Probing cosmic structure formation in the wavelet representation

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outline

- introduction
- phenomenological hierarchical clustering models and scale-scale correlation
- intermittency and small scale problem of LCDM model
- quasi-local evolution
- summary

cosmic random fields

- density distribution $\rho(\mathbf{x}, t)$
- velocity field $\mathbf{v}(\mathbf{x}, t)$
- temperature $T(\mathbf{x}, t)$
- entropy $S(\mathbf{x}, t)$
- …

density distribution of hydrogen gas in the LCDM model $\rho(\mathbf{x}, t)$

Information of structures: position, scale

dark matter **baryon gas**

Feng,Shu, Zhang (2004)

representations

- ***x*-representation** $f(\mathbf{x},t)$

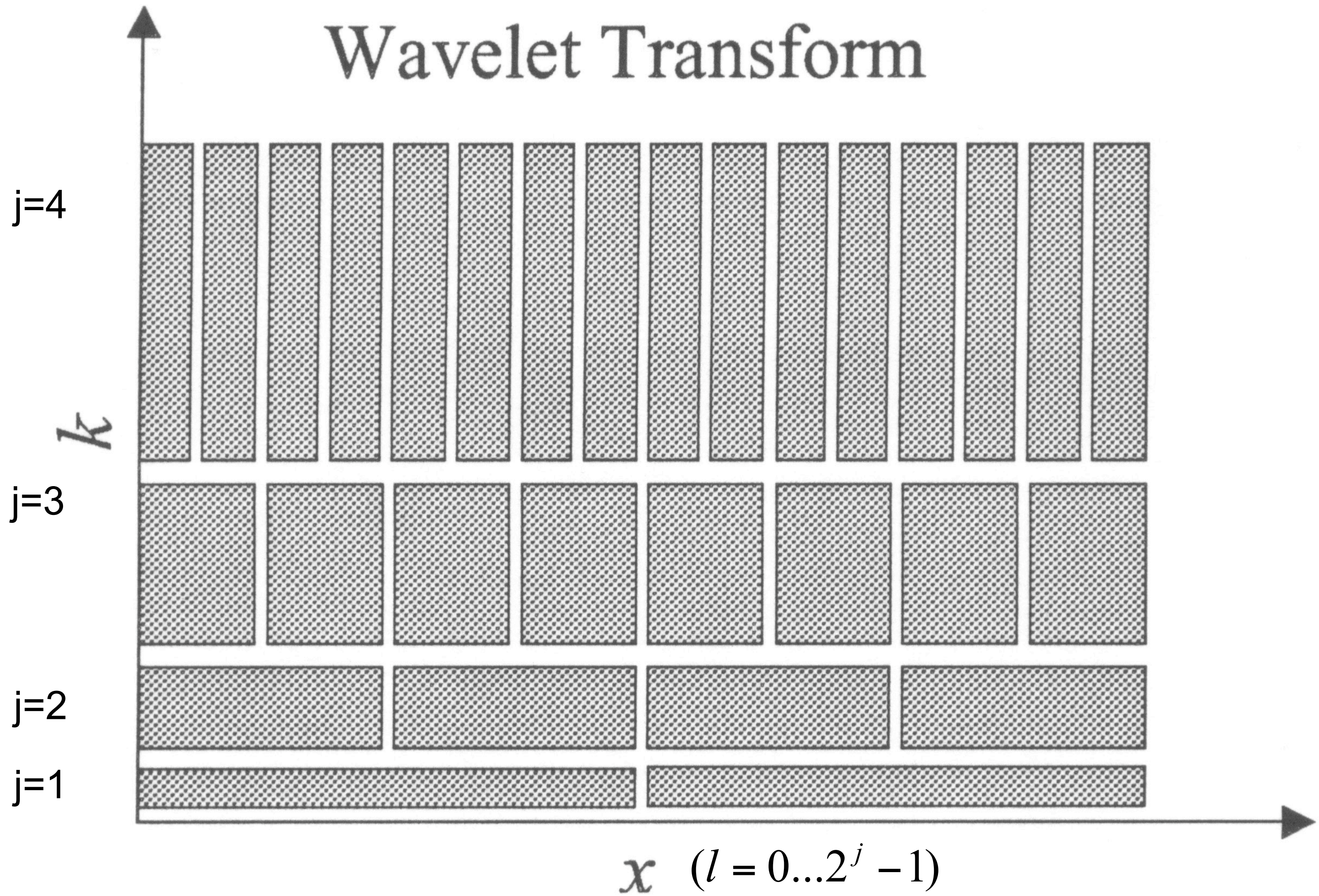
- ***k(p)*-representation** $f(\mathbf{k},t)$, $f(\mathbf{p},t)$,

- **Wigner distribution function (WDF),**
  Wigner representation

  $W(\mathbf{x},\mathbf{k},t)$, $W(\mathbf{x},\mathbf{p},t)$,

phase-space behavior (position and scale)
coherent structure
phase-sensitive effects

WDF is density matrix, not a linear transform of $f(\mathbf{x},t)$.

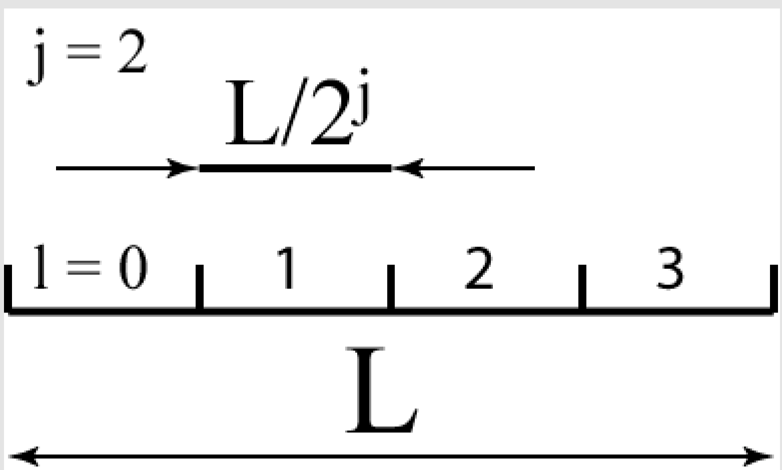


mode (j, l) $\Delta x \Delta k \approx 2\pi$

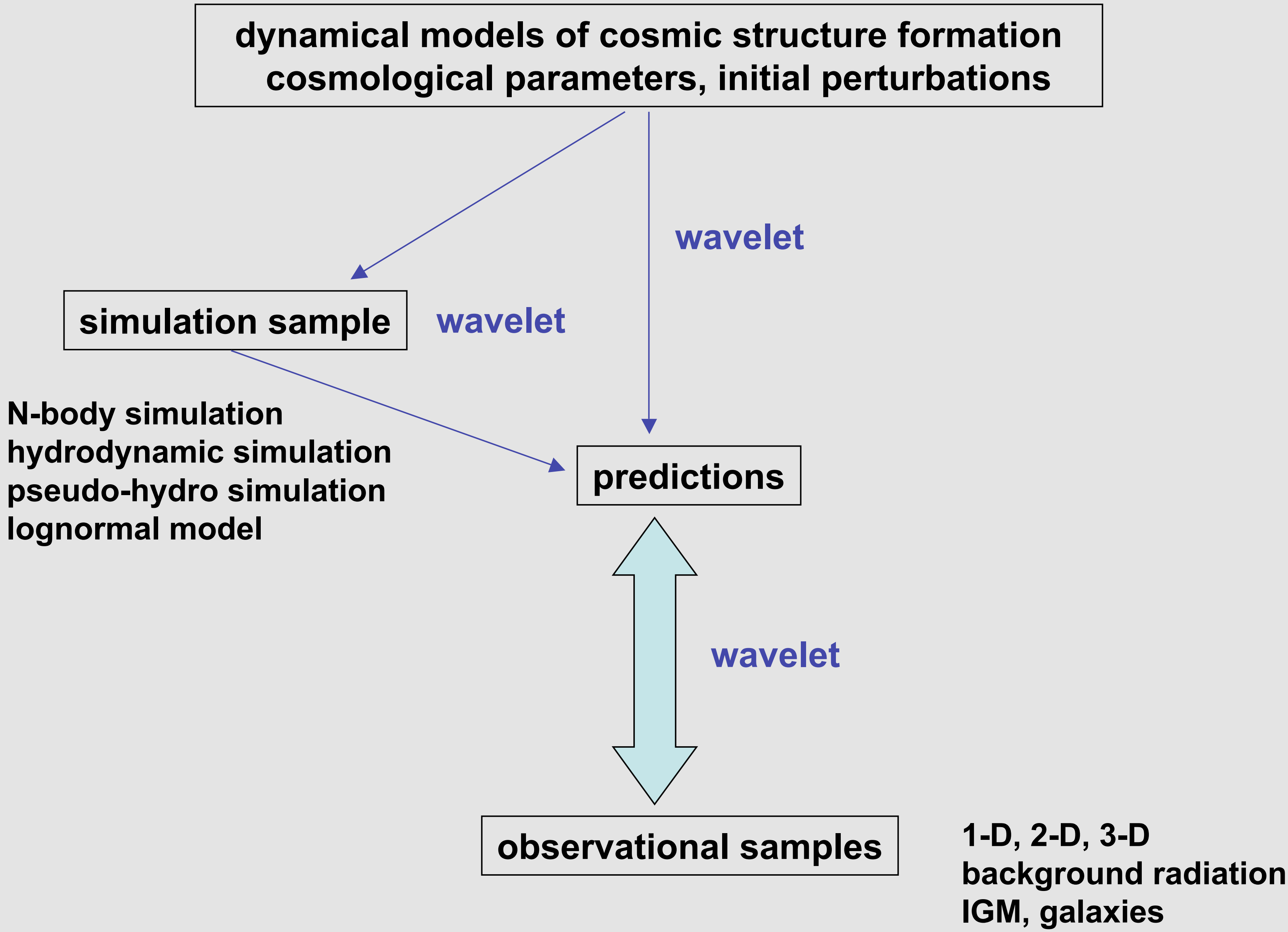
space-scale decomposition by discrete wavelet mode

$$\tilde{\varepsilon}_{jl}(t) = \int_0^L f(x,t)\psi_{jl}(x)dx$$

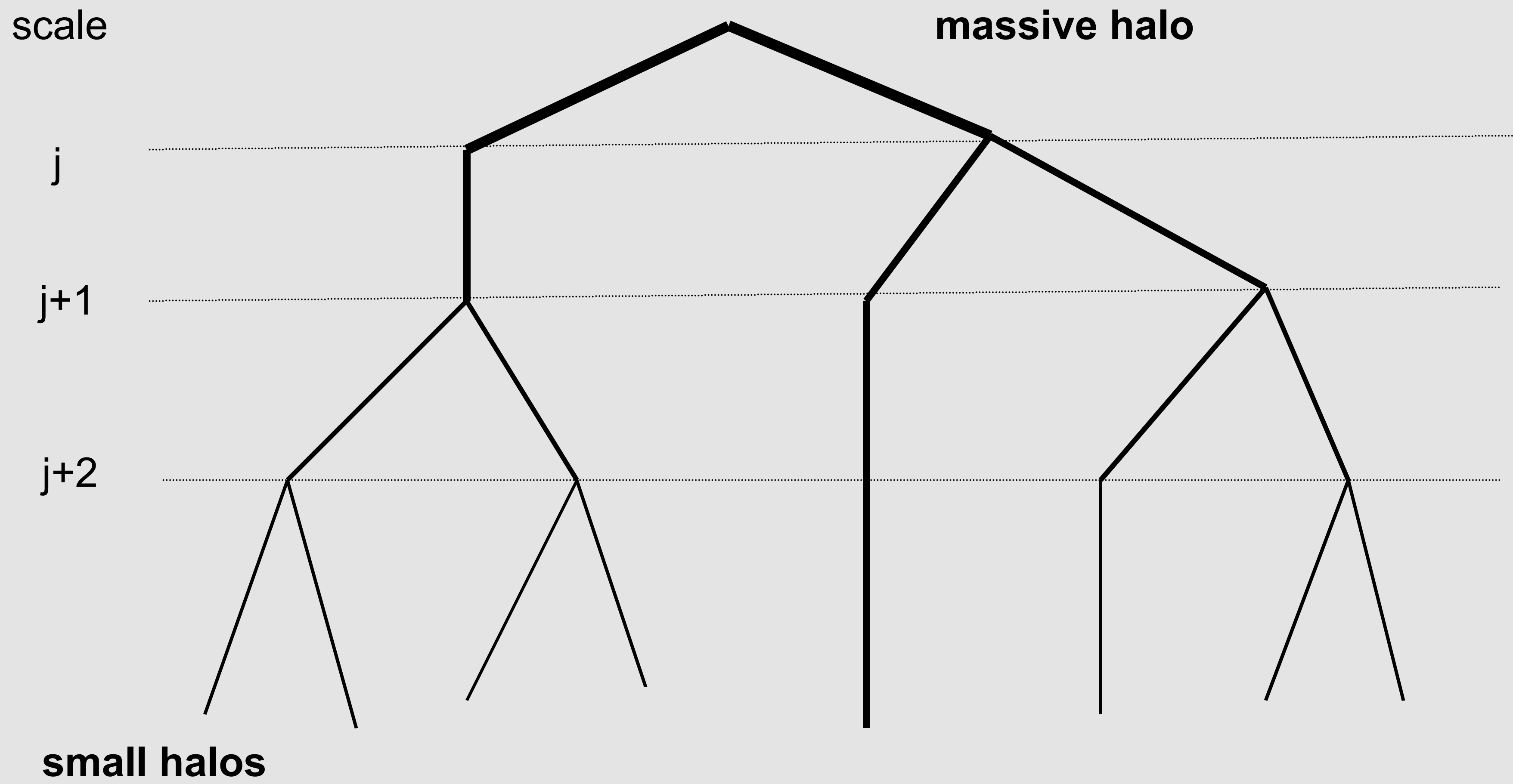
$$f(x,t) = \sum_{j=0}^{\infty} \sum_{l=1}^{2^j-1} \tilde{\varepsilon}_{jl}(t)\psi_{jl}(x)$$



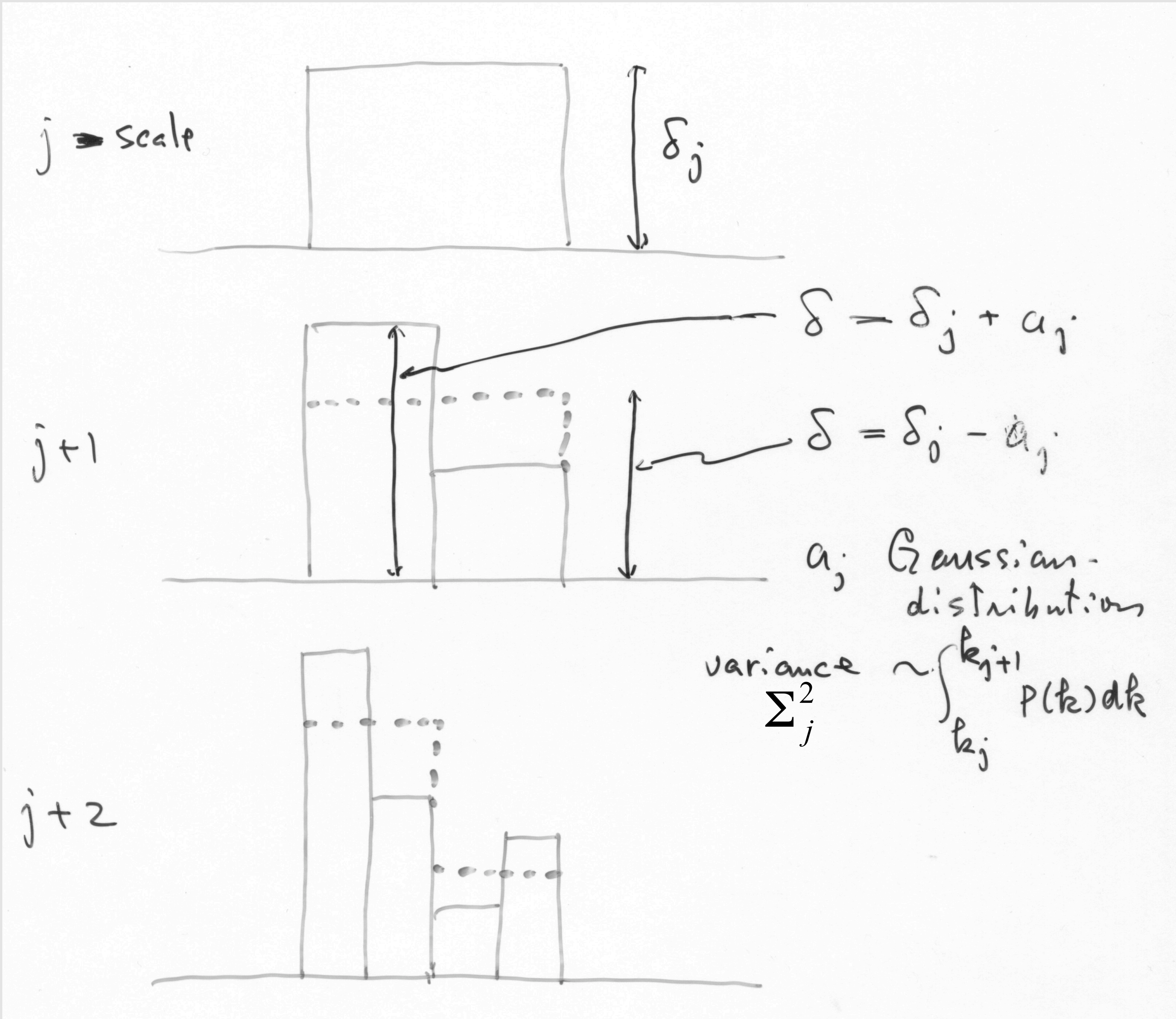
$\tilde{\varepsilon}_{jl}$ WFCs (wavelet function coefficients) of mode (j, l) represents fluctuations on a length scale L/2j


dynamical models of cosmic structure formation
cosmological parameters, initial perturbations
wavelet
simulation sample
wavelet
N-body simulation
hydrodynamic simulation
pseudo-hydro simulation
lognormal model
predictions
wavelet
observational samples
1-D, 2-D, 3-D
background radiation
IGM, galaxies

hierarchical clustering

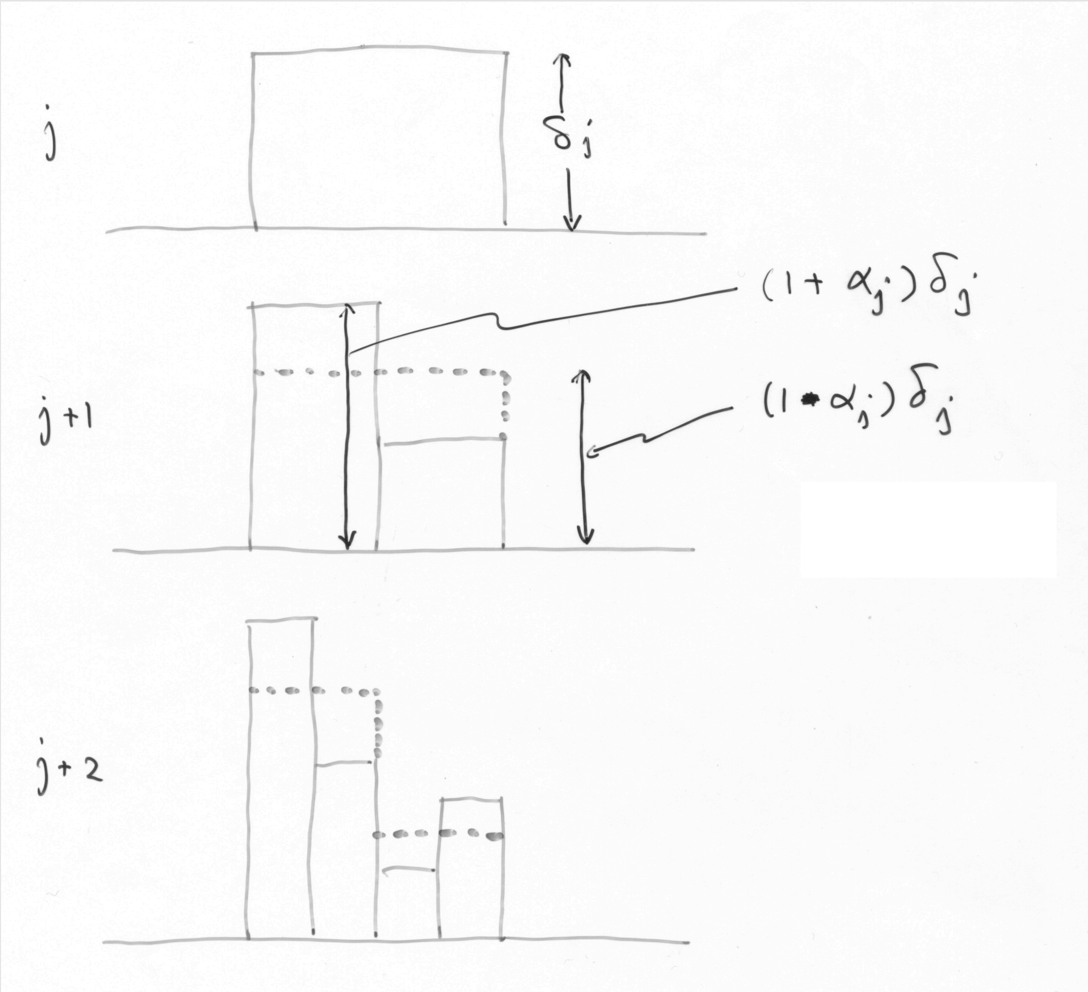


block model



Cole, Kaiser (1988)

branching model



Pando, Lipa, Greiner, Fang (1998)

