Cosmology and Multiscale Geometric Analysis

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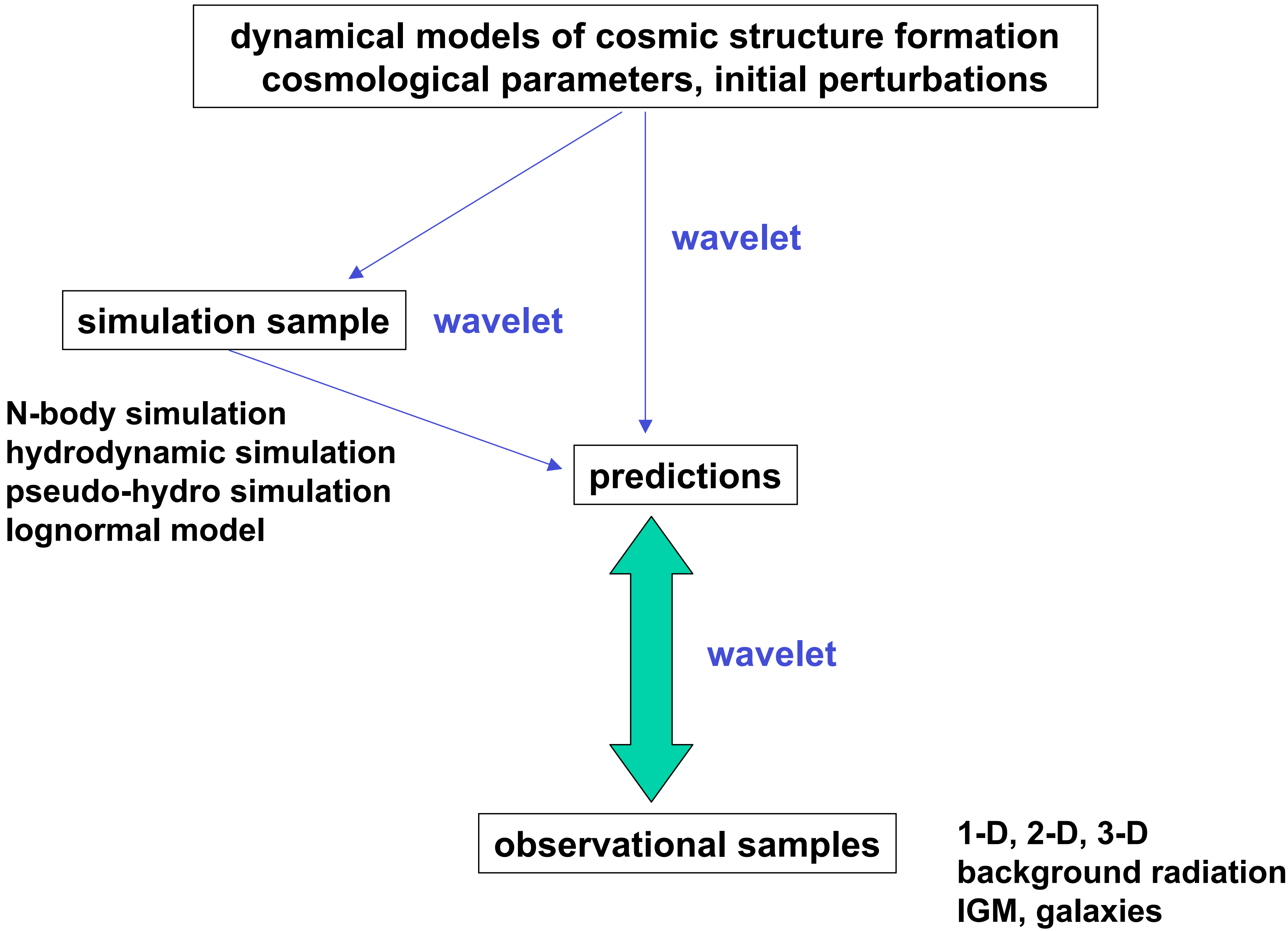
WAVELETS AND COSMOLOGY (Li-Zhi Fang, IPAM, Nov. 10, 2004)



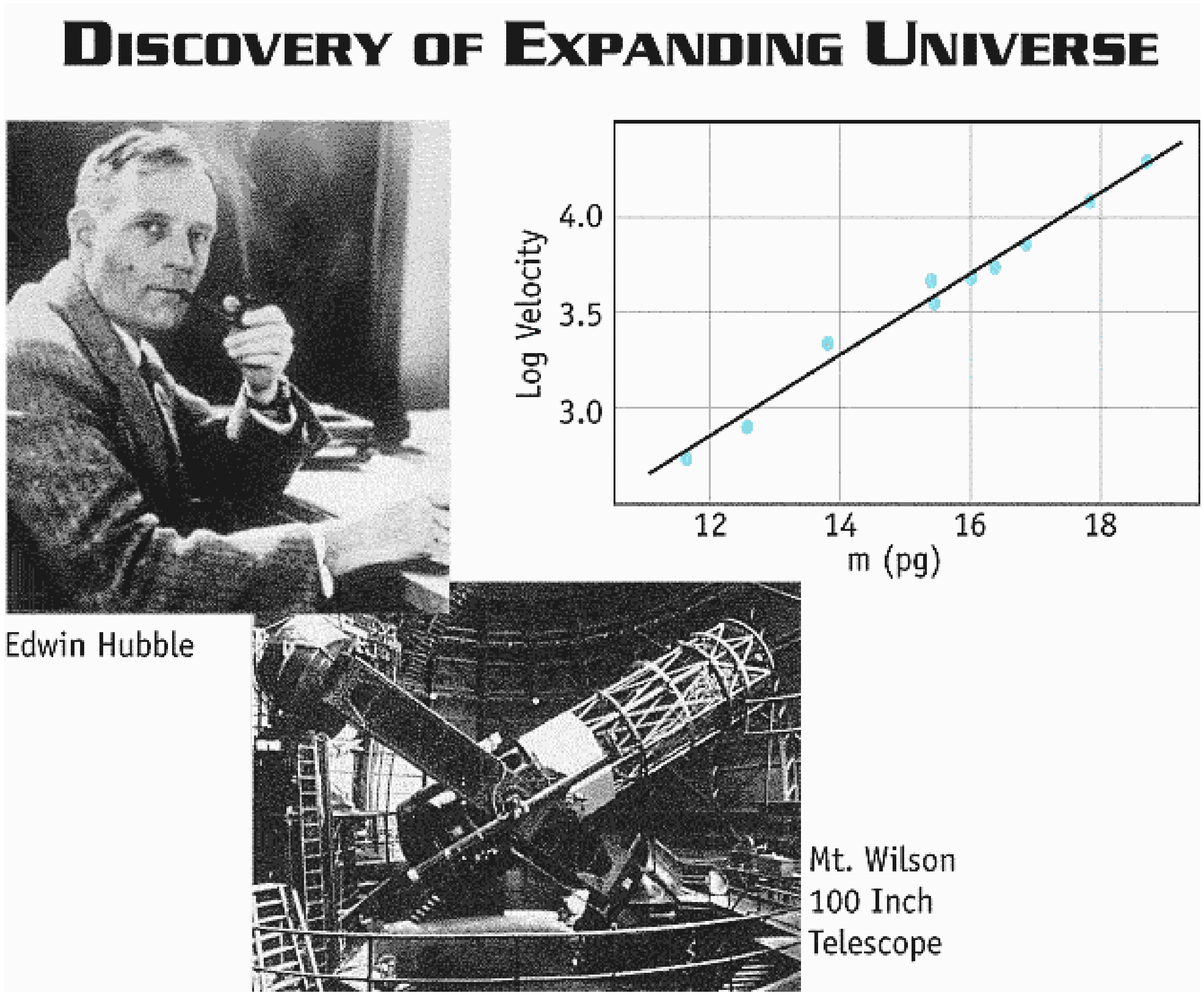
1) Brief introduction

2) The Cosmic Microwave Background (CMB)
with David Donoho and Jiashun Jin (Stanford)
Alain Forni and Nabila Aghanim (Institut d'Astrophysique Spatial, Orsay)

3) The Weak Lensing ==> dark matter mass map reconstruction
with Alexandre Refregier and Sandrine Pires (CEA-Saclay)

4) The Spatial Distribution of galaxies
with David Donoho, Ofer Levi (Stanford)
Vicent Martinez (Observatori de Valencia)

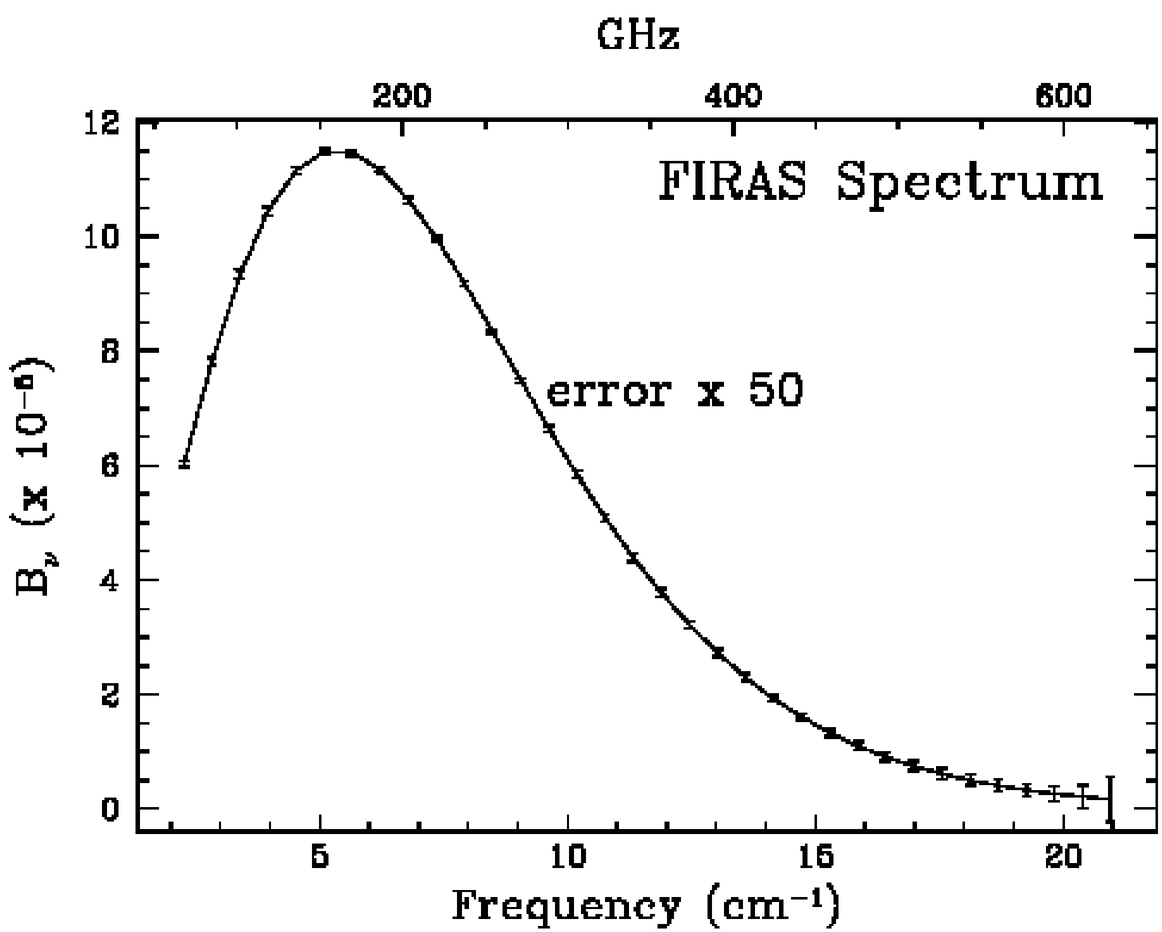
The Big Bang


DISCOVERY OF EXPANDING UNIVERSE
Log Velocity
4.0
3.5
3.0
12
14
16
18
m (pg)
Edwin Hubble
Mt. Wilson
100 Inch
Telescope

The Cosmic Microwave Background

- The Universe is filled with a blackbody radiation field at a temperature of 3K.
- Predicted by G. Gamow in 1948
- Observed for the first time by Penzias and Wilson (1965)
- Confirmed by COBE (1990)

The Cosmic Microwave Background



$T = 2.728 \pm 0.004$ K

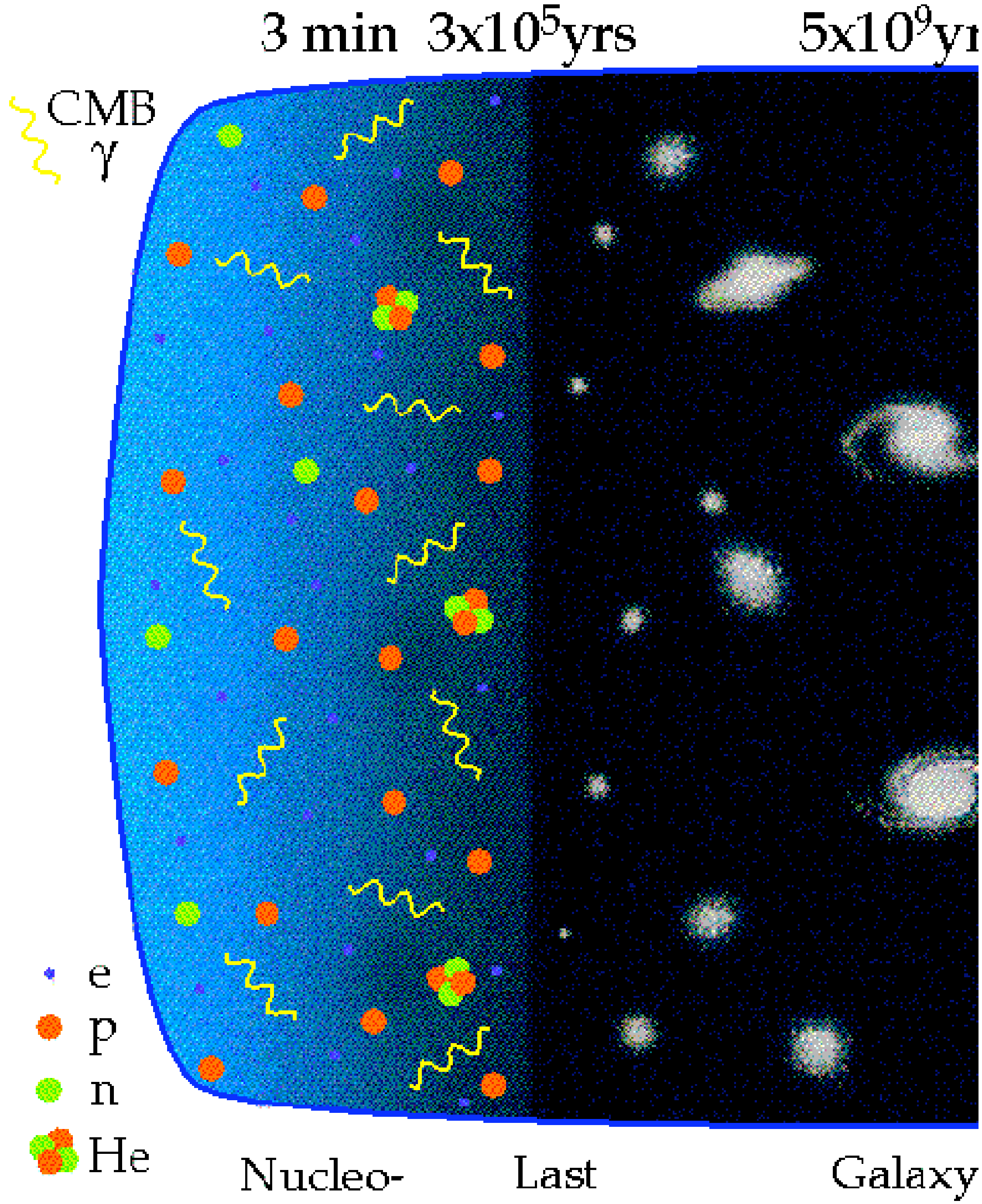
CMB electromagnetic spectrum

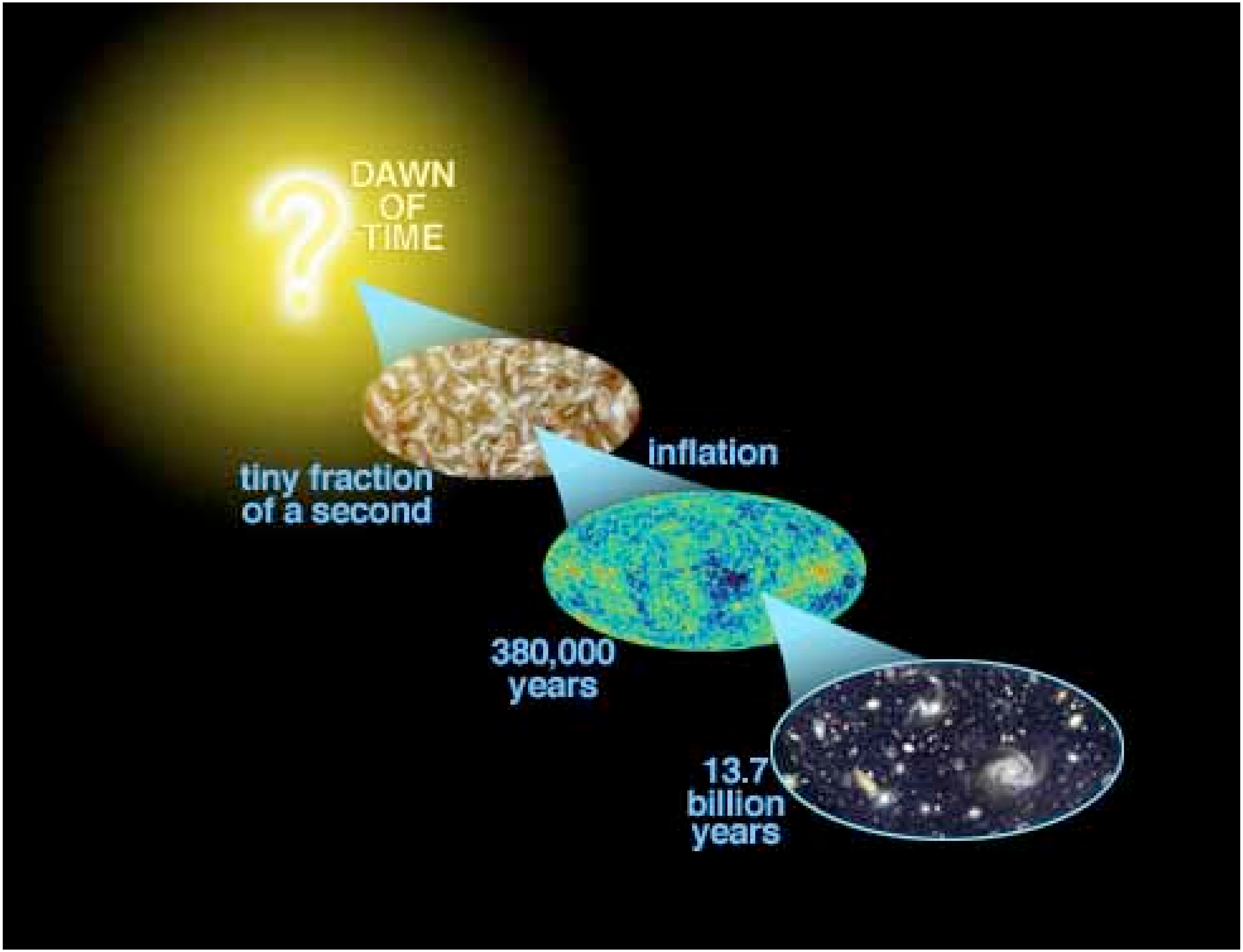
(Very) Brief History

Homogeneous Univers
until recombination
of structures

Last scatering surface
at t=380 000 years

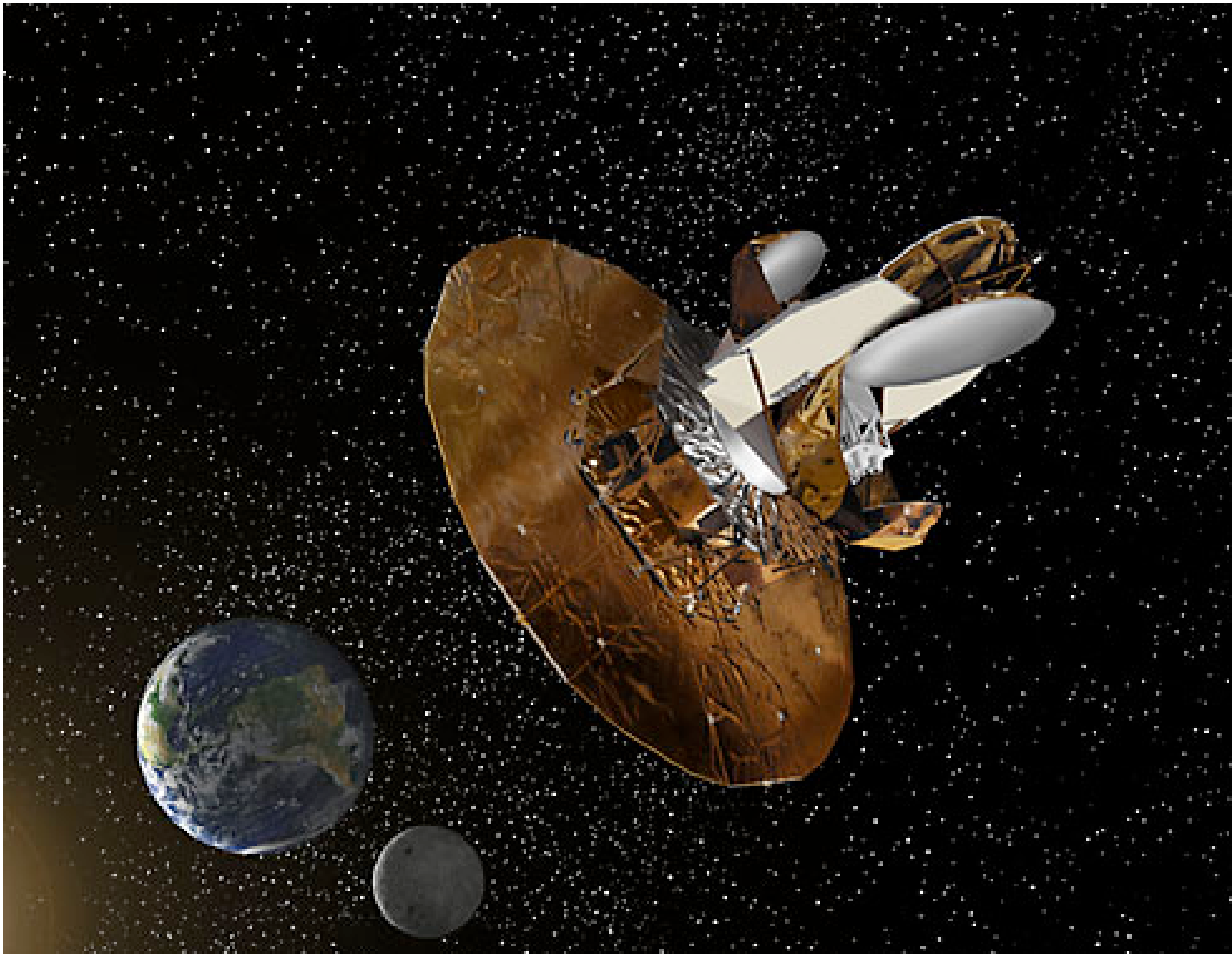
Anisotropies measured
in the CMB




DAWN
OF
TIME
inflation
tiny fraction
of a second
380,000
years
13.7
billion
years

Wilkinson Microwave Anisotropy Probe

A partnership between NASA/GSFC and Princeton



Science Team:

NASA/GSFC

Chuck Bennett (**PI**)
Michael Greason
Bob Hill
Gary Hinshaw
Al Kogut
Michele Limon
Nils Odegard
Janet Weiland
Ed Wollack

Brown

Greg Tucker

UCLA

Ned Wright

UBC

Mark Halpern

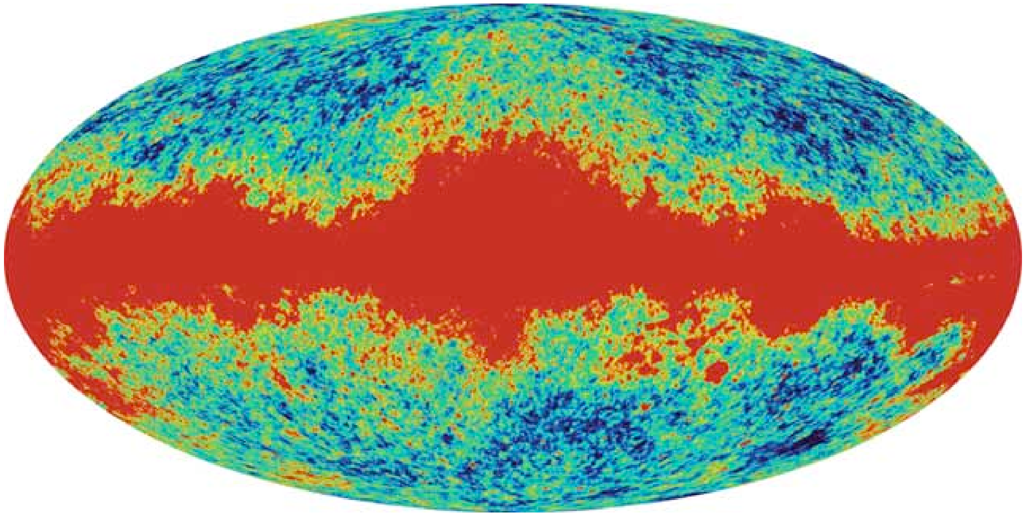
Chicago

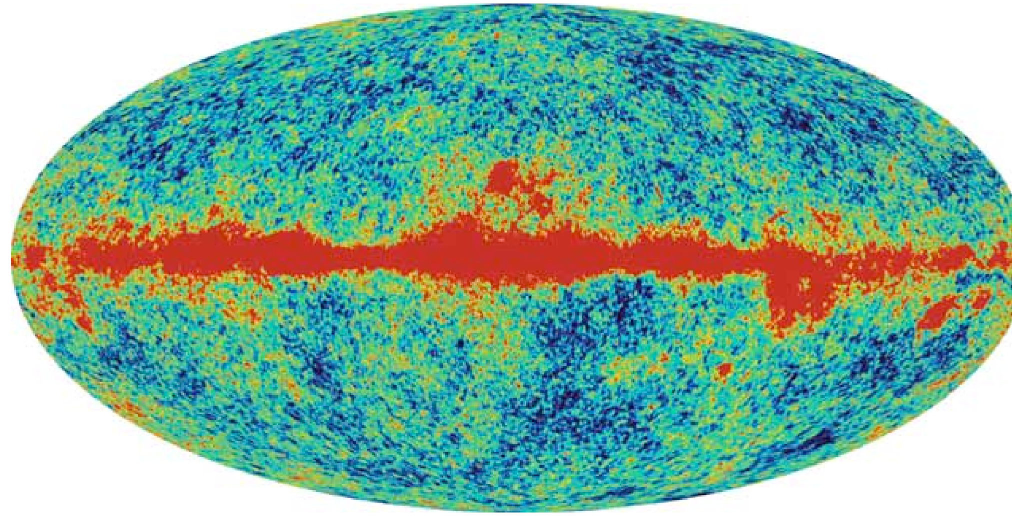
Stephan Meyer

Princeton

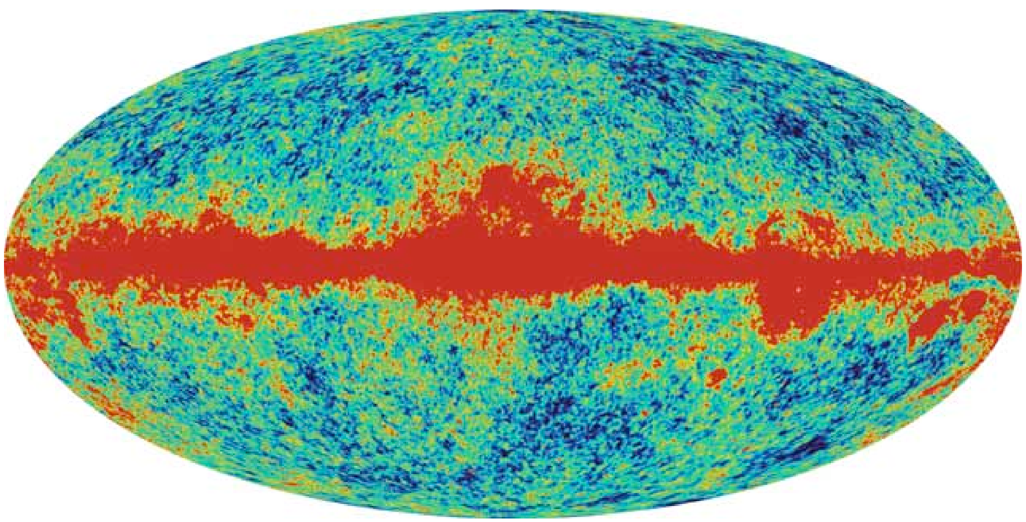
Chris Barnes
Norm Jarosik
Eiichiro Komatsu
Michael Nolta
Lyman Page
Hiranya Peiris
David Spergel
Licia Verde

