

# FLUID

A detailed brown ink drawing on aged, yellowed paper, depicting a complex fluid flow. The central feature is a large, swirling mass of fluid, rendered with dense, intricate lines and shading to suggest turbulence and vorticity. A prominent jet of fluid exits from the right side of the main mass, curving upwards and outwards. The overall shape of the fluid mass is roughly oval, with a smaller, more defined vortex structure visible on the left side. The drawing is executed with fine, consistent lines, characteristic of a technical or scientific illustration from a historical manuscript.[illegible]



# COMPLEXITY

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \boxed{\frac{1}{\text{Re}}} \nabla^2 \mathbf{u}$$

$$\nabla \cdot \mathbf{u} = 0$$

Increasing  
Reynold

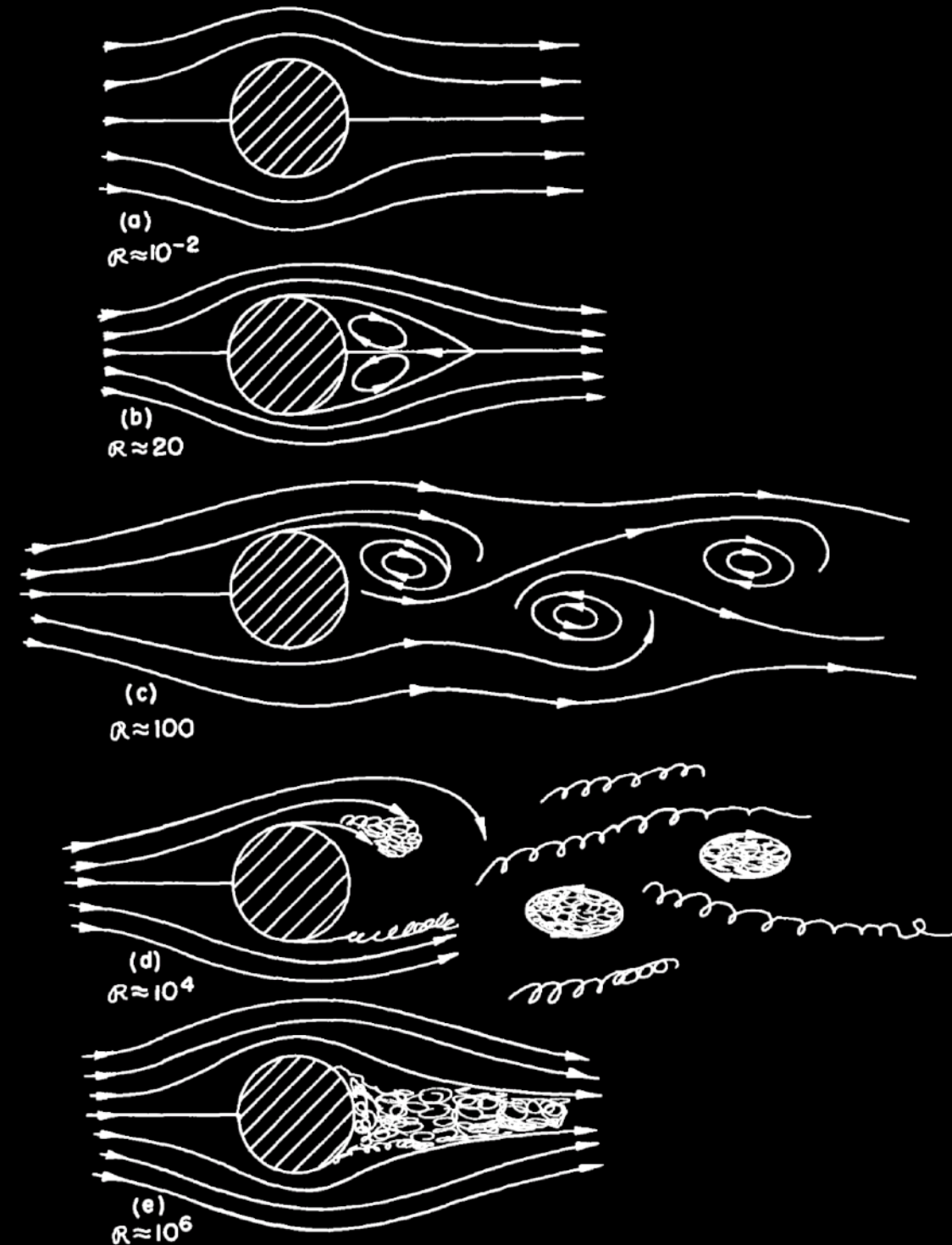
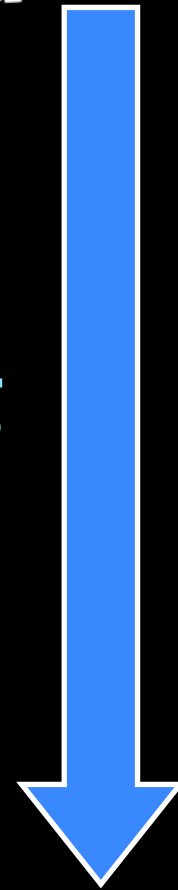


Fig. 41-6. Flow past a cylinder for various Reynolds numbers.

# COMPLEXITY

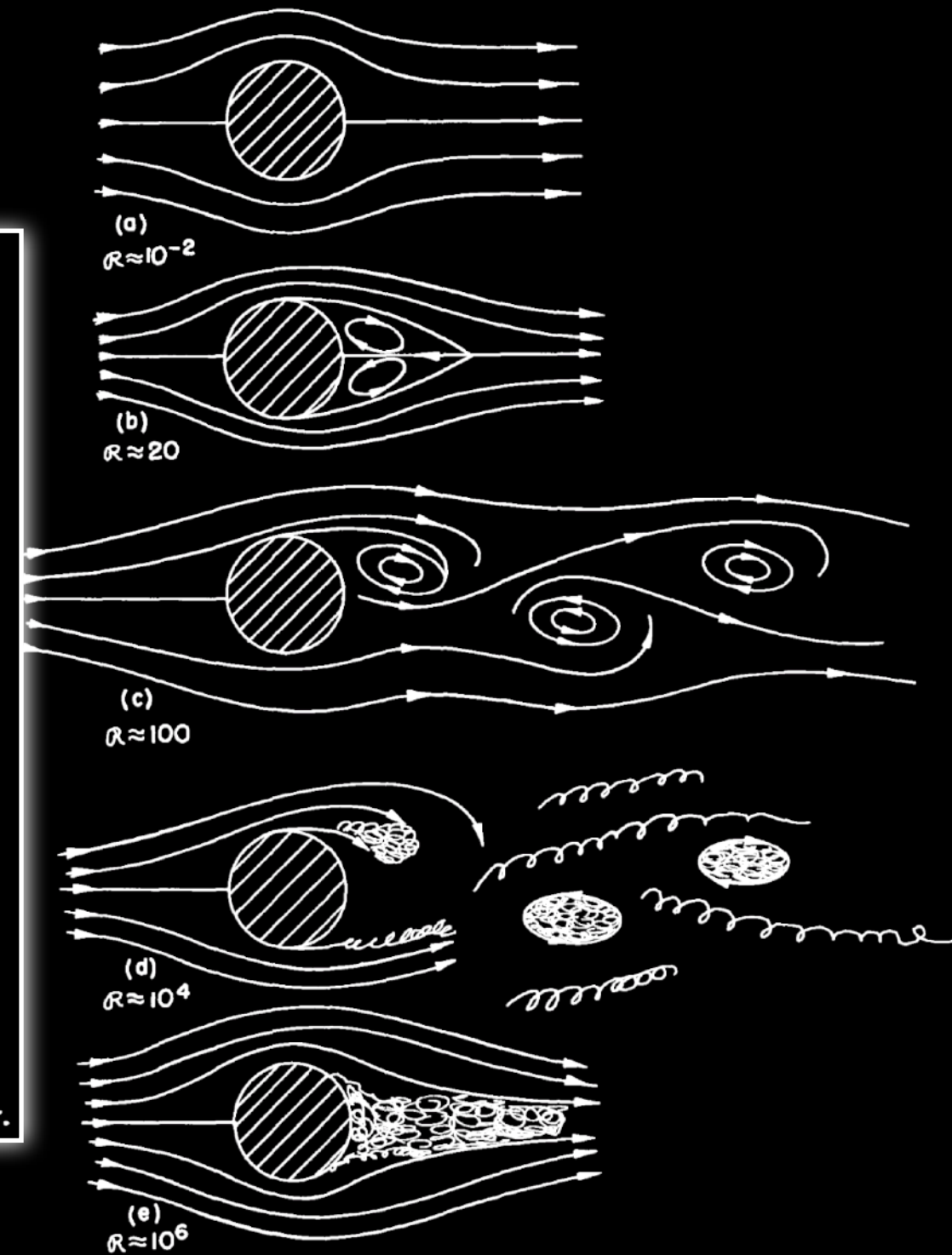
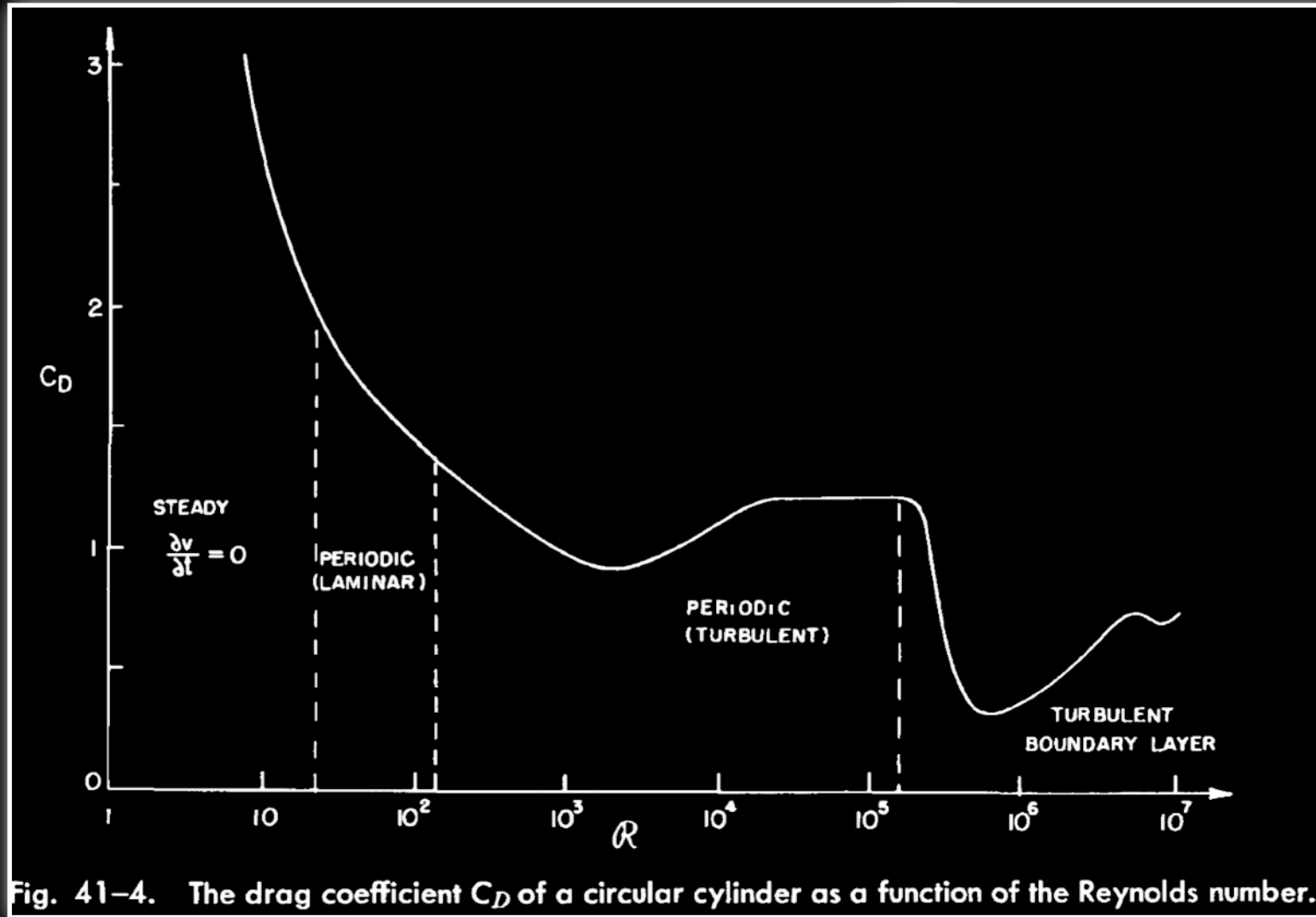
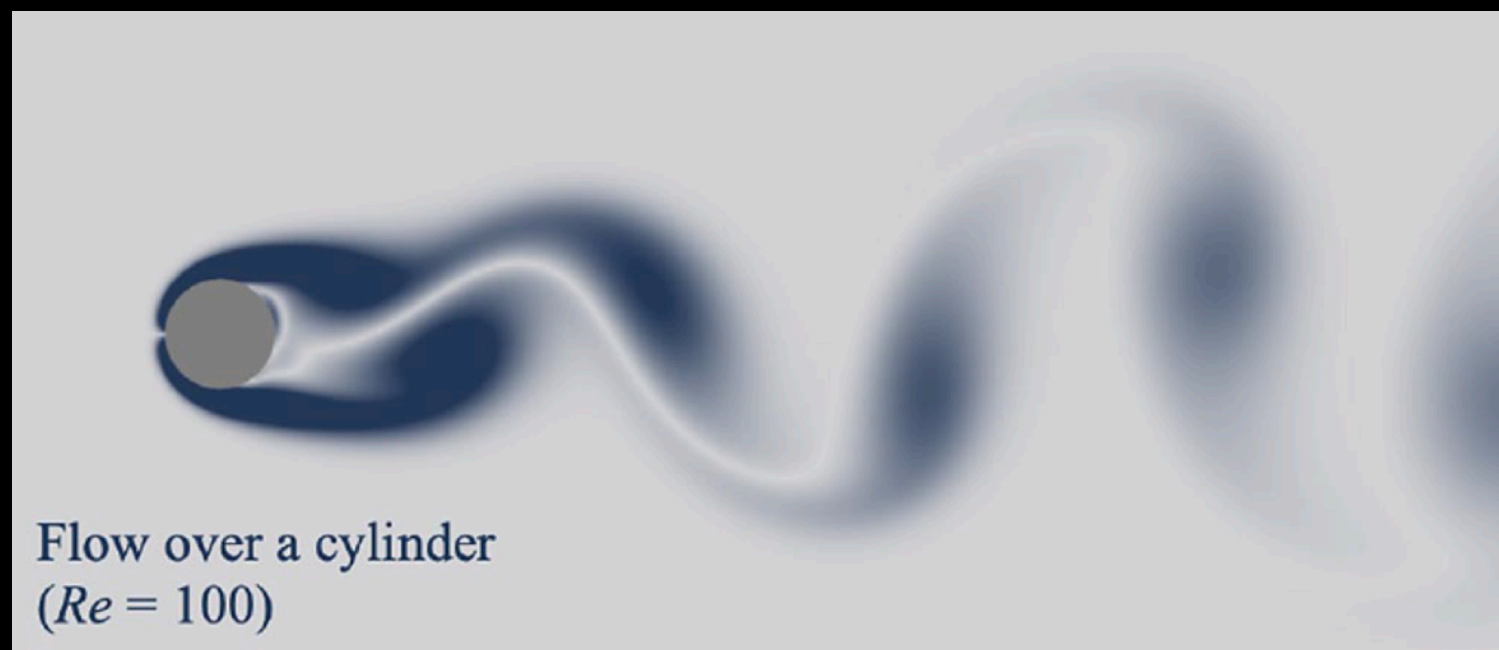
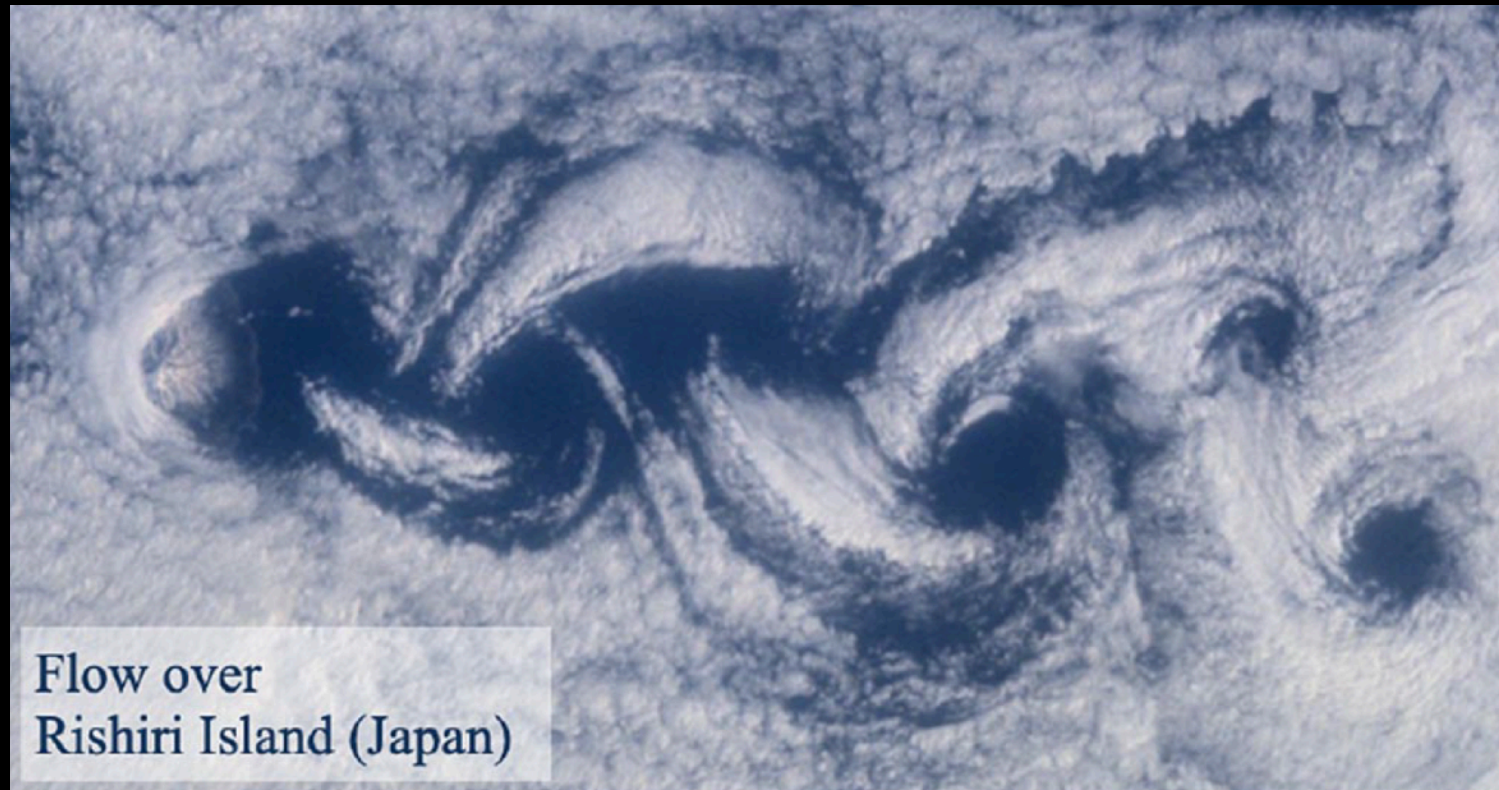


Fig. 41-6. Flow past a cylinder for various Reynolds numbers.



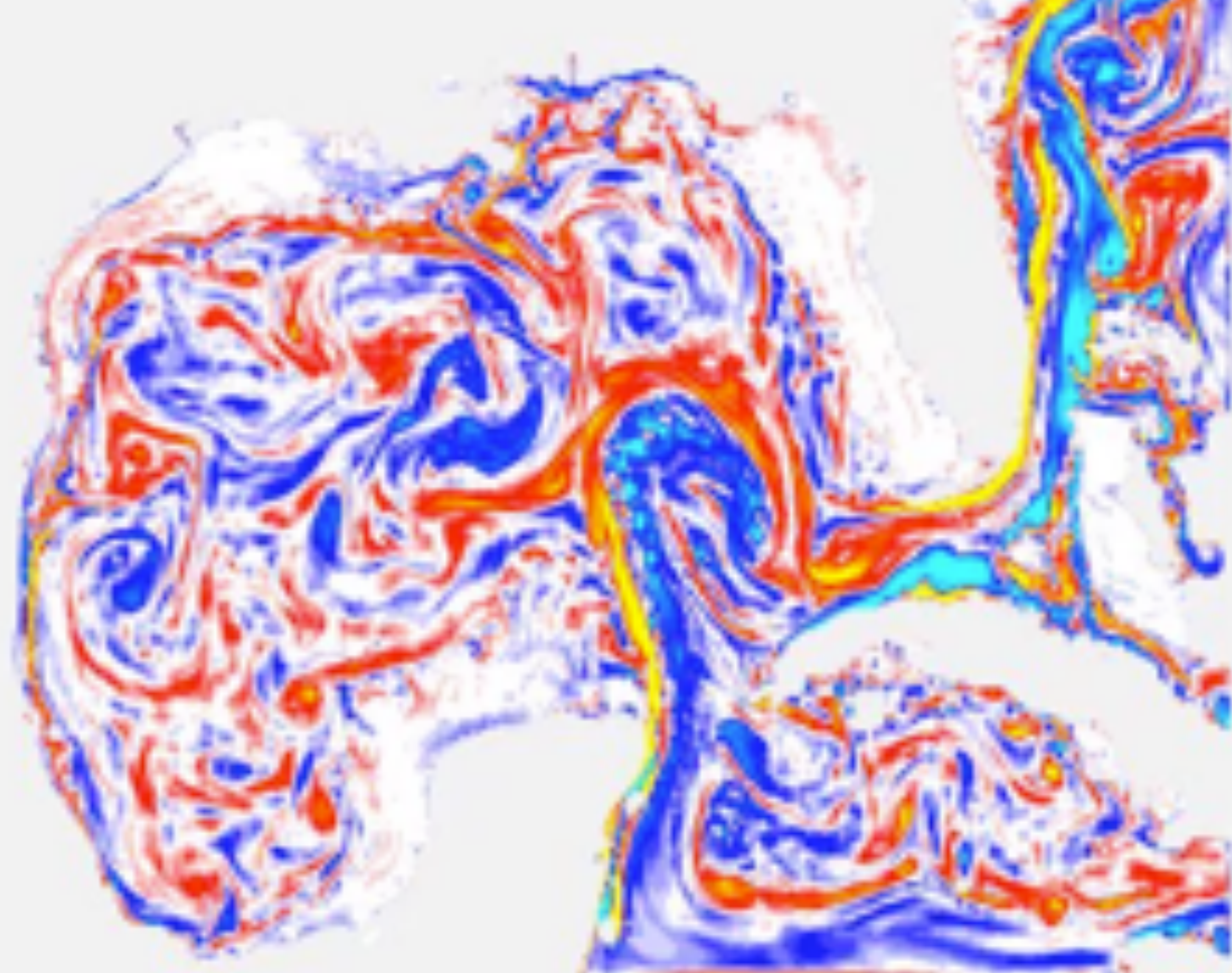






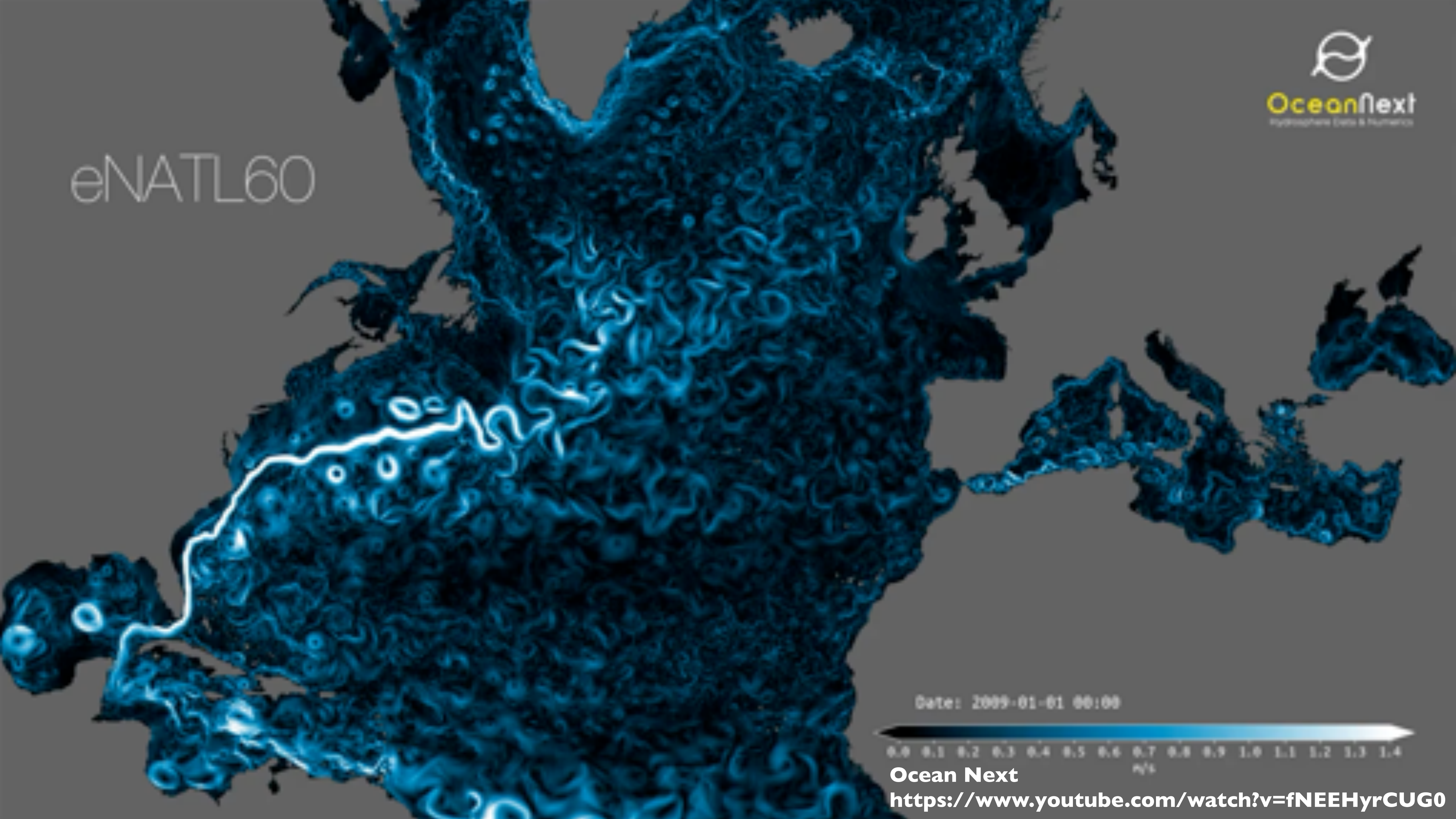




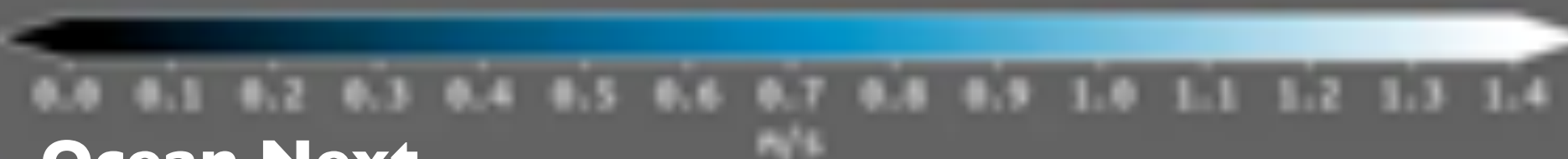




eNATL60



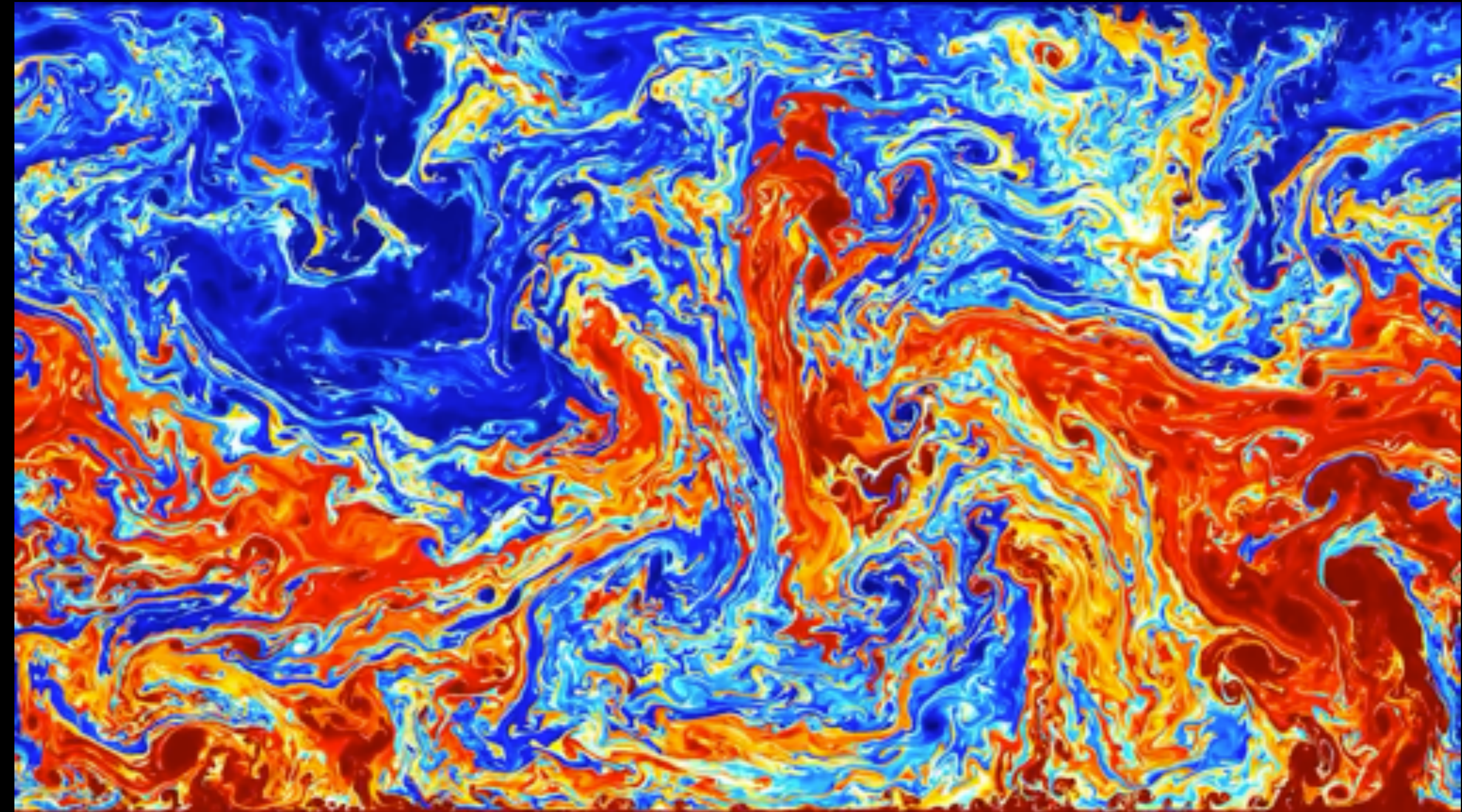
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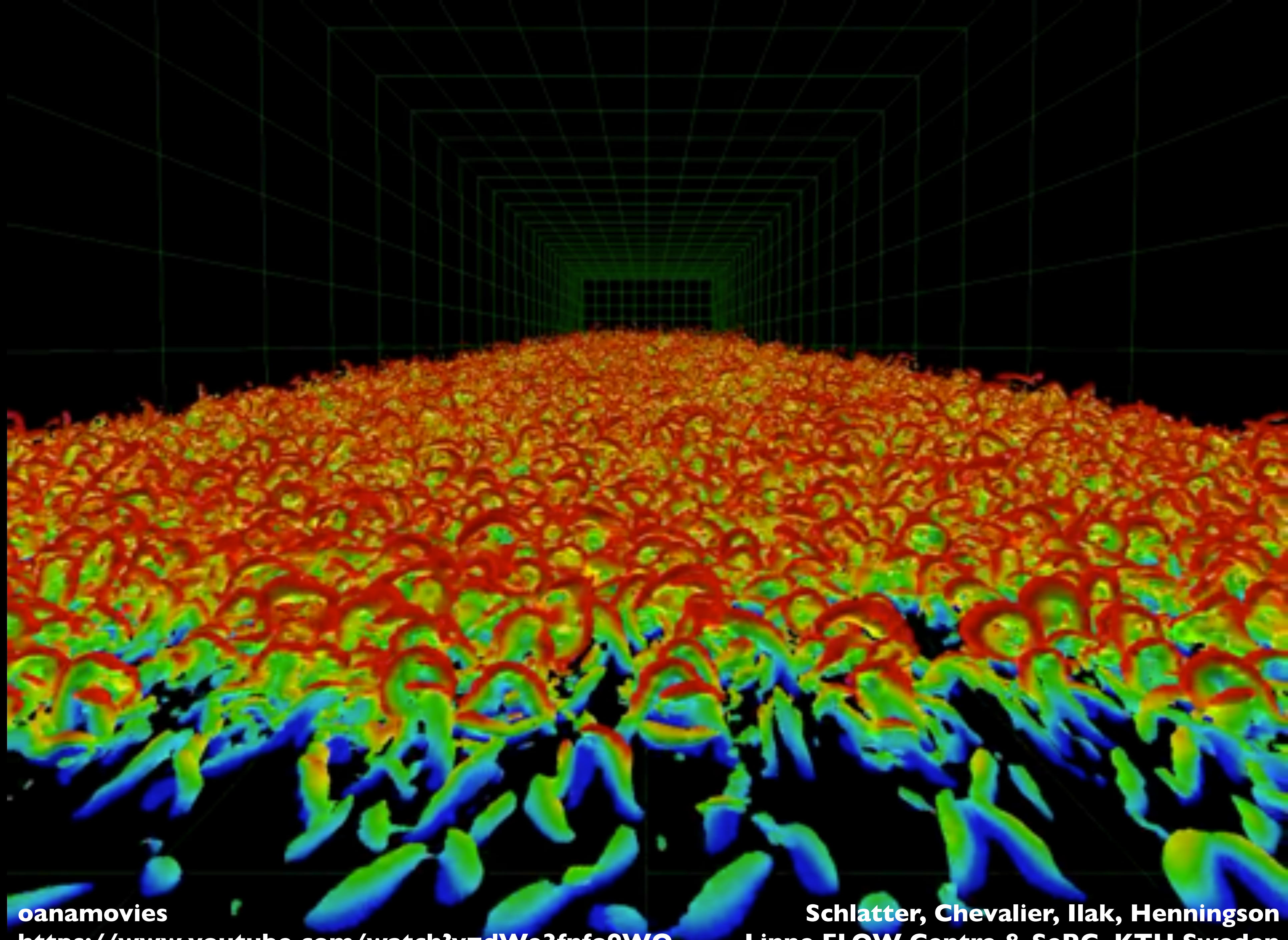
**Ocean Next**

<https://www.youtube.com/watch?v=fNEEHyrCUG0>











We have written the equations of water flow. From experiment, we find a set of concepts and approximations to use to discuss the solution—vortex streets, turbulent wakes, boundary layers. When we have similar equations in a less familiar situation, and one for which we cannot yet experiment, we try to solve the equations in a primitive, halting, and confused way to try to determine what new qualitative features may come out, or what new qualitative forms are a consequence of the equations. Our equations for the sun, for example, as a ball of hydrogen gas, describe a sun without sunspots, without the rice-grain structure of the surface, without prominences, without coronas. Yet, all of these are really in the equations; we just haven't found the way to get them out.

There are those who are going to be disappointed when no life is found on other planets. Not I—I want to be reminded and delighted and surprised once again, through interplanetary exploration, with the infinite variety and novelty of phenomena that can be generated from such simple principles. The test of science is its ability to predict. Had you never visited the earth, could you predict the thunderstorms, the volcanos, the ocean waves, the auroras, and the colorful sunset? A salutary lesson it will be when we learn of all that goes on on each of those dead planets—those eight or ten balls, each agglomerated from the same dust cloud and each obeying exactly the same laws of physics.

The next great era of awakening of human intellect may well produce a method of understanding the *qualitative* content of equations. Today we cannot. Today we cannot see that the water flow equations contain such things as the barber pole structure of turbulence that one sees between rotating cylinders. Today we cannot see whether Schrödinger's equation contains frogs, musical composers, or morality—or whether it does not. We cannot say whether something beyond it like God is needed, or not. And so we can all hold strong opinions either way.