The first and second orders statistical properties of the block model and branching model are the same if

$$\alpha_j^2 = \frac{\Sigma_j^2}{(\bar{\rho}^2 + \Sigma^2)(1 + \alpha_0^2)(1 + \alpha_1^2), \ldots, (1 + \alpha_{j-1}^2)}$$

Pando, Lipa, Greiner, Fang (1998)

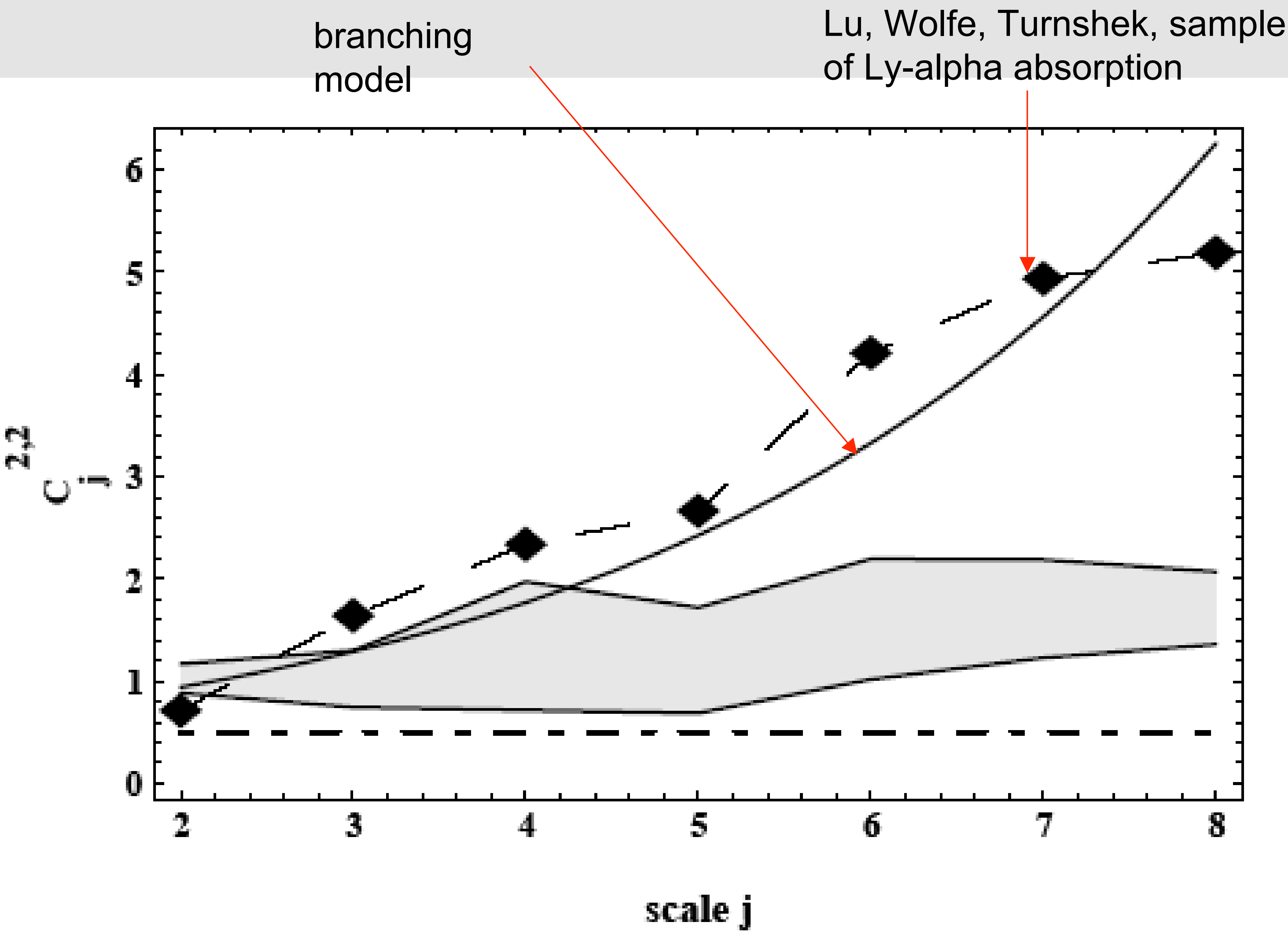
scale-scale correlation

$$C_j^{p,p} = \frac{2^{j+1} \sum_{l=0}^{2^j-1} \langle \tilde{\varepsilon}_{j,l}^{p} \tilde{\varepsilon}_{j+1,2l}^{p} \rangle}{\sum_{l=0}^{2^j-1} \langle \tilde{\varepsilon}_{j,l}^{p} \rangle \sum_{l'=0}^{2^{j+1}-1} \langle \tilde{\varepsilon}_{j+,l'}^{p} \rangle}$$

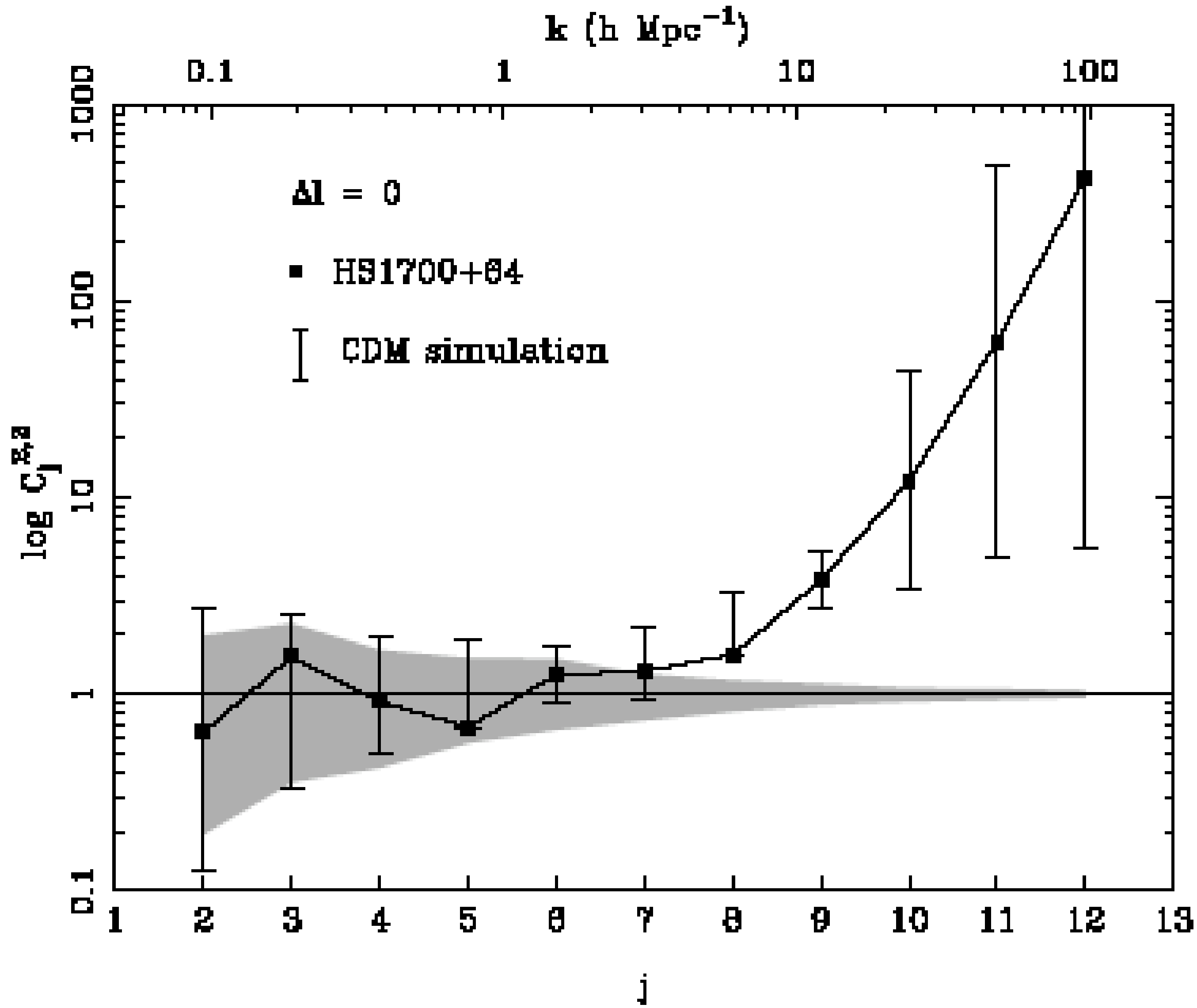
block model $\quad C_j^{p,p} = 1$

branching model $\quad C_j^{2,2} = \frac{(1+6\alpha^2+\alpha^4)^j}{(1+\alpha^2)^{2j}}$

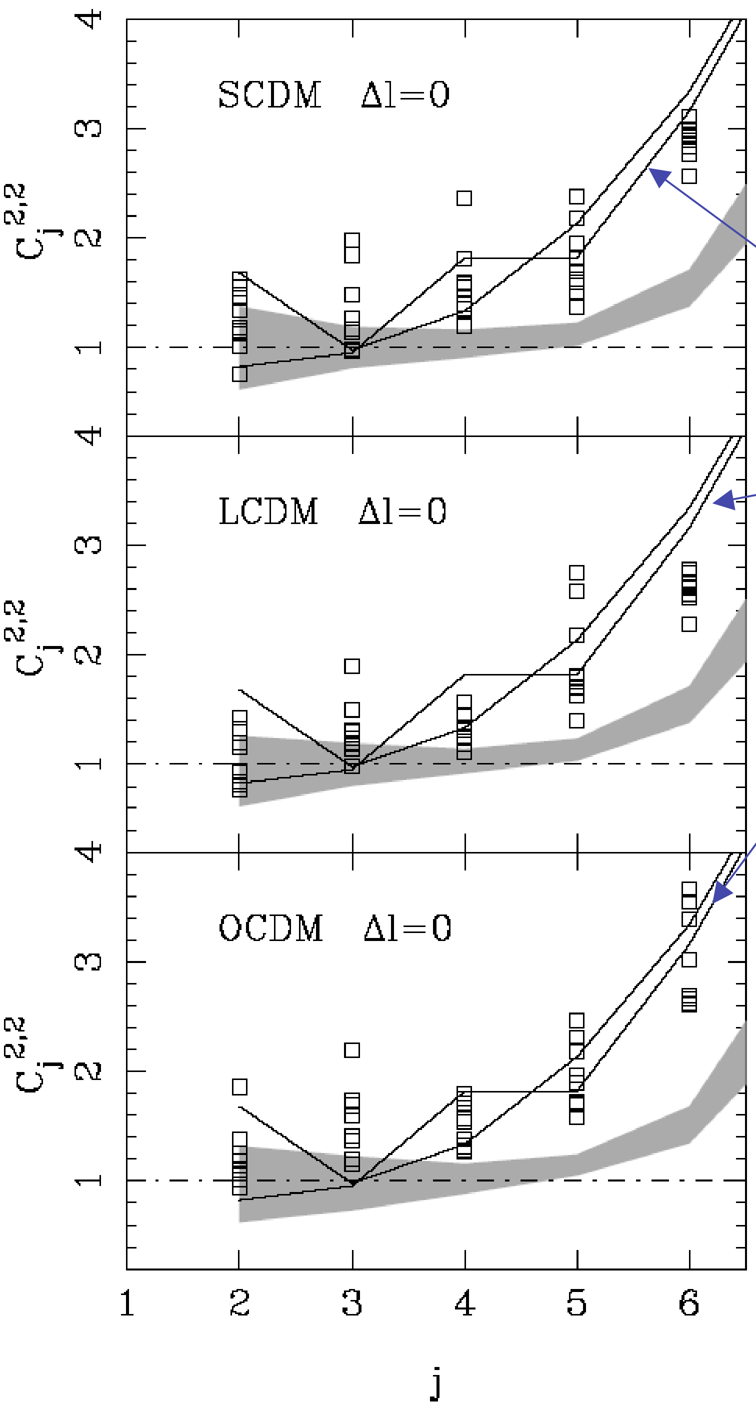
Pando, Lipa, Greiner, Fang (1998)



Pando, Lipa, Greiner, Fang (1998)



Feng, Fang 2000



APM-BGC galaxy sample

□ **mock samples**

Feng, Deng, Fang 2000

problem of the LCDM model small scale powers

reducing the power on small scales relative to the `standard' LCDM model is favored by the analysis of WMAP plus galaxy and Lyman-alpha forest data. It can also solve the halo concentration and substructure problems.

Warm dark matter (WDM) models leads to exponential damping of linear power spectrum, and results in better agreement with observations

intermittency

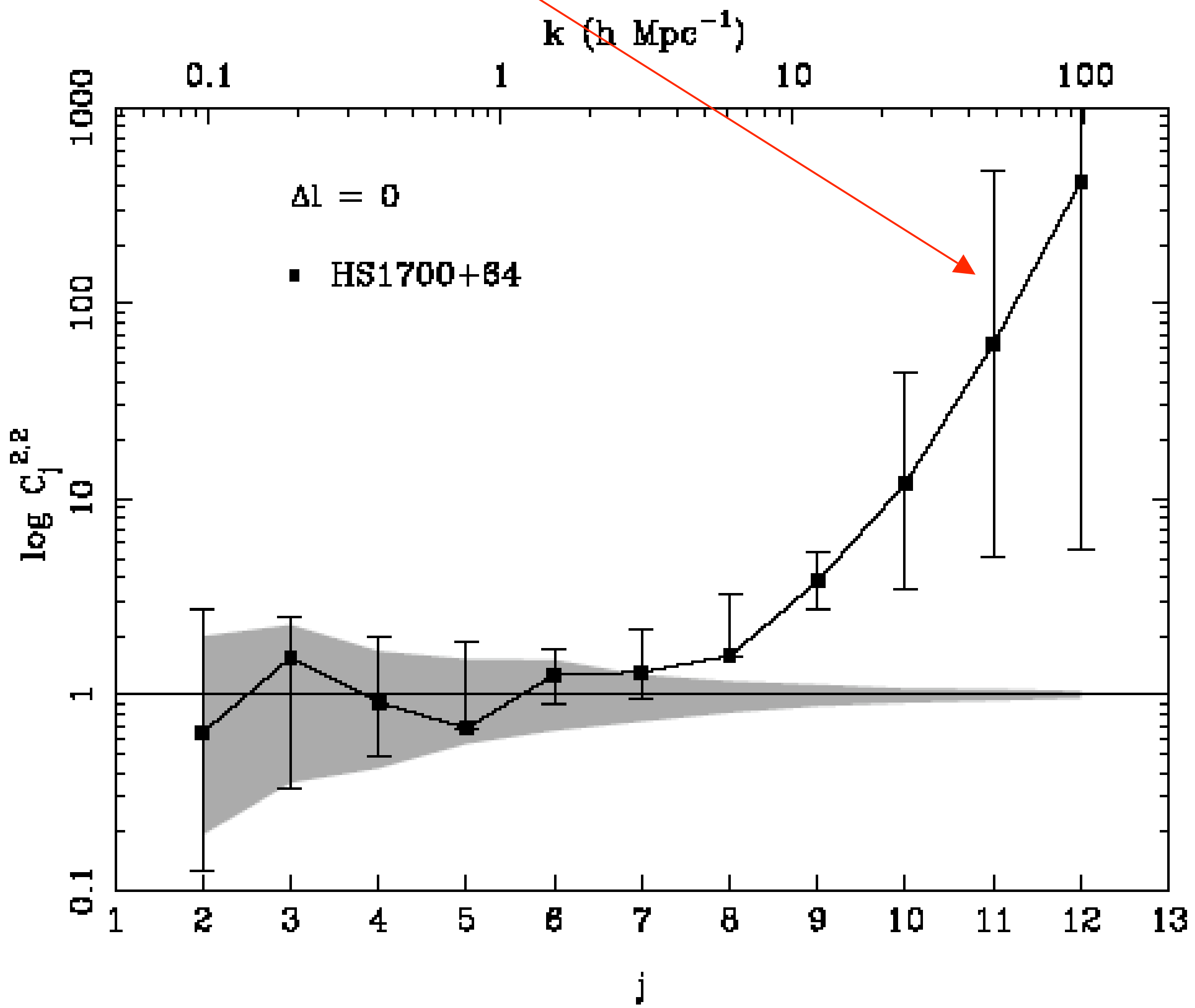
The lognormal model of IGM (baryon gas)

$$\rho_{IGM}(\mathbf{x},t) = \rho_0 e^{[\delta_{Jeans}(\mathbf{x},t) - \frac{\langle \delta^2_{Jeans} \rangle}{2}]}$$

$\delta(\mathbf{x},t)_{Jeans}$ is a Gaussian random field derived from the density contrast δ_{DM} of dark matter filtered on the Jeans scale.

