic reward** functions [[Li et al., 2018](#)]

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