# CANONICAL FLOWS

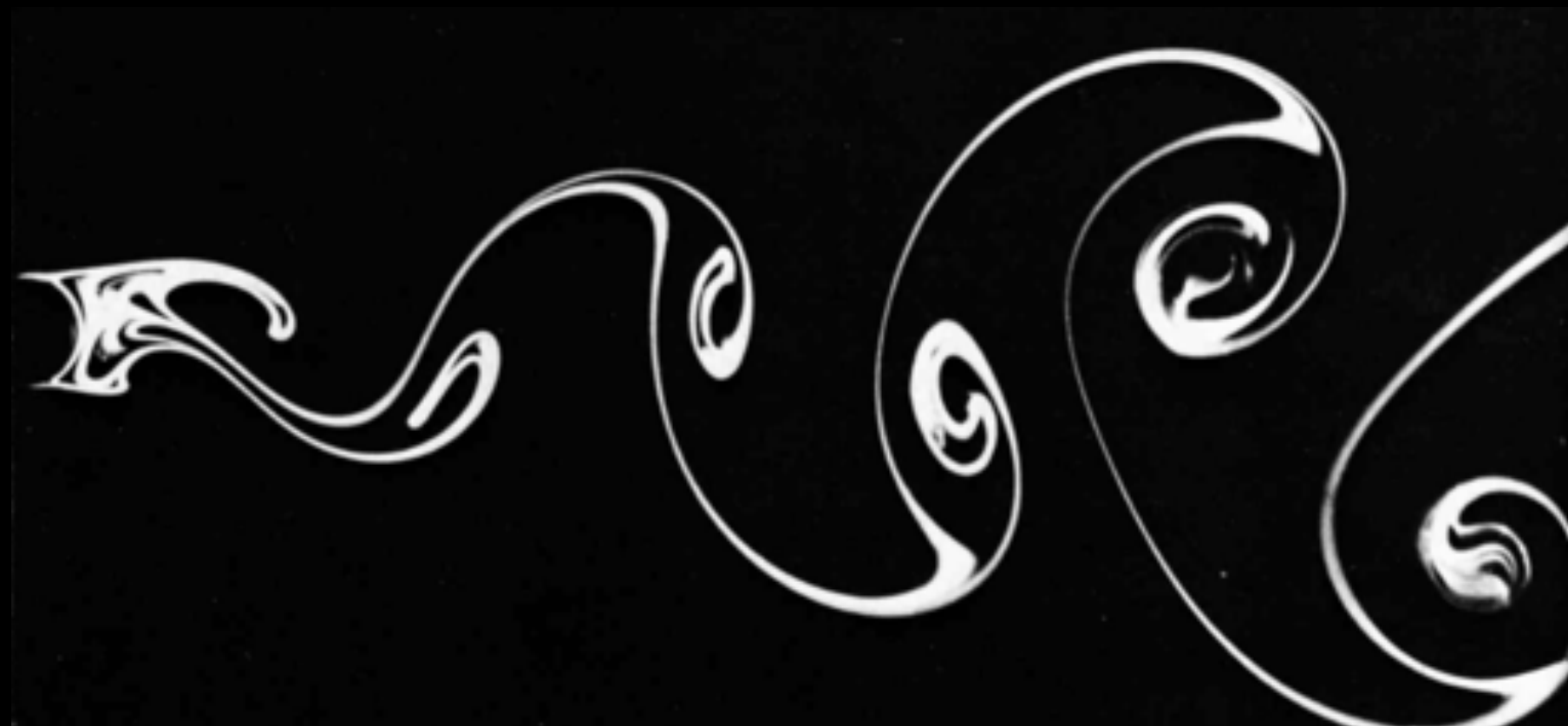
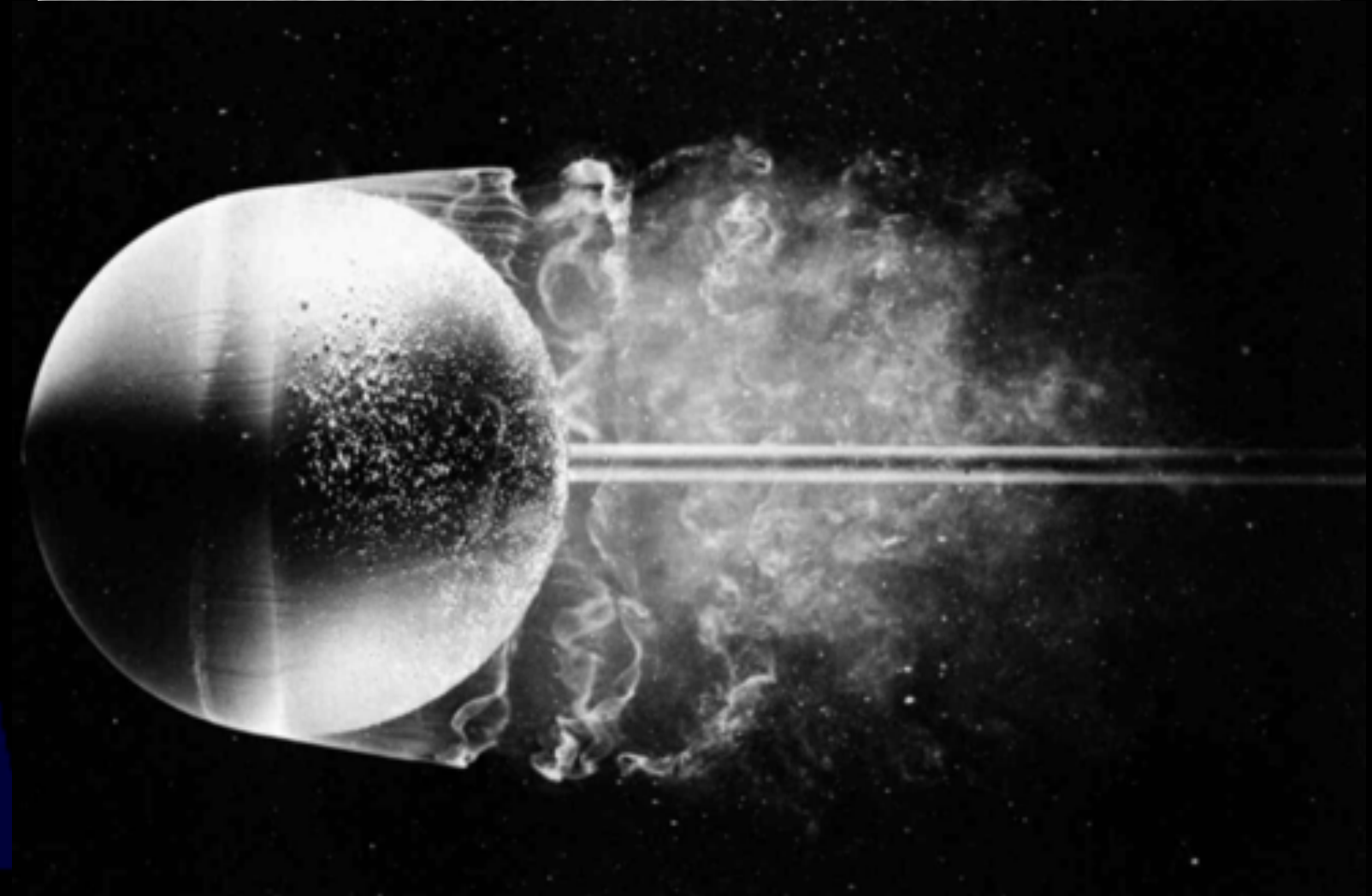
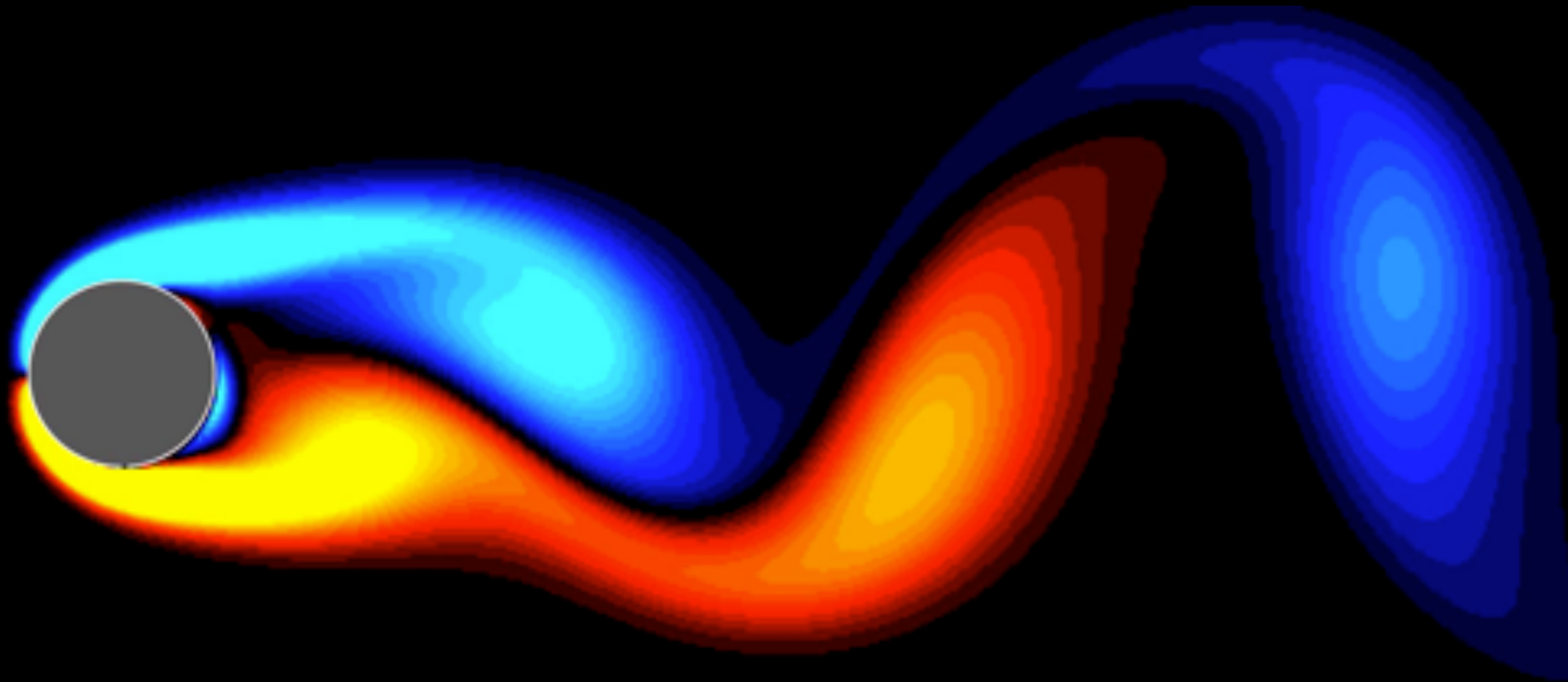
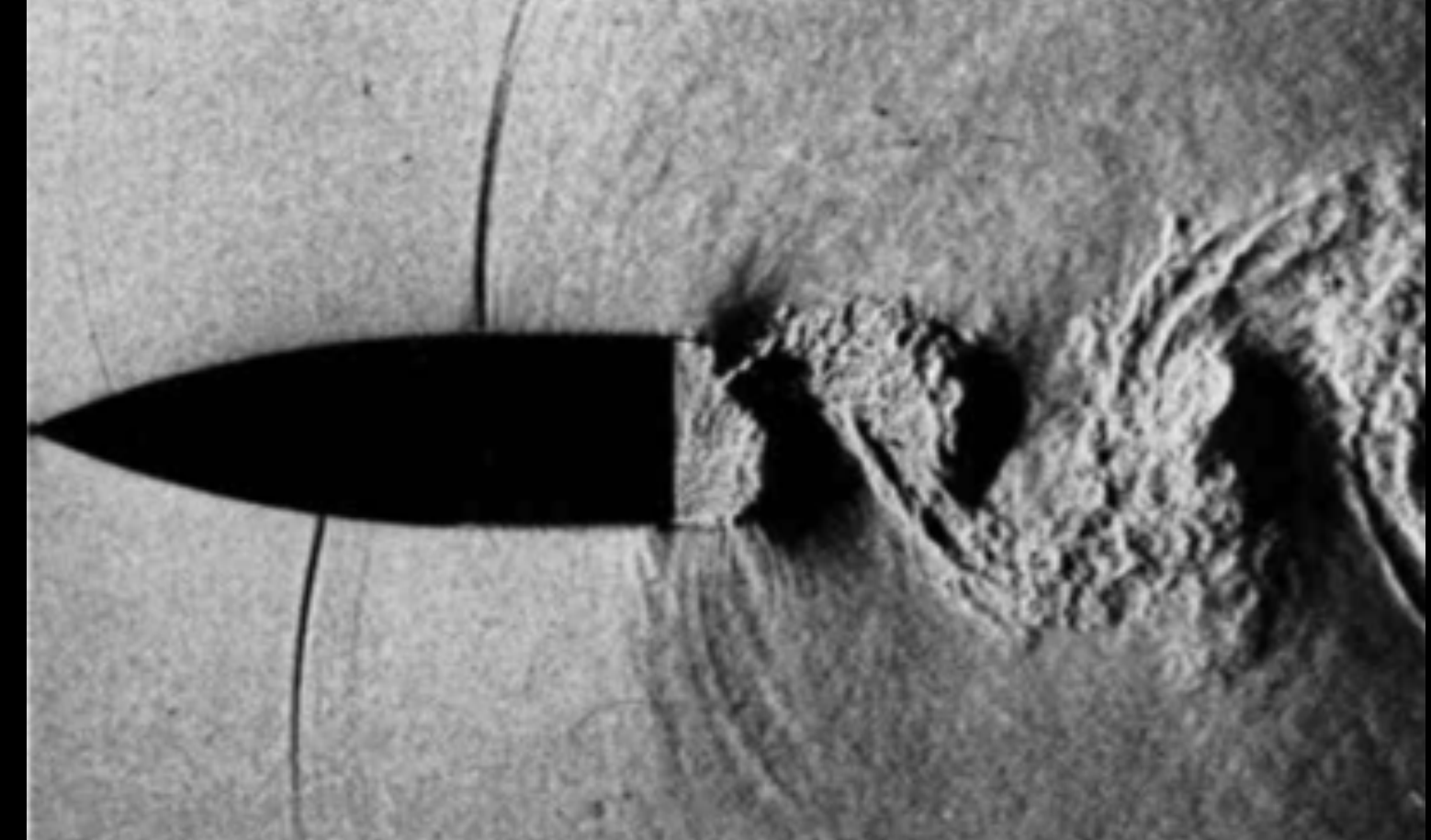
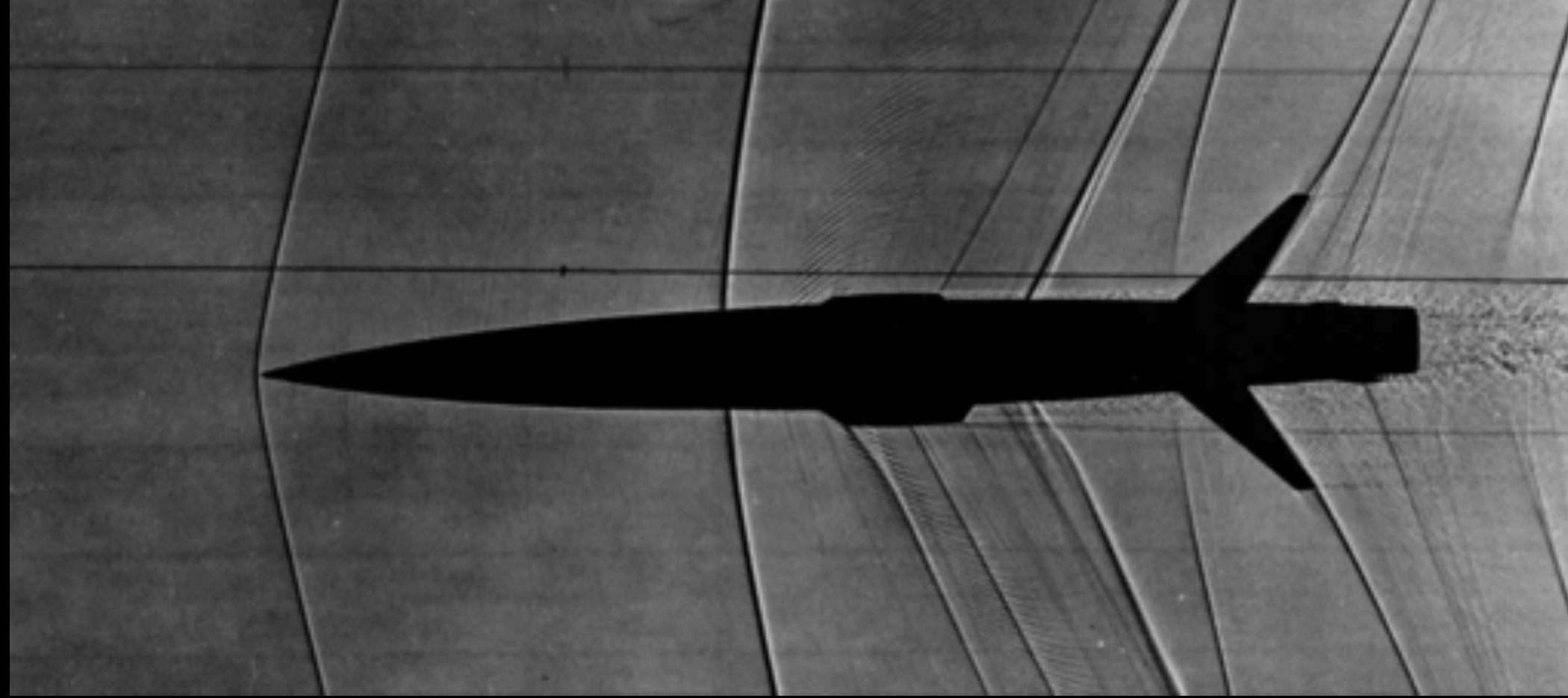
## An Album of Fluid Motion

**Jet**  
**Wake**  
**Cavity**  
**Channel**  
**Pipe flow**  
**Mixing Layer**  
**Boundary Layer**  
**Wall bounded flows**  
**Isotropic turbulence**



**Milton Van Dyke**





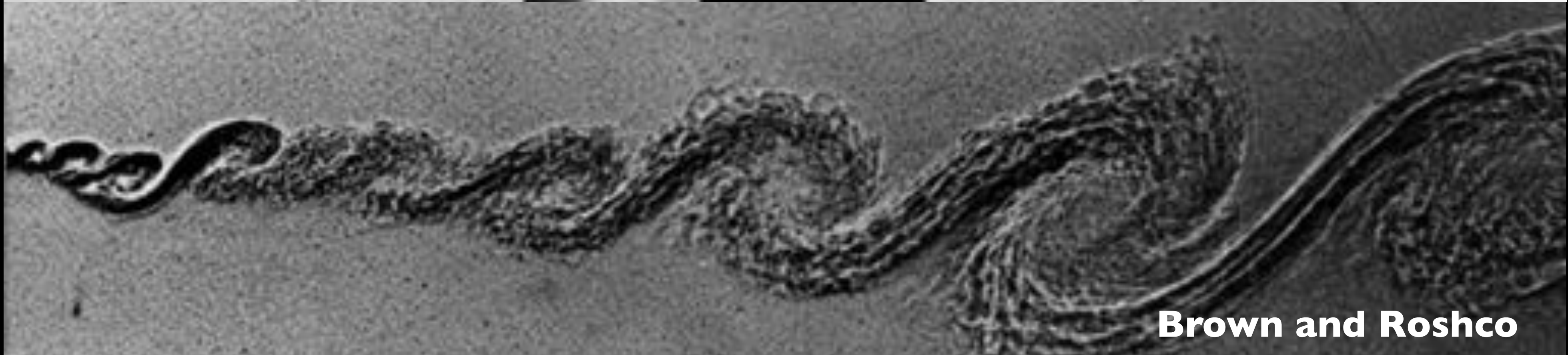
**WAKES**

**Milton Van Dyke**





**Milton Van Dyke**



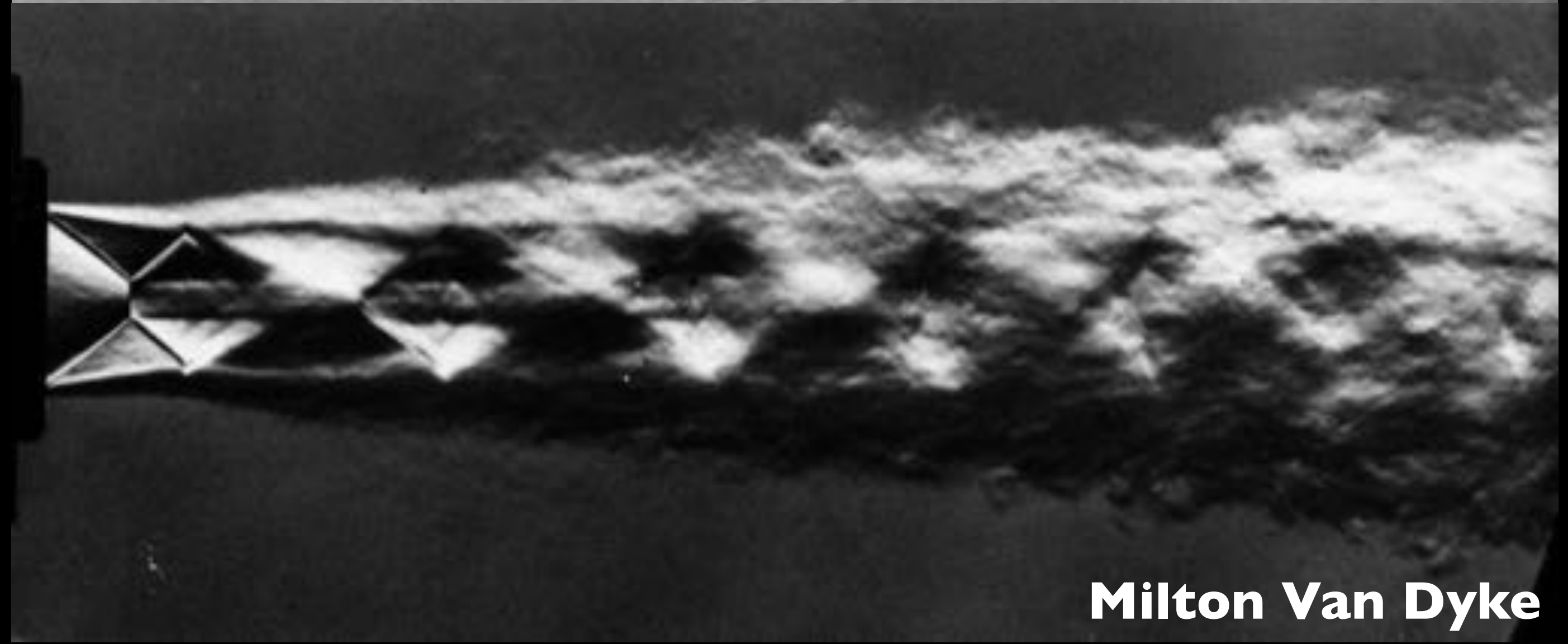
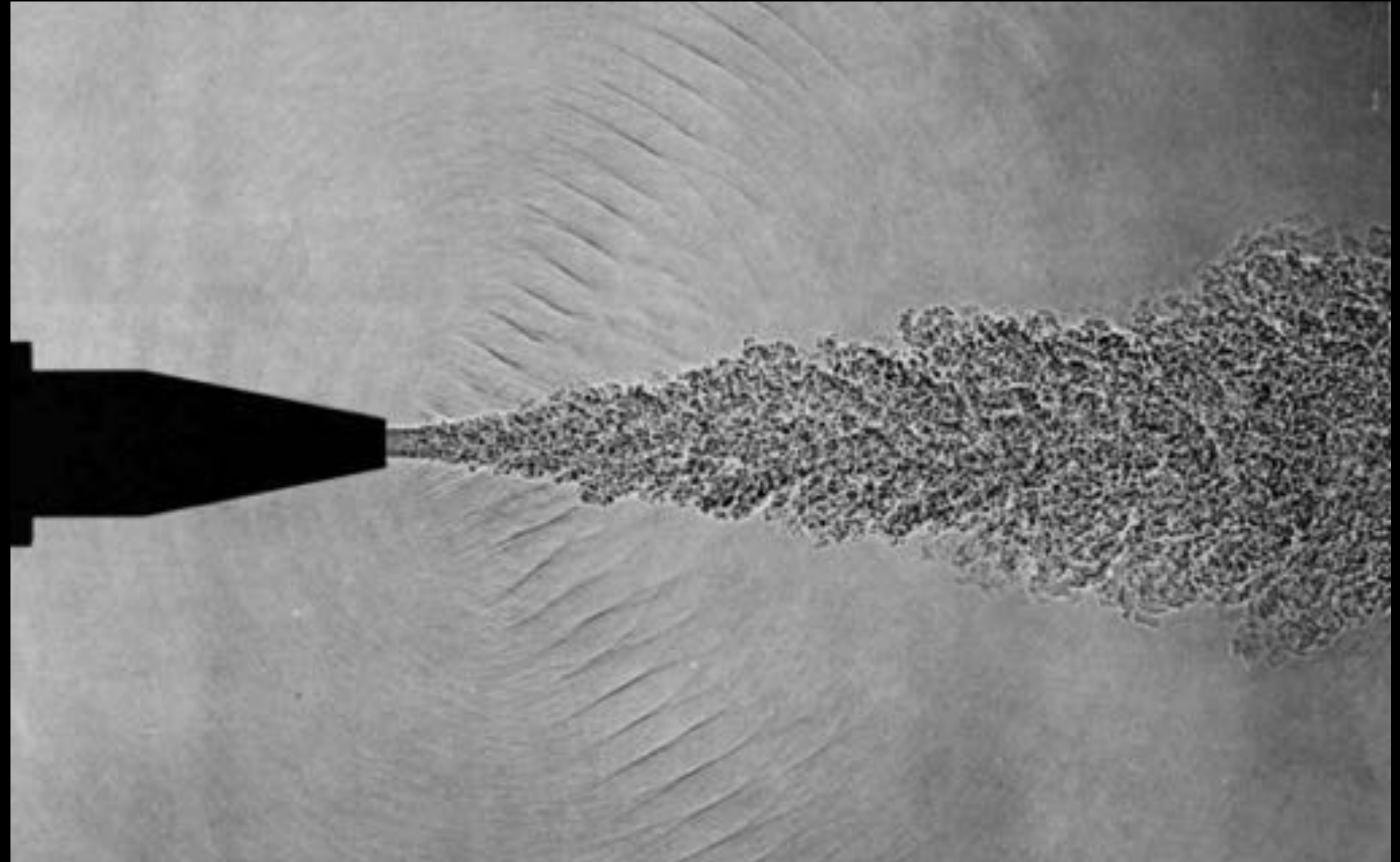
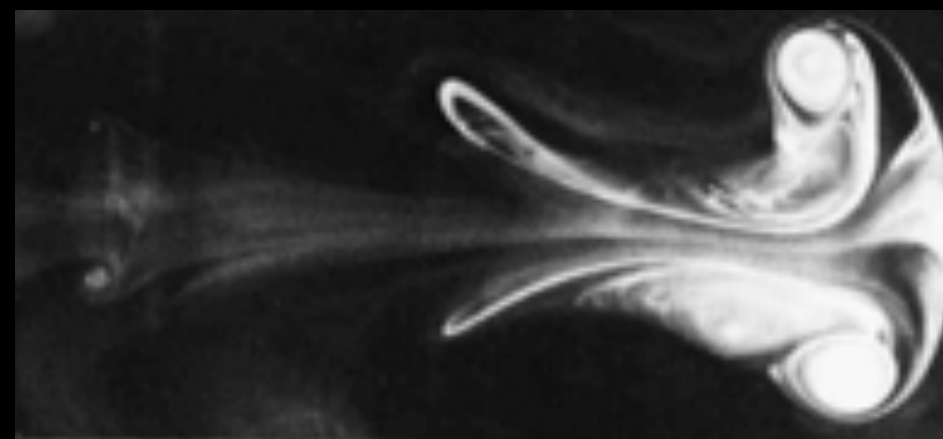
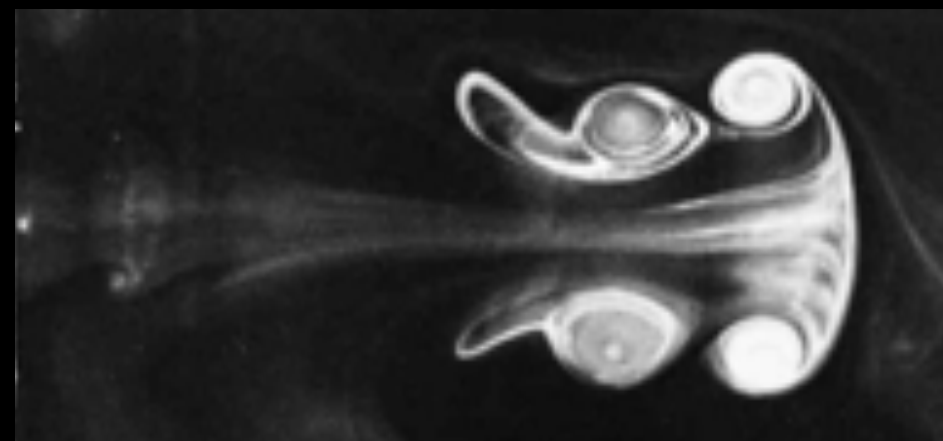
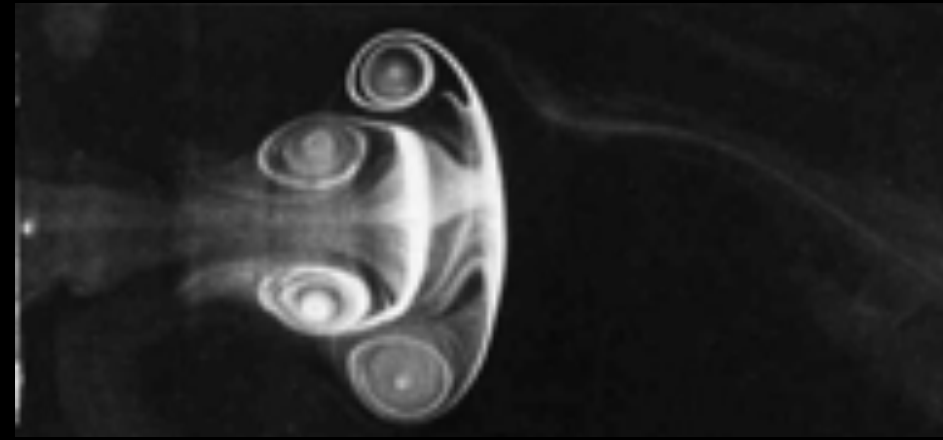
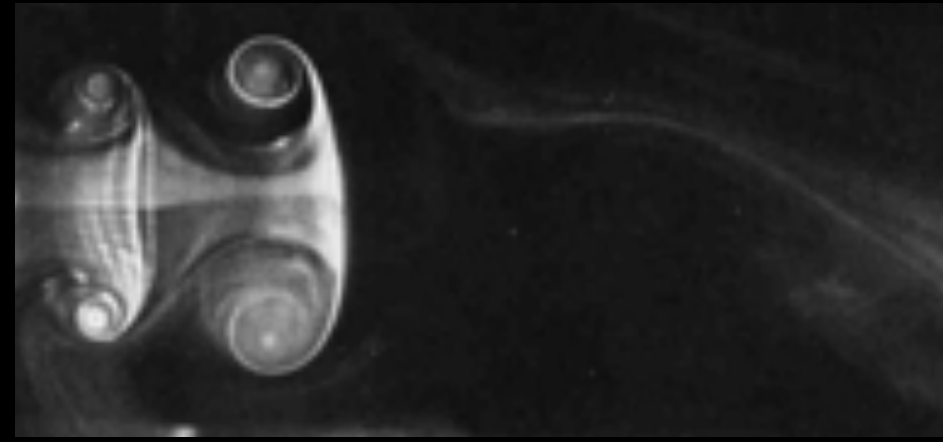
**Brown and Roshco**



**Callaham, Maeda, SLB**

**MIXING  
LAYERS**





**JETS**

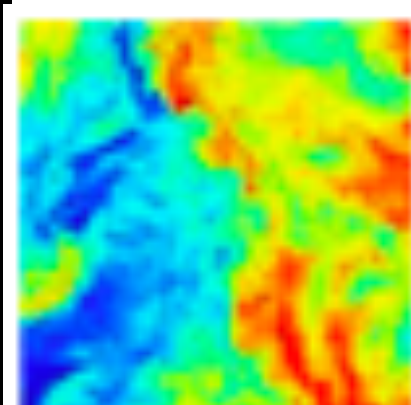
**Milton Van Dyke**



# DATABASES

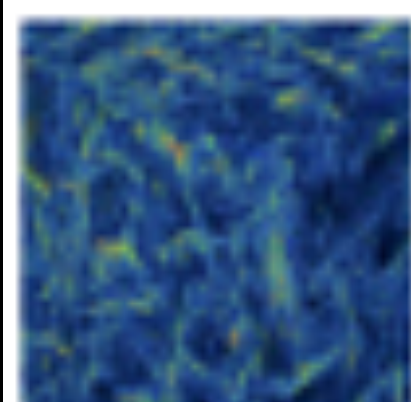


## Dataset descriptions



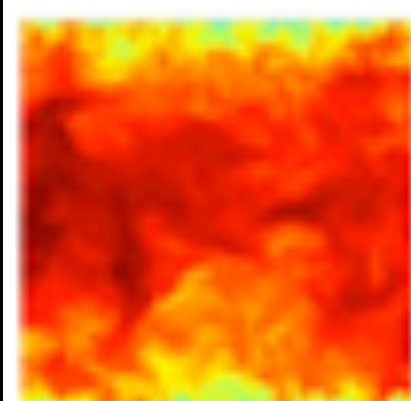
### 1. Forced isotropic turbulence:

Direct numerical simulation (DNS) using  $1,024^3$  nodes. The full time evolution is available, over 5 large-scale turnover times.



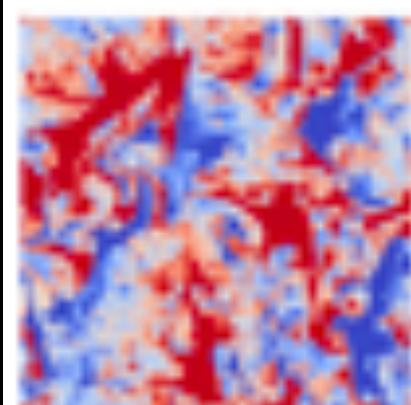
### 2. Forced MHD turbulence:

Direct numerical simulation (DNS) of magneto-hydrodynamic isotropic turbulence using  $1,024^3$  nodes. The full time evolution is available, over about 1 large-scale turnover time.



### 3. Channel flow:

Direct numerical simulation (DNS) of channel flow turbulence in a domain of size  $8\pi \times 2 \times 3\pi$ , using  $2048 \times 512 \times 1536$  nodes. The full time evolution is available, over a flow-through time across across the  $8\pi$  channel

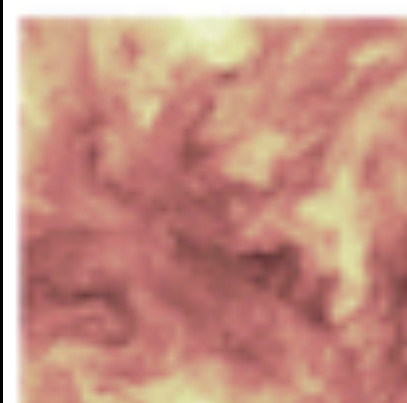


### 4. Homogeneous buoyancy driven turbulence:

Direct Numerical Simulation (DNS) of homogeneous buoyancy driven turbulence in a domain size  $2\pi \times 2\pi \times 2\pi$ , using  $1,024^3$  nodes. The full time evolution is available, covering both the buoyancy driven increase in turbulence intensity as well as the buoyancy mediated turbulence decay.

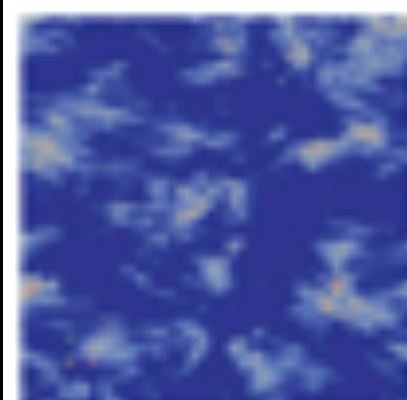


## Dataset descriptions



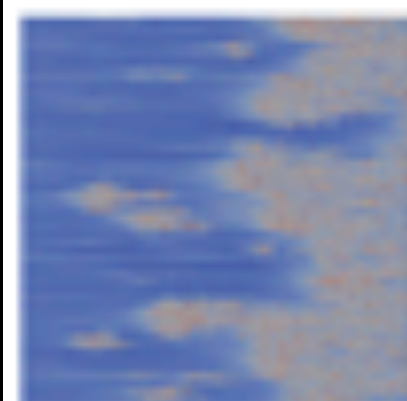
### 5. Forced isotropic turbulence dataset on $4096^3$ Grid:

Direct numerical simulation (DNS) using  $4096^3$  nodes. A single timestep snapshot is available.



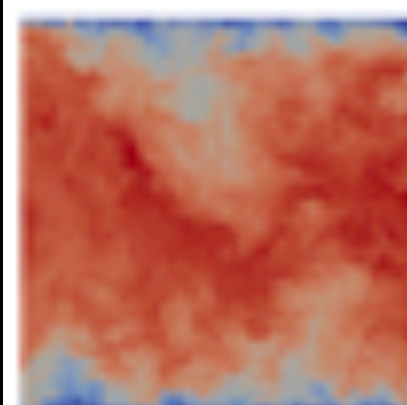
### 6. Rotating stratified turbulence dataset on $4096^3$ Grid:

Direct numerical simulation (DNS) of rotating stratified turbulence using  $4096^3$  nodes. A total of 5 snapshots are available.



### 7. Transitional boundary layer:

Direct numerical simulation (DNS) of a transitional boundary layer using a finite volume DNS code. Data are stored on  $3320 \times 224 \times 2048$  grid points. The full time evolution is available, over about 1 flow-through time across the length of simulation domain.



### 8. Channel flow at $Re_\tau=5200$ :

Direct numerical simulation (DNS) of channel flow turbulence in a domain of size  $8\pi \times 2 \times 3\pi$ , using  $10240 \times 1536 \times 7680$  nodes. A total of 11 snapshots are available.



**Burger, Treib, Westermann, Werner,  
Lalescu, Szalay, Meneveau, Eying**

Entry #: 84174

## Vortices within vortices: hierarchical nature of vortex tubes in turbulence

Kai Bürger<sup>1</sup>, Marc Treib<sup>1</sup>, Rüdiger Westermann<sup>1</sup>,  
Suzanne Werner<sup>2</sup>, Cristian C Lalescu<sup>3</sup>,  
Alexander Szalay<sup>2</sup>, Charles Meneveau<sup>4</sup>, Gregory L Eyink<sup>2,3,4</sup>

<sup>1</sup> Informatik 15 (Computer Graphik & Visualisierung), Technische Universität München

<sup>2</sup> Department of Physics & Astronomy, The Johns Hopkins University

<sup>3</sup> Department of Applied Mathematics & Statistics, The Johns Hopkins University

<sup>4</sup> Department of Mechanical Engineering, The Johns Hopkins University



# MIXING



**Plunge**

## 2D Incompressible Navier-Stokes:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \int_s \mathbf{f}(\xi(s, t)) \delta(\xi - \mathbf{x}) ds$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u}(\xi(s, t)) = \int_{\mathbf{x}} \mathbf{u}(\mathbf{x}) \delta(\mathbf{x} - \xi) d\mathbf{x} = \mathbf{u}_B(\xi(s, t))$$

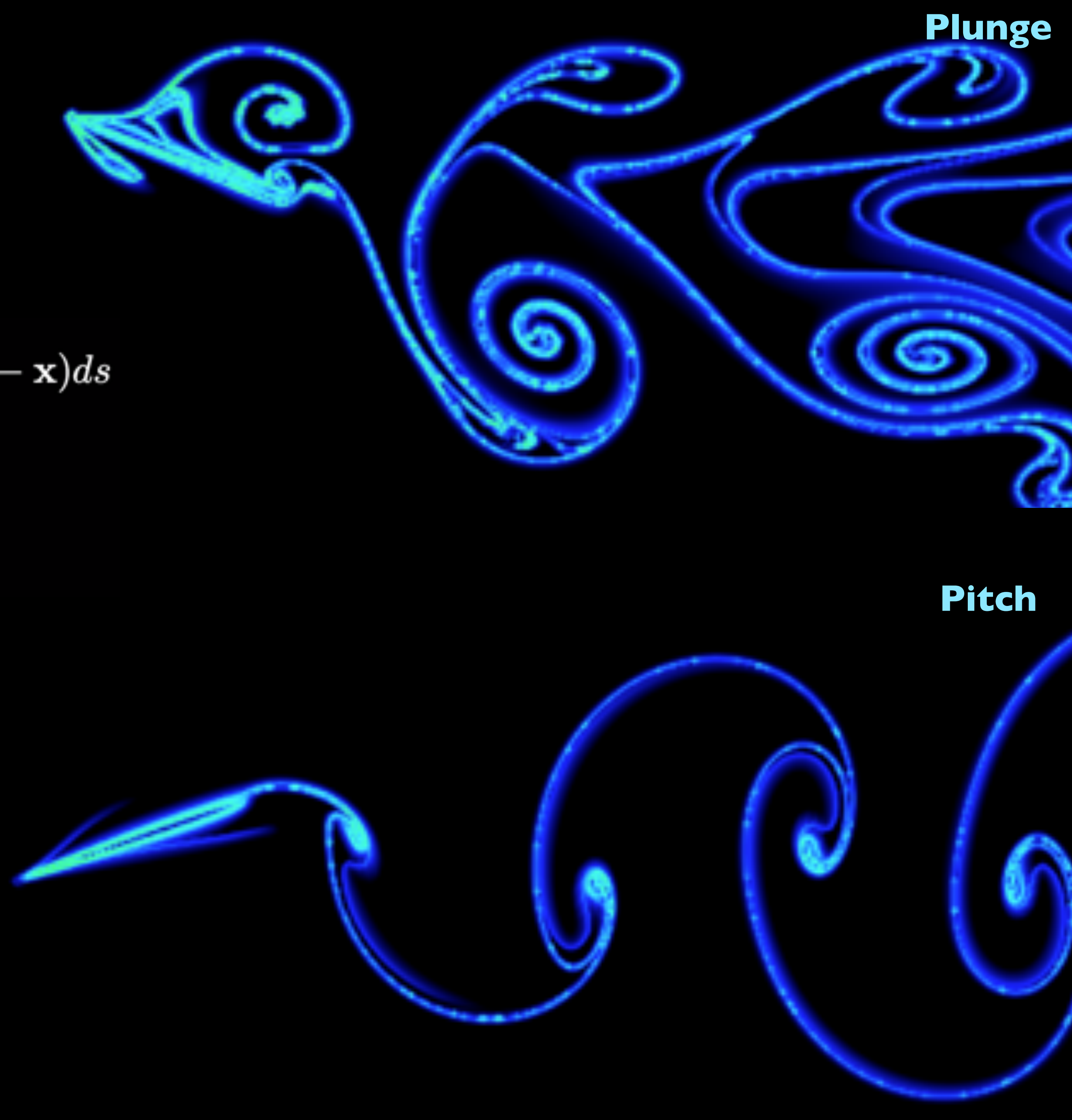
**Pitch**

## Immersed boundary method

Boundary forces computed as Lagrange-multipliers to enforce no slip

**Taira & Colonius, 2007.**

**Colonius & Taira, 2008.**





# Dynamical Systems: Poincare and Geometry

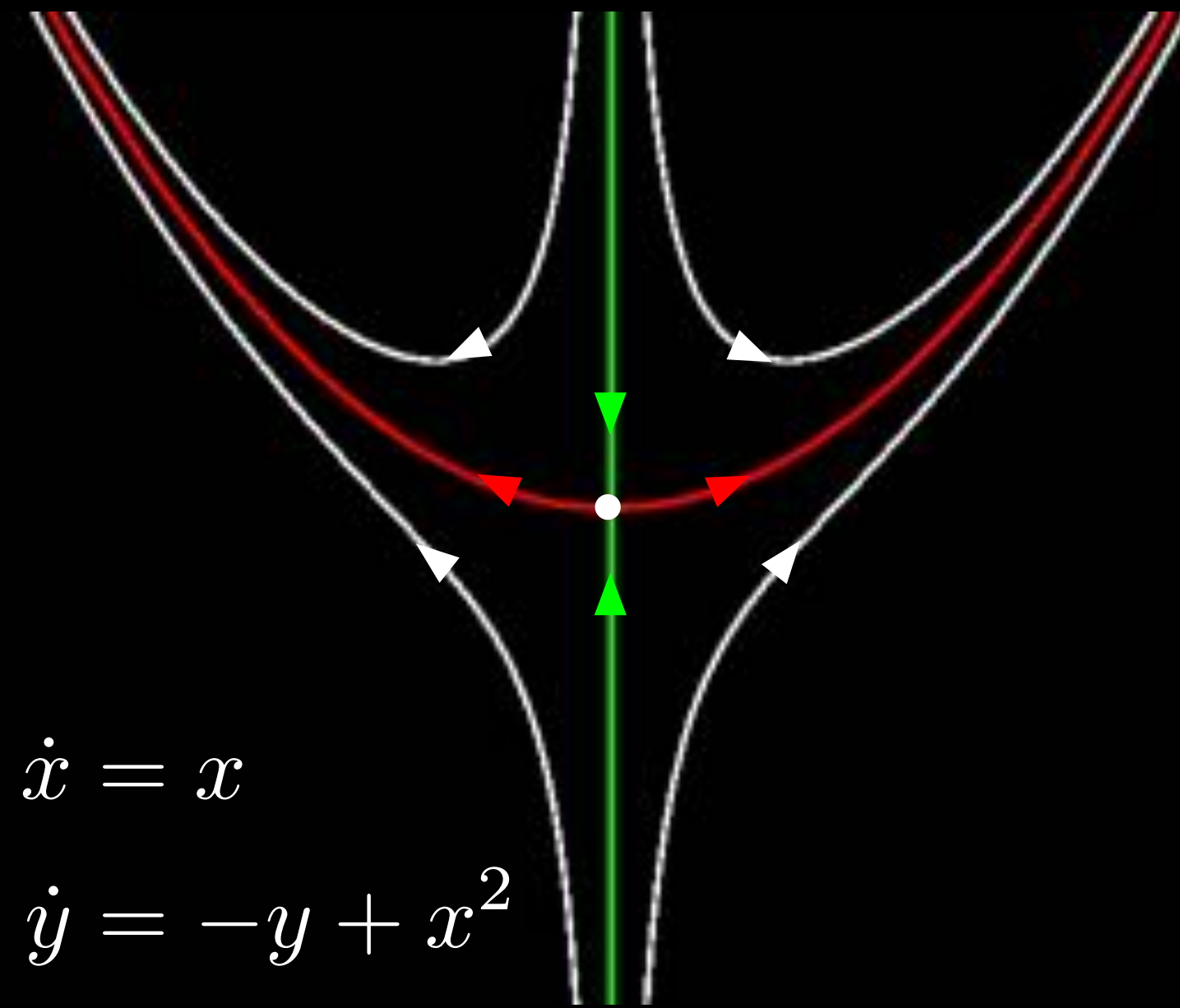
Dynamics

$$\frac{d}{dt}\mathbf{x} = \mathbf{f}(\mathbf{x}), \quad \longrightarrow \quad \mathbf{F}_t(\mathbf{x}(t_0)) = \mathbf{x}(t_0 + t) = \mathbf{x}(t_0) + \int_{t_0}^{t_0+t} \mathbf{f}(\mathbf{x}(\tau)) d\tau.$$

$$\longrightarrow \quad \mathbf{x}_{k+1} = \mathbf{F}_t(\mathbf{x}_k), \quad \text{Discrete-time update}$$