Bi, Davidsen 1997

lognormal model at small scales



Feng, Fang 2000

lognormal field is intermittent

structure function

$$\frac{S_r^{2n}}{[S_r^2]^n} = \frac{\langle[\rho(x+r)-\rho(x)]^{2n}\rangle}{\langle[\rho(x+r)-\rho(x)]^2\rangle^n} \propto \left(\frac{r}{L}\right)^{\varsigma(n)}$$

intermittent exponent

A field is intermittent if $\zeta(n) < 0$

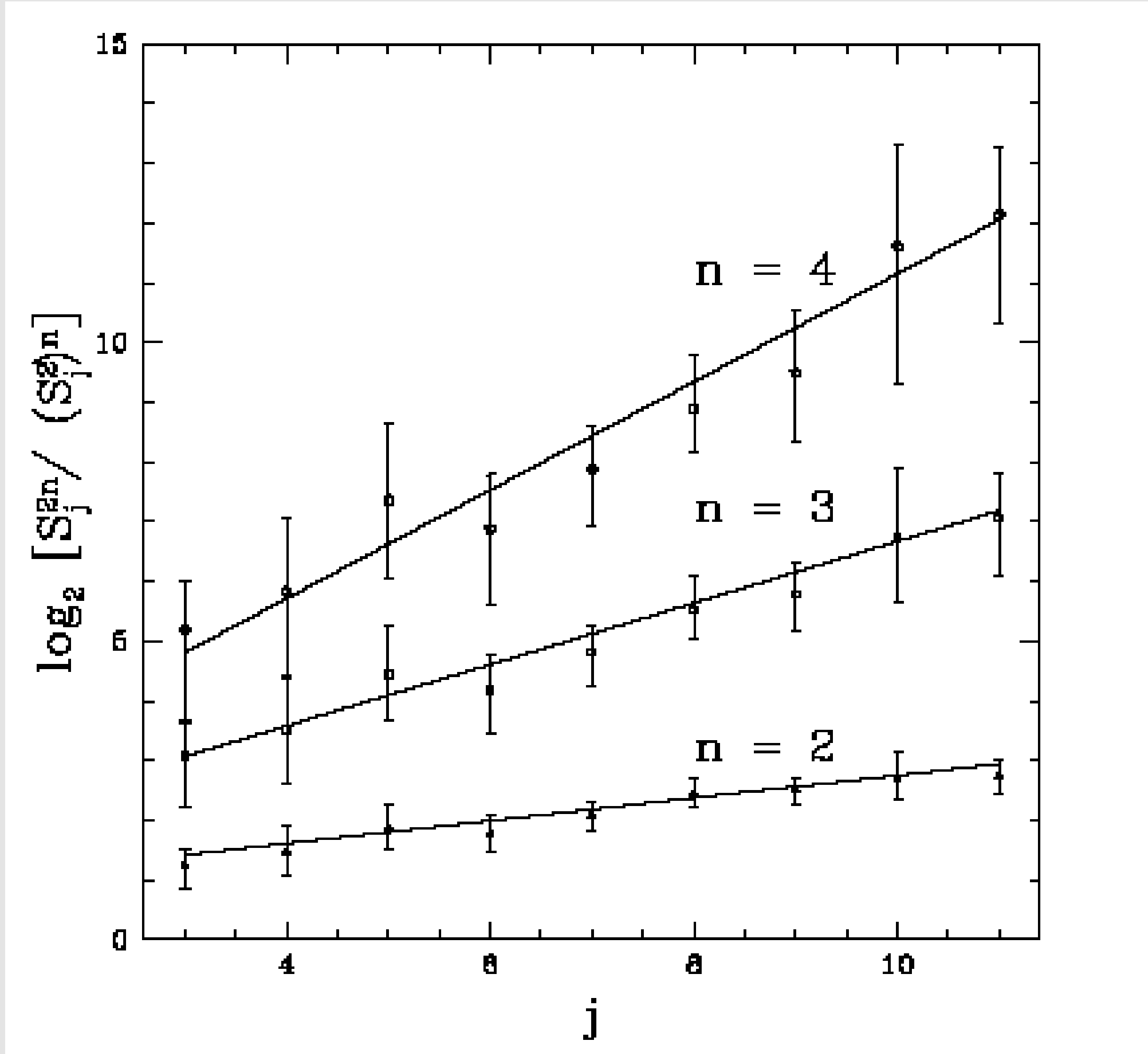
Lognormal Field: $\zeta(n) \propto -n(n-1)$

structure function with DWT

$$S_j^{2n} = \frac{1}{2^j} \sum_{l=0}^{2^j-1} |\,\tilde{\varepsilon}_{j,l}\,|^{2n}$$

$$\zeta(n) - \zeta(1) = -\frac{1}{j} \ln_2 \left[\frac{S_j^{2n}}{(S_j^2)^n} \right]$$

structure functions of Ly-alpha samples



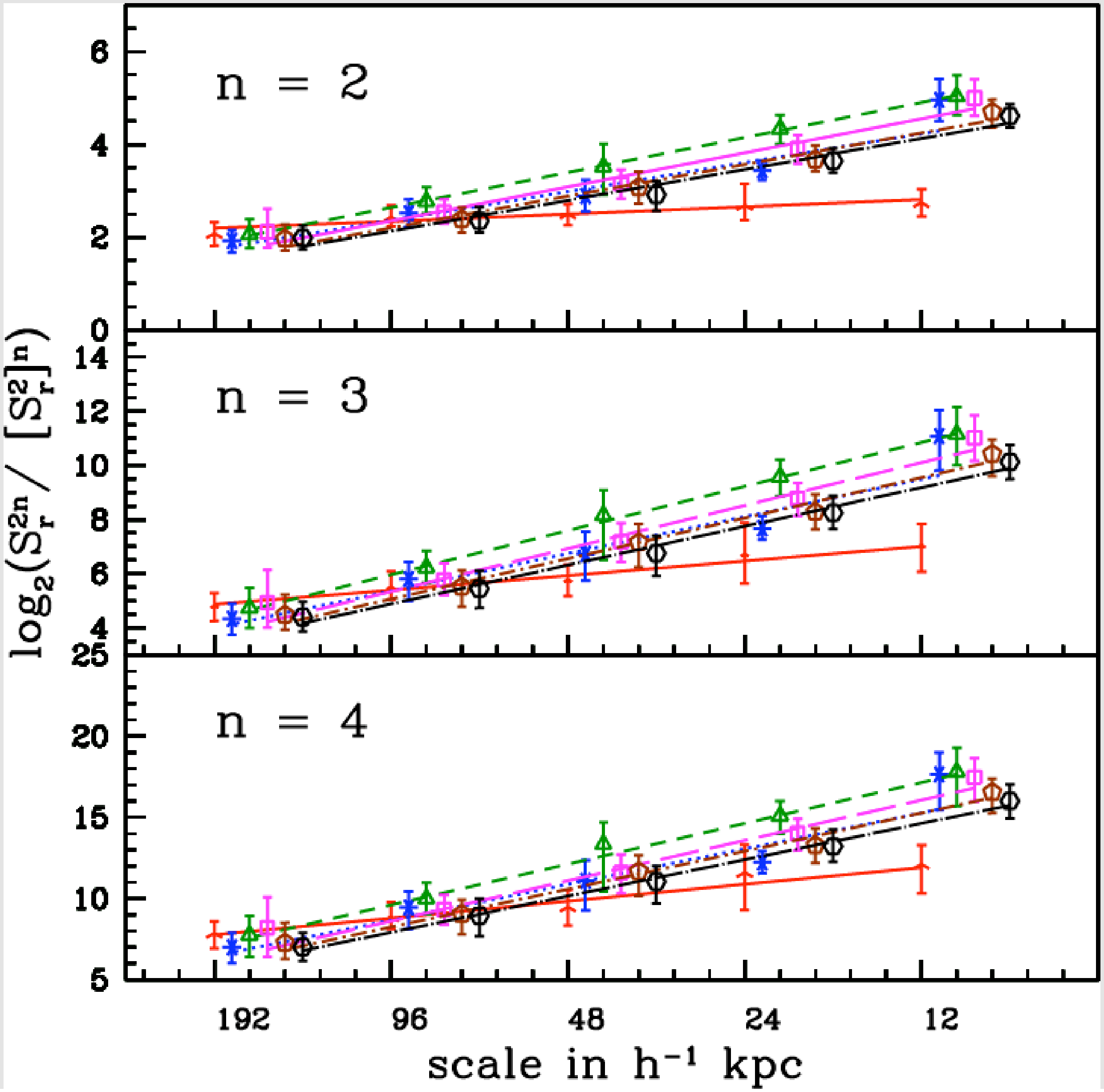
Samples: 28 Keck HIRES QSO spectra. redshift z from 2.19 – 4.11

$$k = 0.4 \times 2^{j-11} \, \text{s/km}$$

$$4 \times 2^{13-j} \, \text{km/s}$$

Pando, Feng, Fang 2002

Structure Functions of Ly-alpha transmission



LCDM

WDM

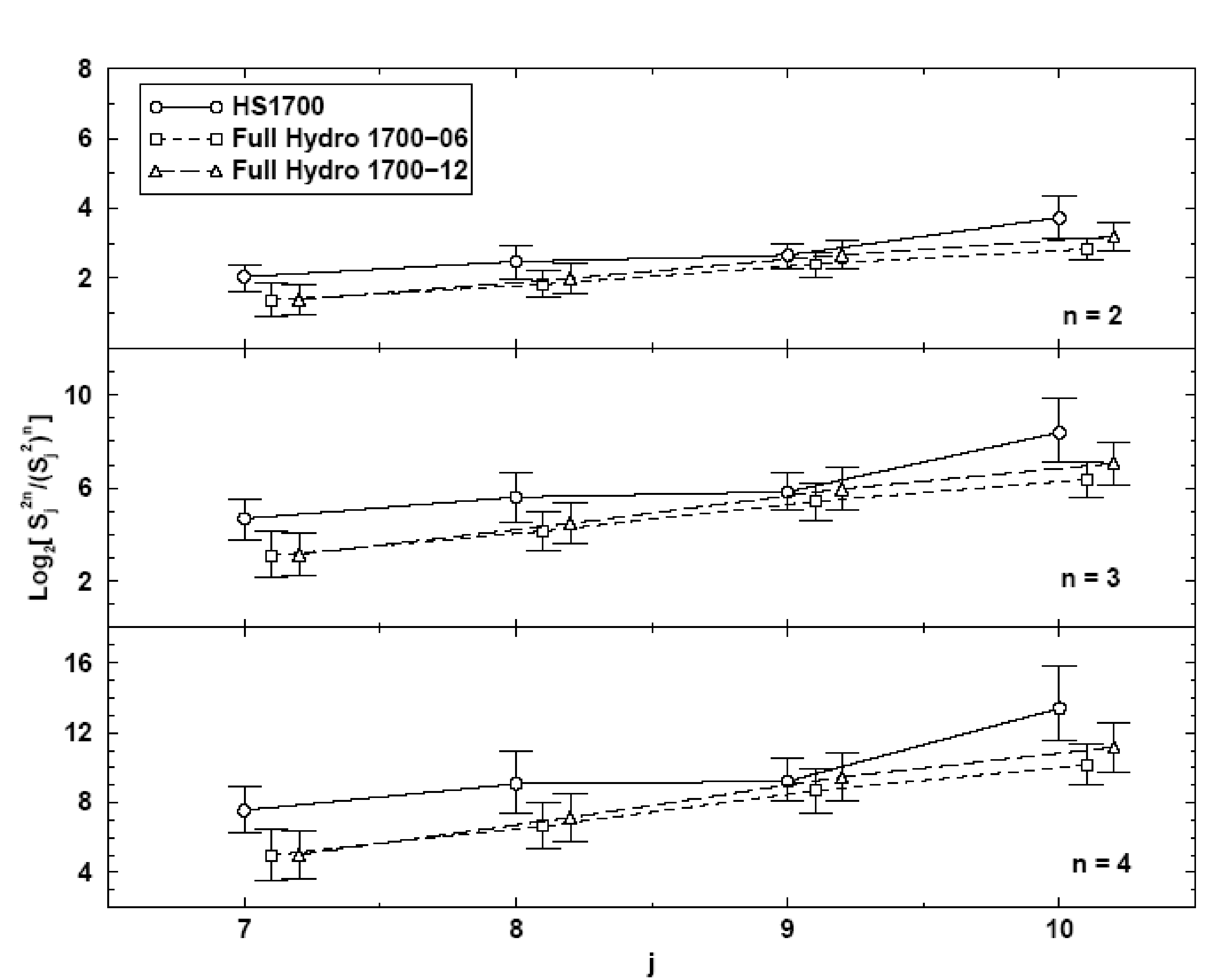
m_W =1000ev

m_W =800ev

m_W =600ev

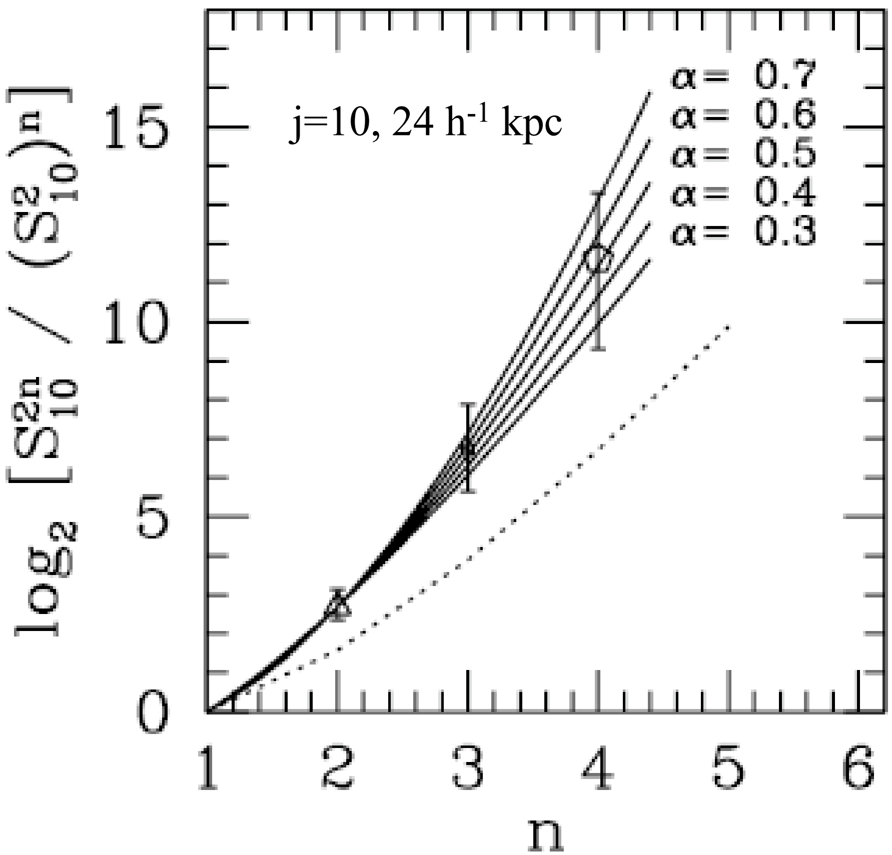
m_W =300ev

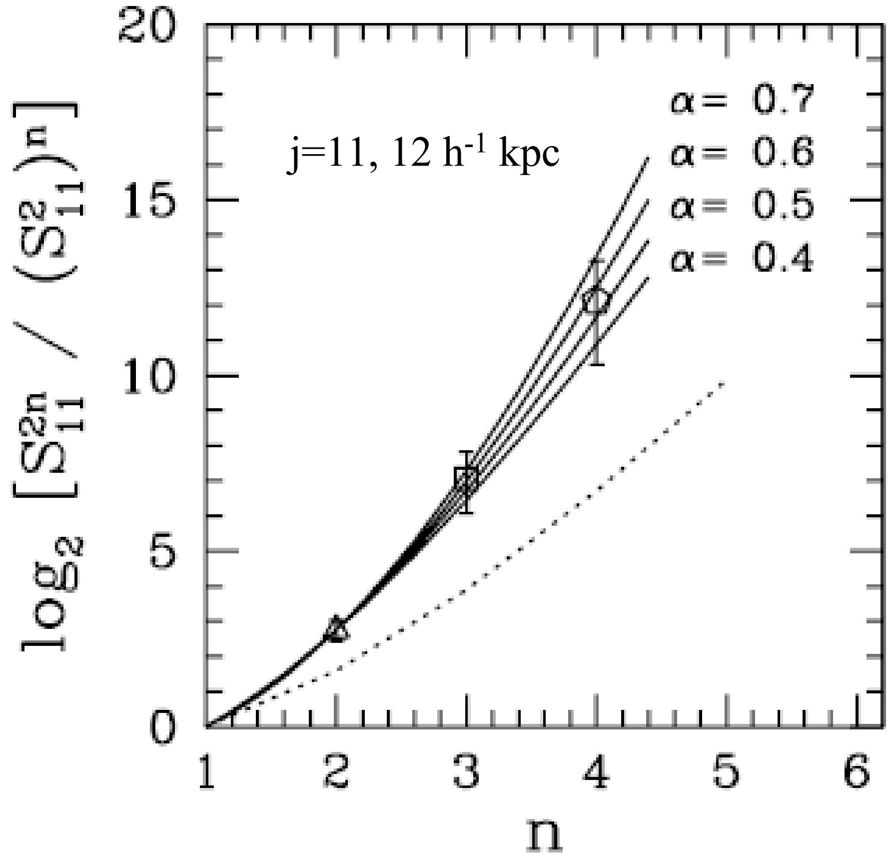
Pando, Feng, Jamkhedkar, Fang, 2002



Feng, Pando, Fang 2003

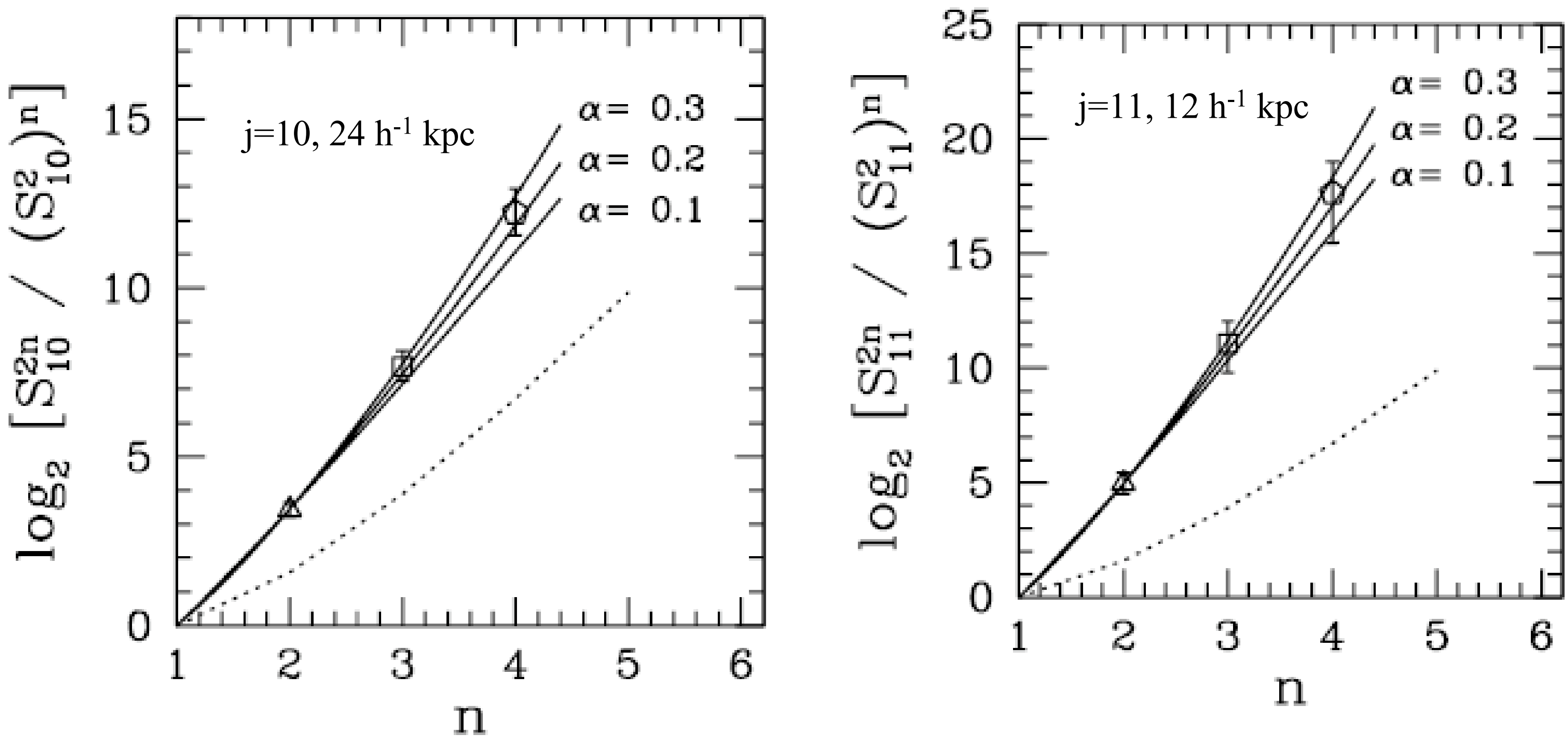
Fit Keck Data to $\log SF \propto n^{\alpha}(n-1)$