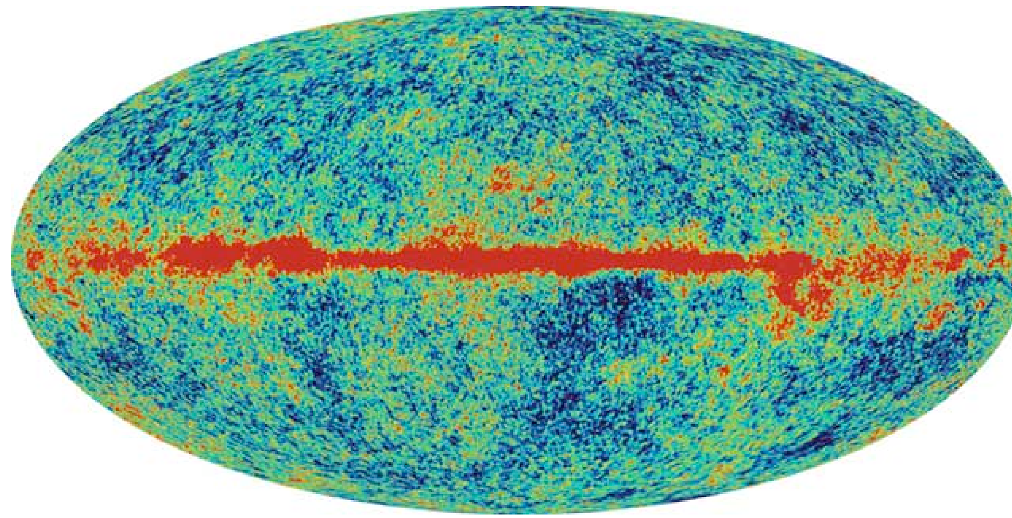
WMAP:five frequency maps

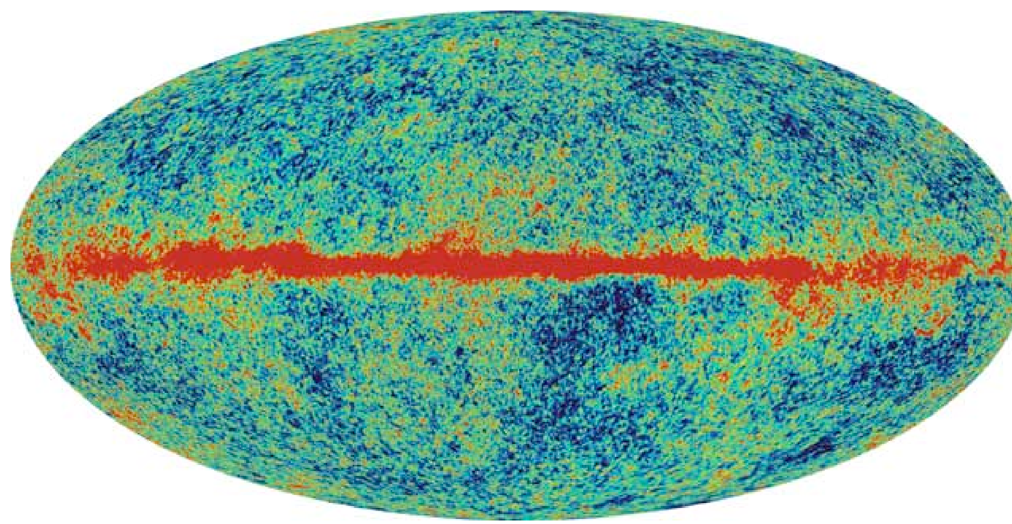


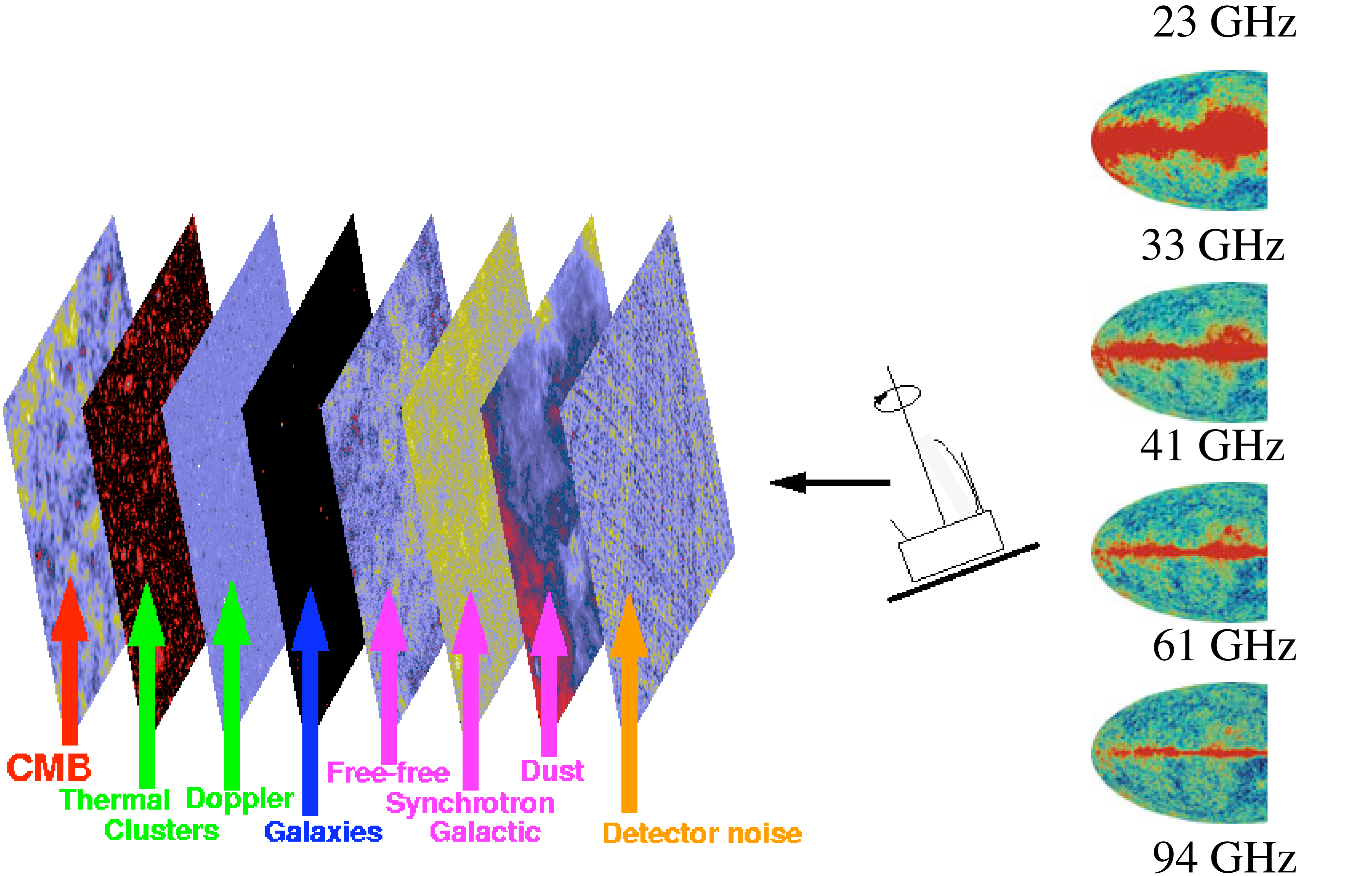


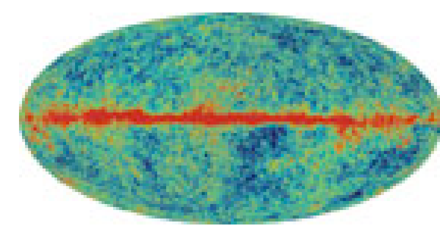
23, 33, 41, 61 and 93 GHz



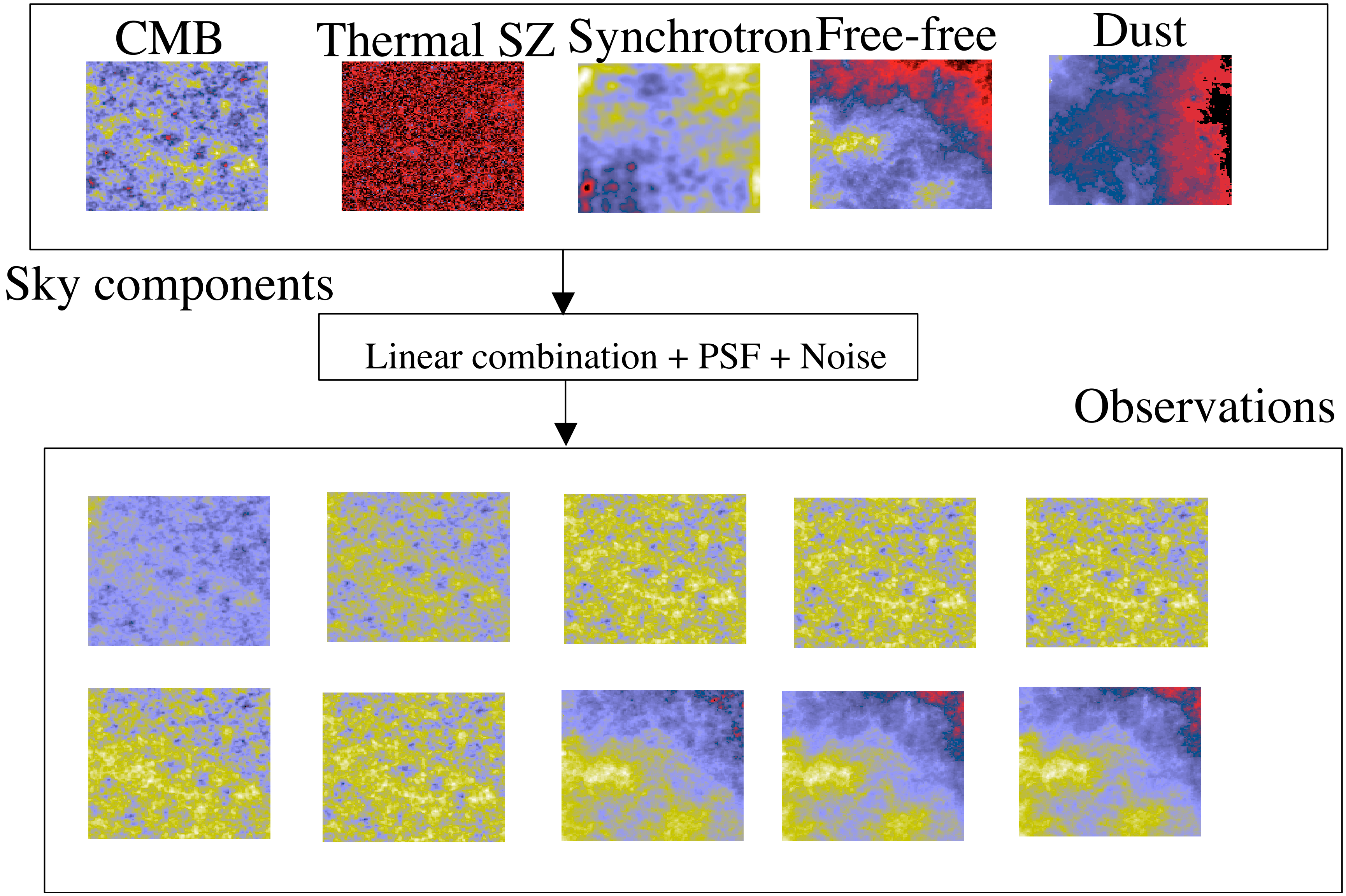






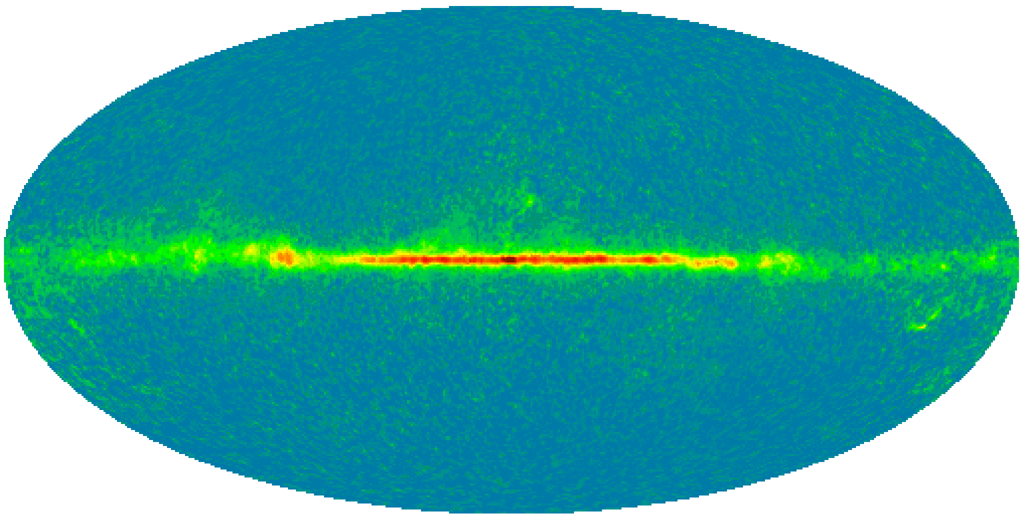


Synchrotron emission due to cosmic rays electrons accelerated into galactic magnetic fields

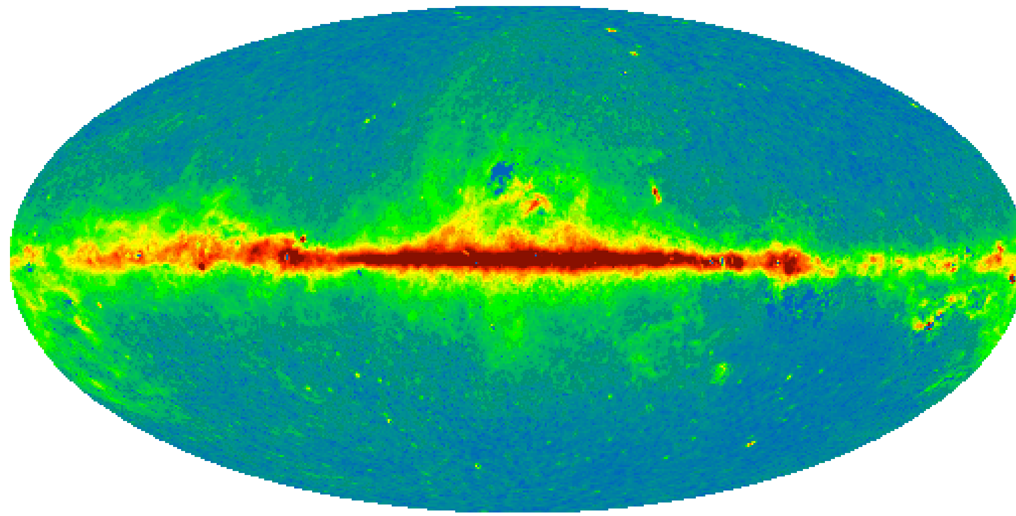


Yassir Moudden et al 2004,
"Blind Source Separation from Gapped Data in Wavelet Space: Application to CMB Analysis, *submitted to Eurasip Journal on Applied Signal Processing, special issue on Applications of Signal Processing in Astrophysics and Cosmology.*