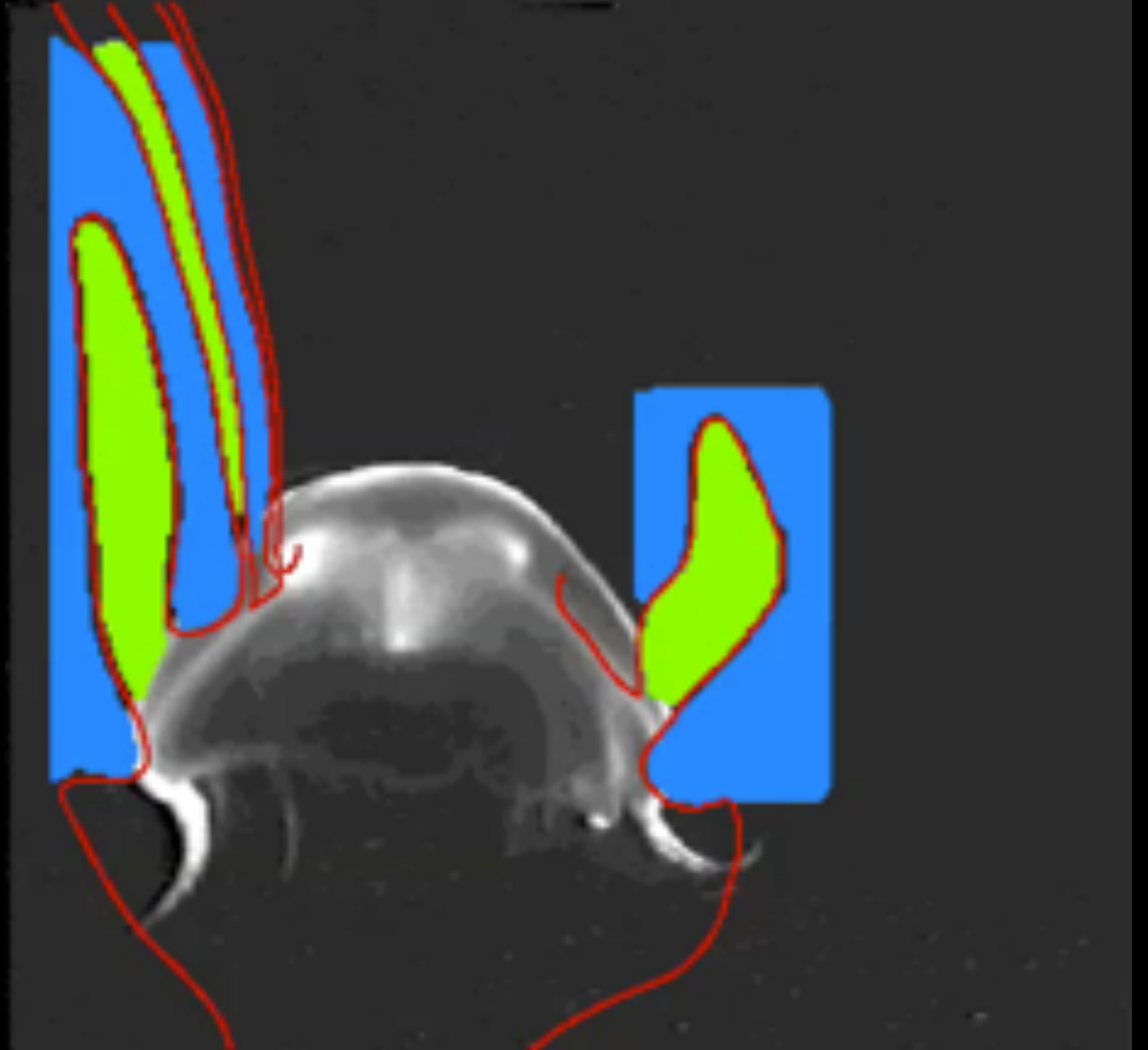
Haller, 2002;  
Shadden et al., 2005

Finite-time Lyapunov exponents (**FTLE**)





# How Jellyfish Eat





Medusoid @1Hz  
(still attached to  
fabrication mold)

**Robotic Jellyfish!**



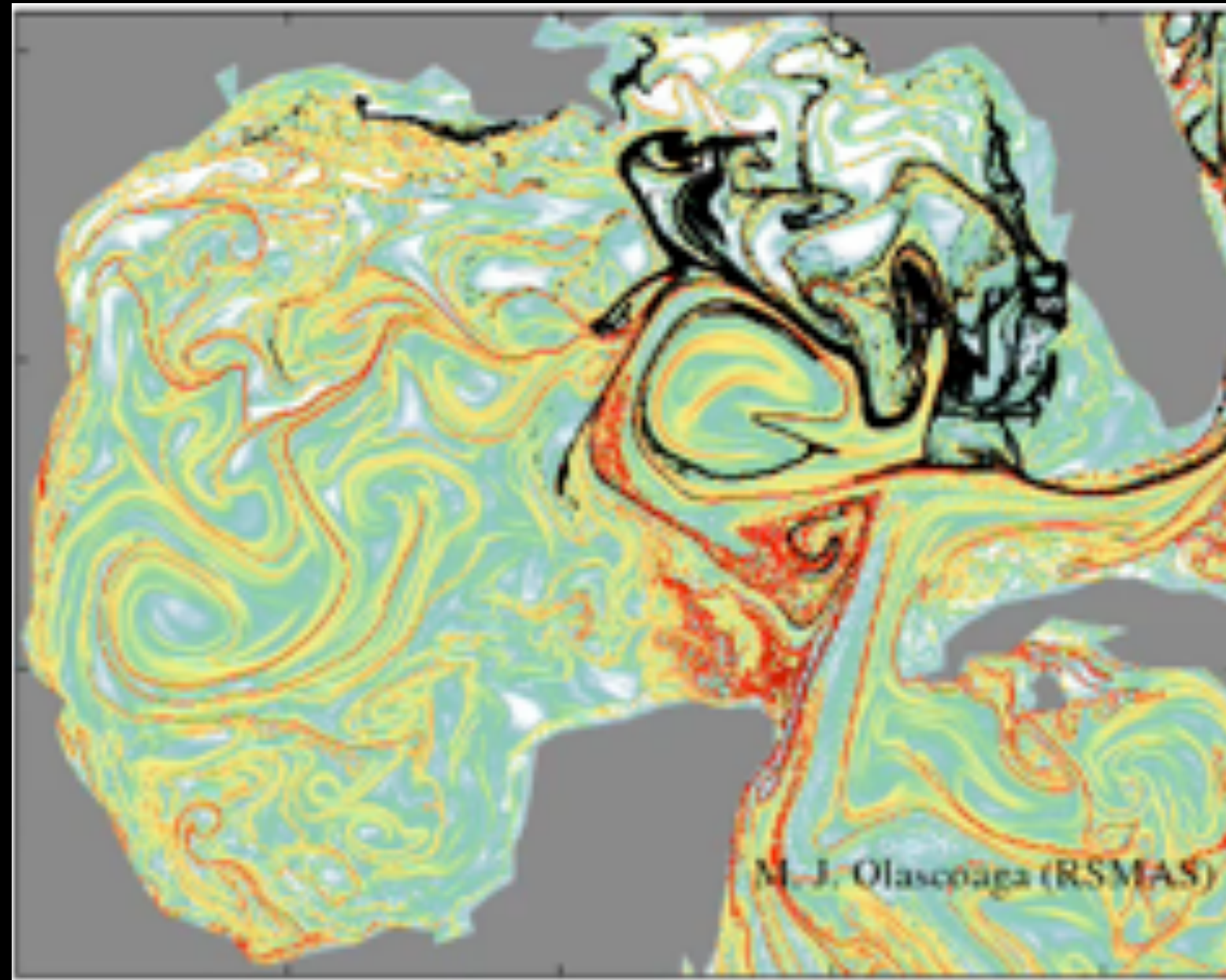
9mm =  $\frac{1}{3}$  inch

**Dabiri & Parker, 2012**



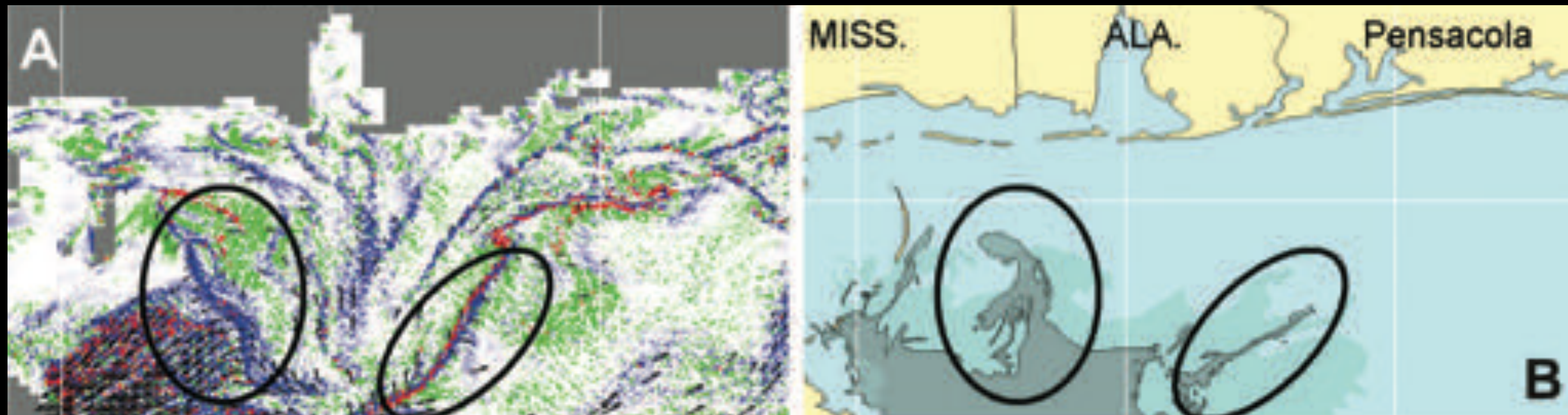
# Contaminate Release in the Ocean

Olascoaga, NOAA 2010

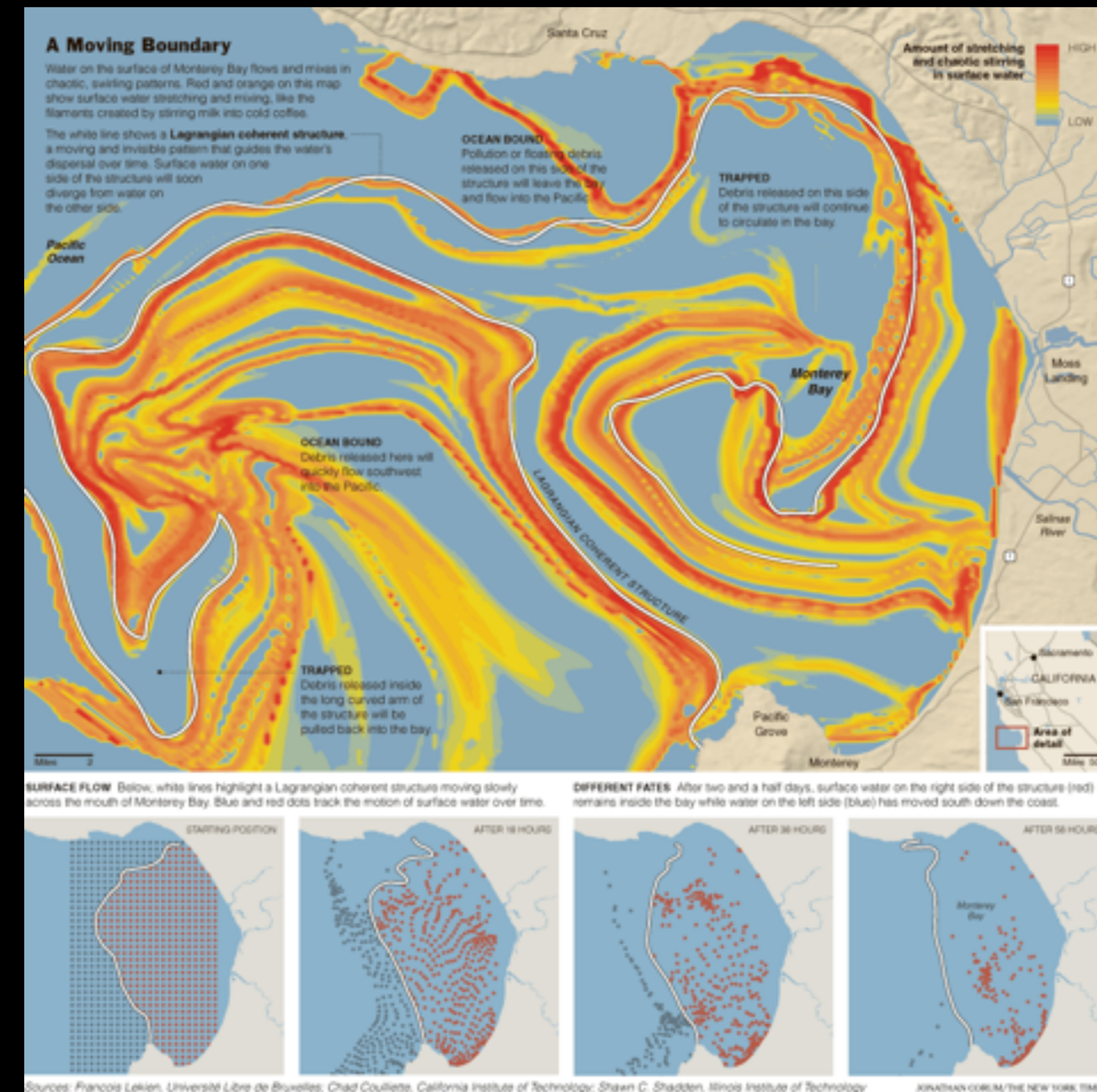


Gulf Oil Spill

Mezic, et al, Science 2010.

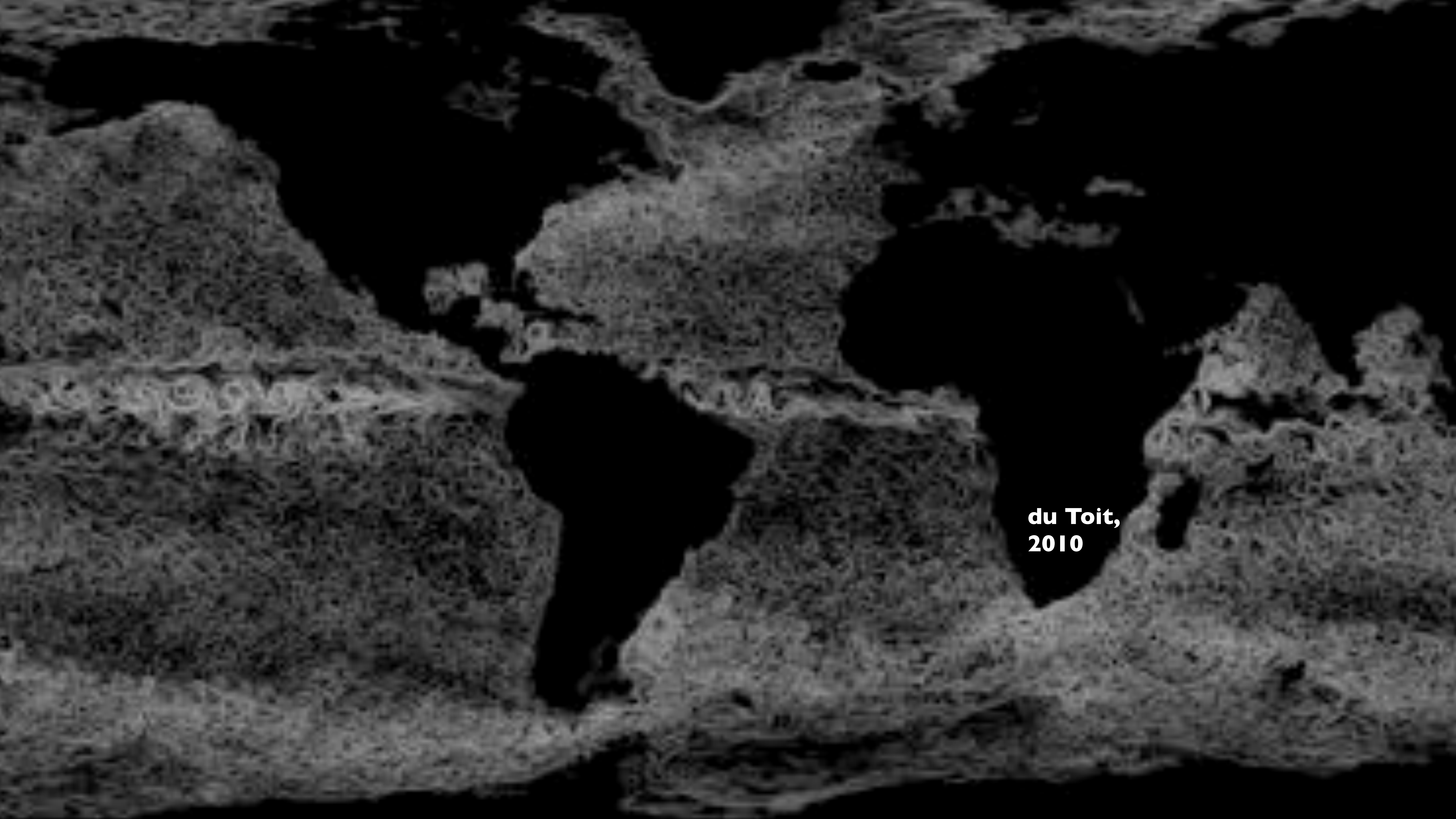


Monterey Bay



Lekien, Coulliette, Shadden  
J. Marsden & N. Leonard, 2005





**du Toit,  
2010**



# Machine Learning for Fluid Mechanics

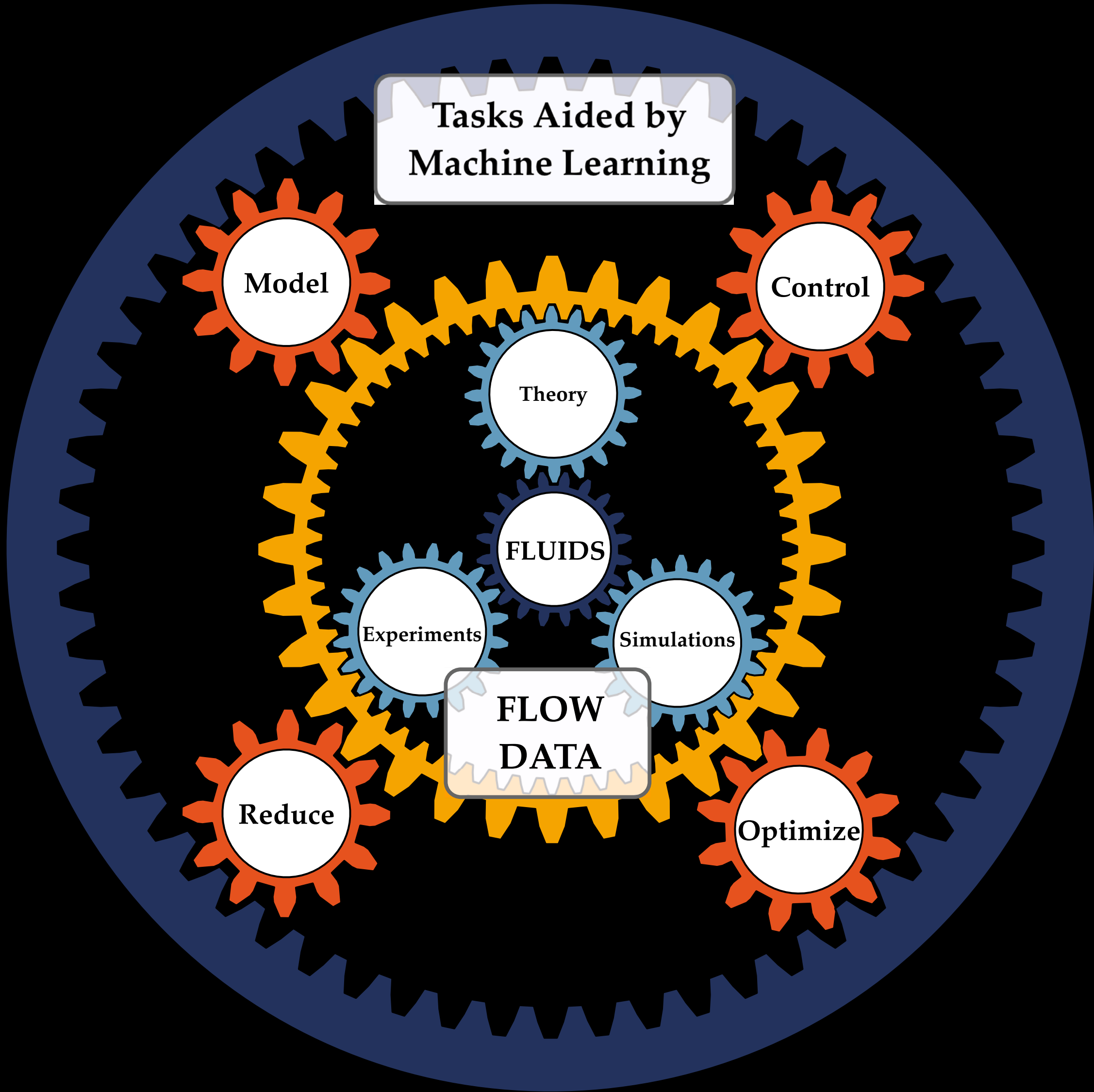
Steven L. Brunton,<sup>1</sup> Bernd R. Noack,<sup>2</sup> and Petros Koumoutsakos<sup>3, 4</sup>

<sup>1</sup>Mechanical Engineering, University of Washington, Seattle, WA, USA, 98115; email: sbrunton@uw.edu

<sup>2</sup>Laboratoire d'Informatique pour la Mécanique et les Sciences de l'Ingénieur, LIMSI-CNRS, Rue John von Neumann, Campus Universitaire d'Orsay, Bât 508, F-91403 Orsay, France

<sup>3</sup>Computational Science and Engineering Laboratory, ETH Zurich, CH-8092, Switzerland

<sup>4</sup>Collegium Helveticum, Zurich, CH-8092, Switzerland



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[https://doi.org/10.1146/\(\(please add article doi\)\)](https://doi.org/10.1146/((please add article doi)))

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## Keywords

machine learning, data-driven modeling, optimization, control

## Abstract

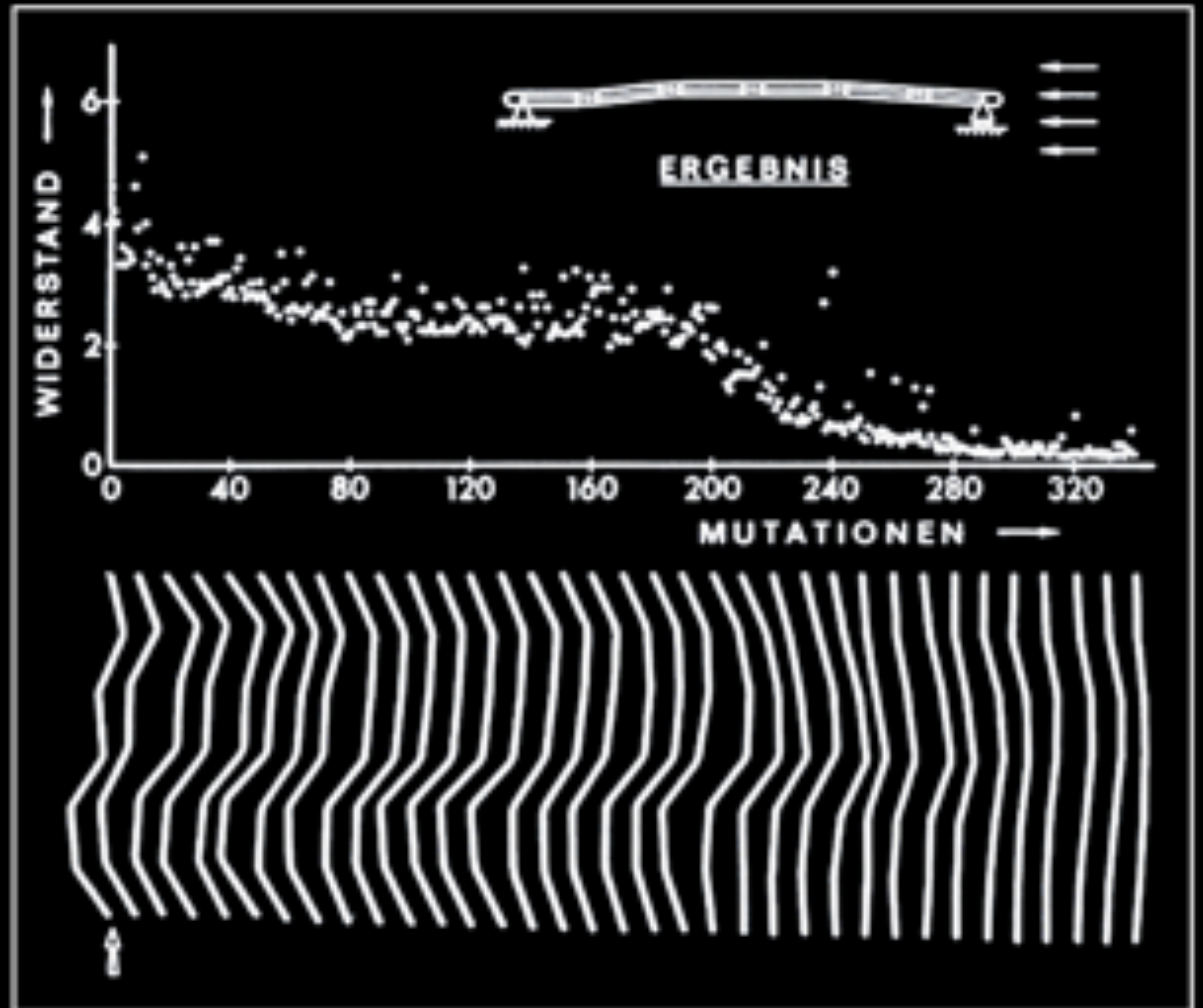
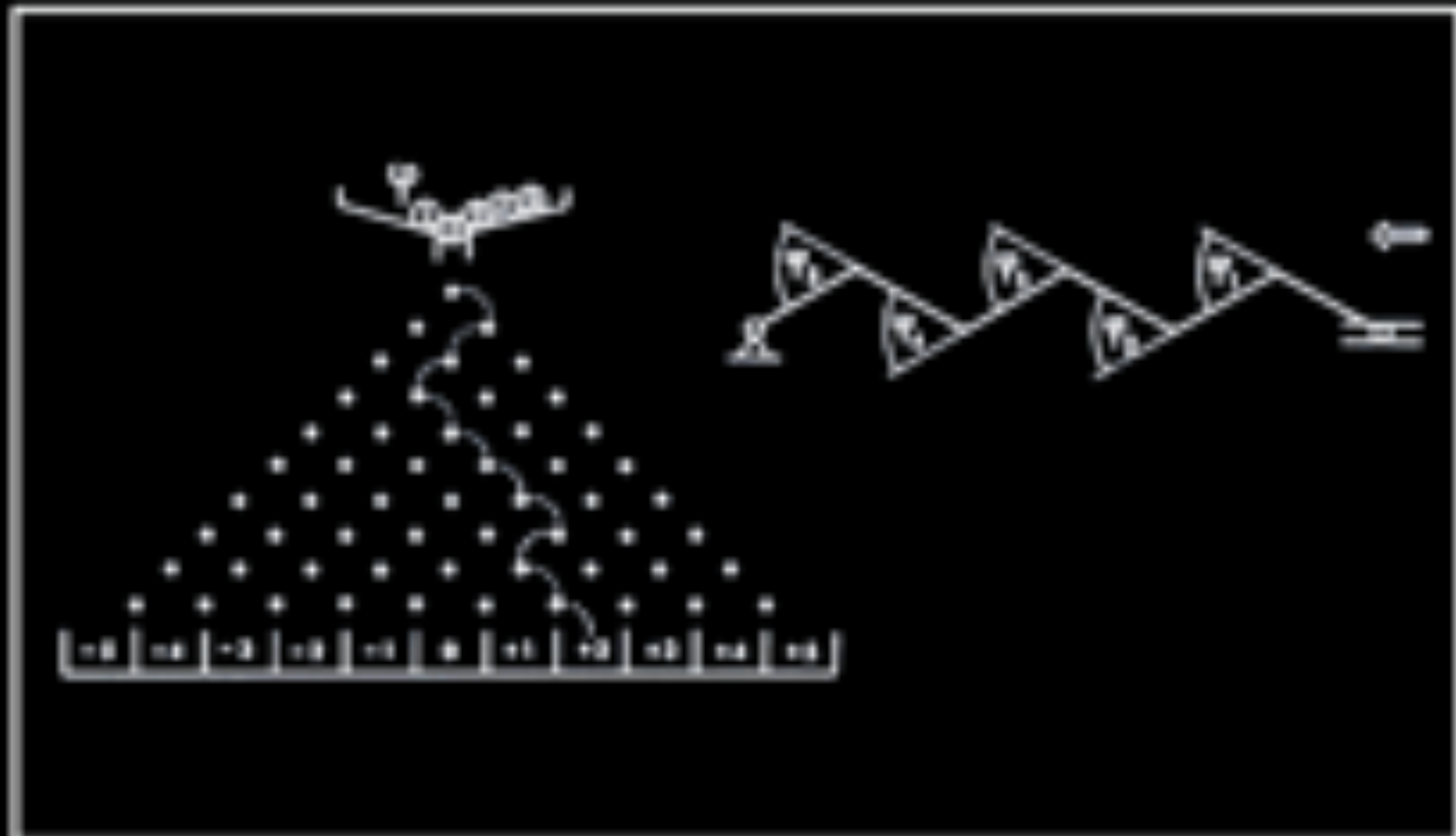
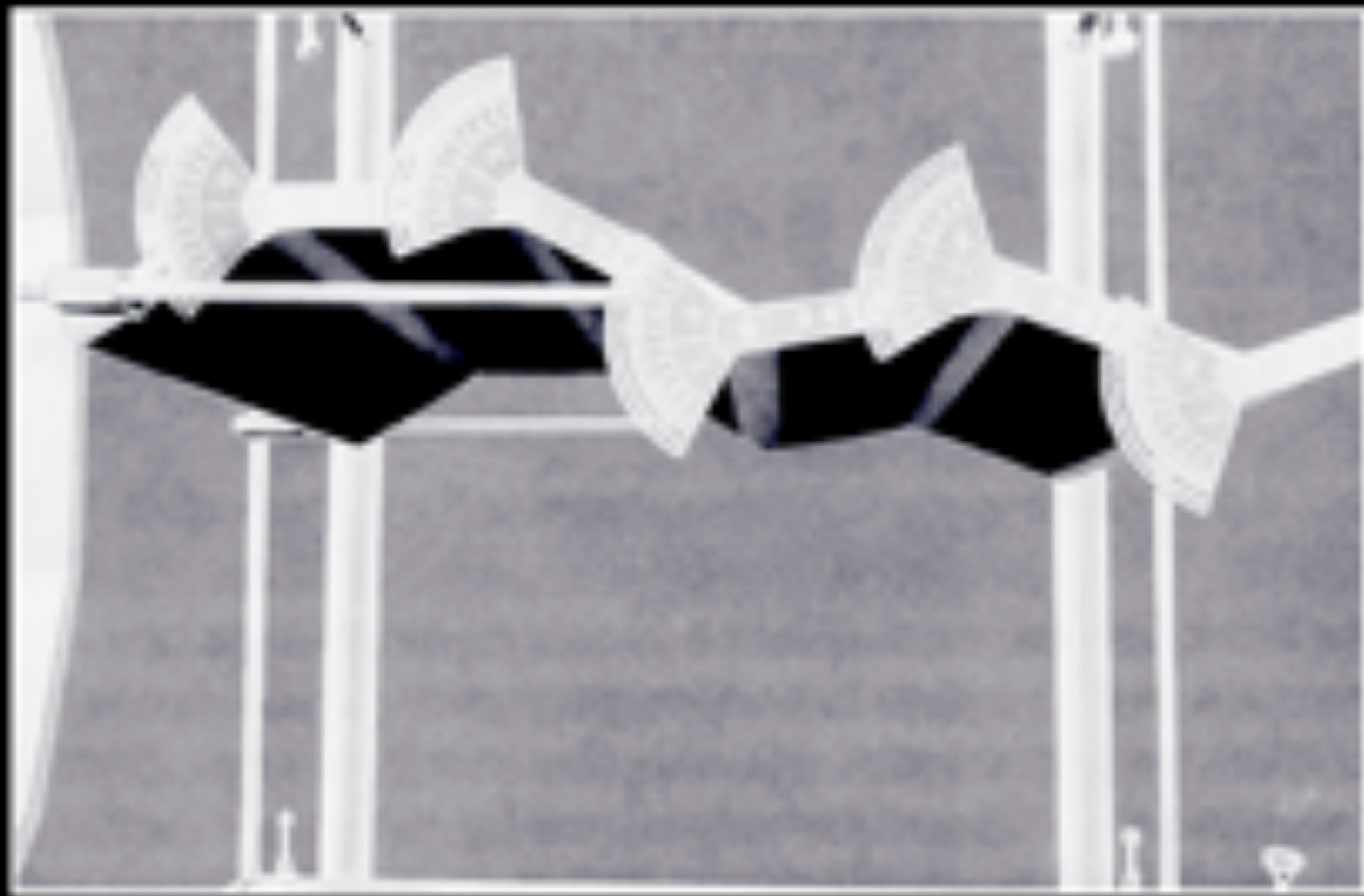
The field of fluid mechanics is rapidly advancing, driven by unprecedented volumes of data from experiments, field measurements, and large-scale simulations at multiple spatiotemporal scales. Machine learning presents us with a wealth of techniques to extract information from data that can be translated into knowledge about the underlying fluid mechanics. Moreover, machine learning algorithms can augment domain knowledge and automate tasks related to flow control and optimization. This article presents an overview of past history, current developments, and emerging opportunities of machine learning for fluid mechanics. We outline fundamental machine learning methodologies and discuss their uses for understanding, modeling, optimizing, and controlling fluid flows. The strengths and limitations of these methods are addressed from the perspective of scientific inquiry that links data with modeling, experiments, and simulations. Machine learning provides a powerful information processing framework that can augment, and possibly even transform, current lines of fluid mechanics research and industrial applications.



# HISTORY

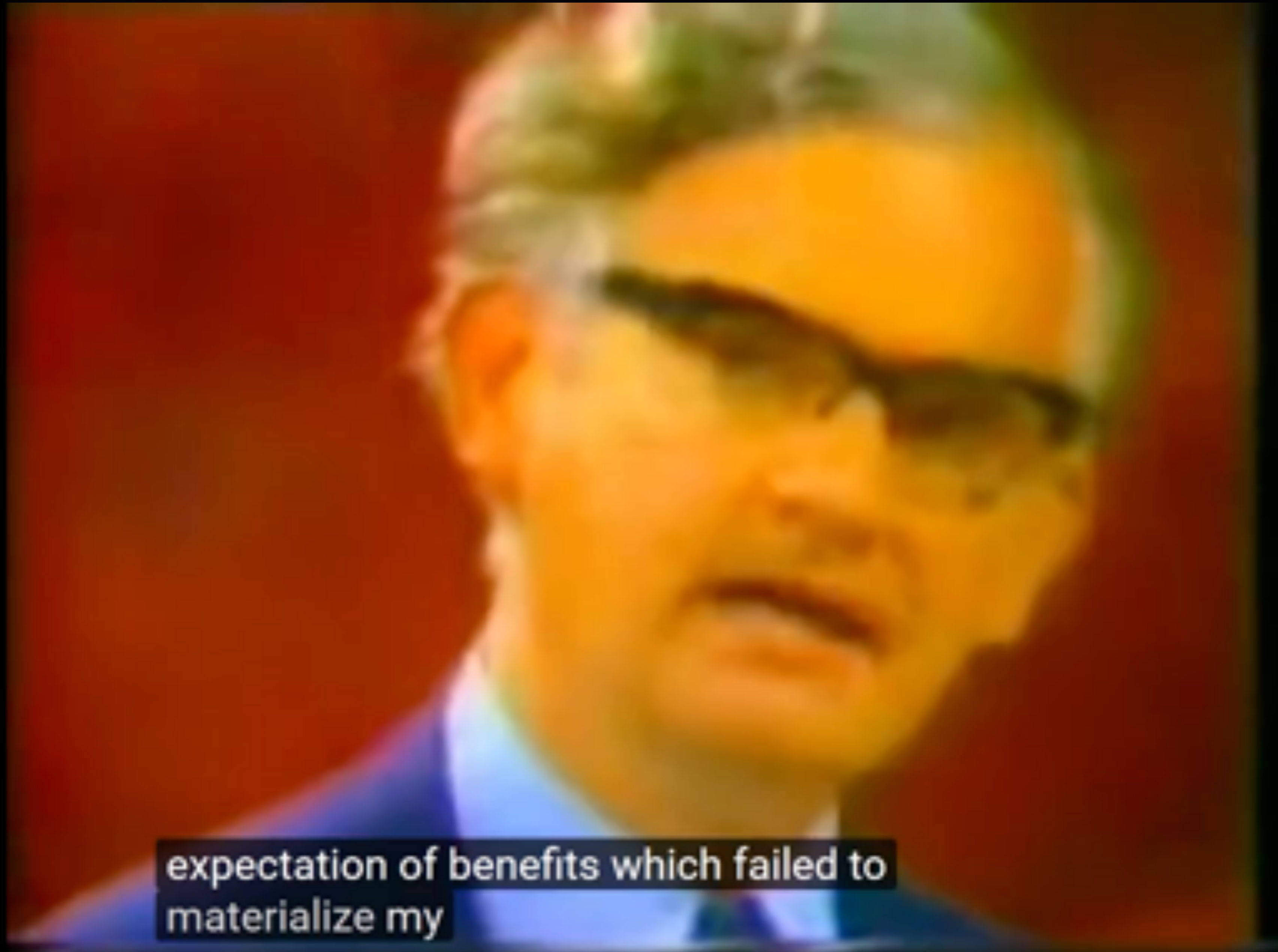
Reichenberg, 1960s-1970s  
Schweifel, 1970s

SLB, Noack, Koumoutsakos,  
*Ann. Rev. Fluid Mech.* 2019





## Sir Lighthill and the AI Winter (1974)



<https://www.youtube.com/watch?v=ug0oZAwjC6g>



# EXPERIMENTAL MEASUREMENTS



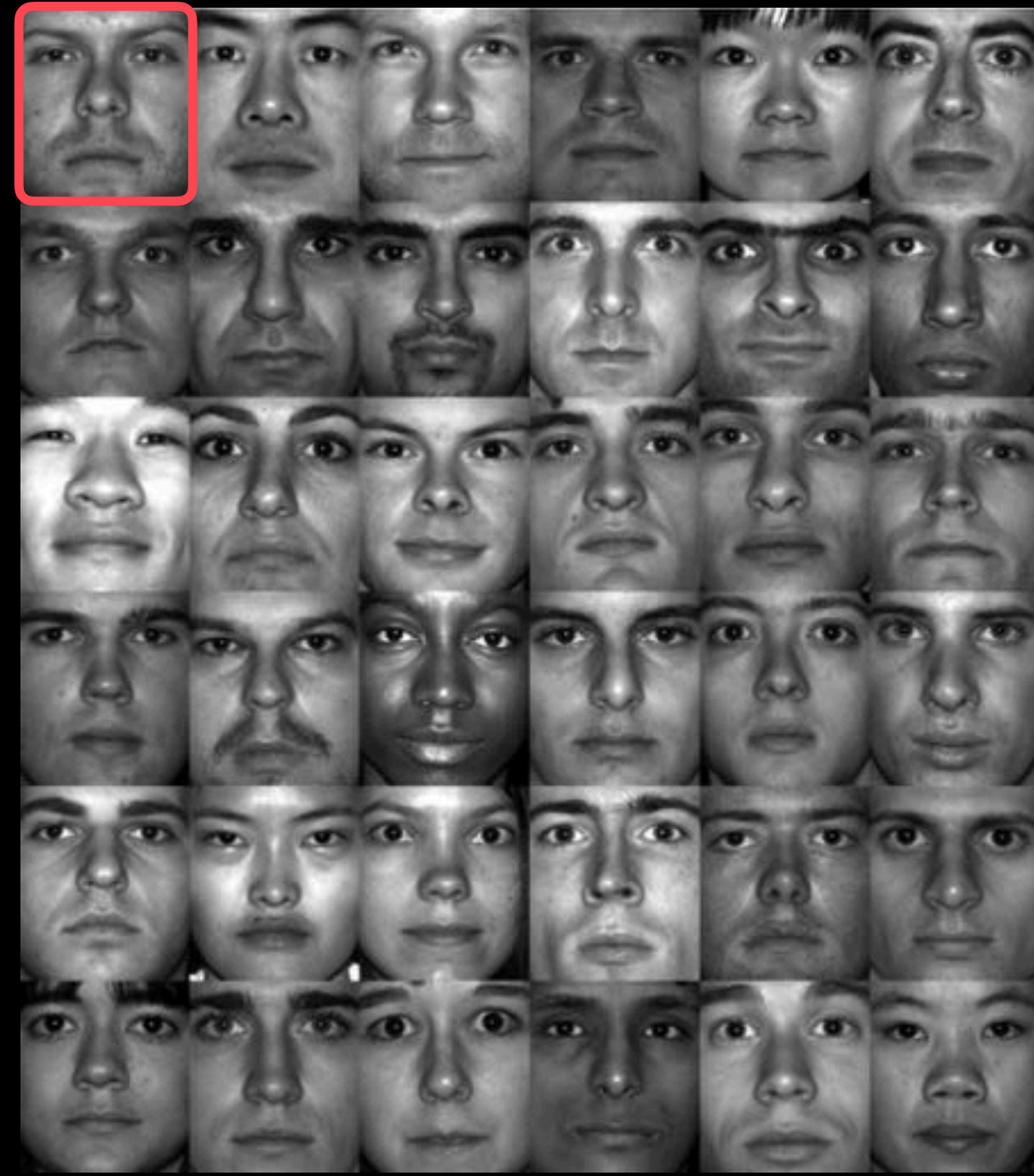
# PARTICLE IMAGE VELOCIMETRY (PIV)