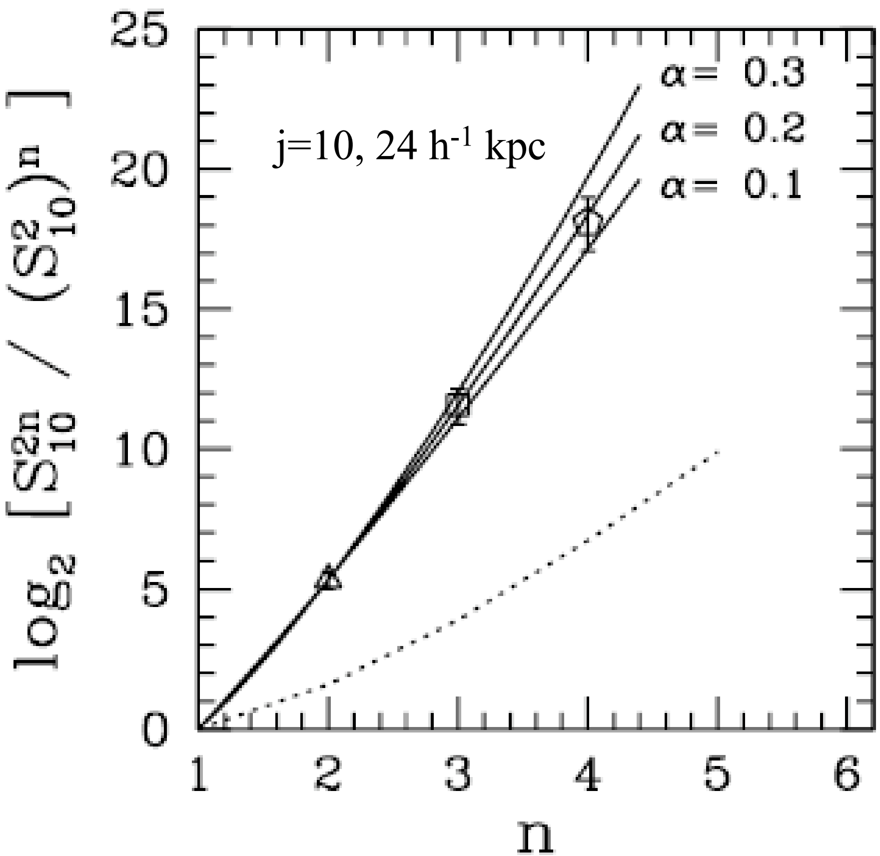
Pando, Feng, Jamkhedkar, Fang, 2002

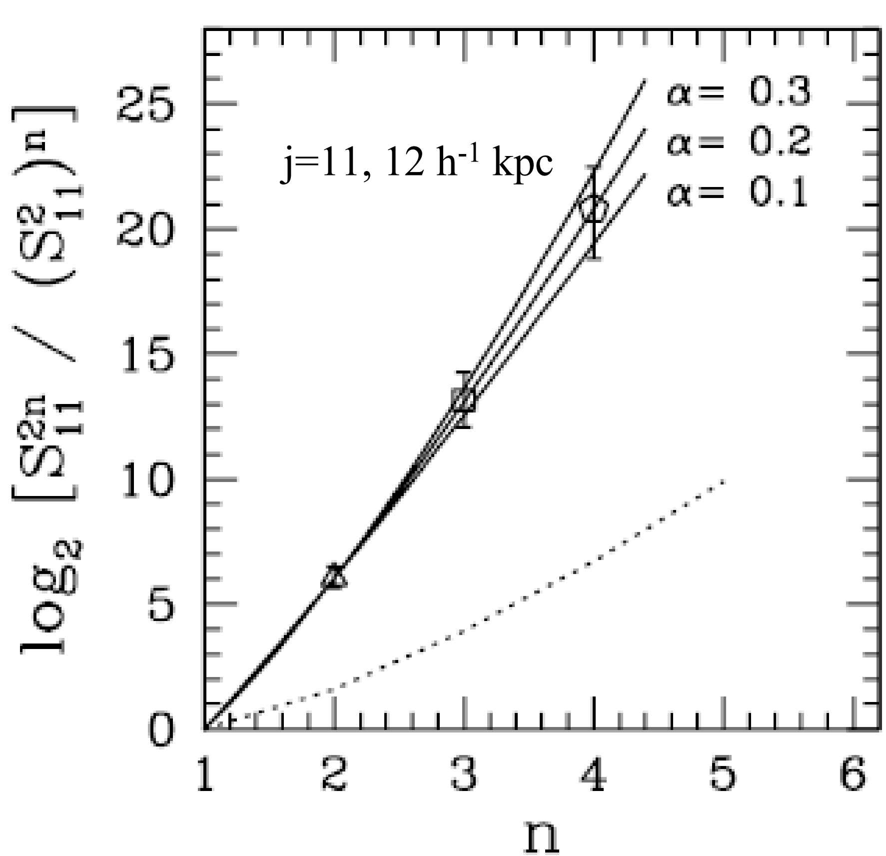
Fit LCDM Data to $\log SF \propto n^{\alpha}(n-1)$



Pando, Feng, Jamkhedkar, Fang, 2002

Fit WDM 300 eV Data to $\log SF \propto n^{\alpha}(n-1)$



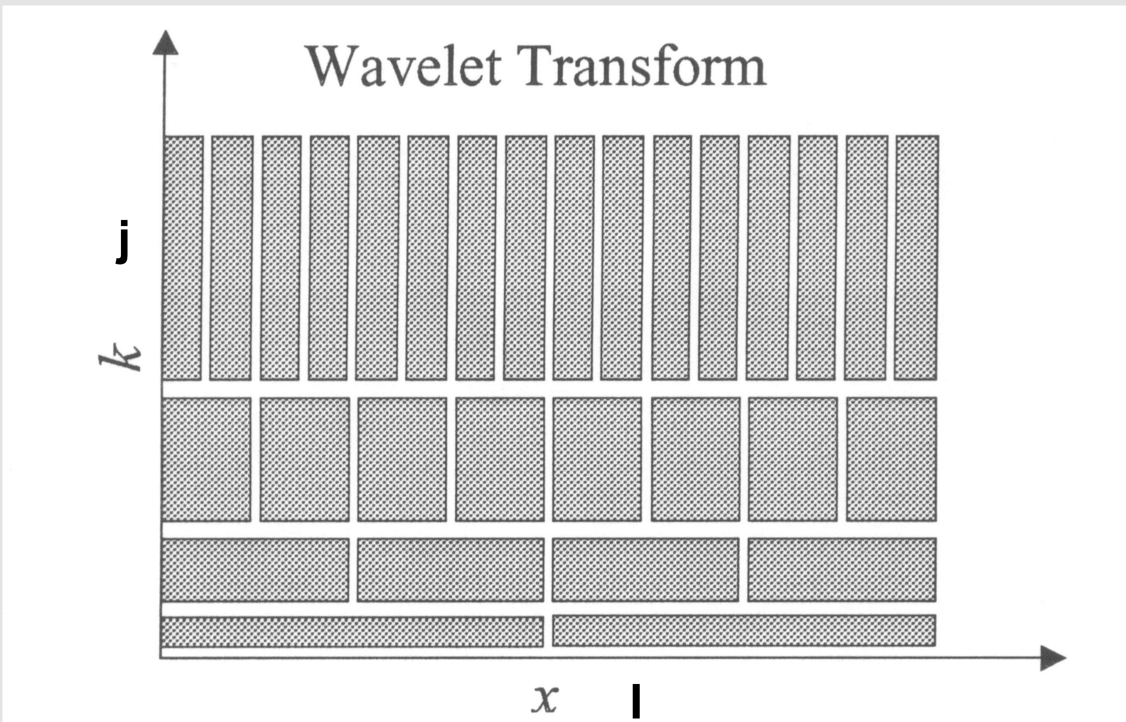


Pando, Feng, Jamkhedkar, Fang, 2002

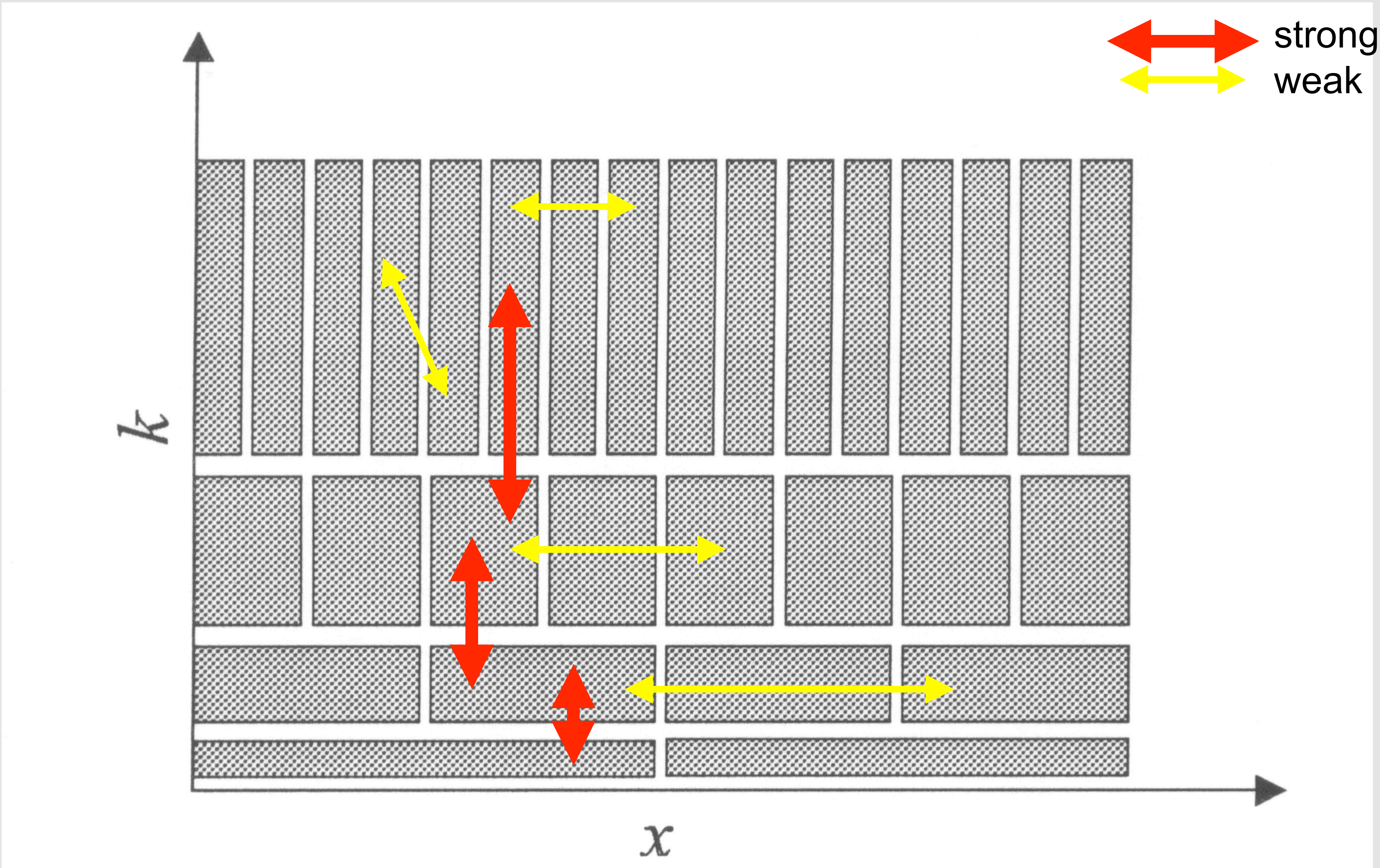
quasi-local evolution of cosmic clustering in the wavelet representation

Gaussian field in the DWT representation

$$\langle \tilde{\varepsilon}_{jl} \tilde{\varepsilon}_{j'l'} \rangle \propto \delta_{jj'} \delta_{ll'}$$



spatial localized correlation



$$\Delta x \Delta k \approx 2\pi$$

quasi-locality
(second order)

$$\kappa_{j,j'}(\Delta l) \equiv \frac{\langle \tilde{\varepsilon}_{jl} \tilde{\varepsilon}_{j'l'} \rangle}{\langle \tilde{\varepsilon}_{jl}^{2} \rangle^{1/2} \langle \tilde{\varepsilon}_{j'l'}^{2} \rangle^{1/2}}$$

$$\kappa_{j,j'}(\Delta l) \approx 0, \quad \text{if } \Delta \mathrm{l} = l \neq l'$$

quasi-locality
(higher order)

$$C^{p,q}_{j,j'}(\Delta l) = \frac{\left\langle \widetilde{\varepsilon}^{\,p}_{j,l}\, \widetilde{\varepsilon}^{\,q}_{j',l'} \right\rangle}{\sum \widetilde{\varepsilon}^{\,p}_{j,l} \sum \widetilde{\varepsilon}^{\,q}_{j',l'}}$$

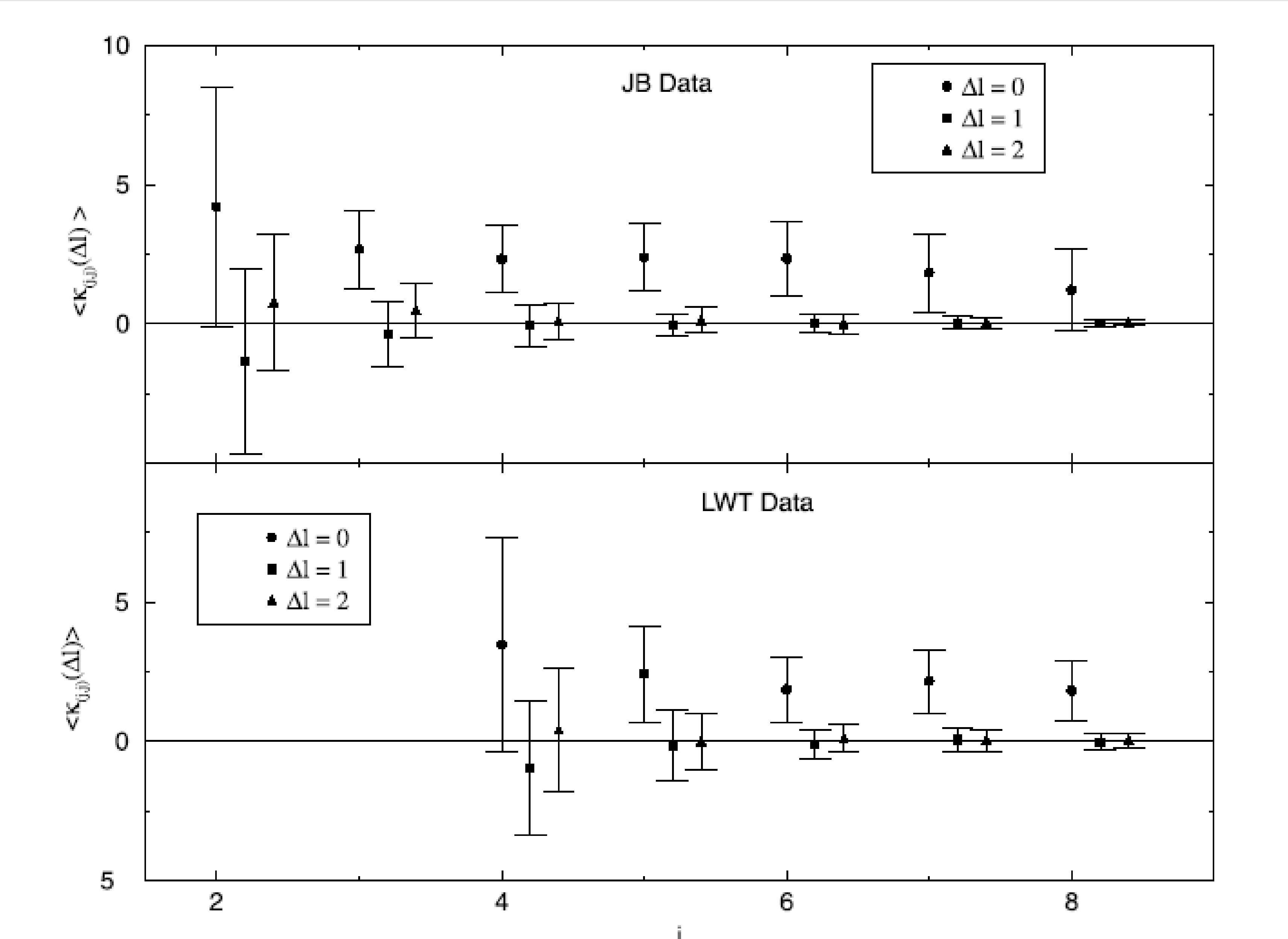
$$C^{p,q}_{j,j'}(\Delta l \neq 0) \approx 1$$

quasi-locality of mass field in Zeld'ovich approximation
(weak linear regime)

If the initial perturbations are Gaussian, the cosmic gravitation clustering in weakly nonlinear regime given by the Zeldovich approximation is quasi-localized in the DWT representation.