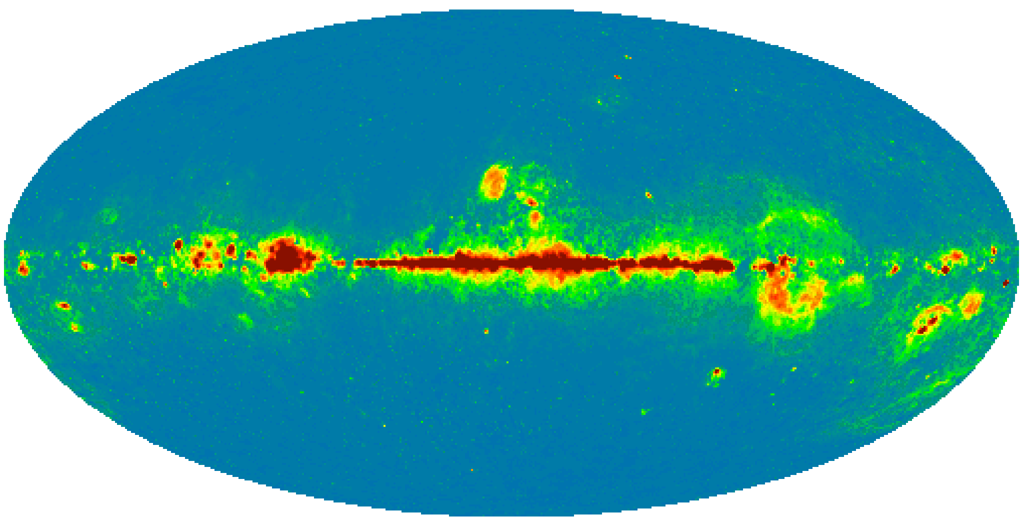
WMAP derived sky maps



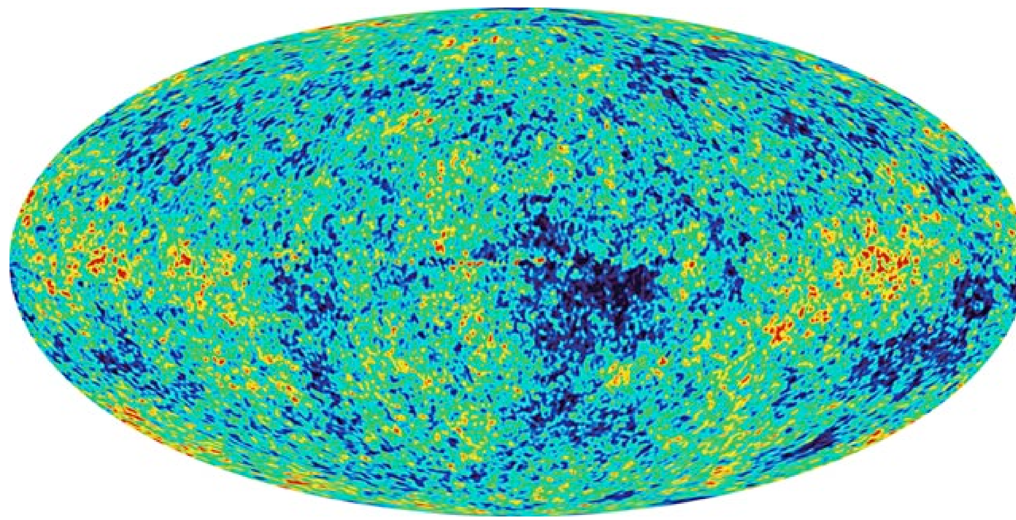
Dust map



Synchrotron map

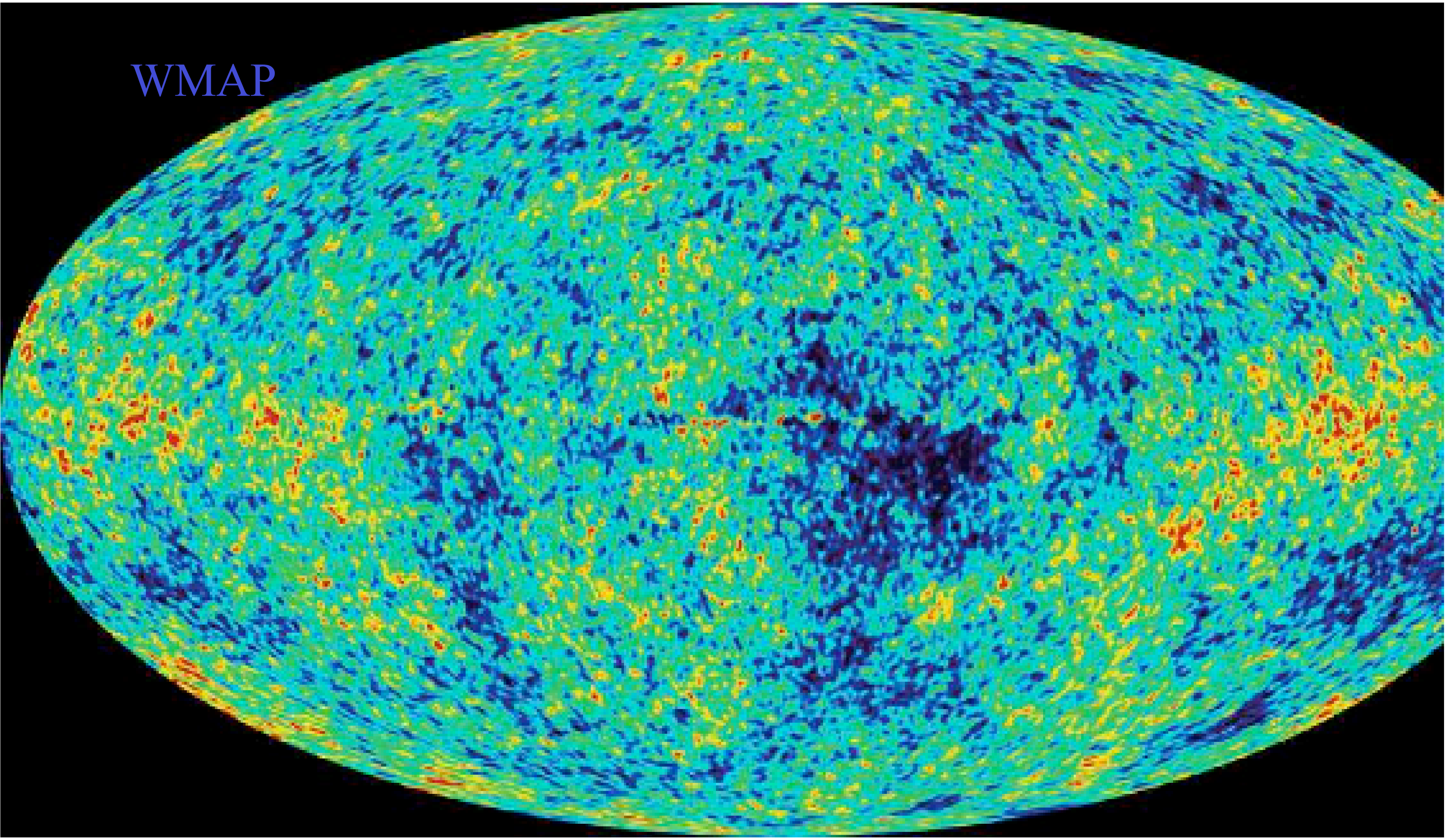


Free-free map

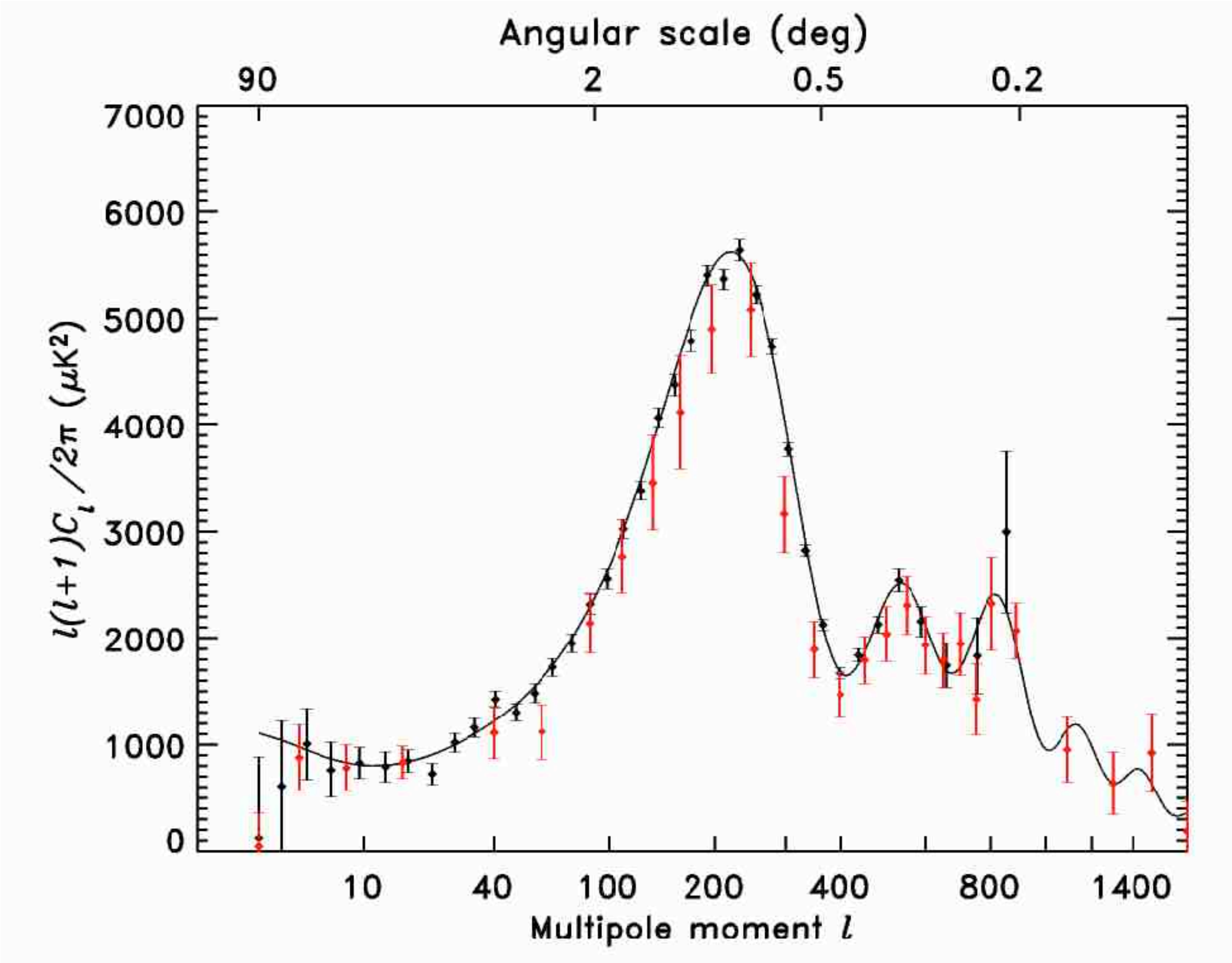


CMB map

The CMB exhibits Fluctuations

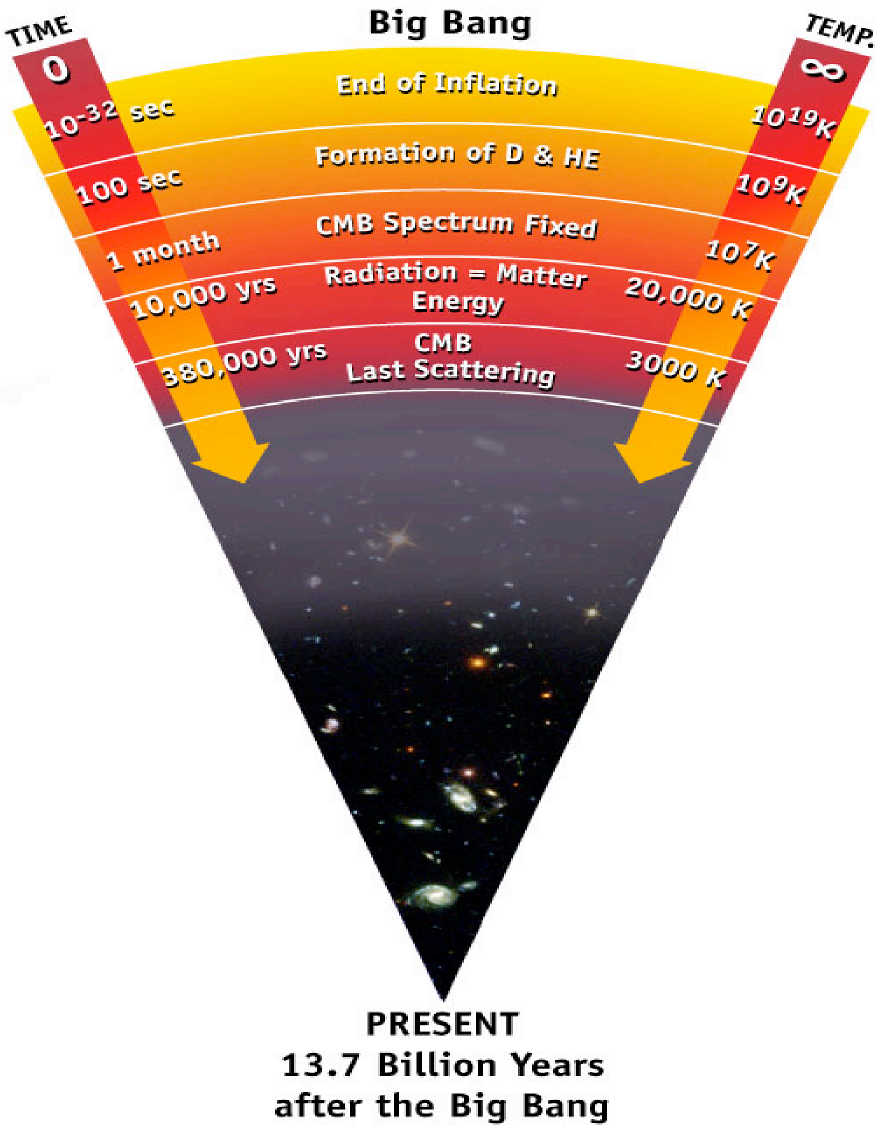


Power spectrum of WMAP

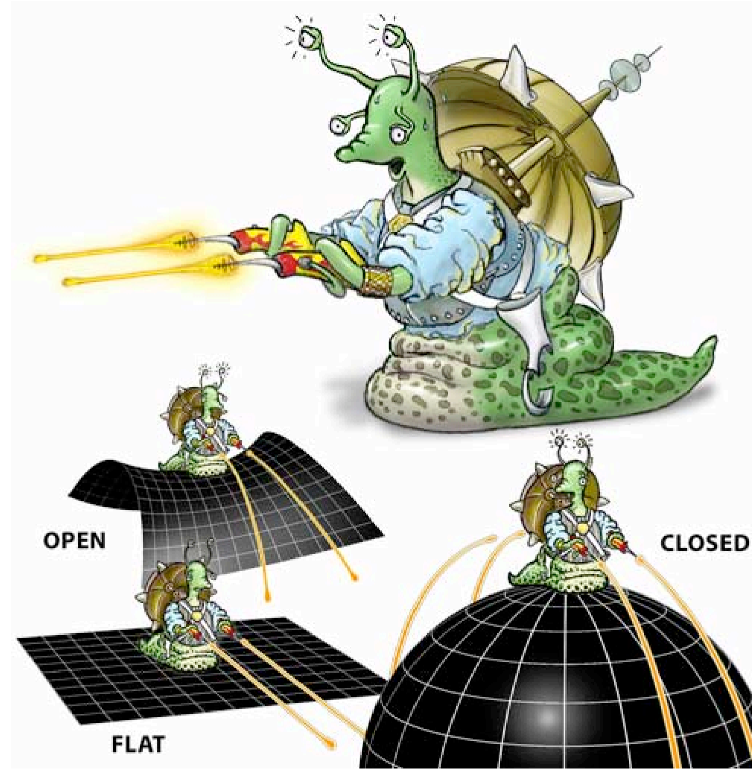


Remarkably consistent with earlier data

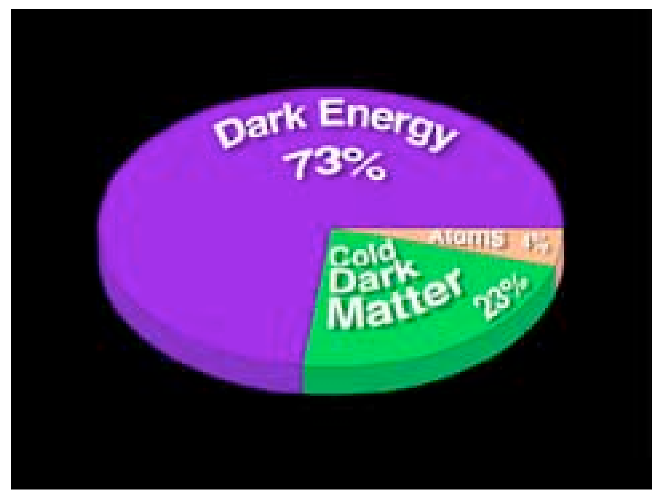
COSMOLOGY



TIME
Big Bang
TEMP.
0
∞
End of Inflation
10-32 sec
1019K
Formation of D & HE
100 sec
109K
CMB Spectrum Fixed
1 month
107K
Radiation = Matter Energy
10,000 yrs
20,000 K
CMB Last Scattering
380,000 yrs
3000 K
PRESENT
13.7 Billion Years
after the Big Bang



OPEN
CLOSED
FLAT



Dark Energy
73%
Atoms
Cold Dark Matter
23%

2004: The current situation:

The triumph of dark stuff in cosmology

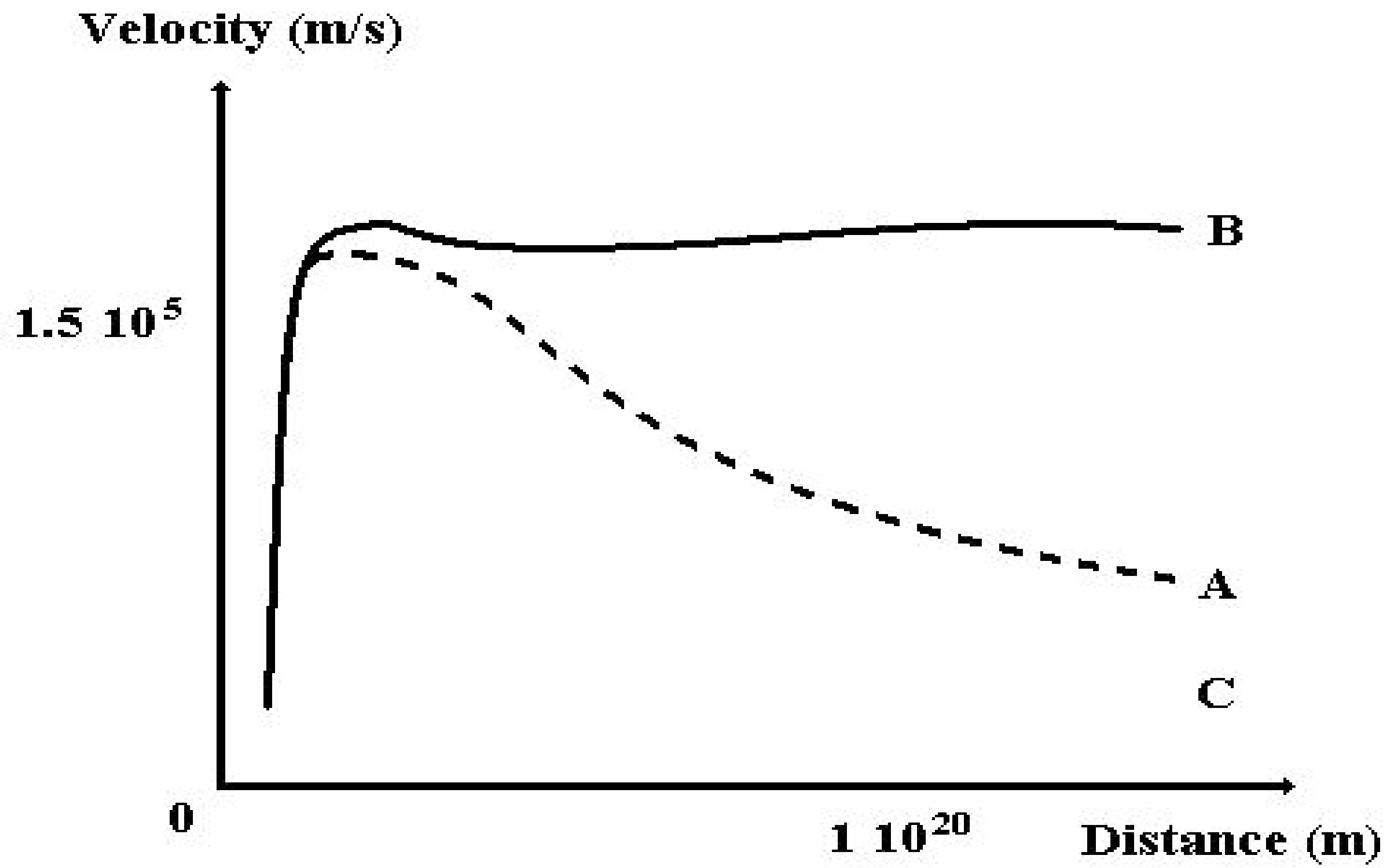
Visible matter cannot account for dynamics; ***dominated*** by DM

Cosmic energy budget:

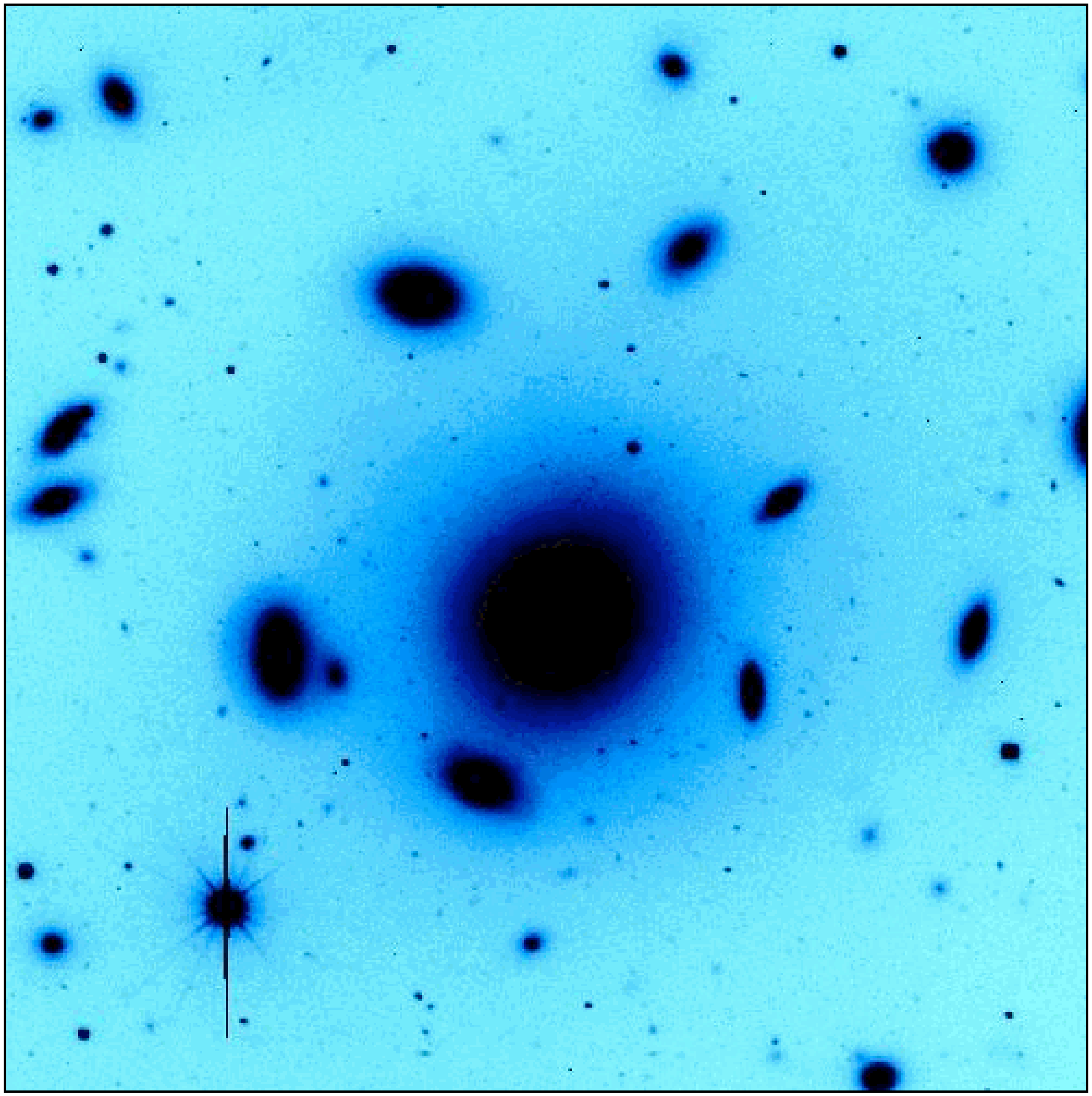
- $\Omega_{\mathrm{visible}} \sim 0.004$
- $\Omega_{\mathrm{baryon}} \sim 0.04$
- $\Omega_{\mathrm{dark\ matter}} \sim 0.3$
- $\Omega_{\mathrm{dark\ energy}} \sim 0.7$

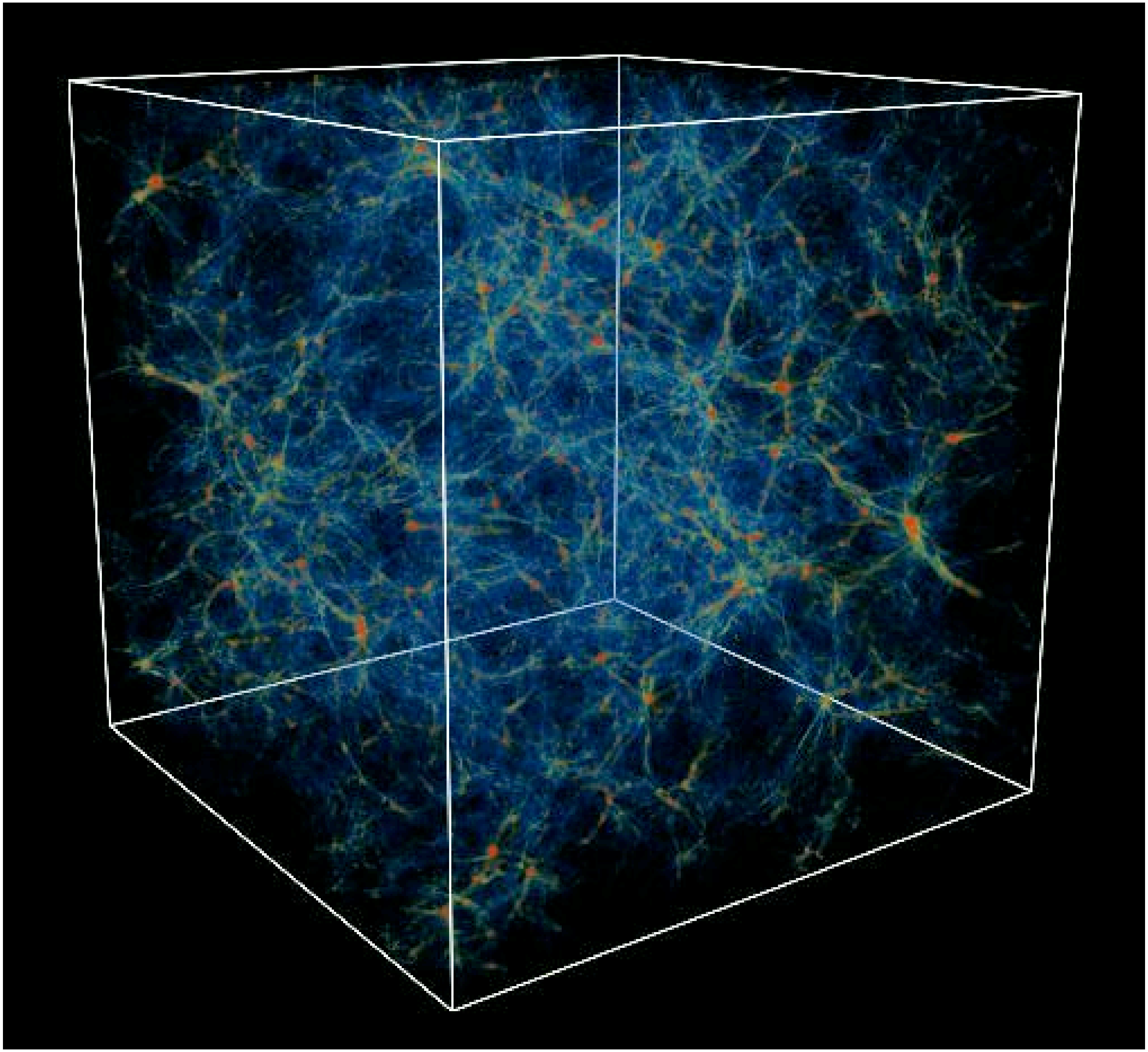
Dark matter/energy, once accepted, works extremely well on large scales.

- Flattened Rotation Curves
- Two Explanations
  - CDM
  - Modify gravity



Clusters of galaxies





Evidence for dark matter

- Rotation curves of galaxies
- Cluster of galaxies
- Inflation predicts Omega=1
- N-body simulation

- **MACHOs** - ***baryonic matter***
(brown dwarfs & black holes)
(**M**assive **A**strophysical **C**ompact **H**alo **O**bjects)
=> NOW EXCLUDED

- **Neutrinos** - ***non baryonic matter***
=> NOW EXCLUDED

- **Les WIMPs** - ***non baryonic matter*** -
(**W**eakly **I**nteractive **M**assive **P**articles)
=> hard to detect

GO BACK TO WORK

PERFORM A DEEP ANALYSIS OF THE DATA

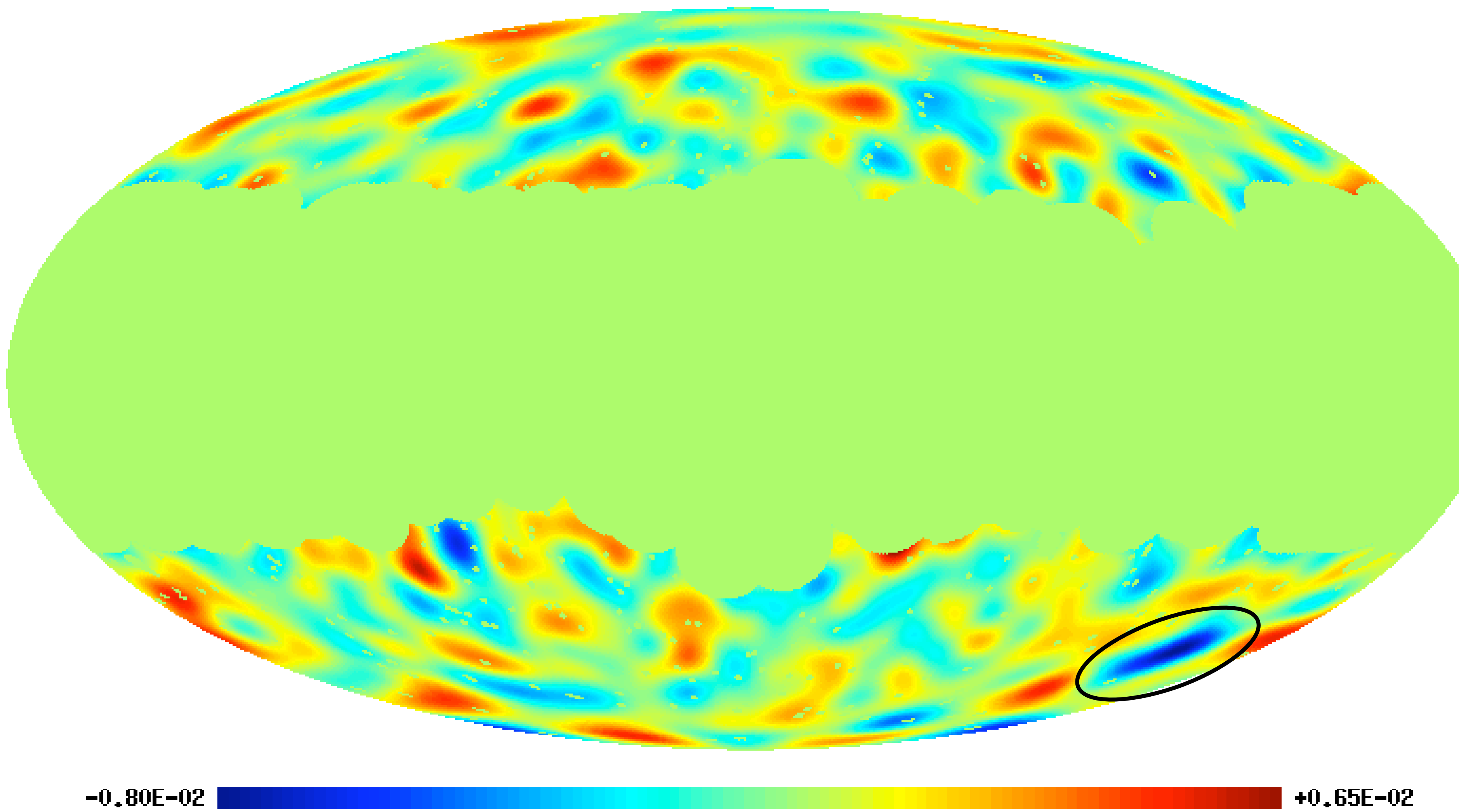
==> to better constrain the models

And try the answer some questions such:

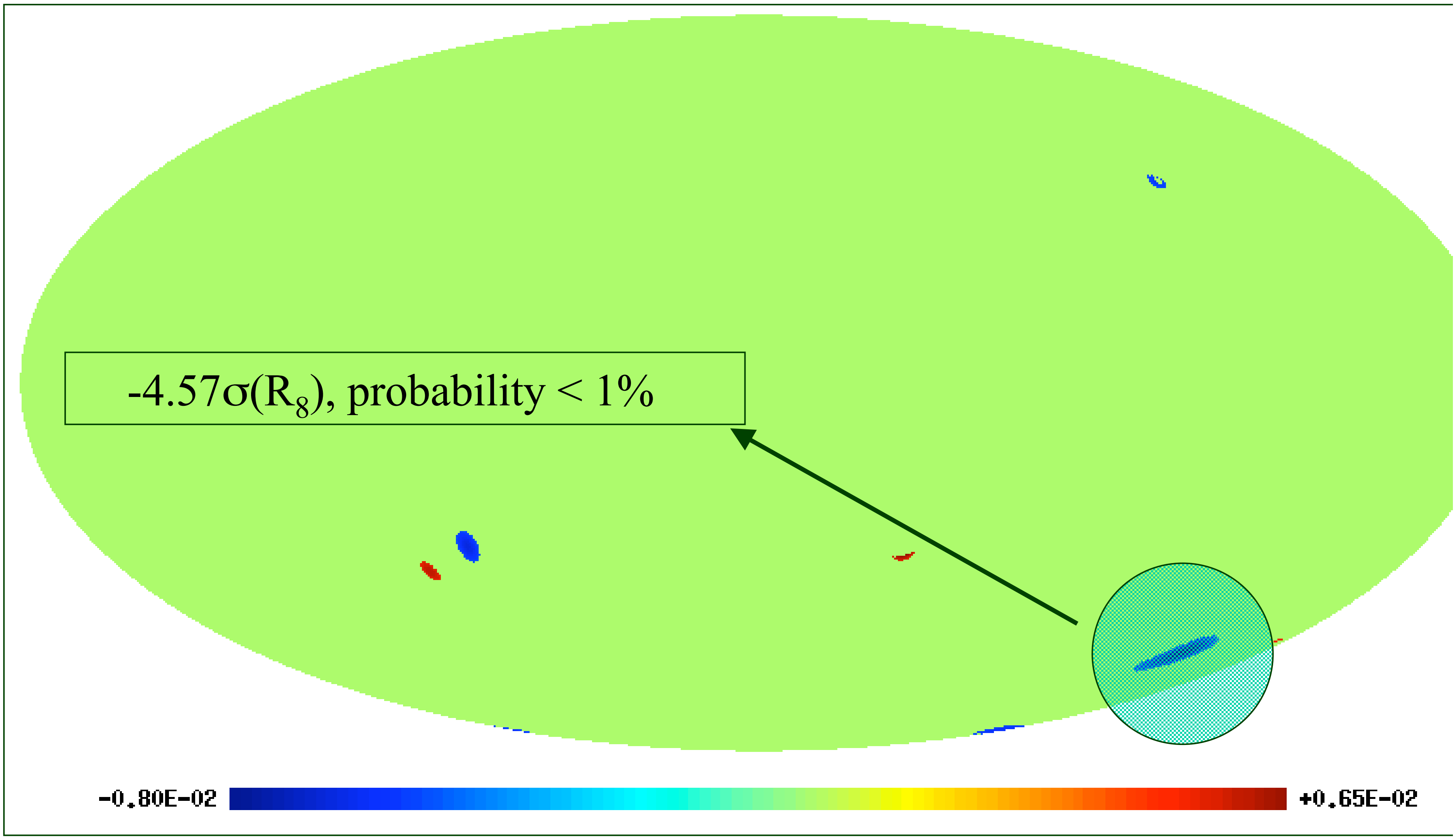
Is the CMB a pure Gaussian Random Field ?
Is the Universe homogeneous at large scales ?
How did the large scale structures form ?
Can we have more information about the dark matter ?

Vielva et al. 2004

SMHW coefficient map at R_8 = 250 arcmin



Vielva et al. 2004



The Cosmic Microwave Background

- The power spectrum gives constraints on the geometry and the physical state of the Universe.

- It supports the hypothesis of a period of rapid expansion of the early Universe: the Inflationary period.

The Primary fluctuations

- This process causes randomly distributed seeds and Gaussian distributed fluctuations.

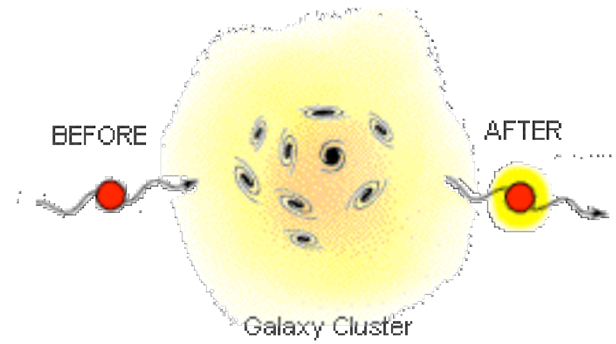
- At the end of this period, topological defects may occur that produce non-Gaussian fluctuations e.g. Cosmic Strings.

The Secondary fluctuations

The secondary fluctuations arise from the interaction of the CMB photons with the cluster of galaxies. It is the Sunyaev Zel'dovich (SZ) effect.