**LaVision PIV**

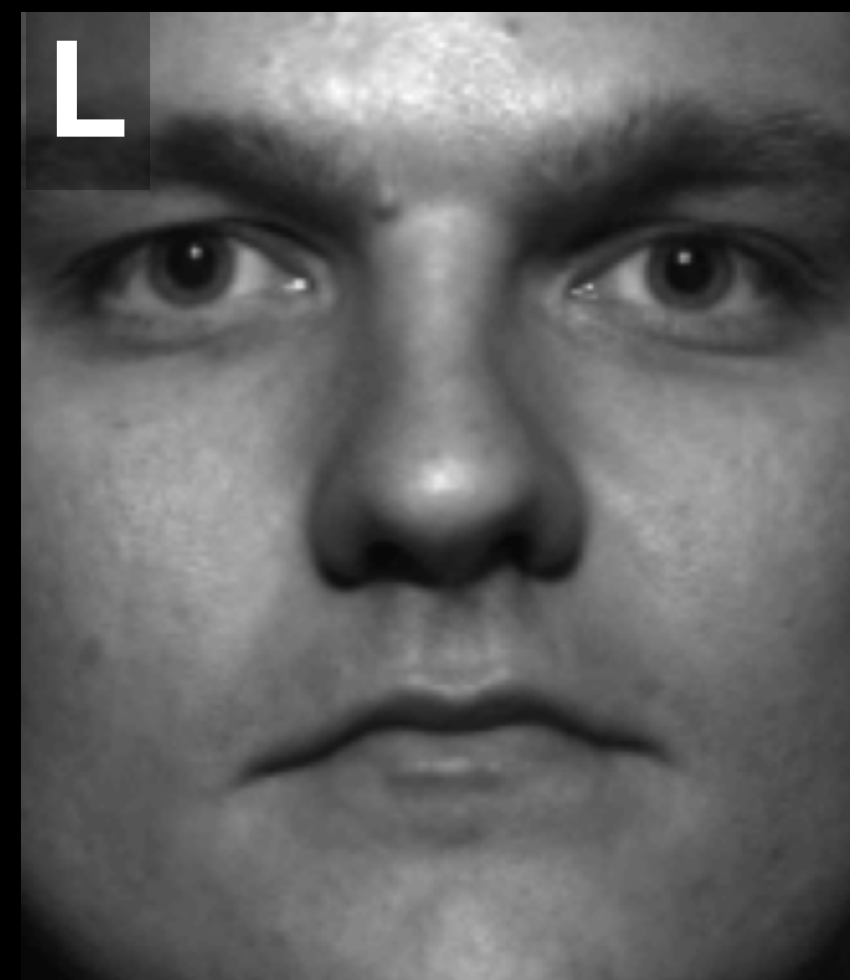




# ROBUST STATISTICS (RPCA)



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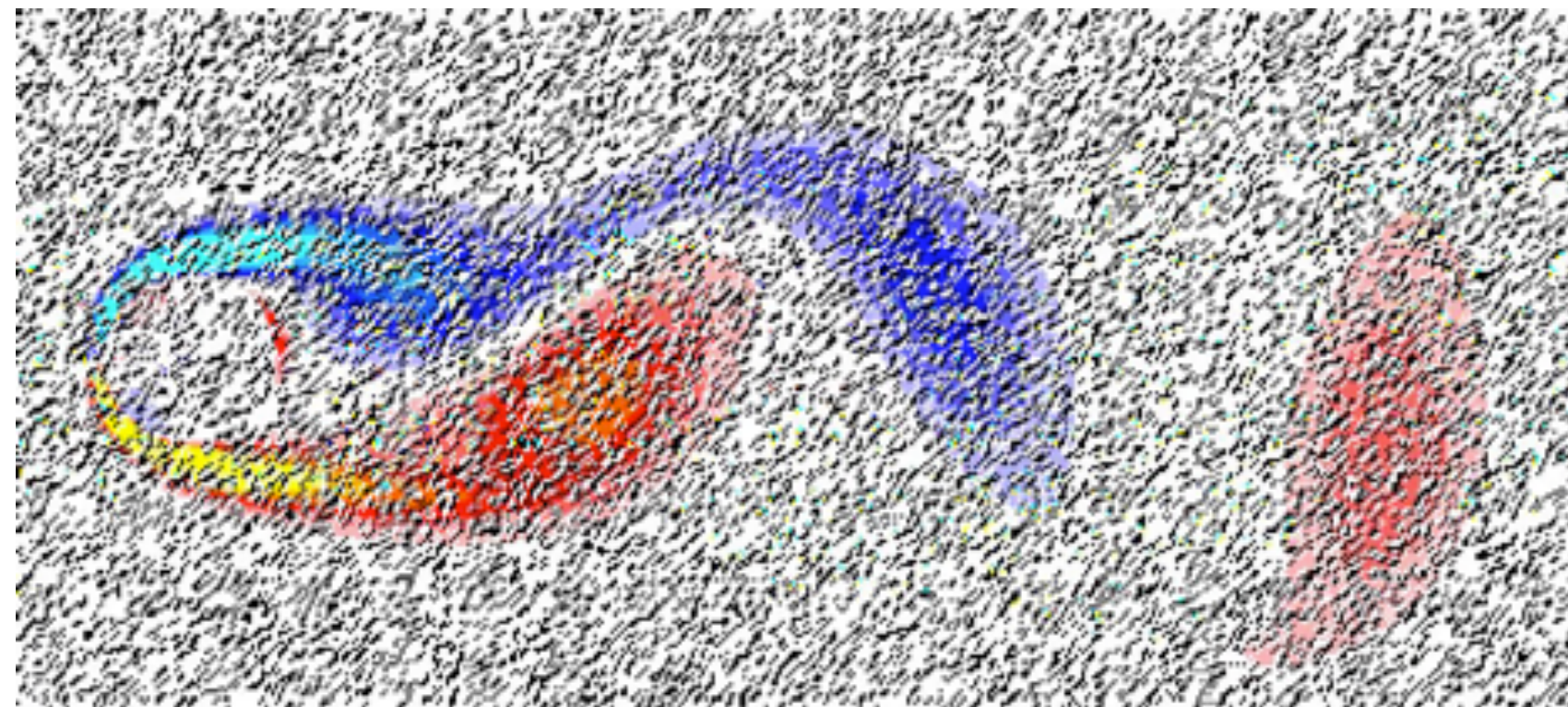


+





# Robust Principal Component Analysis (RPCA)

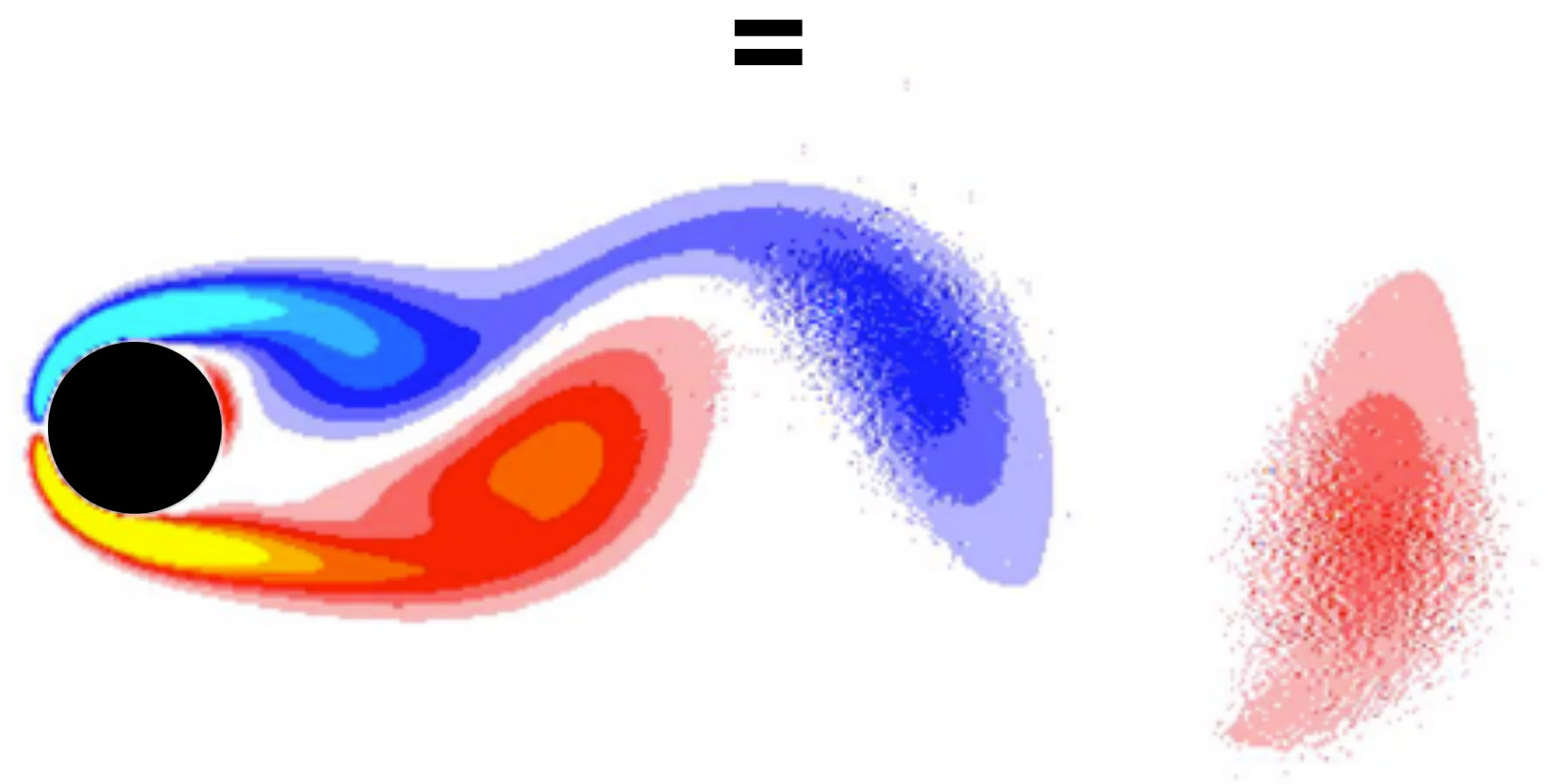


**X**

$$\mathbf{X} = \mathbf{L} + \mathbf{S}.$$

$$\min_{\mathbf{L}, \mathbf{S}} \text{rank}(\mathbf{L}) + \|\mathbf{S}\|_0$$

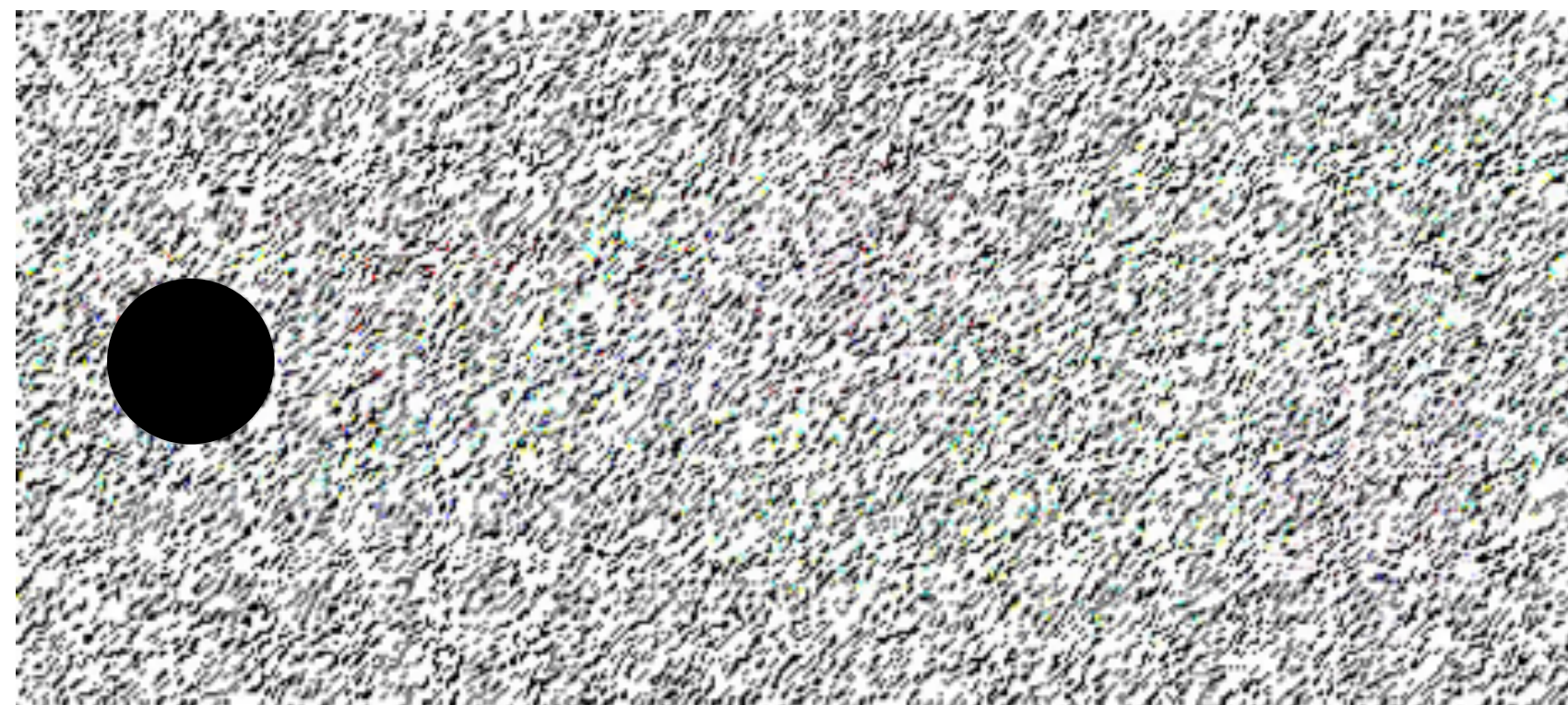
$$\text{subject to } \mathbf{L} + \mathbf{S} = \mathbf{X}.$$



**L**

$$\min_{\mathbf{L}, \mathbf{S}} \|\mathbf{L}\|_* + \lambda_0 \|\mathbf{S}\|_1$$

$$\text{subject to } \mathbf{L} + \mathbf{S} = \mathbf{X}.$$

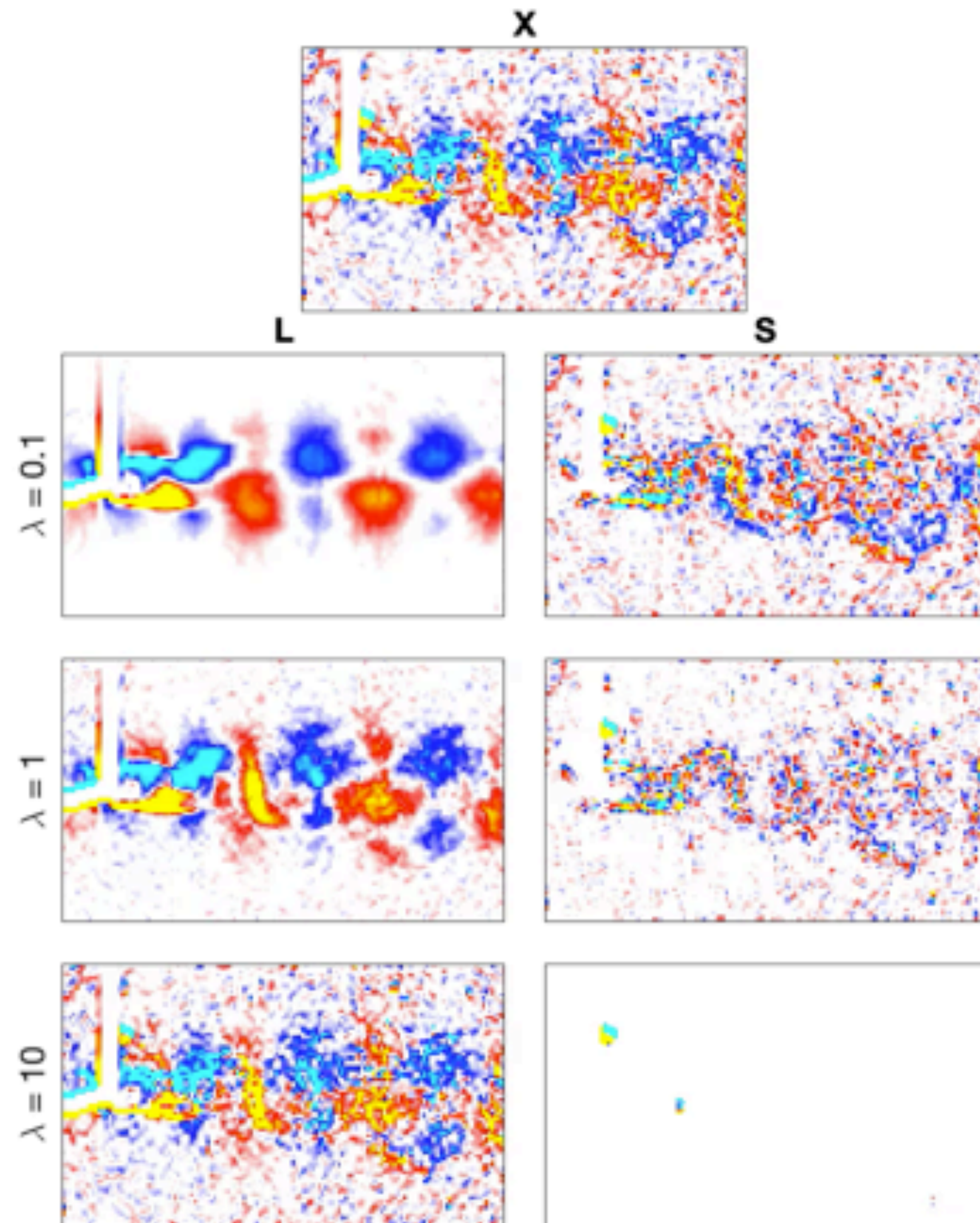


**S**

**Candes et al., J. ACM, 2011**  
**Scherl et al., arXiv:1905.07062, 2019**



# Robust Principal Component Analysis (RPCA)



$$\mathbf{X} = \mathbf{L} + \mathbf{S}.$$

$$\min_{\mathbf{L}, \mathbf{S}} \text{rank}(\mathbf{L}) + \|\mathbf{S}\|_0$$

$$\text{subject to } \mathbf{L} + \mathbf{S} = \mathbf{X}.$$

$$\min_{\mathbf{L}, \mathbf{S}} \|\mathbf{L}\|_* + \lambda_0 \|\mathbf{S}\|_1$$

$$\text{subject to } \mathbf{L} + \mathbf{S} = \mathbf{X}.$$



# SUPER RESOLUTION

LOW-RES



HIGH-RES



Google RAISR



# SUPER RESOLUTION

*Super-resolution reconstruction with machine learning*

3

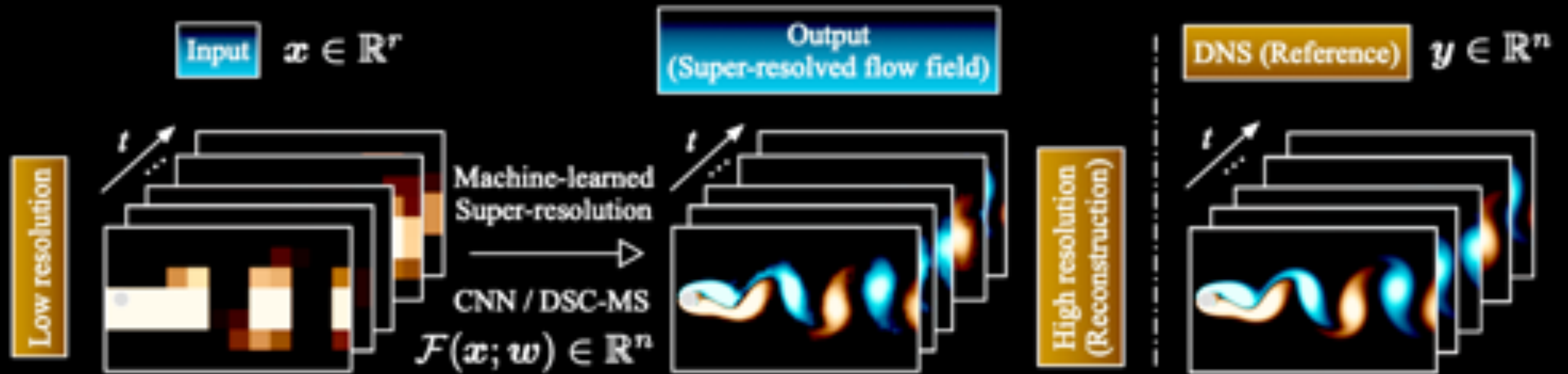


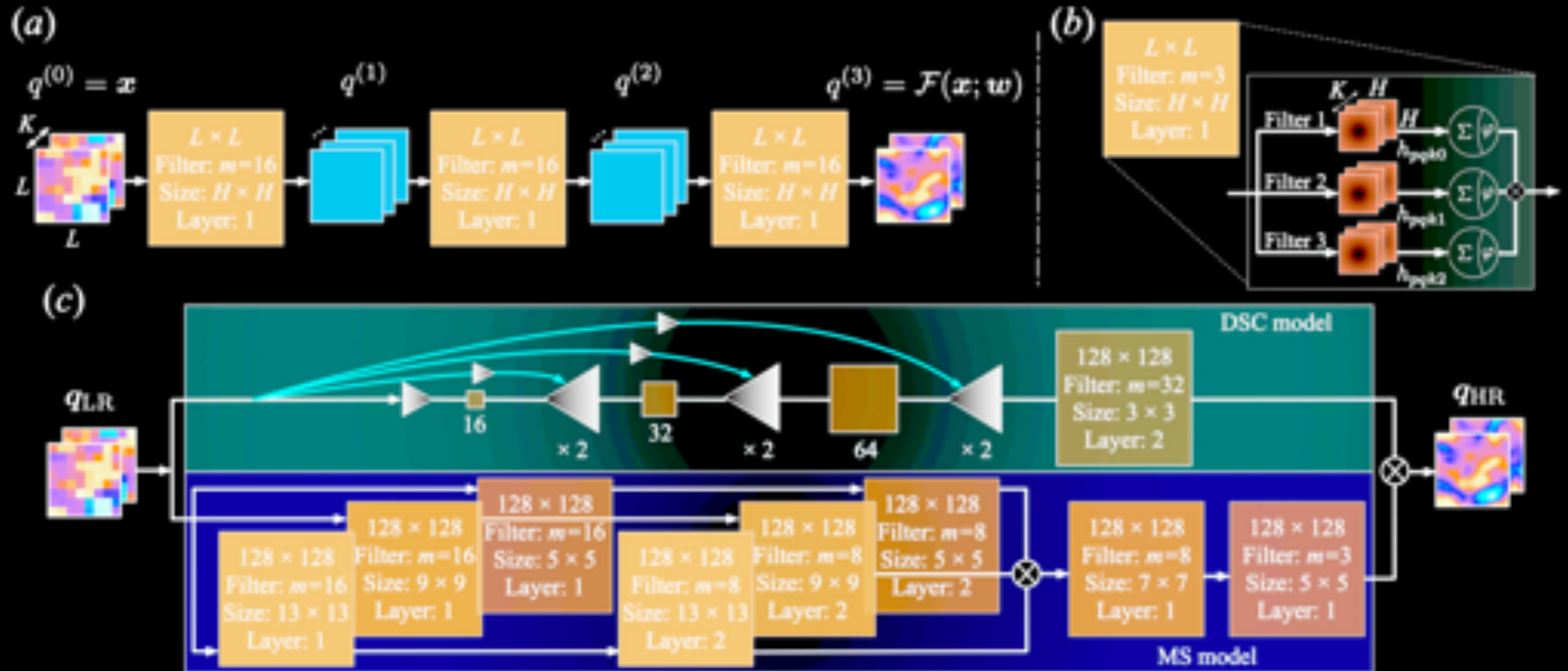
FIGURE 1. An overview of machine-learned super-resolution analysis for cylinder flow.