Pando, Feng, Fang 2001

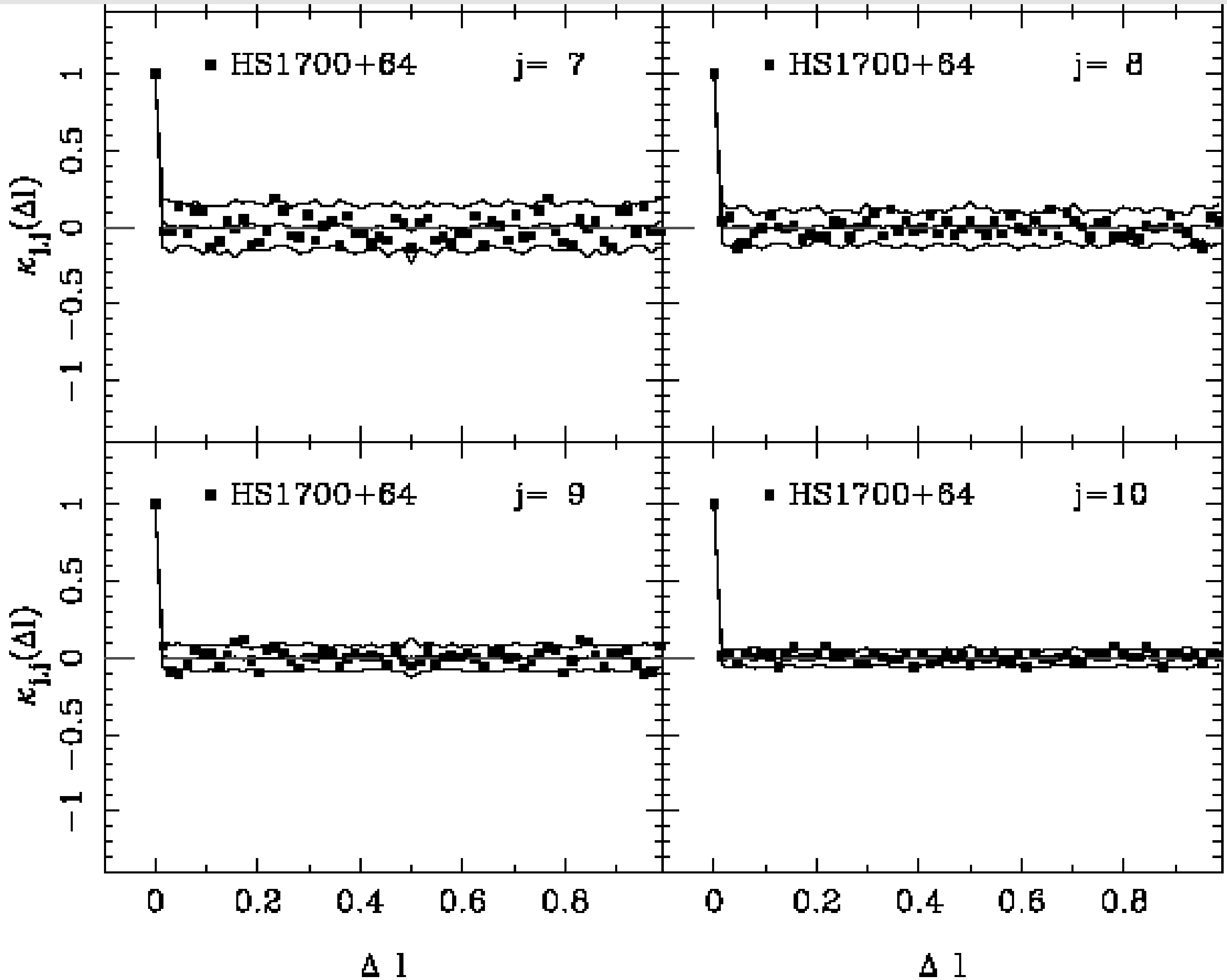
second order quasi-locality : Ly-alpha data



$189/2^{j}\ \ h^{-1}\mathrm{Mpc}$

Pando, Feng, Fang 2001

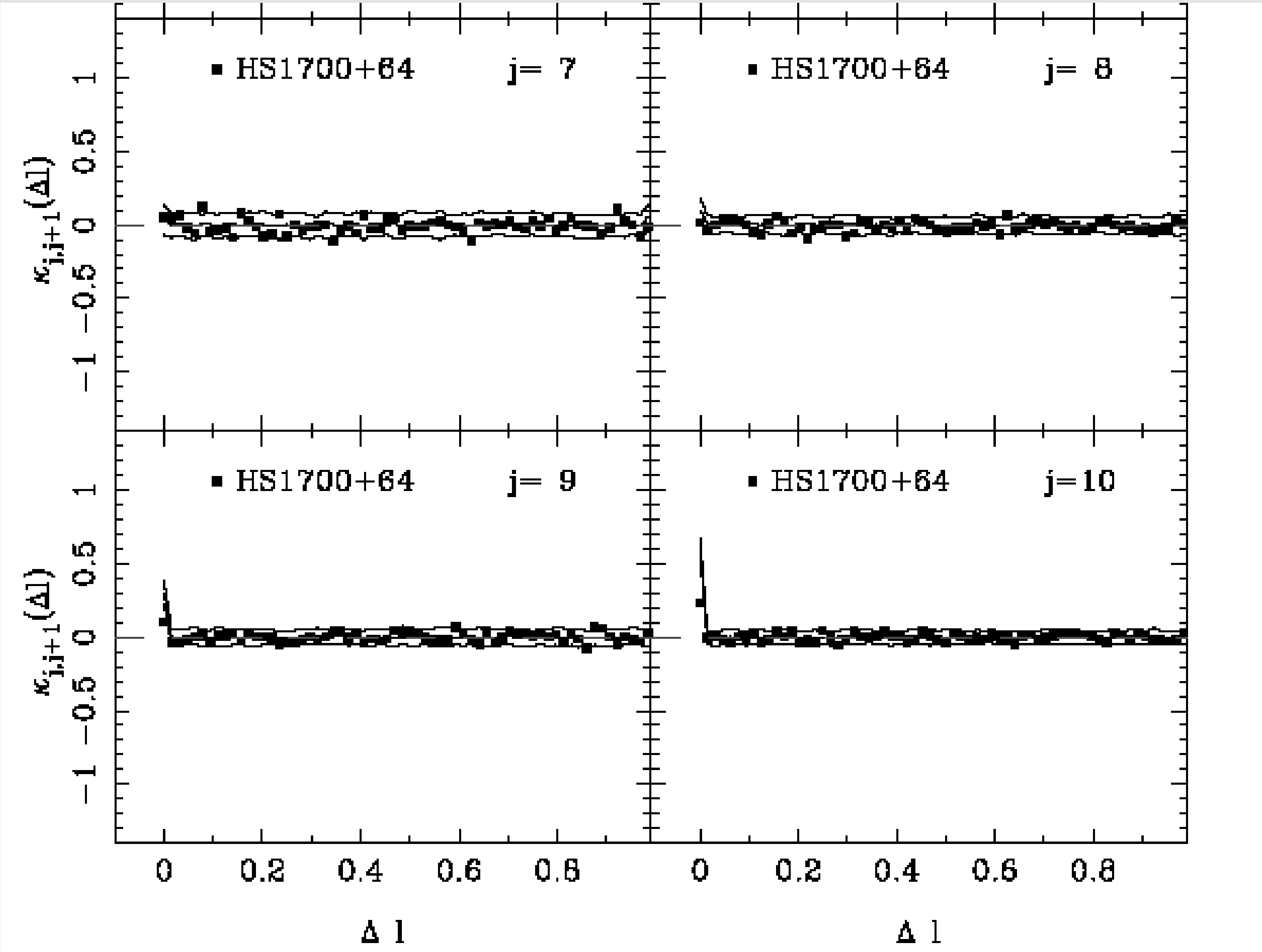
second order quasi-locality : Ly-alpha data



$189/2^j\ \ h^{-1}$Mpc

Pando, Feng, Fang 2001

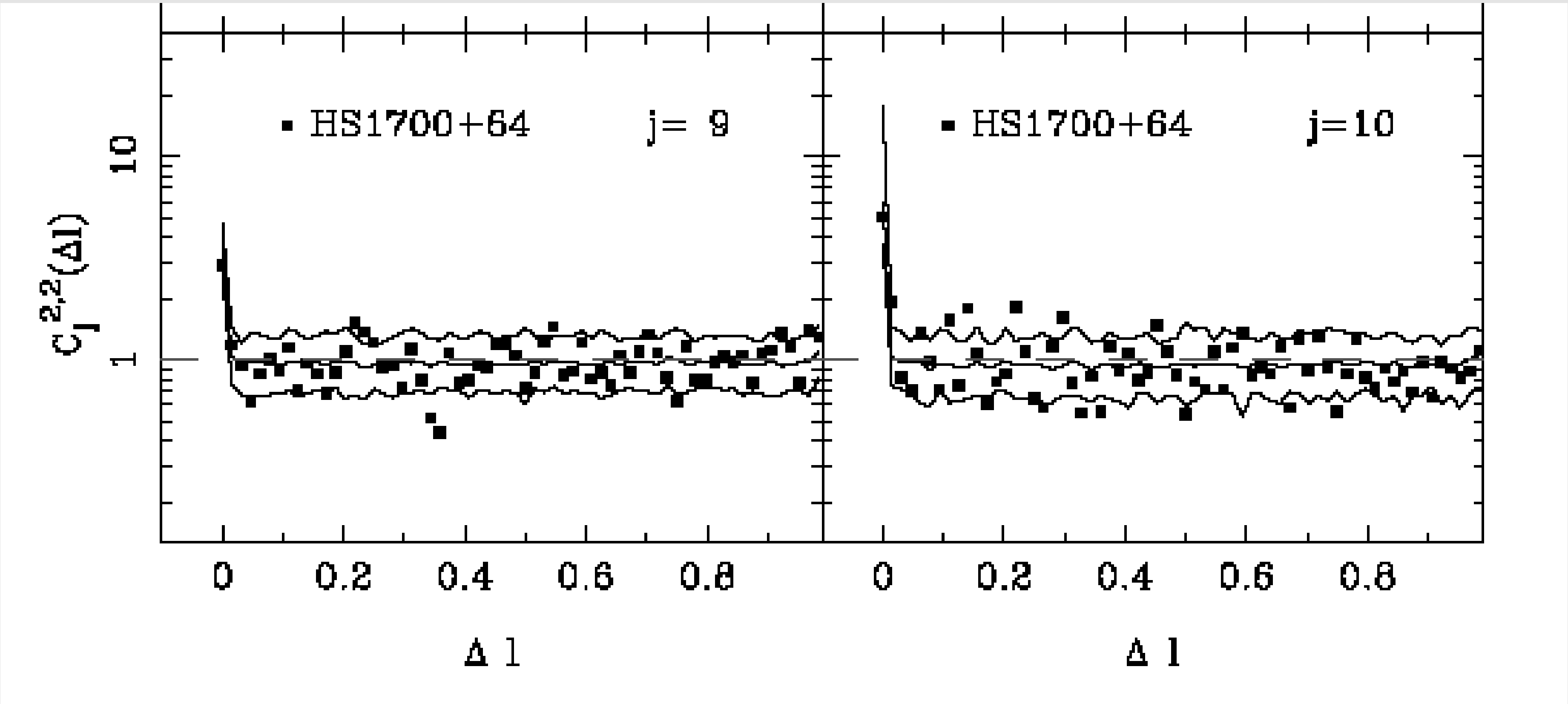
Ly-alpha data: second order



$189/2^j\ \ h^{-1}\mathrm{Mpc}$

Pando, Feng, Fang 2001

4^{th} order locality: Ly-alpha data



$189/2^j \ h^{-1}\mathrm{Mpc}$

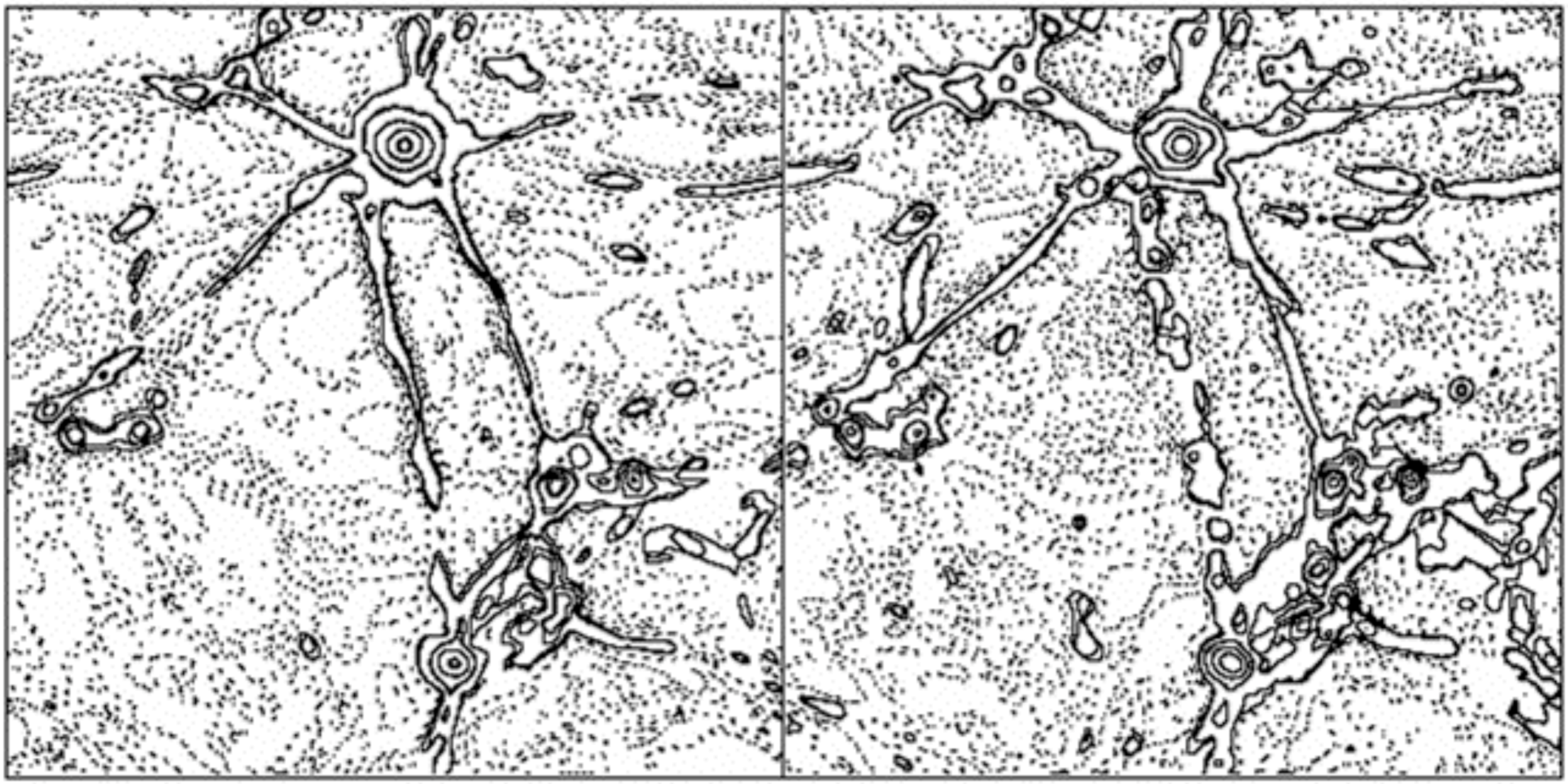
Pando, Feng, Fang 2001

halo model
(highly nonlinear regime)

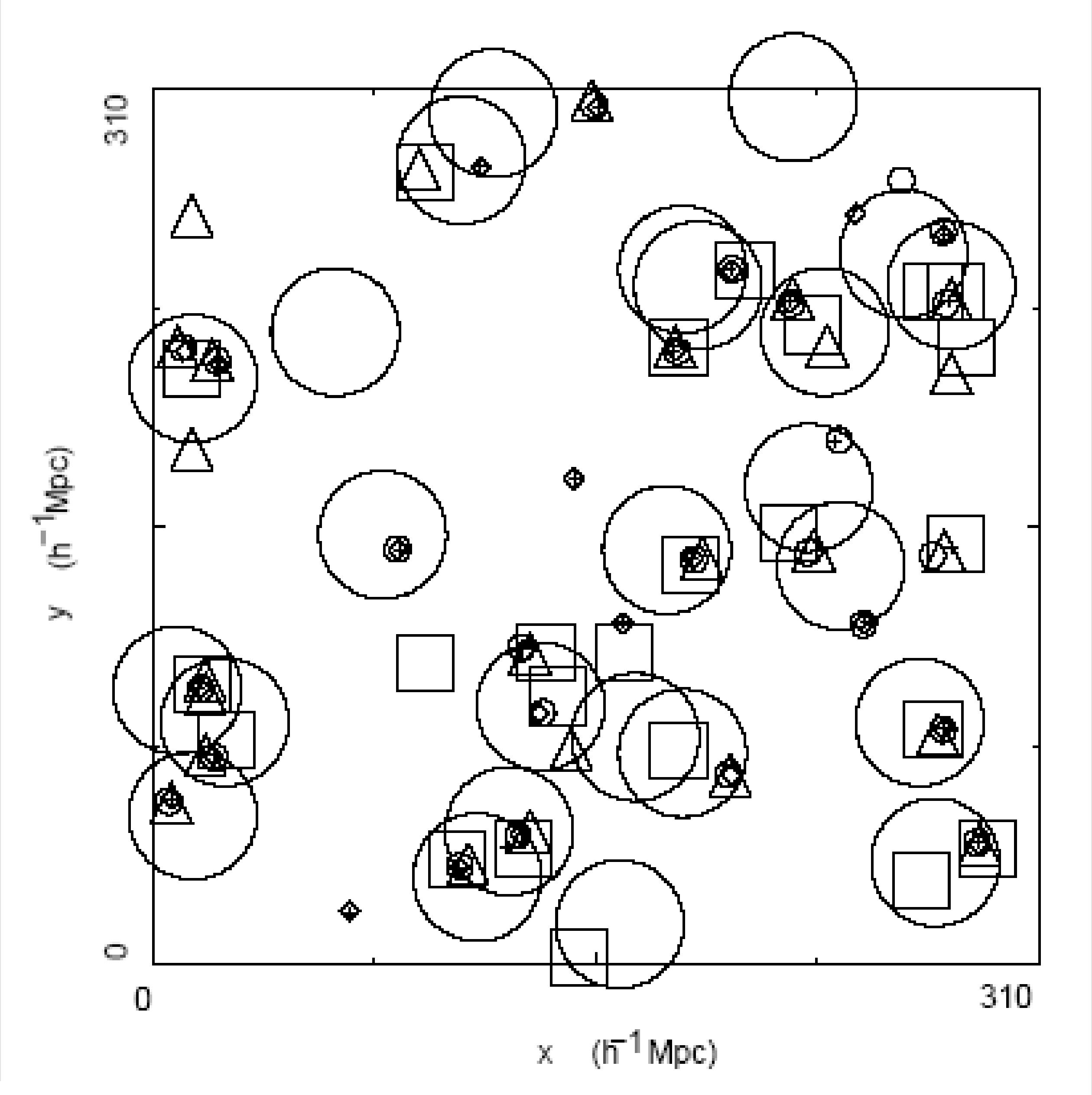
$$\rho(\mathbf{x}) = \sum_i \rho_i(\mathbf{x} - \mathbf{x}_i) = \sum_i m_i u(\mathbf{x} - \mathbf{x}_i, m_i)$$

$$u(r, m) = \frac{\rho_c}{r^\alpha (r + r_s)^\beta}$$

$$\rho_c \propto m^\gamma, \quad r_s \propto m^\delta$$



halo model



Xu, Fang, Wu 2000

quasi-locality of mass field in halo model

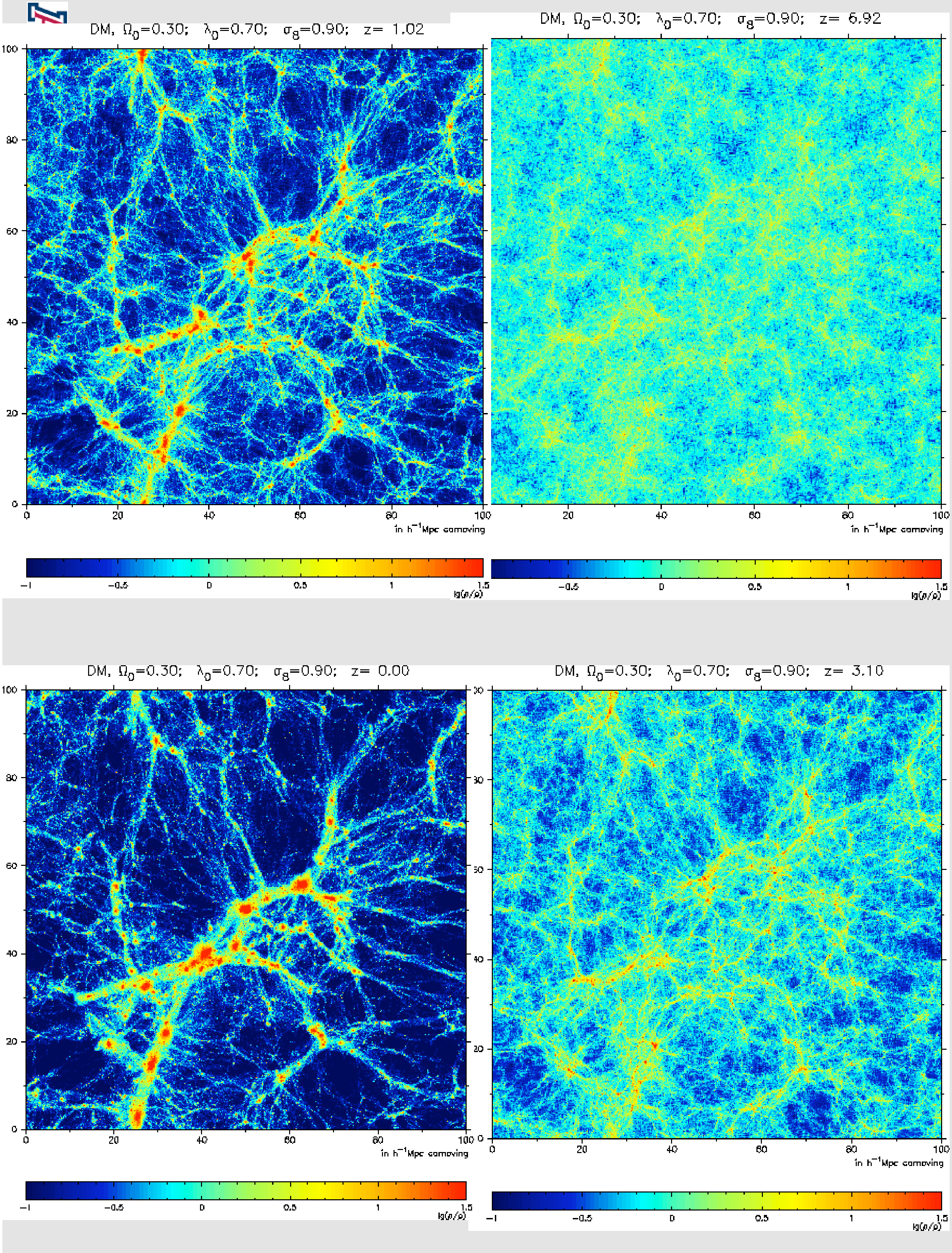
If the initial perturbations is Gaussian (no correlation between modes at different physical positions), the evolved field will also spatially weakly correlated. The nonlinear evolution of gravitational clustering has memory of the initial spatial correlations.

