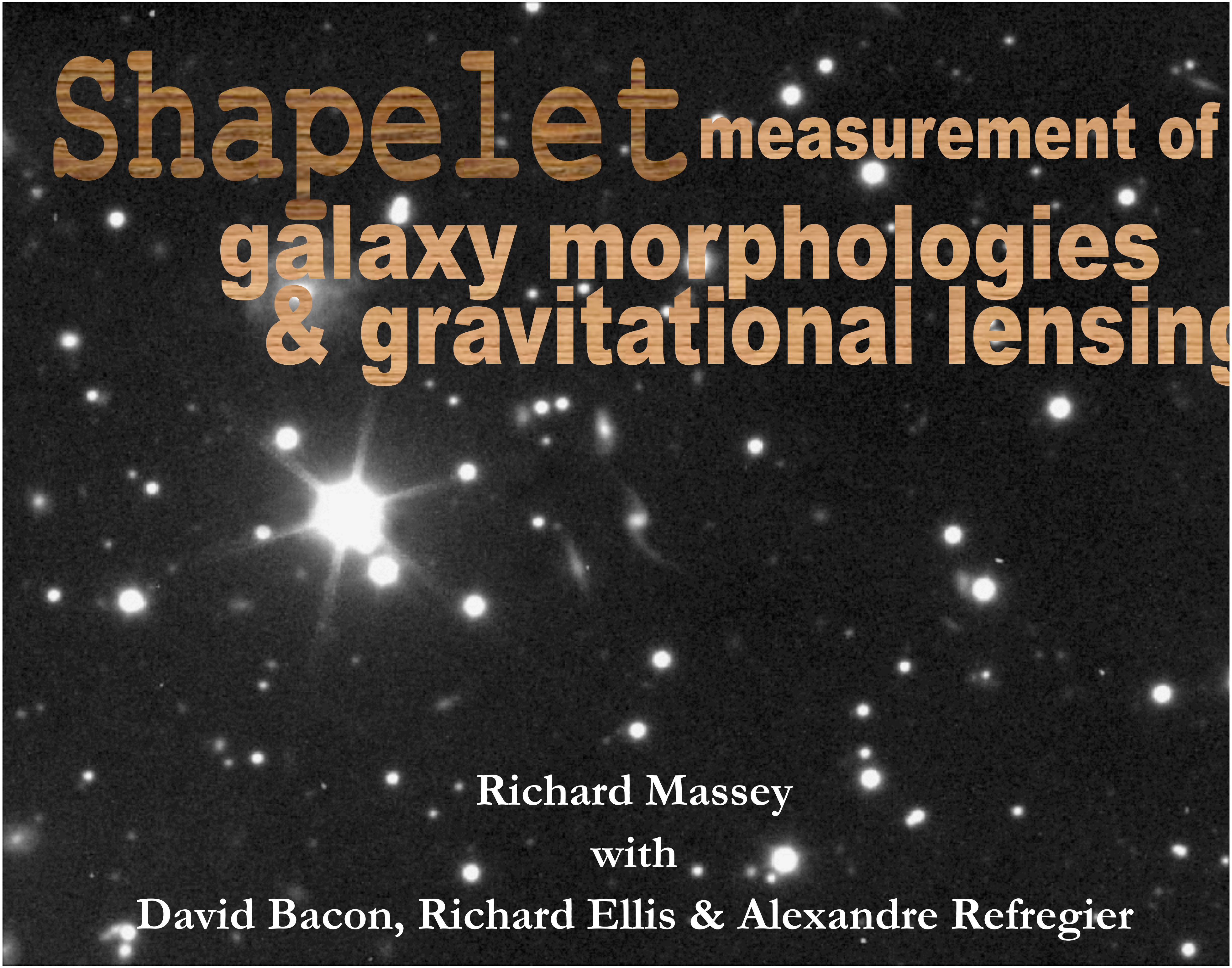
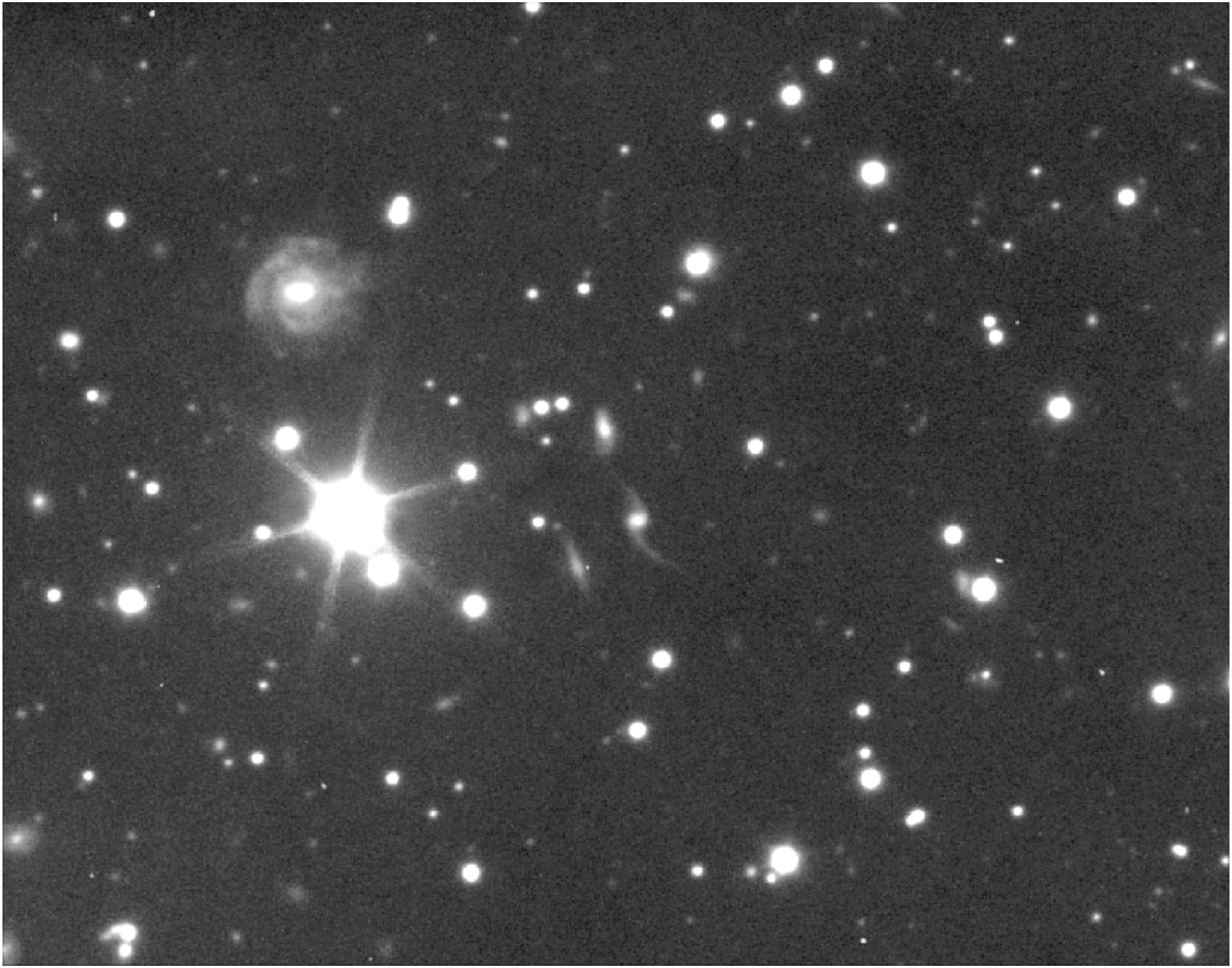

Shapelet measurement of
galaxy morphologies
& gravitational lensing
Richard Massey
with
David Bacon, Richard Ellis & Alexandre Refregier

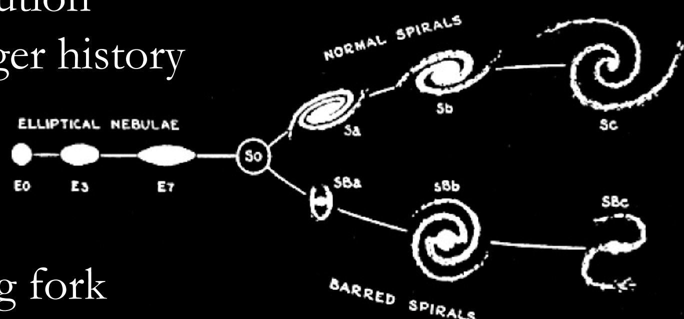


Why measure galaxy shapes?

"Morphology-density" relation

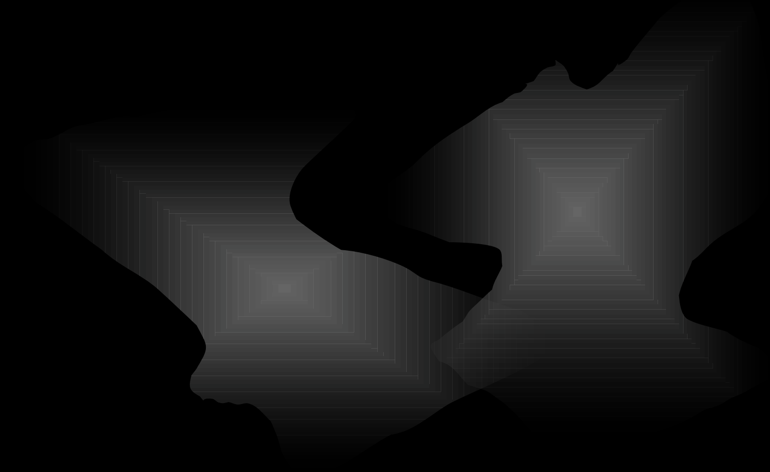
There *must* be some physical information in the diverse range of local galaxy morphologies.

- Star formation rate
- Galaxy evolution
- Recent merger history
- Mass



Hubble tuning fork
Concentration/asymmetry parameters
(rather ad hoc, and not robust)
Certain regions of parameter space populated,
but link to physics is vague.

Need a more complete classification scheme!

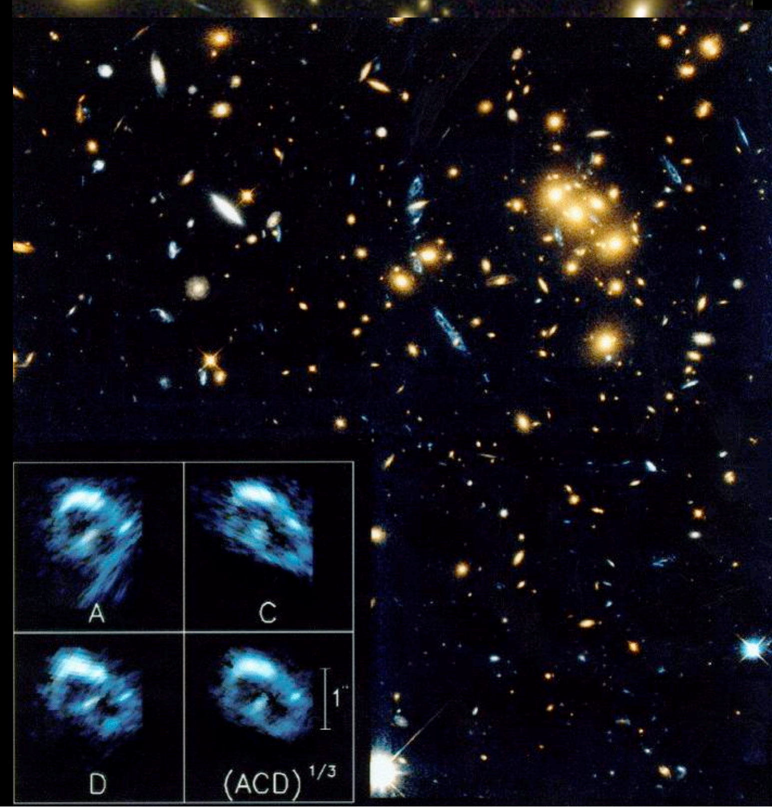


Local environment affects galaxy evolution

But most mass in the universe is dark matter…
…which can be detected via gravitational lensin
of distant galaxies.

Morphology-perturbed morphology relation

Strong Gravitational Lensing



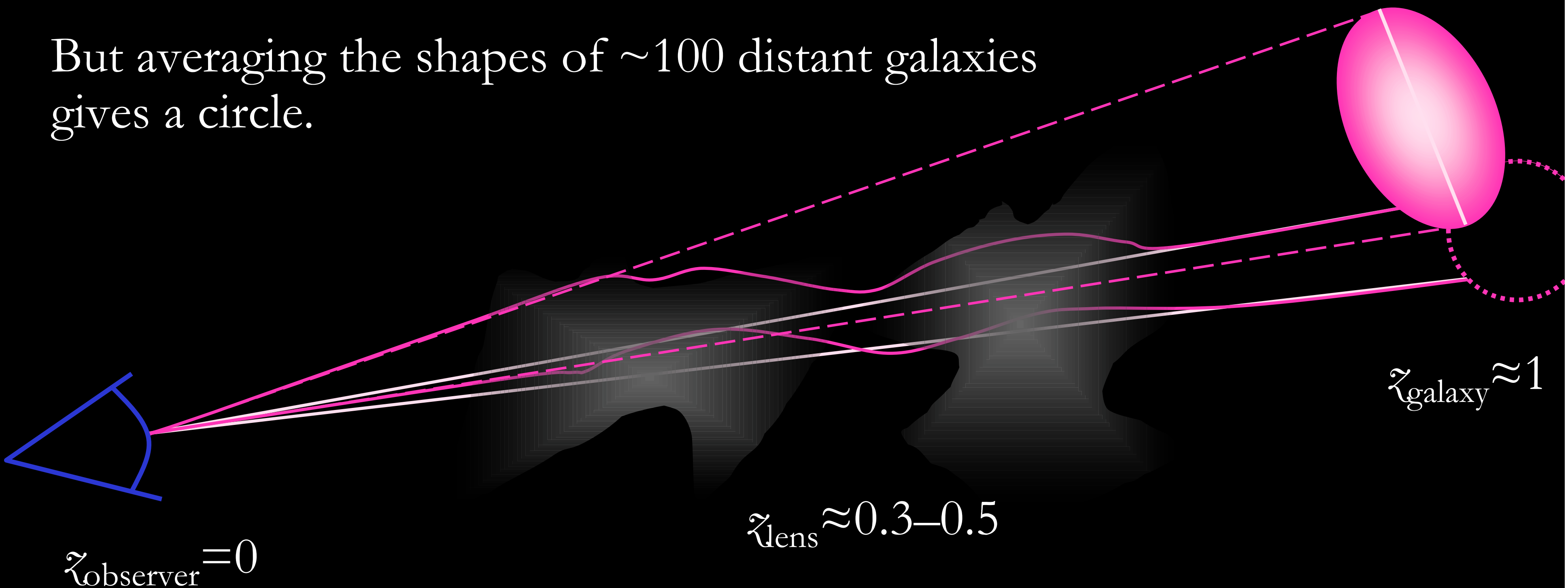
A gravitational field distorts all of space-time like a rubber sheet. Even light rays bend around the massive cores of galaxy clusters. The image of a background galaxy can be strongly lensed into multiple sources (corresponding to multiple paths around the lens) and distorted into arcs.

Gravitational lensing is geometric. It occurs regardless of the nature and state of the foreground mass, and can thus be a bias-free probe of dark matter!

Weak Gravitational Lensing

Most of the universe doesn't contain a massive cluster. Here, the weak lensing effect cannot be measured from any individual galaxy.

But averaging the shapes of ~100 distant galaxies gives a circle.