# SUPER RESOLUTION

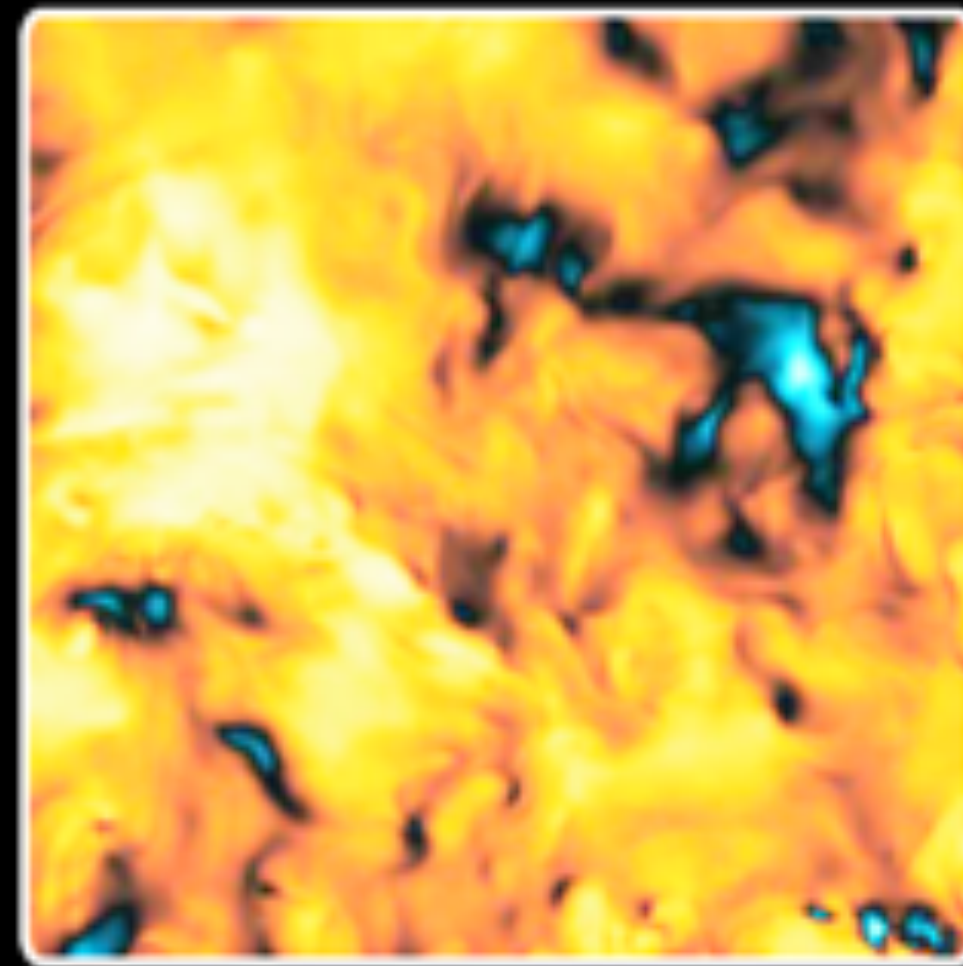
4

*K. Fukami, K. Fukagata and K. Taira*





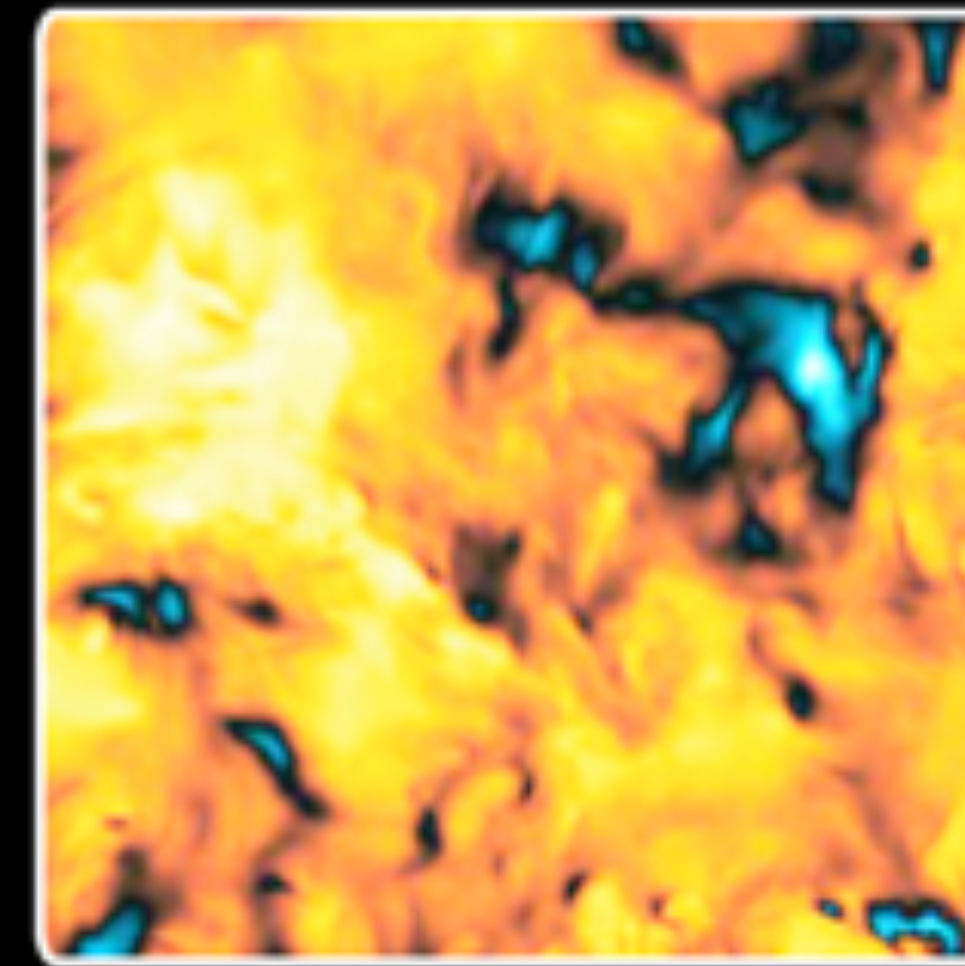
# SUPER RESOLUTION



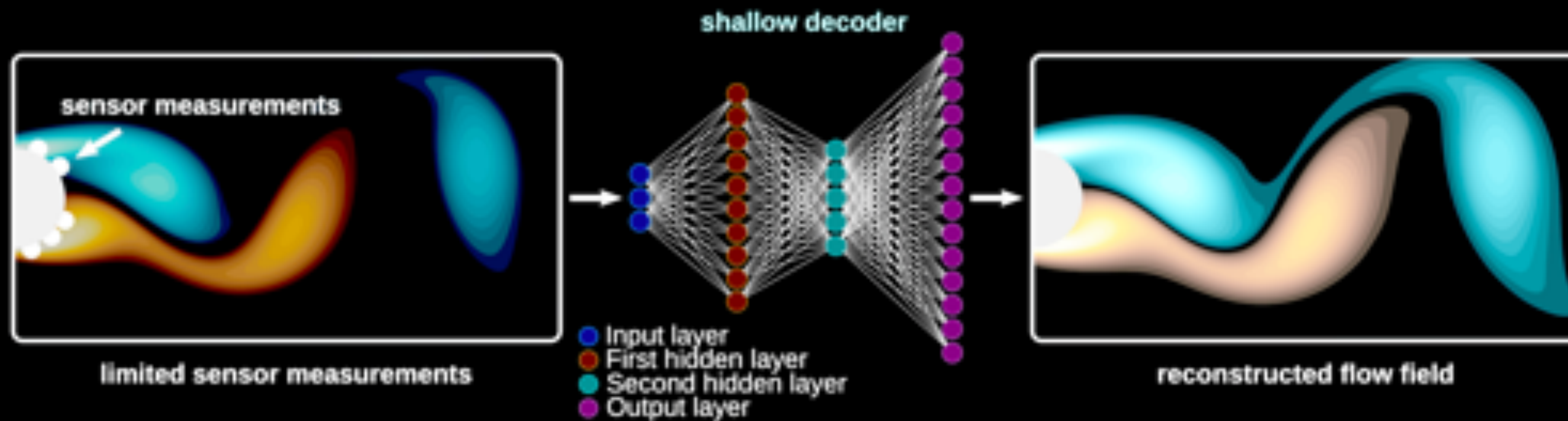
(a) Snapshot



(b) Low resolution



(c) Shallow Decoder





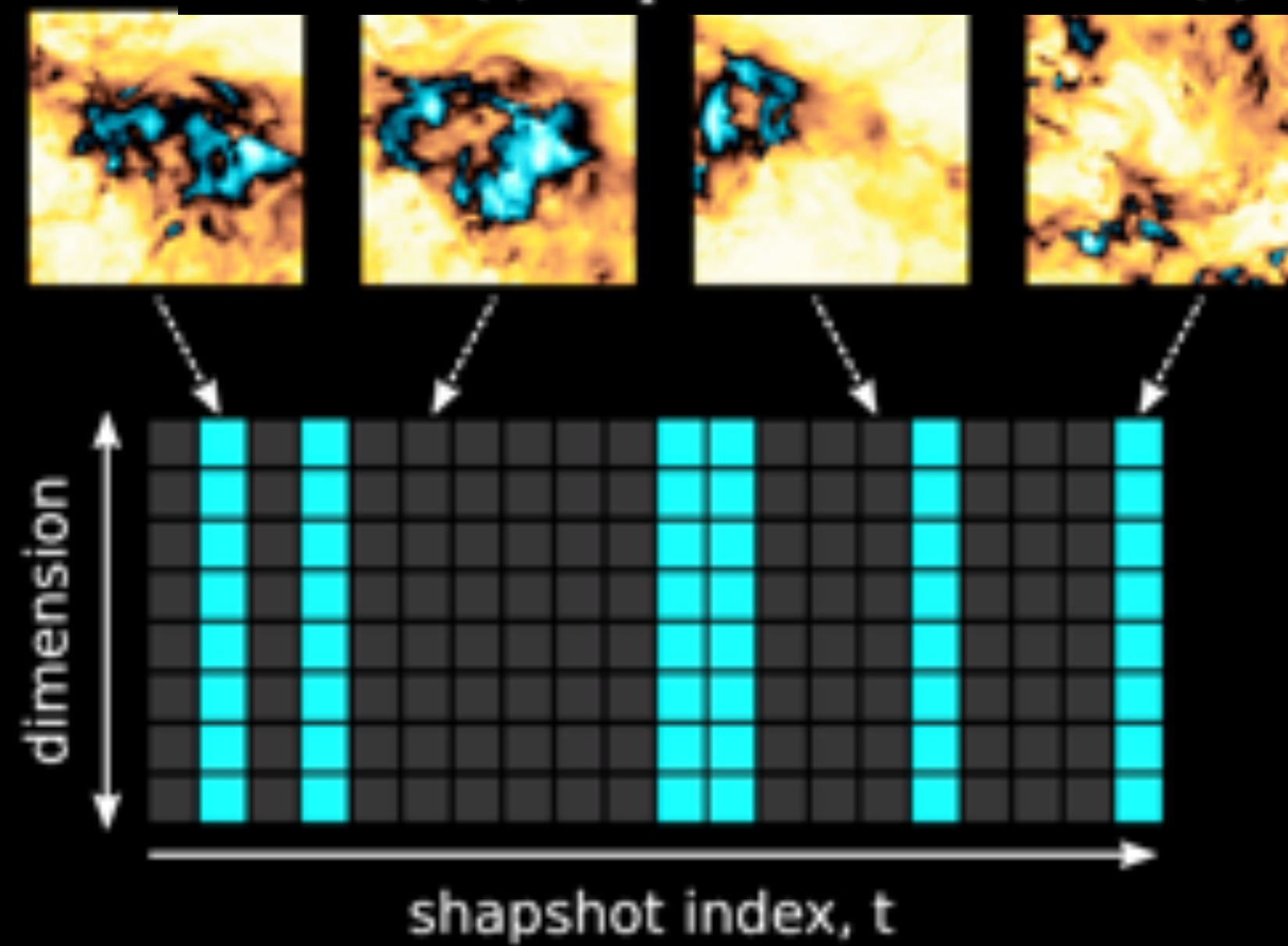
# SUPER RESOLUTION



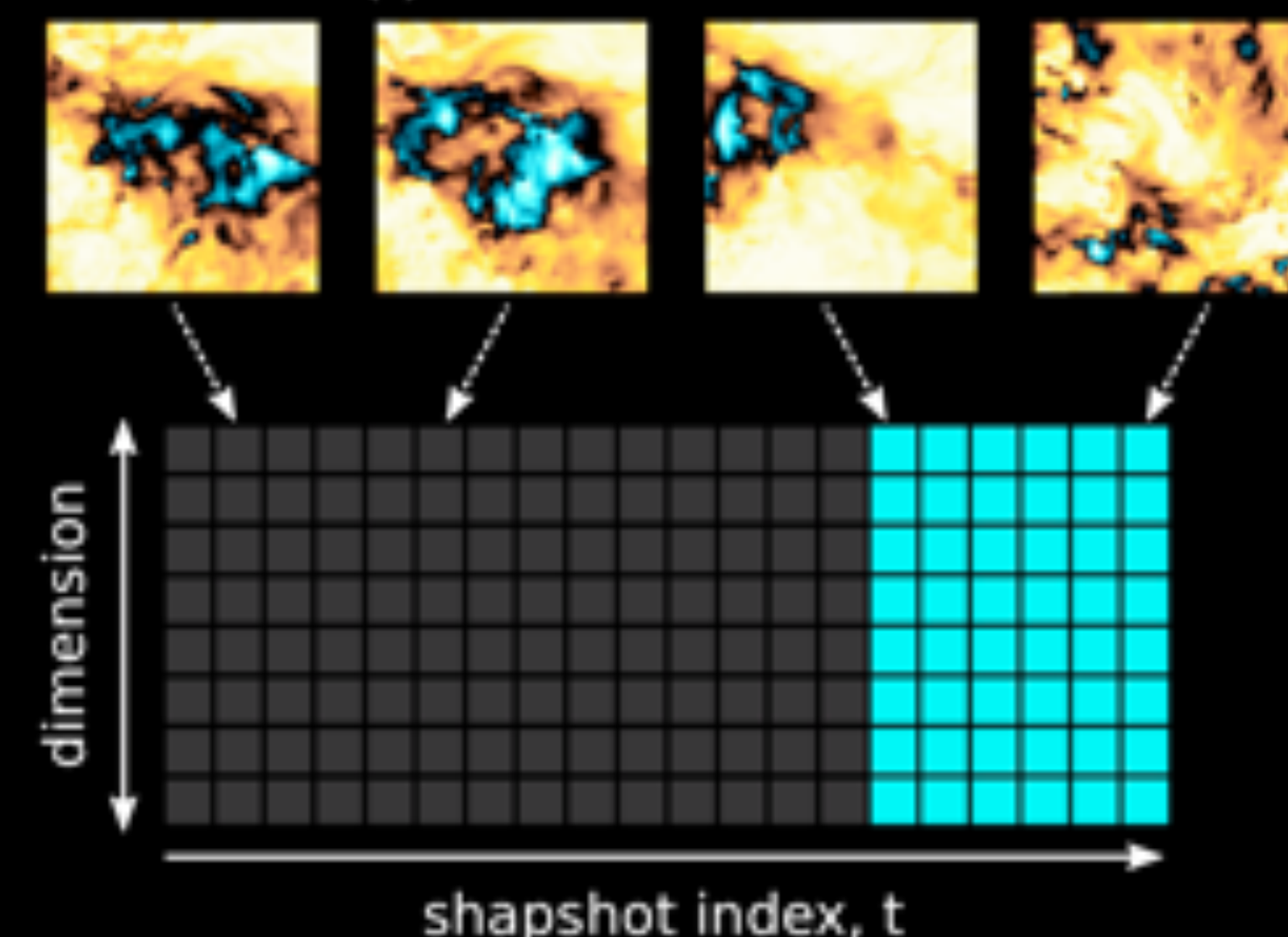
(a) Snapshot

(b) Low resolution

(c) Shallow Decoder



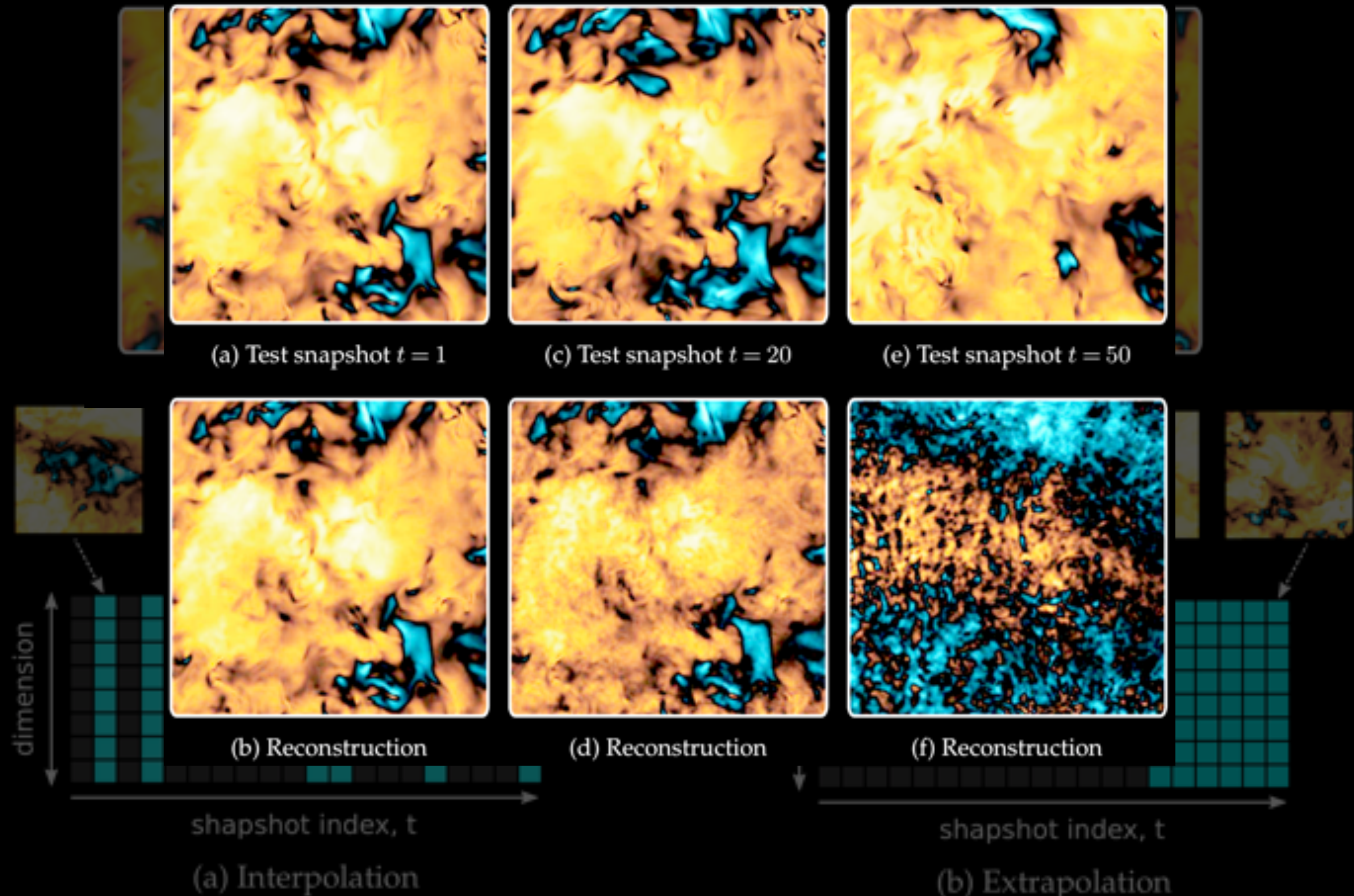
(a) Interpolation



(b) Extrapolation

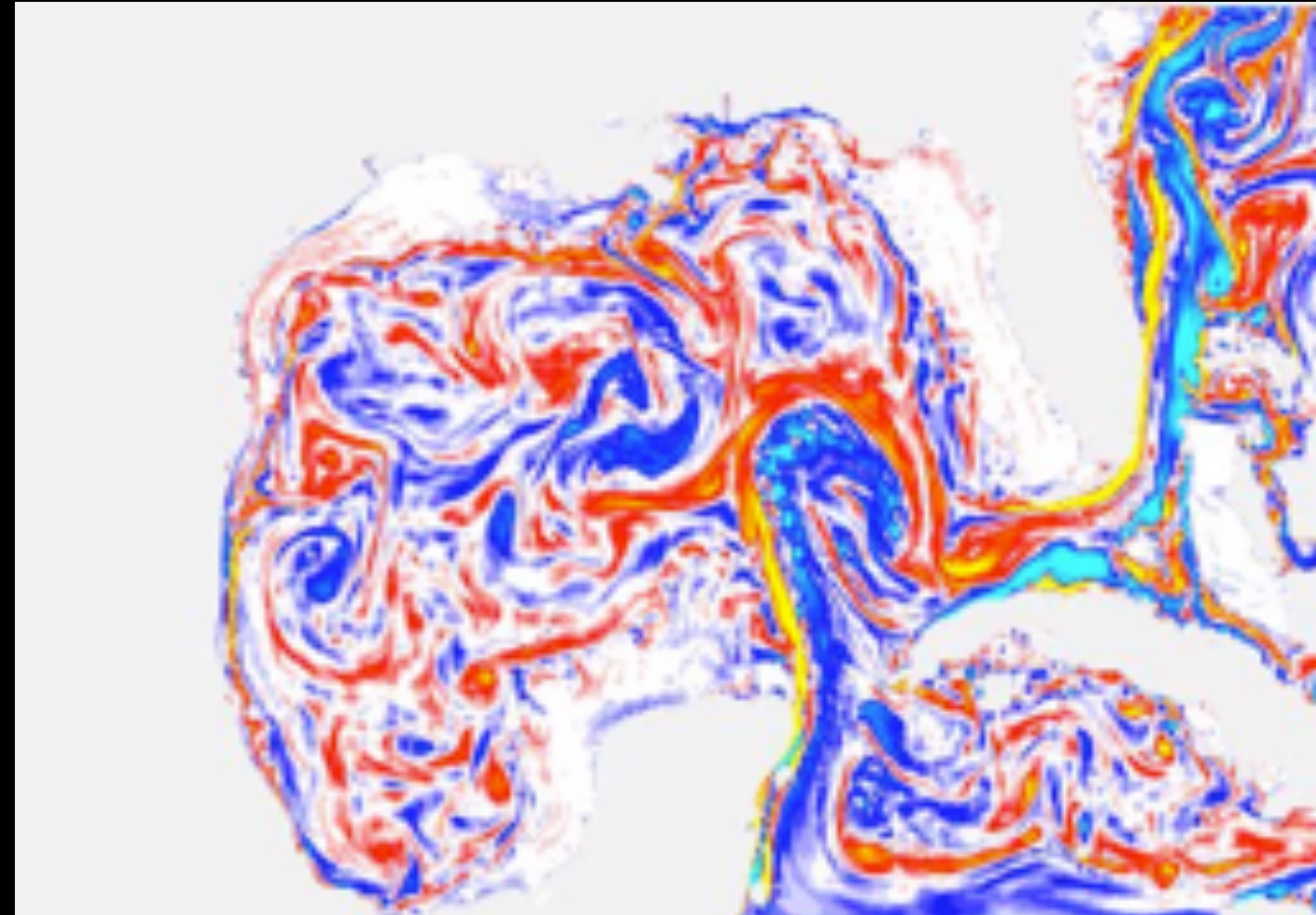
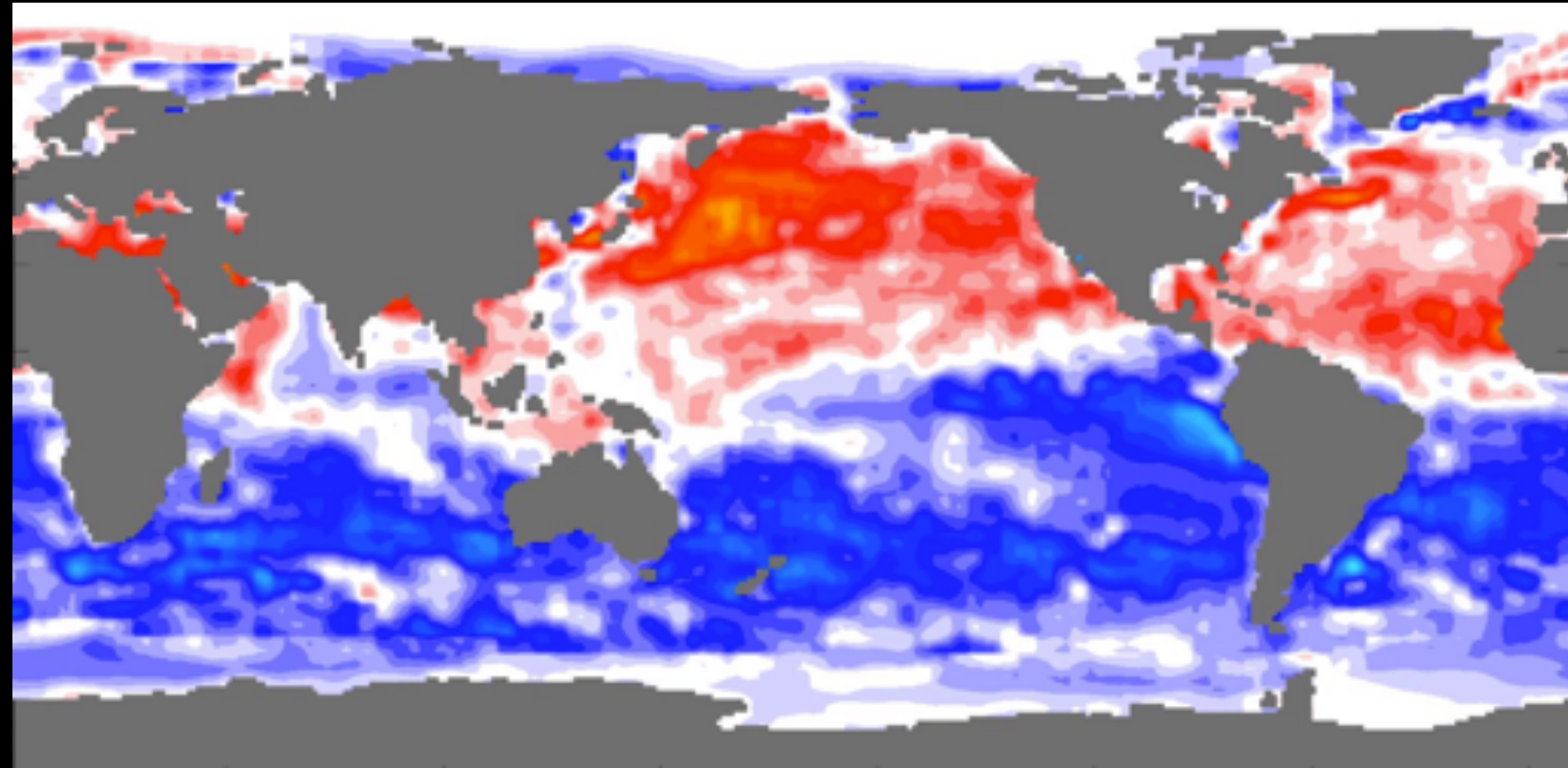
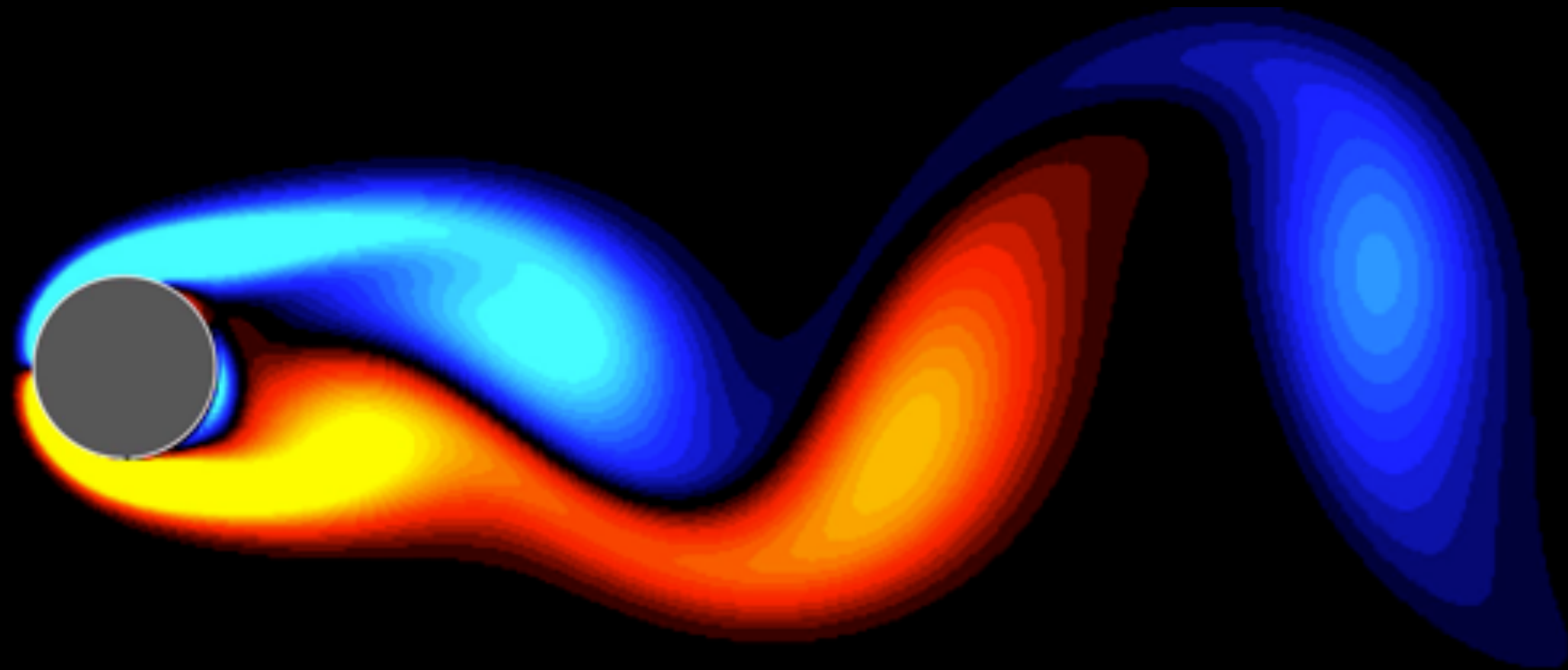


# SUPER RESOLUTION





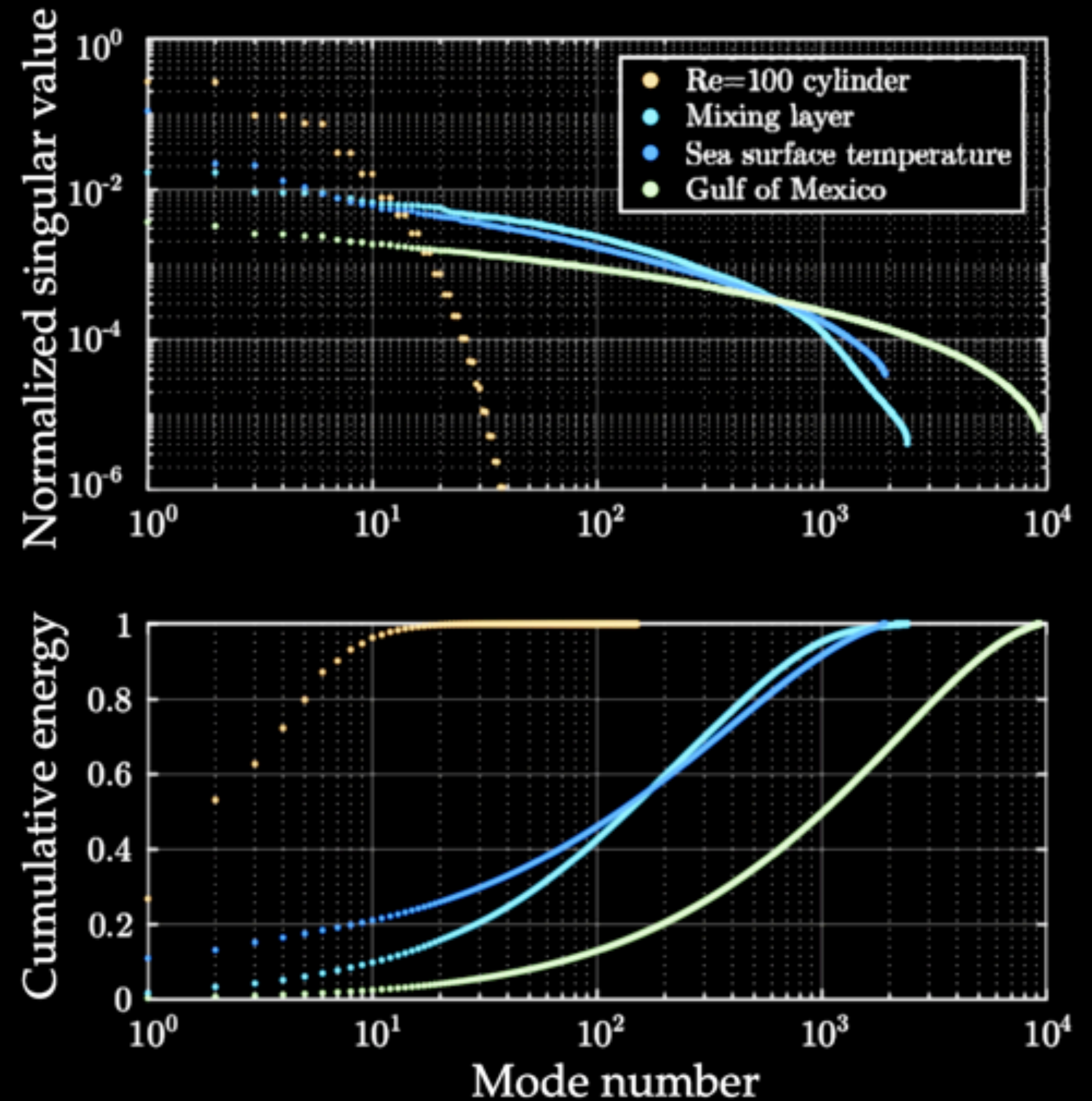
# STATISTICAL STATIONARITY



Callaham, Maeda, SLB, to appear PRF 2019  
[arXiv:1810.06723]

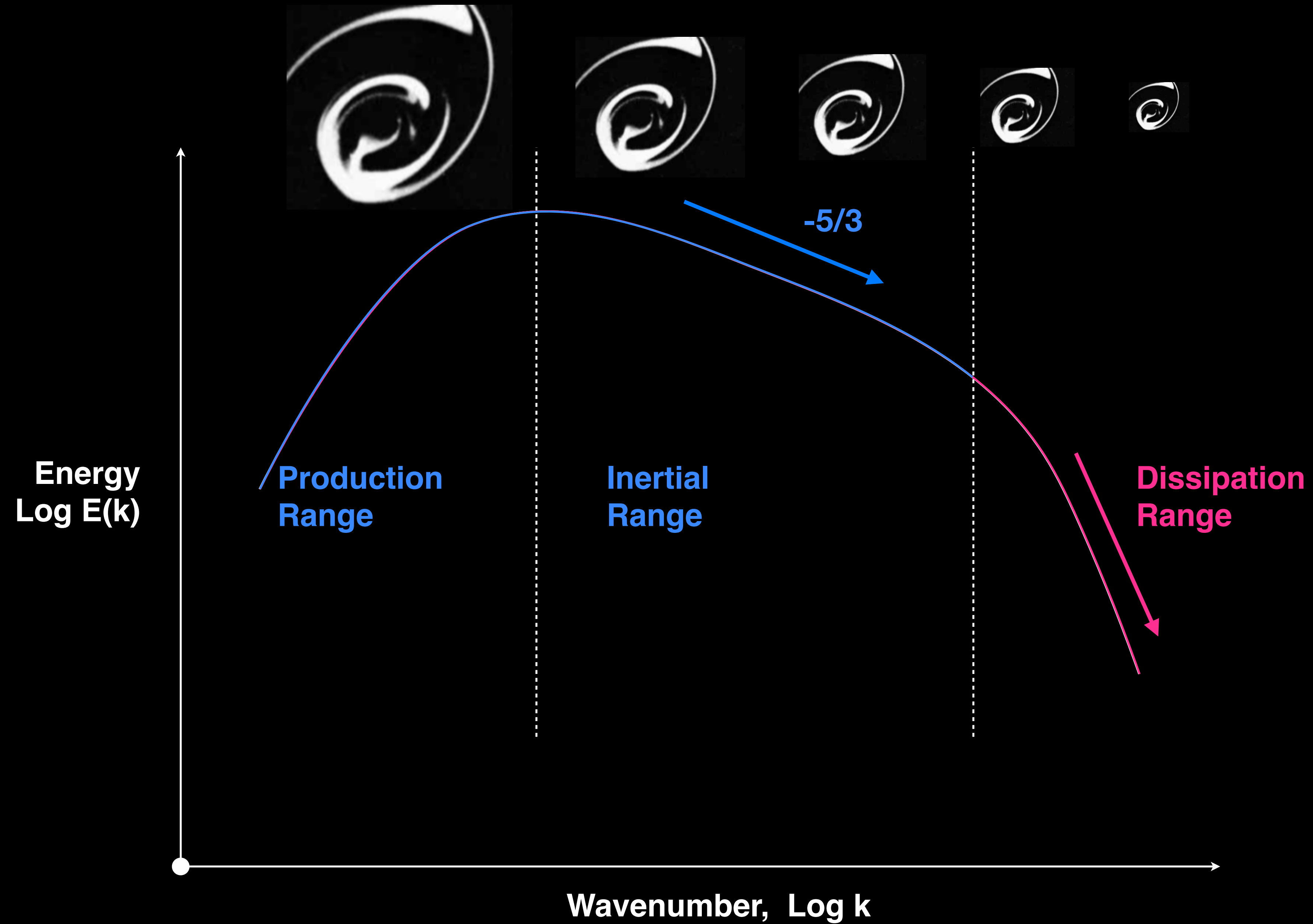


# STATISTICAL STATIONARITY





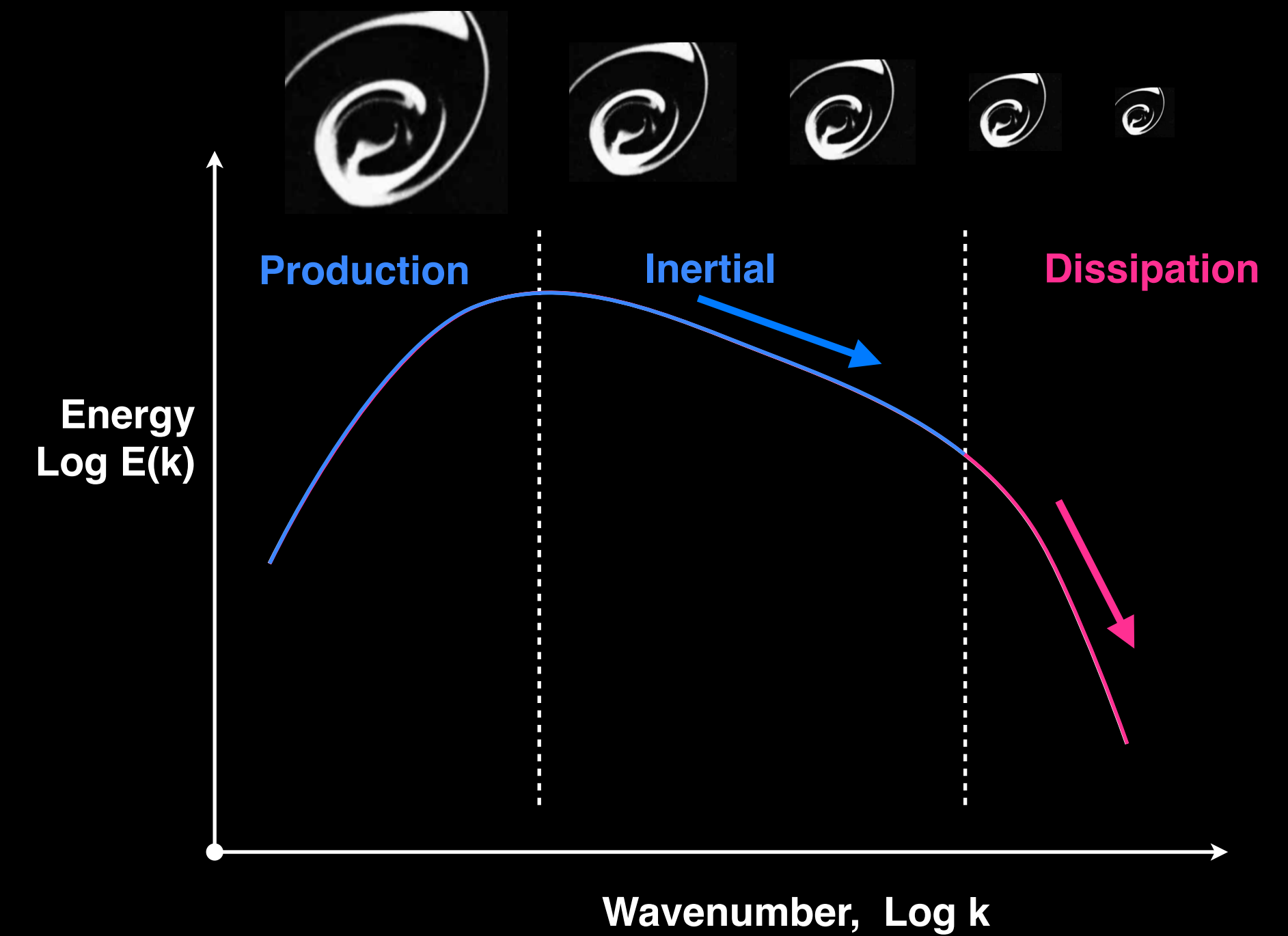
# Kolmogorov Energy Cascade





# Kolmogorov Energy Cascade

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_2, x_3, x_4, \dots) \\ \dot{x}_2 &= f_2(x_1, x_2, x_3, x_4, \dots) \\ \dot{x}_3 &= f_3(x_1, x_2, x_3, x_4, \dots) \\ \dot{x}_4 &= f_4(x_1, x_2, x_3, x_4, \dots) \\ &\vdots\end{aligned}$$

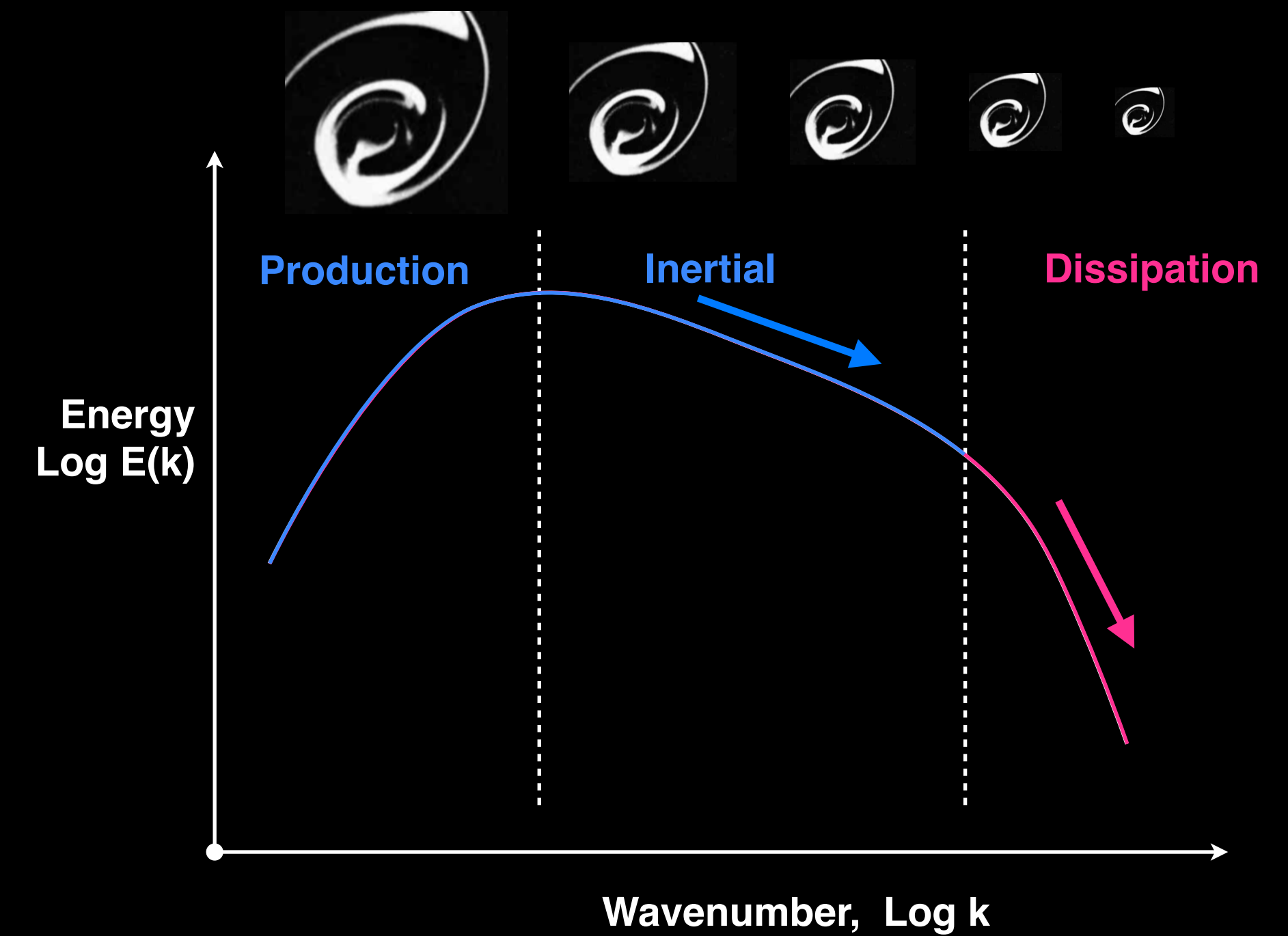


**Can't resolve all scales...**

**... don't want to!**



# Kolmogorov Energy Cascade



$$\dot{\mathbf{x}}_L = f_L(\mathbf{x}_L, \mathbf{x}_H)$$

$$\dot{\mathbf{x}}_H = f_H(\mathbf{x}_L, \mathbf{x}_H)$$

Approximate effect of **fast/small scales**  
on **large scales**

$$\mathbf{x}_H = g(\mathbf{x}_L)$$



$$\dot{\mathbf{x}}_L = f_L(\mathbf{x}_L, g(\mathbf{x}_L))$$

**Closure Model**



# RANS - Reynolds Averaged Navier Stokes

Ling & Templeton 2015,  
Parish & Duraisamy 2016,  
Ling, Kurzawski, Templeton 2016,  
Xiao, Wu, Wang, Sun, Roy 2016,  
Singh, Medida, Duraisamy, 2017,  
Wang, Wu, Xiao, 2017

# LES - Large Eddy Simulation

Maulik, San, Rasheed, Vedula 2019

# Turbulence Modeling in the Age of Data

Karthik Duraisamy<sup>1,†</sup>, Gianluca Iaccarino<sup>2,†</sup>,  
and Heng Xiao<sup>3,†</sup>

<sup>1</sup>Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI 48109; [kdur@umich.edu](mailto:kdur@umich.edu)

<sup>2</sup>Department of Mechanical Engineering, Stanford University, Stanford, CA 94305; [jops@stanford.edu](mailto:jops@stanford.edu)

<sup>3</sup>Kevin T. Crofton Department of Aerospace and Ocean Engineering, Virginia Tech, Blacksburg, VA 24060; [hengxiao@vt.edu](mailto:hengxiao@vt.edu)

<sup>†</sup> Contributed equally. Author list is alphabetical.

Xxxx. Xxx. Xxx. Xxx. YYYY. AA:1-22

[https://doi.org/10.1146/\(\(please add article doi\)\)](https://doi.org/10.1146/((please add article doi)))

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## Keywords

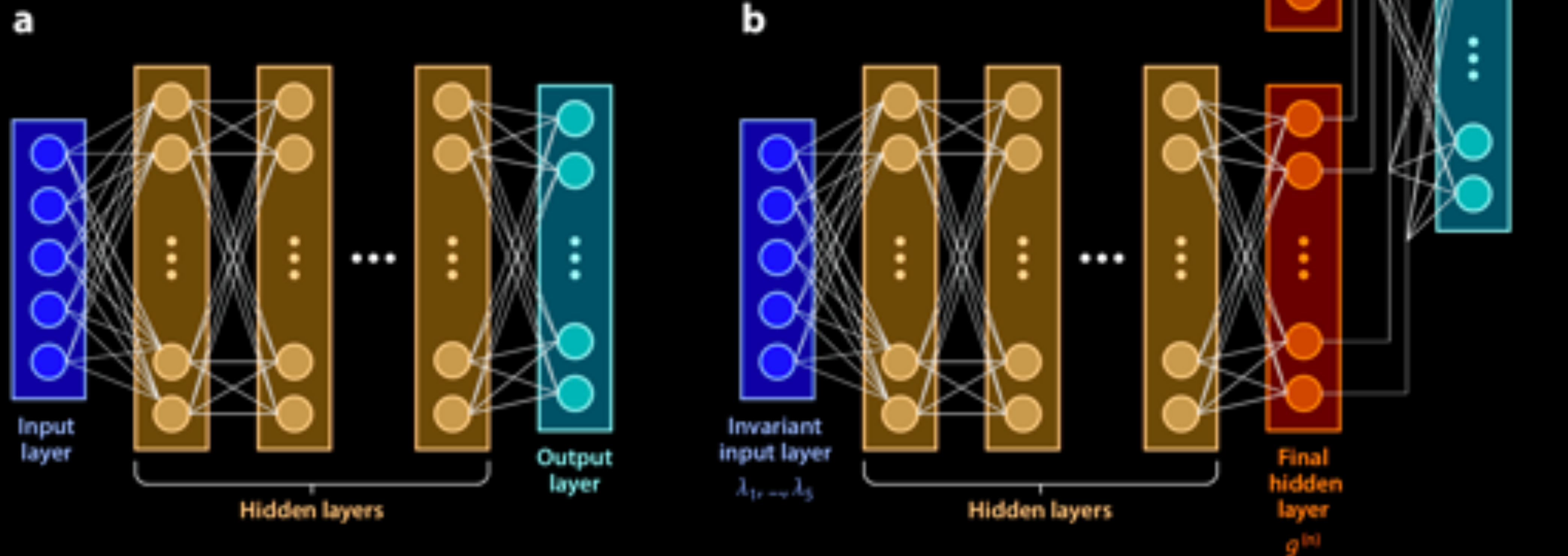
turbulence modeling, statistical inference, machine learning, data-driven modeling, uncertainty quantification

## Abstract

Data from experiments and direct simulations of turbulence have historically been used to calibrate simple engineering models such as those based on the Reynolds-averaged Navier–Stokes (RANS) equations. In the past few years, with the availability of large and diverse datasets, researchers have begun to explore methods to *systematically inform* turbulence models with data, with the goal of quantifying and reducing model uncertainties. This review surveys recent developments in bounding uncertainties in RANS models via physical constraints, in adopting statistical inference to characterize model coefficients and estimate discrepancy, and in using machine learning to improve turbulence models. Key principles, achievements and challenges are discussed. A central perspective advocated in this review is that by exploiting foundational knowledge in turbulence modeling and physical constraints, data-driven approaches can yield useful predictive models.

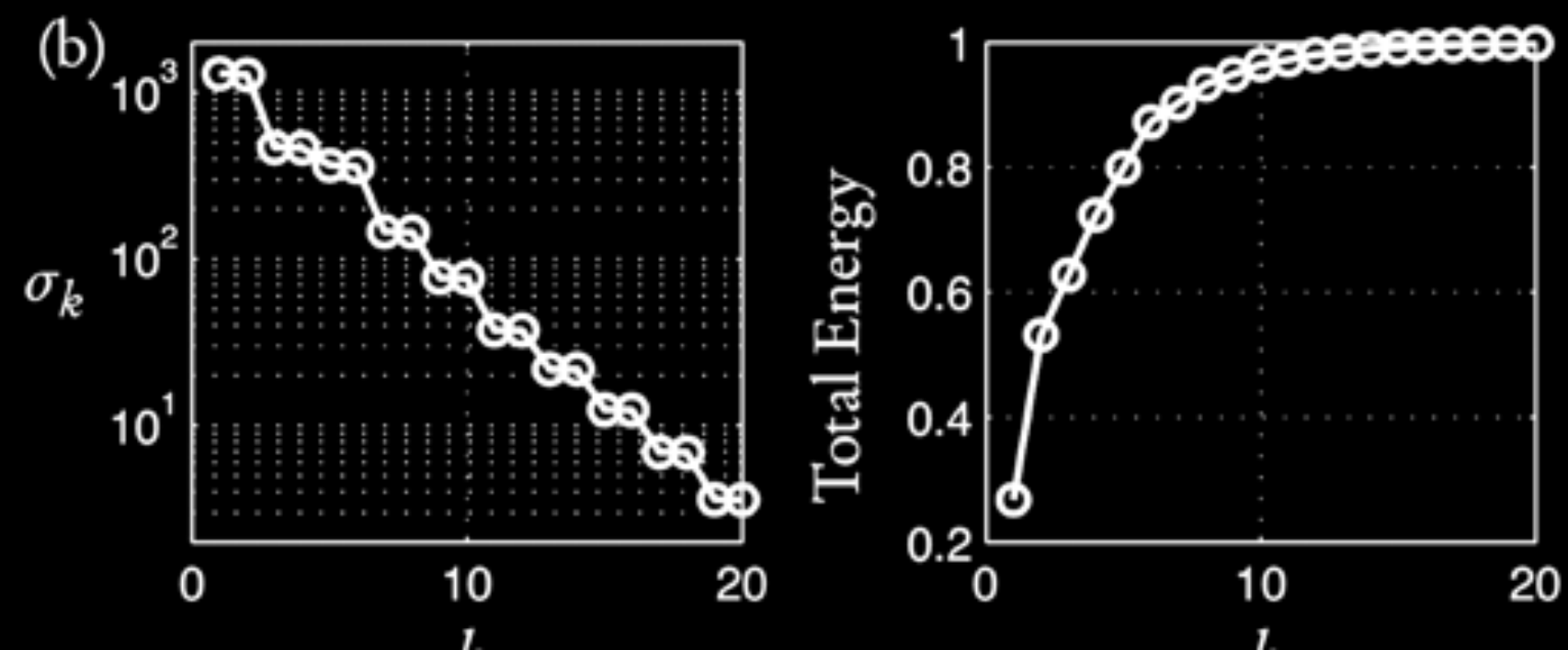
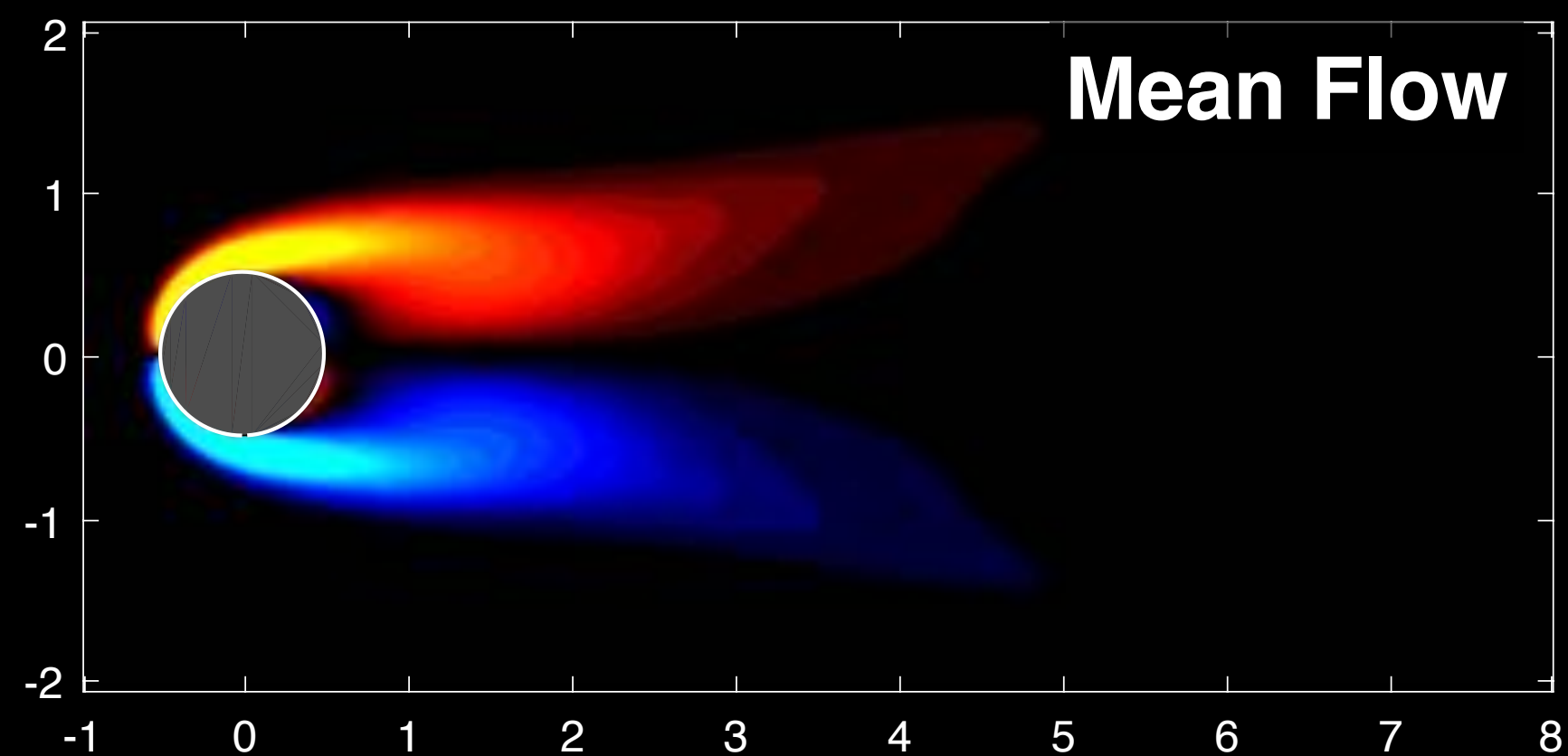
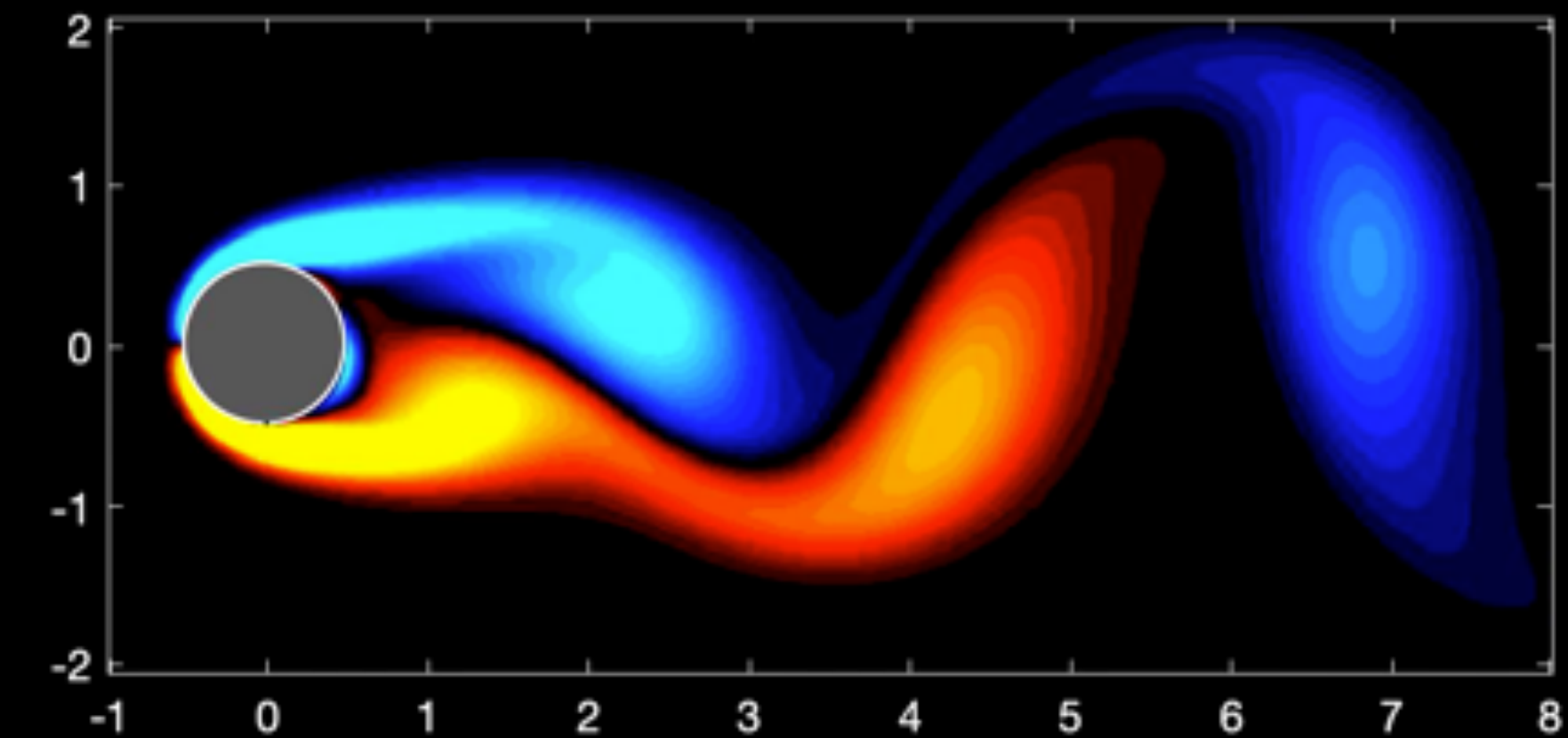