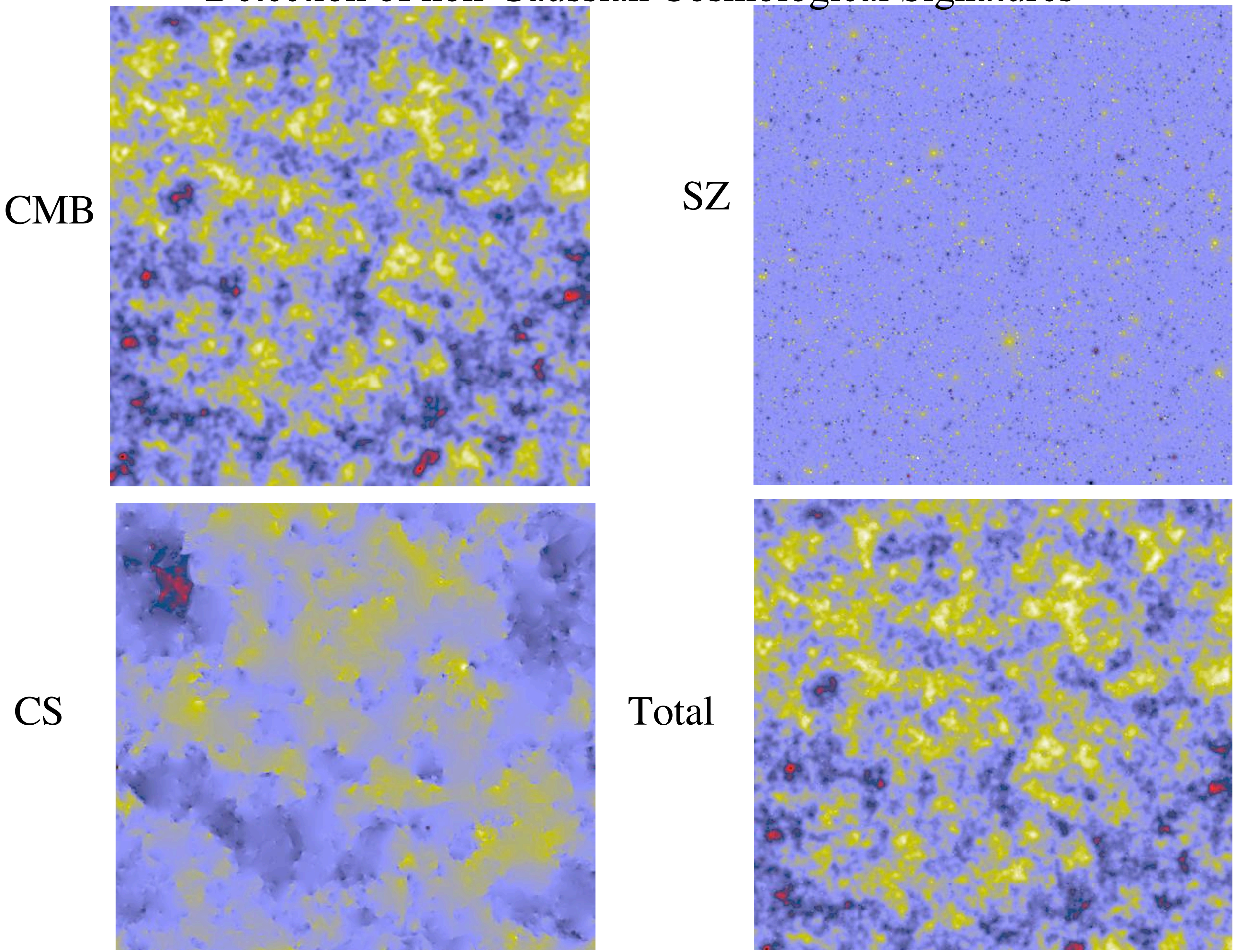
The Sunyaev Zel'dovich effect

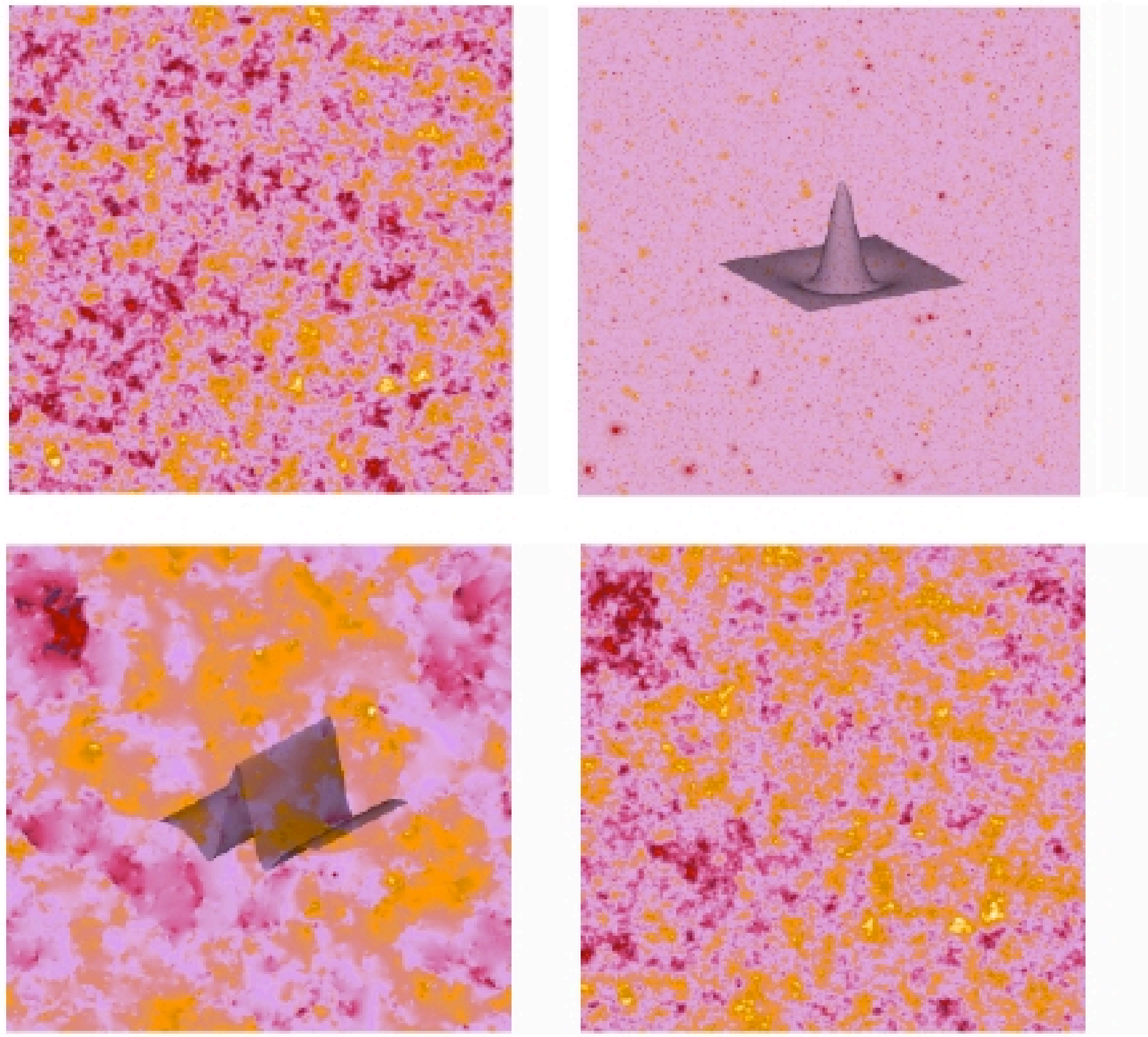
- First predicted by the Russian scientists Sunyaev and Zel'dovich in 1969.
- Galaxy Clusters have hot gas
  - T_{gas}~10-100 million Kelvin
  - Electron scattering from nuclei produces X-rays, thermal bremsstahlung.
- Compton scattering occurs between CMB photons and the hot electrons.
  - ~1% of CMB photons will interact with the hot electrons
    - Energy will be transferred from the hot electrons to the low energy CMB photons, changing the shape of their intensity vs. frequency plot.
      - Measurements made at low frequencies will have a lower intensity, since photons which originally had these energies were scattered to higher energies. This distorts the spectrum by ~0.1%.



The SZ effect comes from the interaction of the cold CMB photons with the hot electrons (TSZ) of moving (KSZ) galaxy clusters

Detection of non-Gaussian Cosmological Signatures

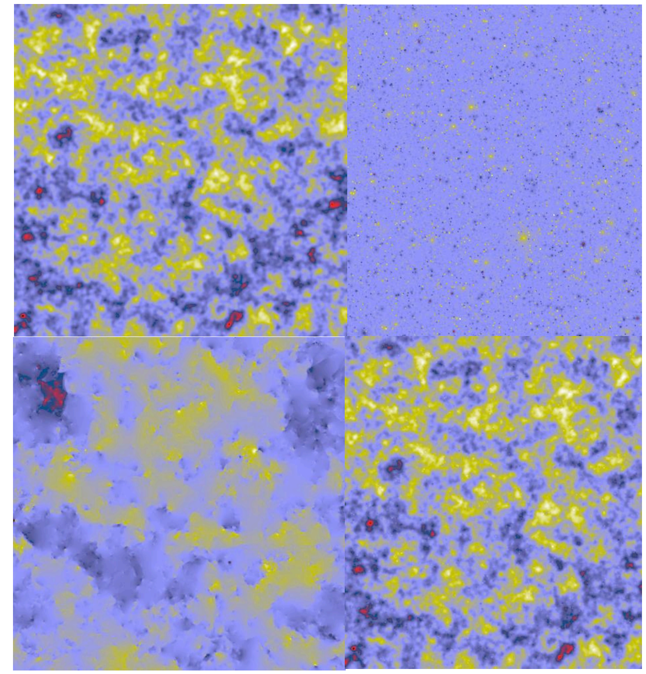




Multiscale Analysis of the CMB

We have applied the following multiscale transforms

- Isotropic wavelet transform
- Bi-orthogonal wavelet transform
- Ridgelets (block size of 16 pixels)
- Ridgelets (block size of 32 pixels)
- Curvelets



On

1) 100 **CMB + KSZ** + 100 Gaussian realizations with the same power spectrum.

$$K_{CMB-SZ}(i,b) \Rightarrow K_{CMB-SZ}(b) = mean(K_{CMB-SZ}(1..100,b)), \overline{K}_{CMB-SZ}(b) = \frac{K_{CMB-SZ}(b)}{K_{CMB}(b)}$$

2) 100 **CMB + CS** + 100 Gaussian realizations with the same power spectrum

3) 100 **CMB + KSZ + CS** + 100 Gaussian realizations with the same power spectrum

We compare the normalized kurtosis for the three data set.

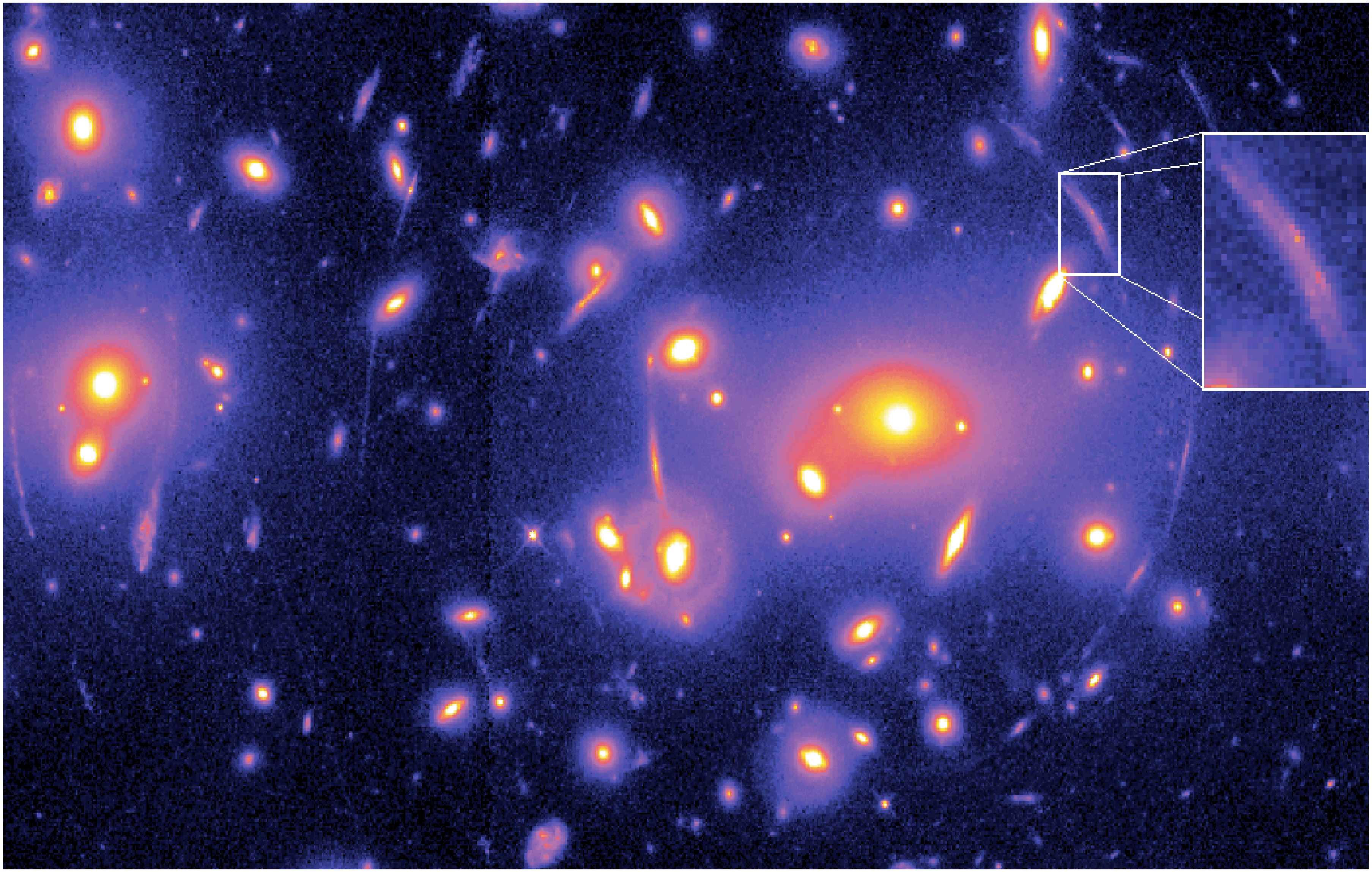
Results

• **Curvelets are NOT sensitive to KSZ but *are* sensitive to cosmic strings**

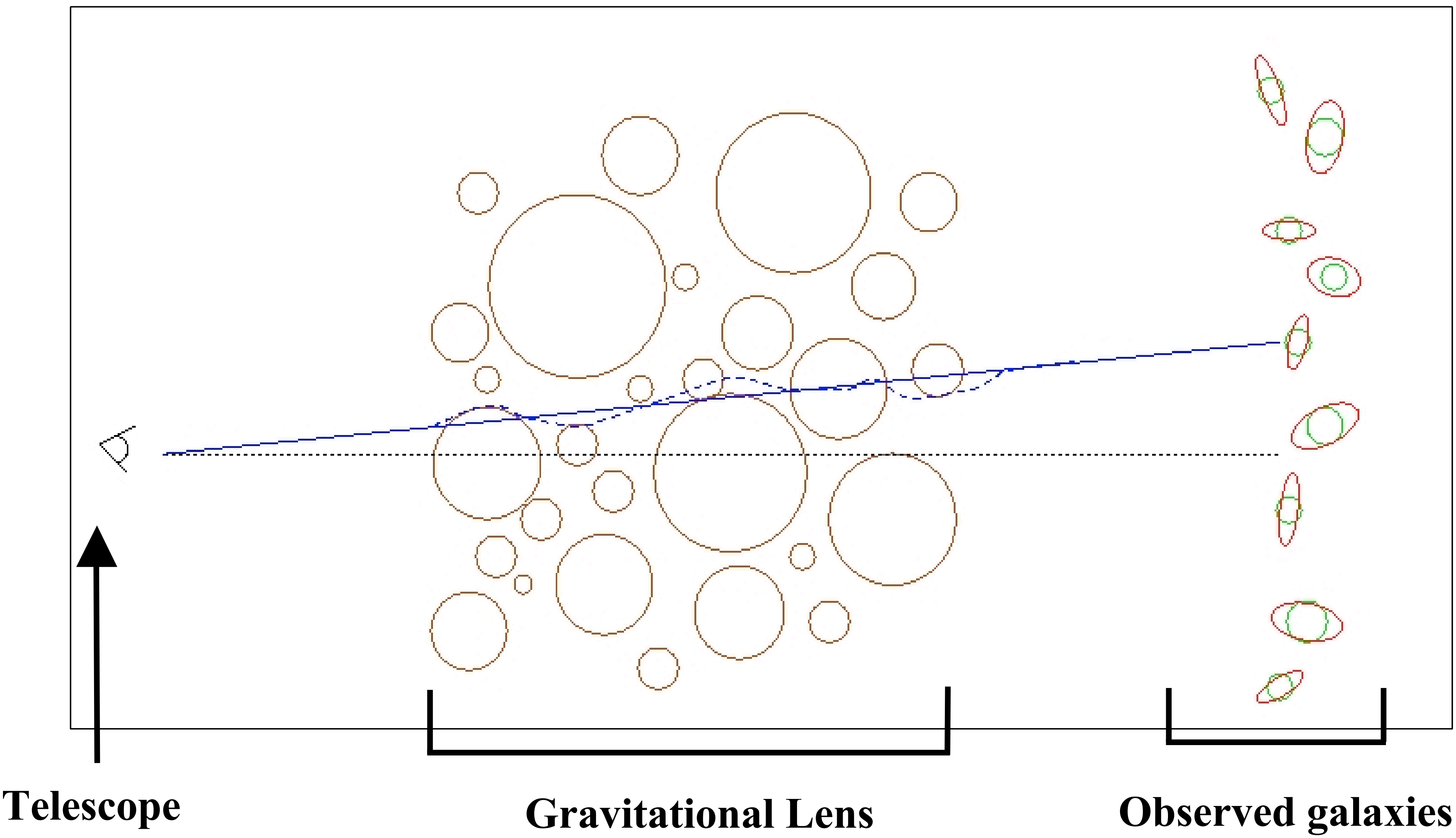
	Bi-orthogonal WT	Ridgelet	Curvelet
CMB+KSZ	1106.	0.1	10.12
CMB+CS	1813.	5.7	198.
CMB+CS+KSZ	1040.	5.9	165.

Detecting cosmological non-Gaussian signatures by multi-scale methods, *Astron. and Astrophys., 416, 9--17, 2004* .

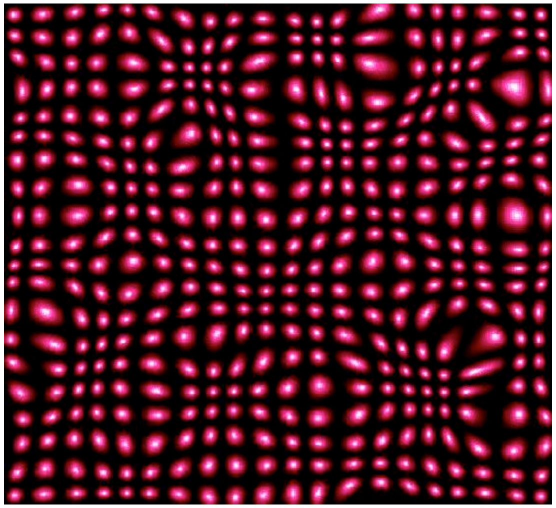
Strong Gravitational Lensing



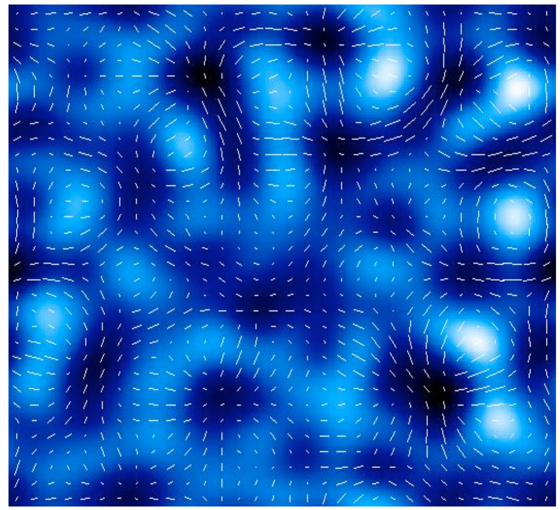
Weak Gravitational Lensing



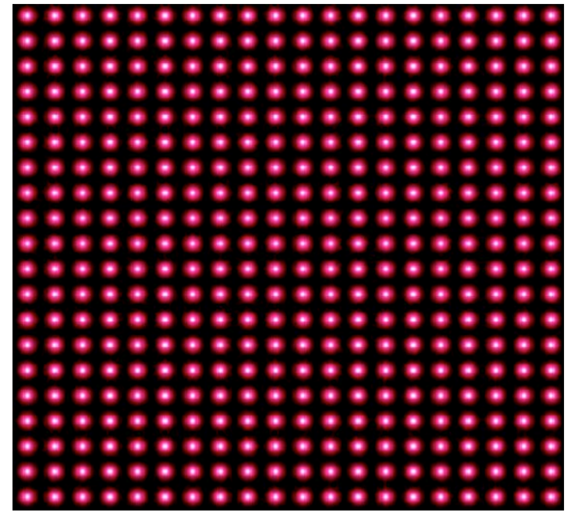
Weak Gravitational Lens



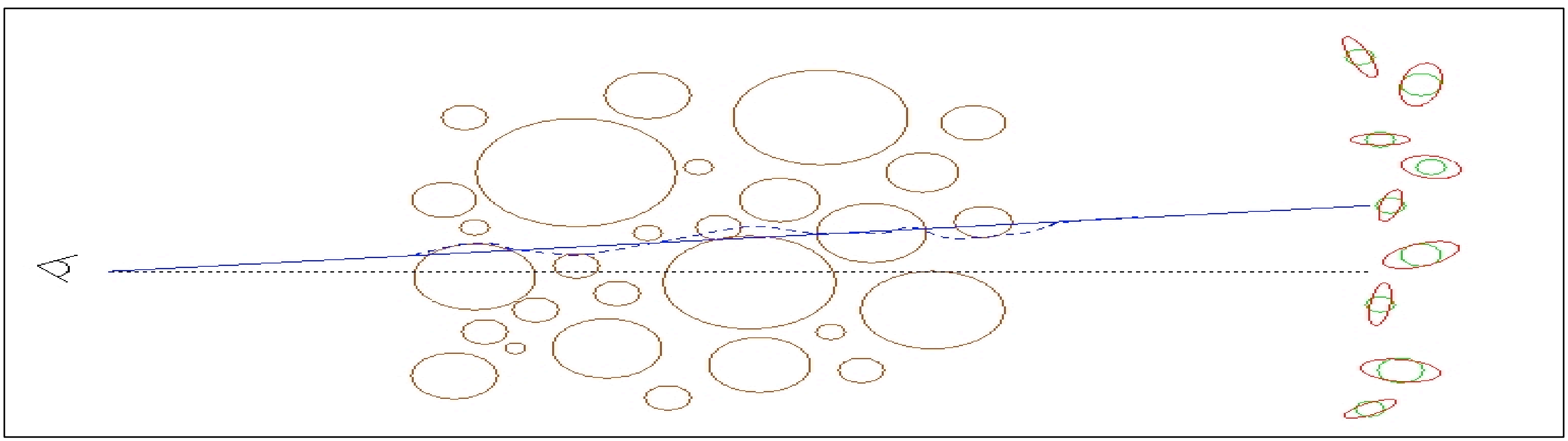
Observation



Dark Mass Map



Original field of galaxies

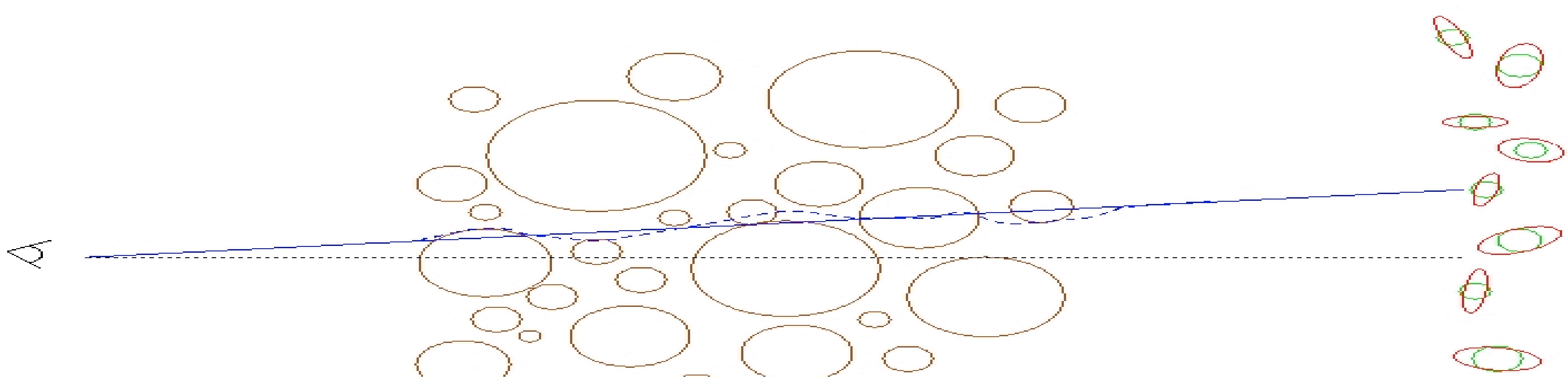


Weak Lensing by Large-Scale Structure

Distortion Matrix Ψ

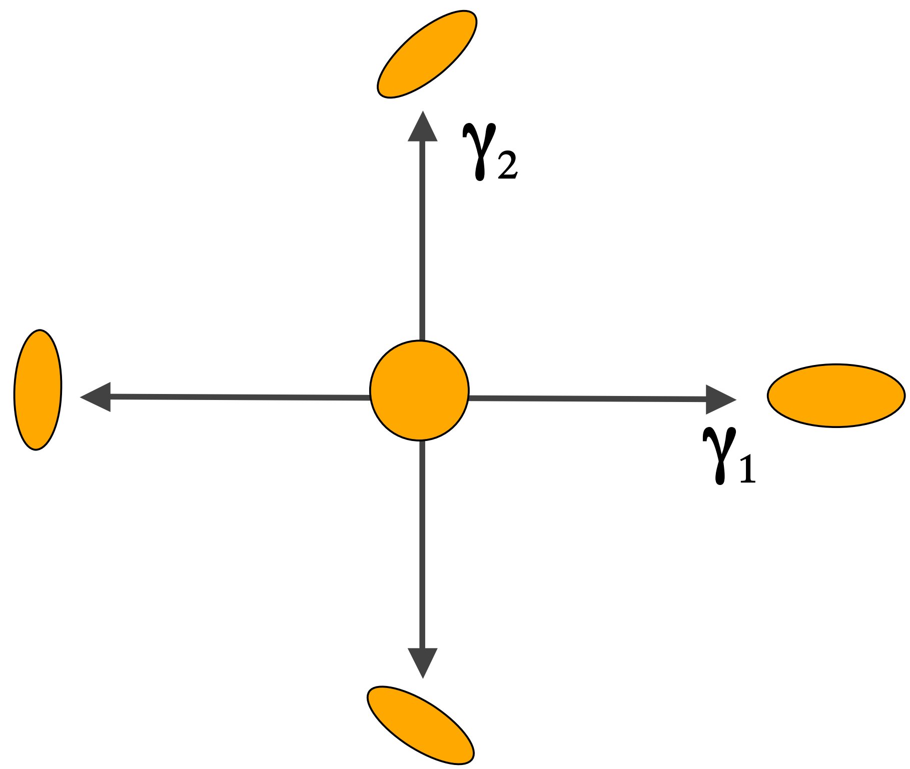
$$\Psi_{ij} = \frac{\partial \delta\theta_i}{\partial \theta_j} = \int dz\, g(z) \frac{\partial^2 \Phi}{\partial\theta_i \partial\theta_j} = \begin{pmatrix} \kappa + \gamma_1 & \gamma_2 \\ \gamma_2 & \kappa - \gamma_1 \end{pmatrix}$$

$\delta\theta_i$ is the deflexion vector produced by lensing on the sky.
Φ is the Newtonian potential, **z** is the distance,
the weight function **g** reflects the fact that a lens is more effective when placed approximately half-way between the source and the observer,
κ is proportial to the projected mass along the line of sight.
The shear (γ_1, γ_2) describes stretches and compression.