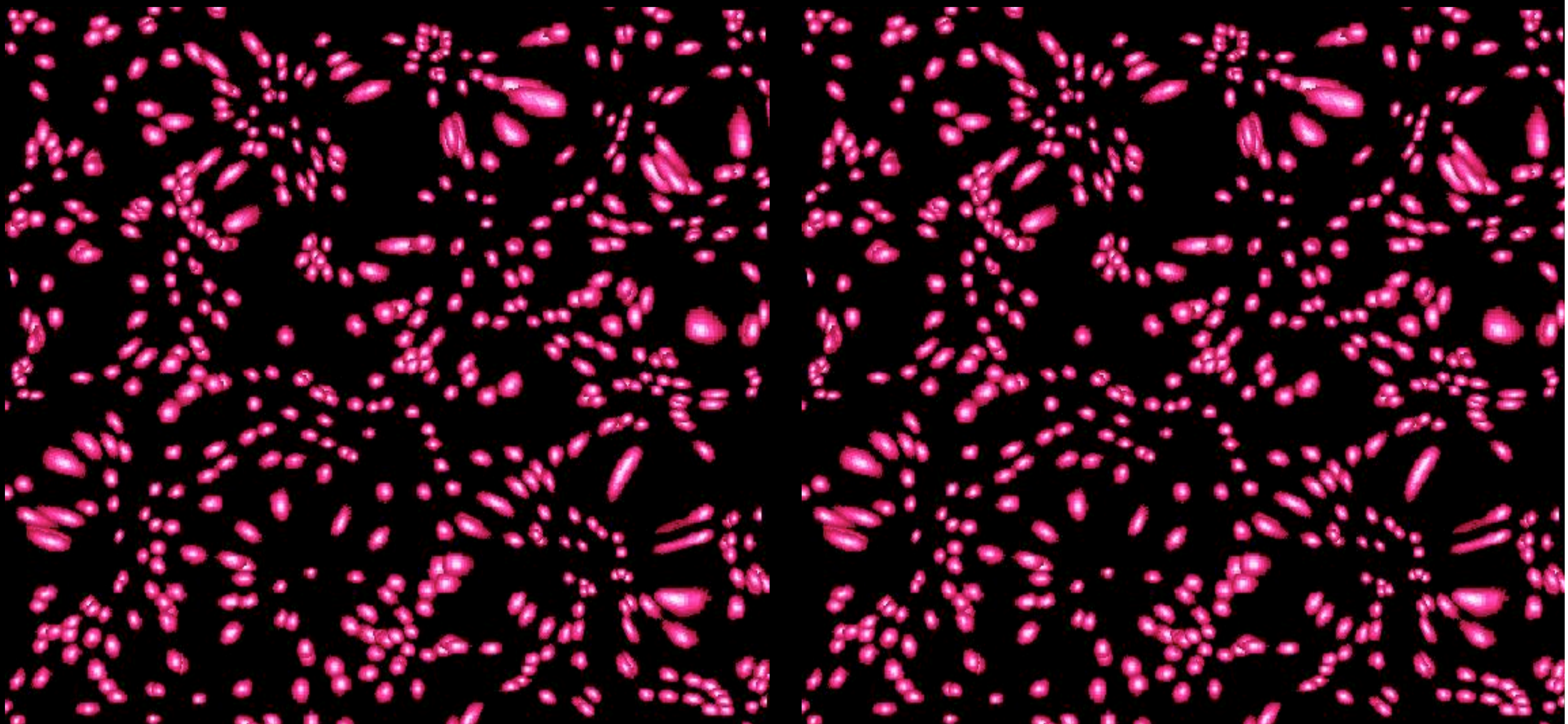


If there is any intervening large-scale structure, light follows the distorted path (exaggerated). Background images are magnified and sheared by ~2%, mapping a circle into an ellipse. Like glass lenses, gravitational lenses are most effective when placed half way between the source and the observer.

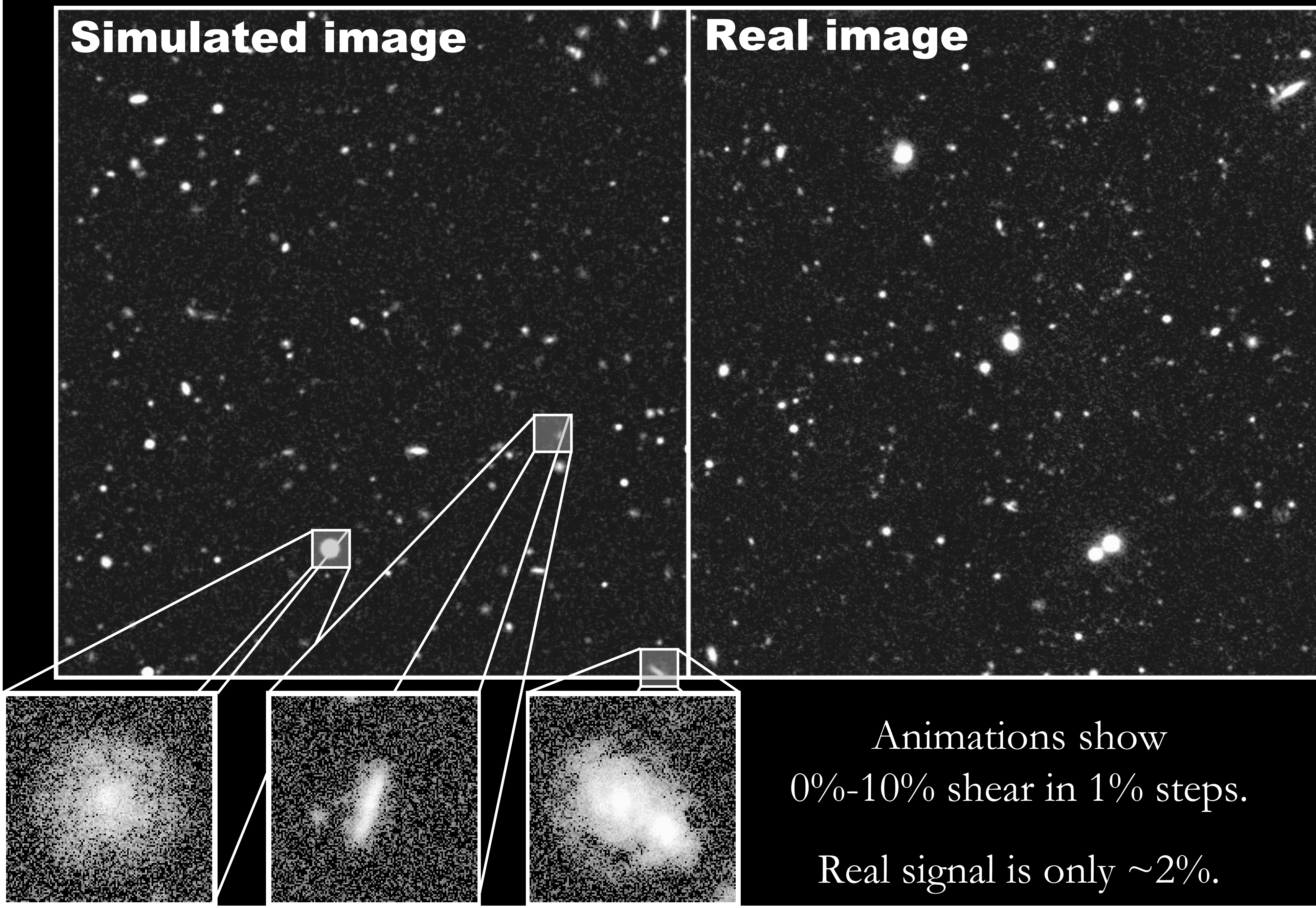
Weak Gravitational Lensing

Gaussian random field in greyscale represents a foreground mass distribution.
Each tick mark shows the (magnitude & direction of the) resulting shear field.

Circles (each representing ~100 background galaxies) are stretched into ellipse



A less cartoon-like illustration



Animations show
0%-10% shear in 1% steps.

Real signal is only ~2%.

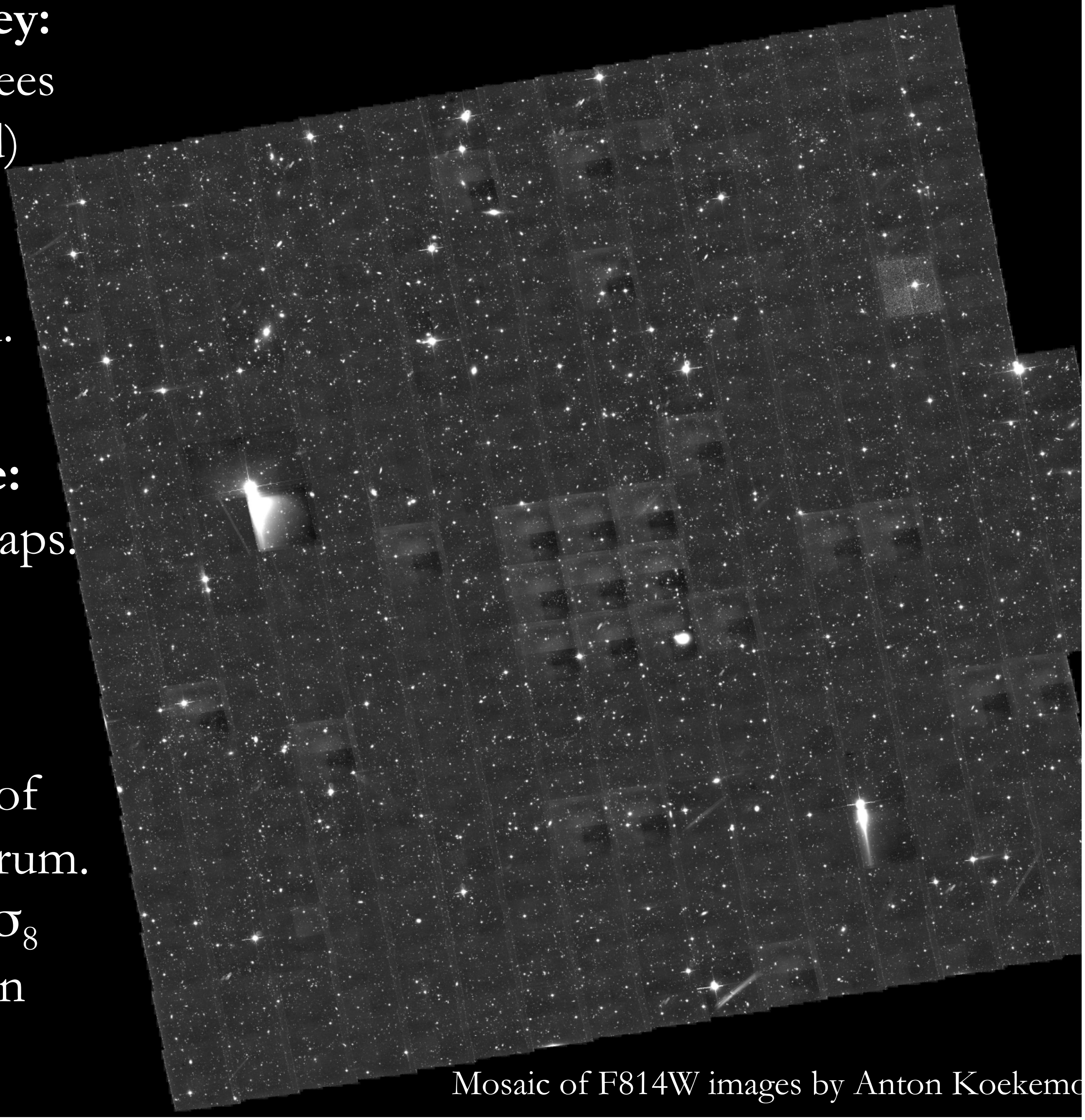
COSMOS: "Cosmic Evolution Survey"

Largest ever HST survey:

2 contiguous square degrees of deep F814W (*I*-band) imaging. Equatorial field chosen for easy follow-up from ground.

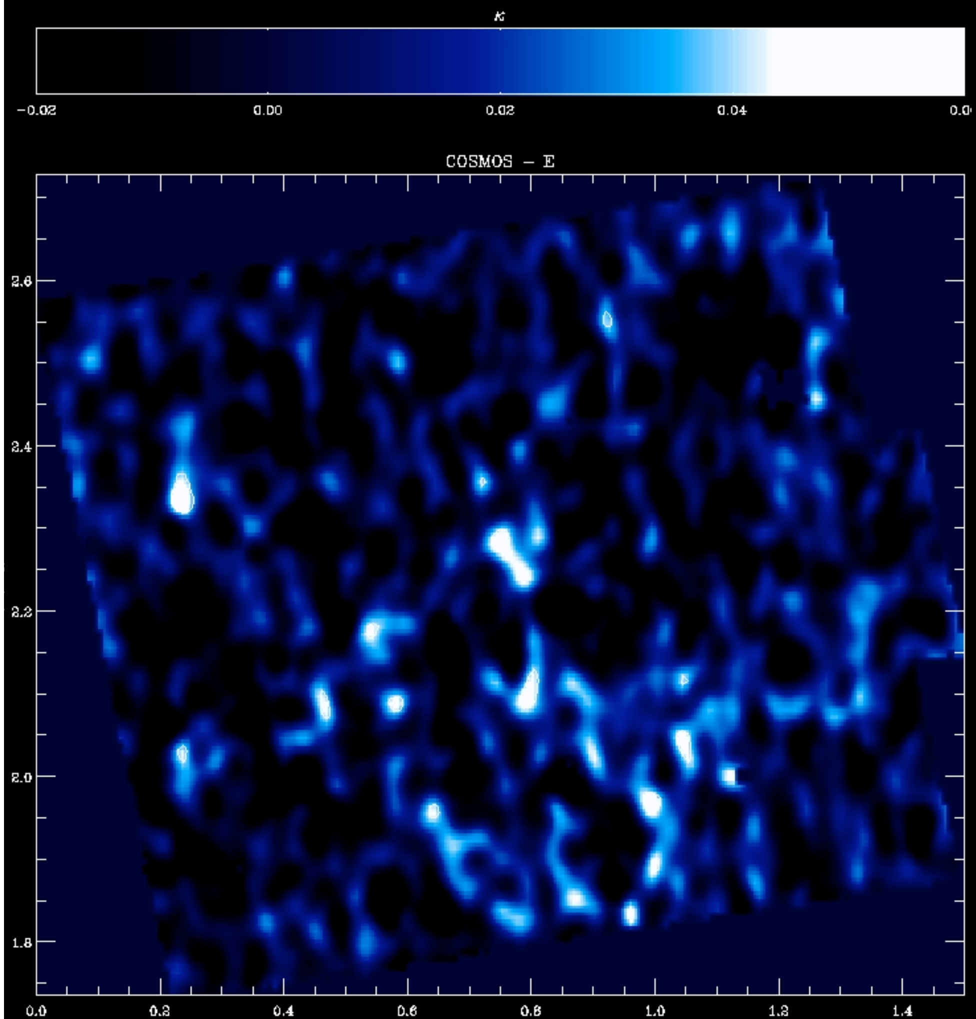
COMOS will provide:

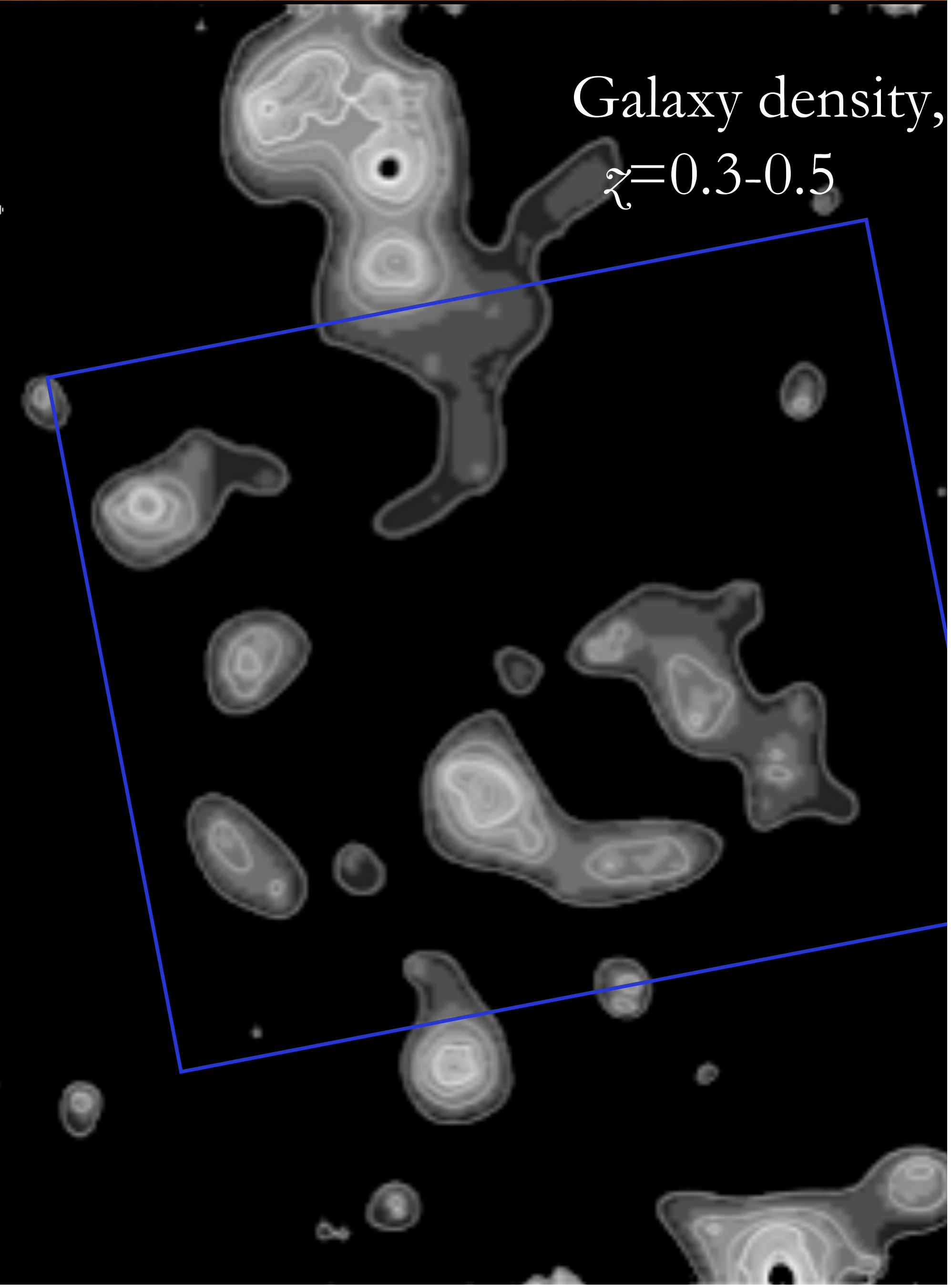
- Unbiased dark matter maps.
- Mass-selected cluster catalogues sensitive to few$\times 10^{14} M_{\odot}$.
- Statistical measurement of dark matter power spectrum.
- Constraints on Ω_M and σ_8 that break degeneracies in other methods.



Mosaic of F814W images by Anton Koekemo

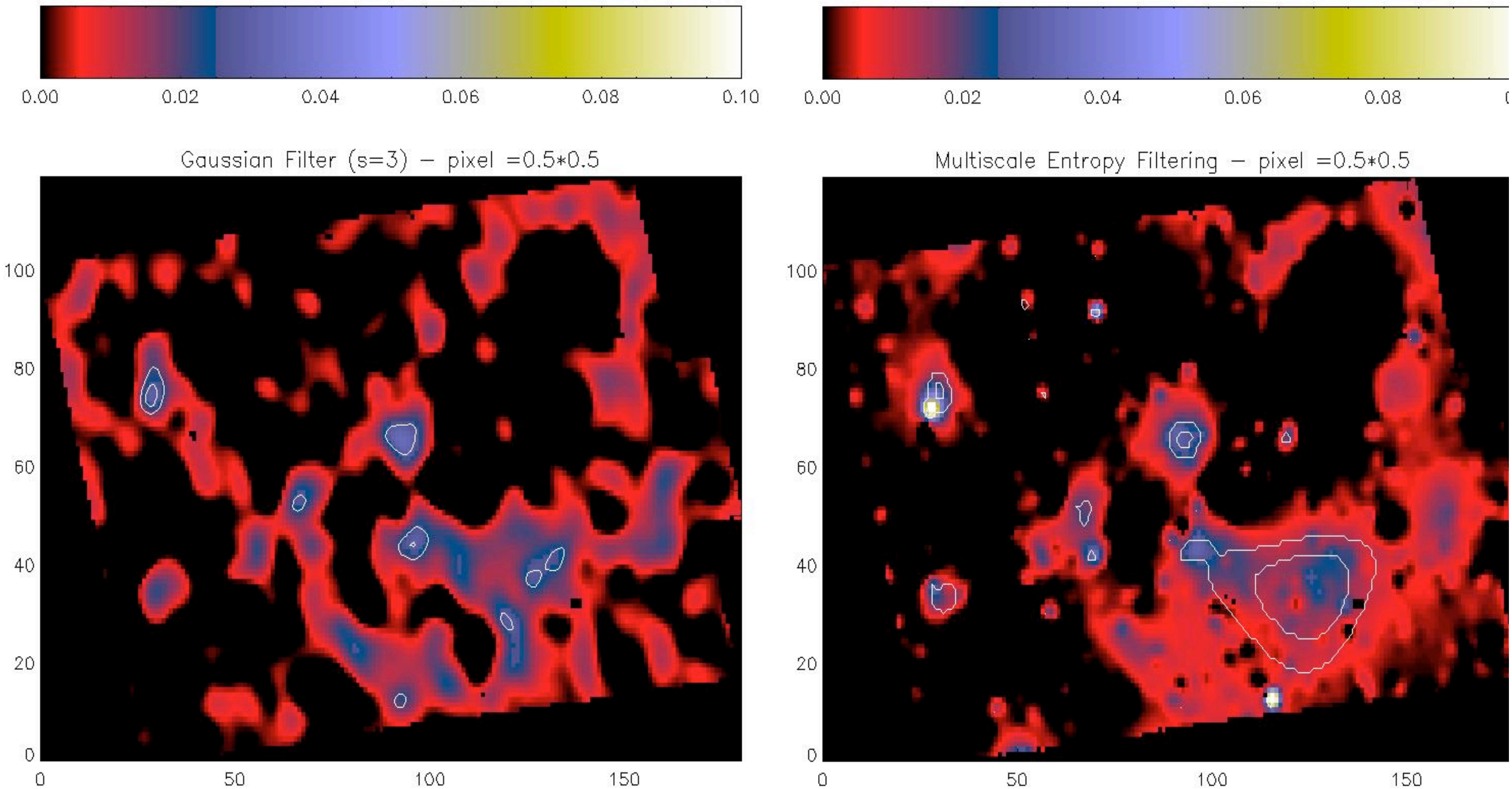
COSMOS mass map versus light map



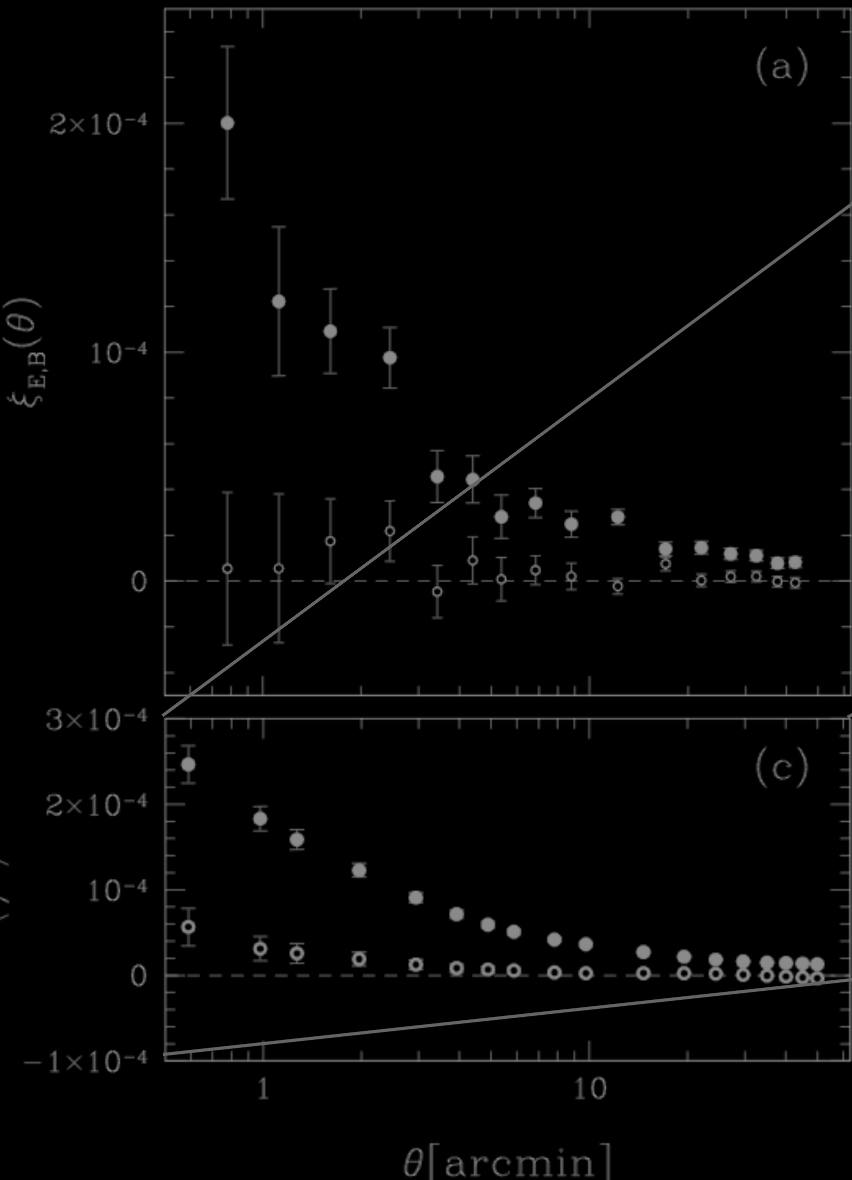


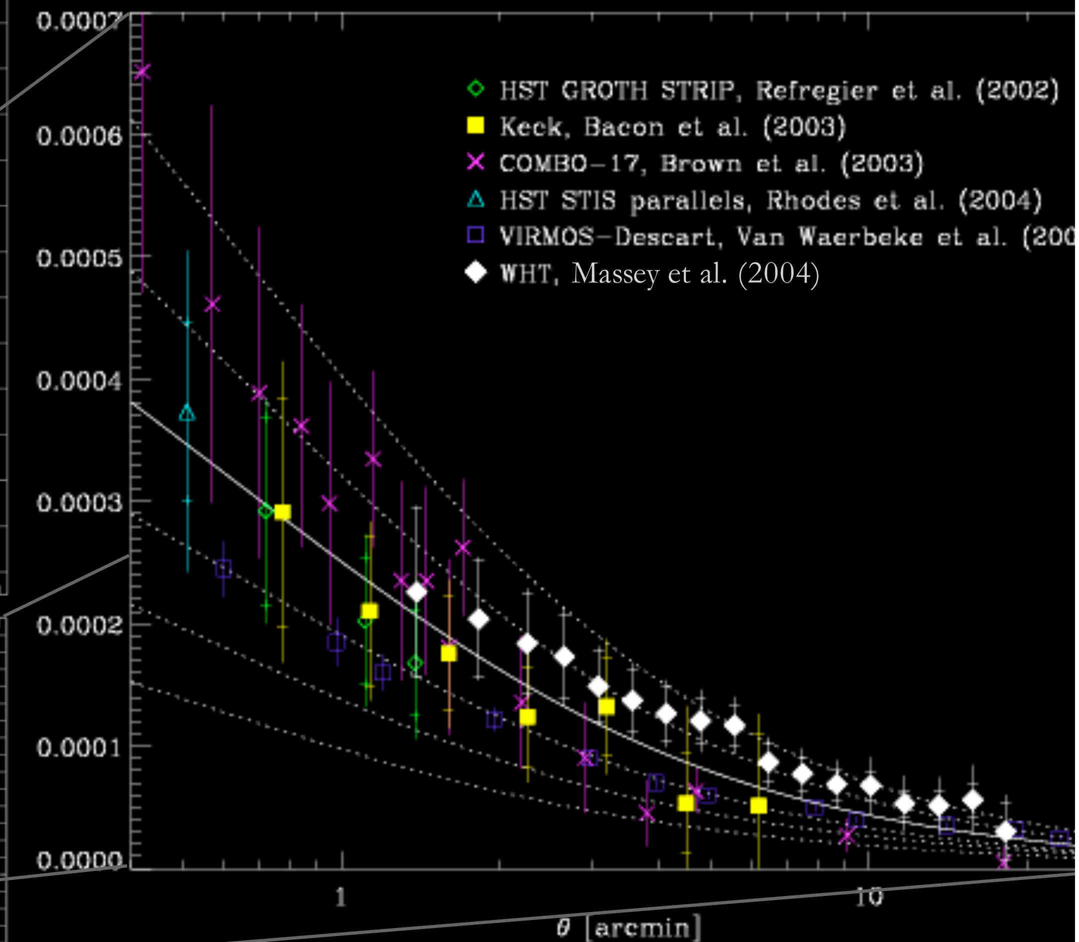
Galaxy density,
z=0.3-0.5

Guassian & Wavelet reconstructions of the dark matter map



Statistical results
(from even larger ground-based surveys)