# RANS CLOSURE MODELS

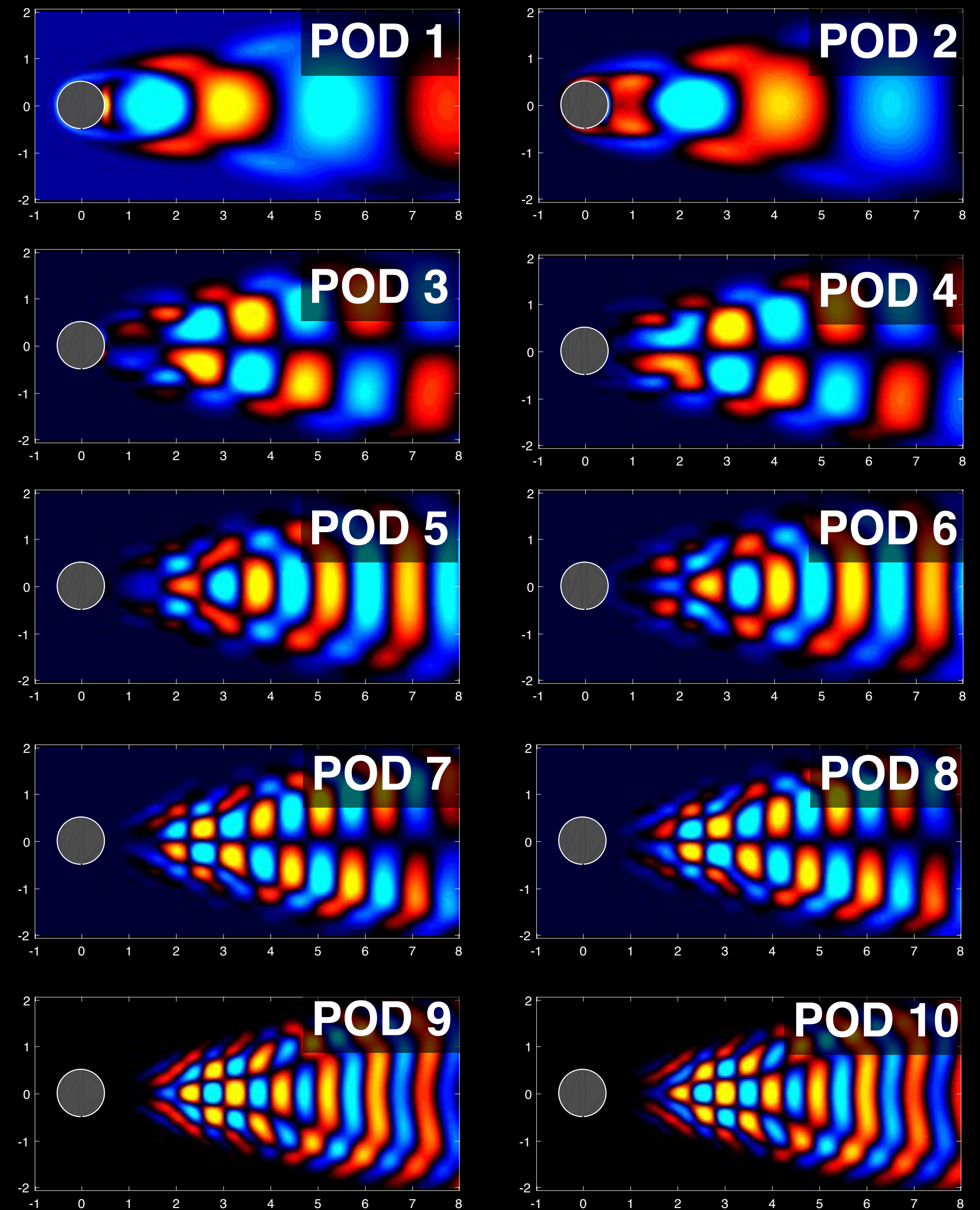




# POD/PCA

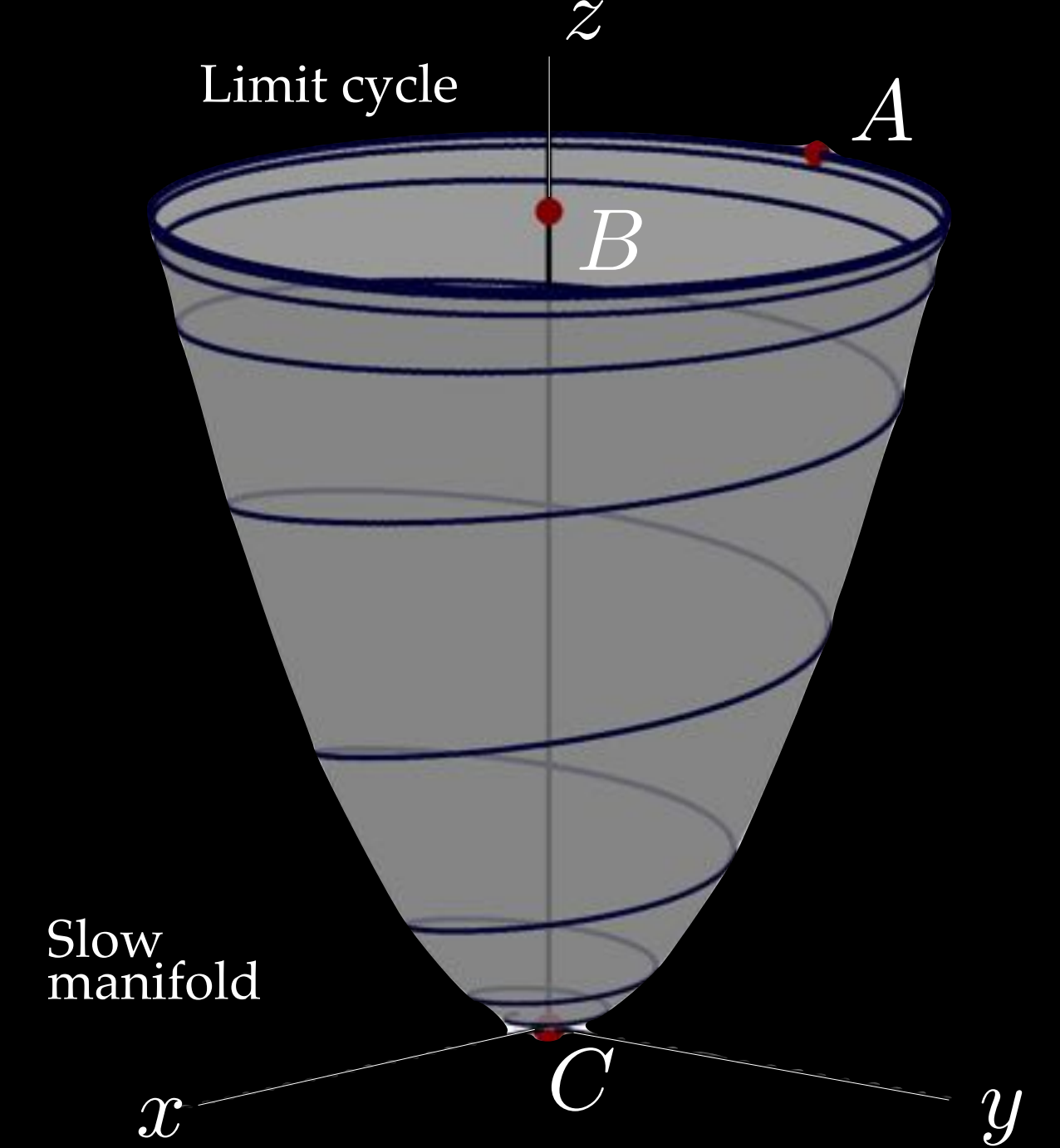


$$\mathbf{u}(\mathbf{x}, t) \approx \bar{\mathbf{u}} + \sum_{k=1}^r \varphi_k(\mathbf{x}) \mathbf{a}_k(t)$$

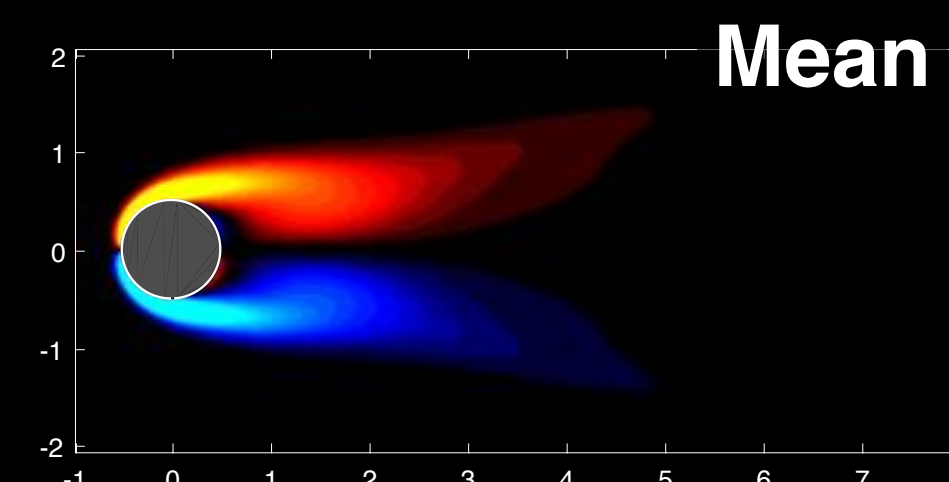
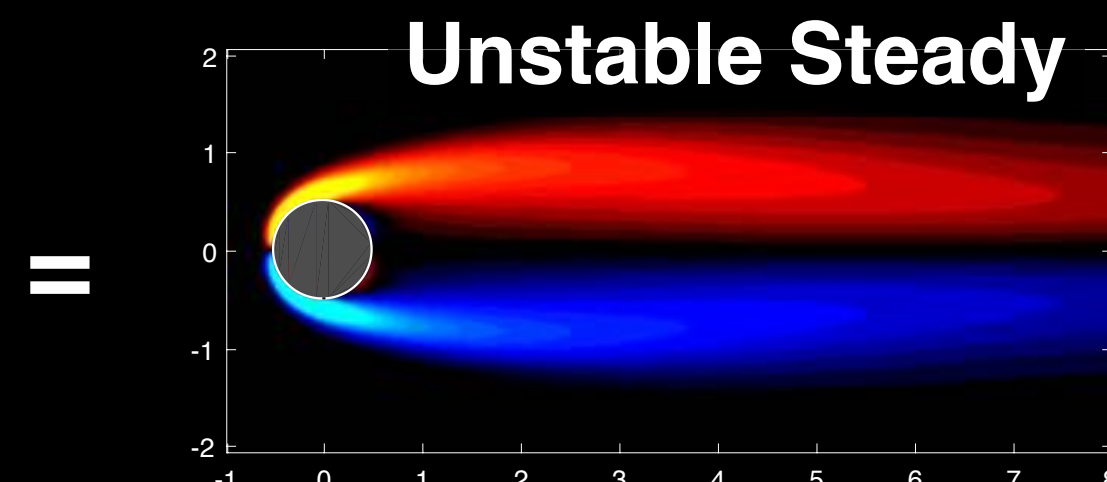
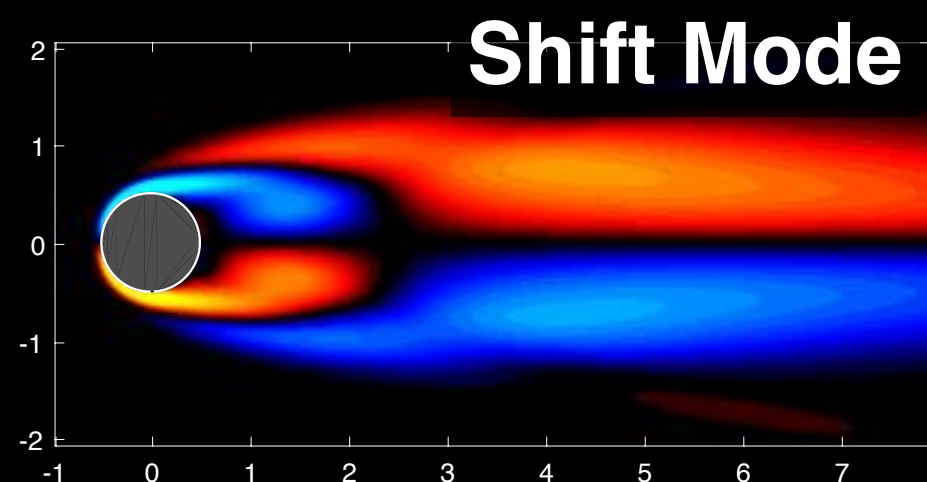
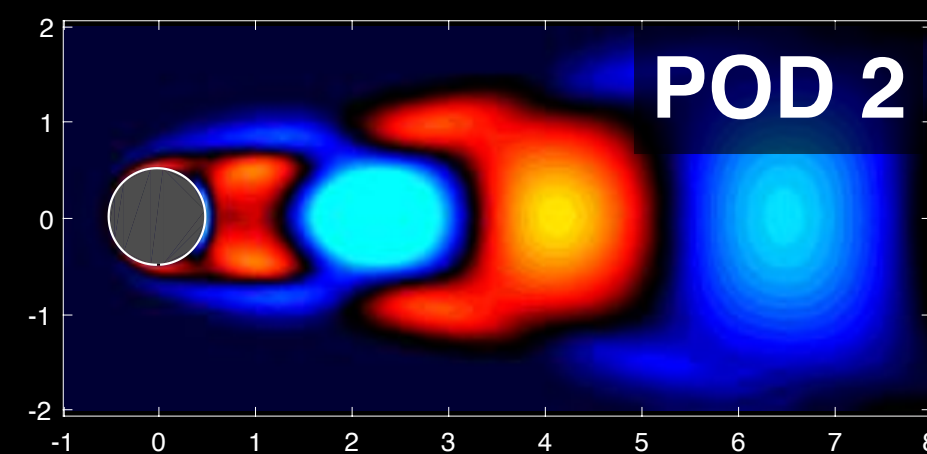
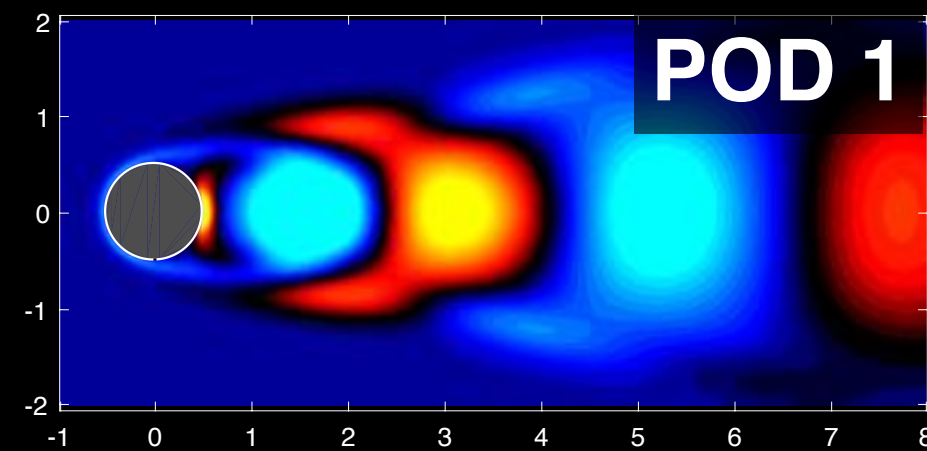
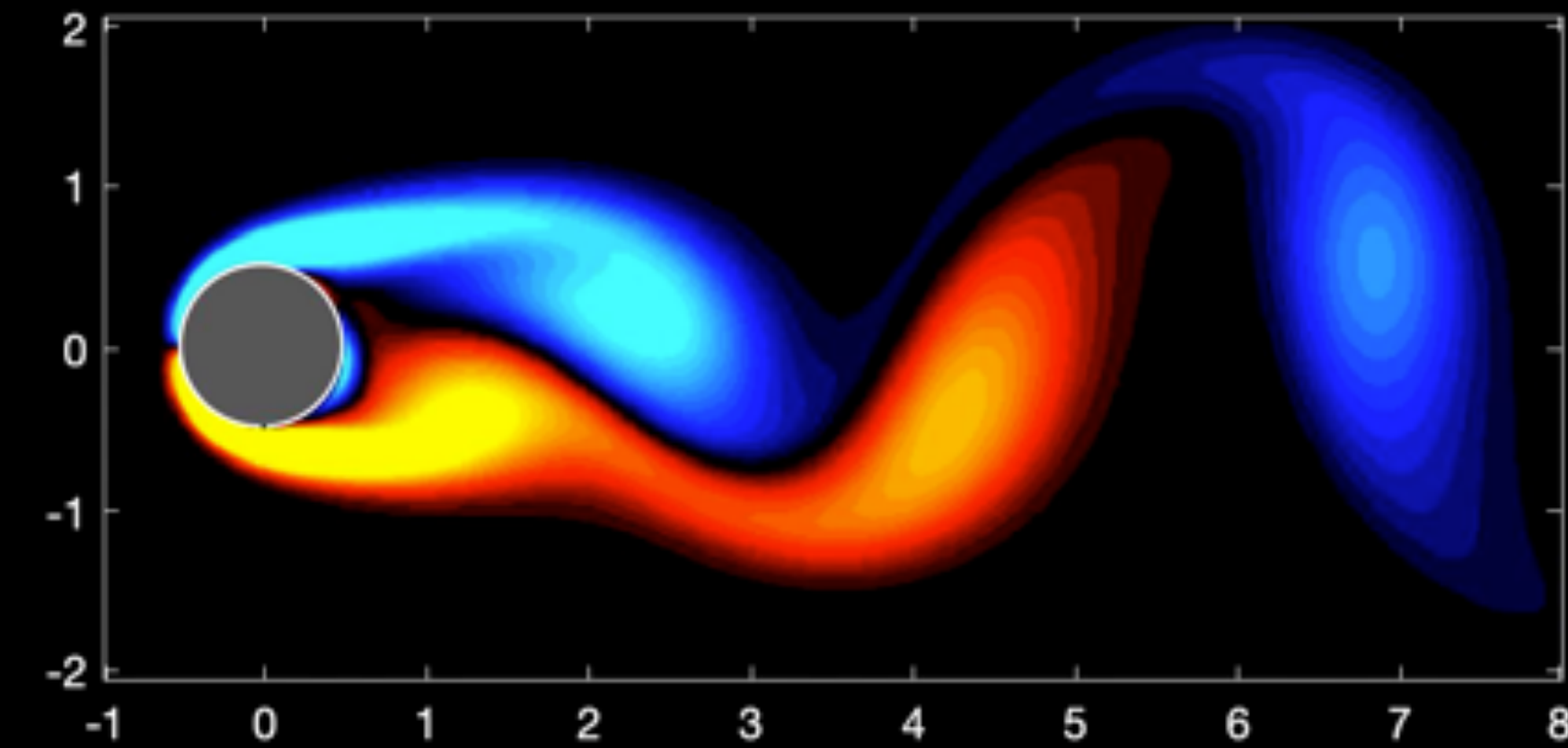




# REDUCED ORDER MODELS



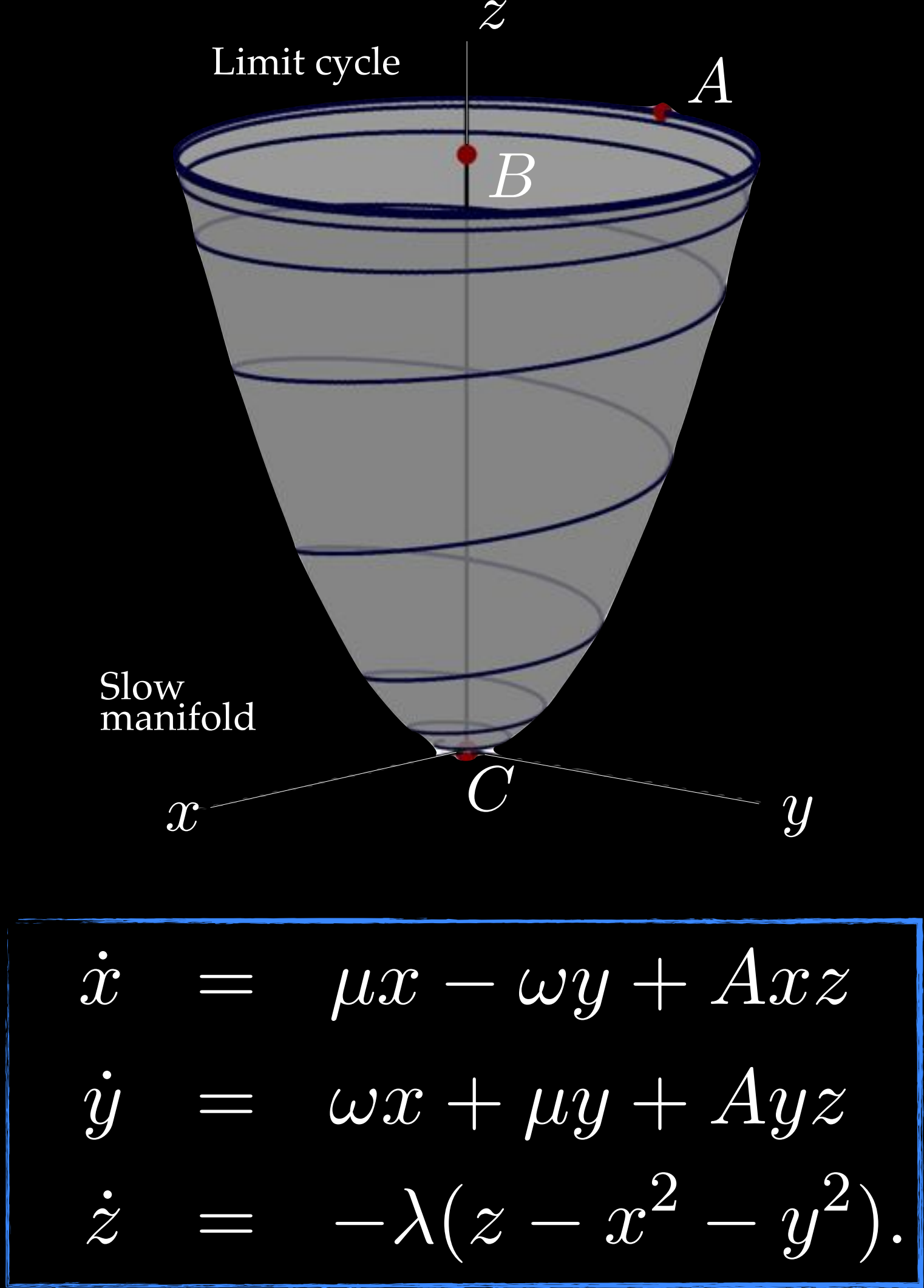
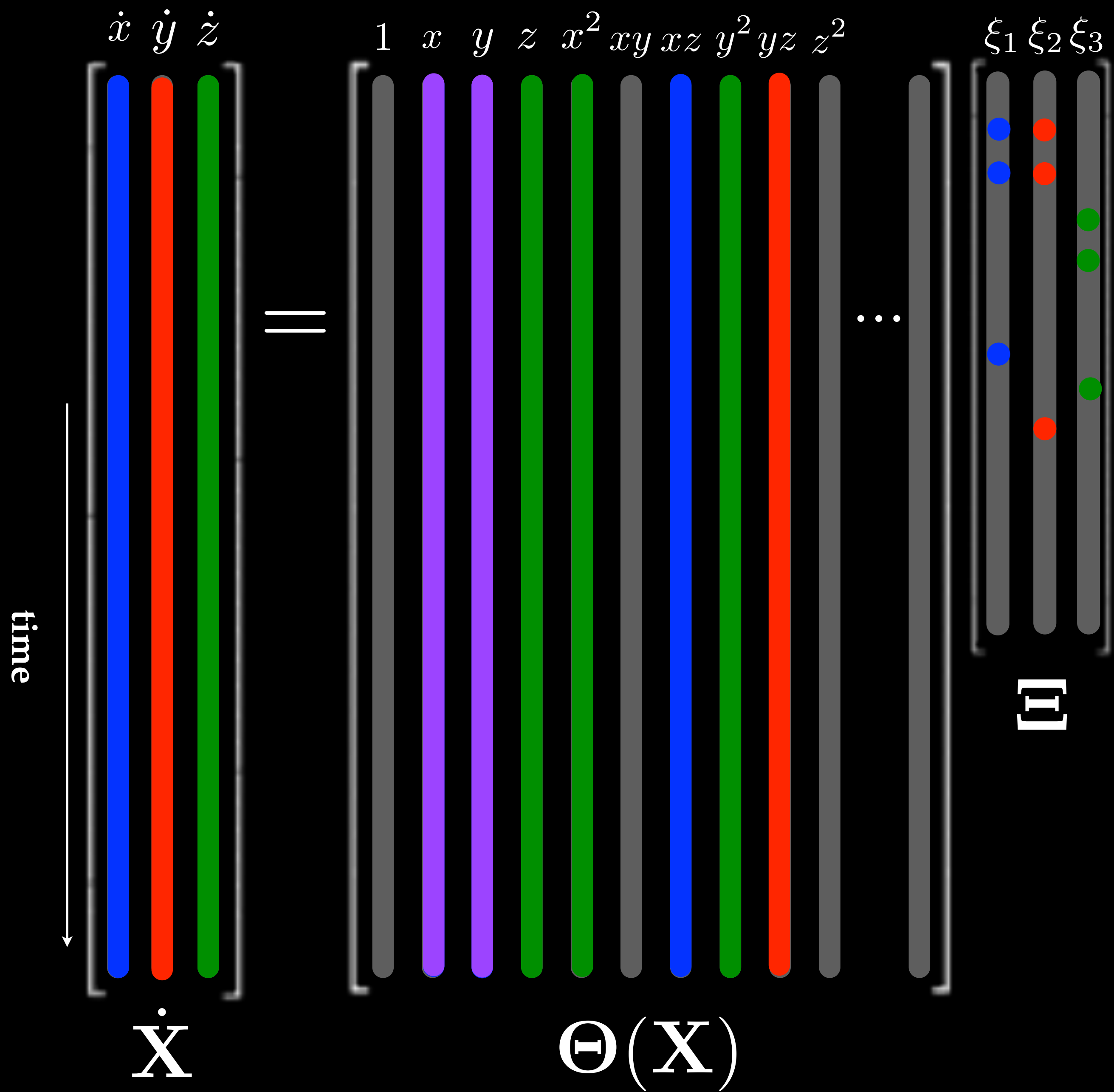
$$\begin{aligned}\dot{x} &= \mu x - \omega y + Axz \\ \dot{y} &= \omega x + \mu y + Ayz \\ \dot{z} &= -\lambda(z - x^2 - y^2).\end{aligned}$$



Ruelle and Takens, 1971  
Noack et al., JFM 2003.

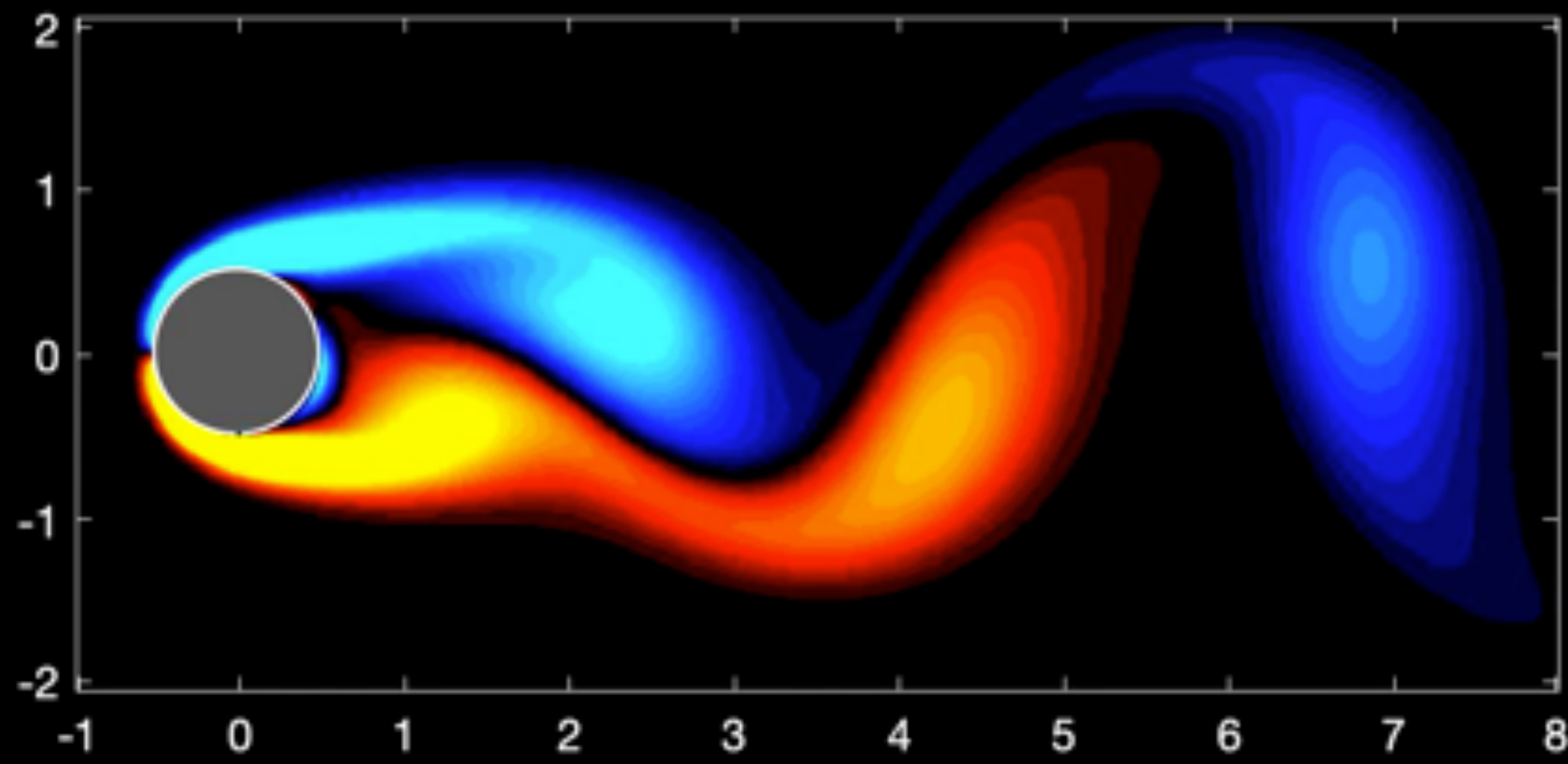
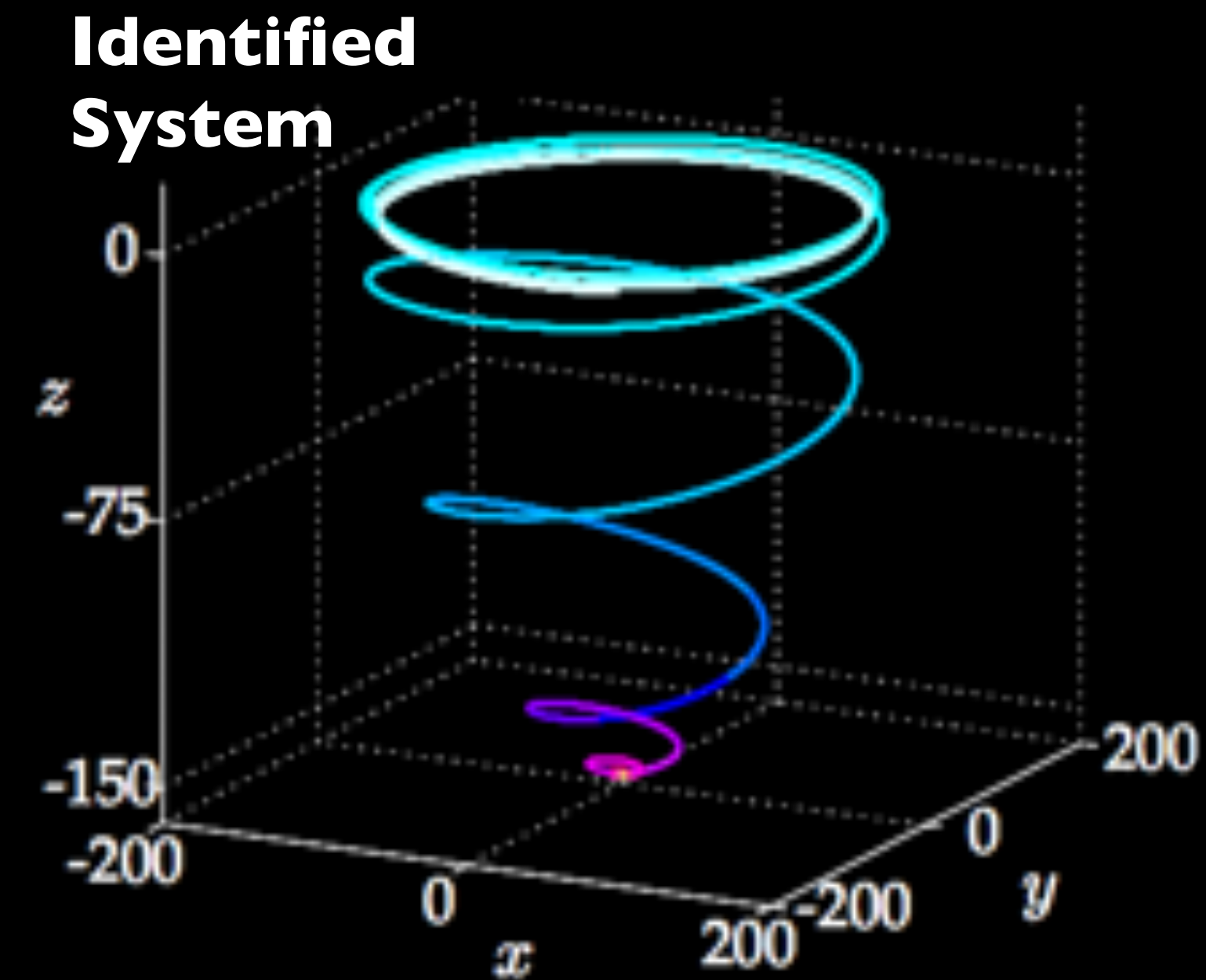
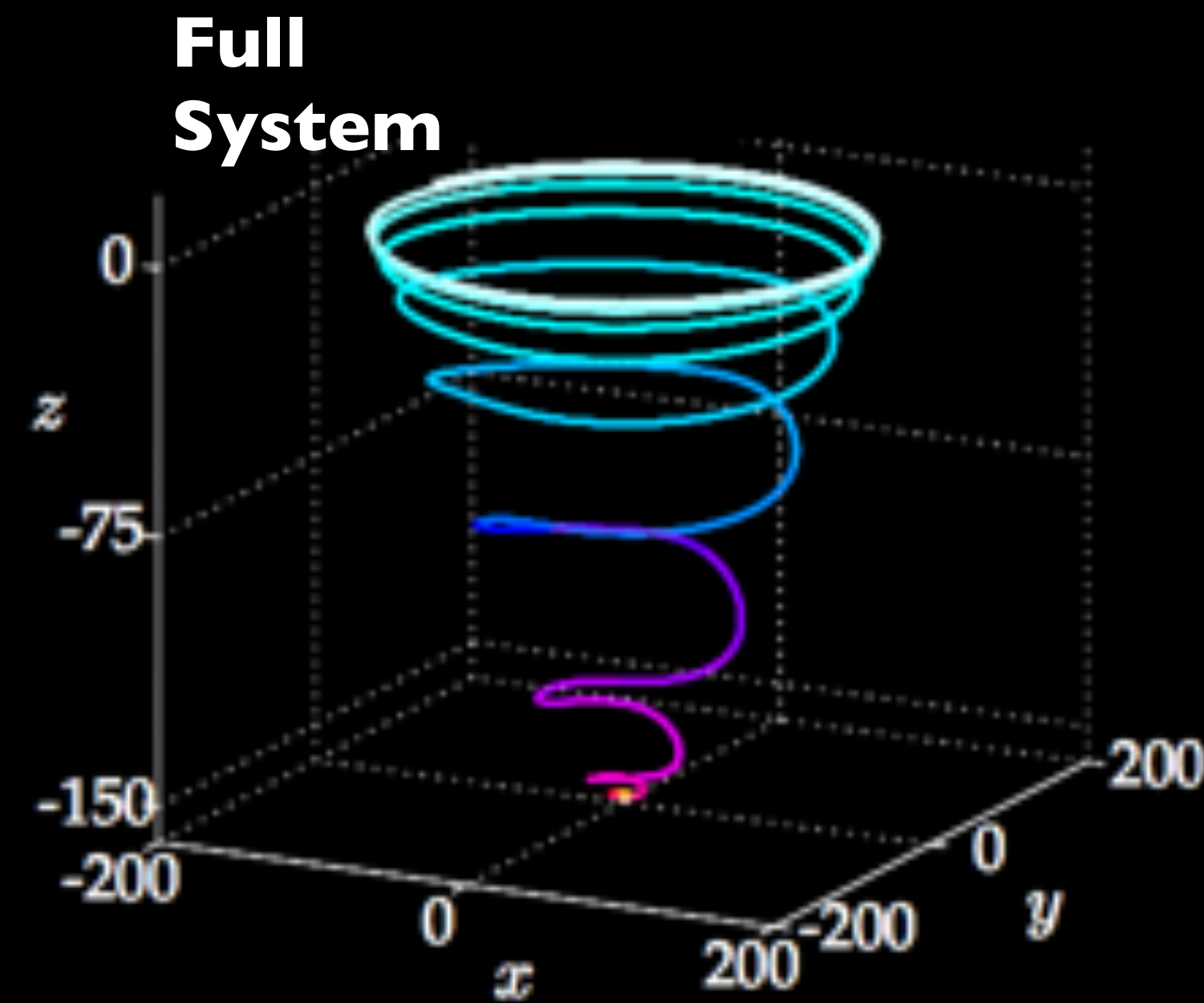
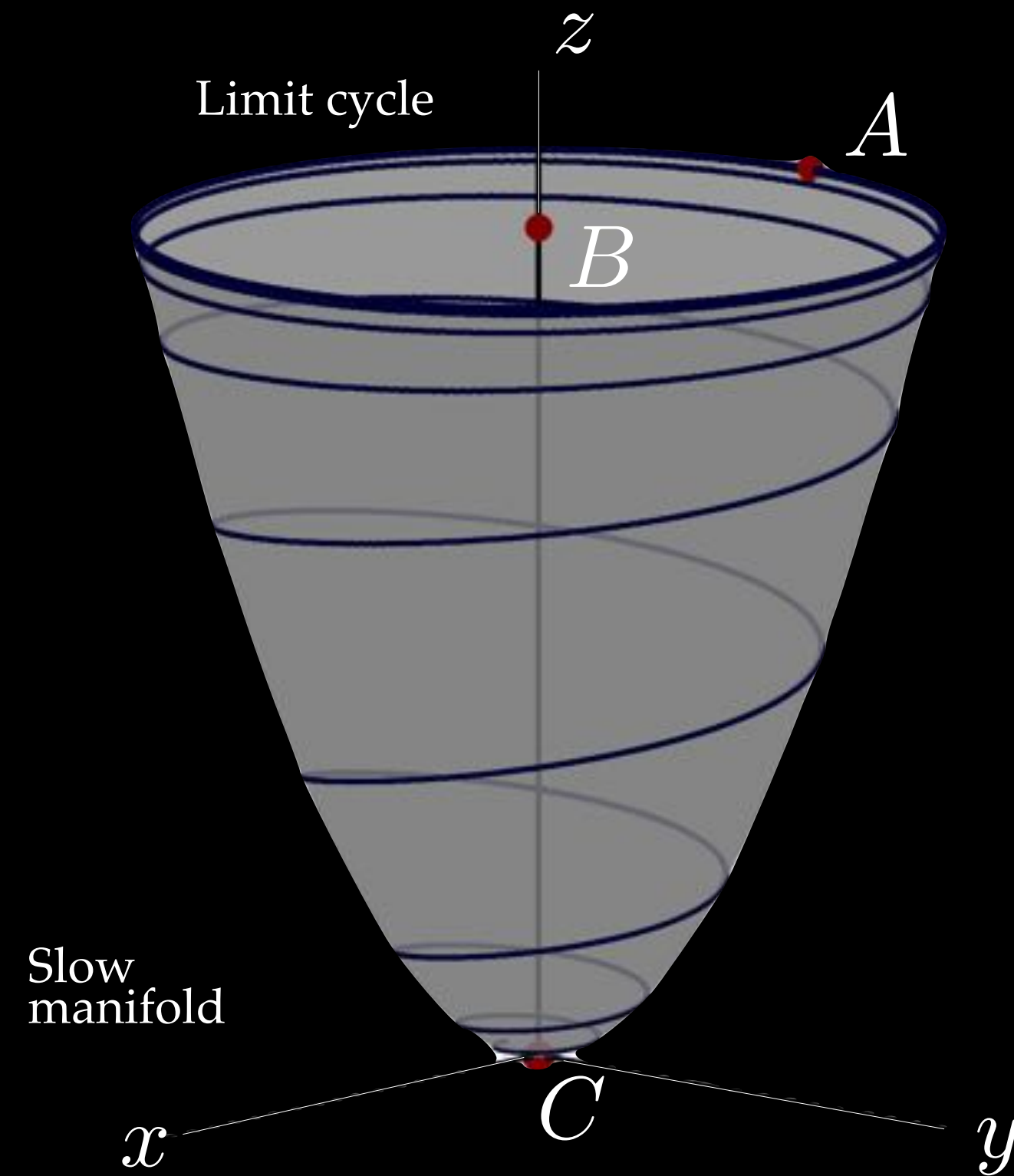


# Sparse Identification of Nonlinear Dynamics (SINDy)





# Constrained Sparse Galerkin Regression



$$\begin{aligned}\dot{x} &= \mu x - \omega y + Axz \\ \dot{y} &= \omega x + \mu y + Ayz \\ \dot{z} &= -\lambda(z - x^2 - y^2).\end{aligned}$$