The shear map (γ_1, γ_2)

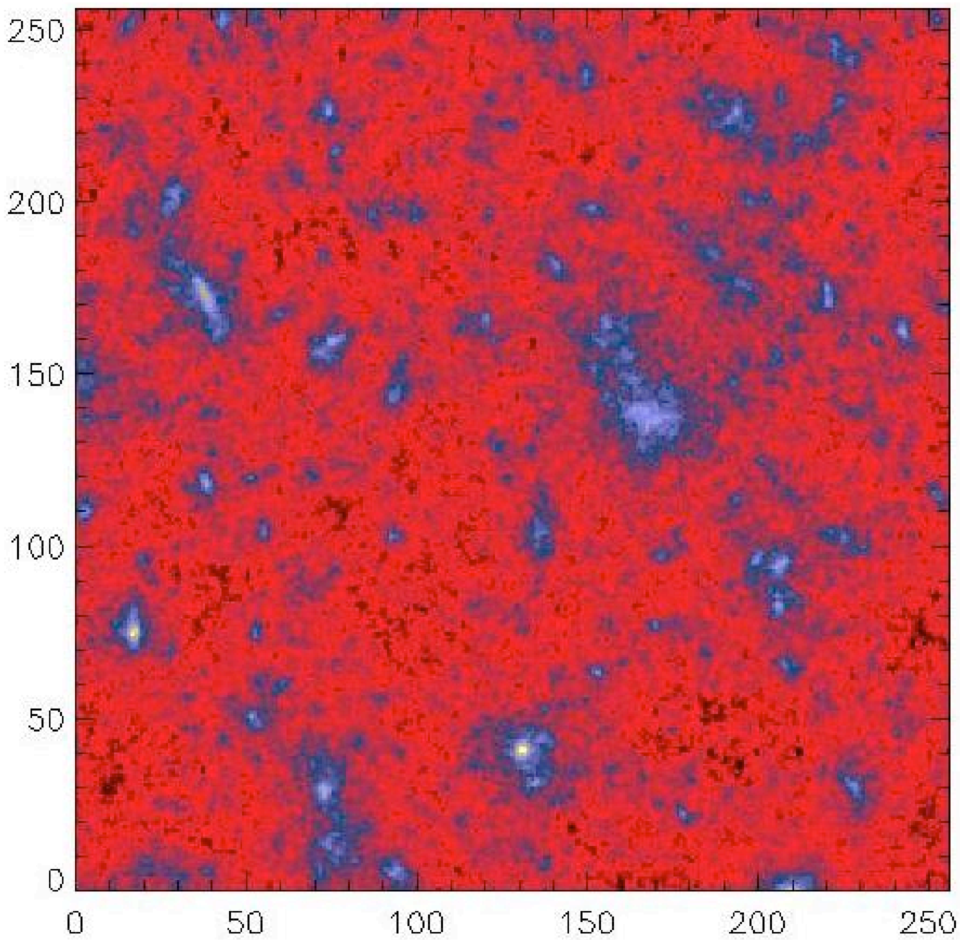
γ_1 = deformation along the x-axis,
and γ_2 at 45 degres from it.

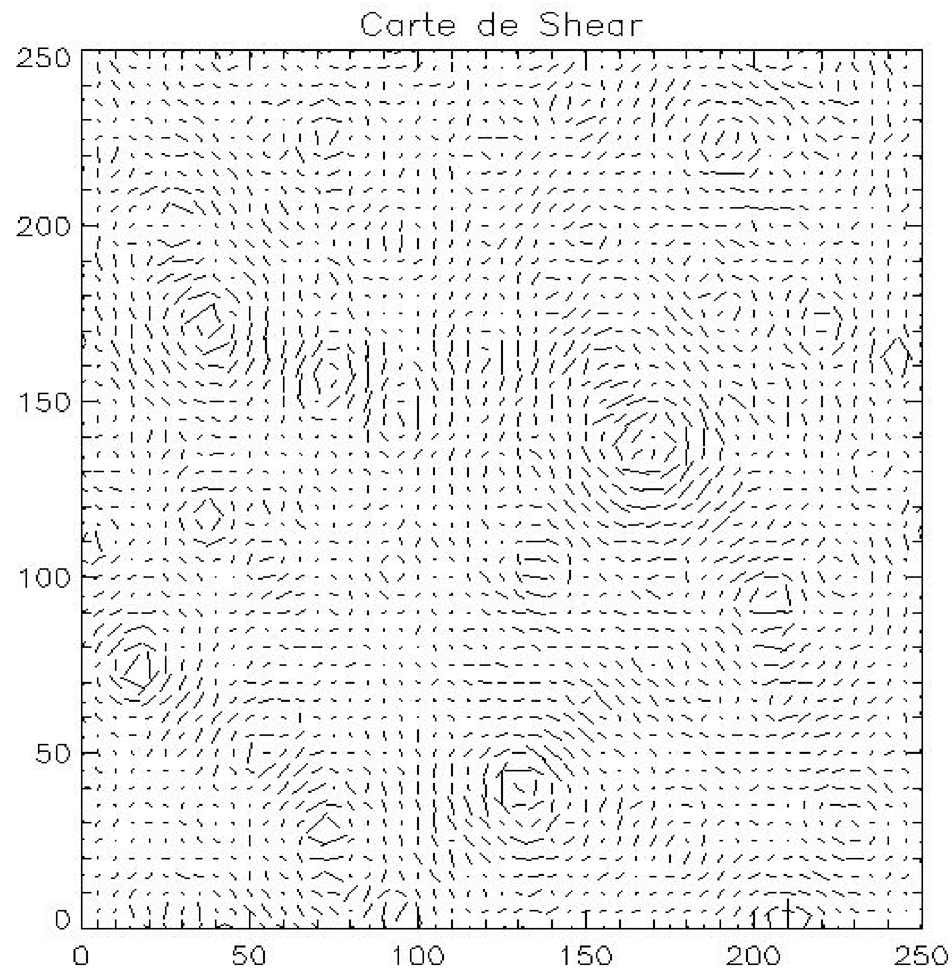


$$\gamma = \gamma_1 + i\gamma_2 = |\gamma| e^{2i\theta}$$

Where the modulus represents the amount of shear and the phase represents its direction.

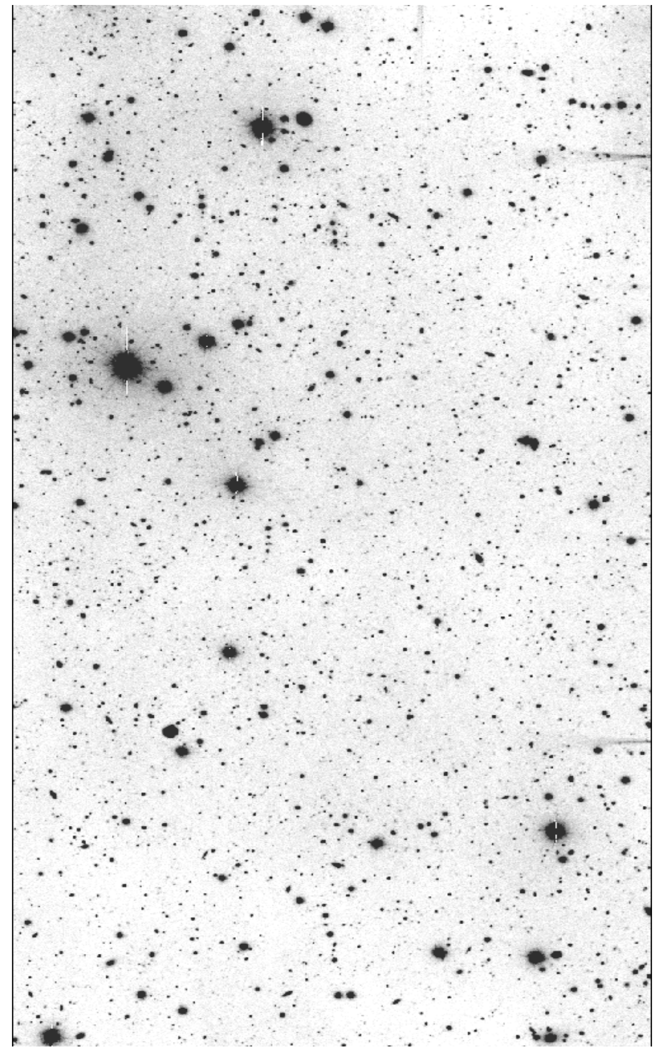
Simulated Mass Map
& Related Shear Mass Map





Deep Optical Images

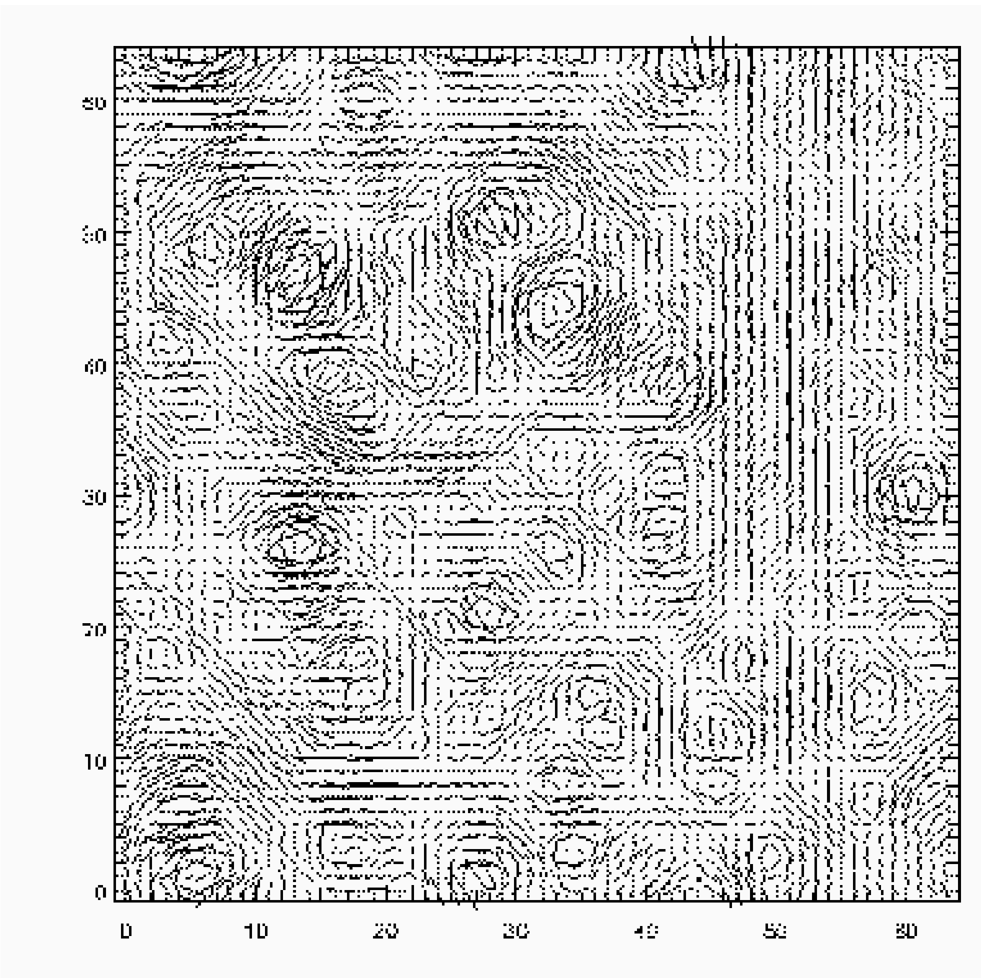
William Herschel Telescope
La Palma, Canaries



16'x8'
R<25.5
30 (15) gals/sq. arcmin

→**Direct measure of the distribution of mass in the universe, as opposed to the distribution of light, as in other methods (eg. Galaxy surveys)**

From the statistics of the shear field, weak lensing provides:



1x1 deg

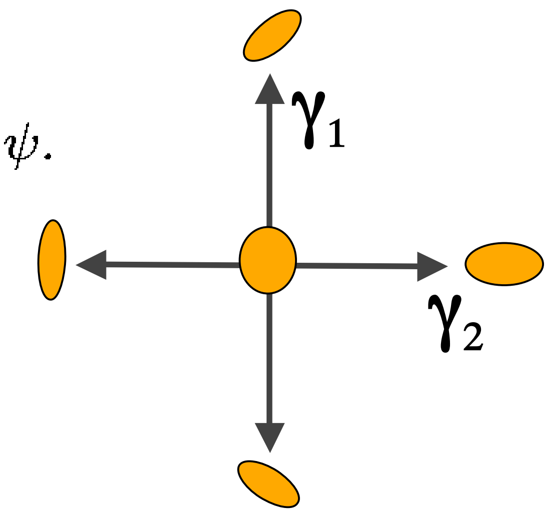
- **Mapping of the distribution of Dark Matter on various scales**
- **Measurement of cosmological parameters.**
- **Measurement of the evolution of structures**
- **a mass-selected cluster catalog**

Relation between the shear maps and the mass map κ

The relation between the weak shear maps γ_1, γ_2 and the mass map κ are:

$$\begin{aligned}
\gamma_1 &= (\partial_1 - \partial_2)\psi \\
\gamma_2 &= 2\partial_1\partial_2\psi
\end{aligned}$$

where ψ is the potential function defined by $\kappa = \nabla^2\psi$.



In Fourier Space

$$\begin{aligned}
\hat{\kappa}(k_1, k_2) &= k^2\hat{\psi}(k_1, k_2) \\
\hat{\gamma_1}(k_1, k_2) &= \frac{k_1^2 - k_2^2}{k^2}\hat{\kappa}(k_1, k_2) \\
\hat{\gamma_2}(k_1, k_2) &= \frac{2k_1k_2}{k^2}\hat{\kappa}(k_1, k_2)
\end{aligned}$$

with $k^2 = k_1^2 + k_2^2$.

An inverse problem

Noting $\hat{P}_1(k_1, k_2) = \frac{k_1^2 - k_2^2}{k^2}$ (with $\hat{P}_1(k_1, k_2) = 0$ when $k_1^2 = k_2^2$) and $\hat{P}_2(k_1, k_2) = \frac{2k_1 k_2}{k^2}$ (with $\hat{P}_2(k_1, k_2) = 0$ when $k_1 = 0$ or $k_2 = 0$), the mass reconstruction consists in **searching κ such that it verifies both $\gamma_1 = P_1 * \kappa$ and $\gamma_2 = P_2 * \kappa$**. In practice, γ_1 and γ_2 are obtained through observations and are contaminated by noise. Then the relations between the observed data γ_{1b}, γ_{2b} and the true mass map κ are given by:

$$\left\{ \begin{array}{lcl} \gamma_{1b} & = & P_1 * \kappa + N_1 \\ \gamma_{2b} & = & P_2 * \kappa + N_2 \end{array} \right.$$

The Inverse Filter: E and B mode

Noticing that $\hat{P_1}^2 + \hat{P_2}^2 = 1$, the least square estimation $\hat{\tilde{\kappa}}_l^{(E)}$ is:

$$\hat{\tilde{\kappa}}_l^{(E)} = \hat{P}_1 \hat{\gamma}_{1b} + \hat{P}_2 \hat{\gamma}_{2b}$$

The relation between this estimation and the true mass map is $\hat{\tilde{\kappa}}_l = \hat{\kappa} + \hat{N}$, where $\hat{N} = \hat{P}_1 \hat{N}_1 + \hat{P}_2 \hat{N}_2$. Another interesting feature is the term $\hat{\tilde{\kappa}}_l^{(B)} = P_2 * \gamma_{1b} - P_1 * \gamma_{2b}$. Indeed it should be free of any contamination from κ and can be used as a test for data quality estimation.

Therefore, the **so called E and B mode** are obtained by:

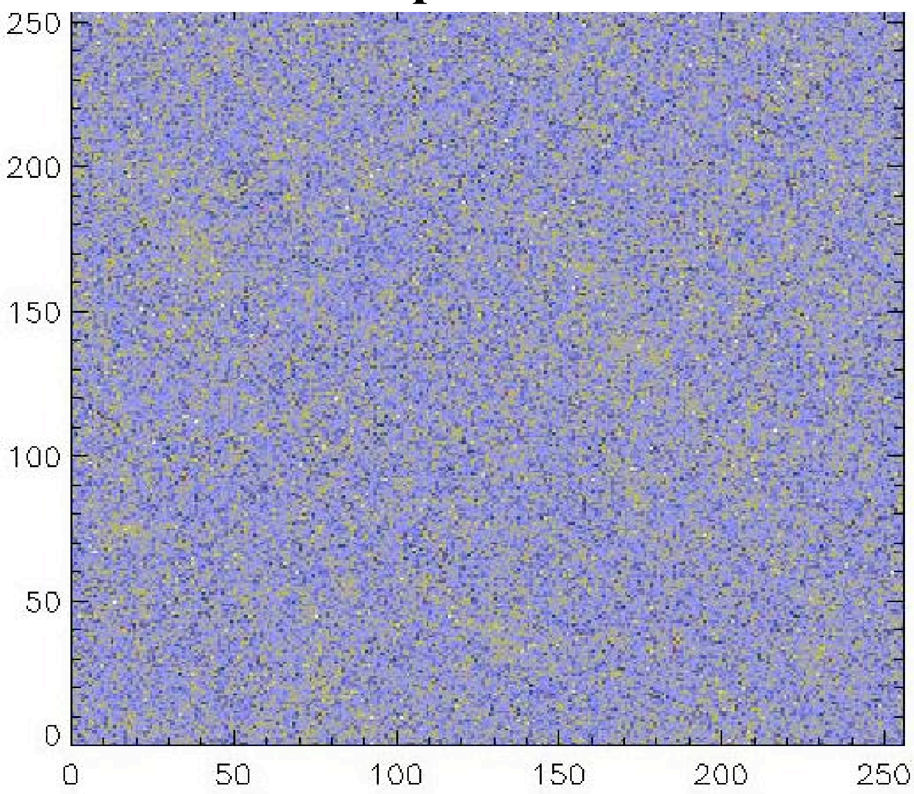
$$\hat{\tilde{\kappa}}_l^{(E)} = P_1 * \gamma_{1b} + P_2 * \gamma_{2b}$$
$$\hat{\tilde{\kappa}}_l^{(B)} = P_2 * \gamma_{1b} - P_1 * \gamma_{2b}$$

Both of them are noisy, and must be filtered before being analysed.

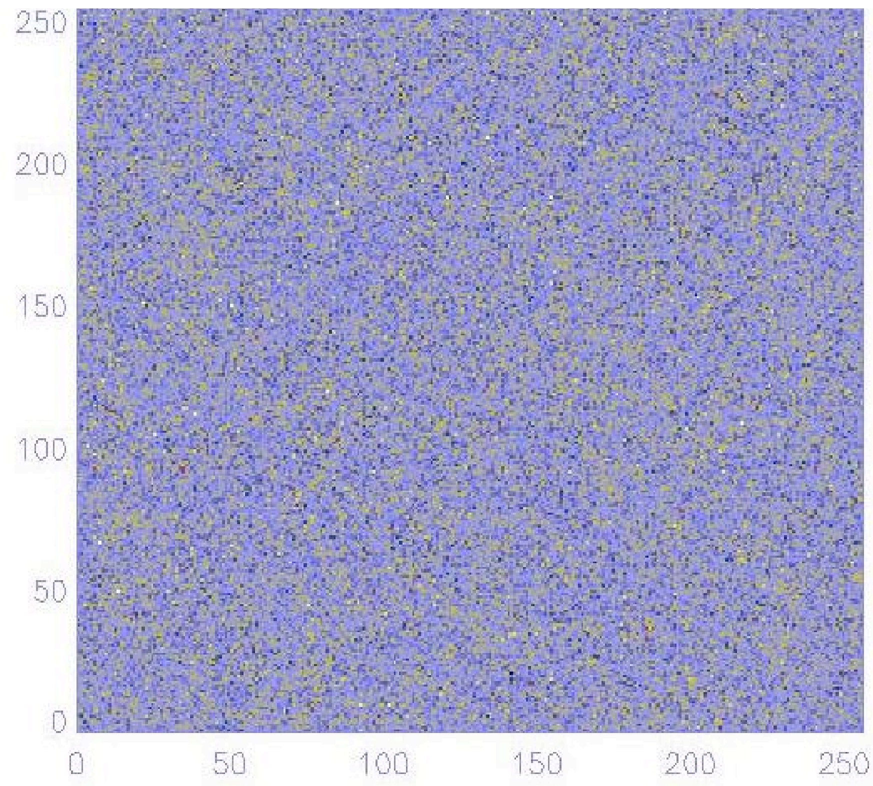
Problem :

maps are very noisy

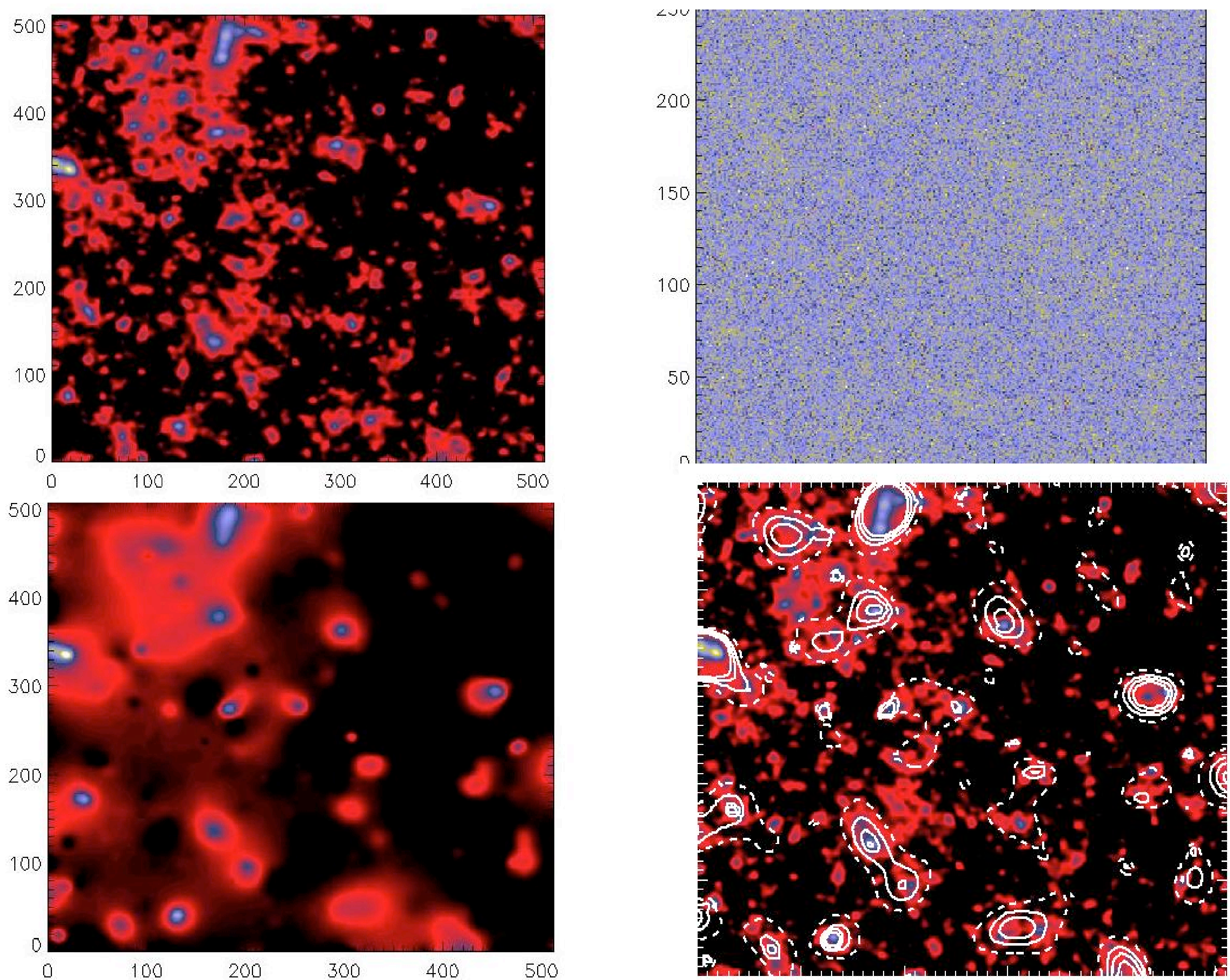
Simulated spatial observation

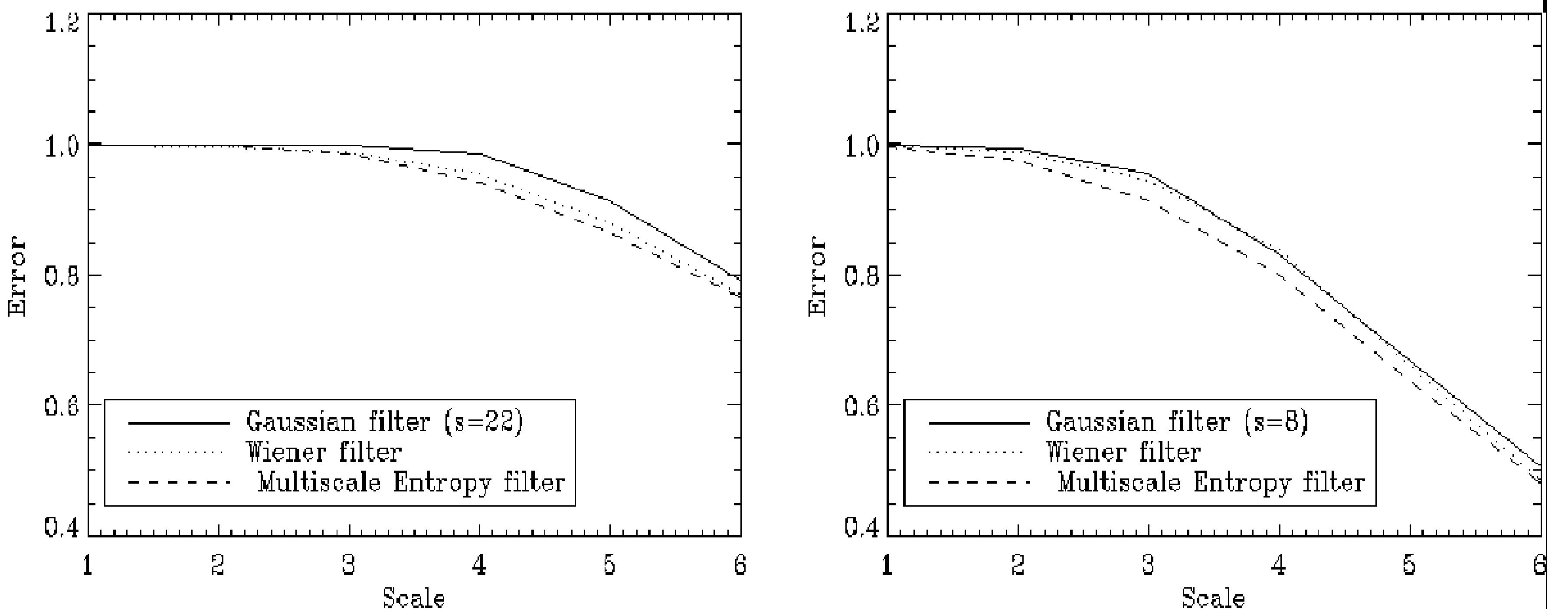


Simulated on ground observation



Reconstructed Dark Matter Map

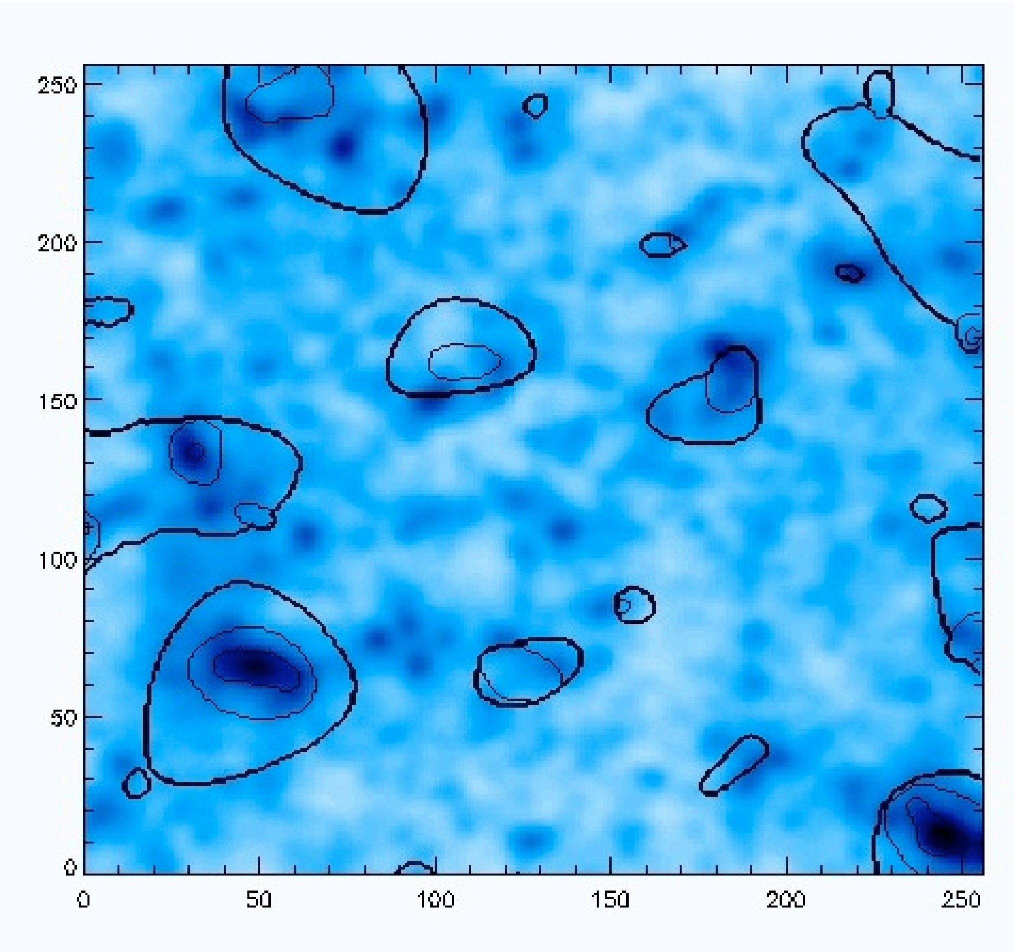




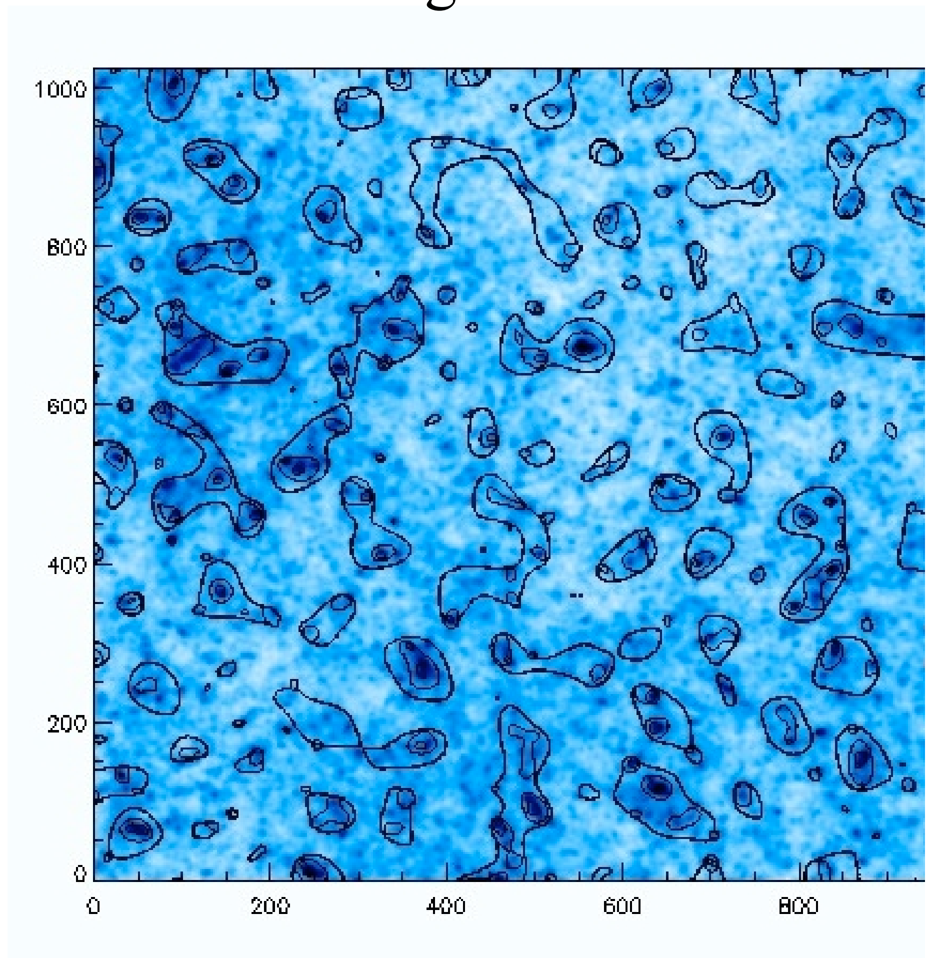
Standard deviation versus scale for the on ground simulation (left) and the spatial simulation (right).