Feng, Fang 2004

N-body simulation

	Box size (Mpc/h)	m_p (M_sun/h)	L_{min} (L_sun/h^2)	Realizations
LCDM	100	6.5×10^8	2×10^8	4
LCDM	300	1.8×10^{10}	3×10^9	4

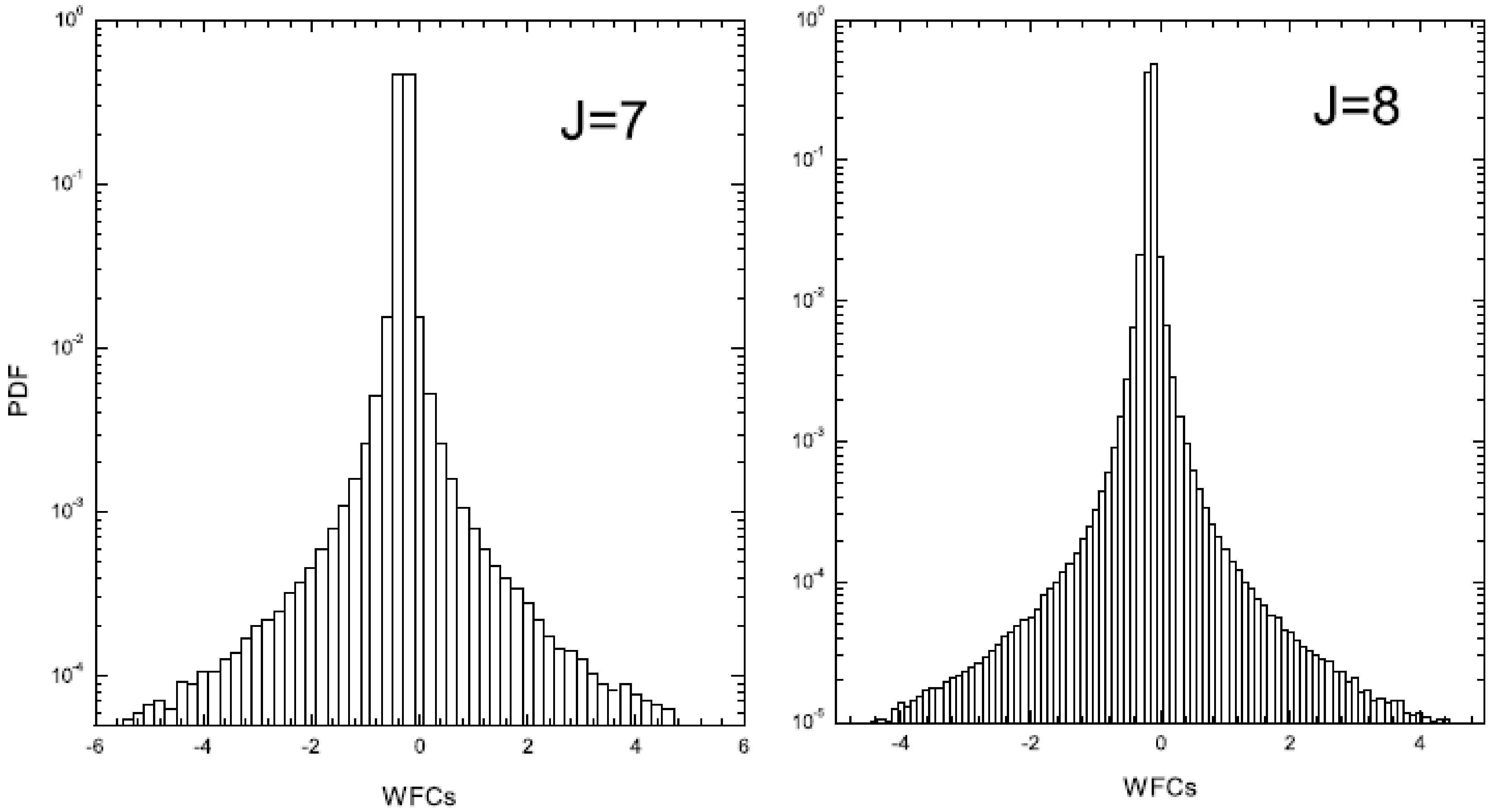
Jing, Suto (2000), Jing (2001)



DM, Ω_0=0.30; λ_0=0.70; σ_8=0.90; z= 1.02
in h^{-1}Mpc comoving
lg($\rho/\bar{\rho}$)
DM, Ω_0=0.30; λ_0=0.70; σ_8=0.90; z= 6.92
in h^{-1}Mpc comoving
lg($\rho/\bar{\rho}$)
DM, Ω_0=0.30; λ_0=0.70; σ_8=0.90; z= 0.00
in h^{-1}Mpc comoving
lg($\rho/\bar{\rho}$)
DM, Ω_0=0.30; λ_0=0.70; σ_8=0.90; z= 3.10
in h^{-1}Mpc comoving
lg($\rho/\bar{\rho}$)

N-body simulation sample (Jing, Suto 2002)

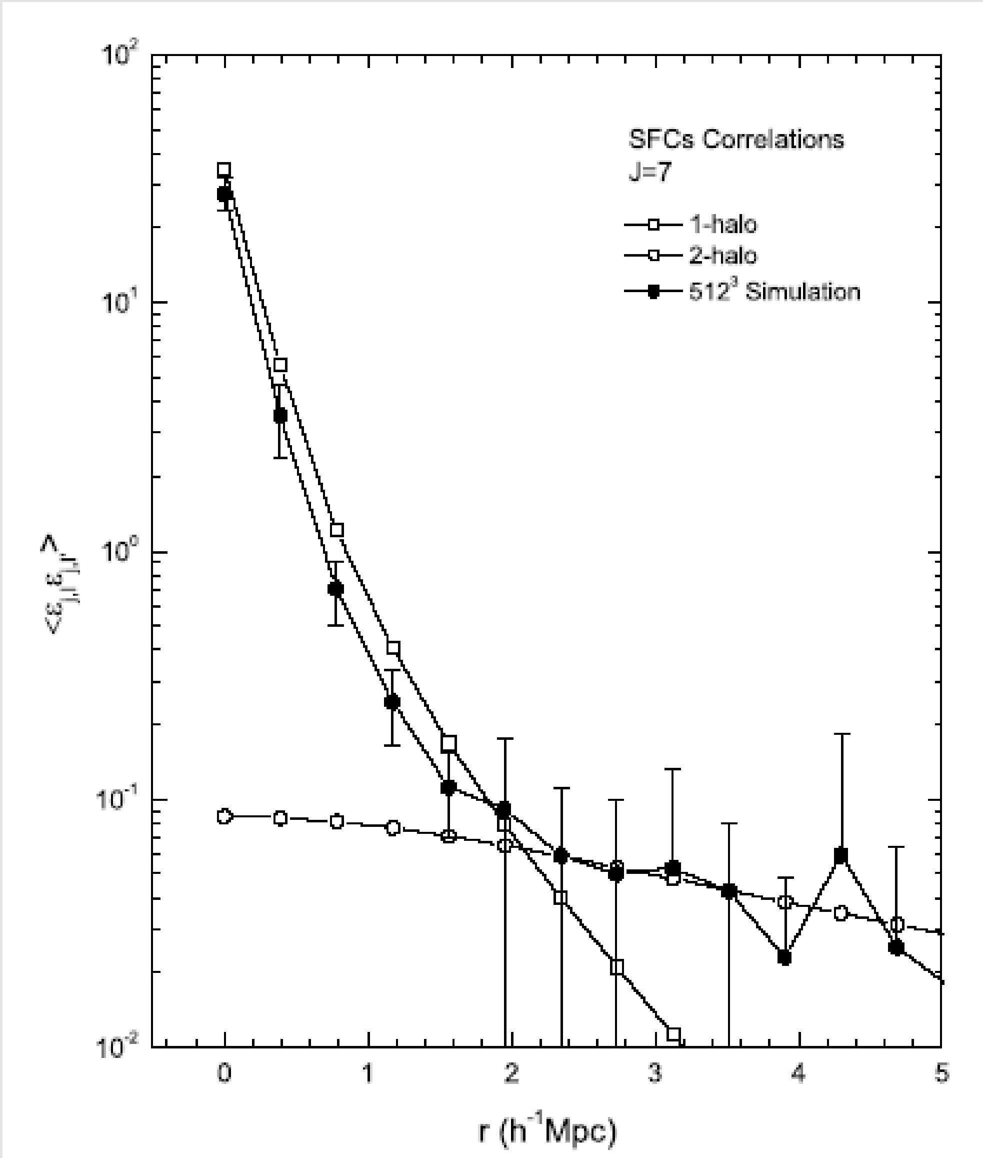
One-point distribution of WFCs



$50/2^j$ h^{-1}Mpc

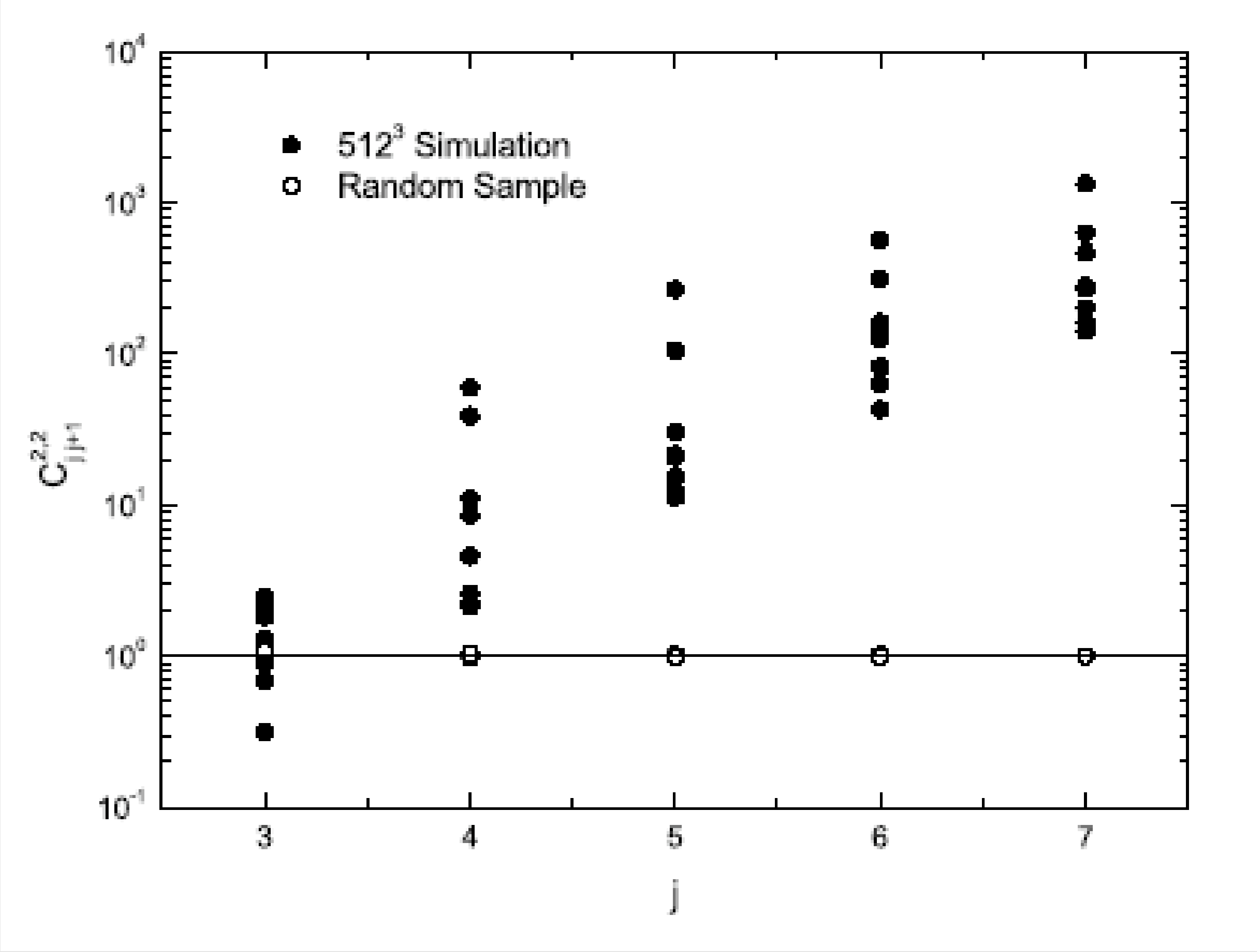
Feng, Fang 2004

two-point correlation functions: N-body simulation sample (Jing, Suto 2002)



Feng, Fang 2004

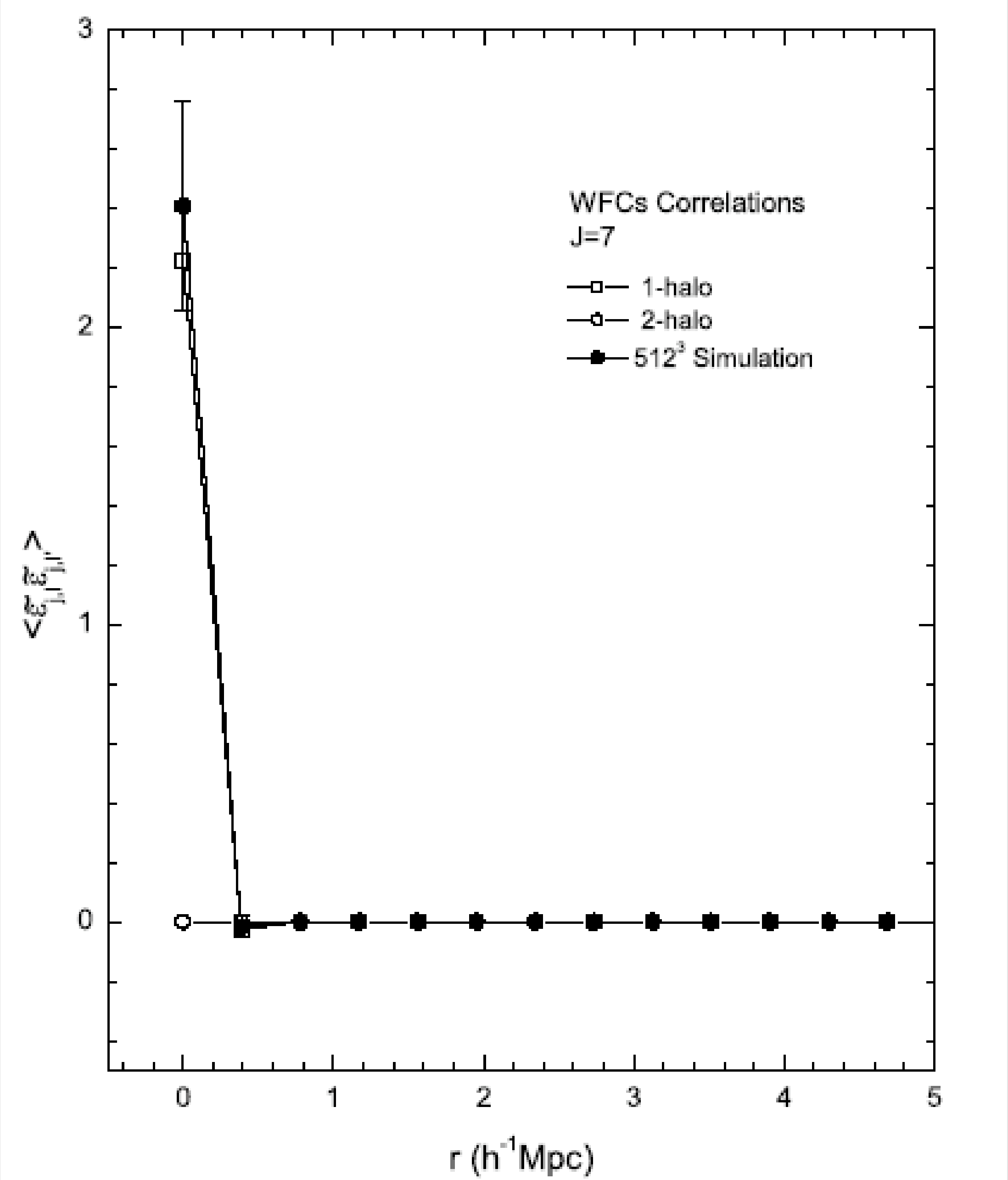
Scale-scale correlation: N-body simulation sample (Jing, Suto 2002)