# Constrained Sparse Galerkin Regression

## Innovation 1: Enforcing known constraints

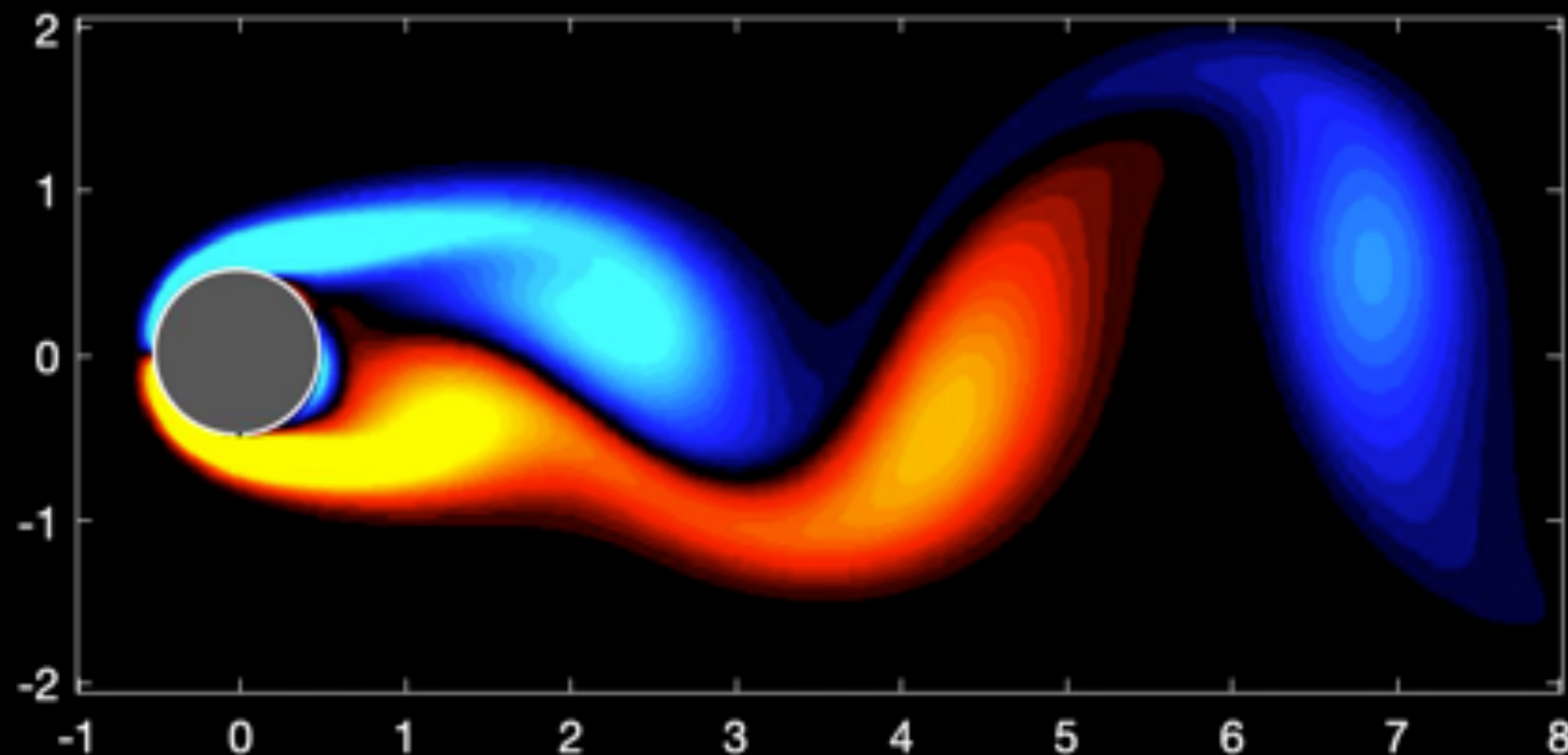
- ▶ Skew-symmetric quadratic nonlinearities to enforce energy conservation
- ▶ Improved stability

$$\min_{\xi, z} \|\Theta(\mathbf{X})\mathbf{E} - \dot{\mathbf{X}}\|_2^2 + \mathbf{z}^T(\mathbf{C}\xi - \mathbf{d})$$

## Innovation 2: Higher-order Nonlinearities

- ▶ Cubic, Quintic, Septic terms approximate truncated terms in Galerkin expansion

$$\begin{aligned}\dot{x} &= \mu x - \omega y + Axz \\ \dot{y} &= \omega x + \mu y + Ayz \\ \dot{z} &= -\lambda(z - x^2 - y^2).\end{aligned}$$



$$\begin{aligned}\dot{x} &= \mu x - \omega y + Axz \\ \dot{y} &= \omega x + \mu y + Ayz \\ \dot{z} &= -\lambda(z - x^2 - y^2).\end{aligned}$$



# Constrained Sparse Galerkin Regression

Innovation 1: Enforcing  
known constraints

$$\int_{\Omega} \mathbf{u} \cdot (\mathbf{u} \cdot \nabla) \mathbf{u} \, d\Omega = 0 \quad \longrightarrow \quad \mathbf{a} \cdot \mathcal{N}(\mathbf{a}) = 0$$

- Skew-symmetric quadratic nonlinearities to enforce energy conservation
- Improved stability

$$0 = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} \xi_4^{(a_1)} a_1 & \xi_5^{(a_1)} a_1 + \xi_7^{(a_1)} a_2 & \xi_6^{(a_1)} a_1 + \xi_9^{(a_1)} a_3 \\ \xi_4^{(a_2)} a_1 + \xi_5^{(a_2)} a_2 & \xi_7^{(a_2)} a_2 & \xi_8^{(a_2)} a_2 + \xi_9^{(a_2)} a_3 \\ \xi_4^{(a_3)} a_1 + \xi_6^{(a_3)} a_3 & \xi_7^{(a_3)} a_2 + \xi_8^{(a_3)} a_3 & \xi_9^{(a_3)} a_3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \\ + \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} \xi_8^{(a_1)} a_2 a_3 \\ \xi_6^{(a_2)} a_1 a_3 \\ \xi_5^{(a_3)} a_1 a_2 \end{bmatrix}.$$



# Constrained Sparse Galerkin Regression

Innovation 1: Enforcing  
known constraints

$$\int_{\Omega} \mathbf{u} \cdot (\mathbf{u} \cdot \nabla) \mathbf{u} \, d\Omega = 0 \quad \longrightarrow \quad \mathbf{a} \cdot \mathcal{N}(\mathbf{a}) = 0$$

- Skew-symmetric quadratic nonlinearities to enforce energy conservation
- Improved stability

$$\min_{\xi, z} \|\hat{\Theta}(\mathbf{X})\mathbf{E} - \dot{\mathbf{X}}\|_2^2 + \mathbf{z}^T (\mathbf{C}\xi - \mathbf{d})$$

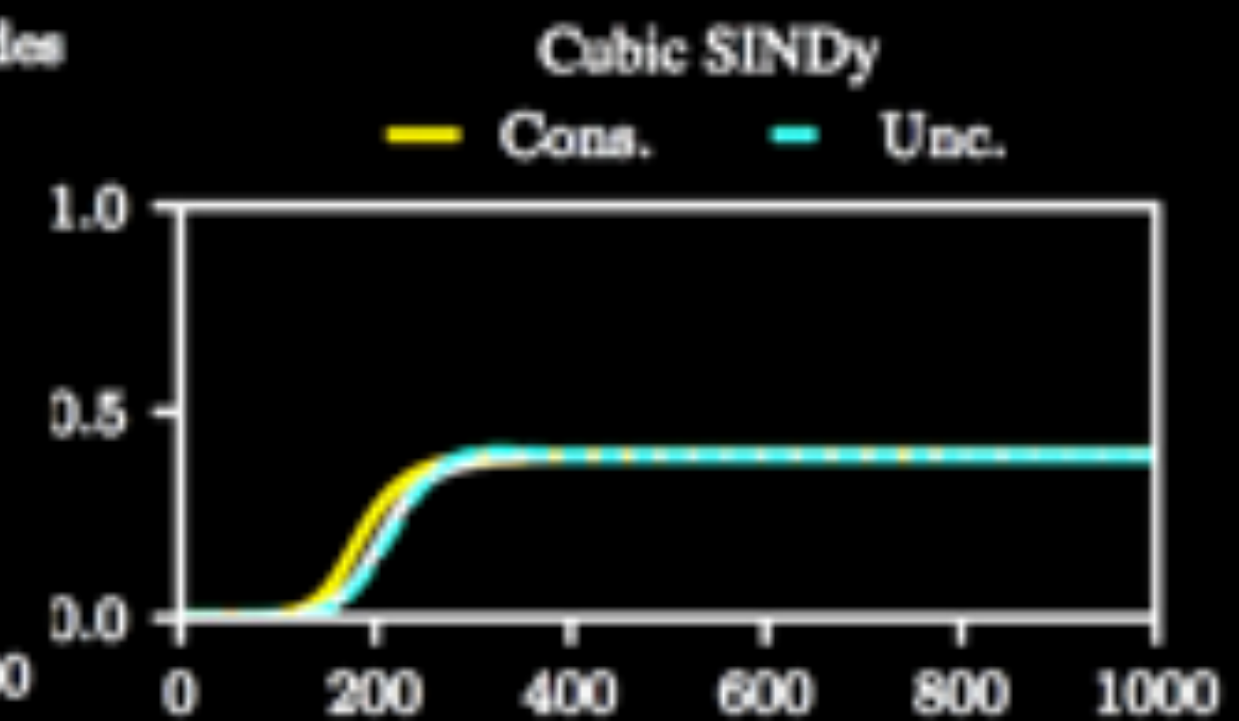
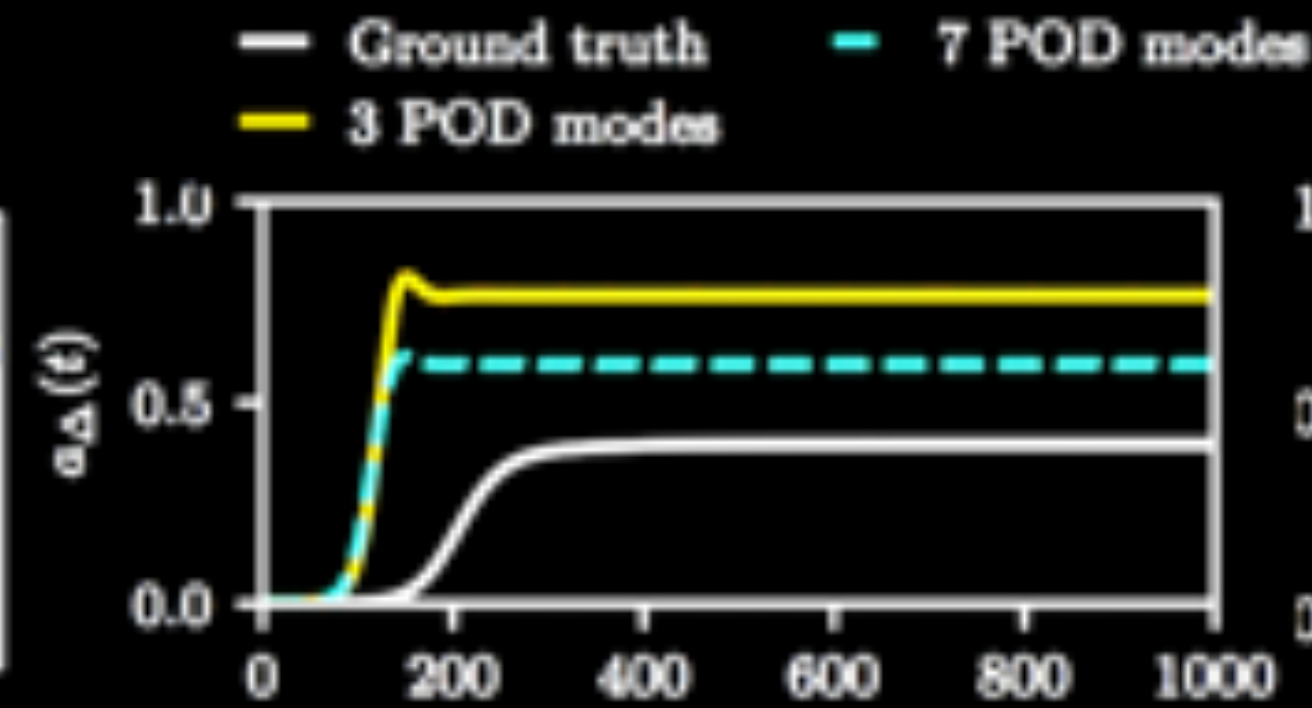
$$\begin{bmatrix} 2\hat{\Theta}(\mathbf{X})^T \hat{\Theta}(\mathbf{X}) & \mathbf{C}^T \\ \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \xi \\ z \end{bmatrix} = \begin{bmatrix} 2\hat{\Theta}(\mathbf{X})^T \dot{\mathbf{X}}(:) \\ \mathbf{d} \end{bmatrix}$$

$$\left. \begin{aligned} \xi_8^{(a_1)} + \xi_6^{(a_2)} + \xi_5^{(a_3)} &= 0, \\ \xi_4^{(a_1)} = \xi_7^{(a_2)} = \xi_9^{(a_3)} &= 0, \\ \xi_5^{(a_1)} &= -\xi_4^{(a_2)}, \\ \xi_7^{(a_1)} &= -\xi_5^{(a_2)}, \\ \xi_6^{(a_1)} &= -\xi_4^{(a_3)}, \\ \xi_9^{(a_1)} &= -\xi_6^{(a_3)}, \\ \xi_8^{(a_2)} &= -\xi_7^{(a_3)}, \\ \xi_9^{(a_2)} &= -\xi_8^{(a_3)}, \end{aligned} \right\}$$

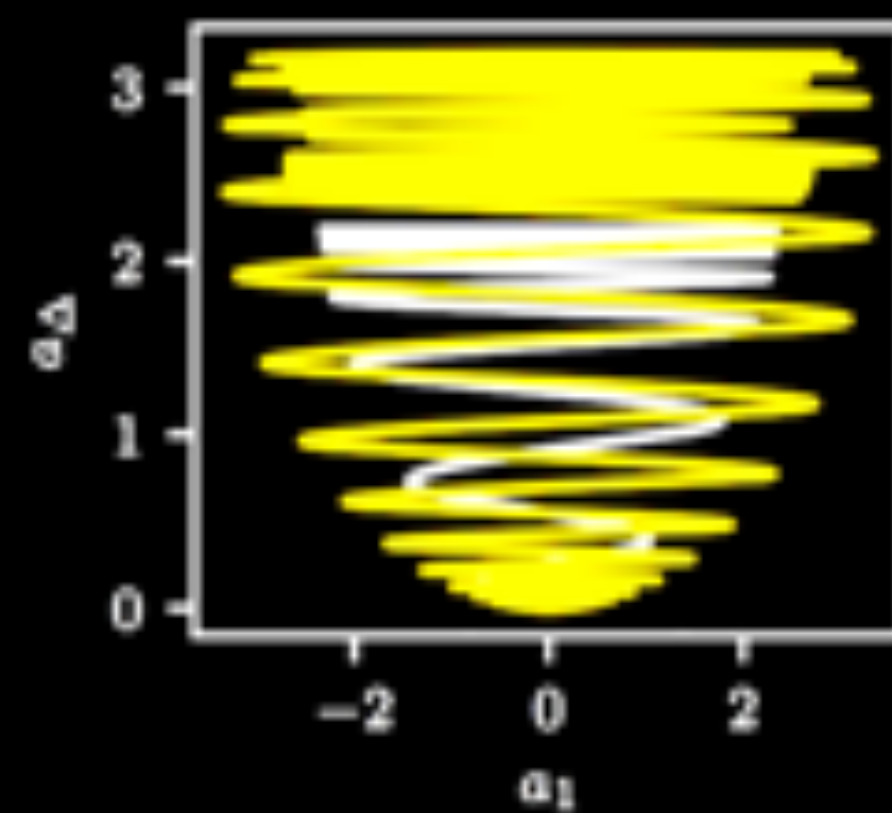
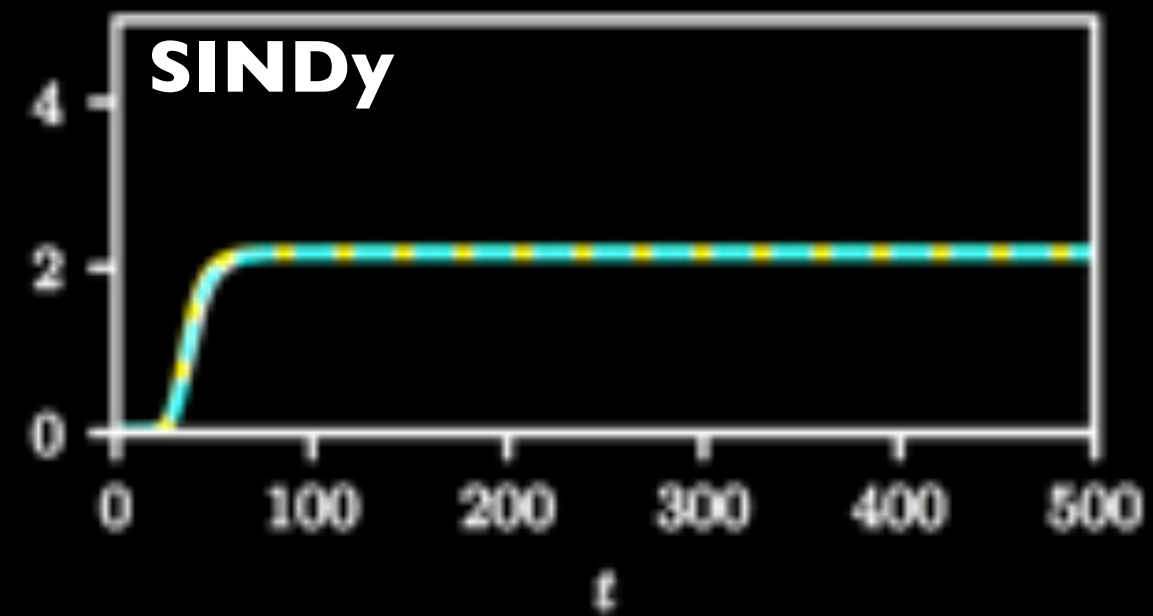
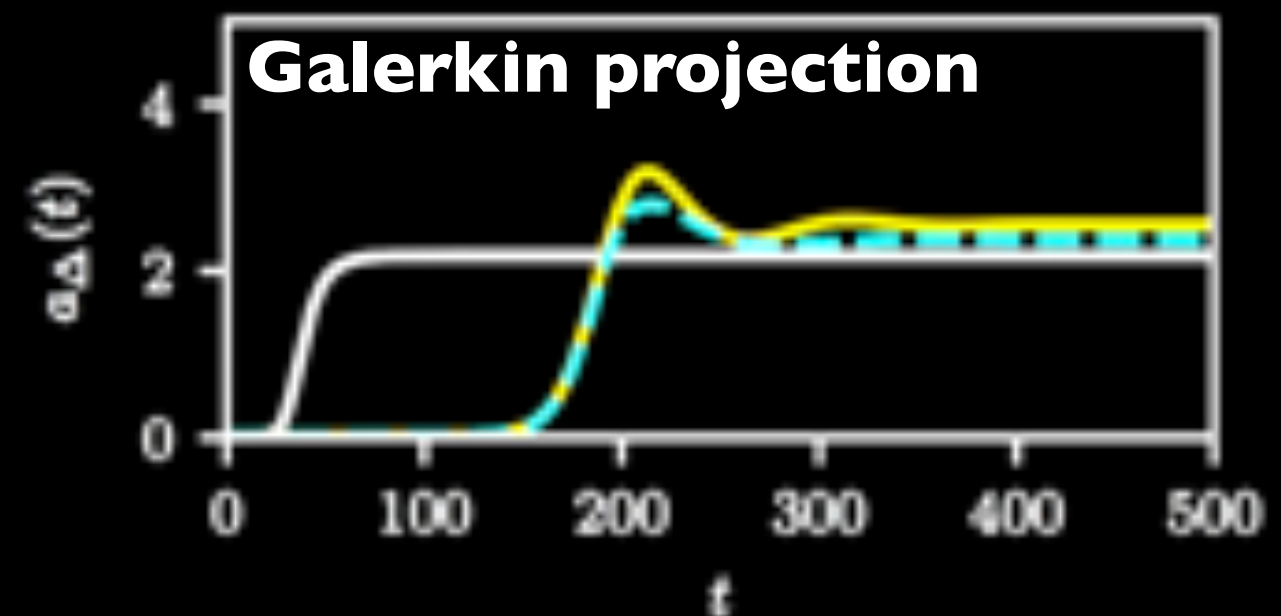
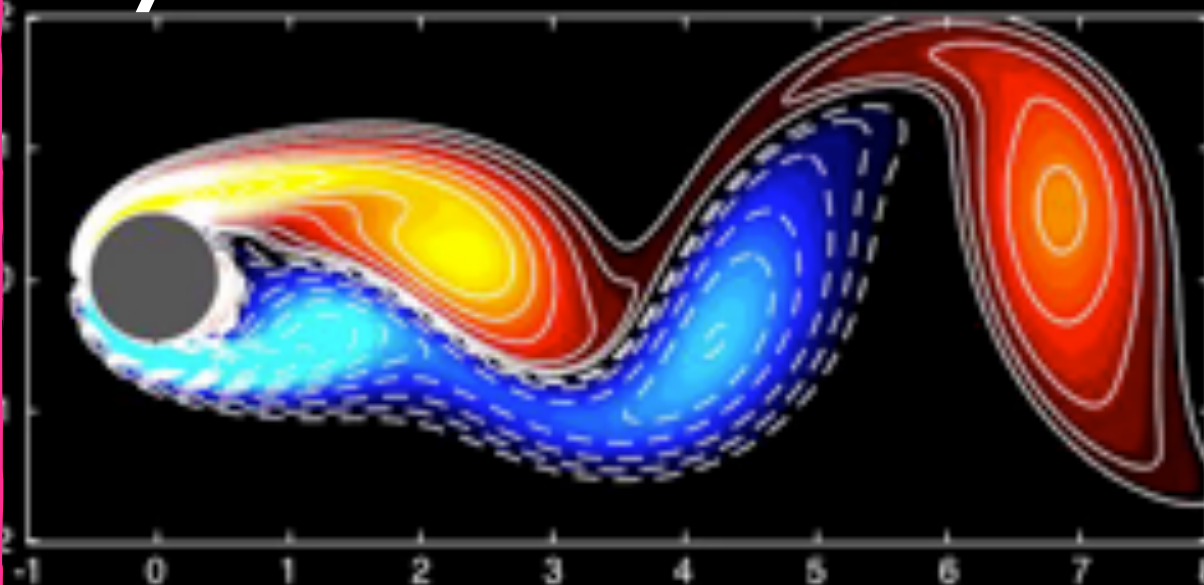


# Constrained Sparse Galerkin Regression

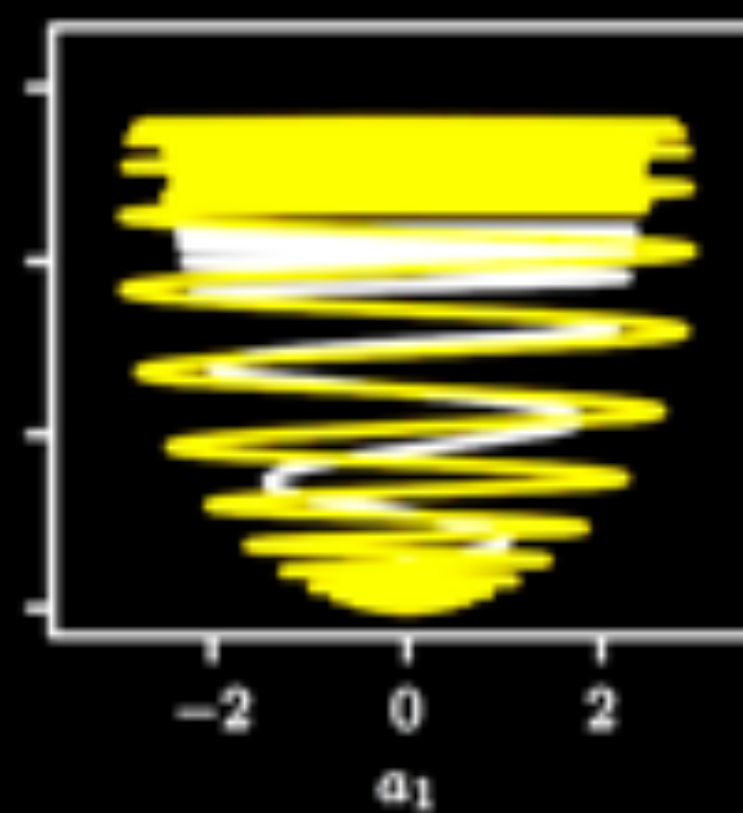
**Cavity flow**



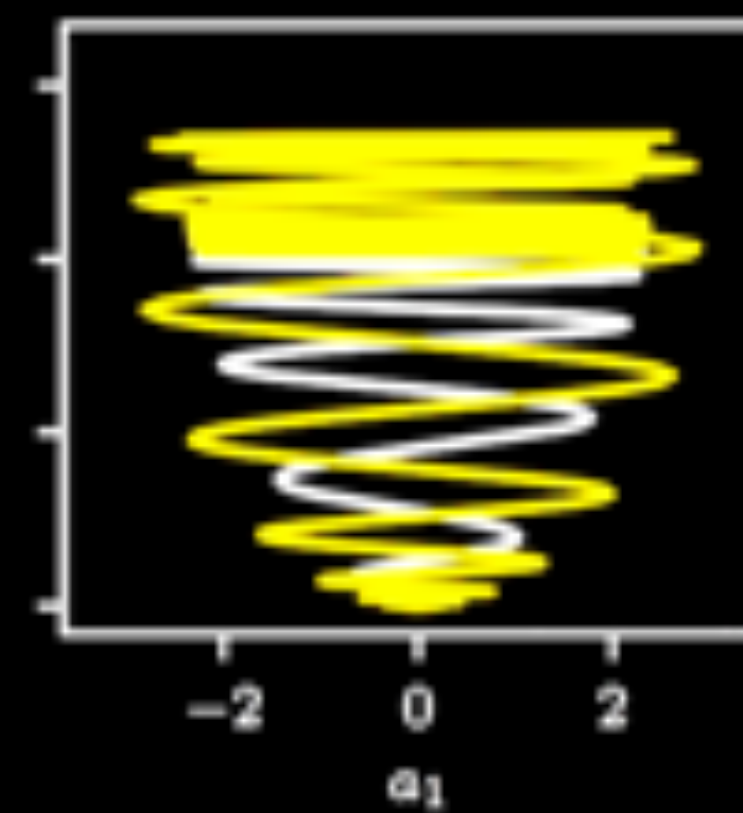
**Cylinder flow**



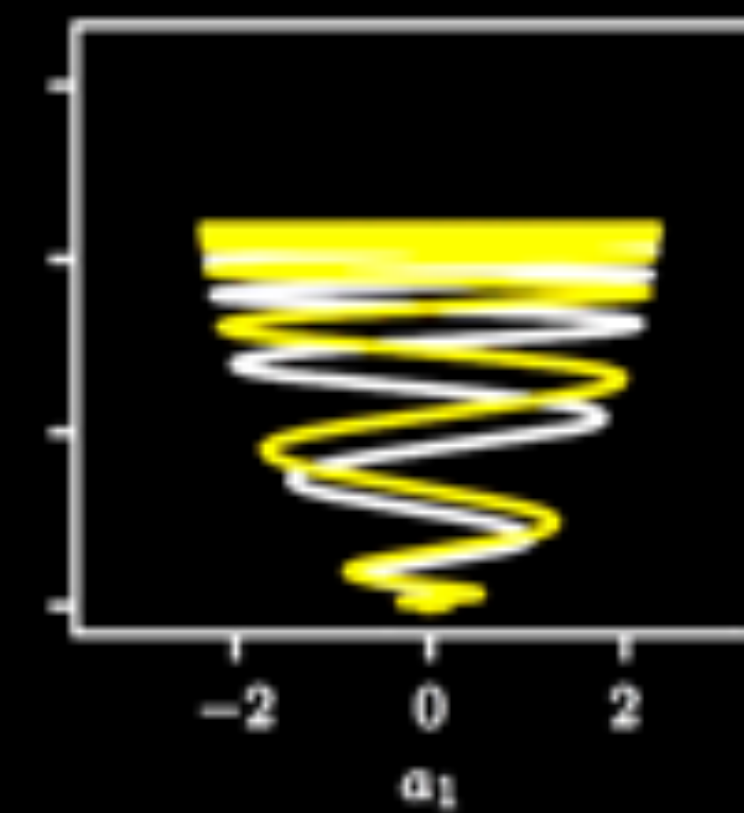
(a) 3 POD modes



(b) 9 POD modes



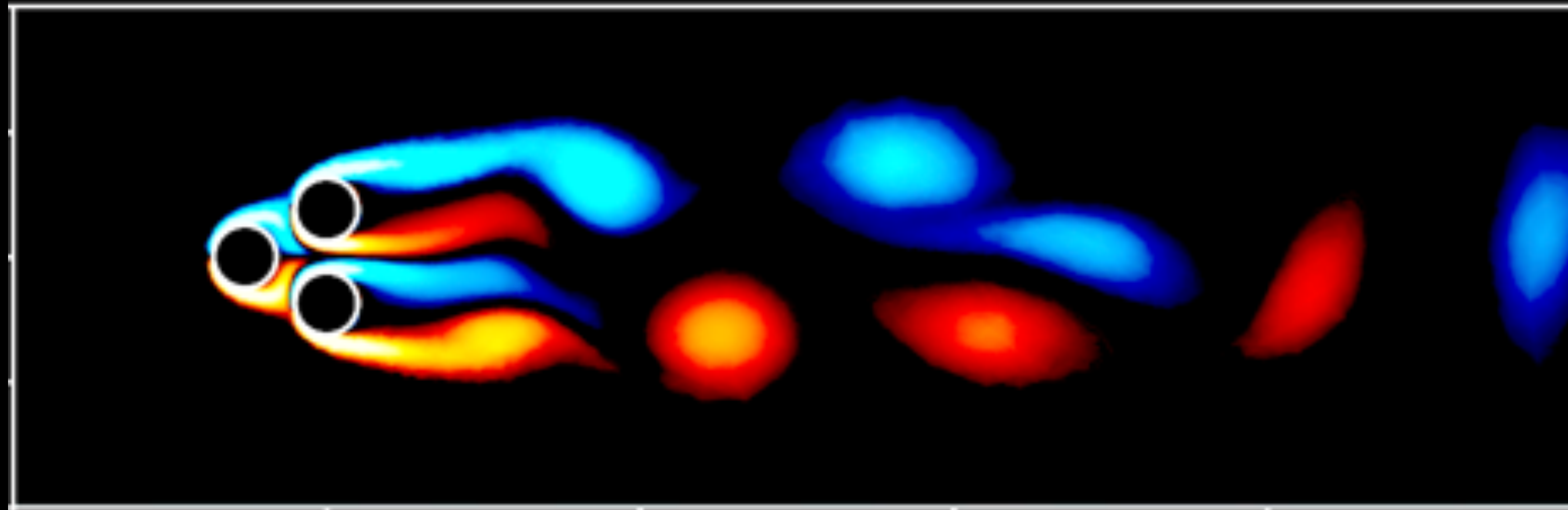
(c) Quadratic SINDy



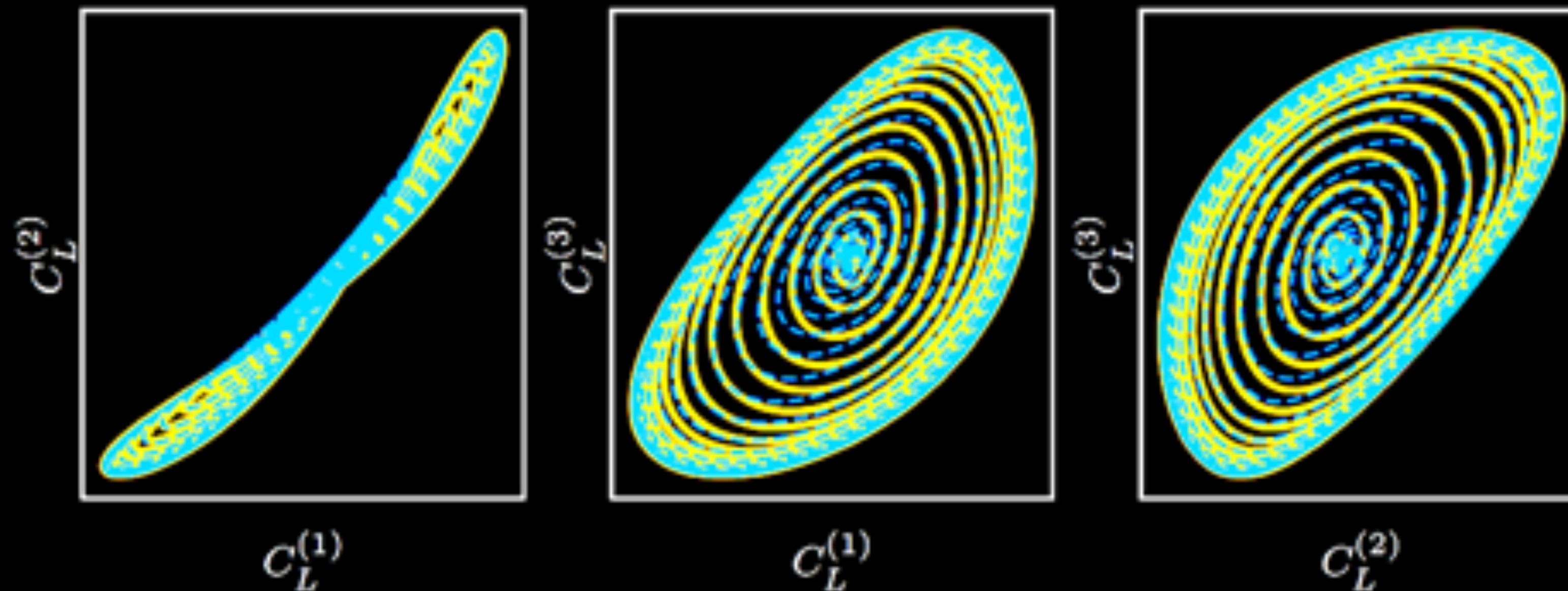
(d) Cubic SINDy



## More Complex Flow: Fluidic Pinball



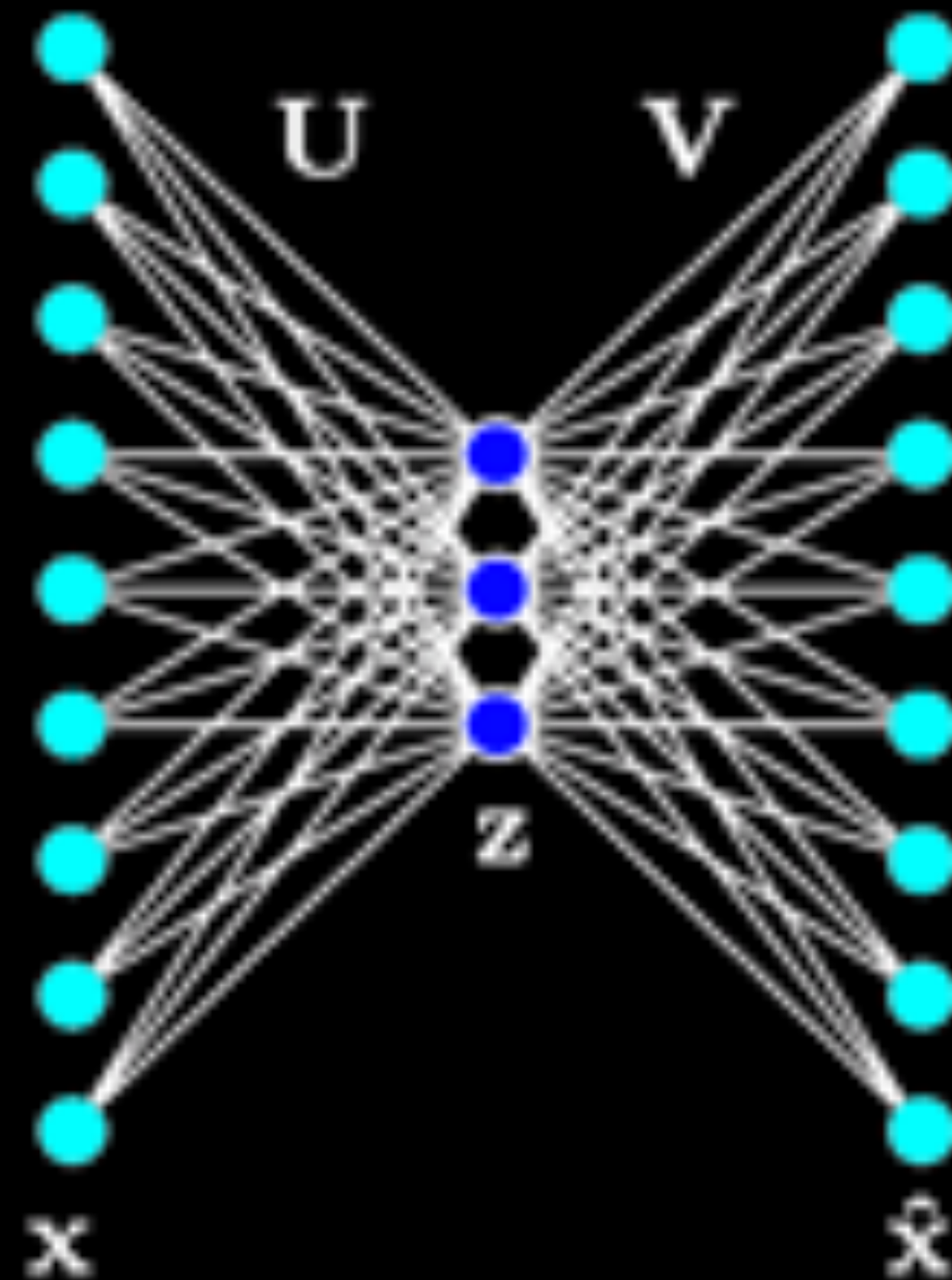
— DNS    --- Low-order model





# POD/PCA

Autoencoder (Shallow, linear)

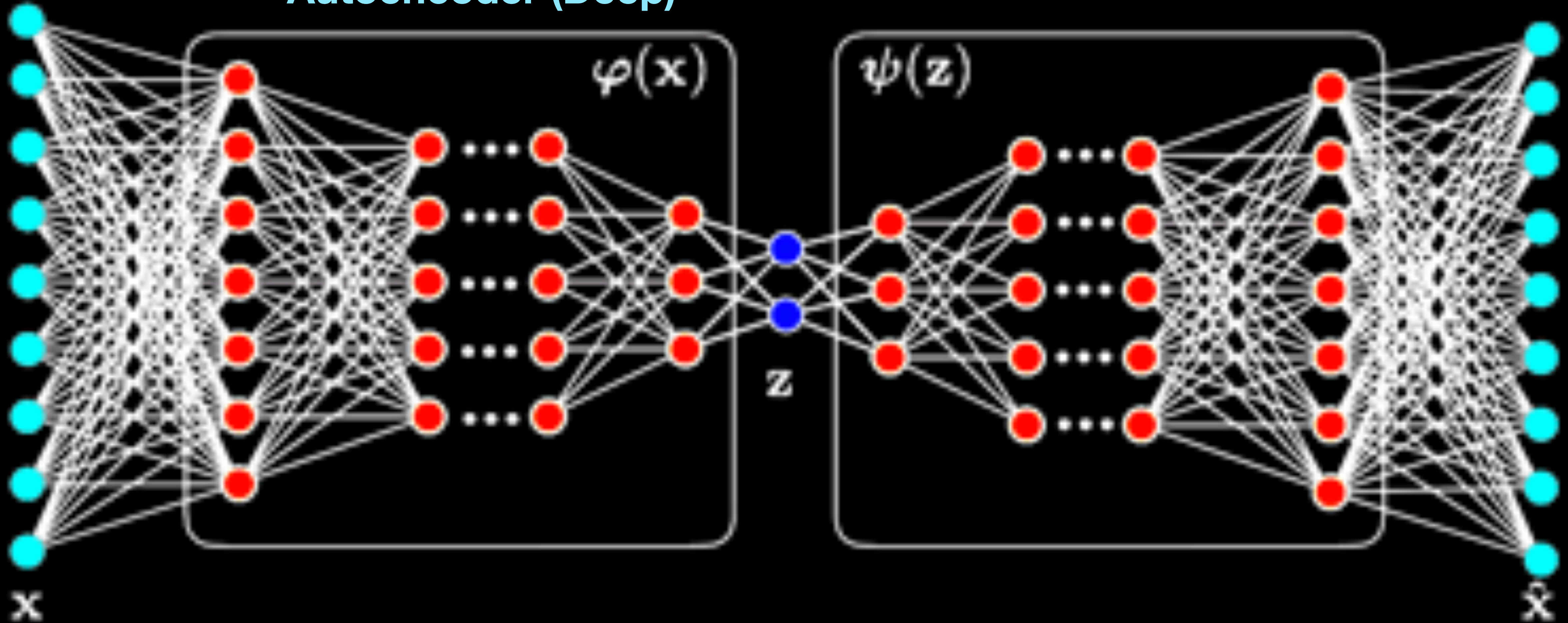


**Principal  
Component  
Analysis**



# BEYOND POD

## Autoencoder (Deep)



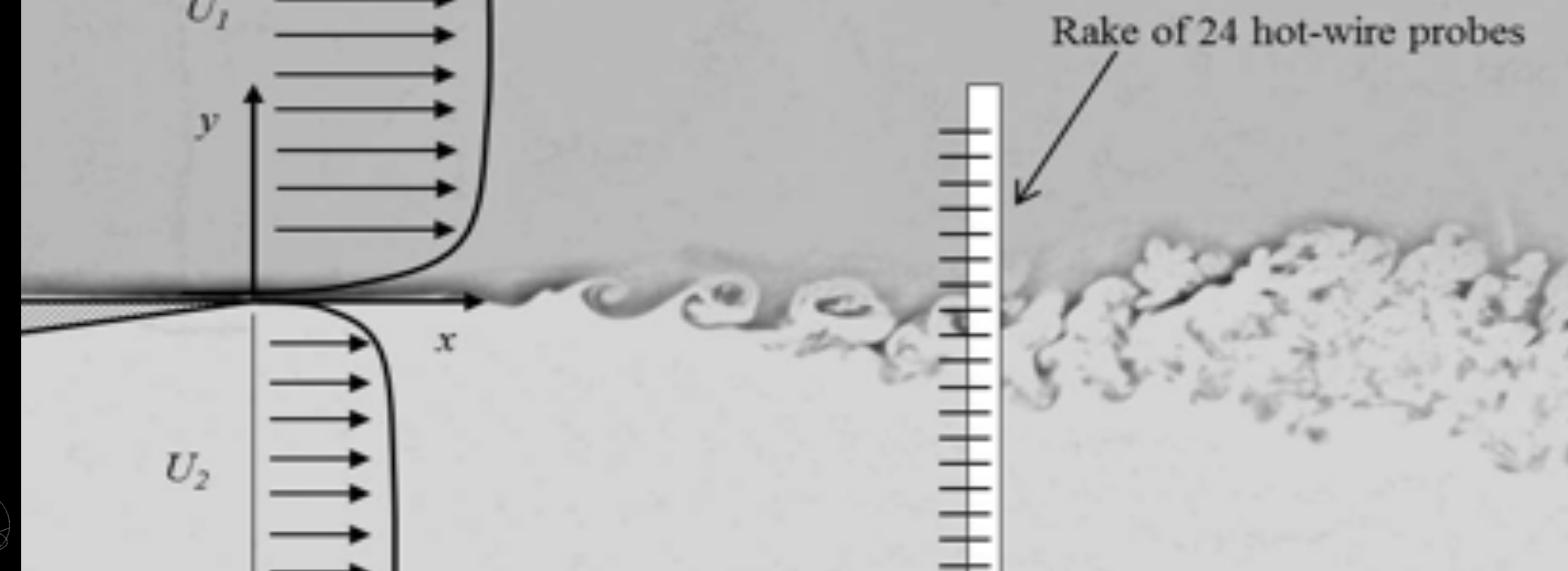
**SLB, Noack, Koumoutsakos,**  
*Ann. Rev. Fluid Mech.* 2019