CLUSTER DETECTION

Ng=20



Ng=100



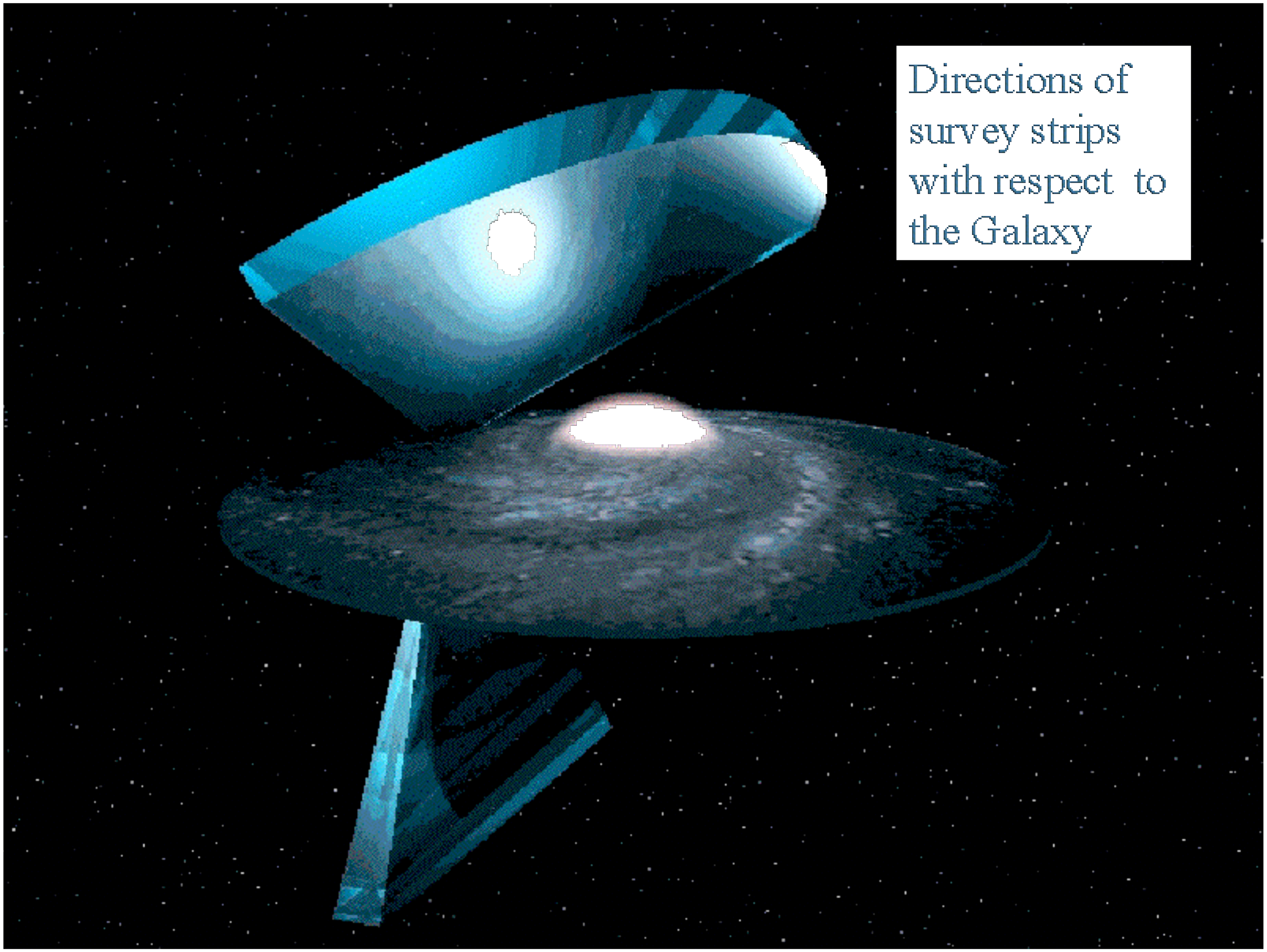
Analysis of catalog of galaxies

The spatial distribution of galaxies allows us to:

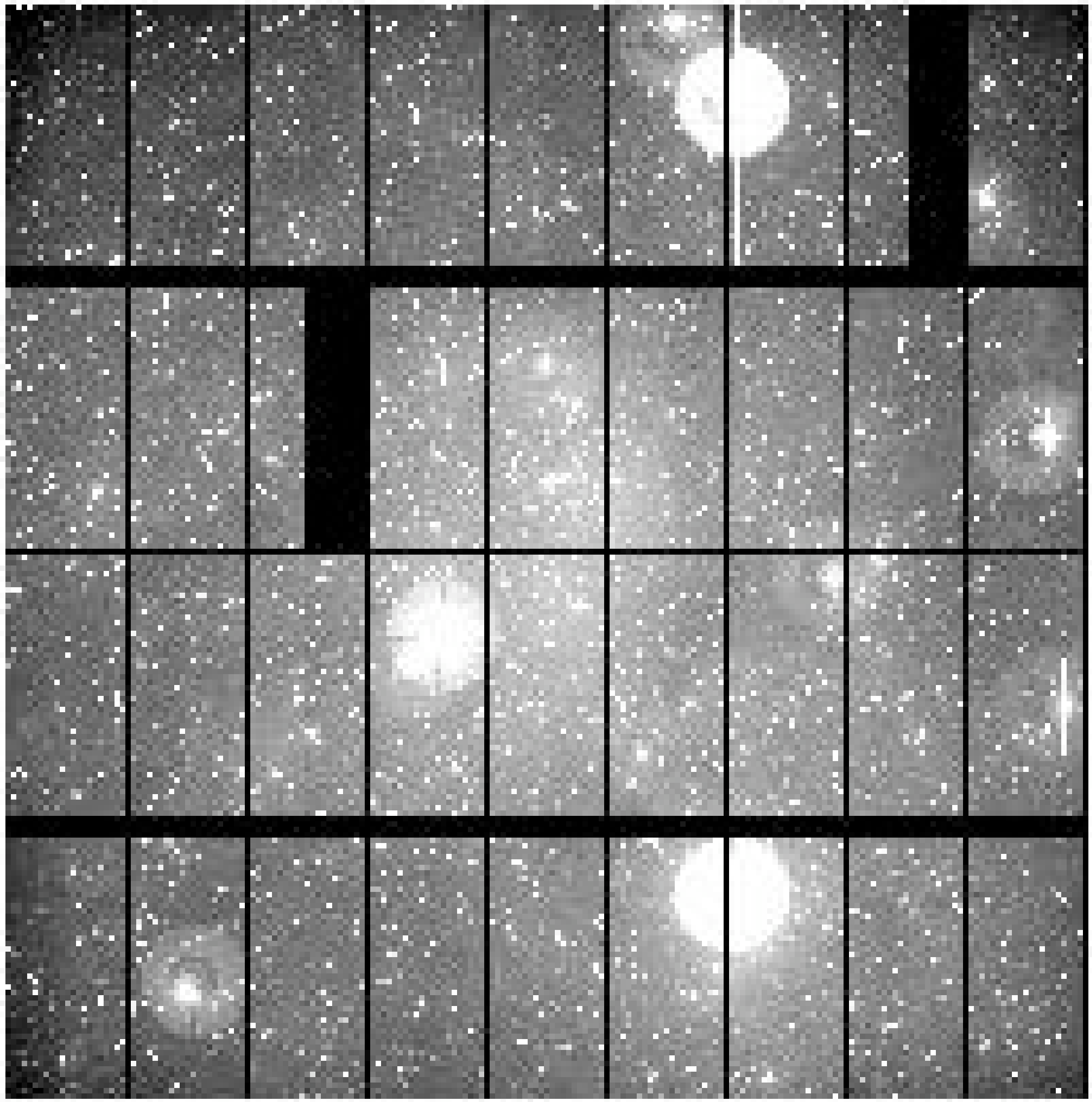
. Check and constraint the cosmological models.

. Study the formation of large scale structures.

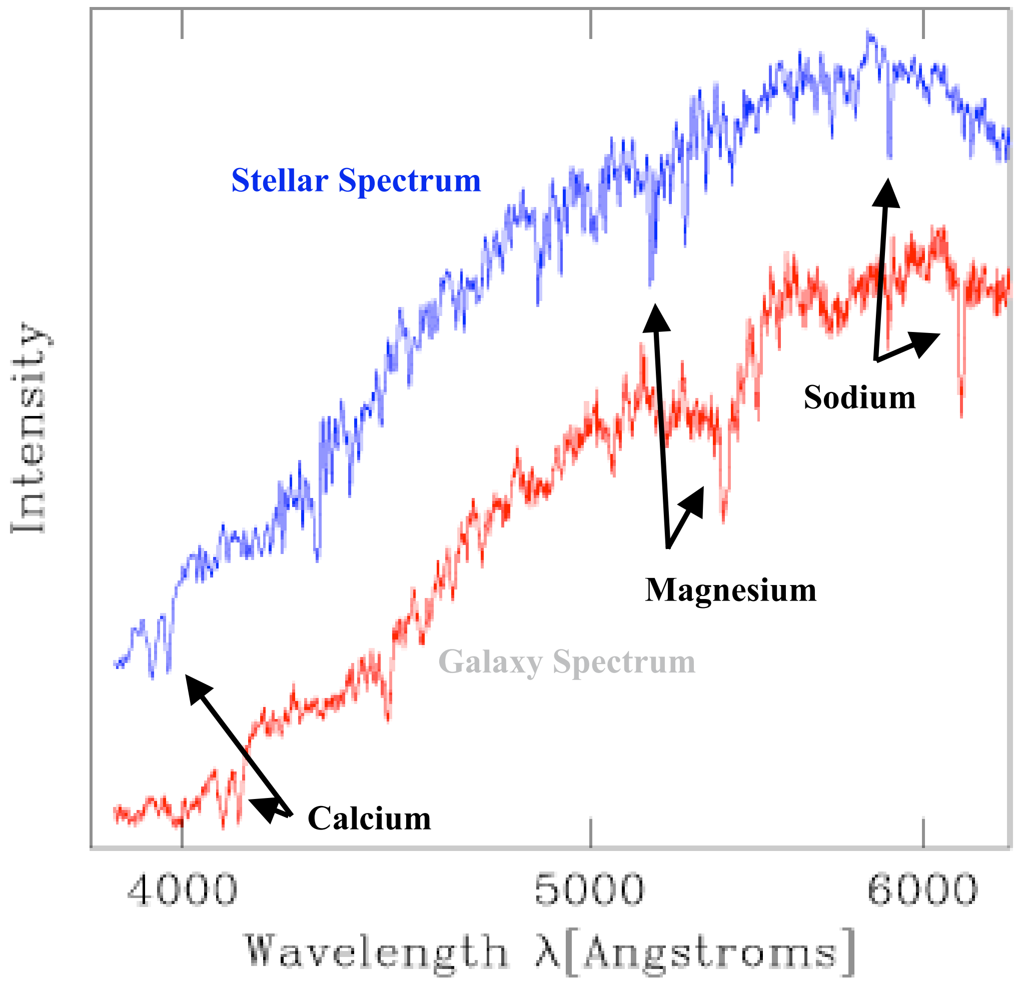
2dF Galaxy Redshift Survey



MEGACAM RAW IMAGE



Galaxy Spectroscopy



- Spectra of a nearby star and a distant galaxy
  - Star is nearby, approximately at rest
  - Galaxy is distant, traveling away from us at 12,000 km/s

- Emission and absorption of light occurs at specific energies for each element
  - creates an elemental fingerprint, recognizable even in light from extremely distant objects
- Expansion of the Universe stretches light wavelengths
  - detected spectrum is shifted to longer wavelengths with respect to emitted spectrum
- Effective recession velocity of object is determined by carefully determining the amount of redshifting

To map out the universe:

1) Measure redshifts of lots of galaxies: $\mathbf{z = \Delta\lambda / \lambda}$

2) Calculate speed from redshift: $\mathbf{V = c\, z}$

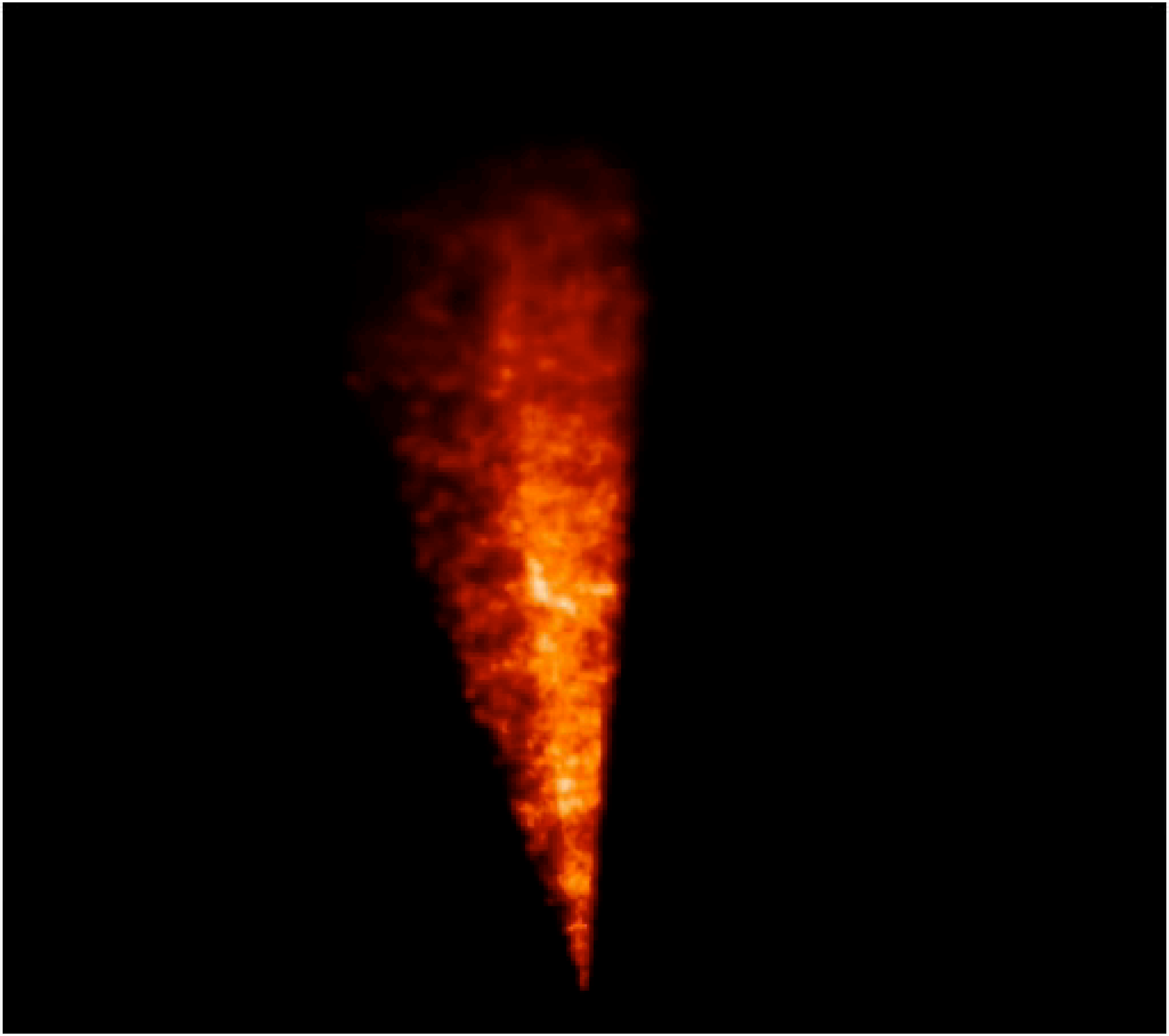
3) Calculate distance from Hubble Law: $\mathbf{V = H\, d}$

V = velocity (in km/s)
d = distance (in megaparsec = 3.08 10^19 km)
H = Hubble constant (around 70 km/s per Mpc)

4) make a map of direction vs. distance

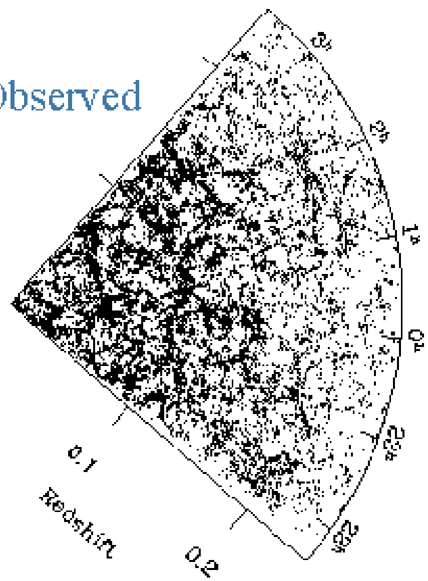
lots of structures
- bubbles and voids
- some structures more than 10 Mpc long
- voids at least that wide across

How do you form such huge things?

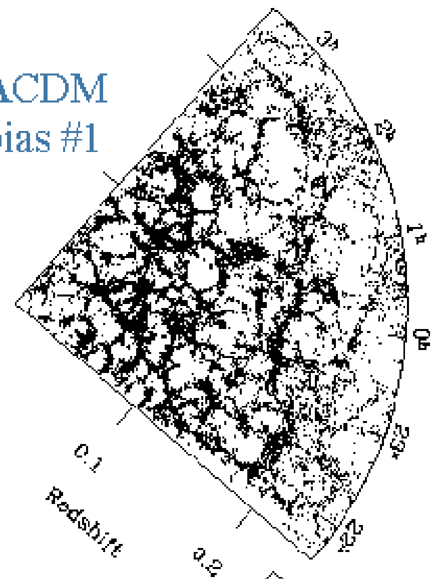


Models vs observations

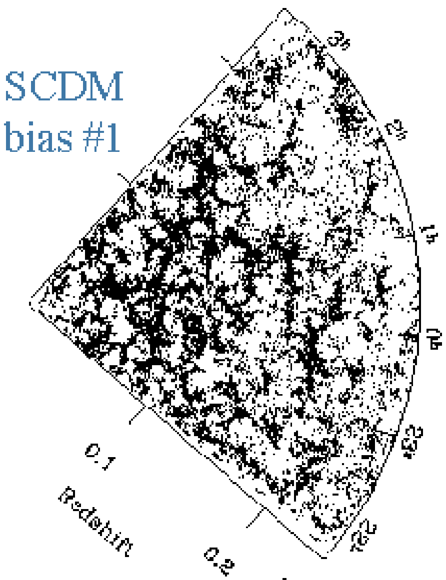
Observed



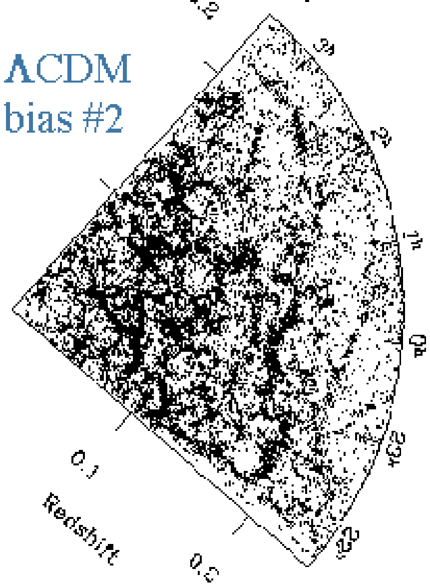
ΛCDM
bias #1



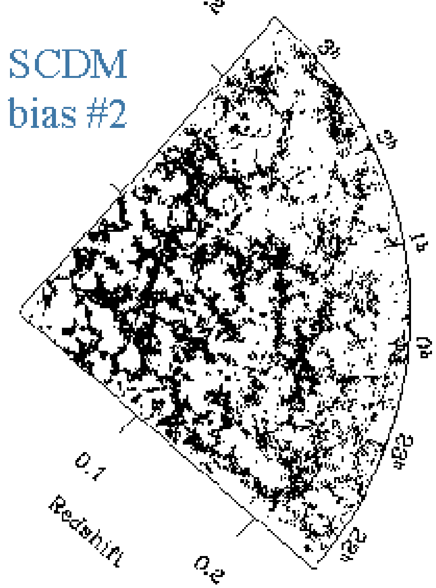
SCDM
bias #1



ΛCDM
bias #2



SCDM
bias #2



Methods

.Two or three point correlation function

. Genus curve

. Voronoi Tessellation

. Minimal spanning trees

. Power spectrum

. Fractals

The Two-Point Correlation Function

A measure of the deviation from randomnes:

$$\xi(r) = \frac{n_{DD}(r)}{n_{RR}(r)} - 1$$

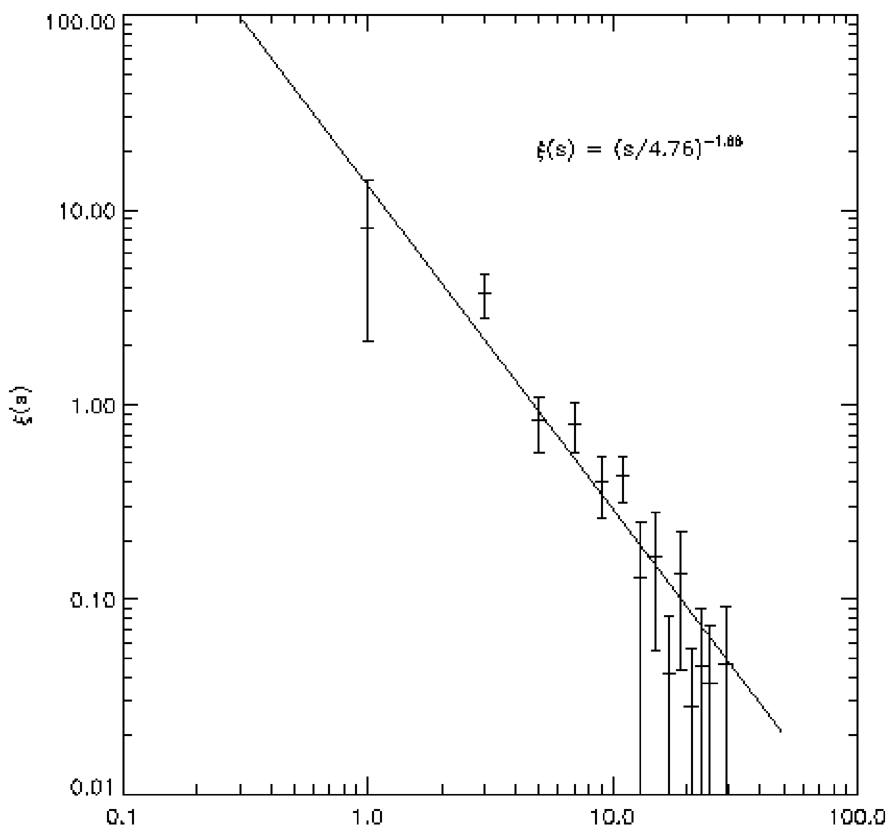
$n_{DD}(r)$ = number of pairs with a separation of r in the data

$n_{RR}(r)$ = number of pairs with a separation of r for a ramdomly distributed data set

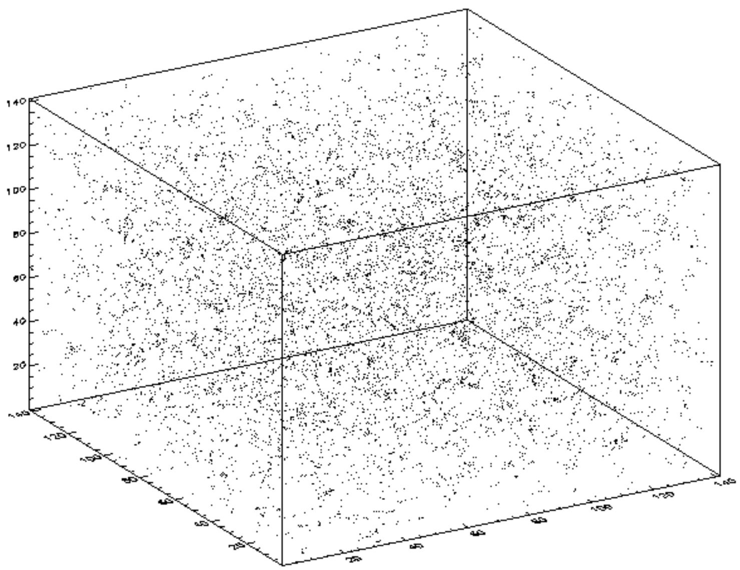
Estimates of the correlation function of the galaxies indicate that it is power law function of the form,

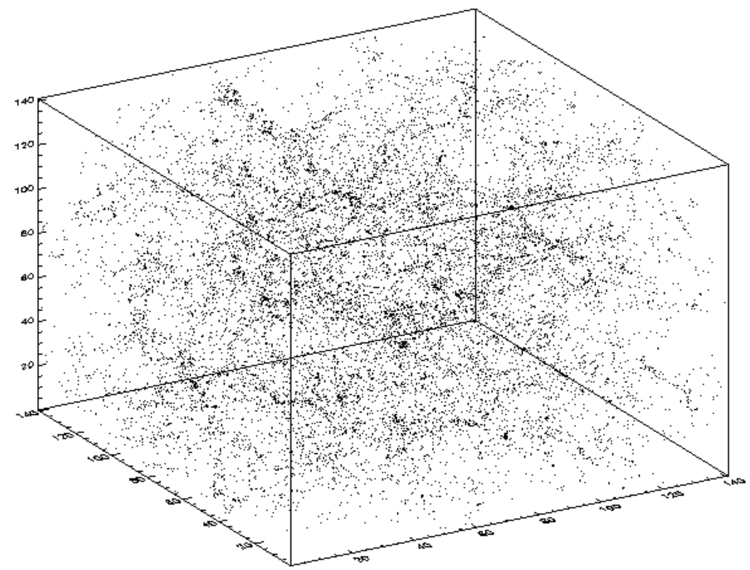
$$\xi(r) = (\frac{r}{r_0})^{-1.8}, r_0 = 5\,Mpc$$

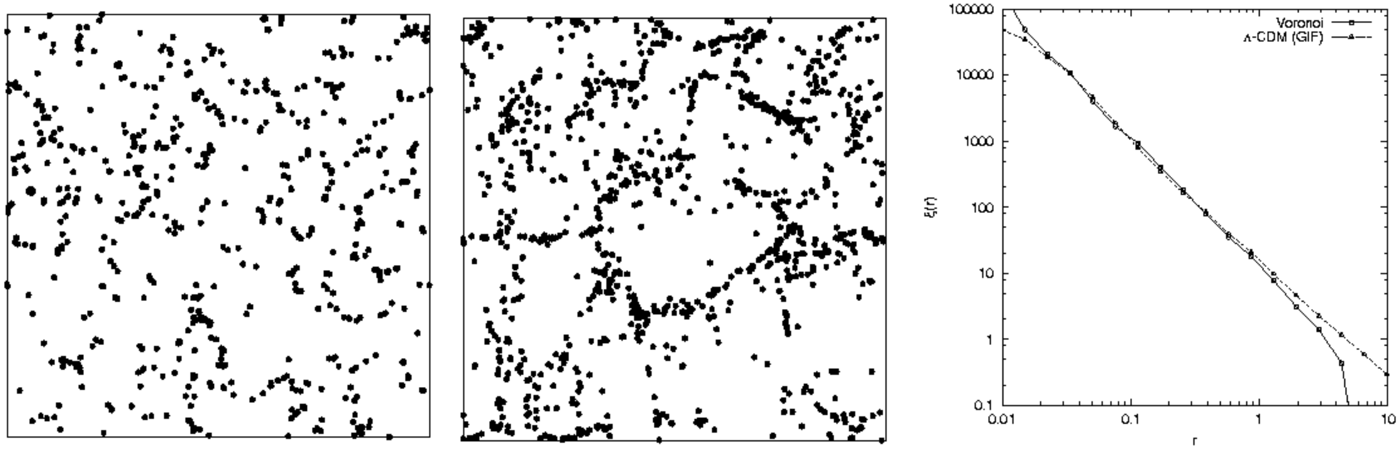
Where r_0 is called the correlation length.