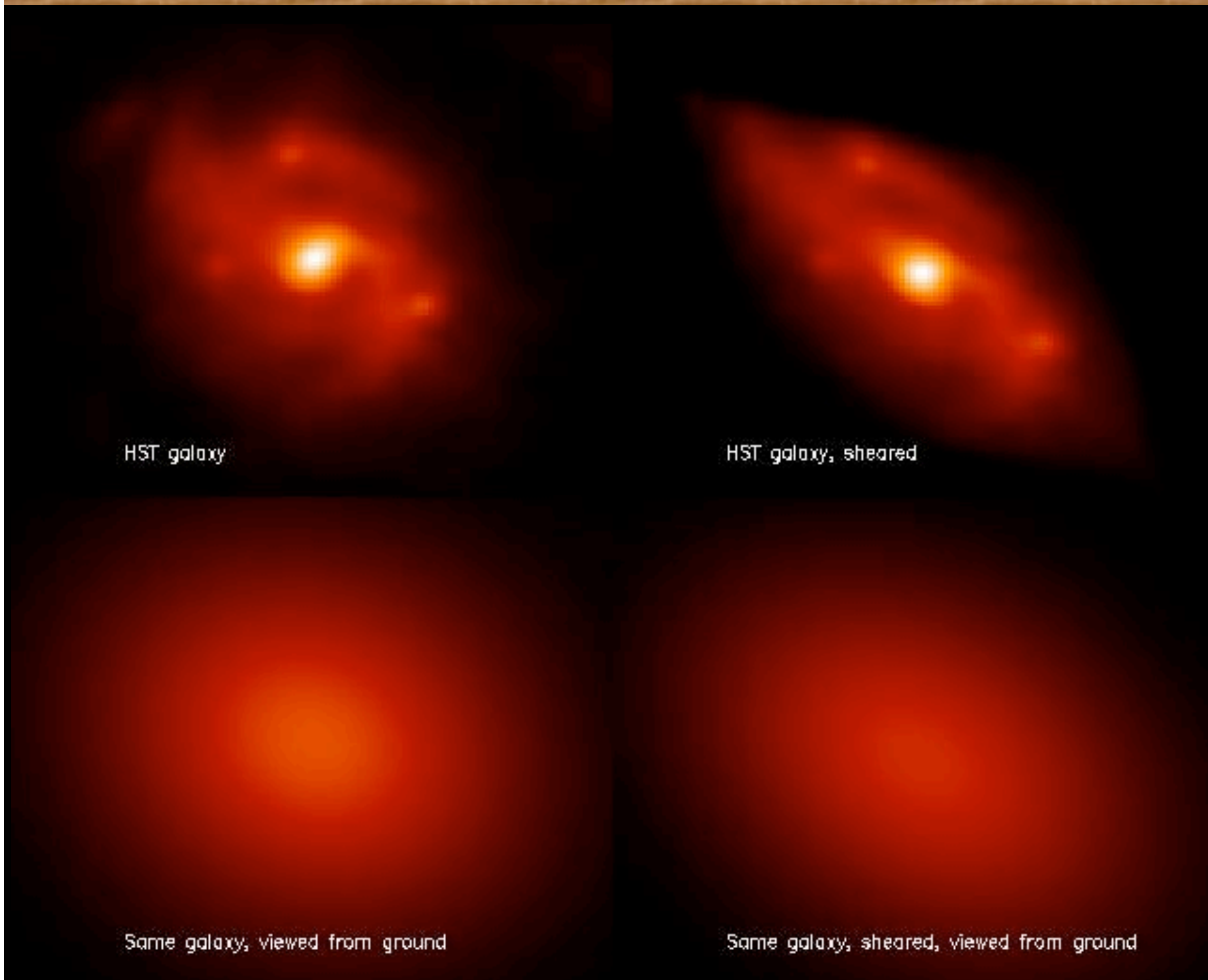


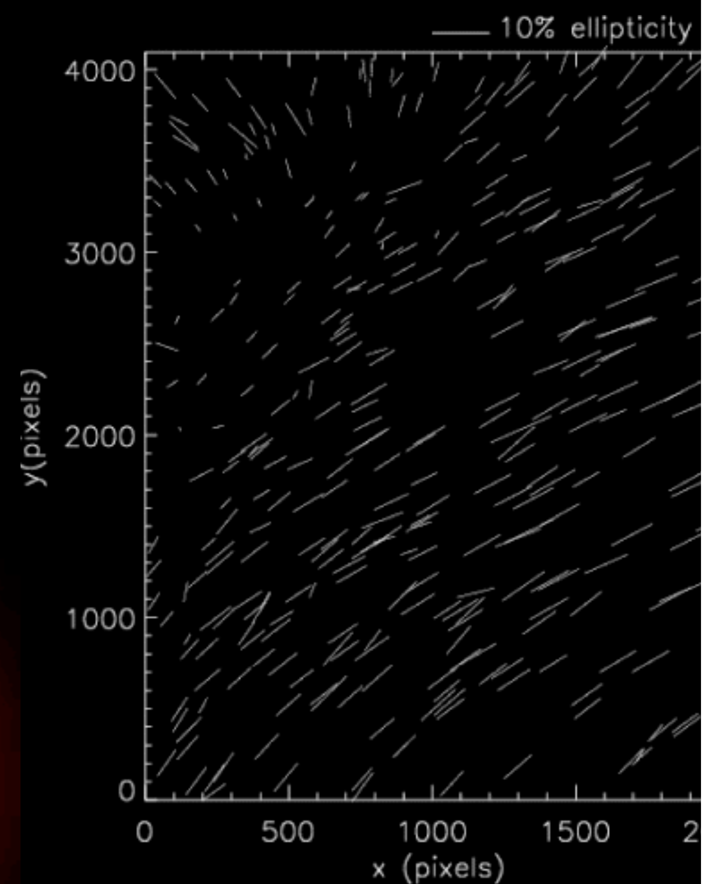


But normalisation(?) difference between groups…

VIRMOS-Descart survey
Van Waerbeke *et al.* (2004)

Why is lensing so hard to measure?
1: Needs PSF deconvolution





Statistical measurement requires 100 *resolved* background galaxies per data poin
Convolution with an isotropic PSF circularises galaxies.
Convolution with an anisotropic PSF also changes their shapes… coherently!
Worst from ground (large PSF, with unpredictable spatial / temporal variation

Why is lensing so hard to measure?
2: shear susceptibility factor

It would be nice if

$$\varepsilon_{\text{observed}} = \varepsilon_{\text{true}} + P^{\gamma}\gamma$$

because, assuming

$$\langle \varepsilon_{\text{true}} \rangle = 0$$

We can form a shear estimator $\tilde{\gamma} = \dfrac{\varepsilon_{\text{observed}}}{P^{\gamma}}$ for each galaxy shape

for which

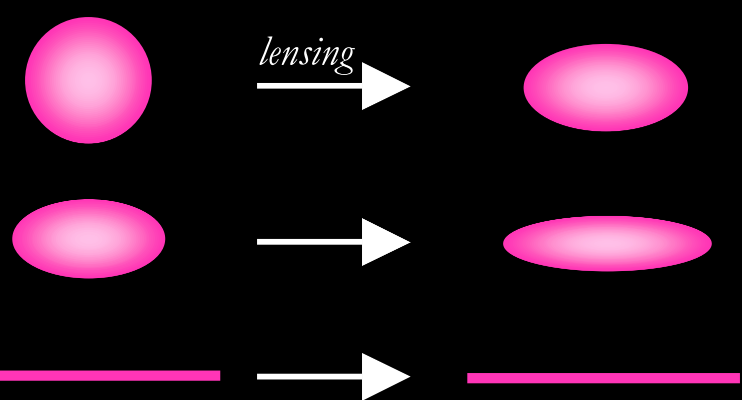
$$\langle \tilde{\gamma} \rangle = \gamma.$$

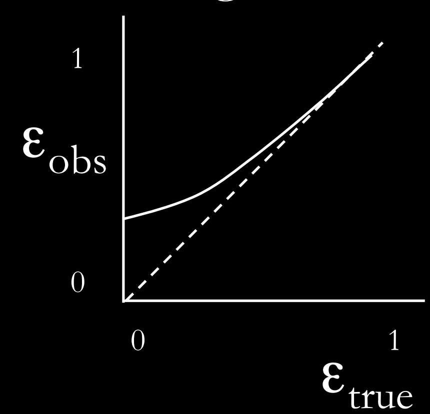
Unfortunately, ellipticities do not add linearly, because they cannot be greater than 1.

$\varepsilon_{\text{true}} = 0$ … $\varepsilon_{\text{obs}} = 0.3$

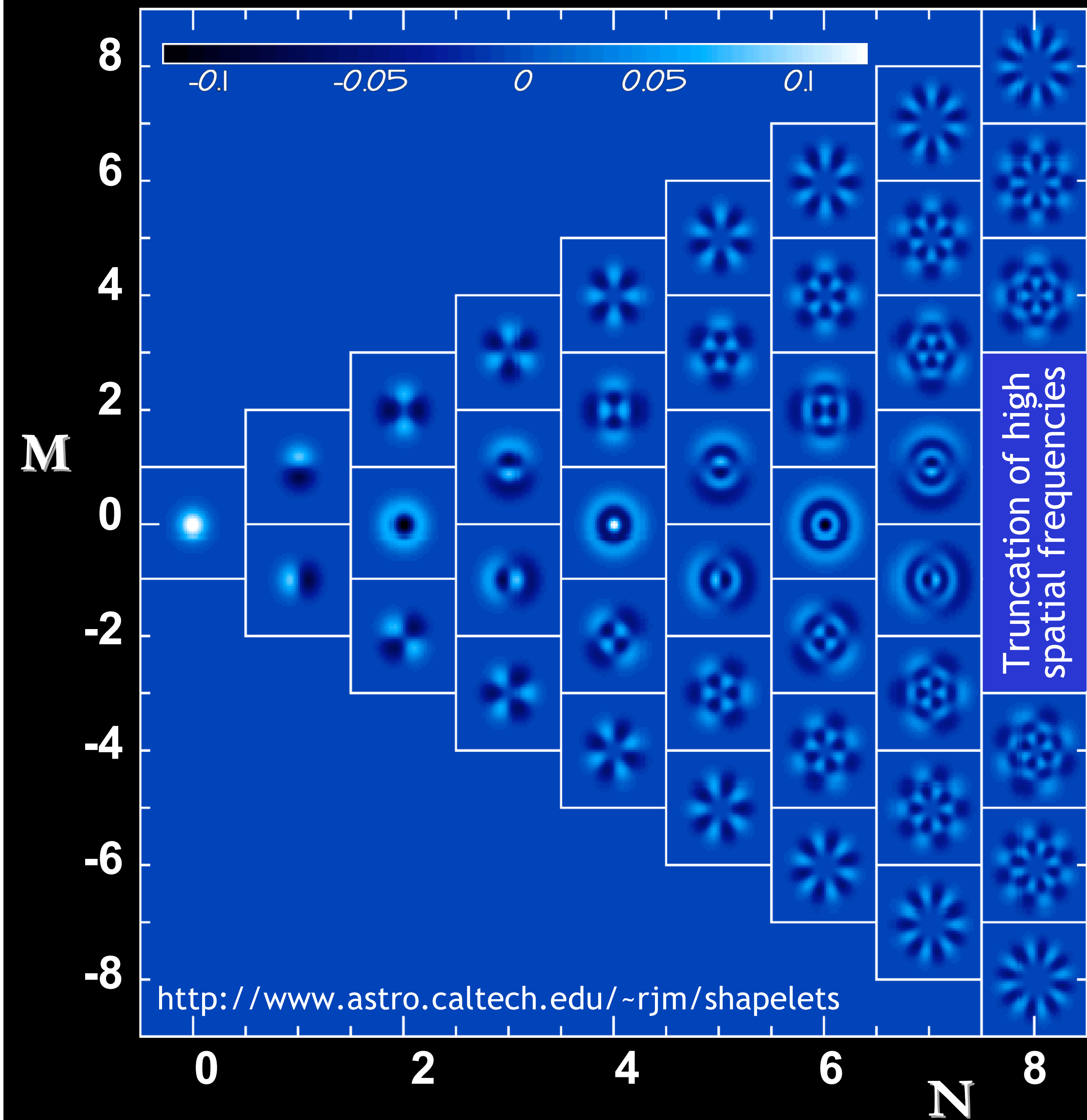
$\varepsilon_{\text{true}} = 0.5$ … $\varepsilon_{\text{obs}} = 0.6$

$\varepsilon_{\text{true}} = 1$ … $\varepsilon_{\text{obs}} = 1$





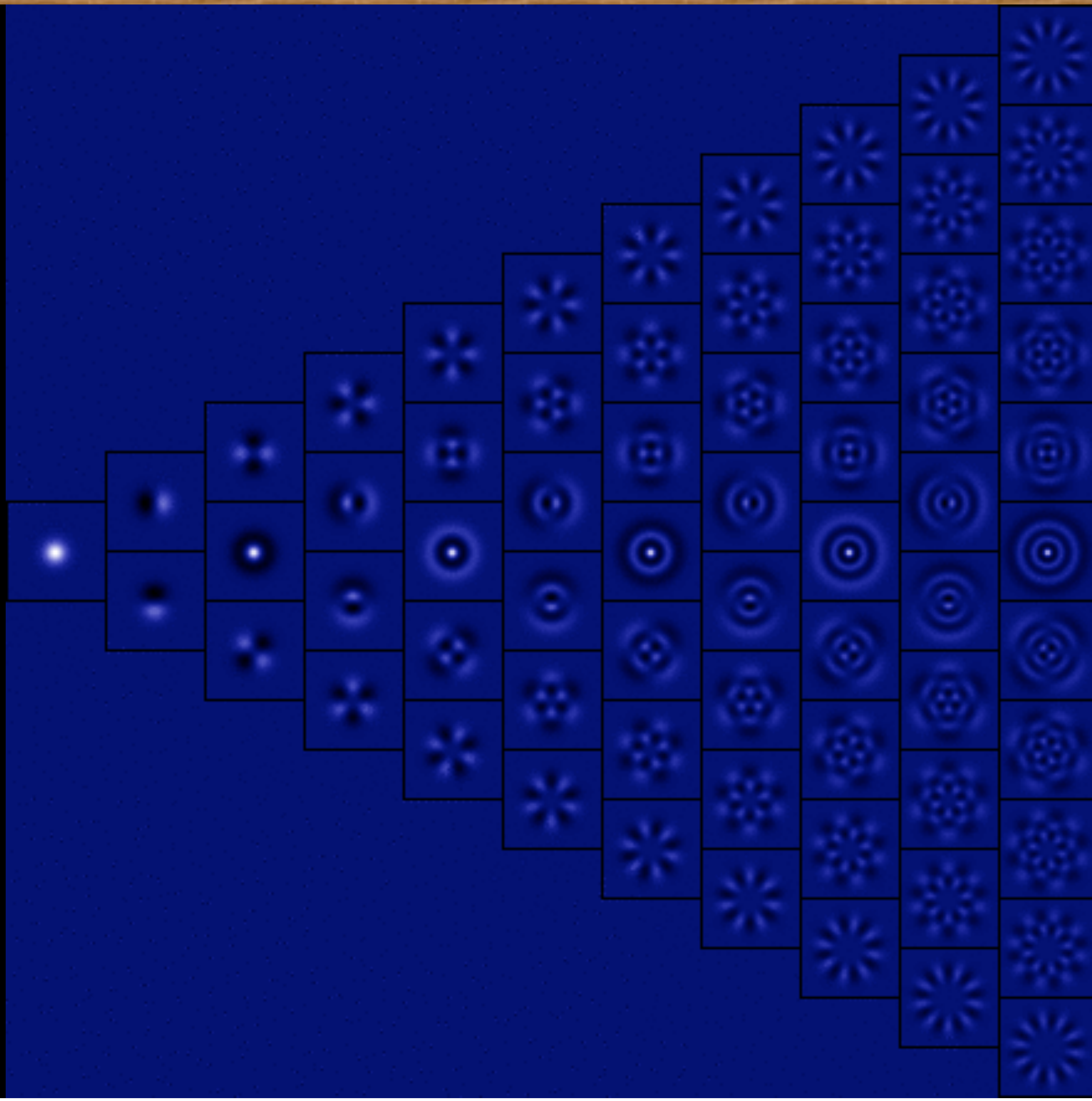
"Shapelets" image analysis method



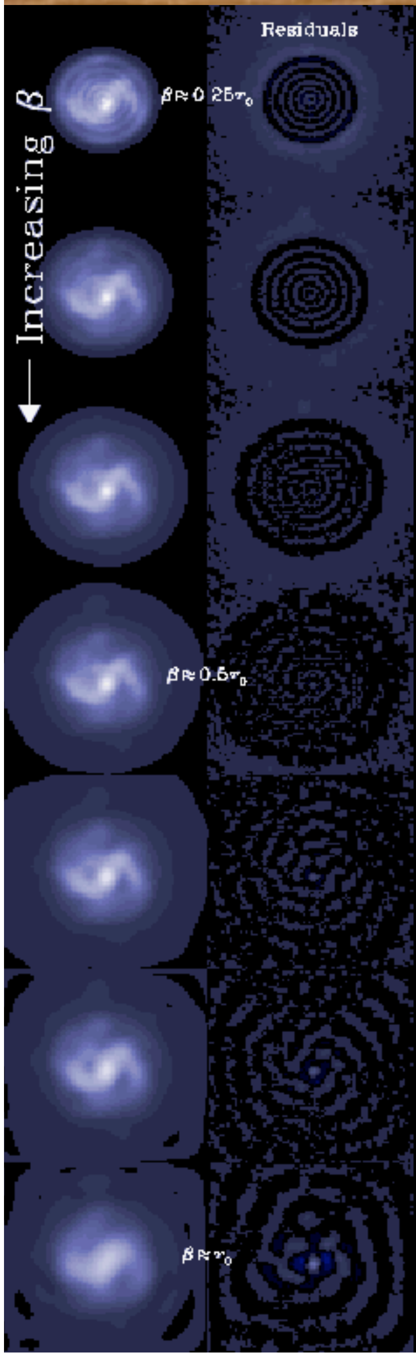
Complete & orthogonal basis
Can represent any isolated imag
as a (unique) weighted, linear
sum of shapelet basis functions
Weights/coefficients can be
calculated via inner product of
image with each basis function.