**Early Work:**  
**Milano and Koumoutsakos,**  
*J. Comp. Phys.* 2002

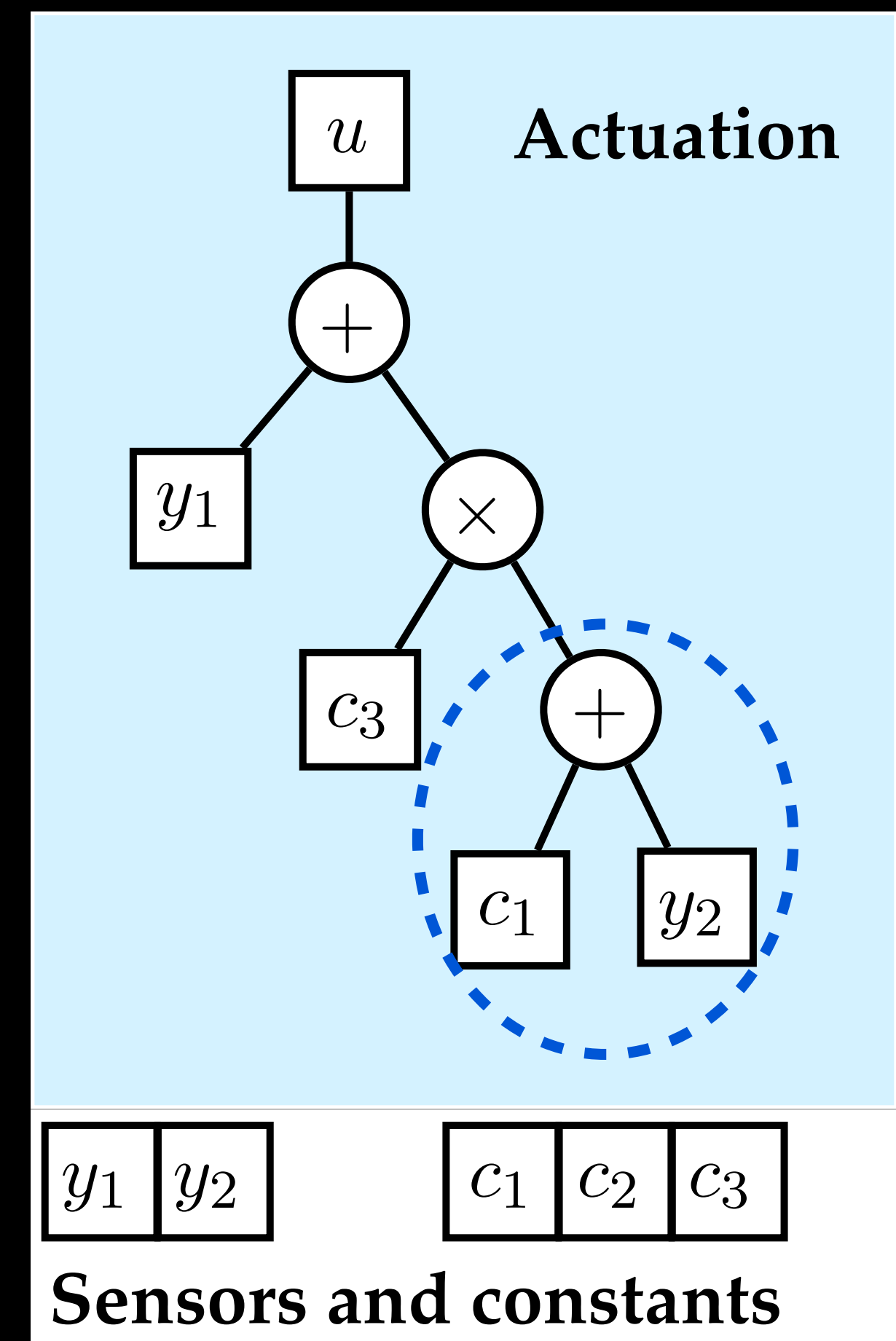
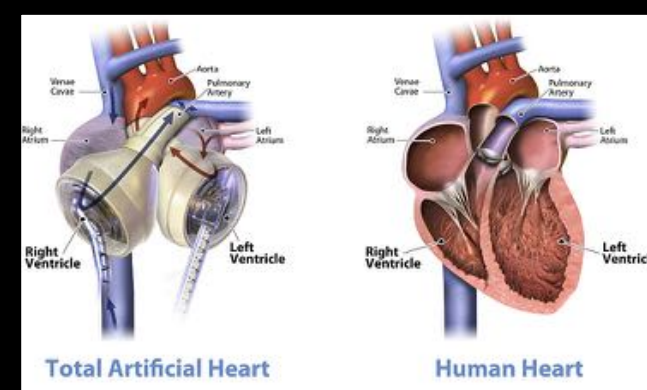
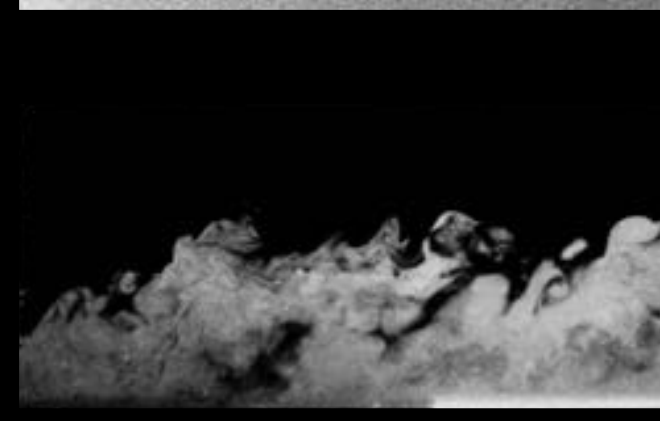
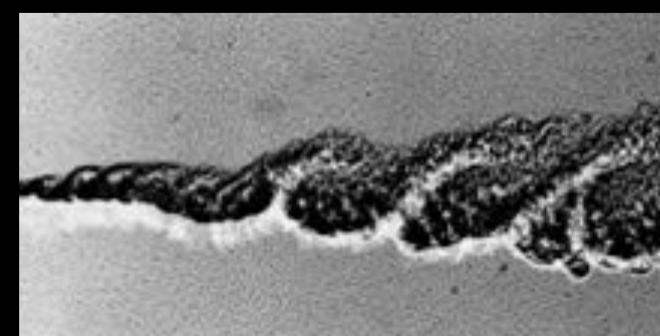
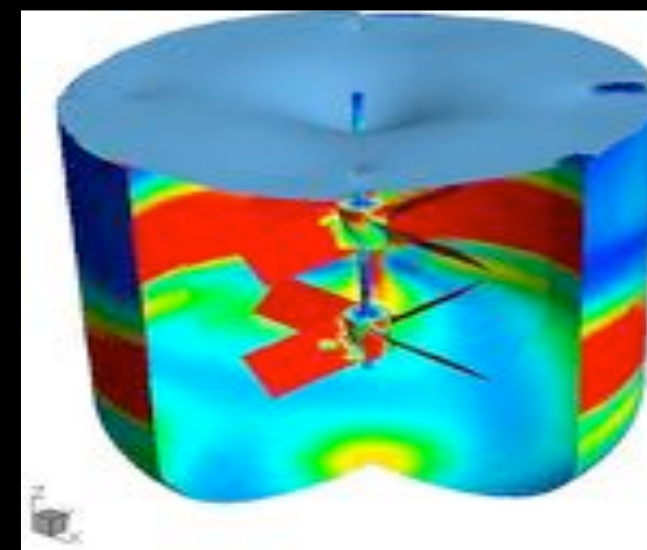
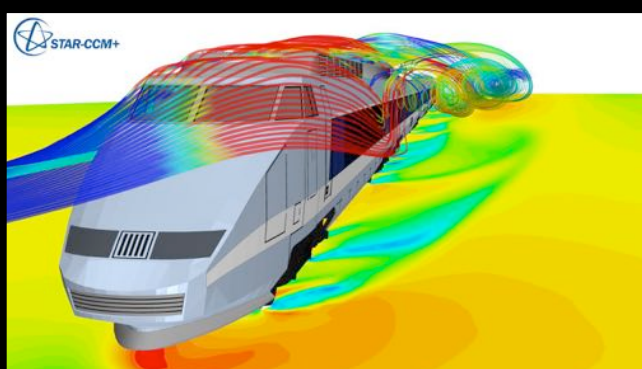
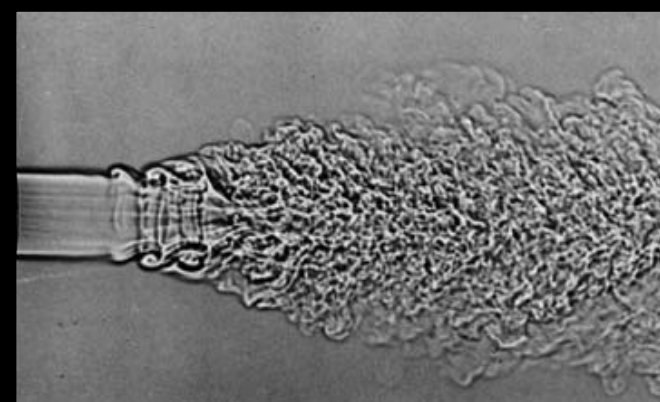
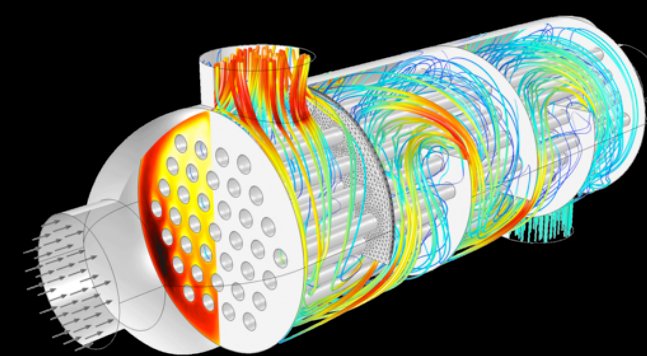
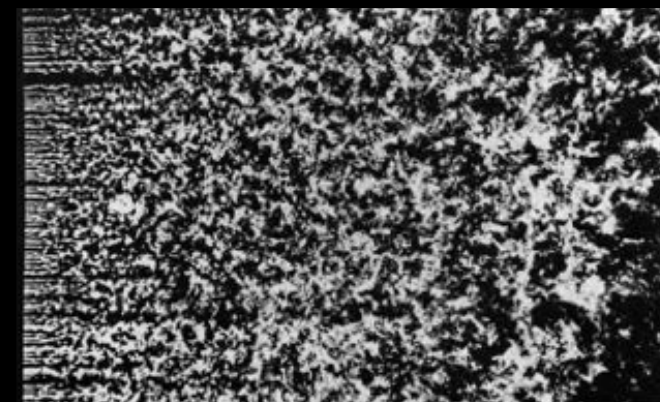


# FLOW CONTROL

SLB, Noack, AMR, 2015  
Duriez, SLB, Noack, Springer 2016



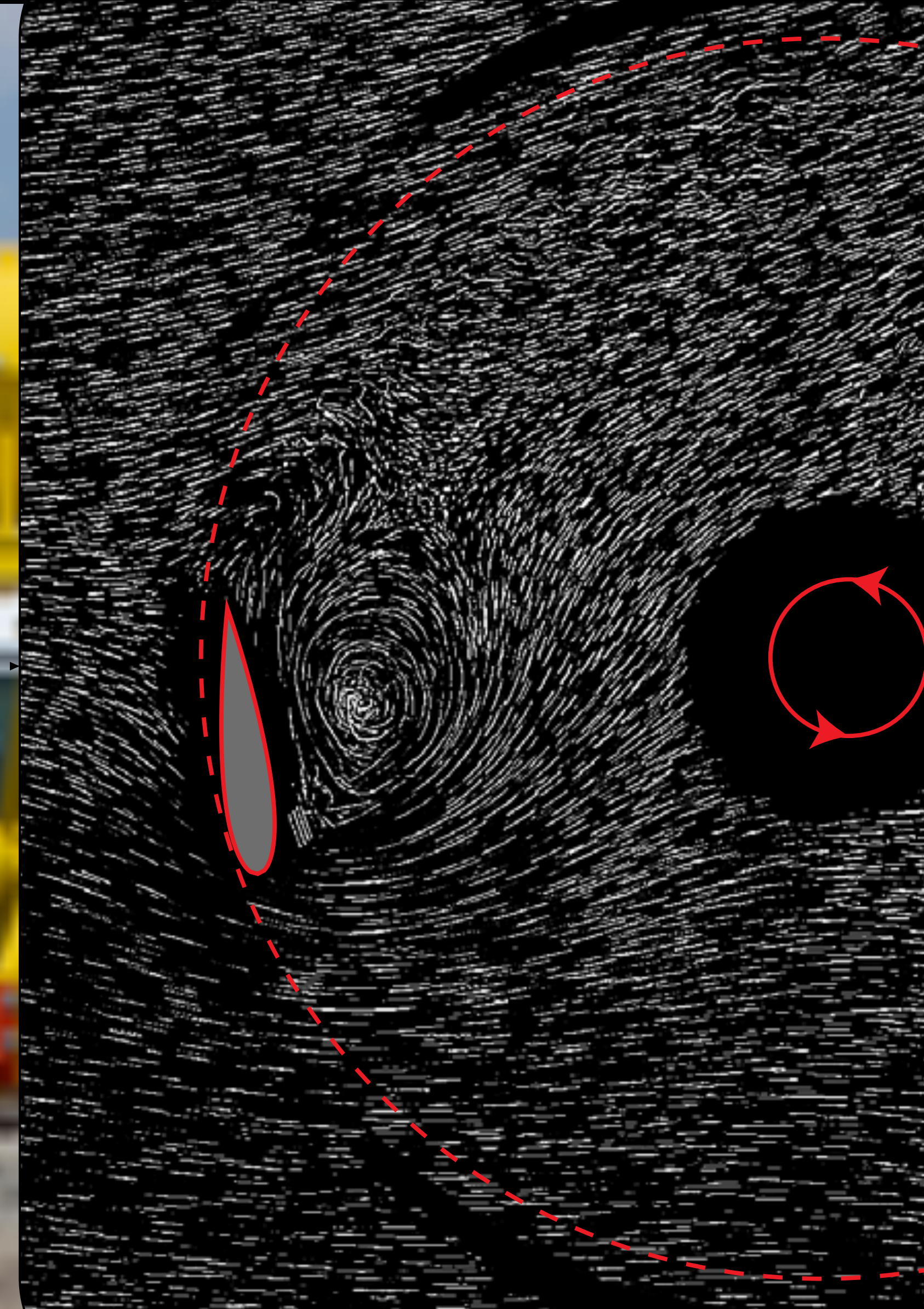
Control law





**59% Power increase in lab-scale cross-flow turbine experiment using gradient simplex optimization**

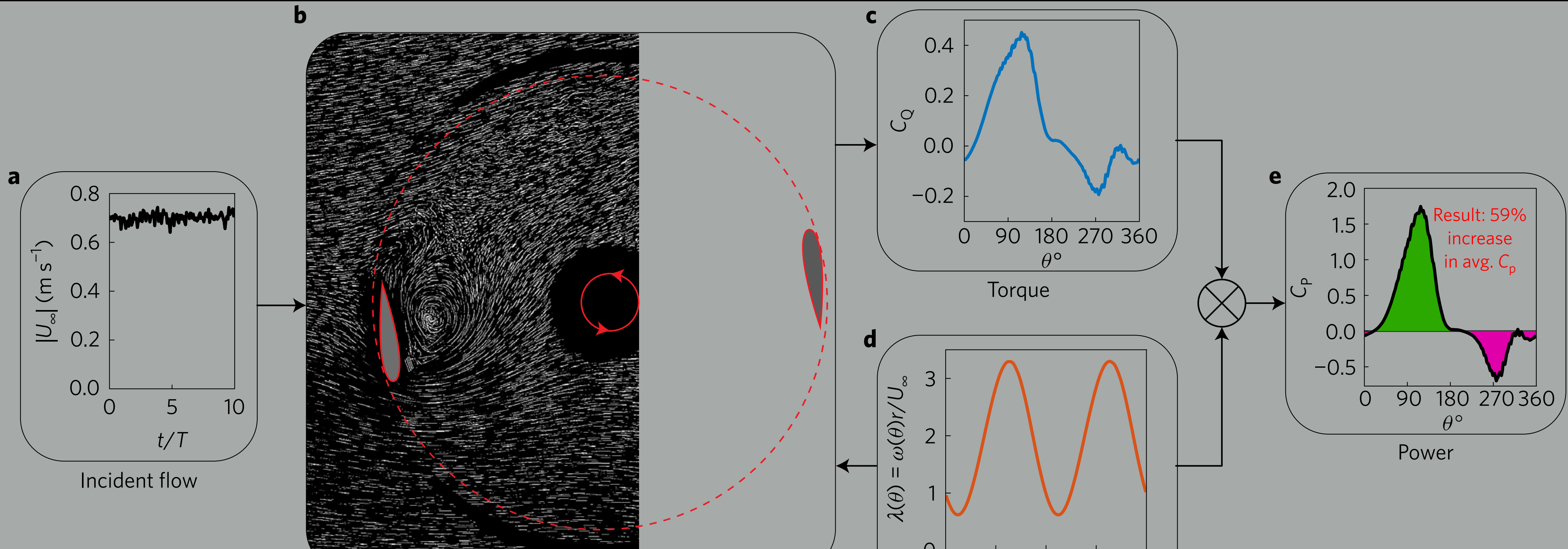
Nature Energy, 2017  
Strom, SLB, Polagye





# 59% Power increase in lab-scale cross-flow turbine experiment using gradient simplex optimization

Nature Energy, 2017  
Strom, SLB, Polagye









**Governing equations are TOO COMPLEX to work with:**

- ▶ **Discover Reduced Order Models with machine learning**

**Dynamics are NONLINEAR and HIGH-DIMENSIONAL:**

- ▶ **Coordinate transformations to linearize dynamics**
- ▶ **Patterns facilitate sparse measurements**

**Proposed approach:**

- ▶ **Learn physics from data: interpretable & generalizable**
- ▶ **Respect known, or partially known, physics**



**Governing equations are TOO COMPLEX to work with:**

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**Dynamics are NONLINEAR and HIGH-DIMENSIONAL:**

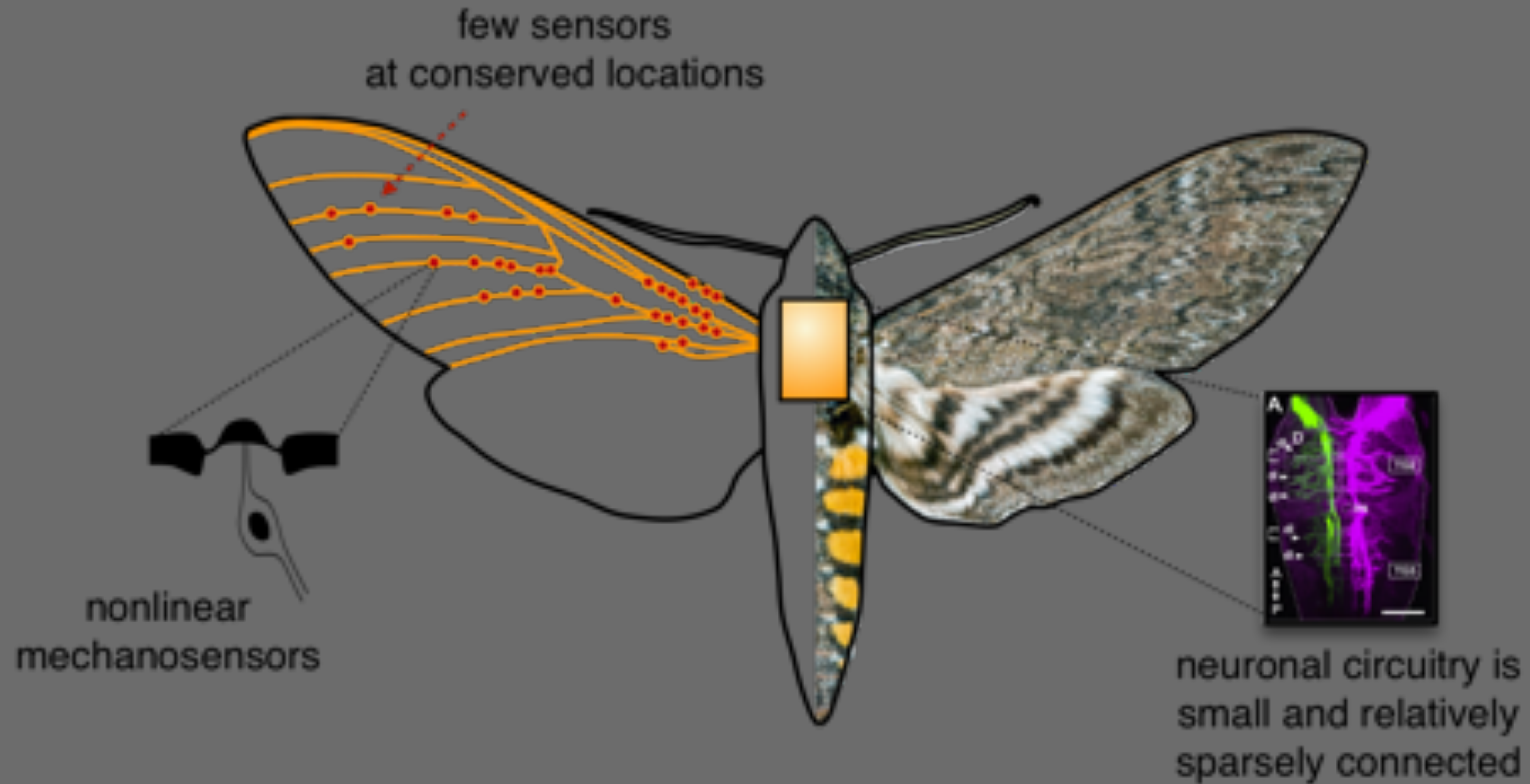
- ▶ **Coordinate transformations to linearize dynamics**
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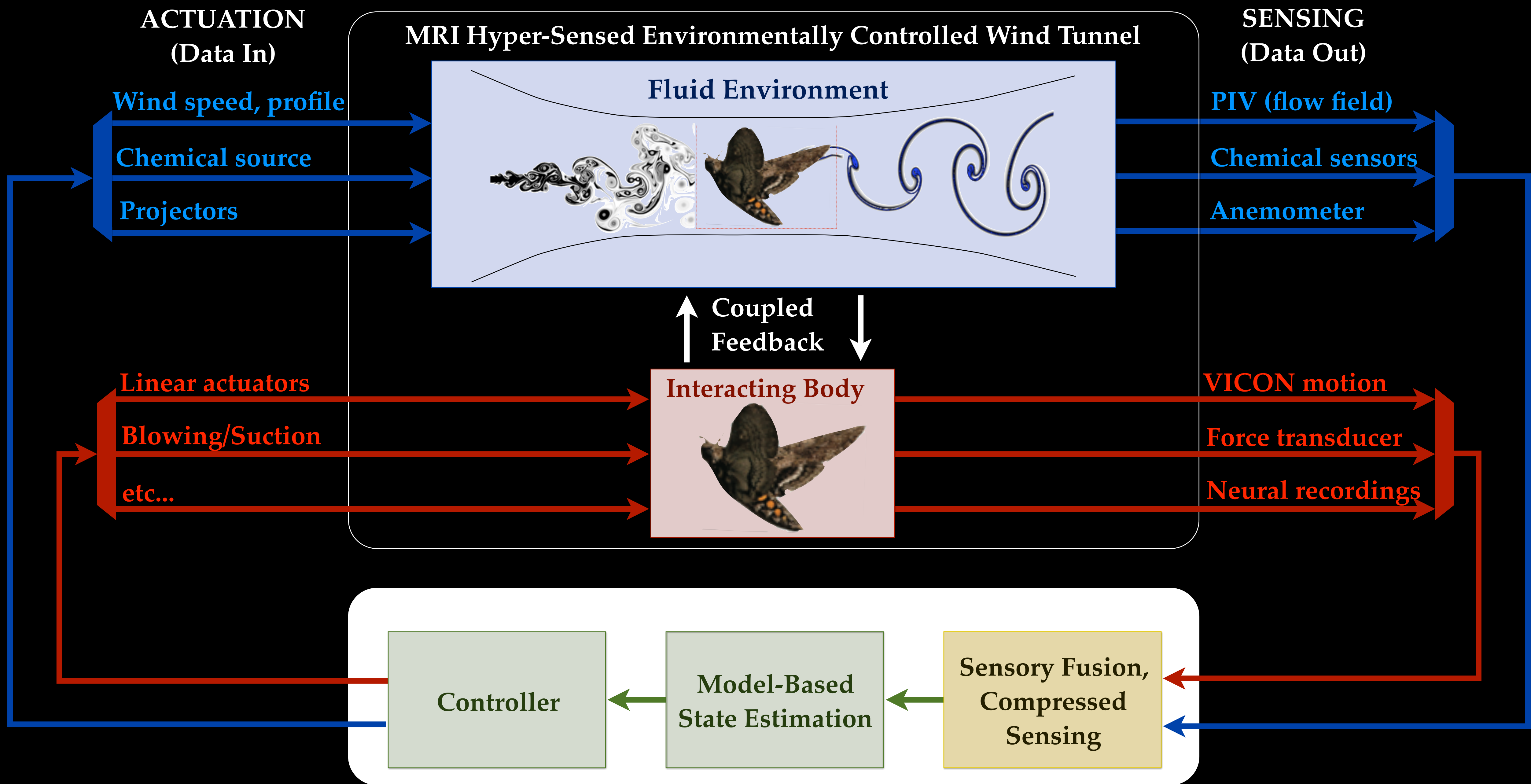
# BIO-INSPIRED



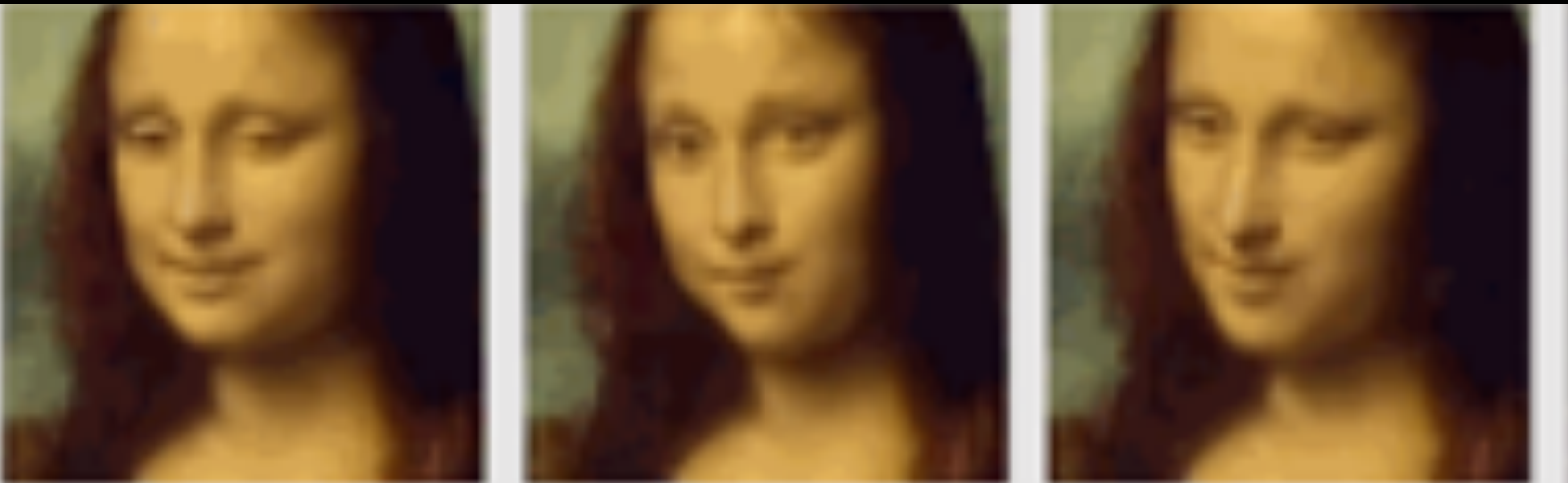
*diagram adapted from Ali Weber*

*anatomy adapted from Ando et al. 2011.*





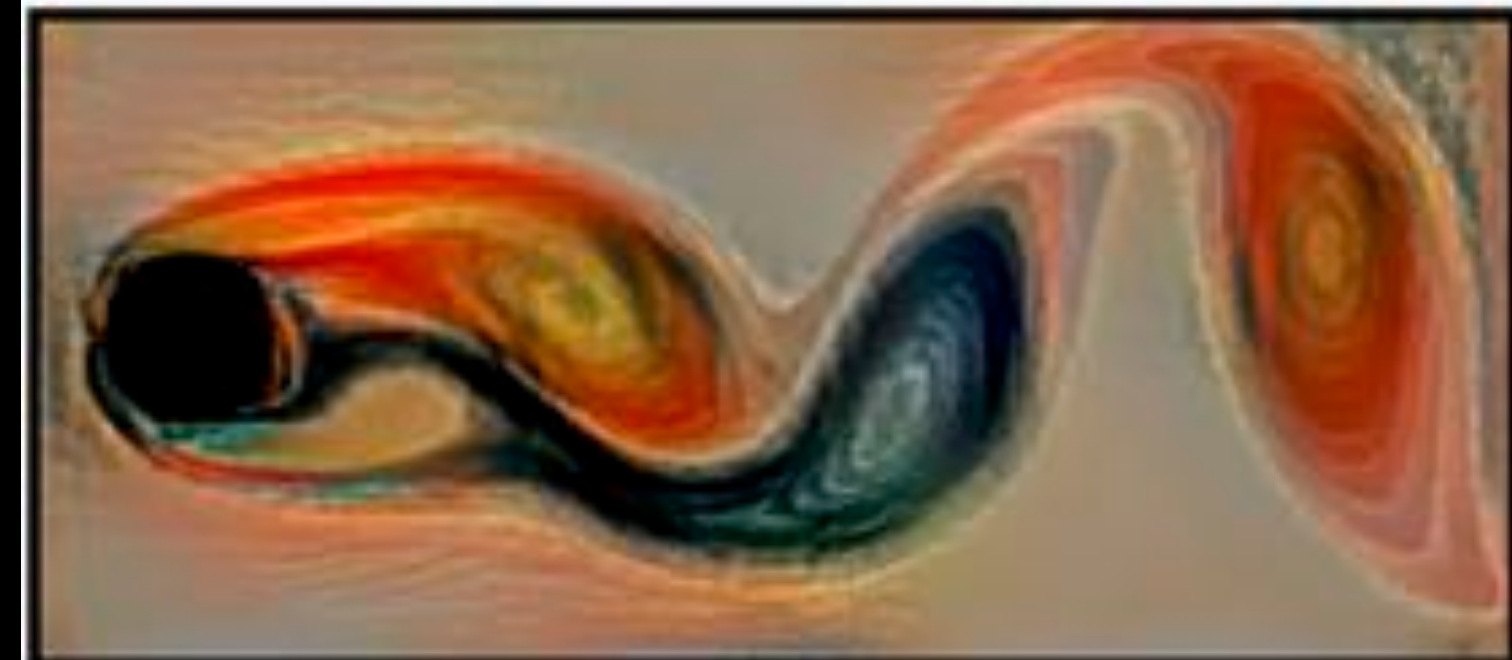
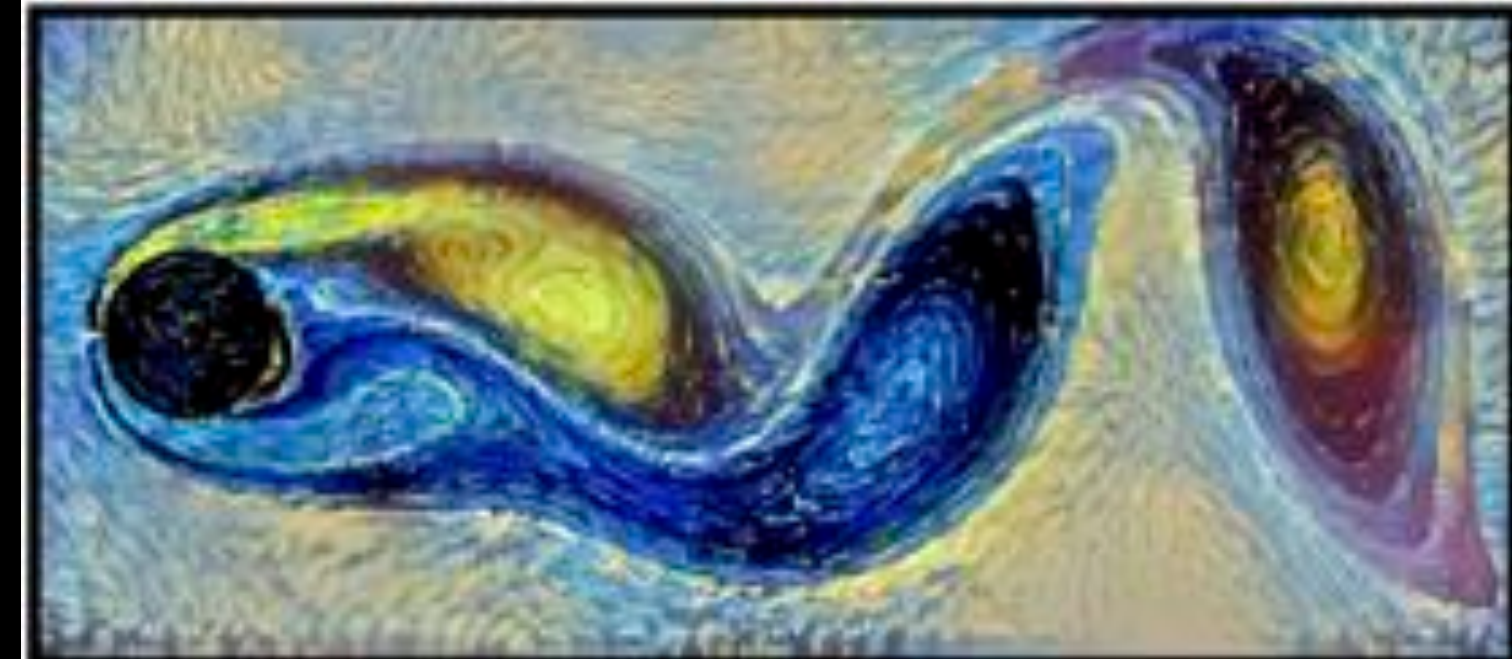
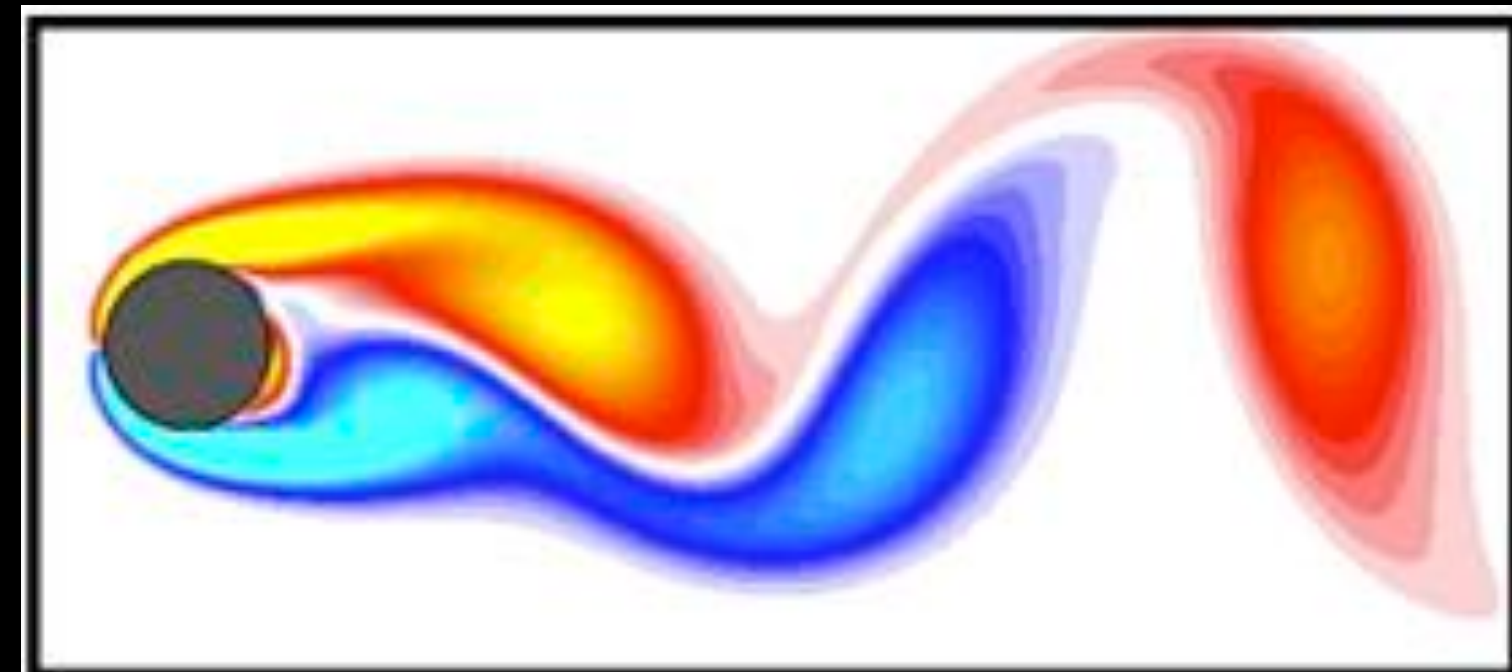




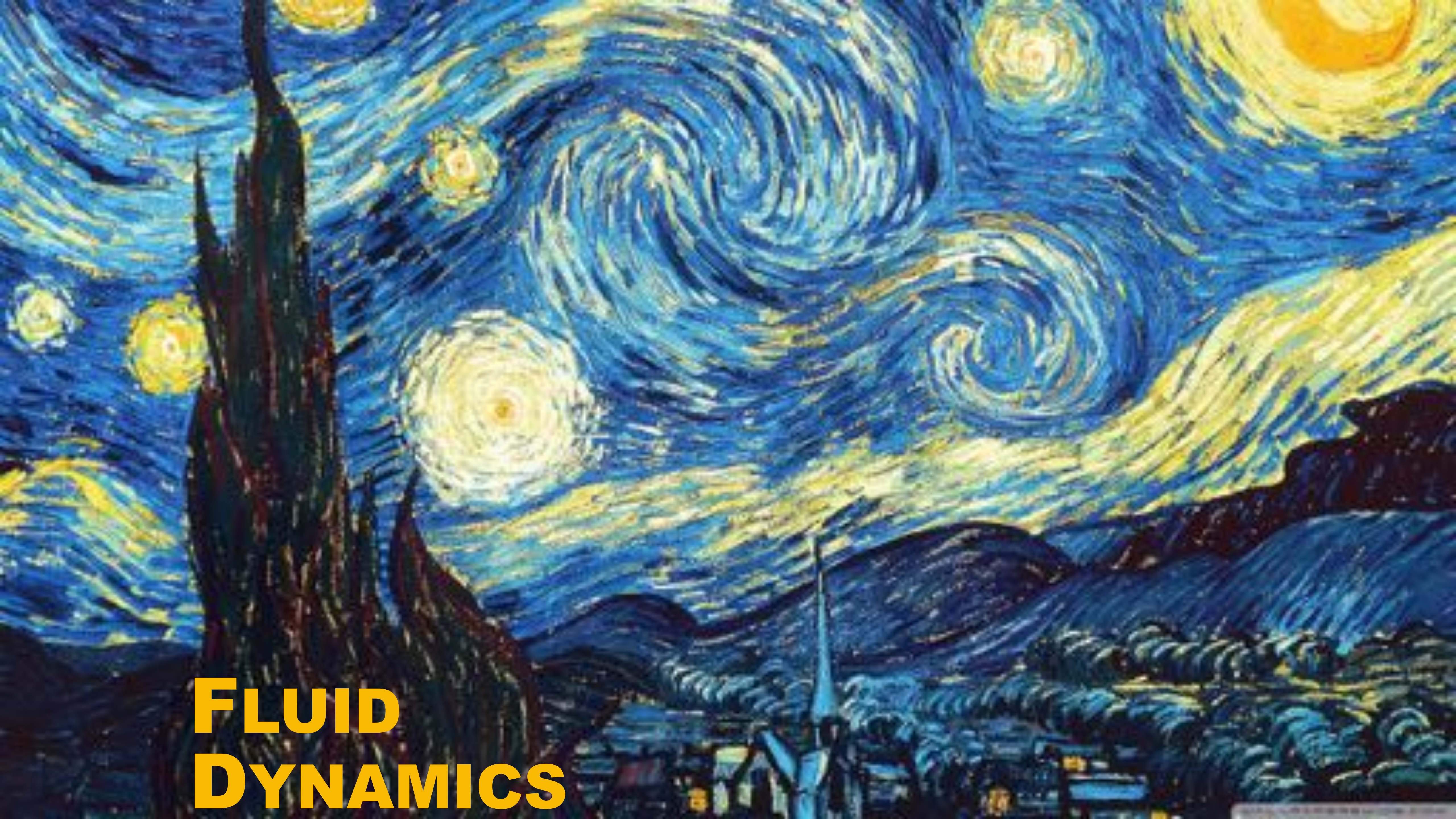




Deep dream of Arcimbaldo's *La Primavera*  
By Calhoun Press







# FLUID DYNAMICS