Simulations






Voronoi
Λ-CDM (GIF)
ξ(r)
r
100000
10000
1000
100
10
1
0.1
0.01
0.1
1
10

GENUS FUNCTION

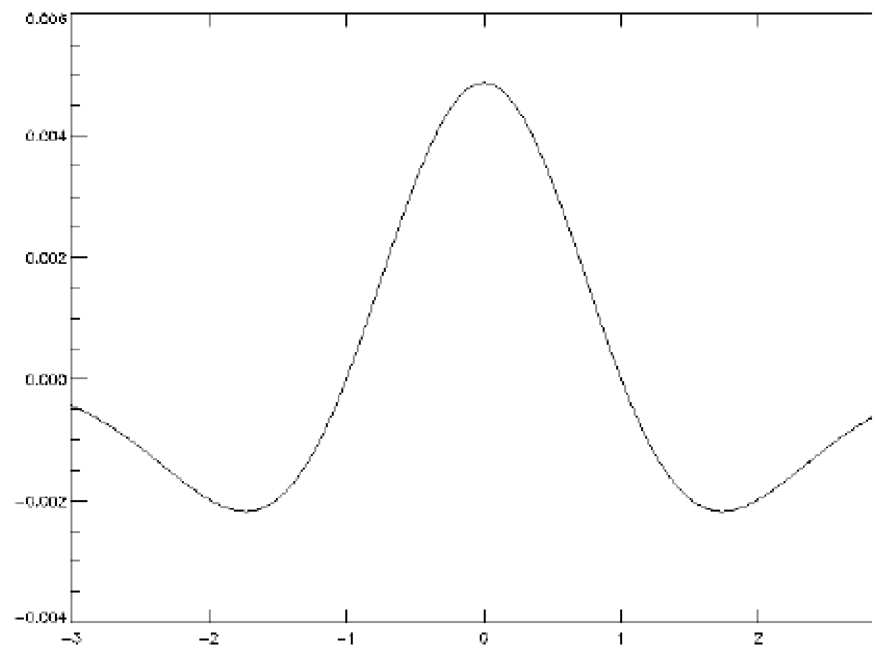
The genus of a surface G is

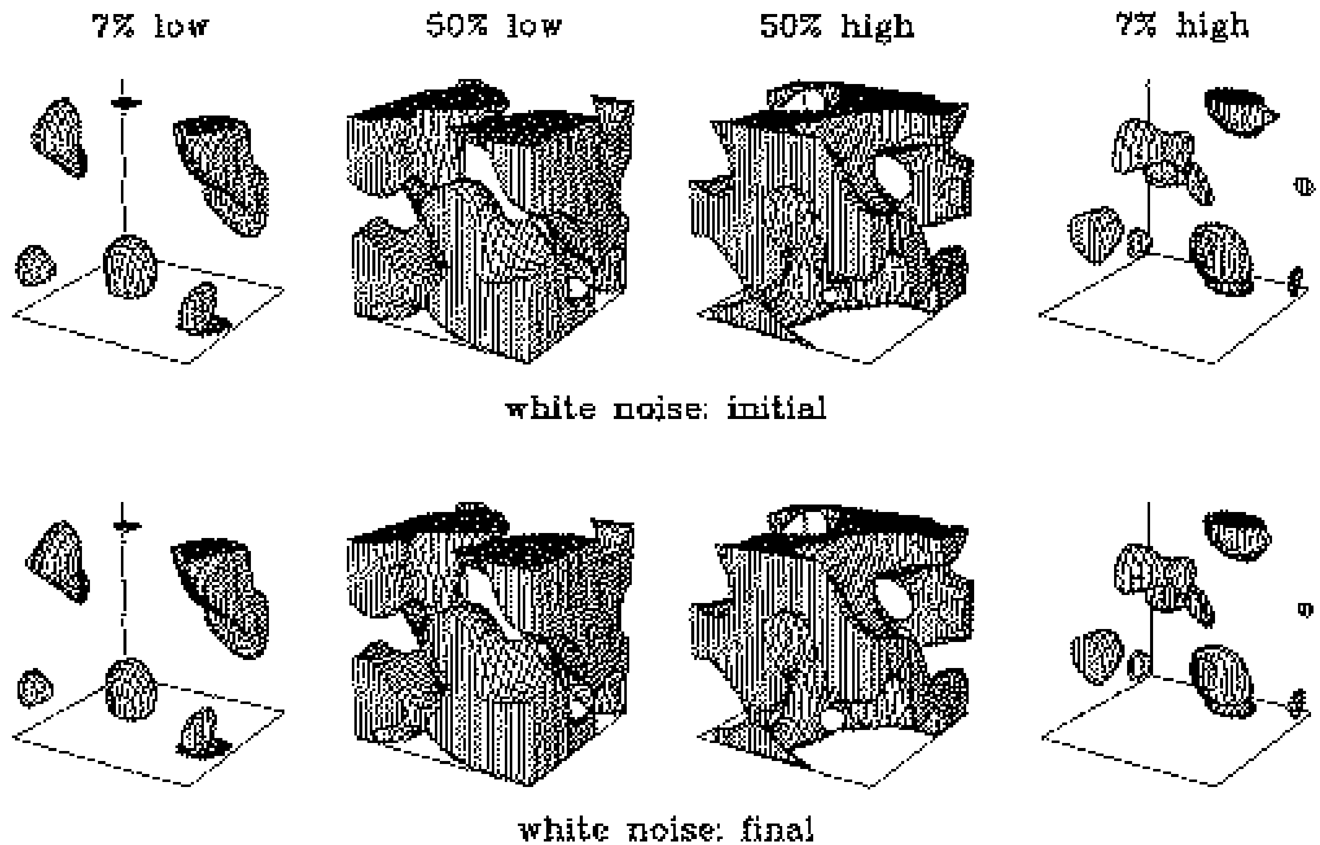
G(T) = (number of holes) - (number of isolated regions) + 1

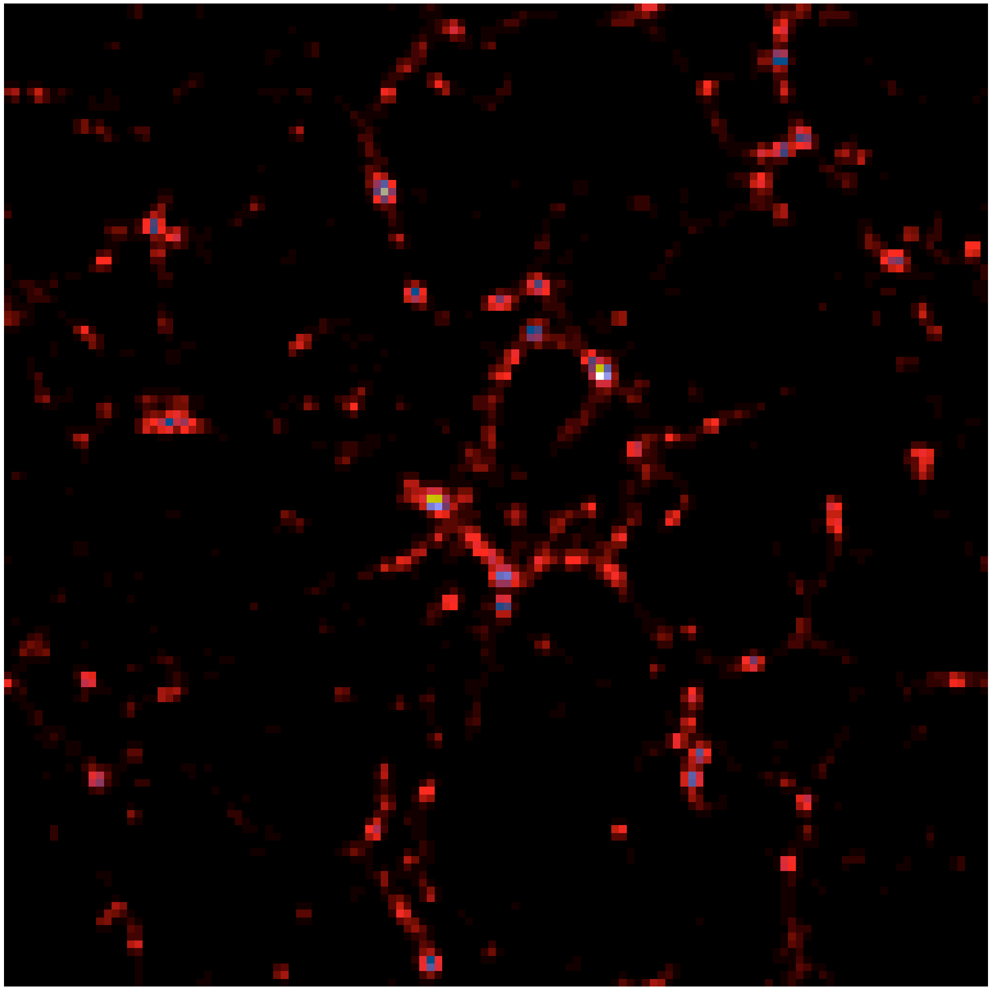
- Convolve the data by a Gaussian
  - Threshold all values under a threshold level T
  - G(T) = (number of holes) - (number of isolated regions) + 1

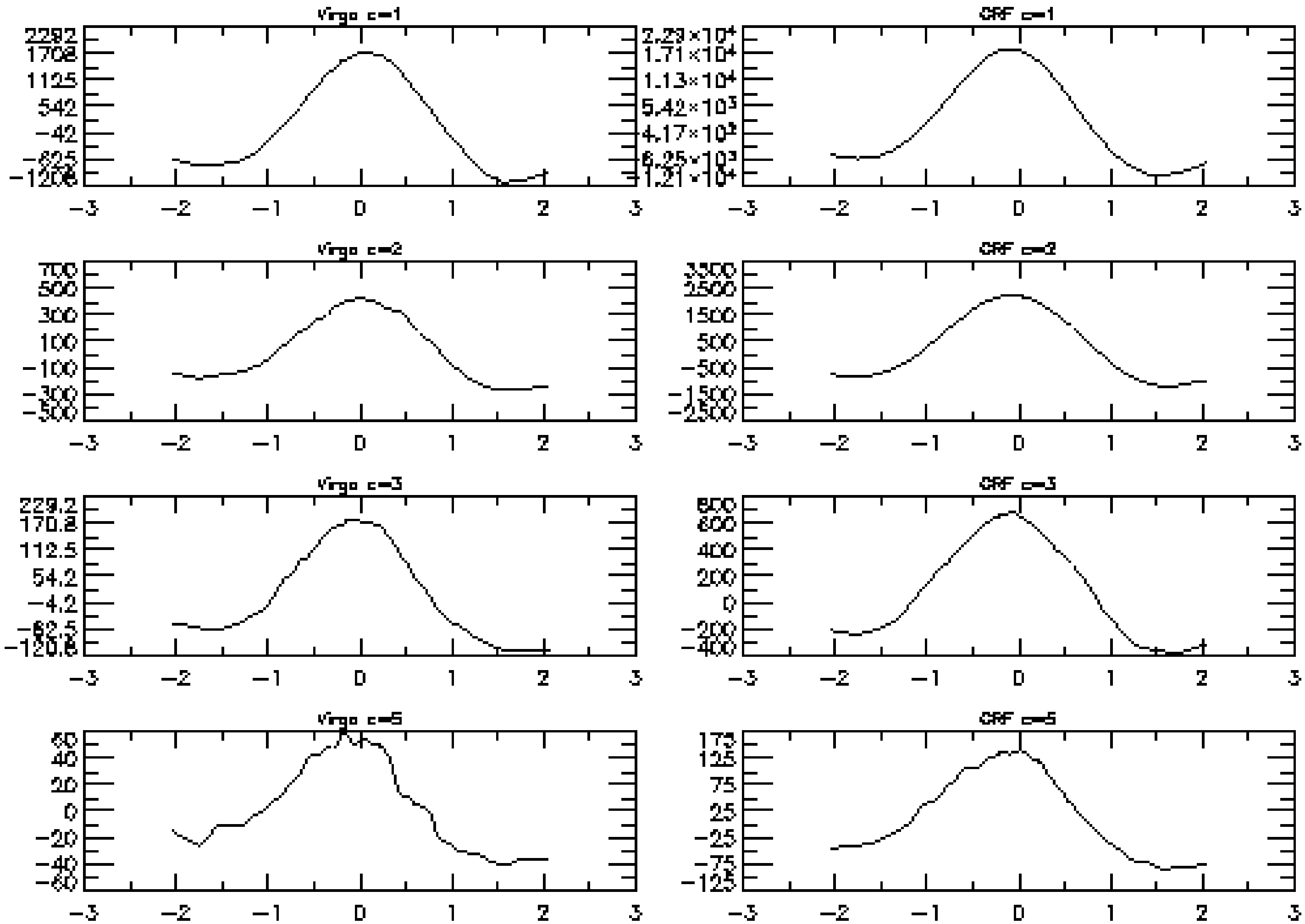
For a Gaussian field, the genus curve is:

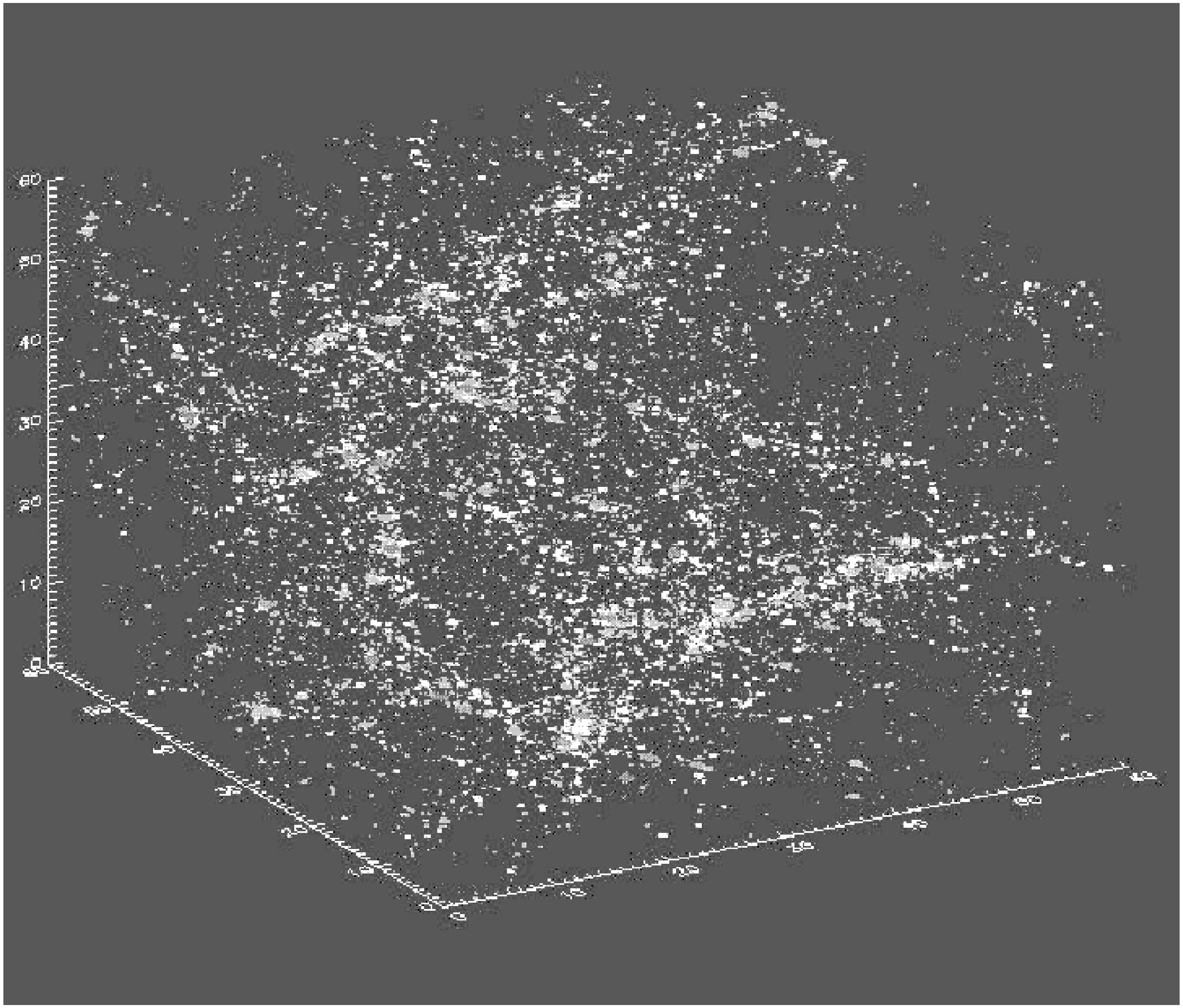
$$g(\nu) = N(1-\nu^2)\exp(-\frac{\nu^2}{2})$$



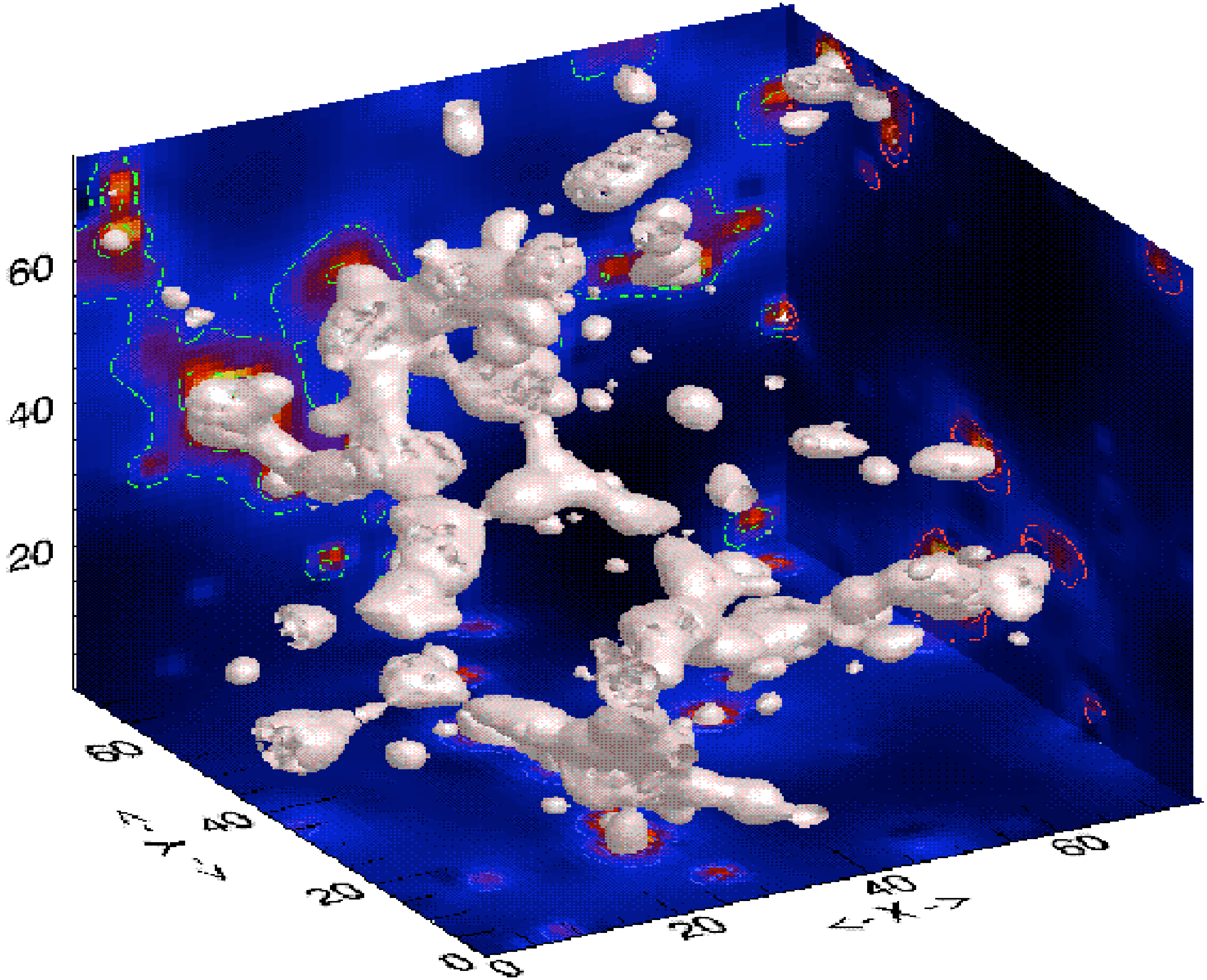

7% low
50% low
50% high
7% high
white noise: initial
white noise: final

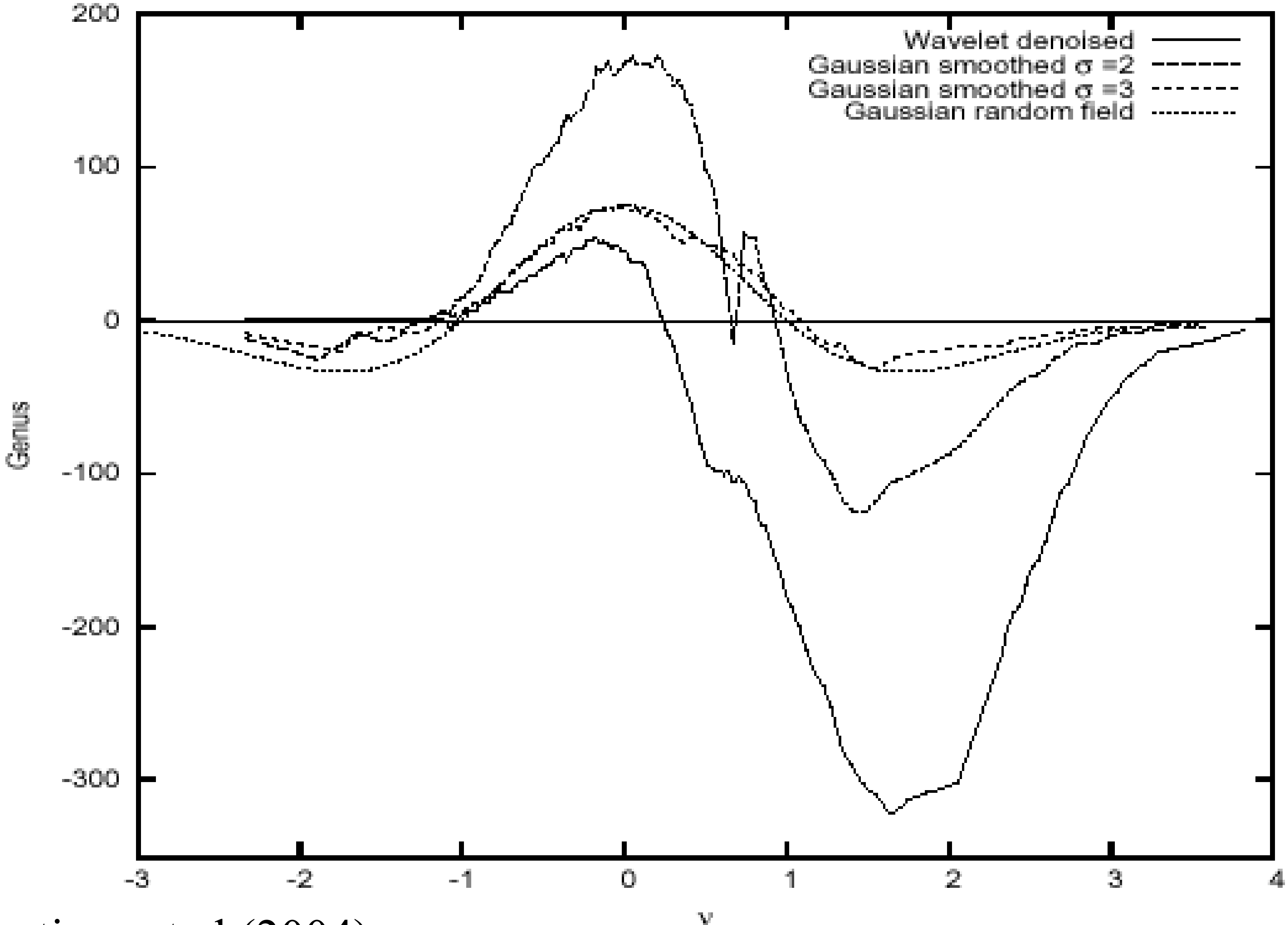



Virgo c=1
CRF c=1
Virgo c=2
CRF c=2
Virgo c=3
CRF c=3
Virgo c=5
CRF c=5



60
50
40
30
20
10
0


60
40
20
60
40
20
0
0
20
40
60
<- Y ->
<- X ->



Martinez et al (2004).

The genus curve of this adaptive reconstructed density field is much more informative because
is unique and does not depend of the particular choice of the filter radius. Additionally, the gen
curves of Gaussian-smoothed density fields mimic those of Gaussian random fields, describi
thus more the properties of the filter than the real morphology of the density distribution.