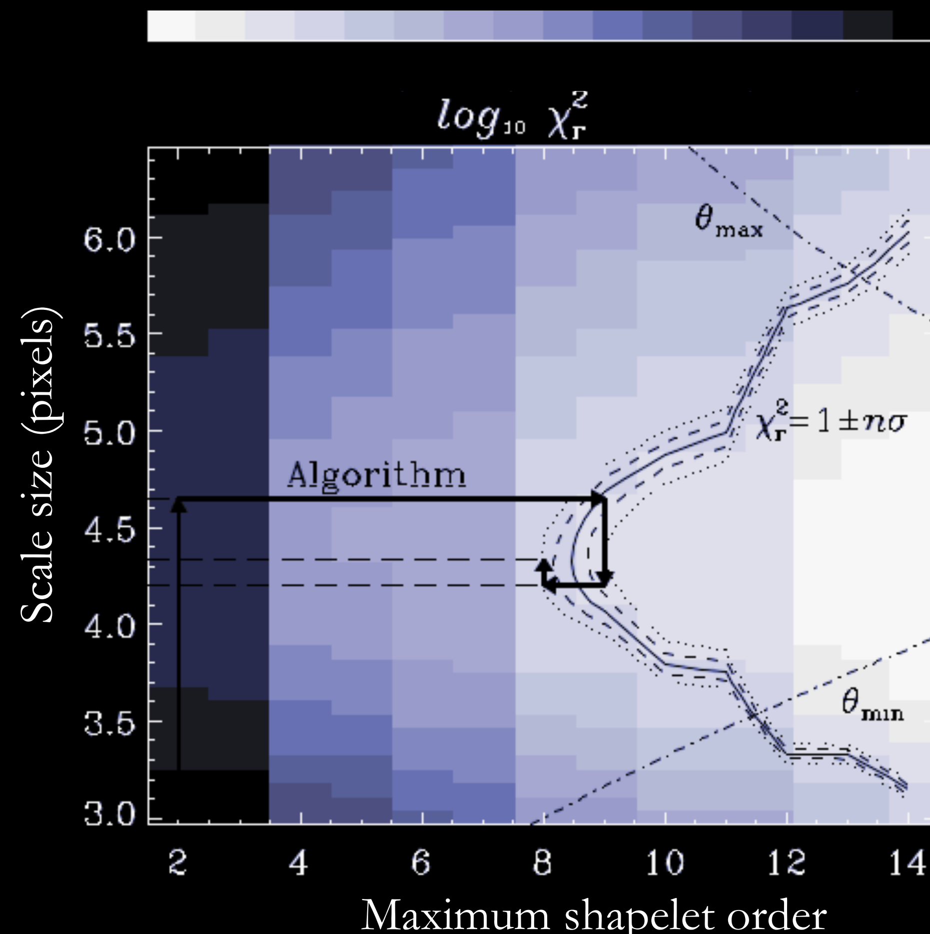
Useful parameterisation:
Mathematically convenient for
analysis and manipulation of
images. Physical interpretation i
often intuitive. Convolution is
matrix multiplication; coordinat
transformations are a coupling c
a minimal number of coefficient
N = radial oscillations
M = rotational symmetry

Animated shapelet decomposition



Optimisation of scale size & truncation parameter



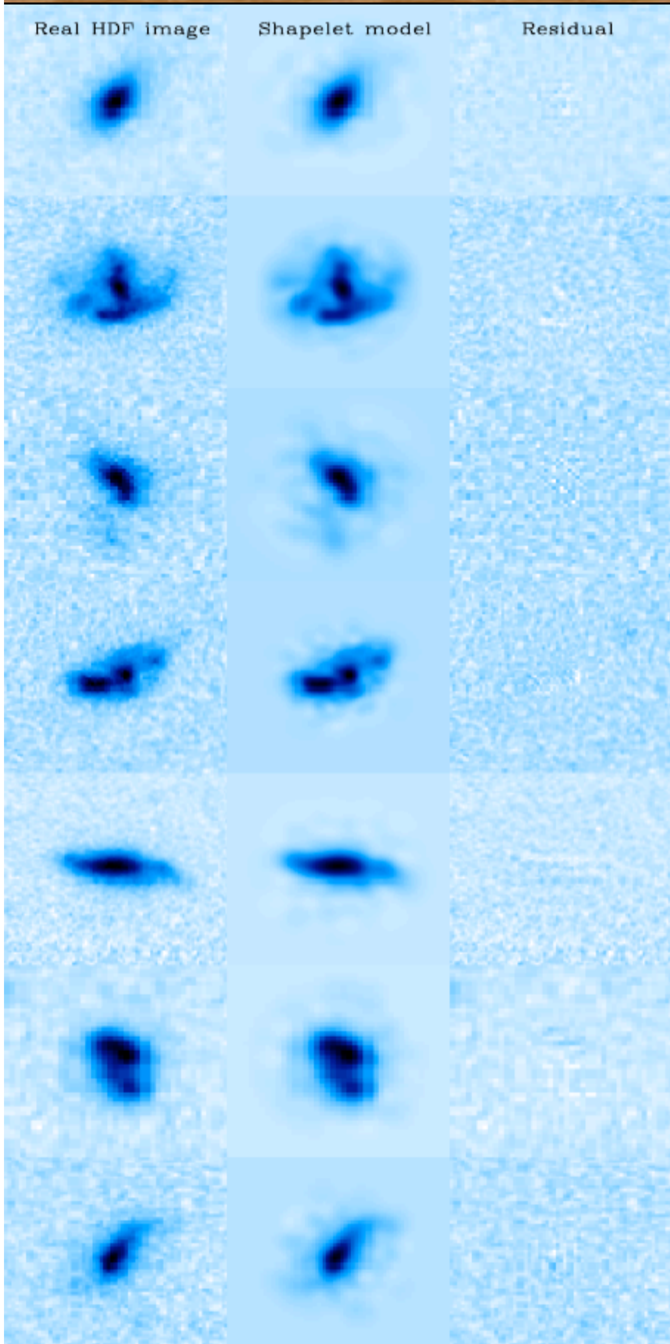


For a given number of coefficients (truncation parameter), there is a clearly preferred scale size for basis functions to get a good fit.

For a given scale size, a certain number of coefficients are needed to ensure $\chi_r^2=1$

Choose scale size and centroid so that residual $\chi_r^2=1$ using smallest possible number of coefficients. De-noise by truncating the rest.

Shapelet decomposition via a least-squares fit



Flux incident at a telescope has been:

- Convolved with PSF.
- Integrated within CCD pixels.

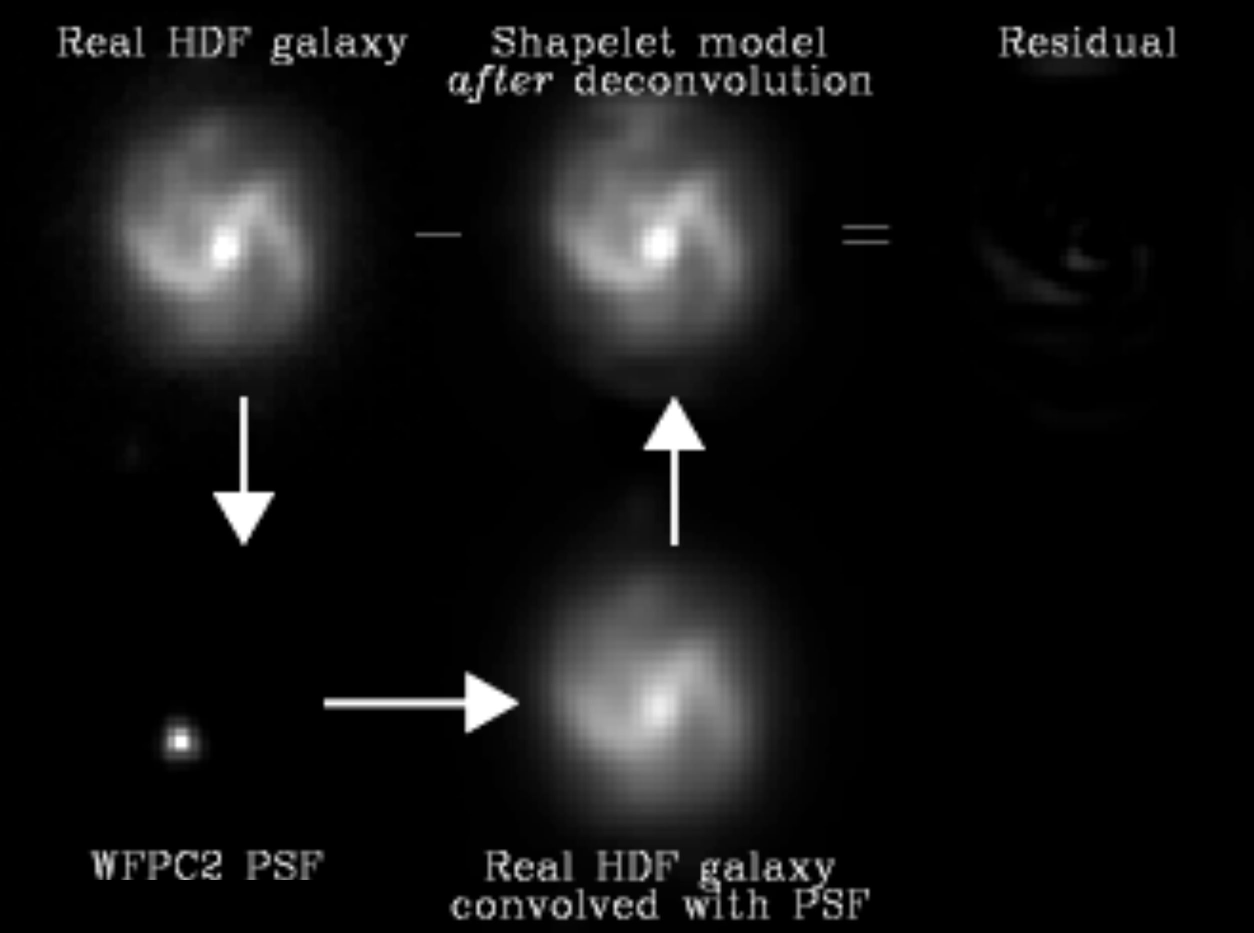
Automatic shapelet decomposition:

- Measure PSF from stars within an image.
- Convolve shapelet basis functions with PSF model.
- Analytically integrate shapelet basis functions within pixels.
- Linearly fit shapelet basis functions to data, iterating their centroid and scale size to optimise image reconstruction.

IDL package for shapelet analysis available from:

www.astro.caltech.edu/~rjm/shapelets

PSF deconvolution



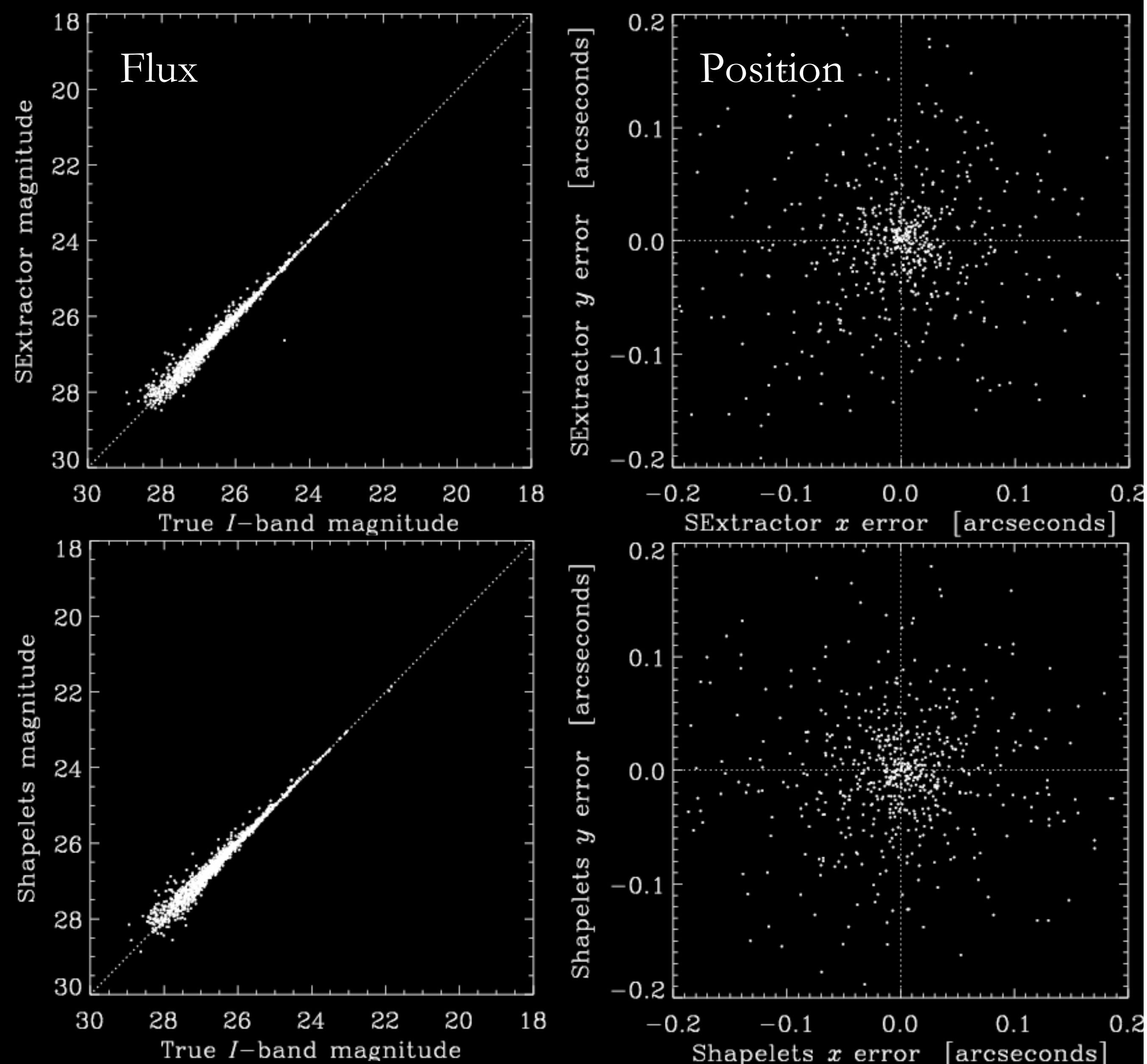
Galaxy models can be analytically deconvolved from a PSF during the shapelet fitting procedure. Results are good as long as the PSF is well-modelled.

Alternatively, PSF convolution is a matrix multipliction in shapelet space. Can also perform deconvolution via matrix inversion.