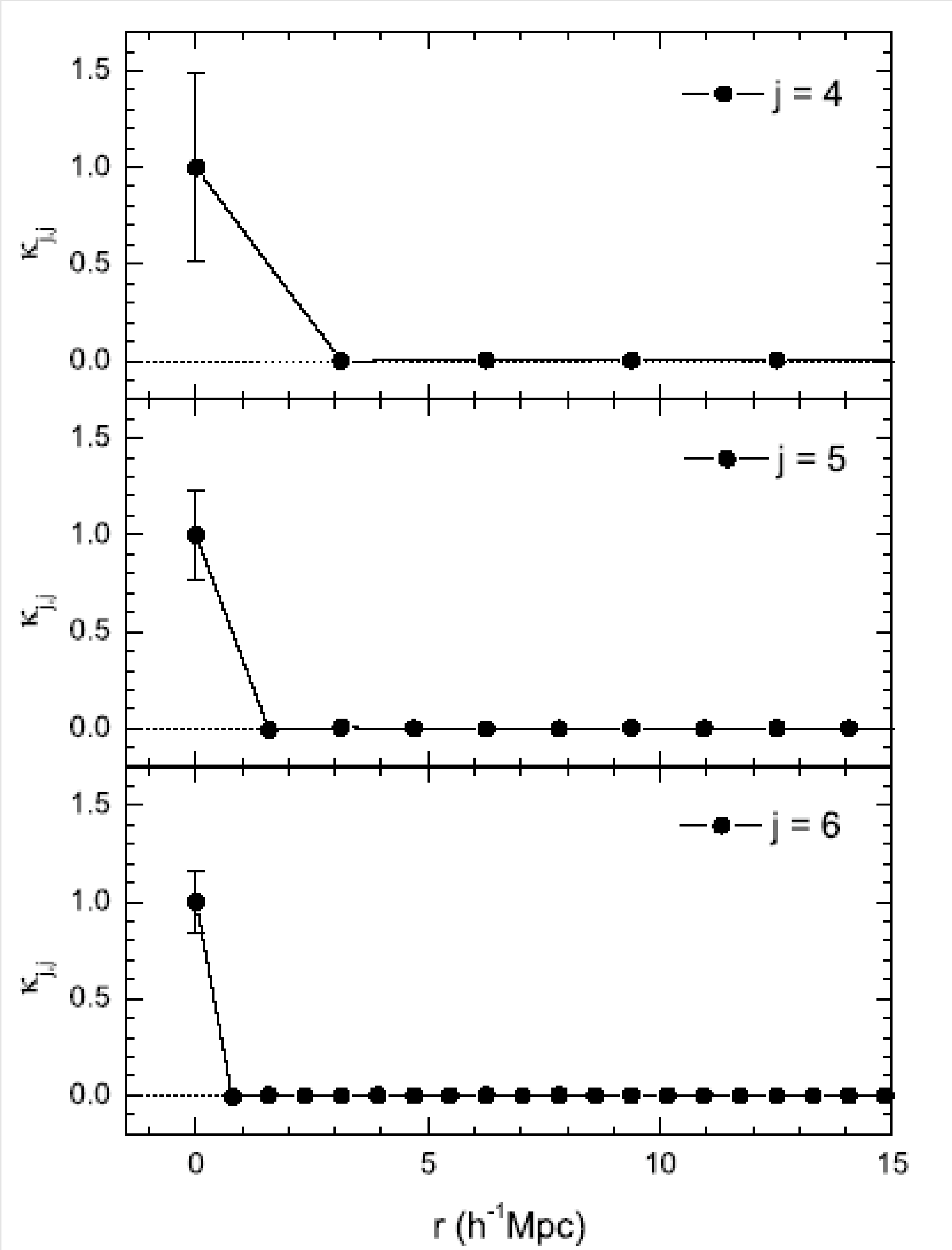
$50/2^j\ h^{-1}\mathrm{Mpc}$

Feng, Fang 2004

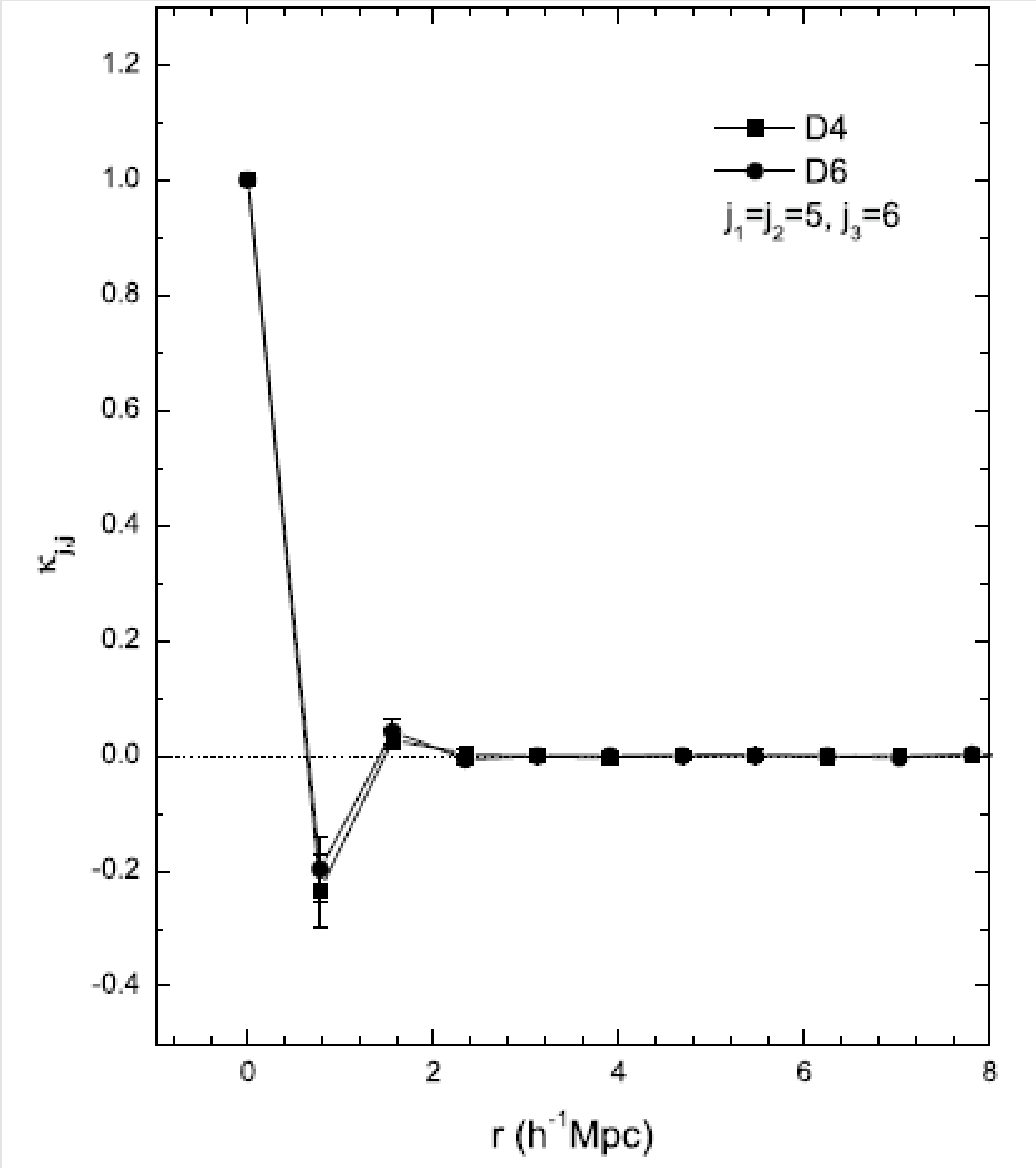
DWT mode-mode correlations: N-body simulation sample (Jing, Suto 2002)



Feng, Fang 2004



Feng, Fang 2004

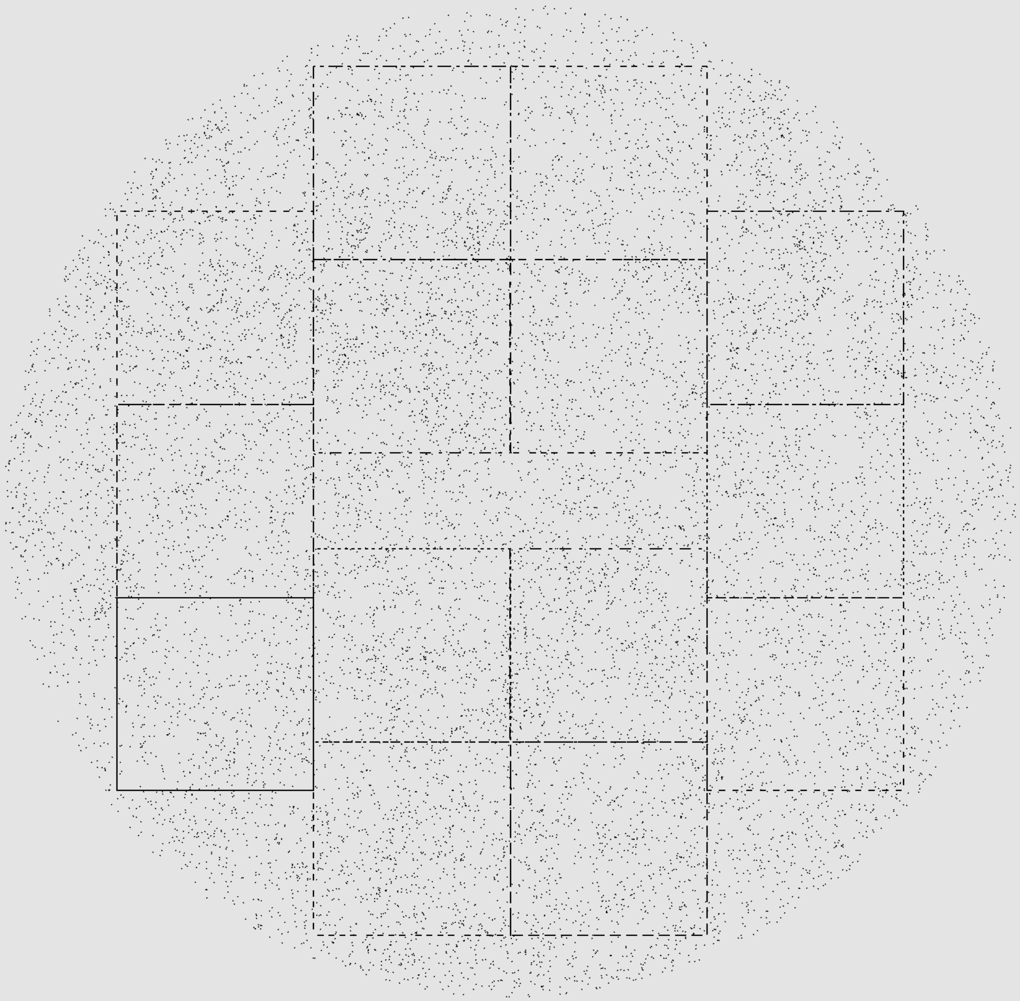


Feng, Fang 2004

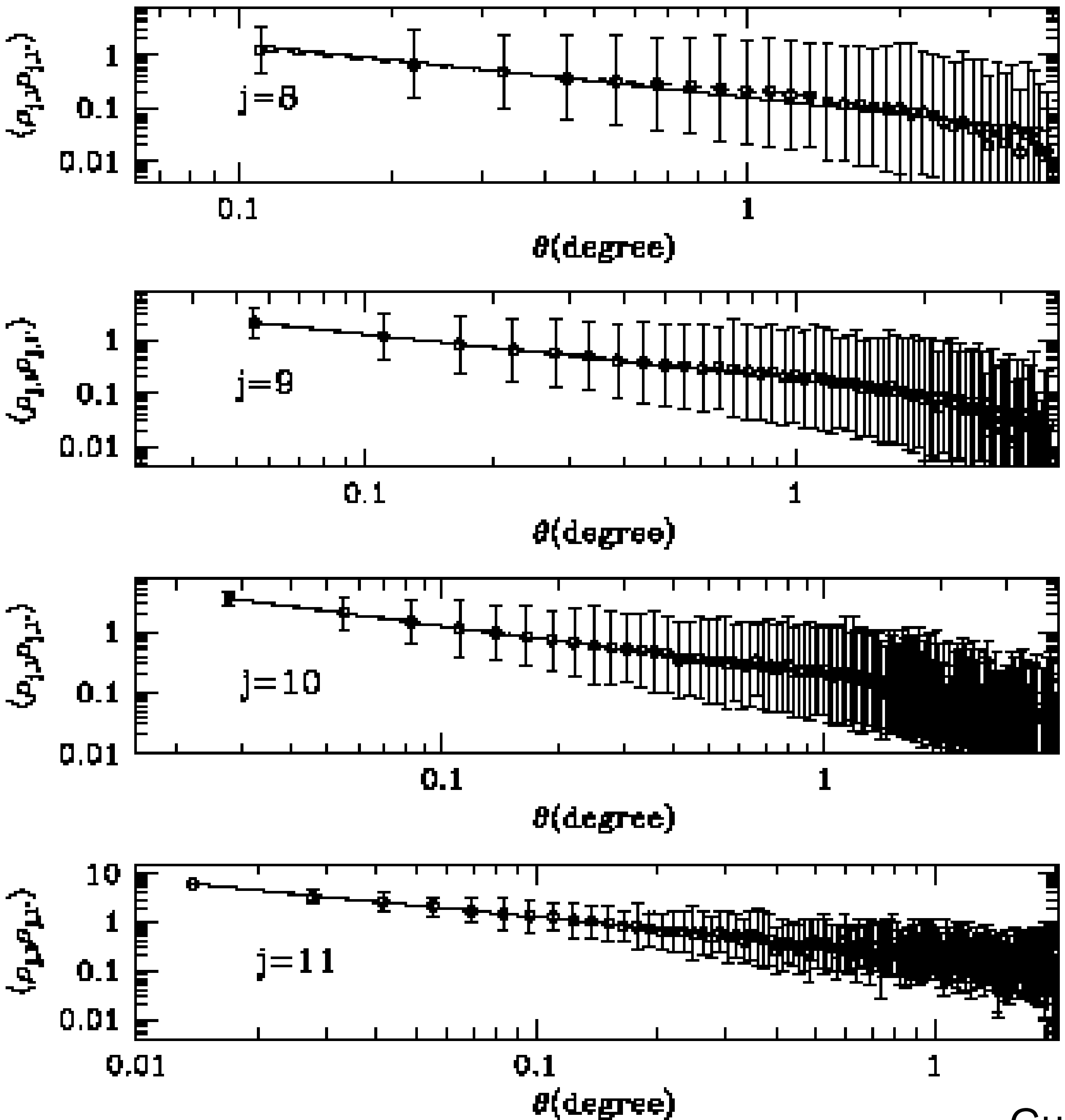
galaxy sample: 2MASS XSC

(Jarrett et al. 2000)