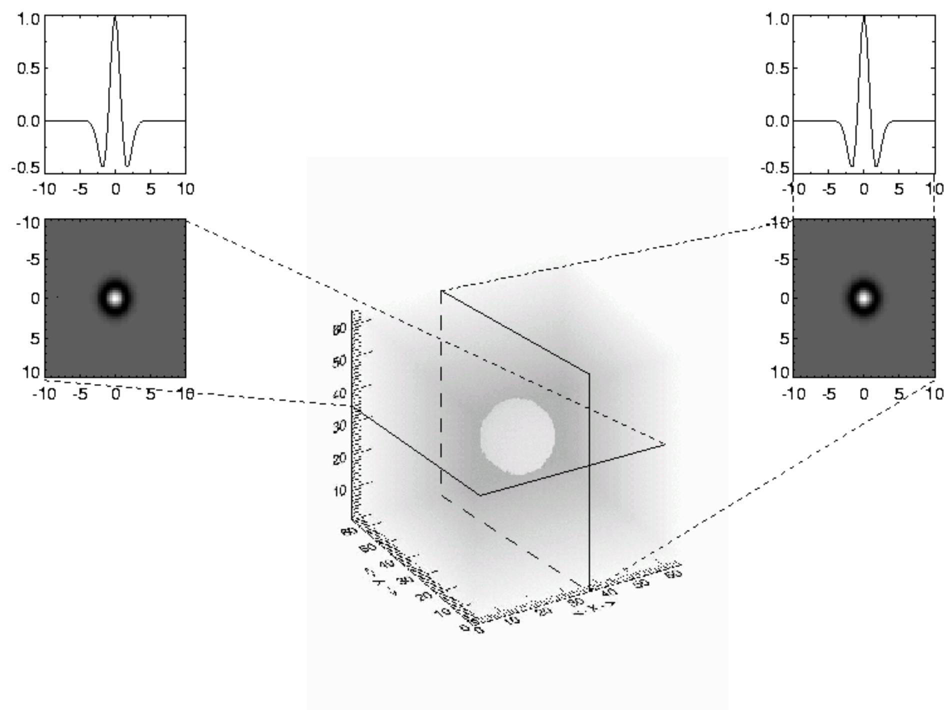
3D MULTISCALE TRANSFORMS

1) **3D WAVELET TRANSFORM: Isotropic Structures**
2) **3D RIDGELET TRANSFORM: Sheet like Structures**
3) **3D BEAMLET TRANSFORM: Filaments**

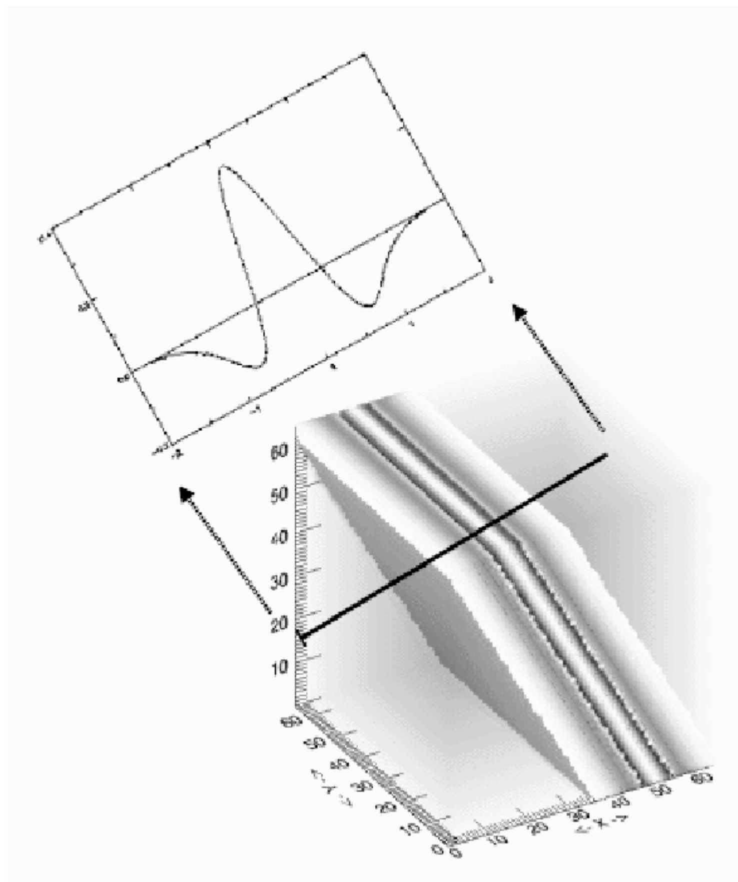
⇒ **Statistical information extraction from all transforms**

Starck et al, "Analysis of the spatial distribution of galaxies by multiscale methods", 2004.

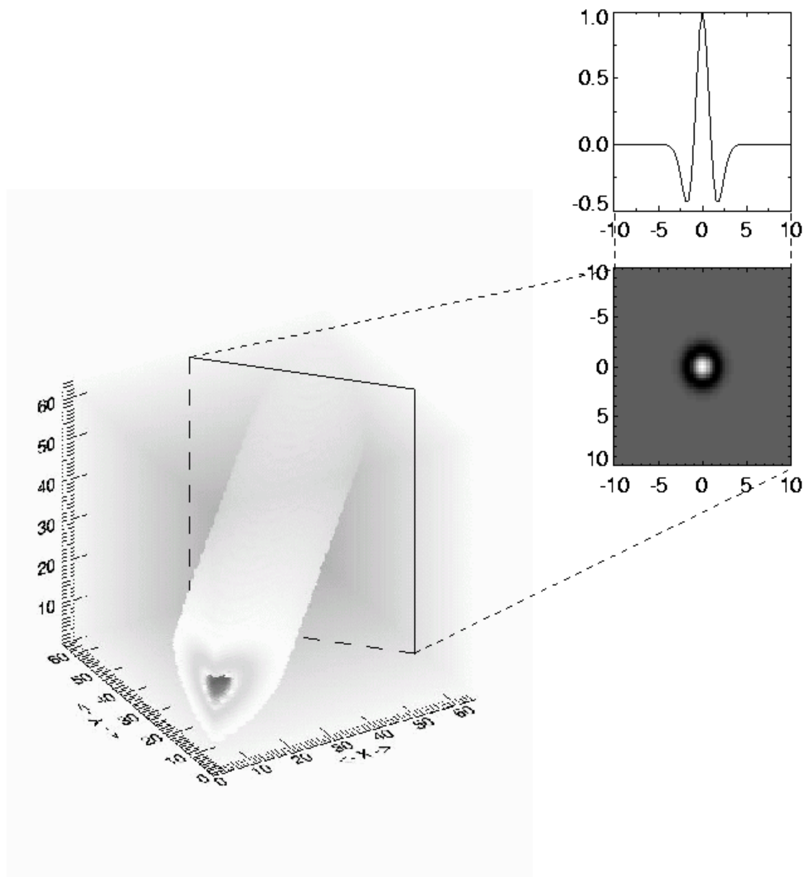
3D Wavelet Function

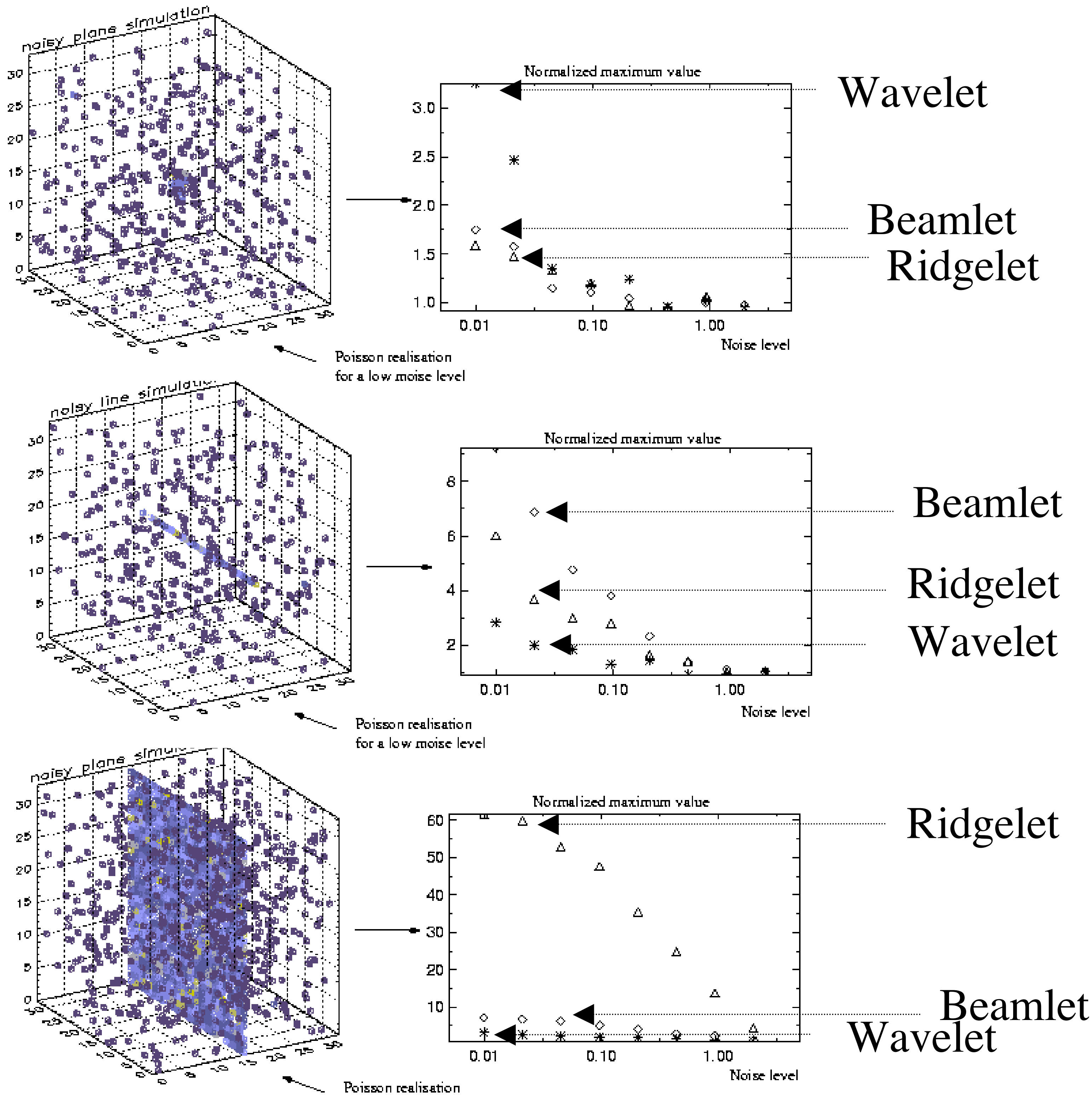


3D Ridgelet Function



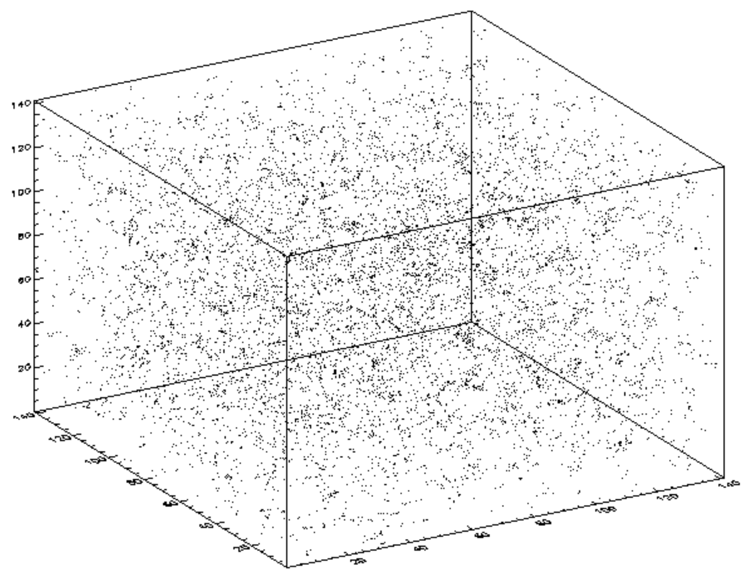
3D Beamlet Function

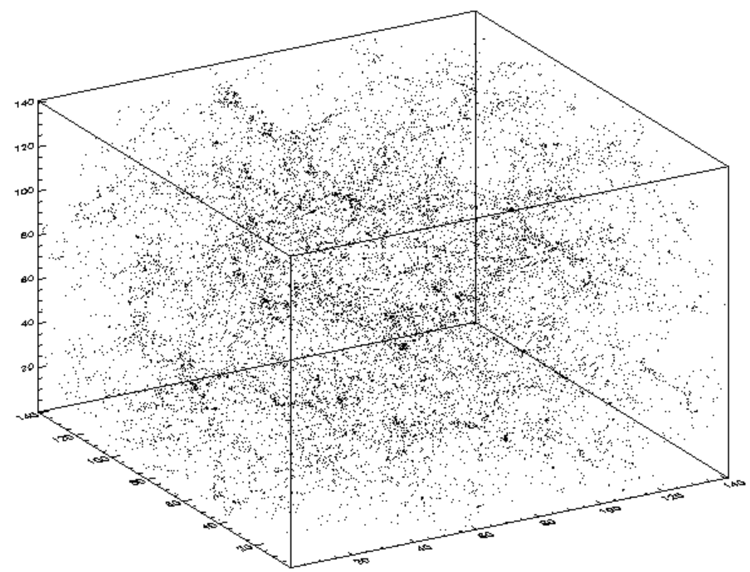




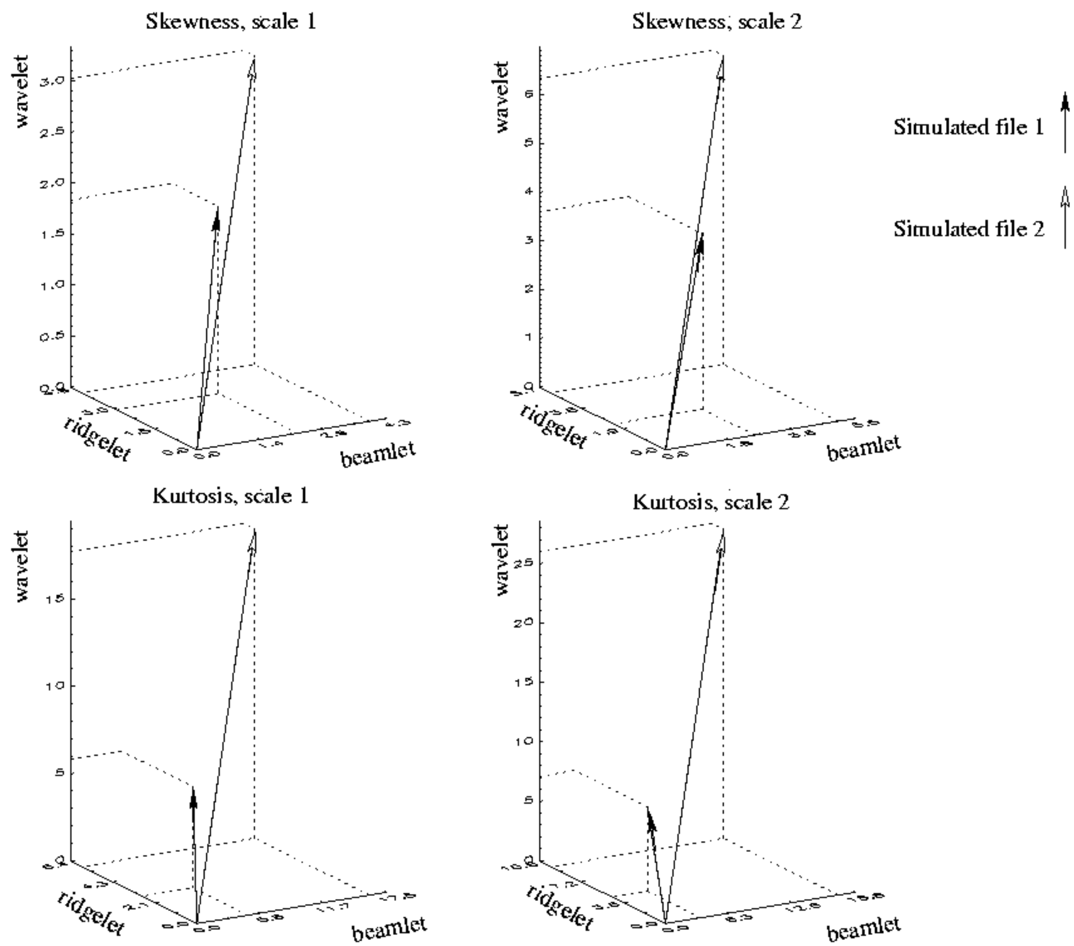
noisy plane simulation
Normalized maximum value
Noise level
Wavelet
Beamlet
Ridgelet
Poisson realisation
for a low moise level
noisy line simulation
Normalized maximum value
Noise level
Beamlet
Ridgelet
Wavelet
Poisson realisation
for a low moise level
noisy plane simulation
Normalized maximum value
Noise level
Ridgelet
Beamlet
Wavelet
Poisson realisation

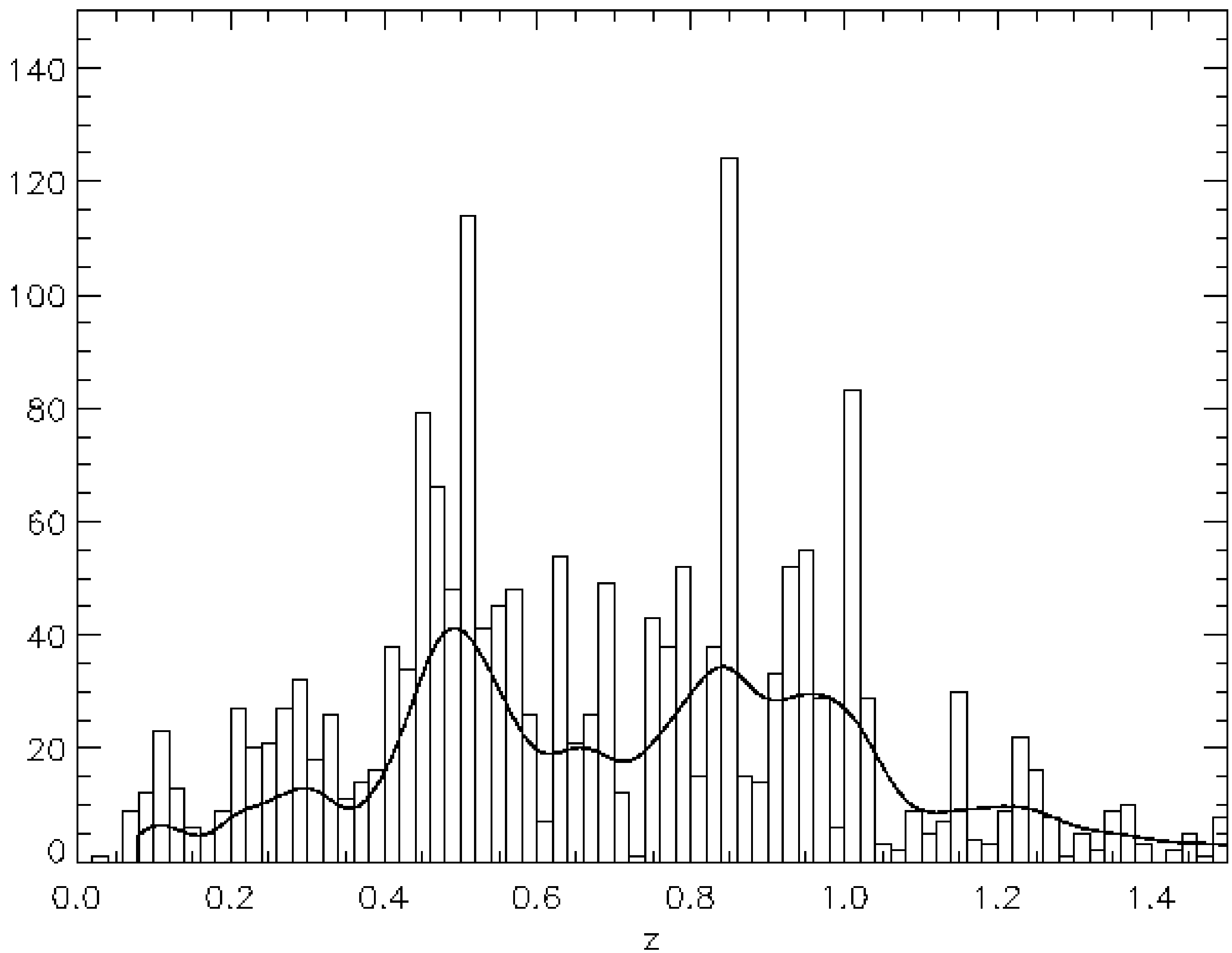
Simulations



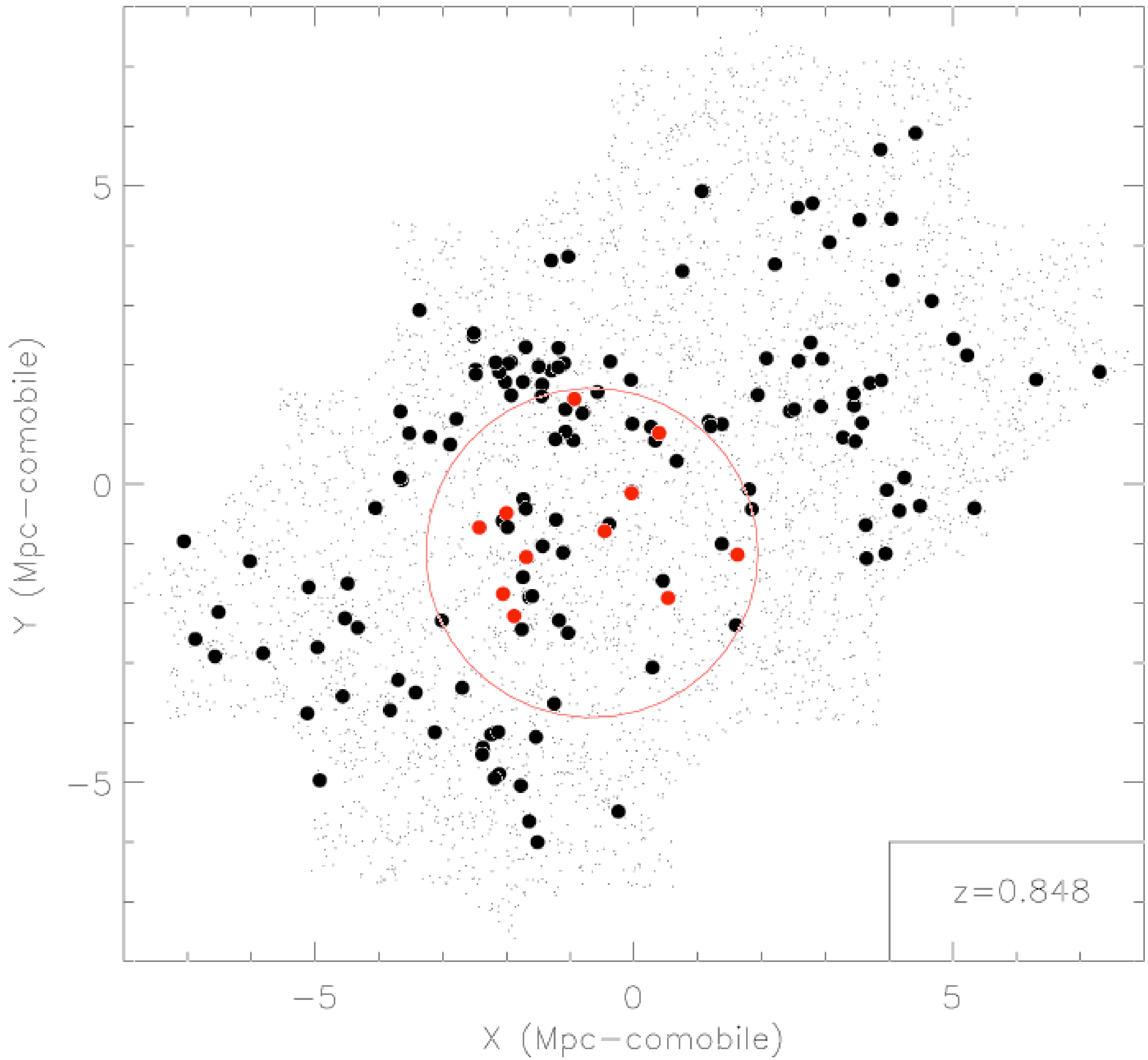


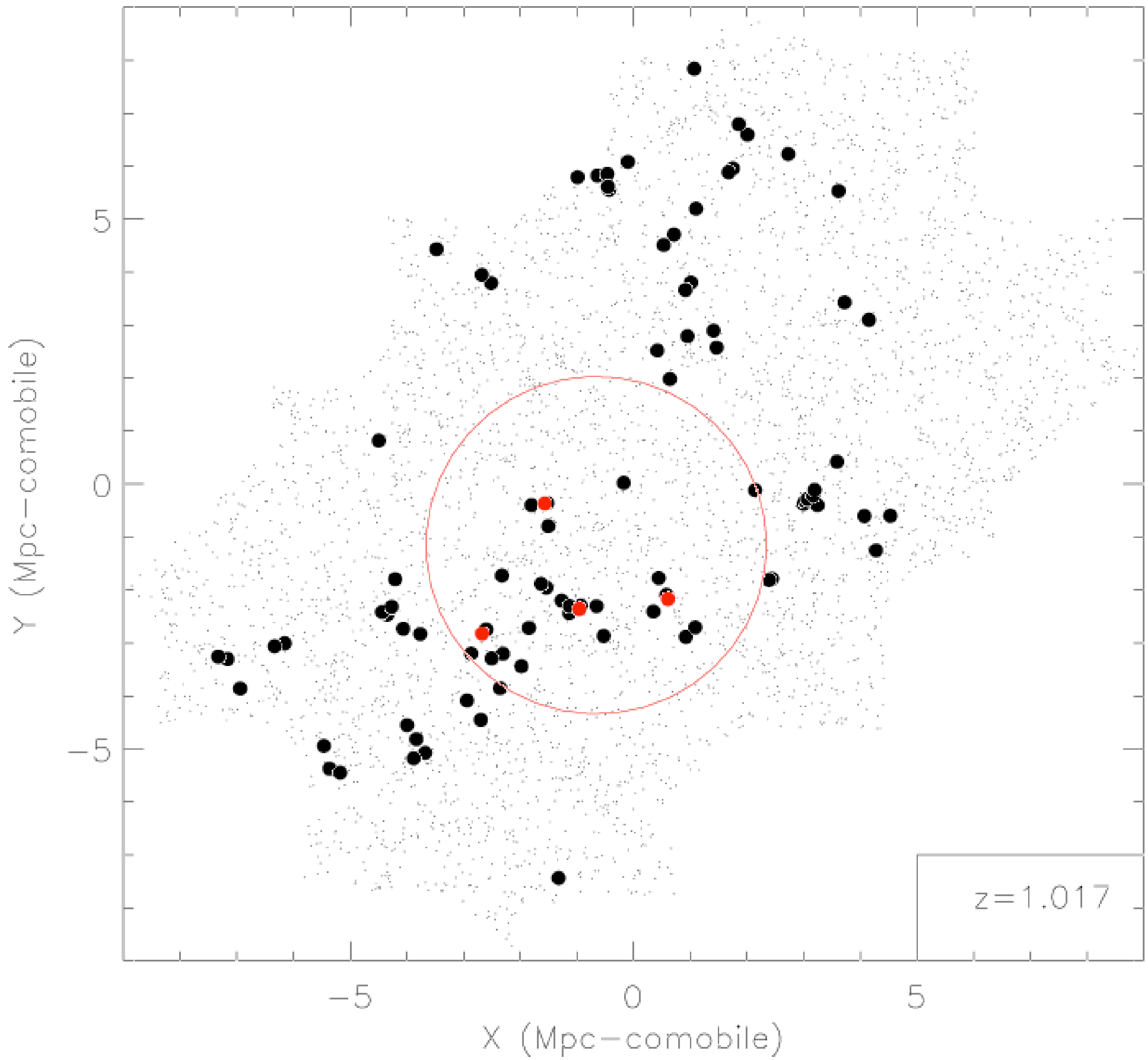
Skewness and Kurtosis





0
20
40
60
80
100
120
140
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
z


Y (Mpc−comobile)
X (Mpc−comobile)
z=0.848
5
0
−5
−5
0
5



5
0
−5
Y (Mpc−comobile)
−5
0
5
X (Mpc−comobile)
z=1.017

Conclusions

- Quantitative descriptors -being reliable, robust, unbiased, and physically interpretable- are needed to extract cosmological information from the data.

- Wavelets are now a standard tool for astronomers.

- Ridgelet, Beamlet, etc, could become also very important in the future.

If you want to know more...