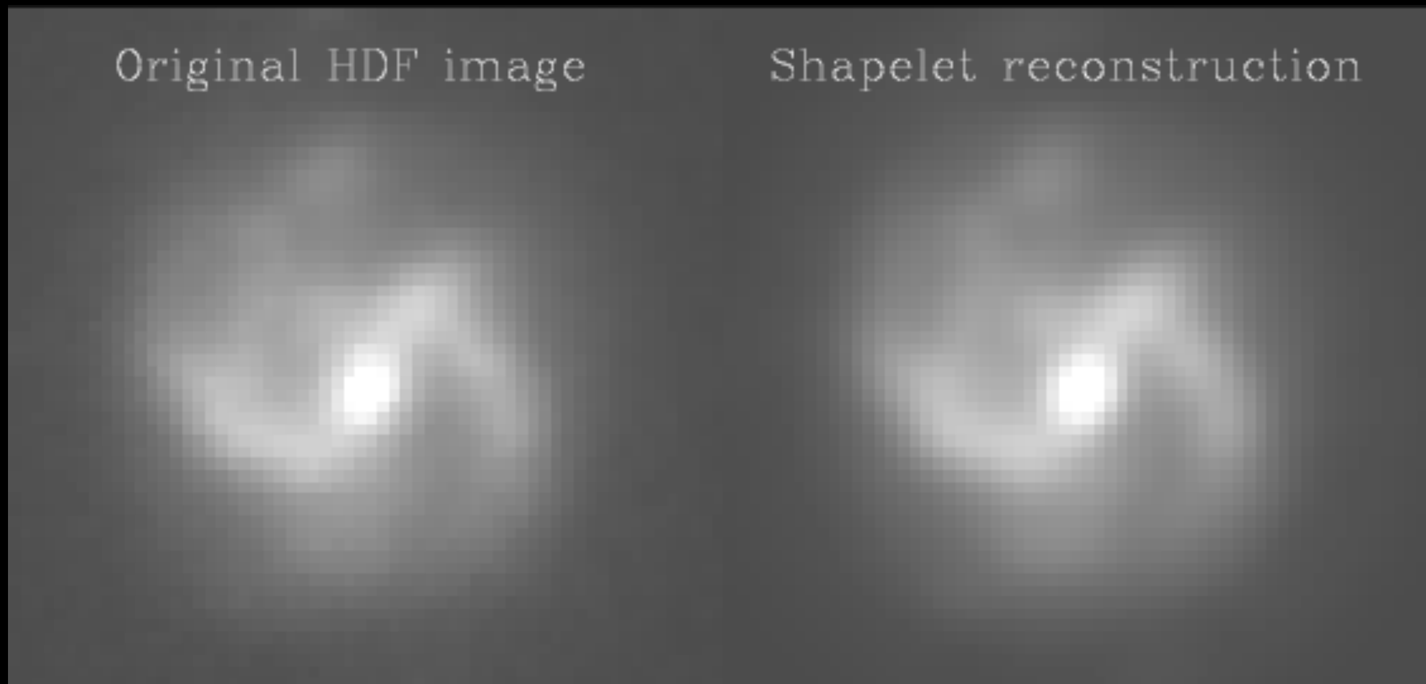
Flux and position estimation

SExtractor
(standard astronomy tool)

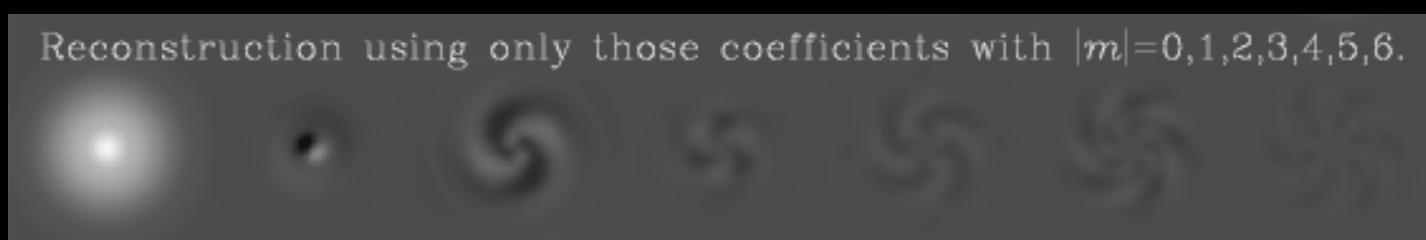
Analytically calculable moments from a shapelet model.



"Standard" galaxy morphology diagnostics



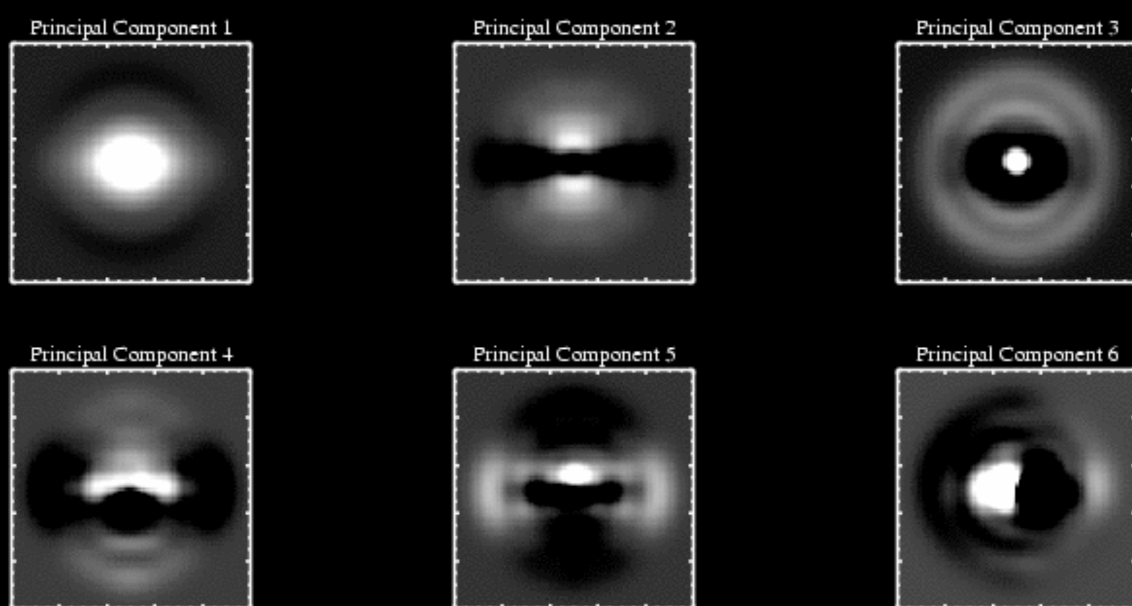
Shapelet decomposition provides an analytic model an image incorporating e.g. proper pixellisation and PSF deconvolution. The model separates naturally into components with differing radial/rotational symmetries, from which concentration/asymmetry/chirality estimators can be robustly calculated.



Parameter-free galaxy morphology diagnostics

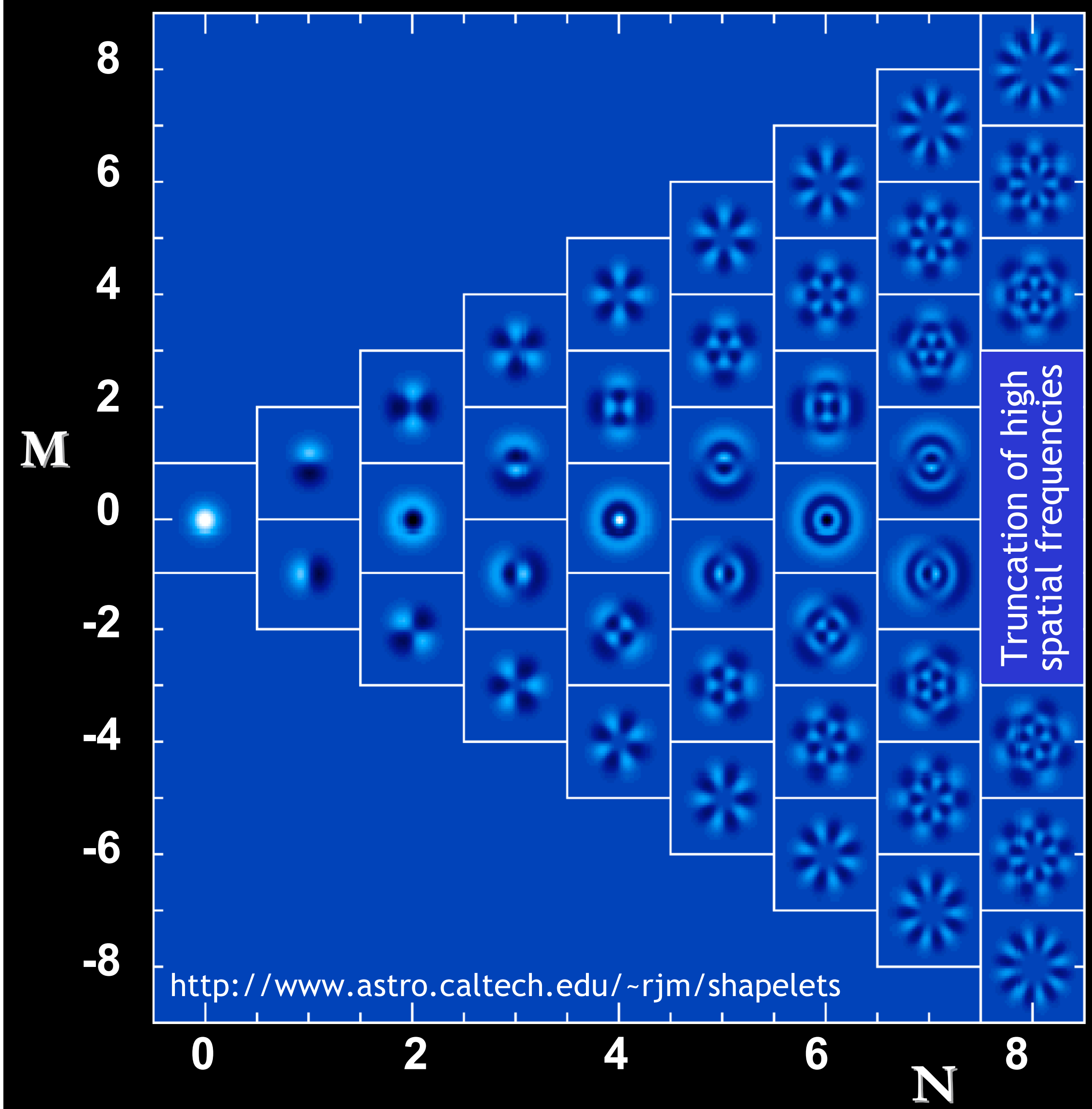
Shapelet coefficients form an high dimensional space, which can be populated by galaxies in the HDFs/COSMOS.

Perform statistical methods (help!) on this ensemble to isolate distinct galaxy populations and to determine their most important differences in morphology.
Can combinations of shapelet coefficients be found to act as morphology diagnostics that correlate optimally with physical/environmental properties? A sub-sample of galaxies with additional spectroscopic data could be obtained as a training set.