987,125 galaxies

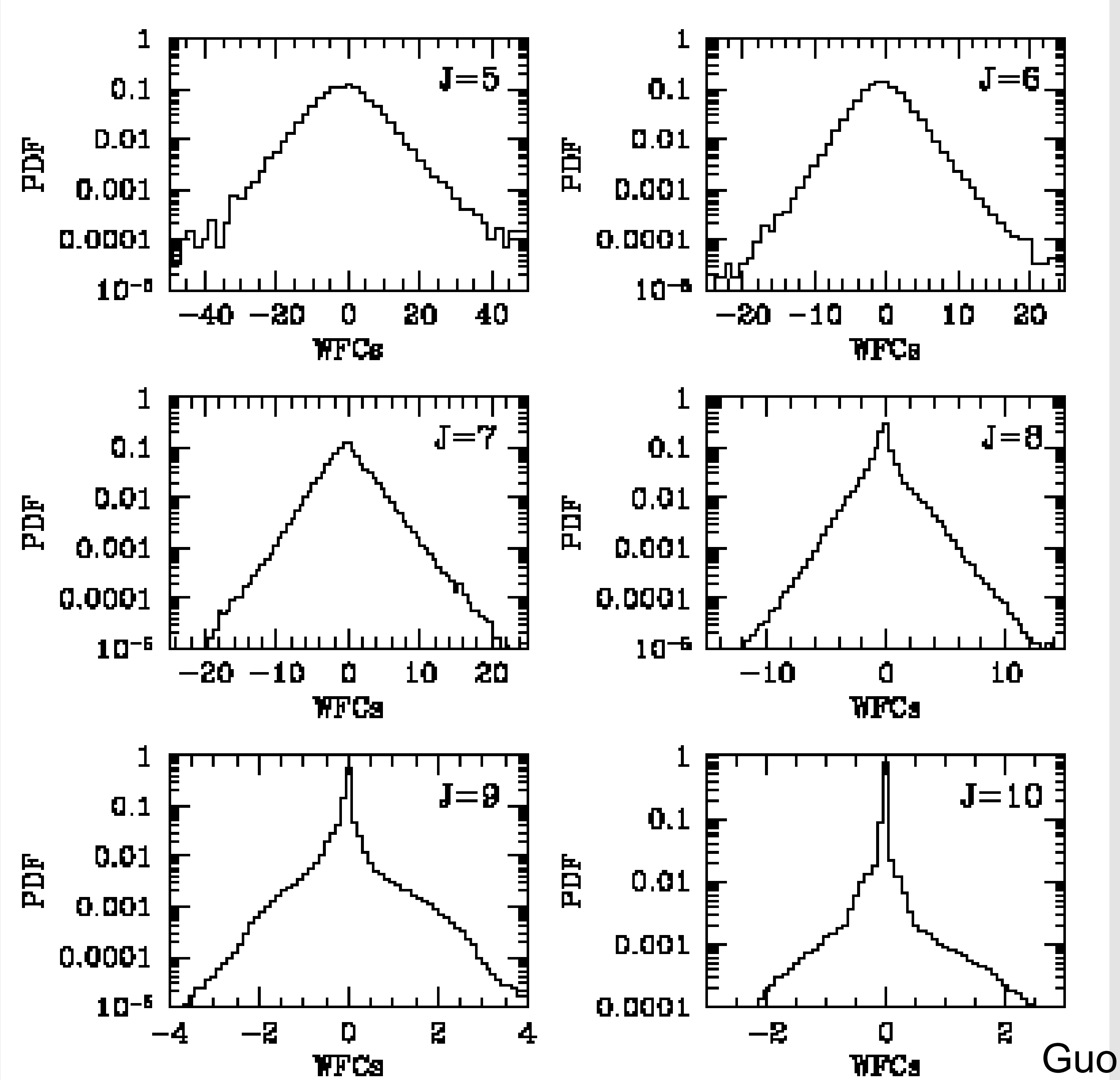


angular correlation functions: 2MASS data



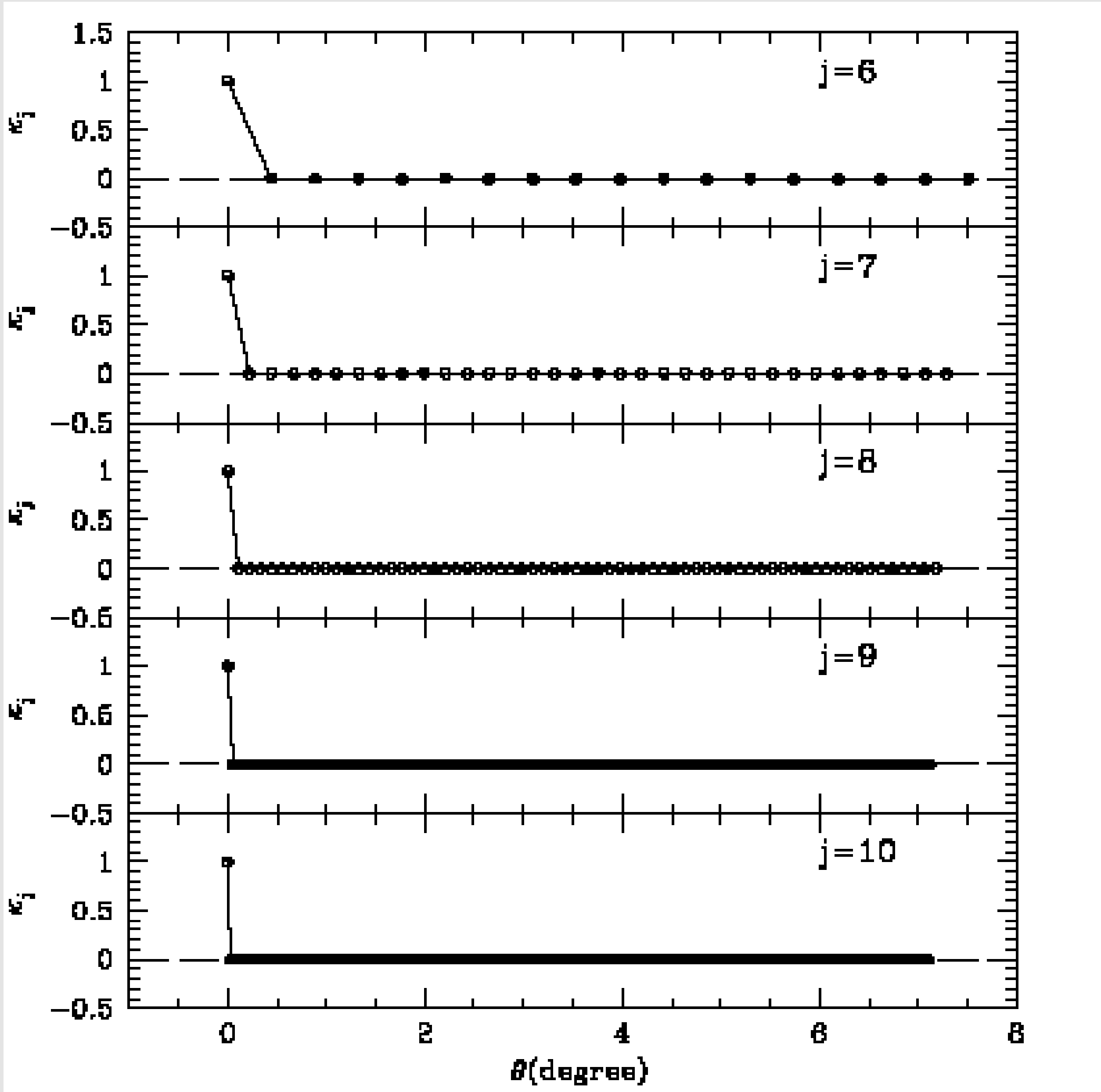
Guo, Chu, Fang 2004

one-point distribution of WFCs: 2MASS data



Guo, Chu, Fang 2004

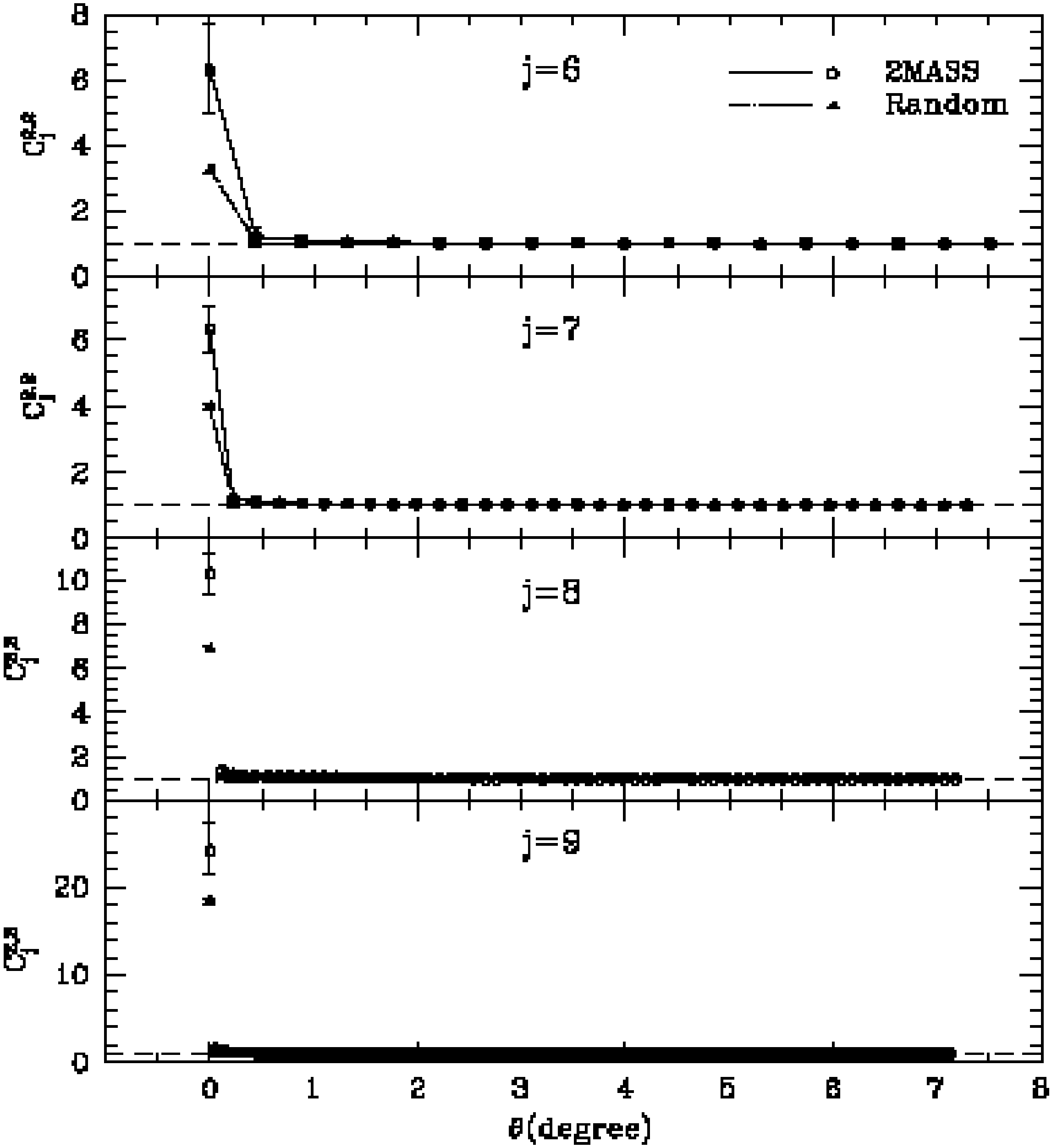
mode-mode correlations of WFCs: 2MASS data



$28 / 2^j$ angular degree

Guo, Chu, Fang 2004

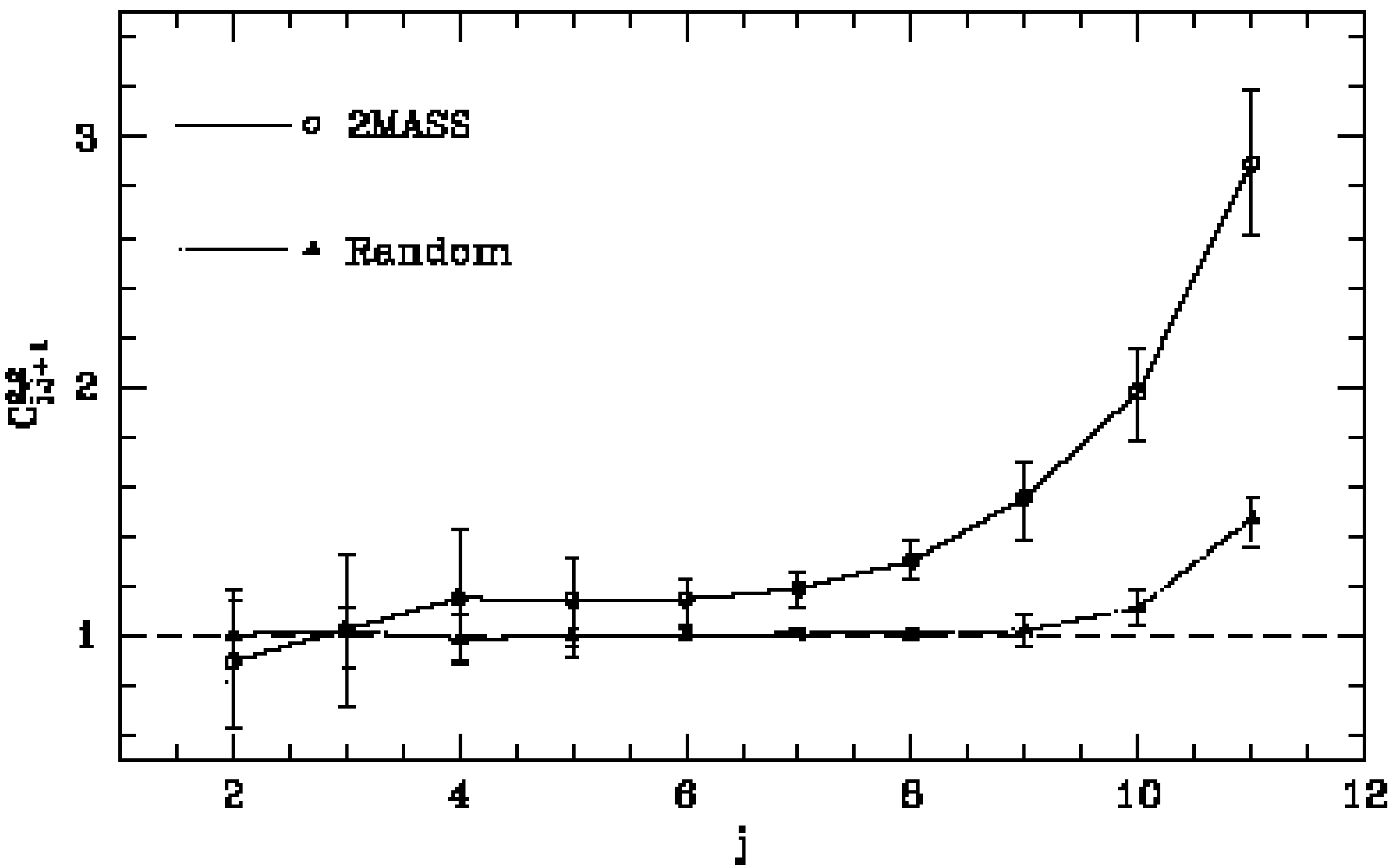
4th order correlations of WFCs: 2MASS data



$28/2^j$ angular degree

Guo, Chu, Fang 2004

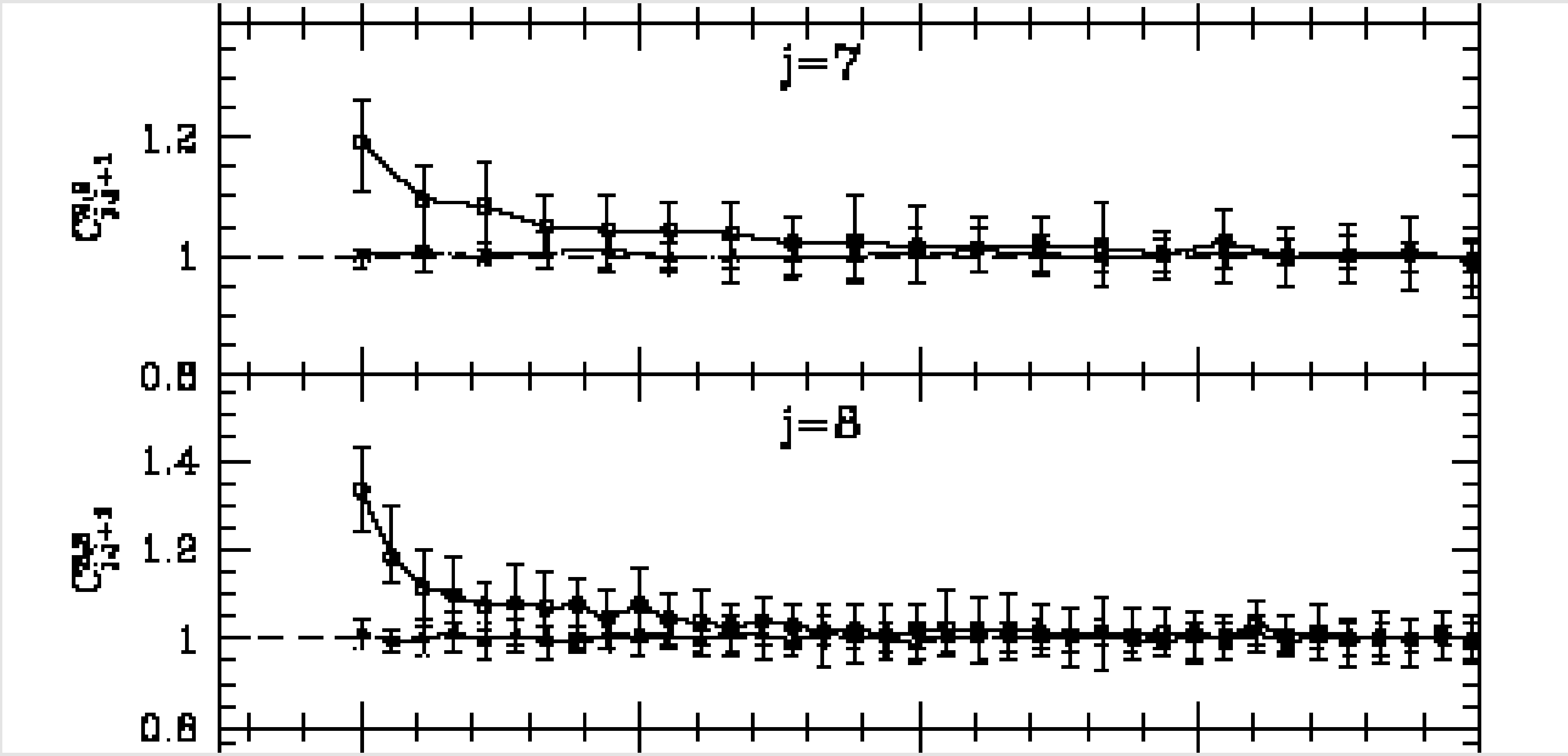
scale-scale angular correlation functions: 2MASS data



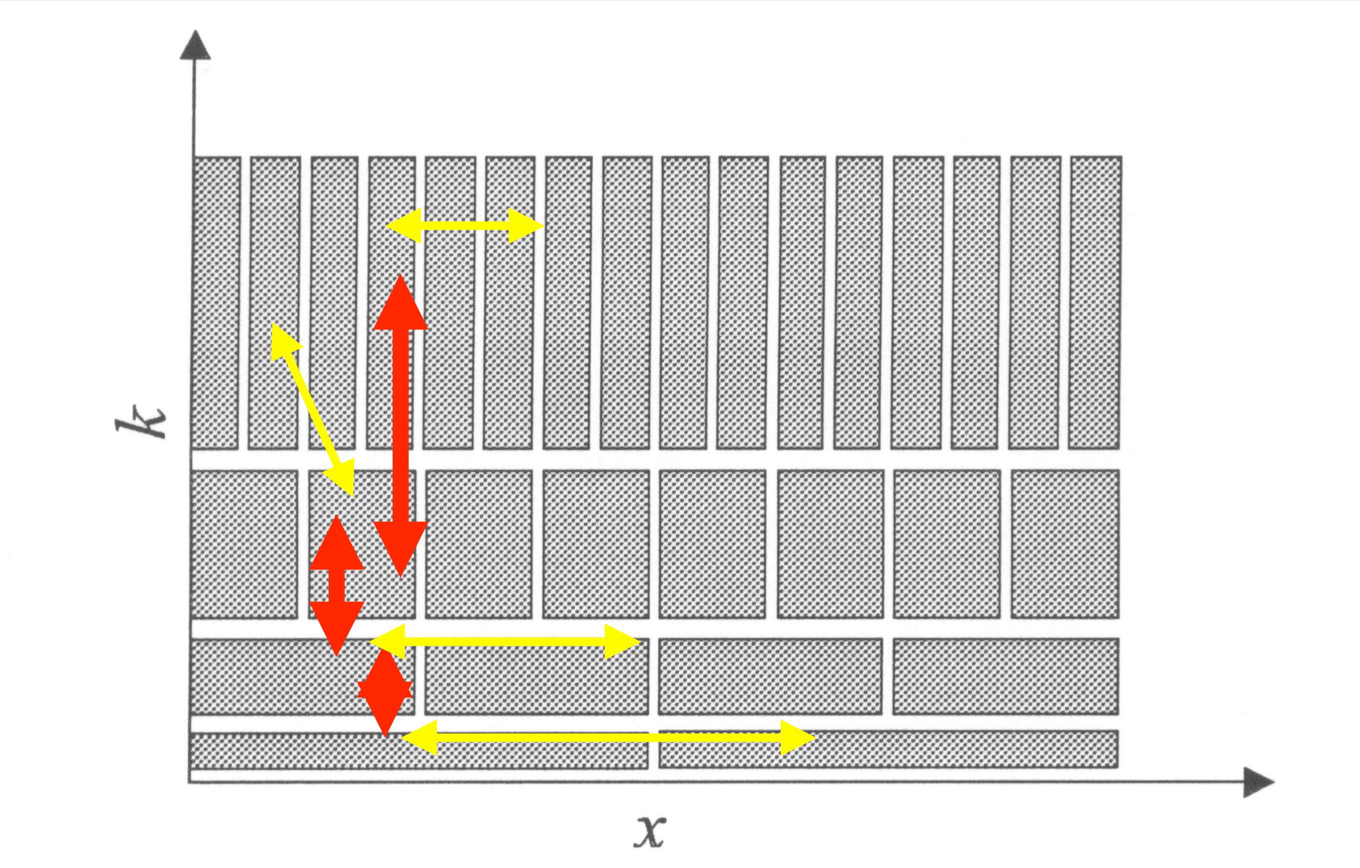
$28 / 2^j$ angular degree

Guo, Chu, Fang 2004

non-local scale-scale angular correlation functions: 2MASS data



Guo, Chu, Fang 2004



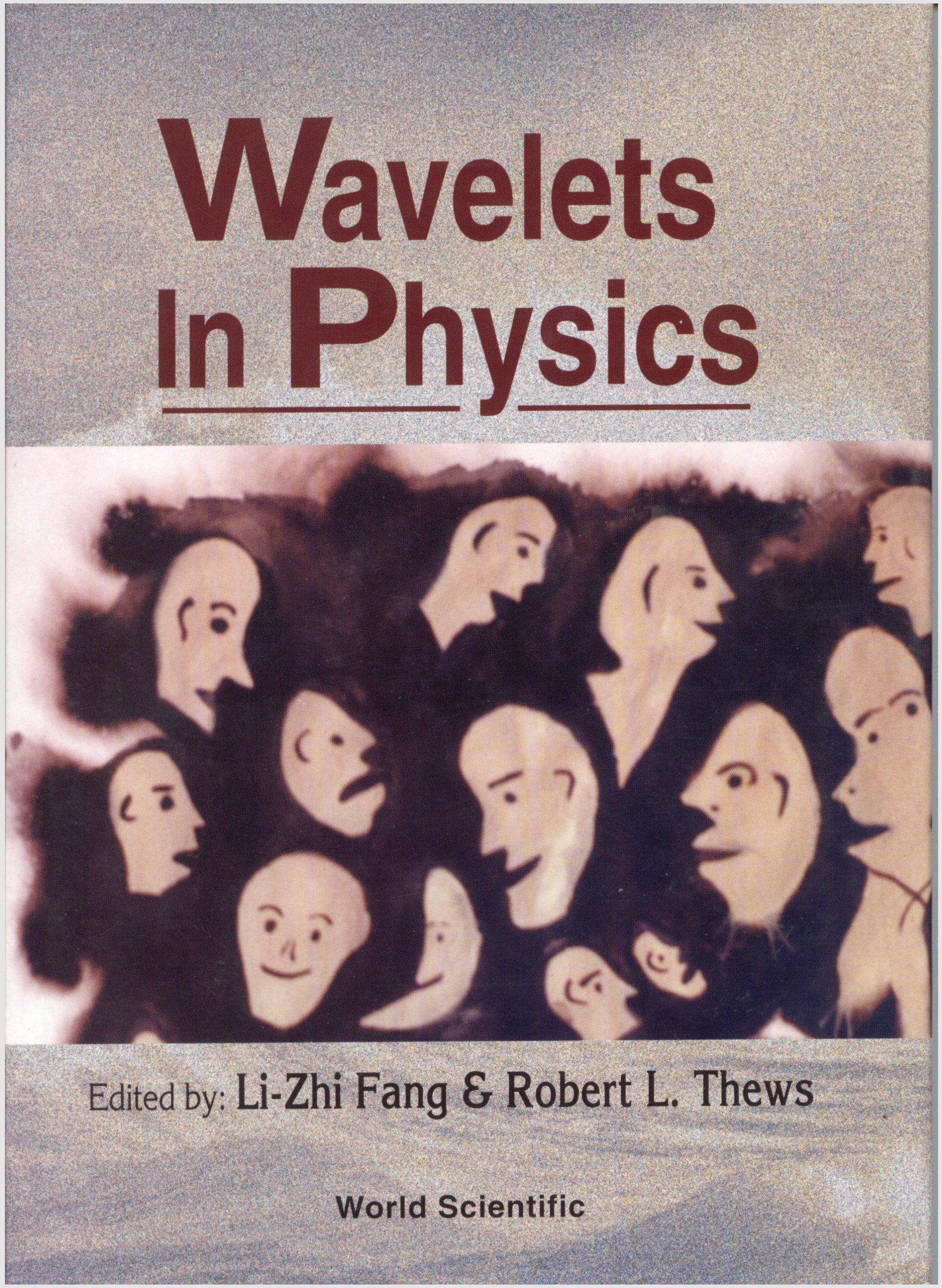
Initial perturbations probably are Gaussian on scales > 0.1 Mpc/h

other topics

- scaling of velocity field
- halos measured by The difference between the Fourier power spectrum and DWT power spectrum
- redshift distortion and power spectrum of non-diagonal DWT modes
- the discrepancy between baryon gas and dark matter

summary

- cosmic gravitational clustering essentially is hierarchical and self-similar. A phase space description of clustering dynamics is necessary.
- The DWT with the features of similarity, admissibility, invertibility, orthogonal and locality in phase space provides an effective representation to describe the nonlinear evolution of the cosmic fields


Wavelets
In Physics
Edited by: Li-Zhi Fang & Robert L. Thews
World Scientific