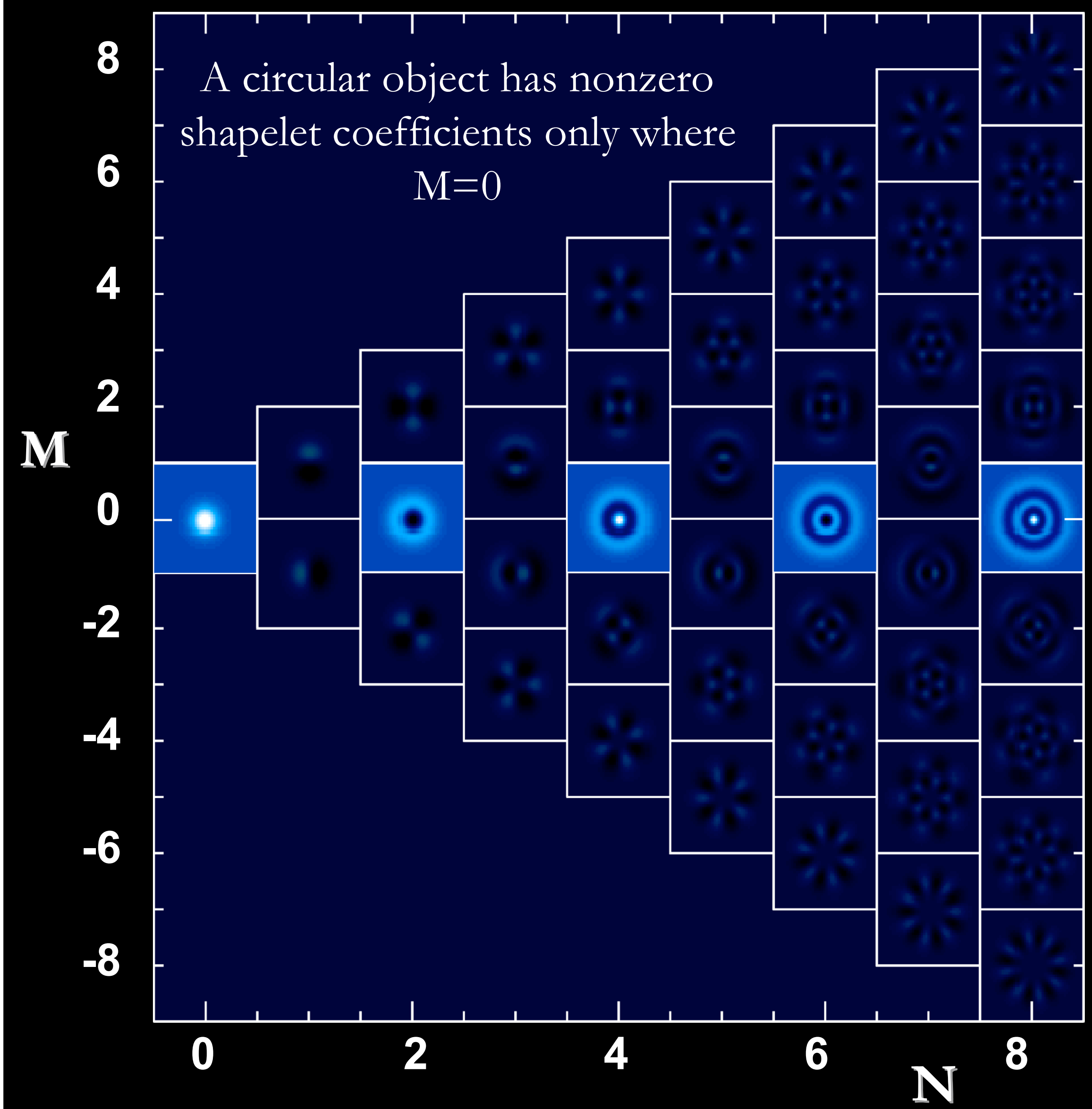


PCA of Sloan DSS b
Kelly & McKay (200

Interpreting shapelet basis functions

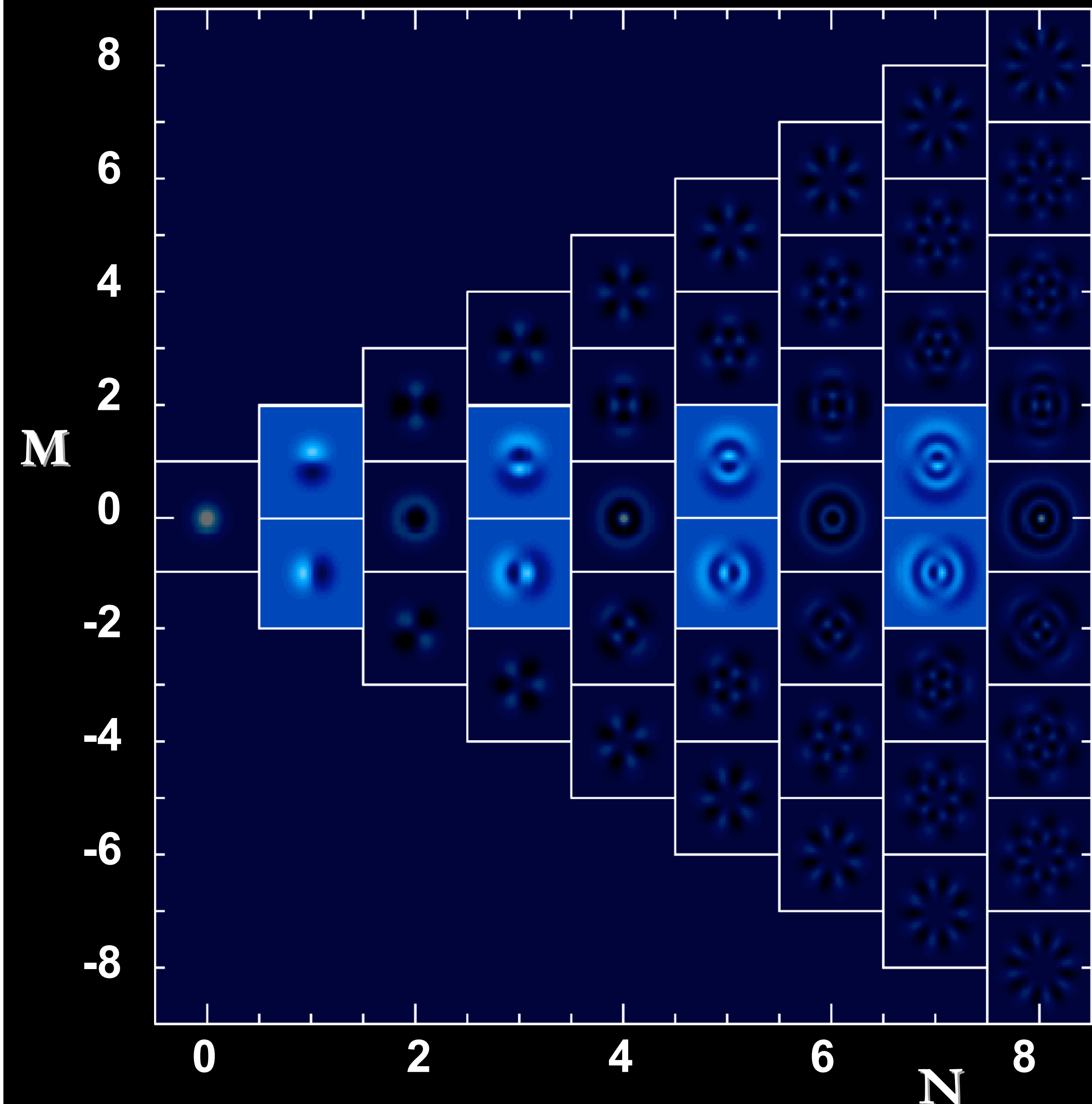


Interpreting shapelet basis functions



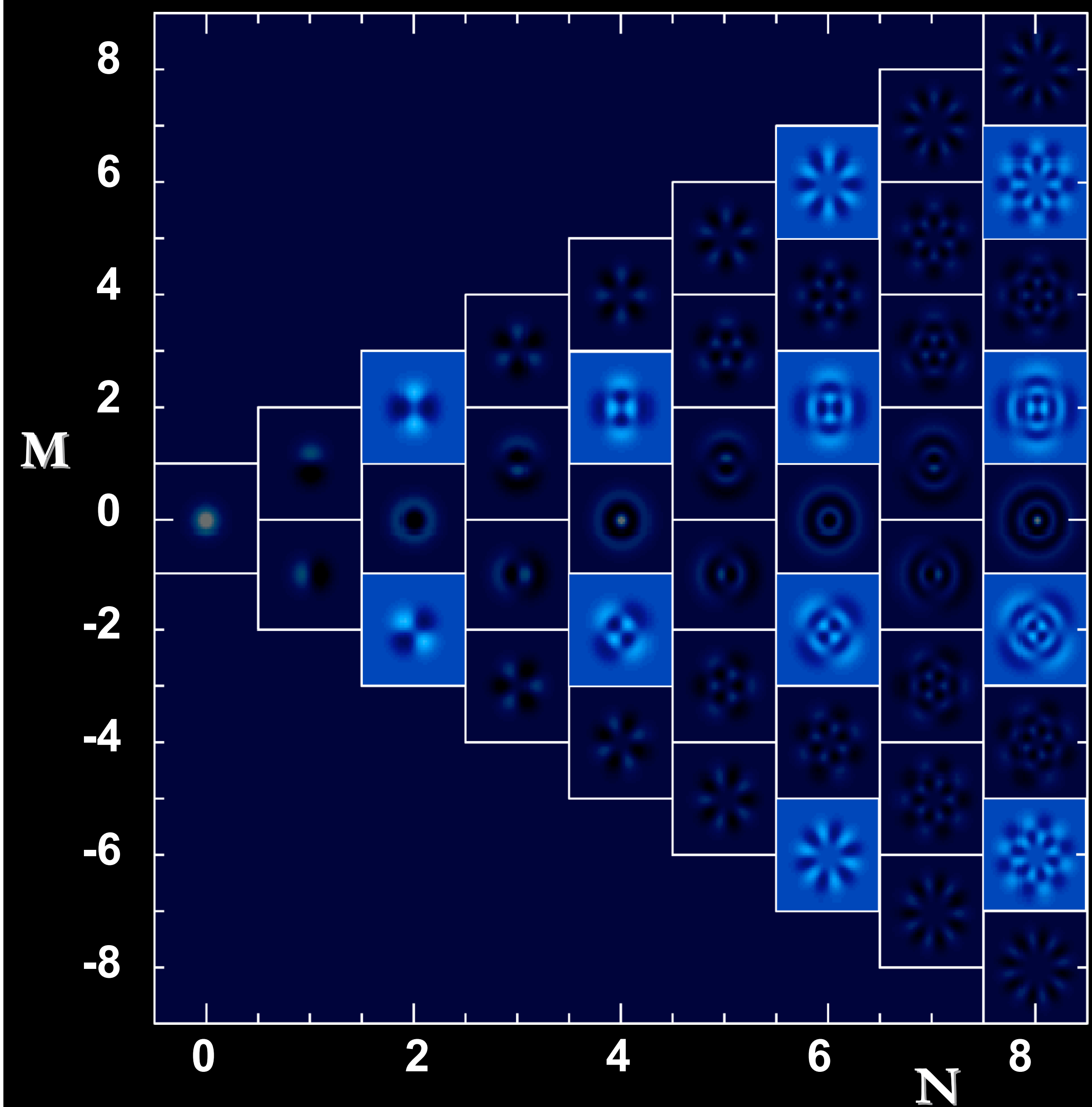
Radial profile (monopole)

Interpreting shapelet basis functions



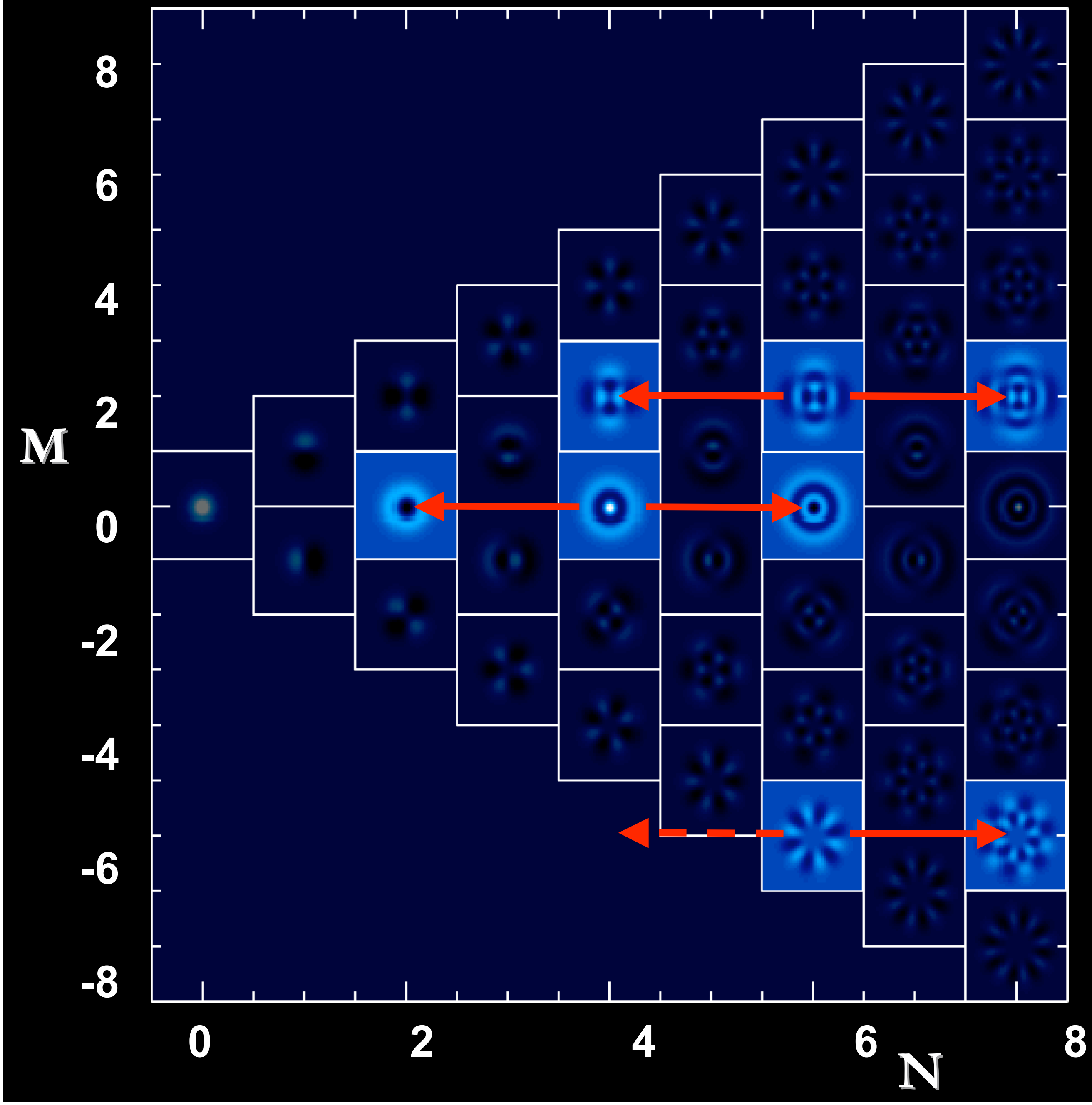
Centroid x_2 (dipole)
Radial profile (monopole)
Centroid x_1 (dipole)

Interpreting shapelet basis functions



Ellipticity?	e_1	(duodecapole
Ellipticity	e_1	(quadrupole)
Centroid	x_2	(dipole)
Radial profile		(monopole)
Centroid	x_1	(dipole)
Ellipticity	e_2	(quadrupole)
Ellipticity?	e_2	(duodecapole

Effect of gravitational lensing
dilation

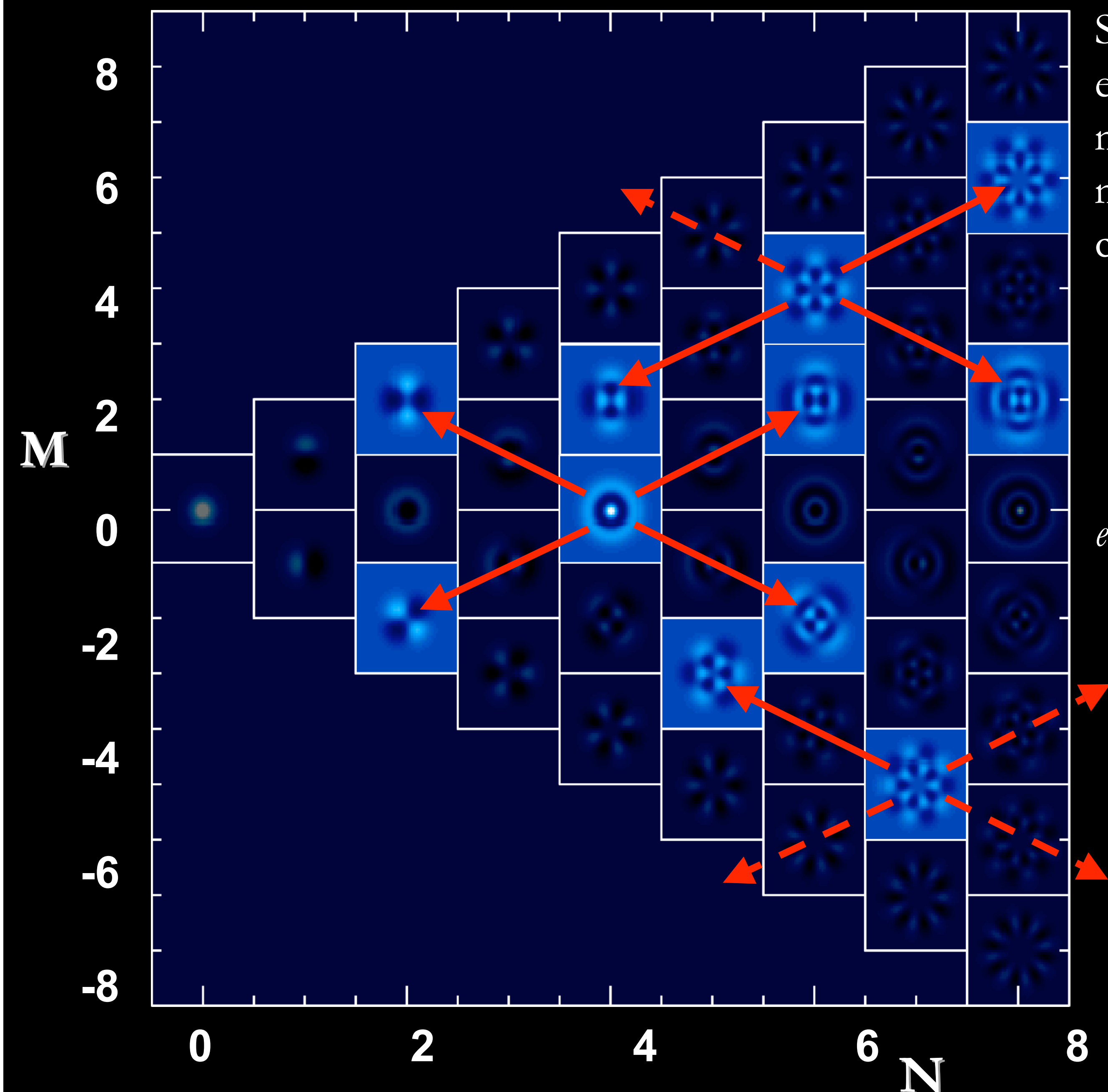


S(O)2 operations simply expressed in shapelet space as mixing of power between a minimal number of adjacent coefficients.

e.g.

$$c'_{20} = c_{20} + \text{constant} \times \gamma \times c$$

Effect of gravitational lensing shear

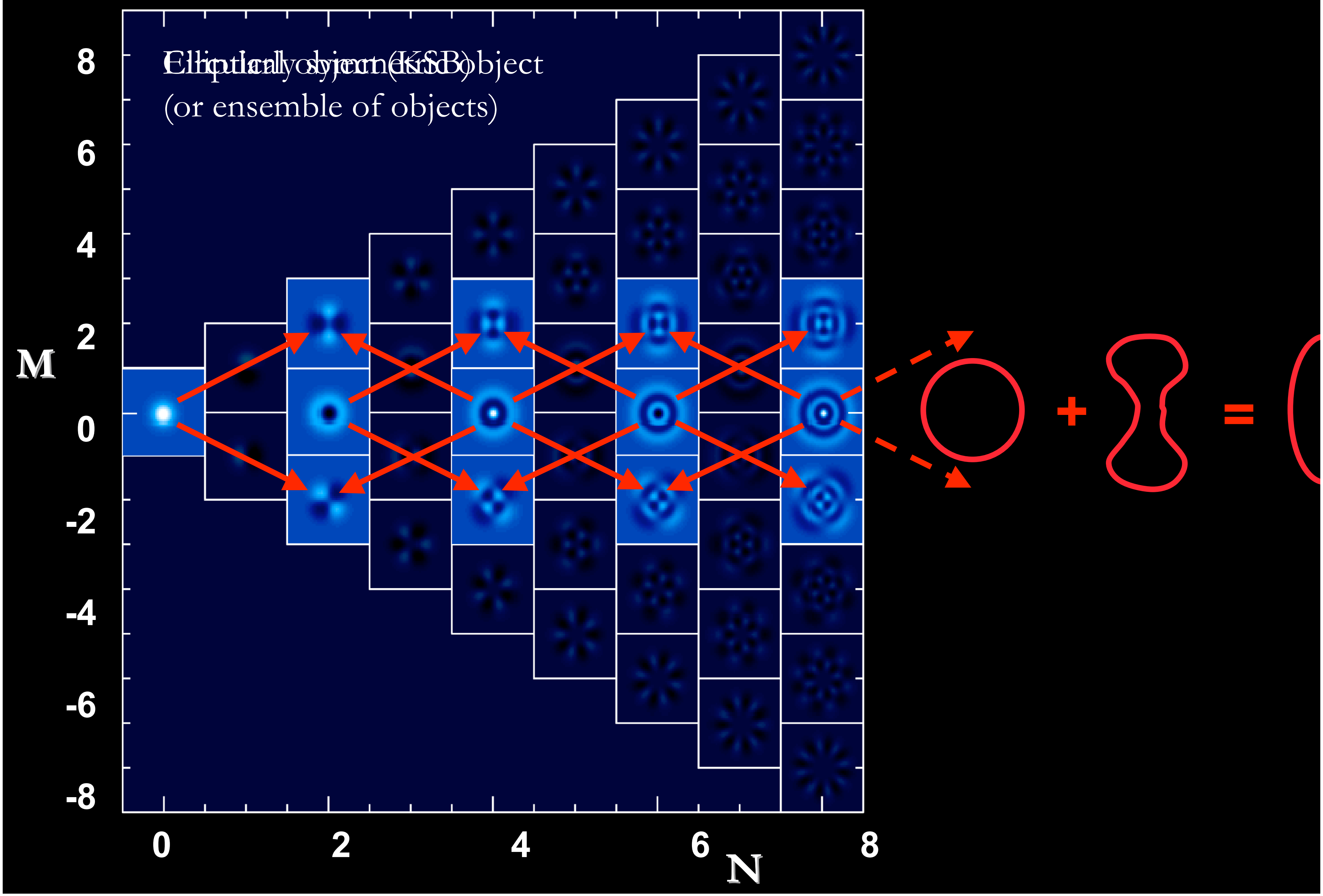


S(O)2 operations simply expressed in shapelet space as mixing of power between a minimal number of adjacent coefficients.

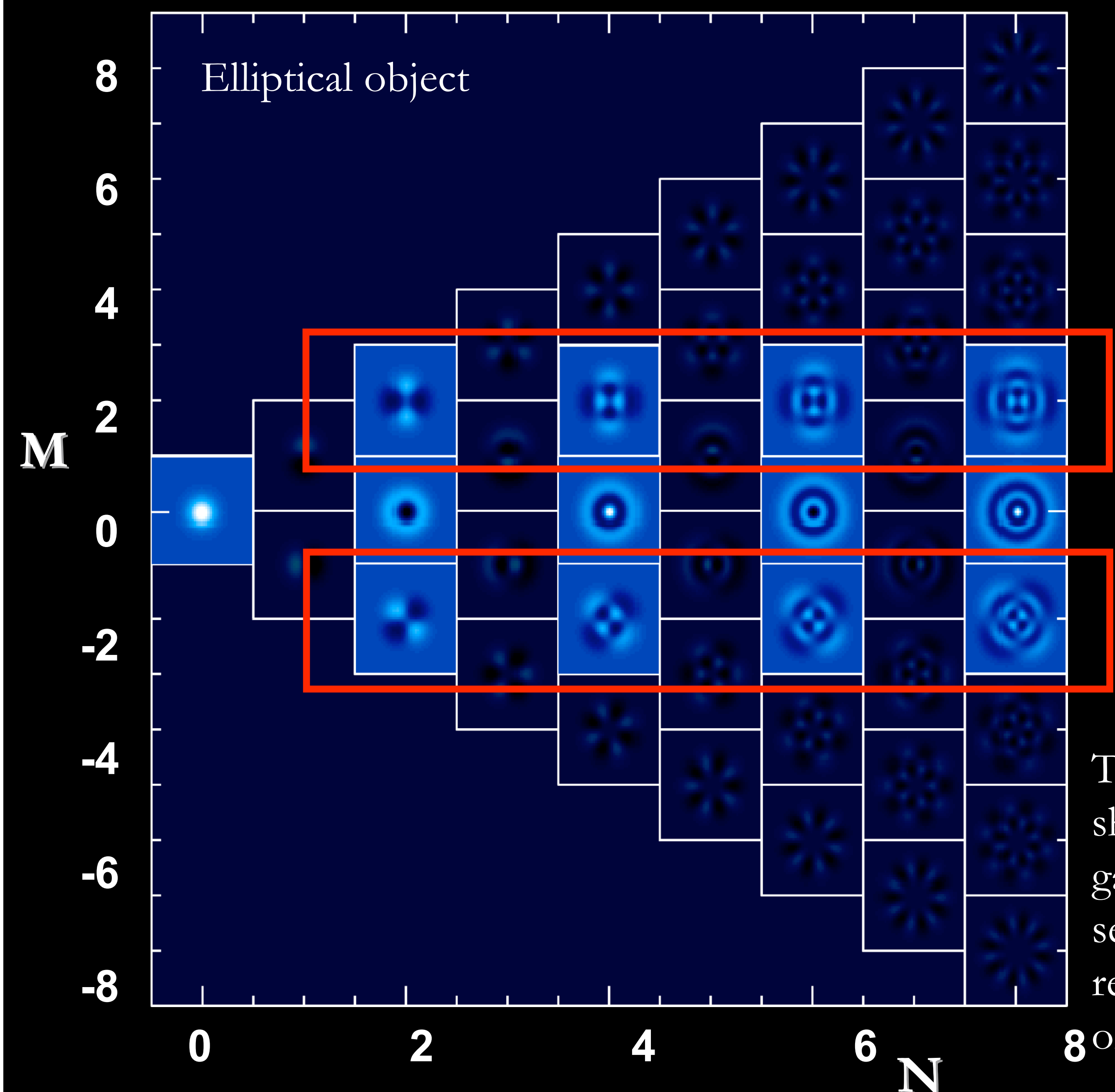
e.g.

$c'_{22} = c_{22} + \text{constant} \times \gamma \times c$

Effect of gravitational lensing shear

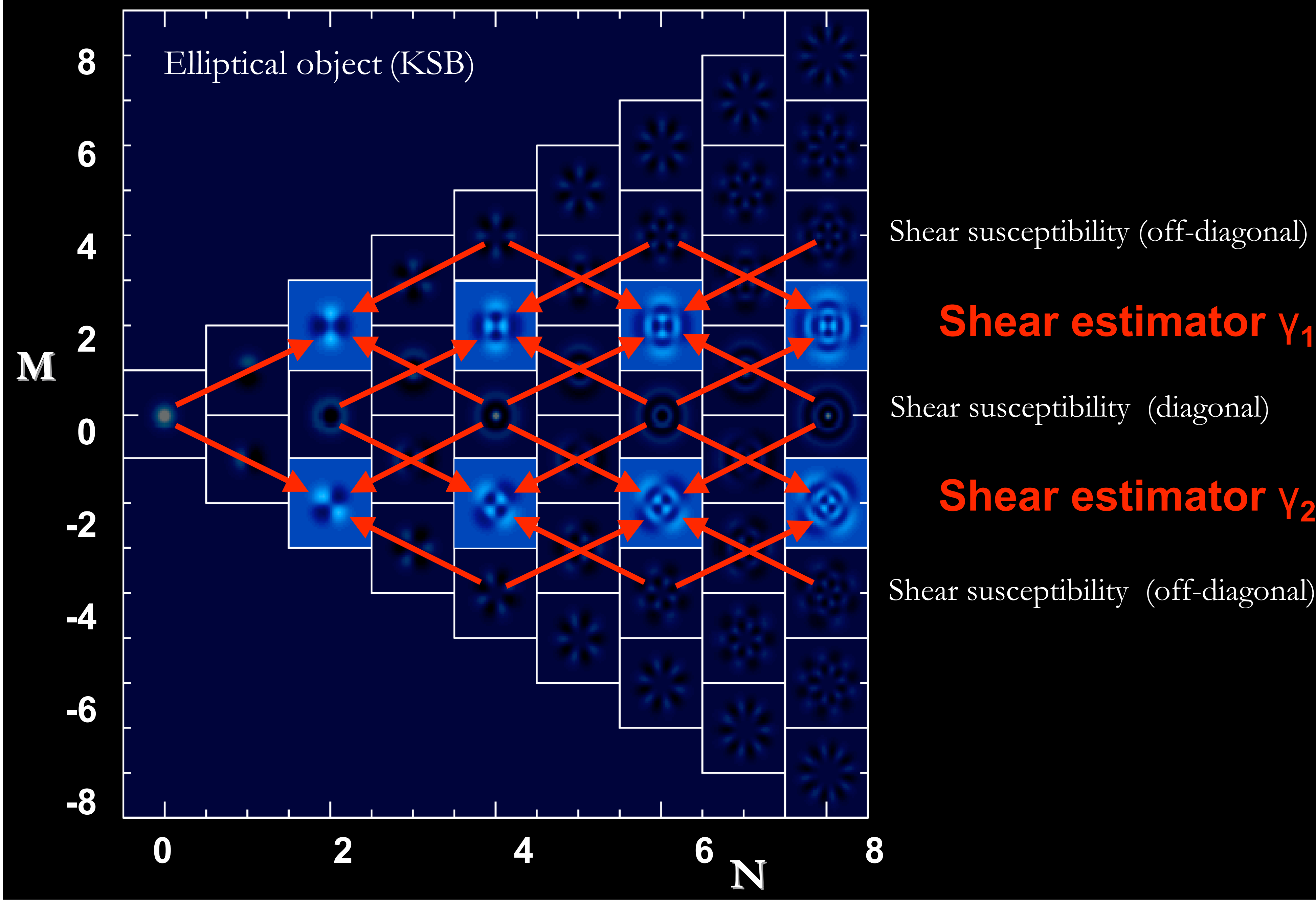


Effect of gravitational lensing shear

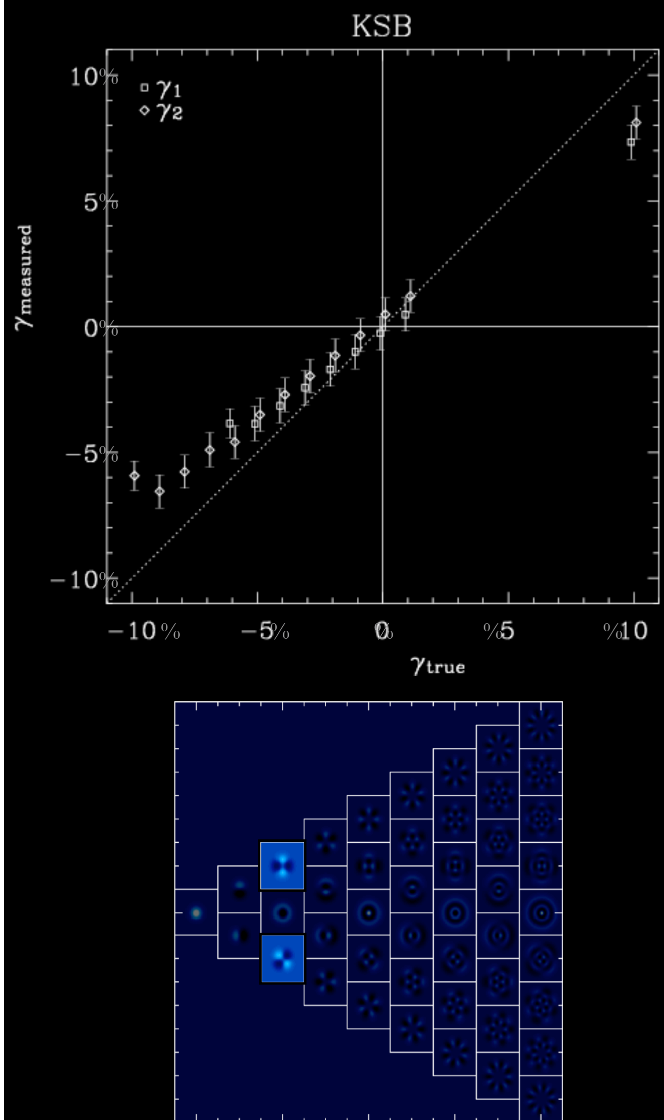


There are *several* independent shear estimators for any given galaxy. These can be used separately, or combined to remove biases - like dependence on the choice of scale size.

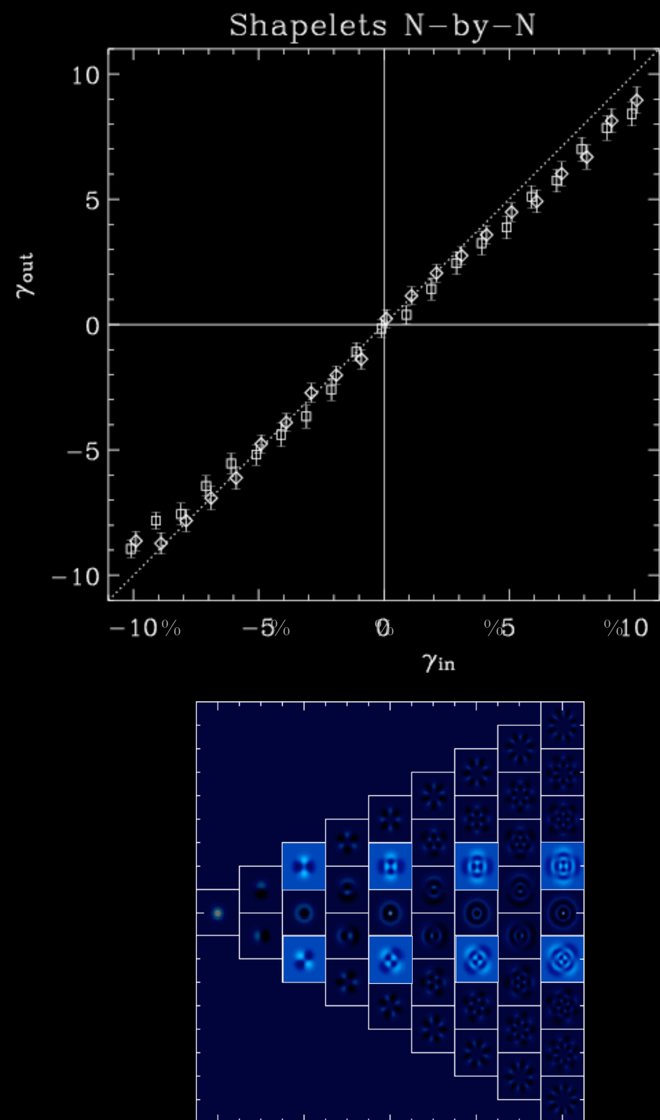
Shear susceptibilty factor



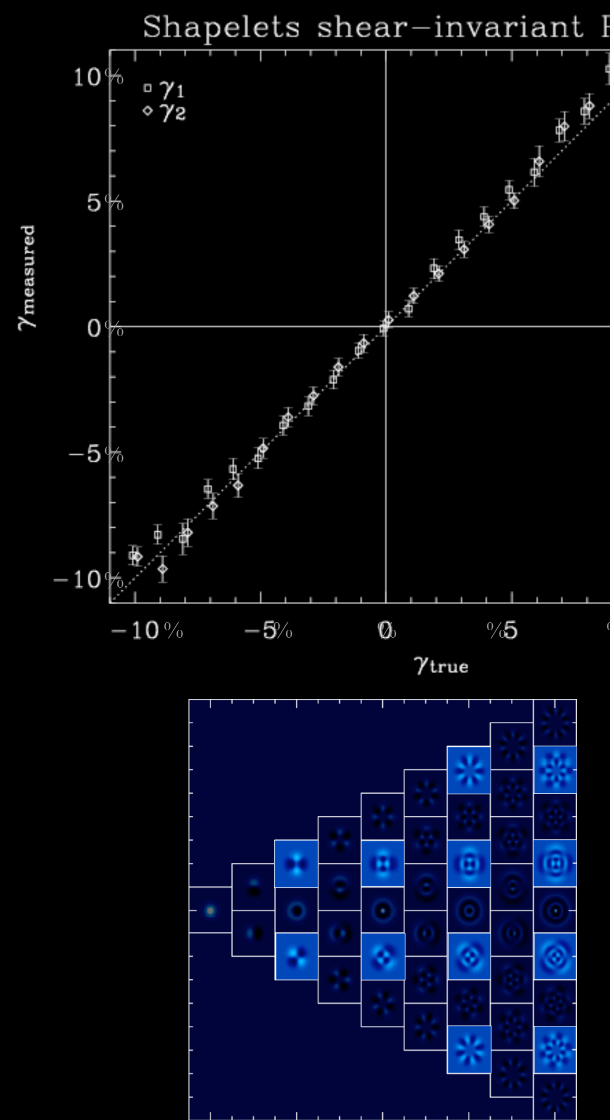
New, improved shear estimators



Given half a chance, astronomers think only to first order.



Invariant of choice of scale size for basis functions



Diagonal shear susceptibility tensor

Conclusions

Astronomical images are largely empty, but with a lot of information in isolated patches (stars and galaxies).

Shapelets are a complete, orthonormal set of basis functions with compact support - higher order multipole moments, weighted by a Gaussian. ~50x data compression for typical galaxies.

Shapelets' Fourier transform invariance renders convolution a matrix multiplication, and PSF *de*convolution more tractable.

Convenient physical interpretation of different basis states suggests several morphology estimators. Almost minimal transfer of power between neighbouring states during translation, dilation, shear, etc, facilitates the measurement of these properties too (and image warping).

http://www.astro.caltech.edu/~rjm/shapelets